Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/29-1.1.3.8-P-x-c-x-^m-a+b-x^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [594]. This is test number [29].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (594)	0.00 (0)
Mathematica	100.00 (594)	0.00 (0)
Maple	97.14 (577)	2.86 (17)
Fricas	89.39 (531)	10.61 (63)
Mupad	75.59 (449)	24.41 (145)
Sympy	72.56 (431)	27.44 (163)
Maxima	71.04 (422)	28.96 (172)
Giac	70.71 (420)	29.29 (174)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

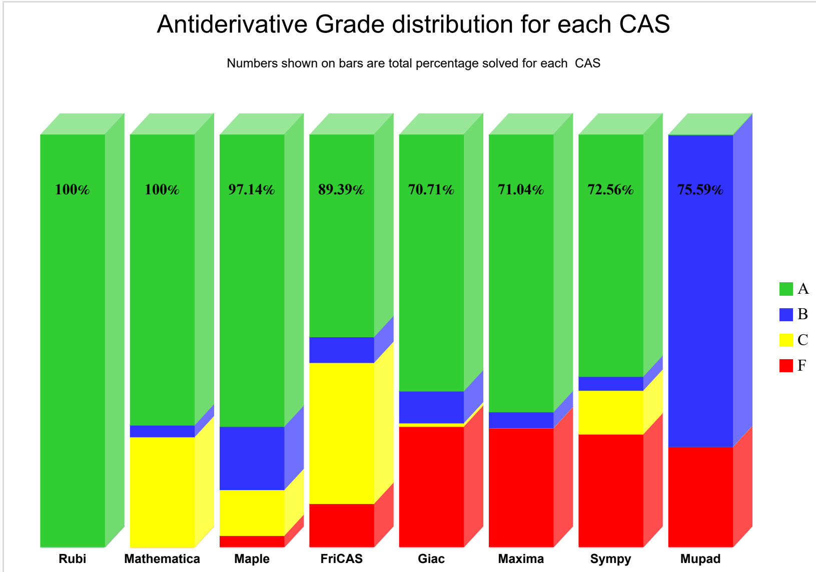
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

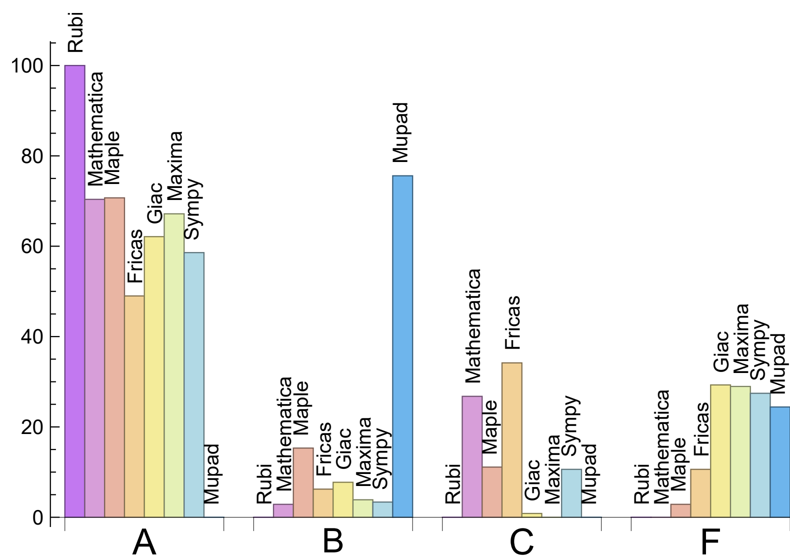
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maple	70.71	15.32	11.11	2.86
Mathematica	70.37	2.86	26.77	0.00
Maxima	67.17	3.87	0.00	28.96
Giac	62.12	7.74	0.84	29.29
Sympy	58.59	3.37	10.61	27.44
Fricas	48.99	6.23	34.18	10.61
Mupad	N/A	75.59	0.00	24.41

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	17	100.00 %	0.00 %	0.00 %
Fricas	63	61.90 %	34.92 %	3.17 %
Giac	174	92.53 %	2.30 %	5.17 %
Maxima	172	100.00 %	0.00 %	0.00 %
Sympy	163	1.23 %	96.93 %	1.84 %
Mupad	145	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

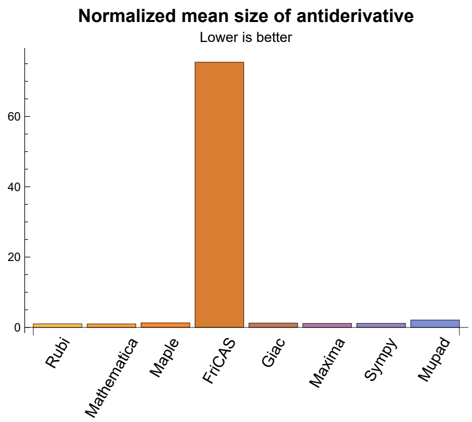
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

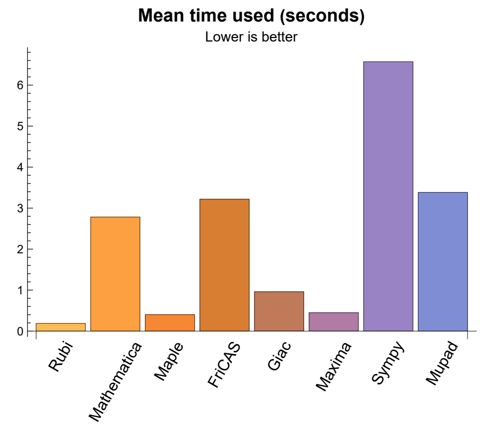
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.19	245.10	1.00	221.50	1.00
Mathematica	2.78	205.55	0.98	170.00	0.96
Maple	0.40	324.78	1.27	224.00	0.98
Maxima	0.45	192.12	1.12	174.00	1.01
Fricas	3.22	17298.67	75.43	208.00	1.03
Sympy	6.57	195.19	1.16	128.00	0.92
Giac	0.96	215.63	1.21	186.50	1.03
Mupad	3.38	440.14	2.08	199.00	1.03

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
60
40
20
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
6
5
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {434, 435, 436, 443, 444, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 581}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

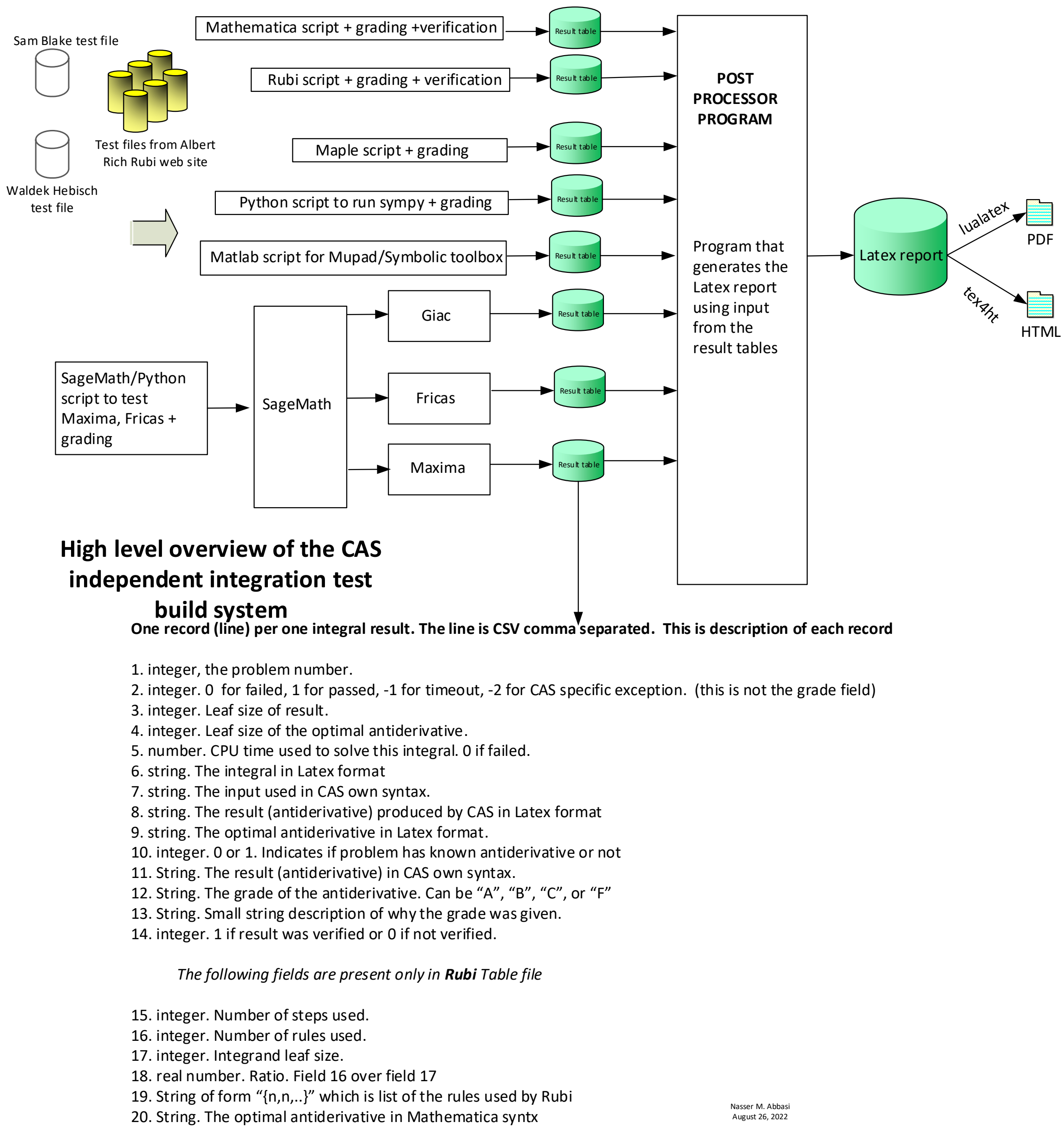
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 37, 38, 39, 40, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 584, 585, 586, 587, 588, 589, 591, 592, 593, 594 }

B grade: { 21, 32, 33, 34, 35, 36, 41, 44, 45, 46, 47, 369, 370, 371, 372, 557, 583 }

C grade: { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 124, 161, 210, 211, 212, 213, 214, 220, 221, 222, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 567, 590 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 39, 42, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 93, 94, 97, 98, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 441, 442, 443, 444, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 589, 593 }

B grade: { 20, 21, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 43, 44, 45, 46, 47, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 123, 150, 168, 369, 370, 371, 372, 440, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 557 }

C grade: { 210, 213, 214, 215, 216, 217, 218, 220, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 592, 594 }

F grade: { 474, 475, 476, 551, 552, 553, 576, 581, 582, 583, 584, 585, 586, 587, 588, 590, 591 }

2.1.4 Maxima

A grade: { 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 36, 39, 42, 44, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 589, 592, 593 }

B grade: { 3, 6, 20, 21, 31, 32, 34, 35, 37, 38, 40, 41, 43, 45, 46, 115, 161, 179, 185, 370, 371, 557, 594 }

C grade: { }

F grade: { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 210, 211, 212, 213, 214, 220, 221, 222, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 576, 581, 582, 583, 584, 585, 586, 587, 588, 590, 591 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 76, 77, 78, 123, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 153, 159, 161, 167, 180, 181, 182, 183, 184, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 477, 478, 479, 480, 481, 482, 483, 484, 495, 496, 497, 498, 499, 506, 507, 508, 509, 510, 511, 512, 513, 514, 525, 526, 527, 528, 529, 530, 531, 532, 533, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 571, 572, 573, 574, 575, 579, 580, 585, 589, 593, 594 }

B grade: { 40, 41, 44, 45, 46, 47, 152, 155, 156, 160, 163, 164, 168, 179, 185, 221, 222, 283, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 557, 569, 570, 577, 578, 591, 592 }

C grade: { 7, 8, 9, 10, 11, 12, 24, 25, 26, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 79, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 95, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 149, 150, 151, 154, 157, 158, 162, 165, 166, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 192, 195, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494 }

F grade: { 80, 82, 92, 94, 96, 98, 112, 114, 186, 187, 188, 189, 190, 191, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 474, 475, 476, 500, 501, 502, 503, 504, 505, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 534, 535, 551, 552, 553, 576, 581, 582, 583, 584, 586, 587, 588, 590 }

2.1.6 Sympy

A grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 29, 31, 37, 38, 39, 42, 43, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 126, 128, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 159, 160, 162, 163, 164, 165, 167, 168, 169, 170, 180, 181, 182, 183, 184, 211, 212, 217, 218, 220, 223, 224, 225, 226, 227, 228, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 251, 252, 253, 254, 260, 262, 264, 266, 267, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 344, 345, 346, 351, 352, 353, 358, 359, 360, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 489, 490, 495, 496, 497, 498, 499, 510, 511, 512, 513, 514, 515, 516, 517, 518, 529, 530, 531, 532, 533, 540, 541, 542, 543, 544, 545, 546, 552, 553, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 580, 581 }

B grade: { 1, 2, 3, 4, 5, 6, 125, 127, 140, 149, 158, 166, 179, 185, 221, 222, 557, 577, 578, 579 }

C grade: { 18, 19, 20, 21, 27, 28, 30, 32, 33, 34, 35, 36, 49, 123, 161, 210, 213, 214, 215, 216, 219, 365, 366, 367, 368, 369, 370, 371, 372, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 534, 535, 536, 537, 538, 539, 547, 548, 549, 550, 576, 582, 584, 586 }

F grade: { 40, 41, 44, 45, 46, 47, 72, 73, 74, 75, 129, 130, 131, 132, 150, 151, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 229, 230, 231, 232, 245, 246, 247, 248, 249, 250, 255, 256, 257, 258, 259, 261, 263, 265, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 341, 342, 343, 347, 348, 349, 350, 354, 355, 356, 357, 361, 362, 363, 364, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 485, 486, 487, 488, 491, 492, 493, 494, 551, 583, 585, 587, 588, 589, 590, 591, 592, 593, 594 }

2.1.7 Giac

A grade: { 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 31, 35, 36, 39, 42, 43, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 116, 118, 120, 122, 124, 126, 128, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 170, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 191, 195, 196, 197, 201, 202, 203, 204, 207, 208, 209, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 477, 478, 479, 480, 481, 482, 483, 484, 487, 488, 489, 490, 491, 492, 493, 494, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 579, 580 }

B grade: { 3, 6, 29, 33, 44, 115, 117, 119, 121, 123, 125, 127, 129, 131, 149, 150, 151, 161, 169, 171, 172, 173, 179, 186, 187, 188, 192, 193, 194, 198, 199, 200, 205, 206, 254, 369, 371, 485, 486, 557, 577, 578, 591, 592, 593, 594 }

C grade: { 30, 32, 34, 45, 370 }

F grade: { 37, 38, 40, 41, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 210, 211, 212, 213, 214, 220, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 576, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 91, 92, 93, 94, 95, 96, 97, 98, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 435, 444, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 536, 548, 549, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 581, 582, 583, 589, 591, 592, 593, 594 }

C grade: { }

F grade: { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 83, 84, 85, 86, 87, 88, 89, 90, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 210, 211, 212, 213, 214, 220, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 550, 551, 552, 553, 576, 584, 585, 586, 587, 588, 590 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	53	75	78	53	223	78	58
N.S.	1	1.00	0.74	1.04	1.08	0.74	3.10	1.08	0.81
time (sec)	N/A	0.023	0.041	0.309	0.502	0.358	5.061	1.341	4.715

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	155	135	238	192	644	237	149
N.S.	1	1.00	0.96	0.84	1.48	1.19	4.00	1.47	0.93
time (sec)	N/A	0.070	0.096	0.468	0.301	0.363	32.930	2.365	4.762

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	355	355	525	457	1406	526	299
N.S.	1	1.00	1.30	1.30	1.92	1.67	5.13	1.92	1.09
time (sec)	N/A	0.125	0.201	0.420	0.308	0.402	74.547	1.981	0.098

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	82	128	129	90	354	129	103
N.S.	1	1.00	0.72	1.12	1.13	0.79	3.11	1.13	0.90
time (sec)	N/A	0.049	0.068	0.307	0.511	0.361	8.808	2.385	4.814

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	320	292	507	417	1365	516	316
N.S.	1	1.00	1.00	0.91	1.58	1.30	4.27	1.61	0.99
time (sec)	N/A	0.158	0.232	0.303	0.305	0.371	76.334	1.735	4.698

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	708	708	913	1254	1373	1221	3691	1414	896
N.S.	1	1.00	1.29	1.77	1.94	1.72	5.21	2.00	1.27
time (sec)	N/A	0.398	0.724	0.312	0.317	0.396	217.621	1.656	0.242

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	124	186	135	1931	76	141	127
N.S.	1	1.00	0.77	1.16	0.84	11.99	0.47	0.88	0.79
time (sec)	N/A	0.074	0.031	0.313	0.502	1.146	0.339	1.456	5.511

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	180	230	169	2088	105	174	169
N.S.	1	1.00	0.95	1.22	0.89	11.05	0.56	0.92	0.89
time (sec)	N/A	0.089	0.101	0.324	1.143	1.086	0.452	1.663	4.872

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	205	274	203	2215	146	194	206
N.S.	1	1.00	0.95	1.27	0.94	10.30	0.68	0.90	0.96
time (sec)	N/A	0.128	0.103	0.325	0.992	1.049	0.600	1.796	0.268

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	229	318	238	2308	185	218	241
N.S.	1	1.00	0.95	1.32	0.99	9.62	0.77	0.91	1.00
time (sec)	N/A	0.153	0.129	0.340	1.258	1.136	0.717	1.801	4.931

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	125	186	105	1961	76	132	127
N.S.	1	1.00	0.78	1.16	0.65	12.18	0.47	0.82	0.79
time (sec)	N/A	0.082	0.038	0.320	0.586	1.125	0.346	1.731	4.847

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	125	188	103	1905	78	115	124
N.S.	1	1.00	0.78	1.17	0.64	11.83	0.48	0.71	0.77
time (sec)	N/A	0.066	0.035	0.316	1.014	1.216	0.345	1.556	0.213

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.010	0.004	0.316	0.936	0.439	0.032	0.811	4.699

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.010	0.004	0.326	1.005	0.442	0.030	0.708	4.670

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	17	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.77	0.73
time (sec)	N/A	0.008	0.003	0.336	0.590	0.441	0.028	0.730	0.063

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	17	19	18
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.77	0.86	0.82
time (sec)	N/A	0.008	0.003	0.354	0.656	0.422	0.028	0.703	0.112

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	32	32	44	33	46
N.S.	1	1.00	1.00	0.80	0.78	0.78	1.07	0.80	1.12
time (sec)	N/A	0.019	0.006	0.354	1.246	0.418	0.052	0.815	0.140

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	35	34	28	54	28	28
N.S.	1	1.00	1.07	1.21	1.17	0.97	1.86	0.97	0.97
time (sec)	N/A	0.015	0.008	0.336	0.603	0.422	0.065	0.966	0.054

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	34	33	26	53	26	28
N.S.	1	1.00	1.00	1.17	1.14	0.90	1.83	0.90	0.97
time (sec)	N/A	0.013	0.006	0.317	0.564	0.464	0.064	1.485	0.045

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	195	163	107	88	48	49
N.S.	1	1.00	0.90	5.00	4.18	2.74	2.26	1.23	1.26
time (sec)	N/A	0.017	0.009	0.375	0.548	0.469	0.116	1.783	4.821

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	129	202	174	114	105	58	49
N.S.	1	1.00	3.15	4.93	4.24	2.78	2.56	1.41	1.20
time (sec)	N/A	0.030	0.033	0.378	0.551	0.453	0.145	1.951	0.231

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	90	94	159	310	26	103	98
N.S.	1	1.00	0.76	0.80	1.35	2.63	0.22	0.87	0.83
time (sec)	N/A	0.086	0.008	0.359	0.524	0.459	0.060	1.611	4.944

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	90	94	159	305	22	115	96
N.S.	1	1.00	0.76	0.80	1.35	2.58	0.19	0.97	0.81
time (sec)	N/A	0.073	0.008	0.355	0.510	0.458	0.064	2.258	5.015

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	124	186	188	1961	76	147	127
N.S.	1	1.00	0.77	1.16	1.17	12.18	0.47	0.91	0.79
time (sec)	N/A	0.112	0.028	0.352	0.543	1.190	0.372	1.389	4.923

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	122	108	96	1076	75	110	158
N.S.	1	1.00	0.91	0.81	0.72	8.03	0.56	0.82	1.18
time (sec)	N/A	0.076	0.015	0.325	0.515	1.200	0.140	1.216	0.189

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	123	110	96	1256	70	95	178
N.S.	1	1.00	0.92	0.82	0.72	9.37	0.52	0.71	1.33
time (sec)	N/A	0.060	0.026	0.329	0.504	1.209	0.196	1.525	5.009

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	72	43	42	36	60	37	84
N.S.	1	1.00	1.95	1.16	1.14	0.97	1.62	1.00	2.27
time (sec)	N/A	0.040	0.015	0.361	0.508	0.434	0.121	1.221	4.809

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	71	44	44	36	60	38	86
N.S.	1	1.00	1.82	1.13	1.13	0.92	1.54	0.97	2.21
time (sec)	N/A	0.028	0.013	0.338	0.508	0.432	0.129	1.732	0.093

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	76	115	47	134	58	115	147
N.S.	1	1.00	1.58	2.40	0.98	2.79	1.21	2.40	3.06
time (sec)	N/A	0.025	0.013	0.342	0.556	0.389	0.150	2.425	5.137

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	72	96	36	40	85	113	145
N.S.	1	1.00	1.53	2.04	0.77	0.85	1.81	2.40	3.09
time (sec)	N/A	0.023	0.017	0.302	0.507	0.415	0.131	1.587	5.024

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	99	120	122	182	58	56	176
N.S.	1	1.00	1.74	2.11	2.14	3.19	1.02	0.98	3.09
time (sec)	N/A	0.046	0.020	0.332	0.506	0.393	0.192	1.303	5.272

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	106	98	93	43	95	84	142
N.S.	1	1.00	2.26	2.09	1.98	0.91	2.02	1.79	3.02
time (sec)	N/A	0.040	0.028	0.297	0.542	0.387	0.156	1.063	0.328

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	146	116	51	52	100	90	172
N.S.	1	1.00	2.92	2.32	1.02	1.04	2.00	1.80	3.44
time (sec)	N/A	0.050	0.033	0.308	0.541	0.394	0.173	1.190	5.098

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	150	119	167	53	110	109	172
N.S.	1	1.00	2.83	2.25	3.15	1.00	2.08	2.06	3.25
time (sec)	N/A	0.053	0.040	0.327	0.518	0.385	0.182	1.038	5.402

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	149	117	168	56	109	91	173
N.S.	1	1.00	2.76	2.17	3.11	1.04	2.02	1.69	3.20
time (sec)	N/A	0.040	0.031	0.301	0.533	0.370	0.186	0.987	5.270

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	147	118	52	53	102	85	171
N.S.	1	1.00	2.77	2.23	0.98	1.00	1.92	1.60	3.23
time (sec)	N/A	0.039	0.029	0.303	0.509	0.394	0.194	1.558	5.191

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	95	112	162	160	70	0	193
N.S.	1	1.00	1.56	1.84	2.66	2.62	1.15	0.00	3.16
time (sec)	N/A	0.027	0.012	0.432	0.509	0.393	0.163	0.000	5.307

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	116	117	173	205	73	0	221
N.S.	1	1.00	1.66	1.67	2.47	2.93	1.04	0.00	3.16
time (sec)	N/A	0.047	0.020	0.426	0.539	0.398	0.192	0.000	5.239

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	50	32	31	31	42	32	46
N.S.	1	1.00	1.25	0.80	0.78	0.78	1.05	0.80	1.15
time (sec)	N/A	0.021	0.010	0.346	0.510	0.364	0.052	3.168	0.155

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	122	217	236	430	0	0	386
N.S.	1	1.00	1.74	3.10	3.37	6.14	0.00	0.00	5.51
time (sec)	N/A	0.045	0.031	0.512	0.524	2.024	0.000	0.000	6.232

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	238	227	252	470	0	0	444
N.S.	1	1.00	2.70	2.58	2.86	5.34	0.00	0.00	5.05
time (sec)	N/A	0.075	0.365	0.511	0.526	1.680	0.000	0.000	6.324

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	12	12	12	12	7	13	12
N.S.	1	1.00	1.09	1.09	1.09	1.09	0.64	1.18	1.09
time (sec)	N/A	0.007	0.002	0.366	0.294	0.362	0.013	1.763	0.036

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	209	210	17	20	16	15
N.S.	1	1.00	1.00	9.95	10.00	0.81	0.95	0.76	0.71
time (sec)	N/A	0.010	0.002	0.371	0.513	0.399	0.061	1.671	4.903

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	247	219	78	429	0	129	436
N.S.	1	1.00	3.48	3.08	1.10	6.04	0.00	1.82	6.14
time (sec)	N/A	0.067	0.180	0.302	0.540	1.163	0.000	1.716	6.077

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	288	224	238	459	0	235	456
N.S.	1	1.00	3.79	2.95	3.13	6.04	0.00	3.09	6.00
time (sec)	N/A	0.072	0.117	0.312	0.519	1.093	0.000	1.811	6.479

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	253	222	239	450	0	133	453
N.S.	1	1.00	3.24	2.85	3.06	5.77	0.00	1.71	5.81
time (sec)	N/A	0.072	0.162	0.300	0.535	1.111	0.000	1.541	6.053

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	244	223	78	450	0	125	435
N.S.	1	1.00	3.25	2.97	1.04	6.00	0.00	1.67	5.80
time (sec)	N/A	0.075	0.168	0.321	0.521	1.134	0.000	2.134	6.357

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	28	26	26	24	27	35
N.S.	1	1.00	0.97	0.88	0.81	0.81	0.75	0.84	1.09
time (sec)	N/A	0.023	0.009	0.329	0.511	0.363	0.191	2.351	4.778

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	62	55	47	47	323	52	87
N.S.	1	1.00	1.13	1.00	0.85	0.85	5.87	0.95	1.58
time (sec)	N/A	0.040	0.023	0.370	0.530	0.365	0.466	2.280	4.948

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75
time (sec)	N/A	0.004	0.001	0.322	0.298	0.371	0.009	2.186	0.023

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	32	32	5	33	63
N.S.	1	1.00	1.00	1.10	1.07	1.07	0.17	1.10	2.10
time (sec)	N/A	0.021	0.006	0.334	0.534	0.361	0.050	1.303	4.930

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	15	17	16
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.83	0.94	0.89
time (sec)	N/A	0.013	0.004	0.347	0.586	0.383	0.032	1.707	0.041

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	113	98	97	97	117	97	97
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.04	0.86	0.86
time (sec)	N/A	0.066	0.003	0.365	0.329	0.353	0.013	1.002	0.061

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	74	74	90	74	74
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.02	0.84	0.84
time (sec)	N/A	0.041	0.003	0.360	0.273	0.379	0.013	1.176	0.036

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	58	50	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.97	0.83	0.83
time (sec)	N/A	0.026	0.002	0.150	0.270	0.371	0.009	1.108	0.028

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.009	0.001	0.313	0.292	0.348	0.013	1.371	0.018

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	124	186	135	1931	76	141	127
N.S.	1	1.00	0.77	1.16	0.84	11.99	0.47	0.88	0.79
time (sec)	N/A	0.068	0.030	0.323	0.529	1.102	0.356	2.026	5.094

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	180	230	169	2088	105	174	169
N.S.	1	1.00	0.95	1.22	0.89	11.05	0.56	0.92	0.89
time (sec)	N/A	0.088	0.099	0.335	0.540	1.098	0.514	1.826	5.084

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	585	585	78	1618	0	117	265	0	-1
N.S.	1	1.00	0.13	2.77	0.00	0.20	0.45	0.00	-0.00
time (sec)	N/A	0.305	7.214	0.346	0.000	0.080	3.053	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	76	1546	0	93	170	0	-1
N.S.	1	1.00	0.14	2.78	0.00	0.17	0.31	0.00	-0.00
time (sec)	N/A	0.227	6.059	0.341	0.000	0.085	2.196	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	75	1480	0	69	163	0	-1
N.S.	1	1.00	0.14	2.82	0.00	0.13	0.31	0.00	-0.00
time (sec)	N/A	0.163	4.481	0.341	0.000	0.075	1.750	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	75	1536	0	43	78	0	-1
N.S.	1	1.00	0.15	3.13	0.00	0.09	0.16	0.00	-0.00
time (sec)	N/A	0.107	10.031	0.329	0.000	0.077	1.943	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	96	1662	0	94	163	0	-1
N.S.	1	1.00	0.18	3.18	0.00	0.18	0.31	0.00	-0.00
time (sec)	N/A	0.177	10.044	0.345	0.000	0.089	4.473	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	123	1782	0	155	163	0	-1
N.S.	1	1.00	0.22	3.22	0.00	0.28	0.29	0.00	-0.00
time (sec)	N/A	0.381	10.064	0.322	0.000	0.077	12.835	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	138	1902	0	214	163	0	-1
N.S.	1	1.00	0.24	3.27	0.00	0.37	0.28	0.00	-0.00
time (sec)	N/A	0.288	10.081	0.325	0.000	0.083	41.481	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	590	590	135	1491	0	87	187	0	-1
N.S.	1	1.00	0.23	2.53	0.00	0.15	0.32	0.00	-0.00
time (sec)	N/A	0.381	10.090	0.349	0.000	0.077	1.861	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	130	1547	0	153	189	0	-1
N.S.	1	1.00	0.22	2.60	0.00	0.26	0.32	0.00	-0.00
time (sec)	N/A	0.292	10.082	0.398	0.000	0.083	7.196	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	170	1673	0	261	209	0	-1
N.S.	1	1.00	0.27	2.66	0.00	0.42	0.33	0.00	-0.00
time (sec)	N/A	0.345	10.130	0.359	0.000	0.083	41.752	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	676	676	196	1793	0	373	231	0	-1
N.S.	1	1.00	0.29	2.65	0.00	0.55	0.34	0.00	-0.00
time (sec)	N/A	0.435	10.180	0.358	0.000	0.081	174.946	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	200	206	192	5014	156	175	357
N.S.	1	1.00	1.08	1.11	1.03	26.96	0.84	0.94	1.92
time (sec)	N/A	0.156	0.055	0.358	0.576	1.143	0.488	1.854	0.262

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	214	228	240	7245	245	214	370
N.S.	1	1.00	0.96	1.03	1.08	32.64	1.10	0.96	1.67
time (sec)	N/A	0.209	0.116	0.342	0.532	1.855	8.251	2.009	5.143

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	280	277	250	303	8787	0	294	513
N.S.	1	0.99	0.98	0.89	1.07	31.16	0.00	1.04	1.82
time (sec)	N/A	0.290	0.165	0.345	0.511	6.256	0.000	2.003	4.973

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	270	269	252	259	12827	0	264	769
N.S.	1	0.99	0.99	0.93	0.95	47.16	0.00	0.97	2.83
time (sec)	N/A	0.327	0.195	0.467	0.568	1.797	0.000	1.848	5.134

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	439	347	462	29479	0	432	1700
N.S.	1	1.00	1.06	0.83	1.11	70.86	0.00	1.04	4.09
time (sec)	N/A	0.455	0.294	0.451	0.590	11.261	0.000	2.288	4.913

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	645	643	678	489	756	47284	0	723	2971
N.S.	1	1.00	1.05	0.76	1.17	73.31	0.00	1.12	4.61
time (sec)	N/A	0.726	0.257	0.475	0.555	97.593	0.000	1.559	5.047

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	54	38	37	37	44	38	49
N.S.	1	1.00	1.26	0.88	0.86	0.86	1.02	0.88	1.14
time (sec)	N/A	0.053	0.009	0.329	0.499	0.387	0.051	2.126	0.098

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	54	38	37	37	46	38	51
N.S.	1	1.00	1.17	0.83	0.80	0.80	1.00	0.83	1.11
time (sec)	N/A	0.054	0.011	0.339	0.522	0.352	0.054	2.322	0.093

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	37	37	48	38	49
N.S.	1	1.00	1.00	0.86	0.84	0.84	1.09	0.86	1.11
time (sec)	N/A	0.027	0.006	0.324	0.529	0.388	0.052	2.099	4.697

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	47	407	0	21	92	0	312
N.S.	1	1.00	0.20	1.77	0.00	0.09	0.40	0.00	1.36
time (sec)	N/A	0.038	10.021	0.411	0.000	0.076	0.953	0.000	0.152

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	43	368	0	1	97	0	342
N.S.	1	1.00	0.17	1.43	0.00	0.00	0.38	0.00	1.33
time (sec)	N/A	0.046	10.026	0.377	0.000	0.087	1.383	0.000	5.138

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	63	407	0	21	82	0	326
N.S.	1	1.00	0.44	2.83	0.00	0.15	0.57	0.00	2.26
time (sec)	N/A	0.021	10.029	0.396	0.000	0.082	1.352	0.000	4.875

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	67	370	0	1	99	0	360
N.S.	1	1.00	0.50	2.74	0.00	0.01	0.73	0.00	2.67
time (sec)	N/A	0.022	10.024	0.354	0.000	0.079	1.041	0.000	4.909

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	90	1003	0	49	122	0	-1
N.S.	1	1.00	0.19	2.14	0.00	0.10	0.26	0.00	-0.00
time (sec)	N/A	0.088	10.057	0.315	0.000	0.081	2.155	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	481	91	949	0	55	128	0	-1
N.S.	1	1.00	0.19	1.97	0.00	0.11	0.27	0.00	-0.00
time (sec)	N/A	0.092	10.043	0.314	0.000	0.077	3.547	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	92	952	0	48	112	0	-1
N.S.	1	1.00	0.34	3.51	0.00	0.18	0.41	0.00	-0.00
time (sec)	N/A	0.049	10.040	0.312	0.000	0.078	3.194	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	93	1012	0	56	129	0	-1
N.S.	1	1.00	0.35	3.80	0.00	0.21	0.48	0.00	-0.00
time (sec)	N/A	0.048	10.033	0.326	0.000	0.076	2.386	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	89	1004	0	53	124	0	-1
N.S.	1	1.00	0.17	1.93	0.00	0.10	0.24	0.00	-0.00
time (sec)	N/A	0.143	10.042	0.325	0.000	0.076	1.332	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	89	950	0	56	129	0	-1
N.S.	1	1.00	0.17	1.78	0.00	0.11	0.24	0.00	-0.00
time (sec)	N/A	0.124	10.036	0.334	0.000	0.078	1.505	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	90	953	0	52	114	0	-1
N.S.	1	1.00	0.35	3.72	0.00	0.20	0.45	0.00	-0.00
time (sec)	N/A	0.057	10.027	0.320	0.000	0.079	1.471	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	92	1013	0	57	131	0	-1
N.S.	1	1.00	0.37	4.04	0.00	0.23	0.52	0.00	-0.00
time (sec)	N/A	0.047	10.031	0.320	0.000	0.086	1.432	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	49	407	0	21	92	0	313
N.S.	1	1.00	0.39	3.20	0.00	0.17	0.72	0.00	2.46
time (sec)	N/A	0.016	10.023	0.414	0.000	0.075	0.952	0.000	0.126

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	45	368	0	1	97	0	343
N.S.	1	1.00	0.32	2.59	0.00	0.01	0.68	0.00	2.42
time (sec)	N/A	0.021	10.018	0.421	0.000	0.078	1.402	0.000	4.738

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	63	407	0	21	82	0	327
N.S.	1	1.00	0.24	1.54	0.00	0.08	0.31	0.00	1.24
time (sec)	N/A	0.037	10.037	0.448	0.000	0.082	1.362	0.000	4.750

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	67	370	0	1	97	0	361
N.S.	1	1.00	0.27	1.50	0.00	0.00	0.39	0.00	1.46
time (sec)	N/A	0.038	10.033	0.352	0.000	0.096	1.100	0.000	4.824

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	47	407	0	21	92	0	312
N.S.	1	1.00	0.37	3.23	0.00	0.17	0.73	0.00	2.48
time (sec)	N/A	0.018	10.024	0.418	0.000	0.095	1.354	0.000	4.820

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	43	368	0	1	97	0	342
N.S.	1	1.00	0.30	2.57	0.00	0.01	0.68	0.00	2.39
time (sec)	N/A	0.021	10.012	0.359	0.000	0.128	1.072	0.000	0.052

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	63	407	0	21	82	0	326
N.S.	1	1.00	0.24	1.55	0.00	0.08	0.31	0.00	1.24
time (sec)	N/A	0.038	10.025	0.395	0.000	0.160	0.993	0.000	0.061

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	67	370	0	1	97	0	360
N.S.	1	1.00	0.27	1.49	0.00	0.00	0.39	0.00	1.45
time (sec)	N/A	0.037	10.016	0.330	0.000	0.080	1.670	0.000	4.900

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	90	1003	0	48	122	0	-1
N.S.	1	1.00	0.35	3.92	0.00	0.19	0.48	0.00	-0.00
time (sec)	N/A	0.034	10.052	0.326	0.000	0.083	2.262	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	90	949	0	56	128	0	-1
N.S.	1	1.00	0.34	3.61	0.00	0.21	0.49	0.00	-0.00
time (sec)	N/A	0.036	10.041	0.329	0.000	0.086	3.053	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	91	952	0	49	112	0	-1
N.S.	1	1.00	0.18	1.92	0.00	0.10	0.23	0.00	-0.00
time (sec)	N/A	0.102	10.033	0.308	0.000	0.078	3.143	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	488	488	93	1012	0	55	128	0	-1
N.S.	1	1.00	0.19	2.07	0.00	0.11	0.26	0.00	-0.00
time (sec)	N/A	0.086	10.039	0.316	0.000	0.079	2.502	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	89	1004	0	52	124	0	-1
N.S.	1	1.00	0.37	4.17	0.00	0.22	0.51	0.00	-0.00
time (sec)	N/A	0.056	10.044	0.323	0.000	0.081	1.345	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	89	950	0	57	129	0	-1
N.S.	1	1.00	0.36	3.83	0.00	0.23	0.52	0.00	-0.00
time (sec)	N/A	0.046	10.047	0.327	0.000	0.085	1.600	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	549	549	90	953	0	53	114	0	-1
N.S.	1	1.00	0.16	1.74	0.00	0.10	0.21	0.00	-0.00
time (sec)	N/A	0.155	10.038	0.311	0.000	0.076	1.448	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	540	540	92	1013	0	56	129	0	-1
N.S.	1	1.00	0.17	1.88	0.00	0.10	0.24	0.00	-0.00
time (sec)	N/A	0.122	10.036	0.314	0.000	0.091	1.559	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	75	720	0	43	78	0	-1
N.S.	1	1.00	0.15	1.47	0.00	0.09	0.16	0.00	-0.00
time (sec)	N/A	0.107	10.026	0.311	0.000	0.073	1.040	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	75	681	0	47	82	0	-1
N.S.	1	1.00	0.15	1.35	0.00	0.09	0.16	0.00	-0.00
time (sec)	N/A	0.108	10.025	0.354	0.000	0.087	1.135	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	76	683	0	43	73	0	-1
N.S.	1	1.00	0.15	1.33	0.00	0.08	0.14	0.00	-0.00
time (sec)	N/A	0.120	10.033	0.346	0.000	0.075	1.137	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	78	726	0	47	83	0	-1
N.S.	1	1.00	0.15	1.43	0.00	0.09	0.16	0.00	-0.00
time (sec)	N/A	0.109	10.031	0.330	0.000	0.072	1.144	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	42	291	0	18	61	0	373
N.S.	1	1.00	0.17	1.18	0.00	0.07	0.25	0.00	1.52
time (sec)	N/A	0.058	10.020	0.398	0.000	0.072	0.867	0.000	4.770

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	38	267	0	1	65	0	406
N.S.	1	1.00	0.14	0.99	0.00	0.00	0.24	0.00	1.50
time (sec)	N/A	0.069	10.014	0.375	0.000	0.075	1.006	0.000	5.070

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	58	291	0	18	56	0	374
N.S.	1	1.00	0.21	1.06	0.00	0.07	0.20	0.00	1.36
time (sec)	N/A	0.064	10.017	0.385	0.000	0.078	0.879	0.000	0.120

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	62	269	0	1	66	0	405
N.S.	1	1.00	0.24	1.03	0.00	0.00	0.25	0.00	1.55
time (sec)	N/A	0.056	10.024	0.340	0.000	0.073	0.921	0.000	4.821

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	134	87	126	39057	126	225	182
N.S.	1	1.00	1.54	1.00	1.45	448.93	1.45	2.59	2.09
time (sec)	N/A	0.043	0.024	0.329	0.511	1.311	0.466	0.804	5.012

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	184	124	207	41851	124	213	160
N.S.	1	1.00	0.84	0.57	0.95	191.10	0.57	0.97	0.73
time (sec)	N/A	0.113	0.059	0.339	0.550	1.467	0.452	1.760	4.798

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	168	128	157	40560	156	254	283
N.S.	1	1.00	1.53	1.16	1.43	368.73	1.42	2.31	2.57
time (sec)	N/A	0.058	0.110	0.378	0.534	1.424	0.612	1.587	4.919

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	224	163	238	43065	155	238	282
N.S.	1	1.00	0.93	0.68	0.99	178.69	0.64	0.99	1.17
time (sec)	N/A	0.131	0.137	0.403	0.559	1.654	0.599	1.689	4.944

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	193	174	186	40637	194	272	315
N.S.	1	1.00	1.42	1.28	1.37	298.80	1.43	2.00	2.32
time (sec)	N/A	0.072	0.097	0.323	0.542	1.492	1.175	1.131	4.979

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	249	207	269	43180	192	256	315
N.S.	1	1.00	0.94	0.78	1.01	162.33	0.72	0.96	1.18
time (sec)	N/A	0.155	0.137	0.327	0.527	2.688	1.143	0.896	4.989

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	217	165	223	40780	231	296	351
N.S.	1	1.00	1.34	1.02	1.38	251.73	1.43	1.83	2.17
time (sec)	N/A	0.093	0.117	0.340	0.536	1.562	0.895	0.691	4.975

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	274	201	304	43302	231	280	350
N.S.	1	1.00	0.94	0.69	1.04	148.80	0.79	0.96	1.20
time (sec)	N/A	0.185	0.193	0.343	0.530	5.088	0.888	0.711	0.309

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	42	40	35	35	313	37	100
N.S.	1	1.00	1.75	1.67	1.46	1.46	13.04	1.54	4.17
time (sec)	N/A	0.013	0.012	0.371	0.531	0.383	0.284	0.665	4.918

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	99	61	86	34376	83	86	71
N.S.	1	1.00	1.01	0.62	0.88	350.78	0.85	0.88	0.72
time (sec)	N/A	0.046	0.056	0.344	0.540	1.319	0.285	0.648	0.092

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	187	139	155	120560	471	263	725
N.S.	1	1.00	1.61	1.20	1.34	1039.31	4.06	2.27	6.25
time (sec)	N/A	0.064	0.034	0.351	0.528	2.519	5.631	0.570	5.143

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	229	226	261	121386	466	275	712
N.S.	1	1.00	0.83	0.82	0.94	438.22	1.68	0.99	2.57
time (sec)	N/A	0.133	0.071	0.339	0.573	2.656	5.576	0.621	5.086

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	211	203	194	116982	508	311	477
N.S.	1	1.00	1.45	1.39	1.33	801.25	3.48	2.13	3.27
time (sec)	N/A	0.084	0.134	0.329	0.527	3.213	41.019	0.563	4.982

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	305	287	299	124258	505	306	472
N.S.	1	1.00	0.99	0.93	0.97	403.44	1.64	0.99	1.53
time (sec)	N/A	0.170	0.214	0.330	0.517	3.620	40.932	0.549	0.333

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	244	273	234	118710	0	340	826
N.S.	1	1.00	1.36	1.53	1.31	663.18	0.00	1.90	4.61
time (sec)	N/A	0.113	0.138	0.345	0.517	5.631	0.000	0.800	5.111

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	337	354	342	124787	0	336	826
N.S.	1	1.00	0.99	1.04	1.00	365.94	0.00	0.99	2.42
time (sec)	N/A	0.206	0.205	0.338	0.528	7.744	0.000	0.757	5.047

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	276	248	284	118903	0	377	874
N.S.	1	1.00	1.31	1.18	1.35	563.52	0.00	1.79	4.14
time (sec)	N/A	0.141	0.171	0.342	0.535	10.420	0.000	0.802	5.220

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	369	334	390	124960	0	373	873
N.S.	1	1.00	0.99	0.90	1.05	335.91	0.00	1.00	2.35
time (sec)	N/A	0.255	0.249	0.325	0.538	16.451	0.000	0.696	5.137

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	24	25	24	27	25	25
N.S.	1	1.00	0.96	0.86	0.89	0.86	0.96	0.89	0.89
time (sec)	N/A	0.010	0.002	0.324	0.285	0.342	0.008	0.626	4.674

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	27	27	27	29	27	27
N.S.	1	1.00	0.97	0.82	0.82	0.82	0.88	0.82	0.82
time (sec)	N/A	0.009	0.002	0.354	0.287	0.370	0.010	0.711	0.034

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	58	50	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.97	0.83	0.83
time (sec)	N/A	0.043	0.002	0.353	0.283	0.363	0.011	0.625	0.025

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	27	27	31	27	27
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.94	0.82	0.82
time (sec)	N/A	0.010	0.001	0.317	0.294	0.385	0.009	0.651	0.039

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	60	50	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.00	0.83	0.83
time (sec)	N/A	0.018	0.002	0.352	0.274	0.362	0.011	0.551	0.025

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	53	61	53	53
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.94	0.82	0.82
time (sec)	N/A	0.066	0.002	0.322	0.298	0.359	0.010	0.708	0.027

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	77	76	76	90	76	76
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.83
time (sec)	N/A	0.036	0.003	0.377	0.282	0.363	0.011	0.797	0.038

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	33	16	16	27	29	16	26
N.S.	1	1.00	1.94	0.94	0.94	1.59	1.71	0.94	1.53
time (sec)	N/A	0.003	0.001	0.317	0.282	0.368	0.009	0.635	0.033

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	60	51	50	50	58	50	50
N.S.	1	1.00	1.33	1.13	1.11	1.11	1.29	1.11	1.11
time (sec)	N/A	0.013	0.002	0.362	0.300	0.345	0.010	0.565	0.024

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	65	54	53	53	60	53	53
N.S.	1	1.00	1.30	1.08	1.06	1.06	1.20	1.06	1.06
time (sec)	N/A	0.013	0.003	0.344	0.297	0.354	0.010	0.553	0.027

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	76	76	88	76	76
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.14	0.99	0.99
time (sec)	N/A	0.036	0.003	0.367	0.294	0.364	0.011	0.615	0.038

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	65	54	53	53	61	53	53
N.S.	1	1.00	1.30	1.08	1.06	1.06	1.22	1.06	1.06
time (sec)	N/A	0.015	0.002	0.327	0.291	0.372	0.010	0.530	0.027

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	76	76	90	76	76
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.17	0.99	0.99
time (sec)	N/A	0.026	0.003	0.352	0.281	0.352	0.023	0.526	0.038

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	79	79	92	79	79
N.S.	1	1.00	1.18	0.98	0.96	0.96	1.12	0.96	0.96
time (sec)	N/A	0.054	0.003	0.362	0.289	0.355	0.032	0.720	0.041

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	105	102	121	105	102
N.S.	1	1.00	1.14	0.94	0.96	0.94	1.11	0.96	0.94
time (sec)	N/A	0.046	0.005	0.368	0.296	0.366	0.035	0.514	4.677

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	154	150	180	154	150
N.S.	1	1.00	1.19	1.00	1.02	0.99	1.19	1.02	0.99
time (sec)	N/A	0.071	0.004	0.371	0.604	0.370	0.021	0.525	4.863

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	220	222	203	117016	520	320	483
N.S.	1	1.00	1.42	1.43	1.31	754.94	3.35	2.06	3.12
time (sec)	N/A	0.085	0.097	0.330	0.695	3.335	45.644	0.661	0.415

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	253	312	253	118761	0	358	832
N.S.	1	1.00	1.35	1.66	1.35	631.71	0.00	1.90	4.43
time (sec)	N/A	0.104	0.136	0.358	0.786	5.915	0.000	0.596	5.185

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	286	254	302	118945	0	395	880
N.S.	1	1.00	1.30	1.15	1.37	540.66	0.00	1.80	4.00
time (sec)	N/A	0.133	0.188	0.338	0.501	10.787	0.000	0.545	5.246

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	78	94	123	279	88	97	36
N.S.	1	1.00	0.77	0.93	1.22	2.76	0.87	0.96	0.36
time (sec)	N/A	0.065	0.023	0.325	0.508	0.390	0.206	0.710	0.125

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	16	15	15	19	15	15
N.S.	1	1.00	1.00	0.73	0.68	0.68	0.86	0.68	0.68
time (sec)	N/A	0.008	0.009	0.347	0.507	0.361	0.030	0.645	4.774

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	107	110	147	12348	88	115	119
N.S.	1	1.00	0.87	0.89	1.20	100.39	0.72	0.93	0.97
time (sec)	N/A	0.073	0.034	0.392	0.495	1.232	0.297	0.582	0.200

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	78	94	123	299	88	97	32
N.S.	1	1.00	0.77	0.93	1.22	2.96	0.87	0.96	0.32
time (sec)	N/A	0.053	0.012	0.336	0.507	0.381	0.174	0.658	4.974

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	113	188	167	2556	68	131	315
N.S.	1	1.00	0.80	1.33	1.18	18.13	0.48	0.93	2.23
time (sec)	N/A	0.067	0.035	0.342	0.510	0.458	0.218	0.577	5.110

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	99	110	147	12741	85	114	162
N.S.	1	1.00	0.80	0.89	1.20	103.59	0.69	0.93	1.32
time (sec)	N/A	0.079	0.030	0.328	0.508	1.252	0.310	0.620	0.217

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	129	203	187	46651	292	143	270
N.S.	1	1.00	0.79	1.25	1.15	286.20	1.79	0.88	1.66
time (sec)	N/A	0.083	0.049	0.342	0.491	1.627	2.758	0.760	5.519

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	9
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.69
time (sec)	N/A	0.003	0.003	0.316	0.265	0.401	0.021	0.867	0.030

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	108	106	149	353	51	109	117
N.S.	1	1.00	0.95	0.93	1.31	3.10	0.45	0.96	1.03
time (sec)	N/A	0.069	0.021	0.336	0.522	0.387	0.181	0.858	0.283

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	65	28	113	27	53	93	25
N.S.	1	1.00	1.81	0.78	3.14	0.75	1.47	2.58	0.69
time (sec)	N/A	0.021	0.025	0.336	0.509	0.385	0.186	0.800	0.056

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	128	121	171	17085	199	125	307
N.S.	1	1.00	0.94	0.89	1.26	125.62	1.46	0.92	2.26
time (sec)	N/A	0.078	0.039	0.338	0.498	1.336	0.891	1.350	5.496

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	108	106	152	299	70	109	117
N.S.	1	1.00	0.95	0.93	1.33	2.62	0.61	0.96	1.03
time (sec)	N/A	0.080	0.017	0.341	0.512	0.410	0.145	0.918	0.374

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	148	199	195	2604	148	137	286
N.S.	1	1.00	0.96	1.29	1.27	16.91	0.96	0.89	1.86
time (sec)	N/A	0.081	0.085	0.327	0.500	0.432	0.856	0.803	5.810

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	125	121	174	18086	189	124	300
N.S.	1	1.00	0.92	0.89	1.28	132.99	1.39	0.91	2.21
time (sec)	N/A	0.095	0.042	0.343	0.514	1.355	0.868	1.210	5.388

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	164	214	207	54479	580	149	1168
N.S.	1	1.00	0.93	1.22	1.18	309.54	3.30	0.85	6.64
time (sec)	N/A	0.096	0.070	0.338	0.508	1.850	5.186	1.045	5.636

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75
time (sec)	N/A	0.005	0.001	0.325	0.287	0.386	0.009	1.107	0.023

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	118	76	150	73	70	156
N.S.	1	1.00	0.94	2.23	1.43	2.83	1.38	1.32	2.94
time (sec)	N/A	0.034	0.026	0.363	0.489	0.370	0.173	1.056	0.402

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	203	150	160	91748	187	290	312
N.S.	1	1.00	1.64	1.21	1.29	739.90	1.51	2.34	2.52
time (sec)	N/A	0.060	0.034	0.365	0.507	1.442	0.947	1.283	5.034

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	283	236	296	96349	187	270	305
N.S.	1	1.00	1.02	0.85	1.07	347.83	0.68	0.97	1.10
time (sec)	N/A	0.133	0.137	0.348	0.510	1.489	1.124	1.804	5.041

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	249	167	204	592528	0	303	2500
N.S.	1	1.00	1.68	1.13	1.38	4003.57	0.00	2.05	16.89
time (sec)	N/A	0.137	0.053	0.336	0.512	29.570	0.000	1.136	5.507

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	221	206	227	334837	0	344	1393
N.S.	1	1.00	1.28	1.20	1.32	1946.73	0.00	2.00	8.10
time (sec)	N/A	0.109	0.219	0.344	0.522	21.208	0.000	1.322	5.560

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	263	244	288	343626	0	393	1002
N.S.	1	1.00	1.19	1.10	1.30	1554.87	0.00	1.78	4.53
time (sec)	N/A	0.177	0.332	0.335	0.504	30.945	0.000	1.061	5.436

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	313	285	350	343822	0	442	1056
N.S.	1	1.00	1.18	1.07	1.32	1292.56	0.00	1.66	3.97
time (sec)	N/A	0.220	0.210	0.352	0.521	45.600	0.000	1.296	5.662

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	311	253	332	622377	0	340	2500
N.S.	1	1.00	0.97	0.79	1.04	1951.03	0.00	1.07	7.84
time (sec)	N/A	0.232	0.218	0.336	0.509	30.942	0.000	2.071	5.588

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	319	291	355	352423	0	365	1383
N.S.	1	1.00	0.94	0.85	1.04	1033.50	0.00	1.07	4.06
time (sec)	N/A	0.210	0.136	0.337	0.506	24.569	0.000	1.456	5.592

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	366	330	418	358509	0	416	1001
N.S.	1	1.00	0.93	0.84	1.06	909.92	0.00	1.06	2.54
time (sec)	N/A	0.291	0.188	0.334	0.507	44.572	0.000	1.165	0.705

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	411	371	479	358702	0	466	1053
N.S.	1	1.00	0.94	0.85	1.10	820.83	0.00	1.07	2.41
time (sec)	N/A	0.354	0.236	0.362	0.500	83.459	0.000	1.311	5.562

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	15	15	15	15	15
N.S.	1	1.00	0.82	0.73	1.36	1.36	1.36	1.36	1.36
time (sec)	N/A	0.009	0.001	0.306	0.281	0.372	0.017	0.876	0.031

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	14	8	12	12	8	12	11
N.S.	1	1.00	1.27	0.73	1.09	1.09	0.73	1.09	1.00
time (sec)	N/A	0.008	0.001	0.309	0.273	0.389	0.014	0.553	0.024

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	5	7	6
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.56	0.78	0.67
time (sec)	N/A	0.005	0.001	0.018	0.288	0.371	0.008	0.797	0.019

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75
time (sec)	N/A	0.005	0.001	0.327	0.277	0.398	0.008	0.936	0.002

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	5	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.71	1.00	1.00
time (sec)	N/A	0.010	0.001	0.353	0.280	0.369	0.024	0.734	0.031

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	12	12	10	7	7
N.S.	1	1.00	0.82	0.73	1.09	1.09	0.91	0.64	0.64
time (sec)	N/A	0.013	0.001	0.418	0.276	0.422	0.032	0.630	4.837

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	17	17	17	7	7
N.S.	1	1.00	0.82	0.73	1.55	1.55	1.55	0.64	0.64
time (sec)	N/A	0.013	0.001	0.429	0.275	0.398	0.039	0.622	4.806

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	256	180	224	0	0	342	2478
N.S.	1	1.00	1.55	1.09	1.36	0.00	0.00	2.07	15.02
time (sec)	N/A	0.178	0.172	0.342	0.510	0.000	0.000	0.722	5.544

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	301	195	240	0	0	533	2500
N.S.	1	1.00	1.60	1.04	1.28	0.00	0.00	2.84	13.30
time (sec)	N/A	0.217	0.235	0.348	0.509	0.000	0.000	0.661	5.074

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	318	210	257	0	0	548	2500
N.S.	1	1.00	1.55	1.02	1.25	0.00	0.00	2.67	12.20
time (sec)	N/A	0.211	0.243	0.369	0.507	0.000	0.000	0.603	5.161

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	342	267	355	0	0	375	2469
N.S.	1	1.00	1.01	0.79	1.05	0.00	0.00	1.11	7.33
time (sec)	N/A	0.265	0.226	0.349	0.511	0.000	0.000	0.618	5.542

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	427	283	399	0	0	554	2500
N.S.	1	1.00	1.11	0.74	1.04	0.00	0.00	1.44	6.51
time (sec)	N/A	0.376	0.202	0.346	0.516	0.000	0.000	0.528	5.054

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	445	299	429	0	0	570	2500
N.S.	1	1.00	1.11	0.74	1.07	0.00	0.00	1.42	6.22
time (sec)	N/A	0.368	0.188	0.365	0.511	0.000	0.000	0.559	5.203

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	257	222	246	710521	0	380	1626
N.S.	1	1.00	1.40	1.21	1.34	3861.53	0.00	2.07	8.84
time (sec)	N/A	0.139	0.107	0.340	0.512	194.957	0.000	0.540	5.615

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	302	241	261	0	0	576	2611
N.S.	1	1.00	1.49	1.19	1.29	0.00	0.00	2.84	12.86
time (sec)	N/A	0.179	0.143	0.364	0.496	0.000	0.000	0.495	5.671

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	338	262	300	0	0	603	2500
N.S.	1	1.00	1.50	1.16	1.33	0.00	0.00	2.68	11.11
time (sec)	N/A	0.212	0.144	0.358	0.501	0.000	0.000	0.600	5.909

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	359	309	379	736847	0	398	1623
N.S.	1	1.00	1.02	0.88	1.07	2087.39	0.00	1.13	4.60
time (sec)	N/A	0.232	0.166	0.350	0.507	207.375	0.000	0.665	5.578

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	415	329	416	0	0	582	2605
N.S.	1	1.00	1.05	0.83	1.05	0.00	0.00	1.47	6.59
time (sec)	N/A	0.323	0.226	0.351	0.511	0.000	0.000	0.718	5.702

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	460	350	458	0	0	608	2500
N.S.	1	1.00	1.10	0.84	1.10	0.00	0.00	1.46	6.00
time (sec)	N/A	0.351	0.209	0.360	0.533	0.000	0.000	0.637	5.844

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	309	267	320	0	0	440	1687
N.S.	1	1.00	1.28	1.11	1.33	0.00	0.00	1.83	7.00
time (sec)	N/A	0.219	0.185	0.353	0.527	0.000	0.000	1.105	5.732

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	359	295	344	0	0	645	2680
N.S.	1	1.00	1.34	1.10	1.28	0.00	0.00	2.41	10.00
time (sec)	N/A	0.281	0.216	0.426	0.499	0.000	0.000	1.074	5.801

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	380	311	378	0	0	677	2696
N.S.	1	1.00	1.33	1.09	1.33	0.00	0.00	2.38	9.46
time (sec)	N/A	0.255	0.172	0.395	0.510	0.000	0.000	1.910	5.910

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	411	353	452	0	0	459	1686
N.S.	1	1.00	1.00	0.85	1.09	0.00	0.00	1.11	4.08
time (sec)	N/A	0.328	0.228	0.355	0.507	0.000	0.000	1.587	5.691

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	473	381	497	0	0	652	2680
N.S.	1	1.00	1.02	0.82	1.07	0.00	0.00	1.41	5.79
time (sec)	N/A	0.456	0.348	0.347	0.520	0.000	0.000	1.174	5.749

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	500	396	535	0	0	684	2695
N.S.	1	1.00	1.04	0.82	1.11	0.00	0.00	1.42	5.61
time (sec)	N/A	0.443	0.275	0.370	0.503	0.000	0.000	1.079	5.788

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	360	313	394	0	0	501	1747
N.S.	1	1.00	1.23	1.07	1.34	0.00	0.00	1.71	5.96
time (sec)	N/A	0.282	0.258	0.346	0.519	0.000	0.000	0.821	5.994

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	422	346	429	0	0	717	2500
N.S.	1	1.00	1.27	1.05	1.30	0.00	0.00	2.17	7.55
time (sec)	N/A	0.378	0.227	0.368	0.531	0.000	0.000	0.569	6.140

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	439	362	463	0	0	749	2500
N.S.	1	1.00	1.26	1.04	1.33	0.00	0.00	2.15	7.16
time (sec)	N/A	0.387	0.228	0.366	0.531	0.000	0.000	0.559	6.396

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	461	399	524	0	0	521	1743
N.S.	1	1.00	1.00	0.86	1.13	0.00	0.00	1.13	3.77
time (sec)	N/A	0.744	0.297	0.342	0.496	0.000	0.000	0.521	6.075

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	516	530	432	581	0	0	725	2500
N.S.	1	1.00	1.03	0.84	1.13	0.00	0.00	1.41	4.84
time (sec)	N/A	0.551	0.440	0.360	0.523	0.000	0.000	0.593	6.084

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	555	448	615	0	0	757	2500
N.S.	1	1.00	1.04	0.84	1.15	0.00	0.00	1.42	4.68
time (sec)	N/A	0.526	0.350	0.354	0.537	0.000	0.000	0.644	6.480

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	79	96	0	72	61	0	-1
N.S.	1	1.00	0.65	0.79	0.00	0.60	0.50	0.00	-0.01
time (sec)	N/A	0.044	10.051	0.321	0.000	0.111	1.261	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	90	0	79	95	0	-1
N.S.	1	1.00	0.93	1.03	0.00	0.91	1.09	0.00	-0.01
time (sec)	N/A	0.040	10.040	0.368	0.000	0.123	1.362	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	83	95	0	73	90	0	-1
N.S.	1	1.00	0.93	1.07	0.00	0.82	1.01	0.00	-0.01
time (sec)	N/A	0.041	10.036	0.371	0.000	0.110	1.408	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	85	101	0	82	66	0	-1
N.S.	1	1.00	0.67	0.80	0.00	0.65	0.52	0.00	-0.01
time (sec)	N/A	0.045	10.046	0.323	0.000	0.109	1.324	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	131	193	0	128	102	0	-1
N.S.	1	1.00	0.51	0.75	0.00	0.50	0.40	0.00	-0.00
time (sec)	N/A	0.083	10.094	0.325	0.000	0.116	1.581	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	195	12	12	80	12	12
N.S.	1	1.00	1.00	13.93	0.86	0.86	5.71	0.86	0.86
time (sec)	N/A	0.004	1.058	0.338	0.323	0.396	3.355	0.618	5.036

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	213	26	34	104	23	23
N.S.	1	1.00	0.93	7.34	0.90	1.17	3.59	0.79	0.79
time (sec)	N/A	0.015	9.775	0.326	0.317	0.397	4.496	0.612	4.905

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	210	23	33	109	22	20
N.S.	1	1.00	1.08	8.40	0.92	1.32	4.36	0.88	0.80
time (sec)	N/A	0.019	10.030	0.357	0.321	0.398	5.938	0.693	4.901

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	228	45	44	133	31	29
N.S.	1	1.00	1.00	6.00	1.18	1.16	3.50	0.82	0.76
time (sec)	N/A	0.021	10.048	0.329	0.322	0.406	7.328	0.630	4.836

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	58	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	4.83	0.83	0.83
time (sec)	N/A	0.002	0.133	0.328	0.573	0.398	1.860	0.579	4.847

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	281	460	0	204	260	0	-1
N.S.	1	1.00	0.73	1.19	0.00	0.53	0.68	0.00	-0.00
time (sec)	N/A	0.292	10.154	0.351	0.000	0.156	3.503	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	51	143	0	835	1287	101	64
N.S.	1	1.00	0.47	1.31	0.00	7.66	11.81	0.93	0.59
time (sec)	N/A	0.047	0.007	0.316	0.000	1.154	0.545	0.541	4.918

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	47	135	0	799	1287	101	65
N.S.	1	1.00	0.43	1.24	0.00	7.33	11.81	0.93	0.60
time (sec)	N/A	0.027	0.007	0.328	0.000	1.172	0.543	0.606	4.981

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	187	236	215	210	216	246	237
N.S.	1	1.00	0.90	1.13	1.03	1.01	1.04	1.18	1.14
time (sec)	N/A	0.209	0.064	0.344	0.283	0.412	0.622	0.609	4.920

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	154	188	174	170	172	197	189
N.S.	1	1.00	0.91	1.11	1.02	1.00	1.01	1.16	1.11
time (sec)	N/A	0.160	0.046	0.369	0.270	0.374	0.576	0.561	4.959

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	119	140	133	130	128	148	141
N.S.	1	1.00	0.90	1.06	1.01	0.98	0.97	1.12	1.07
time (sec)	N/A	0.122	0.038	0.362	0.277	0.387	0.590	0.588	4.927

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	88	96	94	92	88	101	96
N.S.	1	1.00	0.92	1.00	0.98	0.96	0.92	1.05	1.00
time (sec)	N/A	0.088	0.030	0.363	0.291	0.401	0.571	0.528	4.827

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	75	75	79	80	70	79	76
N.S.	1	1.00	0.94	0.94	0.99	1.00	0.88	0.99	0.95
time (sec)	N/A	0.078	0.023	0.360	0.303	0.416	3.025	0.536	4.925

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	75	78	85	70	95	74
N.S.	1	1.00	0.95	0.93	0.96	1.05	0.86	1.17	0.91
time (sec)	N/A	0.076	0.031	0.339	0.305	0.413	50.847	0.686	4.973

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	90	95	101	0	126	92
N.S.	1	1.00	0.93	0.95	1.00	1.06	0.00	1.33	0.97
time (sec)	N/A	0.085	0.048	0.349	0.296	0.442	0.000	0.640	4.993

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	128	120	128	127	0	184	123
N.S.	1	1.00	1.00	0.94	1.00	0.99	0.00	1.44	0.96
time (sec)	N/A	0.107	0.046	0.346	0.294	0.422	0.000	0.616	5.025

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	164	156	170	168	0	235	161
N.S.	1	1.00	1.00	0.95	1.04	1.02	0.00	1.43	0.98
time (sec)	N/A	0.122	0.044	0.349	0.306	0.424	0.000	0.568	5.073

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	194	193	213	210	0	287	200
N.S.	1	1.00	0.95	0.94	1.04	1.02	0.00	1.40	0.98
time (sec)	N/A	0.139	0.100	0.343	0.339	0.433	0.000	0.629	0.257

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	351	309	359	342	469	454	358
N.S.	1	1.00	1.01	0.89	1.03	0.98	1.35	1.30	1.03
time (sec)	N/A	0.219	0.054	0.346	0.529	0.405	1.148	0.640	0.311

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	311	249	320	321	513	441	313
N.S.	1	1.00	0.98	0.79	1.01	1.02	1.62	1.40	0.99
time (sec)	N/A	0.200	0.063	0.362	0.499	0.412	0.983	0.628	5.162

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	306	261	318	304	423	401	311
N.S.	1	1.00	0.98	0.84	1.02	0.97	1.36	1.29	1.00
time (sec)	N/A	0.192	0.063	0.342	0.498	0.408	1.075	0.617	5.187

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	266	210	275	281	469	386	267
N.S.	1	1.00	0.95	0.75	0.99	1.01	1.68	1.38	0.96
time (sec)	N/A	0.181	0.063	0.338	0.484	0.411	0.897	0.586	5.147

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	264	213	273	249	376	346	264
N.S.	1	1.00	0.96	0.78	1.00	0.91	1.37	1.26	0.96
time (sec)	N/A	0.174	0.061	0.340	0.479	0.400	1.069	0.600	5.101

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	231	173	230	568	427	291	225
N.S.	1	1.00	0.94	0.71	0.94	2.32	1.74	1.19	0.92
time (sec)	N/A	0.141	0.103	0.334	0.481	0.412	0.849	0.615	5.135

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	229	170	228	600	342	253	222
N.S.	1	1.00	0.95	0.71	0.95	2.50	1.42	1.05	0.92
time (sec)	N/A	0.096	0.100	0.334	0.509	0.406	1.003	0.634	5.174

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	224	159	221	560	408	269	204
N.S.	1	1.00	0.99	0.70	0.97	2.47	1.80	1.19	0.90
time (sec)	N/A	0.125	0.087	0.413	0.490	0.413	1.440	1.398	5.372

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	218	155	218	565	326	232	201
N.S.	1	1.00	0.97	0.69	0.97	2.52	1.46	1.04	0.90
time (sec)	N/A	0.112	0.081	0.405	0.480	0.400	1.709	1.235	0.284

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	220	159	220	556	411	261	209
N.S.	1	1.00	0.97	0.70	0.97	2.45	1.81	1.15	0.92
time (sec)	N/A	0.124	0.078	0.378	0.501	0.397	4.558	1.515	5.164

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	220	155	217	584	328	220	207
N.S.	1	1.00	0.98	0.69	0.96	2.60	1.46	0.98	0.92
time (sec)	N/A	0.113	0.060	0.367	0.497	0.438	8.249	0.820	5.091

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	231	170	238	610	432	275	219
N.S.	1	1.00	0.95	0.70	0.98	2.52	1.79	1.14	0.90
time (sec)	N/A	0.126	0.078	0.362	0.493	0.399	45.533	1.020	5.200

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	231	170	238	595	0	297	220
N.S.	1	1.00	0.95	0.70	0.98	2.44	0.00	1.22	0.90
time (sec)	N/A	0.116	0.077	0.373	0.497	0.398	0.000	1.275	5.126

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	266	205	265	262	0	376	253
N.S.	1	1.00	0.96	0.74	0.96	0.95	0.00	1.36	0.91
time (sec)	N/A	0.146	0.082	0.364	0.499	0.377	0.000	2.424	5.329

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	266	205	265	295	0	338	253
N.S.	1	1.00	0.95	0.73	0.95	1.05	0.00	1.21	0.90
time (sec)	N/A	0.132	0.081	0.369	0.518	0.384	0.000	1.568	5.153

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	308	239	313	317	0	419	286
N.S.	1	1.00	0.98	0.76	1.00	1.01	0.00	1.34	0.91
time (sec)	N/A	0.160	0.069	0.385	0.524	0.403	0.000	1.381	5.228

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	311	240	313	335	0	393	287
N.S.	1	1.00	0.99	0.76	0.99	1.06	0.00	1.25	0.91
time (sec)	N/A	0.146	0.068	0.385	0.506	0.403	0.000	0.914	5.171

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	346	277	360	355	0	474	323
N.S.	1	1.00	0.99	0.79	1.03	1.01	0.00	1.35	0.92
time (sec)	N/A	0.175	0.076	0.379	0.496	0.378	0.000	0.990	5.164

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	205	217	229	303	236	300	356
N.S.	1	1.00	0.93	0.99	1.04	1.38	1.07	1.36	1.62
time (sec)	N/A	0.234	0.093	0.362	0.272	0.376	15.279	0.963	4.988

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	167	179	186	257	189	248	233
N.S.	1	1.00	0.93	0.99	1.03	1.43	1.05	1.38	1.29
time (sec)	N/A	0.177	0.076	0.360	0.281	0.382	14.499	0.978	4.995

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	129	138	143	202	141	217	155
N.S.	1	1.00	0.92	0.99	1.02	1.44	1.01	1.55	1.11
time (sec)	N/A	0.136	0.063	0.345	0.281	0.399	12.940	0.995	4.930

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	102	102	143	100	206	103
N.S.	1	1.00	0.90	0.99	0.99	1.39	0.97	2.00	1.00
time (sec)	N/A	0.092	0.043	0.367	0.280	0.382	6.806	0.877	0.085

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	95	97	102	145	0	125	100
N.S.	1	1.00	0.95	0.97	1.02	1.45	0.00	1.25	1.00
time (sec)	N/A	0.085	0.085	0.396	0.279	0.421	0.000	0.763	5.033

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	97	101	117	172	0	131	109
N.S.	1	1.00	0.89	0.93	1.07	1.58	0.00	1.20	1.00
time (sec)	N/A	0.094	0.054	0.361	0.284	0.412	0.000	0.882	5.046

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	118	123	141	208	0	201	130
N.S.	1	1.00	0.91	0.95	1.08	1.60	0.00	1.55	1.00
time (sec)	N/A	0.106	0.081	0.350	0.281	0.417	0.000	0.884	5.014

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	160	169	185	261	0	275	175
N.S.	1	1.00	0.91	0.97	1.06	1.49	0.00	1.57	1.00
time (sec)	N/A	0.141	0.073	0.371	0.283	0.422	0.000	1.206	5.084

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	198	209	231	310	0	331	216
N.S.	1	1.00	0.93	0.98	1.08	1.45	0.00	1.55	1.01
time (sec)	N/A	0.164	0.143	0.375	0.279	0.453	0.000	1.016	5.087

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	364	306	378	488	500	451	481
N.S.	1	1.00	0.99	0.83	1.02	1.32	1.36	1.22	1.30
time (sec)	N/A	0.311	0.229	0.365	0.505	0.410	85.915	0.861	0.347

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	319	258	333	455	0	442	362
N.S.	1	1.00	0.95	0.77	0.99	1.36	0.00	1.32	1.08
time (sec)	N/A	0.459	0.108	0.364	0.492	0.394	0.000	1.110	5.278

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	315	258	329	423	449	394	358
N.S.	1	1.00	0.96	0.79	1.00	1.29	1.37	1.20	1.09
time (sec)	N/A	0.247	0.163	0.353	0.502	0.393	76.644	1.008	5.201

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	282	218	284	920	0	344	287
N.S.	1	1.00	0.95	0.73	0.95	3.09	0.00	1.15	0.96
time (sec)	N/A	0.304	0.104	0.358	0.492	0.426	0.000	1.083	5.222

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	277	215	277	946	401	295	280
N.S.	1	1.00	0.96	0.75	0.96	3.28	1.39	1.02	0.97
time (sec)	N/A	0.217	0.105	0.374	0.493	0.410	67.930	1.287	0.311

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	255	197	265	874	0	318	246
N.S.	1	1.00	0.94	0.73	0.98	3.23	0.00	1.17	0.91
time (sec)	N/A	0.411	0.099	0.351	0.514	0.417	0.000	1.372	5.232

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	251	191	260	861	377	273	241
N.S.	1	1.00	0.95	0.72	0.98	3.26	1.43	1.03	0.91
time (sec)	N/A	0.332	0.097	0.364	0.516	0.444	3.666	1.647	5.177

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	255	187	262	860	457	305	244
N.S.	1	1.00	0.96	0.71	0.99	3.25	1.72	1.15	0.92
time (sec)	N/A	0.179	0.104	0.373	0.501	0.421	67.213	1.413	5.390

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	250	183	262	902	0	261	245
N.S.	1	1.00	0.96	0.70	1.01	3.47	0.00	1.00	0.94
time (sec)	N/A	0.169	0.105	0.368	0.510	0.408	0.000	1.486	5.222

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	255	195	271	902	0	310	247
N.S.	1	1.00	0.95	0.72	1.01	3.35	0.00	1.15	0.92
time (sec)	N/A	0.192	0.105	0.362	0.499	0.432	0.000	1.386	5.177

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	253	193	272	897	0	264	248
N.S.	1	1.00	0.94	0.71	1.01	3.32	0.00	0.98	0.92
time (sec)	N/A	0.181	0.106	0.443	0.509	0.439	0.000	0.743	5.127

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	281	215	297	982	0	333	274
N.S.	1	1.00	0.95	0.72	1.00	3.31	0.00	1.12	0.92
time (sec)	N/A	0.253	0.113	0.419	0.497	0.413	0.000	1.009	5.184

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	280	214	297	959	0	347	274
N.S.	1	1.00	0.94	0.72	1.00	3.23	0.00	1.17	0.92
time (sec)	N/A	0.247	0.114	0.375	0.490	0.420	0.000	1.049	5.200

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	319	251	329	442	0	437	310
N.S.	1	1.00	0.96	0.75	0.99	1.32	0.00	1.31	0.93
time (sec)	N/A	0.305	0.120	0.380	0.496	0.424	0.000	1.114	5.406

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	317	250	329	475	0	391	310
N.S.	1	1.00	0.95	0.75	0.98	1.42	0.00	1.17	0.93
time (sec)	N/A	0.290	0.124	0.373	0.525	0.424	0.000	0.862	5.121

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	370	287	381	507	0	482	348
N.S.	1	1.00	0.99	0.77	1.02	1.35	0.00	1.29	0.93
time (sec)	N/A	0.356	0.230	0.383	0.516	0.387	0.000	0.792	5.118

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	246	261	283	396	0	349	449
N.S.	1	1.00	0.92	0.98	1.06	1.49	0.00	1.31	1.69
time (sec)	N/A	0.301	0.096	0.362	0.279	0.372	0.000	0.651	4.958

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	208	223	240	353	0	298	293
N.S.	1	1.00	0.92	0.99	1.06	1.56	0.00	1.32	1.30
time (sec)	N/A	0.226	0.081	0.373	0.284	0.377	0.000	0.604	4.969

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	170	182	197	295	0	236	204
N.S.	1	1.00	0.91	0.98	1.06	1.59	0.00	1.27	1.10
time (sec)	N/A	0.185	0.068	0.354	0.283	0.385	0.000	0.658	4.924

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	145	143	152	225	0	146	152
N.S.	1	1.00	0.99	0.98	1.04	1.54	0.00	1.00	1.04
time (sec)	N/A	0.137	0.051	0.366	0.267	0.392	0.000	0.700	0.105

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	105	109	113	158	0	100	112
N.S.	1	1.00	0.96	1.00	1.04	1.45	0.00	0.92	1.03
time (sec)	N/A	0.095	0.038	0.342	0.283	0.377	0.000	0.617	4.939

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	104	114	131	187	0	128	123
N.S.	1	1.00	0.91	1.00	1.15	1.64	0.00	1.12	1.08
time (sec)	N/A	0.102	0.073	0.345	0.284	0.414	0.000	0.677	0.175

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	121	125	145	250	0	173	135
N.S.	1	1.00	0.90	0.93	1.08	1.87	0.00	1.29	1.01
time (sec)	N/A	0.115	0.063	0.352	0.270	0.399	0.000	0.585	5.068

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	149	153	186	316	0	189	167
N.S.	1	1.00	0.91	0.94	1.14	1.94	0.00	1.16	1.02
time (sec)	N/A	0.132	0.075	0.374	0.276	0.400	0.000	0.562	5.099

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	200	212	237	396	0	324	222
N.S.	1	1.00	0.92	0.97	1.09	1.82	0.00	1.49	1.02
time (sec)	N/A	0.175	0.099	0.366	0.283	0.417	0.000	0.588	5.171

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	238	253	286	448	0	380	265
N.S.	1	1.00	0.92	0.98	1.11	1.74	0.00	1.47	1.03
time (sec)	N/A	0.210	0.127	0.402	0.282	0.437	0.000	0.636	0.307

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	411	344	434	667	0	500	575
N.S.	1	1.00	0.99	0.83	1.04	1.60	0.00	1.20	1.38
time (sec)	N/A	0.482	0.304	0.397	0.503	0.389	0.000	0.555	5.241

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	380	296	389	634	0	491	425
N.S.	1	1.00	0.99	0.77	1.01	1.65	0.00	1.28	1.11
time (sec)	N/A	0.688	0.264	0.359	0.491	0.408	0.000	0.547	5.339

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	362	296	385	602	0	443	420
N.S.	1	1.00	0.97	0.79	1.03	1.61	0.00	1.18	1.12
time (sec)	N/A	0.406	0.239	0.346	0.494	0.420	0.000	0.588	5.351

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	329	256	338	1278	0	391	338
N.S.	1	1.00	0.95	0.74	0.98	3.70	0.00	1.13	0.98
time (sec)	N/A	0.500	0.147	0.356	0.504	0.429	0.000	0.640	5.530

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	323	253	334	1318	0	345	335
N.S.	1	1.00	0.96	0.75	0.99	3.92	0.00	1.03	1.00
time (sec)	N/A	0.331	0.208	0.371	0.504	0.422	0.000	0.827	5.302

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	300	233	318	1224	0	365	295
N.S.	1	1.00	0.95	0.74	1.01	3.87	0.00	1.16	0.93
time (sec)	N/A	0.334	0.135	0.361	0.513	0.431	0.000	1.322	5.271

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	294	227	312	1213	0	319	290
N.S.	1	1.00	0.96	0.74	1.02	3.95	0.00	1.04	0.94
time (sec)	N/A	0.277	0.135	0.352	0.510	0.416	0.000	1.115	5.143

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	284	220	301	1158	0	339	280
N.S.	1	1.00	0.94	0.73	1.00	3.85	0.00	1.13	0.93
time (sec)	N/A	0.250	0.134	0.362	0.506	0.416	0.000	1.213	5.268

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	279	215	296	1184	0	295	275
N.S.	1	1.00	0.96	0.74	1.01	4.05	0.00	1.01	0.94
time (sec)	N/A	0.201	0.128	0.361	0.515	0.405	0.000	1.057	5.196

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	286	218	305	1206	0	341	276
N.S.	1	1.00	0.94	0.72	1.01	3.98	0.00	1.13	0.91
time (sec)	N/A	0.225	0.144	0.370	0.503	0.419	0.000	0.830	5.198

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	283	216	307	1217	0	312	279
N.S.	1	1.00	0.94	0.72	1.02	4.04	0.00	1.04	0.93
time (sec)	N/A	0.221	0.141	0.391	0.509	0.406	0.000	0.979	5.164

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	303	230	322	1254	0	357	293
N.S.	1	1.00	0.96	0.73	1.02	3.96	0.00	1.13	0.92
time (sec)	N/A	0.249	0.150	0.379	0.512	0.419	0.000	1.361	5.232

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	299	228	323	1247	0	310	293
N.S.	1	1.00	0.95	0.72	1.02	3.95	0.00	0.98	0.93
time (sec)	N/A	0.244	0.146	0.381	0.505	0.441	0.000	1.017	5.196

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	328	253	349	1340	0	380	321
N.S.	1	1.00	0.96	0.74	1.02	3.91	0.00	1.11	0.94
time (sec)	N/A	0.387	0.151	0.445	0.502	0.414	0.000	0.681	5.262

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	324	252	349	1317	0	394	321
N.S.	1	1.00	0.95	0.74	1.02	3.86	0.00	1.16	0.94
time (sec)	N/A	0.371	0.152	0.402	0.508	0.439	0.000	0.649	5.217

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	366	289	383	621	0	486	359
N.S.	1	1.00	0.96	0.76	1.01	1.63	0.00	1.28	0.94
time (sec)	N/A	0.472	0.266	0.376	0.511	0.431	0.000	0.571	5.279

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	376	288	383	654	0	440	359
N.S.	1	1.00	0.99	0.76	1.01	1.72	0.00	1.16	0.94
time (sec)	N/A	0.450	0.292	0.374	0.516	0.377	0.000	0.658	5.176

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	419	325	435	686	0	531	397
N.S.	1	1.00	0.99	0.77	1.03	1.62	0.00	1.25	0.94
time (sec)	N/A	0.564	0.315	0.384	0.515	0.404	0.000	0.630	5.304

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	59	45	44	44	53	45	56
N.S.	1	1.00	1.09	0.83	0.81	0.81	0.98	0.83	1.04
time (sec)	N/A	0.048	0.010	0.344	0.503	0.390	0.068	0.782	0.099

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	25	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.83	0.80
time (sec)	N/A	0.026	0.003	0.334	0.499	0.374	0.043	0.626	0.032

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	53	38	37	37	44	38	49
N.S.	1	1.00	1.20	0.86	0.84	0.84	1.00	0.86	1.11
time (sec)	N/A	0.040	0.006	0.356	0.490	0.391	0.060	0.757	4.961

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	50	35	34	34	42	35	63
N.S.	1	1.00	1.22	0.85	0.83	0.83	1.02	0.85	1.54
time (sec)	N/A	0.027	0.006	0.335	0.483	0.375	0.054	0.725	0.080

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	53	37	36	36	46	38	48
N.S.	1	1.00	1.26	0.88	0.86	0.86	1.10	0.90	1.14
time (sec)	N/A	0.034	0.006	0.349	0.488	0.386	0.079	0.781	4.960

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	60	44	43	48	49	45	55
N.S.	1	1.00	1.22	0.90	0.88	0.98	1.00	0.92	1.12
time (sec)	N/A	0.033	0.011	0.349	0.470	0.375	0.108	0.616	0.080

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	28	33	27	29	25
N.S.	1	1.00	1.00	0.84	0.88	1.03	0.84	0.91	0.78
time (sec)	N/A	0.022	0.003	0.340	0.485	0.362	0.038	0.602	0.069

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	47	35	34	34	42	35	63
N.S.	1	1.00	1.15	0.85	0.83	0.83	1.02	0.85	1.54
time (sec)	N/A	0.028	0.006	0.391	0.484	0.368	0.052	0.651	4.956

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	53	33	32	32	41	33	63
N.S.	1	1.00	1.36	0.85	0.82	0.82	1.05	0.85	1.62
time (sec)	N/A	0.026	0.007	0.379	0.483	0.382	0.052	0.595	0.089

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	44	45	43	49	45	43
N.S.	1	1.00	1.00	0.80	0.82	0.78	0.89	0.82	0.78
time (sec)	N/A	0.039	0.003	0.145	0.264	0.368	0.008	0.613	0.028

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	44	45	43	49	45	43
N.S.	1	1.00	1.00	0.80	0.82	0.78	0.89	0.82	0.78
time (sec)	N/A	0.025	0.002	0.137	0.275	0.388	0.008	0.557	0.025

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	41	42	40	46	42	40
N.S.	1	1.00	1.00	0.82	0.84	0.80	0.92	0.84	0.80
time (sec)	N/A	0.017	0.002	0.146	0.276	0.377	0.008	0.540	0.024

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	40	38	44	41	38
N.S.	1	1.00	1.00	0.85	0.87	0.83	0.96	0.89	0.83
time (sec)	N/A	0.018	0.003	0.023	0.291	0.376	0.037	0.758	0.029

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	40	45	41	41	38
N.S.	1	1.00	1.00	0.89	0.91	1.02	0.93	0.93	0.86
time (sec)	N/A	0.023	0.004	0.029	0.276	0.369	0.058	0.758	0.030

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	40	45	44	41	38
N.S.	1	1.00	1.00	0.89	0.91	1.02	1.00	0.93	0.86
time (sec)	N/A	0.023	0.004	0.030	0.282	0.370	0.112	0.780	0.028

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	82	79	92	82	79
N.S.	1	1.00	1.18	0.98	1.00	0.96	1.12	1.00	0.96
time (sec)	N/A	0.042	0.003	0.361	0.280	0.358	0.011	0.630	0.040

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	82	79	94	82	79
N.S.	1	1.00	1.18	0.98	1.00	0.96	1.15	1.00	0.96
time (sec)	N/A	0.034	0.003	0.389	0.276	0.358	0.011	0.651	0.038

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	79	76	88	79	76
N.S.	1	1.00	1.19	1.00	1.03	0.99	1.14	1.03	0.99
time (sec)	N/A	0.042	0.003	0.373	0.282	0.354	0.011	0.664	0.038

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	77	74	88	78	74
N.S.	1	1.00	1.00	0.85	0.88	0.84	1.00	0.89	0.84
time (sec)	N/A	0.034	0.006	0.328	0.267	0.376	0.055	0.685	0.042

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	74	76	81	82	77	73
N.S.	1	1.00	1.00	0.89	0.92	0.98	0.99	0.93	0.88
time (sec)	N/A	0.041	0.006	0.347	0.264	0.370	0.067	0.807	0.042

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	75	77	81	87	78	74
N.S.	1	1.00	1.00	0.89	0.92	0.96	1.04	0.93	0.88
time (sec)	N/A	0.042	0.006	0.498	0.272	0.360	0.128	0.746	0.038

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	139	116	119	115	138	119	115
N.S.	1	1.00	1.26	1.05	1.08	1.05	1.25	1.08	1.05
time (sec)	N/A	0.054	0.003	0.396	0.268	0.376	0.013	0.729	0.078

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	139	116	119	115	138	119	115
N.S.	1	1.00	1.26	1.05	1.08	1.05	1.25	1.08	1.05
time (sec)	N/A	0.048	0.003	0.383	0.288	0.362	0.013	0.717	0.074

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	134	113	116	112	134	116	112
N.S.	1	1.00	1.28	1.08	1.10	1.07	1.28	1.10	1.07
time (sec)	N/A	0.070	0.003	0.383	0.283	0.368	0.013	0.698	0.073

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	110	113	109	131	114	109
N.S.	1	1.00	1.00	0.87	0.89	0.86	1.03	0.90	0.86
time (sec)	N/A	0.051	0.007	0.353	0.287	0.370	0.068	0.535	0.079

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	125	110	113	117	128	114	109
N.S.	1	1.00	1.00	0.88	0.90	0.94	1.02	0.91	0.87
time (sec)	N/A	0.063	0.008	0.344	0.281	0.408	0.079	0.656	0.081

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	126	111	114	117	131	115	110
N.S.	1	1.00	1.00	0.88	0.90	0.93	1.04	0.91	0.87
time (sec)	N/A	0.060	0.007	0.390	0.271	0.386	0.140	0.785	4.896

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	181	152	156	151	184	156	151
N.S.	1	1.00	1.31	1.10	1.13	1.09	1.33	1.13	1.09
time (sec)	N/A	0.068	0.004	0.400	0.264	0.363	0.016	0.741	5.073

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	181	152	156	151	185	156	151
N.S.	1	1.00	1.31	1.10	1.13	1.09	1.34	1.13	1.09
time (sec)	N/A	0.061	0.004	0.397	0.270	0.400	0.022	0.718	0.131

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	173	148	152	147	178	152	147
N.S.	1	1.00	1.33	1.14	1.17	1.13	1.37	1.17	1.13
time (sec)	N/A	0.099	0.003	0.523	0.292	0.385	0.016	0.994	0.152

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	166	145	149	144	175	150	144
N.S.	1	1.00	1.00	0.87	0.90	0.87	1.05	0.90	0.87
time (sec)	N/A	0.070	0.007	0.343	0.273	0.395	0.100	1.381	0.141

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	162	145	149	153	168	150	144
N.S.	1	1.00	1.00	0.90	0.92	0.94	1.04	0.93	0.89
time (sec)	N/A	0.082	0.007	0.349	0.276	0.397	0.100	1.343	4.994

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	166	147	151	153	175	152	146
N.S.	1	1.00	1.00	0.89	0.91	0.92	1.05	0.92	0.88
time (sec)	N/A	0.084	0.007	0.487	0.264	0.374	0.141	1.371	4.993

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	191	227	193	4798	178	208	319
N.S.	1	1.00	0.93	1.11	0.94	23.40	0.87	1.01	1.56
time (sec)	N/A	0.175	0.071	0.376	0.506	1.186	0.886	1.333	5.067

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	184	220	185	4261	150	195	340
N.S.	1	1.00	0.95	1.14	0.96	22.08	0.78	1.01	1.76
time (sec)	N/A	0.165	0.060	0.405	0.508	1.106	0.833	1.076	5.133

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	200	211	177	4628	160	178	266
N.S.	1	1.00	1.09	1.15	0.97	25.29	0.87	0.97	1.45
time (sec)	N/A	0.148	0.038	0.389	0.492	1.140	0.795	0.957	5.162

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	176	200	161	4671	160	163	274
N.S.	1	1.00	0.99	1.13	0.91	26.39	0.90	0.92	1.55
time (sec)	N/A	0.088	0.063	0.379	0.522	1.165	0.787	0.804	0.257

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	176	211	179	4588	0	179	716
N.S.	1	1.00	0.96	1.15	0.97	24.93	0.00	0.97	3.89
time (sec)	N/A	0.141	0.061	0.381	0.498	1.272	0.000	0.696	5.247

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	184	221	189	4524	0	201	723
N.S.	1	1.00	0.96	1.15	0.98	23.56	0.00	1.05	3.77
time (sec)	N/A	0.140	0.163	0.384	0.504	1.256	0.000	1.017	5.056

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	192	232	180	4279	0	204	701
N.S.	1	1.00	0.95	1.14	0.89	21.08	0.00	1.00	3.45
time (sec)	N/A	0.131	0.120	0.385	0.544	1.186	0.000	2.336	0.131

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	174	226	167	2077	110	180	180
N.S.	1	1.00	0.92	1.19	0.88	10.93	0.58	0.95	0.95
time (sec)	N/A	0.112	0.092	0.360	0.511	1.125	1.161	1.131	0.220

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	186	230	189	2358	124	190	194
N.S.	1	1.00	0.93	1.15	0.94	11.79	0.62	0.95	0.97
time (sec)	N/A	0.107	0.118	0.373	0.512	1.165	0.906	1.085	5.167

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	189	245	180	2118	116	184	175
N.S.	1	1.00	0.95	1.23	0.90	10.64	0.58	0.92	0.88
time (sec)	N/A	0.125	0.125	0.376	0.584	1.157	0.670	0.879	0.251

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	199	240	207	5018	0	217	490
N.S.	1	1.00	0.90	1.08	0.93	22.60	0.00	0.98	2.21
time (sec)	N/A	0.400	0.103	0.515	0.509	1.227	0.000	0.809	0.380

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	213	249	226	4976	0	237	488
N.S.	1	1.00	0.92	1.08	0.98	21.54	0.00	1.03	2.11
time (sec)	N/A	0.391	0.176	0.369	0.515	1.313	0.000	0.679	5.468

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	221	262	224	4774	0	248	733
N.S.	1	1.00	0.91	1.08	0.93	19.73	0.00	1.02	3.03
time (sec)	N/A	0.240	0.111	0.348	0.516	1.272	0.000	0.711	5.394

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	225	271	241	5373	0	269	537
N.S.	1	1.00	0.86	1.03	0.92	20.51	0.00	1.03	2.05
time (sec)	N/A	0.274	0.116	0.365	0.526	1.468	0.000	0.687	5.484

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	198	246	208	2163	148	208	216
N.S.	1	1.00	0.92	1.14	0.97	10.06	0.69	0.97	1.00
time (sec)	N/A	0.133	0.124	0.362	0.534	1.238	4.967	0.685	0.232

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	214	250	228	2519	170	215	232
N.S.	1	1.00	0.90	1.05	0.95	10.54	0.71	0.90	0.97
time (sec)	N/A	0.141	0.194	0.406	0.507	1.246	1.867	0.671	0.225

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	213	311	220	2251	163	210	212
N.S.	1	1.00	0.95	1.38	0.98	10.00	0.72	0.93	0.94
time (sec)	N/A	0.129	0.194	0.379	0.492	1.119	1.136	0.818	0.262

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	229	270	251	5229	0	253	540
N.S.	1	1.00	0.89	1.05	0.98	20.35	0.00	0.98	2.10
time (sec)	N/A	0.280	0.121	0.367	0.481	1.288	0.000	0.907	5.440

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	248	279	271	5112	0	273	793
N.S.	1	1.00	0.93	1.04	1.01	19.15	0.00	1.02	2.97
time (sec)	N/A	0.310	0.144	0.363	0.523	1.397	0.000	1.277	5.460

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	253	287	270	4911	0	282	778
N.S.	1	1.00	0.92	1.04	0.98	17.79	0.00	1.02	2.82
time (sec)	N/A	0.337	0.140	0.371	0.506	1.274	0.000	1.311	5.358

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	255	301	289	5550	0	305	870
N.S.	1	1.00	0.86	1.01	0.97	18.62	0.00	1.02	2.92
time (sec)	N/A	0.386	0.241	0.378	0.512	1.567	0.000	1.413	0.465

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	230	268	254	2364	201	242	253
N.S.	1	1.00	0.93	1.08	1.02	9.53	0.81	0.98	1.02
time (sec)	N/A	0.165	0.154	0.355	0.488	1.429	153.881	0.941	0.267

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	241	273	266	2646	214	244	265
N.S.	1	1.00	0.89	1.01	0.99	9.80	0.79	0.90	0.98
time (sec)	N/A	0.174	0.237	0.345	0.503	1.449	6.267	1.558	0.239

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	239	378	255	2344	202	234	247
N.S.	1	1.00	0.96	1.51	1.02	9.38	0.81	0.94	0.99
time (sec)	N/A	0.155	0.152	0.355	0.509	1.148	1.950	1.268	0.279

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	259	306	299	5370	0	290	871
N.S.	1	1.00	0.89	1.05	1.03	18.45	0.00	1.00	2.99
time (sec)	N/A	0.350	0.158	0.385	0.505	1.359	0.000	1.022	5.402

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	279	315	319	5250	0	310	840
N.S.	1	1.00	0.93	1.05	1.06	17.44	0.00	1.03	2.79
time (sec)	N/A	0.389	0.187	0.382	0.509	1.402	0.000	1.053	5.434

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	284	321	318	5049	0	320	825
N.S.	1	1.00	0.92	1.04	1.03	16.29	0.00	1.03	2.66
time (sec)	N/A	0.425	0.179	0.397	0.621	1.281	0.000	1.012	5.375

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	284	336	337	5670	0	333	918
N.S.	1	1.00	0.84	0.99	0.99	16.68	0.00	0.98	2.70
time (sec)	N/A	0.500	0.304	0.388	0.575	1.899	0.000	1.174	0.525

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	57	29	26	26	54	27	26
N.S.	1	1.00	1.97	1.00	0.90	0.90	1.86	0.93	0.90
time (sec)	N/A	0.040	0.010	0.348	0.484	0.389	0.060	1.514	4.970

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	57	29	26	26	54	27	26
N.S.	1	1.00	1.97	1.00	0.90	0.90	1.86	0.93	0.90
time (sec)	N/A	0.023	0.004	0.343	0.520	0.402	0.056	1.190	0.027

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	58	29	28	28	54	29	27
N.S.	1	1.00	1.87	0.94	0.90	0.90	1.74	0.94	0.87
time (sec)	N/A	0.038	0.010	0.379	0.485	0.388	0.059	1.233	4.946

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	58	29	28	28	54	29	27
N.S.	1	1.00	1.87	0.94	0.90	0.90	1.74	0.94	0.87
time (sec)	N/A	0.025	0.005	0.378	0.487	0.390	0.060	1.207	0.030

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	146	116	51	52	100	96	154
N.S.	1	1.00	2.92	2.32	1.02	1.04	2.00	1.92	3.08
time (sec)	N/A	0.062	0.028	0.323	0.494	0.383	0.127	1.339	5.223

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	149	119	166	53	110	165	156
N.S.	1	1.00	2.81	2.25	3.13	1.00	2.08	3.11	2.94
time (sec)	N/A	0.063	0.048	0.316	0.496	0.380	0.140	1.221	5.250

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	148	117	167	56	109	97	155
N.S.	1	1.00	2.74	2.17	3.09	1.04	2.02	1.80	2.87
time (sec)	N/A	0.052	0.027	0.335	0.483	0.390	0.129	1.654	5.224

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	147	118	52	53	102	90	155
N.S.	1	1.00	2.77	2.23	0.98	1.00	1.92	1.70	2.92
time (sec)	N/A	0.052	0.038	0.328	0.481	0.393	0.140	1.339	5.234

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	81	79	90	87	82
N.S.	1	1.00	1.00	0.82	0.84	0.81	0.93	0.90	0.85
time (sec)	N/A	0.083	0.022	1.999	0.264	0.397	0.011	1.178	0.051

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	81	79	90	87	82
N.S.	1	1.00	1.00	0.82	0.84	0.81	0.93	0.90	0.85
time (sec)	N/A	0.078	0.020	1.982	0.278	0.394	0.011	1.115	0.042

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	81	79	90	87	82
N.S.	1	1.00	1.00	0.82	0.84	0.81	0.93	0.90	0.85
time (sec)	N/A	0.064	0.017	1.975	0.268	0.355	0.011	1.111	0.044

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	81	79	90	87	82
N.S.	1	1.00	1.00	0.82	0.84	0.81	0.93	0.90	0.85
time (sec)	N/A	0.055	0.012	2.026	0.358	0.354	0.011	1.176	0.043

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	77	78	76	87	84	79
N.S.	1	1.00	1.00	0.84	0.85	0.83	0.95	0.91	0.86
time (sec)	N/A	0.050	0.010	0.117	0.329	0.370	0.011	1.929	0.041

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	81	76	74	85	83	77
N.S.	1	1.00	1.00	0.92	0.86	0.84	0.97	0.94	0.88
time (sec)	N/A	0.037	0.020	0.031	0.278	0.369	0.066	1.512	0.047

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	81	76	81	82	83	77
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.95	0.97	0.90
time (sec)	N/A	0.045	0.031	0.032	0.319	0.391	0.078	0.785	0.048

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	78	76	81	83	80	76
N.S.	1	1.00	0.91	0.91	0.88	0.94	0.97	0.93	0.88
time (sec)	N/A	0.048	0.045	0.034	0.273	0.366	0.126	0.688	0.043

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	76	76	77	81	83	79	75
N.S.	1	1.00	0.88	0.88	0.90	0.94	0.97	0.92	0.87
time (sec)	N/A	0.048	0.041	0.038	0.288	0.365	0.326	0.643	0.041

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	74	77	81	83	77	74
N.S.	1	1.00	0.90	0.86	0.90	0.94	0.97	0.90	0.86
time (sec)	N/A	0.049	0.043	0.040	0.275	0.383	1.342	0.662	4.976

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	163	152	154	151	167	160	151
N.S.	1	1.00	1.00	0.93	0.94	0.93	1.02	0.98	0.93
time (sec)	N/A	0.144	0.024	2.029	0.285	0.364	0.019	0.586	0.102

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	163	152	154	151	167	160	151
N.S.	1	1.00	1.00	0.93	0.94	0.93	1.02	0.98	0.93
time (sec)	N/A	0.106	0.019	2.066	0.268	0.359	0.019	0.561	0.088

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	150	152	154	151	167	160	151
N.S.	1	1.00	0.95	0.96	0.97	0.96	1.06	1.01	0.96
time (sec)	N/A	0.087	0.051	2.002	0.272	0.392	0.020	0.569	0.091

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	163	152	154	151	167	160	151
N.S.	1	1.00	1.03	0.96	0.97	0.96	1.06	1.01	0.96
time (sec)	N/A	0.088	0.020	2.075	0.279	0.360	0.020	0.555	0.091

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	125	149	151	148	163	157	148
N.S.	1	1.00	0.82	0.97	0.99	0.97	1.07	1.03	0.97
time (sec)	N/A	0.086	0.052	1.889	0.275	0.413	0.018	0.531	0.092

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	154	153	149	146	162	156	146
N.S.	1	1.00	1.03	1.03	1.00	0.98	1.09	1.05	0.98
time (sec)	N/A	0.072	0.030	0.352	0.278	0.389	0.118	0.521	0.096

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	152	152	149	153	156	155	145
N.S.	1	1.00	1.03	1.03	1.01	1.04	1.06	1.05	0.99
time (sec)	N/A	0.087	0.044	0.333	0.287	0.393	0.130	0.584	0.098

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	127	150	149	153	158	153	145
N.S.	1	1.00	0.86	1.02	1.01	1.04	1.07	1.04	0.99
time (sec)	N/A	0.091	0.049	0.327	0.273	0.385	0.171	0.573	5.008

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	123	147	150	153	158	153	145
N.S.	1	1.00	0.81	0.97	0.99	1.01	1.04	1.01	0.95
time (sec)	N/A	0.084	0.053	0.338	0.284	0.407	0.401	0.513	0.076

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	125	144	150	153	156	152	145
N.S.	1	1.00	0.82	0.95	0.99	1.01	1.03	1.00	0.95
time (sec)	N/A	0.082	0.050	0.325	0.274	0.392	1.515	0.533	0.066

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	223	224	221	217	246	233	205
N.S.	1	1.00	1.00	1.00	0.99	0.97	1.10	1.04	0.92
time (sec)	N/A	0.201	0.034	2.006	0.272	0.361	0.025	0.501	0.174

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	223	224	221	217	246	233	205
N.S.	1	1.00	1.00	1.00	0.99	0.97	1.10	1.04	0.92
time (sec)	N/A	0.149	0.032	2.029	0.337	0.371	0.025	0.589	5.165

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	223	224	221	217	246	233	205
N.S.	1	1.00	1.05	1.06	1.04	1.02	1.16	1.10	0.97
time (sec)	N/A	0.118	0.031	2.243	0.297	0.403	0.025	0.498	0.162

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	223	224	221	217	246	233	205
N.S.	1	1.00	1.05	1.06	1.04	1.02	1.16	1.10	0.97
time (sec)	N/A	0.126	0.021	2.091	0.279	0.369	0.025	0.531	0.157

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	170	221	218	214	243	230	202
N.S.	1	1.00	0.82	1.07	1.05	1.03	1.17	1.11	0.98
time (sec)	N/A	0.119	0.069	1.927	0.268	0.358	0.024	0.511	0.157

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	214	224	216	212	240	228	199
N.S.	1	1.00	1.07	1.12	1.08	1.06	1.20	1.14	1.00
time (sec)	N/A	0.091	0.053	0.342	0.282	0.383	0.175	0.486	5.113

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	172	224	216	219	236	228	199
N.S.	1	1.00	0.87	1.13	1.09	1.11	1.19	1.15	1.01
time (sec)	N/A	0.121	0.093	0.329	0.310	0.364	0.188	0.531	5.045

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	174	222	216	219	238	226	199
N.S.	1	1.00	0.88	1.12	1.09	1.11	1.20	1.14	1.01
time (sec)	N/A	0.131	0.084	0.352	0.269	0.393	0.233	0.510	0.137

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	172	218	216	219	236	225	199
N.S.	1	1.00	0.82	1.04	1.03	1.05	1.13	1.08	0.95
time (sec)	N/A	0.119	0.083	0.362	0.282	0.411	0.458	0.501	0.122

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	170	215	216	219	235	224	199
N.S.	1	1.00	0.81	1.03	1.03	1.05	1.12	1.07	0.95
time (sec)	N/A	0.114	0.085	0.358	0.286	0.386	1.574	0.598	5.027

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	334	332	383	15635	0	380	1271
N.S.	1	1.00	1.01	1.00	1.16	47.24	0.00	1.15	3.84
time (sec)	N/A	0.684	0.289	0.380	0.557	1.992	0.000	0.534	5.086

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	299	291	335	15451	0	353	1236
N.S.	1	1.00	0.96	0.93	1.07	49.36	0.00	1.13	3.95
time (sec)	N/A	0.659	0.165	0.378	0.522	1.966	0.000	0.514	4.992

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	290	285	317	14746	0	333	1170
N.S.	1	1.00	0.99	0.97	1.08	50.16	0.00	1.13	3.98
time (sec)	N/A	0.653	0.180	0.366	0.487	1.737	0.000	0.521	5.023

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	272	271	304	14875	0	295	1161
N.S.	1	1.00	0.99	0.99	1.11	54.09	0.00	1.07	4.22
time (sec)	N/A	0.603	0.262	0.372	0.508	1.716	0.000	0.504	4.989

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	257	254	246	268	15235	0	272	1150
N.S.	1	0.99	0.98	0.95	1.03	58.82	0.00	1.05	4.44
time (sec)	N/A	0.251	0.198	0.360	0.494	1.818	0.000	0.563	5.033

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	256	258	259	293	15327	0	281	1731
N.S.	1	0.99	1.00	1.00	1.14	59.41	0.00	1.09	6.71
time (sec)	N/A	0.316	0.140	0.377	0.525	60.849	0.000	0.652	5.097

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	257	260	293	15238	0	277	1802
N.S.	1	1.00	1.02	1.03	1.16	60.23	0.00	1.09	7.12
time (sec)	N/A	0.287	0.191	0.380	0.491	65.199	0.000	0.542	5.087

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	258	257	251	274	15424	0	269	2500
N.S.	1	0.99	0.99	0.97	1.05	59.32	0.00	1.03	9.62
time (sec)	N/A	0.261	0.219	0.418	0.489	35.627	0.000	0.483	5.205

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	274	264	276	306	15204	0	291	1842
N.S.	1	0.99	0.96	1.00	1.11	55.09	0.00	1.05	6.67
time (sec)	N/A	0.301	0.309	0.421	0.503	64.233	0.000	0.538	5.870

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	334	329	371	16147	0	357	1241
N.S.	1	1.00	0.99	0.98	1.10	47.91	0.00	1.06	3.68
time (sec)	N/A	0.465	0.261	0.381	0.511	2.096	0.000	0.558	5.109

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	294	301	330	16285	0	330	1229
N.S.	1	1.00	0.95	0.97	1.06	52.36	0.00	1.06	3.95
time (sec)	N/A	0.414	0.137	0.418	0.501	2.222	0.000	0.543	0.152

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	288	280	278	289	12153	0	307	816
N.S.	1	0.99	0.97	0.96	1.00	41.91	0.00	1.06	2.81
time (sec)	N/A	0.330	0.131	0.410	0.528	1.763	0.000	0.544	0.137

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	285	285	316	12617	0	318	827
N.S.	1	1.00	0.99	0.99	1.09	43.66	0.00	1.10	2.86
time (sec)	N/A	0.340	0.131	0.398	0.526	1.910	0.000	0.458	5.390

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	268	283	291	12636	0	302	835
N.S.	1	1.00	0.97	1.03	1.05	45.78	0.00	1.09	3.03
time (sec)	N/A	0.249	0.113	0.384	0.494	1.789	0.000	0.503	5.540

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	287	269	293	307	12541	0	319	1660
N.S.	1	0.99	0.93	1.01	1.06	43.39	0.00	1.10	5.74
time (sec)	N/A	0.637	0.132	0.432	0.496	20.805	0.000	0.476	5.601

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	285	298	332	12556	0	328	1684
N.S.	1	1.00	0.95	0.99	1.10	41.71	0.00	1.09	5.59
time (sec)	N/A	0.553	0.196	0.427	0.498	21.853	0.000	0.471	5.768

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	304	292	293	320	12231	0	336	1632
N.S.	1	0.99	0.95	0.96	1.05	39.97	0.00	1.10	5.33
time (sec)	N/A	0.385	0.315	0.421	0.497	15.873	0.000	0.462	5.712

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	336	303	332	369	16568	0	363	1924
N.S.	1	0.99	0.90	0.98	1.09	49.02	0.00	1.07	5.69
time (sec)	N/A	0.492	0.304	0.575	0.494	67.333	0.000	0.489	5.958

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	342	339	395	12967	0	385	916
N.S.	1	1.00	0.99	0.98	1.14	37.59	0.00	1.12	2.66
time (sec)	N/A	0.571	0.198	0.397	0.538	2.639	0.000	0.464	0.579

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	315	337	369	12939	0	363	908
N.S.	1	1.00	0.97	1.04	1.14	39.81	0.00	1.12	2.79
time (sec)	N/A	0.424	0.167	0.402	0.534	2.480	0.000	0.509	5.659

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	287	311	314	6926	0	320	627
N.S.	1	1.00	0.97	1.05	1.06	23.32	0.00	1.08	2.11
time (sec)	N/A	0.289	0.157	0.404	0.499	1.929	0.000	0.483	5.690

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	297	316	349	7190	0	340	640
N.S.	1	1.00	0.92	0.98	1.08	22.26	0.00	1.05	1.98
time (sec)	N/A	0.317	0.202	0.409	0.477	2.119	0.000	0.646	5.365

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	295	309	328	6984	0	330	630
N.S.	1	1.00	0.94	0.99	1.05	22.31	0.00	1.05	2.01
time (sec)	N/A	0.291	0.154	0.445	0.491	1.923	0.000	0.476	0.432

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	345	311	339	373	12815	0	376	1716
N.S.	1	0.99	0.90	0.98	1.07	36.93	0.00	1.08	4.95
time (sec)	N/A	0.480	0.181	0.424	0.501	21.812	0.000	0.486	5.705

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	336	345	404	12951	0	390	1747
N.S.	1	1.00	0.93	0.95	1.12	35.78	0.00	1.08	4.83
time (sec)	N/A	0.891	0.352	0.421	0.512	22.793	0.000	0.472	5.747

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	357	337	340	394	12435	0	399	1697
N.S.	1	0.99	0.94	0.94	1.09	34.54	0.00	1.11	4.71
time (sec)	N/A	0.557	0.374	0.402	0.502	17.063	0.000	0.475	5.659

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	392	352	394	448	16697	0	431	1994
N.S.	1	0.99	0.89	1.00	1.13	42.27	0.00	1.09	5.05
time (sec)	N/A	0.667	0.370	0.411	0.497	72.053	0.000	0.503	6.321

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	583	583	132	793	0	81	129	0	-1
N.S.	1	1.00	0.23	1.36	0.00	0.14	0.22	0.00	-0.00
time (sec)	N/A	0.485	10.103	0.382	0.000	0.083	1.912	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	121	773	0	74	107	0	-1
N.S.	1	1.00	0.22	1.38	0.00	0.13	0.19	0.00	-0.00
time (sec)	N/A	0.489	10.066	0.398	0.000	0.084	1.799	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	537	537	114	753	0	66	107	0	-1
N.S.	1	1.00	0.21	1.40	0.00	0.12	0.20	0.00	-0.00
time (sec)	N/A	0.218	10.057	0.396	0.000	0.092	1.769	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	107	735	0	55	105	0	-1
N.S.	1	1.00	0.21	1.44	0.00	0.11	0.21	0.00	-0.00
time (sec)	N/A	0.119	10.040	0.392	0.000	0.096	1.290	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	493	740	0	190	105	0	-1
N.S.	1	1.00	0.95	1.43	0.00	0.37	0.20	0.00	-0.00
time (sec)	N/A	0.137	10.687	0.385	0.000	0.221	2.252	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	547	547	513	759	0	235	107	0	121
N.S.	1	1.00	0.94	1.39	0.00	0.43	0.20	0.00	0.22
time (sec)	N/A	0.234	10.642	0.394	0.000	0.116	1.624	0.000	5.957

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	525	778	0	246	112	0	-1
N.S.	1	1.00	0.92	1.37	0.00	0.43	0.20	0.00	-0.00
time (sec)	N/A	0.309	11.338	0.386	0.000	0.115	1.813	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	134	836	0	138	129	0	-1
N.S.	1	1.00	0.23	1.41	0.00	0.23	0.22	0.00	-0.00
time (sec)	N/A	0.426	10.081	0.398	0.000	0.085	10.643	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	127	817	0	130	129	0	-1
N.S.	1	1.00	0.22	1.42	0.00	0.23	0.22	0.00	-0.00
time (sec)	N/A	0.323	10.074	0.390	0.000	0.095	7.943	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	118	800	0	107	129	0	-1
N.S.	1	1.00	0.22	1.48	0.00	0.20	0.24	0.00	-0.00
time (sec)	N/A	0.214	10.063	0.396	0.000	0.091	5.720	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	107	779	0	99	109	0	-1
N.S.	1	1.00	0.20	1.49	0.00	0.19	0.21	0.00	-0.00
time (sec)	N/A	0.181	10.066	0.436	0.000	0.088	5.006	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	561	561	108	782	0	112	109	0	-1
N.S.	1	1.00	0.19	1.39	0.00	0.20	0.19	0.00	-0.00
time (sec)	N/A	0.226	10.052	0.382	0.000	0.086	4.508	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	109	785	0	98	107	0	-1
N.S.	1	1.00	0.20	1.48	0.00	0.18	0.20	0.00	-0.00
time (sec)	N/A	0.174	10.032	0.364	0.000	0.094	4.061	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	579	579	518	810	0	345	265	0	-1
N.S.	1	1.00	0.89	1.40	0.00	0.60	0.46	0.00	-0.00
time (sec)	N/A	0.268	11.140	0.372	0.000	0.126	6.158	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	607	607	542	825	0	377	267	0	136
N.S.	1	1.00	0.89	1.36	0.00	0.62	0.44	0.00	0.22
time (sec)	N/A	0.374	11.614	0.398	0.000	0.130	6.300	0.000	5.804

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	733	733	172	1674	0	202	238	0	-1
N.S.	1	1.00	0.23	2.28	0.00	0.28	0.32	0.00	-0.00
time (sec)	N/A	1.301	7.923	0.369	0.000	0.088	2.524	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	681	681	158	1197	0	177	223	0	-1
N.S.	1	1.00	0.23	1.76	0.00	0.26	0.33	0.00	-0.00
time (sec)	N/A	0.930	8.199	0.375	0.000	0.089	2.372	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	667	667	143	1311	0	147	223	0	-1
N.S.	1	1.00	0.21	1.97	0.00	0.22	0.33	0.00	-0.00
time (sec)	N/A	0.688	7.848	0.378	0.000	0.088	2.290	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	639	639	135	1557	0	141	194	0	-1
N.S.	1	1.00	0.21	2.44	0.00	0.22	0.30	0.00	-0.00
time (sec)	N/A	0.486	7.532	0.377	0.000	0.080	2.313	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	620	620	714	1118	0	340	235	0	-1
N.S.	1	1.00	1.15	1.80	0.00	0.55	0.38	0.00	-0.00
time (sec)	N/A	0.361	8.268	0.368	0.000	0.221	5.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	810	1248	0	307	236	0	-1
N.S.	1	1.00	1.27	1.96	0.00	0.48	0.37	0.00	-0.00
time (sec)	N/A	0.431	8.177	0.436	0.000	0.227	3.178	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	640	640	962	1529	0	320	255	0	-1
N.S.	1	1.00	1.50	2.39	0.00	0.50	0.40	0.00	-0.00
time (sec)	N/A	0.518	8.872	0.408	0.000	0.217	3.244	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	637	637	769	1114	0	348	265	0	-1
N.S.	1	1.00	1.21	1.75	0.00	0.55	0.42	0.00	-0.00
time (sec)	N/A	0.568	9.076	0.409	0.000	0.361	3.784	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	694	694	855	1286	0	330	274	0	-1
N.S.	1	1.00	1.23	1.85	0.00	0.48	0.39	0.00	-0.00
time (sec)	N/A	0.720	10.450	0.516	0.000	0.378	3.841	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	652	652	934	1571	0	346	240	0	-1
N.S.	1	1.00	1.43	2.41	0.00	0.53	0.37	0.00	-0.00
time (sec)	N/A	0.555	11.598	0.475	0.000	0.247	3.506	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	659	659	800	1180	0	404	304	0	-1
N.S.	1	1.00	1.21	1.79	0.00	0.61	0.46	0.00	-0.00
time (sec)	N/A	0.641	11.777	0.424	0.000	0.153	5.158	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	711	711	892	1376	0	439	308	0	-1
N.S.	1	1.00	1.25	1.94	0.00	0.62	0.43	0.00	-0.00
time (sec)	N/A	0.755	11.524	0.421	0.000	0.141	5.352	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	743	743	979	1679	0	482	304	0	-1
N.S.	1	1.00	1.32	2.26	0.00	0.65	0.41	0.00	-0.00
time (sec)	N/A	0.882	11.769	0.422	0.000	0.116	5.048	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	791	791	179	1764	0	262	512	0	-1
N.S.	1	1.00	0.23	2.23	0.00	0.33	0.65	0.00	-0.00
time (sec)	N/A	1.445	9.865	0.400	0.000	0.088	4.659	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	742	742	162	1269	0	237	525	0	-1
N.S.	1	1.00	0.22	1.71	0.00	0.32	0.71	0.00	-0.00
time (sec)	N/A	1.039	10.115	0.382	0.000	0.097	4.288	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	723	723	148	1383	0	207	525	0	-1
N.S.	1	1.00	0.20	1.91	0.00	0.29	0.73	0.00	-0.00
time (sec)	N/A	0.824	9.736	0.391	0.000	0.087	4.093	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	694	694	139	1629	0	201	444	0	-1
N.S.	1	1.00	0.20	2.35	0.00	0.29	0.64	0.00	-0.00
time (sec)	N/A	0.605	9.325	0.380	0.000	0.085	4.147	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	676	676	753	1188	0	457	473	0	-1
N.S.	1	1.00	1.11	1.76	0.00	0.68	0.70	0.00	-0.00
time (sec)	N/A	0.464	10.143	0.363	0.000	0.232	9.741	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	692	692	817	1317	0	424	474	0	-1
N.S.	1	1.00	1.18	1.90	0.00	0.61	0.68	0.00	-0.00
time (sec)	N/A	0.528	10.258	0.425	0.000	0.248	5.474	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	694	694	952	1613	0	433	462	0	-1
N.S.	1	1.00	1.37	2.32	0.00	0.62	0.67	0.00	-0.00
time (sec)	N/A	0.591	10.606	0.413	0.000	0.256	5.577	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	692	692	813	1193	0	434	484	0	-1
N.S.	1	1.00	1.17	1.72	0.00	0.63	0.70	0.00	-0.00
time (sec)	N/A	0.624	10.340	0.422	0.000	0.378	6.729	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	741	741	878	1342	0	384	495	0	-1
N.S.	1	1.00	1.18	1.81	0.00	0.52	0.67	0.00	-0.00
time (sec)	N/A	0.815	10.188	0.403	0.000	0.371	6.714	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	689	689	949	1606	0	382	476	0	-1
N.S.	1	1.00	1.38	2.33	0.00	0.55	0.69	0.00	-0.00
time (sec)	N/A	0.622	11.648	0.418	0.000	0.249	6.517	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	692	692	805	1196	0	430	524	0	-1
N.S.	1	1.00	1.16	1.73	0.00	0.62	0.76	0.00	-0.00
time (sec)	N/A	0.691	11.806	0.541	0.000	0.373	8.749	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	897	1375	0	446	536	0	-1
N.S.	1	1.00	1.20	1.84	0.00	0.60	0.72	0.00	-0.00
time (sec)	N/A	0.844	11.511	0.454	0.000	0.389	8.980	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	705	705	978	1663	0	470	527	0	-1
N.S.	1	1.00	1.39	2.36	0.00	0.67	0.75	0.00	-0.00
time (sec)	N/A	0.660	11.741	0.428	0.000	0.246	8.300	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	714	714	844	1273	0	525	573	0	-1
N.S.	1	1.00	1.18	1.78	0.00	0.74	0.80	0.00	-0.00
time (sec)	N/A	0.734	11.881	0.404	0.000	0.148	14.680	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	764	764	930	1470	0	559	576	0	-1
N.S.	1	1.00	1.22	1.92	0.00	0.73	0.75	0.00	-0.00
time (sec)	N/A	0.902	11.698	0.441	0.000	0.158	15.160	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	796	796	1017	1773	0	606	541	0	-1
N.S.	1	1.00	1.28	2.23	0.00	0.76	0.68	0.00	-0.00
time (sec)	N/A	1.623	11.802	0.454	0.000	0.129	13.532	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	120	114	0	0	22	112	0	-1
N.S.	1	1.18	1.12	0.00	0.00	0.22	1.10	0.00	-0.01
time (sec)	N/A	0.049	0.601	0.044	0.000	0.379	29.241	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	125	116	0	0	26	114	0	-1
N.S.	1	1.17	1.08	0.00	0.00	0.24	1.07	0.00	-0.01
time (sec)	N/A	0.062	0.588	0.046	0.000	0.386	44.050	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	125	116	0	0	28	114	0	-1
N.S.	1	1.17	1.08	0.00	0.00	0.26	1.07	0.00	-0.01
time (sec)	N/A	0.072	0.604	0.054	0.000	0.405	61.927	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	56	54	63	56	54
N.S.	1	1.00	1.00	0.81	0.82	0.79	0.93	0.82	0.79
time (sec)	N/A	0.026	0.004	0.125	0.312	0.365	0.008	0.590	0.035

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	59	57	66	59	57
N.S.	1	1.00	1.00	0.79	0.81	0.78	0.90	0.81	0.78
time (sec)	N/A	0.043	0.002	0.123	0.289	0.375	0.009	0.543	0.031

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	105	102	121	105	102
N.S.	1	1.00	1.14	0.94	0.96	0.94	1.11	0.96	0.94
time (sec)	N/A	0.047	0.003	0.320	0.294	0.366	0.012	0.481	0.080

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	129	106	108	105	124	108	105
N.S.	1	1.00	1.13	0.93	0.95	0.92	1.09	0.95	0.92
time (sec)	N/A	0.055	0.003	0.364	0.269	0.369	0.013	0.531	0.072

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	154	150	180	154	150
N.S.	1	1.00	1.19	1.00	1.02	0.99	1.19	1.02	0.99
time (sec)	N/A	0.069	0.004	0.314	0.292	0.364	0.017	0.470	0.164

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	185	154	157	153	184	157	153
N.S.	1	1.00	1.19	0.99	1.01	0.98	1.18	1.01	0.98
time (sec)	N/A	0.071	0.004	0.378	0.273	0.371	0.015	0.628	0.161

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	236	199	203	198	241	203	198
N.S.	1	1.00	1.22	1.03	1.05	1.03	1.25	1.05	1.03
time (sec)	N/A	0.098	0.004	0.453	0.279	0.381	0.018	0.513	5.080

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	241	202	206	201	245	206	201
N.S.	1	1.00	1.22	1.02	1.04	1.02	1.24	1.04	1.02
time (sec)	N/A	0.099	0.004	0.380	0.320	0.383	0.018	0.525	0.359

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	214	154	176	241149	0	280	1970
N.S.	1	1.00	1.61	1.16	1.32	1813.15	0.00	2.11	14.81
time (sec)	N/A	0.081	0.044	0.356	0.495	5.023	0.000	0.491	5.658

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	221	176	211	220680	0	328	846
N.S.	1	1.00	1.36	1.09	1.30	1362.22	0.00	2.02	5.22
time (sec)	N/A	0.132	0.056	0.354	0.489	3.951	0.000	0.489	4.846

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	296	240	281	254687	0	290	1952
N.S.	1	1.00	1.01	0.82	0.96	869.24	0.00	0.99	6.66
time (sec)	N/A	0.147	0.144	0.344	0.498	5.075	0.000	0.488	0.927

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	311	261	308	219615	0	308	838
N.S.	1	1.00	0.97	0.81	0.96	684.16	0.00	0.96	2.61
time (sec)	N/A	0.223	0.117	0.354	0.508	3.751	0.000	0.507	4.854

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	315	305	310	124301	517	316	478
N.S.	1	1.00	0.99	0.96	0.97	390.88	1.63	0.99	1.50
time (sec)	N/A	0.178	0.226	0.368	0.481	3.670	43.838	0.595	0.360

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	294	273	297	122993	510	303	559
N.S.	1	1.00	0.95	0.88	0.96	396.75	1.65	0.98	1.80
time (sec)	N/A	0.183	0.177	0.369	0.488	3.712	196.045	0.534	5.097

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	347	391	361	124838	0	354	832
N.S.	1	1.00	0.99	1.11	1.03	355.66	0.00	1.01	2.37
time (sec)	N/A	0.217	0.210	0.362	0.502	7.944	0.000	0.635	5.199

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	329	304	347	124542	0	338	521
N.S.	1	1.00	0.97	0.89	1.02	366.30	0.00	0.99	1.53
time (sec)	N/A	0.221	0.224	0.353	0.487	7.958	0.000	0.731	0.396

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	379	340	409	125011	0	391	879
N.S.	1	1.00	0.99	0.89	1.07	327.25	0.00	1.02	2.30
time (sec)	N/A	0.264	0.248	0.359	0.513	16.513	0.000	0.655	5.255

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	366	334	401	125996	0	380	888
N.S.	1	1.00	0.96	0.88	1.06	331.57	0.00	1.00	2.34
time (sec)	N/A	0.263	0.268	0.367	0.503	17.041	0.000	0.555	0.482

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	202	331	0	214	252	0	-1
N.S.	1	1.00	0.48	0.79	0.00	0.51	0.60	0.00	-0.00
time (sec)	N/A	0.264	10.452	0.373	0.000	0.124	3.711	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	215	321	0	205	212	0	-1
N.S.	1	1.00	0.55	0.81	0.00	0.52	0.54	0.00	-0.00
time (sec)	N/A	0.235	10.417	0.374	0.000	0.122	3.674	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	182	303	0	193	212	0	-1
N.S.	1	1.00	0.49	0.82	0.00	0.52	0.57	0.00	-0.00
time (sec)	N/A	0.202	10.476	0.382	0.000	0.120	3.584	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	211	280	0	170	158	0	-1
N.S.	1	1.00	0.60	0.79	0.00	0.48	0.45	0.00	-0.00
time (sec)	N/A	0.180	10.126	0.436	0.000	0.128	2.463	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	171	257	0	163	156	0	-1
N.S.	1	1.00	0.52	0.78	0.00	0.49	0.47	0.00	-0.00
time (sec)	N/A	0.131	10.096	0.405	0.000	0.118	2.408	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	280	284	0	30	204	0	-1
N.S.	1	1.00	0.81	0.82	0.00	0.09	0.59	0.00	-0.00
time (sec)	N/A	0.170	10.854	0.342	0.000	0.247	4.592	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	267	284	0	30	206	0	-1
N.S.	1	1.00	0.78	0.83	0.00	0.09	0.60	0.00	-0.00
time (sec)	N/A	0.170	12.669	0.418	0.000	0.239	3.176	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	296	304	0	30	230	0	-1
N.S.	1	1.00	0.87	0.89	0.00	0.09	0.67	0.00	-0.00
time (sec)	N/A	0.169	10.478	0.383	0.000	0.229	2.999	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	295	306	0	30	235	0	-1
N.S.	1	1.00	0.83	0.86	0.00	0.08	0.66	0.00	-0.00
time (sec)	N/A	0.196	10.496	0.392	0.000	0.230	3.045	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	267	328	0	30	211	0	-1
N.S.	1	1.00	0.81	1.00	0.00	0.09	0.64	0.00	-0.00
time (sec)	N/A	0.185	11.367	0.406	0.000	0.249	3.137	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	314	346	0	30	216	0	-1
N.S.	1	1.00	0.87	0.96	0.00	0.08	0.60	0.00	-0.00
time (sec)	N/A	0.213	10.496	0.411	0.000	0.248	3.261	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	277	303	0	166	189	0	-1
N.S.	1	1.00	0.79	0.86	0.00	0.47	0.54	0.00	-0.00
time (sec)	N/A	0.211	10.385	0.405	0.000	0.120	2.981	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	283	326	0	173	192	0	-1
N.S.	1	1.00	0.75	0.87	0.00	0.46	0.51	0.00	-0.00
time (sec)	N/A	0.240	10.342	0.407	0.000	0.113	3.095	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	293	348	0	196	246	0	-1
N.S.	1	1.00	0.73	0.87	0.00	0.49	0.62	0.00	-0.00
time (sec)	N/A	0.259	10.385	0.408	0.000	0.123	4.300	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	305	368	0	208	246	0	-1
N.S.	1	1.00	0.72	0.87	0.00	0.49	0.58	0.00	-0.00
time (sec)	N/A	0.296	10.382	0.418	0.000	0.125	4.472	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	225	400	0	264	462	0	-1
N.S.	1	1.00	0.47	0.84	0.00	0.55	0.97	0.00	-0.00
time (sec)	N/A	0.318	10.538	0.372	0.000	0.147	10.229	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	238	372	0	255	398	0	-1
N.S.	1	1.00	0.53	0.82	0.00	0.56	0.88	0.00	-0.00
time (sec)	N/A	0.295	10.449	0.370	0.000	0.117	9.943	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	205	352	0	247	398	0	-1
N.S.	1	1.00	0.48	0.82	0.00	0.58	0.93	0.00	-0.00
time (sec)	N/A	0.241	10.540	0.369	0.000	0.121	9.755	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	196	332	0	224	396	0	-1
N.S.	1	1.00	0.48	0.81	0.00	0.55	0.97	0.00	-0.00
time (sec)	N/A	0.222	10.427	0.376	0.000	0.132	5.578	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	175	309	0	214	394	0	-1
N.S.	1	1.00	0.46	0.81	0.00	0.56	1.03	0.00	-0.00
time (sec)	N/A	0.169	10.389	0.398	0.000	0.121	5.424	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	319	352	0	59	405	0	-1
N.S.	1	1.00	0.79	0.87	0.00	0.15	1.00	0.00	-0.00
time (sec)	N/A	0.226	10.456	0.366	0.000	0.253	14.004	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	328	352	0	59	406	0	-1
N.S.	1	1.00	0.81	0.87	0.00	0.15	1.00	0.00	-0.00
time (sec)	N/A	0.234	10.488	0.447	0.000	0.241	6.528	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	326	350	0	59	377	0	-1
N.S.	1	1.00	0.80	0.86	0.00	0.15	0.93	0.00	-0.00
time (sec)	N/A	0.234	10.551	0.421	0.000	0.252	5.174	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	327	349	0	59	381	0	-1
N.S.	1	1.00	0.80	0.86	0.00	0.14	0.93	0.00	-0.00
time (sec)	N/A	0.230	10.554	0.412	0.000	0.276	5.199	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	329	350	0	59	379	0	-1
N.S.	1	1.00	0.85	0.91	0.00	0.15	0.98	0.00	-0.00
time (sec)	N/A	0.237	10.527	0.429	0.000	0.268	5.740	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	331	350	0	59	386	0	-1
N.S.	1	1.00	0.86	0.90	0.00	0.15	1.00	0.00	-0.00
time (sec)	N/A	0.235	10.520	0.458	0.000	0.263	5.848	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	331	349	0	59	406	0	-1
N.S.	1	1.00	0.84	0.89	0.00	0.15	1.04	0.00	-0.00
time (sec)	N/A	0.234	10.550	0.441	0.000	0.276	5.497	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	330	352	0	59	415	0	-1
N.S.	1	1.00	0.80	0.85	0.00	0.14	1.01	0.00	-0.00
time (sec)	N/A	0.266	10.568	0.411	0.000	0.267	5.713	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	309	357	0	59	444	0	-1
N.S.	1	1.00	0.82	0.95	0.00	0.16	1.18	0.00	-0.00
time (sec)	N/A	0.253	11.774	0.410	0.000	0.272	7.110	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	351	377	0	59	449	0	-1
N.S.	1	1.00	0.87	0.93	0.00	0.15	1.11	0.00	-0.00
time (sec)	N/A	0.291	10.632	0.416	0.000	0.270	7.391	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	314	357	0	217	398	0	-1
N.S.	1	1.00	0.79	0.89	0.00	0.54	1.00	0.00	-0.00
time (sec)	N/A	0.280	10.425	0.419	0.000	0.115	7.306	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	317	380	0	227	401	0	-1
N.S.	1	1.00	0.75	0.90	0.00	0.54	0.95	0.00	-0.00
time (sec)	N/A	0.317	10.428	0.445	0.000	0.117	7.654	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	328	400	0	250	403	0	-1
N.S.	1	1.00	0.73	0.89	0.00	0.56	0.90	0.00	-0.00
time (sec)	N/A	0.332	10.468	0.444	0.000	0.118	12.063	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	339	420	0	258	403	0	-1
N.S.	1	1.00	0.72	0.89	0.00	0.54	0.85	0.00	-0.00
time (sec)	N/A	0.377	10.432	0.461	0.000	0.131	12.572	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	212	279	0	163	177	0	-1
N.S.	1	1.00	0.59	0.77	0.00	0.45	0.49	0.00	-0.00
time (sec)	N/A	0.452	10.118	0.371	0.000	0.127	3.042	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	212	269	0	156	156	0	-1
N.S.	1	1.00	0.63	0.80	0.00	0.46	0.46	0.00	-0.00
time (sec)	N/A	0.192	10.109	0.374	0.000	0.117	2.937	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	193	250	0	147	156	0	-1
N.S.	1	1.00	0.63	0.81	0.00	0.48	0.51	0.00	-0.00
time (sec)	N/A	0.155	10.135	0.375	0.000	0.125	2.814	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	160	230	0	133	129	0	-1
N.S.	1	1.00	0.54	0.77	0.00	0.44	0.43	0.00	-0.00
time (sec)	N/A	0.136	10.076	0.360	0.000	0.114	2.145	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	150	208	0	135	128	0	-1
N.S.	1	1.00	0.54	0.75	0.00	0.49	0.46	0.00	-0.00
time (sec)	N/A	0.100	10.082	0.398	0.000	0.119	1.541	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	235	222	0	38	126	0	-1
N.S.	1	1.00	0.82	0.78	0.00	0.13	0.44	0.00	-0.00
time (sec)	N/A	0.122	10.345	0.354	0.000	0.232	2.470	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	250	241	0	40	128	0	-1
N.S.	1	1.00	0.81	0.78	0.00	0.13	0.41	0.00	-0.00
time (sec)	N/A	0.148	11.324	0.378	0.000	0.229	1.872	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	242	235	0	139	126	0	118
N.S.	1	1.00	0.81	0.78	0.00	0.46	0.42	0.00	0.39
time (sec)	N/A	0.146	10.243	0.380	0.000	0.121	1.785	0.000	5.849

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	249	259	0	136	131	0	-1
N.S.	1	1.00	0.77	0.80	0.00	0.42	0.41	0.00	-0.00
time (sec)	N/A	0.177	10.278	0.384	0.000	0.118	1.916	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	259	279	0	150	158	0	-1
N.S.	1	1.00	0.75	0.81	0.00	0.43	0.46	0.00	-0.00
time (sec)	N/A	0.195	10.340	0.384	0.000	0.114	2.670	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	268	297	0	159	163	0	-1
N.S.	1	1.00	0.71	0.79	0.00	0.42	0.43	0.00	-0.00
time (sec)	N/A	0.222	10.336	0.395	0.000	0.130	2.729	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	220	320	0	242	202	0	-1
N.S.	1	1.00	0.60	0.88	0.00	0.66	0.55	0.00	-0.00
time (sec)	N/A	0.349	10.130	0.402	0.000	0.123	12.623	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	176	301	0	211	172	0	-1
N.S.	1	1.00	0.51	0.88	0.00	0.62	0.50	0.00	-0.00
time (sec)	N/A	0.248	10.113	0.407	0.000	0.117	10.819	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	166	284	0	223	172	0	-1
N.S.	1	1.00	0.53	0.90	0.00	0.71	0.55	0.00	-0.00
time (sec)	N/A	0.187	10.106	0.407	0.000	0.118	8.953	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	297	181	275	0	215	156	0	-1
N.S.	1	0.98	0.60	0.91	0.00	0.71	0.52	0.00	-0.00
time (sec)	N/A	0.141	10.099	0.367	0.000	0.115	7.910	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	165	275	0	201	156	0	-1
N.S.	1	1.00	0.50	0.83	0.00	0.60	0.47	0.00	-0.00
time (sec)	N/A	0.177	10.155	0.372	0.000	0.114	7.319	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	116	250	0	147	133	0	-1
N.S.	1	1.00	0.38	0.83	0.00	0.49	0.44	0.00	-0.00
time (sec)	N/A	0.133	10.077	0.378	0.000	0.089	6.817	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	116	250	0	129	131	0	-1
N.S.	1	1.00	0.42	0.91	0.00	0.47	0.48	0.00	-0.00
time (sec)	N/A	0.076	10.062	0.366	0.000	0.098	6.292	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	225	280	0	191	289	0	-1
N.S.	1	1.00	0.70	0.87	0.00	0.59	0.89	0.00	-0.00
time (sec)	N/A	0.195	11.082	0.372	0.000	0.130	8.279	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	245	298	0	215	291	0	133
N.S.	1	1.00	0.71	0.87	0.00	0.62	0.85	0.00	0.39
time (sec)	N/A	0.253	10.300	0.425	0.000	0.118	8.666	0.000	5.945

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	259	306	0	231	316	0	147
N.S.	1	1.00	0.71	0.83	0.00	0.63	0.86	0.00	0.40
time (sec)	N/A	0.316	10.295	0.425	0.000	0.125	8.063	0.000	6.076

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	267	325	0	216	321	0	-1
N.S.	1	1.00	0.69	0.84	0.00	0.56	0.83	0.00	-0.00
time (sec)	N/A	0.408	10.311	0.434	0.000	0.112	11.339	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	174	0	0	32	0	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.12	0.00	0.00	-0.00
time (sec)	N/A	0.175	0.271	0.043	0.000	0.391	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	170	147	0	0	27	141	0	-1
N.S.	1	1.19	1.03	0.00	0.00	0.19	0.99	0.00	-0.01
time (sec)	N/A	0.097	0.657	0.048	0.000	0.372	20.501	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	145	0	0	33	143	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.19	0.82	0.00	-0.01
time (sec)	N/A	0.123	0.672	0.056	0.000	0.382	47.447	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75
time (sec)	N/A	0.006	0.001	0.391	0.288	0.363	0.009	0.823	0.023

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	9	6
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.90	0.60
time (sec)	N/A	0.009	0.001	0.440	0.274	0.367	0.010	0.665	0.056

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	9	6
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.90	0.60
time (sec)	N/A	0.009	0.001	0.384	0.291	0.351	0.012	0.751	4.992

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	18	17	17	15	15	6
N.S.	1	1.00	2.10	1.80	1.70	1.70	1.50	1.50	0.60
time (sec)	N/A	0.006	0.002	0.359	0.278	0.350	0.029	0.990	0.097

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	16	16	24	16	16
N.S.	1	1.00	1.00	0.71	0.67	0.67	1.00	0.67	0.67
time (sec)	N/A	0.014	0.006	0.367	0.490	0.354	0.040	0.885	0.030

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	38	38	46	39	49
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.92	0.78	0.98
time (sec)	N/A	0.033	0.012	0.368	0.484	0.364	0.071	1.088	0.131

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	39	38	38	46	39	48
N.S.	1	1.00	0.92	0.78	0.76	0.76	0.92	0.78	0.96
time (sec)	N/A	0.032	0.010	0.373	0.502	0.391	0.072	0.801	4.987

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	47	46	46	56	48	52
N.S.	1	1.00	0.93	0.78	0.77	0.77	0.93	0.80	0.87
time (sec)	N/A	0.033	0.009	0.372	0.494	0.365	0.083	0.877	5.010

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	47	46	46	56	48	52
N.S.	1	1.00	0.87	0.78	0.77	0.77	0.93	0.80	0.87
time (sec)	N/A	0.035	0.008	0.364	0.493	0.360	0.087	0.966	4.975

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	38	38	48	35	46
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.96	0.70	0.92
time (sec)	N/A	0.017	0.004	0.357	0.497	0.392	0.052	0.677	0.086

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	38	38	48	39	46
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.96	0.78	0.92
time (sec)	N/A	0.029	0.008	0.498	0.502	0.375	0.061	0.811	0.100

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	100	85	84	115	105	86	100
N.S.	1	1.00	0.91	0.77	0.76	1.05	0.95	0.78	0.91
time (sec)	N/A	0.081	0.061	0.393	0.524	0.375	0.196	0.831	5.098

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	85	84	115	105	86	100
N.S.	1	1.00	0.88	0.77	0.76	1.05	0.95	0.78	0.91
time (sec)	N/A	0.082	0.054	0.385	0.517	0.401	0.200	0.824	0.189

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	122	68	61	91	70	63	52
N.S.	1	1.00	1.51	0.84	0.75	1.12	0.86	0.78	0.64
time (sec)	N/A	0.047	0.284	0.378	0.529	0.363	0.083	0.701	4.923

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	73	74	126	82	76	77
N.S.	1	1.00	0.91	0.79	0.80	1.37	0.89	0.83	0.84
time (sec)	N/A	0.082	0.021	0.385	0.524	0.370	0.138	0.786	0.123

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	119	115	105	256	124	111	120
N.S.	1	1.00	0.80	0.78	0.71	1.73	0.84	0.75	0.81
time (sec)	N/A	0.121	0.041	0.473	0.506	0.413	0.322	1.472	0.190

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	121	115	105	257	124	111	121
N.S.	1	1.00	0.83	0.79	0.72	1.76	0.85	0.76	0.83
time (sec)	N/A	0.122	0.054	0.397	0.520	0.424	0.306	2.105	5.089

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	111	111	95	187	116	106	110
N.S.	1	1.00	0.78	0.78	0.67	1.32	0.82	0.75	0.77
time (sec)	N/A	0.101	0.045	0.397	0.495	0.362	0.280	1.308	5.080

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	111	111	95	187	116	106	111
N.S.	1	1.00	0.78	0.78	0.67	1.32	0.82	0.75	0.78
time (sec)	N/A	0.100	0.043	0.395	0.503	0.360	0.281	1.456	0.189

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	103	102	87	131	110	89	102
N.S.	1	1.00	0.91	0.90	0.77	1.16	0.97	0.79	0.90
time (sec)	N/A	0.054	0.035	0.387	0.516	0.374	0.222	1.526	0.172

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	111	102	95	187	119	99	111
N.S.	1	1.00	0.85	0.78	0.73	1.43	0.91	0.76	0.85
time (sec)	N/A	0.101	0.044	0.401	0.519	0.380	0.288	1.505	0.190

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	91	76	75	75	102	69	91
N.S.	1	1.00	0.92	0.77	0.76	0.76	1.03	0.70	0.92
time (sec)	N/A	0.045	0.011	0.393	0.515	0.370	0.190	0.880	5.096

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	150	0	0	38	654	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.23	4.04	0.00	-0.01
time (sec)	N/A	0.118	0.440	0.073	0.000	0.375	23.932	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	162	118	118	305	1251	392	115
N.S.	1	1.00	1.93	1.40	1.40	3.63	14.89	4.67	1.37
time (sec)	N/A	0.041	0.161	0.345	0.287	0.378	1.004	0.821	5.135

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	99	80	78	160	552	196	76
N.S.	1	1.00	1.62	1.31	1.28	2.62	9.05	3.21	1.25
time (sec)	N/A	0.029	0.109	0.342	0.292	0.390	0.578	0.911	5.058

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	42	43	39	56	163	65	38
N.S.	1	1.00	1.02	1.05	0.95	1.37	3.98	1.59	0.93
time (sec)	N/A	0.016	0.082	0.034	0.307	0.373	0.309	1.434	5.056

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	17	12	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.42	1.00	1.00	1.00
time (sec)	N/A	0.002	0.001	0.020	0.287	0.388	0.007	1.680	5.011

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	48	0	0	21	65	0	43
N.S.	1	1.00	1.14	0.00	0.00	0.50	1.55	0.00	1.02
time (sec)	N/A	0.020	0.084	0.014	0.000	0.380	6.195	0.000	5.326

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	56	0	0	34	299	0	49
N.S.	1	1.00	1.27	0.00	0.00	0.77	6.80	0.00	1.11
time (sec)	N/A	0.020	0.097	0.023	0.000	0.392	16.606	0.000	5.349

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	108	0	0	47	0	0	59
N.S.	1	1.00	2.35	0.00	0.00	1.02	0.00	0.00	1.28
time (sec)	N/A	0.021	0.114	0.035	0.000	0.352	0.000	0.000	5.414

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	399	0	0	0	274	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.90	0.00	-0.00
time (sec)	N/A	0.162	1.569	0.013	0.000	0.000	26.528	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	66	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	1.47	0.00	0.00	-0.02
time (sec)	N/A	0.313	0.673	0.085	0.000	0.383	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	178	0	0	32	248	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.12	0.91	0.00	-0.00
time (sec)	N/A	0.130	0.244	0.093	0.000	0.389	110.508	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	204	0	0	38	0	0	-1
N.S.	1	1.00	0.69	0.00	0.00	0.13	0.00	0.00	-0.00
time (sec)	N/A	0.144	0.756	0.105	0.000	0.381	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	151	0	0	46	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.261	0.021	0.000	0.377	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	20	0	0	20
N.S.	1	1.00	1.00	0.88	0.83	0.83	0.00	0.00	0.83
time (sec)	N/A	0.033	5.750	0.392	0.328	0.370	0.000	0.000	5.587

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	10.128	180.000	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	61	0	228	95
N.S.	1	1.00	1.00	0.00	0.00	2.18	0.00	8.14	3.39
time (sec)	N/A	0.066	0.284	0.148	0.000	0.393	0.000	0.809	5.197

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	138	77	119	0	237	124
N.S.	1	1.00	1.02	3.07	1.71	2.64	0.00	5.27	2.76
time (sec)	N/A	0.102	0.339	0.725	0.415	0.393	0.000	0.886	5.367

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	52	64	54	0	115	76
N.S.	1	1.00	1.00	1.68	2.06	1.74	0.00	3.71	2.45
time (sec)	N/A	0.144	0.625	0.510	0.374	0.396	0.000	0.868	5.298

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	136	94	88	0	155	106
N.S.	1	1.00	0.91	3.02	2.09	1.96	0.00	3.44	2.36
time (sec)	N/A	0.379	0.978	0.791	0.396	0.399	0.000	1.443	5.640

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [594] had the largest ratio of [86]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	20	0.050
2	A	2	1	1.00	22	0.045
3	A	2	1	1.00	22	0.045
4	A	2	1	1.00	25	0.040
5	A	2	1	1.00	27	0.037
6	A	2	1	1.00	27	0.037
7	A	6	6	1.00	15	0.400
8	A	7	7	1.00	15	0.467
9	A	8	7	1.00	15	0.467
10	A	9	7	1.00	15	0.467
11	A	6	6	1.00	15	0.400
12	A	6	6	1.00	16	0.375
13	A	3	3	1.00	11	0.273
14	A	3	3	1.00	15	0.200
15	A	3	3	1.00	13	0.231
16	A	3	3	1.00	13	0.231
17	A	6	6	1.00	15	0.400
18	A	3	3	1.00	19	0.158
19	A	3	3	1.00	21	0.143
20	A	3	3	1.00	31	0.097
21	A	3	3	1.00	36	0.083
22	A	12	10	1.00	35	0.286
23	A	11	9	1.00	33	0.273
24	A	10	8	1.00	36	0.222
25	A	10	10	1.00	19	0.526

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	9	9	1.00	18	0.500
27	A	4	4	1.00	27	0.148
28	A	4	4	1.00	28	0.143
29	A	4	4	1.00	24	0.167
30	A	4	4	1.00	24	0.167
31	A	4	4	1.00	26	0.154
32	A	4	4	1.00	26	0.154
33	A	4	4	1.00	28	0.143
34	A	4	4	1.00	30	0.133
35	A	4	4	1.00	29	0.138
36	A	4	4	1.00	29	0.138
37	A	4	4	1.00	29	0.138
38	A	4	4	1.00	32	0.125
39	A	6	6	1.00	13	0.462
40	A	4	4	1.00	49	0.082
41	A	4	4	1.00	57	0.070
42	A	2	2	1.00	31	0.065
43	A	2	2	1.00	42	0.048
44	A	4	4	1.00	42	0.095
45	A	4	4	1.00	45	0.089
46	A	4	4	1.00	45	0.089
47	A	4	4	1.00	44	0.091
48	A	3	3	1.00	20	0.150
49	A	6	6	1.00	20	0.300
50	A	2	2	1.00	16	0.125
51	A	5	5	1.00	20	0.250
52	A	3	3	1.00	18	0.167
53	A	2	1	1.00	30	0.033
54	A	2	1	1.00	30	0.033
55	A	2	1	1.00	28	0.036
56	A	2	1	1.00	30	0.033
57	A	7	7	1.00	30	0.233
58	A	8	8	1.00	30	0.267
59	A	7	5	1.00	32	0.156
60	A	6	5	1.00	32	0.156

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	5	1.00	32	0.156
62	A	4	4	1.00	32	0.125
63	A	5	5	1.00	32	0.156
64	A	6	5	1.00	32	0.156
65	A	7	5	1.00	32	0.156
66	A	7	6	1.00	32	0.188
67	A	6	6	1.00	32	0.188
68	A	5	5	1.00	32	0.156
69	A	6	6	1.00	32	0.188
70	A	8	8	1.00	17	0.471
71	A	10	9	1.00	17	0.529
72	A	10	9	0.99	17	0.529
73	A	10	9	0.99	22	0.409
74	A	10	9	1.00	22	0.409
75	A	10	9	1.00	22	0.409
76	A	9	8	1.00	17	0.471
77	A	9	8	1.00	19	0.421
78	A	8	7	1.00	18	0.389
79	A	3	3	1.00	18	0.167
80	A	3	3	1.00	22	0.136
81	A	1	1	1.00	20	0.050
82	A	1	1	1.00	20	0.050
83	A	3	3	1.00	33	0.091
84	A	3	3	1.00	35	0.086
85	A	1	1	1.00	36	0.028
86	A	1	1	1.00	36	0.028
87	A	3	3	1.00	30	0.100
88	A	3	3	1.00	32	0.094
89	A	1	1	1.00	33	0.030
90	A	1	1	1.00	33	0.030
91	A	1	1	1.00	20	0.050
92	A	1	1	1.00	24	0.042
93	A	3	3	1.00	22	0.136
94	A	3	3	1.00	22	0.136
95	A	1	1	1.00	20	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	1	1	1.00	20	0.050
97	A	3	3	1.00	18	0.167
98	A	3	3	1.00	22	0.136
99	A	1	1	1.00	35	0.029
100	A	1	1	1.00	37	0.027
101	A	3	3	1.00	38	0.079
102	A	3	3	1.00	38	0.079
103	A	1	1	1.00	32	0.031
104	A	1	1	1.00	34	0.029
105	A	3	3	1.00	35	0.086
106	A	3	3	1.00	35	0.086
107	A	3	3	1.00	17	0.176
108	A	3	3	1.00	18	0.167
109	A	3	3	1.00	19	0.158
110	A	3	3	1.00	20	0.150
111	A	3	3	1.00	15	0.200
112	A	3	3	1.00	17	0.176
113	A	3	3	1.00	15	0.200
114	A	3	3	1.00	17	0.176
115	A	7	5	1.00	16	0.312
116	A	13	9	1.00	15	0.600
117	A	8	6	1.00	16	0.375
118	A	14	10	1.00	15	0.667
119	A	9	6	1.00	16	0.375
120	A	15	10	1.00	15	0.667
121	A	10	6	1.00	16	0.375
122	A	16	10	1.00	15	0.667
123	A	7	5	1.00	15	0.333
124	A	13	9	1.00	13	0.692
125	A	7	5	1.00	21	0.238
126	A	13	9	1.00	20	0.450
127	A	8	6	1.00	21	0.286
128	A	14	10	1.00	20	0.500
129	A	9	6	1.00	21	0.286
130	A	15	10	1.00	20	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	10	6	1.00	21	0.286
132	A	16	10	1.00	20	0.500
133	A	3	2	1.00	11	0.182
134	A	3	2	1.00	12	0.167
135	A	2	1	1.00	15	0.067
136	A	3	2	1.00	14	0.143
137	A	2	1	1.00	17	0.059
138	A	3	2	1.00	19	0.105
139	A	2	1	1.00	20	0.050
140	A	2	2	1.00	14	0.143
141	A	4	3	1.00	17	0.176
142	A	4	3	1.00	19	0.158
143	A	3	2	1.00	20	0.100
144	A	4	3	1.00	21	0.143
145	A	3	2	1.00	22	0.091
146	A	4	3	1.00	24	0.125
147	A	3	2	1.00	25	0.080
148	A	3	2	1.00	25	0.080
149	A	8	6	1.00	26	0.231
150	A	9	7	1.00	26	0.269
151	A	10	7	1.00	26	0.269
152	A	10	7	1.00	11	0.636
153	A	3	3	1.00	12	0.250
154	A	13	9	1.00	15	0.600
155	A	10	7	1.00	14	0.500
156	A	9	6	1.00	17	0.353
157	A	14	10	1.00	19	0.526
158	A	13	9	1.00	20	0.450
159	A	2	2	1.00	14	0.143
160	A	12	8	1.00	17	0.471
161	A	5	5	1.00	19	0.263
162	A	15	11	1.00	20	0.550
163	A	13	9	1.00	21	0.429
164	A	12	8	1.00	22	0.364
165	A	16	12	1.00	24	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	15	11	1.00	25	0.440
167	A	2	2	1.00	19	0.105
168	A	11	8	1.00	17	0.471
169	A	9	7	1.00	20	0.350
170	A	15	11	1.00	19	0.579
171	A	11	8	1.00	31	0.258
172	A	8	6	1.00	31	0.194
173	A	9	7	1.00	31	0.226
174	A	10	8	1.00	31	0.258
175	A	17	12	1.00	30	0.400
176	A	14	10	1.00	30	0.333
177	A	15	11	1.00	30	0.367
178	A	16	12	1.00	30	0.400
179	A	2	2	1.00	21	0.095
180	A	2	2	1.00	21	0.095
181	A	2	1	1.00	19	0.053
182	A	2	2	1.00	19	0.105
183	A	2	2	1.00	21	0.095
184	A	2	2	1.00	21	0.095
185	A	2	2	1.00	21	0.095
186	A	13	9	1.00	36	0.250
187	A	13	9	1.00	41	0.220
188	A	13	9	1.00	46	0.196
189	A	19	13	1.00	35	0.371
190	A	19	13	1.00	40	0.325
191	A	19	13	1.00	45	0.289
192	A	8	6	1.00	36	0.167
193	A	8	6	1.00	41	0.146
194	A	10	8	1.00	46	0.174
195	A	14	10	1.00	35	0.286
196	A	14	10	1.00	40	0.250
197	A	16	12	1.00	45	0.267
198	A	9	7	1.00	36	0.194
199	A	9	7	1.00	41	0.171
200	A	9	7	1.00	46	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	15	11	1.00	35	0.314
202	A	15	11	1.00	40	0.275
203	A	15	11	1.00	45	0.244
204	A	10	8	1.00	36	0.222
205	A	10	8	1.00	41	0.195
206	A	10	8	1.00	46	0.174
207	A	16	12	1.00	35	0.343
208	A	16	12	1.00	40	0.300
209	A	16	12	1.00	45	0.267
210	A	6	5	1.00	17	0.294
211	A	7	6	1.00	18	0.333
212	A	7	6	1.00	19	0.316
213	A	6	5	1.00	20	0.250
214	A	8	7	1.00	22	0.318
215	A	1	1	1.00	23	0.043
216	A	1	1	1.00	26	0.038
217	A	1	1	1.00	28	0.036
218	A	1	1	1.00	31	0.032
219	A	1	1	1.00	15	0.067
220	A	12	10	1.00	42	0.238
221	A	3	2	1.00	11	0.182
222	A	3	2	1.00	15	0.133
223	A	3	2	1.00	30	0.067
224	A	3	2	1.00	30	0.067
225	A	3	2	1.00	30	0.067
226	A	3	2	1.00	30	0.067
227	A	3	2	1.00	30	0.067
228	A	3	2	1.00	30	0.067
229	A	3	2	1.00	30	0.067
230	A	3	2	1.00	30	0.067
231	A	3	2	1.00	30	0.067
232	A	3	2	1.00	30	0.067
233	A	9	8	1.00	30	0.267
234	A	9	8	1.00	30	0.267
235	A	9	8	1.00	30	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	9	8	1.00	30	0.267
237	A	9	8	1.00	30	0.267
238	A	9	8	1.00	28	0.286
239	A	8	7	1.00	27	0.259
240	A	8	7	1.00	30	0.233
241	A	8	7	1.00	30	0.233
242	A	8	7	1.00	30	0.233
243	A	8	7	1.00	30	0.233
244	A	8	7	1.00	30	0.233
245	A	8	7	1.00	30	0.233
246	A	8	7	1.00	30	0.233
247	A	8	7	1.00	30	0.233
248	A	8	7	1.00	30	0.233
249	A	8	7	1.00	30	0.233
250	A	8	7	1.00	30	0.233
251	A	3	2	1.00	30	0.067
252	A	3	2	1.00	30	0.067
253	A	3	2	1.00	30	0.067
254	A	3	2	1.00	30	0.067
255	A	3	2	1.00	30	0.067
256	A	3	2	1.00	30	0.067
257	A	3	2	1.00	30	0.067
258	A	3	2	1.00	30	0.067
259	A	3	2	1.00	30	0.067
260	A	9	8	1.00	30	0.267
261	A	12	10	1.00	30	0.333
262	A	9	8	1.00	30	0.267
263	A	11	10	1.00	30	0.333
264	A	9	8	1.00	30	0.267
265	A	10	9	1.00	28	0.321
266	A	9	9	1.00	27	0.333
267	A	9	8	1.00	30	0.267
268	A	9	8	1.00	30	0.267
269	A	9	8	1.00	30	0.267
270	A	9	8	1.00	30	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	9	8	1.00	30	0.267
272	A	9	8	1.00	30	0.267
273	A	9	8	1.00	30	0.267
274	A	9	8	1.00	30	0.267
275	A	9	8	1.00	30	0.267
276	A	3	2	1.00	30	0.067
277	A	3	2	1.00	30	0.067
278	A	3	2	1.00	30	0.067
279	A	3	2	1.00	30	0.067
280	A	3	2	1.00	30	0.067
281	A	3	2	1.00	30	0.067
282	A	3	2	1.00	30	0.067
283	A	3	2	1.00	30	0.067
284	A	3	2	1.00	30	0.067
285	A	3	2	1.00	30	0.067
286	A	10	9	1.00	30	0.300
287	A	14	10	1.00	30	0.333
288	A	10	9	1.00	30	0.300
289	A	13	10	1.00	30	0.333
290	A	10	9	1.00	30	0.300
291	A	12	10	1.00	30	0.333
292	A	10	10	1.00	30	0.333
293	A	10	10	1.00	28	0.357
294	A	9	9	1.00	27	0.333
295	A	9	9	1.00	30	0.300
296	A	9	9	1.00	30	0.300
297	A	10	9	1.00	30	0.300
298	A	10	9	1.00	30	0.300
299	A	10	8	1.00	30	0.267
300	A	10	8	1.00	30	0.267
301	A	10	8	1.00	30	0.267
302	A	10	8	1.00	30	0.267
303	A	10	8	1.00	30	0.267
304	A	8	7	1.00	16	0.438
305	A	5	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	8	7	1.00	16	0.438
307	A	6	6	1.00	14	0.429
308	A	6	5	1.00	16	0.312
309	A	6	5	1.00	16	0.312
310	A	3	2	1.00	16	0.125
311	A	6	6	1.00	14	0.429
312	A	6	6	1.00	16	0.375
313	A	2	1	1.00	21	0.048
314	A	2	1	1.00	19	0.053
315	A	2	1	1.00	18	0.056
316	A	2	1	1.00	21	0.048
317	A	2	1	1.00	21	0.048
318	A	2	1	1.00	21	0.048
319	A	3	2	1.00	23	0.087
320	A	3	2	1.00	21	0.095
321	A	3	2	1.00	20	0.100
322	A	2	1	1.00	23	0.043
323	A	2	1	1.00	23	0.043
324	A	2	1	1.00	23	0.043
325	A	3	2	1.00	23	0.087
326	A	3	2	1.00	21	0.095
327	A	3	2	1.00	20	0.100
328	A	2	1	1.00	23	0.043
329	A	2	1	1.00	23	0.043
330	A	2	1	1.00	23	0.043
331	A	3	2	1.00	23	0.087
332	A	3	2	1.00	21	0.095
333	A	3	2	1.00	20	0.100
334	A	2	1	1.00	23	0.043
335	A	2	1	1.00	23	0.043
336	A	2	1	1.00	23	0.043
337	A	10	9	1.00	23	0.391
338	A	10	9	1.00	23	0.391
339	A	10	9	1.00	21	0.429
340	A	8	8	1.00	20	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	10	9	1.00	23	0.391
342	A	10	9	1.00	23	0.391
343	A	10	9	1.00	23	0.391
344	A	7	7	1.00	23	0.304
345	A	7	7	1.00	21	0.333
346	A	7	7	1.00	20	0.350
347	A	11	10	1.00	23	0.435
348	A	11	10	1.00	23	0.435
349	A	11	10	1.00	23	0.435
350	A	11	10	1.00	23	0.435
351	A	8	8	1.00	23	0.348
352	A	8	8	1.00	21	0.381
353	A	8	8	1.00	20	0.400
354	A	12	10	1.00	23	0.435
355	A	12	10	1.00	23	0.435
356	A	12	10	1.00	23	0.435
357	A	12	10	1.00	23	0.435
358	A	9	8	1.00	23	0.348
359	A	9	9	1.00	21	0.429
360	A	9	8	1.00	20	0.400
361	A	13	10	1.00	23	0.435
362	A	13	10	1.00	23	0.435
363	A	13	10	1.00	23	0.435
364	A	13	10	1.00	23	0.435
365	A	5	5	1.00	20	0.250
366	A	4	4	1.00	18	0.222
367	A	5	5	1.00	20	0.250
368	A	4	4	1.00	18	0.222
369	A	4	4	1.00	27	0.148
370	A	4	4	1.00	29	0.138
371	A	4	4	1.00	28	0.143
372	A	4	4	1.00	28	0.143
373	A	2	1	1.00	36	0.028
374	A	2	1	1.00	36	0.028
375	A	2	1	1.00	36	0.028

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	2	1	1.00	34	0.029
377	A	2	1	1.00	33	0.030
378	A	2	1	1.00	36	0.028
379	A	2	1	1.00	36	0.028
380	A	2	1	1.00	36	0.028
381	A	2	1	1.00	36	0.028
382	A	2	1	1.00	36	0.028
383	A	2	1	1.00	38	0.026
384	A	2	1	1.00	38	0.026
385	A	3	2	1.00	38	0.053
386	A	3	2	1.00	36	0.056
387	A	3	2	1.00	35	0.057
388	A	3	2	1.00	38	0.053
389	A	3	2	1.00	38	0.053
390	A	3	2	1.00	38	0.053
391	A	2	1	1.00	38	0.026
392	A	2	1	1.00	38	0.026
393	A	2	1	1.00	38	0.026
394	A	2	1	1.00	38	0.026
395	A	3	2	1.00	38	0.053
396	A	3	2	1.00	36	0.056
397	A	3	2	1.00	35	0.057
398	A	3	2	1.00	38	0.053
399	A	3	2	1.00	38	0.053
400	A	3	2	1.00	38	0.053
401	A	2	1	1.00	38	0.026
402	A	2	1	1.00	38	0.026
403	A	13	10	1.00	38	0.263
404	A	13	10	1.00	38	0.263
405	A	13	10	1.00	38	0.263
406	A	13	10	1.00	36	0.278
407	A	10	9	0.99	35	0.257
408	A	10	9	0.99	38	0.237
409	A	10	9	1.00	38	0.237
410	A	10	9	0.99	38	0.237

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	10	9	0.99	38	0.237
412	A	11	10	1.00	38	0.263
413	A	11	10	1.00	38	0.263
414	A	11	10	0.99	38	0.263
415	A	11	10	1.00	36	0.278
416	A	9	9	1.00	35	0.257
417	A	11	10	0.99	38	0.263
418	A	11	10	1.00	38	0.263
419	A	11	10	0.99	38	0.263
420	A	11	10	0.99	38	0.263
421	A	12	11	1.00	38	0.290
422	A	10	10	1.00	38	0.263
423	A	8	8	1.00	38	0.210
424	A	8	8	1.00	36	0.222
425	A	8	8	1.00	35	0.229
426	A	12	10	0.99	38	0.263
427	A	12	10	1.00	38	0.263
428	A	12	10	0.99	38	0.263
429	A	12	10	0.99	38	0.263
430	A	10	7	1.00	25	0.280
431	A	8	7	1.00	25	0.280
432	A	6	6	1.00	23	0.261
433	A	5	5	1.00	22	0.227
434	A	7	7	1.00	25	0.280
435	A	8	8	1.00	25	0.320
436	A	9	8	1.00	25	0.320
437	A	8	7	1.00	25	0.280
438	A	7	7	1.00	25	0.280
439	A	6	6	1.00	25	0.240
440	A	4	4	1.00	25	0.160
441	A	6	6	1.00	23	0.261
442	A	4	4	1.00	22	0.182
443	A	10	10	1.00	25	0.400
444	A	11	11	1.00	25	0.440
445	A	13	9	1.00	35	0.257

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	11	9	1.00	35	0.257
447	A	9	8	1.00	33	0.242
448	A	8	7	1.00	32	0.219
449	A	11	11	1.00	35	0.314
450	A	11	11	1.00	35	0.314
451	A	10	9	1.00	35	0.257
452	A	11	9	1.00	35	0.257
453	A	12	9	1.00	35	0.257
454	A	10	10	1.00	35	0.286
455	A	11	10	1.00	35	0.286
456	A	12	10	1.00	35	0.286
457	A	13	10	1.00	35	0.286
458	A	14	9	1.00	35	0.257
459	A	12	9	1.00	35	0.257
460	A	10	8	1.00	33	0.242
461	A	9	7	1.00	32	0.219
462	A	12	11	1.00	35	0.314
463	A	12	11	1.00	35	0.314
464	A	11	9	1.00	35	0.257
465	A	12	9	1.00	35	0.257
466	A	13	9	1.00	35	0.257
467	A	11	11	1.00	35	0.314
468	A	12	11	1.00	35	0.314
469	A	13	11	1.00	35	0.314
470	A	11	10	1.00	35	0.286
471	A	12	10	1.00	35	0.286
472	A	13	10	1.00	35	0.286
473	A	14	10	1.00	35	0.286
474	A	8	7	1.18	20	0.350
475	A	7	4	1.17	21	0.190
476	A	7	4	1.17	23	0.174
477	A	2	1	1.00	23	0.043
478	A	2	1	1.00	26	0.038
479	A	3	2	1.00	25	0.080
480	A	3	2	1.00	28	0.071

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	3	2	1.00	25	0.080
482	A	3	2	1.00	28	0.071
483	A	3	2	1.00	25	0.080
484	A	3	2	1.00	28	0.071
485	A	9	7	1.00	26	0.269
486	A	12	9	1.00	29	0.310
487	A	15	11	1.00	25	0.440
488	A	18	13	1.00	28	0.464
489	A	14	10	1.00	25	0.400
490	A	14	10	1.00	28	0.357
491	A	15	11	1.00	25	0.440
492	A	15	11	1.00	28	0.393
493	A	16	11	1.00	25	0.440
494	A	16	11	1.00	28	0.393
495	A	14	12	1.00	30	0.400
496	A	13	11	1.00	30	0.367
497	A	12	11	1.00	30	0.367
498	A	12	11	1.00	28	0.393
499	A	11	10	1.00	27	0.370
500	A	14	13	1.00	30	0.433
501	A	14	13	1.00	30	0.433
502	A	14	13	1.00	30	0.433
503	A	15	14	1.00	30	0.467
504	A	13	13	1.00	30	0.433
505	A	14	14	1.00	30	0.467
506	A	12	12	1.00	30	0.400
507	A	13	12	1.00	30	0.400
508	A	14	13	1.00	30	0.433
509	A	15	13	1.00	30	0.433
510	A	16	12	1.00	30	0.400
511	A	15	11	1.00	30	0.367
512	A	14	11	1.00	30	0.367
513	A	14	11	1.00	28	0.393
514	A	13	10	1.00	27	0.370
515	A	16	13	1.00	30	0.433

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	16	14	1.00	30	0.467
517	A	16	15	1.00	30	0.500
518	A	16	14	1.00	30	0.467
519	A	15	15	1.00	30	0.500
520	A	15	15	1.00	30	0.500
521	A	15	15	1.00	30	0.500
522	A	16	16	1.00	30	0.533
523	A	14	13	1.00	30	0.433
524	A	15	14	1.00	30	0.467
525	A	13	12	1.00	30	0.400
526	A	14	12	1.00	30	0.400
527	A	15	13	1.00	30	0.433
528	A	16	13	1.00	30	0.433
529	A	12	10	1.00	30	0.333
530	A	11	9	1.00	30	0.300
531	A	10	9	1.00	30	0.300
532	A	10	9	1.00	28	0.321
533	A	9	8	1.00	27	0.296
534	A	12	11	1.00	30	0.367
535	A	13	12	1.00	30	0.400
536	A	11	10	1.00	30	0.333
537	A	12	10	1.00	30	0.333
538	A	13	11	1.00	30	0.367
539	A	14	11	1.00	30	0.367
540	A	12	11	1.00	30	0.367
541	A	11	10	1.00	30	0.333
542	A	10	9	1.00	30	0.300
543	A	9	8	0.98	30	0.267
544	A	10	9	1.00	30	0.300
545	A	7	6	1.00	28	0.214
546	A	4	4	1.00	27	0.148
547	A	11	10	1.00	30	0.333
548	A	13	12	1.00	30	0.400
549	A	15	12	1.00	30	0.400
550	A	17	13	1.00	30	0.433

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	14	4	1.00	30	0.133
552	A	12	8	1.19	25	0.320
553	A	13	7	1.00	28	0.250
554	A	2	2	1.00	22	0.091
555	A	2	2	1.00	35	0.057
556	A	2	2	1.00	35	0.057
557	A	2	2	1.00	22	0.091
558	A	3	3	1.00	25	0.120
559	A	6	5	1.00	15	0.333
560	A	6	5	1.00	15	0.333
561	A	7	6	1.00	20	0.300
562	A	7	6	1.00	20	0.300
563	A	7	7	1.00	17	0.412
564	A	7	6	1.00	25	0.240
565	A	11	6	1.00	35	0.171
566	A	11	6	1.00	35	0.171
567	A	8	5	1.00	22	0.227
568	A	11	7	1.00	25	0.280
569	A	17	7	1.00	15	0.467
570	A	17	7	1.00	15	0.467
571	A	14	7	1.00	20	0.350
572	A	14	7	1.00	20	0.350
573	A	15	9	1.00	17	0.529
574	A	14	7	1.00	25	0.280
575	A	13	7	1.00	18	0.389
576	A	7	4	1.00	36	0.111
577	A	4	3	1.00	19	0.158
578	A	4	3	1.00	19	0.158
579	A	4	2	1.00	17	0.118
580	A	1	0	1.00	9	0.000
581	A	3	3	1.00	19	0.158
582	A	3	3	1.00	19	0.158
583	A	3	3	1.00	19	0.158
584	A	13	4	1.00	38	0.105
585	A	2	2	1.00	58	0.034

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	10	3	1.00	30	0.100
587	A	13	4	1.00	36	0.111
588	A	4	4	1.00	35	0.114
589	A	1	1	1.00	46	0.022
590	A	10	8	1.00	24	0.333
591	A	1	1	1.00	48	0.021
592	A	1	1	1.00	45	0.022
593	A	1	1	1.00	69	0.014
594	A	1	1	1.00	86	0.012

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=72

$$\frac{2(b^2c-abd+a^2e)\sqrt{a+bx}}{b^3}+\frac{2(bd-2ae)(a+bx)^{3/2}}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}$$

```
[Out] 2/3*(-2*a*e+b*d)*(b*x+a)^(3/2)/b^3+2/5*e*(b*x+a)^(5/2)/b^3+2*(a^2*e-a*b*d+b
^2*c)*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {712}

$$\frac{2\sqrt{a+bx}\,(a^2e-abd+b^2c)}{b^3}+\frac{2(a+bx)^{3/2}(bd-2ae)}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)*Sqrt[a + b*x])/b^3 + (2*(b*d - 2*a*e)*(a + b*x)^
(3/2))/(3*b^3) + (2*e*(a + b*x)^(5/2))/(5*b^3)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx &= \int \left(\frac{b^2c-abd+a^2e}{b^2\sqrt{a+bx}}+\frac{(bd-2ae)\sqrt{a+bx}}{b^2}+\frac{e(a+bx)^{3/2}}{b^2}\right)dx\\ &= \frac{2(b^2c-abd+a^2e)\sqrt{a+bx}}{b^3}+\frac{2(bd-2ae)(a+bx)^{3/2}}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 0.74

$$\frac{2\sqrt{a+bx}\,(8a^2e-2ab(5d+2ex)+b^2(15c+x(5d+3ex)))}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^2*e - 2*a*b*(5*d + 2*e*x) + b^2*(15*c + x*(5*d + 3*e*
x))))/(15*b^3)
```

Maple [A]
time = 0.31, size = 75, normalized size = 1.04

method	result	size
gosper	$\frac{2\sqrt{bx+a}\,(3e\,x^{2}b^{2}-4abex+5b^{2}dx+8a^{2}e-10abd+15b^{2}c)}{15b^{3}}$	53
trager	$\frac{2\sqrt{bx+a}\,(3e\,x^{2}b^{2}-4abex+5b^{2}dx+8a^{2}e-10abd+15b^{2}c)}{15b^{3}}$	53
risch	$\frac{2\sqrt{bx+a}\,(3e\,x^{2}b^{2}-4abex+5b^{2}dx+8a^{2}e-10abd+15b^{2}c)}{15b^{3}}$	53
derivativedivides	$\frac{\frac{2e(bx+a)^{\frac{5}{2}}}{5}-\frac{4ae(bx+a)^{\frac{3}{2}}}{3}+\frac{2bd(bx+a)^{\frac{3}{2}}}{3}+2a^{2}e\sqrt{bx+a}-2abd\sqrt{bx+a}+2b^{2}c\sqrt{bx+a}}{b^{3}}$	75
default	$\frac{\frac{2e(bx+a)^{\frac{5}{2}}}{5}-\frac{4ae(bx+a)^{\frac{3}{2}}}{3}+\frac{2bd(bx+a)^{\frac{3}{2}}}{3}+2a^{2}e\sqrt{bx+a}-2abd\sqrt{bx+a}+2b^{2}c\sqrt{bx+a}}{b^{3}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/5*e*(b*x+a)^(5/2)-2/3*a*e*(b*x+a)^(3/2)+1/3*b*d*(b*x+a)^(3/2)+a^2*
e*(b*x+a)^(1/2)-a*b*d*(b*x+a)^(1/2)+b^2*c*(b*x+a)^(1/2))
```

Maxima [A]
time = 0.50, size = 78, normalized size = 1.08

$$\frac{2\left(15\sqrt{bx+a}\,c+\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)d}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^{2}\right)e}{b^{2}}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + (3
*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2)/b
```

Fricas [A]
time = 0.36, size = 53, normalized size = 0.74

$$\frac{2\,(3\,b^{2}ex^{2}+15\,b^{2}c-10\,abd+8\,a^{2}e+(5\,b^{2}d-4\,abe)x)\sqrt{bx+a}}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*e*x^2 + 15*b^2*c - 10*a*b*d + 8*a^2*e + (5*b^2*d - 4*a*b*e)*x)*
sqrt(b*x + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(73) = 146$.
time = 5.06, size = 223, normalized size = 3.10

$$\begin{cases} \frac{-\frac{2ac}{\sqrt{a+bx}} - \frac{2ad\left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx}\right)}{b} - \frac{2ae\left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b^2} - 2c\left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx}\right) - \frac{2d\left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b} - \frac{2e\left(-\frac{a^3}{\sqrt{a+bx}} - 3a^2\sqrt{a+bx} + a(a+bx)^{\frac{3}{2}} - \frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}}{b} & \text{for } b \neq 0 \\ \frac{cx + \frac{dx^2}{2} + \frac{ex^3}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*c/sqrt(a + b*x) - 2*a*d*(-a/sqrt(a + b*x) - sqrt(a + b*x))
/b - 2*a*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b*
*2 - 2*c*(-a/sqrt(a + b*x) - sqrt(a + b*x)) - 2*d*(a**2/sqrt(a + b*x) + 2*a
*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b - 2*e*(-a**3/sqrt(a + b*x) - 3*a**2*
sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2)/b, Ne(b, 0))
, ((c*x + d*x**2/2 + e*x**3/3)/sqrt(a), True))
```

Giac [A]
time = 1.34, size = 78, normalized size = 1.08

$$\frac{2\left(15\sqrt{bx+a}\,c + \frac{5\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}\,a\right)d}{b} + \frac{\left(3(bx+a)^{\frac{5}{2}} - 10(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+a}\,a^2\right)e}{b^2}\right)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + (3
*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2)/b
```

Mupad [B]
time = 4.72, size = 58, normalized size = 0.81

$$\frac{2\sqrt{a+bx}\,\left(3e(a+bx)^2 + 15b^2c + 15a^2e - 10ae(a+bx) + 5bd(a+bx) - 15abd\right)}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(1/2)*(3*e*(a + b*x)^2 + 15*b^2*c + 15*a^2*e - 10*a*e*(a + b*x
) + 5*b*d*(a + b*x) - 15*a*b*d))/(15*b^3)
```

3.2 $\int \frac{\left(c+dx+ex^2\right)^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=161

$$\frac{2(b^2c-abd+a^2e)^2\sqrt{a+bx}}{b^5}+\frac{4(bd-2ae)\left(b^2c-abd+a^2e\right)(a+bx)^{3/2}}{3b^5}-\frac{2(6abde-6a^2e^2-b^2(d^2+2ce))}{5b^5}$$

```
[Out] 4/3*(-2*a*e+b*d)*(a^2*e-a*b*d+b^2*c)*(b*x+a)^(3/2)/b^5-2/5*(6*a*b*d*e-6*a^2
*e^2-b^2*(2*c*e+d^2))*(b*x+a)^(5/2)/b^5+4/7*e*(-2*a*e+b*d)*(b*x+a)^(7/2)/b^
5+2/9*e^2*(b*x+a)^(9/2)/b^5+2*(a^2*e-a*b*d+b^2*c)^2*(b*x+a)^(1/2)/b^5
```

Rubi [A]
time = 0.07, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{712\}$

$$-\frac{2(a+bx)^{5/2}\left(-6a^2e^2+6abde-\left(b^2(2ce+d^2)\right)\right)}{5b^5}+\frac{4(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)}{3b^5}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)^2}{b^5}+\frac{4e(a+bx)^{7/2}(bd-2ae)}{7b^5}+\frac{2e^2(a+bx)^{9/2}}{9b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^2*Sqrt[a + b*x])/b^5 + (4*(b*d - 2*a*e)*(b^2*c -
 a*b*d + a^2*e)*(a + b*x)^(3/2))/(3*b^5) - (2*(6*a*b*d*e - 6*a^2*e^2 - b^2*
(d^2 + 2*c*e))*(a + b*x)^(5/2))/(5*b^5) + (4*e*(b*d - 2*a*e)*(a + b*x)^(7/2
))/(7*b^5) + (2*e^2*(a + b*x)^(9/2))/(9*b^5)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2\right)^2}{\sqrt{a+bx}}\,dx = \int \left(\frac{\left(b^2c-abd+a^2e\right)^2}{b^4\sqrt{a+bx}}+\frac{2(bd-2ae)\left(b^2c-abd+a^2e\right)\sqrt{a+bx}}{b^4}+\frac{(-6abde+6}{}\right.$$

$$=\frac{2(b^2c-abd+a^2e)^2\sqrt{a+bx}}{b^5}+\frac{4(bd-2ae)\left(b^2c-abd+a^2e\right)(a+bx)^{3/2}}{3b^5}-\frac{2(6ab}{}$$

Mathematica [A]

time = 0.10, size = 155, normalized size = 0.96

$$\frac{2\sqrt{a+bx}\,(128a^4e^2-32a^3be(9d+2ex)+24a^2b^2(7d^2+6dex+2e(7c+ex^2))-4ab^3(21c(5d+2ex)+x(21d^2+27dex+10e^2x^2))+b^4(315c^2+42cx(5d+3ex)+x^2(63d^2+90dex+35e^2x^2)))}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(128*a^4*e^2 - 32*a^3*b*e*(9*d + 2*e*x) + 24*a^2*b^2*(7*d^
2 + 6*d*e*x + 2*e*(7*c + e*x^2)) - 4*a*b^3*(21*c*(5*d + 2*e*x) + x*(21*d^2
+ 27*d*e*x + 10*e^2*x^2)) + b^4*(315*c^2 + 42*c*x*(5*d + 3*e*x) + x^2*(63*d
^2 + 90*d*e*x + 35*e^2*x^2))))/(315*b^5)
```

Maple [A]

time = 0.47, size = 135, normalized size = 0.84

method	result
derivativedivides	$\frac{\frac{2e^2(bx+a)^{\frac{9}{2}}}{9}+\frac{4(-2ae+bd)e(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left(2\left(a^2e-abd+b^2c\right)e+(-2ae+bd)^2\right)(bx+a)^{\frac{5}{2}}}{5}+\frac{4\left(a^2e-abd+b^2c\right)(-2ae+bd)(bx+a)^{\frac{3}{2}}}{3}+2(a^2 \ldots}{b^5}$
default	$\frac{\frac{2e^2(bx+a)^{\frac{9}{2}}}{9}+\frac{4(-2ae+bd)e(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left(2\left(a^2e-abd+b^2c\right)e+(-2ae+bd)^2\right)(bx+a)^{\frac{5}{2}}}{5}+\frac{4\left(a^2e-abd+b^2c\right)(-2ae+bd)(bx+a)^{\frac{3}{2}}}{3}+2(a^2 \ldots}{b^5}$
gosper	$\frac{2\sqrt{bx+a}\,(35e^2x^4b^4-40a\,b^3e^2x^3+90b^4de\,x^3+48a^2b^2e^2x^2-108a\,b^3de\,x^2+126b^4ce\,x^2+63b^4d^2x^2-64a^3b\,e^2x+144a^2b \ldots}{315b^5}$
trager	$\frac{2\sqrt{bx+a}\,(35e^2x^4b^4-40a\,b^3e^2x^3+90b^4de\,x^3+48a^2b^2e^2x^2-108a\,b^3de\,x^2+126b^4ce\,x^2+63b^4d^2x^2-64a^3b\,e^2x+144a^2b \ldots}{315b^5}$
risch	$\frac{2\sqrt{bx+a}\,(35e^2x^4b^4-40a\,b^3e^2x^3+90b^4de\,x^3+48a^2b^2e^2x^2-108a\,b^3de\,x^2+126b^4ce\,x^2+63b^4d^2x^2-64a^3b\,e^2x+144a^2b \ldots}{315b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/9*e^2*(b*x+a)^(9/2)+2/7*(-2*a*e+b*d)*e*(b*x+a)^(7/2)+1/5*(2*(a^2*e
-a*b*d+b^2*c)*e+(-2*a*e+b*d)^2)*(b*x+a)^(5/2)+2/3*(a^2*e-a*b*d+b^2*c)*(-2*a
*e+b*d)*(b*x+a)^(3/2)+(a^2*e-a*b*d+b^2*c)^2*(b*x+a)^(1/2))
```

Maxima [A]

time = 0.30, size = 238, normalized size = 1.48

$$\frac{2\left(315\sqrt{bx+a}\,c^2+42\,c\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)d}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)e}{b^2}\right)+\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)d^2}{b^2}+\frac{18\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)de}{b^3}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)e^2}{b^4}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(b*x + a)*c^2 + 42*c*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a
)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e
```

```
/b^2) + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2
)*d^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/
2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a
)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*
x + a)*a^4)*e^2/b^4)/b
```

Fricas [A]
time = 0.36, size = 192, normalized size = 1.19

$$\frac{2\,{\left(35\,b^{4} e^{2} x^{4} + 315\,b^{4} c^{2} - 420\,a b^{3} c d + 168\,a^{2} b^{2} d^{2} + 128\,a^{4} e^{2} + 10\,{\left(9\,b^{4} d e - 4\,a b^{3} e^{2}\right)} x^{3} + 3\,{\left(21\,b^{4} d^{2} + 16\,a^{2} b^{2} e^{2} + 6\,{\left(7\,b^{4} c - 6\,a b^{3} d\right)} e\right)} x^{2} + 48\,{\left(7\,a^{2} b^{2} c - 6\,a^{3} b d\right)} e + 2\,{\left(105\,b^{4} c d - 42\,a b^{3} d^{2} - 32\,a^{3} b e^{2} - 12\,{\left(7\,a b^{3} c - 6\,a^{2} b^{2} d\right)} e\right)} x\right)} \sqrt{b x + a}}{315\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*e^2*x^4 + 315*b^4*c^2 - 420*a*b^3*c*d + 168*a^2*b^2*d^2 + 128
*a^4*e^2 + 10*(9*b^4*d*e - 4*a*b^3*e^2)*x^3 + 3*(21*b^4*d^2 + 16*a^2*b^2*e^
2 + 6*(7*b^4*c - 6*a*b^3*d)*e)*x^2 + 48*(7*a^2*b^2*c - 6*a^3*b*d)*e + 2*(10
5*b^4*c*d - 42*a*b^3*d^2 - 32*a^3*b*e^2 - 12*(7*a*b^3*c - 6*a^2*b^2*d)*e)*x
)*sqrt(b*x + a)/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(162) = 324$.
time = 32.93, size = 644, normalized size = 4.00

$$\begin{cases} \frac{- \frac{2 a c^{2}}{\sqrt{a + b x}} - \frac{4 a c d \left(- \frac{a}{\sqrt{a + b x}} - \sqrt{a + b x}\right)}{b} - \frac{4 a c e \left(\frac{a^{2}}{\sqrt{a + b x}} + 2 a \sqrt{a + b x} - \frac{\left(a + b x\right)^{\frac{3}{2}}}{3}\right)}{b^{2}} - \frac{2 a d^{2} \left(\frac{a^{2}}{\sqrt{a + b x}} + 2 a \sqrt{a + b x} - \frac{\left(a + b x\right)^{\frac{3}{2}}}{3}\right)}{b^{2}} - \frac{4 a d e \left(- \frac{a^{3}}{\sqrt{a + b x}} - 3 a^{2} \sqrt{a + b x} + a \left(a + b x\right)^{\frac{3}{2}} - \frac{\left(a + b x\right)^{\frac{5}{2}}}{5}\right)}{b^{3}} - \frac{2 a e^{2} \left(\frac{a^{4}}{\sqrt{a + b x}} + 4 a^{3} \sqrt{a + b x} - 2 a^{2} \left(a + b x\right)^{\frac{3}{2}} + \frac{4 a \left(a + b x\right)^{\frac{5}{2}}}{5} - \frac{\left(a + b x\right)^{\frac{7}{2}}}{7}\right)}{b^{4}} - 2 c^{2} \left(- \frac{a}{\sqrt{a + b x}} - \sqrt{a + b x}\right) - \frac{4 c d \left(\frac{a^{2}}{\sqrt{a + b x}} + 2 a \sqrt{a + b x} - \frac{\left(a + b x\right)^{\frac{3}{2}}}{3}\right)}{b} - \frac{4 c e \left(- \frac{a^{3}}{\sqrt{a + b x}} - 3 a^{2} \sqrt{a + b x} + a \left(a + b x\right)^{\frac{3}{2}} - \frac{\left(a + b x\right)^{\frac{5}{2}}}{5}\right)}{b^{2}} - \frac{2 d^{2} \left(- \frac{a^{3}}{\sqrt{a + b x}} - 3 a^{2} \sqrt{a + b x} + a \left(a + b x\right)^{\frac{3}{2}} - \frac{\left(a + b x\right)^{\frac{5}{2}}}{5}\right)}{b^{2}} - \frac{4 d e \left(\frac{a^{4}}{\sqrt{a + b x}} + 4 a^{3} \sqrt{a + b x} - 2 a^{2} \left(a + b x\right)^{\frac{3}{2}} + \frac{4 a \left(a + b x\right)^{\frac{5}{2}}}{5} - \frac{\left(a + b x\right)^{\frac{7}{2}}}{7}\right)}{b^{3}} - \frac{2 e^{2} \left(- \frac{a^{5}}{\sqrt{a + b x}} - 5 a^{4} \sqrt{a + b x} + \frac{10 a^{3} \left(a + b x\right)^{\frac{3}{2}}}{3} - 2 a^{2} \left(a + b x\right)^{\frac{5}{2}} + \frac{5 a \left(a + b x\right)^{\frac{7}{2}}}{7} - \frac{\left(a + b x\right)^{\frac{9}{2}}}{9}\right)}{b^{4}}}{b} & \text{for}\: b \neq 0 \\ \frac{c^{2} x + c d x^{2} + \frac{d e x^{4}}{2} + \frac{e^{2} x^{5}}{5} + \frac{x^{3} \left(2 c e + d^{2}\right)}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*c**2/sqrt(a + b*x) - 4*a*c*d*(-a/sqrt(a + b*x) - sqrt(a +
b*x))/b - 4*a*c*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2
)/3)/b**2 - 2*a*d**2*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(
3/2)/3)/b**2 - 4*a*d*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a +
 b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 - 2*a*e**2*(a**4/sqrt(a + b*x) + 4*
a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a
+ b*x)**(7/2)/7)/b**4 - 2*c**2*(-a/sqrt(a + b*x) - sqrt(a + b*x)) - 4*c*d*(
a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b - 4*c*e*(-a*
*3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(
5/2)/5)/b**2 - 2*d**2*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a +
b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 - 4*d*e*(a**4/sqrt(a + b*x) + 4*a**3
*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*
x)**(7/2)/7)/b**3 - 2*e**2*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10
*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7
 - (a + b*x)**(9/2)/9)/b**4)/b, Ne(b, 0)), ((c**2*x + c*d*x**2 + d*e*x**4/2
 + e**2*x**5/5 + x**3*(2*c*e + d**2)/3)/sqrt(a), True))
```

Giac [A]
time = 2.37, size = 237, normalized size = 1.47

$$\frac{2\left(315\sqrt{bx+a}\,c^{2}+\frac{210\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)cd}{b}+\frac{21\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^{2}\right)d^{2}}{b^{2}}+\frac{42\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^{2}\right)ce}{b^{2}}+\frac{18\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^{2}-35\sqrt{bx+a}\,a^{3}\right)de}{b^{3}}+\frac{\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^{2}-420(bx+a)^{\frac{3}{2}}a^{3}+315\sqrt{bx+a}\,a^{4}\right)e^{2}}{b^{4}}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(b*x + a)*c^2 + 210*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*c*
d/b + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*
d^2/b^2 + 42*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a
^2)*c*e/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(
3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x +
 a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(
b*x + a)*a^4)*e^2/b^4)/b
```

Mupad [B]
time = 4.76, size = 149, normalized size = 0.93

$$\frac{2\,e^2\,{\left(a+b\,x\right)}^{9/2}}{9\,b^5}+\frac{{\left(a+b\,x\right)}^{5/2}\,\left(12\,a^2\,e^2-12\,a\,b\,d\,e+2\,b^2\,d^2+4\,c\,b^2\,e\right)}{5\,b^5}+\frac{2\,\sqrt{a+b\,x}\,{\left(e\,a^2-d\,a\,b+c\,b^2\right)}^2}{b^5}-\frac{\left(8\,a\,e^2-4\,b\,d\,e\right)\,{\left(a+b\,x\right)}^{7/2}}{7\,b^5}-\frac{4\,\left(2\,a\,e-b\,d\right)\,{\left(a+b\,x\right)}^{3/2}\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)^2/(a + b*x)^(1/2),x)
```

```
[Out] (2*e^2*(a + b*x)^(9/2))/(9*b^5) + ((a + b*x)^(5/2)*(12*a^2*e^2 + 2*b^2*d^2
+ 4*b^2*c*e - 12*a*b*d*e))/(5*b^5) + (2*(a + b*x)^(1/2)*(b^2*c + a^2*e - a*
b*d)^2)/b^5 - ((8*a*e^2 - 4*b*d*e)*(a + b*x)^(7/2))/(7*b^5) - (4*(2*a*e - b
*d)*(a + b*x)^(3/2)*(b^2*c + a^2*e - a*b*d))/(3*b^5)
```

3.3 $\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=274

$$\frac{2(b^2c-abd+a^2e)^3\sqrt{a+bx}}{b^7}+\frac{2(bd-2ae)\left(b^2c-abd+a^2e\right)^2(a+bx)^{3/2}}{b^7}-\frac{6(b^2c-abd+a^2e)\left(5abde-5a\right.}{5b^7}$$

```
[Out] 2*(-2*a*e+b*d)*(a^2*e-a*b*d+b^2*c)^2*(b*x+a)^(3/2)/b^7-6/5*(a^2*e-a*b*d+b^2
*c)*(5*a*b*d*e-5*a^2*e^2-b^2*(c*e+d^2))*(b*x+a)^(5/2)/b^7-2/7*(-2*a*e+b*d)*
(10*a*b*d*e-10*a^2*e^2-b^2*(6*c*e+d^2))*(b*x+a)^(7/2)/b^7-2/3*e*(5*a*b*d*e-
5*a^2*e^2-b^2*(c*e+d^2))*(b*x+a)^(9/2)/b^7+6/11*e^2*(-2*a*e+b*d)*(b*x+a)^(1
1/2)/b^7+2/13*e^3*(b*x+a)^(13/2)/b^7+2*(a^2*e-a*b*d+b^2*c)^3*(b*x+a)^(1/2)/
b^7
```

Rubi [A]
time = 0.13, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{712\}$

$$-\frac{2e(a+bx)^{9/2}\left(-5a^2e^2+5abde-\left(b^2(ce+d^2)\right)\right)}{3b^7}-\frac{2(a+bx)^{7/2}(bd-2ae)\left(-10a^2e^2+10abde-\left(b^2(6ce+d^2)\right)\right)}{7b^7}-\frac{6(a+bx)^{5/2}\left(a^2e-abd+b^2c\right)\left(-5a^2e^2+5abde-\left(b^2(ce+d^2)\right)\right)}{5b^7}+\frac{2(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)^2}{b^7}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)^3}{b^7}+\frac{6e^2(a+bx)^{11/2}(bd-2ae)}{11b^7}+\frac{2e^3(a+bx)^{13/2}}{13b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^3*Sqrt[a + b*x])/b^7 + (2*(b*d - 2*a*e)*(b^2*c -
 a*b*d + a^2*e)^2*(a + b*x)^(3/2))/b^7 - (6*(b^2*c - a*b*d + a^2*e)*(5*a*b*
d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(5/2))/(5*b^7) - (2*(b*d - 2*a
*e)*(10*a*b*d*e - 10*a^2*e^2 - b^2*(d^2 + 6*c*e))*(a + b*x)^(7/2))/(7*b^7)
- (2*e*(5*a*b*d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(9/2))/(3*b^7) +
 (6*e^2*(b*d - 2*a*e)*(a + b*x)^(11/2))/(11*b^7) + (2*e^3*(a + b*x)^(13/2))
/(13*b^7)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx = \int \left(\frac{(b^2c-abd+a^2e)^3}{b^6\sqrt{a+bx}} + \frac{3(bd-2ae)\,(b^2c-abd+a^2e)^2\,\sqrt{a+bx}}{b^6} + \frac{3(b^2c-abd+\cdots}{\cdots}\right.$$

$$= \frac{2(b^2c-abd+a^2e)^3\,\sqrt{a+bx}}{b^7} + \frac{2(bd-2ae)\,(b^2c-abd+a^2e)^2\,(a+bx)^{3/2}}{b^7} - \frac{6(b^2c\cdots}{\cdots}$$

Mathematica [A]
time = 0.20, size = 355, normalized size = 1.30

$$\frac{2\sqrt{a+bx}\,(5120a^6e^3-1280a^5be^2(13d+2ex)+128a^4b^2e(143d^2+65dex+e(143c+15ex^2))-16a^3b^3(429d^3+572d^2ex+78de(33c+5ex^2)+4e^2x(143c+25ex^2))+8a^2b^4(3003c^2e+429c(7d^2+6dex+2e^2x^2)+x(429d^3+858d^2ex+650de^2x^2+175e^3x^3))+b^6(15015c^3+3003c^2x(5d+3ex)+143cx^2(63d^2+90dex+35e^2x^2)+5x^3(429d^3+1001d^2ex+819de^2x^2+231e^3x^3))-2ab^5(3003c^2(5d+2ex)+286cx(21d^2+27dex+10e^2x^2)+x^2(1287d^3+2860d^2ex+2275de^2x^2+630e^3x^3)))}{15015b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(5120*a^6*e^3 - 1280*a^5*b*e^2*(13*d + 2*e*x) + 128*a^4*b^
2*e*(143*d^2 + 65*d*e*x + e*(143*c + 15*e*x^2)) - 16*a^3*b^3*(429*d^3 + 572
*d^2*e*x + 78*d*e*(33*c + 5*e*x^2) + 4*e^2*x*(143*c + 25*e*x^2)) + 8*a^2*b^
4*(3003*c^2*e + 429*c*(7*d^2 + 6*d*e*x + 2*e^2*x^2) + x*(429*d^3 + 858*d^2*
e*x + 650*d*e^2*x^2 + 175*e^3*x^3)) + b^6*(15015*c^3 + 3003*c^2*x*(5*d + 3*
e*x) + 143*c*x^2*(63*d^2 + 90*d*e*x + 35*e^2*x^2) + 5*x^3*(429*d^3 + 1001*d
^2*e*x + 819*d*e^2*x^2 + 231*e^3*x^3)) - 2*a*b^5*(3003*c^2*(5*d + 2*e*x) +
286*c*x*(21*d^2 + 27*d*e*x + 10*e^2*x^2) + x^2*(1287*d^3 + 2860*d^2*e*x + 2
275*d*e^2*x^2 + 630*e^3*x^3))))/(15015*b^7)
```

Maple [A]
time = 0.42, size = 355, normalized size = 1.30 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^7*(1/13*e^3*(b*x+a)^(13/2)+3/11*(-2*a*e+b*d)*e^2*(b*x+a)^(11/2)+1/9*((a
^2*e-a*b*d+b^2*c)*e^2+2*(-2*a*e+b*d)^2*e+e*(2*(a^2*e-a*b*d+b^2*c)*e+(-2*a*e
+b*d)^2))*(b*x+a)^(9/2)+1/7*(4*(a^2*e-a*b*d+b^2*c)*(-2*a*e+b*d)*e+(-2*a*e+b
*d)*(2*(a^2*e-a*b*d+b^2*c)*e+(-2*a*e+b*d)^2))*(b*x+a)^(7/2)+1/5*((a^2*e-a*b
*d+b^2*c)*(2*(a^2*e-a*b*d+b^2*c)*e+(-2*a*e+b*d)^2)+2*(-2*a*e+b*d)^2*(a^2*e-
a*b*d+b^2*c)+e*(a^2*e-a*b*d+b^2*c)^2)*(b*x+a)^(5/2)+(a^2*e-a*b*d+b^2*c)^2*(
-2*a*e+b*d)*(b*x+a)^(3/2)+(a^2*e-a*b*d+b^2*c)^3*(b*x+a)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(258) = 516.
time = 0.31, size = 525, normalized size = 1.92

$$\frac{2\left(15015\sqrt{bx+a}\,c^3+3003c^2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)d}{b}+\frac{\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)e}{b^2}\right)+143c\left(\frac{21\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)d^2}{b^2}+\frac{18\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)de}{b^3}+\frac{\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)e^2}{b^4}\right)+\frac{429\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)d^3}{b^3}+\frac{143\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)d^2e}{b^4}+\frac{65\left(63(bx+a)^{\frac{11}{2}}-385(bx+a)^{\frac{9}{2}}a+990(bx+a)^{\frac{7}{2}}a^2-1386(bx+a)^{\frac{5}{2}}a^3+1155(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)de^2}{b^5}+\frac{5\left(231(bx+a)^{\frac{13}{2}}-1638(bx+a)^{\frac{11}{2}}a+5005(bx+a)^{\frac{9}{2}}a^2-8580(bx+a)^{\frac{7}{2}}a^3+9009(bx+a)^{\frac{5}{2}}a^4-6006(bx+a)^{\frac{3}{2}}a^5+3003\sqrt{bx+a}\,a^6\right)e^3}{b^6}\right)}{15015\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(15015*sqrt(b*x + a)*c^3 + 3003*c^2*(5*((b*x + a)^(3/2) - 3*sqrt(b*
x + a)*a)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a
)*a^2)*e/b^2) + 143*c*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sq
rt(b*x + a)*a^2)*d^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 3
5*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2)
 - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^
3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4) + 429*(5*(b*x + a)^(7/2) - 21*(b*x + a)
^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 143*(35
*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b
*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b^4 + 65*(63*(b*x + a)^(11
/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2
)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2/b^5 + 5*(23
1*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8
580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a
^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6)/b
```

Fricas [A]
time = 0.40, size = 457, normalized size = 1.67

$$\frac{2\left(1155\,b^6e^3x^6+15015\,b^6c^3-30030\,ab^5c^2d+24024\,a^2b^4cd^2-6864\,a^3b^3d^3+5120\,a^6e^3+315\left(13\,b^6de^2-4\,ab^5e^3\right)x^5+35\left(143\,b^6d^2e+40\,a^2b^4e^3+13\left(11\,b^6c-10\,ab^5d\right)e^2\right)x^4+5\left(429\,b^6d^3-320\,a^3b^3e^3-104\left(11\,ab^5c-10\,a^2b^4d\right)e^2+286\left(9\,b^6cd-4\,ab^5d^2\right)e\right)x^3+1664\left(11\,a^4b^2c-10\,a^5bd\right)e^2+3\left(3003\,b^6cd^2-858\,ab^5d^3+640\,a^4b^2e^3+208\left(11\,a^2b^4c-10\,a^3b^3d\right)e^2+143\left(21\,b^6c^2-36\,ab^5cd+16\,a^2b^4d^2\right)e\right)x^2+1144\left(21\,a^2b^4c^2-36\,a^3b^3cd+16\,a^4b^2d^2\right)e+\left(15015\,b^6c^2d-12012\,ab^5cd^2+3432\,a^2b^4d^3-2560\,a^5be^3-832\left(11\,a^3b^3c-10\,a^4b^2d\right)e^2-572\left(21\,ab^5c^2-36\,a^2b^4cd+16\,a^3b^3d^2\right)e\right)x\right)\sqrt{bx+a}}{15015\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^6*e^3*x^6 + 15015*b^6*c^3 - 30030*a*b^5*c^2*d + 24024*a^2*b
^4*c*d^2 - 6864*a^3*b^3*d^3 + 5120*a^6*e^3 + 315*(13*b^6*d*e^2 - 4*a*b^5*e^
3)*x^5 + 35*(143*b^6*d^2*e + 40*a^2*b^4*e^3 + 13*(11*b^6*c - 10*a*b^5*d)*e^
2)*x^4 + 5*(429*b^6*d^3 - 320*a^3*b^3*e^3 - 104*(11*a*b^5*c - 10*a^2*b^4*d)
*e^2 + 286*(9*b^6*c*d - 4*a*b^5*d^2)*e)*x^3 + 1664*(11*a^4*b^2*c - 10*a^5*b
*d)*e^2 + 3*(3003*b^6*c*d^2 - 858*a*b^5*d^3 + 640*a^4*b^2*e^3 + 208*(11*a^2
*b^4*c - 10*a^3*b^3*d)*e^2 + 143*(21*b^6*c^2 - 36*a*b^5*c*d + 16*a^2*b^4*d^
2)*e)*x^2 + 1144*(21*a^2*b^4*c^2 - 36*a^3*b^3*c*d + 16*a^4*b^2*d^2)*e + (15
015*b^6*c^2*d - 12012*a*b^5*c*d^2 + 3432*a^2*b^4*d^3 - 2560*a^5*b*e^3 - 832
*(11*a^3*b^3*c - 10*a^4*b^2*d)*e^2 - 572*(21*a*b^5*c^2 - 36*a^2*b^4*c*d + 1
6*a^3*b^3*d^2)*e)*x)*sqrt(b*x + a)/b^7
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1406 vs. $2(277) = 554$.
time = 74.55, size = 1406, normalized size = 5.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
[Out] Piecewise(((-2*a*c**3/sqrt(a + b*x) - 6*a*c**2*d*(-a/sqrt(a + b*x) - sqrt(a
 + b*x))/b - 6*a*c**2*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)
**(3/2)/3)/b**2 - 6*a*c*d**2*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a +
 b*x)**(3/2)/3)/b**2 - 12*a*c*d*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*
x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 - 2*a*d**3*(-a**3/sqrt(a
 + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b
**3 - 6*a*c*e**2*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b
*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 - 6*a*d**2*e
*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a
*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 - 6*a*d*e**2*(-a**5/sqrt(a +
 b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x
)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**5 - 2*a*e**3*(a*
*6/sqrt(a + b*x) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*
(a + b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a
 + b*x)**(11/2)/11)/b**6 - 2*c**3*(-a/sqrt(a + b*x) - sqrt(a + b*x)) - 6*c*
*2*d*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b - 6*c*
*2*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a
+ b*x)**(5/2)/5)/b**2 - 6*c*d**2*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x
) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 - 12*c*d*e*(a**4/sqrt(a +
 b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/
2)/5 - (a + b*x)**(7/2)/7)/b**3 - 2*d**3*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(
a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7
/2)/7)/b**3 - 6*c*e**2*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**
3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (
a + b*x)**(9/2)/9)/b**4 - 6*d**2*e*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b
*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)*
*(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 - 6*d*e**2*(a**6/sqrt(a + b*x) + 6*a**5
*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 15*a**
2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x)**(11/2)/11)/b**5
- 2*e**3*(-a**7/sqrt(a + b*x) - 7*a**6*sqrt(a + b*x) + 7*a**5*(a + b*x)**(3
/2) - 7*a**4*(a + b*x)**(5/2) + 5*a**3*(a + b*x)**(7/2) - 7*a**2*(a + b*x)*
*(9/2)/3 + 7*a*(a + b*x)**(11/2)/11 - (a + b*x)**(13/2)/13)/b**6)/b, Ne(b,
0)), ((c**3*x + 3*c**2*d*x**2/2 + d*e**2*x**6/2 + e**3*x**7/7 + x**5*(3*c*e
**2 + 3*d**2*e)/5 + x**4*(6*c*d*e + d**3)/4 + x**3*(3*c**2*e + 3*c*d**2)/3)
/sqrt(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(258) = 516.
time = 1.98, size = 526, normalized size = 1.92

$$\frac{2\left(15015\sqrt{bx+a}\,c^3 + \frac{15015\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}\,a\right)c^2d}{b} + \frac{3003\left(3(bx+a)^{\frac{5}{2}} - 10(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+a}\,a^2\right)cd^2}{b^2} + \frac{3003\left(3(bx+a)^{\frac{5}{2}} - 10(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+a}\,a^2\right)c^2e}{b^2} + \frac{429\left(5(bx+a)^{\frac{7}{2}} - 21(bx+a)^{\frac{5}{2}}a + 35(bx+a)^{\frac{3}{2}}a^2 - 35\sqrt{bx+a}\,a^3\right)d^3}{b^3} + \frac{2574\left(5(bx+a)^{\frac{7}{2}} - 21(bx+a)^{\frac{5}{2}}a + 35(bx+a)^{\frac{3}{2}}a^2 - 35\sqrt{bx+a}\,a^3\right)cde}{b^3} + \frac{143\left(35(bx+a)^{\frac{9}{2}} - 180(bx+a)^{\frac{7}{2}}a + 378(bx+a)^{\frac{5}{2}}a^2 - 420(bx+a)^{\frac{3}{2}}a^3 + 315\sqrt{bx+a}\,a^4\right)d^2e}{b^4} + \frac{143\left(35(bx+a)^{\frac{9}{2}} - 180(bx+a)^{\frac{7}{2}}a + 378(bx+a)^{\frac{5}{2}}a^2 - 420(bx+a)^{\frac{3}{2}}a^3 + 315\sqrt{bx+a}\,a^4\right)ce^2}{b^4} + \frac{65\left(63(bx+a)^{\frac{11}{2}} - 385(bx+a)^{\frac{9}{2}}a + 990(bx+a)^{\frac{7}{2}}a^2 - 1386(bx+a)^{\frac{5}{2}}a^3 + 1155(bx+a)^{\frac{3}{2}}a^4 - 693\sqrt{bx+a}\,a^5\right)de^2}{b^5} + \frac{5\left(231(bx+a)^{\frac{13}{2}} - 1638(bx+a)^{\frac{11}{2}}a + 5005(bx+a)^{\frac{9}{2}}a^2 - 8580(bx+a)^{\frac{7}{2}}a^3 + 9009(bx+a)^{\frac{5}{2}}a^4 - 6006(bx+a)^{\frac{3}{2}}a^5 + 3003\sqrt{bx+a}\,a^6\right)e^3}{b^6}\right)}{15015\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15015*(15015*sqrt(b*x + a)*c^3 + 15015*((b*x + a)^(3/2) - 3*sqrt(b*x + a)
*a)*c^2*d/b + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x
+ a)*a^2)*c*d^2/b^2 + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*s
qrt(b*x + a)*a^2)*c^2*e/b^2 + 429*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a
 + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 2574*(5*(b*x +
a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)
*a^3)*c*d*e/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*
x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b
^4 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*
a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*e^2/b^4 + 65*(63*(
b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b
*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2
/b^5 + 5*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(
9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x
+ a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6)/b
```

Mupad [B]
time = 0.10, size = 299, normalized size = 1.09

$$\frac{2\,e^3\,{\left(a+b\,x\right)}^{13/2}}{13\,b^7}-\frac{\left(12\,a\,e^3-6\,b\,d\,e^2\right)\,{\left(a+b\,x\right)}^{11/2}}{11\,b^7}+\frac{{\left(a+b\,x\right)}^{9/2}\,\left(30\,a^2\,e^3-30\,a\,b\,d\,e^2+6\,b^2\,d^2\,e+6\,c\,b^2\,e^2\right)}{9\,b^7}+\frac{2\,\sqrt{a+b\,x}\,{\left(e\,a^2-d\,a\,b+c\,b^2\right)}^3}{b^7}+\frac{{\left(a+b\,x\right)}^{5/2}\,\left(30\,a^4\,e^3-60\,a^3\,b\,d\,e^2+36\,a^2\,b^2\,c\,e^2+36\,a^2\,b^2\,d^2\,e-36\,a\,b^3\,c\,d\,e-6\,a\,b^3\,d^3+6\,b^4\,c^2\,e+6\,b^4\,c\,d^2\right)}{5\,b^7}-\frac{2\,\left(2\,a\,e-b\,d\right)\,{\left(a+b\,x\right)}^{7/2}\,\left(10\,a^2\,e^2-10\,a\,b\,d\,e+b^2\,d^2+6\,c\,b^2\,e\right)}{7\,b^7}-\frac{2\,\left(2\,a\,e-b\,d\right)\,{\left(a+b\,x\right)}^{3/2}\,{\left(e\,a^2-d\,a\,b+c\,b^2\right)}^2}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)^3/(a + b*x)^(1/2),x)
```

```
[Out] (2*e^3*(a + b*x)^(13/2))/(13*b^7) - ((12*a*e^3 - 6*b*d*e^2)*(a + b*x)^(11/2
))/(11*b^7) + ((a + b*x)^(9/2)*(30*a^2*e^3 + 6*b^2*c*e^2 + 6*b^2*d^2*e - 30
*a*b*d*e^2))/(9*b^7) + (2*(a + b*x)^(1/2)*(b^2*c + a^2*e - a*b*d)^3)/b^7 +
((a + b*x)^(5/2)*(30*a^4*e^3 - 6*a*b^3*d^3 + 6*b^4*c*d^2 + 6*b^4*c^2*e + 36
*a^2*b^2*c*e^2 + 36*a^2*b^2*d^2*e - 60*a^3*b*d*e^2 - 36*a*b^3*c*d*e))/(5*b^
7) - (2*(2*a*e - b*d)*(a + b*x)^(7/2)*(10*a^2*e^2 + b^2*d^2 + 6*b^2*c*e - 1
0*a*b*d*e))/(7*b^7) - (2*(2*a*e - b*d)*(a + b*x)^(3/2)*(b^2*c + a^2*e - a*b
*d)^2)/b^7
```

3.4 $\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}} \, dx$

Optimal. Leaf size=114

$$\frac{2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx}}{b^4}+\frac{2(b^2d-2abe+3a^2f)(a+bx)^{3/2}}{3b^4}+\frac{2(be-3af)(a+bx)^{5/2}}{5b^4}+\frac{2f(a+b\ldots}{7b^4}$$

```
[Out] 2/3*(3*a^2*f-2*a*b*e+b^2*d)*(b*x+a)^(3/2)/b^4+2/5*(-3*a*f+b*e)*(b*x+a)^(5/2
)/b^4+2/7*f*(b*x+a)^(7/2)/b^4+2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x+a)^(1/2
)/b^4
```

Rubi [A]
time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1864}

$$\frac{2(a+bx)^{3/2}(3a^2f-2abe+b^2d)}{3b^4}+\frac{2\sqrt{a+bx}\,(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4}+\frac{2(a+bx)^{5/2}(be-3af)}{5b^4}+\frac{2f(a+bx)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x])/b^4 + (2*(b^2*d - 2*a
*b*e + 3*a^2*f)*(a + b*x)^(3/2))/(3*b^4) + (2*(b*e - 3*a*f)*(a + b*x)^(5/2)
)/(5*b^4) + (2*f*(a + b*x)^(7/2))/(7*b^4)
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx = \int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}}+\frac{(b^2d-2abe+3a^2f)\sqrt{a+bx}}{b^3}+\frac{(be-3af)(a\ldots}{b^3}\right.$$
$$= \frac{2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx}}{b^4}+\frac{2(b^2d-2abe+3a^2f)(a+bx)^{3/2}}{3b^4}+\frac{2(b\ldots}{}$$

Mathematica [A]
time = 0.07, size = 82, normalized size = 0.72

$$\frac{2\sqrt{a+bx}\,(-48a^3f+8a^2b(7e+3fx)-2ab^2(35d+x(14e+9fx))+b^3(105c+x(35d+3x(7e+5fx))))}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x) - 2*a*b^2*(35*d + x*(14
*e + 9*f*x)) + b^3*(105*c + x*(35*d + 3*x*(7*e + 5*f*x)))))/(105*b^4)
```

Maple [A]
time = 0.31, size = 128, normalized size = 1.12

method	result
gosper	$-\frac{2\sqrt{bx+a}\,(-15f\,x^3b^3+18a\,b^2f\,x^2-21b^3e\,x^2-24a^2bfx+28a\,b^2ex-35b^3dx+48a^3f-56a^2be+70a\,b^2d-105b^3c)}{105b^4}$
trager	$-\frac{2\sqrt{bx+a}\,(-15f\,x^3b^3+18a\,b^2f\,x^2-21b^3e\,x^2-24a^2bfx+28a\,b^2ex-35b^3dx+48a^3f-56a^2be+70a\,b^2d-105b^3c)}{105b^4}$
risch	$-\frac{2\sqrt{bx+a}\,(-15f\,x^3b^3+18a\,b^2f\,x^2-21b^3e\,x^2-24a^2bfx+28a\,b^2ex-35b^3dx+48a^3f-56a^2be+70a\,b^2d-105b^3c)}{105b^4}$
derivativedivides	$\frac{\frac{2f(bx+a)^{\frac{7}{2}}}{7}-\frac{6af(bx+a)^{\frac{5}{2}}}{5}+\frac{2be(bx+a)^{\frac{5}{2}}}{5}+2a^2f(bx+a)^{\frac{3}{2}}-\frac{4abe(bx+a)^{\frac{3}{2}}}{3}+\frac{2b^2d(bx+a)^{\frac{3}{2}}}{3}-2a^3f\sqrt{bx+a}+2a^2be\sqrt{bx-}}{b^4}$
default	$\frac{\frac{2f(bx+a)^{\frac{7}{2}}}{7}-\frac{6af(bx+a)^{\frac{5}{2}}}{5}+\frac{2be(bx+a)^{\frac{5}{2}}}{5}+2a^2f(bx+a)^{\frac{3}{2}}-\frac{4abe(bx+a)^{\frac{3}{2}}}{3}+\frac{2b^2d(bx+a)^{\frac{3}{2}}}{3}-2a^3f\sqrt{bx+a}+2a^2be\sqrt{bx-}}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/7*f*(b*x+a)^(7/2)-3/5*a*f*(b*x+a)^(5/2)+1/5*b*e*(b*x+a)^(5/2)+a^2*
f*(b*x+a)^(3/2)-2/3*a*b*e*(b*x+a)^(3/2)+1/3*b^2*d*(b*x+a)^(3/2)-a^3*f*(b*x+
a)^(1/2)+a^2*b*e*(b*x+a)^(1/2)-a*b^2*d*(b*x+a)^(1/2)+b^3*c*(b*x+a)^(1/2))
```

Maxima [A]
time = 0.51, size = 129, normalized size = 1.13

$$\frac{2\left(105\sqrt{bx+a}\,c+\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)d}{b}+\frac{7\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)e}{b^2}+\frac{3\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)f}{b^3}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b +
 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2
+ 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35
*sqrt(b*x + a)*a^3)*f/b^3)/b
```

Fricas [A]
time = 0.36, size = 90, normalized size = 0.79

$$\frac{2\,(15\,b^3fx^3+105\,b^3c-70\,ab^2d+56\,a^2be-48\,a^3f+3\,(7\,b^3e-6\,ab^2f)x^2+(35\,b^3d-28\,ab^2e+24\,a^2bf)x)\sqrt{bx+a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*f*x^3 + 105*b^3*c - 70*a*b^2*d + 56*a^2*b*e - 48*a^3*f + 3*(7
*b^3*e - 6*a*b^2*f)*x^2 + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x)*sqrt(b*x
+ a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(119) = 238$.
time = 8.81, size = 354, normalized size = 3.11

$$\begin{cases} \frac{-\frac{2ac}{\sqrt{a+bx}} - \frac{2ad\left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx}\right)}{b} - \frac{2ae\left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b^2} - \frac{2af\left(-\frac{a^3}{\sqrt{a+bx}} - 3a^2\sqrt{a+bx} + a(a+bx)^{\frac{3}{2}} - \frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^3} - 2c\left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx}\right) - \frac{2d\left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b} - \frac{2e\left(-\frac{a^3}{\sqrt{a+bx}} - 3a^2\sqrt{a+bx} + a(a+bx)^{\frac{3}{2}} - \frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2} - \frac{2f\left(\frac{a^4}{\sqrt{a+bx}} + 4a^3\sqrt{a+bx} - 2a^2(a+bx)^{\frac{3}{2}} + \frac{4a(a+bx)^{\frac{5}{2}}}{5} - \frac{(a+bx)^{\frac{7}{2}}}{7}\right)}{b^3}}{b} & \text{for } b \neq 0 \\ \frac{cx + \frac{dx^2}{2} + \frac{ex^3}{3} + \frac{fx^4}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*c/sqrt(a + b*x) - 2*a*d*(-a/sqrt(a + b*x) - sqrt(a + b*x))
/b - 2*a*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b*
*2 - 2*a*f*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2)
 - (a + b*x)**(5/2)/5)/b**3 - 2*c*(-a/sqrt(a + b*x) - sqrt(a + b*x)) - 2*d*
(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b - 2*e*(-a**
3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5
/2)/5)/b**2 - 2*f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a +
b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3)/b, Ne(b, 0
)), ((c*x + d*x**2/2 + e*x**3/3 + f*x**4/4)/sqrt(a), True))
```

Giac [A]
time = 2.39, size = 129, normalized size = 1.13

$$\frac{2\left(105\sqrt{bx+a}\,c + \frac{35\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}\,a\right)d}{b} + \frac{7\left(3(bx+a)^{\frac{5}{2}} - 10(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+a}\,a^2\right)e}{b^2} + \frac{3\left(5(bx+a)^{\frac{7}{2}} - 21(bx+a)^{\frac{5}{2}}a + 35(bx+a)^{\frac{3}{2}}a^2 - 35\sqrt{bx+a}\,a^3\right)f}{b^3}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b +
 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2
+ 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35
*sqrt(b*x + a)*a^3)*f/b^3)/b
```

Mupad [B]
time = 4.81, size = 103, normalized size = 0.90

$$\frac{(a+b\,x)^{3/2}\,\left(6\,f\,a^2-4\,e\,a\,b+2\,d\,b^2\right)}{3\,b^4}-\frac{\left(6\,a\,f-2\,b\,e\right)\,{\left(a+b\,x\right)}^{5/2}}{5\,b^4}+\frac{\sqrt{a+b\,x}\,\left(-2\,f\,a^3+2\,e\,a^2\,b-2\,d\,a\,b^2+2\,c\,b^3\right)}{b^4}+\frac{2\,f\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x)^(1/2),x)
```

```
[Out] ((a + b*x)^(3/2)*(2*b^2*d + 6*a^2*f - 4*a*b*e))/(3*b^4) - ((6*a*f - 2*b*e)*
(a + b*x)^(5/2))/(5*b^4) + ((a + b*x)^(1/2)*(2*b^3*c - 2*a^3*f - 2*a*b^2*d
+ 2*a^2*b*e))/b^4 + (2*f*(a + b*x)^(7/2))/(7*b^4)
```

3.5 $\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=320

$$\frac{2(b^3c-ab^2d+a^2be-a^3f)^2\sqrt{a+bx}}{b^7}+\frac{4(b^2d-2abe+3a^2f)(b^3c-ab^2d+a^2be-a^3f)(a+bx)^{3/2}}{3b^7}+\frac{2(b^4(d^2$$

```
[Out] 4/3*(3*a^2*f-2*a*b*e+b^2*d)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x+a)^(3/2)/b^
7+2/5*(b^4*(2*c*e+d^2)-20*a^3*b*e*f+15*a^4*f^2-6*a*b^3*(c*f+d*e)+6*a^2*b^2*
(2*d*f+e^2))*(b*x+a)^(5/2)/b^7+4/7*(10*a^2*b*e*f-10*a^3*f^2+b^3*(c*f+d*e)-2
*a*b^2*(2*d*f+e^2))*(b*x+a)^(7/2)/b^7-2/9*(10*a*b*e*f-15*a^2*f^2-b^2*(2*d*f
+e^2))*(b*x+a)^(9/2)/b^7+4/11*f*(-3*a*f+b*e)*(b*x+a)^(11/2)/b^7+2/13*f^2*(b
*x+a)^(13/2)/b^7+2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)^2*(b*x+a)^(1/2)/b^7
```

Rubi [A]
time = 0.16, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = $\{1864\}$

$$-\frac{2(a+bx)^{9/2}\left(-15a^2f^2+10abef-\left(b^2(2df+e^2)\right)\right)}{9b^7}+\frac{4(a+bx)^{7/2}\left(-10a^3f^2+10a^2bef-2ab^2(2df+e^2)+b^3(cf+de)\right)}{7b^7}+\frac{4(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^7}+\frac{2\sqrt{a+bx}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)^2}{b^7}+\frac{2(a+bx)^{5/2}\left(15a^4f^2-20a^3bef+6a^2b^2(2df+e^2)-6ab^3(cf+de)+b^4(2ce+d^2)\right)}{5b^7}+\frac{4f(a+bx)^{11/2}(be-3af)}{11b^7}+\frac{2f^2(a+bx)^{13/2}}{13b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*Sqrt[a + b*x])/b^7 + (4*(b^2*d - 2
*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a + b*x)^(3/2))/(3*b
^7) + (2*(b^4*(d^2 + 2*c*e) - 20*a^3*b*e*f + 15*a^4*f^2 - 6*a*b^3*(d*e + c*
f) + 6*a^2*b^2*(e^2 + 2*d*f))*(a + b*x)^(5/2))/(5*b^7) + (4*(10*a^2*b*e*f -
 10*a^3*f^2 + b^3*(d*e + c*f) - 2*a*b^2*(e^2 + 2*d*f))*(a + b*x)^(7/2))/(7*
b^7) - (2*(10*a*b*e*f - 15*a^2*f^2 - b^2*(e^2 + 2*d*f))*(a + b*x)^(9/2))/(9
*b^7) + (4*f*(b*e - 3*a*f)*(a + b*x)^(11/2))/(11*b^7) + (2*f^2*(a + b*x)^(1
3/2))/(13*b^7)
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx = \int \left(\frac{(b^3c-ab^2d+a^2be-a^3f)^2}{b^6\sqrt{a+bx}}+\frac{2(b^2d-2abe+3a^2f)(b^3c-ab^2d+a^2be-}{b^6}\right.$$

$$= \frac{2(b^3c-ab^2d+a^2be-a^3f)^2\sqrt{a+bx}}{b^7}+\frac{4(b^2d-2abe+3a^2f)(b^3c-ab^2d+}{3b^7}$$

Mathematica [**A**]
time = 0.23, size = 320, normalized size = 1.00

$$\frac{2\sqrt{a+bx}\left(15360a^6f^2-2560a^5bf(13e+3fx)+128a^4b^2\left(143e^2+130efx+f\left(286d+45fx^2\right)\right)-32a^3b^3\left(1287cf+143d(9e+4fx)+2x\left(143e^2+195efx+75f^2x^2\right)\right)+8a^2b^4\left(3003d^2+858dx(3e+2fx)+858c(7e+3fx)+x^2\left(858e^2+1300efx+525f^2x^2\right)\right)+b^6\left(45045c^2+858cx(35d+3x(7e+5fx))+x^2\left(9009d^2+1430dx(9e+7fx)+35x^2\left(143e^2+234efx+99f^2x^2\right)\right)\right)-4ab^5\left(429c(35d+x(14e+9fx))+x\left(3003d^2+143dx(27e+20fx)+5x^2\left(286e^2+455efx+189f^2x^2\right)\right)\right)\right)}{45045b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(15360*a^6*f^2 - 2560*a^5*b*f*(13*e + 3*f*x) + 128*a^4*b^2
*(143*e^2 + 130*e*f*x + f*(286*d + 45*f*x^2)) - 32*a^3*b^3*(1287*c*f + 143*
d*(9*e + 4*f*x) + 2*x*(143*e^2 + 195*e*f*x + 75*f^2*x^2)) + 8*a^2*b^4*(3003
*d^2 + 858*d*x*(3*e + 2*f*x) + 858*c*(7*e + 3*f*x) + x^2*(858*e^2 + 1300*e*
f*x + 525*f^2*x^2)) + b^6*(45045*c^2 + 858*c*x*(35*d + 3*x*(7*e + 5*f*x)) +
 x^2*(9009*d^2 + 1430*d*x*(9*e + 7*f*x) + 35*x^2*(143*e^2 + 234*e*f*x + 99*
f^2*x^2))) - 4*a*b^5*(429*c*(35*d + x*(14*e + 9*f*x)) + x*(3003*d^2 + 143*d
*x*(27*e + 20*f*x) + 5*x^2*(286*e^2 + 455*e*f*x + 189*f^2*x^2)))))/(45045*b
^7)
```

Maple [**A**]
time = 0.30, size = 292, normalized size = 0.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^7*(1/13*f^2*(b*x+a)^(13/2)+2/11*(-3*a*f+b*e)*f*(b*x+a)^(11/2)+1/9*(2*(3
*a^2*f-2*a*b*e+b^2*d)*f+(-3*a*f+b*e)^2)*(b*x+a)^(9/2)+1/7*(2*(-a^3*f+a^2*b*
e-a*b^2*d+b^3*c)*f+2*(3*a^2*f-2*a*b*e+b^2*d)*(-3*a*f+b*e))*(b*x+a)^(7/2)+1/
5*(2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(-3*a*f+b*e)+(3*a^2*f-2*a*b*e+b^2*d)^2)
*(b*x+a)^(5/2)+2/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(3*a^2*f-2*a*b*e+b^2*d)*(
b*x+a)^(3/2)+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)^2*(b*x+a)^(1/2))
```

Maxima [**A**]
time = 0.31, size = 507, normalized size = 1.58

$$\frac{2\left(45045\sqrt{bx+a}\,c^2+858\,c\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)d}{b}+\frac{7\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)e}{b^2}+\frac{3\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)f}{b^3}\right)+\frac{3003\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)d^2}{b^2}+\frac{286\left(35(bx+a)^{\frac{9}{2}}f-45(4af-be)(bx+a)^{\frac{7}{2}}+189(2a^2f-abe)(bx+a)^{\frac{5}{2}}-105(4a^3f-3a^2be)(bx+a)^{\frac{3}{2}}+315(a^4f-a^3be)\sqrt{bx+a}\right)d}{b^4}+\frac{143\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)e^2}{b^4}+\frac{130\left(63(bx+a)^{\frac{11}{2}}-385(bx+a)^{\frac{9}{2}}a+990(bx+a)^{\frac{7}{2}}a^2-1386(bx+a)^{\frac{5}{2}}a^3+1155(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)fe}{b^5}+\frac{15\left(231(bx+a)^{\frac{13}{2}}-1638(bx+a)^{\frac{11}{2}}a+5005(bx+a)^{\frac{9}{2}}a^2-8580(bx+a)^{\frac{7}{2}}a^3+9009(bx+a)^{\frac{5}{2}}a^4-6006(bx+a)^{\frac{3}{2}}a^5+3003\sqrt{bx+a}\,a^6\right)f^2}{b^6}\right)}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(45045*sqrt(b*x + a)*c^2 + 858*c*(35*((b*x + a)^(3/2) - 3*sqrt(b*x
+ a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a
)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(
3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3) + 3003*(3*(b*x + a)^(5/2) - 10*(b*x
 + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 286*(35*(b*x + a)^(9/2)*f -
 45*(4*a*f - b*e)*(b*x + a)^(7/2) + 189*(2*a^2*f - a*b*e)*(b*x + a)^(5/2) -
 105*(4*a^3*f - 3*a^2*b*e)*(b*x + a)^(3/2) + 315*(a^4*f - a^3*b*e)*sqrt(b*x
```

```
 + a))*d/b^4 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x +
 a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4 +
130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2
- 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a
^5)*f*e/b^5 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*
x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 60
06*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6)/b
```

Fricas [A]
time = 0.37, size = 417, normalized size = 1.30

$$\frac{2\left(3465 b^6 f^2 x^6 + 45045 b^6 c^2 - 60060 a b^5 c d + 24024 a^2 b^4 d^2 + 18304 a^4 b^2 e^2 + 15360 a^6 f^2 + 630 \left(13 b^6 e f - 6 a b^5 f^2\right) x^5 + 35 \left(143 b^6 e^2 + 120 a^2 b^4 f^2 + 26 \left(11 b^6 d - 10 a b^5 e\right) f\right) x^4 + 10 \left(1287 b^6 d e - 572 a b^5 e^2 - 480 a^3 b^3 f^2 + 13 \left(99 b^6 c - 88 a b^5 d + 80 a^2 b^4 e\right) f\right) x^3 + 3 \left(3003 b^6 d^2 + 2288 a^2 b^4 e^2 + 1920 a^4 b^2 f^2 + 858 \left(7 b^6 c - 6 a b^5 d\right) e - 52 \left(99 a b^5 c - 88 a^2 b^4 d + 80 a^3 b^3 e\right) f\right) x^2 + 6864 \left(7 a^2 b^4 c - 6 a^3 b^3 d\right) e - 416 \left(99 a^3 b^3 c - 88 a^4 b^2 d + 80 a^5 b e\right) f + 2 \left(15015 b^6 c d - 6006 a b^5 d^2 - 4576 a^3 b^3 e^2 - 3840 a^5 b f^2 - 1716 \left(7 a b^5 c - 6 a^2 b^4 d\right) e + 104 \left(99 a^2 b^4 c - 88 a^3 b^3 d + 80 a^4 b^2 e\right) f\right) x\right) \sqrt{b x + a}}{45045 b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3465*b^6*f^2*x^6 + 45045*b^6*c^2 - 60060*a*b^5*c*d + 24024*a^2*b^4
*d^2 + 18304*a^4*b^2*e^2 + 15360*a^6*f^2 + 630*(13*b^6*e*f - 6*a*b^5*f^2)*x
^5 + 35*(143*b^6*e^2 + 120*a^2*b^4*f^2 + 26*(11*b^6*d - 10*a*b^5*e)*f)*x^4
+ 10*(1287*b^6*d*e - 572*a*b^5*e^2 - 480*a^3*b^3*f^2 + 13*(99*b^6*c - 88*a*
b^5*d + 80*a^2*b^4*e)*f)*x^3 + 3*(3003*b^6*d^2 + 2288*a^2*b^4*e^2 + 1920*a^
4*b^2*f^2 + 858*(7*b^6*c - 6*a*b^5*d)*e - 52*(99*a*b^5*c - 88*a^2*b^4*d + 8
0*a^3*b^3*e)*f)*x^2 + 6864*(7*a^2*b^4*c - 6*a^3*b^3*d)*e - 416*(99*a^3*b^3*
c - 88*a^4*b^2*d + 80*a^5*b*e)*f + 2*(15015*b^6*c*d - 6006*a*b^5*d^2 - 4576
*a^3*b^3*e^2 - 3840*a^5*b*f^2 - 1716*(7*a*b^5*c - 6*a^2*b^4*d)*e + 104*(99*
a^2*b^4*c - 88*a^3*b^3*d + 80*a^4*b^2*e)*f)*x)*sqrt(b*x + a)/b^7
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. $2(333) = 666$.
time = 76.33, size = 1365, normalized size = 4.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*c**2/sqrt(a + b*x) - 4*a*c*d*(-a/sqrt(a + b*x) - sqrt(a +
b*x))/b - 4*a*c*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2
)/3)/b**2 - 2*a*d**2*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(
3/2)/3)/b**2 - 4*a*c*f*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a +
 b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 - 4*a*d*e*(-a**3/sqrt(a + b*x) - 3*
a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 - 4*a*d*
f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*
a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 - 2*a*e**2*(a**4/sqrt(a + b
*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)
/5 - (a + b*x)**(7/2)/7)/b**4 - 4*a*e*f*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(
```

```
a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a +
b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**5 - 2*a*f**2*(a**6/sqrt(a + b*x) + 6
*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 1
5*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x)**(11/2)/11)/
b**6 - 2*c**2*(-a/sqrt(a + b*x) - sqrt(a + b*x)) - 4*c*d*(a**2/sqrt(a + b*x
) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b - 4*c*e*(-a**3/sqrt(a + b*x)
- 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 - 2*
d**2*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a
+ b*x)**(5/2)/5)/b**2 - 4*c*f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) -
2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3
 - 4*d*e*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/
2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3 - 4*d*f*(-a**5/sqrt(
a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a +
b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 - 2*e**2*(-
a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*
a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 -
 4*e*f*(a**6/sqrt(a + b*x) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2)
 + 4*a**3*(a + b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9
/2)/3 - (a + b*x)**(11/2)/11)/b**5 - 2*f**2*(-a**7/sqrt(a + b*x) - 7*a**6*s
qrt(a + b*x) + 7*a**5*(a + b*x)**(3/2) - 7*a**4*(a + b*x)**(5/2) + 5*a**3*(
a + b*x)**(7/2) - 7*a**2*(a + b*x)**(9/2)/3 + 7*a*(a + b*x)**(11/2)/11 - (a
 + b*x)**(13/2)/13)/b**6)/b, Ne(b, 0)), ((c**2*x + c*d*x**2 + e*f*x**6/3 +
f**2*x**7/7 + x**5*(2*d*f + e**2)/5 + x**4*(2*c*f + 2*d*e)/4 + x**3*(2*c*e
+ d**2)/3)/sqrt(a), True))
```

<u>Giac</u> [A]
time = 1.73, size = 516, normalized size = 1.61

$$\frac{2\left(45045\sqrt{bx+a}\,c^2+\frac{30030\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)cd}{b}+\frac{3003\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)d^2}{b^2}+\frac{6006\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)ce}{b^2}+\frac{2574\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)cf}{b^3}+\frac{2574\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)de}{b^3}+\frac{286\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)df}{b^4}+\frac{143\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)e^2}{b^4}+\frac{130\left(63(bx+a)^{\frac{11}{2}}-385(bx+a)^{\frac{9}{2}}a+990(bx+a)^{\frac{7}{2}}a^2-1386(bx+a)^{\frac{5}{2}}a^3+1155(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)fe}{b^5}+\frac{15\left(231(bx+a)^{\frac{13}{2}}-1638(bx+a)^{\frac{11}{2}}a+5005(bx+a)^{\frac{9}{2}}a^2-8580(bx+a)^{\frac{7}{2}}a^3+9009(bx+a)^{\frac{5}{2}}a^4-6006(bx+a)^{\frac{3}{2}}a^5+3003\sqrt{bx+a}\,a^6\right)f^2}{b^6}\right)}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45045*(45045*sqrt(b*x + a)*c^2 + 30030*((b*x + a)^(3/2) - 3*sqrt(b*x + a)
*a)*c*d/b + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x +
a)*a^2)*d^2/b^2 + 6006*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(
b*x + a)*a^2)*c*e/b^2 + 2574*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35
*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*c*f/b^3 + 2574*(5*(b*x + a)^(7
/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)
*d*e/b^3 + 286*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^
(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d*f/b^4 + 143*
(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420
*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4 + 130*(63*(b*x + a)^(
11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5
/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*f*e/b^5 + 15*(2
```

```
31*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 -
8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*
a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6)/b
```

Mupad [B]
time = 4.70, size = 316, normalized size = 0.99

$$\frac{2\sqrt{a+b\,x}\,{\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}^2}{b^7}+\frac{2\,f^2\,{\left(a+b\,x\right)}^{13/2}}{13\,b^7}-\frac{{\left(a+b\,x\right)}^{7/2}\,\left(40\,a^3\,f^2-40\,a^2\,b\,e\,f+8\,a\,b^2\,e^2+16\,d\,a\,b^2\,f-4\,d\,b^3\,e-4\,c\,b^3\,f\right)}{7\,b^7}+\frac{{\left(a+b\,x\right)}^{9/2}\,\left(30\,a^2\,f^2-20\,a\,b\,e\,f+2\,b^2\,e^2+4\,d\,b^2\,f\right)}{9\,b^7}+\frac{{\left(a+b\,x\right)}^{5/2}\,\left(30\,a^4\,f^2-40\,a^3\,b\,e\,f+24\,a^2\,b^2\,d\,f+12\,a^2\,b^2\,e^2-12\,a\,b^3\,d\,e-12\,c\,a\,b^3\,f+2\,b^4\,d^2+4\,c\,b^4\,e\right)}{5\,b^7}-\frac{\left(12\,a\,f^2-4\,b\,e\,f\right)\,{\left(a+b\,x\right)}^{11/2}}{11\,b^7}+\frac{4\,{\left(a+b\,x\right)}^{3/2}\,\left(3\,f\,a^2-2\,e\,a\,b+d\,b^2\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)^2/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e)^2)/b^7 + (2*f^2*(a +
 b*x)^(13/2))/(13*b^7) - ((a + b*x)^(7/2)*(40*a^3*f^2 + 8*a*b^2*e^2 - 4*b^3
*c*f - 4*b^3*d*e + 16*a*b^2*d*f - 40*a^2*b*e*f))/(7*b^7) + ((a + b*x)^(9/2)
*(30*a^2*f^2 + 2*b^2*e^2 + 4*b^2*d*f - 20*a*b*e*f))/(9*b^7) + ((a + b*x)^(5
/2)*(2*b^4*d^2 + 30*a^4*f^2 + 12*a^2*b^2*e^2 + 4*b^4*c*e - 12*a*b^3*c*f - 1
2*a*b^3*d*e - 40*a^3*b*e*f + 24*a^2*b^2*d*f))/(5*b^7) - ((12*a*f^2 - 4*b*e*
f)*(a + b*x)^(11/2))/(11*b^7) + (4*(a + b*x)^(3/2)*(b^2*d + 3*a^2*f - 2*a*b
*e)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^7)
```

3.6 $\int \frac{\left(c+dx+ex^2+fx^3\right)^3}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=708

$$\frac{2(b^3c-ab^2d+a^2be-a^3f)^3\sqrt{a+bx}}{b^{10}}+\frac{2(b^2d-2abe+3a^2f)\,(b^3c-ab^2d+a^2be-a^3f)^2\,(a+bx)^{3/2}}{b^{10}}+\frac{6(b^3c}{}$$

```
[Out] 2*(3*a^2*f-2*a*b*e+b^2*d)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)^2*(b*x+a)^(3/2)/b^
10+6/5*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b^4*(c*e+d^2)-16*a^3*b*e*f+12*a^4*f^
2-a*b^3*(3*c*f+5*d*e)+a^2*b^2*(9*d*f+5*e^2))*(b*x+a)^(5/2)/b^10-2/7*(168*a^
5*b*e*f^2-84*a^6*f^3-b^6*(3*c^2*f+6*c*d*e+d^3)-105*a^4*b^2*f*(d*f+e^2)+12*a
*b^5*(2*c*d*f+c*e^2+d^2*e)-30*a^2*b^4*(2*c*e*f+d^2*f+d*e^2)+20*a^3*b^3*(3*c
*f^2+6*d*e*f+e^3))*(b*x+a)^(7/2)/b^10+2/3*(70*a^4*b*e*f^2-42*a^5*f^3-35*a^3
*b^2*f*(d*f+e^2)+b^5*(2*c*d*f+c*e^2+d^2*e)-5*a*b^4*(2*c*e*f+d^2*f+d*e^2)+5*
a^2*b^3*(3*c*f^2+6*d*e*f+e^3))*(b*x+a)^(9/2)/b^10-6/11*(56*a^3*b*e*f^2-42*a
^4*f^3-21*a^2*b^2*f*(d*f+e^2)-b^4*(2*c*e*f+d^2*f+d*e^2)+2*a*b^3*(3*c*f^2+6*
d*e*f+e^3))*(b*x+a)^(11/2)/b^10+2/13*(84*a^2*b*e*f^2-84*a^3*f^3-21*a*b^2*f*
(d*f+e^2)+b^3*(3*c*f^2+6*d*e*f+e^3))*(b*x+a)^(13/2)/b^10-2/5*f*(8*a*b*e*f-1
2*a^2*f^2-b^2*(d*f+e^2))*(b*x+a)^(15/2)/b^10+6/17*f^2*(-3*a*f+b*e)*(b*x+a)^
(17/2)/b^10+2/19*f^3*(b*x+a)^(19/2)/b^10+2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)^3
*(b*x+a)^(1/2)/b^10
```

Rubi [A]
time = 0.40, antiderivative size = 708, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1864}

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^3*Sqrt[a + b*x])/b^10 + (2*(b^2*d -
2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*(a + b*x)^(3/2))/b
^10 + (6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(b^4*(d^2 + c*e) - 16*a^3*b*e*
f + 12*a^4*f^2 - a*b^3*(5*d*e + 3*c*f) + a^2*b^2*(5*e^2 + 9*d*f))*(a + b*x)
^(5/2))/(5*b^10) - (2*(168*a^5*b*e*f^2 - 84*a^6*f^3 - b^6*(d^3 + 6*c*d*e +
3*c^2*f) - 105*a^4*b^2*f*(e^2 + d*f) + 12*a*b^5*(d^2*e + c*e^2 + 2*c*d*f) -
 30*a^2*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 20*a^3*b^3*(e^3 + 6*d*e*f + 3*c*f^2
))*(a + b*x)^(7/2))/(7*b^10) + (2*(70*a^4*b*e*f^2 - 42*a^5*f^3 - 35*a^3*b^2
*f*(e^2 + d*f) + b^5*(d^2*e + c*e^2 + 2*c*d*f) - 5*a*b^4*(d*e^2 + d^2*f + 2
*c*e*f) + 5*a^2*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(9/2))/(3*b^10) -
(6*(56*a^3*b*e*f^2 - 42*a^4*f^3 - 21*a^2*b^2*f*(e^2 + d*f) - b^4*(d*e^2 + d
^2*f + 2*c*e*f) + 2*a*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(11/2))/(11*
```

```
b^10) + (2*(84*a^2*b*e*f^2 - 84*a^3*f^3 - 21*a*b^2*f*(e^2 + d*f) + b^3*(e^3
 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(13/2))/(13*b^10) - (2*f*(8*a*b*e*f - 12*a
^2*f^2 - b^2*(e^2 + d*f))*(a + b*x)^(15/2))/(5*b^10) + (6*f^2*(b*e - 3*a*f)
*(a + b*x)^(17/2))/(17*b^10) + (2*f^3*(a + b*x)^(19/2))/(19*b^10)
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx = \int \left(\frac{(b^3c-ab^2d+a^2be-a^3f)^3}{b^9\sqrt{a+bx}} + \frac{3(b^2d-2abe+3a^2f)\,(b^3c-ab^2d+a^2be-}{b^9}\right.$$

$$= \frac{2(b^3c-ab^2d+a^2be-a^3f)^3\sqrt{a+bx}}{b^{10}} + \frac{2(b^2d-2abe+3a^2f)\,(b^3c-ab^2d+}{b^{10}}$$

Mathematica [A]
time = 0.72, size = 913, normalized size = 1.29

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-1376256*a^9*f^3 + 229376*a^8*b*f^2*(19*e + 3*f*x) - 1433
6*a^7*b^2*f*(323*e^2 + 152*e*f*x + f*(323*d + 36*f*x^2)) + 1024*a^6*b^3*(16
15*e^3 + 2261*e^2*f*x + 114*e*f*(85*d + 14*f*x^2) + f^2*(4845*c + 2261*d*x
+ 420*f*x^3)) - 256*a^5*b^4*(20995*d^2*f + 3230*c*f*(13*e + 3*f*x) + 323*d*
(65*e^2 + 60*e*f*x + 21*f^2*x^2) + x*(3230*e^3 + 6783*e^2*f*x + 5320*e*f^2*
x^2 + 1470*f^3*x^3)) + 128*a^4*b^5*(4199*d^2*(11*e + 5*f*x) + 323*c*(143*e^
2 + 130*e*f*x + 45*f^2*x^2) + x^2*(4845*e^3 + 11305*e^2*f*x + 9310*e*f^2*x^
2 + 2646*f^3*x^3) + 323*d*(286*c*f + 5*x*(13*e^2 + 18*e*f*x + 7*f^2*x^2)))
- 16*a^3*b^6*(138567*d^3 + 415701*c^2*f + 8398*d^2*x*(22*e + 15*f*x) + 1292
*c*x*(143*e^2 + 195*e*f*x + 75*f^2*x^2) + x^3*(32300*e^3 + 79135*e^2*f*x +
67032*e*f^2*x^2 + 19404*f^3*x^3) + 323*d*(286*c*(9*e + 4*f*x) + 5*x^2*(78*e
^2 + 120*e*f*x + 49*f^2*x^2))) + b^9*(4849845*c^3 + 138567*c^2*x*(35*d + 3*
x*(7*e + 5*f*x)) + 323*c*x^2*(9009*d^2 + 1430*d*x*(9*e + 7*f*x) + 35*x^2*(1
43*e^2 + 234*e*f*x + 99*f^2*x^2)) + x^3*(692835*d^3 + 146965*d^2*x*(11*e +
9*f*x) + 6783*d*x^2*(195*e^2 + 330*e*f*x + 143*f^2*x^2) + 231*x^3*(1615*e^3
 + 4199*e^2*f*x + 3705*e*f^2*x^2 + 1105*f^3*x^3))) + 8*a^2*b^7*(138567*c^2*
```

```
(7*e + 3*f*x) + 323*c*(3003*d^2 + 858*d*x*(3*e + 2*f*x) + x^2*(858*e^2 + 13
00*e*f*x + 525*f^2*x^2)) + x*(138567*d^3 + 8398*d^2*x*(33*e + 25*f*x) + 323
*d*x^2*(650*e^2 + 1050*e*f*x + 441*f^2*x^2) + 7*x^3*(8075*e^3 + 20349*e^2*f
*x + 17556*e*f^2*x^2 + 5148*f^3*x^3))) - 2*a*b^8*(138567*c^2*(35*d + x*(14*
e + 9*f*x)) + 646*c*x*(3003*d^2 + 143*d*x*(27*e + 20*f*x) + 5*x^2*(286*e^2
+ 455*e*f*x + 189*f^2*x^2)) + x^2*(415701*d^3 + 20995*d^2*x*(44*e + 35*f*x)
 + 2261*d*x^2*(325*e^2 + 540*e*f*x + 231*f^2*x^2) + 21*x^3*(9690*e^3 + 2487
1*e^2*f*x + 21736*e*f^2*x^2 + 6435*f^3*x^3)))))/(4849845*b^10)
```

Maple [A]
time = 0.31, size = 1254, normalized size = 1.77 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^10*(1/19*f^3*(b*x+a)^(19/2)+3/17*(-3*a*f+b*e)*f^2*(b*x+a)^(17/2)+1/15*(
(3*a^2*f-2*a*b*e+b^2*d)*f^2+2*(-3*a*f+b*e)^2*f+f*(2*(3*a^2*f-2*a*b*e+b^2*d)
*f+(-3*a*f+b*e)^2))*(b*x+a)^(15/2)+1/13*((-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*f^2
+2*(3*a^2*f-2*a*b*e+b^2*d)*(-3*a*f+b*e)*f+(-3*a*f+b*e)*(2*(3*a^2*f-2*a*b*e+
b^2*d)*f+(-3*a*f+b*e)^2)+f*(2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*f+2*(3*a^2*f-2
*a*b*e+b^2*d)*(-3*a*f+b*e)))*(b*x+a)^(13/2)+1/11*(2*(-a^3*f+a^2*b*e-a*b^2*d
+b^3*c)*(-3*a*f+b*e)*f+(3*a^2*f-2*a*b*e+b^2*d)*(2*(3*a^2*f-2*a*b*e+b^2*d)*f
+(-3*a*f+b*e)^2)+(-3*a*f+b*e)*(2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*f+2*(3*a^2*
f-2*a*b*e+b^2*d)*(-3*a*f+b*e))+f*(2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(-3*a*f+
b*e)+(3*a^2*f-2*a*b*e+b^2*d)^2))*(b*x+a)^(11/2)+1/9*((-a^3*f+a^2*b*e-a*b^2*
d+b^3*c)*(2*(3*a^2*f-2*a*b*e+b^2*d)*f+(-3*a*f+b*e)^2)+(3*a^2*f-2*a*b*e+b^2*
d)*(2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*f+2*(3*a^2*f-2*a*b*e+b^2*d)*(-3*a*f+b*
e))+(-3*a*f+b*e)*(2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(-3*a*f+b*e)+(3*a^2*f-2*
a*b*e+b^2*d)^2)+2*f*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(3*a^2*f-2*a*b*e+b^2*d))
*(b*x+a)^(9/2)+1/7*((-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(2*(-a^3*f+a^2*b*e-a*b^2
*d+b^3*c)*f+2*(3*a^2*f-2*a*b*e+b^2*d)*(-3*a*f+b*e))+(3*a^2*f-2*a*b*e+b^2*d)
*(2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(-3*a*f+b*e)+(3*a^2*f-2*a*b*e+b^2*d)^2)+
2*(-3*a*f+b*e)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(3*a^2*f-2*a*b*e+b^2*d)+f*(-a
^3*f+a^2*b*e-a*b^2*d+b^3*c)^2)*(b*x+a)^(7/2)+1/5*((-a^3*f+a^2*b*e-a*b^2*d+b
^3*c)*(2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(-3*a*f+b*e)+(3*a^2*f-2*a*b*e+b^2*d
)^2)+2*(3*a^2*f-2*a*b*e+b^2*d)^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)+(-3*a*f+b*e
)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)^2)*(b*x+a)^(5/2)+(-a^3*f+a^2*b*e-a*b^2*d+b
^3*c)^2*(3*a^2*f-2*a*b*e+b^2*d)*(b*x+a)^(3/2)+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)^3*(b*x+a)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. 2(680) = 1360.
time = 0.32, size = 1373, normalized size = 1.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/4849845*(4849845*sqrt(b*x + a)*c^3 + 138567*c^2*(35*((b*x + a)^(3/2) - 3*
sqrt(b*x + a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqr
t(b*x + a)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b
*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3) + 323*c*(3003*(3*(b*x + a)
^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 286*(35*(b*
x + a)^(9/2)*f - 45*(4*a*f - b*e)*(b*x + a)^(7/2) + 189*(2*a^2*f - a*b*e)*(
b*x + a)^(5/2) - 105*(4*a^3*f - 3*a^2*b*e)*(b*x + a)^(3/2) + 315*(a^4*f - a
^3*b*e)*sqrt(b*x + a))*d/b^4 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2
)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)
*a^4)*e^2/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x
 + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693
*sqrt(b*x + a)*a^5)*f*e/b^5 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11
/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a
)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6) +
 138567*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2
- 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 4199*(315*(b*x + a)^(11/2)*f - 385*(5*a*f
 - b*e)*(b*x + a)^(9/2) + 990*(5*a^2*f - 2*a*b*e)*(b*x + a)^(7/2) - 1386*(5
*a^3*f - 3*a^2*b*e)*(b*x + a)^(5/2) + 1155*(5*a^4*f - 4*a^3*b*e)*(b*x + a)^
(3/2) - 3465*(a^5*f - a^4*b*e)*sqrt(b*x + a))*d^2/b^5 + 1615*(231*(b*x + a)
^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x +
a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*s
qrt(b*x + a)*a^6)*e^3/b^6 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13
/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x
 + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6
435*sqrt(b*x + a)*a^7)*f*e^2/b^7 + 133*(6435*(b*x + a)^(17/2) - 58344*(b*x
+ a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 +
 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)
^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*f^2*e/b
^8 + 323*(3003*(b*x + a)^(15/2)*f^2 - 3465*(7*a*f^2 - 2*b*f*e)*(b*x + a)^(1
3/2) + 4095*(21*a^2*f^2 - 12*a*b*f*e + b^2*e^2)*(b*x + a)^(11/2) - 25025*(7
*a^3*f^2 - 6*a^2*b*f*e + a*b^2*e^2)*(b*x + a)^(9/2) + 32175*(7*a^4*f^2 - 8*
a^3*b*f*e + 2*a^2*b^2*e^2)*(b*x + a)^(7/2) - 9009*(21*a^5*f^2 - 30*a^4*b*f*
e + 10*a^3*b^2*e^2)*(b*x + a)^(5/2) + 15015*(7*a^6*f^2 - 12*a^5*b*f*e + 5*a
^4*b^2*e^2)*(b*x + a)^(3/2) - 45045*(a^7*f^2 - 2*a^6*b*f*e + a^5*b^2*e^2)*s
qrt(b*x + a))*d/b^7 + 21*(12155*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*
a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b
*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*
a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqr
t(b*x + a)*a^9)*f^3/b^9)/b
```

Fricas [A]

time = 0.40, size = 1221, normalized size = 1.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/4849845*(255255*b^9*f^3*x^9 + 4849845*b^9*c^3 - 9699690*a*b^8*c^2*d + 775
9752*a^2*b^7*c*d^2 - 2217072*a^3*b^6*d^3 + 1653760*a^6*b^3*e^3 - 1376256*a^
9*f^3 + 45045*(19*b^9*e*f^2 - 6*a*b^8*f^3)*x^8 + 3003*(323*b^9*e^2*f + 96*a
^2*b^7*f^3 + 19*(17*b^9*d - 16*a*b^8*e)*f^2)*x^7 + 231*(1615*b^9*e^3 - 1344
*a^3*b^6*f^3 + 19*(255*b^9*c - 238*a*b^8*d + 224*a^2*b^7*e)*f^2 + 646*(15*b
^9*d*e - 7*a*b^8*e^2)*f)*x^6 + 63*(20995*b^9*d*e^2 - 6460*a*b^8*e^3 + 5376*
a^4*b^5*f^3 - 76*(255*a*b^8*c - 238*a^2*b^7*d + 224*a^3*b^6*e)*f^2 + 323*(6
5*b^9*d^2 + 56*a^2*b^7*e^2 + 10*(13*b^9*c - 12*a*b^8*d)*e)*f)*x^5 + 35*(461
89*b^9*d^2*e + 12920*a^2*b^7*e^3 - 10752*a^5*b^4*f^3 + 4199*(11*b^9*c - 10*
a*b^8*d)*e^2 + 152*(255*a^2*b^7*c - 238*a^3*b^6*d + 224*a^4*b^5*e)*f^2 + 64
6*(143*b^9*c*d - 65*a*b^8*d^2 - 56*a^3*b^6*e^2 - 10*(13*a*b^8*c - 12*a^2*b^
7*d)*e)*f)*x^4 + 5*(138567*b^9*d^3 - 103360*a^3*b^6*e^3 + 86016*a^6*b^3*f^3
 - 33592*(11*a*b^8*c - 10*a^2*b^7*d)*e^2 - 1216*(255*a^3*b^6*c - 238*a^4*b^
5*d + 224*a^5*b^4*e)*f^2 + 92378*(9*b^9*c*d - 4*a*b^8*d^2)*e + 323*(1287*b^
9*c^2 - 2288*a*b^8*c*d + 1040*a^2*b^7*d^2 + 896*a^4*b^5*e^2 + 160*(13*a^2*b
^7*c - 12*a^3*b^6*d)*e)*f)*x^3 + 537472*(11*a^4*b^5*c - 10*a^5*b^4*d)*e^2 +
 19456*(255*a^6*b^3*c - 238*a^7*b^2*d + 224*a^8*b*e)*f^2 + 3*(969969*b^9*c*
d^2 - 277134*a*b^8*d^3 + 206720*a^4*b^5*e^3 - 172032*a^7*b^2*f^3 + 67184*(1
1*a^2*b^7*c - 10*a^3*b^6*d)*e^2 + 2432*(255*a^4*b^5*c - 238*a^5*b^4*d + 224
*a^6*b^3*e)*f^2 + 46189*(21*b^9*c^2 - 36*a*b^8*c*d + 16*a^2*b^7*d^2)*e - 64
6*(1287*a*b^8*c^2 - 2288*a^2*b^7*c*d + 1040*a^3*b^6*d^2 + 896*a^5*b^4*e^2 +
 160*(13*a^3*b^6*c - 12*a^4*b^5*d)*e)*f)*x^2 + 369512*(21*a^2*b^7*c^2 - 36*
a^3*b^6*c*d + 16*a^4*b^5*d^2)*e - 5168*(1287*a^3*b^6*c^2 - 2288*a^4*b^5*c*d
 + 1040*a^5*b^4*d^2 + 896*a^7*b^2*e^2 + 160*(13*a^5*b^4*c - 12*a^6*b^3*d)*e
)*f + (4849845*b^9*c^2*d - 3879876*a*b^8*c*d^2 + 1108536*a^2*b^7*d^3 - 8268
80*a^5*b^4*e^3 + 688128*a^8*b*f^3 - 268736*(11*a^3*b^6*c - 10*a^4*b^5*d)*e^
2 - 9728*(255*a^5*b^4*c - 238*a^6*b^3*d + 224*a^7*b^2*e)*f^2 - 184756*(21*a
*b^8*c^2 - 36*a^2*b^7*c*d + 16*a^3*b^6*d^2)*e + 2584*(1287*a^2*b^7*c^2 - 22
88*a^3*b^6*c*d + 1040*a^4*b^5*d^2 + 896*a^6*b^3*e^2 + 160*(13*a^4*b^5*c - 1
2*a^5*b^4*d)*e)*f)*x)*sqrt(b*x + a)/b^10
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3691 vs. $2(758) = 1516$.
time = 217.62, size = 3691, normalized size = 5.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*c**3/sqrt(a + b*x) - 6*a*c**2*d*(-a/sqrt(a + b*x) - sqrt(a
 + b*x))/b - 6*a*c**2*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)
```

```
**(3/2)/3)/b**2 - 6*a*c*d**2*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a +
 b*x)**(3/2)/3)/b**2 - 6*a*c**2*f*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*
x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 - 12*a*c*d*e*(-a**3/sqrt
(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)
/b**3 - 2*a*d**3*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)*
*(3/2) - (a + b*x)**(5/2)/5)/b**3 - 12*a*c*d*f*(a**4/sqrt(a + b*x) + 4*a**3
*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*
x)**(7/2)/7)/b**4 - 6*a*c*e**2*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) -
 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**
4 - 6*a*d**2*e*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x
)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 - 12*a*c*e*f*(
-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2
*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**5
- 6*a*d**2*f*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x
)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**
(9/2)/9)/b**5 - 6*a*d*e**2*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10
*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7
 - (a + b*x)**(9/2)/9)/b**5 - 6*a*c*f**2*(a**6/sqrt(a + b*x) + 6*a**5*sqrt(
a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 15*a**2*(a +
 b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x)**(11/2)/11)/b**6 - 12*a
*d*e*f*(a**6/sqrt(a + b*x) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2)
 + 4*a**3*(a + b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9
/2)/3 - (a + b*x)**(11/2)/11)/b**6 - 2*a*e**3*(a**6/sqrt(a + b*x) + 6*a**5*
sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 15*a**2
*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x)**(11/2)/11)/b**6 -
 6*a*d*f**2*(-a**7/sqrt(a + b*x) - 7*a**6*sqrt(a + b*x) + 7*a**5*(a + b*x)*
*(3/2) - 7*a**4*(a + b*x)**(5/2) + 5*a**3*(a + b*x)**(7/2) - 7*a**2*(a + b*
x)**(9/2)/3 + 7*a*(a + b*x)**(11/2)/11 - (a + b*x)**(13/2)/13)/b**7 - 6*a*e
**2*f*(-a**7/sqrt(a + b*x) - 7*a**6*sqrt(a + b*x) + 7*a**5*(a + b*x)**(3/2)
 - 7*a**4*(a + b*x)**(5/2) + 5*a**3*(a + b*x)**(7/2) - 7*a**2*(a + b*x)**(9
/2)/3 + 7*a*(a + b*x)**(11/2)/11 - (a + b*x)**(13/2)/13)/b**7 - 6*a*e*f**2*
(a**8/sqrt(a + b*x) + 8*a**7*sqrt(a + b*x) - 28*a**6*(a + b*x)**(3/2)/3 + 5
6*a**5*(a + b*x)**(5/2)/5 - 10*a**4*(a + b*x)**(7/2) + 56*a**3*(a + b*x)**(
9/2)/9 - 28*a**2*(a + b*x)**(11/2)/11 + 8*a*(a + b*x)**(13/2)/13 - (a + b*x
)**(15/2)/15)/b**8 - 2*a*f**3*(-a**9/sqrt(a + b*x) - 9*a**8*sqrt(a + b*x) +
 12*a**7*(a + b*x)**(3/2) - 84*a**6*(a + b*x)**(5/2)/5 + 18*a**5*(a + b*x)*
*(7/2) - 14*a**4*(a + b*x)**(9/2) + 84*a**3*(a + b*x)**(11/2)/11 - 36*a**2*
(a + b*x)**(13/2)/13 + 3*a*(a + b*x)**(15/2)/5 - (a + b*x)**(17/2)/17)/b**9
 - 2*c**3*(-a/sqrt(a + b*x) - sqrt(a + b*x)) - 6*c**2*d*(a**2/sqrt(a + b*x)
 + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b - 6*c**2*e*(-a**3/sqrt(a + b*x
) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 -
6*c*d**2*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) -
 (a + b*x)**(5/2)/5)/b**2 - 6*c**2*f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a +
b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/
7)/b**3 - 12*c*d*e*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a +
```

```
 b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3 - 2*d**3*(
a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(
a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3 - 12*c*d*f*(-a**5/sqrt(a + b*x
) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(
5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 - 6*c*e**2*(-a**5/
sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*
(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 - 6*d*
*2*e*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)
/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)
/b**4 - 12*c*e*f*(a**6/sqrt(a + b*x) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b
*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a
+ b*x)**(9/2)/3 - (a + b*x)**(11/2)/11)/b**5 - 6*d**2*f*(a**6/sqrt(a + b*x)
 + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2)
 - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x)**(11/2)/
11)/b**5 - 6*d*e**2*(a**6/sqrt(a + b*x) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a
+ b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*
(a + b*x)**(9/2)/3 - (a + b*x)**(11/2)/11)/b**5 - 6*c*f**2*(-a**7/sqrt(a +
b*x) - 7*a**6*sqrt(a + b*x) + 7*a**5*(a + b*x)**(3/2) - 7*a**4*(a + b*x)**(
5/2) + 5*a**3*(a + b*x)**(7/2) - 7*a**2*(a + b*x)**(9/2)/3 + 7*a*(a + b*x)*
*(11/2)/11 - (a + b*x)**(13/2)/13)/b**6 - 12*d*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(680) = 1360$.
time = 1.66, size = 1414, normalized size = 2.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/4849845*(4849845*sqrt(b*x + a)*c^3 + 4849845*((b*x + a)^(3/2) - 3*sqrt(b*
x + a)*a)*c^2*d/b + 969969*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*s
qrt(b*x + a)*a^2)*c*d^2/b^2 + 969969*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2
)*a + 15*sqrt(b*x + a)*a^2)*c^2*e/b^2 + 138567*(5*(b*x + a)^(7/2) - 21*(b*x
 + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 41
5701*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 3
5*sqrt(b*x + a)*a^3)*c^2*f/b^3 + 831402*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(
5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*c*d*e/b^3 + 92378*(
35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*
(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*d*f/b^4 + 46189*(35*(b*x + a
)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(
3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b^4 + 20995*(63*(b*x + a)^(11/2) -
385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3
+ 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d^2*f/b^5 + 46189*(35*(
b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x
```

```
 + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*e^2/b^4 + 41990*(63*(b*x + a)^(1
1/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/
2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*c*f*e/b^5 + 4845
*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2
 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/
2)*a^5 + 3003*sqrt(b*x + a)*a^6)*c*f^2/b^6 + 20995*(63*(b*x + a)^(11/2) - 3
85*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 +
 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2/b^5 + 9690*(231*(b
*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*
(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 +
 3003*sqrt(b*x + a)*a^6)*d*f*e/b^6 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x
 + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 3
2175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2
)*a^6 - 6435*sqrt(b*x + a)*a^7)*d*f^2/b^7 + 1615*(231*(b*x + a)^(13/2) - 16
38*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3
 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)
*a^6)*e^3/b^6 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 1228
5*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*
a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x
 + a)*a^7)*f*e^2/b^7 + 133*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*
a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x
 + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 -
 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*f^2*e/b^8 + 21*(121
55*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a
^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*
(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)
*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*f^3/b^9)/b
```

Mupad [B]
time = 0.24, size = 896, normalized size = 1.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)^3/(a + b*x)^(1/2),x)
```

```
[Out] ((a + b*x)^(11/2)*(252*a^4*f^3 - 12*a*b^3*e^3 + 6*b^4*d*e^2 + 6*b^4*d^2*f +
 126*a^2*b^2*d*f^2 + 126*a^2*b^2*e^2*f + 12*b^4*c*e*f - 36*a*b^3*c*f^2 - 33
6*a^3*b*e*f^2 - 72*a*b^3*d*e*f))/(11*b^10) + (2*(a + b*x)^(1/2)*(b^3*c - a^
3*f - a*b^2*d + a^2*b*e)^3)/b^10 + ((a + b*x)^(9/2)*(6*b^5*c*e^2 - 252*a^5*
f^3 + 6*b^5*d^2*e + 30*a^2*b^3*e^3 + 90*a^2*b^3*c*f^2 - 210*a^3*b^2*d*f^2 -
 210*a^3*b^2*e^2*f + 12*b^5*c*d*f - 30*a*b^4*d*e^2 - 30*a*b^4*d^2*f + 420*a
^4*b*e*f^2 + 180*a^2*b^3*d*e*f - 60*a*b^4*c*e*f))/(9*b^10) + (2*f^3*(a + b*
x)^(19/2))/(19*b^10) + ((a + b*x)^(13/2)*(2*b^3*e^3 - 168*a^3*f^3 + 6*b^3*c
*f^2 + 12*b^3*d*e*f - 42*a*b^2*d*f^2 - 42*a*b^2*e^2*f + 168*a^2*b*e*f^2))/(
```

13*b^10) - ((18*a*f^3 - 6*b*e*f^2)*(a + b*x)^(17/2))/(17*b^10) + ((a + b*x)^(15/2)*(72*a^2*f^3 + 6*b^2*d*f^2 + 6*b^2*e^2*f - 48*a*b*e*f^2))/(15*b^10) - ((a + b*x)^(5/2)*(72*a^7*f^3 + 6*a*b^6*d^3 - 6*b^7*c*d^2 - 6*b^7*c^2*e - 30*a^4*b^3*e^3 - 36*a^2*b^5*c*e^2 - 36*a^2*b^5*d^2*e + 60*a^3*b^4*d*e^2 - 90*a^4*b^3*c*f^2 + 60*a^3*b^4*d^2*f + 126*a^5*b^2*d*f^2 + 126*a^5*b^2*e^2*f + 18*a*b^6*c^2*f - 168*a^6*b*e*f^2 - 72*a^2*b^5*c*d*f + 120*a^3*b^4*c*e*f - 180*a^4*b^3*d*e*f + 36*a*b^6*c*d*e))/(5*b^10) + ((a + b*x)^(7/2)*(2*b^6*d^3 + 168*a^6*f^3 + 6*b^6*c^2*f - 40*a^3*b^3*e^3 + 60*a^2*b^4*d*e^2 - 120*a^3*b^3*c*f^2 + 60*a^2*b^4*d^2*f + 210*a^4*b^2*d*f^2 + 210*a^4*b^2*e^2*f + 12*b^6*c*d*e - 24*a*b^5*c*e^2 - 24*a*b^5*d^2*e - 336*a^5*b*e*f^2 + 120*a^2*b^4*c*e*f - 240*a^3*b^3*d*e*f - 48*a*b^5*c*d*f))/(7*b^10) + (2*(a + b*x)^(3/2)*(b^2*d + 3*a^2*f - 2*a*b*e)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e)^2)/b^10

3.7 $\int \frac{c+dx}{a+bx^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}+\frac{\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{6a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1/6*(c-a^(1
/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)-1/
3*(b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(2/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}-\frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3),x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx}{a+bx^3}\,dx = \frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}} + \frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}}$$

$$= \frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{2/3}} + \frac{1}{2}\left(\frac{c}{\sqrt[3]{a}}+\right.$$

$$= \frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}} + \frac{\left(\sqrt[3]{\right.}}{}$$

$$= -\frac{\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}} + \frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}\,c-\sqrt[3]{\right.}}{}$$

Mathematica [A]
time = 0.03, size = 124, normalized size = 0.77

$$\frac{-2\sqrt{3}\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

Maple [A]
time = 0.31, size = 186, normalized size = 1.16

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^2}}{3b}$
default	$c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right.\right.}{6b\left(\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))+d*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1)))
```

Maxima [A]
time = 0.50, size = 135, normalized size = 0.84

$$\frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(d*(a/b)^(1/3) + c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(d*(a/b)^(1/3) - c)*log(x^2 - x*(a/b)^(1/3)
```

```
+ (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(d*(a/b)^(1/3) - c)*log(x + (a/b)^(1/3
))/(b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.15, size = 1931, normalized size = 11.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d
^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*
d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(
2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)
/(a^2*b^2))^(1/3)))*a*b*c^2 + 2*a*c*d^2 + (b*c^3 + a*d^3)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2
))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^
2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^
(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2)
+ (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1
/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a
*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3 + a*d^3)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a^2*b*d + 2*a*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*
b))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^
3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c
^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^
3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d
^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
```

```
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^
2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3
+ a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*
a^2*b*d + 2*a*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^
2*a*b + 16*c*d)/(a*b)))
```

Sympy [A]
time = 0.34, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^3a^2b^2+9tabcd+ad^3-bc^3,\left(t\mapsto t\log\left(x+\frac{9t^2a^2bd+3tabc^2+2acd^2}{ad^3+bc^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lambda(_t, _t*
log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)/(a*d**3 + b*c**3)))
)
```

Giac [A]
time = 1.46, size = 141, normalized size = 0.88

$$-\frac{\sqrt{3}\left(bc-(-ab^2)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}}-\frac{\left(bc+(-ab^2)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}}-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c + (-a*b^2)^(1/3)*d)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(d*(-a/b)^(1/3) + c)*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a
```

Mupad [B]
time = 5.51, size = 127, normalized size = 0.79

$$\sum_{k=1}^{3}\ln\left(b\left(c\,d+d^2\,x+\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)^2\,a\,b\,9+\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)\,b\,c\,x\,3\right)\right)\,\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^3),x)
```

```
[Out] symsum(log(b*(c*d + d^2*x + 9*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b
*c^3, z, k)^2*a*b + 3*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z,
 k)*b*c*x))*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z, k), k, 1,
 3)
```

3.8 $\int \frac{c+dx}{(a+bx^3)^2} dx$

Optimal. Leaf size=189

$$\frac{x(c+dx)}{3a\left(a+bx^3\right)}-\frac{\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\right.}{}$$

```
[Out] 1/3*x*(d*x+c)/a/(b*x^3+a)+1/9*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)
/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/9*(2*b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1869, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,d+2\sqrt[3]{b}\,c\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}}-\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^2,x]
```

```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5
/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = \frac{x(c+dx)}{3a(a+bx^3)} - \frac{\int \frac{-2c-dx}{a+bx^3}\,dx}{3a}$$

$$= \frac{x(c+dx)}{3a(a+bx^3)} - \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}c-\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}} + \frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{\ldots}}}{9a^{5/3}}$$

$$= \frac{x(c+dx)}{3a(a+bx^3)} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2\ldots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\ldots}}{18a^{5/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{3a(a+bx^3)} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\ldots\right.}{18a^{5/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{3a(a+bx^3)} - \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{\ldots}\right.}{9a^{5/3}b^{2/3}}$$

Mathematica [A]
time = 0.10, size = 180, normalized size = 0.95

$$\frac{\frac{6ax(c+dx)}{a+bx^3} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{\left(-2\sqrt[3]{a}\sqrt[3]{b}c+a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}}{18a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^2,x]
```

```
[Out] ((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)*
d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b^(
1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(1/3
)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(
18*a^2)
```

Maple [A]
time = 0.32, size = 230, normalized size = 1.22

method	result
risch	$\frac{\frac{d\,x^2}{3a}+\frac{cx}{3a}}{b\,x^3+a} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{\left(\_R d+2c\right)\ln\left(x-\_R\right)}{\_R^2}}{9ba}$

default	$c\left(\frac{x}{3a(bx^3+a)}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}\right)+d\left(\frac{x^2}{3a(bx^3+a)}+\frac{-\frac{\ln(}{}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/3*x/a/(b*x^3+a)+2/3/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)
^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))+d*(1/3*x^2/a/(b*x^3+a)+1/3/a*(-1/3/b/(a/b
)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 1.14, size = 169, normalized size = 0.89

$$\frac{dx^2+cx}{3\left(abx^3+a^2\right)}+\frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+2c\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(d*x^2 + c*x)/(a*b*x^3 + a^2) + 1/9*sqrt(3)*(d*(a/b)^(1/3) + 2*c)*arcta
n(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) + 1/18*(d*
(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3))
 - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.09, size = 2088, normalized size = 11.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
 a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*
```

(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) + 12*c*x + ((a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) + 3*sqrt(1/3)*(a*b*x^3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d^3)*x + 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b))) + ((a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) - 3*sqrt(1/3)*(a*b*x^3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/

```
2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3
 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b))))/(a*b*x^3 + a^2)
```

Sympy [A]
time = 0.45, size = 105, normalized size = 0.56

$$\operatorname{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{cx + dx^2}{3a^2 + 3abx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(
_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3
 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)
```

Giac [A]
time = 1.66, size = 174, normalized size = 0.92

$$-\frac{\sqrt{3}\left(2\,bc - \left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{\left(2\,bc + \left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{dx^2 + cx}{3\left(bx^3 + a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)^(1
/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(d*x^2 + c*x)/
((b*x^3 + a)*a)
```

Mupad [B]
time = 4.87, size = 169, normalized size = 0.89

$$\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(2\,c\,d + d^2\,x + \mathrm{root}(729\,a^5\,b^2\,z^3 + 54\,a^2\,b\,c\,d\,z - 8\,b\,c^3 + a\,d^3, z, k)^2\,a^3\,b\,81 + \mathrm{root}(729\,a^5\,b^2\,z^3 + 54\,a^2\,b\,c\,d\,z - 8\,b\,c^3 + a\,d^3, z, k)\;a\,b\,c\,x\,18\right)}{a^2\,9}\right)\mathrm{root}\left(729\,a^5\,b^2\,z^3 + 54\,a^2\,b\,c\,d\,z - 8\,b\,c^3 + a\,d^3, z, k\right)\right) + \frac{\frac{d\,x^2}{3\,a} + \frac{c\,x}{3\,a}}{b\,x^3 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z - 8
*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*b*c
*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) + (c*x)/(3*a))/(a
+ b*x^3)
```

3.9 $\int \frac{c+dx}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=215

$$\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}+\frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}-\frac{\left(5\sqrt[3]{b}\,c+2\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{27a^{8/3}b^{2/3}}$$

```
[Out] 1/6*x*(d*x+c)/a/(b*x^3+a)^2+1/18*x*(4*d*x+5*c)/a^2/(b*x^3+a)+1/27*(5*b^(1/3
)*c-2*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(2/3)-1/54*(5*b^(1/3)*c-2*
a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(2/3)-1/27*(
5*b^(1/3)*c+2*a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
a^(8/3)/b^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1869, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(2\sqrt[3]{a}\,d+5\sqrt[3]{b}\,c\right)}{9\sqrt{3}\,a^{8/3}b^{2/3}}-\frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{2/3}}+\frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^3,x]
```

```
[Out] (x*(c + d*x))/(6*a*(a + b*x^3)^2) + (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3))
- ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(1/
3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = \frac{x(c+dx)}{6a\,(a+bx^3)^2} - \frac{\int \frac{-5c-4dx}{(a+bx^3)^2}\,dx}{6a}$$

$$= \frac{x(c+dx)}{6a\,(a+bx^3)^2} + \frac{x(5c+4dx)}{18a^2\,(a+bx^3)} + \frac{\int \frac{10c+4dx}{a+bx^3}\,dx}{18a^2}$$

$$= \frac{x(c+dx)}{6a\,(a+bx^3)^2} + \frac{x(5c+4dx)}{18a^2\,(a+bx^3)} + \frac{\int \frac{\sqrt[3]{a}\left(20\sqrt[3]{b}\,c+4\sqrt[3]{a}\,d\right)+\sqrt[3]{b}\left(-10\sqrt[3]{b}\,c+4\sqrt[3]{a}\,d\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{54a^{8/3}\sqrt[3]{b}} + \frac{(5}{}$$

$$= \frac{x(c+dx)}{6a\,(a+bx^3)^2} + \frac{x(5c+4dx)}{18a^2\,(a+bx^3)} + \frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{}\right.}{}$$

$$= \frac{x(c+dx)}{6a\,(a+bx^3)^2} + \frac{x(5c+4dx)}{18a^2\,(a+bx^3)} + \frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{}\right.}{}$$

$$= \frac{x(c+dx)}{6a\,(a+bx^3)^2} + \frac{x(5c+4dx)}{18a^2\,(a+bx^3)} - \frac{\left(5\sqrt[3]{b}\,c+2\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}\,c-2\right.}{}$$

Mathematica [A]
time = 0.10, size = 205, normalized size = 0.95

$$\frac{\frac{9a^2x(c+dx)}{(a+bx^3)^2}+\frac{3ax(5c+4dx)}{a+bx^3}-\frac{2\sqrt{3}\,\sqrt[3]{a}\left(5\sqrt[3]{b}\,c+2\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{2\left(5\sqrt[3]{a}\,\sqrt[3]{b}\,c-2a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{\left(-5\sqrt[3]{a}\,\sqrt[3]{b}\,c+2a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}}{54a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^3,x]
```

```
[Out] ((9*a^2*x*(c + d*x))/(a + b*x^3)^2 + (3*a*x*(5*c + 4*d*x))/(a + b*x^3) - (2
*Sqrt[3]*a^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(2/3) + (2*(5*a^(1/3)*b^(1/3)*c - 2*a^(2/3)*d)*Log[a^(1/3)
 + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/3)*c + 2*a^(2/3)*d)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(54*a^3)
```

Maple [A]
time = 0.32, size = 274, normalized size = 1.27

method	result

risch	$\frac{\frac{2bd\,x^5}{9a^2}+\frac{5bc\,x^4}{18a^2}+\frac{7d\,x^2}{18a}+\frac{4cx}{9a}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(2\_Rd+5c)\ln(x-\_R)}{\_R^2}}{27a^2b}$
default	$c\left(\frac{x}{6a(b\,x^3+a)^2}+\frac{\frac{5x}{18a(b\,x^3+a)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{6a}}{a}\right)+d\left(\frac{}{6}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/6*x/a/(b*x^3+a)^2+5/6/a*(1/3*x/a/(b*x^3+a)+2/3/a*(1/3/b/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(
a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))))+d*(1/6*x^2/a/
(b*x^3+a)^2+2/3/a*(1/3*x^2/a/(b*x^3+a)+1/3/a*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)
^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(
a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))))
```

Maxima [A]

time = 0.99, size = 203, normalized size = 0.94

$$\frac{4\,bdx^5+5\,bcx^4+7\,adx^2+8\,acx}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)}+\frac{\sqrt{3}\,\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\arctan\left(\frac{\sqrt{3}\,\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b*d*x^5 + 5*b*c*x^4 + 7*a*d*x^2 + 8*a*c*x)/(a^2*b^2*x^6 + 2*a^3*b*x
^3 + a^4) + 1/27*sqrt(3)*(2*d*(a/b)^(1/3) + 5*c)*arctan(1/3*sqrt(3)*(2*x -
```

```
(a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) + 1/54*(2*d*(a/b)^(1/3) - 5*c
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/27*(2*d*(a
/b)^(1/3) - 5*c)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.05, size = 2215, normalized size = 10.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(24*b*d*x^5 + 30*b*c*x^4 + 42*a*d*x^2 + 48*a*c*x - 2*(a^2*b^2*x^6 + 2
*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*
b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt
(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a
^8*b^2))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3
)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(
-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*
d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d - 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125
*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(
1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (
125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 + 40*a*c*d^2 + (125*b*c^3
 + 8*a*d^3)*x) + ((a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(
1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a
^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3))) + 3*sqrt(1/3)*(a^2*b^2*x
^6 + 2*a^3*b*x^3 + a^4)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)
*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d
^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c
^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*
d + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (1
25*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/
(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^
(1/3)))*a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^3 + 8*a*d^3)*x + 3/2*sqrt(1/3)*
(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*(
(125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*
a^6*b*d + 25*a^3*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)
*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b))) + ((a^2*b^2*x^6
```

```
+ 2*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a
^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*s
qrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)
/(a^8*b^2))^(1/3))) - 3*sqrt(1/3)*(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*sqrt(-(
((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
- 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((
125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2
*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c
^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)
^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*
b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d + 25/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^
(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(
a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 - 40*a*c*d^2
+ 2*(125*b*c^3 + 8*a*d^3)*x - 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(
(125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) -
20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2)
 + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^6*b*d + 25*a^3*b*c^2)*sqrt(-(
((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
- 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((
125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2
*a^5*b + 160*c*d)/(a^5*b))))/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)
```

Sympy [**A**]

time = 0.60, size = 146, normalized size = 0.68

$$\operatorname{RootSum}\left(19683t^3a^8b^2+810ta^3bcd+8ad^3-125bc^3,\left(t\mapsto t\log\left(x+\frac{1458t^2a^6bd+675ta^3bc^2+40acd^2}{8ad^3+125bc^3}\right)\right)\right)+\frac{8acx+7adx^2+5bcx^4+4bdx^5}{18a^4+36a^3bx^3+18a^2b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 125*b*c**3,
Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**3*b*c**2 + 40*a*c*d*
*2)/(8*a*d**3 + 125*b*c**3)))) + (8*a*c*x + 7*a*d*x**2 + 5*b*c*x**4 + 4*b*d
*x**5)/(18*a**4 + 36*a**3*b*x**3 + 18*a**2*b**2*x**6)
```

Giac [**A**]

time = 1.80, size = 194, normalized size = 0.90

$$-\frac{\sqrt{3}\left(5\,bc-2\,(-ab^2)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^2}-\frac{\left(5\,bc+2\,(-ab^2)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^2}-\frac{\left(2\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3}+\frac{4\,bdx^5+5\,bcx^4+7\,adx^2+8\,acx}{18\,(bx^3+a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(5*b*c - 2*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(5*b*c + 2*(-a*b^2)^(1/3)
```

```
*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(2
*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/18*(
4*b*d*x^5 + 5*b*c*x^4 + 7*a*d*x^2 + 8*a*c*x)/((b*x^3 + a)^2*a^2)
```

Mupad [B]
time = 0.27, size = 206, normalized size = 0.96

$$\frac{\frac{7\,d\,x^2}{18\,a}+\frac{4\,c\,x}{9\,a}+\frac{5\,b\,c\,x^4}{18\,a^2}+\frac{2\,b\,d\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\left(\sum_{k=1}^{3}\ln\left(\frac{b\,\left(10\,c\,d+4\,d^2\,x+\mathrm{root}(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)^2\,a^5\,b\,729+\mathrm{root}(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)\,a^2\,b\,c\,x\,135\right)}{a^4\,81}\right)\,\mathrm{root}\left(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^3)^3,x)
```

```
[Out] ((7*d*x^2)/(18*a) + (4*c*x)/(9*a) + (5*b*c*x^4)/(18*a^2) + (2*b*d*x^5)/(9*a
^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((b*(10*c*d + 4*d^2*x + 729*ro
ot(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3, z, k)^2*a^5*b
 + 135*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3, z, k
)*a^2*b*c*x))/(81*a^4))*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^
3 + 8*a*d^3, z, k), k, 1, 3)
```

3.10 $\int \frac{c+dx}{(a+bx^3)^4} \, dx$

Optimal. Leaf size=240

$$\frac{x(c+dx)}{9a(a+bx^3)^3}+\frac{x(8c+7dx)}{54a^2(a+bx^3)^2}+\frac{2x(10c+7dx)}{81a^3(a+bx^3)}-\frac{2\left(20\sqrt[3]{b}\,c+7\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{81\sqrt{3}\,a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}\,c-\right.}{}$$

```
[Out] 1/9*x*(d*x+c)/a/(b*x^3+a)^3+1/54*x*(7*d*x+8*c)/a^2/(b*x^3+a)^2+2/81*x*(7*d*
x+10*c)/a^3/(b*x^3+a)+2/243*(20*b^(1/3)*c-7*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x
)/a^(11/3)/b^(2/3)-1/243*(20*b^(1/3)*c-7*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(2/3)-2/243*(20*b^(1/3)*c+7*a^(1/3)*d)*arctan
(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1869, 1874, 31, 648, 631, 210, 642}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}\,d+20\sqrt[3]{b}\,c\right)}{81\sqrt{3}\,a^{11/3}b^{2/3}}-\frac{\left(20\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{11/3}b^{2/3}}+\frac{2x(10c+7dx)}{81a^3(a+bx^3)}+\frac{x(8c+7dx)}{54a^2(a+bx^3)^2}+\frac{x(c+dx)}{9a(a+bx^3)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^4,x]
```

```
[Out] (x*(c + d*x))/(9*a*(a + b*x^3)^3) + (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2
) + (2*x*(10*c + 7*d*x))/(81*a^3*(a + b*x^3)) - (2*(20*b^(1/3)*c + 7*a^(1/3
)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(11/3
)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(243
*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = \frac{x(c+dx)}{9a\,(a+bx^3)^3} - \frac{\int \frac{-8c-7dx}{(a+bx^3)^3}\,dx}{9a}$$

$$= \frac{x(c+dx)}{9a\,(a+bx^3)^3} + \frac{x(8c+7dx)}{54a^2\,(a+bx^3)^2} + \frac{\int \frac{40c+28dx}{(a+bx^3)^2}\,dx}{54a^2}$$

$$= \frac{x(c+dx)}{9a\,(a+bx^3)^3} + \frac{x(8c+7dx)}{54a^2\,(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3\,(a+bx^3)} - \frac{\int \frac{-80c-28dx}{a+bx^3}\,dx}{162a^3}$$

$$= \frac{x(c+dx)}{9a\,(a+bx^3)^3} + \frac{x(8c+7dx)}{54a^2\,(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3\,(a+bx^3)} - \frac{\int \frac{\sqrt[3]{a}\left(-160\sqrt[3]{b}\,c-28\sqrt[3]{a}\,d\right)+\sqrt[3]{b}\left(80\sqrt[3]{\cdots}\right.}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^{2}\cdots}}{486a^{11/3}\sqrt[3]{b}}$$

$$= \frac{x(c+dx)}{9a\,(a+bx^3)^3} + \frac{x(8c+7dx)}{54a^2\,(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3\,(a+bx^3)} + \frac{2\left(20\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{243a^{11/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{9a\,(a+bx^3)^3} + \frac{x(8c+7dx)}{54a^2\,(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3\,(a+bx^3)} + \frac{2\left(20\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{243a^{11/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{9a\,(a+bx^3)^3} + \frac{x(8c+7dx)}{54a^2\,(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3\,(a+bx^3)} - \frac{2\left(20\sqrt[3]{b}\,c+7\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-\cdots}{\sqrt{3}\cdots}\right.}{81\sqrt{3}\,a^{11/3}b^{2/3}}$$

Mathematica [A]
time = 0.13, size = 229, normalized size = 0.95

$$\frac{\frac{54a^3x(c+dx)}{(a+bx^3)^3}+\frac{9a^2x(8c+7dx)}{(a+bx^3)^2}+\frac{12ax(10c+7dx)}{a+bx^3}-\frac{4\sqrt{3}\,\sqrt[3]{a}\left(20\sqrt[3]{b}\,c+7\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{4\left(20\sqrt[3]{a}\,\sqrt[3]{b}\,c-7a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{2\left(-20\sqrt[3]{a}\,\sqrt[3]{b}\,c+7a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}}{486a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^4,x]
```

```
[Out] ((54*a^3*x*(c + d*x))/(a + b*x^3)^3 + (9*a^2*x*(8*c + 7*d*x))/(a + b*x^3)^2
 + (12*a*x*(10*c + 7*d*x))/(a + b*x^3) - (4*Sqrt[3]*a^(1/3)*(20*b^(1/3)*c +
 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*(20
*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (2*(-
20*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/b^(2/3))/(486*a^4)
```

Maple [A]
time = 0.34, size = 318, normalized size = 1.32

method	result
risch	$\frac{\frac{14d\,b^2x^8}{81a^3}+\frac{20c\,b^2x^7}{81a^3}+\frac{77bd\,x^5}{162a^2}+\frac{52bc\,x^4}{81a^2}+\frac{67d\,x^2}{162a}+\frac{41cx}{81a}}{(b\,x^3+a)^3}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(7\_Rd+20c\right)\ln\left(x-\_R\right)}{\_R^2}\right)}{243a^3b}$
default	$c\left(\frac{x}{9a(b\,x^3+a)^3}+\frac{8\left(\frac{4x}{27a\left(b\,x^3+a\right)^2}+\frac{\frac{5x}{18a\left(b\,x^3+a\right)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{6a}}{9a}\right.}{a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
[Out] c*(1/9/a*x/(b*x^3+a)^3+8/9/a*(1/6*x/a/(b*x^3+a)^2+5/6/a*(1/3*x/a/(b*x^3+a)+
```

```
2/3/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))))))+d*(1/9/a*x^2/(b*x^3+a)^3+7/9/a*(1/6*x^2/a/(b*x^3+a)^2+2/3/a*
(1/3*x^2/a/(b*x^3+a)+1/3/a*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b
)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))))
```

Maxima [A]
time = 1.26, size = 238, normalized size = 0.99

$$\frac{28\,b^2dx^8+40\,b^2cx^7+77\,abdx^5+104\,abcx^4+67\,a^2dx^2+82\,a^2cx}{162\,(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6)}+\frac{2\,\sqrt{3}\,\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^3b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^3b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\,\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^3b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/162*(28*b^2*d*x^8 + 40*b^2*c*x^7 + 77*a*b*d*x^5 + 104*a*b*c*x^4 + 67*a^2*
d*x^2 + 82*a^2*c*x)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6) + 2/2
43*sqrt(3)*(7*d*(a/b)^(1/3) + 20*c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) + 1/243*(7*d*(a/b)^(1/3) - 20*c)*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) - 2/243*(7*d*(a/b)^(1/3)
- 20*c)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.14, size = 2308, normalized size = 9.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(168*b^2*d*x^8 + 240*b^2*c*x^7 + 462*a*b*d*x^5 + 624*a*b*c*x^4 + 402*
a^2*d*x^2 + 492*a^2*c*x - 2*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^
6)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*
c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*
b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2
))^(1/3)))*log(7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sq
rt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*
a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d - 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*
b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)
- 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 + 7840*a*c*d^2
 + 4*(8000*b*c^3 + 343*a*d^3)*x) + ((a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*
x^3 + a^6)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +
(8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) +
```

```
1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(
a^11*b^2))^(1/3))) + 3*sqrt(1/3)*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3
 + a^6)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2
) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3
) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^
3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b)))*log(-7/4*(4^(1/3)*(I*s
qrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)
/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 +
 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8
*b*d + 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +
(8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) +
1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(
a^11*b^2))^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2 + 8*(8000*b*c^3 + 343*a*d^3)*x
+ 3/4*sqrt(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sq
rt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*
a*d^3)/(a^11*b^2))^(1/3)))*a^8*b*d + 1600*a^4*b*c^2)*sqrt(-((4^(1/3)*(I*sqr
t(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(
a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 3
43*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b
 + 8960*c*d)/(a^7*b))) + ((a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*
((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))
^(1/3))) - 3*sqrt(1/3)*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)*sq
rt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*
b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^
7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b
^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b)))*log(-7/4*(4^(1/3)*(I*sqrt(3) + 1
)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2
))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3
)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d + 400
*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*
((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))
^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2 + 8*(8000*b*c^3 + 343*a*d^3)*x - 3/4*sqrt
(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8
000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)
/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^
11*b^2))^(1/3)))*a^8*b*d + 1600*a^4*b*c^2)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*
((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))
^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/
(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*
d)/(a^7*b))))/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
```

Sympy [A]
time = 0.72, size = 185, normalized size = 0.77

$$\operatorname{RootSum}\left(14348907t^3a^{11}b^2+408240ta^4bcd+2744ad^3-64000bc^3,\ \left(t\mapsto t\log\left(x+\frac{413343t^2a^8bd+194400ta^4bc^2+7840acd^2}{1372ad^3+32000bc^3}\right)\right)\right)+\frac{82a^2cx+67a^2dx^2+104abcx^4+77abdx^5+40b^2cx^7+28b^2dx^8}{162a^6+486a^5bx^3+486a^4b^2x^6+162a^3b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a*d**3 - 64
000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*d + 194400*_t*a**4*b
*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b*c**3)))) + (82*a**2*c*x + 67*
a**2*d*x**2 + 104*a*b*c*x**4 + 77*a*b*d*x**5 + 40*b**2*c*x**7 + 28*b**2*d*x
**8)/(162*a**6 + 486*a**5*b*x**3 + 486*a**4*b**2*x**6 + 162*a**3*b**3*x**9)
```

Giac [A]
time = 1.80, size = 218, normalized size = 0.91

$$-\frac{2\sqrt{3}\left(20\,bc-7\left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\left(-ab^2\right)^{\frac{2}{3}}a^3}-\frac{\left(20\,bc+7\left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(-ab^2\right)^{\frac{2}{3}}a^3}-\frac{2\left(7\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4}+\frac{28\,b^2dx^8+40\,b^2cx^7+77\,abdx^5+104\,abcx^4+67\,a^2dx^2+82\,a^2cx}{162\left(bx^3+a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -2/243*sqrt(3)*(20*b*c - 7*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/243*(20*b*c + 7*(-a*b^2)^(
1/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 2/2
43*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 +
1/162*(28*b^2*d*x^8 + 40*b^2*c*x^7 + 77*a*b*d*x^5 + 104*a*b*c*x^4 + 67*a^2*
d*x^2 + 82*a^2*c*x)/((b*x^3 + a)^3*a^3)
```

Mupad [B]
time = 4.93, size = 241, normalized size = 1.00

$$\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(560\,c\,d+196\,d^2\,x+\operatorname{root}\left(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k\right)^2\,a^7\,b\,59049+\operatorname{root}\left(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k\right)\,a^3\,b\,c\,x\,9720\right)}{a^6\,6561}\right)\operatorname{root}\left(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k\right)\right)+\frac{\frac{67\,d\,x^2}{162\,a}+\frac{41\,c\,x}{81\,a}+\frac{20\,b^2\,c\,x^7}{81\,a^3}+\frac{14\,b^2\,d\,x^8}{81\,a^3}+\frac{52\,b\,c\,x^4}{81\,a^2}+\frac{77\,b\,d\,x^5}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^3)^4,x)
```

```
[Out] symsum(log((b*(560*c*d + 196*d^2*x + 59049*root(14348907*a^11*b^2*z^3 + 408
240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)^2*a^7*b + 9720*root(14348
907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)*a^3
*b*c*x))/(6561*a^6))*root(14348907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 6400
0*b*c^3 + 2744*a*d^3, z, k), k, 1, 3) + ((67*d*x^2)/(162*a) + (41*c*x)/(81*
a) + (20*b^2*c*x^7)/(81*a^3) + (14*b^2*d*x^8)/(81*a^3) + (52*b*c*x^4)/(81*a
^2) + (77*b*d*x^5)/(162*a^2))/(a^3 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6)
```

3.11 $\int \frac{a+bx}{d+ex^3} dx$

Optimal. Leaf size=161

$$-\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}e^{2/3}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\right.}{6d^{2/3}\sqrt[3]{e}}$$

```
[Out] -1/3*(b*d^(1/3)-a*e^(1/3))*ln(d^(1/3)+e^(1/3)*x)/d^(2/3)/e^(2/3)-1/6*(a-b*d
^(1/3)/e^(1/3))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(1/3)-1
/3*(b*d^(1/3)+a*e^(1/3))*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/
d^(2/3)/e^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1874, 31, 648, 631, 210, 642}

$$-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\text{ArcTan}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}e^{2/3}}-\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(d + e*x^3),x]
```

```
[Out] -(((b*d^(1/3) + a*e^(1/3))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))
])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) - a*e^(1/3))*Log[d^(1/3) + e^(1
/3)*x])/(3*d^(2/3)*e^(2/3)) - ((a - (b*d^(1/3))/e^(1/3))*Log[d^(2/3) - d^(1
/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx}{d+ex^3}\,dx = \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}\,x}\,dx}{3d^{2/3}} + \frac{\int \frac{\sqrt[3]{d}\left(b\sqrt[3]{d}+2a\sqrt[3]{e}\right)+\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\sqrt[3]{e}\,x}{d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2}\,dx}{3d^{2/3}\sqrt[3]{e}}$$

$$= \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{1}{2}\left(\frac{a}{\sqrt[3]{d}}+\frac{b}{\sqrt[3]{e}}\right)\int \frac{1}{d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2}\,dx + \cdots$$

$$= \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}} + \cdots$$

$$= -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}e^{2/3}} + \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3d^{2/3}\sqrt[3]{e}} + \cdots$$

Mathematica [A]
time = 0.04, size = 125, normalized size = 0.78

$$\frac{-2\sqrt{3}\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}\,x}{\sqrt[3]{d}}}{\sqrt{3}}\right)-\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\left(2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)-\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)\right)}{6d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(d + e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b*d^(1/3) + a*e^(1/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt
[3]] - (b*d^(1/3) - a*e^(1/3))*(2*Log[d^(1/3) + e^(1/3)*x] - Log[d^(2/3) -
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))
```

Maple [A]
time = 0.32, size = 186, normalized size = 1.16

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(e\_Z^3+d\right)}\frac{(\_Rb+a)\ln(x-\_R)}{\_R^2}}{3e}$
default	$a\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)+b\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(e*x^3+d),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)
*x+(d/e)^(2/3))+1/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)
*x-1)))+b*(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(d
/e)^(1/3)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d
/e)^(1/3)*x-1)))
```

Maxima [A]
time = 0.59, size = 105, normalized size = 0.65

$$\frac{\sqrt{3}\left(bd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+a\right)\arctan\left(-\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}-2x\right)e^{\frac{1}{3}}}{3d^{\frac{1}{3}}}\right)e^{\left(-\frac{1}{3}\right)}}{3d^{\frac{2}{3}}}+\frac{\left(bd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}-a\right)e^{\left(-\frac{1}{3}\right)}\log\left(-d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)}+x^2+d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6d^{\frac{2}{3}}}-\frac{\left(bd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}-a\right)e^{\left(-\frac{1}{3}\right)}\log\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+x\right)}{3d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b*d^(1/3)*e^(-1/3) + a)*arctan(-1/3*sqrt(3)*(d^(1/3)*e^(-1/3)
- 2*x)*e^(1/3)/d^(1/3))*e^(-1/3)/d^(2/3) + 1/6*(b*d^(1/3)*e^(-1/3) - a)*e^(
-1/3)*log(-d^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2/3))/d^(2/3) - 1/3*(b*d^
(1/3)*e^(-1/3) - a)*e^(-1/3)*log(d^(1/3)*e^(-1/3) + x)/d^(2/3)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.12, size = 1961, normalized size = 12.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3
*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^
3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)
 - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^
3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(
2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)
/(d^2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d + (b^3*d + a^3*e)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2
))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^
2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)
 - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^
3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*e)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^
(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2)
- (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1
/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a
^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d + 2*(b^3*d + a^3*e)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d -
 a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d
+ a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 2*a^2*d*e
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d -
 a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d
+ a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*
e))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*
d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3
*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*
e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3
*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*e)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d -
 a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d
+ a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2
```

```
*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^
2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d + 2*(b^3*d
+ a^3*e)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(
d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3)
 + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*
b*d^2*e + 2*a^2*d*e)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(
d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3)
 + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^
2*d*e + 16*a*b)/(d*e)))
```

Sympy [A]
time = 0.35, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^3d^2e^2 + 9tabde - a^3e + b^3d, \left(t \mapsto t\log\left(x + \frac{9t^2bd^2e + 3ta^2de + 2ab^2d}{a^3e + b^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x**3+d),x)
```

```
[Out] RootSum(27*_t**3*d**2*e**2 + 9*_t*a*b*d*e - a**3*e + b**3*d, Lambda(_t, _t*
log(x + (9*_t**2*b*d**2*e + 3*_t*a**2*d*e + 2*a*b**2*d)/(a**3*e + b**3*d)))
)
```

Giac [A]
time = 1.73, size = 132, normalized size = 0.82

$$-\frac{\sqrt{3}\left(ae - \left(-de^2\right)^{\frac{1}{3}}b\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)}{3\left(-de^2\right)^{\frac{2}{3}}} - \frac{\left(ae + \left(-de^2\right)^{\frac{1}{3}}b\right)\log\left(x^2 + \left(-de^{(-1)}\right)^{\frac{1}{3}}x + \left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6\left(-de^2\right)^{\frac{2}{3}}} - \frac{\left(-de^{(-1)}\right)^{\frac{1}{3}}\left(\left(-de^{(-1)}\right)^{\frac{1}{3}}b + a\right)\log\left(\left|x - \left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(a*e - (-d*e^2)^(1/3)*b)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))
^(1/3))/(-d*e^(-1))^(1/3))/(-d*e^2)^(2/3) - 1/6*(a*e + (-d*e^2)^(1/3)*b)*lo
g(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/(-d*e^2)^(2/3) - 1/3*(-d*e
^(-1))^(1/3)*((-d*e^(-1))^(1/3)*b + a)*log(abs(x - (-d*e^(-1))^(1/3)))/d
```

Mupad [B]
time = 4.85, size = 127, normalized size = 0.79

$$\sum_{k=1}^{3}\ln\left(e\left(a\,b + b^2\,x + \operatorname{root}\left(27\,d^2\,e^2\,z^3 + 9\,a\,b\,d\,e\,z + b^3\,d - a^3\,e, z, k\right)^2\,d\,e\,9 + \operatorname{root}\left(27\,d^2\,e^2\,z^3 + 9\,a\,b\,d\,e\,z + b^3\,d - a^3\,e, z, k\right)\;a\,e\,x\,3\right)\right)\,\operatorname{root}\left(27\,d^2\,e^2\,z^3 + 9\,a\,b\,d\,e\,z + b^3\,d - a^3\,e, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(d + e*x^3),x)
```

```
[Out] symsum(log(e*(a*b + b^2*x + 9*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d - a
^3*e, z, k)^2*d*e + 3*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d - a^3*e, z,
 k)*a*e*x))*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d - a^3*e, z, k), k, 1,
 3)
```

3.12 $\int \frac{a+bx}{d-ex^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}e^{2/3}}-\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{2/3}}+\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(d^{2/3}+\right.}{6d^{2/3}e^{2/}}$$

```
[Out] -1/3*(b*d^(1/3)+a*e^(1/3))*ln(d^(1/3)-e^(1/3)*x)/d^(2/3)/e^(2/3)+1/6*(b*d^(
1/3)+a*e^(1/3))*ln(d^(2/3)+d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(2/3)-1
/3*(b*d^(1/3)-a*e^(1/3))*arctan(1/3*(d^(1/3)+2*e^(1/3)*x)/d^(1/3)*3^(1/2))/
d^(2/3)/e^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1875, 31, 648, 631, 210, 642}

$$-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\text{ArcTan}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}e^{2/3}}+\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(d^{2/3}+\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(d - e*x^3),x]
```

```
[Out] -(((b*d^(1/3) - a*e^(1/3))*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))
])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) + a*e^(1/3))*Log[d^(1/3) - e^(1
/3)*x])/(3*d^(2/3)*e^(2/3)) + ((b*d^(1/3) + a*e^(1/3))*Log[d^(2/3) + d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1875

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Dist[r*((B*r + A*s)/(3*a*
s)), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s*(B*
r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && N
eQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+bx}{d-ex^3}\,dx = \frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e}\,x}\,dx}{3d^{2/3}} - \frac{\int \frac{\sqrt[3]{d}\left(b\sqrt[3]{d}-2a\sqrt[3]{e}\right)-\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\sqrt[3]{e}\,x}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2}\,dx}{3d^{2/3}\sqrt[3]{e}}$$

$$= -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{2/3}} - \frac{1}{2}\left(-\frac{a}{\sqrt[3]{d}}+\frac{b}{\sqrt[3]{e}}\right)\int \frac{1}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2}\,dx$$

$$= -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}} +$$

$$= -\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}e^{2/3}} - \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}+\right.}{}$$

Mathematica [A]
time = 0.03, size = 125, normalized size = 0.78

$$\frac{-2\sqrt{3}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{e}\,x}{\sqrt[3]{d}}}{\sqrt{3}}\right)-\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\left(2\log\left(\sqrt[3]{d}-\sqrt[3]{e}\,x\right)-\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2\right)\right)}{6d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(d - e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b*d^(1/3) - a*e^(1/3))*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqrt
[3]] - (b*d^(1/3) + a*e^(1/3))*(2*Log[d^(1/3) - e^(1/3)*x] - Log[d^(2/3) +
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))
```

Maple [A]
time = 0.32, size = 188, normalized size = 1.17

method	result
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(e\_Z^3-d\right)}\frac{\left(\_Rb+a\right)\ln\left(x-\_R\right)}{\_R^2}}{3e}$
default	$a\left(-\frac{\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^2+\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)+b\left(-\frac{\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-e*x^3+d),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/e/(d/e)^(2/3)*ln(x-(d/e)^(1/3))+1/6/e/(d/e)^(2/3)*ln(x^2+(d/e)^(1/3
)*x+(d/e)^(2/3))+1/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3
)*x+1)))+b*(-1/3/e/(d/e)^(1/3)*ln(x-(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2+(
d/e)^(1/3)*x+(d/e)^(2/3))-1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(
d/e)^(1/3)*x+1)))
```

Maxima [A]
time = 1.01, size = 103, normalized size = 0.64

$$-\frac{\sqrt{3}\left(bd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}-a\right)\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+2x\right)e^{\frac{1}{3}}}{3d^{\frac{1}{3}}}\right)e^{\left(-\frac{1}{3}\right)}}{3d^{\frac{2}{3}}}+\frac{\left(bd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+a\right)e^{\left(-\frac{1}{3}\right)}\log\left(d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)}+x^2+d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6d^{\frac{2}{3}}}-\frac{\left(bd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+a\right)e^{\left(-\frac{1}{3}\right)}\log\left(-d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+x\right)}{3d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-e*x^3+d),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(b*d^(1/3)*e^(-1/3) - a)*arctan(1/3*sqrt(3)*(d^(1/3)*e^(-1/3)
+ 2*x)*e^(1/3)/d^(1/3))*e^(-1/3)/d^(2/3) + 1/6*(b*d^(1/3)*e^(-1/3) + a)*e^(
-1/3)*log(d^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2/3))/d^(2/3) - 1/3*(b*d^(
1/3)*e^(-1/3) + a)*e^(-1/3)*log(-d^(1/3)*e^(-1/3) + x)/d^(2/3)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.22, size = 1905, normalized size = 11.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-e*x^3+d),x, algorithm="fricas")
```

```
[Out] -1/18*(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a
^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(
d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(1/36*(9*(I*sqrt(3) +
 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3
) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*
d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b^
3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sq
rt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^
2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d - (b^3*d - a^3*e)*x) + 1/36*(9*(I*sqrt(
3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^
(1/3) + 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e
^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1
/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d
*e - 144*a*b)/(d*e)) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^
2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(-1/36*(9*(I*sqrt(3) +
1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)
 + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d
 - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b^3
*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqr
t(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2
*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d - 2*(b^3*d - a^3*e)*x + 1/12*sqrt(1/3)*(
(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/
(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^
2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 6*a^2*d*e)*sqrt(-((9
*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d
^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2)
 - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e - 144*a*b)/(d*e))) + 1/36*
(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/
(d^2*e^2))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^
3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) +
1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^
(1/3)))^2*d*e - 144*a*b)/(d*e)) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d +
 a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(-1/36*(9*(I
*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*
e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) -
1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/6*(9*(I*sqrt(3) + 1)*
(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) +
```

```
a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d -
a^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d - 2*(b^3*d - a^3*e)*x - 1/12*
sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*
d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3
*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 6*a^2*d*e
)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d
- a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e
)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e - 144*a*b)/(d*e
)))
```

Sympy [A]

time = 0.34, size = 78, normalized size = 0.48

$$-\operatorname{RootSum}\left(27t^3d^2e^2 - 9tabde - a^3e - b^3d, \left(t \mapsto t\log\left(x + \frac{9t^2bd^2e - 3ta^2de - 2ab^2d}{a^3e - b^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-e*x**3+d),x)
```

```
[Out] -RootSum(27*_t**3*d**2*e**2 - 9*_t*a*b*d*e - a**3*e - b**3*d, Lambda(_t, _t
*log(x + (9*_t**2*b*d**2*e - 3*_t*a**2*d*e - 2*a*b**2*d)/(a**3*e - b**3*d))
))
```

Giac [A]

time = 1.56, size = 115, normalized size = 0.71

$$-\frac{\sqrt{3}\left(bd^{\frac{2}{3}}e^{\frac{4}{3}} - ad^{\frac{1}{3}}e^{\frac{5}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+2x\right)e^{\frac{1}{3}}}{3d^{\frac{1}{3}}}\right)e^{(-2)}}{3d} - \frac{\left(bd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)} + a\right)e^{\left(-\frac{1}{3}\right)}\log\left(\left|-d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)} + x\right|\right)}{3d^{\frac{2}{3}}} + \frac{\left(bd^{\frac{2}{3}}e^{\frac{4}{3}} + ad^{\frac{1}{3}}e^{\frac{5}{3}}\right)e^{(-2)}\log\left(d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)} + x^2 + d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*d^(2/3)*e^(4/3) - a*d^(1/3)*e^(5/3))*arctan(1/3*sqrt(3)*(d^
(1/3)*e^(-1/3) + 2*x)*e^(1/3)/d^(1/3))*e^(-2)/d - 1/3*(b*d^(1/3)*e^(-1/3) +
 a)*e^(-1/3)*log(abs(-d^(1/3)*e^(-1/3) + x))/d^(2/3) + 1/6*(b*d^(2/3)*e^(4/
3) + a*d^(1/3)*e^(5/3))*e^(-2)*log(d^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2
/3))/d
```

Mupad [B]

time = 0.21, size = 124, normalized size = 0.77

$$\sum_{k=1}^{3}\ln\left(e\left(a\,b+b^2\,x-\mathrm{root}\left(27\,d^2\,e^2\,z^3-9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)^2\,d\,e\,9-\mathrm{root}\left(27\,d^2\,e^2\,z^3-9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)\,a\,e\,x\,3\right)\right)\,\mathrm{root}\left(27\,d^2\,e^2\,z^3-9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(d - e*x^3),x)
```

```
[Out] symsum(log(e*(a*b + b^2*x - 9*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d + a
^3*e, z, k)^2*d*e - 3*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d + a^3*e, z,
 k)*a*e*x))*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d + a^3*e, z, k), k, 1,
 3)
```

3.13 $\int \frac{1+x}{1+x^3} \, dx$

Optimal. Leaf size=19

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -2/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1600, 632, 210}

$$-\frac{2\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(1 + x^3),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x}{1+x^3}\,dx &= \int \frac{1}{1-x+x^2}\,dx \\
&= -\left(2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right)\right) \\
&= -\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(1 + x^3),x]
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Maple [A]
time = 0.32, size = 17, normalized size = 0.89

method	result
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$
meijerg	$-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}-\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(\right.}{6(x^3)^{\frac{1}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^3+1),x,method=_RETURNVERBOSE)
[Out] 2/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.94, size = 16, normalized size = 0.84

$$\frac{2}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+1),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

Fricas [A]
time = 0.44, size = 16, normalized size = 0.84

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+1),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

Sympy [A]
time = 0.03, size = 26, normalized size = 1.37

$$\frac{2\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**3+1),x)
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Giac [A]
time = 0.81, size = 16, normalized size = 0.84

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+1),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

Mupad [B]
time = 4.70, size = 16, normalized size = 0.84

$$\frac{2\,\sqrt{3}\ \operatorname{atan}\left(\frac{\sqrt{3}\,(2\,x-1)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x^3 + 1),x)
[Out] (2*3^(1/2)*atan((3^(1/2)*(2*x - 1))/3))/3
```

3.14 $\int \frac{1-x}{1-x^3} \, dx$

Optimal. Leaf size=19

$$\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1600, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 - x^3),x]
```

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\int \frac{1-x}{1-x^3}\,dx = \int \frac{1}{1+x+x^2}\,dx$$
$$= -\left(2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right)\right)$$
$$= \frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(1 - x^3),x]
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Maple [A]
time = 0.33, size = 17, normalized size = 0.89

method	result
default	$\frac{2\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$\frac{2\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\right.}{3(x^3)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(-x^3+1),x,method=_RETURNVERBOSE)
[Out] 2/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 1.00, size = 16, normalized size = 0.84

$$\frac{2}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^3+1),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A]
time = 0.44, size = 16, normalized size = 0.84

$$\frac{2}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^3+1),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Sympy [A]
time = 0.03, size = 26, normalized size = 1.37

$$\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x**3+1),x)
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Giac [A]
time = 0.71, size = 16, normalized size = 0.84

$$\frac{2}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^3+1),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Mupad [B]
time = 4.67, size = 16, normalized size = 0.84

$$\frac{2\,\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,(2\,x+1)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x^3 - 1),x)
[Out] (2*3^(1/2)*atan((3^(1/2)*(2*x + 1))/3))/3
```

3.15 $\int \frac{1+x}{1-x^3} \, dx$

Optimal. Leaf size=22

$$-\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)$$

```
[Out] -2/3*ln(1-x)+1/3*ln(x^2+x+1)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1875, 31, 642}

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(1 - x^3),x]
```

```
[Out] (-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1875

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Dist[r*((B*r + A*s)/(3*a*
s)), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s*(B*
r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && N
eQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{1-x^3}\,dx &= -\left(\frac{1}{3}\int \frac{-1-2x}{1+x+x^2}\,dx\right)+\frac{2}{3}\int \frac{1}{1-x}\,dx\\ &= -\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$-\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(1 - x^3),x]
```

```
[Out] (-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

Maple [A]
time = 0.34, size = 17, normalized size = 0.77

method	result
default	$\frac{\ln(x^2+x+1)}{3}-\frac{2\ln(x-1)}{3}$
norman	$\frac{\ln(x^2+x+1)}{3}-\frac{2\ln(x-1)}{3}$
risch	$\frac{\ln(x^2+x+1)}{3}-\frac{2\ln(x-1)}{3}$
meijerg	$-\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{\ }\right.}{3(x^3)^{\frac{1}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(x^2+x+1)-2/3*ln(x-1)
```

Maxima [A]
time = 0.59, size = 16, normalized size = 0.73

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

Fricas [A]
time = 0.44, size = 16, normalized size = 0.73

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.77

$$-\frac{2\log(x-1)}{3}+\frac{\log(x^2+x+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**3+1),x)
```

```
[Out] -2*log(x - 1)/3 + log(x**2 + x + 1)/3
```

Giac [A]
time = 0.73, size = 17, normalized size = 0.77

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(abs(x - 1))
```

Mupad [B]
time = 0.06, size = 16, normalized size = 0.73

$$\frac{\ln(x^2+x+1)}{3}-\frac{2\,\ln(x-1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 1)/(x^3 - 1),x)
```

```
[Out] log(x + x^2 + 1)/3 - (2*log(x - 1))/3
```

3.16 $\int \frac{1-x}{1+x^3} dx$

Optimal. Leaf size=22

$$\frac{2}{3}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)$$

```
[Out] 2/3*ln(1+x)-1/3*ln(x^2-x+1)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1874, 31, 642}

$$\frac{2}{3}\log(x+1)-\frac{1}{3}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 + x^3),x]
```

```
[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{1+x^3}\,dx &= \frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx+\frac{2}{3}\int \frac{1}{1+x}\,dx\\ &= \frac{2}{3}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{2}{3}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(1 + x^3),x]
[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Maple [A]
time = 0.35, size = 19, normalized size = 0.86

method	result
default	$\frac{2\ln(x+1)}{3}-\frac{\ln(x^2-x+1)}{3}$
norman	$\frac{2\ln(x+1)}{3}-\frac{\ln(x^2-x+1)}{3}$
risch	$\frac{2\ln(x+1)}{3}-\frac{\ln(x^2-x+1)}{3}$
meijerg	$\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}-\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{1}{3}}}+\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}-\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right.}{6(x^3)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x^3+1),x,method=_RETURNVERBOSE)
[Out] 2/3*ln(x+1)-1/3*ln(x^2-x+1)
```

Maxima [A]
time = 0.66, size = 18, normalized size = 0.82

$$-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^3+1),x, algorithm="maxima")
[Out] -1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

Fricas [A]
time = 0.42, size = 18, normalized size = 0.82

$$-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^3+1),x, algorithm="fricas")
[Out] -1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\log{(x+1)}}{3}-\frac{\log{(x^2-x+1)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**3+1),x)
[Out] 2*log(x + 1)/3 - log(x**2 - x + 1)/3
```

Giac [A]
time = 0.70, size = 19, normalized size = 0.86

$$-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^3+1),x, algorithm="giac")
[Out] -1/3*log(x^2 - x + 1) + 2/3*log(abs(x + 1))
```

Mupad [B]
time = 0.11, size = 18, normalized size = 0.82

$$\frac{2\ \ln{(x+1)}}{3}-\frac{\ln{(x^2-x+1)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/(x^3 + 1),x)
[Out] (2*log(x + 1))/3 - log(x^2 - x + 1)/3
```

3.17 $\int \frac{3-x}{1-x^3} \, dx$

Optimal. Leaf size=41

$$\frac{4\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)$$

```
[Out] -2/3*ln(1-x)+1/3*ln(x^2+x+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1875, 31, 648, 632, 210, 642}

$$\frac{4\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x)/(1 - x^3),x]
```

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1 + x + x^2]
/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1875

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Dist[r*((B*r + A*s)/(3*a*
s)), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s*(B*
r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && N
eQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3-x}{1-x^3}\,dx &= -\left(\frac{1}{3}\int \frac{-7-2x}{1+x+x^2}\,dx\right)+\frac{2}{3}\int \frac{1}{1-x}\,dx\\
&= -\frac{2}{3}\log(1-x)+\frac{1}{3}\int \frac{1+2x}{1+x+x^2}\,dx+2\int \frac{1}{1+x+x^2}\,dx\\
&= -\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)-4\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= \frac{4\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$\frac{4\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x)/(1 - x^3),x]
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1 + x + x^2]
/3
```

Maple [A]
time = 0.35, size = 33, normalized size = 0.80

method	result
default	$\frac{\ln(x^2+x+1)}{3}+\frac{4\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{2\ln(x-1)}{3}$
risch	$-\frac{2\ln(x-1)}{3}+\frac{\ln(16x^2+16x+16)}{3}+\frac{4\sqrt{3}\,\arctan\left(\frac{(4x+2)\sqrt{3}}{6}\right)}{3}$
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{(x^3)^{\frac{1}{3}}}+\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\right.}{3(x^3)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-x)/(-x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(x^2+x+1)+4/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-2/3*ln(x-1)
```

Maxima [A]
time = 1.25, size = 32, normalized size = 0.78

$$\frac{4}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{3}\,\log\left(x^2+x+1\right)-\frac{2}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^3+1),x, algorithm="maxima")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log(
x - 1)
```

Fricas [A]
time = 0.42, size = 32, normalized size = 0.78

$$\frac{4}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{3}\,\log\left(x^2+x+1\right)-\frac{2}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^3+1),x, algorithm="fricas")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log(
x - 1)
```

Sympy [A]
time = 0.05, size = 44, normalized size = 1.07

$$-\frac{2\log\left(x-1\right)}{3}+\frac{\log\left(x^2+x+1\right)}{3}+\frac{4\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x**3+1),x)
```

```
[Out] -2*log(x - 1)/3 + log(x**2 + x + 1)/3 + 4*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt
(3)/3)/3
```

Giac [A]
time = 0.81, size = 33, normalized size = 0.80

$$\frac{4}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{3}\,\log\left(x^2+x+1\right)-\frac{2}{3}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log(
abs(x - 1))
```

Mupad [B]
time = 0.14, size = 46, normalized size = 1.12

$$-\frac{2\,\ln\left(x-1\right)}{3}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3)/(x^3 - 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*2i)/3 + 1/3) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*2i)/3 - 1/3) - (2*log(x - 1))/3
```

3.18 $\int \frac{c+dx}{c^3+d^3x^3}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tan^{-1}\left(\frac{c-2dx}{\sqrt{3}\,c}\right)}{\sqrt{3}\,cd}$$

```
[Out] -2/3*arctan(1/3*(-2*d*x+c)/c*3^(1/2))/c/d*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1600, 631, 210}

$$-\frac{2\text{ArcTan}\left(\frac{c-2dx}{\sqrt{3}\,c}\right)}{\sqrt{3}\,cd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(c^3 + d^3*x^3),x]
```

```
[Out] (-2*ArcTan[(c - 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = \int \frac{1}{c^2-cdx+d^2x^2}\,dx$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2dx}{c}\right)}{cd}$$
$$= -\frac{2\tan^{-1}\left(\frac{c-2dx}{\sqrt{3}\,c}\right)}{\sqrt{3}\,cd}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.07

$$\frac{2\tan^{-1}\left(\frac{-c+2dx}{\sqrt{3}\,c}\right)}{\sqrt{3}\,cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(c^3 + d^3*x^3),x]
[Out] (2*ArcTan[(-c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Maple [A]
time = 0.34, size = 35, normalized size = 1.21

method	result	size
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{2d\sqrt{3}\,x}{3c}-\frac{\sqrt{3}}{3}\right)}{3dc}$	29
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2d^2x-cd\right)\sqrt{3}}{3cd}\right)}{3cd}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(d^3*x^3+c^3),x,method=_RETURNVERBOSE)
[Out] 2/3*3^(1/2)/c/d*arctan(1/3*(2*d^2*x-c*d)*3^(1/2)/c/d)
```

Maxima [A]
time = 0.60, size = 34, normalized size = 1.17

$$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2\,d^2x-cd)}{3\,cd}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x - c*d)/(c*d))/(c*d)
```

Fricas [A]
time = 0.42, size = 28, normalized size = 0.97

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2dx-c)}{3c}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.06, size = 54, normalized size = 1.86

$$\frac{-\frac{\sqrt{3}i\log\left(x+\frac{-c-\sqrt{3}ic}{2d}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{-c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(d**3*x**3+c**3),x)
```

```
[Out] (-sqrt(3)*I*log(x + (-c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (-c + s
qrt(3)*I*c)/(2*d))/3)/(c*d)
```

Giac [A]
time = 0.97, size = 28, normalized size = 0.97

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2dx-c)}{3c}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

Mupad [B]
time = 0.05, size = 28, normalized size = 0.97

$$-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,d\,x}{3\,c}\right)}{3\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(c^3 + d^3*x^3),x)
[Out] -(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*d*x)/(3*c)))/(3*c*d)
```

3.19 $\int \frac{c-dx}{c^3-d^3x^3}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}\,c}\right)}{\sqrt{3}\,cd}$$

```
[Out] 2/3*arctan(1/3*(2*d*x+c)/c*3^(1/2))/c/d*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1600, 631, 210}

$$\frac{2\text{ArcTan}\left(\frac{c+2dx}{\sqrt{3}\,c}\right)}{\sqrt{3}\,cd}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x)/(c^3 - d^3*x^3),x]
```

```
[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx = \int \frac{1}{c^2+cdx+d^2x^2}\,dx$$
$$= -\frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2dx}{c}\right)}{cd}$$
$$= \frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}\,c}\right)}{\sqrt{3}\,cd}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}\,c}\right)}{\sqrt{3}\,cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x)/(c^3 - d^3*x^3),x]
```

```
[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Maple [A]
time = 0.32, size = 34, normalized size = 1.17

method	result	size
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{2d\sqrt{3}\,x}{3c}+\frac{\sqrt{3}}{3}\right)}{3dc}$	29
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2d^2x+cd\right)\sqrt{3}}{3cd}\right)}{3cd}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x+c)/(-d^3*x^3+c^3),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*3^(1/2)/c/d*arctan(1/3*(2*d^2*x+c*d)*3^(1/2)/c/d)
```

Maxima [A]
time = 0.56, size = 33, normalized size = 1.14

$$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2\,d^2x+cd)}{3\,cd}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x + c*d)/(c*d))/(c*d)
```

Fricas [A]
time = 0.46, size = 26, normalized size = 0.90

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2dx+c)}{3c}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.06, size = 53, normalized size = 1.83

$$\frac{-\frac{\sqrt{3}i\log\left(x+\frac{c-\sqrt{3}ic}{2d}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(-d**3*x**3+c**3),x)
```

```
[Out] (-sqrt(3)*I*log(x + (c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (c + sqr
t(3)*I*c)/(2*d))/3)/(c*d)
```

Giac [A]
time = 1.49, size = 26, normalized size = 0.90

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2dx+c)}{3c}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)
```

Mupad [B]
time = 0.04, size = 28, normalized size = 0.97

$$\frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}dx}{3c}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x)/(c^3 - d^3*x^3),x)
[Out] (2*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*d*x)/(3*c)))/(3*c*d)
```

3.20 $\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}\,dx$

Optimal. Leaf size=39

$$-\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

```
[Out] -2/3*B*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1600, 631, 210}

$$-\frac{2B\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] (-2*B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}\,\sqrt[3]{b}\,B + b^{2/3}Bx}{a + bx^3}\,dx = \int \frac{1}{\frac{a^{2/3}}{\sqrt[3]{b}\,B} - \frac{\sqrt[3]{a}\,x}{B} + \frac{\sqrt[3]{b}\,x^2}{B}}\,dx$$
$$= \frac{(2B)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$
$$= -\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.90

$$-\frac{2B\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] (-2*B*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(1/3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(28) = 56$.
time = 0.38, size = 195, normalized size = 5.00

method	result
default	$B\,b^{\frac{1}{3}}\left(a^{\frac{1}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right) + b^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] B*b^(1/3)*(a^(1/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1)))+b^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/
b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(30) = 60$.
time = 0.55, size = 163, normalized size = 4.18

$$\frac{\sqrt{3}\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}-Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}-Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*b^(2/3)*(a/b)^(1/3) + B*a^(1/3)*b^(1/3))*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(B*b^(2/3)*(a/b)^(1/
3) - B*a^(1/3)*b^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/
3)) - 1/3*(B*b^(2/3)*(a/b)^(1/3) - B*a^(1/3)*b^(1/3))*log(x + (a/b)^(1/3))/
(b*(a/b)^(2/3))
```

Fricas [A]
time = 0.47, size = 107, normalized size = 2.74

$$\left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^3-3a^{\frac{2}{3}}b^{\frac{1}{3}}x+3\sqrt{\frac{1}{3}}\left(2a^{\frac{2}{3}}b^{\frac{2}{3}}x^2+ab^{\frac{1}{3}}x-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-a}{bx^3+a}\right),\ \frac{2\sqrt{\frac{1}{3}}B\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2b^{\frac{1}{3}}x-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*b*x^3 - 3*a^(2/3)*b^(1/3)*x + 3*sqrt(1
/3)*(2*a^(2/3)*b^(2/3)*x^2 + a*b^(1/3)*x - a^(4/3))*sqrt(-1/a^(2/3)) - a)/(
b*x^3 + a)), 2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*b^(1/3)*x - a^(1/3))/a^(1/3)
)/a^(1/3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.12, size = 88, normalized size = 2.26

$$\frac{B\left(-\frac{\sqrt{3}\,i\log\left(x+\frac{-B\sqrt[3]{a}-\sqrt{3}\,iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3}+\frac{\sqrt{3}\,i\log\left(x+\frac{-B\sqrt[3]{a}+\sqrt{3}\,iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3}\right)}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*b**(1/3)*B+b**(2/3)*B*x)/(b*x**3+a),x)
```

```
[Out] B*(-sqrt(3)*I*log(x + (-B*a**(1/3) - sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3)))/
3 + sqrt(3)*I*log(x + (-B*a**(1/3) + sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3)))/
3)/a**(1/3)
```

Giac [A]
time = 1.78, size = 48, normalized size = 1.23

$$\frac{2\sqrt{3}\,Bb^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,b^{\frac{2}{3}}x-a^{\frac{1}{3}}b^{\frac{1}{3}}\right)}{3\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}\right)}{3\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*B*b^(1/3)*arctan(1/3*sqrt(3)*(2*b^(2/3)*x - a^(1/3)*b^(1/3))/sq
rt(a^(2/3)*b^(2/3)))/sqrt(a^(2/3)*b^(2/3))
```

Mupad [B]
time = 4.82, size = 49, normalized size = 1.26

$$\frac{2\sqrt{3}\,B\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sqrt{b}}{3\sqrt{-b}}-\frac{2\sqrt{3}\,b^{5/6}\,x}{3\,a^{1/3}\,\sqrt{-b}}\right)}{3\,a^{1/3}\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a^(1/3)*b^(1/3) + B*b^(2/3)*x)/(a + b*x^3),x)
```

```
[Out] (2*3^(1/2)*B*b^(1/2)*atanh((3^(1/2)*b^(1/2))/(3*(-b)^(1/2)) - (2*3^(1/2)*b^
(5/6)*x)/(3*a^(1/3)*(-b)^(1/2))))/(3*a^(1/3)*(-b)^(1/2))
```

3.21 $\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx$

Optimal. Leaf size=41

$$\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

```
[Out] 2/3*B*arctan(1/3*(a^(1/3)+2*(-b)^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1600, 631, 210}

$$\frac{2B\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] (2*B*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}dx = \int \frac{1}{-\frac{a^{2/3}(-b)^{2/3}}{bB}+\frac{\sqrt[3]{a}x}{B}+\frac{\sqrt[3]{-b}x^2}{B}}dx$$
$$= -\frac{(2B)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1+\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$
$$= \frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 129 vs. $2(41) = 82$.
time = 0.03, size = 129, normalized size = 3.15

$$\frac{\sqrt[3]{-b}B\left(2\sqrt{3}\left(\sqrt[3]{-b}-\sqrt[3]{b}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(\sqrt[3]{-b}+\sqrt[3]{b}\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)\right)}{6\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] ((-b)^(1/3)*B*(2*Sqrt[3]*((-b)^(1/3) - b^(1/3))*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + ((-b)^(1/3) + b^(1/3))*(2*Log[a^(1/3) + b^(1/3)*x] - Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])))/(6*a^(1/3)*b^(2/3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(30) = 60$.
time = 0.38, size = 202, normalized size = 4.93

method	result
default	$-B\,b^{\frac{1}{3}}\left(-a^{\frac{1}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\right)+(-b)^{\frac{1}{3}}\left(-\frac{\ln\left(x\right.}{3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x,method=_RETURNVERBOSE
)
```

```
[Out] -B*b^(1/3)*(-a^(1/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1)))+(-b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))
+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))*(-1)^(1/3)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(30) = 60$.
time = 0.55, size = 174, normalized size = 4.24

$$-\frac{\sqrt{3}\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}-Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="max
ima")
```

```
[Out] -1/3*sqrt(3)*(B*(-b)^(2/3)*(a/b)^(1/3) - B*a^(1/3)*(-b)^(1/3))*arctan(1/3*s
qrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2/3)) - 1/6*(B*(-b)^(2/3)
*(a/b)^(1/3) + B*a^(1/3)*(-b)^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(b*(a/b)^(2/3)) + 1/3*(B*(-b)^(2/3)*(a/b)^(1/3) + B*a^(1/3)*(-b)^(1/3))*lo
g(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Fricas [**A**]
time = 0.45, size = 114, normalized size = 2.78

$$\left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^3+3a^{\frac{2}{3}}(-b)^{\frac{1}{3}}x-3\sqrt{\frac{1}{3}}\left(2a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x^2-a(-b)^{\frac{1}{3}}x-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-a}{bx^3+a}\right),\ \frac{2\sqrt{\frac{1}{3}}B\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(-b)^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="fri
cas")
```

```
[Out] [sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*a^(2/3)*(-b)^(1/3)*x - 3*sqr
t(1/3)*(2*a^(2/3)*(-b)^(2/3)*x^2 - a*(-b)^(1/3)*x - a^(4/3))*sqrt(-1/a^(2/3
)) - a)/(b*x^3 + a)), 2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*(-b)^(1/3)*x + a^(1
/3))/a^(1/3))/a^(1/3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.14, size = 105, normalized size = 2.56

$$-\frac{B\left(-\frac{\sqrt{3}\,i\log\left(-\frac{\sqrt[3]{a}\,(-b)^{\frac{2}{3}}}{2b}-\frac{\sqrt{3}\,i\sqrt[3]{a}\,(-b)^{\frac{2}{3}}}{2b}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(-\frac{\sqrt[3]{a}\,(-b)^{\frac{2}{3}}}{2b}+\frac{\sqrt{3}\,i\sqrt[3]{a}\,(-b)^{\frac{2}{3}}}{2b}+x\right)}{3}\right)}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*(-b)**(1/3)*B-(-b)**(2/3)*B*x)/(b*x**3+a),x)
```

```
[Out] -B*(-sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) - sqrt(3)*I*a**(1/3)*(-b)**(
2/3)/(2*b) + x)/3 + sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) + sqrt(3)*I*a
**(1/3)*(-b)**(2/3)/(2*b) + x)/3)/a**(1/3)
```

Giac [A]
time = 1.95, size = 58, normalized size = 1.41

$$\frac{2\sqrt{3}\,Bb\arctan\left(-\frac{\sqrt{3}\left(2\,(-b)^{\frac{2}{3}}x+a^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)}{3\sqrt{a^{\frac{2}{3}}(-b)^{\frac{2}{3}}}}\right)}{3\sqrt{a^{\frac{2}{3}}(-b)^{\frac{2}{3}}}\,(-b)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="gia
c")
```

```
[Out] 2/3*sqrt(3)*B*b*arctan(-1/3*sqrt(3)*(2*(-b)^(2/3)*x + a^(1/3)*(-b)^(1/3))/s
qrt(a^(2/3)*(-b)^(2/3)))/(sqrt(a^(2/3)*(-b)^(2/3))*(-b)^(2/3))
```

Mupad [B]
time = 0.23, size = 49, normalized size = 1.20

$$-\frac{2\sqrt{3}\,B\sqrt{-b}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sqrt{-b}}{3\sqrt{b}}-\frac{2\sqrt{3}\,\sqrt{b}\,x}{3\,a^{1/3}\,(-b)^{1/6}}\right)}{3\,a^{1/3}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(B*(-b)^(2/3)*x - B*a^(1/3)*(-b)^(1/3))/(a + b*x^3),x)
```

```
[Out] -(2*3^(1/2)*B*(-b)^(1/2)*atanh((3^(1/2)*(-b)^(1/2))/(3*b^(1/2)) - (2*3^(1/2
)*b^(1/2)*x)/(3*a^(1/3)*(-b)^(1/6))))/(3*a^(1/3)*b^(1/2))
```

3.22 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right) dx$

Optimal. Leaf size=118

$$-\frac{B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}$$

```
[Out] -1/3*B*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(2/3)+1/6*B*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(1/3)/b^(2/3)-1/3*B*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/a^(1/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {266, 1607, 1885, 12, 298, 31, 648, 631, 210, 642}

$$\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}-\frac{B\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] -((B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^
(2/3))) - (B*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)) + (B*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx = -\left(C\int \frac{x^2}{a+bx^3}\,dx\right)+\int \frac{Bx+Cx^2}{a+bx^3}\,dx$$

$$= -\frac{C\log\left(a+bx^3\right)}{3b}+\int \frac{x(B+Cx)}{a+bx^3}\,dx$$

$$= -\frac{C\log\left(a+bx^3\right)}{3b}+C\int \frac{x^2}{a+bx^3}\,dx+\int \frac{Bx}{a+bx^3}\,dx$$

$$= B\int \frac{x}{a+bx^3}\,dx$$

$$= -\frac{B\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}}+\frac{B\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}}$$

$$= -\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{B\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}\,b^{2/3}}+\frac{B\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{\ldots}}}{2\sqrt[3]{b}\ldots}$$

$$= -\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}+\frac{B\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\ldots\right.}{6\sqrt[3]{a}\,b\ldots}$$

Mathematica [**A**]
time = 0.01, size = 90, normalized size = 0.76

$$\frac{B\left(-2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)}{6\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (B*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
+ b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(1/3)*
b^(2/3))
```

Maple [**A**]
time = 0.36, size = 94, normalized size = 0.80

method	result	size
risch	$-\frac{C\ln\left(b\,x^{3}+a\right)}{3b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(C\_R+B\right)\ln\left(x-\_R\right)}{\_R}}{3b}$	47
default	$-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{B\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*B/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*B/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/
3)*x+(a/b)^(2/3))+1/3*B*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))
```

Maxima [A]
time = 0.52, size = 159, normalized size = 1.35

$$-\frac{C\log\left(bx^{3}+a\right)}{3\,b}+\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}+B\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}-B\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,\left(2\,Ca-\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*C*log(b*x^3 + a)/b + 1/6*(2*C*(a/b)^(1/3) + B)*log(x^2 - x*(a/b)^(1/3)
 + (a/b)^(2/3))/(b*(a/b)^(1/3)) + 1/3*(C*(a/b)^(1/3) - B)*log(x + (a/b)^(1/
3))/(b*(a/b)^(1/3)) - 1/9*sqrt(3)*(2*C*a - (3*B*(a/b)^(2/3) + 2*C*a/b)*b)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

Fricas [A]
time = 0.46, size = 310, normalized size = 2.63

$$\left[\frac{3\sqrt{\frac{1}{3}}\,Bab\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^{2}x^{3}-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^{2}\right)^{\frac{2}{3}}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}-3\left(-ab^{2}\right)^{\frac{2}{3}}x}{bx^{3}+a}\right)+\left(-ab^{2}\right)^{\frac{2}{3}}B\log\left(b^{2}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}bx+\left(-ab^{2}\right)^{\frac{2}{3}}\right)-2\left(-ab^{2}\right)^{\frac{2}{3}}B\log\left(bx-\left(-ab^{2}\right)^{\frac{1}{3}}\right)}{6\,ab^{2}},\ \frac{6\sqrt{\frac{1}{3}}\,Bab\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx+\left(-ab^{2}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}}{b}\right)+\left(-ab^{2}\right)^{\frac{2}{3}}B\log\left(b^{2}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}bx+\left(-ab^{2}\right)^{\frac{2}{3}}\right)-2\left(-ab^{2}\right)^{\frac{2}{3}}B\log\left(bx-\left(-ab^{2}\right)^{\frac{1}{3}}\right)}{6\,ab^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*B*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqr
t(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3
```

```
)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*B*log(b^2*x^2 + (-
a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*B*log(b*x - (-a*b^2)^
(1/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*B*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt
(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*
B*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*B*l
og(b*x - (-a*b^2)^(1/3)))/(a*b^2)]
```

Sympy [A]
time = 0.06, size = 26, normalized size = 0.22

$$B\,\mathrm{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x)/(b*x**3+a),x)
```

```
[Out] B*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

Giac [A]
time = 1.61, size = 103, normalized size = 0.87

$$\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}}-\frac{B\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}}-\frac{B\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*B*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(-a*b^2
)^(1/3) - 1/6*B*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(1/3) - 1
/3*B*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a
```

Mupad [B]
time = 4.94, size = 98, normalized size = 0.83

$$-\frac{B\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{1/3}\,b^{2/3}}+\frac{\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2{}\mathrm{i}\right)\,\left(B-\sqrt{3}\,B\,1{}\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}+\frac{\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2{}\mathrm{i}\right)\,\left(B+\sqrt{3}\,B\,1{}\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

```
[Out] (log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i - 2*a^(1/3))*(B - 3^(1/2)*B*1i))/(6*a
^(1/3)*b^(2/3)) - (B*log(b^(1/3)*x + a^(1/3)))/(3*a^(1/3)*b^(2/3)) + (log(3
^(1/2)*a^(1/3)*2i + 4*b^(1/3)*x - 2*a^(1/3))*(B + 3^(1/2)*B*1i))/(6*a^(1/3)
*b^(2/3))
```

3.23 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx$

Optimal. Leaf size=118

$$-\frac{A\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*A*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(1/3)-1/6*A*ln(a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)-1/3*A*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/a^(2/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {266, 1885, 12, 206, 31, 648, 631, 210, 642}

$$-\frac{A\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}}-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

```
[Out] -((A*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^
(1/3))) + (A*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) - (A*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)dx = -\left(C\int \frac{x^2}{a+bx^3}dx\right)+\int \frac{A+Cx^2}{a+bx^3}dx$$

$$= -\frac{C\log(a+bx^3)}{3b}+C\int \frac{x^2}{a+bx^3}dx+\int \frac{A}{a+bx^3}dx$$

$$= A\int \frac{1}{a+bx^3}dx$$

$$= \frac{A\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^{2/3}}+\frac{A\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{3a^{2/3}}$$

$$= \frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{A\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{2\sqrt[3]{a}}-\frac{A\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}}}{6a^{2/3}\sqrt[3]{b}}$$

$$= \frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{A\text{Subst}\left(\int\right.}{}$$

$$= -\frac{A\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\right.}{6a^{2/3}}$$

Mathematica [A]
time = 0.01, size = 90, normalized size = 0.76

$$-\frac{A\left(2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

```
[Out] -1/6*(A*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1
/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(2/3
)*b^(1/3))
```

Maple [A]
time = 0.36, size = 94, normalized size = 0.80

method	result	size

risch	$-\frac{C\ln(b\,x^3+a)}{3b}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(C\_R^2+A)\ln(x-\_R)}{\_R^2}}{3b}$	49
default	$\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{A\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*A/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*A/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+1/3*A/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))
```

Maxima [A]
time = 0.51, size = 159, normalized size = 1.35

$$-\frac{C\log\left(bx^3+a\right)}{3\,b}-\frac{\sqrt{3}\left(2\,Ca-\left(3\,A\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}+\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}-A\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+A\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*C*log(b*x^3 + a)/b - 1/9*sqrt(3)*(2*C*a - (3*A*(a/b)^(1/3) + 2*C*a/b)*
b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*C*(a/
b)^(2/3) - A)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*
(C*(a/b)^(2/3) + A)*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Fricas [A]
time = 0.46, size = 305, normalized size = 2.58

$$\left[\frac{3\sqrt{\frac{1}{3}}\,Aab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-(a^2b)^{\frac{2}{3}}A\log\left(abx^2-(a^2b)^{\frac{2}{3}}x+(a^2b)^{\frac{1}{3}}a\right)+2\,(a^2b)^{\frac{2}{3}}A\log\left(abx+(a^2b)^{\frac{2}{3}}\right)}{6\,a^2b},\ \frac{6\sqrt{\frac{1}{3}}\,Aab\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-(a^2b)^{\frac{2}{3}}A\log\left(abx^2-(a^2b)^{\frac{2}{3}}x+(a^2b)^{\frac{1}{3}}a\right)+2\,(a^2b)^{\frac{2}{3}}A\log\left(abx+(a^2b)^{\frac{2}{3}}\right)}{6\,a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*A*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(
1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*A*log(a*b*x^2 - (a^2
```

```
*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*A*log(a*b*x + (a^2*b)^(2/3
)))/(a^2*b), 1/6*(6*sqrt(1/3)*A*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*
(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^
(2/3)*A*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*
A*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [A]
time = 0.06, size = 22, normalized size = 0.19

$$A\,\mathrm{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+A)/(b*x**3+a),x)
```

```
[Out] A*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

Giac [A]
time = 2.26, size = 115, normalized size = 0.97

$$-\frac{A\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}A\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{\left(-ab^2\right)^{\frac{1}{3}}A\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*A*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*sqrt(3)*(-a*b^2)^(1/
3)*A*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/6*(-a*
b^2)^(1/3)*A*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

Mupad [B]
time = 5.01, size = 96, normalized size = 0.81

$$\frac{A\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{2/3}\,b^{1/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x-\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(A-\sqrt{3}\,A\,1{}\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}-\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(A+\sqrt{3}\,A\,1{}\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

```
[Out] (A*log(b^(1/3)*x + a^(1/3)))/(3*a^(2/3)*b^(1/3)) - (log(a^(1/3) - 2*b^(1/3)
*x - 3^(1/2)*a^(1/3)*1i)*(A - 3^(1/2)*A*1i))/(6*a^(2/3)*b^(1/3)) - (log(2*b
^(1/3)*x - 3^(1/2)*a^(1/3)*1i - a^(1/3))*(A + 3^(1/2)*A*1i))/(6*a^(2/3)*b^(
1/3))
```

3.24 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx$

Optimal. Leaf size=161

$$-\frac{\left(A\sqrt[3]{b}+\sqrt[3]{a}\,B\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}+\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}\,B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(A-\frac{\sqrt[3]{a}\,B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\right.}{6a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*(A*b^(1/3)-a^(1/3)*B)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1/6*(A-a^(1
/3)*B/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)-1/
3*(A*b^(1/3)+a^(1/3)*B)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(2/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {266, 1885, 1874, 31, 648, 631, 210, 642}

$$-\frac{\left(\sqrt[3]{a}\,B+A\sqrt[3]{b}\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}-\frac{\left(A-\frac{\sqrt[3]{a}\,B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}\,B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] -(((A*b^(1/3) + a^(1/3)*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((A*b^(1/3) - a^(1/3)*B)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((A - (a^(1/3)*B)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)dx = -\left(C\int \frac{x^2}{a+bx^3}\,dx\right)+\int \frac{A+Bx+Cx^2}{a+bx^3}\,dx$$

$$= -\frac{C\log(a+bx^3)}{3b}+C\int \frac{x^2}{a+bx^3}\,dx+\int \frac{A+Bx}{a+bx^3}\,dx$$

$$= \frac{\int \frac{\sqrt[3]{a}\left(2A\sqrt[3]{b}+\sqrt[3]{a}\,B\right)+\sqrt[3]{b}\left(-A\sqrt[3]{b}+\sqrt[3]{a}\,B\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(A-\frac{\sqrt[3]{a}\,B}{\sqrt[3]{b}}\right)\int \ldots}{3a^{2/3}\ldots}$$

$$= \frac{\left(A-\frac{\sqrt[3]{a}\,B}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}\,B\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}\ldots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{\ldots}}}{6a^{2/3}b^{2/3}\ldots}$$

$$= \frac{\left(A-\frac{\sqrt[3]{a}\,B}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}\,B\right)\log\left(a^{2/3}-\ldots\right.}{6a^{2/3}b^{2/3}}$$

$$= -\frac{\left(A\sqrt[3]{b}+\sqrt[3]{a}\,B\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}+\frac{\left(A-\frac{\sqrt[3]{a}\,B}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}\ldots\right.}{3a^{2/3}\sqrt[3]{b}}$$

Mathematica [A]
time = 0.03, size = 124, normalized size = 0.77

$$\frac{-2\sqrt{3}\left(A\sqrt[3]{b}+\sqrt[3]{a}\,B\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(A\sqrt[3]{b}-\sqrt[3]{a}\,B\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(A*b^(1/3) + a^(1/3)*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + (A*b^(1/3) - a^(1/3)*B)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

Maple [A]
time = 0.35, size = 186, normalized size = 1.16

method	result
risch	$-\frac{C\ln(b\,x^3+a)}{3b}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\left(C\_R^2+B\_R+A\right)\ln(x-\_R)}{\_R^2}}{3b}$

default	$\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{A\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{6b\left(\frac{a}{b}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*A/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*A/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+1/3*A/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-1/3*B/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*B/b/(a/b)^(1/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*B*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))
```

Maxima [A]
time = 0.54, size = 188, normalized size = 1.17

$$-\frac{C\log\left(bx^{3}+a\right)}{3\,b}-\frac{\sqrt{3}\left(2\,Ca-\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\,A\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}+\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}-A\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}+A\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*C*log(b*x^3 + a)/b - 1/9*sqrt(3)*(2*C*a - (3*B*(a/b)^(2/3) + 3*A*(a/b)
^(1/3) + 2*C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a
*b) + 1/6*(2*C*(a/b)^(2/3) + B*(a/b)^(1/3) - A)*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(C*(a/b)^(2/3) - B*(a/b)^(1/3) + A)*log(x
 + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.19, size = 1961, normalized size = 12.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3
*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^
3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^
3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(
```

```
2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)
/(a^2*b^2))^(1/3)))*A^2*a*b + 2*A*B^2*a + (B^3*a + A^3*b)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2
))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^
2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^
3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*b)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^
(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2)
 - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1
/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A
^3*b)/(a^2*b^2))^(1/3)))*A^2*a*b - 2*A*B^2*a + 2*(B^3*a + A^3*b)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a -
 A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a
+ A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*B*a^2*b + 2*A^2*a*b
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a -
 A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a
+ A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*
b))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*
a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3
*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*
b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3
*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*b)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a -
 A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a
+ A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^
2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*A^2*a*b - 2*A*B^2*a + 2*(B^3*a
+ A^3*b)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(
a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3)
 + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*
B*a^2*b + 2*A^2*a*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(
a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3)
 + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^
2*a*b + 16*A*B)/(a*b)))
```

Sympy [A]
time = 0.37, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^3a^2b^2 + 9tABab - A^3b + B^3a, \left(t \mapsto t\log\left(x + \frac{9t^2Ba^2b + 3tA^2ab + 2AB^2a}{A^3b + B^3a}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x+A)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*A*B*a*b - A**3*b + B**3*a, Lambda(_t, _t*
log(x + (9*_t**2*B*a**2*b + 3*_t*A**2*a*b + 2*A*B**2*a)/(A**3*b + B**3*a)))
)
```

Giac [A]
time = 1.39, size = 147, normalized size = 0.91

$$-\frac{\sqrt{3}\left(Ab-\left(-ab^{2}\right)^{\frac{1}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{2}{3}}}-\frac{\left(Ab+\left(-ab^{2}\right)^{\frac{1}{3}}B\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{2}{3}}}-\frac{\left(Bb\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(A*b - (-a*b^2)^(1/3)*B)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(A*b + (-a*b^2)^(1/3)*B)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(B*b*(-a/b)^(1/3) + A*b)*
(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b)
```

Mupad [B]
time = 4.92, size = 127, normalized size = 0.79

$$\sum_{k=1}^{3}\ln\left(b\left(B^{2}\,x+A\,B+\mathrm{root}\left(27\,a^{2}\,b^{2}\,z^{3}+9\,A\,B\,a\,b\,z+B^{3}\,a-A^{3}\,b,z,k\right)^{2}\,a\,b\,9+A\,\mathrm{root}\left(27\,a^{2}\,b^{2}\,z^{3}+9\,A\,B\,a\,b\,z+B^{3}\,a-A^{3}\,b,z,k\right)\,b\,x\,3\right)\right)\,\mathrm{root}\left(27\,a^{2}\,b^{2}\,z^{3}+9\,A\,B\,a\,b\,z+B^{3}\,a-A^{3}\,b,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

```
[Out] symsum(log(b*(B^2*x + A*B + 9*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a - A
^3*b, z, k)^2*a*b + 3*A*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a - A^3*b,
z, k)*b*x))*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a - A^3*b, z, k), k, 1,
 3)
```

3.25 $\int \frac{bx+cx^2}{d+ex^3}\,dx$

Optimal. Leaf size=134

$$-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,\sqrt[3]{d}\,e^{2/3}}-\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3\sqrt[3]{d}\,e^{2/3}}+\frac{b\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{6\sqrt[3]{d}\,e^{2/3}}+\frac{c\log\left(d+ex^3\right)}{3e}$$

```
[Out] -1/3*b*ln(d^(1/3)+e^(1/3)*x)/d^(1/3)/e^(2/3)+1/6*b*ln(d^(2/3)-d^(1/3)*e^(1/
3)*x+e^(2/3)*x^2)/d^(1/3)/e^(2/3)+1/3*c*ln(e*x^3+d)/e-1/3*b*arctan(1/3*(d^(
1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(1/3)/e^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1607, 1885, 12, 298, 31, 648, 631, 210, 642, 266}

$$-\frac{b\text{ArcTan}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,\sqrt[3]{d}\,e^{2/3}}+\frac{b\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{6\sqrt[3]{d}\,e^{2/3}}-\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3\sqrt[3]{d}\,e^{2/3}}+\frac{c\log\left(d+ex^3\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/(d + e*x^3),x]
```

```
[Out] -((b*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(1/3)*e^
(2/3))) - (b*Log[d^(1/3) + e^(1/3)*x])/(3*d^(1/3)*e^(2/3)) + (b*Log[d^(2/3)
 - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(1/3)*e^(2/3)) + (c*Log[d + e*x^3
])/(3*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{bx+cx^2}{d+ex^3}\,dx = \int \frac{x(b+cx)}{d+ex^3}\,dx$$
$$= c\int \frac{x^2}{d+ex^3}\,dx + \int \frac{bx}{d+ex^3}\,dx$$
$$= \frac{c\log(d+ex^3)}{3e} + b\int \frac{x}{d+ex^3}\,dx$$
$$= \frac{c\log(d+ex^3)}{3e} - \frac{b\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}dx}{3\sqrt[3]{d}\sqrt[3]{e}} + \frac{b\int \frac{\sqrt[3]{d}+\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}dx}{3\sqrt[3]{d}\sqrt[3]{e}}$$
$$= -\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{c\log(d+ex^3)}{3e} + \frac{b\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}dx}{6\sqrt[3]{d}e^{2/3}} + \frac{b\int \frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x\cdots}}{2\sqrt[3]{e}}$$
$$= -\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}} + \frac{c\log(d+ex^3)}{3e} + \frac{b\text{Subst}\left(\int \cdots\right.}{\cdots}$$
$$= -\frac{b\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}} - \frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}} + \frac{c\log\cdots}{\cdots}$$

Mathematica [A]
time = 0.02, size = 122, normalized size = 0.91

$$\frac{-2\sqrt{3}b\sqrt[3]{e}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right) - 2b\sqrt[3]{e}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right) + b\sqrt[3]{e}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right) + 2c\sqrt[3]{d}\log(d+ex^3)}{6\sqrt[3]{d}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(d + e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*b*e^(1/3)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*b*e^(
1/3)*Log[d^(1/3) + e^(1/3)*x] + b*e^(1/3)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x +
 e^(2/3)*x^2] + 2*c*d^(1/3)*Log[d + e*x^3])/(6*d^(1/3)*e)
```

Maple [A]
time = 0.32, size = 108, normalized size = 0.81

method	result	size

risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}e+d\right)}\frac{\left(\_R^{2}c+\_Rb\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3e}$	36
default	$b\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)+\frac{c\ln\left(e\,x^{3}+d\right)}{3e}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(e*x^3+d),x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3
)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3
)*x-1)))+1/3*c*ln(e*x^3+d)/e
```

Maxima [A]
time = 0.51, size = 96, normalized size = 0.72

$$\frac{\sqrt{3}\,b\arctan\left(-\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}-2\,x\right)e^{\frac{1}{3}}}{3\,d^{\frac{1}{3}}}\right)e^{\left(-\frac{2}{3}\right)}}{3\,d^{\frac{1}{3}}}+\frac{\left(2\,cd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+b\right)e^{\left(-\frac{2}{3}\right)}\log\left(-d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)}+x^{2}+d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6\,d^{\frac{1}{3}}}+\frac{\left(cd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}-b\right)e^{\left(-\frac{2}{3}\right)}\log\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+x\right)}{3\,d^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(-1/3*sqrt(3)*(d^(1/3)*e^(-1/3) - 2*x)*e^(1/3)/d^(1/3))
*e^(-2/3)/d^(1/3) + 1/6*(2*c*d^(1/3)*e^(-1/3) + b)*e^(-2/3)*log(-d^(1/3)*x*
e^(-1/3) + x^2 + d^(2/3)*e^(-2/3))/d^(1/3) + 1/3*(c*d^(1/3)*e^(-1/3) - b)*e
^(-2/3)*log(d^(1/3)*e^(-1/3) + x)/d^(1/3)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.20, size = 1076, normalized size = 8.03

$$[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(1/3)*e*sqrt(((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d
*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*e^2 + 4*(3*(I*sqrt(3
```

```
) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1
/3) - 2*c/e)*c*e + 4*c^2)/e^2)*arctan(1/8*sqrt(1/3)*((3*(I*sqrt(3) + 1)*(-1
/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/
e)^2*d*e^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*
(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*d*e - 8*b^2*e*x + 4*c^2*d + 4*sqr
t(-(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3
*e)/(d*e^3))^(1/3) - 2*c/e)^2*b^2*d*e^3*x + 4*b^4*e^2*x^2 - 4*b^2*c^2*d*e*x
 + 4*b^3*c*d*e - 2*(2*b^2*c*d*e^2*x - b^3*d*e^2)*(3*(I*sqrt(3) + 1)*(-1/54*
c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)))
*sqrt(((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d -
 b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*e^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3
 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*e + 4*
c^2)/e^2)/b^3) + 2*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1
/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*e*log(1/4*(3*(I*sqrt(3) + 1)*(-
1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c
/e)^2*d*e^2 + (3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(
c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*d*e + b^2*e*x + c^2*d) - ((3*(I*sq
rt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3)
)^(1/3) - 2*c/e)*e + 6*c)*log(-(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3
/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*b^2*d*e^3*x + 4*b
^4*e^2*x^2 - 4*b^2*c^2*d*e*x + 4*b^3*c*d*e - 2*(2*b^2*c*d*e^2*x - b^3*d*e^2
)*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*
e)/(d*e^3))^(1/3) - 2*c/e)))/e
```

Sympy [A]
time = 0.14, size = 75, normalized size = 0.56

$$\operatorname{RootSum}\left(27t^{3}de^{3} - 27t^{2}cde^{2} + 9tc^{2}de + b^{3}e - c^{3}d, \left(t \mapsto t\log\left(x + \frac{9t^{2}de^{2} - 6tcde + c^{2}d}{b^{2}e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(e*x**3+d),x)
```

```
[Out] RootSum(27*_t**3*d*e**3 - 27*_t**2*c*d*e**2 + 9*_t*c**2*d*e + b**3*e - c**3
*d, Lambda(_t, _t*log(x + (9*_t**2*d*e**2 - 6*_t*c*d*e + c**2*d)/(b**2*e)))
)
```

Giac [A]
time = 1.22, size = 110, normalized size = 0.82

$$\frac{1}{3}\,ce^{(-1)}\log\left(\left|x^{3}e + d\right|\right) + \frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)}{3\left(-de^{2}\right)^{\frac{1}{3}}} - \frac{b\log\left(x^{2} + \left(-de^{(-1)}\right)^{\frac{1}{3}}x + \left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6\left(-de^{2}\right)^{\frac{1}{3}}} - \frac{\left(-de^{(-1)}\right)^{\frac{2}{3}}b\log\left(\left|x - \left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="giac")
```

```
[Out] 1/3*c*e^(-1)*log(abs(x^3*e + d)) + 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x +
(-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))/(-d*e^2)^(1/3) - 1/6*b*log(x^2 + (-d*
e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/(-d*e^2)^(1/3) - 1/3*(-d*e^(-1))^(2/3)
*b*log(abs(x - (-d*e^(-1))^(1/3)))/d
```

Mupad [B]
time = 0.19, size = 158, normalized size = 1.18

$$\sum_{k=1}^{3} \ln\left(-\mathrm{root}\left(27\,d\,e^3\,z^3-27\,c\,d\,e^2\,z^2+9\,c^2\,d\,e\,z+b^3\,e-c^3\,d,z,k\right)\,\left(6\,c\,d\,e-\mathrm{root}\left(27\,d\,e^3\,z^3-27\,c\,d\,e^2\,z^2+9\,c^2\,d\,e\,z+b^3\,e-c^3\,d,z,k\right)\,d\,e^2\,9\right)+c^2\,d+b^2\,e\,x\right)\,\mathrm{root}\left(27\,d\,e^3\,z^3-27\,c\,d\,e^2\,z^2+9\,c^2\,d\,e\,z+b^3\,e-c^3\,d,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)/(d + e*x^3),x)
```

```
[Out] symsum(log(c^2*d - root(27*d*e^3*z^3 - 27*c*d*e^2*z^2 + 9*c^2*d*e*z + b^3*e
 - c^3*d, z, k)*(6*c*d*e - 9*root(27*d*e^3*z^3 - 27*c*d*e^2*z^2 + 9*c^2*d*e
*z + b^3*e - c^3*d, z, k)*d*e^2) + b^2*e*x)*root(27*d*e^3*z^3 - 27*c*d*e^2*
z^2 + 9*c^2*d*e*z + b^3*e - c^3*d, z, k), k, 1, 3)
```

3.26 $\int \frac{a+cx^2}{d-ex^3}\,dx$

Optimal. Leaf size=134

$$\frac{a\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{e}}-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}$$

```
[Out] -1/3*a*ln(d^(1/3)-e^(1/3)*x)/d^(2/3)/e^(1/3)+1/6*a*ln(d^(2/3)+d^(1/3)*e^(1/
3)*x+e^(2/3)*x^2)/d^(2/3)/e^(1/3)-1/3*c*ln(-e*x^3+d)/e+1/3*a*arctan(1/3*(d^
(1/3)+2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(2/3)/e^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1885, 12, 206, 31, 648, 631, 210, 642, 266}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{e}}+\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{3d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(d - e*x^3),x]
```

```
[Out] (a*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(1
/3)) - (a*Log[d^(1/3) - e^(1/3)*x])/(3*d^(2/3)*e^(1/3)) + (a*Log[d^(2/3) +
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3)) - (c*Log[d - e*x^3])/
(3*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{a+cx^2}{d-ex^3}\,dx = c\int \frac{x^2}{d-ex^3}\,dx + \int \frac{a}{d-ex^3}\,dx$$

$$= -\frac{c\log(d-ex^3)}{3e} + a\int \frac{1}{d-ex^3}\,dx$$

$$= -\frac{c\log(d-ex^3)}{3e} + \frac{a\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e}\,x}\,dx}{3d^{2/3}} + \frac{a\int \frac{2\sqrt[3]{d}+\sqrt[3]{e}\,x}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2}\,dx}{3d^{2/3}}$$

$$= -\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{3d^{2/3}\sqrt[3]{e}} - \frac{c\log(d-ex^3)}{3e} + \frac{a\int \frac{1}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2}\,dx}{2\sqrt[3]{d}} + \frac{a\int \frac{\sqrt[3]{d}\sqrt[3]{e}+2e^{\cdots}}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}\,x\cdots}}{6d^{2/3}\sqrt[3]{e}}$$

$$= -\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}} - \frac{c\log(d-ex^3)}{3e} - \frac{a\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{e}} - \frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}} - \frac{c\log\cdots}{\cdots}$$

Mathematica [A]
time = 0.03, size = 123, normalized size = 0.92

$$\frac{2\sqrt{3}\,ae^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{e}\,x}{\sqrt[3]{d}}}{\sqrt{3}}\right) - 2ae^{2/3}\log\left(\sqrt[3]{d}-\sqrt[3]{e}\,x\right) + ae^{2/3}\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2\right) - 2cd^{2/3}\log(d-ex^3)}{6d^{2/3}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(d - e*x^3),x]
```

```
[Out] (2*Sqrt[3]*a*e^(2/3)*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*a*e^(2
/3)*Log[d^(1/3) - e^(1/3)*x] + a*e^(2/3)*Log[d^(2/3) + d^(1/3)*e^(1/3)*x +
e^(2/3)*x^2] - 2*c*d^(2/3)*Log[d - e*x^3])/(6*d^(2/3)*e)
```

Maple [A]
time = 0.33, size = 110, normalized size = 0.82

method	result	size
risch	$-\frac{\sum_{\_R=\text{RootOf}(\_Z^3 e-d)} \frac{(\_R^2 c+a)\ln(x-\_R)}{\_R^2}}{3e}$	36

default	$a\left(-\frac{\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)-\frac{c\ln\left(-e\,x^{3}+d\right)}{3e}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(-e*x^3+d),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/e/(d/e)^(2/3)*ln(x-(d/e)^(1/3))+1/6/e/(d/e)^(2/3)*ln(x^2+(d/e)^(1/3
)*x+(d/e)^(2/3))+1/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3
)*x+1)))-1/3*c*ln(-e*x^3+d)/e
```

Maxima [A]
time = 0.50, size = 96, normalized size = 0.72

$$\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+2\,x\right)e^{\frac{1}{3}}}{3\,d^{\frac{1}{3}}}\right)e^{\left(-\frac{1}{3}\right)}}{3\,d^{\frac{2}{3}}}-\frac{\left(2\,cd^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}-a\right)e^{\left(-\frac{1}{3}\right)}\log\left(d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)}+x^{2}+d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6\,d^{\frac{2}{3}}}-\frac{\left(cd^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}+a\right)e^{\left(-\frac{1}{3}\right)}\log\left(-d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+x\right)}{3\,d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(d^(1/3)*e^(-1/3) + 2*x)*e^(1/3)/d^(1/3))*
e^(-1/3)/d^(2/3) - 1/6*(2*c*d^(2/3)*e^(-2/3) - a)*e^(-1/3)*log(d^(1/3)*x*e^
(-1/3) + x^2 + d^(2/3)*e^(-2/3))/d^(2/3) - 1/3*(c*d^(2/3)*e^(-2/3) + a)*e^(
-1/3)*log(-d^(1/3)*e^(-1/3) + x)/d^(2/3)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.21, size = 1256, normalized size = 9.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(1/3)*e*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2
*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*
(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/
3) + 2*c/e)*c*e + 4*c^2)/e^2)*arctan(-1/8*(2*sqrt(1/3)*sqrt(((1/2)^(1/3)*(I
*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3)
 + 2*c/e)^2*d^2*e^2 + 4*a^2*e^2*x^2 - 4*a*c*d*e*x + 4*c^2*d^2 + 2*(a*d*e^2*
```

```
x - 2*c*d^2*e)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d
^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e))*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3
/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*d*e - 2*
c*d)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 +
 a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*
(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*c*e
+ 4*c^2)/e^2) - sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2
*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*d^2*e^2 - 8*a*c*d*e*x
 + 4*c^2*d^2 + 4*(a*d*e^2*x - c*d^2*e)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^
3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e))*sqrt((((1/
2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*
e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/
(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*c*e + 4*c^2)/e^2))/
(a^3*e)) - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2
 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) +
1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*d
*e + a*e*x + c*d) + (((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) -
(c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e - 6*c)*log(((1/2)^(1/3)*(I*
sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3)
+ 2*c/e)^2*d^2*e^2 + 4*a^2*e^2*x^2 - 4*a*c*d*e*x + 4*c^2*d^2 + 2*(a*d*e^2*x
 - 2*c*d^2*e)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^
2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)))/e
```

Sympy [A]
time = 0.20, size = 70, normalized size = 0.52

$$-\operatorname{RootSum}\left(27t^3d^2e^3 - 27t^2cd^2e^2 + 9tc^2d^2e - a^3e^2 - c^3d^2, \left(t \mapsto t\log\left(x + \frac{-3tde + cd}{ae}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(-e*x**3+d),x)
```

```
[Out] -RootSum(27*_t**3*d**2*e**3 - 27*_t**2*c*d**2*e**2 + 9*_t*c**2*d**2*e - a**
3*e**2 - c**3*d**2, Lambda(_t, _t*log(x + (-3*_t*d*e + c*d)/(a*e))))
```

Giac [A]
time = 1.52, size = 95, normalized size = 0.71

$$-\frac{1}{3}ce^{(-1)}\log\left(\left|x^3e - d\right|\right) + \frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)} + 2x\right)e^{\frac{1}{3}}}{3d^{\frac{1}{3}}}\right)e^{\left(-\frac{1}{3}\right)}}{3d^{\frac{2}{3}}} + \frac{ae^{\left(-\frac{1}{3}\right)}\log\left(d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)} + x^2 + d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6d^{\frac{2}{3}}} - \frac{ae^{\left(-\frac{1}{3}\right)}\log\left(\left|-d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)} + x\right|\right)}{3d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*c*e^(-1)*log(abs(x^3*e - d)) + 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(d^(1/
3)*e^(-1/3) + 2*x)*e^(1/3)/d^(1/3))*e^(-1/3)/d^(2/3) + 1/6*a*e^(-1/3)*log(d
```

```
^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2/3))/d^(2/3) - 1/3*a*e^(-1/3)*log(ab
s(-d^(1/3)*e^(-1/3) + x))/d^(2/3)
```

Mupad [B]
time = 5.01, size = 178, normalized size = 1.33

$$\sum_{k=1}^{3} \ln\left(-\left(c+\mathrm{root}\left(27\,d^2\,e^3\,z^3+27\,c\,d^2\,e^2\,z^2+9\,c^2\,d^2\,e\,z+c^3\,d^2+a^3\,e^2,z,k\right)\,e\,3\right)\,\left(c\,d+\mathrm{root}\left(27\,d^2\,e^3\,z^3+27\,c\,d^2\,e^2\,z^2+9\,c^2\,d^2\,e\,z+c^3\,d^2+a^3\,e^2,z,k\right)\,d\,e\,3+a\,e\,x\right)\right)\,\mathrm{root}\left(27\,d^2\,e^3\,z^3+27\,c\,d^2\,e^2\,z^2+9\,c^2\,d^2\,e\,z+c^3\,d^2+a^3\,e^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/(d - e*x^3),x)
```

```
[Out] symsum(log(-(c + 3*root(27*d^2*e^3*z^3 + 27*c*d^2*e^2*z^2 + 9*c^2*d^2*e*z +
 c^3*d^2 + a^3*e^2, z, k)*e)*(c*d + 3*root(27*d^2*e^3*z^3 + 27*c*d^2*e^2*z^
2 + 9*c^2*d^2*e*z + c^3*d^2 + a^3*e^2, z, k)*d*e + a*e*x))*root(27*d^2*e^3*
z^3 + 27*c*d^2*e^2*z^2 + 9*c^2*d^2*e*z + c^3*d^2 + a^3*e^2, z, k), k, 1, 3)
```

3.27 $\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tan^{-1}\left(\frac{a-2bx}{\sqrt{3}\,a}\right)}{\sqrt{3}\,b}+\frac{\log(a+bx)}{b}$$

```
[Out] ln(b*x+a)/b-2/3*arctan(1/3*(-2*b*x+a)/a*3^(1/2))/b*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1882, 31, 631, 210}

$$\frac{\log(a+bx)}{b}-\frac{2\text{ArcTan}\left(\frac{a-2bx}{\sqrt{3}\,a}\right)}{\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) + Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1882

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
```

```
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx &= \frac{a\int \frac{1}{\frac{a^2}{b^2}-\frac{ax}{b}+x^2}\,dx}{b^2}+\frac{\int \frac{1}{\frac{a}{b}+x}\,dx}{b}\\ &= \frac{\log(a+bx)}{b}+\frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2bx}{a}\right)}{b}\\ &= -\frac{2\tan^{-1}\left(\frac{a-2bx}{\sqrt{3}\,a}\right)}{\sqrt{3}\,b}+\frac{\log(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 72, normalized size = 1.95

$$\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{-a+2bx}{\sqrt{3}\,a}\right)+2\log(a+bx)-\log\left(a^2-abx+b^2x^2\right)+\log\left(a^3+b^3x^3\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*b*x)/(Sqrt[3]*a)] + 2*Log[a + b*x] - Log[a^2 - a*
b*x + b^2*x^2] + Log[a^3 + b^3*x^3])/(3*b)
```

Maple [A]
time = 0.36, size = 43, normalized size = 1.16

method	result	size
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{(2bx-a)\sqrt{3}}{3a}\right)}{3b}+\frac{\ln(bx+a)}{b}$	37
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2b^2x-ab\right)\sqrt{3}}{3ab}\right)}{3b}+\frac{\ln(bx+a)}{b}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*3^(1/2)/b*arctan(1/3*(2*b^2*x-a*b)*3^(1/2)/a/b)+ln(b*x+a)/b
```

Maxima [A]
time = 0.51, size = 42, normalized size = 1.14

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(2\,b^2x-ab)}{3\,ab}\right)}{3\,b}+\frac{\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x - a*b)/(a*b))/b + log(b*x + a)/b
```

Fricas [A]
time = 0.43, size = 36, normalized size = 0.97

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(2\,bx-a)}{3\,a}\right)+3\,\log\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x - a)/a) + 3*log(b*x + a))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.12, size = 60, normalized size = 1.62

$$\frac{-\frac{\sqrt{3}\,i\log\left(x+\frac{-a-\sqrt{3}\,ia}{2b}\right)}{3}+\frac{\sqrt{3}\,i\log\left(x+\frac{-a+\sqrt{3}\,ia}{2b}\right)}{3}+\log\left(\frac{a}{b}+x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a**2)/(b**3*x**3+a**3),x)
```

```
[Out] (-sqrt(3)*I*log(x + (-a - sqrt(3)*I*a)/(2*b))/3 + sqrt(3)*I*log(x + (-a + s
qrt(3)*I*a)/(2*b))/3 + log(a/b + x))/b
```

Giac [A]
time = 1.22, size = 37, normalized size = 1.00

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(2\,bx-a)}{3\,a}\right)}{3\,b}+\frac{\log\left(|bx+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x - a)/a)/b + log(abs(b*x + a))/b
```

Mupad [B]
time = 4.81, size = 84, normalized size = 2.27

$$\frac{\ln\left(a+b\,x\right)}{b}-\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{4\,\sqrt{3}\,a^3\,b^4}{4\,a^3\,b^4+4\,x\,a^2\,b^5}-\frac{4\,\sqrt{3}\,a^2\,b^5\,x}{4\,a^3\,b^4+4\,x\,a^2\,b^5}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x)
```

```
[Out] log(a + b*x)/b - (2*3^(1/2)*atan((4*3^(1/2)*a^3*b^4)/(4*a^3*b^4 + 4*a^2*b^5
*x) - (4*3^(1/2)*a^2*b^5*x)/(4*a^3*b^4 + 4*a^2*b^5*x)))/(3*b)
```

3.28 $\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx$

Optimal. Leaf size=39

$$\frac{2\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}\,a}\right)}{\sqrt{3}\,b}-\frac{\log(a-bx)}{b}$$

```
[Out] -ln(-b*x+a)/b+2/3*arctan(1/3*(2*b*x+a)/a*3^(1/2))/b*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1882, 31, 631, 210}

$$\frac{2\text{ArcTan}\left(\frac{a+2bx}{\sqrt{3}\,a}\right)}{\sqrt{3}\,b}-\frac{\log(a-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]
```

```
[Out] (2*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) - Log[a - b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1882

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
```

```
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx &= \frac{a\int \frac{1}{\frac{a^2}{b^2}+\frac{ax}{b}+x^2}\,dx}{b^2} - \frac{\int \frac{1}{-\frac{a}{b}+x}\,dx}{b} \\
&= -\frac{\log(a-bx)}{b} - \frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2bx}{a}\right)}{b} \\
&= \frac{2\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}\,a}\right)}{\sqrt{3}\,b} - \frac{\log(a-bx)}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 71, normalized size = 1.82

$$
\frac{2\sqrt{3}\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}\,a}\right) - 2\log(a-bx) + \log\left(a^2+abx+b^2x^2\right) - \log\left(a^3-b^3x^3\right)}{3b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)] - 2*Log[a - b*x] + Log[a^2 + a*b
*x + b^2*x^2] - Log[a^3 - b^3*x^3])/(3*b)
```

Maple [A]
time = 0.34, size = 44, normalized size = 1.13

method	result	size
risch	$\frac{2\arctan\left(\frac{(2bx+a)\sqrt{3}}{3a}\right)\sqrt{3}}{3b} - \frac{\ln(bx-a)}{b}$	38
default	$-\frac{\ln(-bx+a)}{b} + \frac{2\sqrt{3}\,\arctan\left(\frac{(2b^2x+ab)\sqrt{3}}{3ab}\right)}{3b}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(-b*x+a)/b+2/3*3^(1/2)/b*arctan(1/3*(2*b^2*x+a*b)*3^(1/2)/a/b)
```

Maxima [A]
time = 0.51, size = 44, normalized size = 1.13

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(2\,b^2x+ab)}{3\,ab}\right)}{3\,b}-\frac{\log\left(bx-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x + a*b)/(a*b))/b - log(b*x - a)/b
```

Fricas [A]
time = 0.43, size = 36, normalized size = 0.92

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(2\,bx+a)}{3\,a}\right)-3\log\left(bx-a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="fricas")
[Out] 1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x + a)/a) - 3*log(b*x - a))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.13, size = 60, normalized size = 1.54

$$-\frac{\frac{\sqrt{3}\,i\log\left(x+\frac{a-\sqrt{3}\,ia}{2b}\right)}{3}-\frac{\sqrt{3}\,i\log\left(x+\frac{a+\sqrt{3}\,ia}{2b}\right)}{3}+\log\left(-\frac{a}{b}+x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a**2)/(-b**3*x**3+a**3),x)
[Out] -(sqrt(3)*I*log(x + (a - sqrt(3)*I*a)/(2*b))/3 - sqrt(3)*I*log(x + (a + sqr
t(3)*I*a)/(2*b))/3 + log(-a/b + x))/b
```

Giac [A]
time = 1.73, size = 38, normalized size = 0.97

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(2\,bx+a)}{3\,a}\right)}{3\,b}-\frac{\log\left(|bx-a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x + a)/a)/b - log(abs(b*x - a))/b
```

Mupad [B]
time = 0.09, size = 86, normalized size = 2.21

$$\frac{2\sqrt{3}\ \mathrm{atan}\left(\frac{4\sqrt{3}\ a^3\,b^4}{4\,a^3\,b^4-4\,a^2\,b^5\,x}+\frac{4\sqrt{3}\ a^2\,b^5\,x}{4\,a^3\,b^4-4\,a^2\,b^5\,x}\right)}{3\,b}-\frac{\ln\left(a-b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x)
```

```
[Out] (2*3^(1/2)*atan((4*3^(1/2)*a^3*b^4)/(4*a^3*b^4 - 4*a^2*b^5*x) + (4*3^(1/2)*
a^2*b^5*x)/(4*a^3*b^4 - 4*a^2*b^5*x)))/(3*b) - log(a - b*x)/b
```

3.29 $\int \frac{8C+b^{2/3}Cx^2}{8+bx^3} dx$

Optimal. Leaf size=48

$$-\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{b}\,x}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{C\log\left(2+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}$$

[Out] C*ln(2+b^(1/3)*x)/b^(1/3)-2/3*C*arctan(1/3*(1-b^(1/3)*x)*3^(1/2))/b^(1/3)*3^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1877, 31, 631, 210}

$$\frac{C\log\left(\sqrt[3]{b}\,x+2\right)}{\sqrt[3]{b}}-\frac{2C\,\text{ArcTan}\left(\frac{1-\sqrt[3]{b}\,x}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]

[Out] (-2*C*ArcTan[(1 - b^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (C*Log[2 + b^(1/3)*x])/b^(1/3)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1877

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx &= \frac{(2C)\int \frac{1}{\frac{4}{b^{2/3}}-\frac{2x}{\sqrt[3]{b}}+x^2}\,dx}{b^{2/3}}+\frac{C\int \frac{1}{\frac{2}{\sqrt[3]{b}}+x}\,dx}{\sqrt[3]{b}}\\ &= \frac{C\log\left(2+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+\frac{(2C)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}\\ &= -\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{b}\,x}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{C\log\left(2+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 76, normalized size = 1.58

$$\frac{C\left(2\sqrt{3}\,\tan^{-1}\left(\frac{-1+\sqrt[3]{b}\,x}{\sqrt{3}}\right)+2\log\left(2+\sqrt[3]{b}\,x\right)-\log\left(4-2\sqrt[3]{b}\,x+b^{2/3}x^2\right)+\log\left(8+bx^3\right)\right)}{3\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*ArcTan[(-1 + b^(1/3)*x)/Sqrt[3]] + 2*Log[2 + b^(1/3)*x] - Log
[4 - 2*b^(1/3)*x + b^(2/3)*x^2] + Log[8 + b*x^3]))/(3*b^(1/3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(37) = 74$.
time = 0.34, size = 115, normalized size = 2.40

method	result	s

default	$C\left(\frac{8^{\frac{1}{3}}\ln\left(x+8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}-\frac{8^{\frac{1}{3}}\ln\left(x^{2}-8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}x+8^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{1}{b}\right)^{\frac{2}{3}}}+\frac{8^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{8^{\frac{2}{3}}x}{4\left(\frac{1}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(b\,x^{3}+8\right)}{3b^{\frac{1}{3}}}\right)$	11
meijerg	$\frac{2C\left(\frac{b^{\frac{1}{3}}x\ln\left(1+\frac{\left(b\,x^{3}\right)^{\frac{1}{3}}}{2}\right)}{\left(b\,x^{3}\right)^{\frac{1}{3}}}-\frac{b^{\frac{1}{3}}x\ln\left(1-\frac{\left(b\,x^{3}\right)^{\frac{1}{3}}}{2}+\frac{\left(b\,x^{3}\right)^{\frac{2}{3}}}{4}\right)}{2\left(b\,x^{3}\right)^{\frac{1}{3}}}+\frac{b^{\frac{1}{3}}x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(b\,x^{3}\right)^{\frac{1}{3}}}{4-\left(b\,x^{3}\right)^{\frac{1}{3}}}\right)}{\left(b\,x^{3}\right)^{\frac{1}{3}}}\right)}{3b^{\frac{1}{3}}}+\frac{C\ln\left(1+\frac{b\,x^{3}}{8}\right)}{3b^{\frac{1}{3}}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x,method=_RETURNVERBOSE)
```

```
[Out] C*(1/3/b*8^(1/3)/(1/b)^(2/3)*ln(x+8^(1/3)*(1/b)^(1/3))-1/6/b*8^(1/3)/(1/b)^
(2/3)*ln(x^2-8^(1/3)*(1/b)^(1/3)*x+8^(2/3)*(1/b)^(2/3))+1/3/b*8^(1/3)/(1/b)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/4*8^(2/3)/(1/b)^(1/3)*x-1))+1/3/b^(1/3
)*ln(b*x^3+8))
```

Maxima [A]
time = 0.56, size = 47, normalized size = 0.98

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(b^{\frac{2}{3}}x-b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{3\,b^{\frac{1}{3}}}+\frac{C\log\left(\frac{b^{\frac{1}{3}}x+2}{b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(b^(2/3)*x - b^(1/3))/b^(1/3))/b^(1/3) + C
*log((b^(1/3)*x + 2)/b^(1/3))/b^(1/3)
```

Fricas [A]
time = 0.39, size = 134, normalized size = 2.79

$$\left[\frac{\sqrt{\frac{1}{3}}\,Cb\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\,\log\left(\frac{bx^{3}+6\sqrt{\frac{1}{3}}\left(bx^{2}+b^{\frac{2}{3}}x-2\,b^{\frac{1}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-6\,b^{\frac{1}{3}}x-4}{bx^{3}+8}\right)+Cb^{\frac{2}{3}}\log\left(bx+2\,b^{\frac{2}{3}}\right)}{b},\ \frac{2\sqrt{\frac{1}{3}}\,Cb^{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{2}{3}}x-b^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}\right)+Cb^{\frac{2}{3}}\log\left(bx+2\,b^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((b*x^3 + 6*sqrt(1/3)*(b*x^2 + b^(2/3)*
x - 2*b^(1/3))*sqrt(-1/b^(2/3)) - 6*b^(1/3)*x - 4)/(b*x^3 + 8)) + C*b^(2/3)
*log(b*x + 2*b^(2/3)))/b, (2*sqrt(1/3)*C*b^(2/3)*arctan(sqrt(1/3)*(b^(2/3)*
x - b^(1/3))/b^(1/3)) + C*b^(2/3)*log(b*x + 2*b^(2/3)))/b]
```

Sympy [A]
time = 0.15, size = 58, normalized size = 1.21

$$\operatorname{RootSum}\left(3t^3b^{\frac{5}{3}} - 3t^2Cb^{\frac{4}{3}} + tC^2b - C^3b^{\frac{2}{3}}, \left(t \mapsto t\log\left(x + \frac{3t\sqrt[3]{b} - C}{C\sqrt[3]{b}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b**(2/3)*C*x**2)/(b*x**3+8),x)
```

```
[Out] RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*b**(2/3),
Lambda(_t, _t*log(x + (3*_t*b**(1/3) - C)/(C*b**(1/3)))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(36) = 72.
time = 2.43, size = 115, normalized size = 2.40

$$\frac{2}{3}\sqrt{3}\,C\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(x+\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{b}\right)^{\frac{1}{3}}}\right) - \frac{1}{3}\left(Cb^{\frac{2}{3}}\left(-\frac{1}{b}\right)^{\frac{2}{3}} + 2\,C\right)\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(\left|x-2\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right|\right) + \frac{1}{3}\left(C\left(-\frac{1}{b}\right)^{\frac{1}{3}} + \frac{C}{b^{\frac{1}{3}}}\right)\log\left(x^2+2\,x\left(-\frac{1}{b}\right)^{\frac{1}{3}} + 4\left(-\frac{1}{b}\right)^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*(-1/b)^(1/3)*arctan(1/3*sqrt(3)*(x + (-1/b)^(1/3))/(-1/b)^(1/
3)) - 1/3*(C*b^(2/3)*(-1/b)^(2/3) + 2*C)*(-1/b)^(1/3)*log(abs(x - 2*(-1/b)^
(1/3))) + 1/3*(C*(-1/b)^(1/3) + C/b^(1/3))*log(x^2 + 2*x*(-1/b)^(1/3) + 4*(
-1/b)^(2/3))
```

Mupad [B]
time = 5.14, size = 147, normalized size = 3.06

$$\sum_{k=1}^{3}\ln\left(-\frac{\left(C-\mathrm{root}\left(27\,b^3\,z^3-27\,C\,b^{8/3}\,z^2+9\,C^2\,b^{7/3}\,z-9\,C^3\,b^2,z,k\right)\,b^{1/3}\,3\right)\,\left(-C+\mathrm{root}\left(27\,b^3\,z^3-27\,C\,b^{8/3}\,z^2+9\,C^2\,b^{7/3}\,z-9\,C^3\,b^2,z,k\right)\,b^{1/3}\,3+C\,b^{1/3}\,x\right)\,8}{b^{5/3}}\right)\,\mathrm{root}\left(27\,b^3\,z^3-27\,C\,b^{8/3}\,z^2+9\,C^2\,b^{7/3}\,z-9\,C^3\,b^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*C + C*b^(2/3)*x^2)/(b*x^3 + 8),x)
```

```
[Out] symsum(log(-(8*(C - 3*root(27*b^3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2*b^(7/3)*z
- 9*C^3*b^2, z, k)*b^(1/3))*(3*root(27*b^3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2*b
^(7/3)*z - 9*C^3*b^2, z, k)*b^(1/3) - C + C*b^(1/3)*x))/b^(5/3))*root(27*b^
3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2*b^(7/3)*z - 9*C^3*b^2, z, k), k, 1, 3)
```

3.30 $\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx$

Optimal. Leaf size=47

$$-\frac{C\tan^{-1}\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}+\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)$$

```
[Out] 1/4*C*ln(a^(1/3)+2*x)-1/6*C*arctan(1/3*(a^(1/3)-4*x)/a^(1/3)*3^(1/2))*3^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1877, 31, 631, 210}

$$\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)-\frac{C\text{ArcTan}\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
```

```
[Out] -1/2*(C*ArcTan[(a^(1/3) - 4*x)/(Sqrt[3]*a^(1/3))])/Sqrt[3] + (C*Log[a^(1/3)
 + 2*x])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1877

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx &= \frac{1}{4}C\int \frac{1}{\frac{\sqrt[3]{a}}{2}+x}\,dx+\frac{1}{8}\left(\sqrt[3]{a}\,C\right)\int \frac{1}{\frac{a^{2/3}}{4}-\frac{\sqrt[3]{a}\,x}{2}+x^2}\,dx\\ &= \frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)+\frac{1}{2}C\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{4x}{\sqrt[3]{a}}\right)\\ &= -\frac{C\tan^{-1}\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}}+\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 72, normalized size = 1.53

$$\frac{1}{12}C\left(-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{a}+2x\right)-\log\left(a^{2/3}-2\sqrt[3]{a}\,x+4x^2\right)+\log\left(a+8x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
[Out] (C*(-2*Sqrt[3]*ArcTan[(1 - (4*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3) + 2*x] -
 Log[a^(2/3) - 2*a^(1/3)*x + 4*x^2] + Log[a + 8*x^3]))/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(34) = 68$.
time = 0.30, size = 96, normalized size = 2.04

method	result
default	$C\left(a^{\frac{2}{3}}\left(\frac{8^{\frac{2}{3}}\ln\left(x+\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}}{8}\right)}{24a^{\frac{2}{3}}}-\frac{8^{\frac{2}{3}}\ln\left(x^2-\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}x}{8}+\frac{8^{\frac{1}{3}}a^{\frac{2}{3}}}{8}\right)}{48a^{\frac{2}{3}}}+\frac{8^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\,8^{\frac{1}{3}}x}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{24a^{\frac{2}{3}}}\right)+\frac{\ln(8x^3+a)}{12}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(a^(2/3)*(1/24*8^(2/3)/a^(2/3)*ln(x+1/8*8^(2/3)*a^(1/3))-1/48*8^(2/3)/a^(
2/3)*ln(x^2-1/8*8^(2/3)*a^(1/3)*x+1/8*8^(1/3)*a^(2/3))+1/24*8^(2/3)/a^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2*8^(1/3)/a^(1/3)*x-1)))+1/12*ln(8*x^3+a))
```

Maxima [A]
time = 0.51, size = 36, normalized size = 0.77

$$\frac{1}{6}\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(4\,x-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)+\frac{1}{4}\,C\log\left(x+\frac{1}{2}\,a^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*C*arctan(1/3*sqrt(3)*(4*x - a^(1/3))/a^(1/3)) + 1/4*C*log(x + 1
/2*a^(1/3))
```

Fricas [A]
time = 0.42, size = 40, normalized size = 0.85

$$\frac{1}{6}\sqrt{3}\,C\arctan\left(\frac{4\sqrt{3}\,a^{\frac{2}{3}}x-\sqrt{3}\,a}{3\,a}\right)+\frac{1}{4}\,C\log\left(2\,x+a^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*C*arctan(1/3*(4*sqrt(3)*a^(2/3)*x - sqrt(3)*a)/a) + 1/4*C*log(2
*x + a^(1/3))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.13, size = 85, normalized size = 1.81

$$C\left(\frac{\log\left(\frac{\sqrt[3]{a}}{2}+x\right)}{4}-\frac{\sqrt{3}\,i\log\left(x+\frac{-C\sqrt[3]{a}-\sqrt{3}\,iC\sqrt[3]{a}}{4C}\right)}{12}+\frac{\sqrt{3}\,i\log\left(x+\frac{-C\sqrt[3]{a}+\sqrt{3}\,iC\sqrt[3]{a}}{4C}\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(2/3)*C+2*C*x**2)/(8*x**3+a),x)
```

```
[Out] C*(log(a**(1/3)/2 + x)/4 - sqrt(3)*I*log(x + (-C*a**(1/3) - sqrt(3)*I*C*a**
(1/3))/(4*C))/12 + sqrt(3)*I*log(x + (-C*a**(1/3) + sqrt(3)*I*C*a**(1/3))/(
4*C))/12)
```

Giac [C] Result contains complex when optimal does not.
time = 1.59, size = 113, normalized size = 2.40

$$-\frac{\sqrt{3}\left(-i\sqrt{3}\,|a|-a\right)C\arctan\left(\frac{\sqrt{3}\left(4\,x+(-a)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)}{12\,a}-\frac{\left(-i\sqrt{3}\,|a|-3\,a\right)C\log\left(x^2+\frac{1}{2}\,(-a)^{\frac{1}{3}}\,x+\frac{1}{4}\,(-a)^{\frac{2}{3}}\right)}{24\,a}-\frac{\left(C(-a)^{\frac{2}{3}}+2\,Ca^{\frac{2}{3}}\right)(-a)^{\frac{1}{3}}\log\left(\left|x-\frac{1}{2}\,(-a)^{\frac{1}{3}}\right|\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*(-I*sqrt(3)*abs(a) - a)*C*arctan(1/3*sqrt(3)*(4*x + (-a)^(1/3
))/(-a)^(1/3))/a - 1/24*(-I*sqrt(3)*abs(a) - 3*a)*C*log(x^2 + 1/2*(-a)^(1/3
)*x + 1/4*(-a)^(2/3))/a - 1/12*(C*(-a)^(2/3) + 2*C*a^(2/3))*(-a)^(1/3)*log(
abs(x - 1/2*(-a)^(1/3)))/a
```

Mupad [B]
time = 5.02, size = 145, normalized size = 3.09

$$\sum_{k=1}^{3}\ln\left(-\frac{a^{2/3}\,(C-12\,\mathrm{root}(1728\,a^2\,z^3-432\,C\,a^2\,z^2+36\,C^2\,a^2\,z-9\,C^3\,a^2,z,k))\;(4\,C\,x-C\,a^{1/3}+\mathrm{root}(1728\,a^2\,z^3-432\,C\,a^2\,z^2+36\,C^2\,a^2\,z-9\,C^3\,a^2,z,k)\;a^{1/3}\,12)}{128}\right)\mathrm{root}\left(1728\,a^2\,z^3-432\,C\,a^2\,z^2+36\,C^2\,a^2\,z-9\,C^3\,a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*a^(2/3) + 2*C*x^2)/(a + 8*x^3),x)
```

```
[Out] symsum(log(-(a^(2/3)*(C - 12*root(1728*a^2*z^3 - 432*C*a^2*z^2 + 36*C^2*a^2
*z - 9*C^3*a^2, z, k))*(4*C*x - C*a^(1/3) + 12*root(1728*a^2*z^3 - 432*C*a^
2*z^2 + 36*C^2*a^2*z - 9*C^3*a^2, z, k)*a^(1/3)))/128)*root(1728*a^2*z^3 -
432*C*a^2*z^2 + 36*C^2*a^2*z - 9*C^3*a^2, z, k), k, 1, 3)
```

3.31 $\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx$

Optimal. Leaf size=57

$$\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{-b}\,x}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{-b}}-\frac{C\log\left(2+\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}$$

```
[Out] -C*ln(2+(-b)^(1/3)*x)/(-b)^(1/3)+2/3*C*arctan(1/3*(1-(-b)^(1/3)*x)*3^(1/2))
/(-b)^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1878, 31, 631, 210}

$$\frac{2C\,\mathrm{ArcTan}\left(\frac{1-\sqrt[3]{-b}\,x}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{-b}}-\frac{C\log\left(\sqrt[3]{-b}\,x+2\right)}{\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (-b)^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*(-b)^(1/3)) - (C*Log[2 +
(-b)^(1/3)*x])/(-b)^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1878

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a)^(1/3)/(-b)^(1/3)},
Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2)
, x], x]] /; EqQ[A*(-b)^(2/3) - (-a)^(1/3)*(-b)^(1/3)*B - 2*(-a)^(2/3)*C, 0
]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx &= -\frac{(2C)\int \frac{1}{\frac{4}{(-b)^{2/3}}-\frac{2x}{\sqrt[3]{-b}}+x^2}\,dx}{(-b)^{2/3}}-\frac{C\int \frac{1}{\frac{2}{\sqrt[3]{-b}}+x}\,dx}{\sqrt[3]{-b}}\\ &= -\frac{C\log\left(2+\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}-\frac{(2C)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}\\ &= \frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{-b}\,x}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{-b}}-\frac{C\log\left(2+\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 99, normalized size = 1.74

$$\frac{C\left(-2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1+\sqrt[3]{b}\,x}{\sqrt{3}}\right)+2b^{2/3}\log\left(2-\sqrt[3]{b}\,x\right)-b^{2/3}\log\left(4+2\sqrt[3]{b}\,x+b^{2/3}x^2\right)+(-b)^{2/3}\log\left(8-bx^3\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 + b^(1/3)*x)/Sqrt[3]] + 2*b^(2/3)*Log[2 -
b^(1/3)*x] - b^(2/3)*Log[4 + 2*b^(1/3)*x + b^(2/3)*x^2] + (-b)^(2/3)*Log[8
- b*x^3]))/(3*b)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(46) = 92$.
time = 0.33, size = 120, normalized size = 2.11

method	result
meijerg	$\frac{2Cx\left(\ln\left(1-\frac{(bx^3)^{\frac{1}{3}}}{2}\right)-\frac{\ln\left(1+\frac{(bx^3)^{\frac{1}{3}}}{2}+\frac{(bx^3)^{\frac{2}{3}}}{4}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(bx^3)^{\frac{1}{3}}}{4+(bx^3)^{\frac{1}{3}}}\right)\right)}{3(bx^3)^{\frac{1}{3}}}-\frac{C\ln\left(1-\frac{bx^3}{8}\right)}{3(-b)^{\frac{1}{3}}}$

default	$C\left(\frac{8^{\frac{1}{3}}\ln\left(x-8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}-\frac{8^{\frac{1}{3}}\ln\left(x^{2}+8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}x+8^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{1}{b}\right)^{\frac{2}{3}}}-\frac{8^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{8^{\frac{2}{3}}x}{4\left(\frac{1}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}+\frac{(-b)^{\frac{2}{3}}\ln\left(b\,x^{3}-8\right)}{3b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x,method=_RETURNVERBOSE)
```

```
[Out] C*(1/3/b*8^(1/3)/(1/b)^(2/3)*ln(x-8^(1/3)*(1/b)^(1/3))-1/6/b*8^(1/3)/(1/b)^
(2/3)*ln(x^2+8^(1/3)*(1/b)^(1/3)*x+8^(2/3)*(1/b)^(2/3))-1/3/b*8^(1/3)/(1/b)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/4*8^(2/3)/(1/b)^(1/3)*x+1))+1/3*(-b)^(
2/3)/b*ln(b*x^3-8))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(45) = 90$.
time = 0.51, size = 122, normalized size = 2.14

$$\frac{\left(C(-b)^{\frac{2}{3}}-Cb^{\frac{2}{3}}\right)\log\left(b^{\frac{2}{3}}x^{2}+2\,b^{\frac{1}{3}}x+4\right)}{3\,b}+\frac{\left(C(-b)^{\frac{2}{3}}+2\,Cb^{\frac{2}{3}}\right)\log\left(\frac{b^{\frac{1}{3}}x-2}{b^{\frac{1}{3}}}\right)}{3\,b}+\frac{2\,\sqrt{3}\,\left(C(-b)^{\frac{2}{3}}\,b^{\frac{4}{3}}-\left(C(-b)^{\frac{2}{3}}\,b^{\frac{1}{3}}+3\,Cb\right)b\right)\arctan\left(\frac{\sqrt{3}\,\left(b^{\frac{2}{3}}x+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{9\,b^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="maxima")
```

```
[Out] 1/3*(C*(-b)^(2/3) - C*b^(2/3))*log(b^(2/3)*x^2 + 2*b^(1/3)*x + 4)/b + 1/3*(
C*(-b)^(2/3) + 2*C*b^(2/3))*log((b^(1/3)*x - 2)/b^(1/3))/b + 2/9*sqrt(3)*(C
*(-b)^(2/3)*b^(4/3) - (C*(-b)^(2/3)*b^(1/3) + 3*C*b)*b)*arctan(1/3*sqrt(3)*
(b^(2/3)*x + b^(1/3))/b^(1/3))/b^(7/3)
```

Fricas [A]
time = 0.39, size = 182, normalized size = 3.19

$$\left[\frac{\sqrt{\frac{1}{3}}\,Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\,\log\left(\frac{bx^{3}-6\,\sqrt{\frac{1}{3}}\,\left(bx^{2}-(-b)^{\frac{2}{3}}x+2\,(-b)^{\frac{1}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+6\,(-b)^{\frac{1}{3}}x+4}{bx^{3}-8}\right)+C(-b)^{\frac{2}{3}}\log\left(bx-2\,(-b)^{\frac{2}{3}}\right)}{b},\,-\frac{2\,\sqrt{\frac{1}{3}}\,Cb\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\,\arctan\left(\sqrt{\frac{1}{3}}\left((-b)^{\frac{2}{3}}x-(-b)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\right)-C(-b)^{\frac{2}{3}}\log\left(bx-2\,(-b)^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((b*x^3 - 6*sqrt(1/3)*(b*x^2 - (-b)^(
2/3)*x + 2*(-b)^(1/3))*sqrt((-b)^(1/3)/b) + 6*(-b)^(1/3)*x + 4)/(b*x^3 - 8)
```

```
) + C*(-b)^(2/3)*log(b*x - 2*(-b)^(2/3)))/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b)^(
1/3)/b)*arctan(sqrt(1/3)*((-b)^(2/3)*x - (-b)^(1/3))*sqrt(-(-b)^(1/3)/b)) -
 C*(-b)^(2/3)*log(b*x - 2*(-b)^(2/3)))/b]
```

Sympy [A]
time = 0.19, size = 58, normalized size = 1.02

$$\operatorname{RootSum}\left(3t^3b^2 - 3t^2Cb(-b)^{\frac{2}{3}} + tC^2(-b)^{\frac{4}{3}} - C^3b, \left(t \mapsto t\log\left(-\frac{3t}{C} + x + \frac{(-b)^{\frac{2}{3}}}{b}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)**(2/3)*C*x**2)/(b*x**3-8),x)
```

```
[Out] RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + _t*C**2*(-b)**(4/3) - C**3
*b, Lambda(_t, _t*log(-3*_t/C + x + (-b)**(2/3)/b)))
```

Giac [A]
time = 1.30, size = 56, normalized size = 0.98

$$-\frac{2\sqrt{3}\,C|b|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,b^{\frac{1}{3}}\left(x + \frac{1}{b^{\frac{1}{3}}}\right)\right)}{3\,b} + \frac{\left(2\,C + \frac{C(-b)^{\frac{2}{3}}}{b^{\frac{2}{3}}}\right)\log\left(\left|x - \frac{2}{b^{\frac{1}{3}}}\right|\right)}{3\,b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*abs(b)^(2/3)*arctan(1/3*sqrt(3)*b^(1/3)*(x + 1/b^(1/3)))/b +
 1/3*(2*C + C*(-b)^(2/3)/b^(2/3))*log(abs(x - 2/b^(1/3)))/b^(1/3)
```

Mupad [B]
time = 5.27, size = 176, normalized size = 3.09

$$\sum_{k=1}^{3}\ln\left(\frac{8\,C^2}{{(-b)}^{5/3}} + \mathrm{root}\left(27\,b^3\,z^3 - 27\,C\,{(-b)}^{8/3}\,z^2 - 9\,C^2\,{(-b)}^{7/3}\,z - 9\,C^3\,b^2, z, k\right)\,\left(-\frac{\mathrm{root}\left(27\,b^3\,z^3 - 27\,C\,{(-b)}^{8/3}\,z^2 - 9\,C^2\,{(-b)}^{7/3}\,z - 9\,C^3\,b^2, z, k\right)\,72}{b} + \frac{48\,C}{{(-b)}^{4/3}} + \frac{24\,C\,x}{b}\right) - \frac{8\,C^2\,x}{{(-b)}^{4/3}}\right)\,\mathrm{root}\left(27\,b^3\,z^3 - 27\,C\,{(-b)}^{8/3}\,z^2 - 9\,C^2\,{(-b)}^{7/3}\,z - 9\,C^3\,b^2, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*C + C*(-b)^(2/3)*x^2)/(b*x^3 - 8),x)
```

```
[Out] symsum(log((8*C^2)/(-b)^(5/3) + root(27*b^3*z^3 - 27*C*(-b)^(8/3)*z^2 - 9*C
^2*(-b)^(7/3)*z - 9*C^3*b^2, z, k)*((48*C)/(-b)^(4/3) - (72*root(27*b^3*z^3
 - 27*C*(-b)^(8/3)*z^2 - 9*C^2*(-b)^(7/3)*z - 9*C^3*b^2, z, k))/b + (24*C*x
)/b) - (8*C^2*x)/(-b)^(4/3))*root(27*b^3*z^3 - 27*C*(-b)^(8/3)*z^2 - 9*C^2*
(-b)^(7/3)*z - 9*C^3*b^2, z, k), k, 1, 3)
```

3.32 $\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx$

Optimal. Leaf size=47

$$\frac{C\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

```
[Out] -1/4*C*ln((-a)^(1/3)+2*x)+1/6*C*arctan(1/3*(1-4*x/(-a)^(1/3))*3^(1/2))*3^(1
/2)
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {1878, 31, 631, 210}

$$\frac{C\text{ArcTan}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]
```

```
[Out] (C*ArcTan[(1 - (4*x)/(-a)^(1/3))/Sqrt[3]])/(2*Sqrt[3]) - (C*Log[(-a)^(1/3)
+ 2*x])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1878

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a)^(1/3)/(-b)^(1/3)},
Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2)
, x], x]] /; EqQ[A*(-b)^(2/3) - (-a)^(1/3)*(-b)^(1/3)*B - 2*(-a)^(2/3)*C, 0
]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx &= -\left(\frac{1}{4}C\int \frac{1}{\frac{\sqrt[3]{-a}}{2}+x}\,dx\right)-\frac{1}{8}\left(\sqrt[3]{-a}\,C\right)\int \frac{1}{\frac{1}{4}(-a)^{2/3}-\frac{1}{2}\sqrt[3]{-a}\,x+x^2}\,dx\\ &= -\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)-\frac{1}{2}C\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{4x}{\sqrt[3]{-a}}\right)\\ &= \frac{C\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 106 vs. 2(47) = 94.
time = 0.03, size = 106, normalized size = 2.26

$$\frac{C\left(2\sqrt{3}\,(-a)^{2/3}\tan^{-1}\left(\frac{1+\frac{4x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2(-a)^{2/3}\log\left(\sqrt[3]{a}-2x\right)+(-a)^{2/3}\log\left(a^{2/3}+2\sqrt[3]{a}\,x+4x^2\right)-a^{2/3}\log\left(-a+8x^3\right)\right)}{12a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(-a)^(2/3)*ArcTan[(1 + (4*x)/a^(1/3))/Sqrt[3]] - 2*(-a)^(2/3)
*Log[a^(1/3) - 2*x] + (-a)^(2/3)*Log[a^(2/3) + 2*a^(1/3)*x + 4*x^2] - a^(2/
3)*Log[-a + 8*x^3]))/(12*a^(2/3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(36) = 72.
time = 0.30, size = 98, normalized size = 2.09

method	result

default	$C\left((-a)^{\frac{2}{3}}\left(-\frac{8^{\frac{2}{3}}\ln\left(x-\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}}{8}\right)}{24a^{\frac{2}{3}}}+\frac{8^{\frac{2}{3}}\ln\left(x^{2}+\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}x}{8}+\frac{8^{\frac{1}{3}}a^{\frac{2}{3}}}{8}\right)}{48a^{\frac{2}{3}}}+\frac{8^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,8^{\frac{1}{3}}x}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{24a^{\frac{2}{3}}}\right)-\frac{\ln(-8x^{3}+}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*((-a)^(2/3)*(-1/24*8^(2/3)/a^(2/3)*ln(x-1/8*8^(2/3)*a^(1/3))+1/48*8^(2/3)
/a^(2/3)*ln(x^2+1/8*8^(2/3)*a^(1/3)*x+1/8*8^(1/3)*a^(2/3))+1/24*8^(2/3)/a^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*8^(1/3)/a^(1/3)*x+1)))-1/12*ln(-8*x^3+a)
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(36) = 72$.
time = 0.54, size = 93, normalized size = 1.98

$$\frac{\sqrt{3}\,C(-a)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(4\,x+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{6\,a^{\frac{2}{3}}}+\frac{\left(C(-a)^{\frac{2}{3}}-Ca^{\frac{2}{3}}\right)\log\left(4\,x^{2}+2\,a^{\frac{1}{3}}x+a^{\frac{2}{3}}\right)}{12\,a^{\frac{2}{3}}}-\frac{\left(2\,C(-a)^{\frac{2}{3}}+Ca^{\frac{2}{3}}\right)\log\left(x-\frac{1}{2}\,a^{\frac{1}{3}}\right)}{12\,a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*C*(-a)^(2/3)*arctan(1/3*sqrt(3)*(4*x + a^(1/3))/a^(1/3))/a^(2/3
) + 1/12*(C*(-a)^(2/3) - C*a^(2/3))*log(4*x^2 + 2*a^(1/3)*x + a^(2/3))/a^(2
/3) - 1/12*(2*C*(-a)^(2/3) + C*a^(2/3))*log(x - 1/2*a^(1/3))/a^(2/3)
```

Fricas [A]
time = 0.39, size = 43, normalized size = 0.91

$$\frac{1}{6}\,\sqrt{3}\,C\arctan\left(\frac{4\,\sqrt{3}\,(-a)^{\frac{2}{3}}\,x+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{4}\,C\log\left(2\,x+(-a)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*C*arctan(1/3*(4*sqrt(3)*(-a)^(2/3)*x + sqrt(3)*a)/a) - 1/4*C*lo
g(2*x + (-a)^(1/3))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.16, size = 95, normalized size = 2.02

$$-C\left(\frac{\log\left(-\frac{a}{2(-a)^{\frac{2}{3}}}+x\right)}{4}+\frac{\sqrt{3}\,i\log\left(\frac{a}{4(-a)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{4(-a)^{\frac{2}{3}}}+x\right)}{12}-\frac{\sqrt{3}\,i\log\left(\frac{a}{4(-a)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{4(-a)^{\frac{2}{3}}}+x\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a)**(2/3)*C+2*C*x**2)/(-8*x**3+a),x)
```

```
[Out] -C*(log(-a/(2*(-a)**(2/3)) + x)/4 + sqrt(3)*I*log(a/(4*(-a)**(2/3)) - sqrt(
3)*I*a/(4*(-a)**(2/3)) + x)/12 - sqrt(3)*I*log(a/(4*(-a)**(2/3)) + sqrt(3)*
I*a/(4*(-a)**(2/3)) + x)/12)
```

Giac [C] Result contains complex when optimal does not.
time = 1.06, size = 84, normalized size = 1.79

$$\frac{1}{6}\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(4\,x+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{\left(-i\,\sqrt{3}\,|a|+3\,a\right)C\log\left(x^2+\frac{1}{2}\,a^{\frac{1}{3}}x+\frac{1}{4}\,a^{\frac{2}{3}}\right)}{24\,a}-\frac{\left(2\,C(-a)^{\frac{2}{3}}+Ca^{\frac{2}{3}}\right)\log\left(\left|x-\frac{1}{2}\,a^{\frac{1}{3}}\right|\right)}{12\,a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*C*arctan(1/3*sqrt(3)*(4*x + a^(1/3))/a^(1/3)) - 1/24*(-I*sqrt(3
)*abs(a) + 3*a)*C*log(x^2 + 1/2*a^(1/3)*x + 1/4*a^(2/3))/a - 1/12*(2*C*(-a)
^(2/3) + C*a^(2/3))*log(abs(x - 1/2*a^(1/3)))/a^(2/3)
```

Mupad [B]
time = 0.33, size = 142, normalized size = 3.02

$$\sum_{k=1}^{3}\ln\left(-\frac{\left(C+12\,\mathrm{root}(1728\,a^2\,z^3+432\,C\,a^2\,z^2+36\,C^2\,a^2\,z+9\,C^3\,a^2,z,k)\right)\left(C\,a+\mathrm{root}(1728\,a^2\,z^3+432\,C\,a^2\,z^2+36\,C^2\,a^2\,z+9\,C^3\,a^2,z,k)\;a\,12+4\,C\,(-a)^{2/3}\,x\right)}{128}\right)\mathrm{root}\left(1728\,a^2\,z^3+432\,C\,a^2\,z^2+36\,C^2\,a^2\,z+9\,C^3\,a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*C*x^2 + C*(-a)^(2/3))/(a - 8*x^3),x)
```

```
[Out] symsum(log(-((C + 12*root(1728*a^2*z^3 + 432*C*a^2*z^2 + 36*C^2*a^2*z + 9*C
^3*a^2, z, k))*(C*a + 12*root(1728*a^2*z^3 + 432*C*a^2*z^2 + 36*C^2*a^2*z +
 9*C^3*a^2, z, k)*a + 4*C*(-a)^(2/3)*x))/128)*root(1728*a^2*z^3 + 432*C*a^2
*z^2 + 36*C^2*a^2*z + 9*C^3*a^2, z, k), k, 1, 3)
```

3.33 $\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=50

$$-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

```
[Out] C*ln((a/b)^(1/3)+x)/b-2/3*C*arctan(1/3*(1-2*x/(a/b)^(1/3))*3^(1/2))/b*3^(1/
2)
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1881, 31, 631, 210}

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2C\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^(
1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1881

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}+x}\,dx}{b}+\frac{\left(\sqrt[3]{\frac{a}{b}}\,C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}\,x+x^2}\,dx}{b}$$

$$= \frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}+\frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$

$$= -\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 146 vs. 2(50) = 100.
time = 0.03, size = 146, normalized size = 2.92

$$\frac{C\left(-2\sqrt{3}\left(\frac{a}{b}\right)^{2/3}b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(\frac{a}{b}\right)^{2/3}b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-\left(\frac{a}{b}\right)^{2/3}b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+a^{2/3}\log\left(a+bx^3\right)\right)}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

[Out] (C*(-2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - (a/b)^(2/3)*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*Log[a + b*x^3]))/(3*a^(2/3)*b)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(43) = 86$.
time = 0.31, size = 116, normalized size = 2.32

method	result	size
default	$C\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\frac{\ln\left(b\,x^{3}+a\right)}{3b}\right)$	116

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] C*(2*(a/b)^(2/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*ln(b*x^3+a)/b)

Maxima [A]
time = 0.54, size = 51, normalized size = 1.02

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}+\frac{C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="maxima")

[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/b + C*log(x + (a/b)^(1/3))/b

Fricas [A]
time = 0.39, size = 52, normalized size = 1.04

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+3\,C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 3*
C*log(x + (a/b)^(1/3)))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.17, size = 100, normalized size = 2.00

$$\frac{C\left(\log\left(\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(a/b)**(2/3)) + x) - sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) - sqrt
(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(43) = 86.
time = 1.19, size = 90, normalized size = 1.80

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/
3*(C*b^2*(-a/b)^(2/3) + 2*(a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a*b^2)
```

Mupad [B]
time = 5.10, size = 172, normalized size = 3.44

$$\sum_{k=1}^{3}\ln\left(-\frac{\left(C-\mathrm{root}\left(27a^2b^3z^3-27Ca^2b^2z^2+9C^2a^2bz-9C^3a^2,z,k\right)b3\right)\left(-Ca+\mathrm{root}\left(27a^2b^3z^3-27Ca^2b^2z^2+9C^2a^2bz-9C^3a^2,z,k\right)ab3+2Cbx\left(\frac{a}{b}\right)^{2/3}\right)}{b^3}\right)\mathrm{root}\left(27a^2b^3z^3-27Ca^2b^2z^2+9C^2a^2bz-9C^3a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2 + 2*C*(a/b)^(2/3))/(a + b*x^3),x)
```

```
[Out] symsum(log(-((C - 3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
- 9*C^3*a^2, z, k)*b)*(3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2
*b*z - 9*C^3*a^2, z, k)*a*b - C*a + 2*C*b*x*(a/b)^(2/3)))/b^3)*root(27*a^2*
b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z - 9*C^3*a^2, z, k), k, 1, 3)
```

3.34 $\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

```
[Out] -C*ln((-a/b)^(1/3)+x)/b+2/3*C*arctan(1/3*(1-2*x/(-a/b)^(1/3))*3^(1/2))/b*3^
(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1881, 31, 631, 210}

$$\frac{2C\,\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(-(a/
b))^(1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1881

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a - bx^3}\,dx = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}+x}\,dx}{b} - \frac{\left(\sqrt[3]{-\frac{a}{b}}\,C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}-\sqrt[3]{-\frac{a}{b}}\,x+x^2}\,dx}{b}$$

$$= -\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b} - \frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$= \frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b} - \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 150 vs. 2(53) = 106.

time = 0.04, size = 150, normalized size = 2.83

$$\frac{C\left(2\sqrt{3}\left(-\frac{a}{b}\right)^{2/3} b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\left(-\frac{a}{b}\right)^{2/3} b^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right) + \left(-\frac{a}{b}\right)^{2/3} b^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right) - a^{2/3}\log\left(a-bx^3\right)\right)}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

[Out] (C*(2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*x] + (-(a/b))^(2/3)*b^(2/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*Log[a - b*x^3]))/(3*a^(2/3)*b)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(46) = 92$.
time = 0.33, size = 119, normalized size = 2.25

method	result
default	$C\left(2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-\frac{\ln\left(-b\,x^{3}+a\right)}{3b}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x,method=_RETURNVERBOSE)

[Out] C*(2*(-a/b)^(2/3)*(-1/3/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(2/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2)))-1/3*ln(-b*x^3+a)/b)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(46) = 92$.
time = 0.52, size = 167, normalized size = 3.15

$$-\frac{2\sqrt{3}\left(Ca-\left(3\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}-\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x^{2}+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="maxima")

[Out] -2/9*sqrt(3)*(C*a - (3*C*(a/b)^(1/3)*(-a/b)^(2/3) + C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b) - 1/3*(C*(a/b)^(2/3) - C*(-a/b)^(2/3))*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(C*(a/b)^(2/3) + 2*C*(-a/b)^(2/3))*log(x - (a/b)^(1/3))/(b*(a/b)^(2/3))

Fricas [A]
time = 0.39, size = 53, normalized size = 1.00

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-3\,C\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt(3)*a)/a) - 3
*C*log(x + (-a/b)^(1/3)))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.18, size = 110, normalized size = 2.08

$$-\frac{C\left(\log\left(-\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(-a/b)**(2/3)) + x) + sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3)) - s
qrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

Giac [C] Result contains complex when optimal does not.
time = 1.04, size = 109, normalized size = 2.06

$$-\frac{\sqrt{3}\left(ab^2-i\sqrt{3}\sqrt{a^2b^4}\right)C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^3}-\frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(a*b^2 - I*sqrt(3)*sqrt(a^2*b^4))*C*arctan(1/3*sqrt(3)*(2*x +
(a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) - 1/3*(C*b^2*(a/b)^(2/3) + 2*(-a*b^2)^(2/
3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/(a*b^2)
```

Mupad [B]
time = 5.40, size = 172, normalized size = 3.25

$$\sum_{k=1}^{3}\ln\left(-\frac{\left(C+\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z+9\,C^3\,a^2,z,k\right)\,b\,3\right)\,\left(C\,a+\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z+9\,C^3\,a^2,z,k\right)\,a\,b\,3+2\,C\,b\,x\,{\left(-\frac{a}{b}\right)}^{2/3}\right)}{b^3}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z+9\,C^3\,a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2 + 2*C*(-a/b)^(2/3))/(a - b*x^3),x)
```

```
[Out] symsum(log(-((C + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
+ 9*C^3*a^2, z, k)*b)*(C*a + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C
^2*a^2*b*z + 9*C^3*a^2, z, k)*a*b + 2*C*b*x*(-a/b)^(2/3)))/b^3)*root(27*a^2
*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z + 9*C^3*a^2, z, k), k, 1, 3)
```

3.35 $\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=54

$$-\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

```
[Out] C*ln((-a/b)^(1/3)-x)/b-2/3*C*arctan(1/3*(1+2*x/(-a/b)^(1/3))*3^(1/2))/b*3^(
1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1883, 31, 631, 210}

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{2C\text{ArcTan}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[-(a
/b))^(1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1883

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Dist[-C/b
, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]
] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &&
PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a+bx^3}\,dx &= -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}\,dx}{b} - \frac{\left(\sqrt[3]{-\frac{a}{b}}\,C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}+\sqrt[3]{-\frac{a}{b}}\,x+x^2}\,dx}{b} \\
&= \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b} + \frac{(2C)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b} \\
&= -\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b} + \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 149 vs. 2(54) = 108.
time = 0.03, size = 149, normalized size = 2.76

$$\frac{C\left(-2\sqrt{3}\left(-\frac{a}{b}\right)^{2/3} b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(-\frac{a}{b}\right)^{2/3} b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-\left(-\frac{a}{b}\right)^{2/3} b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+a^{2/3}\log\left(a+bx^3\right)\right)}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] + 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - (-(a/b))^(2/3)
*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*Log[a + b
*x^3]))/(3*a^(2/3)*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(47) = 94$.
time = 0.30, size = 117, normalized size = 2.17

method	result	size
default	$C\left(2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\frac{\ln(b\,x^{3}+a)}{3b}\right)$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(2*(-a/b)^(2/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1)))+1/3*ln(b*x^3+a)/b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(47) = 94$.
time = 0.53, size = 168, normalized size = 3.11

$$-\frac{2\sqrt{3}\left(Ca-\left(3\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*(C*a - (3*C*(a/b)^(1/3)*(-a/b)^(2/3) + C*a/b)*b)*arctan(1/3*sq
rt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/3*(C*(a/b)^(2/3) - C*(-a/b
)^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(C*(a
/b)^(2/3) + 2*C*(-a/b)^(2/3))*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Fricas [A]
time = 0.37, size = 56, normalized size = 1.04

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+3\,C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 3
*C*log(x - (-a/b)^(1/3)))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.19, size = 109, normalized size = 2.02

$$\frac{C\left(\log\left(\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(-a/b)**(2/3)) + x) - sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3)) - sq
rt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

Giac [A]
time = 0.99, size = 91, normalized size = 1.69

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/
3*(C*b^2*(-a/b)^(2/3) + 2*(-a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a*b^2)
```

Mupad [B]
time = 5.27, size = 173, normalized size = 3.20

$$\sum_{k=1}^{3}\ln\left(-\frac{\left(C-\mathrm{root}\left(27a^2b^3z^3-27Ca^2b^2z^2+9C^2a^2bz-9C^3a^2,z,k\right)b\,3\right)\left(-Ca+\mathrm{root}\left(27a^2b^3z^3-27Ca^2b^2z^2+9C^2a^2bz-9C^3a^2,z,k\right)ab\,3+2Cbx\left(-\frac{a}{b}\right)^{2/3}\right)}{b^3}\right)\mathrm{root}\left(27a^2b^3z^3-27Ca^2b^2z^2+9C^2a^2bz-9C^3a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2 + 2*C*(-a/b)^(2/3))/(a + b*x^3),x)
```

```
[Out] symsum(log(-((C - 3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
- 9*C^3*a^2, z, k)*b)*(3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2
*b*z - 9*C^3*a^2, z, k)*a*b - C*a + 2*C*b*x*(-a/b)^(2/3)))/b^3)*root(27*a^2
*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z - 9*C^3*a^2, z, k), k, 1, 3)
```

3.36 $\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

```
[Out] -C*ln((a/b)^(1/3)-x)/b+2/3*C*arctan(1/3*(1+2*x/(a/b)^(1/3))*3^(1/2))/b*3^(1
/2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1883, 31, 631, 210}

$$\frac{2C\,\text{ArcTan}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^(1
/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1883

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Dist[-C/b
, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]
] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &&
PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx &= \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}\,dx}{b}+\frac{\left(\sqrt[3]{\frac{a}{b}}\,C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}+\sqrt[3]{\frac{a}{b}}\,x+x^2}\,dx}{b}\\
&= -\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}-\frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}\\
&= \frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 147 vs. 2(53) = 106.
time = 0.03, size = 147, normalized size = 2.77

$$\frac{C\left(2\sqrt{3}\left(\frac{a}{b}\right)^{2/3}b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\left(\frac{a}{b}\right)^{2/3}b^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)+\left(\frac{a}{b}\right)^{2/3}b^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-a^{2/3}\log\left(a-bx^3\right)\right)}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

[Out] (C*(2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] - 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*x] + (a/b)^(2/3)*b^(2/3)*L
og[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*Log[a - b*x^3]))/(3
*a^(2/3)*b)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(46) = 92$.
time = 0.30, size = 118, normalized size = 2.23

method	result	siz
default	$C\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-\frac{\ln\left(-b\,x^{3}+a\right)}{3b}\right)$	11

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x,method=_RETURNVERBOSE)

[Out] C*(2*(a/b)^(2/3)*(-1/3/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(2/3)*ln
(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*(1+2/(
a/b)^(1/3)*x)*3^(1/2)))-1/3*ln(-b*x^3+a)/b)

Maxima [A]
time = 0.51, size = 52, normalized size = 0.98

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="maxima")

[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - C*log
(x - (a/b)^(1/3))/b

Fricas [A]
time = 0.39, size = 53, normalized size = 1.00

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-3\,C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) + sqrt(3)*a)/a) - 3*
C*log(x - (a/b)^(1/3)))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.19, size = 102, normalized size = 1.92

$$-\frac{C\left(\log\left(-\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(a/b)**(2/3)) + x) + sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) - sqr
t(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

Giac [A]
time = 1.56, size = 85, normalized size = 1.60

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - 1/3*(
C*b^2*(a/b)^(2/3) + 2*(a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3))
)/(a*b^2)
```

Mupad [B]
time = 5.19, size = 171, normalized size = 3.23

$$\sum_{k=1}^{3}\ln\left(-\frac{\left(C+\mathrm{root}(27a^2b^3z^3+27Ca^2b^2z^2+9C^2a^2bz+9C^3a^2,z,k)\,b\,3\right)\left(Ca+\mathrm{root}(27a^2b^3z^3+27Ca^2b^2z^2+9C^2a^2bz+9C^3a^2,z,k)\,a\,b\,3+2Cbx\left(\frac{a}{b}\right)^{2/3}\right)}{b^3}\right)\mathrm{root}(27a^2b^3z^3+27Ca^2b^2z^2+9C^2a^2bz+9C^3a^2,z,k)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2 + 2*C*(a/b)^(2/3))/(a - b*x^3),x)
```

```
[Out] symsum(log(-((C + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
+ 9*C^3*a^2, z, k)*b)*(C*a + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C
^2*a^2*b*z + 9*C^3*a^2, z, k)*a*b + 2*C*b*x*(a/b)^(2/3)))/b^3)*root(27*a^2*
b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z + 9*C^3*a^2, z, k), k, 1, 3)
```

3.37 $\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=61

$$-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}$$

```
[Out] C*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-2/3*C*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1
/3)*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1877, 31, 631, 210}

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{2C\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3))
+ (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1877

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx &= \frac{\left(\sqrt[3]{a}\,C\right)\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx}{b^{2/3}}+\frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx}{\sqrt[3]{b}}\\ &= \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+\frac{(2C)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}\\ &= -\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 95, normalized size = 1.56

$$\frac{C\left(-2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+\log\left(a+bx^3\right)\right)}{3\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3)
+ b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + Log[a + b*x
^3]))/(3*b^(1/3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(44) = 88$.
time = 0.43, size = 112, normalized size = 1.84

method	result	size

default	$C\left(2a^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\frac{\ln\left(bx^{3}+a\right)}{3b^{\frac{1}{3}}}\right)$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(2*a^(2/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1)))+1/3/b^(1/3)*ln(b*x^3+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(46) = 92$.
time = 0.51, size = 162, normalized size = 2.66

$$-\frac{2\sqrt{3}\left(Cab^{\frac{2}{3}}-\left(3Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{Ca}{b^{\frac{1}{3}}}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab}+\frac{\left(Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-Ca^{\frac{2}{3}}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}+2Ca^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*(C*a*b^(2/3) - (3*C*a^(2/3)*(a/b)^(1/3) + C*a/b^(1/3))*b)*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/3*(C*b^(2/3)*(a/b
)^(2/3) - C*a^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3))
 + 1/3*(C*b^(2/3)*(a/b)^(2/3) + 2*C*a^(2/3))*log(x + (a/b)^(1/3))/(b*(a/b)^
(2/3))
```

Fricas [A]
time = 0.39, size = 160, normalized size = 2.62

$$\left[\frac{\sqrt{\frac{1}{3}}\,Cb\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2bx^{3}-3a^{\frac{2}{3}}b^{\frac{1}{3}}x+3\sqrt{\frac{1}{3}}\left(2a^{\frac{1}{3}}bx^{2}+a^{\frac{2}{3}}b^{\frac{2}{3}}x-ab^{\frac{1}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-a}{bx^{3}+a}\right)+Cb^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b},\ \frac{2\sqrt{\frac{1}{3}}\,Cb^{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2a^{\frac{2}{3}}b^{\frac{2}{3}}x-ab^{\frac{1}{3}}\right)}{ab^{\frac{1}{3}}}\right)+Cb^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((2*b*x^3 - 3*a^(2/3)*b^(1/3)*x + 3*sqr
t(1/3)*(2*a^(1/3)*b*x^2 + a^(2/3)*b^(2/3)*x - a*b^(1/3))*sqrt(-1/b^(2/3)) -
 a)/(b*x^3 + a)) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b, (2*sqrt(1/3)*C*
b^(2/3)*arctan(sqrt(1/3)*(2*a^(2/3)*b^(2/3)*x - a*b^(1/3))/(a*b^(1/3))) + C
*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b]
```

Sympy [A]
time = 0.16, size = 70, normalized size = 1.15

$$\operatorname{RootSum}\left(3t^3b^{\frac{5}{3}} - 3t^2Cb^{\frac{4}{3}} + tC^2b - C^3b^{\frac{2}{3}}, \left(t \mapsto t\log\left(x + \frac{3t\sqrt[3]{a}\,\sqrt[3]{b} - C\sqrt[3]{a}}{2C\sqrt[3]{b}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a**(2/3)*C+b**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*b**(2/3),
Lambda(_t, _t*log(x + (3*_t*a**(1/3)*b**(1/3) - C*a**(1/3))/(2*C*b**(1/3)))
))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.31, size = 193, normalized size = 3.16

$$\sum_{k=1}^{3}\ln\left(-\frac{a^{2/3}\left(C-\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^{8/3}\,z^2+9\,C^2\,a^2\,b^{7/3}\,z-9\,C^3\,a^2\,b^2,z,k\right)\,b^{1/3}\,3\right)\,\left(-C\,a^{1/3}+\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^{8/3}\,z^2+9\,C^2\,a^2\,b^{7/3}\,z-9\,C^3\,a^2\,b^2,z,k\right)\,a^{1/3}\,b^{1/3}\,3+2\,C\,b^{1/3}\,x\right)}{b^{5/3}}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^{8/3}\,z^2+9\,C^2\,a^2\,b^{7/3}\,z-9\,C^3\,a^2\,b^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*C*a^(2/3) + C*b^(2/3)*x^2)/(a + b*x^3),x)
```

```
[Out] symsum(log(-(a^(2/3)*(C - 3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^(8/3)*z^2 + 9*
C^2*a^2*b^(7/3)*z - 9*C^3*a^2*b^2, z, k)*b^(1/3))*(3*root(27*a^2*b^3*z^3 -
27*C*a^2*b^(8/3)*z^2 + 9*C^2*a^2*b^(7/3)*z - 9*C^3*a^2*b^2, z, k)*a^(1/3)*b
^(1/3) - C*a^(1/3) + 2*C*b^(1/3)*x))/b^(5/3))*root(27*a^2*b^3*z^3 - 27*C*a^
2*b^(8/3)*z^2 + 9*C^2*a^2*b^(7/3)*z - 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

3.38 $\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=70

$$-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}$$

```
[Out] C*ln(a^(1/3)-(-b)^(1/3)*x)/(-b)^(1/3)-2/3*C*arctan(1/3*(a^(1/3)+2*(-b)^(1/3
)*x)/a^(1/3)*3^(1/2))/(-b)^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1880, 31, 631, 210}

$$\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}-\frac{2C\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(-b)^(
1/3)) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1880

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/(-b)^(1/3)}, Dis
t[-C/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2),
x], x]] /; EqQ[A*(-b)^(2/3) + a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C, 0]] /; Fr
eeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx &= -\frac{\left(\sqrt[3]{a}\,C\right)\int \frac{1}{\frac{a^{2/3}}{(-b)^{2/3}}+\frac{\sqrt[3]{a}\,x}{\sqrt[3]{-b}}+x^2}\,dx}{(-b)^{2/3}}-\frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{-b}}-x}\,dx}{\sqrt[3]{-b}}\\ &= \frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}+\frac{(2C)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{-b}\,x}{\sqrt[3]{a}}\right)}{\sqrt[3]{-b}}\\ &= -\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{-b}}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 116, normalized size = 1.66

$$-\frac{C\left(-2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+(-b)^{2/3}\log\left(a+bx^3\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] -1/3*(C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2
*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2] + (-b)^(2/3)*Log[a + b*x^3]))/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(53) = 106$.
time = 0.43, size = 117, normalized size = 1.67

method	result

default	$C\left(-2a^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-\frac{(-b)^{\frac{2}{3}}\ln\left(b\,x^{3}+a\right)}{3b}\right)$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-2*a^(2/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1)))-1/3*(-b)^(2/3)*ln(b*x^3+a)/b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(53) = 106$.
time = 0.54, size = 173, normalized size = 2.47

$$\frac{2\sqrt{3}\left(Ca(-b)^{\frac{2}{3}}-\left(3\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{Ca(-b)^{\frac{2}{3}}}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}-\frac{\left(C(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-Ca^{\frac{2}{3}}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(C(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,Ca^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*(C*a*(-b)^(2/3) - (3*C*a^(2/3)*(a/b)^(1/3) + C*a*(-b)^(2/3)/b)*
b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) - 1/3*(C*(-b)^
(2/3)*(a/b)^(2/3) - C*a^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a
/b)^(2/3)) - 1/3*(C*(-b)^(2/3)*(a/b)^(2/3) + 2*C*a^(2/3))*log(x + (a/b)^(1/
3))/(b*(a/b)^(2/3))
```

Fricas [A]
time = 0.40, size = 205, normalized size = 2.93

$$\left[\frac{\sqrt{\frac{1}{3}}\,Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,bx^{3}+3\,a^{\frac{2}{3}}(-b)^{\frac{1}{3}}x-3\sqrt{\frac{1}{3}}\left(2\,a^{\frac{1}{3}}bx^{2}+a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x+a(-b)^{\frac{1}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}-a}{bx^{3}+a}\right)-C(-b)^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}(-b)^{\frac{2}{3}}\right)}{b},-\frac{2\sqrt{\frac{1}{3}}\,Cb\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x+a(-b)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{a}\right)+C(-b)^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}(-b)^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((2*b*x^3 + 3*a^(2/3)*(-b)^(1/3)*x -
3*sqrt(1/3)*(2*a^(1/3)*b*x^2 + a^(2/3)*(-b)^(2/3)*x + a*(-b)^(1/3))*sqrt((-
b)^(1/3)/b) - a)/(b*x^3 + a)) - C*(-b)^(2/3)*log(b*x + a^(1/3)*(-b)^(2/3)))
/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b)^(1/3)/b)*arctan(sqrt(1/3)*(2*a^(2/3)*(-b)^
(2/3)*x + a*(-b)^(1/3))*sqrt(-(-b)^(1/3)/b)/a) + C*(-b)^(2/3)*log(b*x + a^(
1/3)*(-b)^(2/3)))/b]
```

Sympy [A]
time = 0.19, size = 73, normalized size = 1.04

$$-\operatorname{RootSum}\left(3t^3b^2 - 3t^2Cb(-b)^{\frac{2}{3}} + tC^2(-b)^{\frac{4}{3}} - C^3b, \left(t \mapsto t\log\left(\frac{3t\sqrt[3]{a}}{2C} - \frac{\sqrt[3]{a}\,(-b)^{\frac{2}{3}}}{2b} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a**(2/3)*C-(-b)**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] -RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + _t*C**2*(-b)**(4/3) - C**
3*b, Lambda(_t, _t*log(3*_t*a**(1/3)/(2*C) - a**(1/3)*(-b)**(2/3)/(2*b) + x
)))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.24, size = 221, normalized size = 3.16

$$\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,{\left(-b\right)}^{8/3}\,z^2-9\,C^2\,a^2\,{\left(-b\right)}^{7/3}\,z+9\,C^3\,a^2\,b^2,z,k\right)\,\left(\frac{6\,C\,a}{{\left(-b\right)}^{4/3}}+\frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,{\left(-b\right)}^{8/3}\,z^2-9\,C^2\,a^2\,{\left(-b\right)}^{7/3}\,z+9\,C^3\,a^2\,b^2,z,k\right)\,a\,9}{b}-\frac{6\,C\,a^{2/3}\,x}{b}\right)-\frac{C^2\,a}{{\left(-b\right)}^{5/3}}-\frac{2\,C^2\,a^{2/3}\,x}{{\left(-b\right)}^{4/3}}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,{\left(-b\right)}^{8/3}\,z^2-9\,C^2\,a^2\,{\left(-b\right)}^{7/3}\,z+9\,C^3\,a^2\,b^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*C*a^(2/3) + C*(-b)^(2/3)*x^2)/(a + b*x^3),x)
```

```
[Out] symsum(log(root(27*a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 - 9*C^2*a^2*(-b)^(
7/3)*z + 9*C^3*a^2*b^2, z, k)*((6*C*a)/(-b)^(4/3) + (9*root(27*a^2*b^3*z^3
+ 27*C*a^2*(-b)^(8/3)*z^2 - 9*C^2*a^2*(-b)^(7/3)*z + 9*C^3*a^2*b^2, z, k)*a
)/b - (6*C*a^(2/3)*x)/b) - (C^2*a)/(-b)^(5/3) - (2*C^2*a^(2/3)*x)/(-b)^(4/3
))*root(27*a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 - 9*C^2*a^2*(-b)^(7/3)*z +
 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

3.39 $\int \frac{-3+x^2}{-1+x^3}\,dx$

Optimal. Leaf size=40

$$\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\frac{2}{3}\log(1-x)+\frac{5}{6}\log\left(1+x+x^2\right)$$

```
[Out] -2/3*ln(1-x)+5/6*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1889, 31, 648, 632, 210, 642}

$$\sqrt{3}\,\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x^2)/(-1 + x^3),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - (2*Log[1 - x])/3 + (5*Log[1 + x + x^2])
/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q
 + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q -
 C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b
*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
 && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+x^2}{-1+x^3}\,dx &= -\left(\frac{1}{3}\int \frac{-7-5x}{1+x+x^2}\,dx\right)+\frac{2}{3}\int \frac{1}{1-x}\,dx\\ &= -\frac{2}{3}\log(1-x)+\frac{5}{6}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{3}{2}\int \frac{1}{1+x+x^2}\,dx\\ &= -\frac{2}{3}\log(1-x)+\frac{5}{6}\log\left(1+x+x^2\right)-3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= \sqrt{3}\,\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\frac{2}{3}\log(1-x)+\frac{5}{6}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.25

$$\sqrt{3}\,\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\log(1-x)+\frac{1}{2}\log\left(1+x+x^2\right)+\frac{1}{3}\log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x^2)/(-1 + x^3),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[1 - x] + Log[1 + x + x^2]/2 + Log[1
 - x^3]/3
```

Maple [A]
time = 0.35, size = 32, normalized size = 0.80

method	result	size
default	$\frac{5\ln(x^2+x+1)}{6}+\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}-\frac{2\ln(x-1)}{3}$	32
risch	$-\frac{2\ln(x-1)}{3}+\frac{5\ln(9x^2+9x+9)}{6}+\sqrt{3}\ \arctan\left(\frac{2(\frac{3}{2}+3x)\sqrt{3}}{9}\right)$	36
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ (x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{(x^3)^{\frac{1}{3}}}+\frac{\ln(-x^3+1)}{3}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3)/(x^3-1),x,method=_RETURNVERBOSE)
[Out] 5/6*ln(x^2+x+1)+arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-2/3*ln(x-1)
```

Maxima [A]
time = 0.51, size = 31, normalized size = 0.78

$$\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{5}{6}\ \log\left(x^2+x+1\right)-\frac{2}{3}\ \log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)/(x^3-1),x, algorithm="maxima")
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(x -
1)
```

Fricas [A]
time = 0.36, size = 31, normalized size = 0.78

$$\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{5}{6}\ \log\left(x^2+x+1\right)-\frac{2}{3}\ \log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)/(x^3-1),x, algorithm="fricas")
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(x -
1)
```

Sympy [A]
time = 0.05, size = 42, normalized size = 1.05

$$-\frac{2\log\left(x-1\right)}{3}+\frac{5\log\left(x^2+x+1\right)}{6}+\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3)/(x**3-1),x)
```

```
[Out] -2*log(x - 1)/3 + 5*log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt
(3)/3)
```

Giac [A]
time = 3.17, size = 32, normalized size = 0.80

$$\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)/(x^3-1),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(abs(
x - 1))
```

Mupad [B]
time = 0.16, size = 46, normalized size = 1.15

$$-\frac{2\ln(x-1)}{3}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(-\frac{5}{6}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{5}{6}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 3)/(x^3 - 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/2 + 5/6) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*1i)/2 - 5/6) - (2*log(x - 1))/3
```

3.40 $\int \frac{\sqrt[3]{a}\,\sqrt[3]{b}\,B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=70

$$-\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}}+\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}$$

```
[Out] C*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-2/3*(B/a^(1/3)+C/b^(1/3))*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.082$, Rules used = {1877, 31, 631, 210}

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)*C*x^2)/(a + b*
x^3),x]
```

```
[Out] (-2*(B/a^(1/3) + C/b^(1/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/Sqrt[3] + (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1877

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx=\frac{C\int\frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx}{\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}B+\sqrt[3]{a}C\right)\int\frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx}{b^{2/3}}$$
$$=\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\left(2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\right)\text{Subst}\left(\int -\right.$$
$$=-\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}+\frac{C\log\left(\sqrt[3]{a}+\right.}{\sqrt[3]{b}}$$

Mathematica [A]
time = 0.03, size = 122, normalized size = 1.74

$$\frac{-2\sqrt{3}\left(\sqrt[3]{b}B+\sqrt[3]{a}C\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\sqrt[3]{a}C\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\log\left(a+bx^3\right)\right)}{3\sqrt[3]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)*C*x^2)/(
a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*B + a^(1/3)*C)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + a^(1/3)*C*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2] + Log[a + b*x^3]))/(3*a^(1/3)*b^(1/3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(51) = 102$.
time = 0.51, size = 217, normalized size = 3.10

method	result

default	$\left(a^{\frac{1}{3}} b^{\frac{1}{3}} B + 2a^{\frac{2}{3}} C\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right) + b^{\frac{2}{3}} B\left(-\frac{\ln}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+a),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] (a^(1/3)*b^(1/3)*B+2*a^(2/3)*C)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/
(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+b^(2/3)*B*(-1/3/b/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/
b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*C/b^(1/3)*ln(b*x
^3+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(53) = 106$.
time = 0.52, size = 236, normalized size = 3.37

$$-\frac{\sqrt{3}\left(2\,Cab^{\frac{2}{3}}-\left(6\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+3\,Ba^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}+\frac{2\,Ca}{b}\right)b^{\frac{2}{3}}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}-\frac{\left(2\,Ca^{\frac{2}{3}}+Ba^{\frac{1}{3}}b^{\frac{1}{3}}-\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{\frac{2}{3}}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2\,Ca^{\frac{2}{3}}+Ba^{\frac{1}{3}}b^{\frac{1}{3}}+\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+
a),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*C*a*b^(2/3) - (6*C*a^(2/3)*(a/b)^(1/3) + 3*B*a^(1/3)*b^(1/3
)*(a/b)^(1/3) + (3*B*(a/b)^(2/3) + 2*C*a/b)*b^(2/3))*b)*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) - 1/6*(2*C*a^(2/3) + B*a^(1/3)*b^(1/
3) - (2*C*(a/b)^(2/3) + B*(a/b)^(1/3))*b^(2/3))*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(2*C*a^(2/3) + B*a^(1/3)*b^(1/3) + (C*(a/
b)^(2/3) - B*(a/b)^(1/3))*b^(2/3))*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(53) = 106$.
time = 2.02, size = 430, normalized size = 6.14

$$\left[\frac{\sqrt{\frac{1}{3}}\,b\sqrt{-\frac{C^{2}ab^{\frac{1}{3}}+2\,BCa^{\frac{2}{3}}b^{\frac{2}{3}}+B^{2}a^{\frac{1}{3}}b}{ab}}\log\left(-\frac{C^{3}a^{2}+B^{3}ab-2\left(C^{3}ab+B^{3}b^{2}\right)x^{3}+3\left(C^{3}a+B^{3}b\right)a^{\frac{2}{3}}b^{\frac{1}{3}}x-3\sqrt{\frac{1}{3}}\left(\left(2\,B^{2}bx^{2}+C^{2}ax+BCa\right)a^{\frac{2}{3}}b^{\frac{2}{3}}+\left(2\,C^{2}abx^{2}-BCabx-B^{2}ab\right)a^{\frac{1}{3}}-\left(2\,BCabx^{2}-B^{2}abx+C^{2}a^{2}\right)b^{\frac{1}{3}}\right)\sqrt{-\frac{C^{2}ab^{\frac{1}{3}}+2\,BCa^{\frac{2}{3}}b^{\frac{2}{3}}+B^{2}a^{\frac{1}{3}}b}{ab}}}{bx^{3}+a}\right)+Cb^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b},\ \frac{2\sqrt{\frac{1}{3}}\,b\sqrt{\frac{C^{2}ab^{\frac{1}{3}}+2\,BCa^{\frac{2}{3}}b^{\frac{2}{3}}+B^{2}a^{\frac{1}{3}}b}{ab}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(\left(2\,C^{2}x+BC\right)a^{\frac{2}{3}}b^{\frac{2}{3}}-\left(2\,BCbx+B^{2}b\right)a^{\frac{1}{3}}+\left(2\,B^{2}bx-C^{2}a\right)b^{\frac{1}{3}}\right)\sqrt{\frac{C^{2}ab^{\frac{1}{3}}+2\,BCa^{\frac{2}{3}}b^{\frac{2}{3}}+B^{2}a^{\frac{1}{3}}b}{ab}}}{C^{3}a+B^{3}b}\right)+Cb^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+
a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*b*sqrt(-(C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)
/(a*b))*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 + 3*(C^3*a + B^
3*b)*a^(2/3)*b^(1/3)*x - 3*sqrt(1/3)*((2*B^2*b*x^2 + C^2*a*x + B*C*a)*a^(2/
3)*b^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*a^(1/3) - (2*B*C*a*b*x^2
 - B^2*a*b*x + C^2*a^2)*b^(1/3))*sqrt(-(C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/
3) + B^2*a^(1/3)*b)/(a*b)))/(b*x^3 + a)) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2
/3)))/b, (2*sqrt(1/3)*b*sqrt((C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a
^(1/3)*b)/(a*b))*arctan(sqrt(1/3)*((2*C^2*x + B*C)*a^(2/3)*b^(2/3) - (2*B*C
*b*x + B^2*b)*a^(1/3) + (2*B^2*b*x - C^2*a)*b^(1/3))*sqrt((C^2*a*b^(1/3) +
2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)/(a*b))/(C^3*a + B^3*b)) + C*b^(2/3)*
log(b*x + a^(1/3)*b^(2/3)))/b]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*b**(1/3)*B+2*a**(2/3)*C+b**(2/3)*B*x+b**(2/3)*C*x**2)/(
b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+
a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.23, size = 386, normalized size = 5.51

$$\sum_{k=1}^{3}\ln\left(-\frac{x\,\left(2\,C^2\,a^{2/3}\,b^{2/3}-B^2\,b^{4/3}+B\,C\,a^{1/3}\,b\right)}{b^2}+\frac{a^{1/3}\,{\left(B\,b^{1/3}+C\,a^{1/3}\right)}^2}{b^{5/3}}+\frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^{8/3}\,z^2+18\,B\,C\,a^{5/3}\,b^{8/3}\,z+9\,C^2\,a^2\,b^{7/3}\,z+9\,B^2\,a^{4/3}\,b^3\,z-18\,B\,C^2\,a^{5/3}\,b^{7/3}-9\,B^2\,C\,a^{4/3}\,b^{8/3}-9\,C^3\,a^2\,b^2,z,k\right)\,\left(-6\,C\,a+\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^{8/3}\,z^2+18\,B\,C\,a^{5/3}\,b^{8/3}\,z+9\,C^2\,a^2\,b^{7/3}\,z+9\,B^2\,a^{4/3}\,b^3\,z-18\,B\,C^2\,a^{5/3}\,b^{7/3}-9\,B^2\,C\,a^{4/3}\,b^{8/3}-9\,C^3\,a^2\,b^2,z,k\right)\,a\,b^{1/3}\,9+3\,B\,a^{1/3}\,b^{2/3}\,x+6\,C\,a^{2/3}\,b^{1/3}\,x\right)}{b^{4/3}}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^{8/3}\,z^2+18\,B\,C\,a^{5/3}\,b^{8/3}\,z+9\,C^2\,a^2\,b^{7/3}\,z+9\,B^2\,a^{4/3}\,b^3\,z-18\,B\,C^2\,a^{5/3}\,b^{7/3}-9\,B^2\,C\,a^{4/3}\,b^{8/3}-9\,C^3\,a^2\,b^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*C*a^(2/3) + B*a^(1/3)*b^(1/3) + C*b^(2/3)*x^2 + B*b^(2/3)*x)/(a + b*
x^3),x)
```

```
[Out] symsum(log((a^(1/3)*(B*b^(1/3) + C*a^(1/3))^2)/b^(5/3) - (x*(2*C^2*a^(2/3)*
b^(2/3) - B^2*b^(4/3) + B*C*a^(1/3)*b))/b^2 + (root(27*a^2*b^3*z^3 - 27*C*a
^2*b^(8/3)*z^2 + 18*B*C*a^(5/3)*b^(8/3)*z + 9*C^2*a^2*b^(7/3)*z + 9*B^2*a^(
4/3)*b^3*z - 18*B*C^2*a^(5/3)*b^(7/3) - 9*B^2*C*a^(4/3)*b^(8/3) - 9*C^3*a^2
*b^2, z, k)*(9*root(27*a^2*b^3*z^3 - 27*C*a^2*b^(8/3)*z^2 + 18*B*C*a^(5/3)*
b^(8/3)*z + 9*C^2*a^2*b^(7/3)*z + 9*B^2*a^(4/3)*b^3*z - 18*B*C^2*a^(5/3)*b^
(7/3) - 9*B^2*C*a^(4/3)*b^(8/3) - 9*C^3*a^2*b^2, z, k)*a*b^(1/3) - 6*C*a +
3*B*a^(1/3)*b^(2/3)*x + 6*C*a^(2/3)*b^(1/3)*x))/b^(4/3))*root(27*a^2*b^3*z^
3 - 27*C*a^2*b^(8/3)*z^2 + 18*B*C*a^(5/3)*b^(8/3)*z + 9*C^2*a^2*b^(7/3)*z +
 9*B^2*a^(4/3)*b^3*z - 18*B*C^2*a^(5/3)*b^(7/3) - 9*B^2*C*a^(4/3)*b^(8/3) -
 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

3.41 $\int \frac{\sqrt[3]{a}\,\sqrt[3]{-b}\,B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=88

$$\frac{2\left(bB+\sqrt[3]{a}\,(-b)^{2/3}C\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}$$

```
[Out] C*ln(a^(1/3)-(-b)^(1/3)*x)/(-b)^(1/3)+2/3*(b*B+a^(1/3)*(-b)^(2/3)*C)*arctan
(1/3*(a^(1/3)+2*(-b)^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {1880, 31, 631, 210}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,(-b)^{2/3}C+bB\right)}{\sqrt{3}\,\sqrt[3]{a}\,b}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-b)^(2/3)*C*x^2
)/(a + b*x^3),x]
```

```
[Out] (2*(b*B + a^(1/3)*(-b)^(2/3)*C)*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(1/3)*b) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1880

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/(-b)^(1/3)}, Dis
t[-C/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2),
x], x]] /; EqQ[A*(-b)^(2/3) + a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C, 0]] /; Fr
eeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx=-\frac{C\int\frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{-b}}-x}\,dx}{\sqrt[3]{-b}}+\frac{\left(\sqrt[3]{-b}B-\sqrt[3]{a}C\right)\int\frac{\quad}{\frac{a^{2/3}}{(-b)^{2/3}}}}{(-b)^{2/3}}$$

$$=\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}-\left(2\left(\frac{B}{\sqrt[3]{a}}+\frac{bC}{(-b)^{4/3}}\right)\right)$$

$$=\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{bC}{(-b)^{4/3}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}+\frac{C\log}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 238 vs. 2(88) = 176.
time = 0.36, size = 238, normalized size = 2.70

$$\frac{2\sqrt{3}\sqrt[3]{b}\left(\left((-b)^{2/3}-\sqrt[3]{-b^2}\right)B+2\sqrt[3]{a}\sqrt[3]{b}C\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\frac{-2b\left(\left(-(-b)^{2/3}+b^{2/3}\right)B+2\sqrt[3]{a}\sqrt[3]{-b}C\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\left((-b)^{5/3}B+b^{5/3}B+2\sqrt[3]{a}\sqrt[3]{-b}bC\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{a}(-b)^{2/3}\sqrt[3]{-b^2}C\log(a+bx^3)}{\sqrt[3]{-b^2}}}{6\sqrt[3]{a}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-b)^(2/3)
*C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[3]*b^(1/3)*(((-b)^(2/3) - (-b^2)^(1/3))*B + 2*a^(1/3)*b^(1/3)*C)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (-2*b*((-(-b)^(2/3) + b^(2/3))*
B + 2*a^(1/3)*(-b)^(1/3)*C)*Log[a^(1/3) + b^(1/3)*x] + ((-b)^(5/3)*B + b^(5
/3)*B + 2*a^(1/3)*(-b)^(1/3)*b*C)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2] - 2*a^(1/3)*(-b)^(2/3)*(-b^2)^(1/3)*C*Log[a + b*x^3])/(-b^2)^(1/3))/(
6*a^(1/3)*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(67) = 134.
time = 0.51, size = 227, normalized size = 2.58

method	result
default	$\left(a^{\frac{1}{3}}(-b)^{\frac{1}{3}} B-2a^{\frac{2}{3}}C\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-(-b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2)/(b*x
^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6
/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-(-b)^(2/3)*B*(-1/3/b/(a/b)^(1/3)*l
n(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^
(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-1/3*C*(-b)^(2/
3)*ln(b*x^3+a)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(67) = 134$.
time = 0.53, size = 252, normalized size = 2.86

$$\frac{\sqrt{3}\left(2\,Ca(-b)^{\frac{2}{3}}-\left(6\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}-3\,Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}+\frac{2\,Ca}{b}\right)(-b)^{\frac{2}{3}}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}+\frac{\left(2\,Ca^{\frac{2}{3}}-Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}-\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)(-b)^{\frac{2}{3}}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2\,Ca^{\frac{2}{3}}-Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}+\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)(-b)^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2
)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*(2*C*a*(-b)^(2/3) - (6*C*a^(2/3)*(a/b)^(1/3) - 3*B*a^(1/3)*(-b)
^(1/3)*(a/b)^(1/3) + (3*B*(a/b)^(2/3) + 2*C*a/b)*(-b)^(2/3))*b)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*C*a^(2/3) - B*a^(1/
3)*(-b)^(1/3) - (2*C*(a/b)^(2/3) + B*(a/b)^(1/3))*(-b)^(2/3))*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(2*C*a^(2/3) - B*a^(1/3)*(-
b)^(1/3) + (C*(a/b)^(2/3) - B*(a/b)^(1/3))*(-b)^(2/3))*log(x + (a/b)^(1/3))
/(b*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(67) = 134$.
time = 1.68, size = 470, normalized size = 5.34

$$\left[\frac{\sqrt{\frac{1}{3}}\,b\sqrt{\frac{C^{2}a(-b)^{\frac{1}{3}}-2\,BCa^{\frac{2}{3}}(-b)^{\frac{2}{3}}-B^{2}a^{\frac{1}{3}}b}{ab}}\log\left(-\frac{C^{3}a^{2}+B^{3}ab-2\left(C^{3}ab+B^{3}b^{2}\right)x^{3}-3\left(C^{3}a+B^{3}b\right)a^{\frac{2}{3}}(-b)^{\frac{1}{3}}x+3\sqrt{\frac{1}{3}}\left(\left(2B^{2}bx^{2}+C^{2}ax+BCa\right)a^{\frac{2}{3}}(-b)^{\frac{2}{3}}+\left(2C^{2}abx^{2}-BCabx-B^{2}ab\right)a^{\frac{1}{3}}+\left(2BCabx^{2}-B^{2}abx+C^{2}a^{2}\right)(-b)^{\frac{1}{3}}\right)\sqrt{\frac{C^{2}a(-b)^{\frac{1}{3}}-2\,BCa^{\frac{2}{3}}(-b)^{\frac{2}{3}}-B^{2}a^{\frac{1}{3}}b}{ab}}}{bx^{3}+a}\right)-C(-b)^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}(-b)^{\frac{2}{3}}\right)}{b},-\frac{2\sqrt{\frac{1}{3}}\,b\sqrt{-\frac{C^{2}a(-b)^{\frac{1}{3}}-2\,BCa^{\frac{2}{3}}(-b)^{\frac{2}{3}}-B^{2}a^{\frac{1}{3}}b}{ab}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(\left(2C^{2}x+BC\right)a^{\frac{2}{3}}(-b)^{\frac{2}{3}}-\left(2BCbx+B^{2}b\right)a^{\frac{1}{3}}-\left(2B^{2}bx-C^{2}a\right)(-b)^{\frac{1}{3}}\right)\sqrt{-\frac{C^{2}a(-b)^{\frac{1}{3}}-2\,BCa^{\frac{2}{3}}(-b)^{\frac{2}{3}}-B^{2}a^{\frac{1}{3}}b}{ab}}}{C^{3}a+B^{3}b}\right)+C(-b)^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}(-b)^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2
)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*b*sqrt((C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/
3)*b)/(a*b))*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 - 3*(C^3*a
 + B^3*b)*a^(2/3)*(-b)^(1/3)*x + 3*sqrt(1/3)*((2*B^2*b*x^2 + C^2*a*x + B*C*
a)*a^(2/3)*(-b)^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*a^(1/3) + (2*
B*C*a*b*x^2 - B^2*a*b*x + C^2*a^2)*(-b)^(1/3))*sqrt((C^2*a*(-b)^(1/3) - 2*B
*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b)))/(b*x^3 + a)) - C*(-b)^(2/3)*
log(b*x + a^(1/3)*(-b)^(2/3)))/b, -(2*sqrt(1/3)*b*sqrt(-(C^2*a*(-b)^(1/3) -
 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b))*arctan(sqrt(1/3)*((2*C^2*
x + B*C)*a^(2/3)*(-b)^(2/3) - (2*B*C*b*x + B^2*b)*a^(1/3) - (2*B^2*b*x - C^
2*a)*(-b)^(1/3))*sqrt(-(C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a
^(1/3)*b)/(a*b))/(C^3*a + B^3*b)) + C*(-b)^(2/3)*log(b*x + a^(1/3)*(-b)^(2/
3)))/b]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*(-b)**(1/3)*B-2*a**(2/3)*C-(-b)**(2/3)*B*x-(-b)**(2/3)*
C*x**2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2
)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 6.32, size = 444, normalized size = 5.05

$$\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,{\left(-b\right)}^{8/3}\,z^2+18\,B\,C\,a^{5/3}\,{\left(-b\right)}^{8/3}\,z+9\,B^2\,a^{4/3}\,b^3\,z-9\,C^2\,a^2\,{\left(-b\right)}^{7/3}\,z-18\,B\,C^2\,a^{5/3}\,{\left(-b\right)}^{7/3}+9\,B^2\,C\,a^{4/3}\,{\left(-b\right)}^{8/3}+9\,C^3\,a^2\,b^2,z,k\right)\,\left(\frac{6\,C\,a}{{\left(-b\right)}^{4/3}}-\frac{x\,\left(3\,B\,a^{1/3}\,{\left(-b\right)}^{4/3}+6\,C\,a^{2/3}\,b\right)}{b^2}+\frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,{\left(-b\right)}^{8/3}\,z^2+18\,B\,C\,a^{5/3}\,{\left(-b\right)}^{8/3}\,z+9\,B^2\,a^{4/3}\,b^3\,z-9\,C^2\,a^2\,{\left(-b\right)}^{7/3}\,z-18\,B\,C^2\,a^{5/3}\,{\left(-b\right)}^{7/3}+9\,B^2\,C\,a^{4/3}\,{\left(-b\right)}^{8/3}+9\,C^3\,a^2\,b^2,z,k\right)\,a\,9}{b}\right)+\frac{B^2\,a^{1/3}\,b^2+C^2\,a\,{\left(-b\right)}^{4/3}-2\,B\,C\,a^{2/3}\,{\left(-b\right)}^{5/3}}{b^3}-\frac{x\,\left(2\,C^2\,a^{2/3}\,{\left(-b\right)}^{2/3}-B^2\,{\left(-b\right)}^{4/3}+B\,C\,a^{1/3}\,b\right)}{b^2}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,{\left(-b\right)}^{8/3}\,z^2+18\,B\,C\,a^{5/3}\,{\left(-b\right)}^{8/3}\,z+9\,B^2\,a^{4/3}\,b^3\,z-9\,C^2\,a^2\,{\left(-b\right)}^{7/3}\,z-18\,B\,C^2\,a^{5/3}\,{\left(-b\right)}^{7/3}+9\,B^2\,C\,a^{4/3}\,{\left(-b\right)}^{8/3}+9\,C^3\,a^2\,b^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*C*a^(2/3) + B*(-b)^(2/3)*x - B*a^(1/3)*(-b)^(1/3) + C*(-b)^(2/3)*x^
2)/(a + b*x^3),x)
```

```
[Out] symsum(log(root(27*a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 + 18*B*C*a^(5/3)*(
-b)^(8/3)*z + 9*B^2*a^(4/3)*b^3*z - 9*C^2*a^2*(-b)^(7/3)*z - 18*B*C^2*a^(5/
3)*(-b)^(7/3) + 9*B^2*C*a^(4/3)*(-b)^(8/3) + 9*C^3*a^2*b^2, z, k)*((6*C*a)/
(-b)^(4/3) - (x*(3*B*a^(1/3)*(-b)^(4/3) + 6*C*a^(2/3)*b))/b^2 + (9*root(27*
a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 + 18*B*C*a^(5/3)*(-b)^(8/3)*z + 9*B^2
*a^(4/3)*b^3*z - 9*C^2*a^2*(-b)^(7/3)*z - 18*B*C^2*a^(5/3)*(-b)^(7/3) + 9*B
^2*C*a^(4/3)*(-b)^(8/3) + 9*C^3*a^2*b^2, z, k)*a)/b) + (B^2*a^(1/3)*b^2 + C
^2*a*(-b)^(4/3) - 2*B*C*a^(2/3)*(-b)^(5/3))/b^3 - (x*(2*C^2*a^(2/3)*(-b)^(2
/3) - B^2*(-b)^(4/3) + B*C*a^(1/3)*b))/b^2)*root(27*a^2*b^3*z^3 + 27*C*a^2*
(-b)^(8/3)*z^2 + 18*B*C*a^(5/3)*(-b)^(8/3)*z + 9*B^2*a^(4/3)*b^3*z - 9*C^2*
a^2*(-b)^(7/3)*z - 18*B*C^2*a^(5/3)*(-b)^(7/3) + 9*B^2*C*a^(4/3)*(-b)^(8/3)
 + 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

3.42 $\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3}\,dx$

Optimal. Leaf size=11

$$\frac{\log(B-Cx)}{C}$$

```
[Out] ln(-C*x+B)/C
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1600, 31}

$$\frac{\log(B-Cx)}{C}$$

Antiderivative was successfully verified.

```
[In] Int[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]
```

```
[Out] Log[B - C*x]/C
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3}\,dx &= \int \frac{1}{-B+Cx}\,dx\\ &= \frac{\log(B-Cx)}{C}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.09

$$\frac{\log(-B+Cx)}{C}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]
```

```
[Out] Log[-B + C*x]/C
```

Maple [A]
time = 0.37, size = 12, normalized size = 1.09

method	result	size
default	$\frac{\ln(-Cx+B)}{C}$	12
norman	$\frac{\ln(-Cx+B)}{C}$	12
risch	$\frac{\ln(-Cx+B)}{C}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x,method=_RETURNVERBOSE)
```

```
[Out] ln(-C*x+B)/C
```

Maxima [A]
time = 0.29, size = 12, normalized size = 1.09

$$\frac{\log(Cx - B)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="maxima")
```

```
[Out] log(C*x - B)/C
```

Fricas [A]
time = 0.36, size = 12, normalized size = 1.09

$$\frac{\log(Cx - B)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="fricas")
```

```
[Out] log(C*x - B)/C
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.64

$$\frac{\log(-B + Cx)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C**2*x**2+B*C*x+B**2)/(C**3*x**3-B**3),x)
```

```
[Out] log(-B + C*x)/C
```

Giac [A]
time = 1.76, size = 13, normalized size = 1.18

$$\frac{\log\left(|Cx - B|\right)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="giac")
```

```
[Out] log(abs(C*x - B))/C
```

Mupad [B]
time = 0.04, size = 12, normalized size = 1.09

$$\frac{\ln\left(C\,x - B\right)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(B^2 + C^2*x^2 + B*C*x)/(B^3 - C^3*x^3),x)
```

```
[Out] log(C*x - B)/C
```

3.43 $\int \frac{a^{2/3}C-\sqrt[3]{a}\,\sqrt[3]{b}\,Cx+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=21

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}$$

```
[Out] C*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1600, 31}

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^{2/3}C-\sqrt[3]{a}\,\sqrt[3]{b}\,Cx+b^{2/3}Cx^2}{a+bx^3}\,dx &= \int \frac{1}{\frac{\sqrt[3]{a}}{C}+\frac{\sqrt[3]{b}\,x}{C}}\,dx\\ &= \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(15) = 30.
time = 0.37, size = 209, normalized size = 9.95

method	result
risch	$\frac{C\ln\left(a^{\frac{1}{3}}b^{\frac{2}{3}}+bx\right)}{b^{\frac{1}{3}}}$
default	$C\left(a^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-a^{\frac{1}{3}}b^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x,method=_RETUR
NVERBOSE)
```

```
[Out] C*(a^(2/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1)))-a^(1/3)*b^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/
b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3/b^(1/3)*ln(b*x^3+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(15) = 30.
time = 0.51, size = 210, normalized size = 10.00

$$-\frac{\sqrt{3}\left(2\,Cab^{\frac{2}{3}}+\left(3\,Ca^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-3\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{2\,Ca}{b^{\frac{1}{3}}}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}+\frac{\left(2\,Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-Ca^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}-Ca^{\frac{2}{3}}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}+Ca^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+Ca^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algori
thm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*C*a*b^(2/3) + (3*C*a^(1/3)*b^(1/3)*(a/b)^(2/3) - 3*C*a^(2/3
)*(a/b)^(1/3) - 2*C*a/b^(1/3))*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a
/b)^(1/3))/(a*b) + 1/6*(2*C*b^(2/3)*(a/b)^(2/3) - C*a^(1/3)*b^(1/3)*(a/b)^(
1/3) - C*a^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) +
1/3*(C*b^(2/3)*(a/b)^(2/3) + C*a^(1/3)*b^(1/3)*(a/b)^(1/3) + C*a^(2/3))*log
(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Fricas [A]
time = 0.40, size = 17, normalized size = 0.81

$$\frac{C\log\left(bx + a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algori
thm="fricas")
```

```
[Out] C*log(b*x + a^(1/3)*b^(2/3))/b^(1/3)
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.95

$$\frac{C\log\left(\sqrt[3]{a}\,b^{\frac{2}{3}} + bx\right)}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(2/3)*C-a**(1/3)*b**(1/3)*C*x+b**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] C*log(a**(1/3)*b**(2/3) + b*x)/b**(1/3)
```

Giac [A]
time = 1.67, size = 16, normalized size = 0.76

$$\frac{C\log\left(\left|b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right|\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algori
thm="giac")
```

```
[Out] C*log(abs(b^(1/3)*x + a^(1/3)))/b^(1/3)
```

Mupad [B]
time = 4.90, size = 15, normalized size = 0.71

$$\frac{C\,\ln\left(x+\frac{a^{1/3}}{b^{1/3}}\right)}{b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*a^(2/3) + C*b^(2/3)*x^2 - C*a^(1/3)*b^(1/3)*x)/(a + b*x^3),x)
[Out] (C*log(x + a^(1/3)/b^(1/3)))/b^(1/3)
```

3.44 $\int \frac{\sqrt[3]{\frac{a}{b}}\,B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=71

$$-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B+\sqrt[3]{\frac{a}{b}}\,C\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,a}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

```
[Out] C*ln((a/b)^(1/3)+x)/b-2/3*(a/b)^(2/3)*(B+(a/b)^(1/3)*C)*arctan(1/3*(1-2*x/(
a/b)^(1/3))*3^(1/2))/a*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1881, 31, 631, 210}

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2\left(\frac{a}{b}\right)^{2/3}\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)\left(C\sqrt[3]{\frac{a}{b}}+B\right)}{\sqrt{3}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*(a/b)^(2/3)*(B + (a/b)^(1/3)*C)*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]]
)/(Sqrt[3]*a) + (C*Log[(a/b)^(1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1881

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx=\frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}+x}\,dx}{b}+\frac{\left(B+\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$

$$=\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}+\left(2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a}+\frac{C}{b}\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.$$

$$=-\frac{2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a}+\frac{C}{b}\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 247 vs. 2(71) = 142.
time = 0.18, size = 247, normalized size = 3.48

$$\frac{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}B+\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\left(B+2\sqrt[3]{\frac{a}{b}}C\right)\right)\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)+2\sqrt[3]{b}\left(-a^{2/3}B+\sqrt[3]{a}\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\left(B+2\sqrt[3]{\frac{a}{b}}C\right)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\sqrt[3]{b}\left(a^{2/3}B-\sqrt[3]{a}\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\left(B+2\sqrt[3]{\frac{a}{b}}C\right)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2aC\log\left(a+bx^3\right)}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(1
/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 2*b^(1/3)*(-(a
```

```
^(2/3)*B) + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(1/3)*C))*Log[a^(1/3)
+ b^(1/3)*x] + b^(1/3)*(a^(2/3)*B - a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B + 2*(a/b
)^(1/3)*C))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*C*Log[a +
b*x^3])/(6*a*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(60) = 120$.
time = 0.30, size = 219, normalized size = 3.08

method	result
default	$\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}} C+\left(\frac{a}{b}\right)^{\frac{1}{3}} B\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+B\left(-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x,method=_RETURNVER
BOSE)
```

```
[Out] (2*(a/b)^(2/3)*C+(a/b)^(1/3)*B)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/
(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+B*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3
))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^
(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*C/b*ln(b*x^3+a)
```

Maxima [A]
time = 0.54, size = 78, normalized size = 1.10

$$\frac{C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}-\frac{2\sqrt{3}\,\left(Ca-\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}+\frac{4\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm=
"maxima")
```

```
[Out] C*log(x + (a/b)^(1/3))/b - 2/9*sqrt(3)*(C*a - (3*B*(a/b)^(2/3) + 4*C*a/b)*b
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(60) = 120$.

time = 1.16, size = 429, normalized size = 6.04

$$\left[\frac{C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+\sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}\log\left(-\frac{C^3a^2+B^3ab-2\left(C^3ab+B^3b^2\right)x^3+3\left(C^3ab+B^3b^2\right)x\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}\left(2\,BCabx^2-B^2abx+C^2a^2-\left(2\,B^2b^2x^2+C^2abx+BCab\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}-\left(2\,C^2abx^2-BCabx-B^2ab\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}}{bx^3+a}\right)}{b},\ \frac{2\sqrt{\frac{1}{3}}\sqrt{\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,B^2bx-C^2a+\left(2\,C^2bx+BCb\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}-\left(2\,BCbx+B^2b\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}}{C^3a+B^3b}\right)+C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm=
"fricas")
```

```
[Out] [(C*log(x + (a/b)^(1/3)) + sqrt(1/3)*sqrt(-(2*B*C*b*(a/b)^(2/3) + B^2*b*(a/
b)^(1/3) + C^2*a)/a)*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 +
3*(C^3*a*b + B^3*b^2)*x*(a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a*b*
x + C^2*a^2 - (2*B^2*b^2*x^2 + C^2*a*b*x + B*C*a*b)*(a/b)^(2/3) - (2*C^2*a*
b*x^2 - B*C*a*b*x - B^2*a*b)*(a/b)^(1/3))*sqrt(-(2*B*C*b*(a/b)^(2/3) + B^2*
b*(a/b)^(1/3) + C^2*a)/a))/(b*x^3 + a)))/b, (2*sqrt(1/3)*sqrt((2*B*C*b*(a/b
)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)*arctan(sqrt(1/3)*(2*B^2*b*x - C^2*a
 + (2*C^2*b*x + B*C*b)*(a/b)^(2/3) - (2*B*C*b*x + B^2*b)*(a/b)^(1/3))*sqrt(
(2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)/(C^3*a + B^3*b)) + C*l
og(x + (a/b)^(1/3)))/b]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(60) = 120.
time = 1.72, size = 129, normalized size = 1.82

$$-\frac{2\sqrt{3}\left(Cab+\left(ab^2\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(ab^2\right)^{\frac{1}{3}}Bb+2\left(ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm=
"giac")
```

```
[Out] -2/3*sqrt(3)*(C*a*b + (a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(-a/b)^(2/3) + B*b^2*(-a/b)^(1/3) +
(a*b^2)^(1/3)*B*b + 2*(a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3
)))/(a*b^2)
```

Mupad [B]
time = 6.08, size = 436, normalized size = 6.14

$$\sum_{k=1}^{3}\ln\left(\frac{C^2\,a+B^2\,b\,\left(\frac{a}{b}\right)^{1/3}+2\,B\,C\,b\,\left(\frac{a}{b}\right)^{2/3}}{b^3}+\frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+18\,B\,C\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{2/3}+9\,B^2\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,\left(\frac{a}{b}\right)^{2/3}-9\,B^2\,C\,a\,b\,\left(\frac{a}{b}\right)^{1/3}-9\,C^3\,a^2,z,k\right)\,\left(-6\,C\,a+\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+18\,B\,C\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{2/3}+9\,B^2\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,\left(\frac{a}{b}\right)^{2/3}-9\,B^2\,C\,a\,b\,\left(\frac{a}{b}\right)^{1/3}-9\,C^3\,a^2,z,k\right)\,a\,b\,9+3\,B\,b\,x\,\left(\frac{a}{b}\right)^{1/3}+6\,C\,b\,x\,\left(\frac{a}{b}\right)^{2/3}\right)}{b^2}-\frac{x\,\left(2\,C^2\,\left(\frac{a}{b}\right)^{2/3}-B^2+B\,C\,\left(\frac{a}{b}\right)^{1/3}\right)}{b^2}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+18\,B\,C\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{2/3}+9\,B^2\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,\left(\frac{a}{b}\right)^{2/3}-9\,B^2\,C\,a\,b\,\left(\frac{a}{b}\right)^{1/3}-9\,C^3\,a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x + C*x^2 + B*(a/b)^(1/3) + 2*C*(a/b)^(2/3))/(a + b*x^3),x)
```

```
[Out] symsum(log((C^2*a + B^2*b*(a/b)^(1/3) + 2*B*C*b*(a/b)^(2/3))/b^3 + (root(27
*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*
z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(a/b)^(2/3) - 9*B^2*C*a*b*(a/b
)^(1/3) - 9*C^3*a^2, z, k)*(9*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*B
*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C
^2*a*b*(a/b)^(2/3) - 9*B^2*C*a*b*(a/b)^(1/3) - 9*C^3*a^2, z, k)*a*b - 6*C*a
 + 3*B*b*x*(a/b)^(1/3) + 6*C*b*x*(a/b)^(2/3)))/b^2 - (x*(2*C^2*(a/b)^(2/3)
- B^2 + B*C*(a/b)^(1/3)))/b^2)*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*
B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*
C^2*a*b*(a/b)^(2/3) - 9*B^2*C*a*b*(a/b)^(1/3) - 9*C^3*a^2, z, k), k, 1, 3)
```

3.45 $\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=76

$$\frac{2\left(B+\sqrt[3]{-\frac{a}{b}}\,C\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{-\frac{a}{b}}\,b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

```
[Out] -C*ln((-a/b)^(1/3)+x)/b+2/3*(B+(-a/b)^(1/3)*C)*arctan(1/3*(1-2*x/(-a/b)^(1/
3))*3^(1/2))/(-a/b)^(1/3)/b*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {1881, 31, 631, 210}

$$\frac{2\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)\left(C\sqrt[3]{-\frac{a}{b}}+B\right)}{\sqrt{3}\,b\sqrt[3]{-\frac{a}{b}}}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (2*(B + (-(a/b))^(1/3)*C)*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt
[3]*(-(a/b))^(1/3)*b) - (C*Log[(-(a/b))^(1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1881

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\int \frac{\sqrt[3]{-\frac{a}{b}}\,B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx=-\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}+x}\,dx}{b}-\frac{\left(B+\sqrt[3]{-\frac{a}{b}}\,C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}-\sqrt[3]{-\frac{a}{b}}\,x+x^2}}{b}$$

$$=-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}-\frac{\left(2\left(\frac{B}{\sqrt[3]{-\frac{a}{b}}}+C\right)\right)\text{Subst}\left(\int \frac{1}{-3-}\right.}{b}$$

$$=\frac{2\left(\frac{B}{\sqrt[3]{-\frac{a}{b}}}+C\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 288 vs. 2(76) = 152.
time = 0.12, size = 288, normalized size = 3.79

$$-\frac{\left(a^{2/3}B-\sqrt[3]{a}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}B-2\sqrt[3]{a}\left(-\frac{a}{b}\right)^{2/3}\sqrt[3]{b}C\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,ab^{2/3}}-\frac{\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}B+2\sqrt[3]{a}\left(-\frac{a}{b}\right)^{2/3}\sqrt[3]{b}C\right)\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3ab^{2/3}}-\frac{\left(-a^{2/3}B-\sqrt[3]{a}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}B-2\sqrt[3]{a}\left(-\frac{a}{b}\right)^{2/3}\sqrt[3]{b}C\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6ab^{2/3}}-\frac{C\log\left(a-bx^3\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3)
,x]
```

```
[Out] -(((a^(2/3)*B - a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B - 2*a^(1/3)*(-(a/b))^(2/3)
*b^(1/3)*C)*ArcTan[(a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a*b
^(2/3))) - ((a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B + 2*a^(1/3)*(-(a/
b))^(2/3)*b^(1/3)*C)*Log[a^(1/3) - b^(1/3)*x])/(3*a*b^(2/3)) - ((-(a^(2/3)*
B) - a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B - 2*a^(1/3)*(-(a/b))^(2/3)*b^(1/3)*C)
*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a*b^(2/3)) - (C*Log[a -
 b*x^3])/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(65) = 130$.
time = 0.31, size = 224, normalized size = 2.95

method	result
default	$\left(2\left(-\frac{a}{b}\right)^{\frac{2}{3}} C+\left(-\frac{a}{b}\right)^{\frac{1}{3}} B\right)\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\left(1+\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) \sqrt{3}}{3}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+B$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x,method=_RETURN
VERBOSE)
```

```
[Out] (2*(-a/b)^(2/3)*C+(-a/b)^(1/3)*B)*(-1/3/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6
/b/(a/b)^(2/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2)))+B*(-1/3/b/(a/b)^(1/3)*ln(x-(a/b)^(
1/3))+1/6/b/(a/b)^(1/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*3^(1/2)/b/(a/
b)^(1/3)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2)))-1/3*C*ln(-b*x^3+a)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(65) = 130$.
time = 0.52, size = 238, normalized size = 3.13

$$-\frac{\sqrt{3}\left(2 C a-\left(6 C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}-3 B\left(\frac{a}{b}\right)^{\frac{2}{3}}+3 B\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2 C a}{b}\right) b\right) \arctan\left(\frac{\sqrt{3}\left(2 x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 a b}-\frac{\left(2 C\left(\frac{a}{b}\right)^{\frac{2}{3}}-2 C\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}-B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) \log\left(x^{2}+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+2 C\left(-\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}+B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) \log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*C*a - (6*C*(a/b)^(1/3)*(-a/b)^(2/3) - 3*B*(a/b)^(2/3) + 3*B
*(a/b)^(1/3)*(-a/b)^(1/3) + 2*C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/
```

```
3))/(a/b)^(1/3))/(a*b) - 1/6*(2*C*(a/b)^(2/3) - 2*C*(-a/b)^(2/3) - B*(a/b)^
(1/3) - B*(-a/b)^(1/3))*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/
3)) - 1/3*(C*(a/b)^(2/3) + 2*C*(-a/b)^(2/3) + B*(a/b)^(1/3) + B*(-a/b)^(1/3
))*log(x - (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(65) = 130$.
time = 1.09, size = 459, normalized size = 6.04

$$\left[-\frac{C\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)-\sqrt{\frac{1}{3}}\sqrt{\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}\log\left(-\frac{C^3a^2-B^3ab+2\left(C^3ab-B^3b^2\right)x^3-3\left(C^3ab-B^3b^2\right)x\left(-\frac{a}{b}\right)^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}\left(2\,BCabx^2-B^2abx-C^2a^2+\left(2\,B^2b^2x^2-C^2abx-BCab\right)\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\left(2\,C^2abx^2-BCabx-B^2ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}}{bx^3-a}\right)}{b}, -\frac{2\sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2\,B^2bx+C^2a+\left(2\,C^2bx+BCb\right)\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\left(2\,BCbx+B^2b\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}}{C^3a-B^3b}\right)+C\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="fricas")
```

```
[Out] [-(C*log(x + (-a/b)^(1/3)) - sqrt(1/3)*sqrt((2*B*C*b*(-a/b)^(2/3) + B^2*b*(
-a/b)^(1/3) - C^2*a)/a)*log(-(C^3*a^2 - B^3*a*b + 2*(C^3*a*b - B^3*b^2)*x^3
 - 3*(C^3*a*b - B^3*b^2)*x*(-a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*
a*b*x - C^2*a^2 + (2*B^2*b^2*x^2 - C^2*a*b*x - B*C*a*b)*(-a/b)^(2/3) - (2*C
^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(-a/b)^(1/3))*sqrt((2*B*C*b*(-a/b)^(2/3)
+ B^2*b*(-a/b)^(1/3) - C^2*a)/a))/(b*x^3 - a)))/b, -(2*sqrt(1/3)*sqrt(-(2*B
*C*b*(-a/b)^(2/3) + B^2*b*(-a/b)^(1/3) - C^2*a)/a)*arctan(-sqrt(1/3)*(2*B^2
*b*x + C^2*a + (2*C^2*b*x + B*C*b)*(-a/b)^(2/3) - (2*B*C*b*x + B^2*b)*(-a/b
)^(1/3))*sqrt(-(2*B*C*b*(-a/b)^(2/3) + B^2*b*(-a/b)^(1/3) - C^2*a)/a)/(C^3*
a - B^3*b)) + C*log(x + (-a/b)^(1/3)))/b]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [C] Result contains complex when optimal does not.
time = 1.81, size = 235, normalized size = 3.09

$$-\frac{\left(2\,Cab-\left(-a^2b^4\right)^{\frac{1}{3}}B\right)\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab^2+i\,\sqrt{3}\,\sqrt{a^2b^4}}-\frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}Bb+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}+\frac{\sqrt{3}\left(\left(9\left(-a^2b^4\right)^{\frac{1}{3}}ab^2+27^{\frac{5}{6}}\left(-a^2b^4\right)^{\frac{5}{6}}\right)B-18\left(a^2b^3+i\,\sqrt{3}\,\sqrt{a^4b^6}\right)C\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{54\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="giac")
```

```
[Out] -(2*C*a*b - (-a^2*b^4)^(1/3)*B)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(3*a
*b^2 + I*sqrt(3)*sqrt(a^2*b^4)) - 1/3*(C*b^2*(a/b)^(2/3) + B*b^2*(a/b)^(1/3
) + (-a*b^2)^(1/3)*B*b + 2*(-a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^
(1/3)))/(a*b^2) + 1/54*sqrt(3)*((9*(-a^2*b^4)^(1/3)*a*b^2 + 27^(5/6)*(-a^2*
b^4)^(5/6))*B - 18*(a^2*b^3 + I*sqrt(3)*sqrt(a^4*b^6))*C)*arctan(1/3*sqrt(3
)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4)
```

Mupad [B]
time = 6.48, size = 456, normalized size = 6.00

$$\sum_{k=1}^{3}\ln\left(\frac{B^2\,b\,{\left(-\frac{a}{b}\right)}^{1/3}-C^2\,a+2\,B\,C\,b\,{\left(-\frac{a}{b}\right)}^{2/3}}{b^3}-\frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2-18\,B\,C\,a\,b^2\,z\,{\left(-\frac{a}{b}\right)}^{2/3}-9\,B^2\,a\,b^2\,z\,{\left(-\frac{a}{b}\right)}^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,{\left(-\frac{a}{b}\right)}^{2/3}-9\,B^2\,C\,a\,b\,{\left(-\frac{a}{b}\right)}^{1/3}+9\,C^3\,a^2,z,k\right)\,\left(6\,C\,a+\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2-18\,B\,C\,a\,b^2\,z\,{\left(-\frac{a}{b}\right)}^{2/3}-9\,B^2\,a\,b^2\,z\,{\left(-\frac{a}{b}\right)}^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,{\left(-\frac{a}{b}\right)}^{2/3}-9\,B^2\,C\,a\,b\,{\left(-\frac{a}{b}\right)}^{1/3}+9\,C^3\,a^2,z,k\right)\,a\,b\,9+3\,B\,b\,x\,{\left(-\frac{a}{b}\right)}^{1/3}+6\,C\,b\,x\,{\left(-\frac{a}{b}\right)}^{2/3}\right)}{b^2}-\frac{x\,\left(2\,C^2\,{\left(-\frac{a}{b}\right)}^{2/3}-B^2+B\,C\,{\left(-\frac{a}{b}\right)}^{1/3}\right)}{b^2}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2-18\,B\,C\,a\,b^2\,z\,{\left(-\frac{a}{b}\right)}^{2/3}-9\,B^2\,a\,b^2\,z\,{\left(-\frac{a}{b}\right)}^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,{\left(-\frac{a}{b}\right)}^{2/3}-9\,B^2\,C\,a\,b\,{\left(-\frac{a}{b}\right)}^{1/3}+9\,C^3\,a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x + C*x^2 + B*(-a/b)^(1/3) + 2*C*(-a/b)^(2/3))/(a - b*x^3),x)
```

```
[Out] symsum(log((B^2*b*(-a/b)^(1/3) - C^2*a + 2*B*C*b*(-a/b)^(2/3))/b^3 - (root(
27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 - 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b
^2*z*(-a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) - 9*B^2*C*a*b
*(-a/b)^(1/3) + 9*C^3*a^2, z, k)*(6*C*a + 9*root(27*a^2*b^3*z^3 + 27*C*a^2*
b^2*z^2 - 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9*C^2*
a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) - 9*B^2*C*a*b*(-a/b)^(1/3) + 9*C^3*a^2,
 z, k)*a*b + 3*B*b*x*(-a/b)^(1/3) + 6*C*b*x*(-a/b)^(2/3)))/b^2 - (x*(2*C^2*
(-a/b)^(2/3) - B^2 + B*C*(-a/b)^(1/3)))/b^2)*root(27*a^2*b^3*z^3 + 27*C*a^2
*b^2*z^2 - 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9*C^2
*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) - 9*B^2*C*a*b*(-a/b)^(1/3) + 9*C^3*a^2
, z, k), k, 1, 3)
```

3.46 $\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=78

$$\frac{2\left(B-\sqrt[3]{-\frac{a}{b}}\,C\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{-\frac{a}{b}}\,b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

```
[Out] C*ln((-a/b)^(1/3)-x)/b+2/3*(B-(-a/b)^(1/3)*C)*arctan(1/3*(1+2*x/(-a/b)^(1/3
))*3^(1/2))/(-a/b)^(1/3)/b*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {1883, 31, 631, 210}

$$\frac{2\text{ArcTan}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)\left(B-C\sqrt[3]{-\frac{a}{b}}\right)}{\sqrt{3}\,b\sqrt[3]{-\frac{a}{b}}}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*(B - (-(a/b))^(1/3)*C)*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt
[3]*(-(a/b))^(1/3)*b) + (C*Log[(-(a/b))^(1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1883

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Dist[-C/b
, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]
] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &&
PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}\, B+2\left(-\frac{a}{b}\right)^{2/3} C+Bx+Cx^2}{a+bx^3}\, dx = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}\, dx}{b}+\frac{\left(B-\sqrt[3]{-\frac{a}{b}}\, C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}+\sqrt[3]{-\frac{a}{b}}\, x+x^2}}{b}$$

$$=\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{\left(2\left(B-\sqrt[3]{-\frac{a}{b}}\, C\right)\right)\text{Subst}\left(\int \frac{1}{-3-x}\right.}{\sqrt[3]{-\frac{a}{b}}\, b}$$

$$=\frac{2\left(B-\sqrt[3]{-\frac{a}{b}}\, C\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{-\frac{a}{b}}\, b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right.}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 253 vs. 2(78) = 156.
time = 0.16, size = 253, normalized size = 3.24

$$\frac{2\sqrt{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}\, B+\sqrt[3]{-\frac{a}{b}}\,\sqrt[3]{b}\left(-B+2\sqrt[3]{-\frac{a}{b}}\, C\right)\right)\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\, x}{\sqrt{3}\,\sqrt[3]{a}}\right)-2\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\,\sqrt[3]{-\frac{a}{b}}\,\sqrt[3]{b}\left(B-2\sqrt[3]{-\frac{a}{b}}\, C\right)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right)+\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\,\sqrt[3]{-\frac{a}{b}}\,\sqrt[3]{b}\left(B-2\sqrt[3]{-\frac{a}{b}}\, C\right)\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\, x+b^{2/3}x^2\right)+2aC\log\left(a+bx^3\right)}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a + b*x
^3),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (-(a/b))^(1/3)*b^(1/3)*(-B + 2*(-(a
/b))^(1/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] - 2*b^(1/
3)*(a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*(B - 2*(-(a/b))^(1/3)*C))*Lo
g[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3
)*(B - 2*(-(a/b))^(1/3)*C))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
+ 2*a*C*Log[a + b*x^3])/(6*a*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(67) = 134$.
time = 0.30, size = 222, normalized size = 2.85

method	result
default	$\left(2\left(-\frac{a}{b}\right)^{\frac{2}{3}} C-\left(-\frac{a}{b}\right)^{\frac{1}{3}} B\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+B$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x,method=_RETURN
VERBOSE)
```

```
[Out] (2*(-a/b)^(2/3)*C-(-a/b)^(1/3)*B)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/
b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+B*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1
/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*C/b*ln(b*x^3+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(68) = 136$.
time = 0.53, size = 239, normalized size = 3.06

$$-\frac{\sqrt{3}\left(2\,Ca-\left(6\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}+3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}-3\,B\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}+\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}+B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}-B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorit
hm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*C*a - (6*C*(a/b)^(1/3)*(-a/b)^(2/3) + 3*B*(a/b)^(2/3) - 3*B
*(a/b)^(1/3)*(-a/b)^(1/3) + 2*C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/
```

```
3))/(a/b)^(1/3))/(a*b) + 1/6*(2*C*(a/b)^(2/3) - 2*C*(-a/b)^(2/3) + B*(a/b)^
(1/3) + B*(-a/b)^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/
3)) + 1/3*(C*(a/b)^(2/3) + 2*C*(-a/b)^(2/3) - B*(a/b)^(1/3) - B*(-a/b)^(1/3
))*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(68) = 136$.
time = 1.11, size = 450, normalized size = 5.77

$$\left[\frac{C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+\sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}\log\left(-\frac{C^3a^2+B^3ab-2\left(C^3ab+B^3b^2\right)x^3+3\left(C^3ab+B^3b^2\right)x\left(-\frac{a}{b}\right)^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}\left(2\,BCabx^2-B^2abx+C^2a^2-\left(2\,B^2b^2x^2+C^2abx+BCab\right)\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\left(2\,C^2abx^2-BCabx-B^2ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}}{bx^3+a}\right)}{b},\ \frac{2\sqrt{\frac{1}{3}}\sqrt{\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,B^2bx-C^2a+\left(2\,C^2bx+BCb\right)\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\left(2\,BCbx+B^2b\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}}{C^3a+B^3b}\right)+C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorit
hm="fricas")
```

```
[Out] [(C*log(x - (-a/b)^(1/3)) + sqrt(1/3)*sqrt(-(2*B*C*b*(-a/b)^(2/3) - B^2*b*(
-a/b)^(1/3) + C^2*a)/a)*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3
 + 3*(C^3*a*b + B^3*b^2)*x*(-a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*
a*b*x + C^2*a^2 - (2*B^2*b^2*x^2 + C^2*a*b*x + B*C*a*b)*(-a/b)^(2/3) + (2*C
^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(-a/b)^(1/3))*sqrt(-(2*B*C*b*(-a/b)^(2/3)
 - B^2*b*(-a/b)^(1/3) + C^2*a)/a))/(b*x^3 + a)))/b, (2*sqrt(1/3)*sqrt((2*B*
C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a)*arctan(sqrt(1/3)*(2*B^2*b
*x - C^2*a + (2*C^2*b*x + B*C*b)*(-a/b)^(2/3) + (2*B*C*b*x + B^2*b)*(-a/b)^
(1/3))*sqrt((2*B*C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a)/(C^3*a +
 B^3*b)) + C*log(x - (-a/b)^(1/3)))/b]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.54, size = 133, normalized size = 1.71

$$-\frac{2\sqrt{3}\left(Cab+\left(-ab^2\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}Bb+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorit
hm="giac")
```

```
[Out] -2/3*sqrt(3)*(C*a*b + (-a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(-a/b)^(2/3) + B*b^2*(-a/b)^(1/3) -
 (-a*b^2)^(1/3)*B*b + 2*(-a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(
1/3)))/(a*b^2)
```

Mupad [B]
time = 6.05, size = 453, normalized size = 5.81

$$\sum_{k=1}^{3}\ln\left(\frac{C^2\,a-B^2\,b\,{\left(-\frac{a}{b}\right)}^{1/3}+2\,B\,C\,b\,{\left(-\frac{a}{b}\right)}^{2/3}}{b^3}-\frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+18\,B\,C\,a\,b^2\,z\,{\left(-\frac{a}{b}\right)}^{2/3}-9\,B^2\,a\,b^2\,z\,{\left(-\frac{a}{b}\right)}^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,{\left(-\frac{a}{b}\right)}^{2/3}+9\,B^2\,C\,a\,b\,{\left(-\frac{a}{b}\right)}^{1/3}-9\,C^3\,a^2,z,k\right)\,\left(6\,C\,a-\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+18\,B\,C\,a\,b^2\,z\,{\left(-\frac{a}{b}\right)}^{2/3}-9\,B^2\,a\,b^2\,z\,{\left(-\frac{a}{b}\right)}^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,{\left(-\frac{a}{b}\right)}^{2/3}+9\,B^2\,C\,a\,b\,{\left(-\frac{a}{b}\right)}^{1/3}-9\,C^3\,a^2,z,k\right)\,a\,b\,9+3\,B\,b\,x\,{\left(-\frac{a}{b}\right)}^{1/3}-6\,C\,b\,x\,{\left(-\frac{a}{b}\right)}^{2/3}\right)}{b^2}+\frac{x\,\left(B^2-2\,C^2\,{\left(-\frac{a}{b}\right)}^{2/3}+B\,C\,{\left(-\frac{a}{b}\right)}^{1/3}\right)}{b^2}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+18\,B\,C\,a\,b^2\,z\,{\left(-\frac{a}{b}\right)}^{2/3}-9\,B^2\,a\,b^2\,z\,{\left(-\frac{a}{b}\right)}^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,{\left(-\frac{a}{b}\right)}^{2/3}+9\,B^2\,C\,a\,b\,{\left(-\frac{a}{b}\right)}^{1/3}-9\,C^3\,a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x + C*x^2 - B*(-a/b)^(1/3) + 2*C*(-a/b)^(2/3))/(a + b*x^3),x)
```

```
[Out] symsum(log((C^2*a - B^2*b*(-a/b)^(1/3) + 2*B*C*b*(-a/b)^(2/3))/b^3 - (root(
27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b
^2*z*(-a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) + 9*B^2*C*a*b
*(-a/b)^(1/3) - 9*C^3*a^2, z, k)*(6*C*a - 9*root(27*a^2*b^3*z^3 - 27*C*a^2*
b^2*z^2 + 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9*C^2*
a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) + 9*B^2*C*a*b*(-a/b)^(1/3) - 9*C^3*a^2,
 z, k)*a*b + 3*B*b*x*(-a/b)^(1/3) - 6*C*b*x*(-a/b)^(2/3)))/b^2 + (x*(B^2 -
2*C^2*(-a/b)^(2/3) + B*C*(-a/b)^(1/3)))/b^2)*root(27*a^2*b^3*z^3 - 27*C*a^2
*b^2*z^2 + 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9*C^2
*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) + 9*B^2*C*a*b*(-a/b)^(1/3) - 9*C^3*a^2
, z, k), k, 1, 3)
```

3.47 $\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=75

$$-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B-\sqrt[3]{\frac{a}{b}}\,C\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,a}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

```
[Out] -C*ln((a/b)^(1/3)-x)/b-2/3*(a/b)^(2/3)*(B-(a/b)^(1/3)*C)*arctan(1/3*(1+2*x/
(a/b)^(1/3))*3^(1/2))/a*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1883, 31, 631, 210}

$$-\frac{2\left(\frac{a}{b}\right)^{2/3}\text{ArcTan}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)\left(B-C\sqrt[3]{\frac{a}{b}}\right)}{\sqrt{3}\,a}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (-2*(a/b)^(2/3)*(B - (a/b)^(1/3)*C)*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]]
)/(Sqrt[3]*a) - (C*Log[(a/b)^(1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1883

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Dist[-C/b
, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]
] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &&
PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{-\sqrt[3]{\frac{a}{b}}\,B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}\,dx}{b} - \frac{\left(B-\sqrt[3]{\frac{a}{b}}\,C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}+\sqrt[3]{\frac{a}{b}}\,x+x^2}\,dx}{b}$$

$$= -\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b} + \left(2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a}-\frac{C}{b}\right)\right)\text{Subst}\left(\int \frac{1}{-3-\right.$$

$$= -\frac{2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a}-\frac{C}{b}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(75) = 150.
time = 0.17, size = 244, normalized size = 3.25

$$\frac{-2\sqrt{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}\,B+\sqrt[3]{\frac{a}{b}}\,\sqrt[3]{b}\left(B-2\sqrt[3]{\frac{a}{b}}\,C\right)\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\,\sqrt[3]{\frac{a}{b}}\,\sqrt[3]{b}\left(-B+2\sqrt[3]{\frac{a}{b}}\,C\right)\right)\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)+\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\,\sqrt[3]{\frac{a}{b}}\,\sqrt[3]{b}\left(-B+2\sqrt[3]{\frac{a}{b}}\,C\right)\right)\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2aC\log\left(a-bx^3\right)}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B - 2*(a/b)^(
1/3)*C))*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*(a^(2/3)*B
```

```
 + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^(1/3)*C))*Log[a^(1/3) - b^(1/3
)*x] + b^(1/3)*(a^(2/3)*B + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^(1/3)
*C))*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*C*Log[a - b*x^3])
/(6*a*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(64) = 128$.
time = 0.32, size = 223, normalized size = 2.97

method	result
default	$\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}} C-\left(\frac{a}{b}\right)^{\frac{1}{3}} B\right)\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+B\left(-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x,method=_RETURNV
ERBOSE)
```

```
[Out] (2*(a/b)^(2/3)*C-(a/b)^(1/3)*B)*(-1/3/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6/b
/(a/b)^(2/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2)))+B*(-1/3/b/(a/b)^(1/3)*ln(x-(a/b)^(1/
3))+1/6/b/(a/b)^(1/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*3^(1/2)/b/(a/b)
^(1/3)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2)))-1/3*C*ln(-b*x^3+a)/b
```

Maxima [A]
time = 0.52, size = 78, normalized size = 1.04

$$-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}-\frac{2\sqrt{3}\left(Ca+\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}-\frac{4\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorith
m="maxima")
```

```
[Out] -C*log(x - (a/b)^(1/3))/b - 2/9*sqrt(3)*(C*a + (3*B*(a/b)^(2/3) - 4*C*a/b)*
b)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(65) = 130$.

time = 1.13, size = 450, normalized size = 6.00

$$\left[-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\sqrt{\frac{1}{3}}\sqrt{\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}\log\left(-\frac{C^3a^2-B^3ab+2\left(C^3ab-B^3b^2\right)x^3-3\left(C^3ab-B^3b^2\right)x\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}\left(2\,BCabx^2-B^2abx-C^2a^2+\left(2\,B^2b^2x^2-C^2abx-BCab\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}+\left(2\,C^2abx^2-BCabx-B^2ab\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}}{bx^3-a}\right)}{b},-\frac{2\sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2\,B^2bx+C^2a+\left(2\,C^2bx+BCb\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}+\left(2\,BCbx+B^2b\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}}{C^3a-B^3b}\right)+C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorith
m="fricas")
```

```
[Out] [-(C*log(x - (a/b)^(1/3)) - sqrt(1/3)*sqrt((2*B*C*b*(a/b)^(2/3) - B^2*b*(a/
b)^(1/3) - C^2*a)/a)*log(-(C^3*a^2 - B^3*a*b + 2*(C^3*a*b - B^3*b^2)*x^3 -
3*(C^3*a*b - B^3*b^2)*x*(a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a*b*
x - C^2*a^2 + (2*B^2*b^2*x^2 - C^2*a*b*x - B*C*a*b)*(a/b)^(2/3) + (2*C^2*a*
b*x^2 - B*C*a*b*x - B^2*a*b)*(a/b)^(1/3))*sqrt((2*B*C*b*(a/b)^(2/3) - B^2*b
*(a/b)^(1/3) - C^2*a)/a))/(b*x^3 - a)))/b, -(2*sqrt(1/3)*sqrt(-(2*B*C*b*(a/
b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a)*arctan(-sqrt(1/3)*(2*B^2*b*x + C^2
*a + (2*C^2*b*x + B*C*b)*(a/b)^(2/3) + (2*B*C*b*x + B^2*b)*(a/b)^(1/3))*sqr
t(-(2*B*C*b*(a/b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a)/(C^3*a - B^3*b)) +
C*log(x - (a/b)^(1/3)))/b]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.13, size = 125, normalized size = 1.67

$$\frac{2\sqrt{3}\left(Cab-\left(ab^2\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}-\frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-\left(ab^2\right)^{\frac{1}{3}}Bb+2\left(ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorith
m="giac")
```

```
[Out] 2/3*sqrt(3)*(C*a*b - (a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3)
)/(a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(a/b)^(2/3) + B*b^2*(a/b)^(1/3) - (a*b^
2)^(1/3)*B*b + 2*(a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/(a*
b^2)
```

Mupad [B]

time = 6.36, size = 435, normalized size = 5.80

$$\sum_{k=1}^{3}\ln\left(-\frac{C^2\,a+B^2\,b\,\left(\frac{a}{b}\right)^{1/3}-2\,B\,C\,b\,\left(\frac{a}{b}\right)^{2/3}}{b^3}-\frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2-18\,B\,C\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{2/3}+9\,B^2\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,\left(\frac{a}{b}\right)^{2/3}+9\,B^2\,C\,a\,b\,\left(\frac{a}{b}\right)^{1/3}+9\,C^3\,a^2,z,k\right)\,\left(6\,C\,a+\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2-18\,B\,C\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{2/3}+9\,B^2\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,\left(\frac{a}{b}\right)^{2/3}+9\,B^2\,C\,a\,b\,\left(\frac{a}{b}\right)^{1/3}+9\,C^3\,a^2,z,k\right)\,a\,b\,9-3\,B\,b\,x\,\left(\frac{a}{b}\right)^{1/3}+6\,C\,b\,x\,\left(\frac{a}{b}\right)^{2/3}\right)}{b^2}+\frac{x\,\left(B^2-2\,C^2\,\left(\frac{a}{b}\right)^{2/3}+B\,C\,\left(\frac{a}{b}\right)^{1/3}\right)}{b^2}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2-18\,B\,C\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{2/3}+9\,B^2\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,\left(\frac{a}{b}\right)^{2/3}+9\,B^2\,C\,a\,b\,\left(\frac{a}{b}\right)^{1/3}+9\,C^3\,a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x + C*x^2 - B*(a/b)^(1/3) + 2*C*(a/b)^(2/3))/(a - b*x^3),x)
```

```
[Out] symsum(log((x*(B^2 - 2*C^2*(a/b)^(2/3) + B*C*(a/b)^(1/3)))/b^2 - (root(27*a
^2*b^3*z^3 + 27*C*a^2*b^2*z^2 - 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*
(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(a/b)^(2/3) + 9*B^2*C*a*b*(a/b)^
(1/3) + 9*C^3*a^2, z, k)*(6*C*a + 9*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2
- 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z -
18*B*C^2*a*b*(a/b)^(2/3) + 9*B^2*C*a*b*(a/b)^(1/3) + 9*C^3*a^2, z, k)*a*b -
 3*B*b*x*(a/b)^(1/3) + 6*C*b*x*(a/b)^(2/3)))/b^2 - (C^2*a + B^2*b*(a/b)^(1/
3) - 2*B*C*b*(a/b)^(2/3))/b^3)*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 - 18*
B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*
C^2*a*b*(a/b)^(2/3) + 9*B^2*C*a*b*(a/b)^(1/3) + 9*C^3*a^2, z, k), k, 1, 3)
```

3.48 $\int \frac{a+ax+cx^2}{1-x^3} \, dx$

Optimal. Leaf size=32

$$-\frac{1}{3}(2a+c)\log(1-x)+\frac{1}{3}(a-c)\log\left(1+x+x^2\right)$$

```
[Out] -1/3*(2*a+c)*ln(1-x)+1/3*(a-c)*ln(x^2+x+1)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1889, 31, 642}

$$\frac{1}{3}(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}(2a+c)\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x + c*x^2)/(1 - x^3),x]
```

```
[Out] -1/3*((2*a + c)*Log[1 - x]) + ((a - c)*Log[1 + x + x^2])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q
 + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q -
 C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b
*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
 && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+ax+cx^2}{1-x^3}\,dx &= \frac{1}{3}\int \frac{a-c+(2a-2c)x}{1+x+x^2}\,dx+\frac{1}{3}(2a+c)\int \frac{1}{1-x}\,dx\\ &= -\frac{1}{3}(2a+c)\log(1-x)+\frac{1}{3}(a-c)\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.97

$$\frac{1}{3}\left(-((2a+c)\log(1-x))+(a-c)\log\left(1+x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x + c*x^2)/(1 - x^3),x]
```

```
[Out] (-((2*a + c)*Log[1 - x]) + (a - c)*Log[1 + x + x^2])/3
```

Maple [A]
time = 0.33, size = 28, normalized size = 0.88

method	result
default	$\frac{(a-c)\ln(x^2+x+1)}{3}+\left(-\frac{2a}{3}-\frac{c}{3}\right)\ln(x-1)$
norman	$\left(-\frac{2a}{3}-\frac{c}{3}\right)\ln(x-1)+\left(\frac{a}{3}-\frac{c}{3}\right)\ln(x^2+x+1)$
risch	$-\frac{2\ln(x-1)a}{3}-\frac{\ln(x-1)c}{3}+\frac{\ln(-x^2-x-1)a}{3}-\frac{\ln(-x^2-x-1)c}{3}$
meijerg	$-\frac{c\ln(-x^3+1)}{3}-\frac{a\,x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}-\frac{ax\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a*x+a)/(-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a-c)*ln(x^2+x+1)+(-2/3*a-1/3*c)*ln(x-1)
```

Maxima [A]
time = 0.51, size = 26, normalized size = 0.81

$$\frac{1}{3}\,(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}\,(2\,a+c)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

Fricas [A]
time = 0.36, size = 26, normalized size = 0.81

$$\frac{1}{3}\,(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}\,(2\,a+c)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

Sympy [A]
time = 0.19, size = 24, normalized size = 0.75

$$\frac{(a-c)\log\left(x^2+x+1\right)}{3}-\frac{(2a+c)\log\left(x-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a*x+a)/(-x**3+1),x)
```

```
[Out] (a - c)*log(x**2 + x + 1)/3 - (2*a + c)*log(x - 1)/3
```

Giac [A]
time = 2.35, size = 27, normalized size = 0.84

$$\frac{1}{3}\,(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}\,(2\,a+c)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(abs(x - 1))
```

Mupad [B]
time = 4.78, size = 35, normalized size = 1.09

$$\frac{a\,\ln\left(x^2+x+1\right)}{3}-\frac{c\,\ln\left(x-1\right)}{3}-\frac{2\,a\,\ln\left(x-1\right)}{3}-\frac{c\,\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + a*x + c*x^2)/(x^3 - 1),x)
```

```
[Out] (a*log(x + x^2 + 1))/3 - (c*log(x - 1))/3 - (2*a*log(x - 1))/3 - (c*log(x +
 x^2 + 1))/3
```

3.49 $\int \frac{a+bx+cx^2}{1-x^3} \, dx$

Optimal. Leaf size=55

$$\frac{(a-b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}(a+b+c)\log(1-x)+\frac{1}{6}(a+b-2c)\log\left(1+x+x^2\right)$$

```
[Out] -1/3*(a+b+c)*ln(1-x)+1/6*(a+b-2*c)*ln(x^2+x+1)+1/3*(a-b)*arctan(1/3*(1+2*x)
*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1889, 31, 648, 632, 210, 642}

$$\frac{(a-b)\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^2+x+1\right)(a+b-2c)-\frac{1}{3}\log(1-x)(a+b+c)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(1 - x^3),x]
```

```
[Out] ((a - b)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - ((a + b + c)*Log[1 - x])/3 +
((a + b - 2*c)*Log[1 + x + x^2])/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q
 + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q -
 C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b
*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
 && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{1-x^3}\,dx &= \frac{1}{3}\int \frac{2a-b-c+(a+b-2c)x}{1+x+x^2}\,dx+\frac{1}{3}(a+b+c)\int \frac{1}{1-x}\,dx\\ &= -\frac{1}{3}(a+b+c)\log(1-x)+\frac{1}{2}(a-b)\int \frac{1}{1+x+x^2}\,dx+\frac{1}{6}(a+b-2c)\int \frac{1+2x}{1+x+x^2}\\ &= -\frac{1}{3}(a+b+c)\log(1-x)+\frac{1}{6}(a+b-2c)\log\left(1+x+x^2\right)+(-a+b)\mathrm{Subst}\left(\int \frac{1}{-3-}\right.\\ &= \frac{(a-b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}(a+b+c)\log(1-x)+\frac{1}{6}(a+b-2c)\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 1.13

$$\frac{1}{6}\left(2\sqrt{3}\,(a-b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-2(a+b)\log(1-x)+(a+b)\log\left(1+x+x^2\right)-2c\log\left(1-x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(1 - x^3),x]
```

```
[Out] (2*Sqrt[3]*(a - b)*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*(a + b)*Log[1 - x] + (a +
b)*Log[1 + x + x^2] - 2*c*Log[1 - x^3])/6
```

Maple [A]
time = 0.37, size = 55, normalized size = 1.00

method	result
default	$\frac{(a+b-2c)\ln(x^2+x+1)}{6}+\frac{2\left(\frac{3a}{2}-\frac{3b}{2}\right)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\left(-\frac{c}{3}-\frac{b}{3}-\frac{a}{3}\right)\ln\left(x-1\right)$
meijerg	$-\frac{c\ln(-x^3+1)}{3}-\frac{b\,x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}-\frac{ax\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x\right.}{}\right.}{}$
risch	$\frac{a\ln(4a^2x^2+4ab\,x^2+4b^2x^2+4a^2x+4abx+4b^2x+4a^2+4ab+4b^2)}{6}+\frac{b\ln(4a^2x^2+4ab\,x^2+4b^2x^2+4a^2x+4abx+4b^2x+4a^2+4ab+4b^2)}{6}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(a+b-2*c)*ln(x^2+x+1)+2/9*(3/2*a-3/2*b)*arctan(1/3*(2*x+1)*3^(1/2))*3^(
1/2)+(-1/3*c-1/3*b-1/3*a)*ln(x-1)
```

Maxima [A]
time = 0.53, size = 47, normalized size = 0.85

$$\frac{1}{3}\,\sqrt{3}\,(a-b)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\,(a+b-2\,c)\log\left(x^2+x+1\right)-\frac{1}{3}\,(a+b+c)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*c)*log(x
^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)
```

Fricas [A]
time = 0.37, size = 47, normalized size = 0.85

$$\frac{1}{3}\,\sqrt{3}\,(a-b)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\,(a+b-2\,c)\log\left(x^2+x+1\right)-\frac{1}{3}\,(a+b+c)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*c)*log(x
^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 323, normalized size = 5.87

$$-\frac{(a+b+c)\log\left(x+\frac{a^2c-a^2(a+b+c)-2ab^2+bc^2-2bc(a+b+c)+b(a+b+c)^2}{a^3-b^3}\right)}{3}-\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}\,i(a-b)}{6}\right)\log\left(x+\frac{a^2c-3a^2\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}\,i(a-b)}{6}\right)-2ab^2+bc^2-6bc\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}\,i(a-b)}{6}\right)+9b\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}\,i(a-b)}{6}\right)^2}{a^3-b^3}\right)-\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}\,i(a-b)}{6}\right)\log\left(x+\frac{a^2c-3a^2\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}\,i(a-b)}{6}\right)-2ab^2+bc^2-6bc\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}\,i(a-b)}{6}\right)+9b\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}\,i(a-b)}{6}\right)^2}{a^3-b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-x**3+1),x)
```

```
[Out] -(a + b + c)*log(x + (a**2*c - a**2*(a + b + c) - 2*a*b**2 + b*c**2 - 2*b*c
*(a + b + c) + b*(a + b + c)**2)/(a**3 - b**3))/3 - (-a/6 - b/6 + c/3 - sqr
t(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a
 - b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a - b)/
6) + 9*b*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a - b)/6)**2)/(a**3 - b**3)) - (-a/
6 - b/6 + c/3 + sqrt(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-a/6 - b/6 +
 c/3 + sqrt(3)*I*(a - b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6 + c/3 +
 sqrt(3)*I*(a - b)/6) + 9*b*(-a/6 - b/6 + c/3 + sqrt(3)*I*(a - b)/6)**2)/(a
**3 - b**3))
```

Giac [A]
time = 2.28, size = 52, normalized size = 0.95

$$\frac{1}{3}\left(\sqrt{3}\,a-\sqrt{3}\,b\right)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\,(a+b-2\,c)\log\left(x^2+x+1\right)-\frac{1}{3}\,(a+b+c)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(3)*a - sqrt(3)*b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*
c)*log(x^2 + x + 1) - 1/3*(a + b + c)*log(abs(x - 1))
```

Mupad [B]
time = 4.95, size = 87, normalized size = 1.58

$$\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{a}{6}+\frac{b}{6}-\frac{c}{3}-\frac{\sqrt{3}\,a\,1{}\mathrm{i}}{6}+\frac{\sqrt{3}\,b\,1{}\mathrm{i}}{6}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{a}{6}+\frac{b}{6}-\frac{c}{3}+\frac{\sqrt{3}\,a\,1{}\mathrm{i}}{6}-\frac{\sqrt{3}\,b\,1{}\mathrm{i}}{6}\right)-\ln\left(x-1\right)\left(\frac{a}{3}+\frac{b}{3}+\frac{c}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x + c*x^2)/(x^3 - 1),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*(a/6 + b/6 - c/3 - (3^(1/2)*a*1i)/6 + (3^(1/2
)*b*1i)/6) + log(x + (3^(1/2)*1i)/2 + 1/2)*(a/6 + b/6 - c/3 + (3^(1/2)*a*1i
)/6 - (3^(1/2)*b*1i)/6) - log(x - 1)*(a/3 + b/3 + c/3)
```

3.50 $\int \frac{1+x+x^2}{1-x^3} dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

[Out] -ln(1-x)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1600, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)/(1 - x^3),x]

[Out] -Log[1 - x]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x+x^2}{1-x^3} \, dx &= \int \frac{1}{1-x} \, dx \\ &= -\log(1-x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\log(1-x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2)/(1 - x^3),x]

```
[Out] -Log[1 - x]
```

Maple [A]
time = 0.32, size = 7, normalized size = 0.88

method	result
default	$-\ln\left(x-1\right)$
norman	$-\ln\left(x-1\right)$
risch	$-\ln\left(x-1\right)$
meijerg	$-\frac{\ln\left(-x^{3}+1\right)}{3}-\frac{x^{2}\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{3}\right)^{\frac{1}{3}}}{2+\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^{3}\right)^{\frac{2}{3}}}-\frac{x\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
[Out] -ln(x-1)
```

Maxima [A]
time = 0.30, size = 6, normalized size = 0.75

$$-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="maxima")
[Out] -log(x - 1)
```

Fricas [A]
time = 0.37, size = 6, normalized size = 0.75

$$-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="fricas")
[Out] -log(x - 1)
```

Sympy [A]
time = 0.01, size = 5, normalized size = 0.62

$$-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(-x**3+1),x)
```

```
[Out] -log(x - 1)
```

Giac [**A**]
time = 2.19, size = 7, normalized size = 0.88

$$-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="giac")
```

```
[Out] -log(abs(x - 1))
```

Mupad [**B**]
time = 0.02, size = 6, normalized size = 0.75

$$-\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2 + 1)/(x^3 - 1),x)
```

```
[Out] -log(x - 1)
```

3.51 $\int \frac{1-x+3x^2}{1-x^3}\,dx$

Optimal. Leaf size=30

$$\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1-x^3\right)$$

```
[Out] -ln(-x^3+1)+2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1885, 1600, 632, 210, 266}

$$\frac{2\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + 3*x^2)/(1 - x^3),x]
```

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x+3x^2}{1-x^3}\,dx &= 3\int \frac{x^2}{1-x^3}\,dx+\int \frac{1-x}{1-x^3}\,dx\\ &= -\log\left(1-x^3\right)+\int \frac{1}{1+x+x^2}\,dx\\ &= -\log\left(1-x^3\right)-2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= \frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1-x^3\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 3*x^2)/(1 - x^3),x]
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

Maple [A]
time = 0.33, size = 33, normalized size = 1.10

method	result
default	$-\ln\left(x^2+x+1\right)+\frac{2\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\ln\left(x-1\right)$
risch	$\frac{2\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\ln\left(4x^2+4x+4\right)-\ln\left(x-1\right)$

meijerg	$-\ln(-x^3+1)+\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln(1}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
[Out] -ln(x^2+x+1)+2/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-ln(x-1)
```

Maxima [A]
time = 0.53, size = 32, normalized size = 1.07

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\log\left(x^2+x+1\right)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(x - 1)
```

Fricas [A]
time = 0.36, size = 32, normalized size = 1.07

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\log\left(x^2+x+1\right)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(x - 1)
```

Sympy [A]
time = 0.05, size = 5, normalized size = 0.17

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+1)/(-x**3+1),x)
[Out] -log(x - 1)
```

Giac [A]
time = 1.30, size = 33, normalized size = 1.10

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\log\left(x^2+x+1\right)-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(abs(x -
1))
```

Mupad [B]
time = 4.93, size = 63, normalized size = 2.10

$$-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)-\ln\left(x-1\right)-\frac{\sqrt{3}\ \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\ 1\mathrm{i}}{3}+\frac{\sqrt{3}\ \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\ 1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^2 - x + 1)/(x^3 - 1),x)
```

```
[Out] (3^(1/2)*log(x + (3^(1/2)*1i)/2 + 1/2)*1i)/3 - log(x + (3^(1/2)*1i)/2 + 1/2
) - log(x - 1) - (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/3 - log(x - (3^
(1/2)*1i)/2 + 1/2)
```

3.52 $\int \frac{1+x+4x^2}{1-x^3} \, dx$

Optimal. Leaf size=18

$$-2\log(1-x)-\log\left(1+x+x^2\right)$$

```
[Out] -2*ln(1-x)-ln(x^2+x+1)
```

Rubi [A]

time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1889, 31, 642}

$$-\log\left(x^2+x+1\right)-2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 4*x^2)/(1 - x^3),x]
```

```
[Out] -2*Log[1 - x] - Log[1 + x + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q
 + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q -
 C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b
*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
 && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+4x^2}{1-x^3} \, dx &= \frac{1}{3}\int \frac{-3-6x}{1+x+x^2} \, dx+2\int \frac{1}{1-x} \, dx\\ &= -2\log(1-x)-\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$-2\log(1-x)-\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 4*x^2)/(1 - x^3),x]
```

```
[Out] -2*Log[1 - x] - Log[1 + x + x^2]
```

Maple [A]
time = 0.35, size = 17, normalized size = 0.94

method	result
default	$-\ln\left(x^2+x+1\right)-2\ln\left(x-1\right)$
norman	$-\ln\left(x^2+x+1\right)-2\ln\left(x-1\right)$
risch	$-\ln\left(x^2+x+1\right)-2\ln\left(x-1\right)$
meijerg	$-\frac{4\ln\left(-x^3+1\right)}{3}-\frac{x^2\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^3\right)^{\frac{1}{3}}}{2+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^3\right)^{\frac{2}{3}}}-\frac{x\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x^2+x+1)-2*ln(x-1)
```

Maxima [A]
time = 0.59, size = 16, normalized size = 0.89

$$-\log\left(x^2+x+1\right)-2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] -log(x^2 + x + 1) - 2*log(x - 1)
```

Fricas [A]
time = 0.38, size = 16, normalized size = 0.89

$$-\log\left(x^2+x+1\right)-2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] -log(x^2 + x + 1) - 2*log(x - 1)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.83

$$-2\log\left(x-1\right)-\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+x+1)/(-x**3+1),x)
[Out] -2*log(x - 1) - log(x**2 + x + 1)
```

Giac [A]
time = 1.71, size = 17, normalized size = 0.94

$$-\log\left(x^2+x+1\right)-2\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="giac")
[Out] -log(x^2 + x + 1) - 2*log(abs(x - 1))
```

Mupad [B]
time = 0.04, size = 16, normalized size = 0.89

$$-\ln\left(x^2+x+1\right)-2\,\ln\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 4*x^2 + 1)/(x^3 - 1),x)
[Out] - log(x + x^2 + 1) - 2*log(x - 1)
```

3.53 $\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4) \, dx$

Optimal. Leaf size=113

$$a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

```
[Out] a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*a^3*b*d*x^5+6/7*a^2*b^2*c*x^7+3/4*a^2
*b^2*d*x^8+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/13*b^4*c*x^13+1/14*b^4*d*x^
14
```

Rubi [A]
time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{1864\}$

$$a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4) \, dx = \int (a^4c+a^4dx+4a^3bcx^3+4a^3bdx^4+6a^2b^2cx^6+6a^2b^2dx^7+$$
$$= a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+$$

Mathematica [A]
time = 0.00, size = 113, normalized size = 1.00

$$a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14
```

Maple [A]
time = 0.36, size = 98, normalized size = 0.87

method	result
default	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}a^3bd\,x^5+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}a^2b^2d\,x^8+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}a\,b^3d\,x^{11}+\frac{1}{13}b^4c\,x$
norman	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}a^3bd\,x^5+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}a^2b^2d\,x^8+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}a\,b^3d\,x^{11}+\frac{1}{13}b^4c\,x$
risch	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}a^3bd\,x^5+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}a^2b^2d\,x^8+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}a\,b^3d\,x^{11}+\frac{1}{13}b^4c\,x$
gosper	$\frac{x(1430b^4d\,x^{13}+1540b^4c\,x^{12}+7280a\,b^3d\,x^{10}+8008a\,b^3c\,x^9+15015a^2b^2d\,x^7+17160a^2b^2c\,x^6+16016a^3bd\,x^4+20020a^3bc\,x^3+10010a^4d}{20020}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*a^3*b*d*x^5+6/7*a^2*b^2*c*x^7+3/4*a^2
*b^2*d*x^8+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/13*b^4*c*x^13+1/14*b^4*d*x^
14
```

Maxima [A]
time = 0.33, size = 97, normalized size = 0.86

$$\frac{1}{14}\,b^4dx^{14}+\frac{1}{13}\,b^4cx^{13}+\frac{4}{11}\,ab^3dx^{11}+\frac{2}{5}\,ab^3cx^{10}+\frac{3}{4}\,a^2b^2dx^8+\frac{6}{7}\,a^2b^2cx^7+\frac{4}{5}\,a^3bdx^5+a^3bcx^4+\frac{1}{2}\,a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/2
*a^4*d*x^2 + a^4*c*x
```

Fricas [A]
time = 0.35, size = 97, normalized size = 0.86

$$\frac{1}{14}\,b^4dx^{14}+\frac{1}{13}\,b^4cx^{13}+\frac{4}{11}\,ab^3dx^{11}+\frac{2}{5}\,ab^3cx^{10}+\frac{3}{4}\,a^2b^2dx^8+\frac{6}{7}\,a^2b^2cx^7+\frac{4}{5}\,a^3bdx^5+a^3bcx^4+\frac{1}{2}\,a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/2
*a^4*d*x^2 + a^4*c*x
```

Sympy [A]
time = 0.01, size = 117, normalized size = 1.04

$$a^4cx + \frac{a^4dx^2}{2} + a^3bcx^4 + \frac{4a^3bdx^5}{5} + \frac{6a^2b^2cx^7}{7} + \frac{3a^2b^2dx^8}{4} + \frac{2ab^3cx^{10}}{5} + \frac{4ab^3dx^{11}}{11} + \frac{b^4cx^{13}}{13} + \frac{b^4dx^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**3*b*c*x**4 + 4*a**3*b*d*x**5/5 + 6*a**2*b**2*
c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/11
+ b**4*c*x**13/13 + b**4*d*x**14/14
```

Giac [A]
time = 1.00, size = 97, normalized size = 0.86

$$\frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{2}a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/2
*a^4*d*x^2 + a^4*c*x
```

Mupad [B]
time = 0.06, size = 97, normalized size = 0.86

$$\frac{d\,a^4\,x^2}{2} + c\,a^4\,x + \frac{4\,d\,a^3\,b\,x^5}{5} + c\,a^3\,b\,x^4 + \frac{3\,d\,a^2\,b^2\,x^8}{4} + \frac{6\,c\,a^2\,b^2\,x^7}{7} + \frac{4\,d\,a\,b^3\,x^{11}}{11} + \frac{2\,c\,a\,b^3\,x^{10}}{5} + \frac{d\,b^4\,x^{14}}{14} + \frac{c\,b^4\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

```
[Out] (a^4*d*x^2)/2 + (b^4*c*x^13)/13 + (b^4*d*x^14)/14 + a^4*c*x + (6*a^2*b^2*c*
x^7)/7 + (3*a^2*b^2*d*x^8)/4 + a^3*b*c*x^4 + (2*a*b^3*c*x^10)/5 + (4*a^3*b*
d*x^5)/5 + (4*a*b^3*d*x^11)/11
```

3.54 $\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4) \, dx$

Optimal. Leaf size=88

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

```
[Out] a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x^5+3/7*a*b^2*c*x^7+3/8*a
*b^2*d*x^8+1/10*b^3*c*x^10+1/11*b^3*d*x^11
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1864}

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4) \, dx = \int (a^3c+a^3dx+3a^2bcx^3+3a^2bdx^4+3ab^2cx^6+3ab^2dx^7+b$$
$$= a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+$$

Mathematica [A]
time = 0.00, size = 88, normalized size = 1.00

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11
```

Maple [A]
time = 0.36, size = 75, normalized size = 0.85

method	result	size
default	$a^{3} c x +\frac{1}{2} a^{3} d \,x^{2}+\frac{3}{4} a^{2} b c \,x^{4}+\frac{3}{5} a^{2} b d \,x^{5}+\frac{3}{7} a \,b^{2} c \,x^{7}+\frac{3}{8} a \,b^{2} d \,x^{8}+\frac{1}{10} b^{3} c \,x^{10}+\frac{1}{11} b^{3} d \,x^{11}$	75
norman	$a^{3} c x +\frac{1}{2} a^{3} d \,x^{2}+\frac{3}{4} a^{2} b c \,x^{4}+\frac{3}{5} a^{2} b d \,x^{5}+\frac{3}{7} a \,b^{2} c \,x^{7}+\frac{3}{8} a \,b^{2} d \,x^{8}+\frac{1}{10} b^{3} c \,x^{10}+\frac{1}{11} b^{3} d \,x^{11}$	75
risch	$a^{3} c x +\frac{1}{2} a^{3} d \,x^{2}+\frac{3}{4} a^{2} b c \,x^{4}+\frac{3}{5} a^{2} b d \,x^{5}+\frac{3}{7} a \,b^{2} c \,x^{7}+\frac{3}{8} a \,b^{2} d \,x^{8}+\frac{1}{10} b^{3} c \,x^{10}+\frac{1}{11} b^{3} d \,x^{11}$	75
gosper	$\frac{x \left(280 b^{3} d \,x^{10}+308 b^{3} c \,x^{9}+1155 a \,b^{2} d \,x^{7}+1320 a \,b^{2} c \,x^{6}+1848 a^{2} b d \,x^{4}+2310 a^{2} b c \,x^{3}+1540 a^{3} d x +3080 c \,a^{3}\right)}{3080}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x^5+3/7*a*b^2*c*x^7+3/8*a
*b^2*d*x^8+1/10*b^3*c*x^10+1/11*b^3*d*x^11
```

Maxima [A]
time = 0.27, size = 74, normalized size = 0.84

$$\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/5
*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

Fricas [A]
time = 0.38, size = 74, normalized size = 0.84

$$\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/5
*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

Sympy [A]
time = 0.01, size = 90, normalized size = 1.02

$$a^3cx+\frac{a^3dx^2}{2}+\frac{3a^2bcx^4}{4}+\frac{3a^2bdx^5}{5}+\frac{3ab^2cx^7}{7}+\frac{3ab^2dx^8}{8}+\frac{b^3cx^{10}}{10}+\frac{b^3dx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*x**5/5 + 3*a*b**2
*c*x**7/7 + 3*a*b**2*d*x**8/8 + b**3*c*x**10/10 + b**3*d*x**11/11
```

Giac [A]
time = 1.18, size = 74, normalized size = 0.84

$$\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/5
*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B]
time = 0.04, size = 74, normalized size = 0.84

$$\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{3\,d\,a^2\,b\,x^5}{5}+\frac{3\,c\,a^2\,b\,x^4}{4}+\frac{3\,d\,a\,b^2\,x^8}{8}+\frac{3\,c\,a\,b^2\,x^7}{7}+\frac{d\,b^3\,x^{11}}{11}+\frac{c\,b^3\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

```
[Out] (a^3*d*x^2)/2 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + a^3*c*x + (3*a^2*b*c*x^
4)/4 + (3*a*b^2*c*x^7)/7 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*d*x^8)/8
```

3.55 $\int (a+bx^3) (ac+adx+bcx^3+bdx^4) \, dx$

Optimal. Leaf size=60

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/2*a*b*c*x^4+2/5*a*b*d*x^5+1/7*b^2*c*x^7+1/8*b^2*d*x
^8
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{1864\}$

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx^3) (ac+adx+bcx^3+bdx^4) \, dx &= \int (a^2c+a^2dx+2abcx^3+2abdx^4+b^2cx^6+b^2dx^7) \, dx\\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8\end{aligned}$$

Mathematica [A]
time = 0.00, size = 60, normalized size = 1.00

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8
```

Maple [A]
time = 0.15, size = 51, normalized size = 0.85

method	result	size
default	$a^2cx+\frac{1}{2}a^2d\,x^2+\frac{1}{2}abc\,x^4+\frac{2}{5}abd\,x^5+\frac{1}{7}b^2c\,x^7+\frac{1}{8}b^2d\,x^8$	51
norman	$a^2cx+\frac{1}{2}a^2d\,x^2+\frac{1}{2}abc\,x^4+\frac{2}{5}abd\,x^5+\frac{1}{7}b^2c\,x^7+\frac{1}{8}b^2d\,x^8$	51
risch	$a^2cx+\frac{1}{2}a^2d\,x^2+\frac{1}{2}abc\,x^4+\frac{2}{5}abd\,x^5+\frac{1}{7}b^2c\,x^7+\frac{1}{8}b^2d\,x^8$	51
gosper	$\frac{x(35b^2d\,x^7+40b^2c\,x^6+112abd\,x^4+140abc\,x^3+140a^2dx+280a^2c)}{280}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/2*a*b*c*x^4+2/5*a*b*d*x^5+1/7*b^2*c*x^7+1/8*b^2*d*x
^8
```

Maxima [A]
time = 0.27, size = 50, normalized size = 0.83

$$\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*x
^2 + a^2*c*x
```

Fricas [A]
time = 0.37, size = 50, normalized size = 0.83

$$\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*x
^2 + a^2*c*x
```

Sympy [A]
time = 0.01, size = 58, normalized size = 0.97

$$a^2cx+\frac{a^2dx^2}{2}+\frac{abcx^4}{2}+\frac{2abdx^5}{5}+\frac{b^2cx^7}{7}+\frac{b^2dx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a*b*c*x**4/2 + 2*a*b*d*x**5/5 + b**2*c*x**7/7 +
b**2*d*x**8/8
```

Giac [**A**]
time = 1.11, size = 50, normalized size = 0.83

$$\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*x
^2 + a^2*c*x
```

Mupad [**B**]
time = 0.03, size = 50, normalized size = 0.83

$$\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{2\,d\,a\,b\,x^5}{5}+\frac{c\,a\,b\,x^4}{2}+\frac{d\,b^2\,x^8}{8}+\frac{c\,b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + a^2*c*x + (a*b*c*x^4)/2 + (
2*a*b*d*x^5)/5
```

3.56 $\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3} dx$

Optimal. Leaf size=12

$$cx + \frac{dx^2}{2}$$

```
[Out] c*x+1/2*d*x^2
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1600}

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]
```

```
[Out] c*x + (d*x^2)/2
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx &= \int (c+dx)\,dx\\ &= cx+\frac{dx^2}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]
```

```
[Out] c*x + (d*x^2)/2
```

Maple [A]
time = 0.31, size = 11, normalized size = 0.92

method	result	size
gosper	$\frac{x(dx+2c)}{2}$	11
default	$\frac{1}{2}d\,x^2+cx$	11
norman	$\frac{1}{2}d\,x^2+cx$	11
risch	$\frac{1}{2}d\,x^2+cx$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d*x^2+c*x
```

Maxima [A]
time = 0.29, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^2+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + c*x
```

Fricas [A]
time = 0.35, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^2+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/2*d*x^2 + c*x
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.67

$$cx+\frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a),x)
```

```
[Out] c*x + d*x**2/2
```

Giac [A]
time = 1.37, size = 10, normalized size = 0.83

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + c*x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.83

$$\frac{d\,x^2}{2} + c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x)
```

```
[Out] c*x + (d*x^2)/2
```

3.57 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}+\frac{\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{6a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1/6*(c-a^(1
/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)-1/
3*(b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(2/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$,
Rules used = {1600, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}-\frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx = \int \frac{c+dx}{a+bx^3}\,dx$$

$$= \frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}} + \frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,d}{3a^{2/3}}$$

$$= \frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,d}{6a^{2/3}b^{2/3}}$$

$$= \frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+\right.}{6a^{2/3}b^{2/3}}$$

$$= -\frac{\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}} + \frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}$$

Mathematica [A]
time = 0.03, size = 124, normalized size = 0.77

$$\frac{-2\sqrt{3}\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
[Out] (-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

Maple [A]
time = 0.32, size = 186, normalized size = 1.16

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^2}}{3b}$

default	$c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\right.\right.}{6}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))+d*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1)))
```

Maxima [A]
time = 0.53, size = 135, normalized size = 0.84

$$\frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(d*(a/b)^(1/3) + c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(d*(a/b)^(1/3) - c)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(d*(a/b)^(1/3) - c)*log(x + (a/b)^(1/3
))/(b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.10, size = 1931, normalized size = 11.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d
^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*
d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
```

```
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(
2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)
/(a^2*b^2))^(1/3)))*a*b*c^2 + 2*a*c*d^2 + (b*c^3 + a*d^3)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2
))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^
2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^
(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2)
+ (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1
/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a
*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3 + a*d^3)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a^2*b*d + 2*a*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*
b))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^
3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c
^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^
3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d
^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^
2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3
+ a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*
a^2*b*d + 2*a*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^
2*a*b + 16*c*d)/(a*b)))
```

Sympy [A]

time = 0.36, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^3a^2b^2 + 9tabcd + ad^3 - bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd + 3tabc^2 + 2acd^2}{ad^3 + bc^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lambda(_t, _t*
log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)/(a*d**3 + b*c**3)))
)
```

Giac [A]
time = 2.03, size = 141, normalized size = 0.88

$$-\frac{\sqrt{3}\left(bc - \left(-ab^2\right)^{\frac{1}{3}} d\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(bc + \left(-ab^2\right)^{\frac{1}{3}} d\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c + (-a*b^2)^(1/3)*d)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(d*(-a/b)^(1/3) + c)*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a
```

Mupad [B]
time = 5.09, size = 127, normalized size = 0.79

$$\sum_{k=1}^{3} \ln\left(b\left(c\,d + d^2\,x + \operatorname{root}\left(27\,a^2\,b^2\,z^3 + 9\,a\,b\,c\,d\,z + a\,d^3 - b\,c^3, z, k\right)^2 a\,b\,9 + \operatorname{root}\left(27\,a^2\,b^2\,z^3 + 9\,a\,b\,c\,d\,z + a\,d^3 - b\,c^3, z, k\right)\,b\,c\,x\,3\right)\right)\operatorname{root}\left(27\,a^2\,b^2\,z^3 + 9\,a\,b\,c\,d\,z + a\,d^3 - b\,c^3, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x)
```

```
[Out] symsum(log(b*(c*d + d^2*x + 9*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b
*c^3, z, k)^2*a*b + 3*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z,
 k)*b*c*x))*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z, k), k, 1,
 3)
```

3.58 $\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=189

$$\frac{x(c+dx)}{3a\left(a+bx^3\right)}-\frac{\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\right.}{}$$

```
[Out] 1/3*x*(d*x+c)/a/(b*x^3+a)+1/9*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)
/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/9*(2*b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1600, 1869, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,d+2\sqrt[3]{b}\,c\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}}-\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5
/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx = \int \frac{c+dx}{(a+bx^3)^2}\,dx$$
$$= \frac{x(c+dx)}{3a(a+bx^3)} - \frac{\int \frac{-2c-dx}{a+bx^3}\,dx}{3a}$$
$$= \frac{x(c+dx)}{3a(a+bx^3)} - \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}} + \frac{\left(2c-\frac{\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{}$$
$$= \frac{x(c+dx)}{3a(a+bx^3)} + \frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\int}{18a}$$
$$= \frac{x(c+dx)}{3a(a+bx^3)} + \frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log}{}$$
$$= \frac{x(c+dx)}{3a(a+bx^3)} - \frac{\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)}{9a^{5/}}$$

Mathematica [A]
time = 0.10, size = 180, normalized size = 0.95

$$\frac{\frac{6ax(c+dx)}{a+bx^3} - \frac{2\sqrt{3}\,\sqrt[3]{a}\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{2\left(2\sqrt[3]{a}\,\sqrt[3]{b}\,c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}} + \frac{\left(-2\sqrt[3]{a}\,\sqrt[3]{b}\,c+a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}}{18a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

```
[Out] ((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)*
d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b^(
1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(1/3
)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(
18*a^2)
```

Maple [A]
time = 0.34, size = 230, normalized size = 1.22

method	result

risch	$\frac{\frac{d x^2}{3a}+\frac{cx}{3a}}{b x^3+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_Rd+2c\right)\ln\left(x-\_R\right)}{\_R^2}}{9ba}$
default	$c\left(\frac{x}{3a(b x^3+a)}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}\right)+d\left(\frac{x^2}{3a(b x^3+a)}+\frac{-\frac{\ln(}{}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/3*x/a/(b*x^3+a)+2/3/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)
^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))+d*(1/3*x^2/a/(b*x^3+a)+1/3/a*(-1/3/b/(a/b
)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.54, size = 169, normalized size = 0.89

$$\frac{dx^2+cx}{3\left(abx^3+a^2\right)}+\frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+2c\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(d*x^2 + c*x)/(a*b*x^3 + a^2) + 1/9*sqrt(3)*(d*(a/b)^(1/3) + 2*c)*arcta
n(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) + 1/18*(d*
(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3))
 - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.10, size = 2088, normalized size = 11.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/36*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
 a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*
(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^
5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^
5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3)
- 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/
3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2
) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/
(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*
a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) + 12*c*x + ((a*b*x^3 + a^2)*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) + 3*sqrt(1/3)*(a*b*x^
3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) +
 (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^
3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a
^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(
I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3
)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqr
t(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2)
)^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d^3)*x + 3/4*sqrt(1/3)*(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c
^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8
*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b +
32*c*d)/(a^3*b))) + ((a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3))) - 3*sqrt(1/3)*(a*b*x^3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt
(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)
+ 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (
8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/
(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*
(8*b*c^3 + a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
```

```
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1
)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/
2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3
 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b))))/(a*b*x^3 + a^2)
```

Sympy [A]
time = 0.51, size = 105, normalized size = 0.56

$$\operatorname{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{cx + dx^2}{3a^2 + 3abx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(
_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3
 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)
```

Giac [A]
time = 1.83, size = 174, normalized size = 0.92

$$-\frac{\sqrt{3}\left(2\,bc - \left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{\left(2\,bc + \left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{dx^2 + cx}{3\,(bx^3 + a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)^(1
/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(d*x^2 + c*x)/
((b*x^3 + a)*a)
```

Mupad [B]
time = 5.08, size = 169, normalized size = 0.89

$$\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(2\,c\,d + d^2\,x + \mathrm{root}(729\,a^5\,b^2\,z^3 + 54\,a^2\,b\,c\,d\,z - 8\,b\,c^3 + a\,d^3, z, k)^2\,a^3\,b\,81 + \mathrm{root}(729\,a^5\,b^2\,z^3 + 54\,a^2\,b\,c\,d\,z - 8\,b\,c^3 + a\,d^3, z, k)\;a\,b\,c\,x\,18\right)}{a^2\,9}\right)\mathrm{root}\left(729\,a^5\,b^2\,z^3 + 54\,a^2\,b\,c\,d\,z - 8\,b\,c^3 + a\,d^3, z, k\right)\right) + \frac{\frac{d\,x^2}{3\,a} + \frac{c\,x}{3\,a}}{b\,x^3 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x)
```

```
[Out] symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z - 8
*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*b*c
*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) + (c*x)/(3*a))/(a
+ b*x^3)
```

3.59 $\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right) \, dx$

Optimal. Leaf size=585

$$\frac{810a^3d\sqrt{a+bx^3}}{1729b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{54a^2(1729cx+935dx^2)\sqrt{a+bx^3}}{323323}+\frac{30a(247cx+187dx^2)\left(a+bx^3\right)^{3/2}}{46189}$$

```
[Out] 30/46189*a*(187*d*x^2+247*c*x)*(b*x^3+a)^(3/2)+2/323*(17*d*x^2+19*c*x)*(b*x
^3+a)^(5/2)+54/323323*a^2*(935*d*x^2+1729*c*x)*(b*x^3+a)^(1/2)+810/1729*a^3
*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-405/1729*3^(1/4)
*a^(10/3)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)+54/323323*3^(3/4)*a^3*(a^(1/3)+b^(1/3)*x)*EllipticF(
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1729*b^(1/3)*c-935*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1866, 1867, 1892, 224, 1891}

$$-\frac{405\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{10/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{810a^3d\sqrt{a+bx^3}}{1729b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{54a^2\sqrt{a+bx^3}\,(1729cx+935dx^2)}{323323}+\frac{54\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(1729\sqrt[3]{b}\,c-935\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{323323b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{30a\left(a+bx^3\right)^{3/2}(247cx+187dx^2)}{46189}+\frac{2}{323}\left(a+bx^3\right)^{5/2}(19cx+17dx^2)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] (810*a^3*d*Sqrt[a + b*x^3])/(1729*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)) + (54*a^2*(1729*c*x + 935*d*x^2)*Sqrt[a + b*x^3])/323323 + (30*a*(247*c
*x + 187*d*x^2)*(a + b*x^3)^(3/2))/46189 + (2*(19*c*x + 17*d*x^2)*(a + b*x^
3)^(5/2))/323 - (405*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*d*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(2/3)*Sqrt[(a^(1
```

```
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (54*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(1729*b^(1/3)*c - 935*(1 - Sqrt
[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(323323*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1866

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[PolynomialQuoti
ent[Pq, a + b*x^n, x]*(a + b*x^n)^(p + 1), x] /; FreeQ[{a, b, p}, x] && Pol
yQ[Pq, x] && IGtQ[n, 0] && GeQ[Expon[Pq, x], n] && EqQ[PolynomialRemainder[
Pq, a + b*x^n, x], 0]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right)\, dx = \int (c+dx)\left(a+bx^3\right)^{5/2}\, dx$$

$$= \frac{2}{323}\left(19cx+17dx^2\right)\left(a+bx^3\right)^{5/2} + \frac{1}{2}(15a)\int \left(\frac{2c}{17}+\frac{2dx}{19}\right)($$

$$= \frac{30a(247cx+187dx^2)\left(a+bx^3\right)^{3/2}}{46189} + \frac{2}{323}\left(19cx+17dx^2\right)(a+$$

$$= \frac{54a^2(1729cx+935dx^2)\sqrt{a+bx^3}}{323323} + \frac{30a(247cx+187dx^2)(a}{46189}$$

$$= \frac{54a^2(1729cx+935dx^2)\sqrt{a+bx^3}}{323323} + \frac{30a(247cx+187dx^2)(a}{46189}$$

$$= \frac{810a^3d\sqrt{a+bx^3}}{1729b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{54a^2(1729cx+935dx}{323323}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.21, size = 78, normalized size = 0.13

$$\frac{a^2x\sqrt{a+bx^3}\left(2c\,{}_2F_1\left(-\frac{5}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(-\frac{5}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
[Out] (a^2*x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-5/2, 1/3, 4/3, -((b*x^3)/a)]
 + d*x*Hypergeometric2F1[-5/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3
)/a])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1617 vs. $2(441) = 882$.
time = 0.35, size = 1618, normalized size = 2.77 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] b*d*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729*a^2*x^
2*(b*x^3+a)^(1/2)/b+72/1729*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))))+b*c*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935*a
^2*x*(b*x^3+a)^(1/2)/b+36/935*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))+a*d*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*
x^3+a)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(
-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(
-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+a*c*(
2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)/b*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
```

```
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 117, normalized size = 0.20

$$\frac{2\left(140049\,a^3\sqrt{b}\,c\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-75735\,a^3\sqrt{b}\,d\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(17017\,b^3dx^8+19019\,b^3cx^7+53669\,ab^2dx^5+63973\,ab^2cx^4+61897\,a^2bdx^2+91637\,a^2bcx\right)\sqrt{bx^3+a}\right)}{323323\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas"
)
```

```
[Out] 2/323323*(140049*a^3*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 75735*a^
3*sqrt(b)*d*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) +
 (17017*b^3*d*x^8 + 19019*b^3*c*x^7 + 53669*a*b^2*d*x^5 + 63973*a*b^2*c*x^4
 + 61897*a^2*b*d*x^2 + 91637*a^2*b*c*x)*sqrt(b*x^3 + a))/b
```

Sympy [A]
time = 3.05, size = 265, normalized size = 0.45

$$\frac{a^{\frac{5}{2}}cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{5}{2}}dx^2\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{2a^{\frac{3}{2}}bcx^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2a^{\frac{3}{2}}bdx^5\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{\sqrt{a}\,b^2cx^7\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt{a}\,b^2dx^8\Gamma\left(\frac{8}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**(5/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(5/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a**(3/2)*b*c*x**4*gamma(4/3)*hyp
er((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(3/
2)*b*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(8/3)) + sqrt(a)*b**2*c*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b**2*d*x**8*gamma(8/3
)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^3+a\right)^{3/2}\,\left(b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
[Out] int((a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x)
```

3.60 $\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx$

Optimal. Leaf size=556

$$\frac{54a^2d\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{18a\left(91cx+55dx^2\right)\sqrt{a+bx^3}}{5005}+\frac{2}{143}\left(13cx+11dx^2\right)\left(a+bx^3\right)^{3/2}-\frac{27\sqrt[4]{3}}{\quad}$$

```
[Out] 2/143*(11*d*x^2+13*c*x)*(b*x^3+a)^(3/2)+18/5005*a*(55*d*x^2+91*c*x)*(b*x^3+
a)^(1/2)+54/91*a^2*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)-27/91*3^(1/4)*a^(7/3)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/
2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+18/5005*3^(3/4)*a^2*(a^(1/3)+b^(1/3)*x
)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(91*b^(1/3)*c-55*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1866, 1867, 1892, 224, 1891}

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{54a^2d\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(91\sqrt[3]{b}\,c-55\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{18a\sqrt{a+bx^3}\left(91cx+55dx^2\right)}{5005}+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] (54*a^2*d*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 + (18*a*(91*c*x + 55*d*x^2)*Sqrt[a + b*x^3])/5005 + (2*(13*c*x + 11*d*x^2)
*(a + b*x^3)^(3/2))/143 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*d*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(91*b^(1/3)*c - 55*(1 - S
```

```
qrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(5005*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1866

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[PolynomialQuoti
ent[Pq, a + b*x^n, x]*(a + b*x^n)^(p + 1), x] /; FreeQ[{a, b, p}, x] && Pol
yQ[Pq, x] && IGtQ[n, 0] && GeQ[Expon[Pq, x], n] && EqQ[PolynomialRemainder[
Pq, a + b*x^n, x], 0]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx = \int (c+dx)\left(a+bx^3\right)^{3/2}\,dx$$
$$= \frac{2}{143}\left(13cx+11dx^2\right)\left(a+bx^3\right)^{3/2}+\frac{1}{2}(9a)\int\left(\frac{2c}{11}+\frac{2dx}{13}\right)\sqrt{a+$$
$$= \frac{18a\left(91cx+55dx^2\right)\sqrt{a+bx^3}}{5005}+\frac{2}{143}\left(13cx+11dx^2\right)\left(a+bx^3\right)^{$$
$$= \frac{18a\left(91cx+55dx^2\right)\sqrt{a+bx^3}}{5005}+\frac{2}{143}\left(13cx+11dx^2\right)\left(a+bx^3\right)^{$$
$$= \frac{54a^2d\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{18a\left(91cx+55dx^2\right)\sqrt{a+b}}{5005}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.06, size = 76, normalized size = 0.14

$$\frac{ax\sqrt{a+bx^3}\left(2c\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
[Out] (a*x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] +
 d*x*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3)/
a])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1545 vs. 2(416) = 832.
time = 0.34, size = 1546, normalized size = 2.78 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] b*d*(2/13*x^5*(b*x^3+a)^(1/2)+6/91*a*x^2*(b*x^3+a)^(1/2)/b+8/91*I/b^2*a^2*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+b*c*(2/11*x^4*(b*x^3+a)^(1/2)+6/5
5*a*x*(b*x^3+a)^(1/2)/b+4/55*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)))+a*d*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+a*c*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2
)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 93, normalized size = 0.17

$$\frac{2\left(2457\,a^2\sqrt{b}\,c\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-1485\,a^2\sqrt{b}\,d\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(385\,b^2dx^5+455\,b^2cx^4+880\,abdx^2+1274\,abcx\right)\sqrt{bx^3+a}\right)}{5005\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas"
)
```

```
[Out] 2/5005*(2457*a^2*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 1485*a^2*sqr
t(b)*d*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (385
*b^2*d*x^5 + 455*b^2*c*x^4 + 880*a*b*d*x^2 + 1274*a*b*c*x)*sqrt(b*x^3 + a))
/b
```

Sympy [A]
time = 2.20, size = 170, normalized size = 0.31

$$\frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{3}{2}}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt{a}\,bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}\,bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**(3/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(3/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x**4*gamma(4/3)*hyper(
(-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d
*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*ga
mma(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b\,x^3+a}\,\left(b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
[Out] int((a + b*x^3)^(1/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x)
```

3.61 $\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=525

$$\frac{6ad\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2}{35}\left(7cx+5dx^2\right)\sqrt{a+bx^3}-\frac{3\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}\cdots}{\left(\left(1\cdots\right.\right.}}}{7b^{2/3}\sqrt{\frac{\cdots}{\left(\left(\cdots\right.\right.}}}$$

```
[Out] 2/35*(5*d*x^2+7*c*x)*(b*x^3+a)^(1/2)+6/7*a*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))-3/7*3^(1/4)*a^(4/3)*d*(a^(1/3)+b^(1/3)*x)*Ellipti
cE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/35*3^(3/4)*a*(
a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*c-5*a^(1/3)*d*(1-3^(1/2)))*(1/2
*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1600, 1867, 1892, 224, 1891}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(7\sqrt[3]{b}\,c-5\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{6ad\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2}{35}\sqrt{a+bx^3}\left(7cx+5dx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] (6*a*d*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (
2*(7*c*x + 5*d*x^2)*Sqrt[a + b*x^3])/35 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4
/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(7*b^(1/3)*c -
 5*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
```

```
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(35*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx = \int (c+dx)\sqrt{a+bx^3}\,dx$$
$$= \frac{2}{35}\left(7cx+5dx^2\right)\sqrt{a+bx^3} + \frac{1}{2}(3a)\int \frac{\frac{2c}{5}+\frac{2dx}{7}}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{2}{35}\left(7cx+5dx^2\right)\sqrt{a+bx^3} + \frac{(3ad)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{7\sqrt[3]{b}} + \frac{1}{35}\left(3a\left(7c\right.\right.$$
$$3\sqrt[4]{3}\sqrt{2-}$$
$$= \frac{6ad\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{2}{35}\left(7cx+5dx^2\right)\sqrt{a+bx^3} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.48, size = 75, normalized size = 0.14

$$\frac{x\sqrt{a+bx^3}\left(2c\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)] + d
*x*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3)/a]
)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1479 vs. 2(389) = 778.
time = 0.34, size = 1480, normalized size = 2.82 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b*d*(2/7*x^2*(b*x^3+a)^(1/2)/b+8/21*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
```

```
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))))+b*c*(2/5*x*(b*x^3+a)^(1/2)/b+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*I*a*d*3^(1/2)/b*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)))-2/3*I*a*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 69, normalized size = 0.13

$$\frac{2\left(21\,a\sqrt{b}\,c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-15\,a\sqrt{b}\,d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(5\,bdx^2+7\,bcx\right)\sqrt{bx^3+a}\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/35*(21*a*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 15*a*sqrt(b)*d*wei
erstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*b*d*x^2 + 7
*b*c*x)*sqrt(b*x^3 + a))/b
```

Sympy [A]
time = 1.75, size = 163, normalized size = 0.31

$$\frac{\sqrt{a}\,cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{a}\,dx^2\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)}+\frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) + sqrt(a)*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((1/2, 4/3),
(7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + b*d*x**5*gamma(5
/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/
3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(1/2),x)
```

```
[Out] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(1/2), x)
```

3.62 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=490

$$\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\Bigg(\text{si}}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a}}$$

```
[Out] 2*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)
*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c-a^(1/3)*d*(1
-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)
/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1600, 1892, 224, 1891}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{b}\,c-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
```

```
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx=\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$$
$$=\frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}}+\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$
$$=\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\ }}{b^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 75, normalized size = 0.15

$$\frac{x\sqrt{1+\frac{bx^3}{a}}\left(2c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1535 vs. 2(364) = 728.
time = 0.33, size = 1536, normalized size = 3.13

method	result

elliptic	$-\frac{2ic\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{}}{}\right.}{(-}}}{3b\sqrt{b}}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b*d*(-2/3/b*x^2/((x^3+a/b)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))))+b*c*(-2/3/b*x/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^
(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+a*d*(2/3/a*x^2/((x^3+a/b)*b)^
(1/2)+2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
```

```
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+a*c*(2/3/a*x/((x
^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(
-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 43, normalized size = 0.09

$$\frac{2\left(\sqrt{b}\,c\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{b}\,d\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 2*(sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - sqrt(b)*d*weierstrassZeta(
0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [**A**]
time = 1.94, size = 78, normalized size = 0.16

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(3/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c}{{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x)
```

```
[Out] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2), x)
```

3.63 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=522

$$\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}}}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{\right.}}}}$$

```
[Out] 2/3*x*(d*x+c)/a/(b*x^3+a)^(1/2)-2/3*d*(b*x^3+a)^(1/2)/a/b^(2/3)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))+1/3*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/9*(a^(1/3)+b^(1/3)*
x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(b^(1/3)*c+a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2)
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(3/4)/a/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1600, 1869, 1892, 224, 1891}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*(c + d*x))/(3*a*Sqrt[a + b*x^3]) - (2*d*Sqrt[a + b*x^3])/(3*a*b^(2/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c + (1 - Sqrt[3])*a^(1
/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
```

```
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \int \frac{c+dx}{(a+bx^3)^{3/2}}\,dx$$
$$= \frac{2x(c+dx)}{3a\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{c}{2}+\frac{dx}{2}}{\sqrt{a+bx^3}}\,dx}{3a}$$
$$= \frac{2x(c+dx)}{3a\sqrt{a+bx^3}} - \frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{3a\sqrt[3]{b}} + \frac{\left(c+\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{}{\sqrt{a}}}{3a}$$
$$= \frac{2x(c+dx)}{3a\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 96, normalized size = 0.18

$$\frac{x\left(4c+2c\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+3dx\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{6a\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (x*(4*c + 2*c*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3
)/a)] + 3*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3
)/a)]))/(6*a*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1661 vs. 2(389) = 778.
time = 0.34, size = 1662, normalized size = 3.18

method	result

elliptic	$-\frac{2b\left(-\frac{d\,x^2}{3ab}-\frac{cx}{3ba}\right)}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2ic\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b*d*(-2/9*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+8/27/b/a*x^2/((x^3+a/b)*b)^(1
/2)+8/81*I/b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+b*c*(-2/9*x/b^3
*(b*x^3+a)^(1/2)/(x^3+a/b)^2+4/27/b/a*x/((x^3+a/b)*b)^(1/2)-4/81*I/b^2/a*3^
(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+a*d*(2/9/a*x^2
/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+10/27/a^2*x^2/((x^3+a/b)*b)^(1/2)+10/81*I/
a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
```

```
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+a*c*(2/9/a*x/b^2*(b*x^3+a)
^(1/2)/(x^3+a/b)^2+14/27/a^2*x/((x^3+a/b)*b)^(1/2)-14/81*I/a^2*3^(1/2)/b*(-
a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 94, normalized size = 0.18

$$\frac{2\left((bcx^3+ac)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+(bdx^3+ad)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(bdx^2+bcx)\sqrt{bx^3+a}\right)}{3\,(ab^2x^3+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*((b*c*x^3 + a*c)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (b*d*x^3 +
 a*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
 + (b*d*x^2 + b*c*x)*sqrt(b*x^3 + a))/(a*b^2*x^3 + a^2*b)
```

Sympy [A]

time = 4.47, size = 163, normalized size = 0.31

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(5/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 5/2)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(7/3)) + b*d*x**5*gamm
a(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamm
a(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c}{{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x)
```

```
[Out] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2), x)
```

3.64 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx$

Optimal. Leaf size=554

$$\frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}}+\frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}}-\frac{10d\sqrt{a+bx^3}}{27a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{5\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\ldots}{\left(\left(\ldots\right.\right.}}}{9\;3^{3/4}a^{5\ldots}}$$

```
[Out] 2/9*x*(d*x+c)/a/(b*x^3+a)^(3/2)+2/27*x*(5*d*x+7*c)/a^2/(b*x^3+a)^(1/2)-10/2
7*d*(b*x^3+a)^(1/2)/a^2/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+5/27*d*(a^(
1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5
/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)+2/81*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*c+
5*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^2/b^(2/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)
```

Rubi [A]

time = 0.38, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1600, 1869, 1892, 224, 1891}

$$\frac{5\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{9\;3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{10d\sqrt{a+bx^3}}{27a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(5\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d+7\sqrt[3]{b}\,c\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,a^2b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (2*x*(c + d*x))/(9*a*(a + b*x^3)^(3/2)) + (2*x*(7*c + 5*d*x))/(27*a^2*Sqrt[
a + b*x^3]) - (10*d*Sqrt[a + b*x^3])/(27*a^2*b^(2/3)*((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)) + (5*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
```

```
]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*c + 5*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^
2*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^{7/2}}\,dx = \int \frac{c+dx}{\left(a+bx^3\right)^{5/2}}\,dx$$

$$= \frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}} - \frac{2\int \frac{-\frac{7c}{2}-\frac{5dx}{2}}{\left(a+bx^3\right)^{3/2}}\,dx}{9a}$$

$$= \frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}} + \frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}} + \frac{4\int \frac{\frac{7c}{4}-\frac{5dx}{4}}{\sqrt{a+bx^3}}\,dx}{27a^2}$$

$$= \frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}} + \frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}} - \frac{(5d)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{27a^2\sqrt[3]{b}} + \frac{\Big(7c\ \cdots}{\cdots}$$

$$= \frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}} + \frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}} - \frac{10d\sqrt{a+bx^3}}{27a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 123, normalized size = 0.22

$$\frac{4cx\left(10a+7bx^3\right)+14cx\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+27dx^2\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)}{54a^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (4*c*x*(10*a + 7*b*x^3) + 14*c*x*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 27*d*x^2*(a + b*x^3)*Sqrt[1 + (b*x^3
)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(54*a^2*(a + b*x^3)^(3
/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1781 vs. 2(414) = 828.
time = 0.32, size = 1782, normalized size = 3.22

method	result
elliptic	$\frac{\left(\frac{2d\,x^2}{9a\,b^2}+\frac{2cx}{9a\,b^2}\right)\sqrt{b\,x^3+a}}{\left(x^3+\frac{a}{b}\right)^2}-\frac{2b\left(-\frac{5d\,x^2}{27a^2b}-\frac{7cx}{27a^2b}\right)}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{14ic\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] b*d*(-2/15*x^2/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^3+8/135/a*x^2/b^3*(b*x^3+a)^(1
/2)/(x^3+a/b)^2+8/81/b/a^2*x^2/((x^3+a/b)*b)^(1/2)+8/243*I/a^2/b^2*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+b*c*(-2/15*x/b^4*(b*x^3+a)^(1/2)/(x^3+a/
b)^3+4/135/a*x/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+28/405/b/a^2*x/((x^3+a/b)*b)
^(1/2)-28/1215*I/a^2/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
```

```
3)))^(1/2)))+a*d*(2/15/a*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^3+22/135/a^2*x^2
/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+22/81/a^3*x^2/((x^3+a/b)*b)^(1/2)+22/243*I
/a^3*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(
b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+a*c*(2/15/a*x/b^3*(b*x^3+
a)^(1/2)/(x^3+a/b)^3+26/135/a^2*x/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+182/405/a
^3*x/((x^3+a/b)*b)^(1/2)-182/1215*I/a^3*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 155, normalized size = 0.28

$$\frac{2\left(7\left(b^{2}cx^{6}+2\,abcx^{3}+a^{2}c\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+5\left(b^{2}dx^{6}+2\,abdx^{3}+a^{2}d\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(5\,b^{2}dx^{5}+7\,b^{2}cx^{4}+8\,abdx^{2}+10\,abcx\right)\sqrt{bx^{3}+a}\right)}{27\left(a^{2}b^{3}x^{6}+2\,a^{3}b^{2}x^{3}+a^{4}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="fricas"
)
```

```
[Out] 2/27*(7*(b^2*c*x^6 + 2*a*b*c*x^3 + a^2*c)*sqrt(b)*weierstrassPInverse(0, -4
*a/b, x) + 5*(b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d)*sqrt(b)*weierstrassZeta(0, -
4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*b^2*d*x^5 + 7*b^2*c*x^4 + 8*
```

```
a*b*d*x^2 + 10*a*b*c*x)*sqrt(b*x^3 + a))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4
*b)
```

Sympy [A]
time = 12.83, size = 163, normalized size = 0.29

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{2}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{7}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{7}{2}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(7/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 7/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 7/2), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(5/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 7/2)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)*gamma(7/3)) + b*d*x**5*gamm
a(5/3)*hyper((5/3, 7/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)*gamm
a(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c}{{\left(b\,x^3+a\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x)
```

```
[Out] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2), x)
```

3.65 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx$

Optimal. Leaf size=581

$$\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}+\frac{2x(13c+11dx)}{135a^2\left(a+bx^3\right)^{3/2}}+\frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}}-\frac{22d\sqrt{a+bx^3}}{81a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{11\sqrt{2-\sqrt{3}}$$

```
[Out] 2/15*x*(d*x+c)/a/(b*x^3+a)^(5/2)+2/135*x*(11*d*x+13*c)/a^2/(b*x^3+a)^(3/2)+
2/405*x*(55*d*x+91*c)/a^3/(b*x^3+a)^(1/2)-22/81*d*(b*x^3+a)^(1/2)/a^3/b^(2/
3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+11/81*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b
^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I
)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/b^(2/3)/(b*x^3+a)^(1/2)/
(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/121
5*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(91*b^(1/3)*c+55*a^(1/3)*d*(1-3^(1/2)))
*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^3/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1600, 1869, 1892, 224, 1891}

$$\frac{11\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{22d\sqrt{a+bx^3}}{81a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}}+\frac{2x(13c+11dx)}{135a^2\left(a+bx^3\right)^{3/2}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(55\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d+91\sqrt[3]{b}\,c\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{405\sqrt[4]{3}\,a^3b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x]
```

```
[Out] (2*x*(c + d*x))/(15*a*(a + b*x^3)^(5/2)) + (2*x*(13*c + 11*d*x))/(135*a^2*(
a + b*x^3)^(3/2)) + (2*x*(91*c + 55*d*x))/(405*a^3*Sqrt[a + b*x^3]) - (22*d
*Sqrt[a + b*x^3])/(81*a^3*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (1
1*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(27*3^(3/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
```

```
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 +
Sqrt[3]]*(91*b^(1/3)*c + 55*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(2/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx=\int \frac{c+dx}{(a+bx^3)^{7/2}}\,dx$$

$$=\frac{2x(c+dx)}{15a\,(a+bx^3)^{5/2}}-\frac{2\int \frac{-\frac{13c}{2}-\frac{11dx}{2}}{(a+bx^3)^{5/2}}\,dx}{15a}$$

$$=\frac{2x(c+dx)}{15a\,(a+bx^3)^{5/2}}+\frac{2x(13c+11dx)}{135a^2\,(a+bx^3)^{3/2}}+\frac{4\int \frac{\frac{91c}{4}+\frac{55dx}{4}}{(a+bx^3)^{3/2}}\,dx}{135a^2}$$

$$=\frac{2x(c+dx)}{15a\,(a+bx^3)^{5/2}}+\frac{2x(13c+11dx)}{135a^2\,(a+bx^3)^{3/2}}+\frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}}-\frac{8\int \frac{-\frac{91c}{8}+\frac{55dx}{8}}{\sqrt{a+bx^3}}}{405a^3}$$

$$=\frac{2x(c+dx)}{15a\,(a+bx^3)^{5/2}}+\frac{2x(13c+11dx)}{135a^2\,(a+bx^3)^{3/2}}+\frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}}-\frac{(11d)\int \frac{\left(1-\sqrt{\ }\right.}{\sqrt{\ }}}{81a}$$

$$=\frac{2x(c+dx)}{15a\,(a+bx^3)^{5/2}}+\frac{2x(13c+11dx)}{135a^2\,(a+bx^3)^{3/2}}+\frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}}-\frac{22a}{81a^3b^{2/3}\left(\left(1-\right.\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 138, normalized size = 0.24

$$\frac{4cx(157a^2+221abx^3+91b^2x^6)+182cx(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+405dx^2(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{2}{3},\frac{7}{2};\frac{5}{3};-\frac{bx^3}{a}\right)}{810a^3\,(a+bx^3)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x]
```

```
[Out] (4*c*x*(157*a^2 + 221*a*b*x^3 + 91*b^2*x^6) + 182*c*x*(a + b*x^3)^2*Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 405*d*x^2*(a
+ b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 7/2, 5/3, -((b*x^3)/a
)])/(810*a^3*(a + b*x^3)^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1901 vs. 2(437) = 874.
time = 0.32, size = 1902, normalized size = 3.27

method	result
elliptic	$\frac{\left(\frac{2d\,x^2}{15a\,b^3}+\frac{2cx}{15a\,b^3}\right)\sqrt{b\,x^3+a}}{\left(x^3+\frac{a}{b}\right)^3}+\frac{\left(\frac{22d\,x^2}{135b^2a^2}+\frac{26cx}{135b^2a^2}\right)\sqrt{b\,x^3+a}}{\left(x^3+\frac{a}{b}\right)^2}-\frac{2b\left(-\frac{11d\,x^2}{81a^3b}-\frac{91cx}{405a^3b}\right)}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{182ic\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\right.}{}}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] b*d*(-2/21*x^2/b^5*(b*x^3+a)^(1/2)/(x^3+a/b)^4+8/315/a*x^2/b^4*(b*x^3+a)^(1
/2)/(x^3+a/b)^3+88/2835/a^2*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+88/1701/b/a
^3*x^2/((x^3+a/b)*b)^(1/2)+88/5103*I/a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))))+b*c*(-2/21*x/b^5*(b*x^3+a)^(1/2)/(x^3+a/b)^4+4/315/a*x/b^4*(b*
x^3+a)^(1/2)/(x^3+a/b)^3+52/2835/a^2*x/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+52/1
215/b/a^3*x/((x^3+a/b)*b)^(1/2)-52/3645*I/a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
```

```
b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+a*d*(2/21/a*x^2/b^4*(b*x^3+a)^(1/2)/(
x^3+a/b)^4+34/315/a^2*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^3+374/2835/a^3*x^2/
b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+374/1701/a^4*x^2/((x^3+a/b)*b)^(1/2)+374/51
03*I/a^4*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+a*c*(2/21/a*x/b^4*(b*
x^3+a)^(1/2)/(x^3+a/b)^4+38/315/a^2*x/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^3+494/2
835/a^3*x/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+494/1215/a^4*x/((x^3+a/b)*b)^(1/2
)-494/3645*I/a^4*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 214, normalized size = 0.37

$$\frac{2\left(91\,(b^3cx^9+3\,ab^2cx^6+3\,a^2bcx^3+a^3c)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+55\,(b^3dx^9+3\,ab^2dx^6+3\,a^2bdx^3+a^3d)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(55\,b^3dx^8+91\,b^3cx^7+143\,ab^2dx^5+221\,ab^2cx^4+115\,a^2bdx^2+157\,a^2bcx)\sqrt{bx^3+a}\right)}{405\,(a^3b^4x^9+3\,a^4b^3x^6+3\,a^5b^2x^3+a^6b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="fricas"
)
```

```
[Out] 2/405*(91*(b^3*c*x^9 + 3*a*b^2*c*x^6 + 3*a^2*b*c*x^3 + a^3*c)*sqrt(b)*weier
strassPInverse(0, -4*a/b, x) + 55*(b^3*d*x^9 + 3*a*b^2*d*x^6 + 3*a^2*b*d*x^
3 + a^3*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b
, x)) + (55*b^3*d*x^8 + 91*b^3*c*x^7 + 143*a*b^2*d*x^5 + 221*a*b^2*c*x^4 +
115*a^2*b*d*x^2 + 157*a^2*b*c*x)*sqrt(b*x^3 + a))/(a^3*b^4*x^9 + 3*a^4*b^3*
x^6 + 3*a^5*b^2*x^3 + a^6*b)
```

Sympy [A]
time = 41.48, size = 163, normalized size = 0.28

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{9}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{9}{2}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{bcx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},\frac{9}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{9}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{bdx^{5}\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{3},\frac{9}{2}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{9}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(9/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 9/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7
/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 9/2), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(7/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 9/2)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(9/2)*gamma(7/3)) + b*d*x**5*gamm
a(5/3)*hyper((5/3, 9/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(9/2)*gamm
a(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c}{{\left(b\,x^3+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x)
```

```
[Out] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2), x)
```

3.66 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=590

$$\frac{2e\sqrt{a+bx^3}}{3b}+\frac{2fx\sqrt{a+bx^3}}{5b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}+\frac{2(7bd-4ag)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,(7b\cdots}{\cdots}$$

```
[Out] 2/3*e*(b*x^3+a)^(1/2)/b+2/5*f*x*(b*x^3+a)^(1/2)/b+2/7*g*x^2*(b*x^3+a)^(1/2)
/b+2/7*(-4*a*g+7*b*d)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))-1/7*3^(1/4)*a^(1/3)*(-4*a*g+7*b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/105*(a^(1/3)+b^(1/3)*x)*
EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*(7*b^(1/3)*(-2*a*f+5*b*c)-5*a^(1/3)*(-4*a*g+7*b*d)*(1-3^(1/2)
))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1902, 1900, 267, 1892, 224, 1891}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}\,(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}\,(7bd-4ag)\right)}{35\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7bd-4ag)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}\,(7bd-4ag)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2e\sqrt{a+bx^3}}{3b}+\frac{2fx\sqrt{a+bx^3}}{5b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*f*x*Sqrt[a + b*x^3])/(5*b) + (2*g*x^2*Sqrt
[a + b*x^3])/(7*b) + (2*(7*b*d - 4*a*g)*Sqrt[a + b*x^3])/(7*b^(5/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*b*d -
 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
```

```
^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(5*b*c - 2*
a*f) - 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
```

```
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx = \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{2\int \frac{\frac{7bc}{2}+\frac{1}{2}(7bd-4ag)x+\frac{7}{2}bex^2+\frac{7}{2}bfx^3}{\sqrt{a+bx^3}}\,dx}{7b}$$

$$= \frac{2fx\sqrt{a+bx^3}}{5b} + \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{4\int \frac{\frac{7}{4}b(5bc-2af)+\frac{5}{4}b(7bd-4ag)x+\frac{35}{4}b^2ex^2}{\sqrt{a+bx^3}}\,dx}{35b^2}$$

$$= \frac{2fx\sqrt{a+bx^3}}{5b} + \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{4\int \frac{\frac{7}{4}b(5bc-2af)+\frac{5}{4}b(7bd-4ag)x}{\sqrt{a+bx^3}}\,dx}{35b^2} + e\int$$

$$= \frac{2e\sqrt{a+bx^3}}{3b} + \frac{2fx\sqrt{a+bx^3}}{5b} + \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{(7bd-4ag)\int \frac{\left(1-\sqrt{}\right.}{}}{7b^{4/}}$$

$$= \frac{2e\sqrt{a+bx^3}}{3b} + \frac{2fx\sqrt{a+bx^3}}{5b} + \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{2(7bd-4ag)\sqrt{}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt{}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.09, size = 135, normalized size = 0.23

$$\frac{4(a+bx^3)\left(35e+3x(7f+5gx)\right)+42(5bc-2af)x\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+15(7bd-4ag)x^2\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{210b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] (4*(a + b*x^3)*(35*e + 3*x*(7*f + 5*g*x)) + 42*(5*b*c - 2*a*f)*x*Sqrt[1 + (
b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 15*(7*b*d - 4*a*
g)*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/
(210*b*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1490 vs. $2(446) = 892$.
time = 0.35, size = 1491, normalized size = 2.53

method	result
elliptic	$\frac{2 g x^2 \sqrt{b x^3+a}}{7 b}+\frac{2 f x \sqrt{b x^3+a}}{5 b}+\frac{2 e \sqrt{b x^3+a}}{3 b}-\frac{2 i \left(c-\frac{2 a f}{5 b}\right) \sqrt{3}\, (-a b^2)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{(-a b^2)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\, (-a b^2)^{\frac{1}{3}}}{2 b}\right.}{(-a b^2)^{\frac{1}{3}}}}}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/7*x^2*(b*x^3+a)^(1/2)/b+8/21*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)))+f*(2/5*x*(b*x^3+a)^(1/2)/b+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I
```

(x+1/2/b(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/3*e*(b*x^3+a)^(1/2)/b-2/3*I*d*3^(1/
2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)/sqrt(b*x^3 + a), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 87, normalized size = 0.15

$$\frac{2\left(21\,(5\,bc-2\,af)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-15\,(7\,bd-4\,ag)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(15\,bgx^2+21\,bfx+35\,be)\sqrt{bx^3+a}\right)}{105\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] 2/105*(21*(5*b*c - 2*a*f)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - 15*(7
*b*d - 4*a*g)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*
a/b, x)) + (15*b*g*x^2 + 21*b*f*x + 35*b*e)*sqrt(b*x^3 + a))/b^2

Sympy [A]
time = 1.86, size = 187, normalized size = 0.32

$$e\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for } b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right)+\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)}+\frac{gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True))
 + c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + f*x**4*gamma(4/3)*hyper((1/2, 4/3),
 (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + g*x**5*gamma(5/
3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(1/2),x)
```

```
[Out] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(1/2), x)
```

3.67 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=594

$$\frac{2x(bc-af+(bd-ag)x+bex^2)}{3ab\sqrt{a+bx^3}}-\frac{2e\sqrt{a+bx^3}}{3ab}-\frac{2(bd-4ag)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt{2-\sqrt{3}}\,(bd-4ag}{}$$

```
[Out] 2/3*x*(b*c-a*f+(-a*g+b*d)*x+b*e*x^2)/a/b/(b*x^3+a)^(1/2)-2/3*e*(b*x^3+a)^(1
/2)/a/b-2/3*(-4*a*g+b*d)*(b*x^3+a)^(1/2)/a/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))+1/3*(-4*a*g+b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2
*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)
*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(b^(1/3)*(2*a*f+b*c)+a^(1/3)*(-4*a*g+b*d)*(1-3^(1/2)))*(1/2*
6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1872, 1900, 267, 1892, 224, 1891}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{b}\,(2af+bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}\,(bd-4ag)\right)}{3\sqrt[4]{3}\,ab^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(bd-4ag)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{a+bx^3}\,(bd-4ag)}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2x(x(bd-ag)-af+bc+bex^2)}{3ab\sqrt{a+bx^3}}-\frac{2e\sqrt{a+bx^3}}{3ab}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(3*a*b*Sqrt[a + b*x^3]) - (2*e*
Sqrt[a + b*x^3])/(3*a*b) - (2*(b*d - 4*a*g)*Sqrt[a + b*x^3])/(3*a*b^(5/3)*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*(b*d - 4*a*g)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^
(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
```

```
 b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*(b*c + 2*a*
f) + (1 - Sqrt[3])*a^(1/3)*(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(5/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \frac{2x(bc-af+(bd-ag)x+bex^2)}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{1}{2}b(bc+2af)+\frac{1}{2}b(bd-4ag)x+\frac{3}{2}b^2ex^2}{\sqrt{a+bx^3}}\,dx}{3ab^2}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{1}{2}b(bc+2af)+\frac{1}{2}b(bd-4ag)x}{\sqrt{a+bx^3}}\,dx}{3ab^2} - \frac{e\int}{}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{3ab\sqrt{a+bx^3}} - \frac{2e\sqrt{a+bx^3}}{3ab} - \frac{(bd-4ag)\int \frac{\left(1-\sqrt{3}\right.}{\sqrt{}}}{3ab^{4/}}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{3ab\sqrt{a+bx^3}} - \frac{2e\sqrt{a+bx^3}}{3ab} - \frac{2(bd-4ag)\sqrt{}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 130, normalized size = 0.22

$$\frac{4bcx-4a(e+x(f-3gx))+2(bc+2af)x\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+3(bd-4ag)x^2\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)}{6ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (4*b*c*x - 4*a*(e + x*(f - 3*g*x)) + 2*(b*c + 2*a*f)*x*Sqrt[1 + (b*x^3)/a]*
Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 3*(b*d - 4*a*g)*x^2*Sqrt[1
 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqrt[a
 + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1546 vs. $2(457) = 914$.
time = 0.40, size = 1547, normalized size = 2.60

method	result
elliptic	$-\frac{2b\left(\frac{(ag-bd)x^2}{3a\,b^2}+\frac{(af-bc)x}{3b^2a}+\frac{e}{3b^2}\right)}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\left(\frac{f}{b}-\frac{af-bc}{3ab}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x}{-\frac{3(-a\,b^2)}{2b}}}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(-2/3/b*x^2/((x^3+a/b)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))))+f*(-2/3/b*x/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
```

```
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*e/b/(b*x^3+a)^(1/2)+d*(2/3/a*
x^2/((x^3+a/b)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
)+c*(2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 153, normalized size = 0.26

$$\frac{2\left(\left((b^2c+2\,abf)x^3+abc+2\,a^2f\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left((b^2d-4\,abg)x^3+abd-4\,a^2g\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\sqrt{bx^3+a}\,\left(abe-(b^2d-abg)x^2-(b^2c-abf)x\right)\right)}{3\,(ab^3x^3+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(((b^2*c + 2*a*b*f)*x^3 + a*b*c + 2*a^2*f)*sqrt(b)*weierstrassPInverse(
0, -4*a/b, x) + ((b^2*d - 4*a*b*g)*x^3 + a*b*d - 4*a^2*g)*sqrt(b)*weierstra
```

```
ssZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - sqrt(b*x^3 + a)*(a*b
*e - (b^2*d - a*b*g)*x^2 - (b^2*c - a*b*f)*x))/(a*b^3*x^3 + a^2*b^2)
```

Sympy [A]
time = 7.20, size = 189, normalized size = 0.32

$$e\left(\begin{cases}-\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0\\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] e*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True
)) + c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + f*x**4*gamma(4/3)*hyper((4/3, 3
/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + g*x**5*gam
ma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gam
ma(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x)
```

```
[Out] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2), x)
```

3.68 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=628

$$\frac{2x(bc-af+(bd-ag)x+bex^2)}{9ab\,(a+bx^3)^{3/2}}-\frac{2(5bd+4ag)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2(3ae-x(7bc+2af+(5bd+4ag$$

```
[Out] 2/9*x*(b*c-a*f+(-a*g+b*d)*x+b*e*x^2)/a/b/(b*x^3+a)^(3/2)-2/27*(3*a*e-x*(7*b
*c+2*a*f+(4*a*g+5*b*d)*x))/a^2/b/(b*x^3+a)^(1/2)-2/27*(4*a*g+5*b*d)*(b*x^3+
a)^(1/2)/a^2/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/27*(4*a*g+5*b*d)*(a^
(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(
5/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)+2/81*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*(2*
a*f+7*b*c)+a^(1/3)*(4*a*g+5*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)*3^(3/4)/a^2/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1872, 1868, 1892, 224, 1891}

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(4ag+5bd)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{a+bx^3}(4ag+5bd)}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2(3ae-x(x(4ag+5bd)+2af+7bc))}{27a^2b\sqrt{a+bx^3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{b}(2af+7bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+5bd)\right)}{27\sqrt[4]{3}\,a^2b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2x(x(bd-ag)-af+bc+bex^2)}{9ab\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(9*a*b*(a + b*x^3)^(3/2)) - (2*
(5*b*d + 4*a*g)*Sqrt[a + b*x^3])/(27*a^2*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)) - (2*(3*a*e - x*(7*b*c + 2*a*f + (5*b*d + 4*a*g)*x)))/(27*a^2*b*
Sqrt[a + b*x^3]) + (Sqrt[2 - Sqrt[3]]*(5*b*d + 4*a*g)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(5/3)*S
```

```
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*(7*b*c + 2*a*f) + (1 - Sqr
t[3])*a^(1/3)*(5*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = \frac{2x(bc-af+(bd-ag)x+bex^2)}{9ab\,(a+bx^3)^{3/2}} - \frac{2\int \frac{-\frac{1}{2}b(7bc+2af)-\frac{1}{2}b(5bd+4ag)x-\frac{3}{2}b^2ex^2}{(a+bx^3)^{3/2}}\,d}{9ab^2}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{9ab\,(a+bx^3)^{3/2}} - \frac{2(3ae-x(7bc+2af+(5bd+4ag)}{27a^2b\sqrt{a+bx^3}}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{9ab\,(a+bx^3)^{3/2}} - \frac{2(3ae-x(7bc+2af+(5bd+4ag)}{27a^2b\sqrt{a+bx^3}}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{9ab\,(a+bx^3)^{3/2}} - \frac{2(5bd+4ag)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.13, size = 170, normalized size = 0.27

$$\frac{140b^2cx^4+40abx(5c+fx^3)-4a^2(15e+x(5f+27gx))+10(7bc+2af)x(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+27(5bd+4ag)x^2(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)}{270a^2b\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (140*b^2*c*x^4 + 40*a*b*x*(5*c + f*x^3) - 4*a^2*(15*e + x*(5*f + 27*g*x)) +
 10*(7*b*c + 2*a*f)*x*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3
, 1/2, 4/3, -((b*x^3)/a)] + 27*(5*b*d + 4*a*g)*x^2*(a + b*x^3)*Sqrt[1 + (b*
x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(270*a^2*b*(a + b*x
^3)^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1672 vs. 2(488) = 976.
time = 0.36, size = 1673, normalized size = 2.66

method	result
elliptic	$\frac{\left(-\frac{2(ag-bd)x^2}{9b^3a}-\frac{2(af-bc)x}{9b^3a}-\frac{2e}{9b^3}\right)\sqrt{b\,x^3+a}}{\left(x^3+\frac{a}{b}\right)^2}-\frac{2b\left(-\frac{(4ag+5bd)x^2}{27a^2b^2}-\frac{(2af+7bc)x}{27a^2b^2}\right)}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i(2af+7bc)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a}{}\right.}{}}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(-2/9*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+8/27/b/a*x^2/((x^3+a/b)*b)^(1/2
)+8/81*I/b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+f*(-2/9*x/b^3*(b*
x^3+a)^(1/2)/(x^3+a/b)^2+4/27/b/a*x/((x^3+a/b)*b)^(1/2)-4/81*I/b^2/a*3^(1/2
)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
```

```
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/9*e/b/(b*x^3+a)^
(3/2)+d*(2/9/a*x^2/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+10/27/a^2*x^2/((x^3+a/b)
*b)^(1/2)+10/81*I/a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*
b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(
-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*
b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+c*(2/9/a
*x/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+14/27/a^2*x/((x^3+a/b)*b)^(1/2)-14/81*I/
a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)/(b*x^3 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 261, normalized size = 0.42

$$\frac{2\left(\left(\left(7\,b^3c+2\,ab^2f\right)x^6+7\,a^2bc+2\,a^3f+2\left(7\,ab^2c+2\,a^2bf\right)x^3\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(\left(5\,b^3d+4\,ab^2g\right)x^6+5\,a^2bd+4\,a^3g+2\left(5\,ab^2d+4\,a^2bg\right)x^3\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(\left(5\,b^3d+4\,ab^2g\right)x^5+\left(7\,b^3c+2\,ab^2f\right)x^4-3\,a^2be+\left(8\,ab^2d+a^2bg\right)x^2+\left(10\,ab^2c-a^2bf\right)x\right)\sqrt{bx^3+a}\right)}{27\left(a^2b^4x^6+2\,a^3b^3x^3+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*(((7*b^3*c + 2*a*b^2*f)*x^6 + 7*a^2*b*c + 2*a^3*f + 2*(7*a*b^2*c + 2*a
^2*b*f)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + ((5*b^3*d + 4*a*b^
2*g)*x^6 + 5*a^2*b*d + 4*a^3*g + 2*(5*a*b^2*d + 4*a^2*b*g)*x^3)*sqrt(b)*wei
erstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + ((5*b^3*d + 4*
a*b^2*g)*x^5 + (7*b^3*c + 2*a*b^2*f)*x^4 - 3*a^2*b*e + (8*a*b^2*d + a^2*b*g
```

```
)*x^2 + (10*a*b^2*c - a^2*b*f)*x)*sqrt(b*x^3 + a))/(a^2*b^4*x^6 + 2*a^3*b^3
*x^3 + a^4*b^2)
```

Sympy [A]
time = 41.75, size = 209, normalized size = 0.33

$$e\left(\begin{cases} -\frac{2}{9ab\sqrt{a+bx^3}+9b^2x^3\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^3}{3a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{3},\frac{5}{2}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(5/2),x)
```

```
[Out] e*Piecewise((-2/(9*a*b*sqrt(a + b*x**3) + 9*b**2*x**3*sqrt(a + b*x**3)), Ne
(b, 0)), (x**3/(3*a**(5/2)), True)) + c*x*gamma(1/3)*hyper((1/3, 5/2), (4/3
,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(4/3)) + d*x**2*gamma(2/3)*h
yper((2/3, 5/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(5/3))
+ f*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(5/2)*gamma(7/3)) + g*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(5/2)*gamma(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)/(b*x^3 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{{\left(b\,x^3+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x)
```

```
[Out] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2), x)
```

3.69 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx$

Optimal. Leaf size=676

$$\frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}}+\frac{2x(7(13bc+2af)+5(11bd+4ag)x)}{405a^3b\sqrt{a+bx^3}}-\frac{2(11bd+4ag)\sqrt{a+bx^3}}{81a^3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}$$

```
[Out] 2/15*x*(b*c-a*f+(-a*g+b*d)*x+b*e*x^2)/a/b/(b*x^3+a)^(5/2)-2/135*(9*a*e-x*(1
3*b*c+2*a*f+(4*a*g+11*b*d)*x))/a^2/b/(b*x^3+a)^(3/2)+2/405*x*(14*a*f+91*b*c
+5*(4*a*g+11*b*d)*x)/a^3/b/(b*x^3+a)^(1/2)-2/81*(4*a*g+11*b*d)*(b*x^3+a)^(1
/2)/a^3/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/81*(4*a*g+11*b*d)*(a^(1/3
)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)
/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)+2/1215*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*(2*
a*f+13*b*c)+5*a^(1/3)*(4*a*g+11*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)*3^(3/4)/a^3/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1872, 1868, 1869, 1892, 224, 1891}

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4ag+11bd)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\,3^{3/4}a^{8/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{a+bx^3}(4ag+11bd)}{81a^3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(7(2af+13bc)+5x(4ag+11bd))}{405a^3b\sqrt{a+bx^3}}-\frac{2(9ae-x(x(4ag+11bd)+2af+13bc))}{135a^2b\left(a+bx^3\right)^{3/2}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(2af+13bc)+5\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+11bd)\right)}{405\sqrt[4]{3}a^3b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(15*a*b*(a + b*x^3)^(5/2)) + (2
*x*(7*(13*b*c + 2*a*f) + 5*(11*b*d + 4*a*g)*x))/(405*a^3*b*Sqrt[a + b*x^3])
 - (2*(11*b*d + 4*a*g)*Sqrt[a + b*x^3])/(81*a^3*b^(5/3)*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) - (2*(9*a*e - x*(13*b*c + 2*a*f + (11*b*d + 4*a*g)*x)))/(
135*a^2*b*(a + b*x^3)^(3/2)) + (Sqrt[2 - Sqrt[3]]*(11*b*d + 4*a*g)*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
```

```
])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(3/4)*a^(
8/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(13*b*c +
2*a*f) + 5*(1 - Sqrt[3])*a^(1/3)*(11*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(5/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \frac{2x(bc-af+(bd-ag)x+bex^2)}{15ab\,(a+bx^3)^{5/2}} - \frac{2\int \frac{-\frac{1}{2}b(13bc+2af)-\frac{1}{2}b(11bd+4ag)x-\frac{9}{2}b^2ex^2}{(a+bx^3)^{5/2}}}{15ab^2}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{15ab\,(a+bx^3)^{5/2}} - \frac{2(9ae-x(13bc+2af+(11bd+4a}{135a^2b\,(a+bx^3)^{3/2}}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{15ab\,(a+bx^3)^{5/2}} + \frac{2x(7(13bc+2af)+5(11bd+4ag)x}{405a^3b\sqrt{a+bx^3}}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{15ab\,(a+bx^3)^{5/2}} + \frac{2x(7(13bc+2af)+5(11bd+4ag)x}{405a^3b\sqrt{a+bx^3}}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{15ab\,(a+bx^3)^{5/2}} + \frac{2x(7(13bc+2af)+5(11bd+4ag)x}{405a^3b\sqrt{a+bx^3}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.18, size = 196, normalized size = 0.29

$$\frac{4004b^3cx^7+44ab^2x^4(221c+14fx^3)+44a^2bx(157c+34fx^3)-4a^3(297e+x(77f+405gx))+154(13bc+2af)x(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+405(11bd+4ag)x^2(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{2}{3},\frac{7}{2};\frac{5}{3};-\frac{bx^3}{a}\right)}{8910a^3b\left(a+bx^3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (4004*b^3*c*x^7 + 44*a*b^2*x^4*(221*c + 14*f*x^3) + 44*a^2*b*x*(157*c + 34*
f*x^3) - 4*a^3*(297*e + x*(77*f + 405*g*x)) + 154*(13*b*c + 2*a*f)*x*(a + b
*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
+ 405*(11*b*d + 4*a*g)*x^2*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[2/3, 7/2, 5/3, -((b*x^3)/a)])/(8910*a^3*b*(a + b*x^3)^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1792 vs. 2(529) = 1058.
time = 0.36, size = 1793, normalized size = 2.65

method	result	size
elliptic	Expression too large to display	921
default	Expression too large to display	1793

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(-2/15*x^2/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^3+8/135/a*x^2/b^3*(b*x^3+a)^(1/2
)/(x^3+a/b)^2+8/81/b/a^2*x^2/((x^3+a/b)*b)^(1/2)+8/243*I/a^2/b^2*3^(1/2)*(-
a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+f*(-2/15*x/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^3
+4/135/a*x/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+28/405/b/a^2*x/((x^3+a/b)*b)^(1/
2)-28/1215*I/a^2/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
```

```
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)))-2/15*e/b/(b*x^3+a)^(5/2)+d*(2/15/a*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+a/
b)^3+22/135/a^2*x^2/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+22/81/a^3*x^2/((x^3+a/b
)*b)^(1/2)+22/243*I/a^3*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-
a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-
a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+c*(2/1
5/a*x/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^3+26/135/a^2*x/b^2*(b*x^3+a)^(1/2)/(x^3
+a/b)^2+182/405/a^3*x/((x^3+a/b)*b)^(1/2)-182/1215*I/a^3*3^(1/2)/b*(-a*b^2)
^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)/(b*x^3 + a)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 373, normalized size = 0.55

$$\frac{2\left(7\left((13b^4c+2ab^3f)x^9+3(13ab^3c+2a^2b^2f)x^6+13a^3bc+2a^4f+3(13a^2b^2c+2a^3bf)x^3\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+5\left((11b^4d+4ab^3g)x^9+3(11ab^3d+4a^2b^2g)x^6+11a^3bd+4a^4g+3(11a^2b^2d+4a^3bg)x^3\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(5(11b^4d+4ab^3g)x^8+7(13b^4c+2ab^3f)x^7+13(11ab^3d+4a^2b^2g)x^5-27a^3be+17(13ab^3c+2a^2b^2f)x^4+5(23a^2b^2d+a^3bg)x^2+(157a^2b^2c-7a^3bf)x\right)\sqrt{bx^3+a}\right)}{405\left(a^3b^5x^9+3a^4b^4x^6+3a^5b^3x^3+a^6b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/405*(7*((13*b^4*c + 2*a*b^3*f)*x^9 + 3*(13*a*b^3*c + 2*a^2*b^2*f)*x^6 + 1
3*a^3*b*c + 2*a^4*f + 3*(13*a^2*b^2*c + 2*a^3*b*f)*x^3)*sqrt(b)*weierstrass
PInverse(0, -4*a/b, x) + 5*((11*b^4*d + 4*a*b^3*g)*x^9 + 3*(11*a*b^3*d + 4*
a^2*b^2*g)*x^6 + 11*a^3*b*d + 4*a^4*g + 3*(11*a^2*b^2*d + 4*a^3*b*g)*x^3)*s
```

```
qrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*(
11*b^4*d + 4*a*b^3*g)*x^8 + 7*(13*b^4*c + 2*a*b^3*f)*x^7 + 13*(11*a*b^3*d +
 4*a^2*b^2*g)*x^5 - 27*a^3*b*e + 17*(13*a*b^3*c + 2*a^2*b^2*f)*x^4 + 5*(23*
a^2*b^2*d + a^3*b*g)*x^2 + (157*a^2*b^2*c - 7*a^3*b*f)*x)*sqrt(b*x^3 + a))/
(a^3*b^5*x^9 + 3*a^4*b^4*x^6 + 3*a^5*b^3*x^3 + a^6*b^2)
```

Sympy [**A**]
time = 174.95, size = 231, normalized size = 0.34

$$e\left(\begin{cases}-\frac{2}{15a^2b\sqrt{a+bx^3}+30ab^2x^3\sqrt{a+bx^3}+15b^3x^6\sqrt{a+bx^3}} & \text{for } b\neq 0\\ \frac{x^3}{3a^{\frac{7}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{7}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{7}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(7/2),x)
```

```
[Out] e*Piecewise((-2/(15*a**2*b*sqrt(a + b*x**3) + 30*a*b**2*x**3*sqrt(a + b*x**
3) + 15*b**3*x**6*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(7/2)), True))
+ c*x*gamma(1/3)*hyper((1/3, 7/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(7/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 7/2), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(7/2)*gamma(5/3)) + f*x**4*gamma(4/3)*hyper((4/3, 7/2)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)*gamma(7/3)) + g*x**5*gamma(
5/3)*hyper((5/3, 7/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)*gamma(
8/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)/(b*x^3 + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{{\left(b\,x^3+a\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x)
```

```
[Out] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2), x)
```

3.70 $\int \frac{(a+bx)^2}{c+dx^3} \, dx$

Optimal. Leaf size=186

$$-\frac{a\left(2b\sqrt[3]{c}+a\sqrt[3]{d}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{2/3}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{2/3}}+\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log}{6c}$$

```
[Out] -1/3*a*(2*b*c^(1/3)-a*d^(1/3))*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^(2/3)+1/6*a*
(2*b*c^(1/3)-a*d^(1/3))*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/d
^(2/3)+1/3*b^2*ln(d*x^3+c)/d-1/3*a*(2*b*c^(1/3)+a*d^(1/3))*arctan(1/3*(c^(1
/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/d^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{a\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{2/3}}+\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x^3),x]
```

```
[Out] -((a*(2*b*c^(1/3) + a*d^(1/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1
/3))])/(Sqrt[3]*c^(2/3)*d^(2/3))) - (a*(2*b*c^(1/3) - a*d^(1/3))*Log[c^(1/3
) + d^(1/3)*x])/(3*c^(2/3)*d^(2/3)) + (a*(2*b*c^(1/3) - a*d^(1/3))*Log[c^(2
/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(2/3)) + (b^2*Log[c +
d*x^3])/(3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = b^2\int \frac{x^2}{c+dx^3}\,dx + \int \frac{a^2+2abx}{c+dx^3}\,dx$$

$$= \frac{b^2\log(c+dx^3)}{3d} + \frac{\int \frac{\sqrt[3]{c}\left(2ab\sqrt[3]{c}+2a^2\sqrt[3]{d}\right)+\left(2ab\sqrt[3]{c}-a^2\sqrt[3]{d}\right)\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{3c^{2/3}\sqrt[3]{d}} - \frac{\left(2ab\sqrt[3]{c}-a^2\sqrt[3]{d}\right.}{3c^{2/3}}$$

$$= -\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{2/3}} + \frac{b^2\log(c+dx^3)}{3d} + \frac{1}{2}\left(a\left(\frac{a}{\sqrt[3]{c}}+\frac{2b}{\sqrt[3]{d}}\right)\right)\int \frac{}{c^2}$$

$$= -\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{2/3}} + \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/}\right.}{6c^{2/3}d^{2/3}}$$

$$= -\frac{a\left(2b\sqrt[3]{c}+a\sqrt[3]{d}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{2/3}} - \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{2/3}} + \frac{a(}{}$$

Mathematica [A]
time = 0.06, size = 200, normalized size = 1.08

$$\frac{\left(2abc^{2/3}+a^2\sqrt[3]{c}\,\sqrt[3]{d}\right)\tan^{-1}\left(\frac{-\sqrt[3]{c}+2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,cd^{2/3}} + \frac{\left(-2abc^{2/3}+a^2\sqrt[3]{c}\,\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3cd^{2/3}} - \frac{\left(-2abc^{2/3}+a^2\sqrt[3]{c}\,\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6cd^{2/3}} + \frac{b^2\log(c+dx^3)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x^3),x]
```

```
[Out] ((2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1/3))*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*c*d^(2/3)) + ((-2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1/3
))*Log[c^(1/3) + d^(1/3)*x])/(3*c*d^(2/3)) - ((-2*a*b*c^(2/3) + a^2*c^(1/3)
*d^(1/3))*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c*d^(2/3)) + (
b^2*Log[c + d*x^3])/(3*d)
```

Maple [A]
time = 0.36, size = 206, normalized size = 1.11

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^3 d+c)} \frac{\left(\_R^2 b^2+2\_R ab+a^2\right)\ln\left(x-\_R\right)}{\_R^2}}{3d}$

default	$a^2\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)+2ab\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/
3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/
3)*x-1)))+2*a*b*(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(
x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)
*(2/(c/d)^(1/3)*x-1)))+1/3*b^2*ln(d*x^3+c)/d
```

Maxima [A]
time = 0.58, size = 192, normalized size = 1.03

$$-\frac{\sqrt{3}\left(2\,b^2c-\left(6\,ab\left(\frac{c}{d}\right)^{\frac{2}{3}}+3\,a^2\left(\frac{c}{d}\right)^{\frac{1}{3}}+\frac{2b^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd}+\frac{\left(2\,b^2\left(\frac{c}{d}\right)^{\frac{2}{3}}+2\,ab\left(\frac{c}{d}\right)^{\frac{1}{3}}-a^2\right)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\left(b^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,ab\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^2\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*b^2*c - (6*a*b*(c/d)^(2/3) + 3*a^2*(c/d)^(1/3) + 2*b^2*c/d)
*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d) + 1/6*(2*b^2*
(c/d)^(2/3) + 2*a*b*(c/d)^(1/3) - a^2)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3
))/(d*(c/d)^(2/3)) + 1/3*(b^2*(c/d)^(2/3) - 2*a*b*(c/d)^(1/3) + a^2)*log(x
+ (c/d)^(1/3))/(d*(c/d)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.14, size = 5014, normalized size = 26.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3)
 + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*
b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^
```

(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*
b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(
3) + 1) - 2*b^2/d)*d*log(2*b^5*c^2 + 7*a^3*b^2*c*d + 1/2*(2*(1/2)^(2/3)*(b^
4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c
 + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*
a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c
 + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*
a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)^2*b*c^2*
d^2 + 1/2*(4*b^3*c^2*d - a^3*c*d^2)*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^
3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^
2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2
)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^
2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2
)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d) + (8*a^2*b^3*c*d + a^5*d^2)*x
) - (6*b^2 + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqr
t(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b
*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1
/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b
*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*s
qrt(3) + 1) - 2*b^2/d)*d + 3*sqrt(1/3)*d*sqrt(-(4*b^4*c + 32*a^3*b*d + 4*(2
(1/2)^(2/3)(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^
6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3)
 + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^
6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3)
 + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2
*b^2/d)*b^2*c*d + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-
I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*
a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)
 + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*
a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)
*(I*sqrt(3) + 1) - 2*b^2/d)^2*c*d^2)/(c*d^2)))*log(-2*b^5*c^2 - 7*a^3*b^2*c
d - 1/2(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3)
 + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*
b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^
(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*
b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(
3) + 1) - 2*b^2/d)^2*b*c^2*d^2 - 1/2*(4*b^3*c^2*d - a^3*c*d^2)*(2*(1/2)^(2/
3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8
*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^
2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8
*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^
2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d) +
2*(8*a^2*b^3*c*d + a^5*d^2)*x + 3/2*sqrt(1/3)*(2*b^3*c^2*d + a^3*c*d^2 + (2
(1/2)^(2/3)(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^
6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3)

```
 + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^
6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3)
 + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2
*b^2/d)*b*c^2*d^2)*sqrt(-(4*b^4*c + 32*a^3*b*d + 4*(2*(1/2)^(2/3)*(b^4/d^2
- (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3
*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^
3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3
*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^
3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)*b^2*c*d + (2*(
1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/
d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) +
 (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/
d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) +
 (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b
^2/d)^2*c*d^2)/(c*d^2))) - (6*b^2 + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^
3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^
2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2
)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^
2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c...
```

Sympy [A]
time = 0.49, size = 156, normalized size = 0.84

$$\operatorname{RootSum}\left(27t^3c^2d^3 - 27t^2b^2c^2d^2 + t\left(18a^3bcd^2 + 9b^4c^2d\right) - a^6d^2 + 2a^3b^3cd - b^6c^2, \left(t \mapsto t\log\left(x + \frac{18t^2bc^2d^2 + 3ta^3cd^2 - 12tb^3c^2d + 7a^3b^2cd + 2b^5c^2}{a^5d^2 + 8a^2b^3cd}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x**3+c),x)
```

```
[Out] RootSum(27*_t**3*c**2*d**3 - 27*_t**2*b**2*c**2*d**2 + _t*(18*a**3*b*c*d**2
 + 9*b**4*c**2*d) - a**6*d**2 + 2*a**3*b**3*c*d - b**6*c**2, Lambda(_t, _t*
log(x + (18*_t**2*b*c**2*d**2 + 3*_t*a**3*c*d**2 - 12*_t*b**3*c**2*d + 7*a*
*3*b**2*c*d + 2*b**5*c**2)/(a**5*d**2 + 8*a**2*b**3*c*d))))
```

Giac [A]
time = 1.85, size = 175, normalized size = 0.94

$$\frac{b^2\log\left(|dx^3 + c|\right)}{3\,d} - \frac{\sqrt{3}\left(a^2d - 2\left(-cd^2\right)^{\frac{1}{3}}ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}} - \frac{\left(a^2d + 2\left(-cd^2\right)^{\frac{1}{3}}ab\right)\log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}} - \frac{\left(2\,abd\left(-\frac{c}{d}\right)^{\frac{1}{3}} + a^2d\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*log(abs(d*x^3 + c))/d - 1/3*sqrt(3)*(a^2*d - 2*(-c*d^2)^(1/3)*a*b)*
arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(-c*d^2)^(2/3) - 1/6*
(a^2*d + 2*(-c*d^2)^(1/3)*a*b)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(-c
```

```
*d^2)^(2/3) - 1/3*(2*a*b*d*(-c/d)^(1/3) + a^2*d)*(-c/d)^(1/3)*log(abs(x - (
-c/d)^(1/3)))/(c*d)
```

Mupad [B]
time = 0.26, size = 357, normalized size = 1.92

$$\sum_{k=1}^{3}\ln\left(b^4\,c+\mathrm{root}\left(27\,c^2\,d^3\,z^3-27\,b^2\,c^2\,d^2\,z^2+18\,a^3\,b\,c\,d^2\,z+9\,b^4\,c^2\,d\,z+2\,a^3\,b^3\,c\,d-b^6\,c^2-a^6\,d^2,z,k\right)^2\,c\,d^2\,9+2\,a^3\,b\,d-\mathrm{root}\left(27\,c^2\,d^3\,z^3-27\,b^2\,c^2\,d^2\,z^2+18\,a^3\,b\,c\,d^2\,z+9\,b^4\,c^2\,d\,z+2\,a^3\,b^3\,c\,d-b^6\,c^2-a^6\,d^2,z,k\right)\,b^2\,c\,d\,6+\mathrm{root}\left(27\,c^2\,d^3\,z^3-27\,b^2\,c^2\,d^2\,z^2+18\,a^3\,b\,c\,d^2\,z+9\,b^4\,c^2\,d\,z+2\,a^3\,b^3\,c\,d-b^6\,c^2-a^6\,d^2,z,k\right)\,a^2\,d^2\,x\,3+3\,a^2\,b^2\,d\,x\right)\,\mathrm{root}\left(27\,c^2\,d^3\,z^3-27\,b^2\,c^2\,d^2\,z^2+18\,a^3\,b\,c\,d^2\,z+9\,b^4\,c^2\,d\,z+2\,a^3\,b^3\,c\,d-b^6\,c^2-a^6\,d^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x^3),x)
```

```
[Out] symsum(log(b^4*c + 9*root(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*
d^2*z + 9*b^4*c^2*d*z + 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)^2*c*d^2 +
2*a^3*b*d - 6*root(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*z +
 9*b^4*c^2*d*z + 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)*b^2*c*d + 3*root(
27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*z + 9*b^4*c^2*d*z + 2*
a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)*a^2*d^2*x + 3*a^2*b^2*d*x)*root(27*c
^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*z + 9*b^4*c^2*d*z + 2*a^3*
b^3*c*d - b^6*c^2 - a^6*d^2, z, k), k, 1, 3)
```

3.71 $\int \frac{(a+bx)^3}{c+dx^3} \, dx$

Optimal. Leaf size=222

$$\frac{b^3x}{d}+\frac{\left(b^3c-3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{4/3}}-\frac{\left(b^3c+3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}}+\frac{(b^3c}{}$$

```
[Out] b^3*x/d-1/3*(b^3*c+3*a^2*b*c^(1/3)*d^(2/3)-a^3*d)*ln(c^(1/3)+d^(1/3)*x)/c^(
2/3)/d^(4/3)+1/6*(b^3*c+3*a^2*b*c^(1/3)*d^(2/3)-a^3*d)*ln(c^(2/3)-c^(1/3)*d
^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/d^(4/3)+a*b^2*ln(d*x^3+c)/d+1/3*(b^3*c-3*a^2*
b*c^(1/3)*d^(2/3)-a^3*d)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/
c^(2/3)/d^(4/3)*3^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\left(a^3(-d)-3a^2b\sqrt[3]{c}\,d^{2/3}+b^3c\right)\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{4/3}}+\frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}\,d^{2/3}+b^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}\,d^{2/3}+b^3c\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}}+\frac{ab^2\log\left(c+dx^3\right)}{d}+\frac{b^3x}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x^3),x]
```

```
[Out] (b^3*x)/d + ((b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*ArcTan[(c^(1/3) - 2*
d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(4/3)) - ((b^3*c + 3*a^2*
b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(4/3)) +
((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x
+ d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (a*b^2*Log[c + d*x^3])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \int \left(\frac{b^3}{d} - \frac{b^3c-a^3d-3a^2bdx-3ab^2dx^2}{d\,(c+dx^3)}\right)\,dx$$

$$= \frac{b^3x}{d} - \frac{\int \frac{b^3c-a^3d-3a^2bdx-3ab^2dx^2}{c+dx^3}\,dx}{d}$$

$$= \frac{b^3x}{d} + (3ab^2)\int \frac{x^2}{c+dx^3}\,dx - \frac{\int \frac{b^3c-a^3d-3a^2bdx}{c+dx^3}\,dx}{d}$$

$$= \frac{b^3x}{d} + \frac{ab^2\log(c+dx^3)}{d} - \frac{\int \frac{\sqrt[3]{c}\left(-3a^2b\sqrt[3]{c}\,d+2\sqrt[3]{d}\,(b^3c-a^3d)\right)+\sqrt[3]{d}\left(-3a^2b\sqrt[3]{c}\,d-\sqrt[3]{d}\,(b^3c-a^3d)\right)x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}}{3c^{2/3}d^{4/3}}$$

$$= \frac{b^3x}{d} - \frac{\left(b^3c+3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}} + \frac{ab^2\log(c+dx^3)}{d} - \frac{\left(b^3c-3a^2b\sqrt[3]{c}\right.}{}$$

$$= \frac{b^3x}{d} - \frac{\left(b^3c+3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}} + \frac{\left(b^3c+3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\log\left(c^{2}\right.}{6c^{2/3}d^{4/3}}$$

$$= \frac{b^3x}{d} + \frac{\left(b^3c-3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{4/3}} - \frac{\left(b^3c+3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\log\left(\right.}{3c^{2/3}d^{4/3}}$$

Mathematica [A]
time = 0.12, size = 214, normalized size = 0.96

$$\frac{6b^3c^{2/3}\sqrt[3]{d}\,x + 2\sqrt{3}\left(b^3c-3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right) - 2\left(b^3c+3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right) + \left(b^3c+3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right) + 6ab^2c^{2/3}\sqrt[3]{d}\,\log(c+dx^3)}{6c^{2/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x^3),x]
```

```
[Out] (6*b^3*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3
*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] - 2*(b^3*c + 3*a^2*b*c^(1/3
)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x] + (b^3*c + 3*a^2*b*c^(1/3)*d^(2
/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2] + 6*a*b^2*c^(2/
3)*d^(1/3)*Log[c + d*x^3])/(6*c^(2/3)*d^(4/3))
```

Maple [A]
time = 0.34, size = 228, normalized size = 1.03

method	result

risch	$\frac{b^3x}{d}+\frac{\sum_{\_R=\text{RootOf}(\_Z^3d+c)}\frac{\left(3\_R^2ab^2d+3a^2bd\_R+a^3d-b^3c\right)\ln\left(x-\_R\right)}{\_R^2}}{3d^2}$
default	$\frac{b^3x}{d}+\frac{(a^3d-b^3c)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)+3da^2b\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x\right.}{}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] b^3*x/d+((a^3*d-b^3*c)*(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/
3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(c/d)^(1/3)*x-1)))+3*d*a^2*b*(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3)
)+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(
1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))+a*b^2*ln(d*x^3+c))/d
```

Maxima [A]
time = 0.53, size = 240, normalized size = 1.08

$$\frac{b^3x}{d}-\frac{\sqrt{3}\left(\left(b^3\left(\frac{c}{d}\right)^{\frac{1}{3}}+2\,ab^2\right)c-\left(3\,a^2b\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^3\left(\frac{c}{d}\right)^{\frac{1}{3}}+\frac{2\,ab^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd}+\frac{\left(b^3c+\left(6\,ab^2\left(\frac{c}{d}\right)^{\frac{2}{3}}+3\,a^2b\left(\frac{c}{d}\right)^{\frac{1}{3}}-a^3\right)d\right)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\left(b^3c-\left(3\,ab^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-3\,a^2b\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^3\right)d\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] b^3*x/d - 1/3*sqrt(3)*((b^3*(c/d)^(1/3) + 2*a*b^2)*c - (3*a^2*b*(c/d)^(2/3)
 + a^3*(c/d)^(1/3) + 2*a*b^2*c/d)*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))
/(c/d)^(1/3))/(c*d) + 1/6*(b^3*c + (6*a*b^2*(c/d)^(2/3) + 3*a^2*b*(c/d)^(1/
3) - a^3)*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(d^2*(c/d)^(2/3)) - 1/3
*(b^3*c - (3*a*b^2*(c/d)^(2/3) - 3*a^2*b*(c/d)^(1/3) + a^3)*d)*log(x + (c/d
)^(1/3))/(d^2*(c/d)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.85, size = 7245, normalized size = 32.64

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x^3+c),x, algorithm="fricas")

[Out] 1/12*(12*b^3*x - 2*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/
(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^
2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^
4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1
/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*
a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^
2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4)
)^(1/3)*(I*sqrt(3) + 1))*d*log(-3*a*b^8*c^3 + 15*a^4*b^5*c^2*d + 15*a^7*b^2
*c*d^2 + 3/4*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2
))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d
^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (
b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) -
6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/
(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4)
 - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3
)*(I*sqrt(3) + 1))^2*a^2*b*c^2*d^3 - 1/2*(b^6*c^3*d - 20*a^3*b^3*c^2*d^2 +
a^6*c*d^3)*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))
*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3
) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^
9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*
a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c
*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) -
 (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*
(I*sqrt(3) + 1)) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)
*x) + (18*a*b^2 + (6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(
c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2
/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4
) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/
3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a
*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2
*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))
^(1/3)*(I*sqrt(3) + 1))*d + 3*sqrt(1/3)*d*sqrt((12*a^2*b^4*c - 48*a^5*b*d -
 12*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sq
rt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^
9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 -
 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d
 + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) -
 (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c
^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt
(3) + 1))*a*b^2*c*d - (6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*
d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a
*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2
*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))
```

```
^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*
d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/
(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d
^4))^(1/3)*(I*sqrt(3) + 1))^2*c*d^2)/(c*d^2)))*log(3*a*b^8*c^3 - 15*a^4*b^5
*c^2*d - 15*a^7*b^2*c*d^2 - 3/4*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*
c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c +
a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9
*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/
(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*
c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 -
 a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d
^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))^2*a^2*b*c^2*d^3 + 1/2*(b^6*c^3*d - 20
*a^3*b^3*c^2*d^2 + a^6*c*d^3)*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c
+ a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^
5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d
^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c
^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c
+ a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a
^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3
)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1)) - 2*(b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6
*b^3*c*d^2 - a^9*d^3)*x + 3/4*sqrt(1/3)*(2*b^6*c^3*d + 14*a^3*b^3*c^2*d^2 +
 2*a^6*c*d^3 + 3*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c
*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/
(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^...
```

Sympy [A]
time = 8.25, size = 245, normalized size = 1.10

$$\frac{b^3x}{d} + \operatorname{RootSum}\left(27t^3c^2d^4 - 81t^2ab^2c^2d^3 + t\left(27a^5bcd^3 + 54a^2b^4c^2d^2\right) - a^9d^3 + 3a^6b^3cd^2 - 3a^3b^6c^2d + b^9c^3, \left(t \mapsto t\log\left(x + \frac{27t^2a^2bc^2d^3 + 3ta^6cd^3 - 60ta^3b^3c^2d^2 + 3tb^6c^3d + 15a^7b^2cd^2 + 15a^4b^5c^2d - 3ab^8c^3}{a^9d^3 + 24a^6b^3cd^2 + 3a^3b^6c^2d - b^9c^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x**3+c),x)
```

```
[Out] b**3*x/d + RootSum(27*_t**3*c**2*d**4 - 81*_t**2*a*b**2*c**2*d**3 + _t*(27*
a**5*b*c*d**3 + 54*a**2*b**4*c**2*d**2) - a**9*d**3 + 3*a**6*b**3*c*d**2 -
3*a**3*b**6*c**2*d + b**9*c**3, Lambda(_t, _t*log(x + (27*_t**2*a**2*b*c**2
*d**3 + 3*_t*a**6*c*d**3 - 60*_t*a**3*b**3*c**2*d**2 + 3*_t*b**6*c**3*d + 1
5*a**7*b**2*c*d**2 + 15*a**4*b**5*c**2*d - 3*a*b**8*c**3)/(a**9*d**3 + 24*a
**6*b**3*c*d**2 + 3*a**3*b**6*c**2*d - b**9*c**3))))
```

Giac [A]
time = 2.01, size = 214, normalized size = 0.96

$$\frac{b^3x}{d} + \frac{ab^2\log\left(|dx^3+c|\right)}{d} + \frac{\sqrt{3}\left(b^3c - a^3d + 3\left(-cd^2\right)^{\frac{1}{3}}a^2b\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}} + \frac{\left(b^3c - a^3d - 3\left(-cd^2\right)^{\frac{1}{3}}a^2b\right)\log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}} - \frac{\left(3a^2bd^3\left(-\frac{c}{d}\right)^{\frac{1}{3}} - b^3cd^2 + a^3d^3\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x^3+c),x, algorithm="giac")
```

```
[Out] b^3*x/d + a*b^2*log(abs(d*x^3 + c))/d + 1/3*sqrt(3)*(b^3*c - a^3*d + 3*(-c*
d^2)^(1/3)*a^2*b)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(-c
*d^2)^(2/3) + 1/6*(b^3*c - a^3*d - 3*(-c*d^2)^(1/3)*a^2*b)*log(x^2 + x*(-c/
d)^(1/3) + (-c/d)^(2/3))/(-c*d^2)^(2/3) - 1/3*(3*a^2*b*d^3*(-c/d)^(1/3) - b
^3*c*d^2 + a^3*d^3)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^3)
```

Mupad [B]
time = 5.14, size = 370, normalized size = 1.67

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,c^2\,d^4\,z^3-81\,a\,b^2\,c^2\,d^3\,z^2+54\,a^2\,b^4\,c^2\,d^2\,z+27\,a^5\,b\,c\,d^3\,z+3\,a^6\,b^3\,c\,d^2-3\,a^3\,b^6\,c^2\,d+b^9\,c^3-a^9\,d^3,z,k\right)\,\left(x\,\left(3\,a^3\,d^2-3\,b^3\,c\,d\right)+\mathrm{root}\left(27\,c^2\,d^4\,z^3-81\,a\,b^2\,c^2\,d^3\,z^2+54\,a^2\,b^4\,c^2\,d^2\,z+27\,a^5\,b\,c\,d^3\,z+3\,a^6\,b^3\,c\,d^2-3\,a^3\,b^6\,c^2\,d+b^9\,c^3-a^9\,d^3,z,k\right)\,c\,d^2\,9-18\,a\,b^2\,c\,d\right)+x\,\left(6\,d\,a^4\,b^2+3\,c\,a\,b^5\right)+6\,a^2\,b^4\,c+3\,a^5\,b\,d\right)\,\mathrm{root}\left(27\,c^2\,d^4\,z^3-81\,a\,b^2\,c^2\,d^3\,z^2+54\,a^2\,b^4\,c^2\,d^2\,z+27\,a^5\,b\,c\,d^3\,z+3\,a^6\,b^3\,c\,d^2-3\,a^3\,b^6\,c^2\,d+b^9\,c^3-a^9\,d^3,z,k\right)\right)+\frac{b^3\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x^3),x)
```

```
[Out] symsum(log(root(27*c^2*d^4*z^3 - 81*a*b^2*c^2*d^3*z^2 + 54*a^2*b^4*c^2*d^2*
z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^2 - 3*a^3*b^6*c^2*d + b^9*c^3 - a^9*d^
3, z, k)*(x*(3*a^3*d^2 - 3*b^3*c*d) + 9*root(27*c^2*d^4*z^3 - 81*a*b^2*c^2*
d^3*z^2 + 54*a^2*b^4*c^2*d^2*z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^2 - 3*a^3
*b^6*c^2*d + b^9*c^3 - a^9*d^3, z, k)*c*d^2 - 18*a*b^2*c*d) + x*(6*a^4*b^2*
d + 3*a*b^5*c) + 6*a^2*b^4*c + 3*a^5*b*d)*root(27*c^2*d^4*z^3 - 81*a*b^2*c^
2*d^3*z^2 + 54*a^2*b^4*c^2*d^2*z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^2 - 3*a
^3*b^6*c^2*d + b^9*c^3 - a^9*d^3, z, k), k, 1, 3) + (b^3*x)/d
```

3.72 $\int \frac{(a+bx)^4}{c+dx^3}\,dx$

Optimal. Leaf size=282

$$\frac{4ab^3x}{d}+\frac{b^4x^2}{2d}+\frac{\left(b^4c^{4/3}+4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}\,d-a^4d^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{5/3}}+\frac{\left(b\sqrt[3]{c}\,(b^3c-4a^3d)-\sqrt[3]{d}\right.}{3}$$

```
[Out] 4*a*b^3*x/d+1/2*b^4*x^2/d+1/3*(b*c^(1/3)*(-4*a^3*d+b^3*c)-d^(1/3)*(-a^4*d+4
*a*b^3*c))*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^(5/3)-1/6*(b*c^(1/3)*(-4*a^3*d+b
^3*c)-d^(1/3)*(-a^4*d+4*a*b^3*c))*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)
/c^(2/3)/d^(5/3)+2*a^2*b^2*ln(d*x^3+c)/d+1/3*(b^4*c^(4/3)+4*a*b^3*c*d^(1/3)
-4*a^3*b*c^(1/3)*d-a^4*d^(4/3))*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^
(1/2))/c^(2/3)/d^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 280, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{2a^2b^2\log\left(c+dx^3\right)}{d}+\frac{\left(a^4\left(-d^{4/3}\right)-4a^3b\sqrt[3]{c}\,d+4ab^3c\sqrt[3]{d}+b^4c^{4/3}\right)\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{5/3}}+\frac{\left(a^4(-d)-\frac{b\sqrt[3]{c}\,(b^3c-4a^3d)}{\sqrt[3]{d}}+4ab^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}+\frac{\left(b\sqrt[3]{c}\,(b^3c-4a^3d)-\sqrt[3]{d}\,(4ab^3c-a^4d)\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{5/3}}+\frac{4ab^3x}{d}+\frac{b^4x^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x^3),x]
```

```
[Out] (4*a*b^3*x)/d + (b^4*x^2)/(2*d) + ((b^4*c^(4/3) + 4*a*b^3*c*d^(1/3) - 4*a^3
*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3)
)])/(Sqrt[3]*c^(2/3)*d^(5/3)) + ((b*c^(1/3)*(b^3*c - 4*a^3*d) - d^(1/3)*(4*
a*b^3*c - a^4*d))*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(5/3)) + ((4*a*b^3
*c - a^4*d - (b*c^(1/3)*(b^3*c - 4*a^3*d))/d^(1/3))*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (2*a^2*b^2*Log[c + d*x^3])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = \int \left(\frac{4ab^3}{d} + \frac{b^4x}{d} - \frac{4ab^3c - a^4d + b(b^3c - 4a^3d)\,x - 6a^2b^2dx^2}{d\,(c+dx^3)}\right)dx$$

$$= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} - \frac{\int \frac{4ab^3c-a^4d+b(b^3c-4a^3d)x-6a^2b^2dx^2}{c+dx^3}\,dx}{d}$$

$$= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} + (6a^2b^2)\int \frac{x^2}{c+dx^3}\,dx - \frac{\int \frac{4ab^3c-a^4d+b(b^3c-4a^3d)x}{c+dx^3}\,dx}{d}$$

$$= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} + \frac{2a^2b^2\log(c+dx^3)}{d} - \frac{\int \frac{\sqrt[3]{c}\left(b\sqrt[3]{c}\,(b^3c-4a^3d)+2\sqrt[3]{d}\,(4ab^3c-a^4d)\right)+\sqrt[3]{d}\left(b\sqrt[3]{c}\,(\right.}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}}{3c^{2/3}d^{4/3}}$$

$$= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} - \frac{\left(4ab^3c - a^4d - \frac{b\sqrt[3]{c}\,(b^3c-4a^3d)}{\sqrt[3]{d}}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}} + \frac{2a^2b^2\log(c+dx^3}{d}$$

$$= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} - \frac{\left(4ab^3c - a^4d - \frac{b\sqrt[3]{c}\,(b^3c-4a^3d)}{\sqrt[3]{d}}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}} + \frac{\left(4ab^3c - a^4d - \right.}{}$$

$$= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} + \frac{\left(b^4c^{4/3}+4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}\,d-a^4d^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{5/3}} - \frac{(4a}{}$$

Mathematica [A]
time = 0.16, size = 277, normalized size = 0.98

$$\frac{24ab^3d^{2/3}x + 3b^4d^{2/3}x^2 + \frac{2\sqrt{3}\left(b^4c^{4/3}+4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}\,d-a^4d^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}} + \frac{2\left(b^4c^{4/3}-4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}\,d+a^4d^{4/3}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{2/3}} - \frac{\left(b^4c^{4/3}-4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}\,d+a^4d^{4/3}\right)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{2/3}} + 12a^2b^2d^{2/3}\log(c+dx^3)}{6d^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x^3),x]
```

```
[Out] (24*a*b^3*d^(2/3)*x + 3*b^4*d^(2/3)*x^2 + (2*Sqrt[3]*(b^4*c^(4/3) + 4*a*b^3
*c*d^(1/3) - 4*a^3*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(1 - (2*d^(1/3)*x)/c^(
1/3))/Sqrt[3]])/c^(2/3) + (2*(b^4*c^(4/3) - 4*a*b^3*c*d^(1/3) - 4*a^3*b*c^(
1/3)*d + a^4*d^(4/3))*Log[c^(1/3) + d^(1/3)*x])/c^(2/3) - ((b^4*c^(4/3) - 4
*a*b^3*c*d^(1/3) - 4*a^3*b*c^(1/3)*d + a^4*d^(4/3))*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/c^(2/3) + 12*a^2*b^2*d^(2/3)*Log[c + d*x^3])/(6*d^
(5/3))
```

Maple [A]
time = 0.34, size = 250, normalized size = 0.89

method	result
risch	$\frac{b^{4}x^{2}}{2d}+\frac{4a\,b^{3}x}{d}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}d+c\right)}\frac{\left(6a^{2}b^{2}d\,\_R^{2}+b\left(4a^{3}d-b^{3}c\right)\_R+d\,a^{4}-4a\,b^{3}c\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3d^{2}}$
default	$\frac{b^{3}\left(\frac{1}{2}b\,x^{2}+4ax\right)}{d}+\frac{\left(d\,a^{4}-4a\,b^{3}c\right)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)+\left(4d\,a^{3}b-c\,b^{4}\right)\left(-\cdots\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] b^3/d*(1/2*b*x^2+4*a*x)+((a^4*d-4*a*b^3*c)*(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1
/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))+(4*a^3*b*d-b^4*c)*(-1/3/d/
(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^
(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))+2
*a^2*b^2*ln(d*x^3+c))/d
```

Maxima [A]

time = 0.51, size = 303, normalized size = 1.07

$$-\frac{\sqrt{3}\left(\left(b^{4}\left(\frac{c}{d}\right)^{\frac{2}{3}}+4ab^{3}\left(\frac{c}{d}\right)^{\frac{1}{3}}+4a^{2}b^{2}\right)c-\left(4a^{3}b\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^{4}\left(\frac{c}{d}\right)^{\frac{1}{3}}+\frac{4a^{2}b^{2}c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3cd}+\frac{b^{4}x^{2}+8ab^{3}x}{2d}-\frac{\left(\left(b^{4}\left(\frac{c}{d}\right)^{\frac{1}{3}}-4ab^{3}\right)c-\left(12a^{2}b^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}+4a^{3}b\left(\frac{c}{d}\right)^{\frac{1}{3}}-a^{4}\right)d\right)\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\left(\left(b^{4}\left(\frac{c}{d}\right)^{\frac{1}{3}}-4ab^{3}\right)c+\left(6a^{2}b^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}-4a^{3}b\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^{4}\right)d\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*((b^4*(c/d)^(2/3) + 4*a*b^3*(c/d)^(1/3) + 4*a^2*b^2)*c - (4*a^
3*b*(c/d)^(2/3) + a^4*(c/d)^(1/3) + 4*a^2*b^2*c/d)*d)*arctan(1/3*sqrt(3)*(2
*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d) + 1/2*(b^4*x^2 + 8*a*b^3*x)/d - 1/6*((
b^4*(c/d)^(1/3) - 4*a*b^3)*c - (12*a^2*b^2*(c/d)^(2/3) + 4*a^3*b*(c/d)^(1/3
) - a^4)*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(d^2*(c/d)^(2/3)) + 1/3*
((b^4*(c/d)^(1/3) - 4*a*b^3)*c + (6*a^2*b^2*(c/d)^(2/3) - 4*a^3*b*(c/d)^(1/
3) + a^4)*d)*log(x + (c/d)^(1/3))/(d^2*(c/d)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.

time = 6.26, size = 8787, normalized size = 31.16

too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="fricas")

[Out] 1/12*(6*b^4*x^2 + 48*a*b^3*x + 2*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))*d*log(-8*a*b^11*c^4 - 66*a^4*b^8*c^3*d + 48*a^7*b^5*c^2*d^2 + 26*a^10*b^2*c*d^3 - 1/4*(b^4*c^3*d^3 - 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))^2 + 1/2*(28*a^2*b^6*c^3*d^2 - 56*a^5*b^3*c^2*d^3 + a^8*c*d^4)*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1)) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)*x) + (36*a^2*b^2 - (12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))*d + 3*sqrt(1/3)*d*sqrt(-(64*a*b^7*c^2 - 128*a^4*b^4*c*d + 64*a^7*b*d^2 - 24*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d

```
^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b
^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a
^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^
6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432
*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d
^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5)
 + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12
*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))*a^2*b^2*c*d^2 + (12*a^2*b^2/d - 2*(
1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(
c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*
d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^
3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2
*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*
b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) -
 (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b
^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)
/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))^2*c*d^3)/(c*d^3)))*log(8*a*b^11*c^4 + 66
*a^4*b^8*c^3*d - 48*a^7*b^5*c^2*d^2 - 26*a^10*b^2*c*d^3 + 1/4*(b^4*c^3*d^3
- 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7
*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6
/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b
^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12
*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c
^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^
4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^
9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 -...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.00, size = 294, normalized size = 1.04

$$\frac{2\,a^2b^2\log\left(|dx^3+c|\right)}{d}+\frac{\sqrt{3}\left(4\,ab^3cd-a^4d^2-\left(-cd^2\right)^{\frac{1}{3}}b^4c+4\left(-cd^2\right)^{\frac{1}{3}}a^3bd\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}d}+\frac{\left(4\,ab^3cd-a^4d^2+\left(-cd^2\right)^{\frac{1}{3}}b^4c-4\left(-cd^2\right)^{\frac{1}{3}}a^3bd\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}d}+\frac{b^4dx^2+8\,ab^3dx}{2\,d^2}+\frac{\left(b^4cd^4\left(-\frac{c}{d}\right)^{\frac{1}{3}}-4\,a^3bd^5\left(-\frac{c}{d}\right)^{\frac{1}{3}}+4\,ab^3cd^4-a^4d^5\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 2*a^2*b^2*log(abs(d*x^3 + c))/d + 1/3*sqrt(3)*(4*a*b^3*c*d - a^4*d^2 - (-c*
d^2)^(1/3)*b^4*c + 4*(-c*d^2)^(1/3)*a^3*b*d)*arctan(1/3*sqrt(3)*(2*x + (-c/
d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*d) + 1/6*(4*a*b^3*c*d - a^4*d^2 + (
-c*d^2)^(1/3)*b^4*c - 4*(-c*d^2)^(1/3)*a^3*b*d)*log(x^2 + x*(-c/d)^(1/3) +
(-c/d)^(2/3))/((-c*d^2)^(2/3)*d) + 1/2*(b^4*d*x^2 + 8*a*b^3*d*x)/d^2 + 1/3*
(b^4*c*d^4*(-c/d)^(1/3) - 4*a^3*b*d^5*(-c/d)^(1/3) + 4*a*b^3*c*d^4 - a^4*d^
5)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^5)
```

Mupad [B]
time = 4.97, size = 513, normalized size = 1.82

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,c^2\,d^5\,z^3-162\,a^2\,b^2\,c^2\,d^4\,z^2+171\,a^4\,b^4\,c^2\,d^3\,z+36\,a\,b^7\,c^3\,d^2\,z+36\,a^7\,b\,c\,d^4\,z-6\,a^6\,b^6\,c^2\,d^2+4\,a^9\,b^3\,c\,d^3+4\,a^3\,b^9\,c^3\,d-b^{12}\,c^4-a^{12}\,d^4,z,k\right)\,\left(\frac{x\,\left(3\,a^4\,d^3-12\,a\,b^3\,c\,d^2\right)}{d}+\mathrm{root}\left(27\,c^2\,d^5\,z^3-162\,a^2\,b^2\,c^2\,d^4\,z^2+171\,a^4\,b^4\,c^2\,d^3\,z+36\,a\,b^7\,c^3\,d^2\,z+36\,a^7\,b\,c\,d^4\,z-6\,a^6\,b^6\,c^2\,d^2+4\,a^9\,b^3\,c\,d^3+4\,a^3\,b^9\,c^3\,d-b^{12}\,c^4-a^{12}\,d^4,z,k\right)\,c\,d^2\,9-36\,a^2\,b^2\,c\,d\right)+\frac{4\,a^7\,b\,d^2+19\,a^4\,b^4\,c\,d+4\,a\,b^7\,c^2}{d}+\frac{x\,\left(10\,a^6\,b^2\,d^2+16\,a^3\,b^5\,c\,d+b^8\,c^2\right)}{d}\right)\,\mathrm{root}\left(27\,c^2\,d^5\,z^3-162\,a^2\,b^2\,c^2\,d^4\,z^2+171\,a^4\,b^4\,c^2\,d^3\,z+36\,a\,b^7\,c^3\,d^2\,z+36\,a^7\,b\,c\,d^4\,z-6\,a^6\,b^6\,c^2\,d^2+4\,a^9\,b^3\,c\,d^3+4\,a^3\,b^9\,c^3\,d-b^{12}\,c^4-a^{12}\,d^4,z,k\right)\right)+\frac{b^4\,x^2}{2\,d}+\frac{4\,a\,b^3\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x^3),x)
```

```
[Out] symsum(log(root(27*c^2*d^5*z^3 - 162*a^2*b^2*c^2*d^4*z^2 + 171*a^4*b^4*c^2*
d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7*b*c*d^4*z - 6*a^6*b^6*c^2*d^2 + 4*a^9*b
^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*c^4 - a^12*d^4, z, k)*((x*(3*a^4*d^3 - 12
*a*b^3*c*d^2))/d + 9*root(27*c^2*d^5*z^3 - 162*a^2*b^2*c^2*d^4*z^2 + 171*a^
4*b^4*c^2*d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7*b*c*d^4*z - 6*a^6*b^6*c^2*d^2
 + 4*a^9*b^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*c^4 - a^12*d^4, z, k)*c*d^2 - 3
6*a^2*b^2*c*d) + (4*a*b^7*c^2 + 4*a^7*b*d^2 + 19*a^4*b^4*c*d)/d + (x*(b^8*c
^2 + 10*a^6*b^2*d^2 + 16*a^3*b^5*c*d))/d)*root(27*c^2*d^5*z^3 - 162*a^2*b^2
*c^2*d^4*z^2 + 171*a^4*b^4*c^2*d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7*b*c*d^4*
z - 6*a^6*b^6*c^2*d^2 + 4*a^9*b^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*c^4 - a^12
*d^4, z, k), k, 1, 3) + (b^4*x^2)/(2*d) + (4*a*b^3*x)/d
```

3.73 $\int \frac{(a+bx+cx^2)^2}{d+ex^3} \, dx$

Optimal. Leaf size=272

$$\frac{2bcx}{e}+\frac{c^2x^2}{2e}+\frac{\left(c^2d^{4/3}+2bcd\sqrt[3]{e}-a\left(2b\sqrt[3]{d}+a\sqrt[3]{e}\right)e\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}e^{5/3}}-\frac{\left(\sqrt[3]{e}\,(2bcd-a^2e)-\sqrt[3]{d}\,(c^2\right.}{3d^2}$$

```
[Out] 2*b*c*x/e+1/2*c^2*x^2/e-1/3*(e^(1/3)*(-a^2*e+2*b*c*d)-d^(1/3)*(-2*a*b*e+c^2
*d))*ln(d^(1/3)+e^(1/3)*x)/d^(2/3)/e^(5/3)+1/6*(e^(1/3)*(-a^2*e+2*b*c*d)-d^
(1/3)*(-2*a*b*e+c^2*d))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e
^(5/3)+1/3*(2*a*c+b^2)*ln(e*x^3+d)/e+1/3*(c^2*d^(4/3)+2*b*c*d*e^(1/3)-a*(2*
b*d^(1/3)+a*e^(1/3))*e)*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d
^(2/3)/e^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 270, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)\left(a^2(-e)-\frac{\sqrt[3]{d}\,(c^2d-2abe)}{\sqrt[3]{e}}+2bcd\right)}{6d^{2/3}e^{4/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\left(\sqrt[3]{e}\,(2bcd-a^2e)-\sqrt[3]{d}\,(c^2d-2abe)\right)}{3d^{2/3}e^{5/3}}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)\left(-ae\left(a\sqrt[3]{e}+2b\sqrt[3]{d}\right)+2bcd\sqrt[3]{e}+c^2d^{4/3}\right)}{\sqrt{3}\,d^{2/3}e^{5/3}}+\frac{(2ac+b^2)\log\left(d+ex^3\right)}{3e}+\frac{2bcx}{e}+\frac{c^2x^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

```
[Out] (2*b*c*x)/e + (c^2*x^2)/(2*e) + ((c^2*d^(4/3) + 2*b*c*d*e^(1/3) - a*(2*b*d^
(1/3) + a*e^(1/3))*e)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(S
qrt[3]*d^(2/3)*e^(5/3)) - ((e^(1/3)*(2*b*c*d - a^2*e) - d^(1/3)*(c^2*d - 2*
a*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(5/3)) + ((2*b*c*d - a^2*e -
 (d^(1/3)*(c^2*d - 2*a*b*e))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(
2/3)*x^2])/(6*d^(2/3)*e^(4/3)) + ((b^2 + 2*a*c)*Log[d + e*x^3])/(3*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \int \left(\frac{2bc}{e}+\frac{c^2x}{e}-\frac{2bcd-a^2e+(c^2d-2abe)\,x-(b^2+2ac)\,ex^2}{e\,(d+ex^3)}\right) dx$$

$$= \frac{2bcx}{e}+\frac{c^2x^2}{2e}-\frac{\int \frac{2bcd-a^2e+(c^2d-2abe)x-(b^2+2ac)ex^2}{d+ex^3}\,dx}{e}$$

$$= \frac{2bcx}{e}+\frac{c^2x^2}{2e}-(-b^2-2ac)\int \frac{x^2}{d+ex^3}\,dx-\frac{\int \frac{2bcd-a^2e+(c^2d-2abe)x}{d+ex^3}\,dx}{e}$$

$$= \frac{2bcx}{e}+\frac{c^2x^2}{2e}+\frac{(b^2+2ac)\log(d+ex^3)}{3e}-\frac{\int \frac{\sqrt[3]{d}\left(2\sqrt[3]{e}\,(2bcd-a^2e)+\sqrt[3]{d}\,(c^2d-2abe)\right)+\sqrt[3]{e}}{d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x}}{3d^{2/3}e^{4/}}$$

$$= \frac{2bcx}{e}+\frac{c^2x^2}{2e}-\frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}\,(c^2d-2abe)}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{4/3}}+\frac{(b^2+2ac)\log(}{3e}$$

$$= \frac{2bcx}{e}+\frac{c^2x^2}{2e}-\frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}\,(c^2d-2abe)}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{4/3}}+\frac{\left(2bcd-a^2e-\right.}{}$$

$$= \frac{2bcx}{e}+\frac{c^2x^2}{2e}+\frac{\left(c^2d^{4/3}+2bcd\sqrt[3]{e}-a\left(2b\sqrt[3]{d}+a\sqrt[3]{e}\right)e\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}e^{5/3}}-$$

<u>Mathematica</u> [A]
time = 0.20, size = 269, normalized size = 0.99

$$\frac{12bce^{2/3}x+3c^2e^{2/3}x^2+\frac{2\sqrt{3}\,(cd^{2/3}-ae^{2/3})\left(cd^{2/3}+2b\sqrt[3]{d}\,\sqrt[3]{e}+ae^{2/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}\,x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{2/3}}+\frac{2\left(c^2d^{4/3}-2bcd\sqrt[3]{e}+a\left(-2b\sqrt[3]{d}+a\sqrt[3]{e}\right)e\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{d^{2/3}}-\frac{\left(c^2d^{4/3}-2bcd\sqrt[3]{e}+a\left(-2b\sqrt[3]{d}+a\sqrt[3]{e}\right)e\right)\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{d^{2/3}}+2(b^2+2ac)e^{2/3}\log(d+ex^3)}{6e^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

```
[Out] (12*b*c*e^(2/3)*x + 3*c^2*e^(2/3)*x^2 + (2*Sqrt[3]*(c*d^(2/3) - a*e^(2/3))*
(c*d^(2/3) + 2*b*d^(1/3)*e^(1/3) + a*e^(2/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(
1/3))/Sqrt[3]])/d^(2/3) + (2*(c^2*d^(4/3) - 2*b*c*d*e^(1/3) + a*(-2*b*d^(1/
3) + a*e^(1/3))*e)*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - ((c^2*d^(4/3) - 2*b*
c*d*e^(1/3) + a*(-2*b*d^(1/3) + a*e^(1/3))*e)*Log[d^(2/3) - d^(1/3)*e^(1/3)
*x + e^(2/3)*x^2])/d^(2/3) + 2*(b^2 + 2*a*c)*e^(2/3)*Log[d + e*x^3])/(6*e^(
5/3))
```

<u>Maple</u> [A]
time = 0.47, size = 252, normalized size = 0.93

method	result
risch	$\frac{c^2x^2}{2e}+\frac{2bcx}{e}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^3e+d)}\frac{\left(e\left(2ac+b^2\right)\_R^2+\left(2abe-c^2d\right)\_R+a^2e-2bcd\right)\ln\left(x-\_R\right)}{\_R^2}}{3e^2}$
default	$\frac{c\left(\frac{1}{2}cx^2+2bx\right)}{e}+\frac{\left(a^2e-2bcd\right)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)+\left(2abe-c^2d\right)\left(-\frac{\ln\left(\right.}{}\right.}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x^3+d),x,method=_RETURNVERBOSE)
```

```
[Out] c/e*(1/2*c*x^2+2*b*x)+((a^2*e-2*b*c*d)*(1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))
-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))+1/3/e/(d/e)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1)))+(2*a*b*e-c^2*d)*(-1/3/e/(d/e)^
(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))
+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1)))+1/3*(2*
a*c*e+b^2*e)/e*ln(e*x^3+d))/e
```

Maxima [A]
time = 0.57, size = 259, normalized size = 0.95

$$\frac{\sqrt{3}\left(6\,abd^{\frac{2}{3}}e^{\frac{1}{3}}+3\,a^2d^{\frac{1}{3}}e^{\frac{2}{3}}+2\,b^2d+4\,acd-\left(3\,c^2d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}+2\,b^2+2\left(3\,bd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+2\,a\right)c\right)d\right)\arctan\left(-\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}-2\,x\right)e^{\frac{1}{3}}}{3\,d^{\frac{1}{3}}}\right)e^{(-1)}}{9\,d}+\frac{\left(2\,b^2d^{\frac{2}{3}}e^{\frac{1}{3}}+4\,acd^{\frac{2}{3}}e^{\frac{1}{3}}+2\,abd^{\frac{1}{3}}e^{\frac{2}{3}}-a^2e-\left(c^2d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}-2\,bc\right)d\right)e^{\left(-\frac{4}{3}\right)}\log\left(-d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)}+x^2+d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6\,d^{\frac{2}{3}}}+\frac{\left(b^2d^{\frac{2}{3}}e^{\frac{1}{3}}+2\,acd^{\frac{2}{3}}e^{\frac{1}{3}}-2\,abd^{\frac{1}{3}}e^{\frac{2}{3}}+a^2e+\left(c^2d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}-2\,bc\right)d\right)e^{\left(-\frac{4}{3}\right)}\log\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+x\right)}{3\,d^{\frac{2}{3}}}+\frac{1}{2}\left(c^2x^2+4\,bcx\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*(6*a*b*d^(2/3)*e^(1/3) + 3*a^2*d^(1/3)*e^(2/3) + 2*b^2*d + 4*a*
c*d - (3*c^2*d^(2/3)*e^(-2/3) + 2*b^2 + 2*(3*b*d^(1/3)*e^(-1/3) + 2*a)*c)*d
)*arctan(-1/3*sqrt(3)*(d^(1/3)*e^(-1/3) - 2*x)*e^(1/3)/d^(1/3))*e^(-1)/d +
1/6*(2*b^2*d^(2/3)*e^(1/3) + 4*a*c*d^(2/3)*e^(1/3) + 2*a*b*d^(1/3)*e^(2/3)
- a^2*e - (c^2*d^(1/3)*e^(-1/3) - 2*b*c)*d)*e^(-4/3)*log(-d^(1/3)*x*e^(-1/3
) + x^2 + d^(2/3)*e^(-2/3))/d^(2/3) + 1/3*(b^2*d^(2/3)*e^(1/3) + 2*a*c*d^(2
/3)*e^(1/3) - 2*a*b*d^(1/3)*e^(2/3) + a^2*e + (c^2*d^(1/3)*e^(-1/3) - 2*b*c
)*d)*e^(-4/3)*log(d^(1/3)*e^(-1/3) + x)/d^(2/3) + 1/2*(c^2*x^2 + 4*b*c*x)*e
^(-1)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.80, size = 12827, normalized size = 47.16

too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="fricas")

[Out] 1/12*(6*c^2*x^2 + 24*b*c*x - 2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)*e*log(-4*b*c^5*d^4 - (5*b^4*c^2 - 4*a*b^2*c^3 + 2*a^2*c^4)*d^3*e + 2*(a*b^5 - 2*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^2 + (7*a^4*b^2 - 2*a^5*c)*d*e^3 - 1/4*(c^2*d^3*e^3 - 2*a*b*d^2*e^4)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)^2 - 1/2*(a^4*d*e^4 + 2*(3*b^2*c^2 + 2*a*c^3)*d^3*e^2 - 4*(a*b^3 + 3*a^2*b*c)*d^2*e^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d

```
*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3
*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)*x) + (6*b^2 + 12
*a*c + (2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2
+ b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 -
3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^
4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a
^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e -
 b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^
4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*
(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c
^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e +
b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^
2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^
6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*
c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)*e + 3*sqrt(1/3)*e*sqrt(-(32
*b*c^3*d^2 + 32*a^3*b*e^2 + (2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^
2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(
b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2
)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*
b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a
^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*
(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^
2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4
 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3
+ a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x**3+d),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.85, size = 264, normalized size = 0.97

$$\frac{1}{3}\left(b^2+2ac\right)e^{(-1)}\log\left(\left|x^3e+d\right|\right)+\frac{\sqrt{3}\left(2bcde-\left(-de^2\right)^{\frac{1}{3}}c^2d+2\left(-de^2\right)^{\frac{1}{3}}abe-a^2e^2\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-1)}}{3\left(-de^2\right)^{\frac{2}{3}}}+\frac{\left(2bcde+\left(-de^2\right)^{\frac{1}{3}}c^2d-2\left(-de^2\right)^{\frac{1}{3}}abe-a^2e^2\right)e^{(-1)}\log\left(x^2+\left(-de^{(-1)}\right)^{\frac{1}{3}}x+\left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6\left(-de^2\right)^{\frac{2}{3}}}+\frac{\left(\left(-de^{(-1)}\right)^{\frac{1}{3}}c^2de^4+2bcde^4-2\left(-de^{(-1)}\right)^{\frac{1}{3}}abe^5-a^2e^5\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-5)}\log\left(\left|x-\left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3d}+\frac{1}{2}\left(c^2x^2e+4bcxe\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="giac")
```

```
[Out] 1/3*(b^2 + 2*a*c)*e^(-1)*log(abs(x^3*e + d)) + 1/3*sqrt(3)*(2*b*c*d*e - (-d
*e^2)^(1/3)*c^2*d + 2*(-d*e^2)^(1/3)*a*b*e - a^2*e^2)*arctan(1/3*sqrt(3)*(2
*x + (-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/(-d*e^2)^(2/3) + 1/6*(2*b
*c*d*e + (-d*e^2)^(1/3)*c^2*d - 2*(-d*e^2)^(1/3)*a*b*e - a^2*e^2)*e^(-1)*lo
g(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/(-d*e^2)^(2/3) + 1/3*((-d*
e^(-1))^(1/3)*c^2*d*e^4 + 2*b*c*d*e^4 - 2*(-d*e^(-1))^(1/3)*a*b*e^5 - a^2*e
^5)*(-d*e^(-1))^(1/3)*e^(-5)*log(abs(x - (-d*e^(-1))^(1/3)))/d + 1/2*(c^2*x
^2*e + 4*b*c*x*e)*e^(-2)
```

Mupad [B]
time = 5.13, size = 769, normalized size = 2.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^2/(d + e*x^3),x)
```

```
[Out] symsum(log((2*a^3*b*e^2 + 2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e)/e + (x*(c^
4*d^2 - 2*a^3*c*e^2 + 3*a^2*b^2*e^2 + 2*b^3*c*d*e))/e - 3*root(27*d^2*e^5*z
^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*d^2*e^4*z^2 + 27*a^2*c^2*d^2*e^3*z + 18*b*
c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z + 9*b^4*d^2*e^3*z + 6*a*b^4*c*d^2*e^2 - 9*
a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c*d*e^3 - 6*a*b*c^4*d^3*e - 2*a^3*c^3*d^2*e^2
 + 2*b^3*c^3*d^3*e + 2*a^3*b^3*d*e^3 - b^6*d^2*e^2 - c^6*d^4 - a^6*e^4, z,
k)*e*(2*b^2*d - 3*root(27*d^2*e^5*z^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*d^2*e^4
*z^2 + 27*a^2*c^2*d^2*e^3*z + 18*b*c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z + 9*b^4
*d^2*e^3*z + 6*a*b^4*c*d^2*e^2 - 9*a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c*d*e^3 -
6*a*b*c^4*d^3*e - 2*a^3*c^3*d^2*e^2 + 2*b^3*c^3*d^3*e + 2*a^3*b^3*d*e^3 - b
^6*d^2*e^2 - c^6*d^4 - a^6*e^4, z, k)*d*e + 4*a*c*d - a^2*e*x + 2*b*c*d*x))
*root(27*d^2*e^5*z^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*d^2*e^4*z^2 + 27*a^2*c^2
*d^2*e^3*z + 18*b*c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z + 9*b^4*d^2*e^3*z + 6*a*
b^4*c*d^2*e^2 - 9*a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c*d*e^3 - 6*a*b*c^4*d^3*e -
 2*a^3*c^3*d^2*e^2 + 2*b^3*c^3*d^3*e + 2*a^3*b^3*d*e^3 - b^6*d^2*e^2 - c^6*
d^4 - a^6*e^4, z, k), k, 1, 3) + (c^2*x^2)/(2*e) + (2*b*c*x)/e
```

3.74 $\int \frac{(a+bx+cx^2)^3}{d+ex^3} \, dx$

Optimal. Leaf size=416

$$-\frac{(c^3d - b^3e - 6abce)\,x}{e^2} + \frac{3c(b^2+ac)\,x^2}{2e} + \frac{bc^2x^3}{e} + \frac{c^3x^4}{4e} - \frac{\left(c^3d^2 - 3b^2cd^{4/3}e^{2/3} - 3ac^2d^{4/3}e^{2/3} - b^3de - 6abcd\right.}{\sqrt{3}\,d^{2/3}}$$

```
[Out] -(-6*a*b*c*e-b^3*e+c^3*d)*x/e^2+3/2*c*(a*c+b^2)*x^2/e+b*c^2*x^3/e+1/4*c^3*x
^4/e+1/3*(c^3*d^2-6*a*b*c*d*e-e*(-a^3*e+b^3*d)+3*d^(1/3)*e^(2/3)*(-a^2*b*e+
a*c^2*d+b^2*c*d))*ln(d^(1/3)+e^(1/3)*x)/d^(2/3)/e^(7/3)-1/6*(c^3*d^2-6*a*b*
c*d*e-e*(-a^3*e+b^3*d)+3*d^(1/3)*e^(2/3)*(-a^2*b*e+a*c^2*d+b^2*c*d))*ln(d^(
2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(7/3)-(-a^2*c*e-a*b^2*e+b*c^2
*d)*ln(e*x^3+d)/e^2-1/3*(c^3*d^2-3*b^2*c*d^(4/3)*e^(2/3)-3*a*c^2*d^(4/3)*e^
(2/3)-b^3*d*e-6*a*b*c*d*e+3*a^2*b*d^(1/3)*e^(5/3)+a^3*e^2)*arctan(1/3*(d^(1
/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(2/3)/e^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\log(d+ex^3)\,(a^2(-c)e - ab^2e + bc^2d)}{e^2} - \frac{\text{ArcTan}\left(\frac{\sqrt[3]{d} - 2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)\left(a^3e^2 + 3a^2b\sqrt[3]{d}\,e^{5/3} - 6abcde - 3ac^2d^{4/3}e^{2/3} - b^3de - 3b^2cd^{4/3}e^{2/3} + c^3d^2\right)}{\sqrt{3}\,d^{2/3}e^{7/3}} - \frac{\log\left(d^{2/3} - \sqrt[3]{d}\,\sqrt[3]{e}\,x + e^{2/3}x^2\right)\left(-e(b^3d - a^3e) + 3\sqrt[3]{d}\,e^{2/3}(a^2(-b)e + ac^2d + b^2cd) - 6abcde + c^3d^2\right)}{6d^{2/3}e^{7/3}} + \frac{\log\left(\sqrt[3]{d} + \sqrt[3]{e}\,x\right)\left(-e(b^3d - a^3e) + 3\sqrt[3]{d}\,e^{2/3}(a^2(-b)e + ac^2d + b^2cd) - 6abcde + c^3d^2\right)}{3d^{2/3}e^{7/3}} - \frac{x(-6abce + b^3(-e) + c^3d)}{e^2} + \frac{3cx^2(ac+b^2)}{2e} + \frac{bc^2x^3}{e} + \frac{c^3x^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

```
[Out] -(((c^3*d - b^3*e - 6*a*b*c*e)*x)/e^2) + (3*c*(b^2 + a*c)*x^2)/(2*e) + (b*c
^2*x^3)/e + (c^3*x^4)/(4*e) - ((c^3*d^2 - 3*b^2*c*d^(4/3)*e^(2/3) - 3*a*c^2
*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e + 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^
2)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(7
/3)) + ((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2
*c*d + a*c^2*d - a^2*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/3)) -
((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2*c*d +
a*c^2*d - a^2*b*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2
/3)*e^(7/3)) - ((b*c^2*d - a*b^2*e - a^2*c*e)*Log[d + e*x^3])/e^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = \int \left(-\frac{c^3d-b^3e-6abce}{e^2}+\frac{3c(b^2+ac)\,x}{e}+\frac{3bc^2x^2}{e}+\frac{c^3x^3}{e}+\frac{c^3d^2-6abcde-e(b^3$$

$$= -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\int \frac{c^3d^2-6abcde-e(b^3d-a^3e)}{}}{}$$

$$= -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\int \frac{c^3d^2-6abcde-e(b^3d-a^3e)}{d+ex}}{e^2}$$

$$= -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}-\frac{(bc^2d-ab^2e-a^2ce)\,\mathrm{lo}}{e^2}$$

$$= -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\left(c^3d^2-6abcde-e(b^3\right.}{}$$

$$= -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\left(c^3d^2-6abcde-e(b^3\right.}{}$$

$$= -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}-\frac{\left(c^3d^2-3b^2cd^{4/3}e^{2/3}-\right.}{}$$

Mathematica [A]
time = 0.29, size = 439, normalized size = 1.06

$$\frac{12\sqrt[3]{e}\,(-c^3d+b^3e+6abce)\,x+18c(b^2+ac)\,e^{4/3}x^2+12bc^2e^{4/3}x^3+3c^3e^{4/3}x^4-\frac{4\sqrt{3}\left(c^3d^2-3ac^2d^{4/3}e^{2/3}+e\left(-b^3d+3a^2b\sqrt[3]{d}\,e^{2/3}+a^3e\right)-3c\left(b^2d^{4/3}e^{2/3}+2abde\right)\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}\,x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{2/3}}+\frac{4\left(c^3d^2+3b^2cd^{4/3}e^{2/3}+3ac^2d^{4/3}e^{2/3}-b^3de-6abcde-3a^2b\sqrt[3]{d}\,e^{5/3}+a^3e^2\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{d^{2/3}}-\frac{2\left(c^3d^2+3b^2cd^{4/3}e^{2/3}+3ac^2d^{4/3}e^{2/3}-b^3de-6abcde-3a^2b\sqrt[3]{d}\,e^{5/3}+a^3e^2\right)\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{d^{2/3}}+12\sqrt[3]{e}\,(-bc^2d+ab^2e+a^2ce)\log(d+ex^3)}{12e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

```
[Out] (12*e^(1/3)*(-(c^3*d) + b^3*e + 6*a*b*c*e)*x + 18*c*(b^2 + a*c)*e^(4/3)*x^2
 + 12*b*c^2*e^(4/3)*x^3 + 3*c^3*e^(4/3)*x^4 - (4*Sqrt[3]*(c^3*d^2 - 3*a*c^2
*d^(4/3)*e^(2/3) + e*(-(b^3*d) + 3*a^2*b*d^(1/3)*e^(2/3) + a^3*e) - 3*c*(b^
2*d^(4/3)*e^(2/3) + 2*a*b*d*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]]
)/d^(2/3) + (4*(c^3*d^2 + 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c^2*d^(4/3)*e^(2/3)
 - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*Log[d^(1/3) +
 e^(1/3)*x])/d^(2/3) - (2*(c^3*d^2 + 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c^2*d^(4
/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*Lo
g[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3) + 12*e^(1/3)*(-(b*c^2
*d) + a*b^2*e + a^2*c*e)*Log[d + e*x^3])/(12*e^(7/3))
```

Maple [A]

time = 0.45, size = 347, normalized size = 0.83

method	result
risch	$\frac{c^3x^4}{4e}+\frac{b\,c^2x^3}{e}+\frac{3a\,c^2x^2}{2e}+\frac{3b^2c\,x^2}{2e}+\frac{6abcx}{e}+\frac{b^3x}{e}-\frac{c^3dx}{e^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3e+d\right)}\frac{\left(3e\left(a^2ce+a\,b^2e-b\,c^2d\right)\_R^2+3e\left(a^2be-\right.\right.}{}}{3e^3}$
default	$\frac{\frac{1}{4}c^3x^4e+b\,c^2x^3e+\frac{3}{2}a\,c^2e\,x^2+\frac{3}{2}b^2ce\,x^2+6abcex+b^3ex-c^3dx}{e^2}+\frac{\left(a^3e^2-6abcde-b^3de+c^3d^2\right)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x^3+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/4*c^3*x^4*e+b*c^2*x^3*e+3/2*a*c^2*e*x^2+3/2*b^2*c*e*x^2+6*a*b*c*e*
x+b^3*e*x-c^3*d*x)+((a^3*e^2-6*a*b*c*d*e-b^3*d*e+c^3*d^2)*(1/3/e/(d/e)^(2/3
)*ln(x+(d/e)^(1/3))-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))+1/3
/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1)))+(3*a^2*b*e^
2-3*a*c^2*d*e-3*b^2*c*d*e)*(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e
)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(
1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1)))+1/3*(3*a^2*c*e^2+3*a*b^2*e^2-3*b*c^2*d*e)
/e*ln(e*x^3+d))/e^2
```

Maxima [A]

time = 0.59, size = 462, normalized size = 1.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(2*a*b^2*d*e + 2*a^2*c*d*e + 3*a^2*b*d^(2/3)*e^(4/3) + a^3*d^(1
/3)*e^(5/3) + (c^3*d^(1/3)*e^(-1/3) + 2*b*c^2)*d^2 - (b^3*d^(1/3)*e^(2/3) +
 2*a*b^2*e + (3*a*d^(2/3)*e^(1/3) + 2*b*d)*c^2 + (3*b^2*d^(2/3)*e^(1/3) + 6
*a*b*d^(1/3)*e^(2/3) + 2*a^2*e)*c)*d)*arctan(-1/3*sqrt(3)*(d^(1/3)*e^(-1/3)
 - 2*x)*e^(1/3)/d^(1/3))*e^(-2)/d - 1/6*(c^3*d^2 - 6*a*b^2*d^(2/3)*e^(4/3)
- 6*a^2*c*d^(2/3)*e^(4/3) - 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2 - (b^3*e - 3*
(2*b*d^(2/3)*e^(1/3) + a*d^(1/3)*e^(2/3))*c^2 - 3*(b^2*d^(1/3)*e^(2/3) - 2*
```

```
a*b*e)*c)*d)*e^(-7/3)*log(-d^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2/3))/d^(
2/3) + 1/3*(c^3*d^2 + 3*a*b^2*d^(2/3)*e^(4/3) + 3*a^2*c*d^(2/3)*e^(4/3) - 3
*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2 - (b^3*e + 3*(b*d^(2/3)*e^(1/3) - a*d^(1/3
)*e^(2/3))*c^2 - 3*(b^2*d^(1/3)*e^(2/3) - 2*a*b*e)*c)*d)*e^(-7/3)*log(d^(1/
3)*e^(-1/3) + x)/d^(2/3) + 1/4*(c^3*x^4*e + 4*b*c^2*x^3*e + 6*(b^2*c*e + a*
c^2*e)*x^2 - 4*(c^3*d - b^3*e - 6*a*b*c*e)*x)*e^(-2)
```

Fricas [**C**] Result contains complex when optimal does not.
time = 11.26, size = 29479, normalized size = 70.86

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] 1/12*(3*c^3*e*x^4 + 12*b*c^2*e*x^3 + 18*(b^2*c + a*c^2)*e*x^2 - 2*((-I*sqrt
(3) + 1)*(3*(b*c^2*d - a*b^2*e - a^2*c*e)^2/e^4 - (2*b^2*c^4*d^3 + b^5*c*d^
2*e - a^3*b^2*c*d*e^2 + 2*a^4*c^2*d*e^2 + a^5*b*e^3 + (b*c^3*d^2*e + 2*b^4*
d*e^2)*a^2 - (c^5*d^3 - b^3*c^2*d^2*e)*a)/(d*e^4))/(-(b*c^2*d - a*b^2*e - a
^2*c*e)^3/e^6 + 1/2*(2*b^2*c^4*d^3 + b^5*c*d^2*e - a^3*b^2*c*d*e^2 + 2*a^4*
c^2*d*e^2 + a^5*b*e^3 + (b*c^3*d^2*e + 2*b^4*d*e^2)*a^2 - (c^5*d^3 - b^3*c^
2*d^2*e)*a)*(b*c^2*d - a*b^2*e - a^2*c*e)/(d*e^6) + 1/54*(c^9*d^6 - 3*b^3*c
^6*d^5*e + 3*b^6*c^3*d^4*e^2 - b^9*d^3*e^3 + 27*a^5*b^2*c^2*d^2*e^4 + 9*a^7
*b*c*d*e^5 + a^9*e^6 + 3*(c^3*d^2*e^4 - b^3*d*e^5)*a^6 + 18*(b*c^4*d^3*e^3
- b^4*c*d^2*e^4)*a^4 + 3*(c^6*d^4*e^2 + 7*b^3*c^3*d^3*e^3 + b^6*d^2*e^4)*a^
3 + 27*(b^2*c^5*d^4*e^2 - b^5*c^2*d^3*e^3)*a^2 + 9*(b*c^7*d^5*e - 2*b^4*c^4
*d^4*e^2 + b^7*c*d^3*e^3)*a)/(d^2*e^7) + 1/54*(c^9*d^6 + a^9*e^6 - 3*(b^3*c
^6 + 6*a*b*c^7)*d^5*e - 3*(8*b^6*c^3 + 15*a*b^4*c^4 - 9*a^2*b^2*c^5 + 8*a^3
*c^6)*d^4*e^2 - (b^9 + 18 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x**3+d),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.29, size = 432, normalized size = 1.04

$$-\left(bc^2d-ab^2e-a^2ce\right)e^{(-2)}\log\left(\left|x^3e+d\right|\right)-\frac{\sqrt{3}\left(c^3d^2-b^3de-6\,abcde+3\left(-de^2\right)^{\frac{1}{3}}b^2cd+3\left(-de^2\right)^{\frac{1}{3}}ac^2d-3\left(-de^2\right)^{\frac{1}{3}}a^2be+a^3e^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-1)}}{3\left(-de^2\right)^{\frac{2}{3}}}-\frac{\left(c^3d^2-b^3de-6\,abcde-3\left(-de^2\right)^{\frac{1}{3}}b^2cd-3\left(-de^2\right)^{\frac{1}{3}}ac^2d+3\left(-de^2\right)^{\frac{1}{3}}a^2be+a^3e^2\right)e^{(-1)}\log\left(x^2+\left(-de^{(-1)}\right)^{\frac{1}{3}}x+\left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6\left(-de^2\right)^{\frac{2}{3}}}-\frac{\left(c^3d^2e^7-3\left(-de^{(-1)}\right)^{\frac{1}{3}}b^2cde^8-3\left(-de^{(-1)}\right)^{\frac{1}{3}}ac^2de^8-b^3de^8-6\,abcde^8+3\left(-de^{(-1)}\right)^{\frac{1}{3}}a^2be^9+a^3e^9\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-9)}\log\left(\left|x-\left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3\,d}+\frac{1}{4}\left(c^3x^4e^3+4\,bc^2x^3e^3+6\,b^2cx^2e^3+6\,ac^2x^2e^3-4\,c^3dxe^2+4\,b^3xe^3+24\,abcxe^3\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="giac")
```

```
[Out] -(b*c^2*d - a*b^2*e - a^2*c*e)*e^(-2)*log(abs(x^3*e + d)) - 1/3*sqrt(3)*(c^
3*d^2 - b^3*d*e - 6*a*b*c*d*e + 3*(-d*e^2)^(1/3)*b^2*c*d + 3*(-d*e^2)^(1/3)
*a*c^2*d - 3*(-d*e^2)^(1/3)*a^2*b*e + a^3*e^2)*arctan(1/3*sqrt(3)*(2*x + (-
d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/(-d*e^2)^(2/3) - 1/6*(c^3*d^2 -
b^3*d*e - 6*a*b*c*d*e - 3*(-d*e^2)^(1/3)*b^2*c*d - 3*(-d*e^2)^(1/3)*a*c^2*d
 + 3*(-d*e^2)^(1/3)*a^2*b*e + a^3*e^2)*e^(-1)*log(x^2 + (-d*e^(-1))^(1/3)*x
 + (-d*e^(-1))^(2/3))/(-d*e^2)^(2/3) - 1/3*(c^3*d^2*e^7 - 3*(-d*e^(-1))^(1/
3)*b^2*c*d*e^8 - 3*(-d*e^(-1))^(1/3)*a*c^2*d*e^8 - b^3*d*e^8 - 6*a*b*c*d*e^
8 + 3*(-d*e^(-1))^(1/3)*a^2*b*e^9 + a^3*e^9)*(-d*e^(-1))^(1/3)*e^(-9)*log(a
bs(x - (-d*e^(-1))^(1/3)))/d + 1/4*(c^3*x^4*e^3 + 4*b*c^2*x^3*e^3 + 6*b^2*c
*x^2*e^3 + 6*a*c^2*x^2*e^3 - 4*c^3*d*x*e^2 + 4*b^3*x*e^3 + 24*a*b*c*x*e^3)*
e^(-4)
```

Mupad [B]
time = 4.91, size = 1700, normalized size = 4.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^3/(d + e*x^3),x)
```

```
[Out] x*((b^3 + 6*a*b*c)/e - (c^3*d)/e^2) + symsum(log(root(27*d^2*e^7*z^3 + 81*b
*c^2*d^3*e^5*z^2 - 81*a^2*c*d^2*e^6*z^2 - 81*a*b^2*d^2*e^6*z^2 - 27*a^3*b^2
*c*d^2*e^5*z + 27*a^2*b*c^3*d^3*e^4*z + 27*a*b^3*c^2*d^3*e^4*z + 54*b^2*c^4
*d^4*e^3*z + 54*a^4*c^2*d^2*e^5*z + 54*a^2*b^4*d^2*e^5*z + 27*b^5*c*d^3*e^4
*z - 27*a*c^5*d^4*e^3*z + 27*a^5*b*d*e^6*z + 18*a^4*b^4*c*d^2*e^4 - 18*a^4*
b*c^4*d^3*e^3 + 18*a*b^4*c^4*d^4*e^2 - 9*a*b^7*c*d^3*e^3 - 27*a^5*b^2*c^2*d
^2*e^4 + 27*a^2*b^5*c^2*d^3*e^3 - 27*a^2*b^2*c^5*d^4*e^2 - 21*a^3*b^3*c^3*d
^3*e^3 - 9*a^7*b*c*d*e^5 - 9*a*b*c^7*d^5*e - 3*b^6*c^3*d^4*e^2 - 3*a^6*c^3*
d^2*e^4 - 3*a^3*c^6*d^4*e^2 - 3*a^3*b^6*d^2*e^4 + 3*b^3*c^6*d^5*e + 3*a^6*b
^3*d*e^5 + b^9*d^3*e^3 - c^9*d^6 - a^9*e^6, z, k)*((3*x*(a^3*e^4 - b^3*d*e^
3 + c^3*d^2*e^2 - 6*a*b*c*d*e^3))/e^2 - (3*(6*a*b^2*d*e^3 - 6*b*c^2*d^2*e^2
 + 6*a^2*c*d*e^3))/e^2 + 9*root(27*d^2*e^7*z^3 + 81*b*c^2*d^3*e^5*z^2 - 81*
a^2*c*d^2*e^6*z^2 - 81*a*b^2*d^2*e^6*z^2 - 27*a^3*b^2*c*d^2*e^5*z + 27*a^2*
b*c^3*d^3*e^4*z + 27*a*b^3*c^2*d^3*e^4*z + 54*b^2*c^4*d^4*e^3*z + 54*a^4*c^
2*d^2*e^5*z + 54*a^2*b^4*d^2*e^5*z + 27*b^5*c*d^3*e^4*z - 27*a*c^5*d^4*e^3*
z + 27*a^5*b*d*e^6*z + 18*a^4*b^4*c*d^2*e^4 - 18*a^4*b*c^4*d^3*e^3 + 18*a*b
^4*c^4*d^4*e^2 - 9*a*b^7*c*d^3*e^3 - 27*a^5*b^2*c^2*d^2*e^4 + 27*a^2*b^5*c^
2*d^3*e^3 - 27*a^2*b^2*c^5*d^4*e^2 - 21*a^3*b^3*c^3*d^3*e^3 - 9*a^7*b*c*d*e
^5 - 9*a*b*c^7*d^5*e - 3*b^6*c^3*d^4*e^2 - 3*a^6*c^3*d^2*e^4 - 3*a^3*c^6*d^
4*e^2 - 3*a^3*b^6*d^2*e^4 + 3*b^3*c^6*d^5*e + 3*a^6*b^3*d*e^5 + b^9*d^3*e^3
 - c^9*d^6 - a^9*e^6, z, k)*d*e^2) + (3*(a^5*b*e^3 - a*c^5*d^3 + 2*b^2*c^4*
d^3 + 2*a^2*b^4*d*e^2 + 2*a^4*c^2*d*e^2 + b^5*c*d^2*e + a*b^3*c^2*d^2*e + a
^2*b*c^3*d^2*e - a^3*b^2*c*d*e^2))/e^2 + (3*x*(b*c^5*d^3 - a^5*c*e^3 + 2*a^
```

```
4*b^2*e^3 + 2*a^2*c^4*d^2*e + 2*b^4*c^2*d^2*e + a*b^5*d*e^2 - a*b^2*c^3*d^2
*e + a^2*b^3*c*d*e^2 + a^3*b*c^2*d*e^2))/e^2)*root(27*d^2*e^7*z^3 + 81*b*c^
2*d^3*e^5*z^2 - 81*a^2*c*d^2*e^6*z^2 - 81*a*b^2*d^2*e^6*z^2 - 27*a^3*b^2*c*
d^2*e^5*z + 27*a^2*b*c^3*d^3*e^4*z + 27*a*b^3*c^2*d^3*e^4*z + 54*b^2*c^4*d^
4*e^3*z + 54*a^4*c^2*d^2*e^5*z + 54*a^2*b^4*d^2*e^5*z + 27*b^5*c*d^3*e^4*z
- 27*a*c^5*d^4*e^3*z + 27*a^5*b*d*e^6*z + 18*a^4*b^4*c*d^2*e^4 - 18*a^4*b*c
^4*d^3*e^3 + 18*a*b^4*c^4*d^4*e^2 - 9*a*b^7*c*d^3*e^3 - 27*a^5*b^2*c^2*d^2*
e^4 + 27*a^2*b^5*c^2*d^3*e^3 - 27*a^2*b^2*c^5*d^4*e^2 - 21*a^3*b^3*c^3*d^3*
e^3 - 9*a^7*b*c*d*e^5 - 9*a*b*c^7*d^5*e - 3*b^6*c^3*d^4*e^2 - 3*a^6*c^3*d^2
*e^4 - 3*a^3*c^6*d^4*e^2 - 3*a^3*b^6*d^2*e^4 + 3*b^3*c^6*d^5*e + 3*a^6*b^3*
d*e^5 + b^9*d^3*e^3 - c^9*d^6 - a^9*e^6, z, k), k, 1, 3) + (c^3*x^4)/(4*e)
+ (b*c^2*x^3)/e + (3*c*x^2*(a*c + b^2))/(2*e)
```

3.75 $\int \frac{(a+bx+cx^2)^4}{d+ex^3} dx$

Optimal. Leaf size=645

$$-\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)\,x}{e^2}-\frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x^2}{2e^2}-\frac{c(c^3d-4b^3e-12abce)\,x^3}{3e^2}+$$

```
[Out] -2*(-6*a^2*b*c*e-2*a*b^3*e+2*a*c^3*d+3*b^2*c^2*d)*x/e^2-1/2*(-6*a^2*c^2*e-1
2*a*b^2*c*e-b^4*e+4*b*c^3*d)*x^2/e^2-1/3*c*(-12*a*b*c*e-4*b^3*e+c^3*d)*x^3/
e^2+1/2*c^2*(2*a*c+3*b^2)*x^4/e+4/5*b*c^3*x^5/e+1/6*c^4*x^6/e+1/3*(e^(1/3)*
(a^4*e^2-12*a^2*b*c*d*e-4*a*b^3*d*e+4*a*c^3*d^2+6*b^2*c^2*d^2)+d^(1/3)*(b^4
*d*e+12*a*b^2*c*d*e+6*a^2*c^2*d*e-4*b*(a^3*e^2+c^3*d^2)))*ln(d^(1/3)+e^(1/3
)*x)/d^(2/3)/e^(8/3)-1/6*(e^(1/3)*(a^4*e^2-12*a^2*b*c*d*e-4*a*b^3*d*e+4*a*c
^3*d^2+6*b^2*c^2*d^2)+d^(1/3)*(b^4*d*e+12*a*b^2*c*d*e+6*a^2*c^2*d*e-4*b*(a^
3*e^2+c^3*d^2)))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(8/3)+
1/3*(c^4*d^2-12*a*b*c^2*d*e+6*a^2*b^2*e^2-4*c*e*(-a^3*e+b^3*d))*ln(e*x^3+d)
/e^3-1/3*(b*d^(1/3)+a*e^(1/3))*(4*c^3*d^2+6*c^2*(b*d^(5/3)*e^(1/3)-a*d^(4/3
)*e^(2/3))-12*a*b*c*d*e-e*(b^3*d+3*a*b^2*d^(2/3)*e^(1/3)-3*a^2*b*d^(1/3)*e^
(2/3)-a^3*e))*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(2/3)/e^(
8/3)*3^(1/2)
```

Rubi [A]

time = 0.73, antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

```
[Out] (-2*(3*b^2*c^2*d + 2*a*c^3*d - 2*a*b^3*e - 6*a^2*b*c*e)*x)/e^2 - ((4*b*c^3*
d - b^4*e - 12*a*b^2*c*e - 6*a^2*c^2*e)*x^2)/(2*e^2) - (c*(c^3*d - 4*b^3*e
- 12*a*b*c*e)*x^3)/(3*e^2) + (c^2*(3*b^2 + 2*a*c)*x^4)/(2*e) + (4*b*c^3*x^5
)/(5*e) + (c^4*x^6)/(6*e) - ((b*d^(1/3) + a*e^(1/3))*(4*c^3*d^2 + 6*c^2*(b*
d^(5/3)*e^(1/3) - a*d^(4/3)*e^(2/3)) - 12*a*b*c*d*e - e*(b^3*d + 3*a*b^2*d^
(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3) - a^3*e))*ArcTan[(d^(1/3) - 2*e^(1/
3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(8/3)) + ((e^(1/3)*(6*b^2*c^2*
d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2) + d^(1/3)*(b^4*
d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))*Log[d^(1/3
) + e^(1/3)*x])/(3*d^(2/3)*e^(8/3)) - ((6*b^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b
^3*d*e - 12*a^2*b*c*d*e + a^4*e^2 + (d^(1/3)*(b^4*d*e + 12*a*b^2*c*d*e + 6*
a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1
```

```
/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(7/3)) + ((c^4*d^2 - 12*a*b*c^2*d*e + 6*
a^2*b^2*e^2 - 4*c*e*(b^3*d - a^3*e))*Log[d + e*x^3])/(3*e^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \int \left(-\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x}{e^2} - \right.$$

$$= -\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)\,x}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x^2}{2e^2} - \frac{c(}{}$$

$$= -\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)\,x}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x^2}{2e^2} - \frac{c(}{}$$

$$= -\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)\,x}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x^2}{2e^2} - \frac{c(}{}$$

$$= -\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)\,x}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x^2}{2e^2} - \frac{c(}{}$$

$$= -\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)\,x}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x^2}{2e^2} - \frac{c(}{}$$

$$= -\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)\,x}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x^2}{2e^2} - \frac{c(}{}$$

Mathematica [A]
time = 0.26, size = 678, normalized size = 1.05

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

```
[Out] (60*e^(2/3)*(-3*b^2*c^2*d - 2*a*c^3*d + 2*a*b^3*e + 6*a^2*b*c*e)*x + 15*e^(
2/3)*(-4*b*c^3*d + b^4*e + 12*a*b^2*c*e + 6*a^2*c^2*e)*x^2 + 10*c*e^(2/3)*(
-(c^3*d) + 4*b^3*e + 12*a*b*c*e)*x^3 + 15*c^2*(3*b^2 + 2*a*c)*e^(5/3)*x^4 +
 24*b*c^3*e^(5/3)*x^5 + 5*c^4*e^(5/3)*x^6 + (10*Sqrt[3]*(b*d^(1/3) + a*e^(1
/3))*(-4*c^3*d^2 + c^2*(-6*b*d^(5/3)*e^(1/3) + 6*a*d^(4/3)*e^(2/3)) + 12*a*
b*c*d*e + e*(b^3*d + 3*a*b^2*d^(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3) - a^
3*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (10*(4*a*c^3*d
^2*e^(1/3) + b^4*d^(4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a*b^3*d*e^(4/3) + a^4*
e^(7/3) + 6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e) - 4*b*(c^3*d^(7/3) + 3*
a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - (5*
(4*a*c^3*d^2*e^(1/3) + b^4*d^(4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a*b^3*d*e^(4
/3) + a^4*e^(7/3) + 6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e) - 4*b*(c^3*d^
(7/3) + 3*a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[d^(2/3) - d^(1/3)*e^(1/3)
*x + e^(2/3)*x^2])/d^(2/3) + (10*(c^4*d^2 - 12*a*b*c^2*d*e + 6*a^2*b^2*e^2
+ 4*c*e*(-(b^3*d) + a^3*e))*Log[d + e*x^3])/e^(1/3))/(30*e^(8/3))
```

Maple [A]
time = 0.48, size = 489, normalized size = 0.76

method	result
risch	$\frac{c^4x^6}{6e}+\frac{4b\,c^3x^5}{5e}+\frac{a\,c^3x^4}{e}+\frac{3b^2c^2x^4}{2e}+\frac{4ab\,c^2x^3}{e}+\frac{4b^3c\,x^3}{3e}-\frac{c^4d\,x^3}{3e^2}+\frac{3a^2c^2x^2}{e}+\frac{6a\,b^2c\,x^2}{e}+\frac{b^4x^2}{2e}-\frac{2b\,c^3d\,x^2}{e^2}+1$
default	$\frac{\frac{1}{6}c^4x^6e+\frac{4}{5}x^5b\,c^3e+a\,c^3e\,x^4+\frac{3}{2}b^2c^2e\,x^4+4ab\,c^2e\,x^3+\frac{4}{3}b^3ce\,x^3-\frac{1}{3}c^4d\,x^3+3a^2c^2e\,x^2+6a\,b^2ce\,x^2+\frac{1}{2}b^4e\,x^2-2b\,c^3d\,x^2+12a^2bcex+4a\,b}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x^3+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/6*c^4*x^6*e+4/5*x^5*b*c^3*e+a*c^3*e*x^4+3/2*b^2*c^2*e*x^4+4*a*b*c^
2*e*x^3+4/3*b^3*c*e*x^3-1/3*c^4*d*x^3+3*a^2*c^2*e*x^2+6*a*b^2*c*e*x^2+1/2*b
^4*e*x^2-2*b*c^3*d*x^2+12*a^2*b*c*e*x+4*a*b^3*e*x-4*a*c^3*d*x-6*x*b^2*c^2*d
)+((a^4*e^2-12*a^2*b*c*d*e-4*a*b^3*d*e+4*a*c^3*d^2+6*b^2*c^2*d^2)*(1/3/e/(d
/e)^(2/3)*ln(x+(d/e)^(1/3))-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2
/3))+1/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1)))+(4*
a^3*b*e^2-6*a^2*c^2*d*e-12*a*b^2*c*d*e-b^4*d*e+4*b*c^3*d^2)*(-1/3/e/(d/e)^(
1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))+
1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1)))+1/3*(4*a
```

```
^3*c*e^2+6*a^2*b^2*e^2-12*a*b*c^2*d*e-4*b^3*c*d*e+c^4*d^2)/e*ln(e*x^3+d))/e
^2
```

Maxima [A]
time = 0.56, size = 756, normalized size = 1.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*c^4*d^3 - 12*a^2*b^2*d*e^2 - 8*a^3*c*d*e^2 - 12*a^3*b*d^(2/
3)*e^(7/3) - 3*a^4*d^(1/3)*e^(8/3) - 2*(c^4*d + 4*b^3*c*e + 6*(b*d^(2/3)*e^
(1/3) + a*d^(1/3)*e^(2/3))*c^3 + 3*(3*b^2*d^(1/3)*e^(2/3) + 4*a*b*e)*c^2)*d
^2 + (3*b^4*d^(2/3)*e^(4/3) + 12*a*b^3*d^(1/3)*e^(5/3) + 12*a^2*b^2*e^2 + 6
*(4*a*b*d*e + 3*a^2*d^(2/3)*e^(4/3))*c^2 + 4*(2*b^3*d*e + 9*a*b^2*d^(2/3)*e
^(4/3) + 9*a^2*b*d^(1/3)*e^(5/3) + 2*a^3*e^2)*c)*d)*arctan(-1/3*sqrt(3)*(d^
(1/3)*e^(-1/3) - 2*x)*e^(1/3)/d^(1/3))*e^(-3)/d + 1/6*(12*a^2*b^2*d^(2/3)*e
^(4/3) + 8*a^3*c*d^(2/3)*e^(4/3) + 4*a^3*b*d^(1/3)*e^(5/3) - a^4*e^2 + 2*(c
^4*d^(2/3)*e^(-2/3) - 3*b^2*c^2 + 2*(b*d^(1/3)*e^(-1/3) - a)*c^3)*d^2 - (b^
4*d^(1/3)*e^(2/3) - 4*a*b^3*e + 6*(4*a*b*d^(2/3)*e^(1/3) + a^2*d^(1/3)*e^(2
/3))*c^2 + 4*(2*b^3*d^(2/3)*e^(1/3) + 3*a*b^2*d^(1/3)*e^(2/3) - 3*a^2*b*e)*
c)*d)*e^(-7/3)*log(-d^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2/3))/d^(2/3) +
1/3*(6*a^2*b^2*d^(2/3)*e^(4/3) + 4*a^3*c*d^(2/3)*e^(4/3) - 4*a^3*b*d^(1/3)*
e^(5/3) + a^4*e^2 + (c^4*d^(2/3)*e^(-2/3) + 6*b^2*c^2 - 4*(b*d^(1/3)*e^(-1/
3) - a)*c^3)*d^2 + (b^4*d^(1/3)*e^(2/3) - 4*a*b^3*e - 6*(2*a*b*d^(2/3)*e^(1
/3) - a^2*d^(1/3)*e^(2/3))*c^2 - 4*(b^3*d^(2/3)*e^(1/3) - 3*a*b^2*d^(1/3)*e
^(2/3) + 3*a^2*b*e)*c)*d)*e^(-7/3)*log(d^(1/3)*e^(-1/3) + x)/d^(2/3) + 1/30
*(5*c^4*x^6*e + 24*b*c^3*x^5*e + 15*(3*b^2*c^2*e + 2*a*c^3*e)*x^4 - 10*(c^4
*d - 4*b^3*c*e - 12*a*b*c^2*e)*x^3 - 15*(4*b*c^3*d - b^4*e - 12*a*b^2*c*e -
 6*a^2*c^2*e)*x^2 + 60*(2*a*b^3*e + 6*a^2*b*c*e - (3*b^2*c^2 + 2*a*c^3)*d)*
x)*e^(-2)
```

Fricas [C] Result contains complex when optimal does not.
time = 97.59, size = 47284, normalized size = 73.31

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] 1/60*(10*c^4*e^2*x^6 + 48*b*c^3*e^2*x^5 + 30*(3*b^2*c^2 + 2*a*c^3)*e^2*x^4
- 10*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((c^4*d^2 - 4*b^3*c*d*e - 12*a*b*c^2*d
*e + 6*a^2*b^2*e^2 + 4*a^3*c*e^2)^2/e^6 - (c^8*d^5 + 16*b^3*c^5*d^4*e + 10*
b^6*c^2*d^3*e^2 - 12*a^5*b^2*c*d*e^4 + 10*a^6*c^2*d*e^4 + 4*a^7*b*e^5 - (4*
b*c^3*d^2*e^3 - 19*b^4*d*e^4)*a^4 - 16*(c^5*d^3*e^2 - b^3*c^2*d^2*e^3)*a^3
```

```
+ 12*(2*b^2*c^4*d^3*e^2 + b^5*c*d^2*e^3)*a^2 - 4*(2*b*c^6*d^4*e - b^4*c^3*d
^3*e^2 - b^7*d^2*e^3)*a)/(d*e^6))/(2*(c^4*d^2 - 4*b^3*c*d*e - 12*a*b*c^2*d*
e + 6*a^2*b^2*e^2 + 4*a^3*c*e^2)^3/e^9 - 3*(c^8*d^5 + 16*b^3*c^5*d^4*e + 10
*b^6*c^2*d^3*e^2 - 12*a^5*b^2*c*d*e^4 + 10*a^6*c^2*d*e^4 + 4*a^7*b*e^5 - (4
*b*c^3*d^2*e^3 - 19*b^4*d*e^4)*a^4 - 16*(c^5*d^3*e^2 - b^3*c^2*d^2*e^3)*a^3
 + 12*(2*b^2*c^4*d^3*e^2 + b^5*c*d^2*e^3)*a^2 - 4*(2*b*c^6*d^4*e - b^4*c^3*
d^3*e^2 - b^7*d^2*e^3)*a)*(c^4*d^2 - 4*b^3*c*d*e - 12*a*b*c^2*d*e + 6*a^2*b
^2*e^2 + 4*a^3*c*e^2)/(d*e^9) + (64*b^3*c^9*d^7 + a^12*e^7 + 8*(21*b^6*c^6
- 18*a*b^4*c^7 + 8*a^3*c^ ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x**3+d),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.56, size = 723, normalized size = 1.12

$$\frac{1}{3}\left(c^4d^2-4b^3cde-12abc^2de+6a^2b^2e^2+4a^3ce^2\right)e^{(-3)}\log\left(\left|x^3e+d\right|\right)-\ldots$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="giac")
```

```
[Out] 1/3*(c^4*d^2 - 4*b^3*c*d*e - 12*a*b*c^2*d*e + 6*a^2*b^2*e^2 + 4*a^3*c*e^2)*
e^(-3)*log(abs(x^3*e + d)) - 1/3*sqrt(3)*(6*b^2*c^2*d^2*e + 4*a*c^3*d^2*e -
 4*(-d*e^2)^(1/3)*b*c^3*d^2 + (-d*e^2)^(1/3)*b^4*d*e + 12*(-d*e^2)^(1/3)*a*
b^2*c*d*e + 6*(-d*e^2)^(1/3)*a^2*c^2*d*e - 4*a*b^3*d*e^2 - 12*a^2*b*c*d*e^2
 - 4*(-d*e^2)^(1/3)*a^3*b*e^2 + a^4*e^3)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-
1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-2)/(-d*e^2)^(2/3) - 1/6*(6*b^2*c^2*d^2*e
+ 4*a*c^3*d^2*e + 4*(-d*e^2)^(1/3)*b*c^3*d^2 - (-d*e^2)^(1/3)*b^4*d*e - 12*
(-d*e^2)^(1/3)*a*b^2*c*d*e - 6*(-d*e^2)^(1/3)*a^2*c^2*d*e - 4*a*b^3*d*e^2 -
 12*a^2*b*c*d*e^2 + 4*(-d*e^2)^(1/3)*a^3*b*e^2 + a^4*e^3)*e^(-2)*log(x^2 +
(-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/(-d*e^2)^(2/3) - 1/3*(4*(-d*e^(-1)
)^(1/3)*b*c^3*d^2*e^11 + 6*b^2*c^2*d^2*e^11 + 4*a*c^3*d^2*e^11 - (-d*e^(-1)
)^(1/3)*b^4*d*e^12 - 12*(-d*e^(-1))^(1/3)*a*b^2*c*d*e^12 - 6*(-d*e^(-1))^(1
/3)*a^2*c^2*d*e^12 - 4*a*b^3*d*e^12 - 12*a^2*b*c*d*e^12 + 4*(-d*e^(-1))^(1/
3)*a^3*b*e^13 + a^4*e^13)*(-d*e^(-1))^(1/3)*e^(-13)*log(abs(x - (-d*e^(-1))
^(1/3)))/d + 1/30*(5*c^4*x^6*e^5 + 24*b*c^3*x^5*e^5 + 45*b^2*c^2*x^4*e^5 +
30*a*c^3*x^4*e^5 - 10*c^4*d*x^3*e^4 + 40*b^3*c*x^3*e^5 + 120*a*b*c^2*x^3*e^
5 - 60*b*c^3*d*x^2*e^4 + 15*b^4*x^2*e^5 + 180*a*b^2*c*x^2*e^5 + 90*a^2*c^2*
```

```
x^2*e^5 - 180*b^2*c^2*d*x*e^4 - 120*a*c^3*d*x*e^4 + 120*a*b^3*x*e^5 + 360*a
^2*b*c*x*e^5)*e^(-6)
```

Mupad [B]
time = 5.05, size = 2971, normalized size = 4.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^4/(d + e*x^3),x)
```

```
[Out] x^2*((b^4 + 6*a^2*c^2 + 12*a*b^2*c)/(2*e) - (2*b*c^3*d)/e^2) - x^3*((c^4*d)
/(3*e^2) - (4*b*c*(3*a*c + b^2))/(3*e)) + symsum(log(root(27*d^2*e^9*z^3 +
324*a*b*c^2*d^3*e^7*z^2 + 108*b^3*c*d^3*e^7*z^2 - 108*a^3*c*d^2*e^8*z^2 - 1
62*a^2*b^2*d^2*e^8*z^2 - 27*c^4*d^4*e^6*z^2 - 72*a*b*c^6*d^5*e^4*z + 216*a^
2*b^2*c^4*d^4*e^5*z + 144*a^3*b^3*c^2*d^3*e^6*z - 108*a^5*b^2*c*d^2*e^7*z +
 108*a^2*b^5*c*d^3*e^6*z - 36*a^4*b*c^3*d^3*e^6*z + 36*a*b^4*c^3*d^4*e^5*z
+ 144*b^3*c^5*d^5*e^4*z + 90*b^6*c^2*d^4*e^5*z - 144*a^3*c^5*d^4*e^5*z + 90
*a^6*c^2*d^2*e^7*z + 171*a^4*b^4*d^2*e^7*z + 36*a*b^7*d^3*e^6*z + 36*a^7*b*
d*e^8*z + 9*c^8*d^6*e^3*z + 36*a^7*b^4*c*d^2*e^6 - 36*a^7*b*c^4*d^3*e^5 - 3
6*a^4*b^7*c*d^3*e^5 - 36*a^4*b*c^7*d^5*e^3 - 36*a*b^7*c^4*d^5*e^3 + 36*a*b^
4*c^7*d^6*e^2 + 12*a*b^10*c*d^4*e^4 + 108*a^5*b^5*c^2*d^3*e^5 - 108*a^5*b^2
*c^5*d^4*e^4 + 108*a^2*b^5*c^5*d^5*e^3 - 96*a^6*b^3*c^3*d^3*e^5 + 96*a^3*b^
6*c^3*d^4*e^4 - 96*a^3*b^3*c^6*d^5*e^3 - 54*a^8*b^2*c^2*d^2*e^6 - 54*a^2*b^
8*c^2*d^4*e^4 - 54*a^2*b^2*c^8*d^6*e^2 - 9*a^4*b^4*c^4*d^4*e^4 - 12*a^10*b*
c*d*e^7 - 12*a*b*c^10*d^7*e - 6*b^6*c^6*d^6*e^2 + 4*b^9*c^3*d^5*e^3 - 6*a^6
*c^6*d^4*e^4 - 4*a^9*c^3*d^2*e^6 - 4*a^3*c^9*d^6*e^2 - 6*a^6*b^6*d^2*e^6 +
4*a^3*b^9*d^3*e^5 + 4*b^3*c^9*d^7*e + 4*a^9*b^3*d*e^7 - b^12*d^4*e^4 - c^12
*d^8 - a^12*e^8, z, k)*((x*(3*a^4*e^5 + 12*a*c^3*d^2*e^3 + 18*b^2*c^2*d^2*e
^3 - 12*a*b^3*d*e^4 - 36*a^2*b*c*d*e^4))/e^3 - (6*c^4*d^3*e^3 + 36*a^2*b^2*
d*e^5 - 24*b^3*c*d^2*e^4 + 24*a^3*c*d*e^5 - 72*a*b*c^2*d^2*e^4)/e^4 + 9*roo
t(27*d^2*e^9*z^3 + 324*a*b*c^2*d^3*e^7*z^2 + 108*b^3*c*d^3*e^7*z^2 - 108*a^
3*c*d^2*e^8*z^2 - 162*a^2*b^2*d^2*e^8*z^2 - 27*c^4*d^4*e^6*z^2 - 72*a*b*c^6
*d^5*e^4*z + 216*a^2*b^2*c^4*d^4*e^5*z + 144*a^3*b^3*c^2*d^3*e^6*z - 108*a^
5*b^2*c*d^2*e^7*z + 108*a^2*b^5*c*d^3*e^6*z - 36*a^4*b*c^3*d^3*e^6*z + 36*a
*b^4*c^3*d^4*e^5*z + 144*b^3*c^5*d^5*e^4*z + 90*b^6*c^2*d^4*e^5*z - 144*a^3
*c^5*d^4*e^5*z + 90*a^6*c^2*d^2*e^7*z + 171*a^4*b^4*d^2*e^7*z + 36*a*b^7*d^
3*e^6*z + 36*a^7*b*d*e^8*z + 9*c^8*d^6*e^3*z + 36*a^7*b^4*c*d^2*e^6 - 36*a^
7*b*c^4*d^3*e^5 - 36*a^4*b^7*c*d^3*e^5 - 36*a^4*b*c^7*d^5*e^3 - 36*a*b^7*c^
4*d^5*e^3 + 36*a*b^4*c^7*d^6*e^2 + 12*a*b^10*c*d^4*e^4 + 108*a^5*b^5*c^2*d^
3*e^5 - 108*a^5*b^2*c^5*d^4*e^4 + 108*a^2*b^5*c^5*d^5*e^3 - 96*a^6*b^3*c^3*
d^3*e^5 + 96*a^3*b^6*c^3*d^4*e^4 - 96*a^3*b^3*c^6*d^5*e^3 - 54*a^8*b^2*c^2*
d^2*e^6 - 54*a^2*b^8*c^2*d^4*e^4 - 54*a^2*b^2*c^8*d^6*e^2 - 9*a^4*b^4*c^4*d
^4*e^4 - 12*a^10*b*c*d*e^7 - 12*a*b*c^10*d^7*e - 6*b^6*c^6*d^6*e^2 + 4*b^9*
c^3*d^5*e^3 - 6*a^6*c^6*d^4*e^4 - 4*a^9*c^3*d^2*e^6 - 4*a^3*c^9*d^6*e^2 - 6
```

```
*a^6*b^6*d^2*e^6 + 4*a^3*b^9*d^3*e^5 + 4*b^3*c^9*d^7*e + 4*a^9*b^3*d*e^7 -
b^12*d^4*e^4 - c^12*d^8 - a^12*e^8, z, k)*d*e^2) + (c^8*d^5 + 4*a^7*b*e^5 +
 4*a*b^7*d^2*e^3 + 19*a^4*b^4*d*e^4 + 10*a^6*c^2*d*e^4 + 16*b^3*c^5*d^4*e -
 16*a^3*c^5*d^3*e^2 + 10*b^6*c^2*d^3*e^2 - 8*a*b*c^6*d^4*e + 24*a^2*b^2*c^4
*d^3*e^2 + 16*a^3*b^3*c^2*d^2*e^3 - 12*a^5*b^2*c*d*e^4 + 4*a*b^4*c^3*d^3*e^
2 + 12*a^2*b^5*c*d^2*e^3 - 4*a^4*b*c^3*d^2*e^3)/e^4 + (x*(10*a^6*b^2*e^4 -
4*a^7*c*e^4 - 4*a*c^7*d^4 + 10*b^2*c^6*d^4 + b^8*d^2*e^2 + 16*a^3*b^5*d*e^3
 + 16*b^5*c^3*d^3*e + 19*a^4*c^4*d^2*e^2 + 24*a^2*b^4*c^2*d^2*e^2 - 16*a^3*
b^2*c^3*d^2*e^2 - 4*a*b^3*c^4*d^3*e + 8*a*b^6*c*d^2*e^2 + 12*a^2*b*c^5*d^3*
e - 4*a^4*b^3*c*d*e^3 + 12*a^5*b*c^2*d*e^3))/e^3)*root(27*d^2*e^9*z^3 + 324
*a*b*c^2*d^3*e^7*z^2 + 108*b^3*c*d^3*e^7*z^2 - 108*a^3*c*d^2*e^8*z^2 - 162*
a^2*b^2*d^2*e^8*z^2 - 27*c^4*d^4*e^6*z^2 - 72*a*b*c^6*d^5*e^4*z + 216*a^2*b
^2*c^4*d^4*e^5*z + 144*a^3*b^3*c^2*d^3*e^6*z - 108*a^5*b^2*c*d^2*e^7*z + 10
8*a^2*b^5*c*d^3*e^6*z - 36*a^4*b*c^3*d^3*e^6*z + 36*a*b^4*c^3*d^4*e^5*z + 1
44*b^3*c^5*d^5*e^4*z + 90*b^6*c^2*d^4*e^5*z - 144*a^3*c^5*d^4*e^5*z + 90*a^
6*c^2*d^2*e^7*z + 171*a^4*b^4*d^2*e^7*z + 36*a*b^7*d^3*e^6*z + 36*a^7*b*d*e
^8*z + 9*c^8*d^6*e^3*z + 36*a^7*b^4*c*d^2*e^6 - 36*a^7*b*c^4*d^3*e^5 - 36*a
^4*b^7*c*d^3*e^5 - 36*a^4*b*c^7*d^5*e^3 - 36*a*b^7*c^4*d^5*e^3 + 36*a*b^4*c
^7*d^6*e^2 + 12*a*b^10*c*d^4*e^4 + 108*a^5*b^5*c^2*d^3*e^5 - 108*a^5*b^2*c^
5*d^4*e^4 + 108*a^2*b^5*c^5*d^5*e^3 - 96*a^6*b^3*c^3*d^3*e^5 + 96*a^3*b^6*c
^3*d^4*e^4 - 96*a^3*b^3*c^6*d^5*e^3 - 54*a^8*b^2*c^2*d^2*e^6 - 54*a^2*b^8*c
^2*d^4*e^4 - 54*a^2*b^2*c^8*d^6*e^2 - 9*a^4*b^4*c^4*d^4*e^4 - 12*a^10*b*c*d
*e^7 - 12*a*b*c^10*d^7*e - 6*b^6*c^6*d^6*e^2 + 4*b^9*c^3*d^5*e^3 - 6*a^6*c^
6*d^4*e^4 - 4*a^9*c^3*d^2*e^6 - 4*a^3*c^9*d^6*e^2 - 6*a^6*b^6*d^2*e^6 + 4*a
^3*b^9*d^3*e^5 + 4*b^3*c^9*d^7*e + 4*a^9*b^3*d*e^7 - b^12*d^4*e^4 - c^12*d^
8 - a^12*e^8, z, k), k, 1, 3) - x*((d*(4*a*c^3 + 6*b^2*c^2))/e^2 - (4*a*b*(
3*a*c + b^2))/e) + (c^4*x^6)/(6*e) + (x^4*(4*a*c^3 + 6*b^2*c^2))/(4*e) + (4
*b*c^3*x^5)/(5*e)
```

3.76 $\int \frac{2x^2+x^4}{1+x^3} dx$

Optimal. Leaf size=43

$$\frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\frac{1}{2}\log\left(1-x+x^2\right)$$

```
[Out] 1/2*x^2+ln(1+x)+1/2*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1607, 1901, 1888, 31, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^2}{2}+\frac{1}{2}\log\left(x^2-x+1\right)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x^2 + x^4)/(1 + x^3),x]
```

```
[Out] x^2/2 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x] + Log[1 - x + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2x^2+x^4}{1+x^3}\,dx &= \int \frac{x^2(2+x^2)}{1+x^3}\,dx \\
&= \int \left(x+\frac{x(-1+2x)}{1+x^3}\right)dx \\
&= \frac{x^2}{2}+\int \frac{x(-1+2x)}{1+x^3}\,dx \\
&= \frac{x^2}{2}+\frac{1}{3}\int \frac{-3+3x}{1-x+x^2}\,dx+\int \frac{1}{1+x}\,dx \\
&= \frac{x^2}{2}+\log(1+x)-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx \\
&= \frac{x^2}{2}+\log(1+x)+\frac{1}{2}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&= \frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.26

$$\frac{1}{6}\left(3x^2-2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)+2\log(1+x)-\log\left(1-x+x^2\right)+4\log\left(1+x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x^2 + x^4)/(1 + x^3),x]
```

```
[Out] (3*x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Log[1 - x +
x^2] + 4*Log[1 + x^3])/6
```

Maple [A]
time = 0.33, size = 38, normalized size = 0.88

method	result	size
default	$\frac{x^2}{2}+\ln(x+1)+\frac{\ln(x^2-x+1)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	38
risch	$\frac{x^2}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{\ln(4x^2-4x+4)}{2}+\ln(x+1)$	40

meijerg	$\frac{x^2}{2} - \frac{x^2\left(-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{(x^3)^{\frac{2}{3}}} + \frac{\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2(x^3)^{\frac{2}{3}}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{(x^3)^{\frac{2}{3}}}\right)}{3} + \frac{2\ln(x^3+1)}{3}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x^2)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+ln(x+1)+1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 37, normalized size = 0.86

$$\frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right) + \frac{1}{2}\,\log\left(x^2-x+1\right) + \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
+ log(x + 1)
```

Fricas [A]
time = 0.39, size = 37, normalized size = 0.86

$$\frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right) + \frac{1}{2}\,\log\left(x^2-x+1\right) + \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
+ log(x + 1)
```

Sympy [A]
time = 0.05, size = 44, normalized size = 1.02

$$\frac{x^2}{2} + \log\left(x+1\right) + \frac{\log\left(x^2-x+1\right)}{2} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x**2)/(x**3+1),x)
```

```
[Out] x**2/2 + log(x + 1) + log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sq
rt(3)/3)/3
```

Giac [A]
time = 2.13, size = 38, normalized size = 0.88

$$\frac{1}{2}\,x^2-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
+ log(abs(x + 1))
```

Mupad [B]
time = 0.10, size = 49, normalized size = 1.14

$$\ln\left(x+1\right)+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + x^4)/(x^3 + 1),x)
```

```
[Out] log(x + 1) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/2) - log(x +
 (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/2) + x^2/2
```

3.77 $\int \frac{2x^2+x^4}{1-x^3}\,dx$

Optimal. Leaf size=46

$$-\frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(1-x)-\frac{1}{2}\log\left(1+x+x^2\right)$$

```
[Out] -1/2*x^2-ln(1-x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1607, 1901, 1889, 31, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x^2 + x^4)/(1 - x^3),x]
```

```
[Out] -1/2*x^2 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x] - Log[1 + x + x^2
]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q
 + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q -
 C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b
*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
 && LtQ[a/b, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2x^2+x^4}{1-x^3}\,dx &= \int \frac{x^2(2+x^2)}{1-x^3}\,dx \\
&= \int \left(-x+\frac{x(1+2x)}{1-x^3}\right)\,dx \\
&= -\frac{x^2}{2}+\int \frac{x(1+2x)}{1-x^3}\,dx \\
&= -\frac{x^2}{2}+\frac{1}{3}\int \frac{-3-3x}{1+x+x^2}\,dx+\int \frac{1}{1-x}\,dx \\
&= -\frac{x^2}{2}-\log(1-x)-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx \\
&= -\frac{x^2}{2}-\log(1-x)-\frac{1}{2}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right) \\
&= -\frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(1-x)-\frac{1}{2}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.17

$$
\frac{1}{6}\left(-3x^2-2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-2\log(1-x)+\log\left(1+x+x^2\right)-4\log\left(1-x^3\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x^2 + x^4)/(1 - x^3),x]
```

```
[Out] (-3*x^2 - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 - x] + Log[1 + x + x^2] - 4*Log[1 - x^3])/6
```

Maple [A]
time = 0.34, size = 38, normalized size = 0.83

method	result	size
risch	$-\frac{x^2}{2}-\ln\left(x-1\right)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\ln\left(x^2+x+1\right)}{2}$	36
default	$-\frac{x^2}{2}-\frac{\ln\left(x^2+x+1\right)}{2}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\ln\left(x-1\right)$	38

meijerg	$\frac{(-1)^{\frac{1}{3}}\left(\frac{3x^2(-1)^{\frac{2}{3}}}{2}+\frac{x^2(-1)^{\frac{2}{3}}\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{(x^3)^{\frac{2}{3}}}\right)}{3}-\frac{2\ln(-x^3+1)}{3}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x^2)/(-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-ln(x-1)
```

Maxima [A]
time = 0.52, size = 37, normalized size = 0.80

$$-\frac{1}{2}x^2-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
 - log(x - 1)
```

Fricas [A]
time = 0.35, size = 37, normalized size = 0.80

$$-\frac{1}{2}x^2-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
 - log(x - 1)
```

Sympy [A]
time = 0.05, size = 46, normalized size = 1.00

$$-\frac{x^2}{2}-\log(x-1)-\frac{\log(x^2+x+1)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x**2)/(-x**3+1),x)
```

```
[Out] -x**2/2 - log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + s
qrt(3)/3)/3
```

Giac [A]
time = 2.32, size = 38, normalized size = 0.83

$$-\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log\left(x^2+x+1\right) - \log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
 - log(abs(x - 1))
```

Mupad [B]
time = 0.09, size = 51, normalized size = 1.11

$$-\ln(x-1) + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 + x^4)/(x^3 - 1),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/2) - log(x - 1) - log(x +
 (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/2) - x^2/2
```

3.78 $\int \frac{1-x+4x^3}{1+x^3}\,dx$

Optimal. Leaf size=44

$$4x+\frac{4\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)$$

```
[Out] 4*x-2/3*ln(1+x)+1/3*ln(x^2-x+1)+4/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1901, 1874, 31, 648, 632, 210, 642}

$$\frac{4\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(x^2-x+1\right)+4x-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + 4*x^3)/(1 + x^3),x]
```

```
[Out] 4*x + (4*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x
+ x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x+4x^3}{1+x^3}\,dx &= \int \left(4-\frac{3+x}{1+x^3}\right)\,dx\\
&= 4x-\int \frac{3+x}{1+x^3}\,dx\\
&= 4x-\frac{1}{3}\int \frac{7-2x}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1}{1+x}\,dx\\
&= 4x-\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-1+2x}{1-x+x^2}\,dx-2\int \frac{1}{1-x+x^2}\,dx\\
&= 4x-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)+4\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= 4x+\frac{4\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.00

$$4x-\frac{4\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 4*x^3)/(1 + x^3),x]
```

```
[Out] 4*x - (4*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x
 + x^2]/3
```

Maple [A]
time = 0.32, size = 38, normalized size = 0.86

method	result
default	$4x-\frac{2\ln\left(x+1\right)}{3}+\frac{\ln\left(x^{2}-x+1\right)}{3}-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2x-1\right)\sqrt{3}}{3}\right)}{3}$
risch	$4x+\frac{\ln\left(4x^{2}-4x+4\right)}{3}-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2x-1\right)\sqrt{3}}{3}\right)}{3}-\frac{2\ln\left(x+1\right)}{3}$
meijerg	$4x-\frac{4x\left(\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}-\frac{\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2\left(x^{3}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{3}\right)^{\frac{1}{3}}}{2-\left(x^{3}\right)^{\frac{1}{3}}}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}\right)}{3}+\frac{x^{2}\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{3\left(x^{3}\right)^{\frac{2}{3}}}-\frac{x^{2}\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\cdots}\right.}{6\left(x^{3}\right)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-x+1)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 4*x-2/3*ln(x+1)+1/3*ln(x^2-x+1)-4/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.53, size = 37, normalized size = 0.84

$$-\frac{4}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x-1\right)\right)+4\,x+\frac{1}{3}\,\log\left(x^{2}-x+1\right)-\frac{2}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2
/3*log(x + 1)
```

Fricas [A]
time = 0.39, size = 37, normalized size = 0.84

$$-\frac{4}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x-1\right)\right)+4\,x+\frac{1}{3}\,\log\left(x^{2}-x+1\right)-\frac{2}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2
/3*log(x + 1)
```

Sympy [A]
time = 0.05, size = 48, normalized size = 1.09

$$4x - \frac{2\log{(x+1)}}{3} + \frac{\log{(x^2 - x + 1)}}{3} - \frac{4\sqrt{3}\operatorname{atan}{\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-x+1)/(x**3+1),x)
```

```
[Out] 4*x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3 - 4*sqrt(3)*atan(2*sqrt(3)*x/3 -
 sqrt(3)/3)/3
```

Giac [A]
time = 2.10, size = 38, normalized size = 0.86

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 4x + \frac{1}{3}\log\left(x^2 - x + 1\right) - \frac{2}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2
/3*log(abs(x + 1))
```

Mupad [B]
time = 4.70, size = 49, normalized size = 1.11

$$4x - \frac{2\ln(x+1)}{3} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{3} + \frac{\sqrt{3}\,2\mathrm{i}}{3}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{3} + \frac{\sqrt{3}\,2\mathrm{i}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3 - x + 1)/(x^3 + 1),x)
```

```
[Out] 4*x - (2*log(x + 1))/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*2i)/3 + 1/
3) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*2i)/3 - 1/3)
```

3.79 $\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=230

$$\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{4\sqrt[4]{3}\sqrt{2+\cdots}}{\cdots}$$

```
[Out] 2*(x^3+1)^(1/2)/(1+x+3^(1/2))-3^(1/4)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3^
(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)
^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1+x)*Elliptic
F((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^
2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1892, 224, 1891}

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4
*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx + \int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 47, normalized size = 0.20

$$\left(1+\sqrt{3}\right)x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (x^2*Hypergeometri
c2F1[1/2, 2/3, 5/3, -x^3])/2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(190) = 380.
time = 0.41, size = 407, normalized size = 1.77

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)+\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{2}+\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)$
elliptic	$\frac{2\left(1+\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*
I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((x+1)/(3/2-1/
2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))+2*3
^(1/2)*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*
3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 21, normalized size = 0.09

$$2\left(\sqrt{3}+1\right)\mathrm{weierstrassPInverse}(0,-4,x)-2\,\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(3) + 1)*weierstrassPInverse(0, -4, x) - 2*weierstrassZeta(0, -4, we
ierstrassPInverse(0, -4, x))
```

Sympy [A]
time = 0.95, size = 92, normalized size = 0.40

$$\frac{x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/
3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma
(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi
))/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Mupad [B]
time = 0.15, size = 312, normalized size = 1.36

$$\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)-\frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}+\frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/(x^3 + 1)^(1/2),x)
```

```
[Out] 3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, -x^3) - (6*((x + (3^(1/2)*1i)/2 - 1/2)
/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^
(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
```

```
 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) + (6*((x + (3^(1/2)*1i)/2
- 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((
(x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1
i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1)
- ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.80 $\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=257

$$-\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}+\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}-\frac{4\sqrt[4]{3}\,\sqrt{2}}{}$$

[Out] -2*(-x^3+1)^(1/2)/(1-x+3^(1/2))+3^(1/4)*(1-x)*EllipticE((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-4*3^(1/4)*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1892, 224, 1891}

$$-\frac{4\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\,\sqrt{1-x^3}}+\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\,\sqrt{1-x^3}}-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] - x)/Sqrt[1 - x^3],x]

[Out] (-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}\,dx + \int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$$
$$= -\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 43, normalized size = 0.17

$$\left(1+\sqrt{3}\right)x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] - (x^2*Hypergeometric
2F1[1/2, 2/3, 5/3, x^3])/2
```

Maple [A]
time = 0.38, size = 368, normalized size = 1.43

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)-\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2}+\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)$
elliptic	$-\frac{2i\left(1+\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\sqrt{3}\right.}{3\sqrt{-x^3+1}}$
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x\right.}}{}\right.}{3\sqrt{-x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(-x^3+1)^(1/2)*((-3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+Elliptic
F(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*
I*3^(1/2)))^(1/2))-2*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/
2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

[Out] -integrate((x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)

Fricas [F]
time = 0.09, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [A]
time = 1.38, size = 97, normalized size = 0.38

$$-\frac{x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*
gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar
(2*I*pi))/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Mupad [B]
time = 5.14, size = 342, normalized size = 1.33

$$\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)+\frac{6\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}}-\frac{6\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) - x + 1)/(1 - x^3)^(1/2),x)
```

```
[Out] 3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, x^3) + (6*(x^3 - 1)^(1/2)*(-(x - (3^(1
/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellip
ticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)
/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^
3)^(1/2)) - (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-
(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 -
x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.81 $\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=144

$$\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] 2*(x^3-1)^(1/2)/(1-x-3^(1/2))-3^(1/4)*(1-x)*EllipticE((1-x+3^(1/2))/(1-x-3^
(1/2)),2*I-I*3^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^
(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1893}

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*S
qrt[-1 + x^3])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
```

```
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 63, normalized size = 0.44

$$-\frac{x\sqrt{1-x^3}\left(-2\left(1+\sqrt{3}\right){}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)\right)}{2\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3],x]
[Out] -1/2*(x*Sqrt[1 - x^3]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^
3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/Sqrt[-1 + x^3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(120) = 240.
time = 0.40, size = 407, normalized size = 2.83

method	result
meijerg	$\frac{\sqrt{-\mathrm{signum}\left(x^3-1\right)}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)}{\sqrt{\mathrm{signum}\left(x^3-1\right)}} - \frac{\sqrt{-\mathrm{signum}\left(x^3-1\right)}\,x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2\sqrt{\mathrm{signum}\left(x^3-1\right)}} + \frac{\sqrt{3}}{}$
elliptic	$\frac{2\left(1+\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right.}{\sqrt{x^3-1}}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-\frac{3}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I*
3^(1/2))*EllipticE(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I*3^(1/2))*EllipticF(((x-1)/(-3/2-1/2
*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))+2*3^(1
/2)*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 21, normalized size = 0.15

$$2\left(\sqrt{3}+1\right)\text{weierstrassPInverse}(0,4,x)+2\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(3) + 1)*weierstrassPInverse(0, 4, x) + 2*weierstrassZeta(0, 4, weie
rstrassPInverse(0, 4, x))
```

Sympy [A]
time = 1.35, size = 82, normalized size = 0.57

$$\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - sqrt(3)*
I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - I*x*gamma(1
/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Mupad [B]
time = 4.87, size = 326, normalized size = 2.26

$$\frac{\sqrt{3}\,x\,\sqrt{1-x^3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)}{\sqrt{x^3-1}}+\frac{6\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{6\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) - x + 1)/(x^3 - 1)^(1/2),x)
```

```
[Out] (3^(1/2)*x*(1 - x^3)^(1/2)*hypergeom([1/3, 1/2], 4/3, x^3))/(x^3 - 1)^(1/2)
 + (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x
^3)^(1/2) - (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(
(x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) + x^3)^(1/2)
```

3.82 $\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}} dx$

Optimal. Leaf size=135

$$-\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

```
[Out] -2*(-x^3-1)^(1/2)/(1+x-3^(1/2))+3^(1/4)*(1+x)*EllipticE((1+x+3^(1/2))/(1+x-
3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*
2^(1/2))/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1893}

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/
(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
```

```
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-\right.}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 67, normalized size = 0.50

$$\frac{x\sqrt{1+x^3}\left(2\left(1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3],x]
[Out] (x*Sqrt[1 + x^3]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] +
x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(115) = 230.
time = 0.35, size = 370, normalized size = 2.74

method	result
meijerg	$-ix\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)-\frac{ix^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{2}-i\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],\right.$
elliptic	$-\frac{2i\left(1+\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}}{}\right.}{3\sqrt{-x^3-1}}$

default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*
((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3-1)^(1/2)*((3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1
/2)))^(1/2)))-2*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2
+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Fricas [F]
time = 0.08, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [A]
time = 1.04, size = 99, normalized size = 0.73

$$-\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma
(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*
pi))/(3*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_pol
ar(I*pi))/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Mupad [B]
time = 4.91, size = 360, normalized size = 2.67

$$\frac{\sqrt{3}\,x\,\sqrt{x^3+1}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)}{\sqrt{-x^3-1}}-\frac{6\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}+\frac{6\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/(- x^3 - 1)^(1/2),x)
```

```
[Out] (3^(1/2)*x*(x^3 + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -x^3))/(- x^3 - 1)^(1
/2) - (6*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1
/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) + (6*(x^3 + 1)^(1/2)*((x + (3
^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ell
ipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2
)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2))^(1/2))
```

3.83 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=468

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\sin^{\cdots}\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a\cdots}}$$

```
[Out] 2*(b*x^3+a)^(1/2)/b^(1/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)*(
a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(
b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)+4*3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1892, 224, 1891}

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[
3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
```

```
3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx = \left(2\sqrt{3}\,\sqrt[3]{a}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx + \int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\cdots}}{\sqrt[3]{b}\sqrt{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 90, normalized size = 0.19

$$\frac{x\sqrt{1+\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+\sqrt[3]{b}\,x\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2,
 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(346) = 692$.
time = 0.32, size = 1003, normalized size = 2.14

method	result	size
default	Expression too large to display	1003

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
```

```
^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2*I*a^(1/3)/b*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*a^(1/3)*3^(1/2)/b*(
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 49, normalized size = 0.10

$$\frac{2\left(a^{\frac{1}{3}}\sqrt{b}\left(\sqrt{3}+1\right)\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-b^{\frac{5}{6}}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2*(a^(1/3)*sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - b^(5/6
)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A]
time = 2.16, size = 122, normalized size = 0.26

$$\frac{\sqrt[3]{b}\,x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3,
 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}+1\right)}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(a + b*x^3)^(1/2),x)
```

```
[Out] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(a + b*x^3)^(1/2), x)
```

3.84 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=481

$$-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\left(\ldots\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\ldots}$$

```
[Out] -2*(-b*x^3+a)^(1/2)/b^(1/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+3^(1/4)*a^(1/3
)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(
1/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)-4*3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)
*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b
^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1892, 224, 1891}

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt
[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2
```

```
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
- b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{a-bx^3}}\,dx = \left(2\sqrt{3}\sqrt[3]{a}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx + \int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{a-bx^3}}\,dx$$

$$= -\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 91, normalized size = 0.19

$$\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)-\sqrt[3]{b}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2,
 4/3, (b*x^3)/a] - b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/
(2*Sqrt[a - b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 948 vs. 2(359) = 718.
time = 0.31, size = 949, normalized size = 1.97

method	result	size
default	Expression too large to display	949

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b
^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(
```

```
1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^
2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I/b^(2/3)*3^(1/2
)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x
+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3)
)^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*
(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)
,(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^
(1/3)))^(1/2)))+2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b
*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)
*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)
^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1
/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(
1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 55, normalized size = 0.11

$$-\frac{2\left(a^{\frac{1}{3}}\sqrt{-b}\left(\sqrt{3}+1\right)\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+\sqrt{-b}\,b^{\frac{1}{3}}\mathrm{weierstrassZeta}\left(0,\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -2*(a^(1/3)*sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + sqrt(
-b)*b^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A]
time = 3.55, size = 128, normalized size = 0.27

$$-\frac{\sqrt[3]{b}\,x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*pi
)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3
*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper(
(1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}+1\right)}{\sqrt{a-b\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(a - b*x^3)^(1/2),x)
```

```
[Out] -int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(a - b*x^3)^(1/2), x)
```

3.85 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=271

$$\frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\Bigg(\sin}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a}}$$

```
[Out] 2*(b*x^3-a)^(1/2)/b^(1/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-3^(1/4)*a^(1/3)*
(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+
a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(1/
3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1893}

$$\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{-a+bx^3}}\,dx=\frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\cdots}}{\sqrt[3]{b}\sqrt{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 92, normalized size = 0.34

$$\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)-\sqrt[3]{b}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2,
 4/3, (b*x^3)/a] - b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/
(2*Sqrt[-a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 951 vs. $2(206) = 412$.
time = 0.31, size = 952, normalized size = 3.51

method	result	size
default	Expression too large to display	952

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b
^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1
/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2
)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b
*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I/b^(2/3)*3^(1/2)
*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^
(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1
/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^
(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*
b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(
-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1
/3)))^(1/2)))+2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(
a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(
I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(
1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)
/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 48, normalized size = 0.18

$$\frac{2\left(a^{\frac{1}{3}}\sqrt{b}\left(\sqrt{3}+1\right)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+b^{\frac{5}{6}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2*(a^(1/3)*sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + b^(5/6)
*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [**A**]
time = 3.19, size = 112, normalized size = 0.41

$$\frac{i\sqrt[3]{b}\,x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*g
amma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*
a**(1/6)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(
3*a**(1/6)*gamma(4/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}+1\right)}{\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(b*x^3 - a)^(1/2),x)
```

```
[Out] -int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(b*x^3 - a)^(1/2), x)
```

3.86 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=266

$$-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{si}\right.}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-}}$$

```
[Out] -2*(-b*x^3-a)^(1/2)/b^(1/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+3^(1/4)*a^(1/3)
*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a
^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(1/3)
/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = $\{1893\}$

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[
ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{-a-bx^3}}\,dx=-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\,\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 93, normalized size = 0.35

$$\frac{x\sqrt{1+\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+\sqrt[3]{b}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2,
 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2*Sqrt[-a - b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1011 vs. 2(201) = 402.
time = 0.33, size = 1012, normalized size = 3.80

method	result	size
default	Expression too large to display	1012

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2*I*a^(1/3)/b*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*a^(1/3)*3^(1/2)/b
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 56, normalized size = 0.21

$$-\frac{2\left(a^{\frac{1}{3}}\sqrt{-b}\left(\sqrt{3}+1\right)\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{-b}\,b^{\frac{1}{3}}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -2*(a^(1/3)*sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - sqrt
(-b)*b^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))
/b
```

Sympy [A]
time = 2.39, size = 129, normalized size = 0.48

$$-\frac{i\sqrt[3]{b}\,x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3
,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) - I*x*gamma(1/3)*hype
r((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}+1\right)}{\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(- a - b*x^3)^(1/2),x)
```

```
[Out] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(- a - b*x^3)^(1/2), x)
```

3.87 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=520

$$\frac{2\sqrt[3]{\frac{b}{a}}\,\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\Big(}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a}}$$

```
[Out] 2*(b/a)^(1/3)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/
4)*a^(1/3)*(b/a)^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(-a
^(1/3)*(b/a)^(1/3)*(1-3^(1/2))+b^(1/3)*(1+3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2
))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1892, 224, 1891}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{2\sqrt[3]{\frac{b}{a}}\,\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(b/a)^(1/3)*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
```

```
2*Sqrt[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b/a)^(1
/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3
^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} + \left(1+\sqrt{3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx}}$$

$$= \frac{2\sqrt[3]{\frac{b}{a}}\,\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^2}{((1}}}{b^{2/3}\sqrt{\frac{}{((1}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 89, normalized size = 0.17

$$\frac{x\sqrt{1+\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right){}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -(
(b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])
)/(2*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(386) = 772.
time = 0.32, size = 1004, normalized size = 1.93

method	result	size
default	Expression too large to display	1004

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
```

```
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*
I*(b/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2*I/b*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 53, normalized size = 0.10

$$\frac{2\left(\sqrt{b}\left(\sqrt{3}+1\right)\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] 2*(sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - sqrt(b)*(b/a)^
(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A]
time = 1.33, size = 124, normalized size = 0.24

$$\frac{x^2\sqrt[3]{\frac{b}{a}}\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a)**(1/2),x)
```

```
[Out] x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**
3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1
/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{3}+x\,{\left(\frac{b}{a}\right)}^{1/3}+1}{\sqrt{b\,x^3+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/(a + b*x^3)^(1/2),x)
```

```
[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/(a + b*x^3)^(1/2), x)
```

3.88 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=533

$$-\frac{2\sqrt[3]{\frac{b}{a}}\,\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}}$$

```
[Out] -2*(b/a)^(1/3)*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+3^
(1/4)*a^(1/3)*(b/a)^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-
1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/
(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-2/3*(a^(1/3)-b^(1/3)*x)*EllipticF
((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(-a^(1/3)*(b/a)^(1/3)*(1-3^(1/2))+b^(1/3)*(1+3^(1/2)))*(1/2*6^(1/2)+
1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(
1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1892, 224, 1891}

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a-bx^3}}+\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a-bx^3}}-\frac{2\sqrt[3]{\frac{b}{a}}\,\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(1/3)*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) -
(2*Sqrt[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b/a)^(
```

```
1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^
(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a-bx^3}}\,dx = \frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{a-bx^3}}\,dx}{\sqrt[3]{b}} - \left(-1-\sqrt{3}+\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}\right)\int \frac{}{\sqrt{a-}}$$

$$= -\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{}}{b^{2/3}\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 89, normalized size = 0.17

$$-\frac{x\sqrt{1-\frac{bx^3}{a}}\left(-2\left(1+\sqrt{3}\right){}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - (b*x^3)/a]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4
/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/Sqrt[a - b*x^3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 949 vs. 2(399) = 798.
time = 0.33, size = 950, normalized size = 1.78

method	result	size
default	Expression too large to display	950

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
```

```
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)
^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/
2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*
3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^
(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)
*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2
)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^
2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(
x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3
))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(a*b^2)^(1/3)))^(1/2)))+2*I/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(
1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2
/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1
/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b
^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 56, normalized size = 0.11

$$-\frac{2\left(\sqrt{-b}\left(\sqrt{3}+1\right)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + sqrt(-b)*(b/a
)^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A]

time = 1.50, size = 129, normalized size = 0.24

$$-\frac{x^2\sqrt[3]{\frac{b}{a}}\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*
I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*
x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyp
er((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{3}-x\,{\left(\frac{b}{a}\right)}^{1/3}+1}{\sqrt{a-b\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) - x*(b/a)^(1/3) + 1)/(a - b*x^3)^(1/2),x)
```

```
[Out] int((3^(1/2) - x*(b/a)^(1/3) + 1)/(a - b*x^3)^(1/2), x)
```

3.89 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=256

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a+bx^3}}{b\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}\,x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}\right)\right.}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] 2*(b/a)^(2/3)*(b*x^3-a)^(1/2)/b/(1-(b/a)^(1/3)*x-3^(1/2))-3^(1/4)*(1-(b/a)^
(1/3)*x)*EllipticE((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2)),2*I-
I*3^(1/2))*((1+(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(1-(b/a)^(1/3)*x-3^(1/2))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(b/a)^(1/3)/(b*x^3-a)^(1/2)/((-1+(b/a)^(1/3
)*x)/(1-(b/a)^(1/3)*x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {1893}

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{bx^3-a}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{b}{a}}\,x+\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}\,x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*(b/a)^(2/3)*Sqrt[-a + b*x^3])/(b*(1 - Sqrt[3] - (b/a)^(1/3)*x)) - (3^(1/
4)*Sqrt[2 + Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^(2
/3)*x^2)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (
b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3)
```

```
*Sqrt[-((1 - (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2)]*Sqrt[-a + b*x
^3])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a+bx^3}}{b\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)} - \frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}\,x+\left(\frac{b}{a}\right)^{2/3}\cdots}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\cdots\right.}}}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-\sqrt[3]{\frac{b}{a}}\,x\cdots}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\cdots\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 90, normalized size = 0.35

$$-\frac{x\sqrt{1-\frac{bx^3}{a}}\left(-2\left(1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - (b*x^3)/a]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4
/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/Sqrt[-a + b*x^3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 952 vs. 2(211) = 422.
time = 0.32, size = 953, normalized size = 3.72

method	result	size
default	Expression too large to display	953

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^
(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b
^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2
/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3
^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(
1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*(
(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*
(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)
^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+
1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))
^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a
*b^2)^(1/3)))^(1/2)))+2*I/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/
3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b
*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2
)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)
^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 52, normalized size = 0.20

$$\frac{2\left(\sqrt{b}\left(\sqrt{3}+1\right)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + sqrt(b)*(b/a)^(
1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A]
time = 1.47, size = 114, normalized size = 0.45

$$\frac{ix^{2}\sqrt[3]{\frac{b}{a}}\,\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^{3}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-a)**(1/2),x)
```

```
[Out] I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(
a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)
/(3*sqrt(a)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a
)/(3*sqrt(a)*gamma(4/3))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) - x*(b/a)^(1/3) + 1)/(b*x^3 - a)^(1/2),x)
```

```
[Out] int((3^(1/2) - x*(b/a)^(1/3) + 1)/(b*x^3 - a)^(1/2), x)
```

3.90 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=251

$$-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)}+\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}\,x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}\right)\ldots\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1+\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] -2*(b/a)^(2/3)*(-b*x^3-a)^(1/2)/b/(1+(b/a)^(1/3)*x-3^(1/2))+3^(1/4)*(1+(b/a
)^(1/3)*x)*EllipticE((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2)),2*
I-I*3^(1/2))*((1-(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(1+(b/a)^(1/3)*x-3^(1/2))^2
)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(b/a)^(1/3)/(-b*x^3-a)^(1/2)/((-1-(b/a)^(
1/3)*x)/(1+(b/a)^(1/3)*x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {1893}

$$\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{\frac{b}{a}}\,x+\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}\,x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}\sqrt{-a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(2/3)*Sqrt[-a - b*x^3])/(b*(1 - Sqrt[3] + (b/a)^(1/3)*x)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)^(
2/3)*x^2)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] +
```

```
(b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3
)*Sqrt[-((1 + (b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2)]*Sqrt[-a - b*
x^3])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)} + \frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}\,x+\left(\frac{b}{a}\right)^{2/\cdots}}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,\cdots\right.}}}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1+\sqrt[3]{\frac{b}{a}}\,\cdots}{\left(1-\sqrt{3}+\sqrt[3]{\cdots}\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 92, normalized size = 0.37

$$\frac{x\sqrt{1+\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -(
(b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])
)/(2*Sqrt[-a - b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1012 vs. 2(208) = 416.
time = 0.32, size = 1013, normalized size = 4.04

method	result	size
default	Expression too large to display	1013

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3
*I*(b/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2*I/b*(-a*b^2)^
(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 57, normalized size = 0.23

$$-\frac{2\left(\sqrt{-b}\left(\sqrt{3}+1\right)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="fricas")
[Out] -2*(sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - sqrt(-b)*(b/
a)^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A]
time = 1.43, size = 131, normalized size = 0.52

$$-\frac{ix^{2}\sqrt[3]{\frac{b}{a}}\,\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3-a)**(1/2),x)
[Out] -I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2),
(4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) - I*x*gamma(1/3)*h
yper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/(- a - b*x^3)^(1/2),x)
[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/(- a - b*x^3)^(1/2), x)
```

3.91 $\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=127

$$\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 2*(x^3+1)^(1/2)/(1+x+3^(1/2))-3^(1/4)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3^
(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)
^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1891}

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx=\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 49, normalized size = 0.39

$$\left(1-\sqrt{3}\right)x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/Sqrt[1 + x^3],x]
[Out] (1 - Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (x^2*Hypergeometri
c2F1[1/2, 2/3, 5/3, -x^3])/2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(104) = 208.
time = 0.41, size = 407, normalized size = 3.20

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)+\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{2}-\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right.$
elliptic	$\frac{2\left(1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*
I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((x+1)/(3/2-1/
2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))-2*3
^(1/2)*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*
3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 21, normalized size = 0.17

$$-2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,-4,x)-2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(3) - 1)*weierstrassPInverse(0, -4, x) - 2*weierstrassZeta(0, -4, w
eierstrassPInverse(0, -4, x))
```

Sympy [A]
time = 0.95, size = 92, normalized size = 0.72

$$\frac{x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/
3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/
(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi
))/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Mupad [B]
time = 0.13, size = 313, normalized size = 2.46

$$-\sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)-\frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}+\frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/(x^3 + 1)^(1/2),x)
```

```
[Out] (6*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*
1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1
/2) - (6*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/
2))^(1/2) - 3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, -x^3)
```

3.92 $\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=142

$$-\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2*(-x^3+1)^(1/2)/(1-x+3^(1/2))+3^(1/4)*(1-x)*EllipticE((1-x-3^(1/2))/(1-x+
3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^
2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1891}

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt
[1 - x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 45, normalized size = 0.32

$$\left(1-\sqrt{3}\right)x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (1 - Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] - (x^2*Hypergeometric
2F1[1/2, 2/3, 5/3, x^3])/2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(119) = 238.
time = 0.42, size = 368, normalized size = 2.59

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)-\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2}-\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)$
elliptic	$-\frac{2i\left(1-\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\sqrt{\right.\right.}{3\sqrt{-x^3+1}}$

default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(-x^3+1)^(1/2)*((-3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+Elliptic
F(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*
I*3^(1/2)))^(1/2)))+2*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/
2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Fricas [F]
time = 0.08, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [A]
time = 1.40, size = 97, normalized size = 0.68

$$-\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*
pi))/(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar
(2*I*pi))/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Mupad [B]
time = 4.74, size = 343, normalized size = 2.42

$$-\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)+\frac{6\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{6\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/(1 - x^3)^(1/2),x)
```

```
[Out] (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)*(
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - 3^(1/2)*x*hypergeom([1/3, 1/2],
4/3, x^3) - (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-
(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 -
x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.93 $\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=264

$$\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}-\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}+\frac{4\sqrt[4]{3}\,\sqrt{2-\cdots}}{\cdots}$$

```
[Out] 2*(x^3-1)^(1/2)/(1-x-3^(1/2))+4*3^(1/4)*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-
3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^
2)^(1/2)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-3^(1/4)*(1-x)*Ellipti
cE((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(
1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1894, 225, 1893}

$$\frac{4\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}-\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}+\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*S
qrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -
7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = -\left(\left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 63, normalized size = 0.24

$$-\frac{x\sqrt{1-x^3}\left(2\left(-1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)\right)}{2\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3],x]
[Out] -1/2*(x*Sqrt[1 - x^3]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^
3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/Sqrt[-1 + x^3]
```

Maple [A]
time = 0.45, size = 407, normalized size = 1.54

method	result
meijerg	$\frac{\sqrt{-\mathrm{signum}\left(x^{3}-1\right)}\, x\, \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^{3}\right)}{\sqrt{\mathrm{signum}\left(x^{3}-1\right)}}-\frac{\sqrt{-\mathrm{signum}\left(x^{3}-1\right)}\, x^{2}\, \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^{3}\right)}{2\sqrt{\mathrm{signum}\left(x^{3}-1\right)}}-\sqrt{3}$
elliptic	$\frac{2\left(1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}-\frac{2\left(-\frac{3}{2}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I*
3^(1/2))*EllipticE(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I*3^(1/2))*EllipticF(((x-1)/(-3/2-1/2
*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))-2*3^(1
/2)*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 21, normalized size = 0.08

$$-2\left(\sqrt{3}-1\right)\mathrm{weierstrassPInverse}(0,4,x)+2\,\mathrm{weierstrassZeta}(0,4,\mathrm{weierstrassPInverse}(0,4,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(3) - 1)*weierstrassPInverse(0, 4, x) + 2*weierstrassZeta(0, 4, wei
erstrassPInverse(0, 4, x))
```

Sympy [A]
time = 1.36, size = 82, normalized size = 0.31

$$\frac{ix^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^{3}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^{3}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^{3}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - I*x*gamm
a(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + sqrt(3)*I*x*gamma(1
/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Mupad [B]
time = 4.75, size = 327, normalized size = 1.24

$$-\frac{\sqrt{3}\,x\,\sqrt{1-x^3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)}{\sqrt{x^3-1}}+\frac{6\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{6\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/(x^3 - 1)^(1/2),x)
```

```
[Out] (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2
)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)
```

```
^(1/2) - (3^(1/2)*x*(1 - x^3)^(1/2)*hypergeom([1/3, 1/2], 4/3, x^3))/(x^3 -
 1)^(1/2) - (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(
(x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) + x^3)^(1/2)
```

3.94 $\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=247

$$-\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}-\frac{4\sqrt[4]{3}\sqrt{2\cdots}}{\cdots}$$

```
[Out] -2*(-x^3-1)^(1/2)/(1+x-3^(1/2))-4*3^(1/4)*(1+x)*EllipticF((1+x+3^(1/2))/(1+
x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2)
)^2)^(1/2)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+3^(1/4)*(1+x)*Elli
pticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2
)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(
1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1894, 225, 1893}

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/
(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(
1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)],
-7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\left(\left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx\right)+\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$$

$$= -\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7\right.}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 67, normalized size = 0.27

$$\frac{x\sqrt{1+x^3}\left(-2\left(-1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

[Out] (x*Sqrt[1 + x^3]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])

Maple [A]
time = 0.35, size = 370, normalized size = 1.50

method	result
meijerg	$-ix\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)-\frac{ix^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{2}+i\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],\right.$
elliptic	$-\frac{2i\left(1-\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\right.}{3\sqrt{-x^3-1}}$
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x-3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)))+2*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Fricas [F]
time = 0.10, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [A]
time = 1.10, size = 97, normalized size = 0.39

$$-\frac{ix^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(-x**3-1)**(1/2),x)
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma
(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*
gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_pol
ar(I*pi))/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Mupad [B]
time = 4.82, size = 361, normalized size = 1.46

$$-\frac{\sqrt{3}\,x\,\sqrt{x^3+1}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)}{\sqrt{-x^3-1}}-\frac{6\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}+\frac{6\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/(- x^3 - 1)^(1/2),x)
```

```
[Out] (6*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x
^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (6*(x^3 + 1)^(1/2)*((x + (3^(1/2)
*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE
(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^
(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^
(1/2)) - (3^(1/2)*x*(x^3 + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -x^3))/(- x^
3 - 1)^(1/2)
```

3.95 $\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=126

$$-\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -2*(x^3+1)^(1/2)/(1+x+3^(1/2))+3^(1/4)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3
^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2
)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1891}

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[3] - x)/Sqrt[1 + x^3],x]
```

```
[Out] (-2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt
[1 + x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = -\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} + \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 47, normalized size = 0.37

$$\left(-1+\sqrt{3}\right)x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[3] - x)/Sqrt[1 + x^3],x]
```

```
[Out] (-1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] - (x^2*Hypergeometr
ic2F1[1/2, 2/3, 5/3, -x^3])/2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(103) = 206.
time = 0.42, size = 407, normalized size = 3.23

method	result
meijerg	$-x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)-\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{2}+\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-\right.$
elliptic	$\frac{2\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-$
default	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-2\left(\frac{3}{2}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/
2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*
((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2
*I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2
))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((x+1)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))+2*
3^(1/2)*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I
*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^
(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-
3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 21, normalized size = 0.17

$$2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,-4,x)+2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(3) - 1)*weierstrassPInverse(0, -4, x) + 2*weierstrassZeta(0, -4, we
ierstrassPInverse(0, -4, x))
```

Sympy [A]
time = 1.35, size = 92, normalized size = 0.73

$$-\frac{x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5
/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamm
a(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*p
i))/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Mupad [B]
time = 4.82, size = 312, normalized size = 2.48

$$\sqrt{3}\ x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)+\frac{6\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\ \mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)}}-\frac{6\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\ \mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 3^(1/2) + 1)/(x^3 + 1)^(1/2),x)
```

```
[Out] 3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, -x^3) + (6*((x + (3^(1/2)*1i)/2 - 1/2)
/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^
(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (6*((x + (3^(1/2)*1i)/2
- 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((
(x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1
i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1)
- ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.96 $\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}} \, dx$

Optimal. Leaf size=143

$$\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] 2*(-x^3+1)^(1/2)/(1-x+3^(1/2))-3^(1/4)*(1-x)*EllipticE((1-x-3^(1/2))/(1-x+3
^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2
)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1891}

$$\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3],x]
```

```
[Out] (2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sq
rt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1
 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = \frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 43, normalized size = 0.30

$$\frac{1}{2}x\left(2\left(-1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3],x]
```

```
[Out] (x*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + x*Hypergeometr
ic2F1[1/2, 2/3, 5/3, x^3]))/2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(120) = 240.
time = 0.36, size = 368, normalized size = 2.57

method	result
meijerg	$-x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)+\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2}+\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)$
elliptic	$-\frac{2i\left(\sqrt{3}-1\right)\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\right.}{3\sqrt{-x^3+1}}$

default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}\right.}}{}\right.}{3\sqrt{-x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)
*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(-x^3+1)^(1/2)*((-3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+EllipticF
(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I
*3^(1/2)))^(1/2)))-2*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2
+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2
)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Fricas [F]
time = 0.13, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [A]
time = 1.07, size = 97, normalized size = 0.68

$$\frac{x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma(
5/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*g
amma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(
2*I*pi))/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Mupad [B]
time = 0.05, size = 342, normalized size = 2.39

$$\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)-\frac{6\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}+\frac{6\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) - 1)/(1 - x^3)^(1/2),x)
```

```
[Out] 3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, x^3) - (6*(x^3 - 1)^(1/2)*(-(x - (3^(1
/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellip
ticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)
/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^
3)^(1/2)) + (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-
(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 -
x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.97 $\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=263

$$-\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}-\frac{4\sqrt[4]{3}\sqrt{2 \cdots}}{\cdots}$$

```
[Out] -2*(x^3-1)^(1/2)/(1-x-3^(1/2))-4*3^(1/4)*(1-x)*EllipticF((1-x+3^(1/2))/(1-x
-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))
^2)^(1/2)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)+3^(1/4)*(1-x)*Ellipt
icE((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2
)
```

Rubi [A]

time = 0.04, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1894, 225, 1893}

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[3] + x)/Sqrt[-1 + x^3],x]
```

```
[Out] (-2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/
(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*
Sqrt[-1 + x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx-\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx\\ &= -\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 63, normalized size = 0.24

$$\frac{x\sqrt{1-x^3}\left(2\left(-1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)\right)}{2\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[3] + x)/Sqrt[-1 + x^3],x]
```

```
[Out] (x*Sqrt[1 - x^3]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] +
x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])
```

Maple [A]
time = 0.40, size = 407, normalized size = 1.55

method	result
meijerg	$-\frac{\sqrt{-\mathrm{signum}\left(x^3-1\right)}\, x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)}{\sqrt{\mathrm{signum}\left(x^3-1\right)}}+\frac{\sqrt{-\mathrm{signum}\left(x^3-1\right)}\, x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2\sqrt{\mathrm{signum}\left(x^3-1\right)}}+\sqrt{3}$
elliptic	$\frac{2\left(\sqrt{3}-1\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3
/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*
((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I
*3^(1/2))*EllipticE(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))
/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I*3^(1/2))*EllipticF(((x-1)/(-3/2-1/
2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))+2*3^(
1/2)*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 21, normalized size = 0.08

$$2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,4,x)-2\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(3) - 1)*weierstrassPInverse(0, 4, x) - 2*weierstrassZeta(0, 4, weie
rstrassPInverse(0, 4, x))
```

Sympy [A]
time = 0.99, size = 82, normalized size = 0.31

$$-\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - sqrt(3)
*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + I*x*gamma(
1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Mupad [B]
time = 0.06, size = 326, normalized size = 1.24

$$\frac{\sqrt{3}\,x\,\sqrt{1-x^3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)}{\sqrt{x^3-1}}-\frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}+\frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) - 1)/(x^3 - 1)^(1/2),x)
```

```
[Out] (3^(1/2)*x*(1 - x^3)^(1/2)*hypergeom([1/3, 1/2], 4/3, x^3))/(x^3 - 1)^(1/2)
 - (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x
^3)^(1/2) + (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(
(x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) + x^3)^(1/2)
```

3.98 $\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=248

$$\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{4\sqrt[4]{3}\sqrt{2-}\cdots}{\cdots}$$

```
[Out] 2*(-x^3-1)^(1/2)/(1+x-3^(1/2))+4*3^(1/4)*(1+x)*EllipticF((1+x+3^(1/2))/(1+x
-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))
^2)^(1/2)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)-3^(1/4)*(1+x)*Ellip
ticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)
^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1
/2)
```

Rubi [A]
time = 0.04, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1894, 225, 1893}

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3],x]
```

```
[Out] (2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*S
qrt[-1 - x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx - \int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx\\ &= \frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7\right.}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 67, normalized size = 0.27

$$-\frac{x\sqrt{1+x^3}\left(-2\left(-1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 + x^3]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -
x^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/Sqrt[-1 - x^3]
```

Maple [A]
time = 0.33, size = 370, normalized size = 1.49

method	result
meijerg	$ix\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)+\frac{ix^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{2}-i\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-\cdots\right.$
elliptic	$-\frac{2i\left(\sqrt{3}-1\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\cdots}{\cdots}\right.}{3\sqrt{-x^3-1}}$
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\cdots\right.}}{\cdots}\right.}{3\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(
(x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
-x^3-1)^(1/2)*((3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*
3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/
2)))^(1/2)))-2*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Fricas [F]
time = 0.08, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [A]
time = 1.67, size = 97, normalized size = 0.39

$$\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(
5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*p
i))/(3*gamma(4/3)) + I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_pola
r(I*pi))/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Mupad [B]
time = 4.90, size = 360, normalized size = 1.45

$$\frac{\sqrt{3}\,x\,\sqrt{x^3+1}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)}{\sqrt{-x^3-1}}+\frac{6\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{6\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 3^(1/2) + 1)/(- x^3 - 1)^(1/2),x)
```

```
[Out] (3^(1/2)*x*(x^3 + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -x^3))/(- x^3 - 1)^(1
/2) + (6*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1
/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (6*(x^3 + 1)^(1/2)*((x + (3
^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ell
ipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2
)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2))^(1/2))
```

3.99 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=256

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\sin^{-1}\cdots\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a-\cdots}}$$

```
[Out] 2*(b*x^3+a)^(1/2)/b^(1/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)*(
a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(
b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1891}

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\,\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\cdots}}{\sqrt[3]{b}\,\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 90, normalized size = 0.35

$$\frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+\sqrt[3]{b}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/
2, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3
)/a)]))/(2*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. 2(189) = 378.
time = 0.33, size = 1003, normalized size = 3.92

method	result	size
default	Expression too large to display	1003

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I/b
^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(
b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*I*a^(1/3)*3^(1/2)/b*(-
a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 48, normalized size = 0.19

$$-\frac{2\left(a^{\frac{1}{3}}\sqrt{b}\left(\sqrt{3}-1\right)\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+b^{\frac{5}{6}}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -2*(a^(1/3)*sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + b^(5/
6)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A]
time = 2.26, size = 122, normalized size = 0.48

$$\frac{\sqrt[3]{b}\,x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b
*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + x*gamma(1/3)*hyper((1/3,
 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}-1\right)}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(a + b*x^3)^(1/2),x)
```

```
[Out] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(a + b*x^3)^(1/2), x)
```

3.100 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=263

$$-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\Big(\mathrm{si}}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{a}}$$

```
[Out] -2*(-b*x^3+a)^(1/2)/b^(1/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+3^(1/4)*a^(1/3
)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(
1/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {1891}

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\left(\mathrm{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{a-bx^3}}\,dx=-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 90, normalized size = 0.34

$$-\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+\sqrt[3]{b}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3
, 1/2, 4/3, (b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)
/a]))/Sqrt[a - b*x^3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 948 vs. 2(196) = 392.
time = 0.33, size = 949, normalized size = 3.61

method	result	size
default	Expression too large to display	949

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^
(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b
^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2
/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I/b^(2/3)*3^(1/
2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*
3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2
/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(
x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3
))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b
*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2
),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)
^(1/3)))^(1/2)))+2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/
b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2
)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(
1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^
(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 56, normalized size = 0.21

$$\frac{2\left(a^{\frac{1}{3}}\sqrt{-b}\left(\sqrt{3}-1\right)\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}\,b^{\frac{1}{3}}\mathrm{weierstrassZeta}\left(0,\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2*(a^(1/3)*sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - sqrt(-
b)*b^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A]
time = 3.05, size = 128, normalized size = 0.49

$$-\frac{\sqrt[3]{b}\,x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*pi
)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,)
, b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + x*gamma(1/3)*hyper(
(1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}-1\right)}{\sqrt{a-b\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(a - b*x^3)^(1/2),x)
```

```
[Out] int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(a - b*x^3)^(1/2), x)
```

3.101 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=497

$$\frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\left(\sin\right.}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a}}$$

```
[Out] 2*(b*x^3-a)^(1/2)/b^(1/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+4*3^(1/4)*a^(1/3
)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*
x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^(
1/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(
2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/
2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(1/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(
1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {1894, 225, 1893}

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{bx^3-a}}+\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) + (4*3^(1/4)*Sqrt[2 -
Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x +
```

```
b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*
Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{-a+bx^3}}\,dx = -\left(\left(2\sqrt{3}\,\sqrt[3]{a}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx\right)+\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{-a+bx^3}}\,dx$$

$$= \frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\cdots}}{\sqrt[3]{b}\sqrt{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 91, normalized size = 0.18

$$-\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+\sqrt[3]{b}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3
, 1/2, 4/3, (b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)
/a]))/Sqrt[-a + b*x^3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 951 vs. 2(375) = 750.
time = 0.31, size = 952, normalized size = 1.92

method	result	size
default	Expression too large to display	952

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(
```

```
1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^
2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I/b^(2/3)*3^(1/2
)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b
*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+
1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))
^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a
*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),
(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(
1/3)))^(1/2)))+2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*
(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*
(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^
(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2
)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/
2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 49, normalized size = 0.10

$$-\frac{2\left(a^{\frac{1}{3}}\sqrt{b}\left(\sqrt{3}-1\right)\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-b^{\frac{5}{6}}\mathrm{weierstrassZeta}\left(0,\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -2*(a^(1/3)*sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - b^(5/6
)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A]
time = 3.14, size = 112, normalized size = 0.23

$$\frac{i\sqrt[3]{b}\,x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^{3}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*g
amma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*a**(1/6)
*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(
3*a**(1/6)*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}-1\right)}{\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(b*x^3 - a)^(1/2),x)
```

```
[Out] int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(b*x^3 - a)^(1/2), x)
```

3.102 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=488

$$-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\Big(}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}$$

```
[Out] -2*(-b*x^3-a)^(1/2)/b^(1/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-4*3^(1/4)*a^(1/
3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^(1/
3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1
/2*6^(1/2)+1/2*2^(1/2))/b^(1/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)
*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {1894, 225, 1893}

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[
ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) - (4*3^(1/4)*Sqrt[2 -
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
```

```
b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4
*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{-a-bx^3}}\,dx = -\left(\left(2\sqrt{3}\sqrt[3]{a}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx\right) + \int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{-a-bx^3}}\,dx$$

$$= -\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\ \cdots}{\sqrt[3]{b}\ \cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 93, normalized size = 0.19

$$\frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+\sqrt[3]{b}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/
2, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3
)/a)]))/(2*Sqrt[-a - b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1011 vs. 2(366) = 732.
time = 0.32, size = 1012, normalized size = 2.07

method	result	size
default	Expression too large to display	1012

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-
```

```
b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I/
b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*I*a^(1/3)*3^(1/2)/b*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="ma
xima")
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 55, normalized size = 0.11

$$\frac{2\left(a^{\frac{1}{3}}\sqrt{-b}\,\left(\sqrt{3}\,-1\right)\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{-b}\,b^{\frac{1}{3}}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fr
icas")
[Out] 2*(a^(1/3)*sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + sqrt(
-b)*b^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/
b
```

Sympy [A]
time = 2.50, size = 128, normalized size = 0.26

$$-\frac{i\sqrt[3]{b}\,x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hype
r((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}-1\right)}{\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(- a - b*x^3)^(1/2),x)
```

```
[Out] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(- a - b*x^3)^(1/2), x)
```

3.103 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\, x}{\sqrt{a+bx^3}}\, dx$

Optimal. Leaf size=241

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}\,x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}\right)\right.}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

```
[Out] 2*(b/a)^(2/3)*(b*x^3+a)^(1/2)/b/(1+(b/a)^(1/3)*x+3^(1/2))-3^(1/4)*(1+(b/a)^
(1/3)*x)*EllipticE((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2)),I*3^
(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((1-(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(1+
(b/a)^(1/3)*x+3^(1/2))^2)^(1/2)/(b/a)^(1/3)/(b*x^3+a)^(1/2)/((1+(b/a)^(1/3)
*x)/(1+(b/a)^(1/3)*x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1891}

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{\frac{b}{a}}\,x-\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}\,x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}\,\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(b/a)^(2/3)*Sqrt[a + b*x^3])/(b*(1 + Sqrt[3] + (b/a)^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)^(2/
3)*x^2)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + (b
```

```
/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1/3)*
Sqrt[(1 + (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a+bx^3}}\,dx=\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}\,x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)^2}}\cdots}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)^2}}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 89, normalized size = 0.37

$$\frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2\left(-1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)
]))/(2*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(196) = 392.
time = 0.32, size = 1004, normalized size = 4.17

method	result	size
default	Expression too large to display	1004

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*
I*(b/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+2*I/b*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 52, normalized size = 0.22

$$-\frac{2\left(\sqrt{b}\left(\sqrt{3}-1\right)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + sqrt(b)*(b/a)
^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A]
time = 1.34, size = 124, normalized size = 0.51

$$\frac{x^{2}\sqrt[3]{\frac{b}{a}}\,\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a)**(1/2),x)
```

```
[Out] x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + x*gamma(1/3)*hyper((1
/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(\frac{b}{a}\right)}^{1/3}-\sqrt{3}+1}{\sqrt{b\,x^3+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(a + b*x^3)^(1/2),x)
```

```
[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(a + b*x^3)^(1/2), x)
```

3.104 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\, x}{\sqrt{a-bx^3}}\, dx$

Optimal. Leaf size=248

$$-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\, x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}\, x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}\, x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\, x\right)^2}}\, E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\, x}{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\, x}\right)\right|\left.-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}\, x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\, x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] -2*(b/a)^(2/3)*(-b*x^3+a)^(1/2)/b/(1-(b/a)^(1/3)*x+3^(1/2))+3^(1/4)*(1-(b/a
)^(1/3)*x)*EllipticE((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2)),I*
3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(
1-(b/a)^(1/3)*x+3^(1/2))^2)^(1/2)/(b/a)^(1/3)/(-b*x^3+a)^(1/2)/((1-(b/a)^(1
/3)*x)/(1-(b/a)^(1/3)*x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1891}

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}\, E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{b}{a}}\, x-\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}\, x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(2/3)*Sqrt[a - b*x^3])/(b*(1 + Sqrt[3] - (b/a)^(1/3)*x)) + (3^(1/
4)*Sqrt[2 - Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^(2
/3)*x^2)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - (
b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1/3)
*Sqrt[(1 - (b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a-bx^3}}\,dx = -\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}\,x+\left(\frac{b}{a}\right)^{2/\cdots}}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\cdots\right.}}}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}\cdots}{\left(1+\sqrt{3}-\sqrt[3]{\cdots}\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 89, normalized size = 0.36

$$-\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(-1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4
/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/Sqrt[a - b*x^3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 949 vs. 2(203) = 406.
time = 0.33, size = 950, normalized size = 3.83

method	result	size
default	Expression too large to display	950

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)
^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/
2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*
3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^
(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)
*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2
)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^
2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(
x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3
))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(a*b^2)^(1/3)))^(1/2)))-2*I/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(
1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2
/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1
/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b
^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 57, normalized size = 0.23

$$\frac{2\left(\sqrt{-b}\left(\sqrt{3}-1\right)\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\mathrm{weierstrassZeta}\left(0,\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - sqrt(-b)*(b/a)
^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A]
time = 1.60, size = 129, normalized size = 0.52

$$-\frac{x^2\sqrt[3]{\frac{b}{a}}\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*
I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4
/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + x*gamma(1/3)*hyp
er((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\sqrt{3}+x\,{\left(\frac{b}{a}\right)}^{1/3}-1}{\sqrt{a-b\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(a - b*x^3)^(1/2),x)
```

```
[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(a - b*x^3)^(1/2), x)
```

3.105 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=549

$$\frac{2\sqrt[3]{\frac{b}{a}}\,\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{}}$$

```
[Out] 2*(b/a)^(1/3)*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-2/3*
(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+
a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*(1-3^(1/2))-a^(1/3)*(b/a)^(1/3
)*(1+3^(1/2)))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(b*x^3-a)^
(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/
2)-3^(1/4)*a^(1/3)*(b/a)^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3
)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*
(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3
)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 549, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1894, 225, 1893}

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{bx^3-a}}+\frac{2\sqrt[3]{\frac{b}{a}}\,\sqrt{bx^3-a}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*(b/a)^(1/3)*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(
1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]
) - (2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/3)*(b/
```

```
a)^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]
])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a+bx^3}}\,dx = \frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{-a+bx^3}}\,dx}{\sqrt[3]{b}} - \left(-1+\sqrt{3}+\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a \cdots}}$$

$$= \frac{2\sqrt[3]{\frac{b}{a}}\,\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^2 \cdots}{\left(\left(\cdots\right.\right.}}}{b^{2/3}\sqrt{-\frac{\cdots}{\left(\left(\cdots\right.\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 90, normalized size = 0.16

$$-\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(-1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4
/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/Sqrt[-a + b*x^3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 952 vs. 2(415) = 830.
time = 0.31, size = 953, normalized size = 1.74

method	result	size
default	Expression too large to display	953

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
```

```
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^
(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b
^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2
/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3
^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(
1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*(
(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*
(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)
^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+
1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))
^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a
*b^2)^(1/3)))^(1/2)))-2*I/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/
3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b
*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2
)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)
^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="maxima")
[Out] -integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 53, normalized size = 0.10

$$-\frac{2\left(\sqrt{b}\left(\sqrt{3}-1\right)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="fricas")
[Out] -2*(sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - sqrt(b)*(b/a)^
(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A]
time = 1.45, size = 114, normalized size = 0.21

$$\frac{ix^2\sqrt[3]{\frac{b}{a}}\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-a)**(1/2),x)
```

```
[Out] I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(
a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt
(a)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a
)/(3*sqrt(a)*gamma(4/3))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(b*x^3 - a)^(1/2),x)
```

```
[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(b*x^3 - a)^(1/2), x)
```

3.106 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=540

$$-\frac{2\sqrt[3]{\frac{b}{a}}\,\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}$$

```
[Out] -2*(b/a)^(1/3)*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+2/3
*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a
^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*(1-3^(1/2))-a^(1/3)*(b/a)^(1/3)
*(1+3^(1/2)))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(-b*x^3-a)^(
1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
+3^(1/4)*a^(1/3)*(b/a)^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*
6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)
/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 540, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1894, 225, 1893}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a-bx^3}}-\frac{2\sqrt[3]{\frac{b}{a}}\,\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(1/3)*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3
```

```
]) + (2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/3)*(b
/a)^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3
]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \frac{\sqrt[3]{\frac{b}{a}}\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx}{\sqrt[3]{b}} + \left(1-\sqrt{3}-\frac{(1+\sqrt{3})\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a-\ldots}}$$

$$= -\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\ldots}}{b^{2/3}\sqrt{-\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 92, normalized size = 0.17

$$\frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2\left(-1+\sqrt{3}\right){}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)
]))/(2*Sqrt[-a - b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1012 vs. 2(406) = 812.
time = 0.31, size = 1013, normalized size = 1.88

method	result	size
default	Expression too large to display	1013

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
```

```
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3
*I*(b/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+2*I/b*(-a*b^2)^
(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="maxima")
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 56, normalized size = 0.10

$$\frac{2\left(\sqrt{-b}\left(\sqrt{3}-1\right)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="fricas")
[Out] 2*(sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + sqrt(-b)*(b/a
)^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A]
time = 1.56, size = 129, normalized size = 0.24

$$-\frac{ix^{2}\sqrt[3]{\frac{b}{a}}\,\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,),
b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*h
yper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(\frac{b}{a}\right)}^{1/3}-\sqrt{3}+1}{\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(- a - b*x^3)^(1/2),x)
```

```
[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(- a - b*x^3)^(1/2), x)
```

3.107 $\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=490

$$\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\sin\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a}}$$

```
[Out] 2*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)
*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c-a^(1/3)*d*(1
-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)
/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1892, 224, 1891}

$$\frac{2\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{b}\,c-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
```

```
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} + \left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\cdots\right.}{\left(\left(1+\sqrt{3}\right.\cdots\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 75, normalized size = 0.15

$$\frac{x\sqrt{1+\frac{bx^3}{a}}\left(2c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[a + b*x^3])
```

Maple [A]
time = 0.31, size = 720, normalized size = 1.47 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/
3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

[Out] integrate((d*x + c)/sqrt(b*x^3 + a), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 43, normalized size = 0.09

$$\frac{2\left(\sqrt{b}\,c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{b}\,d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] 2*(sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - sqrt(b)*d*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b

Sympy [A]
time = 1.04, size = 78, normalized size = 0.16

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x**3+a)**(1/2),x)

[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate((d*x + c)/sqrt(b*x^3 + a), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c+d\,x}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)/(a + b*x^3)^(1/2),x)

[Out] int((c + d*x)/(a + b*x^3)^(1/2), x)

3.108 $\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=503

$$\frac{2d\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\Big(\sin}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a-}$$

```
[Out] 2*d*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/
3)*d*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/
3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3
)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/
b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)-2/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c+a^(
1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1892, 224, 1891}

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a-bx^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a-bx^3}}+\frac{2d\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (2*d*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*
(b^(1/3)*c + (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) +
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*El
```

```
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) -
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = -\frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{a-bx^3}}\,dx}{\sqrt[3]{b}} - \left(-c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx$$

$$= \frac{2d\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{\cdots}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{\cdots}\right.}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 75, normalized size = 0.15

$$\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2c\,_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+dx\,_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + d
*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[a - b*x^3])
```

Maple [A]
time = 0.35, size = 681, normalized size = 1.35 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*d*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+
a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/
2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b
^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^
```

```
(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*c*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/
b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/
2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/
3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b
*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2
),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)
^(1/3)))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^3 + a), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 47, normalized size = 0.09

$$-\frac{2\left(\sqrt{-b}\,c\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}\,d\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(-b)*c*weierstrassPInverse(0, 4*a/b, x) - sqrt(-b)*d*weierstrassZet
a(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [**A**]
time = 1.14, size = 82, normalized size = 0.16

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**3+a)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqr
t(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_p
olar(2*I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^3 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c+d\,x}{\sqrt{a-b\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a - b*x^3)^(1/2),x)
```

```
[Out] int((c + d*x)/(a - b*x^3)^(1/2), x)
```

3.109 $\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=515

$$-\frac{2d\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\left(\cdots\right.}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\ \cdots}$$

```
[Out] -2*d*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-2/3*(a^(1/3)-
b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(
1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*c+a^(1/3)*d*(1+3^(1/2)))*((a^(2/3)+a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6
^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1
/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*d*(a^(1/3)
-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*
(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(b*x^
3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2
)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1894, 225, 1893}

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{bx^3-a}}+\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{bx^3-a}}-\frac{2d\sqrt{bx^3-a}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (-2*d*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/
((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) - (2*Sqrt[2 - Sqr
t[3]]*(b^(1/3)*c + (1 + Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
```

```
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = -\frac{d\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt{-a+bx^3}}\,dx}{\sqrt[3]{b}} - \left(-c-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx$$

$$= -\frac{2d\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{}}{\left(\left(1-\sqrt{}\right.\right.}}}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}}{\left(\left(1-\right.\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 76, normalized size = 0.15

$$\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + d
*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[-a + b*x^3])
```

Maple [A]
time = 0.35, size = 683, normalized size = 1.33 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*d*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a
)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*
b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^
2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(
```

```
1/2)/b*(a*b^2)^(1/3)))^(1/2))+2/3*I*c*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b
*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2
)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3
)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*
b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),
(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(
1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^3 - a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 43, normalized size = 0.08

$$\frac{2\left(\sqrt{b}\, c \mathrm{weierstrassPInverse}\left(0, \frac{4a}{b}, x\right) - \sqrt{b}\, d \mathrm{weierstrassZeta}\left(0, \frac{4a}{b}, \mathrm{weierstrassPInverse}\left(0, \frac{4a}{b}, x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(b)*c*weierstrassPInverse(0, 4*a/b, x) - sqrt(b)*d*weierstrassZeta(0
, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A]
time = 1.14, size = 73, normalized size = 0.14

$$-\frac{icx\Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3-a)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt(a)*gamma(4/3)
) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*gamm
a(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^3 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c+d\,x}{\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(b*x^3 - a)^(1/2),x)
```

```
[Out] int((c + d*x)/(b*x^3 - a)^(1/2), x)
```

3.110 $\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=508

$$-\frac{2d\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\Big(\mathrm{s}}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-}}$$

```
[Out] -2*d*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+2/3*(a^(1/3)+
b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-
3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*c-a^(1/3)*d*(1+3^(1/2)))*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1
/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*d*(a^(1/3)+b^
(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^
(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-b*x^3-a)^
(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)
```

Rubi [A]

time = 0.11, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1894, 225, 1893}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{b}\,c-\left(1+\sqrt{3}\right)\sqrt[3]{a}\,d\right)F\left(\mathrm{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a-bx^3}}-\frac{2d\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*d*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) + (2*Sqrt[2 - Sqr
t[3]]*(b^(1/3)*c - (1 + Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
```

```
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = \frac{d\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{-a-bx^3}}\,dx}{\sqrt[3]{b}} + \left(c-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx$$

$$= -\frac{2d\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\cdots}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\cdots}{\left(\left(1-\sqrt{\cdots}\right.\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 78, normalized size = 0.15

$$\frac{x\sqrt{1+\frac{bx^3}{a}}\left(2c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[-a - b*x^3])
```

Maple [A]
time = 0.33, size = 726, normalized size = 1.43 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
```

```
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*c*3^(1/2)/b*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^3 - a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 47, normalized size = 0.09

$$-\frac{2\left(\sqrt{-b}\,c\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{-b}\,d\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(-b)*c*weierstrassPInverse(0, -4*a/b, x) - sqrt(-b)*d*weierstrassZe
ta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A]
time = 1.14, size = 83, normalized size = 0.16

$$-\frac{icx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}-\frac{idx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)/sqrt(-b*x^3 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c+d\,x}{\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(- a - b*x^3)^(1/2),x)
[Out] int((c + d*x)/(- a - b*x^3)^(1/2), x)
```

3.111 $\int \frac{c+dx}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=246

$$\frac{2d\sqrt{1+x^3}}{1+\sqrt{3}+x}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{2\sqrt{2+}}{}$$

```
[Out] 2*d*(x^3+1)^(1/2)/(1+x+3^(1/2))-3^(1/4)*d*(1+x)*EllipticE((1+x-3^(1/2))/(1+
x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2)
)^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+2/3*(1+x)*EllipticF(
(1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(c-d*(1-3^(1/2)))*(1/2*6^(1/2)+1
/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/
(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1892, 224, 1891}

$$\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2d\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[1 + x^3],x]
```

```
[Out] (2*d*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d*(1 + x
)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x
)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]]*(c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1
- x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqr
t[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqr
t[1 + x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = d\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx + \left(c-\left(1-\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2d\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 42, normalized size = 0.17

$$cx\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{1}{2}dx^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[1 + x^3],x]
```

[Out] c*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (d*x^2*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3])/2

Maple [A]
time = 0.40, size = 291, normalized size = 1.18

method	result
meijerg	$\frac{d\,x^{2}\,\text{hypergeom}([\frac{1}{2},\frac{2}{3}],[\frac{5}{3}],-x^{3})}{2}+c x\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^{3}\right)$
default	$\frac{2d\big(\frac{3}{2}-\frac{i\sqrt{3}}{2}\big)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\Big(\big(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\big)\text{EllipticE}\Big(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{\sqrt{x^{3}+1}}$
elliptic	$\frac{2d\big(\frac{3}{2}-\frac{i\sqrt{3}}{2}\big)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\Big(\big(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\big)\text{EllipticE}\Big(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{\sqrt{x^{3}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*d*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))+2*c*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(x^3 + 1), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 18, normalized size = 0.07

$$2\,c\text{weierstrassPInverse}(0,-4,x)-2\,d\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*c*weierstrassPInverse(0, -4, x) - 2*d*weierstrassZeta(0, -4, weierstrassP
Inverse(0, -4, x))
```

Sympy [A]
time = 0.87, size = 61, normalized size = 0.25

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{dx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3
)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*g
amma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(x^3 + 1), x)
```

Mupad [B]
time = 4.77, size = 373, normalized size = 1.52

$$-\frac{2d\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{E}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}}{\sqrt{x^{3}+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}+\frac{2c\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^{3}+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(x^3 + 1)^(1/2),x)
```

```
[Out] (2*c*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*d*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(
```

```
((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*
1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1
/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*
((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2
)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.112 $\int \frac{c+dx}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=271

$$\frac{2d\sqrt{1-x^3}}{1+\sqrt{3}-x}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}\cdots}{\cdots}$$

```
[Out] 2*d*(-x^3+1)^(1/2)/(1-x+3^(1/2))-3^(1/4)*d*(1-x)*EllipticE((1-x-3^(1/2))/(1
-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2
))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-2/3*(1-x)*Elliptic
F((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(c+d-d*3^(1/2))*(1/2*6^(1/2)+1
/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)
/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {1892, 224, 1891}

$$-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}\,d+d\right)F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{2d\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[1 - x^3],x]
```

```
[Out] (2*d*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d*(1 - x
)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*S
qrt[1 - x^3]) - (2*Sqrt[2 + Sqrt[3]]*(c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 +
x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[
3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = -\left(d\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx\right)+\left(c+d-\sqrt{3}\,d\right)\int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= \frac{2d\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 38, normalized size = 0.14

$$cx\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+\frac{1}{2}dx^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[1 - x^3],x]
```

```
[Out] c*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + (d*x^2*Hypergeometric2F1[1/2, 2
/3, 5/3, x^3])/2
```

Maple [A]
time = 0.38, size = 267, normalized size = 0.99

method	result
meijerg	$\frac{d\,x^{2} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^{3}\right)}{2}+c x \operatorname{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^{3}\right)$
default	$-\frac{2 i d \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \left(\left(-\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) \operatorname{EllipticE}\right.}{3 \sqrt{}}$
elliptic	$-\frac{2 i d \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \left(\left(-\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) \operatorname{EllipticE}\right.}{3 \sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*((
-3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))
^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+EllipticF(1/3*3^(1/2)*(I*(x+
1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)))-
2/3*I*c*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-x^3 + 1), x)
```

Fricas [F]
time = 0.08, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [A]
time = 1.01, size = 65, normalized size = 0.24

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4
/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/
(3*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-x^3 + 1), x)
```

Mupad [B]
time = 5.07, size = 406, normalized size = 1.50

$$-\frac{2c\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{2d\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(1 - x^3)^(1/2),x)
```

```
[Out] - (2*c*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/
((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 +
```

```
 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) -
x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2*d*
(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3
/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)
/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(
-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*
(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.113 $\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=275

$$-\frac{2d\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,d(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}-\frac{2\sqrt{2-}}{}$$

```
[Out] -2*d*(x^3-1)^(1/2)/(1-x-3^(1/2))-2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(
1/2)),2*I-I*3^(1/2))*(c+d+d*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(
1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)+
3^(1/4)*d*(1-x)*EllipticE((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*((x^2+
x+1)/(1-x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(x^3-1)^(1/2)/((-1+x)
/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1894, 225, 1893}

$$-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,d(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2d\sqrt{x^3-1}}{-x-\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[-1 + x^3],x]
```

```
[Out] (-2*d*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*d*(1 -
 x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
 x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^
2)]*Sqrt[-1 + x^3]) - (2*Sqrt[2 - Sqrt[3]]*(c + d + Sqrt[3]*d)*(1 - x)*Sqrt
[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 -
 Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^
2)]*Sqrt[-1 + x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = -\left(d\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx\right)+\left(c+d+\sqrt{3}\,d\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2d\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,d(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 58, normalized size = 0.21

$$\frac{x\sqrt{1-x^3}\left(2c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+dx\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)\right)}{2\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[-1 + x^3],x]
```

```
[Out] (x*Sqrt[1 - x^3]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + d*x*Hypergeom
etric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])
```

Maple [A]
time = 0.38, size = 291, normalized size = 1.06

method	result
meijerg	$\frac{d\sqrt{-\mathrm{signum}\left(x^3-1\right)}\, x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2\sqrt{\mathrm{signum}\left(x^3-1\right)}}+\frac{c\sqrt{-\mathrm{signum}\left(x^3-1\right)}\, x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)}{\sqrt{\mathrm{signum}\left(x^3-1\right)}}$
default	$\frac{2d\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+i\sqrt{}}{\frac{3}{2}-i\sqrt{}}}\right.\right.}{\sqrt{x^3-1}}$
elliptic	$\frac{2d\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+i\sqrt{}}{\frac{3}{2}-i\sqrt{}}}\right.\right.}{\sqrt{x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I*3^(1/2))*EllipticE(((x-1)/(-3/2-1/2*I*3
^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I
*3^(1/2))*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))
/(3/2-1/2*I*3^(1/2)))^(1/2)))+2*c*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3
^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/
2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(x^3 - 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 18, normalized size = 0.07

$$2\,c\mathrm{weierstrassPInverse}(0,4,x)-2\,d\mathrm{weierstrassZeta}(0,4,\mathrm{weierstrassPInverse}(0,4,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*c*weierstrassPInverse(0, 4, x) - 2*d*weierstrassZeta(0, 4, weierstrassPIn
verse(0, 4, x))
```

Sympy [A]
time = 0.88, size = 56, normalized size = 0.20

$$-\frac{icx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^{3}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{idx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^{3}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - I*d*x**2
*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(x^3 - 1), x)
```

Mupad [B]
time = 0.12, size = 374, normalized size = 1.36

$$-\frac{2\,c\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}}-\frac{2\,d\,\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)\right)\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(x^3 - 1)^(1/2),x)
```

```
[Out] - (2*c*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
- 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*d*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(
asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^
```

```
(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))
*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

3.114 $\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=261

$$-\frac{2d\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,d(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{2\sqrt{2-}}{}$$

[Out] -2*d*(-x^3-1)^(1/2)/(1+x-3^(1/2))+2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(c-d*(1+3^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+3^(1/4)*d*(1+x)*EllipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1894, 225, 1893}

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,d(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2d\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/Sqrt[-1 - x^3],x]

[Out] (-2*d*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*d*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (2*Sqrt[2 - Sqrt[3]]*(c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = d\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx + \left(c-\left(1+\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= -\frac{2d\sqrt{-1-x^3}}{1-\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,d(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\right|}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 62, normalized size = 0.24

$$\frac{x\sqrt{1+x^3}\left(2c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+dx\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[-1 - x^3],x]
```

```
[Out] (x*Sqrt[1 + x^3]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + d*x*Hypergeo
metric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

Maple [A]
time = 0.34, size = 269, normalized size = 1.03

method	result
meijerg	$-\frac{i d x^{2} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^{3}\right)}{2}-i c x \operatorname{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^{3}\right)$
default	$-\frac{2 i d \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \left(\left(\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) \operatorname{EllipticE}\left(\sqrt{\cdots}\right.\right.}{3 \sqrt{-\cdots}}$
elliptic	$-\frac{2 i d \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \left(\left(\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) \operatorname{EllipticE}\left(\sqrt{\cdots}\right.\right.}{3 \sqrt{-\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3
/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)))-2/3*
I*c*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/
2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I
*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-x^3 - 1), x)
```

Fricas [F]
time = 0.07, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [A]
time = 0.92, size = 66, normalized size = 0.25

$$-\frac{icx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{idx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(
4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))
/(3*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-x^3 - 1), x)
```

Mupad [B]
time = 4.82, size = 405, normalized size = 1.55

$$\frac{2\,c\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\,d\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(- x^3 - 1)^(1/2),x)
```

```
[Out] (2*c*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2
```

```
)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*d*(((
3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))
, -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*
ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x +
(3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/
((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
+ 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.115 $\int \frac{c+dx}{a-bx^4}\, dx$

Optimal. Leaf size=87

$$\frac{c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

```
[Out] 1/2*c*arctan(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(1/4)+1/2*c*arctanh(b^(1/4)*x/a^(
1/4))/a^(3/4)/b^(1/4)+1/2*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1890, 218, 214, 211, 281}

$$\frac{c\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - b*x^4),x]
```

```
[Out] (c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (c*ArcTanh[(b^(1/4)*x
)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqr
t[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{a-bx^4}\,dx &= \int \left(\frac{c}{a-bx^4}+\frac{dx}{a-bx^4}\right)dx\\
&= c\int \frac{1}{a-bx^4}\,dx + d\int \frac{x}{a-bx^4}\,dx\\
&= \frac{c\int \frac{1}{\sqrt{a}-\sqrt{b}\,x^2}\,dx}{2\sqrt{a}} + \frac{c\int \frac{1}{\sqrt{a}+\sqrt{b}\,x^2}\,dx}{2\sqrt{a}} + \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)\\
&= \frac{c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 134, normalized size = 1.54

$$
\frac{2\sqrt[4]{b}\,c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\left(\sqrt[4]{b}\,c+\sqrt[4]{a}\,d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+\sqrt[4]{b}\,c\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)-\sqrt[4]{a}\,d\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+\sqrt[4]{a}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{4a^{3/4}\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - b*x^4),x]
```

```
[Out] (2*b^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(1/4)*c + a^(1/4)*d)*Log[a^(1
/4) - b^(1/4)*x] + b^(1/4)*c*Log[a^(1/4) + b^(1/4)*x] - a^(1/4)*d*Log[a^(1/
4) + b^(1/4)*x] + a^(1/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*a^(3/4)*Sqrt[b])
```

Maple [A]
time = 0.33, size = 87, normalized size = 1.00

method	result	size

risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	34
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(
1/4)))+1/4*d/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(57) = 114$.
time = 0.51, size = 126, normalized size = 1.45

$$\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}}-\frac{d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}}-\frac{c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + 1/4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*d*log(sqrt(b)
*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*c*log((sqrt(b)*x - sqrt(sqrt(a)*sqr
t(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.31, size = 39057, normalized size = 448.93

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/24*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((a*b*sqrt(c^2/(a^2*b*sqrt(1/(a*b))))
*sqrt(1/(a*b)) + d)^2/(a*b) + 3*((d^2*sqrt(1/(a*b)) - 2*d*sqrt(c^2/(a^2*b*s
qrt(1/(a*b)))))*a - c^2)/(a^2*b*sqrt(1/(a*b))))/(9*(a*b*sqrt(c^2/(a^2*b*sqr
t(1/(a*b))))*sqrt(1/(a*b)) + d)*((d^2*sqrt(1/(a*b)) - 2*d*sqrt(c^2/(a^2*b*s
qrt(1/(a*b)))))*a - c^2)/(a^2*b) + 27*(a^2*b^2*(c^2/(a^2*b*sqrt(1/(a*b))))^
```

```
(3/2)*sqrt(1/(a*b)) + a*b*d^2*sqrt(c^2/(a^2*b*sqrt(1/(a*b))))*sqrt(1/(a*b))
 - b*c^2*d*sqrt(1/(a*b)) - d^3)/(a^2*b^2*sqrt(1/(a*b))) + 2*(a*b*sqrt(c^2/(
a^2*b*sqrt(1/(a*b))))*sqrt(1/(a*b)) + d)^3/(a^3*b^3*(1/(a*b))^(3/2)) + sqrt
(108*b^3*c^6*(1/(a*b))^(3/2) + 36*(43*a^2*b^3*(1/(a*b))^(5/2) + 6*a*b^2*(1/
(a*b))^(3/2) - b*sqrt(1/(a*b)))*c^2*d^4 - 48*(8*a^4*b^4*(1/(a*b))^(7/2) + 3
*a^3*b^3*(1/(a*b))^(5/2) - 12*a^2*b^2*(1/(a*b))^(3/2) + a*b*sqrt(1/(a*b)))*
d^5*sqrt(c^2/(a^2*b*sqrt(1/(a*b)))) - 27*b*c^6/(a^2*sqrt(1/(a*b))) - 4*(648
*b*c^2*sqrt(c^2/(a^2*b*sqrt(1/(a*b)))) + (56*a^6*b^6*(1/(a*b))^(9/2) - 216*
a^5*b^5*(1/(a*b))^(7/2) + ...
```

Sympy [A]
time = 0.47, size = 126, normalized size = 1.45

$$-\operatorname{RootSum}\left(256t^4a^3b^2 - 32t^2a^2bd^2 - 16tabc^2d + ad^4 - bc^4, \left(t \mapsto t\log\left(x + \frac{-128t^3a^3bd^2 + 16t^2a^2bc^2d + 8ta^2d^4 - 4tabc^4 + 5ac^2d^3}{4acd^4 + bc^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a*
d**4 - b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 + 16*_t**2*a**
2*b*c**2*d + 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4 +
b*c**5))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(57) = 114.
time = 0.80, size = 225, normalized size = 2.59

$$\frac{\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2 + \sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2 - \sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8\,ab} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}\,bd + \left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}\,bd + \left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))
/(a*b) - 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sq
rt(-a/b))/(a*b) + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d + (-a*b^3)^(1/4)*b*c)
*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2) + 1/
4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d + (-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2)
```

Mupad [B]
time = 5.01, size = 182, normalized size = 2.09

$$\begin{cases} \frac{2\,c+3\,d\,x}{6\,b\,x^3} & \text{if}\ \ a=0 \\ \frac{\operatorname{atan}\left(\frac{\sqrt{2}\,{\left(-b\right)}^{1/4}\,x}{a^{1/4}}-1\right)\,\left(2\,a^{1/4}\,d+\sqrt{2}\,{\left(-b\right)}^{1/4}\,c\right)}{4\,a^{3/4}\,\sqrt{-b}} - \frac{\operatorname{atan}\left(\frac{\sqrt{2}\,{\left(-b\right)}^{1/4}\,x}{a^{1/4}}+1\right)\,\left(4\,a^{1/4}\,d-2\,\sqrt{2}\,{\left(-b\right)}^{1/4}\,c\right)}{8\,a^{3/4}\,\sqrt{-b}} + \frac{\sqrt{2}\,c\,\ln\left(\frac{\sqrt{-b}\,x^2+\sqrt{a}+\sqrt{2}\,a^{1/4}\,{\left(-b\right)}^{1/4}\,x}{\sqrt{-b}\,x^2+\sqrt{a}-\sqrt{2}\,a^{1/4}\,{\left(-b\right)}^{1/4}\,x}\right)}{8\,a^{3/4}\,{\left(-b\right)}^{1/4}} & \text{if}\ \ a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)/(a - b*x^4),x)

[Out] piecewise(a == 0, (2*c + 3*d*x)/(6*b*x^3), a ~= 0, (atan((2^(1/2)*(-b)^(1/4
)*x)/a^(1/4) - 1)*(2*a^(1/4)*d + 2^(1/2)*(-b)^(1/4)*c))/(4*a^(3/4)*(-b)^(1/
2)) - (atan((2^(1/2)*(-b)^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*d - 2*2^(1/2)*(-
b)^(1/4)*c))/(8*a^(3/4)*(-b)^(1/2)) + (2^(1/2)*c*log(((-b)^(1/2)*x^2 + a^(1
/2) + 2^(1/2)*a^(1/4)*(-b)^(1/4)*x)/((-b)^(1/2)*x^2 + a^(1/2) - 2^(1/2)*a^(
1/4)*(-b)^(1/4)*x)))/(8*a^(3/4)*(-b)^(1/4)))

3.116 $\int \frac{c+dx}{a+bx^4} \, dx$

Optimal. Leaf size=219

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{c\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}-\frac{c\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x\right.}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

```
[Out] 1/4*c*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)*2^(1/2)+1/4*c*ar
ctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)*2^(1/2)-1/8*c*ln(-a^(1/4)
*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)+1/8*c*ln(a^
(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)+1/2*d*
arctan(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$-\frac{c\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{c\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}-\frac{c\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (c*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*ArcTan[1 + (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (c*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1
/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx}{a+bx^4}\,dx = \int \left(\frac{c}{a+bx^4}+\frac{dx}{a+bx^4}\right)dx$$
$$= c\int \frac{1}{a+bx^4}\,dx + d\int \frac{x}{a+bx^4}\,dx$$
$$= \frac{c\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}dx}{2\sqrt{a}} + \frac{c\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}dx}{2\sqrt{a}} + \frac{1}{2}d\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)$$
$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}} + \frac{c\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}dx}{4\sqrt{a}\,\sqrt{b}} + \frac{c\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}dx}{4\sqrt{a}\,\sqrt{b}} - \frac{c\int \frac{\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{b}}\ldots}{-\frac{\sqrt{a}}{\sqrt{b}}-\ldots}}{4\sqrt{2}\,a\ldots}$$
$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}} - \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\ldots\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$
$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}} - \frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}} - \frac{c\log\left(\sqrt{a}-\ldots\right)}{4\ldots}$$

Mathematica [A]
time = 0.06, size = 184, normalized size = 0.84

$$\frac{-2\left(\sqrt{2}\sqrt[4]{b}\,c+2\sqrt[4]{a}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{b}\,c-2\sqrt[4]{a}\,d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\sqrt{2}\sqrt[4]{b}\,c\left(-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)\right)}{8a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*b^(1/4)*c + 2*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] + 2*(Sqrt[2]*b^(1/4)*c - 2*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)] + Sqrt[2]*b^(1/4)*c*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a^(3/
4)*Sqrt[b])
```

Maple [A]
time = 0.34, size = 124, normalized size = 0.57

method	result	size
risch	$\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^4+a\right)} \frac{\left(\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	32

default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\arctan\left(x^{2}\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^
2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))
```

Maxima [A]
time = 0.55, size = 207, normalized size = 0.95

$$\frac{\sqrt{2}\,c\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,c\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}c-2\,\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{1}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}c+2\,\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - 1/8*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + s
qrt(a))/(a^(3/4)*b^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(1/4)*c - 2*sqrt(a)*d)*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(1/4
)*c + 2*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4
))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.47, size = 41851, normalized size = 191.10

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(d*sqrt(-1/(a*b)) + sqrt(-c^2/(a^2*b*sqrt(-1/(a*b)))))*log(2*(d*sqrt(-
1/(a*b)) + sqrt(-c^2/(a^2*b*sqrt(-1/(a*b)))))^3*a^3*b*d^2 - (d*sqrt(-1/(a*b
)) + sqrt(-c^2/(a^2*b*sqrt(-1/(a*b)))))^2*a^2*b*c^2*d + 5*a*c^2*d^3 + (a*b*
c^4 + 2*a^2*d^4)*(d*sqrt(-1/(a*b)) + sqrt(-c^2/(a^2*b*sqrt(-1/(a*b))))) - (
b*c^5 - 4*a*c*d^4)*x) + 1/24*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((a*b*sqrt(-c^
2/(a^2*b*sqrt(-1/(a*b))))*sqrt(-1/(a*b)) - d)^2/(a*b) + 3*((d^2*sqrt(-1/(a*
b)) - 2*d*sqrt(-c^2/(a^2*b*sqrt(-1/(a*b)))))*a - c^2)/(a^2*b*sqrt(-1/(a*b))
))/(9*(a*b*sqrt(-c^2/(a^2*b*sqrt(-1/(a*b))))*sqrt(-1/(a*b)) - d)*((d^2*sqrt
```

```
(-1/(a*b)) - 2*d*sqrt(-c^2/(a^2*b*sqrt(-1/(a*b)))))*a - c^2)/(a^2*b) + 27*(
a^2*b^2*(-c^2/(a^2*b*sqrt(-1/(a*b))))^(3/2)*sqrt(-1/(a*b)) - a*b*d^2*sqrt(-
c^2/(a^2*b*sqrt(-1/(a*b))))*sqrt(-1/(a*b)) + b*c^2*d*sqrt(-1/(a*b)) - d^3)/
(a^2*b^2*sqrt(-1/(a*b))) + 2*(a*b*sqrt(-c^2/(a^2*b*sqrt(-1/(a*b))))*sqrt(-1
/(a*b)) - d)^3/(a^3*b^3*(-1/(a*b))^(3/2)) + sqrt(-108*b^3*c^6*(-1/(a*b))^(3
/2) - 36*(43*a^2*b^3*(-1/ ...
```

Sympy [A]
time = 0.45, size = 124, normalized size = 0.57

$$\operatorname{RootSum}\left(256t^4a^3b^2 + 32t^2a^2bd^2 - 16tabc^2d + ad^4 + bc^4, \left(t \mapsto t\log\left(x + \frac{-128t^3a^3bd^2 - 16t^2a^2bc^2d - 8ta^2d^4 - 4tabc^4 + 5ac^2d^3}{4acd^4 - bc^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a*d
**4 + b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 - 16*_t**2*a**2
*b*c**2*d - 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4 - b
*c**5))))
```

Giac [A]
time = 1.76, size = 213, normalized size = 0.97

$$\frac{\sqrt{2}\,(ab^3)^{\frac{1}{4}}c\log\left(x^2 + \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\,(ab^3)^{\frac{1}{4}}c\log\left(x^2 - \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{ab}\,bd - (ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} - \frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{ab}\,bd - (ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
*b) - 1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a*b) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) - 1/4*sqrt(2)*
(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2)
```

Mupad [B]
time = 4.80, size = 160, normalized size = 0.73

$$\begin{cases} -\frac{2\,c+3\,d\,x}{6\,b\,x^3} & \text{if } a=0\\ \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{a^{1/4}}-1\right)\left(2\,a^{1/4}\,d+\sqrt{2}\,b^{1/4}\,c\right)}{4\,a^{3/4}\,\sqrt{b}} - \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{a^{1/4}}+1\right)\left(4\,a^{1/4}\,d-2\,\sqrt{2}\,b^{1/4}\,c\right)}{8\,a^{3/4}\,\sqrt{b}} + \frac{\sqrt{2}\,c\,\ln\left(\frac{\sqrt{a}+\sqrt{b}\,x^2+\sqrt{2}\,a^{1/4}\,b^{1/4}\,x}{\sqrt{a}+\sqrt{b}\,x^2-\sqrt{2}\,a^{1/4}\,b^{1/4}\,x}\right)}{8\,a^{3/4}\,b^{1/4}} & \text{if } a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^4),x)
```

```
[Out] piecewise(a == 0, -(2*c + 3*d*x)/(6*b*x^3), a ~= 0, (atan((2^(1/2)*b^(1/4)*
x)/a^(1/4) - 1)*(2*a^(1/4)*d + 2^(1/2)*b^(1/4)*c))/(4*a^(3/4)*b^(1/2)) - (a
tan((2^(1/2)*b^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*d - 2*2^(1/2)*b^(1/4)*c))/(
8*a^(3/4)*b^(1/2)) + (2^(1/2)*c*log((a^(1/2) + b^(1/2)*x^2 + 2^(1/2)*a^(1/4
)*b^(1/4)*x)/(a^(1/2) + b^(1/2)*x^2 - 2^(1/2)*a^(1/4)*b^(1/4)*x)))/(8*a^(3/
4)*b^(1/4)))
```

3.117 $\int \frac{c+dx}{\left(a-bx^4\right)^2}\,dx$

Optimal. Leaf size=110

$$\frac{x(c+dx)}{4a\left(a-bx^4\right)}+\frac{3c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

```
[Out] 1/4*x*(d*x+c)/a/(-b*x^4+a)+3/8*c*arctan(b^(1/4)*x/a^(1/4))/a^(7/4)/b^(1/4)+
3/8*c*arctanh(b^(1/4)*x/a^(1/4))/a^(7/4)/b^(1/4)+1/4*d*arctanh(x^2*b^(1/2)/
a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1869, 1890, 218, 214, 211, 281}

$$\frac{3c\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x(c+dx)}{4a\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - b*x^4)^2,x]
```

```
[Out] (x*(c + d*x))/(4*a*(a - b*x^4)) + (3*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7
/4)*b^(1/4)) + (3*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(1/4)) + (d*
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{\left(a-bx^4\right)^2}\,dx &= \frac{x(c+dx)}{4a\left(a-bx^4\right)}-\frac{\int \frac{-3c-2dx}{a-bx^4}\,dx}{4a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)}-\frac{\int \left(-\frac{3c}{a-bx^4}-\frac{2dx}{a-bx^4}\right)\,dx}{4a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)}+\frac{(3c)\int \frac{1}{a-bx^4}\,dx}{4a}+\frac{d\int \frac{x}{a-bx^4}\,dx}{2a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)}+\frac{(3c)\int \frac{1}{\sqrt{a}-\sqrt{b}\,x^2}\,dx}{8a^{3/2}}+\frac{(3c)\int \frac{1}{\sqrt{a}+\sqrt{b}\,x^2}\,dx}{8a^{3/2}}+\frac{d\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x\right.}{4a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)}+\frac{3c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 168, normalized size = 1.53

$$\frac{\frac{4ax(c+dx)}{a-bx^4}+\frac{6\sqrt[4]{a}\,c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}-\frac{\left(3\sqrt[4]{a}\,\sqrt[4]{b}\,c+2\sqrt{a}\,d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)}{\sqrt{b}}+\frac{\left(3\sqrt[4]{a}\,\sqrt[4]{b}\,c-2\sqrt{a}\,d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{\sqrt{b}}+\frac{2\sqrt{a}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt{b}}}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*x*(c + d*x))/(a - b*x^4) + (6*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/
b^(1/4) - ((3*a^(1/4)*b^(1/4)*c + 2*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sq
rt[b] + ((3*a^(1/4)*b^(1/4)*c - 2*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/Sqrt
[b] + (2*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)
```

Maple [A]
time = 0.38, size = 128, normalized size = 1.16

method	result	size
risch	$\frac{\frac{d\,x^2}{4a}+\frac{cx}{4a}}{-b\,x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(2\_Rd+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$	69
default	$c\left(\frac{x}{4a(-b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{16a^2}\right)+d\left(\frac{x^2}{4a(-b\,x^4+a)}+\frac{\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)$	128

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/4*x/a/(-b*x^4+a)+3/16/a^2*(a/b)^(1/4)*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/
4)))+2*arctan(x/(a/b)^(1/4))))+d*(1/4*x^2/a/(-b*x^4+a)+1/8/a/(a*b)^(1/2)*ln
((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2))))
```

Maxima [A]
time = 0.53, size = 157, normalized size = 1.43

$$-\frac{dx^2+cx}{4\,(abx^4-a^2)}+\frac{\frac{6\,c\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,d\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{2\,d\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{3\,c\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(d*x^2 + c*x)/(a*b*x^4 - a^2) + 1/16*(6*c*arctan(sqrt(b)*x/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*d*log(sqrt(b)*x^2 + sqrt(a)
)/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 3*
c*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b)
)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))))/a
```

Fricas [C] Result contains complex when optimal does not.
time = 1.42, size = 40560, normalized size = 368.73

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] -1/192*(48*d*x^2 + 2*(a*b*x^4 - a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((3*a^
2*b*sqrt(1/(a*b))*sqrt(c^2/(a^4*b*sqrt(1/(a*b)))) + 2*d)^2/(a^3*b) + 3*(4*a
*d^2*sqrt(1/(a*b)) - 12*a^2*d*sqrt(c^2/(a^4*b*sqrt(1/(a*b)))) - 9*c^2)/(a^4
*b*sqrt(1/(a*b))))/(9*(3*a^2*b*sqrt(1/(a*b))*sqrt(c^2/(a^4*b*sqrt(1/(a*b)))
) + 2*d)*(4*a*d^2*sqrt(1/(a*b)) - 12*a^2*d*sqrt(c^2/(a^4*b*sqrt(1/(a*b))))
- 9*c^2)/(a^5*b) + 27*(27*a^5*b^2*sqrt(1/(a*b))*(c^2/(a^4*b*sqrt(1/(a*b))))
^(3/2) + 12*a^2*b*d^2*sqrt(1/(a*b))*sqrt(c^2/(a^4*b*sqrt(1/(a*b)))) - 18*b*
c^2*d*sqrt(1/(a*b)) - 8*d^3)/(a^5*b^2*sqrt(1/(a*b))) + 2*(3*a^2*b*sqrt(1/(a
*b))*sqrt(c^2/(a^4*b*sqrt(1/(a*b)))) + 2*d)^3/(a^6*b^3*(1/(a*b))^(3/2)) + 3
*sqrt(8748*b^3*c^6*(1/(a*b))^(3/2) + 576*(43*a^2*b^3*(1/(a*b))^(5/2) + 6*a*
b^2*(1/(a*b))^(3/2) - b*sqrt(1/(a*b)))*c^2*d^4 - 512*(8*a^5*b^4*(1/(a*b))^(
7/2) + 3*a^4*b^3*(1/(a*b))^(5/2) - 12*a^3*b^2*(1/(a*b))^(3/2) + a^2*b*sqrt(
1/(a*b)))*d^5*sqrt(c^2/(a^4*b*sqrt(1/(a*b)))) - 2187*b*c^6/(a^2*sqrt(1/(a*b
))) - 96*(648*a*b*c^2*sqr ...
```

Sympy [A]
time = 0.61, size = 156, normalized size = 1.42

$$\mathrm{RootSum}\left(65536t^4a^7b^2-2048t^2a^4bd^2+1152ta^2bc^2d+16ad^4-81bc^4,\left(t\mapsto t\log\left(x+\frac{32768t^3a^6bd^2+4608t^2a^4bc^2d-512ta^3d^4+1296ta^2bc^4+360ac^2d^3}{192acd^4+243bc^5}\right)\right)\right)+\frac{-cx-dx^2}{-4a^2+4abx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**2 - 2048*_t**2*a**4*b*d**2 + 1152*_t*a**2*b*c**
2*d + 16*a*d**4 - 81*b*c**4, Lambda(_t, _t*log(x + (32768*_t**3*a**6*b*d**2
 + 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 + 1296*_t*a**2*b*c**4 + 360*
a*c**2*d**3)/(192*a*c*d**4 + 243*b*c**5)))) + (-c*x - d*x**2)/(-4*a**2 + 4*
a*b*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(79) = 158.
time = 1.59, size = 254, normalized size = 2.31

$$\frac{3\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b}-\frac{3\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b}-\frac{dx^2+cx}{4\left(bx^4-a\right)a}-\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}\,bd-3\left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2}-\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}\,bd-3\left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b)
)/(a^2*b) - 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4)
+ sqrt(-a/b))/(a^2*b) - 1/4*(d*x^2 + c*x)/((b*x^4 - a)*a) - 1/16*sqrt(2)*(2
*sqrt(2)*sqrt(-a*b)*b*d - 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + s
```

```
qrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^2) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt
(-a*b)*b*d - 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)
^(1/4))/(-a/b)^(1/4))/(a^2*b^2)
```

Mupad [B]
time = 4.92, size = 283, normalized size = 2.57

$$\left(\sum_{k=1}^{4}\ln\left(-\frac{b^2\left(3\,c\,d^2+2\,d^3\,x+\mathrm{root}(65536\,a^7\,b^2\,z^4-2048\,a^4\,b\,d^2\,z^2+1152\,a^2\,b\,c^2\,d\,z-81\,b\,c^4+16\,a\,d^4,z,k)^2\,a^3\,b\,c\,192-\mathrm{root}(65536\,a^7\,b^2\,z^4-2048\,a^4\,b\,d^2\,z^2+1152\,a^2\,b\,c^2\,d\,z-81\,b\,c^4+16\,a\,d^4,z,k)^2\,a^3\,b\,d\,x\,128+\mathrm{root}(65536\,a^7\,b^2\,z^4-2048\,a^4\,b\,d^2\,z^2+1152\,a^2\,b\,c^2\,d\,z-81\,b\,c^4+16\,a\,d^4,z,k)\,a\,b\,c^2\,x\,36\right)}{a^3\,16}\right)\mathrm{root}\left(65536\,a^7\,b^2\,z^4-2048\,a^4\,b\,d^2\,z^2+1152\,a^2\,b\,c^2\,d\,z-81\,b\,c^4+16\,a\,d^4,z,k\right)\right)+\frac{\frac{d\,x^2}{4\,a}+\frac{c\,x}{4\,a}}{a-b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log(-(b^2*(3*c*d^2 + 2*d^3*x + 192*root(65536*a^7*b^2*z^4 - 2048*a^4
*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b*c^4 + 16*a*d^4, z, k)^2*a^3*b*c - 12
8*root(65536*a^7*b^2*z^4 - 2048*a^4*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b*c
^4 + 16*a*d^4, z, k)^2*a^3*b*d*x + 36*root(65536*a^7*b^2*z^4 - 2048*a^4*b*d
^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b*c^4 + 16*a*d^4, z, k)*a*b*c^2*x))/(16*a^
3))*root(65536*a^7*b^2*z^4 - 2048*a^4*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b
*c^4 + 16*a*d^4, z, k), k, 1, 4) + ((d*x^2)/(4*a) + (c*x)/(4*a))/(a - b*x^4
)
```

3.118 $\int \frac{c+dx}{(a+bx^4)^2} \, dx$

Optimal. Leaf size=241

$$\frac{x(c+dx)}{4a(a+bx^4)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{3c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{3c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{3c\log\left(\sqrt{a}-\sqrt{\ldots}\right.}{16\sqrt{\ldots}}$$

```
[Out] 1/4*x*(d*x+c)/a/(b*x^4+a)+3/16*c*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/
4)/b^(1/4)*2^(1/2)+3/16*c*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(1/
4)*2^(1/2)-3/32*c*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4
)/b^(1/4)*2^(1/2)+3/32*c*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/
a^(7/4)/b^(1/4)*2^(1/2)+1/4*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1869, 1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$-\frac{3c\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{3c\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{3c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{3c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{x(c+dx)}{4a(a+bx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3
/2)*Sqrt[b]) - (3*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*b^(1/4)) + (3*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*b^(1/4)) - (3*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/(16*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^4\right)^2}\,dx = \frac{x(c+dx)}{4a\left(a+bx^4\right)} - \frac{\int \frac{-3c-2dx}{a+bx^4}\,dx}{4a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} - \frac{\int \left(-\frac{3c}{a+bx^4} - \frac{2dx}{a+bx^4}\right)\,dx}{4a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{(3c)\int \frac{1}{a+bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a+bx^4}\,dx}{2a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{(3c)\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{8a^{3/2}} + \frac{(3c)\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{8a^{3/2}} + \frac{d\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x\right.}{4a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{(3c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}\sqrt{b}} + \frac{(3c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}}}{16a^{3/2}\sqrt{b}}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{3c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{b}} + \frac{3c\log\left(\right.}{}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{3c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}} + \frac{3c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}$$

Mathematica [A]
time = 0.14, size = 224, normalized size = 0.93

$$\frac{\frac{8a^{3/4}x(c+dx)}{a+bx^4} - \frac{2\left(3\sqrt{2}\sqrt[4]{b}\,c+4\sqrt[4]{a}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{2\left(3\sqrt{2}\sqrt[4]{b}\,c-4\sqrt[4]{a}\,d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt{b}} - \frac{3\sqrt{2}\,c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{\sqrt[4]{b}} + \frac{3\sqrt{2}\,c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{\sqrt[4]{b}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^4)^2,x]
```

```
[Out] ((8*a^(3/4)*x*(c + d*x))/(a + b*x^4) - (2*(3*Sqrt[2]*b^(1/4)*c + 4*a^(1/4)*
d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] + (2*(3*Sqrt[2]*b^(1/4)
*c - 4*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] - (3*Sqr
```

```
t[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (3
*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))
/(32*a^(7/4))
```

Maple [A]
time = 0.40, size = 163, normalized size = 0.68

method	result
risch	$\frac{\frac{d x^2}{4a}+\frac{cx}{4a}}{b x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(2\_Rd+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(b x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+d\left(\frac{}{4a(}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/4*x/a/(b*x^4+a)+3/32/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^
(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2
)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))+d*(1/4*x^2/a/(b*x^4+
a)+1/4/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2)))
```

Maxima [A]
time = 0.56, size = 238, normalized size = 0.99

$$\frac{dx^2+cx}{4\,(abx^4+a^2)}+\frac{\frac{3\sqrt{2}\,c\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac14}}-\frac{3\sqrt{2}\,c\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac14}}+\frac{2\left(3\sqrt{2}\,a^{\frac14}b^{\frac14}c-4\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac34}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac14}}+\frac{2\left(3\sqrt{2}\,a^{\frac14}b^{\frac14}c+4\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac34}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac14}}}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(d*x^2 + c*x)/(a*b*x^4 + a^2) + 1/32*(3*sqrt(2)*c*log(sqrt(b)*x^2 + sqr
t(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - 3*sqrt(2)*c*log(sqrt(
b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(3*sqrt
(2)*a^(1/4)*b^(1/4)*c - 4*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt
(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*
b^(1/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(1/4)*c + 4*sqrt(a)*d)*arctan(1/2*sqrt(2)
*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sq
rt(sqrt(a)*sqrt(b))*b^(1/4)))/a
```

Fricas [C] Result contains complex when optimal does not.
time = 1.65, size = 43065, normalized size = 178.69

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(48*d*x^2 - 12*(a*b*x^4 + a^2)*(2*d*sqrt(-1/(a*b))/a + 3*sqrt(-c^2/(a
^4*b*sqrt(-1/(a*b)))))*log(8*a^6*b*d^2*(2*d*sqrt(-1/(a*b))/a + 3*sqrt(-c^2/
(a^4*b*sqrt(-1/(a*b)))))^3 - 18*a^4*b*c^2*d*(2*d*sqrt(-1/(a*b))/a + 3*sqrt(
-c^2/(a^4*b*sqrt(-1/(a*b)))))^2 + 360*a*c^2*d^3 - 3*(81*b*c^5 - 64*a*c*d^4)
*x + (81*a^2*b*c^4 + 32*a^3*d^4)*(2*d*sqrt(-1/(a*b))/a + 3*sqrt(-c^2/(a^4*b
*sqrt(-1/(a*b)))))) + 2*(a*b*x^4 + a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((3
*a^2*b*sqrt(-1/(a*b))*sqrt(-c^2/(a^4*b*sqrt(-1/(a*b)))) - 2*d)^2/(a^3*b) +
3*(4*a*d^2*sqrt(-1/(a*b)) - 12*a^2*d*sqrt(-c^2/(a^4*b*sqrt(-1/(a*b)))) - 9*
c^2)/(a^4*b*sqrt(-1/(a*b))))/(9*(3*a^2*b*sqrt(-1/(a*b))*sqrt(-c^2/(a^4*b*sq
rt(-1/(a*b)))) - 2*d)*(4*a*d^2*sqrt(-1/(a*b)) - 12*a^2*d*sqrt(-c^2/(a^4*b*s
qrt(-1/(a*b)))) - 9*c^2)/(a^5*b) + 27*(27*a^5*b^2*sqrt(-1/(a*b))*(-c^2/(a^4
*b*sqrt(-1/(a*b))))^(3/2) - 12*a^2*b*d^2*sqrt(-1/(a*b))*sqrt(-c^2/(a^4*b*sq
rt(-1/(a*b)))) + 18*b*c^2*d*sqrt(-1/(a*b)) - 8*d^3)/(a^5*b^2*sqrt(-1/(a*b))
) + 2*(3*a^2*b*sqrt(-1/(a ...
```

Sympy [A]
time = 0.60, size = 155, normalized size = 0.64

$$\operatorname{RootSum}\left(65536t^4a^7b^2+2048t^2a^4bd^2-1152ta^2bc^2d+16ad^4+81bc^4,\left(t\mapsto t\log\left(x+\frac{-32768t^3a^6bd^2-4608t^2a^4bc^2d-512ta^3d^4-1296ta^2bc^4+360ac^2d^3}{192acd^4-243bc^5}\right)\right)\right)+\frac{cx+dx^2}{4a^2+4abx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**2 + 2048*_t**2*a**4*b*d**2 - 1152*_t*a**2*b*c**
2*d + 16*a*d**4 + 81*b*c**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*b*d**
2 - 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 - 1296*_t*a**2*b*c**4 + 360
*a*c**2*d**3)/(192*a*c*d**4 - 243*b*c**5)))) + (c*x + d*x**2)/(4*a**2 + 4*a
*b*x**4)
```

Giac [A]
time = 1.69, size = 238, normalized size = 0.99

$$\frac{3\sqrt{2}\,(ab^3)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b}-\frac{3\sqrt{2}\,(ab^3)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b}+\frac{dx^2+cx}{4\,(bx^4+a)a}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,bd+3\,(ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,bd+3\,(ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(
a^2*b) - 3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(a^2*b) + 1/4*(d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(
2)*sqrt(a*b)*b*d + 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b*d
```

```
+ 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/(a^2*b^2)
```

Mupad [B]
time = 4.94, size = 282, normalized size = 1.17

$$\left(\sum_{k=1}^{4}\ln\left(\frac{b^2\left(3\,c\,d^2+2\,d^3\,x-\mathrm{root}(65536\,a^7\,b^2\,z^4+2048\,a^4\,b\,d^2\,z^2-1152\,a^2\,b\,c^2\,d\,z+81\,b\,c^4+16\,a\,d^4,z,k)^2\,a^3\,b\,c\,192+\mathrm{root}(65536\,a^7\,b^2\,z^4+2048\,a^4\,b\,d^2\,z^2-1152\,a^2\,b\,c^2\,d\,z+81\,b\,c^4+16\,a\,d^4,z,k)^2\,a^3\,b\,d\,x\,128-\mathrm{root}(65536\,a^7\,b^2\,z^4+2048\,a^4\,b\,d^2\,z^2-1152\,a^2\,b\,c^2\,d\,z+81\,b\,c^4+16\,a\,d^4,z,k)\,a\,b\,c^2\,x\,36\right)}{a^3\,16}\right)\mathrm{root}\left(65536\,a^7\,b^2\,z^4+2048\,a^4\,b\,d^2\,z^2-1152\,a^2\,b\,c^2\,d\,z+81\,b\,c^4+16\,a\,d^4,z,k\right)\right)+\frac{\frac{d\,x^2}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log((b^2*(3*c*d^2 + 2*d^3*x - 192*root(65536*a^7*b^2*z^4 + 2048*a^4*
b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*c^4 + 16*a*d^4, z, k)^2*a^3*b*c + 128
*root(65536*a^7*b^2*z^4 + 2048*a^4*b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*c^
4 + 16*a*d^4, z, k)^2*a^3*b*d*x - 36*root(65536*a^7*b^2*z^4 + 2048*a^4*b*d^
2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*c^4 + 16*a*d^4, z, k)*a*b*c^2*x))/(16*a^3
))*root(65536*a^7*b^2*z^4 + 2048*a^4*b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*
c^4 + 16*a*d^4, z, k), k, 1, 4) + ((d*x^2)/(4*a) + (c*x)/(4*a))/(a + b*x^4)
```

3.119 $\int \frac{c+dx}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=136

$$\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{21c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{21c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

```
[Out] 1/8*x*(d*x+c)/a/(-b*x^4+a)^2+1/32*x*(6*d*x+7*c)/a^2/(-b*x^4+a)+21/64*c*arct
an(b^(1/4)*x/a^(1/4))/a^(11/4)/b^(1/4)+21/64*c*arctanh(b^(1/4)*x/a^(1/4))/a
^(11/4)/b^(1/4)+3/16*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1869, 1890, 218, 214, 211, 281}

$$\frac{21c\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{21c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - b*x^4)^3,x]
```

```
[Out] (x*(c + d*x))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a - b*x^4))
+ (21*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (21*c*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{\left(a-bx^4\right)^3}\,dx &= \frac{x(c+dx)}{8a\left(a-bx^4\right)^2}-\frac{\int \frac{-7c-6dx}{\left(a-bx^4\right)^2}\,dx}{8a}\\
&= \frac{x(c+dx)}{8a\left(a-bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{\int \frac{21c+12dx}{a-bx^4}\,dx}{32a^2}\\
&= \frac{x(c+dx)}{8a\left(a-bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{\int \left(\frac{21c}{a-bx^4}+\frac{12dx}{a-bx^4}\right)\,dx}{32a^2}\\
&= \frac{x(c+dx)}{8a\left(a-bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{(21c)\int \frac{1}{a-bx^4}\,dx}{32a^2}+\frac{(3d)\int \frac{x}{a-bx^4}\,dx}{8a^2}\\
&= \frac{x(c+dx)}{8a\left(a-bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{(21c)\int \frac{1}{\sqrt{a}-\sqrt{b}\,x^2}\,dx}{64a^{5/2}}+\frac{(21c)\int \frac{1}{\sqrt{a}+\sqrt{b}\,x^2}\,dx}{64a^{5/2}}+\frac{(3d}{}\\
&= \frac{x(c+dx)}{8a\left(a-bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{21c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{21c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{3d\tanh^{-1}}{16a^5}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 193, normalized size = 1.42

$$\frac{\frac{16a^2x(c+dx)}{\left(a-bx^4\right)^2}+\frac{4ax(7c+6dx)}{a-bx^4}+\frac{42\sqrt[4]{a}\,c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}-\frac{3\left(7\sqrt[4]{a}\,\sqrt[4]{b}\,c+4\sqrt{a}\,d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)}{\sqrt{b}}+\frac{3\left(7\sqrt[4]{a}\,\sqrt[4]{b}\,c-4\sqrt{a}\,d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{\sqrt{b}}+\frac{12\sqrt{a}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt{b}}}{128a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - b*x^4)^3,x]
```

```
[Out] ((16*a^2*x*(c + d*x))/(a - b*x^4)^2 + (4*a*x*(7*c + 6*d*x))/(a - b*x^4) + (
42*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(1/4) - (3*(7*a^(1/4)*b^(1/4)*c
 + 4*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sqrt[b] + (3*(7*a^(1/4)*b^(1/4)*c
 - 4*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/Sqrt[b] + (12*Sqrt[a]*d*Log[Sqrt[
a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Maple [A]
time = 0.32, size = 174, normalized size = 1.28

method	result
risch	$\frac{-\frac{3bd\,x^6}{16a^2}-\frac{7bc\,x^5}{32a^2}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}}{(-b\,x^4+a)^2}-\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(4\_Rd+7c\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{128a^2b}$
default	$c\left(\frac{x}{8a(-b\,x^4+a)^2}+\frac{\frac{7x}{32a\left(-b\,x^4+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{128a^2}}{a}\right)+d\left(\frac{x^2}{8a(-b\,x^4+a)^2}+\frac{\frac{3x^2}{16a\left(-b\,x^4+a\right)}+\cdots}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/8*x/a/(-b*x^4+a)^2+7/8/a*(1/4*x/a/(-b*x^4+a)+3/16/a^2*(a/b)^(1/4)*(ln(
(x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))))+d*(1/8*x^2/a/(-
b*x^4+a)^2+3/4/a*(1/4*x^2/a/(-b*x^4+a)+1/8/a/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1
/2))/(a-x^2*(a*b)^(1/2)))))
```

Maxima [A]
time = 0.54, size = 186, normalized size = 1.37

$$-\frac{6\,bdx^6+7\,bcx^5-10\,adx^2-11\,acx}{32\,(a^2b^2x^8-2\,a^3bx^4+a^4)}+\frac{3\left(\frac{14\,c\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{4\,d\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{4\,d\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{7\,c\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{128\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(6*b*d*x^6 + 7*b*c*x^5 - 10*a*d*x^2 - 11*a*c*x)/(a^2*b^2*x^8 - 2*a^3*
b*x^4 + a^4) + 3/128*(14*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)
```

```
*sqrt(sqrt(a)*sqrt(b))) + 4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b))
- 4*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 7*c*log((sqrt(b)*x - s
qrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(b))))/a^2
```

Fricas [C] Result contains complex when optimal does not.
time = 1.49, size = 40637, normalized size = 298.80

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] -1/65536*(12288*b*d*x^6 + 14336*b*c*x^5 - 20480*a*d*x^2 - 22528*a*c*x + 2*(
a^2*b^2*x^8 - 2*a^3*b*x^4 + a^4)*((-I*sqrt(3) + 1)*((7*a^3*b*sqrt(1/(a*b))*
sqrt(c^2/(a^6*b*sqrt(1/(a*b)))) + 4*d)^2/(a^5*b) - 3*(56*a^3*d*sqrt(c^2/(a^
6*b*sqrt(1/(a*b)))) - 16*a*d^2*sqrt(1/(a*b)) + 49*c^2)/(a^6*b*sqrt(1/(a*b))
))/(-9/4194304*(7*a^3*b*sqrt(1/(a*b))*sqrt(c^2/(a^6*b*sqrt(1/(a*b)))) + 4*d
)*(56*a^3*d*sqrt(c^2/(a^6*b*sqrt(1/(a*b)))) - 16*a*d^2*sqrt(1/(a*b)) + 49*c
^2)/(a^8*b) + 27/4194304*(343*a^8*b^2*sqrt(1/(a*b))*(c^2/(a^6*b*sqrt(1/(a*b
))))^(3/2) + 112*a^3*b*d^2*sqrt(1/(a*b))*sqrt(c^2/(a^6*b*sqrt(1/(a*b)))) -
196*b*c^2*d*sqrt(1/(a*b)) - 64*d^3)/(a^8*b^2*sqrt(1/(a*b))) + 1/2097152*(7*
a^3*b*sqrt(1/(a*b))*sqrt(c^2/(a^6*b*sqrt(1/(a*b)))) + 4*d)^3/(a^9*b^3*(1/(a
*b))^(3/2)) + 1/4194304*sqrt(12706092*b^3*c^6*(1/(a*b))^(3/2) + 451584*(43*
a^2*b^3*(1/(a*b))^(5/2) + 6*a*b^2*(1/(a*b))^(3/2) - b*sqrt(1/(a*b)))*c^2*d^
4 - 344064*(8*a^6*b^4*(1/(a*b))^(7/2) + 3*a^5*b^3*(1/(a*b))^(5/2) - 12*a^4*
b^2*(1/(a*b))^(3/2) + a^3 ...
```

Sympy [A]
time = 1.18, size = 194, normalized size = 1.43

$$-\operatorname{RootSum}\left(268435456t^4a^{11}b^2 - 4718592t^2a^6bd^2 - 2709504ta^3bc^2d + 20736ad^4 - 194481bc^4, \left(t \mapsto t\log\left(x + \frac{-67108864t^3a^9bd^2 + 9633792t^2a^6bc^2d + 589824ta^4d^4 - 2765952ta^3bc^4 + 423360ac^2d^3}{193536acd^4 + 453789bc^5}\right)\right)\right) - \frac{-11acx - 10adx^2 + 7bcx^5 + 6bdx^6}{32a^4 - 64a^3bx^4 + 32a^2b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] -RootSum(268435456*_t**4*a**11*b**2 - 4718592*_t**2*a**6*b*d**2 - 2709504*_
t*a**3*b*c**2*d + 20736*a*d**4 - 194481*b*c**4, Lambda(_t, _t*log(x + (-671
08864*_t**3*a**9*b*d**2 + 9633792*_t**2*a**6*b*c**2*d + 589824*_t*a**4*d**4
 - 2765952*_t*a**3*b*c**4 + 423360*a*c**2*d**3)/(193536*a*c*d**4 + 453789*b
*c**5)))) - (-11*a*c*x - 10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a**4 -
64*a**3*b*x**4 + 32*a**2*b**2*x**8)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(104) = 208.
time = 1.13, size = 272, normalized size = 2.00

$$\frac{21\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256a^3b} - \frac{21\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256a^3b} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{-ab}bd + 7\left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^2} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{-ab}bd + 7\left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^2} - \frac{6bdx^6 + 7bcx^5 - 10adx^2 - 11acx}{32\left(bx^4 - a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/
b))/(a^3*b) - 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1
/4) + sqrt(-a/b))/(a^3*b) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d + 7*(-a
*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/(a^3*b^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d + 7*(-a*b^3)^(1/4)*
b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^2
) - 1/32*(6*b*d*x^6 + 7*b*c*x^5 - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a)^2*a^2
)
```

Mupad [B]
time = 4.98, size = 315, normalized size = 2.32

$$\frac{\frac{5\,d\,x^2}{16\,a}+\frac{11\,c\,x}{32\,a}-\frac{7\,b\,c\,x^5}{32\,a^2}-\frac{3\,b\,d\,x^6}{16\,a^2}}{a^2-2\,a\,b\,x^4+b^2\,x^8}+\left(\sum_{k=1}^{4}\ln\left(-\frac{b^2\,\left(63\,c\,d^2+36\,d^3\,x+\mathrm{root}(268435456\,a^{11}\,b^2\,z^4-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z-194481\,b\,c^4+20736\,a\,d^4,z,k)^2\,a^5\,b\,c\,7168+\mathrm{root}(268435456\,a^{11}\,b^2\,z^4-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z-194481\,b\,c^4+20736\,a\,d^4,z,k)\,a^2\,b\,c^2\,x\,1176-\mathrm{root}(268435456\,a^{11}\,b^2\,z^4-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z-194481\,b\,c^4+20736\,a\,d^4,z,k)^2\,a^5\,b\,d\,x\,4096\right)\,3}{a^6\,2048}\right)\,\mathrm{root}\left(268435456\,a^{11}\,b^2\,z^4-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z-194481\,b\,c^4+20736\,a\,d^4,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a - b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) + (11*c*x)/(32*a) - (7*b*c*x^5)/(32*a^2) - (3*b*d*x^6)/(1
6*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(-(3*b^2*(63*c*d^2 + 36*d^3
*x + 7168*root(268435456*a^11*b^2*z^4 - 4718592*a^6*b*d^2*z^2 + 2709504*a^3
*b*c^2*d*z - 194481*b*c^4 + 20736*a*d^4, z, k)^2*a^5*b*c + 1176*root(268435
456*a^11*b^2*z^4 - 4718592*a^6*b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194481*b
*c^4 + 20736*a*d^4, z, k)*a^2*b*c^2*x - 4096*root(268435456*a^11*b^2*z^4 -
4718592*a^6*b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194481*b*c^4 + 20736*a*d^4,
 z, k)^2*a^5*b*d*x))/(2048*a^6))*root(268435456*a^11*b^2*z^4 - 4718592*a^6*
b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194481*b*c^4 + 20736*a*d^4, z, k), k, 1
, 4)
```

3.120 $\int \frac{c+dx}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=266

$$\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{21c\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21c\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}$$

```
[Out] 1/8*x*(d*x+c)/a/(b*x^4+a)^2+1/32*x*(6*d*x+7*c)/a^2/(b*x^4+a)+21/128*c*arcta
n(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/b^(1/4)*2^(1/2)+21/128*c*arctan(1+
b^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/b^(1/4)*2^(1/2)-21/256*c*ln(-a^(1/4)*b^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)+21/256*c*ln(a
^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)+3/16
*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1869, 1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$-\frac{21c\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21c\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{3d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{21c\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21c\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}+\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^4)^3,x]
```

```
[Out] (x*(c + d*x))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a + b*x^4))
+ (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - (21*c*ArcTan[1
 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) + (21*c*ArcT
an[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) - (21*c*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/
4)*b^(1/4)) + (21*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(128*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{\left(a+bx^4\right)^3}\,dx &= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2}-\frac{\int \frac{-7c-6dx}{\left(a+bx^4\right)^2}\,dx}{8a}\\
&= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}+\frac{\int \frac{21c+12dx}{a+bx^4}\,dx}{32a^2}\\
&= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}+\frac{\int \left(\frac{21c}{a+bx^4}+\frac{12dx}{a+bx^4}\right)\,dx}{32a^2}\\
&= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}+\frac{(21c)\int \frac{1}{a+bx^4}\,dx}{32a^2}+\frac{(3d)\int \frac{x}{a+bx^4}\,dx}{8a^2}\\
&= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}+\frac{(21c)\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{64a^{5/2}}+\frac{(21c)\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{64a^{5/2}}+\frac{(3d)}{}\\
&= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{(21c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{128a^{5/2}\sqrt{b}}+\\
&= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{21c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\right.}{128\sqrt{2}\,a^{11/4}\sqrt[4]{b}}\\
&= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{21c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21}{}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 249, normalized size = 0.94

$$
\frac{\frac{32a^{7/4}x(c+dx)}{\left(a+bx^4\right)^2}+\frac{8a^{3/4}x(7c+6dx)}{a+bx^4}-\frac{6\left(7\sqrt{2}\sqrt[4]{b}\,c+8\sqrt[4]{a}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt{b}}+\frac{6\left(7\sqrt{2}\sqrt[4]{b}\,c-8\sqrt[4]{a}\,d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt{b}}-\frac{21\sqrt{2}\,c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{\sqrt[4]{b}}+\frac{21\sqrt{2}\,c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{\sqrt[4]{b}}}{256a^{11/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^4)^3,x]
```

```
[Out] ((32*a^(7/4)*x*(c + d*x))/(a + b*x^4)^2 + (8*a^(3/4)*x*(7*c + 6*d*x))/(a +
b*x^4) - (6*(7*Sqrt[2]*b^(1/4)*c + 8*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/Sqrt[b] + (6*(7*Sqrt[2]*b^(1/4)*c - 8*a^(1/4)*d)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] - (21*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (21*Sqrt[2]*c*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))/(256*a^(11/4))
```

Maple [A]
time = 0.33, size = 207, normalized size = 0.78

method	result
risch	$\frac{\frac{3bd\,x^{6}}{16a^{2}}+\frac{7bc\,x^{5}}{32a^{2}}+\frac{5d\,x^{2}}{16a}+\frac{11cx}{32a}}{\left(b\,x^{4}+a\right)^{2}}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{4}+a\right)}\frac{\left(4\_Rd+7c\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{128a^{2}b}$
default	$c\left(\frac{x}{8a\left(b\,x^{4}+a\right)^{2}}+\frac{\frac{7x}{32a\left(b\,x^{4}+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^{2}}}{a}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/8*x/a/(b*x^4+a)^2+7/8/a*(1/4*x/a/(b*x^4+a)+3/32/a^2*(a/b)^(1/4)*2^(1/2
)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a
/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*
x-1))))+d*(1/8*x^2/a/(b*x^4+a)^2+3/4/a*(1/4*x^2/a/(b*x^4+a)+1/4/a/(a*b)^(1/
2)*arctan(x^2*(b/a)^(1/2))))
```

Maxima [A]
time = 0.53, size = 269, normalized size = 1.01

$$\frac{6\,bdx^{6}+7\,bcx^{5}+10\,adx^{2}+11\,acx}{32\left(a^{2}b^{2}x^{8}+2\,a^{3}bx^{4}+a^{4}\right)}+\frac{3\left(\frac{7\sqrt{2}\,c\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{7\sqrt{2}\,c\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\left(7\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}c-8\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{1}{4}}}+\frac{2\left(7\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}c+8\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{1}{4}}}\right)}{256\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(6*b*d*x^6 + 7*b*c*x^5 + 10*a*d*x^2 + 11*a*c*x)/(a^2*b^2*x^8 + 2*a^3*b
*x^4 + a^4) + 3/256*(7*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*
x + sqrt(a))/(a^(3/4)*b^(1/4)) - 7*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a^(1
```

```
/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(7*sqrt(2)*a^(1/4)*b^(1/4)*c
 - 8*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/
sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 2*(7*sqrt(
2)*a^(1/4)*b^(1/4)*c + 8*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b
^(1/4)))/a^2
```

Fricas [C] Result contains complex when optimal does not.
time = 2.69, size = 43180, normalized size = 162.33

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/65536*(12288*b*d*x^6 + 14336*b*c*x^5 + 20480*a*d*x^2 + 22528*a*c*x - 1536
*(a^2*b^2*x^8 + 2*a^3*b*x^4 + a^4)*(7*sqrt(-c^2/(a^6*b*sqrt(-1/(a*b)))) + 4
*d*sqrt(-1/(a*b))/a^2)*log(864*a^9*b*d^2*(7*sqrt(-c^2/(a^6*b*sqrt(-1/(a*b))
)) + 4*d*sqrt(-1/(a*b))/a^2)^3 - 5292*a^6*b*c^2*d*(7*sqrt(-c^2/(a^6*b*sqrt(
-1/(a*b)))) + 4*d*sqrt(-1/(a*b))/a^2)^2 + 423360*a*c^2*d^3 - 189*(2401*b*c^
5 - 1024*a*c*d^4)*x + 27*(2401*a^3*b*c^4 + 512*a^4*d^4)*(7*sqrt(-c^2/(a^6*b
*sqrt(-1/(a*b)))) + 4*d*sqrt(-1/(a*b))/a^2)) + 2*(a^2*b^2*x^8 + 2*a^3*b*x^4
 + a^4)*((-I*sqrt(3) + 1)*((7*a^3*b*sqrt(-1/(a*b))*sqrt(-c^2/(a^6*b*sqrt(-1
/(a*b)))) - 4*d)^2/(a^5*b) - 3*(56*a^3*d*sqrt(-c^2/(a^6*b*sqrt(-1/(a*b))))
- 16*a*d^2*sqrt(-1/(a*b)) + 49*c^2)/(a^6*b*sqrt(-1/(a*b))))/(-9/4194304*(7*
a^3*b*sqrt(-1/(a*b))*sqrt(-c^2/(a^6*b*sqrt(-1/(a*b)))) - 4*d)*(56*a^3*d*sqr
t(-c^2/(a^6*b*sqrt(-1/(a*b)))) - 16*a*d^2*sqrt(-1/(a*b)) + 49*c^2)/(a^8*b)
+ 27/4194304*(343*a^8*b^2*sqrt(-1/(a*b))*(-c^2/(a^6*b*sqrt(-1/(a*b))))^(3/2
) - 112*a^3*b*d^2*sqrt(-1 ...
```

Sympy [A]
time = 1.14, size = 192, normalized size = 0.72

$$\operatorname{RootSum}\left(268435456t^4a^{11}b^2 + 4718592t^2a^6bd^2 - 2709504ta^3bc^2d + 20736ad^4 + 194481bc^4, \left(t \mapsto t\log\left(x + \frac{-67108864t^3a^9bd^2 - 9633792t^2a^6bc^2d - 589824ta^4d^4 - 2765952ta^3bc^4 + 423360ac^2d^3}{193536acd^4 - 453789bc^5}\right)\right)\right) + \frac{11acx + 10adx^2 + 7bcx^5 + 6bdx^6}{32a^4 + 64a^3bx^4 + 32a^2b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*b**2 + 4718592*_t**2*a**6*b*d**2 - 2709504*_t
*a**3*b*c**2*d + 20736*a*d**4 + 194481*b*c**4, Lambda(_t, _t*log(x + (-6710
8864*_t**3*a**9*b*d**2 - 9633792*_t**2*a**6*b*c**2*d - 589824*_t*a**4*d**4
- 2765952*_t*a**3*b*c**4 + 423360*a*c**2*d**3)/(193536*a*c*d**4 - 453789*b*
c**5)))) + (11*a*c*x + 10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a**4 + 64
*a**3*b*x**4 + 32*a**2*b**2*x**8)
```

Giac [A]
time = 0.90, size = 256, normalized size = 0.96

$$\frac{21\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b}-\frac{21\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b}+\frac{3\,\sqrt{2}\left(4\,\sqrt{2}\,\sqrt{ab}\,bd+7\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2}+\frac{3\,\sqrt{2}\left(4\,\sqrt{2}\,\sqrt{ab}\,bd+7\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2}+\frac{6\,bdx^6+7\,bcx^5+10\,adx^2+11\,acx}{32\,(bx^4+a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(a^3*b) - 21/256*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a^3*b) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b*d + 7*(a*b^3)^(1
/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b
^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b*d + 7*(a*b^3)^(1/4)*b*c)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^2) + 1/32*(6*b*
d*x^6 + 7*b*c*x^5 + 10*a*d*x^2 + 11*a*c*x)/((b*x^4 + a)^2*a^2)
```

Mupad [B]
time = 4.99, size = 315, normalized size = 1.18

$$\frac{\frac{5\,d\,x^2}{16\,a}+\frac{11\,c\,x}{32\,a}+\frac{7\,b\,c\,x^5}{32\,a^2}+\frac{3\,b\,d\,x^6}{16\,a^2}}{a^2+2\,a\,b\,x^4+b^2\,x^8}+\left(\sum_{k=1}^{4}\ln\left(\frac{b^2\left(63\,c\,d^2+36\,d^3\,x-\mathrm{root}(268435456\,a^{11}\,b^2\,z^4+4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+194481\,b\,c^4+20736\,a\,d^4,z,k)^2\,a^5\,b\,c\,7168-\mathrm{root}(268435456\,a^{11}\,b^2\,z^4+4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+194481\,b\,c^4+20736\,a\,d^4,z,k)\,a^2\,b\,c^2\,x\,1176+\mathrm{root}(268435456\,a^{11}\,b^2\,z^4+4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+194481\,b\,c^4+20736\,a\,d^4,z,k)^2\,a^5\,b\,d\,x\,4096\right)3}{a^6\,2048}\right)\mathrm{root}(268435456\,a^{11}\,b^2\,z^4+4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+194481\,b\,c^4+20736\,a\,d^4,z,k)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) + (11*c*x)/(32*a) + (7*b*c*x^5)/(32*a^2) + (3*b*d*x^6)/(1
6*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log((3*b^2*(63*c*d^2 + 36*d^3*
x - 7168*root(268435456*a^11*b^2*z^4 + 4718592*a^6*b*d^2*z^2 - 2709504*a^3*
b*c^2*d*z + 194481*b*c^4 + 20736*a*d^4, z, k)^2*a^5*b*c - 1176*root(2684354
56*a^11*b^2*z^4 + 4718592*a^6*b*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 194481*b*
c^4 + 20736*a*d^4, z, k)*a^2*b*c^2*x + 4096*root(268435456*a^11*b^2*z^4 + 4
718592*a^6*b*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 194481*b*c^4 + 20736*a*d^4,
z, k)^2*a^5*b*d*x))/(2048*a^6))*root(268435456*a^11*b^2*z^4 + 4718592*a^6*b
*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 194481*b*c^4 + 20736*a*d^4, z, k), k, 1,
 4)
```

3.121 $\int \frac{c+dx}{(a-bx^4)^4}\,dx$

Optimal. Leaf size=162

$$\frac{x(c+dx)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{77c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{77c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{5d\tanh^{-1}\left(\ldots\right.}{32a^{7/2}\sqrt{\ldots}}$$

```
[Out] 1/12*x*(d*x+c)/a/(-b*x^4+a)^3+1/96*x*(10*d*x+11*c)/a^2/(-b*x^4+a)^2+1/384*x
*(60*d*x+77*c)/a^3/(-b*x^4+a)+77/256*c*arctan(b^(1/4)*x/a^(1/4))/a^(15/4)/b
^(1/4)+77/256*c*arctanh(b^(1/4)*x/a^(1/4))/a^(15/4)/b^(1/4)+5/32*d*arctanh(
x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1869, 1890, 218, 214, 211, 281}

$$\frac{77c\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{77c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(c+dx)}{12a\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - b*x^4)^4,x]
```

```
[Out] (x*(c + d*x))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a - b*x^4
)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a - b*x^4)) + (77*c*ArcTan[(b^(1/4)*x)
/a^(1/4)])/(256*a^(15/4)*b^(1/4)) + (77*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(25
6*a^(15/4)*b^(1/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt
[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{\left(a-bx^4\right)^4}\,dx &= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}-\frac{\int \frac{-11c-10dx}{\left(a-bx^4\right)^3}\,dx}{12a}\\
&= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{\int \frac{77c+60dx}{\left(a-bx^4\right)^2}\,dx}{96a^2}\\
&= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}-\frac{\int \frac{-231c-120dx}{a-bx^4}\,dx}{384a^3}\\
&= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}-\frac{\int \left(-\frac{231c}{a-bx^4}-\frac{120dx}{a-bx^4}\right)\,dx}{384a^3}\\
&= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{(77c)\int \frac{1}{a-bx^4}\,dx}{128a^3}+\frac{(5d)\int \frac{x}{a-bx^4}\,d
\end{aligned}
$$

$$
= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{(77c)\int \frac{1}{\sqrt{a}-\sqrt{b}\,x^2}\,dx}{256a^{7/2}}+\frac{(77c)\int
$$

$$
= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{77c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{77c\tanh^{-1}}{256a^{15}}
$$

Mathematica [A]
time = 0.12, size = 217, normalized size = 1.34

$$\frac{\frac{128a^3x(c+dx)}{(a-bx^4)^3}+\frac{16a^2x(11c+10dx)}{(a-bx^4)^2}+\frac{4ax(77c+60dx)}{a-bx^4}+\frac{462\sqrt[4]{a}\,c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}-\frac{3\left(77\sqrt[4]{a}\sqrt[4]{b}\,c+40\sqrt{a}\,d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)}{\sqrt{b}}+\frac{3\left(77\sqrt[4]{a}\sqrt[4]{b}\,c-40\sqrt{a}\,d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{\sqrt{b}}+\frac{120\sqrt{a}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt{b}}}{1536a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - b*x^4)^4,x]
```

```
[Out] ((128*a^3*x*(c + d*x))/(a - b*x^4)^3 + (16*a^2*x*(11*c + 10*d*x))/(a - b*x^
4)^2 + (4*a*x*(77*c + 60*d*x))/(a - b*x^4) + (462*a^(1/4)*c*ArcTan[(b^(1/4)
*x)/a^(1/4)])/b^(1/4) - (3*(77*a^(1/4)*b^(1/4)*c + 40*Sqrt[a]*d)*Log[a^(1/4
) - b^(1/4)*x])/Sqrt[b] + (3*(77*a^(1/4)*b^(1/4)*c - 40*Sqrt[a]*d)*Log[a^(1
/4) + b^(1/4)*x])/Sqrt[b] + (120*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt
[b])/(1536*a^4)
```

Maple [A]
time = 0.34, size = 165, normalized size = 1.02

method	result	size
risch	$\frac{\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{\left(-b\,x^4+a\right)^3}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(40\_Rd+77c\right)\ln\left(x-\_R\right)}{\_R^3}}{512a^3b}$	113
default	$\frac{\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{\left(-b\,x^4+a\right)^3}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{10d\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{\sqrt{ab}}}{128a^3}$	165

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (5/32/a^3*d*b^2*x^10+77/384*c/a^3*b^2*x^9-5/12/a^2*b*d*x^6-33/64*b*c/a^2*x^
5+11/32*d/a*x^2+51/128/a*c*x)/(-b*x^4+a)^3+1/128/a^3*(77/4*c*(a/b)^(1/4)/a*
(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))+10*d/(a*b)^(1
/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2))))
```

Maxima [A]
time = 0.54, size = 223, normalized size = 1.38

$$-\frac{60\,b^2dx^{10}+77\,b^2cx^9-160\,abdx^6-198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\left(a^3b^3x^{12}-3\,a^4b^2x^8+3\,a^5bx^4-a^6\right)}+\frac{\frac{154\,c\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{40\,d\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{77\,c\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{512\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 - 160*a*b*d*x^6 - 198*a*b*c*x^5 + 132*
a^2*d*x^2 + 153*a^2*c*x)/(a^3*b^3*x^12 - 3*a^4*b^2*x^8 + 3*a^5*b*x^4 - a^6)
 + 1/512*(154*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(b))) + 40*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d*log
(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 77*c*log((sqrt(b)*x - sqrt(sqrt
(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(b))))/a^3
```

Fricas [C] Result contains complex when optimal does not.
time = 1.56, size = 40780, normalized size = 251.73

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] -1/9437184*(1474560*b^2*d*x^10 + 1892352*b^2*c*x^9 - 3932160*a*b*d*x^6 - 48
66048*a*b*c*x^5 + 3244032*a^2*d*x^2 + 3760128*a^2*c*x + 2*(a^3*b^3*x^12 - 3
*a^4*b^2*x^8 + 3*a^5*b*x^4 - a^6)*((-I*sqrt(3) + 1)*((77*a^4*b*sqrt(1/(a*b)
)*sqrt(c^2/(a^8*b*sqrt(1/(a*b)))) + 40*d)^2/(a^7*b) - 3*(6160*a^4*d*sqrt(c^
2/(a^8*b*sqrt(1/(a*b)))) - 1600*a*d^2*sqrt(1/(a*b)) + 5929*c^2)/(a^8*b*sqrt
(1/(a*b))))/(-1/805306368*(77*a^4*b*sqrt(1/(a*b))*sqrt(c^2/(a^8*b*sqrt(1/(a
*b)))) + 40*d)*(6160*a^4*d*sqrt(c^2/(a^8*b*sqrt(1/(a*b)))) - 1600*a*d^2*sqr
t(1/(a*b)) + 5929*c^2)/(a^11*b) + 1/268435456*(456533*a^11*b^2*sqrt(1/(a*b)
)*(c^2/(a^8*b*sqrt(1/(a*b))))^(3/2) + 123200*a^4*b*d^2*sqrt(1/(a*b))*sqrt(c
^2/(a^8*b*sqrt(1/(a*b)))) - 237160*b*c^2*d*sqrt(1/(a*b)) - 64000*d^3)/(a^11
*b^2*sqrt(1/(a*b))) + 1/3623878656*(77*a^4*b*sqrt(1/(a*b))*sqrt(c^2/(a^8*b*
sqrt(1/(a*b)))) + 40*d)^3/(a^12*b^3*(1/(a*b))^(3/2)) + 1/7247757312*sqrt(22
509617049612*b^3*c^6*(1/(a*b))^(3/2) + 546416640000*(43*a^2*b^3*(1/(a*b))^(
5/2) + 6*a*b^2*(1/(a*b))^ ...
```

Sympy [A]
time = 0.90, size = 231, normalized size = 1.43

$$\operatorname{RootSum}\left(68719476736t^4a^{15}b^2 - 838860800t^2a^8bd^2 + 485703680ta^4bc^2d + 2560000ad^4 - 35153041bc^4, \left(t \mapsto t\log\left(x + \frac{429496729600t^3a^{12}bd^2 + 62170071040t^2a^8bc^2d - 2621440000ta^5d^4 + 17998356992ta^4bc^4 + 1897280000ac^2d^3}{788480000acd^4 + 2706784157bc^5}\right)\right)\right) + \frac{-153a^2cx - 132a^2dx^2 + 198abcx^5 + 160abdx^6 - 77b^2cx^9 - 60b^2dx^{10}}{-384a^6 + 1152a^5bx^4 - 1152a^4b^2x^8 + 384a^3b^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**2 - 838860800*_t**2*a**8*b*d**2 + 485703
680*_t*a**4*b*c**2*d + 2560000*a*d**4 - 35153041*b*c**4, Lambda(_t, _t*log(
x + (429496729600*_t**3*a**12*b*d**2 + 62170071040*_t**2*a**8*b*c**2*d - 26
21440000*_t*a**5*d**4 + 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2*d**3
)/(788480000*a*c*d**4 + 2706784157*b*c**5)))) + (-153*a**2*c*x - 132*a**2*d
*x**2 + 198*a*b*c*x**5 + 160*a*b*d*x**6 - 77*b**2*c*x**9 - 60*b**2*d*x**10)
/(-384*a**6 + 1152*a**5*b*x**4 - 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(129) = 258.

time = 0.69, size = 296, normalized size = 1.83

$$\frac{77\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}c\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^{4}b}-\frac{77\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}c\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^{4}b}-\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}\,bd-77\left(-ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^{4}b^{2}}-\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}\,bd-77\left(-ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^{4}b^{2}}-\frac{60\,b^{2}dx^{10}+77\,b^{2}cx^{9}-160\,abdx^{6}-198\,abcx^{5}+132\,a^{2}dx^{2}+153\,a^{2}cx}{384\,\left(bx^{4}-a\right)^{3}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a
/b))/(a^4*b) - 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^
(1/4) + sqrt(-a/b))/(a^4*b) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d - 77
*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)
^(1/4))/(a^4*b^2) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d - 77*(-a*b^3)^
(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a
^4*b^2) - 1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 - 160*a*b*d*x^6 - 198*a*b*c*x
^5 + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)
```

Mupad [B]

time = 4.97, size = 351, normalized size = 2.17

$$\left(\sum_{k=1}^{4}\ln\left(-\frac{b^{2}\left(1925\,c\,d^{2}+1000\,d^{3}x+\mathrm{root}\left(68719476736\,a^{15}b^{2}z^{4}-838860800\,a^{8}b\,d^{2}z^{2}+485703680\,a^{4}b\,c^{2}d\,z-35153041\,b\,c^{4}+2560000\,a\,d^{4},z,k\right)^{2}a^{7}b\,c\,315392+\mathrm{root}\left(68719476736\,a^{15}b^{2}z^{4}-838860800\,a^{8}b\,d^{2}z^{2}+485703680\,a^{4}b\,c^{2}d\,z-35153041\,b\,c^{4}+2560000\,a\,d^{4},z,k\right)a^{3}b\,c^{2}x\,47432-\mathrm{root}\left(68719476736\,a^{15}b^{2}z^{4}-838860800\,a^{8}b\,d^{2}z^{2}+485703680\,a^{4}b\,c^{2}d\,z-35153041\,b\,c^{4}+2560000\,a\,d^{4},z,k\right)^{2}a^{7}b\,d\,x\,163840\right)}{a^{9}\,32768}\right)\mathrm{root}\left(68719476736\,a^{15}b^{2}z^{4}-838860800\,a^{8}b\,d^{2}z^{2}+485703680\,a^{4}b\,c^{2}d\,z-35153041\,b\,c^{4}+2560000\,a\,d^{4},z,k\right)\right)+\frac{\frac{11\,d\,x^{2}}{32\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^{2}c\,x^{9}}{384\,a^{3}}+\frac{5\,b^{2}d\,x^{10}}{32\,a^{3}}-\frac{33\,b\,c\,x^{5}}{64\,a^{2}}-\frac{5\,b\,d\,x^{6}}{12\,a^{2}}}{a^{3}-3\,a^{2}b\,x^{4}+3\,a\,b^{2}x^{8}-b^{3}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a - b*x^4)^4,x)
```

```
[Out] symsum(log(-(b^2*(1925*c*d^2 + 1000*d^3*x + 315392*root(68719476736*a^15*b^
2*z^4 - 838860800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4
+ 2560000*a*d^4, z, k)^2*a^7*b*c + 47432*root(68719476736*a^15*b^2*z^4 - 83
8860800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560000*
a*d^4, z, k)*a^3*b*c^2*x - 163840*root(68719476736*a^15*b^2*z^4 - 838860800
*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560000*a*d^4,
z, k)^2*a^7*b*d*x))/(32768*a^9))*root(68719476736*a^15*b^2*z^4 - 838860800*
a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560000*a*d^4, z
, k), k, 1, 4) + ((11*d*x^2)/(32*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9)/(38
4*a^3) + (5*b^2*d*x^10)/(32*a^3) - (33*b*c*x^5)/(64*a^2) - (5*b*d*x^6)/(12*
a^2))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

3.122 $\int \frac{c+dx}{(a+bx^4)^4} dx$

Optimal. Leaf size=291

$$\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2}+\frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}-\frac{77c\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{77c\,\mathrm{ta}}{}$$

```
[Out] 1/12*x*(d*x+c)/a/(b*x^4+a)^3+1/96*x*(10*d*x+11*c)/a^2/(b*x^4+a)^2+1/384*x*(
60*d*x+77*c)/a^3/(b*x^4+a)+77/512*c*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^
(15/4)/b^(1/4)*2^(1/2)+77/512*c*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(15/4
)/b^(1/4)*2^(1/2)-77/1024*c*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))/a^(15/4)/b^(1/4)*2^(1/2)+77/1024*c*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*b^(1/2))/a^(15/4)/b^(1/4)*2^(1/2)+5/32*d*arctan(x^2*b^(1/2)/a^(1/2))/a
^(7/2)/b^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1869, 1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$-\frac{77c\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{77c\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{5d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}-\frac{77c\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{77c\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)}+\frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2}+\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^4)^4,x]
```

```
[Out] (x*(c + d*x))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a + b*x^4
)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a + b*x^4)) + (5*d*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - (77*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) - (77*c*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b
^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx = \frac{x(c+dx)}{12a\,(a+bx^4)^3} - \frac{\int \frac{-11c-10dx}{(a+bx^4)^3}\,dx}{12a}$$

$$= \frac{x(c+dx)}{12a\,(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2\,(a+bx^4)^2} + \frac{\int \frac{77c+60dx}{(a+bx^4)^2}\,dx}{96a^2}$$

$$= \frac{x(c+dx)}{12a\,(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3\,(a+bx^4)} - \frac{\int \frac{-231c-120dx}{a+bx^4}\,dx}{384a^3}$$

$$= \frac{x(c+dx)}{12a\,(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3\,(a+bx^4)} - \frac{\int \left(-\frac{231c}{a+bx^4} - \frac{120dx}{a+bx^4}\right)\,dx}{384a^3}$$

$$= \frac{x(c+dx)}{12a\,(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3\,(a+bx^4)} + \frac{(77c)\int \frac{1}{a+bx^4}\,dx}{128a^3} + \frac{(5d)\int \frac{x}{a+bx^4}\,d}{16a^3}$$

$$= \frac{x(c+dx)}{12a\,(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3\,(a+bx^4)} + \frac{(77c)\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{256a^{7/2}} + \frac{(77c)\int}{}$$

$$= \frac{x(c+dx)}{12a\,(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3\,(a+bx^4)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} + \frac{(77c)\int}{5}$$

$$= \frac{x(c+dx)}{12a\,(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3\,(a+bx^4)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{77c\log\left(\sqrt{}\right.}{}$$

$$= \frac{x(c+dx)}{12a\,(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3\,(a+bx^4)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{77c\tan^{-1}\left(\right.}{256\sqrt{}}$$

Mathematica [A]

time = 0.19, size = 274, normalized size = 0.94

$$\frac{\frac{256a^{11/4}x(c+dx)}{(a+bx^4)^3}+\frac{32a^{7/4}x(11c+10dx)}{(a+bx^4)^2}+\frac{8a^{3/4}x(77c+60dx)}{a+bx^4}-\frac{6\left(77\sqrt{2}\sqrt[4]{b}c+80\sqrt[4]{a}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}}+\frac{6\left(77\sqrt{2}\sqrt[4]{b}c-80\sqrt[4]{a}d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}}-\frac{231\sqrt{2}c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{\sqrt[4]{b}}+\frac{231\sqrt{2}c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{\sqrt[4]{b}}}{3072a^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^4)^4,x]
```

```
[Out] ((256*a^(11/4)*x*(c + d*x))/(a + b*x^4)^3 + (32*a^(7/4)*x*(11*c + 10*d*x))/
(a + b*x^4)^2 + (8*a^(3/4)*x*(77*c + 60*d*x))/(a + b*x^4) - (6*(77*Sqrt[2]*
b^(1/4)*c + 80*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b]
+ (6*(77*Sqrt[2]*b^(1/4)*c - 80*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/Sqrt[b] - (231*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2])/b^(1/4) + (231*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/b^(1/4))/(3072*a^(15/4))
```

Maple [A]
time = 0.34, size = 201, normalized size = 0.69

method	result
risch	$\frac{\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(40\_Rd+77c\right)\ln\left(x-\_R\right)}{\_R^3}}{512a^3b}$
default	$\frac{\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(b\,x^4+a)^3}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan}{8a}}{128a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (5/32/a^3*d*b^2*x^10+77/384*c/a^3*b^2*x^9+5/12/a^2*b*d*x^6+33/64*b*c/a^2*x^
5+11/32*d/a*x^2+51/128/a*c*x)/(b*x^4+a)^3+1/128/a^3*(77/8*c*(a/b)^(1/4)/a*2
^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/
2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(
1/4)*x-1))+20*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2)))
```

Maxima [A]
time = 0.53, size = 304, normalized size = 1.04

$$\frac{60\,b^2dx^{10}+77\,b^2cx^9+160\,abdx^6+198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\left(a^3b^3x^{12}+3\,a^4b^2x^8+3\,a^5bx^4+a^6\right)}+\frac{\frac{77\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{77\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\left(77\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-80\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{1}{4}}}+\frac{2\left(77\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c+80\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{1}{4}}}}{1024\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 132*a
^2*d*x^2 + 153*a^2*c*x)/(a^3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6)
+ 1/1024*(77*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a
))/(a^(3/4)*b^(1/4)) - 77*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(1/4)*c - 80*sq
rt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sq
rt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 2*(77*sqrt(2)*a^(
1/4)*b^(1/4)*c + 80*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^
(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4
)))/a^3
```

Fricas [C] Result contains complex when optimal does not.
time = 5.09, size = 43302, normalized size = 148.80

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] 1/9437184*(1474560*b^2*d*x^10 + 1892352*b^2*c*x^9 + 3932160*a*b*d*x^6 + 486
6048*a*b*c*x^5 + 3244032*a^2*d*x^2 + 3760128*a^2*c*x - 18432*(a^3*b^3*x^12
+ 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6)*(77*sqrt(-c^2/(a^8*b*sqrt(-1/(a*b))))
+ 40*d*sqrt(-1/(a*b))/a^3)*log(3200*a^12*b*d^2*(77*sqrt(-c^2/(a^8*b*sqrt(-1
/(a*b)))) + 40*d*sqrt(-1/(a*b))/a^3)^3 - 237160*a^8*b*c^2*d*(77*sqrt(-c^2/(
a^8*b*sqrt(-1/(a*b)))) + 40*d*sqrt(-1/(a*b))/a^3)^2 + 1897280000*a*c^2*d^3
- 77*(35153041*b*c^5 - 10240000*a*c*d^4)*x + (35153041*a^4*b*c^4 + 5120000*
a^5*d^4)*(77*sqrt(-c^2/(a^8*b*sqrt(-1/(a*b)))) + 40*d*sqrt(-1/(a*b))/a^3))
+ 2*(a^3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6)*((-I*sqrt(3) + 1)*((
77*a^4*b*sqrt(-1/(a*b))*sqrt(-c^2/(a^8*b*sqrt(-1/(a*b)))) - 40*d)^2/(a^7*b)
 - 3*(6160*a^4*d*sqrt(-c^2/(a^8*b*sqrt(-1/(a*b)))) - 1600*a*d^2*sqrt(-1/(a*
b)) + 5929*c^2)/(a^8*b*sqrt(-1/(a*b))))/(-1/805306368*(77*a^4*b*sqrt(-1/(a*
b))*sqrt(-c^2/(a^8*b*sqrt(-1/(a*b)))) - 40*d)*(6160*a^4*d*sqrt(-c^2/(a^8*b*
sqrt(-1/(a*b)))) - 1600*a ...
```

Sympy [A]
time = 0.89, size = 231, normalized size = 0.79

$$\operatorname{RootSum}\left(68719476736t^4a^{15}b^2 + 838860800t^2a^8bd^2 - 485703680ta^4bc^2d + 2560000ad^4 + 35153041bc^4, \left(t \mapsto t\log\left(x + \frac{-429496729600t^3a^{12}bd^2 - 62170071040t^2a^8bc^2d - 2621440000ta^5d^4 - 17998356992ta^4bc^4 + 1897280000ac^2d^3}{788480000acd^4 - 2706784157bc^5}\right)\right)\right) + \frac{153a^2cx + 132a^2dx^2 + 198abcx^5 + 160abdx^6 + 77b^2cx^9 + 60b^2dx^{10}}{384a^6 + 1152a^5bx^4 + 1152a^4b^2x^8 + 384a^3b^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**2 + 838860800*_t**2*a**8*b*d**2 - 485703
680*_t*a**4*b*c**2*d + 2560000*a*d**4 + 35153041*b*c**4, Lambda(_t, _t*log(
x + (-429496729600*_t**3*a**12*b*d**2 - 62170071040*_t**2*a**8*b*c**2*d - 2
621440000*_t*a**5*d**4 - 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2*d**
```

```
3)/(788480000*a*c*d**4 - 2706784157*b*c**5)))) + (153*a**2*c*x + 132*a**2*d
*x**2 + 198*a*b*c*x**5 + 160*a*b*d*x**6 + 77*b**2*c*x**9 + 60*b**2*d*x**10)
/(384*a**6 + 1152*a**5*b*x**4 + 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

Giac [A]

time = 0.71, size = 280, normalized size = 0.96

$$\frac{77\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b}-\frac{77\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b}+\frac{\sqrt{2}\left(40\,\sqrt{2}\,\sqrt{ab}\,bd+77\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2}+\frac{\sqrt{2}\left(40\,\sqrt{2}\,\sqrt{ab}\,bd+77\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2}+\frac{60\,b^2dx^{10}+77\,b^2cx^9+160\,abdx^6+198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\,(bx^4+a)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(a^4*b) - 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4)
 + sqrt(a/b))/(a^4*b) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*b^3
)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^4*b^2) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*b^3)^(1/4)*b*c)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^2) + 1/38
4*(60*b^2*d*x^10 + 77*b^2*c*x^9 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 132*a^2*d
*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

Mupad [B]

time = 0.31, size = 350, normalized size = 1.20

$$\left(\sum_{k=1}^{4}\ln\left(\frac{b^2\left(1925\,c\,d^2+1000\,d^3\,x-\mathrm{root}(68719476736\,a^{15}\,b^2\,z^4+838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\,b\,c^2\,d\,z+35153041\,b\,c^4+2560000\,a\,d^4,z,k)^2\,a^7\,b\,c\,315392-\mathrm{root}(68719476736\,a^{15}\,b^2\,z^4+838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\,b\,c^2\,d\,z+35153041\,b\,c^4+2560000\,a\,d^4,z,k)\,a^3\,b\,c^2\,x\,47432+\mathrm{root}(68719476736\,a^{15}\,b^2\,z^4+838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\,b\,c^2\,d\,z+35153041\,b\,c^4+2560000\,a\,d^4,z,k)^2\,a^7\,b\,d\,x\,163840\right)}{a^9\,32768}\right)\mathrm{root}(68719476736\,a^{15}\,b^2\,z^4+838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\,b\,c^2\,d\,z+35153041\,b\,c^4+2560000\,a\,d^4,z,k)\right)+\frac{\frac{11\,d\,x^2}{32\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{33\,b\,c\,x^5}{64\,a^2}+\frac{5\,b\,d\,x^6}{12\,a^2}}{a^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^8+b^3\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^4)^4,x)
```

```
[Out] symsum(log((b^2*(1925*c*d^2 + 1000*d^3*x - 315392*root(68719476736*a^15*b^2
*z^4 + 838860800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 +
 2560000*a*d^4, z, k)^2*a^7*b*c - 47432*root(68719476736*a^15*b^2*z^4 + 838
860800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 2560000*a
*d^4, z, k)*a^3*b*c^2*x + 163840*root(68719476736*a^15*b^2*z^4 + 838860800*
a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 2560000*a*d^4, z
, k)^2*a^7*b*d*x))/(32768*a^9))*root(68719476736*a^15*b^2*z^4 + 838860800*a
^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 2560000*a*d^4, z,
 k), k, 1, 4) + ((11*d*x^2)/(32*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9)/(384
*a^3) + (5*b^2*d*x^10)/(32*a^3) + (33*b*c*x^5)/(64*a^2) + (5*b*d*x^6)/(12*a
^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

3.123 $\int \frac{c+dx}{1-x^4} dx$

Optimal. Leaf size=24

$$\frac{1}{2}c\tan^{-1}(x)+\frac{1}{2}c\tanh^{-1}(x)+\frac{1}{2}d\tanh^{-1}\left(x^2\right)$$

```
[Out] 1/2*c*arctan(x)+1/2*c*arctanh(x)+1/2*d*arctanh(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1890, 218, 212, 209, 281}

$$\frac{1}{2}c\text{ArcTan}(x)+\frac{1}{2}c\tanh^{-1}(x)+\frac{1}{2}d\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(1 - x^4),x]
```

```
[Out] (c*ArcTan[x])/2 + (c*ArcTanh[x])/2 + (d*ArcTanh[x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{1-x^4}\,dx &= \int \left(\frac{c}{1-x^4}+\frac{dx}{1-x^4}\right)dx\\ &= c\int \frac{1}{1-x^4}\,dx+d\int \frac{x}{1-x^4}\,dx\\ &= \frac{1}{2}c\int \frac{1}{1-x^2}\,dx+\frac{1}{2}c\int \frac{1}{1+x^2}\,dx+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}c\tan^{-1}(x)+\frac{1}{2}c\tanh^{-1}(x)+\frac{1}{2}d\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.75

$$\frac{1}{4}\left(2c\tan^{-1}(x)-(c+d)\log(1-x)+c\log(1+x)-d\log(1+x)+d\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(1 - x^4),x]
```

```
[Out] (2*c*ArcTan[x] - (c + d)*Log[1 - x] + c*Log[1 + x] - d*Log[1 + x] + d*Log[1
 + x^2])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(18) = 36.
time = 0.37, size = 40, normalized size = 1.67

method	result
default	$\frac{(-c-d)\ln(x-1)}{4}-\frac{(-c+d)\ln(x+1)}{4}+\frac{d\ln(x^2+1)}{4}+\frac{c\arctan(x)}{2}$
meijerg	$\frac{d\,\text{arctanh}(x^2)}{2}-\frac{cx\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}$
risch	$\frac{d\ln(c^4x^2+4d^4x^2+c^4+4d^4)}{4}+\frac{c\arctan\left(\frac{c^4x}{c^4+4d^4}+\frac{4d^4x}{c^4+4d^4}\right)}{2}+\frac{c\arctan\left(\frac{2cd}{c^2-2d^2}\right)}{2}+\frac{\ln(-1-x)c}{4}-\frac{\ln(-1-x)d}{4}-\frac{\ln(x-1)c}{4}-$ l

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-c-d)*ln(x-1)-1/4*(-c+d)*ln(x+1)+1/4*d*ln(x^2+1)+1/2*c*arctan(x)
```

Maxima [A]
time = 0.53, size = 35, normalized size = 1.46

$$\frac{1}{2}\,c\arctan\left(x\right)+\frac{1}{4}\,d\log\left(x^{2}+1\right)+\frac{1}{4}\left(c-d\right)\log\left(x+1\right)-\frac{1}{4}\left(c+d\right)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1/4*(c + d)
*log(x - 1)
```

Fricas [A]
time = 0.38, size = 35, normalized size = 1.46

$$\frac{1}{2}\,c\arctan\left(x\right)+\frac{1}{4}\,d\log\left(x^{2}+1\right)+\frac{1}{4}\left(c-d\right)\log\left(x+1\right)-\frac{1}{4}\left(c+d\right)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1/4*(c + d)
*log(x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.28, size = 313, normalized size = 13.04

$$\frac{(c-d)\log\left(x+\frac{c^{4}(c-d)+5c^{2}d^{3}+c^{2}d(c-d)^{2}-2d^{4}(c-d)+2d^{2}(c-d)^{3}}{c^{5}+4cd^{4}}\right)}{4}-\frac{(c+d)\log\left(x+\frac{-c^{4}(c+d)+5c^{2}d^{3}+c^{2}d(c+d)^{2}+2d^{4}(c+d)-2d^{2}(c+d)^{3}}{c^{5}+4cd^{4}}\right)}{4}-\left(-\frac{ic}{4}-\frac{d}{4}\right)\log\left(x+\frac{-4c^{4}\left(-\frac{ic}{4}-\frac{d}{4}\right)+5c^{2}d^{3}+16c^{2}d\left(-\frac{ic}{4}-\frac{d}{4}\right)^{2}+8d^{4}\left(-\frac{ic}{4}-\frac{d}{4}\right)-128d^{2}\left(-\frac{ic}{4}-\frac{d}{4}\right)^{3}}{c^{5}+4cd^{4}}\right)-\left(\frac{ic}{4}-\frac{d}{4}\right)\log\left(x+\frac{-4c^{4}\left(\frac{ic}{4}-\frac{d}{4}\right)+5c^{2}d^{3}+16c^{2}d\left(\frac{ic}{4}-\frac{d}{4}\right)^{2}+8d^{4}\left(\frac{ic}{4}-\frac{d}{4}\right)-128d^{2}\left(\frac{ic}{4}-\frac{d}{4}\right)^{3}}{c^{5}+4cd^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x**4+1),x)
```

```
[Out] (c - d)*log(x + (c**4*(c - d) + 5*c**2*d**3 + c**2*d*(c - d)**2 - 2*d**4*(c
 - d) + 2*d**2*(c - d)**3)/(c**5 + 4*c*d**4))/4 - (c + d)*log(x + (-c**4*(c
 + d) + 5*c**2*d**3 + c**2*d*(c + d)**2 + 2*d**4*(c + d) - 2*d**2*(c + d)**
3)/(c**5 + 4*c*d**4))/4 - (-I*c/4 - d/4)*log(x + (-4*c**4*(-I*c/4 - d/4) +
5*c**2*d**3 + 16*c**2*d*(-I*c/4 - d/4)**2 + 8*d**4*(-I*c/4 - d/4) - 128*d**
2*(-I*c/4 - d/4)**3)/(c**5 + 4*c*d**4)) - (I*c/4 - d/4)*log(x + (-4*c**4*(I
*c/4 - d/4) + 5*c**2*d**3 + 16*c**2*d*(I*c/4 - d/4)**2 + 8*d**4*(I*c/4 - d/
4) - 128*d**2*(I*c/4 - d/4)**3)/(c**5 + 4*c*d**4))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.
time = 0.67, size = 37, normalized size = 1.54

$$\frac{1}{2}c\arctan(x) + \frac{1}{4}d\log(x^2+1) + \frac{1}{4}(c-d)\log(|x+1|) - \frac{1}{4}(c+d)\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(abs(x + 1)) - 1/4*(c
 + d)*log(abs(x - 1))
```

Mupad [B]
time = 4.92, size = 100, normalized size = 4.17

$$-\frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{3/4}\sqrt{2}\,x+1\right)\left(\sqrt{2}\,c+2\,(-1)^{1/4}\,d\right)}{4}-\frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{3/4}\sqrt{2}\,x-1\right)\left(2\sqrt{2}\,c-4\,(-1)^{1/4}\,d\right)}{8}+\frac{(-1)^{1/4}\sqrt{2}\,c\,\ln\left(\frac{x^2+(-1)^{1/4}\sqrt{2}\,x+1\mathrm{i}}{x^2-(-1)^{1/4}\sqrt{2}\,x+1\mathrm{i}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(c + d*x)/(x^4 - 1),x)
```

```
[Out] ((-1)^(1/4)*2^(1/2)*c*log((x^2 + (-1)^(1/4)*2^(1/2)*x + 1i)/(x^2 - (-1)^(1/
4)*2^(1/2)*x + 1i)))/8 - ((-1)^(1/4)*atan((-1)^(3/4)*2^(1/2)*x - 1)*(2*2^(1
/2)*c - 4*(-1)^(1/4)*d))/8 - ((-1)^(1/4)*atan((-1)^(3/4)*2^(1/2)*x + 1)*(2^
(1/2)*c + 2*(-1)^(1/4)*d))/4
```

3.124 $\int \frac{c+dx}{1+x^4}\,dx$

Optimal. Leaf size=98

$$\frac{1}{2}d\tan^{-1}\left(x^2\right)-\frac{c\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{c\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}}-\frac{c\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}+\frac{c\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}$$

```
[Out] 1/2*d*arctan(x^2)+1/4*c*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*c*arctan(1+x*2^(1/
2))*2^(1/2)-1/8*c*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/8*c*ln(1+x^2+x*2^(1/2))*2^(
1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {1890, 217, 1179, 642, 1176, 631, 210, 281, 209}

$$-\frac{c\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{c\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{2\sqrt{2}}+\frac{1}{2}d\text{ArcTan}\left(x^2\right)-\frac{c\log\left(x^2-\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{c\log\left(x^2+\sqrt{2}\,x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(1 + x^4),x]
```

```
[Out] (d*ArcTan[x^2])/2 - (c*ArcTan[1 - Sqrt[2]*x])/(2*Sqrt[2]) + (c*ArcTan[1 + S
qrt[2]*x])/(2*Sqrt[2]) - (c*Log[1 - Sqrt[2]*x + x^2])/(4*Sqrt[2]) + (c*Log[
1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx}{1+x^4}\,dx = \int \left(\frac{c}{1+x^4}+\frac{dx}{1+x^4}\right)dx$$
$$= c\int \frac{1}{1+x^4}\,dx + d\int \frac{x}{1+x^4}\,dx$$
$$= \frac{1}{2}c\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{2}c\int \frac{1+x^2}{1+x^4}\,dx + \frac{1}{2}d\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}d\tan^{-1}\left(x^2\right) + \frac{1}{4}c\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{1}{4}c\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx - \frac{c\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{4\sqrt{2}} -$$
$$= \frac{1}{2}d\tan^{-1}\left(x^2\right) - \frac{c\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}} + \frac{c\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}} + \frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x,\right.}{2\sqrt{2}}$$
$$= \frac{1}{2}d\tan^{-1}\left(x^2\right) - \frac{c\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}} + \frac{c\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}} - \frac{c\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}} + \frac{c}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 99, normalized size = 1.01

$$\frac{1}{4}\left(-\left(\left(\sqrt[4]{-1}\,c+id\right)\log\left(\sqrt[4]{-1}\,-x\right)\right)+\left(-(-1)^{3/4}c+id\right)\log\left((-1)^{3/4}-x\right)+\left(\sqrt[4]{-1}\,c-id\right)\log\left(\sqrt[4]{-1}\,+x\right)+\left((-1)^{3/4}c+id\right)\log\left((-1)^{3/4}+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(1 + x^4),x]
```

```
[Out] (-(((-1)^(1/4)*c + I*d)*Log[(-1)^(1/4) - x]) + (-((-1)^(3/4)*c) + I*d)*Log[
(-1)^(3/4) - x] + ((-1)^(1/4)*c - I*d)*Log[(-1)^(1/4) + x] + ((-1)^(3/4)*c
+ I*d)*Log[(-1)^(3/4) + x])/4
```

Maple [A]
time = 0.34, size = 61, normalized size = 0.62

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+1\right)}\frac{\left(\_R d+c\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{4}$
default	$\frac{c\sqrt{2}\,\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}+\frac{d\arctan\left(x^2\right)}{2}$

meijerg	$\frac{d\arctan(x^2)}{2}+\frac{c\left(-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{2(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^4)^{\frac{1}{4}}}\right)}{(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{2(x^4)^{\frac{1}{4}}}+\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c*2^(1/2)*(ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))+2*arctan(x*2^(1/2)+1
)+2*arctan(x*2^(1/2)-1))+1/2*d*arctan(x^2)
```

Maxima [A]
time = 0.54, size = 86, normalized size = 0.88

$$\frac{1}{8}\sqrt{2}\,c\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{8}\sqrt{2}\,c\log\left(x^2-\sqrt{2}\,x+1\right)+\frac{1}{4}\left(\sqrt{2}\,c-2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{4}\left(\sqrt{2}\,c+2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 - sqrt(2)*x
+ 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqr
t(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.32, size = 34376, normalized size = 350.78

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/3145728*((-12884901888*I*sqrt(2)*c*d + (32768*I*sqrt(2)*c + 32768*sqrt(sq
rt(2)*(sqrt(2)*c^2 - 2*sqrt(2)*d^2 + 4*I*c*d)))^2 - 6442450944*I*sqrt(2)*sq
rt(sqrt(2)*(sqrt(2)*c^2 - 2*sqrt(2)*d^2 + 4*I*c*d))*c - 12884901888*d^2)*(-
I*sqrt(3) + 1)/((32768*I*sqrt(2)*c + 32768*sqrt(sqrt(2)*(sqrt(2)*c^2 - 2*sq
rt(2)*d^2 + 4*I*c*d)))^3 + 3799912185593856*c^2*d - 316659348799488*(-2*I*s
qrt(2)*c*d - I*sqrt(2)*sqrt(sqrt(2)*(sqrt(2)*c^2 - 2*sqrt(2)*d^2 + 4*I*c*d)
)*c - 2*d^2)*(-I*sqrt(2)*c - sqrt(sqrt(2)*(sqrt(2)*c^2 - 2*sqrt(2)*d^2 + 4*
I*c*d))) + 1899956092796928*I*sqrt(2)*(c^3 - c*d^2) - 949978046398464*(3*c^
2 - 4*d^2)*sqrt(sqrt(2)*(sqrt(2)*c^2 - 2*sqrt(2)*d^2 + 4*I*c*d)) + 47498902
3199232*(sqrt(2)*(sqrt(2)*c^2 - 2*sqrt(2)*d^2 + 4*I*c*d))^(3/2) + 1179648*s
qrt(4611686018427387904*I*sqrt(2)*c*d^5 - 6341068275337658368*I*sqrt(2)*sqr
t(sqrt(2)*(sqrt(2)*c^2 - 2*sqrt(2)*d^2 + 4*I*c*d))*c^5 - 14411518807585587
```

```
00/3*c^6 + 4611686018427387904/3*d^6 + 15492382718154506240/3*sqrt(2)*(sqrt
(2)*c^2 - 2*sqrt(2)*d^2 + ...
```

Sympy [A]
time = 0.28, size = 83, normalized size = 0.85

$$\operatorname{RootSum}\left(256t^4 + 32t^2d^2 - 16tc^2d + c^4 + d^4, \left(t \mapsto t\log\left(x + \frac{128t^3d^2 + 16t^2c^2d + 4tc^4 + 8td^4 - 5c^2d^3}{c^5 - 4cd^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x**4+1),x)
[Out] RootSum(256*_t**4 + 32*_t**2*d**2 - 16*_t*c**2*d + c**4 + d**4, Lambda(_t,
_t*log(x + (128*_t**3*d**2 + 16*_t**2*c**2*d + 4*_t*c**4 + 8*_t*d**4 - 5*c*
*2*d**3)/(c**5 - 4*c*d**4))))
```

Giac [A]
time = 0.65, size = 86, normalized size = 0.88

$$\frac{1}{8}\sqrt{2}\,c\log\left(x^2 + \sqrt{2}\,x + 1\right) - \frac{1}{8}\sqrt{2}\,c\log\left(x^2 - \sqrt{2}\,x + 1\right) + \frac{1}{4}\left(\sqrt{2}\,c - 2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x + \sqrt{2}\right)\right) + \frac{1}{4}\left(\sqrt{2}\,c + 2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x - \sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^4+1),x, algorithm="giac")
[Out] 1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 - sqrt(2)*x
+ 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqr
t(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

Mupad [B]
time = 0.09, size = 71, normalized size = 0.72

$$\operatorname{atan}\left(\sqrt{2}\,x - 1\right)\left(\frac{d}{2} + \frac{\sqrt{2}\,c}{4}\right) - \operatorname{atan}\left(\sqrt{2}\,x + 1\right)\left(\frac{d}{2} - \frac{\sqrt{2}\,c}{4}\right) + \frac{\sqrt{2}\,c\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(x^4 + 1),x)
[Out] atan(2^(1/2)*x - 1)*(d/2 + (2^(1/2)*c)/4) - atan(2^(1/2)*x + 1)*(d/2 - (2^(
1/2)*c)/4) + (2^(1/2)*c*log((2^(1/2)*x + x^2 + 1)/(x^2 - 2^(1/2)*x + 1)))/8
```

3.125 $\int \frac{c+dx+ex^2}{a-bx^4} \, dx$

Optimal. Leaf size=116

$$\frac{\left(\sqrt{b}\, c-\sqrt{a}\, e\right) \tan^{-1}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{a}}\right)}{2 a^{3/4} b^{3/4}}+\frac{\left(\sqrt{b}\, c+\sqrt{a}\, e\right) \tanh^{-1}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{a}}\right)}{2 a^{3/4} b^{3/4}}+\frac{d \tanh^{-1}\left(\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right)}{2 \sqrt{a}\, \sqrt{b}}$$

```
[Out] 1/2*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)+1/2*arctan(b^(1/4)*x/a^(
1/4))*(-e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*arctanh(b^(1/4)*x/a^(1/4))
*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

Rubi [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{a}}\right) \left(\sqrt{b}\, c-\sqrt{a}\, e\right)}{2 a^{3/4} b^{3/4}}+\frac{\left(\sqrt{a}\, e+\sqrt{b}\, c\right) \tanh^{-1}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{a}}\right)}{2 a^{3/4} b^{3/4}}+\frac{d \tanh^{-1}\left(\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right)}{2 \sqrt{a}\, \sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
 + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx = \int \left(\frac{dx}{a-bx^4}+\frac{c+ex^2}{a-bx^4}\right)dx$$
$$= d\int \frac{x}{a-bx^4}\,dx+\int \frac{c+ex^2}{a-bx^4}\,dx$$
$$= \frac{1}{2}d\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)+\frac{1}{2}\left(-\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{1}{-\sqrt{a}\,\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.$$
$$= \frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{2\sqrt{a}\,\sqrt{b}}$$

Mathematica [A]
time = 0.03, size = 187, normalized size = 1.61

$$\frac{2\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}\,c+\sqrt[4]{a}\,\sqrt[4]{b}\,d+\sqrt{a}\,e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+\sqrt{b}\,c\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)-\sqrt[4]{a}\,\sqrt[4]{b}\,d\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+\sqrt{a}\,e\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+\sqrt[4]{a}\,\sqrt[4]{b}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + a^(1/
4)*b^(1/4)*d + Sqrt[a]*e)*Log[a^(1/4) - b^(1/4)*x] + Sqrt[b]*c*Log[a^(1/4)
+ b^(1/4)*x] - a^(1/4)*b^(1/4)*d*Log[a^(1/4) + b^(1/4)*x] + Sqrt[a]*e*Log[a
^(1/4) + b^(1/4)*x] + a^(1/4)*b^(1/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*a^(3
/4)*b^(3/4))
```

Maple [A]
time = 0.35, size = 139, normalized size = 1.20

method	result	size
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(\_R^2e+\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	39
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(
1/4)))+1/4*d/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2)))-1/4*e/
b/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4))))
```

Maxima [A]
time = 0.53, size = 155, normalized size = 1.34

$$\frac{d\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{4\sqrt{a}\,\sqrt{b}}-\frac{d\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{4\sqrt{a}\,\sqrt{b}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{2\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{4\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*d*log(sqrt(b)*x^2
- sqrt(a))/(sqrt(a)*sqrt(b)) + 1/2*(sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt
(b)*c + sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqr
t(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
```

Fricas [C] Result contains complex when optimal does not.
time = 2.52, size = 120560, normalized size = 1039.31

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/24*(2*(1/4)^(2/3)*(-I*sqrt(3) + 1)*((sqrt(2)*a*b*sqrt(1/(a*b))*sqrt((2*a
*b*c*e*sqrt(1/(a*b)) + b*c^2 + a*e^2)/(a^2*b^2*sqrt(1/(a*b)))) + sqrt(2)*d)
^2/(a*b) + 3*sqrt(2)*(2*sqrt(2)*a*b*c*e*sqrt(1/(a*b)) - 2*sqrt(2)*a*b*d*sqr
t((2*a*b*c*e*sqrt(1/(a*b)) + b*c^2 + a*e^2)/(a^2*b^2*sqrt(1/(a*b)))) - sqrt
```

```
(2)*(b*c^2 - (b*d^2*sqrt(1/(a*b)) - e^2)*a))/(a^2*b^2*sqrt(1/(a*b)))))/(9*(2
*sqrt(2)*a*b*c*e*sqrt(1/(a*b)) - 2*sqrt(2)*a*b*d*sqrt((2*a*b*c*e*sqrt(1/(a*
b)) + b*c^2 + a*e^2)/(a^2*b^2*sqrt(1/(a*b)))) - sqrt(2)*(b*c^2 - (b*d^2*sqr
t(1/(a*b)) - e^2)*a))*(sqrt(2)*a*b*sqrt(1/(a*b))*sqrt((2*a*b*c*e*sqrt(1/(a*
b)) + b*c^2 + a*e^2)/(a^2*b^2*sqrt(1/(a*b)))) + sqrt(2)*d)/(a^2*b^2) + 27*s
qrt(2)*(sqrt(2)*a^2*b^2*sqrt(1/(a*b))*((2*a*b*c*e*sqrt(1/(a*b)) + b*c^2 + a
*e^2)/(a^2*b^2*sqrt(1/(a*b))))^(3/2) + 2*sqrt(2)*c*d*e + (sqrt(2)*d^2*sqrt(
(2*a*b*c*e*sqrt(1/(a*b)) + b*c^2 + a*e^2)/(a^2*b^2*sqrt(1/(a*b)))) - 4*sqrt
(2)*c*e*sqrt((2*a*b*c*e*sqrt(1/(a*b)) + b*c^2 + a*e^2)/(a^2*b^2*sqrt(1/(a*b
)))))*a*b*sqrt(1/(a*b)) - ...
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(105) = 210$.
time = 5.63, size = 471, normalized size = 4.06

$$-\operatorname{RootSum}\left(256t^4a^3b^3 + t^2(-64a^2b^2ce - 32a^2b^2d^2) + t(-16a^2bde^2 - 16ab^2c^2d) - a^2e^4 + 2abc^2e^2 - 4abcd^2e + abd^4 - b^2c^4, \left(t \mapsto t\log\left(x + \frac{-64t^3a^4b^2e^3 - 64t^3a^3b^3c^2e + 128t^3a^3b^3cd^2 + 48t^2a^3b^2cde^2 - 32t^2a^3b^2d^3e - 16t^2a^2b^3c^3d + 12ta^3bce^4 + 12ta^3bd^2e^3 + 16ta^2b^2c^3e^2 - 36ta^2b^2c^2d^2e - 8ta^2b^2cd^4 + 4tab^3c^5 + 3a^3de^5 - 5a^2bcd^3e^2 + 2a^2bd^5e + 5ab^2c^4de - 5ab^2c^3d^3}{a^3e^6 + a^2bc^2e^4 - 8a^2bcd^2e^3 + 4a^2bd^4e^2 - ab^2c^4e^2 + 8ab^2c^3d^2e - 4ab^2c^2d^4 - b^3c^6}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 + _t**2*(-64*a**2*b**2*c*e - 32*a**2*b**2*d**2
) + _t*(-16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) - a**2*e**4 + 2*a*b*c**2*e**2
 - 4*a*b*c*d**2*e + a*b*d**4 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*
a**4*b**2*e**3 - 64*_t**3*a**3*b**3*c**2*e + 128*_t**3*a**3*b**3*c*d**2 + 4
8*_t**2*a**3*b**2*c*d*e**2 - 32*_t**2*a**3*b**2*d**3*e - 16*_t**2*a**2*b**3
*c**3*d + 12*_t*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 + 16*_t*a**2*b**2*c*
*3*e**2 - 36*_t*a**2*b**2*c**2*d**2*e - 8*_t*a**2*b**2*c*d**4 + 4*_t*a*b**3
*c**5 + 3*a**3*d*e**5 - 5*a**2*b*c*d**3*e**2 + 2*a**2*b*d**5*e + 5*a*b**2*c
**4*d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 + a**2*b*c**2*e**4 - 8*a**2*b*c*d*
*2*e**3 + 4*a**2*b*d**4*e**2 - a*b**2*c**4*e**2 + 8*a*b**2*c**3*d**2*e - 4*
a*b**2*c**2*d**4 - b**3*c**6))))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(80) = 160$.
time = 0.57, size = 263, normalized size = 2.27

$$-\frac{\sqrt{2}\left(b^2c - \sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd + \sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c + \sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd - \sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c - \sqrt{-ab}\,be\right)\log\left(x^2 + \sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}} + \frac{\sqrt{2}\left(b^2c - \sqrt{-ab}\,be\right)\log\left(x^2 - \sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*
sqrt(2)*(b^2*c + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*sqrt(
```

```
2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/
(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

Mupad [B]
time = 5.14, size = 725, normalized size = 6.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a - b*x^4),x)
[Out] symsum(log(b^2*c^2*e - b^2*c*d^2 - b^2*d^3*x - a*b*e^3 - 16*root(256*a^3*b^
3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b
^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z
, k)^2*a*b^3*c - 4*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d
^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^
2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)*b^3*c^2*x + 16*root(256*a^3*b^3*z^4
- 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2
*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)^2
*a*b^3*d*x - 4*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z
^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 +
a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)*a*b^2*e^2*x + 8*root(256*a^3*b^3*z^4 - 6
4*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*
z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)*a*b^
2*d*e + 2*b^2*c*d*e*x)*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b
^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^
2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k), k, 1, 4)
```

3.126 $\int \frac{c+dx+ex^2}{a+bx^4}\,dx$

Optimal. Leaf size=277

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{(\sqrt{}}{}$$

```
[Out] 1/2*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)-1/8*ln(-a^(1/4)*b^(1/4)*x
*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2
)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+c*b^(1/
2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(
1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1
/4))*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e+\sqrt{b}\,c\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e+\sqrt{b}\,c\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4))
+ ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sq
rt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2}{a+bx^4}\,dx &= \int \left(\frac{dx}{a+bx^4}+\frac{c+ex^2}{a+bx^4}\right)dx \\
&= d\int \frac{x}{a+bx^4}\,dx+\int \frac{c+ex^2}{a+bx^4}\,dx \\
&= \frac{1}{2}d\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \ldots}{2b} \\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b}+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{\ldots}{\frac{\sqrt{a}}{\sqrt{b}}+\ldots}}{4b} \\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\ldots\right.}{\ldots} \\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\ldots}{2\sqrt{2}\,a^{3/4}\ldots}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 229, normalized size = 0.83

$$
\frac{-2\left(\sqrt{2}\,\sqrt{b}\,c+2\sqrt[4]{a}\,\sqrt[4]{b}\,d+\sqrt{2}\,\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\,\sqrt{b}\,c-2\sqrt[4]{a}\,\sqrt[4]{b}\,d+\sqrt{2}\,\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\sqrt{2}\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\left(\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)-\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)\right)}{8a^{3/4}b^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1
- (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)*d
 + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(Sq
rt[b]*c - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a^(3/4)*b^(
3/4))
```

Maple [A]
time = 0.34, size = 226, normalized size = 0.82

method	result

risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)} \frac{\left(\_R^2 e+\_R d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{e\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^
2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1
/8*e/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2
+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*ar
ctan(2^(1/2)/(a/b)^(1/4)*x-1))
```

Maxima [A]
time = 0.57, size = 261, normalized size = 0.94

$$\frac{\sqrt{2}\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e-2\sqrt{a}\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e+2\sqrt{a}\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*e)*log
(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 1/4
*(sqrt(2)*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e - 2*sqrt(a)*sqrt(b)
*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)
*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 1/4*(sqrt(2)*a^(1/4)*b
^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e + 2*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt
(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)
*sqrt(sqrt(a)*sqrt(b))*b^(3/4))
```

Fricas [C] Result contains complex when optimal does not.
time = 2.66, size = 121386, normalized size = 438.22

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/288*((-I*sqrt(3) + 1)*((a*b*sqrt(-1/(a*b))*sqrt(-(2*a*b*c*e*sqrt(-1/(a*b)
) + b*c^2 - a*e^2)/(a^2*b^2*sqrt(-1/(a*b)))) - d)^2/(a*b) - 3*(b*c^2 - (((d
^2 + 2*c*e)*sqrt(-1/(a*b)) - 2*d*sqrt(-(2*a*b*c*e*sqrt(-1/(a*b)) + b*c^2 -
a*e^2)/(a^2*b^2*sqrt(-1/(a*b)))))*b + e^2)*a)/(a^2*b^2*sqrt(-1/(a*b))))/(-1
/384*(b*c^2 - (((d^2 + 2*c*e)*sqrt(-1/(a*b)) - 2*d*sqrt(-(2*a*b*c*e*sqrt(-1
/(a*b)) + b*c^2 - a*e^2)/(a^2*b^2*sqrt(-1/(a*b)))))*b + e^2)*a)*(a*b*sqrt(-
1/(a*b))*sqrt(-(2*a*b*c*e*sqrt(-1/(a*b)) + b*c^2 - a*e^2)/(a^2*b^2*sqrt(-1/
(a*b)))) - d)/(a^2*b^2) + 1/128*(a^2*b^2*sqrt(-1/(a*b))*(-(2*a*b*c*e*sqrt(-
1/(a*b)) + b*c^2 - a*e^2)/(a^2*b^2*sqrt(-1/(a*b))))^(3/2) + b*c^2*d*sqrt(-1
/(a*b)) - d^3 + 2*c*d*e - (d*e^2*sqrt(-1/(a*b)) + (d^2*sqrt(-(2*a*b*c*e*sqr
t(-1/(a*b)) + b*c^2 - a*e^2)/(a^2*b^2*sqrt(-1/(a*b)))) - 4*c*e*sqrt(-(2*a*b
*c*e*sqrt(-1/(a*b)) + b*c^2 - a*e^2)/(a^2*b^2*sqrt(-1/(a*b)))))*b*sqrt(-1/(
a*b)))*a)/(a^2*b^2*sqrt(-1/(a*b))) + 1/1728*(a*b*sqrt(-1/(a*b))*sqrt(-(2*a*
b*c*e*sqrt(-1/(a*b)) + b* ...
```

Sympy [A]
time = 5.58, size = 466, normalized size = 1.68

$$\mathrm{RootSum}\left(256t^4a^3b^3+t^2\cdot\left(64a^2b^2ce+32a^2b^2d^2\right)+t\left(16a^2bde^2-16ab^2c^2d\right)+a^2e^4+2abc^2e^2-4abcd^2e+abd^4+b^2c^4,\left(t\mapsto t\log\left(x+\frac{64t^3a^4b^2e^3-64t^3a^3b^3c^2e+128t^3a^3b^3cd^2+48t^2a^3b^2cde^2-32t^2a^3b^2d^3e+16t^2a^2b^3c^3d+12ta^3bce^4+12ta^3bd^2e^3-16ta^2b^2c^3e^2+36ta^2b^2c^2d^2e+8ta^2b^2cd^4+4tab^3c^5+3a^3de^5+5a^2bcd^3e^2-2a^2bd^5e+5ab^2c^4de-5ab^2c^3d^3}{a^3e^6-a^2bc^2e^4+8a^2bcd^2e^3-4a^2bd^4e^2-ab^2c^4e^2+8ab^2c^3d^2e-4ab^2c^2d^4+b^3c^6}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + _t**2*(64*a**2*b**2*c*e + 32*a**2*b**2*d**2)
+ _t*(16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) + a**2*e**4 + 2*a*b*c**2*e**2 -
4*a*b*c*d**2*e + a*b*d**4 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**4
*b**2*e**3 - 64*_t**3*a**3*b**3*c**2*e + 128*_t**3*a**3*b**3*c*d**2 + 48*_t
**2*a**3*b**2*c*d*e**2 - 32*_t**2*a**3*b**2*d**3*e + 16*_t**2*a**2*b**3*c**
3*d + 12*_t*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 - 16*_t*a**2*b**2*c**3*e
**2 + 36*_t*a**2*b**2*c**2*d**2*e + 8*_t*a**2*b**2*c*d**4 + 4*_t*a*b**3*c**
5 + 3*a**3*d*e**5 + 5*a**2*b*c*d**3*e**2 - 2*a**2*b*d**5*e + 5*a*b**2*c**4*
d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 - a**2*b*c**2*e**4 + 8*a**2*b*c*d**2*e
**3 - 4*a**2*b*d**4*e**2 - a*b**2*c**4*e**2 + 8*a*b**2*c**3*d**2*e - 4*a*b*
*2*c**2*d**4 + b**3*c**6))))
```

Giac [A]
time = 0.62, size = 275, normalized size = 0.99

$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d-\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d-\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)
*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1
```

```
/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8
*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/
4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

Mupad [B]
time = 5.09, size = 712, normalized size = 2.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^4),x)
```

```
[Out] symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - a*b*e^3 - 16*root(256*a^3*b^
3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b
^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z
, k)^2*a*b^3*c - 4*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d
^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^
2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)*b^3*c^2*x + 16*root(256*a^3*b^3*z^4
+ 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2
*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)^2
*a*b^3*d*x + 4*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z
^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 +
a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)*a*b^2*e^2*x - 8*root(256*a^3*b^3*z^4 + 6
4*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*
z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)*a*b^
2*d*e - 2*b^2*c*d*e*x)*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b
^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^
2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k), k, 1, 4)
```

3.127 $\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx$

Optimal. Leaf size=146

$$\frac{x(c+dx+ex^2)}{4a\,(a-bx^4)}+\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

```
[Out] 1/4*x*(e*x^2+d*x+c)/a/(-b*x^4+a)+1/4*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(3/2)
/b^(1/2)+1/8*arctan(b^(1/4)*x/a^(1/4))*(-e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(
3/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/4)
```

Rubi [A]
time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1869, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x(c+dx+ex^2)}{4a\,(a-bx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*ArcTan
[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*Arc
Tanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^2}\,dx &= \frac{x(c+dx+ex^2)}{4a\left(a-bx^4\right)} - \frac{\int \frac{-3c-2dx-ex^2}{a-bx^4}\,dx}{4a} \\
&= \frac{x(c+dx+ex^2)}{4a\left(a-bx^4\right)} - \frac{\int \left(-\frac{2dx}{a-bx^4}+\frac{-3c-ex^2}{a-bx^4}\right)dx}{4a} \\
&= \frac{x(c+dx+ex^2)}{4a\left(a-bx^4\right)} - \frac{\int \frac{-3c-ex^2}{a-bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a-bx^4}\,dx}{2a} \\
&= \frac{x(c+dx+ex^2)}{4a\left(a-bx^4\right)} + \frac{d\,\mathrm{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{4a} - \frac{\left(\frac{3\sqrt{b}\,c}{\sqrt{a}}-e\right)\int \frac{1}{-\sqrt{a}\,\sqrt{b}-bx^2}\,dx}{8a} + \frac{(3
\end{aligned}$$

$$= \frac{x(c+dx+ex^2)}{4a\left(a-bx^4\right)} + \frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{8a^{7/4}b^{3/4}}$$

Mathematica [A]
time = 0.13, size = 211, normalized size = 1.45

$$\frac{\frac{4ax(c+x(d+ex))}{a-bx^4} - \frac{2\sqrt[4]{a}\left(-3\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}} - \frac{\left(3\sqrt[4]{a}\,\sqrt{b}\,c+2\sqrt{a}\,\sqrt[4]{b}\,d+a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)}{b^{3/4}} + \frac{\left(3\sqrt[4]{a}\,\sqrt{b}\,c-2\sqrt{a}\,\sqrt[4]{b}\,d+a^{3/4}e\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{b^{3/4}} + \frac{2\sqrt{a}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt{b}}}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*x*(c + x*(d + e*x)))/(a - b*x^4) - (2*a^(1/4)*(-3*Sqrt[b]*c + Sqrt[a]
*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((3*a^(1/4)*Sqrt[b]*c + 2*Sqrt[a
]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((3*a^(1/4)*Sq
rt[b]*c - 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4
) + (2*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)
```

Maple [A]
time = 0.33, size = 203, normalized size = 1.39

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{cx}{4a}}{-b\,x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(\_R^2e+2\_Rd+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(-b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{16a^2}\right)+d\left(\frac{x^2}{4a(-b\,x^4+a)}+\frac{\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)+e\left(\frac{}{4a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/4*x/a/(-b*x^4+a)+3/16/a^2*(a/b)^(1/4)*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/
4)))+2*arctan(x/(a/b)^(1/4))))+d*(1/4*x^2/a/(-b*x^4+a)+1/8/a/(a*b)^(1/2)*ln
((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2))))+e*(1/4*x^3/a/(-b*x^4+a)-1/16/a/b
/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))))
```

Maxima [A]
time = 0.53, size = 194, normalized size = 1.33

$$-\frac{x^3e+dx^2+cx}{4\,(abx^4-a^2)}+\frac{\frac{2\,d\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{2\,d\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(3\,\sqrt{b}\,c-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(3\,\sqrt{b}\,c+\sqrt{a}\,e\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(x^3*e + d*x^2 + c*x)/(a*b*x^4 - a^2) + 1/16*(2*d*log(sqrt(b)*x^2 + sq
rt(a))/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b))
 + 2*(3*sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*sqrt(b)*c + sqrt(a)*e)*log((sqrt(b
)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*
sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a
```

Fricas [C] Result contains complex when optimal does not.
time = 3.21, size = 116982, normalized size = 801.25

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] -1/9216*(2304*e*x^3 + 2304*d*x^2 + 2*(a*b*x^4 - a^2)*((-I*sqrt(3) + 1)*((a^
2*b*sqrt(1/(a*b))*sqrt((6*a*b*c*e*sqrt(1/(a*b)) + 9*b*c^2 + a*e^2)/(a^4*b^2
*sqrt(1/(a*b)))) + 2*d)^2/(a^3*b) - 3*(4*a^2*b*d*sqrt((6*a*b*c*e*sqrt(1/(a*
b)) + 9*b*c^2 + a*e^2)/(a^4*b^2*sqrt(1/(a*b)))) + 9*b*c^2 - (2*(2*d^2 + 3*c
*e)*b*sqrt(1/(a*b)) - e^2)*a)/(a^4*b^2*sqrt(1/(a*b))))/(-1/24576*(4*a^2*b*d
*sqrt((6*a*b*c*e*sqrt(1/(a*b)) + 9*b*c^2 + a*e^2)/(a^4*b^2*sqrt(1/(a*b))))
+ 9*b*c^2 - (2*(2*d^2 + 3*c*e)*b*sqrt(1/(a*b)) - e^2)*a)*(a^2*b*sqrt(1/(a*b
))*sqrt((6*a*b*c*e*sqrt(1/(a*b)) + 9*b*c^2 + a*e^2)/(a^4*b^2*sqrt(1/(a*b)))
) + 2*d)/(a^5*b^2) + 1/8192*(a^5*b^2*sqrt(1/(a*b))*((6*a*b*c*e*sqrt(1/(a*b)
) + 9*b*c^2 + a*e^2)/(a^4*b^2*sqrt(1/(a*b))))^(3/2) + 4*(d^2*sqrt((6*a*b*c*
e*sqrt(1/(a*b)) + 9*b*c^2 + a*e^2)/(a^4*b^2*sqrt(1/(a*b)))) - 3*c*e*sqrt((6
*a*b*c*e*sqrt(1/(a*b)) + 9*b*c^2 + a*e^2)/(a^4*b^2*sqrt(1/(a*b)))))*a^2*b*s
qrt(1/(a*b)) - 18*b*c^2*d*sqrt(1/(a*b)) - 2*a*d*e^2*sqrt(1/(a*b)) - 8*d^3 +
 12*c*d*e)/(a^5*b^2*sqrt( ...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(131) = 262$.
time = 41.02, size = 508, normalized size = 3.48

$$\operatorname{RootSum}\left(65536t^4a^7b^3 + t^2\left(-3072a^4b^2ce - 2048a^4b^2d^2\right) + t\left(128a^3bde^2 + 1152a^2b^2c^2d\right) - a^2e^4 + 18abc^2e^2 - 48abcd^2e + 16abd^4 - 81b^2c^4, \left(t \mapsto t\log\left(x + \frac{4096t^3a^7b^2e^3 + 36864t^3a^6b^3c^2e - 98304t^3a^6b^3cd^2 + 4608t^2a^5b^2cde^2 - 4096t^2a^5b^2d^3e - 13824t^2a^4b^3c^3d - 144ta^4bce^4 - 192ta^4bd^2e^3 - 1728ta^3b^2c^3e^2 + 5184ta^3b^2c^2d^2e + 1536ta^3b^2cd^4 - 3888ta^2b^3c^5 + 6a^3de^5 - 120a^2bcd^3e^2 + 64a^2bd^5e + 810ab^2c^4de - 1080ab^2c^3d^3}{a^3e^6 + 9a^2bc^2e^4 - 96a^2bcd^2e^3 + 64a^2bd^4e^2 - 81ab^2c^4e^2 + 864ab^2c^3d^2e - 576ab^2c^2d^4 - 729b^3c^6}\right)\right)\right) + \frac{-cx - dx^2 - ex^3}{-4a^2 + 4abx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**2
*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*a*
b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _t*l
og(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 98304*_t
**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2
*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_t*a**4
*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e
+ 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 - 120*a
**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c*
*3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3 + 64*a**2*
b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**
2*d**4 - 729*b**3*c**6)))) + (-c*x - d*x**2 - e*x**3)/(-4*a**2 + 4*a*b*x**4
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(110) = 220.
time = 0.56, size = 311, normalized size = 2.13

$$-\frac{\sqrt{2}\left(3\,b^{2}c-2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c+2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c-\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^{3}\right)^{\frac{3}{4}}a}+\frac{\sqrt{2}\left(3\,b^{2}c-\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{x^{3}e+dx^{2}+cx}{4\left(bx^{4}-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*
a) - 1/16*sqrt(2)*(3*b^2*c + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3
/4)*a) - 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)
^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b
)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) -
1/4*(x^3*e + d*x^2 + c*x)/((b*x^4 - a)*a)
```

Mupad [B]
time = 4.98, size = 477, normalized size = 3.27

$$\frac{\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{a-b\,x^4}+\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(65536\,a^7\,b^3\,z^4-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4-81\,b^2\,c^4-a^2\,e^4,z,k\right)\left(\mathrm{root}\left(65536\,a^7\,b^3\,z^4-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4-81\,b^2\,c^4-a^2\,e^4,z,k\right)\left(12\,b^3\,c-8\,b^3\,d\,x\right)+\frac{x\left(4\,a^2\,b^2\,e^2+36\,a\,b^3\,c^2\right)}{16\,a^3}-\frac{b^2\,d\,e}{a}\right)-\frac{-9\,b^2\,c^2\,e+12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3}-\frac{x\left(2\,b^2\,d^3-3\,b^2\,c\,d\,e\right)}{16\,a^3}\right)\mathrm{root}\left(65536\,a^7\,b^3\,z^4-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4-81\,b^2\,c^4-a^2\,e^4,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a - b*x^4)^2,x)
```

```
[Out] ((d*x^2)/(4*a) + (e*x^3)/(4*a) + (c*x)/(4*a))/(a - b*x^4) + symsum(log(- ro
ot(65536*a^7*b^3*z^4 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a
^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a
*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k)*(root(65536*a^7*b^3*z^4 - 3072*a^4*b^2
*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z
- 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k
)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 + 4*a^2*b^2*e^2))/(16*a^3) - (b
^2*d*e)/a) - (12*b^2*c*d^2 - 9*b^2*c^2*e + a*b*e^3)/(64*a^3) - (x*(2*b^2*d^
3 - 3*b^2*c*d*e))/(16*a^3))*root(65536*a^7*b^3*z^4 - 3072*a^4*b^2*c*e*z^2 -
 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c
*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k), k, 1, 4
)
```

3.128 $\int \frac{c+dx+ex^2}{(a+bx^4)^2} \, dx$

Optimal. Leaf size=308

$$\frac{x(c+dx+ex^2)}{4a\left(a+bx^4\right)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1\right.}{8\sqrt{2}\,a^{7/4}b^{3/4}}$$

```
[Out] 1/4*x*(e*x^2+d*x+c)/a/(b*x^4+a)+1/4*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b
^(1/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+
3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*b^(1/2))*(-e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^(1/2)+1/16*arct
an(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^
(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^(7
/4)/b^(3/4)*2^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)}{8\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)}{8\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{x(c+dx+ex^2)}{4a\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])
/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) - ((3*Sq
rt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
```

```
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx = \frac{x(c+dx+ex^2)}{4a(a+bx^4)} - \frac{\int \frac{-3c-2dx-ex^2}{a+bx^4}\,dx}{4a}$$

$$= \frac{x(c+dx+ex^2)}{4a(a+bx^4)} - \frac{\int \left(-\frac{2dx}{a+bx^4} + \frac{-3c-ex^2}{a+bx^4}\right)dx}{4a}$$

$$= \frac{x(c+dx+ex^2)}{4a(a+bx^4)} - \frac{\int \frac{-3c-ex^2}{a+bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a+bx^4}\,dx}{2a}$$

$$= \frac{x(c+dx+ex^2)}{4a(a+bx^4)} + \frac{d\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4a} + \frac{\left(\frac{3\sqrt{b}\,c}{\sqrt{a}} - e\right)\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx}{8ab} + \cdots$$

$$= \frac{x(c+dx+ex^2)}{4a(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{\left(\frac{3\sqrt{b}\,c}{\sqrt{a}} + e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx}{16ab} + \cdots$$

$$= \frac{x(c+dx+ex^2)}{4a(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}\,c - \sqrt{a}\,e\right)\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x + \cdots\right.}{16\sqrt{2}\,a^{7/4}b^{3/4}} \cdots$$

$$= \frac{x(c+dx+ex^2)}{4a(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}\,c + \sqrt{a}\,e\right)\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{3/4}} + \cdots$$

Mathematica [A]
time = 0.21, size = 305, normalized size = 0.99

$$\frac{\frac{8ax(c+x(d+ex))}{a+bx^4} - \frac{2\sqrt[4]{a}\left(3\sqrt{2}\,\sqrt{b}\,c+4\sqrt[4]{a}\,\sqrt[4]{b}\,d+\sqrt{2}\,\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{2\sqrt[4]{a}\left(3\sqrt{2}\,\sqrt{b}\,c-4\sqrt[4]{a}\,\sqrt[4]{b}\,d+\sqrt{2}\,\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{\sqrt{2}\left(-3\sqrt[4]{a}\,\sqrt{b}\,c+a^{3/4}e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{b^{3/4}} + \frac{\sqrt{2}\left(3\sqrt[4]{a}\,\sqrt{b}\,c-a^{3/4}e\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{b^{3/4}}}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

```
[Out] ((8*a*x*(c + x*(d + e*x)))/(a + b*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c +
4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/b^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c - 4*a^(1/4)*b^(1/4)*d + Sq
rt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2
]*(-3*a^(1/4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(32*a^2)
```

Maple [A]
time = 0.33, size = 287, normalized size = 0.93

method	result
risch	$\frac{\frac{e x^3}{4a}+\frac{d x^2}{4a}+\frac{cx}{4a}}{b x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(\_R^2 e+2\_R d+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(b x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+d\Bigg($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/4*x/a/(b*x^4+a)+3/32/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^
(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2
)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))+d*(1/4*x^2/a/(b*x^4+
a)+1/4/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2)))+e*(1/4*x^3/a/(b*x^4+a)+1/32/a
/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/
b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.52, size = 299, normalized size = 0.97

$$\frac{x^3e+dx^2+cx}{4\left(abx^4+a^2\right)}+\frac{\frac{\sqrt{2}\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e-4\sqrt{a}\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e+4\sqrt{a}\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x^3*e + d*x^2 + c*x)/(a*b*x^4 + a^2) + 1/32*(sqrt(2)*(3*sqrt(b)*c - sq
rt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^
(3/4)) - sqrt(2)*(3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*c +
```

```
 sqrt(2)*a^(3/4)*b^(1/4)*e - 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqr
t(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt
(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^
(1/4)*e + 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^
(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4
)))/a
```

Fricas [C] Result contains complex when optimal does not.
time = 3.62, size = 124258, normalized size = 403.44

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9216*(2304*e*x^3 + 2304*d*x^2 + 2*(a*b*x^4 + a^2)*((-I*sqrt(3) + 1)*((a^2
*b*sqrt(-1/(a*b))*sqrt(-(6*a*b*c*e*sqrt(-1/(a*b)) + 9*b*c^2 - a*e^2)/(a^4*b
^2*sqrt(-1/(a*b)))) - 2*d)^2/(a^3*b) - 3*(4*a^2*b*d*sqrt(-(6*a*b*c*e*sqrt(-
1/(a*b)) + 9*b*c^2 - a*e^2)/(a^4*b^2*sqrt(-1/(a*b)))) + 9*b*c^2 - (2*(2*d^2
 + 3*c*e)*b*sqrt(-1/(a*b)) + e^2)*a)/(a^4*b^2*sqrt(-1/(a*b))))/(-1/24576*(4
*a^2*b*d*sqrt(-(6*a*b*c*e*sqrt(-1/(a*b)) + 9*b*c^2 - a*e^2)/(a^4*b^2*sqrt(-
1/(a*b)))) + 9*b*c^2 - (2*(2*d^2 + 3*c*e)*b*sqrt(-1/(a*b)) + e^2)*a)*(a^2*b
*sqrt(-1/(a*b))*sqrt(-(6*a*b*c*e*sqrt(-1/(a*b)) + 9*b*c^2 - a*e^2)/(a^4*b^2
*sqrt(-1/(a*b)))) - 2*d)/(a^5*b^2) + 1/8192*(a^5*b^2*sqrt(-1/(a*b))*(-(6*a*
b*c*e*sqrt(-1/(a*b)) + 9*b*c^2 - a*e^2)/(a^4*b^2*sqrt(-1/(a*b))))^(3/2) - 4
*(d^2*sqrt(-(6*a*b*c*e*sqrt(-1/(a*b)) + 9*b*c^2 - a*e^2)/(a^4*b^2*sqrt(-1/(
a*b)))) - 3*c*e*sqrt(-(6*a*b*c*e*sqrt(-1/(a*b)) + 9*b*c^2 - a*e^2)/(a^4*b^2
*sqrt(-1/(a*b)))))*a^2*b*sqrt(-1/(a*b)) + 18*b*c^2*d*sqrt(-1/(a*b)) - 2*a*d
*e^2*sqrt(-1/(a*b)) - 8*d ...
```

Sympy [A]
time = 40.93, size = 505, normalized size = 1.64

$$\operatorname{RootSum}\left(65536t^4a^7b^3 + t^2\cdot\left(3072a^4b^2ce + 2048a^4b^2d^2\right) + t\left(128a^3bde^2 - 1152a^2b^2c^2d\right) + a^2e^4 + 18abc^2e^2 - 48abcd^2e + 16abd^4 + 81b^2c^4, \left(t \mapsto t\log\left(x + \frac{4096t^3a^7b^2e^3 - 36864t^3a^6b^3c^2e + 98304t^3a^6b^3cd^2 + 4608t^2a^5b^2cde^2 - 4096t^2a^5b^2d^3e + 13824t^2a^4b^3c^3d + 144ta^4bce^4 + 192ta^4bd^2e^3 - 1728ta^3b^2c^3e^2 + 5184ta^3b^2c^2d^2e + 1536ta^3b^2cd^4 + 3888ta^2b^3c^5 + 6a^3de^5 + 120a^2bcd^3e^2 - 64a^2bd^5e + 810ab^2c^4de - 1080ab^2c^3d^3}{a^3e^6 - 9a^2bc^2e^4 + 96a^2bcd^2e^3 - 64a^2bd^4e^2 - 81ab^2c^4e^2 + 864ab^2c^3d^2e - 576ab^2c^2d^4 + 729b^3c^6}\right)\right)\right) + \frac{cx + dx^2 + ex^3}{4a^2 + 4abx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2*
d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a*b
*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, Lambda(_t, _t*lo
g(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304*_t*
*3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*
d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t*a**4*
b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e +
 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 + 120*a*
```

```
*2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c**
3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 - 64*a**2*b
*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2
*d**4 + 729*b**3*c**6)))) + (c*x + d*x**2 + e*x**3)/(4*a**2 + 4*a*b*x**4)
```

Giac [A]
time = 0.55, size = 306, normalized size = 0.99

$$\frac{x^3e+dx^2+cx}{4\,(bx^4+a)a}+\frac{\sqrt{2}\,\left(2\,\sqrt{2}\,\sqrt{ab}\,b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\,\left(2x+\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\,\left(2\,\sqrt{2}\,\sqrt{ab}\,b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\,\left(2x-\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\,\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}-\frac{\sqrt{2}\,\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^3*e + d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*
b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sq
rt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^
3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(
x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

Mupad [B]
time = 0.33, size = 472, normalized size = 1.53

$$\frac{\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a}+\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\right)\left(\mathrm{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\right)\left(12\,b^3\,c-8\,b^3\,d\,x\right)+\frac{x\,\left(36\,a\,b^3\,c^2-4\,a^2\,b^2\,e^2\right)}{16\,a^3}+\frac{b^2\,d\,e}{a}\right)-\frac{9\,b^2\,c^2\,e-12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3}+\frac{x\,\left(2\,b^2\,d^3-3\,b^2\,c\,d\,e\right)}{16\,a^3}\right)\,\mathrm{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^4)^2,x)
```

```
[Out] ((d*x^2)/(4*a) + (e*x^3)/(4*a) + (c*x)/(4*a))/(a + b*x^4) + symsum(log((x*(
2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3) - (9*b^2*c^2*e - 12*b^2*c*d^2 + a*b*e^3)
/(64*a^3) - root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^
2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*
c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)*(root(65536*a^7*b^3*z^4
+ 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*
a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 +
 a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 - 4*a^2*b^2*e^2))
/(16*a^3) + (b^2*d*e)/a))*root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 + 2
048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d
^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k), k, 1, 4)
```

3.129 $\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=179

$$\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}+\frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)}+\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}\,c+5\sqrt{a}\,e\right)\tanh^{-1}\left(\cdots\right.}{64a^{11/4}b^{3/4}}$$

```
[Out] 1/8*x*(e*x^2+d*x+c)/a/(-b*x^4+a)^2+1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(-b*x^4+a
)+3/16*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)+1/64*arctan(b^(1/4)*x
/a^(1/4))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)+1/64*arctanh(b^(1/4)
*x/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)
```

Rubi [A]
time = 0.11, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1869, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)}+\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32
*a^2*(a - b*x^4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4
)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh[(b^(1/4)*
x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(
16*a^(5/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx &= \frac{x(c+dx+ex^2)}{8a(a-bx^4)^2} - \frac{\int \frac{-7c-6dx-5ex^2}{(a-bx^4)^2}\,dx}{8a} \\
&= \frac{x(c+dx+ex^2)}{8a(a-bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)} + \frac{\int \frac{21c+12dx+5ex^2}{a-bx^4}\,dx}{32a^2} \\
&= \frac{x(c+dx+ex^2)}{8a(a-bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)} + \frac{\int \left(\frac{12dx}{a-bx^4} + \frac{21c+5ex^2}{a-bx^4}\right)dx}{32a^2} \\
&= \frac{x(c+dx+ex^2)}{8a(a-bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)} + \frac{\int \frac{21c+5ex^2}{a-bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a-bx^4}\,dx}{8a^2} \\
&= \frac{x(c+dx+ex^2)}{8a(a-bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)} + \frac{(3d)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{16a^2} - \frac{\left(\frac{21\sqrt{b}\,c}{\sqrt{a}} - \right.}{} \\
&= \frac{x(c+dx+ex^2)}{8a(a-bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)} + \frac{\left(\frac{21\sqrt{b}\,c}{\sqrt{a}} - 5e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{3/4}} + \frac{\left(21\sqrt{b}\,c\right.}{}
\end{aligned}$$

Mathematica [A]

time = 0.14, size = 244, normalized size = 1.36

$$\frac{\frac{16a^2x(c+x(d+ex))}{(a-bx^4)^2}+\frac{4ax(7c+x(6d+5ex))}{a-bx^4}+\frac{2\sqrt[4]{a}\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{\left(21\sqrt[4]{a}\,\sqrt{b}\,c+12\sqrt{a}\,\sqrt[4]{b}\,d+5a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)}{b^{3/4}}+\frac{\left(21\sqrt[4]{a}\,\sqrt{b}\,c-12\sqrt{a}\,\sqrt[4]{b}\,d+5a^{3/4}e\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{b^{3/4}}+\frac{12\sqrt{a}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt{b}}}{128a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

```
[Out] ((16*a^2*x*(c + x*(d + e*x)))/(a - b*x^4)^2 + (4*a*x*(7*c + x*(6*d + 5*e*x)
))/(a - b*x^4) + (2*a^(1/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)
/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*Sqrt[b]*c + 12*Sqrt[a]*b^(1/4)*d + 5*a^(3
/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*Sqrt[b]*c - 12*Sqrt
[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (12*Sqrt[a
]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Maple [A]
time = 0.34, size = 273, normalized size = 1.53

method	result
risch	$\frac{-\frac{5be\,x^7}{32a^2}-\frac{3bd\,x^6}{16a^2}-\frac{7bc\,x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}}{\left(-b\,x^4+a\right)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(5\_R^2e+12\_Rd+21c\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2b}$
default	$c\left(\frac{x}{8a\left(-b\,x^4+a\right)^2}+\frac{\frac{7x}{32a\left(-b\,x^4+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{128a^2}}{a}\right)+d\left(\frac{x^2}{8a\left(-b\,x^4+a\right)^2}+\frac{\frac{3x^2}{16a\left(-b\,x^4+a\right)}+\frac{3\ln}{}}{a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/8*x/a/(-b*x^4+a)^2+7/8/a*(1/4*x/a/(-b*x^4+a)+3/16/a^2*(a/b)^(1/4)*(ln(
(x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))))+d*(1/8*x^2/a/(-
b*x^4+a)^2+3/4/a*(1/4*x^2/a/(-b*x^4+a)+1/8/a/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1
/2))/(a-x^2*(a*b)^(1/2)))))+e*(1/8*x^3/a/(-b*x^4+a)^2+5/8/a*(1/4*x^3/a/(-b*
x^4+a)-1/16/a/b/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-
(a/b)^(1/4))))))
```

Maxima [A]
time = 0.52, size = 234, normalized size = 1.31

$$-\frac{5\,bx^7e+6\,bdx^6+7\,bcx^5-9\,ax^3e-10\,adx^2-11\,acx}{32\,\left(a^2b^2x^8-2\,a^3bx^4+a^4\right)}+\frac{\frac{12\,d\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{12\,d\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\,\left(21\,\sqrt{b}\,c-5\,\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(21\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{128\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(5*b*x^7*e + 6*b*d*x^6 + 7*b*c*x^5 - 9*a*x^3*e - 10*a*d*x^2 - 11*a*c*
x)/(a^2*b^2*x^8 - 2*a^3*b*x^4 + a^4) + 1/128*(12*d*log(sqrt(b)*x^2 + sqrt(a
))/(sqrt(a)*sqrt(b)) - 12*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) +
2*(21*sqrt(b)*c - 5*sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*sqrt(b)*c + 5*sqrt(a)*e)*log((sqr
t(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^2
```

Fricas [C] Result contains complex when optimal does not.
time = 5.63, size = 118710, normalized size = 663.18

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] -1/589824*(92160*b*e*x^7 + 110592*b*d*x^6 + 129024*b*c*x^5 - 165888*a*e*x^3
 - 184320*a*d*x^2 - 202752*a*c*x + 2*(a^2*b^2*x^8 - 2*a^3*b*x^4 + a^4)*((-I
*sqrt(3) + 1)*((a^3*b*sqrt(1/(a*b))*sqrt((210*a*b*c*e*sqrt(1/(a*b)) + 441*b
*c^2 + 25*a*e^2)/(a^6*b^2*sqrt(1/(a*b)))) + 12*d)^2/(a^5*b) - 3*(24*a^3*b*d
*sqrt((210*a*b*c*e*sqrt(1/(a*b)) + 441*b*c^2 + 25*a*e^2)/(a^6*b^2*sqrt(1/(a
*b)))) + 441*b*c^2 - (6*(24*d^2 + 35*c*e)*b*sqrt(1/(a*b)) - 25*e^2)*a)/(a^6
*b^2*sqrt(1/(a*b))))/(-1/12582912*(24*a^3*b*d*sqrt((210*a*b*c*e*sqrt(1/(a*b
)) + 441*b*c^2 + 25*a*e^2)/(a^6*b^2*sqrt(1/(a*b)))) + 441*b*c^2 - (6*(24*d^
2 + 35*c*e)*b*sqrt(1/(a*b)) - 25*e^2)*a)*(a^3*b*sqrt(1/(a*b))*sqrt((210*a*b
*c*e*sqrt(1/(a*b)) + 441*b*c^2 + 25*a*e^2)/(a^6*b^2*sqrt(1/(a*b)))) + 12*d)
/(a^8*b^2) + 1/4194304*(a^8*b^2*sqrt(1/(a*b))*((210*a*b*c*e*sqrt(1/(a*b)) +
 441*b*c^2 + 25*a*e^2)/(a^6*b^2*sqrt(1/(a*b))))^(3/2) + 12*(12*d^2*sqrt((21
0*a*b*c*e*sqrt(1/(a*b)) + 441*b*c^2 + 25*a*e^2)/(a^6*b^2*sqrt(1/(a*b)))) -
35*c*e*sqrt((210*a*b*c*e* ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(143) = 286.

time = 0.80, size = 340, normalized size = 1.90

$$-\frac{\sqrt{2}\left(21\,b^{2}c-12\,\sqrt{2}\,(-ab^{3})^{\frac{1}{4}}\,bd+5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\,(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{128\,(-ab^{3})^{\frac{3}{4}}\,a^{2}}-\frac{\sqrt{2}\left(21\,b^{2}c+12\,\sqrt{2}\,(-ab^{3})^{\frac{1}{4}}\,bd-5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\,(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{128\,(-ab^{3})^{\frac{3}{4}}\,a^{2}}-\frac{\sqrt{2}\left(21\,b^{2}c-5\,\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^{3})^{\frac{3}{4}}\,a^{2}}+\frac{\sqrt{2}\left(21\,b^{2}c-5\,\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^{3})^{\frac{3}{4}}\,a^{2}}-\frac{5\,bx^{7}e+6\,bdx^{6}+7\,bcx^{5}-9\,ax^{3}e-10\,adx^{2}-11\,acx}{32\,(bx^{4}-a)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^2*c - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqrt(-a*b)*b*e
)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(
3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 5*sqr
t(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/
((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2
+ sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) + 1/256*sqrt(2)
*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b
))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b*x^7*e + 6*b*d*x^6 + 7*b*c*x^5 - 9*a*x^3
*e - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a)^2*a^2)
```

Mupad [B]

time = 5.11, size = 826, normalized size = 4.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a - b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) - (7*b*c*x^5)/(32*a^
2) - (3*b*d*x^6)/(16*a^2) - (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^
4) + symsum(log(-(b*(125*a*e^3 + 3024*b*c*d^2 - 2205*b*c^2*e + 1728*b*d^3*x
 + 344064*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a
^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b
*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4
, z, k)^2*a^5*b^2*c + 3200*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*
e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*
e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4
 - 194481*b^2*c^4, z, k)*a^3*b*e^2*x - 2520*b*c*d*e*x + 56448*root(26843545
6*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 270950
4*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^
2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^2*b^2*c^2*x -
 196608*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6
*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c
*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4,
z, k)^2*a^5*b^2*d*x - 15360*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c
*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d
*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^
4 - 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435456*a^11*b^3*
```

```
z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c
^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 207
36*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k), k, 1, 4)
```

3.130 $\int \frac{c+dx+ex^2}{(a+bx^4)^3} \, dx$

Optimal. Leaf size=341

$$\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}+\frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{\left(21\sqrt{b}\,c+5\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}b^{3/4}}+\frac{(2}{}$$

```
[Out] 1/8*x*(e*x^2+d*x+c)/a/(b*x^4+a)^2+1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(b*x^4+a)+
3/16*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)-1/256*ln(-a^(1/4)*b^(1/4
)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/
4)*2^(1/2)+1/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*e*a^
(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x*2^(1
/2)/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)*2^(1/2)+1/128*arct
an(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)
*2^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)}{64\sqrt{2}\,a^{11/4}b^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)}{64\sqrt{2}\,a^{11/4}b^{3/4}}+\frac{3d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}b^{3/4}}+\frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)}+\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32
*a^2*(a + b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]
) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/
(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*Sqrt[b]*c
- 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128
*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^3}\,dx &= \frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2} - \frac{\int \frac{-7c-6dx-5ex^2}{\left(a+bx^4\right)^2}\,dx}{8a} \\
&= \frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} + \frac{\int \frac{21c+12dx+5ex^2}{a+bx^4}\,dx}{32a^2} \\
&= \frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} + \frac{\int \left(\frac{12dx}{a+bx^4} + \frac{21c+5ex^2}{a+bx^4}\right)\,dx}{32a^2} \\
&= \frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} + \frac{\int \frac{21c+5ex^2}{a+bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a+bx^4}\,dx}{8a^2} \\
&= \frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} + \frac{(3d)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{16a^2} + \frac{\left(\frac{21\sqrt{b}\,c}{\sqrt{a}} - 5 \cdots\right.}{\cdots} \\
&= \frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{b}\,c}{\sqrt{a}} - 5e\right)\int \cdots}{128\sqrt{2}\,a \cdots} \\
&= \frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{b}\,c}{\sqrt{a}} - 5e\right)\log\left(\cdots\right.}{128 \cdots} \\
&= \frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21\sqrt{b}\,c + 5\sqrt{a}\,e\right)\tan \cdots}{64\sqrt{2}\,a \cdots}
\end{aligned}
$$

Mathematica [A]

time = 0.20, size = 337, normalized size = 0.99

$$
\frac{\frac{32a^2x(c+x(d+ex))}{\left(a+bx^4\right)^2} + \frac{8ax(7c+x(6d+5ex))}{a+bx^4} - \frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}\,c+24\sqrt[4]{a}\sqrt[4]{b}\,d+5\sqrt{2}\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}\,c-24\sqrt[4]{a}\sqrt[4]{b}\,d+5\sqrt{2}\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{\sqrt{2}\left(-21\sqrt[4]{a}\sqrt{b}\,c+5a^{3/4}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{b^{3/4}} + \frac{\sqrt{2}\left(21\sqrt[4]{a}\sqrt{b}\,c-5a^{3/4}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{b^{3/4}}}{256a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```

```
[Out] ((32*a^2*x*(c + x*(d + e*x)))/(a + b*x^4)^2 + (8*a*x*(7*c + x*(6*d + 5*e*x)
))/(a + b*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1/4)*b^(1/4)*d +
5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (2*
a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*(-21*a^(1/4)*S
qrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4)*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(256*a^3)
```

Maple [A]
time = 0.34, size = 354, normalized size = 1.04

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{7bc\,x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}}{\left(b\,x^4+a\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(5\_R^2e+12\_Rd+21c\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2b}$
default	$c\left(\frac{x}{8a\left(b\,x^4+a\right)^2}+\frac{\frac{7x}{32a\left(b\,x^4+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/8*x/a/(b*x^4+a)^2+7/8/a*(1/4*x/a/(b*x^4+a)+3/32/a^2*(a/b)^(1/4)*2^(1/2
)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a
/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*
x-1))))+d*(1/8*x^2/a/(b*x^4+a)^2+3/4/a*(1/4*x^2/a/(b*x^4+a)+1/4/a/(a*b)^(1/
2)*arctan(x^2*(b/a)^(1/2))))+e*(1/8*x^3/a/(b*x^4+a)^2+5/8/a*(1/4*x^3/a/(b*x
^4+a)+1/32/a/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/
2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x
+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))))
```

Maxima [A]
time = 0.53, size = 342, normalized size = 1.00

$$\frac{5\,bx^7e+6\,bdx^6+7\,bcx^5+9\,ax^3e+10\,adx^2+11\,acx}{32\,\left(a^2b^2x^8+2\,a^3bx^4+a^4\right)}+\frac{\frac{\sqrt{2}\left(21\,\sqrt{b}\,c-5\,\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\,\sqrt{b}\,c-5\,\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+5\,\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e-24\,\sqrt{a}\,\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+5\,\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e+24\,\sqrt{a}\,\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{3}{4}}}}{256\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*b*x^7*e + 6*b*d*x^6 + 7*b*c*x^5 + 9*a*x^3*e + 10*a*d*x^2 + 11*a*c*x
)/(a^2*b^2*x^8 + 2*a^3*b*x^4 + a^4) + 1/256*(sqrt(2)*(21*sqrt(b)*c - 5*sqrt
(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3
/4)) - sqrt(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(3/4)*c
 + 5*sqrt(2)*a^(3/4)*b^(1/4)*e - 24*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(
2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt
(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(3/4)*c + 5*sqrt(2)*a^
(3/4)*b^(1/4)*e + 24*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - s
qrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b
))*b^(3/4)))/a^2
```

Fricas [C] Result contains complex when optimal does not.
time = 7.74, size = 124787, normalized size = 365.94

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/589824*(92160*b*e*x^7 + 110592*b*d*x^6 + 129024*b*c*x^5 + 165888*a*e*x^3
+ 184320*a*d*x^2 + 202752*a*c*x + 2*(a^2*b^2*x^8 + 2*a^3*b*x^4 + a^4)*((-I*
sqrt(3) + 1)*((a^3*b*sqrt(-1/(a*b))*sqrt(-(210*a*b*c*e*sqrt(-1/(a*b)) + 441
*b*c^2 - 25*a*e^2)/(a^6*b^2*sqrt(-1/(a*b)))) - 12*d)^2/(a^5*b) - 3*(24*a^3*
b*d*sqrt(-(210*a*b*c*e*sqrt(-1/(a*b)) + 441*b*c^2 - 25*a*e^2)/(a^6*b^2*sqrt
(-1/(a*b)))) + 441*b*c^2 - (6*(24*d^2 + 35*c*e)*b*sqrt(-1/(a*b)) + 25*e^2)*
a)/(a^6*b^2*sqrt(-1/(a*b))))/(-1/12582912*(24*a^3*b*d*sqrt(-(210*a*b*c*e*sq
rt(-1/(a*b)) + 441*b*c^2 - 25*a*e^2)/(a^6*b^2*sqrt(-1/(a*b)))) + 441*b*c^2
- (6*(24*d^2 + 35*c*e)*b*sqrt(-1/(a*b)) + 25*e^2)*a)*(a^3*b*sqrt(-1/(a*b))*
sqrt(-(210*a*b*c*e*sqrt(-1/(a*b)) + 441*b*c^2 - 25*a*e^2)/(a^6*b^2*sqrt(-1/
(a*b)))) - 12*d)/(a^8*b^2) + 1/4194304*(a^8*b^2*sqrt(-1/(a*b))*(-(210*a*b*c
*e*sqrt(-1/(a*b)) + 441*b*c^2 - 25*a*e^2)/(a^6*b^2*sqrt(-1/(a*b))))^(3/2) -
 12*(12*d^2*sqrt(-(210*a*b*c*e*sqrt(-1/(a*b)) + 441*b*c^2 - 25*a*e^2)/(a^6*
b^2*sqrt(-1/(a*b)))) - 35 ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.76, size = 336, normalized size = 0.99

$$\frac{5\,bx^7e+6\,bdx^6+7\,bcx^5+9\,ax^3e+10\,adx^2+11\,acx}{32\,(bx^4+a)^2a^2}+\frac{\sqrt{2}\left(12\,\sqrt{2}\,\sqrt{ab}\,b^2d+21\,(ab^3)^{\frac{1}{4}}\,b^2c+5\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(12\,\sqrt{2}\,\sqrt{ab}\,b^2d+21\,(ab^3)^{\frac{1}{4}}\,b^2c+5\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^2c-5\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}-\frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^2c-5\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*b*x^7*e + 6*b*d*x^6 + 7*b*c*x^5 + 9*a*x^3*e + 10*a*d*x^2 + 11*a*c*x
)/((b*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b
^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d
 + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)
*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^
2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3)
```

Mupad [B]
time = 5.05, size = 826, normalized size = 2.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) + (7*b*c*x^5)/(32*a^
2) + (3*b*d*x^6)/(16*a^2) + (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^
4) + symsum(log(-(b*(125*a*e^3 - 3024*b*c*d^2 + 2205*b*c^2*e - 1728*b*d^3*x
 + 344064*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a
^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b
*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4
, z, k)^2*a^5*b^2*c - 3200*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*
e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*
e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4
 + 194481*b^2*c^4, z, k)*a^3*b*e^2*x + 2520*b*c*d*e*x + 56448*root(26843545
6*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 270950
4*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^
2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^2*b^2*c^2*x -
 196608*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6
*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c
*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4,
z, k)^2*a^5*b^2*d*x + 15360*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c
*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d
*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^
```

```
4 + 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435456*a^11*b^3*
z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c
^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 207
36*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k), k, 1, 4)
```

3.131 $\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^4}\,dx$

Optimal. Leaf size=211

$$\frac{x(c+dx+ex^2)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx+9ex^2)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a-bx^4\right)}+\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+ \cdots$$

```
[Out] 1/12*x*(e*x^2+d*x+c)/a/(-b*x^4+a)^3+1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(-b*x^
4+a)^2+1/384*x*(45*e*x^2+60*d*x+77*c)/a^3/(-b*x^4+a)+5/32*d*arctanh(x^2*b^(
1/2)/a^(1/2))/a^(7/2)/b^(1/2)+1/256*arctan(b^(1/4)*x/a^(1/4))*(-15*e*a^(1/2
)+77*c*b^(1/2))/a^(15/4)/b^(3/4)+1/256*arctanh(b^(1/4)*x/a^(1/4))*(15*e*a^(
1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)
```

Rubi [A]

time = 0.14, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1869, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a-bx^4\right)}+\frac{x(11c+10dx+9ex^2)}{96a^2\left(a-bx^4\right)^2}+\frac{x(c+dx+ex^2)}{12a\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/
(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a - b*x^4
)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15
/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*
Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^4}\,dx = \frac{x(c+dx+ex^2)}{12a\left(a-bx^4\right)^3} - \frac{\int \frac{-11c-10dx-9ex^2}{\left(a-bx^4\right)^3}\,dx}{12a}$$

$$= \frac{x(c+dx+ex^2)}{12a\left(a-bx^4\right)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2\left(a-bx^4\right)^2} + \frac{\int \frac{77c+60dx+45ex^2}{\left(a-bx^4\right)^2}\,dx}{96a^2}$$

$$= \frac{x(c+dx+ex^2)}{12a\left(a-bx^4\right)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2\left(a-bx^4\right)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\left(a-bx^4\right)} - \frac{\int \frac{-231c-120dx-\cdots}{a-bx^4}}{384a^3}$$

$$= \frac{x(c+dx+ex^2)}{12a\left(a-bx^4\right)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2\left(a-bx^4\right)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\left(a-bx^4\right)} - \frac{\int \left(-\frac{120dx}{a-bx^4} + \cdots\right.}{38\cdots}$$

$$= \frac{x(c+dx+ex^2)}{12a\left(a-bx^4\right)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2\left(a-bx^4\right)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\left(a-bx^4\right)} - \frac{\int \frac{-231c-45ex^2}{a-bx^4}\,d\cdots}{384a^3}$$

$$= \frac{x(c+dx+ex^2)}{12a\left(a-bx^4\right)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2\left(a-bx^4\right)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\left(a-bx^4\right)} + \frac{(5d)\text{Subst}\left(\int \cdots\right.}{32\cdots}$$

$$= \frac{x(c+dx+ex^2)}{12a\left(a-bx^4\right)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2\left(a-bx^4\right)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\left(a-bx^4\right)} + \frac{\left(77\sqrt{b}\,c-15\cdots\right.}{25\cdots}$$

Mathematica [A]
time = 0.17, size = 276, normalized size = 1.31

$$\frac{\frac{128a^3x(c+x(d+ex))}{(a-bx^4)^3}+\frac{4ax(77c+15x(4d+3ex))}{a-bx^4}+\frac{16a^2x(11c+x(10d+9ex))}{(a-bx^4)^2}+\frac{6\sqrt[4]{a}\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{3\left(77\sqrt[4]{a}\sqrt{b}\,c+40\sqrt{a}\sqrt[4]{b}\,d+15a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)}{b^{3/4}}+\frac{3\left(77\sqrt[4]{a}\sqrt{b}\,c-40\sqrt{a}\sqrt[4]{b}\,d+15a^{3/4}e\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{b^{3/4}}+\frac{120\sqrt{a}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt{b}}}{1536a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

```
[Out] ((128*a^3*x*(c + x*(d + e*x)))/(a - b*x^4)^3 + (4*a*x*(77*c + 15*x*(4*d + 3
*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a - b*x^4)^2 +
(6*a^(1/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/
4) - (3*(77*a^(1/4)*Sqrt[b]*c + 40*Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^
(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*Sqrt[b]*c - 40*Sqrt[a]*b^(1/4)
*d + 15*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (120*Sqrt[a]*d*Log[S
qrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

Maple [A]
time = 0.34, size = 248, normalized size = 1.18

method	result

risch	$\frac{\frac{15 e b^2 x^{11}}{128a^3}+\frac{5 d b^2 x^{10}}{32a^3}+\frac{77 c b^2 x^9}{384a^3}-\frac{21 b e x^7}{64a^2}-\frac{5 b d x^6}{12a^2}-\frac{33 b c x^5}{64a^2}+\frac{113 e x^3}{384a}+\frac{11 d x^2}{32a}+\frac{51 c x}{128a}}{(-b x^4+a)^3}-\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^4-a)}\frac{(15\_R^2 e+40\_R d+77c)}{\_R^3}}{512a^3 b}$
default	$\frac{\frac{15 e b^2 x^{11}}{128a^3}+\frac{5 d b^2 x^{10}}{32a^3}+\frac{77 c b^2 x^9}{384a^3}-\frac{21 b e x^7}{64a^2}-\frac{5 b d x^6}{12a^2}-\frac{33 b c x^5}{64a^2}+\frac{113 e x^3}{384a}+\frac{11 d x^2}{32a}+\frac{51 c x}{128a}}{(-b x^4+a)^3}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32/a^3*d*b^2*x^10+77/384*c/a^3*b^2*x^9-21/64*b*e/a
^2*x^7-5/12/a^2*b*d*x^6-33/64*b*c/a^2*x^5+113/384/a*e*x^3+11/32*d/a*x^2+51/
128/a*c*x)/(-b*x^4+a)^3+1/128/a^3*(77/4*c*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))
/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))+10*d/(a*b)^(1/2)*ln((a+x^2*(a*b)
^(1/2))/(a-x^2*(a*b)^(1/2)))-15/4*e/b/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-
ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))))
```

Maxima [A]
time = 0.54, size = 284, normalized size = 1.35

$$-\frac{45\,b^2x^{11}e+60\,b^2dx^{10}+77\,b^2cx^9-126\,abx^7e-160\,abdx^6-198\,abcx^5+113\,a^2x^3e+132\,a^2dx^2+153\,a^2cx}{384\,(a^3b^3x^{12}-3\,a^4b^2x^8+3\,a^5bx^4-a^6)}+\frac{\frac{40\,d\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(77\sqrt{b}\,c+15\sqrt{a}\,e\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{512\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(45*b^2*x^11*e + 60*b^2*d*x^10 + 77*b^2*c*x^9 - 126*a*b*x^7*e - 160*
a*b*d*x^6 - 198*a*b*c*x^5 + 113*a^2*x^3*e + 132*a^2*d*x^2 + 153*a^2*c*x)/(a
^3*b^3*x^12 - 3*a^4*b^2*x^8 + 3*a^5*b*x^4 - a^6) + 1/512*(40*d*log(sqrt(b)*
x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)
*sqrt(b)) + 2*(77*sqrt(b)*c - 15*sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*sqrt(b)*c + 15*sqrt(
a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqr
t(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^3
```

Fricas [C] Result contains complex when optimal does not.
time = 10.42, size = 118903, normalized size = 563.52

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] -1/9437184*(1105920*b^2*e*x^11 + 1474560*b^2*d*x^10 + 1892352*b^2*c*x^9 - 3
096576*a*b*e*x^7 - 3932160*a*b*d*x^6 - 4866048*a*b*c*x^5 + 2777088*a^2*e*x^
3 + 3244032*a^2*d*x^2 + 3760128*a^2*c*x + 2*(a^3*b^3*x^12 - 3*a^4*b^2*x^8 +
 3*a^5*b*x^4 - a^6)*((-I*sqrt(3) + 1)*((a^4*b*sqrt(1/(a*b))*sqrt((2310*a*b*
c*e*sqrt(1/(a*b)) + 5929*b*c^2 + 225*a*e^2)/(a^8*b^2*sqrt(1/(a*b)))) + 40*d
)^2/(a^7*b) - 3*(80*a^4*b*d*sqrt((2310*a*b*c*e*sqrt(1/(a*b)) + 5929*b*c^2 +
 225*a*e^2)/(a^8*b^2*sqrt(1/(a*b)))) + 5929*b*c^2 - 5*(2*(160*d^2 + 231*c*e
)*b*sqrt(1/(a*b)) - 45*e^2)*a)/(a^8*b^2*sqrt(1/(a*b))))/(-1/805306368*(80*a
^4*b*d*sqrt((2310*a*b*c*e*sqrt(1/(a*b)) + 5929*b*c^2 + 225*a*e^2)/(a^8*b^2*
sqrt(1/(a*b)))) + 5929*b*c^2 - 5*(2*(160*d^2 + 231*c*e)*b*sqrt(1/(a*b)) - 4
5*e^2)*a)*(a^4*b*sqrt(1/(a*b))*sqrt((2310*a*b*c*e*sqrt(1/(a*b)) + 5929*b*c^
2 + 225*a*e^2)/(a^8*b^2*sqrt(1/(a*b)))) + 40*d)/(a^11*b^2) + 1/268435456*(a
^11*b^2*sqrt(1/(a*b))*((2310*a*b*c*e*sqrt(1/(a*b)) + 5929*b*c^2 + 225*a*e^2
)/(a^8*b^2*sqrt(1/(a*b))) ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(175) = 350.
time = 0.80, size = 377, normalized size = 1.79

$$-\frac{\sqrt{2}\left(77\,b^2c-40\,\sqrt{2}\,(-ab^3)^{\frac{1}{4}}\,bd+15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\,(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}-\frac{\sqrt{2}\left(77\,b^2c+40\,\sqrt{2}\,(-ab^3)^{\frac{1}{4}}\,bd-15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\,(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}-\frac{\sqrt{2}\left(77\,b^2c-15\,\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}+\frac{\sqrt{2}\left(77\,b^2c-15\,\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}-\frac{45\,b^2x^{11}e+60\,b^2dx^{10}+77\,b^2cx^9-126\,abx^7e-160\,abdx^6-198\,abcx^5+113\,a^2x^3e+132\,a^2dx^2+153\,a^2cx}{384\,(bx^4-a)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sqrt(-a*b)*b*
e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^
(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 15*s
qrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4)
)/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(
x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) + 1/1024*sq
rt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqr
t(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(45*b^2*x^11*e + 60*b^2*d*x^10 + 77*b
^2*c*x^9 - 126*a*b*x^7*e - 160*a*b*d*x^6 - 198*a*b*c*x^5 + 113*a^2*x^3*e +
132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)
```

Mupad [B]
time = 5.22, size = 874, normalized size = 4.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a - b*x^4)^4,x)
```

```
[Out] ((11*d*x^2)/(32*a) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9
)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) - (33*b*c
*x^5)/(64*a^2) - (5*b*d*x^6)/(12*a^2) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^3*x
^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log(-(b*(3375*a*e^3 + 123200*b*c*
d^2 - 88935*b*c^2*e + 64000*b*d^3*x + 20185088*root(68719476736*a^15*b^3*z^
4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*
b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)^2*a^7*b^2
*c + 115200*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 83
8860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*
z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*
b^2*c^4 - 50625*a^2*e^4, z, k)*a^4*b*e^2*x - 92400*b*c*d*e*x + 3035648*root
(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*
d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*
c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*
a^2*e^4, z, k)*a^3*b^2*c^2*x - 10485760*root(68719476736*a^15*b^3*z^4 - 121
1105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2
*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 +
 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x -
614400*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 8388608
00*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7
392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c
^4 - 50625*a^2*e^4, z, k)*a^4*b*d*e))/(2097152*a^9))*root(68719476736*a^15*
b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 48570368
0*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*
a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k), k,
 1, 4)
```

3.132 $\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^4}\,dx$

Optimal. Leaf size=372

$$\frac{x(c+dx+ex^2)}{12a\left(a+bx^4\right)^3}+\frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}-\frac{\left(77\sqrt{b}\,c+15\sqrt{a}\right.}{256}$$

```
[Out] 1/12*x*(e*x^2+d*x+c)/a/(b*x^4+a)^3+1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(b*x^4+
a)^2+1/384*x*(45*e*x^2+60*d*x+77*c)/a^3/(b*x^4+a)+5/32*d*arctan(x^2*b^(1/2)
/a^(1/2))/a^(7/2)/b^(1/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
b^(1/2))*(-15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1/1024*ln(a^
(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-15*e*a^(1/2)+77*c*b^(1/2))/a
^(15/4)/b^(3/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(15*e*a^
(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1/512*arctan(1+b^(1/4)*x*2^(1/
2)/a^(1/4))*(15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)}{256\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)}{256\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{5d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}-\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)}+\frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2}+\frac{x(c+dx+ex^2)}{12a\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/
(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a + b*x^4
)) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*Sqrt[b
]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a
^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*Sqrt[a
]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a
^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx = \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} - \frac{\int \frac{-11c-10dx-9ex^2}{(a+bx^4)^3}\,dx}{12a}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{\int \frac{77c+60dx+45ex^2}{(a+bx^4)^2}\,dx}{96a^2}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} - \frac{\int \frac{-231c-120dx-}{a+bx^4}}{384a^3}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} - \frac{\int \left(-\frac{120dx}{a+bx^4} + \right.}{38}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} - \frac{\int \frac{-231c-45ex^2}{a+bx^4}}{384a^3}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} + \frac{(5d)\mathrm{Subst}\left(\int \right.}{32}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{32a^{7/2}\sqrt{b}}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{32a^{7/2}\sqrt{b}}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{32a^{7/2}\sqrt{b}}$$

Mathematica [A]

time = 0.25, size = 369, normalized size = 0.99

$$\frac{\frac{256a^3x(c+x(d+ex))}{(a+bx^4)^3}+\frac{8ax(77c+15x(4d+3ex))}{a+bx^4}+\frac{32a^2x(11c+x(10d+9ex))}{(a+bx^4)^2}-\frac{6\sqrt[4]{a}\left(77\sqrt{2}\sqrt{b}c+80\sqrt[4]{a}\sqrt[4]{b}d+15\sqrt{2}\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{6\sqrt[4]{a}\left(77\sqrt{2}\sqrt{b}c-80\sqrt[4]{a}\sqrt[4]{b}d+15\sqrt{2}\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{3\sqrt{2}\left(-77\sqrt[4]{a}\sqrt{b}c+15a^{3/4}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{b^{3/4}}+\frac{3\sqrt{2}\left(77\sqrt[4]{a}\sqrt{b}c-15a^{3/4}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{b^{3/4}}}{3072a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

```
[Out] ((256*a^3*x*(c + x*(d + e*x)))/(a + b*x^4)^3 + (8*a*x*(77*c + 15*x*(4*d + 3
*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a + b*x^4)^2 -
(6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a
]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (6*a^(1/4)*(77*Sqrt
[2]*Sqrt[b]*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^(1/4)*Sqrt[b]*c + 15
*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4)
 + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(3072*a^4)
```

Maple [A]

time = 0.32, size = 334, normalized size = 0.90

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(15\_R^2e+40\_Rd+77c\right)}{\_R^3}}{512a^3b}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(b\,x^4+a)^3}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+}{8a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32/a^3*d*b^2*x^10+77/384*c/a^3*b^2*x^9+21/64*b*e/a
^2*x^7+5/12/a^2*b*d*x^6+33/64*b*c/a^2*x^5+113/384/a*e*x^3+11/32*d/a*x^2+51/
128/a*c*x)/(b*x^4+a)^3+1/128/a^3*(77/8*c*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/
b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*
arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+20*d/(a*
b)^(1/2)*arctan(x^2*(b/a)^(1/2))+15/8*e/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b
)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]

time = 0.54, size = 390, normalized size = 1.05

$$\frac{45\,b^2x^{11}e+60\,b^2dx^{10}+77\,b^2cx^9+126\,abx^7e+160\,abdx^6+198\,abcx^5+113\,a^2x^3e+132\,a^2dx^2+153\,a^2cx}{384\,(a^3b^3x^{12}+3\,a^4b^2x^8+3\,a^5bx^4+a^6)}+\frac{\frac{\sqrt{2}\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+15\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-80\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(77\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+15\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e+80\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{1024\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/384*(45*b^2*x^11*e + 60*b^2*d*x^10 + 77*b^2*c*x^9 + 126*a*b*x^7*e + 160*a
*b*d*x^6 + 198*a*b*c*x^5 + 113*a^2*x^3*e + 132*a^2*d*x^2 + 153*a^2*c*x)/(a^
3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6) + 1/1024*(sqrt(2)*(77*sqrt(
b)*c - 15*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))
/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*sqrt(b)*c - 15*sqrt(a)*e)*log(sqrt(b)*x^2
- sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^
(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e - 80*sqrt(a)*sqrt(b)*d)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(3/4)*
c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e + 80*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)
*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sq
rt(sqrt(a)*sqrt(b))*b^(3/4)))/a^3
```

Fricas [C] Result contains complex when optimal does not.
time = 16.45, size = 124960, normalized size = 335.91

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] 1/9437184*(1105920*b^2*e*x^11 + 1474560*b^2*d*x^10 + 1892352*b^2*c*x^9 + 30
96576*a*b*e*x^7 + 3932160*a*b*d*x^6 + 4866048*a*b*c*x^5 + 2777088*a^2*e*x^3
 + 3244032*a^2*d*x^2 + 3760128*a^2*c*x + 2*(a^3*b^3*x^12 + 3*a^4*b^2*x^8 +
3*a^5*b*x^4 + a^6)*((-I*sqrt(3) + 1)*((a^4*b*sqrt(-1/(a*b))*sqrt(-(2310*a*b
*c*e*sqrt(-1/(a*b)) + 5929*b*c^2 - 225*a*e^2)/(a^8*b^2*sqrt(-1/(a*b)))) - 4
0*d)^2/(a^7*b) - 3*(80*a^4*b*d*sqrt(-(2310*a*b*c*e*sqrt(-1/(a*b)) + 5929*b*
c^2 - 225*a*e^2)/(a^8*b^2*sqrt(-1/(a*b)))) + 5929*b*c^2 - 5*(2*(160*d^2 + 2
31*c*e)*b*sqrt(-1/(a*b)) + 45*e^2)*a)/(a^8*b^2*sqrt(-1/(a*b))))/(-1/8053063
68*(80*a^4*b*d*sqrt(-(2310*a*b*c*e*sqrt(-1/(a*b)) + 5929*b*c^2 - 225*a*e^2)
/(a^8*b^2*sqrt(-1/(a*b)))) + 5929*b*c^2 - 5*(2*(160*d^2 + 231*c*e)*b*sqrt(-
1/(a*b)) + 45*e^2)*a)*(a^4*b*sqrt(-1/(a*b))*sqrt(-(2310*a*b*c*e*sqrt(-1/(a*
b)) + 5929*b*c^2 - 225*a*e^2)/(a^8*b^2*sqrt(-1/(a*b)))) - 40*d)/(a^11*b^2)
+ 1/268435456*(a^11*b^2*sqrt(-1/(a*b))*(-(2310*a*b*c*e*sqrt(-1/(a*b)) + 592
9*b*c^2 - 225*a*e^2)/(a^8 ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.70, size = 373, normalized size = 1.00

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}-\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}+\frac{45\,b^2x^{11}e+60\,b^2dx^{10}+77\,b^2cx^9+126\,abx^7e+160\,abdx^6+198\,abcx^5+113\,a^2x^3e+132\,a^2dx^2+153\,a^2cx}{384\,(bx^4+a)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a*
b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2
*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqr
t(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b
)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^2*x^11*e + 60*b^2*d*x^10 + 77*
b^2*c*x^9 + 126*a*b*x^7*e + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 113*a^2*x^3*e +
 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

Mupad [**B**]
time = 5.14, size = 873, normalized size = 2.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^4)^4,x)
```

```
[Out] ((11*d*x^2)/(32*a) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9
)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (33*b*c
*x^5)/(64*a^2) + (5*b*d*x^6)/(12*a^2) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^3*x
^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log(-(b*(3375*a*e^3 - 123200*b*c*
d^2 + 88935*b*c^2*e - 64000*b*d^3*x + 20185088*root(68719476736*a^15*b^3*z^
4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*
b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)^2*a^7*b^2
*c - 115200*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 83
8860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*
z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*
b^2*c^4 + 50625*a^2*e^4, z, k)*a^4*b*e^2*x + 92400*b*c*d*e*x + 3035648*root
(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*
d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*
c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*
```

```
a^2*e^4, z, k)*a^3*b^2*c^2*x - 10485760*root(68719476736*a^15*b^3*z^4 + 121
1105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2
*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 +
 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x +
614400*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 8388608
00*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7
392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c
^4 + 50625*a^2*e^4, z, k)*a^4*b*d*e))/(2097152*a^9))*root(68719476736*a^15*
b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 48570368
0*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*
a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k), k,
 1, 4)
```

3.133 $\int a(e+fx^4)^2\,dx$

Optimal. Leaf size=28

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9$$

```
[Out] a*e^2*x+2/5*a*e*f*x^5+1/9*a*f^2*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 200}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[a*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int a\left(e+fx^4\right)^2\,dx&=a\int\left(e+fx^4\right)^2\,dx\\&=a\int\left(e^2+2efx^4+f^2x^8\right)\,dx\\&=ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 0.96

$$a\left(e^2x+\frac{2}{5}efx^5+\frac{f^2x^9}{9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*(e + f*x^4)^2,x]
```

```
[Out] a*(e^2*x + (2*e*f*x^5)/5 + (f^2*x^9)/9)
```

Maple [A]
time = 0.32, size = 24, normalized size = 0.86

method	result	size
default	$\left(\frac{1}{9}f^{2}x^{9}+\frac{2}{5}ef\,x^{5}+e^{2}x\right)a$	24
norman	$a\,e^{2}x+\frac{2}{5}aef\,x^{5}+\frac{1}{9}a\,f^{2}x^{9}$	25
risch	$a\,e^{2}x+\frac{2}{5}aef\,x^{5}+\frac{1}{9}a\,f^{2}x^{9}$	25
gosper	$\frac{x\left(5f^{2}x^{8}+18ef\,x^{4}+45e^{2}\right)a}{45}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/9*f^2*x^9+2/5*e*f*x^5+e^2*x)*a
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.89

$$\frac{1}{45}\left(5\,f^{2}x^{9}+18\,fx^{5}e+45\,xe^{2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/45*(5*f^2*x^9 + 18*f*x^5*e + 45*x*e^2)*a
```

Fricas [A]
time = 0.34, size = 24, normalized size = 0.86

$$\frac{1}{9}\,af^{2}x^{9}+\frac{2}{5}\,aefx^{5}+ae^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/9*a*f^2*x^9 + 2/5*a*e*f*x^5 + a*e^2*x
```

Sympy [A]
time = 0.01, size = 27, normalized size = 0.96

$$ae^{2}x+\frac{2aefx^{5}}{5}+\frac{af^{2}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9
```

Giac [A]
time = 0.63, size = 25, normalized size = 0.89

$$\frac{1}{45}\left(5\,f^{2}x^{9}+18\,fx^{5}e+45\,xe^{2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/45*(5*f^2*x^9 + 18*f*x^5*e + 45*x*e^2)*a
```

Mupad [B]
time = 4.67, size = 25, normalized size = 0.89

$$\frac{a\,x\,\left(45\,e^2+18\,e\,f\,x^4+5\,f^2\,x^8\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*(e + f*x^4)^2,x)
```

```
[Out] (a*x*(45*e^2 + 5*f^2*x^8 + 18*e*f*x^4))/45
```

3.134 $\int bx(e+fx^4)^2\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

```
[Out] 1/2*b*e^2*x^2+1/3*b*e*f*x^6+1/10*b*f^2*x^10
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {12, 276}

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[b*x*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int bx\left(e+fx^4\right)^2\,dx&=b\int x\left(e+fx^4\right)^2\,dx\\&=b\int\left(e^2x+2efx^5+f^2x^9\right)dx\\&=\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 32, normalized size = 0.97

$$b\left(\frac{e^2x^2}{2}+\frac{1}{3}efx^6+\frac{f^2x^{10}}{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[b*x*(e + f*x^4)^2,x]
```

```
[Out] b*((e^2*x^2)/2 + (e*f*x^6)/3 + (f^2*x^10)/10)
```

Maple [A]
time = 0.35, size = 27, normalized size = 0.82

method	result	size
default	$\left(\frac{1}{10}f^2x^{10}+\frac{1}{3}ef\,x^6+\frac{1}{2}e^2x^2\right)b$	27
gosper	$\frac{x^2\left(3f^2x^8+10ef\,x^4+15e^2\right)b}{30}$	28
norman	$\frac{1}{2}b\,e^2x^2+\frac{1}{3}bef\,x^6+\frac{1}{10}b\,f^2x^{10}$	28
risch	$\frac{1}{2}b\,e^2x^2+\frac{1}{3}bef\,x^6+\frac{1}{10}b\,f^2x^{10}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/10*f^2*x^10+1/3*e*f*x^6+1/2*e^2*x^2)*b
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.82

$$\frac{1}{30}\left(3\,f^2x^{10}+10\,fx^6e+15\,x^2e^2\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(3*f^2*x^10 + 10*f*x^6*e + 15*x^2*e^2)*b
```

Fricas [A]
time = 0.37, size = 27, normalized size = 0.82

$$\frac{1}{10}\,bf^2x^{10}+\frac{1}{3}\,befx^6+\frac{1}{2}\,be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/10*b*f^2*x^10 + 1/3*b*e*f*x^6 + 1/2*b*e^2*x^2
```

Sympy [A]
time = 0.01, size = 29, normalized size = 0.88

$$\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10
```

Giac [A]
time = 0.71, size = 27, normalized size = 0.82

$$\frac{1}{30}\left(3\,f^2x^{10}+10\,fx^6e+15\,x^2e^2\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/30*(3*f^2*x^10 + 10*f*x^6*e + 15*x^2*e^2)*b
```

Mupad [B]
time = 0.03, size = 27, normalized size = 0.82

$$\frac{b\,x^2\,\left(15\,e^2+10\,e\,f\,x^4+3\,f^2\,x^8\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x*(e + f*x^4)^2,x)
```

```
[Out] (b*x^2*(15*e^2 + 3*f^2*x^8 + 10*e*f*x^4))/30
```

3.135 $\int (a+bx)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=60

$$ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}$$

```
[Out] a*e^2*x+1/2*b*e^2*x^2+2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2*
x^10
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = $\{1864\}$

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(e+fx^4\right)^2\,dx&=\int \left(ae^2+be^2x+2aefx^4+2befx^5+af^2x^8+bf^2x^9\right)\,dx\\&=ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 60, normalized size = 1.00

$$ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10
```

Maple [A]
time = 0.35, size = 51, normalized size = 0.85

method	result	size
gosper	$a\,e^{2} x+\frac{1}{2} b\,e^{2} x^{2}+\frac{2}{5} a e f\,x^{5}+\frac{1}{3} b e f\,x^{6}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{10} b\,f^{2} x^{10}$	51
default	$a\,e^{2} x+\frac{1}{2} b\,e^{2} x^{2}+\frac{2}{5} a e f\,x^{5}+\frac{1}{3} b e f\,x^{6}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{10} b\,f^{2} x^{10}$	51
norman	$a\,e^{2} x+\frac{1}{2} b\,e^{2} x^{2}+\frac{2}{5} a e f\,x^{5}+\frac{1}{3} b e f\,x^{6}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{10} b\,f^{2} x^{10}$	51
risch	$a\,e^{2} x+\frac{1}{2} b\,e^{2} x^{2}+\frac{2}{5} a e f\,x^{5}+\frac{1}{3} b e f\,x^{6}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{10} b\,f^{2} x^{10}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*e^2*x+1/2*b*e^2*x^2+2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2*
x^10
```

Maxima [A]
time = 0.28, size = 50, normalized size = 0.83

$$\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{3}\,bfx^6e+\frac{2}{5}\,afx^5e+\frac{1}{2}\,bx^2e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*f*x^6*e + 2/5*a*f*x^5*e + 1/2*b*x^2
*e^2 + a*x*e^2
```

Fricas [A]
time = 0.36, size = 50, normalized size = 0.83

$$\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{2}\,be^2x^2+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/2*b*e^2
*x^2 + a*e^2*x
```

Sympy [A]
time = 0.01, size = 58, normalized size = 0.97

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
b*f**2*x**10/10
```

Giac [A]
time = 0.62, size = 50, normalized size = 0.83

$$\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{3}\,bfx^6e+\frac{2}{5}\,afx^5e+\frac{1}{2}\,bx^2e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*f*x^6*e + 2/5*a*f*x^5*e + 1/2*b*x^2
*e^2 + a*x*e^2
```

Mupad [B]
time = 0.02, size = 50, normalized size = 0.83

$$\frac{b\,e^2\,x^2}{2}+a\,e^2\,x+\frac{b\,e\,f\,x^6}{3}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{b\,f^2\,x^{10}}{10}+\frac{a\,f^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^4)^2*(a + b*x),x)
```

```
[Out] (b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + a*e^2*x + (2*a*e*f*x^5)/5
 + (b*e*f*x^6)/3
```

3.136 $\int cx^2(e+fx^4)^2\,dx$

Optimal. Leaf size=33

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

```
[Out] 1/3*c*e^2*x^3+2/7*c*e*f*x^7+1/11*c*f^2*x^11
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {12, 276}

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[c*x^2*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int cx^2\left(e+fx^4\right)^2\,dx&=c\int x^2\left(e+fx^4\right)^2\,dx\\&=c\int\left(e^2x^2+2efx^6+f^2x^{10}\right)\,dx\\&=\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 1.00

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[c*x^2*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

Maple [A]
time = 0.32, size = 27, normalized size = 0.82

method	result	size
default	$\left(\frac{1}{11}f^{2}x^{11}+\frac{2}{7}ef\,x^{7}+\frac{1}{3}e^{2}x^{3}\right)c$	27
gosper	$\frac{x^{3}\left(21f^{2}x^{8}+66ef\,x^{4}+77e^{2}\right)c}{231}$	28
norman	$\frac{1}{3}c\,e^{2}x^{3}+\frac{2}{7}cef\,x^{7}+\frac{1}{11}c\,f^{2}x^{11}$	28
risch	$\frac{1}{3}c\,e^{2}x^{3}+\frac{2}{7}cef\,x^{7}+\frac{1}{11}c\,f^{2}x^{11}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^2*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/11*f^2*x^11+2/7*e*f*x^7+1/3*e^2*x^3)*c
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.82

$$\frac{1}{231}\left(21\,f^{2}x^{11}+66\,fx^{7}e+77\,x^{3}e^{2}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/231*(21*f^2*x^11 + 66*f*x^7*e + 77*x^3*e^2)*c
```

Fricas [A]
time = 0.39, size = 27, normalized size = 0.82

$$\frac{1}{11}\,cf^{2}x^{11}+\frac{2}{7}\,cefx^{7}+\frac{1}{3}\,ce^{2}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*c*f^2*x^11 + 2/7*c*e*f*x^7 + 1/3*c*e^2*x^3
```

Sympy [A]
time = 0.01, size = 31, normalized size = 0.94

$$\frac{ce^{2}x^{3}}{3}+\frac{2cefx^{7}}{7}+\frac{cf^{2}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**2*(f*x**4+e)**2,x)
```

```
[Out] c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

Giac [A]
time = 0.65, size = 27, normalized size = 0.82

$$\frac{1}{231}\left(21\,f^{2}x^{11}+66\,fx^{7}e+77\,x^{3}e^{2}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/231*(21*f^2*x^11 + 66*f*x^7*e + 77*x^3*e^2)*c
```

Mupad [B]
time = 0.04, size = 27, normalized size = 0.82

$$\frac{c\,x^3\,\left(77\,e^2+66\,e\,f\,x^4+21\,f^2\,x^8\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^2*(e + f*x^4)^2,x)
```

```
[Out] (c*x^3*(77*e^2 + 21*f^2*x^8 + 66*e*f*x^4))/231
```

3.137 $\int (a+cx^2)(e+fx^4)^2 \, dx$

Optimal. Leaf size=60

$$ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}$$

```
[Out] a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2*
x^11
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1168}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (a+cx^2)(e+fx^4)^2 \, dx &= \int \left(ae^2+ce^2x^2+2aefx^4+2cefx^6+af^2x^8+cf^2x^{10}\right) dx\\ &= ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 60, normalized size = 1.00

$$ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11
```

Maple [A]
time = 0.35, size = 51, normalized size = 0.85

method	result	size
gosper	$a\,e^{2} x +\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51
default	$a\,e^{2} x +\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51
norman	$a\,e^{2} x +\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51
risch	$a\,e^{2} x +\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2*
x^11
```

Maxima [A]
time = 0.27, size = 50, normalized size = 0.83

$$\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cfx^7e+\frac{2}{5}\,afx^5e+\frac{1}{3}\,cx^3e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*f*x^7*e + 2/5*a*f*x^5*e + 1/3*c*x^3
*e^2 + a*x*e^2
```

Fricas [A]
time = 0.36, size = 50, normalized size = 0.83

$$\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cefx^7+\frac{2}{5}\,aefx^5+\frac{1}{3}\,ce^2x^3+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/3*c*e^2
*x^3 + a*e^2*x
```

Sympy [A]
time = 0.01, size = 60, normalized size = 1.00

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7
+ c*f**2*x**11/11
```

Giac [A]
time = 0.55, size = 50, normalized size = 0.83

$$\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cfx^7e+\frac{2}{5}\,afx^5e+\frac{1}{3}\,cx^3e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*f*x^7*e + 2/5*a*f*x^5*e + 1/3*c*x^3
*e^2 + a*x*e^2
```

Mupad [B]
time = 0.03, size = 50, normalized size = 0.83

$$\frac{c\,e^2\,x^3}{3}+a\,e^2\,x+\frac{2\,c\,e\,f\,x^7}{7}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{c\,f^2\,x^{11}}{11}+\frac{a\,f^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(e + f*x^4)^2,x)
```

```
[Out] (a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (c*f^2*x^11)/11 + a*e^2*x + (2*a*e*f*x^5)/5
 + (2*c*e*f*x^7)/7
```

3.138 $\int (bx+cx^2)(e+fx^4)^2 \, dx$

Optimal. Leaf size=65

$$\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$$

```
[Out] 1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/3*b*e*f*x^6+2/7*c*e*f*x^7+1/10*b*f^2*x^10+1/1
1*c*f^2*x^11
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1607, 1634}

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int (bx+cx^2)(e+fx^4)^2 \, dx &= \int x(b+cx)(e+fx^4)^2 \, dx \\
&= \int \left(be^2x+ce^2x^2+2befx^5+2cefx^6+bf^2x^9+cf^2x^{10}\right) dx \\
&= \frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 65, normalized size = 1.00

$$\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11
```

Maple [A]
time = 0.32, size = 54, normalized size = 0.83

method	result	size
gosper	$\frac{x^2(210c\,f^2x^9+231b\,f^2x^8+660cef\,x^5+770bef\,x^4+770c\,e^2x+1155b\,e^2)}{2310}$	54
default	$\frac{1}{2}b\,e^2x^2+\frac{1}{3}c\,e^2x^3+\frac{1}{3}bef\,x^6+\frac{2}{7}cef\,x^7+\frac{1}{10}b\,f^2x^{10}+\frac{1}{11}c\,f^2x^{11}$	54
norman	$\frac{1}{2}b\,e^2x^2+\frac{1}{3}c\,e^2x^3+\frac{1}{3}bef\,x^6+\frac{2}{7}cef\,x^7+\frac{1}{10}b\,f^2x^{10}+\frac{1}{11}c\,f^2x^{11}$	54
risch	$\frac{1}{2}b\,e^2x^2+\frac{1}{3}c\,e^2x^3+\frac{1}{3}bef\,x^6+\frac{2}{7}cef\,x^7+\frac{1}{10}b\,f^2x^{10}+\frac{1}{11}c\,f^2x^{11}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/3*b*e*f*x^6+2/7*c*e*f*x^7+1/10*b*f^2*x^10+1/1
1*c*f^2*x^11
```

Maxima [A]
time = 0.30, size = 53, normalized size = 0.82

$$\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{2}{7}\,cfx^7e+\frac{1}{3}\,bfx^6e+\frac{1}{3}\,cx^3e^2+\frac{1}{2}\,bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*f*x^7*e + 1/3*b*f*x^6*e + 1/3*c*x
^3*e^2 + 1/2*b*x^2*e^2
```

Fricas [A]
time = 0.36, size = 53, normalized size = 0.82

$$\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/3*c*e
^2*x^3 + 1/2*b*e^2*x^2
```

Sympy [A]
time = 0.01, size = 61, normalized size = 0.94

$$\frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x*
*7/7 + c*f**2*x**11/11
```

Giac [A]
time = 0.71, size = 53, normalized size = 0.82

$$\frac{1}{11}\,cf^2x^{11} + \frac{1}{10}\,bf^2x^{10} + \frac{2}{7}\,cfx^7e + \frac{1}{3}\,bfx^6e + \frac{1}{3}\,cx^3e^2 + \frac{1}{2}\,bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*f*x^7*e + 1/3*b*f*x^6*e + 1/3*c*x
^3*e^2 + 1/2*b*x^2*e^2
```

Mupad [B]
time = 0.03, size = 53, normalized size = 0.82

$$\frac{c\,e^2\,x^3}{3} + \frac{b\,e^2\,x^2}{2} + \frac{2\,c\,e\,f\,x^7}{7} + \frac{b\,e\,f\,x^6}{3} + \frac{c\,f^2\,x^{11}}{11} + \frac{b\,f^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)*(e + f*x^4)^2,x)
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (b*e*f*
x^6)/3 + (2*c*e*f*x^7)/7
```

3.139 $\int \left(a+bx+cx^2\right)\left(e+fx^4\right)^2 \, dx$

Optimal. Leaf size=92

$$ae^2x+\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$$

```
[Out] a*e^2*x+1/2*b*e^2*x^2+1/3*c*e^2*x^3+2/5*a*e*f*x^5+1/3*b*e*f*x^6+2/7*c*e*f*x
^7+1/9*a*f^2*x^9+1/10*b*f^2*x^10+1/11*c*f^2*x^11
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{1671\}$

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 +
 (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(e+fx^4\right)^2 \, dx &= \int \left(ae^2+be^2x+ce^2x^2+2aefx^4+2befx^5+2cefx^6+af^2x^8+bf^2x^9+\ldots\right.\\ &= ae^2x+\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{10}b\ldots\end{aligned}$$

Mathematica [A]
time = 0.00, size = 92, normalized size = 1.00

$$ae^2x+\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 +
 (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

Maple [A]
time = 0.38, size = 77, normalized size = 0.84

method	result	siz
gosper	$a e^2x+\frac{1}{2}b e^2x^2+\frac{1}{3}c e^2x^3+\frac{2}{5}aef x^5+\frac{1}{3}bef x^6+\frac{2}{7}cef x^7+\frac{1}{9}a f^2x^9+\frac{1}{10}b f^2x^{10}+\frac{1}{11}c f^2x^{11}$	77
default	$a e^2x+\frac{1}{2}b e^2x^2+\frac{1}{3}c e^2x^3+\frac{2}{5}aef x^5+\frac{1}{3}bef x^6+\frac{2}{7}cef x^7+\frac{1}{9}a f^2x^9+\frac{1}{10}b f^2x^{10}+\frac{1}{11}c f^2x^{11}$	77
norman	$a e^2x+\frac{1}{2}b e^2x^2+\frac{1}{3}c e^2x^3+\frac{2}{5}aef x^5+\frac{1}{3}bef x^6+\frac{2}{7}cef x^7+\frac{1}{9}a f^2x^9+\frac{1}{10}b f^2x^{10}+\frac{1}{11}c f^2x^{11}$	77
risch	$a e^2x+\frac{1}{2}b e^2x^2+\frac{1}{3}c e^2x^3+\frac{2}{5}aef x^5+\frac{1}{3}bef x^6+\frac{2}{7}cef x^7+\frac{1}{9}a f^2x^9+\frac{1}{10}b f^2x^{10}+\frac{1}{11}c f^2x^{11}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*e^2*x+1/2*b*e^2*x^2+1/3*c*e^2*x^3+2/5*a*e*f*x^5+1/3*b*e*f*x^6+2/7*c*e*f*x
^7+1/9*a*f^2*x^9+1/10*b*f^2*x^10+1/11*c*f^2*x^11
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.83

$$\frac{1}{11}cf^2x^{11}+\frac{1}{10}bf^2x^{10}+\frac{1}{9}af^2x^9+\frac{2}{7}cfx^7e+\frac{1}{3}bfx^6e+\frac{2}{5}afx^5e+\frac{1}{3}cx^3e^2+\frac{1}{2}bx^2e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*f*x^7*e + 1/3*b*f
*x^6*e + 2/5*a*f*x^5*e + 1/3*c*x^3*e^2 + 1/2*b*x^2*e^2 + a*x*e^2
```

Fricas [A]
time = 0.36, size = 76, normalized size = 0.83

$$\frac{1}{11}cf^2x^{11}+\frac{1}{10}bf^2x^{10}+\frac{1}{9}af^2x^9+\frac{2}{7}cefx^7+\frac{1}{3}befx^6+\frac{2}{5}aefx^5+\frac{1}{3}ce^2x^3+\frac{1}{2}be^2x^2+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 1/3*b*e
*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2 + a*e^2*x
```

Sympy [A]
time = 0.01, size = 90, normalized size = 0.98

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

Giac [A]
time = 0.80, size = 76, normalized size = 0.83

$$\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cfx^7e+\frac{1}{3}\,bfx^6e+\frac{2}{5}\,afx^5e+\frac{1}{3}\,cx^3e^2+\frac{1}{2}\,bx^2e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*f*x^7*e + 1/3*b*f
*x^6*e + 2/5*a*f*x^5*e + 1/3*c*x^3*e^2 + 1/2*b*x^2*e^2 + a*x*e^2
```

Mupad [B]
time = 0.04, size = 76, normalized size = 0.83

$$\frac{c\,e^2\,x^3}{3}+\frac{b\,e^2\,x^2}{2}+a\,e^2\,x+\frac{2\,c\,e\,f\,x^7}{7}+\frac{b\,e\,f\,x^6}{3}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{c\,f^2\,x^{11}}{11}+\frac{b\,f^2\,x^{10}}{10}+\frac{a\,f^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^4)^2*(a + b*x + c*x^2),x)
```

```
[Out] (b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (c*f^2*x^
11)/11 + a*e^2*x + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7
```

3.140 $\int dx^3(e+fx^4)^2\,dx$

Optimal. Leaf size=17

$$\frac{d(e+fx^4)^3}{12f}$$

```
[Out] 1/12*d*(f*x^4+e)^3/f
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {12, 267}

$$\frac{d(e+fx^4)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[d*x^3*(e + f*x^4)^2,x]
```

```
[Out] (d*(e + f*x^4)^3)/(12*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int dx^3\left(e+fx^4\right)^2\,dx &= d\int x^3\left(e+fx^4\right)^2\,dx\\ &= \frac{d(e+fx^4)^3}{12f}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 1.94

$$\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[d*x^3*(e + f*x^4)^2,x]
```

```
[Out] (d*e^2*x^4)/4 + (d*e*f*x^8)/4 + (d*f^2*x^12)/12
```

Maple [A]
time = 0.32, size = 16, normalized size = 0.94

method	result	size
default	$\frac{d\left(f\,x^{4}+e\right)^{3}}{12f}$	16
gosper	$\frac{x^{4}\left(f^{2}x^{8}+3ef\,x^{4}+3e^{2}\right)d}{12}$	27
norman	$\frac{1}{4}d\,e^{2}x^{4}+\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{4}def\,x^{8}$	28
risch	$\frac{d\,f^{2}x^{12}}{12}+\frac{def\,x^{8}}{4}+\frac{d\,e^{2}x^{4}}{4}+\frac{d\,e^{3}}{12f}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x^3*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d*(f*x^4+e)^3/f
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.94

$$\frac{(fx^4+e)^3d}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(f*x^4 + e)^3*d/f
```

Fricas [A]
time = 0.37, size = 27, normalized size = 1.59

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{4}\,defx^8+\frac{1}{4}\,de^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*d*f^2*x^12 + 1/4*d*e*f*x^8 + 1/4*d*e^2*x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.
time = 0.01, size = 29, normalized size = 1.71

$$\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x**3*(f*x**4+e)**2,x)
```

```
[Out] d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Giac [A]
time = 0.64, size = 16, normalized size = 0.94

$$\frac{\left(f x^{4}+e\right)^{3} d}{12\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*(f*x^4 + e)^3*d/f
```

Mupad [B]
time = 0.03, size = 26, normalized size = 1.53

$$\frac{d\, x^4\, \left(3\, e^2+3\, e\, f\, x^4+f^2\, x^8\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x^3*(e + f*x^4)^2,x)
```

```
[Out] (d*x^4*(3*e^2 + f^2*x^8 + 3*e*f*x^4))/12
```

3.141 $\int (a+dx^3)(e+fx^4)^2 \, dx$

Optimal. Leaf size=45

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{d(e+fx^4)^3}{12f}$$

[Out] a*e^2*x+2/5*a*e*f*x^5+1/9*a*f^2*x^9+1/12*d*(f*x^4+e)^3/f

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1596, 12, 200}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{d(e+fx^4)^3}{12f}$$

Antiderivative was successfully verified.

[In] Int[(a + d*x^3)*(e + f*x^4)^2,x]

[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9 + (d*(e + f*x^4)^3)/(12*f)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\int (a+dx^3)(e+fx^4)^2\,dx = \frac{d(e+fx^4)^3}{12f} + \int a(e+fx^4)^2\,dx$$
$$= \frac{d(e+fx^4)^3}{12f} + a\int (e+fx^4)^2\,dx$$
$$= \frac{d(e+fx^4)^3}{12f} + a\int (e^2+2efx^4+f^2x^8)\,dx$$
$$= ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{d(e+fx^4)^3}{12f}$$

Mathematica [A]
time = 0.00, size = 60, normalized size = 1.33

$$ae^2x + \frac{1}{4}de^2x^4 + \frac{2}{5}aefx^5 + \frac{1}{4}defx^8 + \frac{1}{9}af^2x^9 + \frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 +
 (d*f^2*x^12)/12
```

Maple [A]
time = 0.36, size = 51, normalized size = 1.13

method	result	size
gosper	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+a\,e^2x$	51
default	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+a\,e^2x$	51
norman	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+a\,e^2x$	51
risch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+a\,e^2x$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d*f^2*x^12+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/5*a*e*f*x^5+1/4*d*e^2*x^4+a*e
^2*x
```

Maxima [A]
time = 0.30, size = 50, normalized size = 1.11

$$\frac{1}{12}\,df^2x^{12} + \frac{1}{9}\,af^2x^9 + \frac{1}{4}\,dfx^8e + \frac{2}{5}\,afx^5e + \frac{1}{4}\,dx^4e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="maxima")

[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 2/5*a*f*x^5*e + 1/4*d*x^4 *e^2 + a*x*e^2

Fricas [A]
time = 0.35, size = 50, normalized size = 1.11

$$\frac{1}{12}\,df^2x^{12} + \frac{1}{9}\,af^2x^9 + \frac{1}{4}\,defx^8 + \frac{2}{5}\,aefx^5 + \frac{1}{4}\,de^2x^4 + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="fricas")

[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 + 1/4*d*e^2 *x^4 + a*e^2*x

Sympy [A]
time = 0.01, size = 58, normalized size = 1.29

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+a)*(f*x**4+e)**2,x)

[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12

Giac [A]
time = 0.56, size = 50, normalized size = 1.11

$$\frac{1}{12}\,df^2x^{12} + \frac{1}{9}\,af^2x^9 + \frac{1}{4}\,dfx^8e + \frac{2}{5}\,afx^5e + \frac{1}{4}\,dx^4e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="giac")

[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 2/5*a*f*x^5*e + 1/4*d*x^4 *e^2 + a*x*e^2

Mupad [B]
time = 0.02, size = 50, normalized size = 1.11

$$\frac{d\,e^2\,x^4}{4} + a\,e^2\,x + \frac{d\,e\,f\,x^8}{4} + \frac{2\,a\,e\,f\,x^5}{5} + \frac{d\,f^2\,x^{12}}{12} + \frac{a\,f^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + d*x^3)*(e + f*x^4)^2,x)

[Out] (a*f^2*x^9)/9 + (d*e^2*x^4)/4 + (d*f^2*x^12)/12 + a*e^2*x + (2*a*e*f*x^5)/5 + (d*e*f*x^8)/4

3.142 $\int (bx+dx^3)(e+fx^4)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}$$

```
[Out] 1/2*b*e^2*x^2+1/3*b*e*f*x^6+1/10*b*f^2*x^10+1/12*d*(f*x^4+e)^3/f
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {1596, 12, 276}

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\int \left(bx+dx^3\right)\left(e+fx^4\right)^2\,dx = \frac{d(e+fx^4)^3}{12f}+\int bx\left(e+fx^4\right)^2\,dx$$
$$= \frac{d(e+fx^4)^3}{12f}+b\int x\left(e+fx^4\right)^2\,dx$$
$$= \frac{d(e+fx^4)^3}{12f}+b\int \left(e^2x+2efx^5+f^2x^9\right)\,dx$$
$$= \frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}$$

Mathematica [A]
time = 0.00, size = 65, normalized size = 1.30

$$\frac{1}{2}be^2x^2+\frac{1}{4}de^2x^4+\frac{1}{3}befx^6+\frac{1}{4}defx^8+\frac{1}{10}bf^2x^{10}+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4 + (b*f^2*x^10
)/10 + (d*f^2*x^12)/12
```

Maple [A]
time = 0.34, size = 54, normalized size = 1.08

method	result	size
default	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{1}{4}d\,e^2x^4+\frac{1}{2}b\,e^2x^2$	54
norman	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{1}{4}d\,e^2x^4+\frac{1}{2}b\,e^2x^2$	54
risch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{1}{4}d\,e^2x^4+\frac{1}{2}b\,e^2x^2$	54
gosper	$\frac{x^2\left(5d\,f^2x^{10}+6b\,f^2x^8+15def\,x^6+20bef\,x^4+15d\,e^2x^2+30b\,e^2\right)}{60}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/4*d*e*f*x^8+1/3*b*e*f*x^6+1/4*d*e^2*x^4+1
/2*b*e^2*x^2
```

Maxima [A]
time = 0.30, size = 53, normalized size = 1.06

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{4}\,dfx^8e+\frac{1}{3}\,bfx^6e+\frac{1}{4}\,dx^4e^2+\frac{1}{2}\,bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*f*x^8*e + 1/3*b*f*x^6*e + 1/4*d*x
^4*e^2 + 1/2*b*x^2*e^2
```

Fricas [A]
time = 0.35, size = 53, normalized size = 1.06

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{4}\,defx^8+\frac{1}{3}\,befx^6+\frac{1}{4}\,de^2x^4+\frac{1}{2}\,be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6 + 1/4*d*e
^2*x^4 + 1/2*b*e^2*x^2
```

Sympy [A]
time = 0.01, size = 60, normalized size = 1.20

$$\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**8
/4 + d*f**2*x**12/12
```

Giac [A]
time = 0.55, size = 53, normalized size = 1.06

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{4}\,dfx^8e+\frac{1}{3}\,bfx^6e+\frac{1}{4}\,dx^4e^2+\frac{1}{2}\,bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*f*x^8*e + 1/3*b*f*x^6*e + 1/4*d*x
^4*e^2 + 1/2*b*x^2*e^2
```

Mupad [B]
time = 0.03, size = 53, normalized size = 1.06

$$\frac{d\,e^2\,x^4}{4}+\frac{b\,e^2\,x^2}{2}+\frac{d\,e\,f\,x^8}{4}+\frac{b\,e\,f\,x^6}{3}+\frac{d\,f^2\,x^{12}}{12}+\frac{b\,f^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + d*x^3)*(e + f*x^4)^2,x)
```

```
[Out] (b*e^2*x^2)/2 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (d*f^2*x^12)/12 + (b*e*f*
x^6)/3 + (d*e*f*x^8)/4
```

3.143 $\int \left(a+bx+dx^3\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=77

$$ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}$$

```
[Out] a*e^2*x+1/2*b*e^2*x^2+2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2*
x^10+1/12*d*(f*x^4+e)^3/f
```

Rubi [A]

time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1596, 1864}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(a+bx+dx^3\right)\left(e+fx^4\right)^2\,dx = \frac{d(e+fx^4)^3}{12f} + \int (a+bx)\left(e+fx^4\right)^2\,dx$$

$$= \frac{d(e+fx^4)^3}{12f} + \int \left(ae^2+be^2x+2aefx^4+2befx^5+af^2x^8+bf^2x^9\right)dx$$

$$= ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)}{12f}$$

Mathematica [**A**]
time = 0.00, size = 92, normalized size = 1.19

$$ae^2x+\frac{1}{2}be^2x^2+\frac{1}{4}de^2x^4+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{4}defx^8+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 +
 (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*f^2*x^12)/12
```

Maple [**A**]
time = 0.37, size = 77, normalized size = 1.00

method	result	si
gosper	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+\frac{1}{2}b\,e^2x^2+a\,e^2x$	7
default	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+\frac{1}{2}b\,e^2x^2+a\,e^2x$	7
norman	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+\frac{1}{2}b\,e^2x^2+a\,e^2x$	7
risch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+\frac{1}{2}b\,e^2x^2+a\,e^2x$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/9*a*f^2*x^9+1/4*d*e*f*x^8+1/3*b*e*f*x^6+2
/5*a*e*f*x^5+1/4*d*e^2*x^4+1/2*b*e^2*x^2+a*e^2*x
```

Maxima [**A**]
time = 0.29, size = 76, normalized size = 0.99

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,dfx^8e+\frac{1}{3}\,bfx^6e+\frac{2}{5}\,afx^5e+\frac{1}{4}\,dx^4e^2+\frac{1}{2}\,bx^2e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 1/3*b*f
*x^6*e + 2/5*a*f*x^5*e + 1/4*d*x^4*e^2 + 1/2*b*x^2*e^2 + a*x*e^2
```

Fricas [A]
time = 0.36, size = 76, normalized size = 0.99

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{2}\,be^2x^2+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 1/3*b*e
*f*x^6 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2 + a*e^2*x
```

Sympy [A]
time = 0.01, size = 88, normalized size = 1.14

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Giac [A]
time = 0.62, size = 76, normalized size = 0.99

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,dfx^8e+\frac{1}{3}\,bfx^6e+\frac{2}{5}\,afx^5e+\frac{1}{4}\,dx^4e^2+\frac{1}{2}\,bx^2e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 1/3*b*f
*x^6*e + 2/5*a*f*x^5*e + 1/4*d*x^4*e^2 + 1/2*b*x^2*e^2 + a*x*e^2
```

Mupad [B]
time = 0.04, size = 76, normalized size = 0.99

$$\frac{d\,e^2\,x^4}{4}+\frac{b\,e^2\,x^2}{2}+a\,e^2\,x+\frac{d\,e\,f\,x^8}{4}+\frac{b\,e\,f\,x^6}{3}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{d\,f^2\,x^{12}}{12}+\frac{b\,f^2\,x^{10}}{10}+\frac{a\,f^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^4)^2*(a + b*x + d*x^3),x)
```

```
[Out] (b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (d*f^2*x^
12)/12 + a*e^2*x + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4
```

3.144 $\int (cx^2 + dx^3)(e + fx^4)^2 \, dx$

Optimal. Leaf size=50

$$\frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11} + \frac{d(e+fx^4)^3}{12f}$$

```
[Out] 1/3*c*e^2*x^3+2/7*c*e*f*x^7+1/11*c*f^2*x^11+1/12*d*(f*x^4+e)^3/f
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1596, 12, 276}

$$\frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11} + \frac{d(e+fx^4)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\begin{aligned}\int\left(cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx&=\frac{d(e+fx^4)^3}{12f}+\int cx^2\left(e+fx^4\right)^2\,dx\\&=\frac{d(e+fx^4)^3}{12f}+c\int x^2\left(e+fx^4\right)^2\,dx\\&=\frac{d(e+fx^4)^3}{12f}+c\int\left(e^2x^2+2efx^6+f^2x^{10}\right)\,dx\\&=\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 65, normalized size = 1.30

$$\frac{1}{3}ce^2x^3+\frac{1}{4}de^2x^4+\frac{2}{7}cefx^7+\frac{1}{4}defx^8+\frac{1}{11}cf^2x^{11}+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 + (c*f^2*x^
11)/11 + (d*f^2*x^12)/12
```

Maple [A]
time = 0.33, size = 54, normalized size = 1.08

method	result	size
gosper	$\frac{x^3\left(77d\,f^2x^9+84c\,f^2x^8+231def\,x^5+264cef\,x^4+231d\,e^2x+308c\,e^2\right)}{924}$	54
default	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{11}c\,f^2x^{11}+\frac{1}{4}def\,x^8+\frac{2}{7}cef\,x^7+\frac{1}{4}d\,e^2x^4+\frac{1}{3}c\,e^2x^3$	54
norman	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{11}c\,f^2x^{11}+\frac{1}{4}def\,x^8+\frac{2}{7}cef\,x^7+\frac{1}{4}d\,e^2x^4+\frac{1}{3}c\,e^2x^3$	54
risch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{11}c\,f^2x^{11}+\frac{1}{4}def\,x^8+\frac{2}{7}cef\,x^7+\frac{1}{4}d\,e^2x^4+\frac{1}{3}c\,e^2x^3$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/4*d*e*f*x^8+2/7*c*e*f*x^7+1/4*d*e^2*x^4+1
/3*c*e^2*x^3
```

Maxima [A]
time = 0.29, size = 53, normalized size = 1.06

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{4}\,dfx^8e+\frac{2}{7}\,cfx^7e+\frac{1}{4}\,dx^4e^2+\frac{1}{3}\,cx^3e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*f*x^8*e + 2/7*c*f*x^7*e + 1/4*d*x
^4*e^2 + 1/3*c*x^3*e^2
```

Fricas [A]
time = 0.37, size = 53, normalized size = 1.06

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*e*f*x^8 + 2/7*c*e*f*x^7 + 1/4*d*e
^2*x^4 + 1/3*c*e^2*x^3
```

Sympy [A]
time = 0.01, size = 61, normalized size = 1.22

$$\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2)*(f*x**4+e)**2,x)
```

```
[Out] c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x*
*8/4 + d*f**2*x**12/12
```

Giac [A]
time = 0.53, size = 53, normalized size = 1.06

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{4}\,dfx^8e+\frac{2}{7}\,cfx^7e+\frac{1}{4}\,dx^4e^2+\frac{1}{3}\,cx^3e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*f*x^8*e + 2/7*c*f*x^7*e + 1/4*d*x
^4*e^2 + 1/3*c*x^3*e^2
```

Mupad [B]
time = 0.03, size = 53, normalized size = 1.06

$$\frac{d\,e^2\,x^4}{4}+\frac{c\,e^2\,x^3}{3}+\frac{d\,e\,f\,x^8}{4}+\frac{2\,c\,e\,f\,x^7}{7}+\frac{d\,f^2\,x^{12}}{12}+\frac{c\,f^2\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^4)^2*(c*x^2 + d*x^3),x)
```

```
[Out] (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12 + (2*c*e*
f*x^7)/7 + (d*e*f*x^8)/4
```

3.145 $\int (a+cx^2+dx^3)(e+fx^4)^2\,dx$

Optimal. Leaf size=77

$$ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

```
[Out] a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2*
x^11+1/12*d*(f*x^4+e)^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1596, 1168}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\int \left(a+cx^2+dx^3\right)\left(e+fx^4\right)^2 dx = \frac{d(e+fx^4)^3}{12f} + \int \left(a+cx^2\right)\left(e+fx^4\right)^2 dx$$
$$= \frac{d(e+fx^4)^3}{12f} + \int \left(ae^2+ce^2x^2+2aefx^4+2cefx^6+af^2x^8+cf^2x^{10}\right)$$
$$= ae^2x + \frac{1}{3}ce^2x^3 + \frac{2}{5}aefx^5 + \frac{2}{7}cefx^7 + \frac{1}{9}af^2x^9 + \frac{1}{11}cf^2x^{11} + \frac{d(e+fx}{12f}$$

Mathematica [**A**]
time = 0.00, size = 92, normalized size = 1.19

$$ae^2x+\frac{1}{3}ce^2x^3+\frac{1}{4}de^2x^4+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{4}defx^8+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7
 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

Maple [**A**]
time = 0.35, size = 77, normalized size = 1.00

method	result	si
gosper	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{11}c\,f^2x^{11}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{7}cef\,x^7+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+\frac{1}{3}c\,e^2x^3+a\,e^2x$	7
default	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{11}c\,f^2x^{11}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{7}cef\,x^7+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+\frac{1}{3}c\,e^2x^3+a\,e^2x$	7
norman	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{11}c\,f^2x^{11}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{7}cef\,x^7+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+\frac{1}{3}c\,e^2x^3+a\,e^2x$	7
risch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{11}c\,f^2x^{11}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{7}cef\,x^7+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+\frac{1}{3}c\,e^2x^3+a\,e^2x$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/7*c*e*f*x^7+2
/5*a*e*f*x^5+1/4*d*e^2*x^4+1/3*c*e^2*x^3+a*e^2*x
```

Maxima [**A**]
time = 0.28, size = 76, normalized size = 0.99

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,dfx^8e+\frac{2}{7}\,cfx^7e+\frac{2}{5}\,afx^5e+\frac{1}{4}\,dx^4e^2+\frac{1}{3}\,cx^3e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 2/7*c*f
*x^7*e + 2/5*a*f*x^5*e + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^2 + a*x*e^2
```

Fricas [A]
time = 0.35, size = 76, normalized size = 0.99

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/7*c*e
*f*x^7 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + a*e^2*x
```

Sympy [A]
time = 0.02, size = 90, normalized size = 1.17

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7
+ c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Giac [A]
time = 0.53, size = 76, normalized size = 0.99

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,dfx^8e+\frac{2}{7}\,cfx^7e+\frac{2}{5}\,afx^5e+\frac{1}{4}\,dx^4e^2+\frac{1}{3}\,cx^3e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 2/7*c*f
*x^7*e + 2/5*a*f*x^5*e + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^2 + a*x*e^2
```

Mupad [B]
time = 0.04, size = 76, normalized size = 0.99

$$\frac{d\,e^2\,x^4}{4}+\frac{c\,e^2\,x^3}{3}+a\,e^2\,x+\frac{d\,e\,f\,x^8}{4}+\frac{2\,c\,e\,f\,x^7}{7}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{d\,f^2\,x^{12}}{12}+\frac{c\,f^2\,x^{11}}{11}+\frac{a\,f^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^4)^2*(a + c*x^2 + d*x^3),x)
```

```
[Out] (a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (c*f^2*x^11)/11 + (d*f^2*x^
12)/12 + a*e^2*x + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4
```

3.146 $\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

```
[Out] 1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/3*b*e*f*x^6+2/7*c*e*f*x^7+1/10*b*f^2*x^10+1/1
1*c*f^2*x^11+1/12*d*(f*x^4+e)^3/f
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {1596, 1607, 1634}

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \left(bx+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{d(e+fx^4)^3}{12f}+\int \left(bx+cx^2\right)\left(e+fx^4\right)^2\,dx$$
$$=\frac{d(e+fx^4)^3}{12f}+\int x(b+cx)\left(e+fx^4\right)^2\,dx$$
$$=\frac{d(e+fx^4)^3}{12f}+\int \left(be^2x+ce^2x^2+2befx^5+2cefx^6+bf^2x^9+cf^2x^{10}\right)$$
$$=\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}+\frac{d(e+}{12}$$

Mathematica [A]
time = 0.00, size = 97, normalized size = 1.18

$$\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{4}de^2x^4+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{4}defx^8+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (2*c*e*f*x^
7)/7 + (d*e*f*x^8)/4 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

Maple [A]
time = 0.36, size = 80, normalized size = 0.98

method	result
gosper	$\frac{x^2\left(385d f^2x^{10}+420c f^2x^9+462b f^2x^8+1155def x^6+1320cef x^5+1540bef x^4+1155d e^2x^2+1540c e^2x+2310b e^2\right)}{4620}$
default	$\frac{1}{12}d f^2x^{12}+\frac{1}{11}c f^2x^{11}+\frac{1}{10}b f^2x^{10}+\frac{1}{4}def x^8+\frac{2}{7}cef x^7+\frac{1}{3}bef x^6+\frac{1}{4}d e^2x^4+\frac{1}{3}c e^2x^3+\frac{1}{2}b e^2x^2$
norman	$\frac{1}{12}d f^2x^{12}+\frac{1}{11}c f^2x^{11}+\frac{1}{10}b f^2x^{10}+\frac{1}{4}def x^8+\frac{2}{7}cef x^7+\frac{1}{3}bef x^6+\frac{1}{4}d e^2x^4+\frac{1}{3}c e^2x^3+\frac{1}{2}b e^2x^2$
risch	$\frac{1}{12}d f^2x^{12}+\frac{1}{11}c f^2x^{11}+\frac{1}{10}b f^2x^{10}+\frac{1}{4}def x^8+\frac{2}{7}cef x^7+\frac{1}{3}bef x^6+\frac{1}{4}d e^2x^4+\frac{1}{3}c e^2x^3+\frac{1}{2}b e^2x^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/10*b*f^2*x^10+1/4*d*e*f*x^8+2/7*c*e*f*x^7
+1/3*b*e*f*x^6+1/4*d*e^2*x^4+1/3*c*e^2*x^3+1/2*b*e^2*x^2
```

Maxima [A]
time = 0.29, size = 79, normalized size = 0.96

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{4}\,dfx^8e+\frac{2}{7}\,cfx^7e+\frac{1}{3}\,bfx^6e+\frac{1}{4}\,dx^4e^2+\frac{1}{3}\,cx^3e^2+\frac{1}{2}\,bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*f*x^8*e + 2/7*c
*f*x^7*e + 1/3*b*f*x^6*e + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^2 + 1/2*b*x^2*e^2
```

Fricas [A]
time = 0.36, size = 79, normalized size = 0.96

$$\frac{1}{12}\,df^{2}x^{12}+\frac{1}{11}\,cf^{2}x^{11}+\frac{1}{10}\,bf^{2}x^{10}+\frac{1}{4}\,defx^{8}+\frac{2}{7}\,cefx^{7}+\frac{1}{3}\,befx^{6}+\frac{1}{4}\,de^{2}x^{4}+\frac{1}{3}\,ce^{2}x^{3}+\frac{1}{2}\,be^{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 2/7*c
*e*f*x^7 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
```

Sympy [A]
time = 0.03, size = 92, normalized size = 1.12

$$\frac{be^{2}x^{2}}{2}+\frac{befx^{6}}{3}+\frac{bf^{2}x^{10}}{10}+\frac{ce^{2}x^{3}}{3}+\frac{2cefx^{7}}{7}+\frac{cf^{2}x^{11}}{11}+\frac{de^{2}x^{4}}{4}+\frac{defx^{8}}{4}+\frac{df^{2}x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x*
*7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Giac [A]
time = 0.72, size = 79, normalized size = 0.96

$$\frac{1}{12}\,df^{2}x^{12}+\frac{1}{11}\,cf^{2}x^{11}+\frac{1}{10}\,bf^{2}x^{10}+\frac{1}{4}\,dfx^{8}e+\frac{2}{7}\,cfx^{7}e+\frac{1}{3}\,bfx^{6}e+\frac{1}{4}\,dx^{4}e^{2}+\frac{1}{3}\,cx^{3}e^{2}+\frac{1}{2}\,bx^{2}e^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*f*x^8*e + 2/7*c
*f*x^7*e + 1/3*b*f*x^6*e + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^2 + 1/2*b*x^2*e^2
```

Mupad [B]
time = 0.04, size = 79, normalized size = 0.96

$$\frac{d\,e^2\,x^4}{4}+\frac{c\,e^2\,x^3}{3}+\frac{b\,e^2\,x^2}{2}+\frac{d\,e\,f\,x^8}{4}+\frac{2\,c\,e\,f\,x^7}{7}+\frac{b\,e\,f\,x^6}{3}+\frac{d\,f^2\,x^{12}}{12}+\frac{c\,f^2\,x^{11}}{11}+\frac{b\,f^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^4)^2*(b*x + c*x^2 + d*x^3),x)
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (c*f^2*x^
11)/11 + (d*f^2*x^12)/12 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4
```

3.147 $\int (c+dx+ex^2+fx^3) (a+bx^4)^2 \, dx$

Optimal. Leaf size=109

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{f(a+bx^4)^3}{12b}$$

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/3*a^2*e*x^3+2/5*a*b*c*x^5+1/3*a*b*d*x^6+2/7*a*b*e*x
^7+1/9*b^2*c*x^9+1/10*b^2*d*x^10+1/11*b^2*e*x^11+1/12*f*(b*x^4+a)^3/b
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1596, 1671}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f(a+bx^4)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 +
 (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f*(
a + b*x^4)^3)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = \frac{f(a+bx^4)^3}{12b} + \int \left(c+dx+ex^2\right)\left(a+bx^4\right)^2\,dx$$
$$= \frac{f(a+bx^4)^3}{12b} + \int \left(a^2c+a^2dx+a^2ex^2+2abcx^4+2abdx^5+2abe$$
$$= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{9}b^2cx^9$$

Mathematica [A]
time = 0.01, size = 124, normalized size = 1.14

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b^2fx^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5 +
 (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x
^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

Maple [A]
time = 0.37, size = 103, normalized size = 0.94

method	result
gosper	$\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}x^9b^2c+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}x^5abc+\frac{1}{4}a^2f\,x^4+$
default	$\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}x^9b^2c+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}x^5abc+\frac{1}{4}a^2f\,x^4+$
norman	$\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}x^9b^2c+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}x^5abc+\frac{1}{4}a^2f\,x^4+$
risch	$\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}x^9b^2c+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}x^5abc+\frac{1}{4}a^2f\,x^4+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*x^9*b^2*c+1/4*a*b*f*x^8
+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*x^5*a*b*c+1/4*a^2*f*x^4+1/3*a^2*e*x^3+1/2*
a^2*d*x^2+a^2*c*x
```

Maxima [A]
time = 0.30, size = 105, normalized size = 0.96

$$\frac{1}{12}b^2fx^{12}+\frac{1}{11}b^2x^{11}e+\frac{1}{10}b^2dx^{10}+\frac{1}{9}b^2cx^9+\frac{1}{4}abfx^8+\frac{2}{7}abx^7e+\frac{1}{3}abdx^6+\frac{2}{5}abcx^5+\frac{1}{4}a^2fx^4+\frac{1}{3}a^2x^3e+\frac{1}{2}a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

Fricas [A]
time = 0.37, size = 102, normalized size = 0.94

$$\frac{1}{12}\,b^{2}fx^{12}+\frac{1}{11}\,b^{2}ex^{11}+\frac{1}{10}\,b^{2}dx^{10}+\frac{1}{9}\,b^{2}cx^{9}+\frac{1}{4}\,abfx^{8}+\frac{2}{7}\,abex^{7}+\frac{1}{3}\,abdx^{6}+\frac{2}{5}\,abcx^{5}+\frac{1}{4}\,a^{2}fx^{4}+\frac{1}{3}\,a^{2}ex^{3}+\frac{1}{2}\,a^{2}dx^{2}+a^{2}cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Sympy [A]
time = 0.04, size = 121, normalized size = 1.11

$$a^{2}cx+\frac{a^{2}dx^{2}}{2}+\frac{a^{2}ex^{3}}{3}+\frac{a^{2}fx^{4}}{4}+\frac{2abcx^{5}}{5}+\frac{abdx^{6}}{3}+\frac{2abex^{7}}{7}+\frac{abfx^{8}}{4}+\frac{b^{2}cx^{9}}{9}+\frac{b^{2}dx^{10}}{10}+\frac{b^{2}ex^{11}}{11}+\frac{b^{2}fx^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5 +
 a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x**1
0/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

Giac [A]
time = 0.51, size = 105, normalized size = 0.96

$$\frac{1}{12}\,b^{2}fx^{12}+\frac{1}{11}\,b^{2}x^{11}e+\frac{1}{10}\,b^{2}dx^{10}+\frac{1}{9}\,b^{2}cx^{9}+\frac{1}{4}\,abfx^{8}+\frac{2}{7}\,abx^{7}e+\frac{1}{3}\,abdx^{6}+\frac{2}{5}\,abcx^{5}+\frac{1}{4}\,a^{2}fx^{4}+\frac{1}{3}\,a^{2}x^{3}e+\frac{1}{2}\,a^{2}dx^{2}+a^{2}cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

Mupad [B]
time = 4.68, size = 102, normalized size = 0.94

$$\frac{f\,a^2\,x^4}{4}+\frac{e\,a^2\,x^3}{3}+\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{f\,a\,b\,x^8}{4}+\frac{2\,e\,a\,b\,x^7}{7}+\frac{d\,a\,b\,x^6}{3}+\frac{2\,c\,a\,b\,x^5}{5}+\frac{f\,b^2\,x^{12}}{12}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*c*x^9)/9 + (a^2*e*x^3)/3 + (b^2*d*x^10)/10 + (a^2*f*x^
4)/4 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12 + a^2*c*x + (2*a*b*c*x^5)/5 + (a*b
*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4
```

3.148 $\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx$

Optimal. Leaf size=151

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}$$

```
[Out] a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+3/5*a^2*b*c*x^5+1/2*a^2*b*d*x^6+3/7*a^2
*b*e*x^7+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/11*a*b^2*e*x^11+1/13*b^3*c*x^1
3+1/14*b^3*d*x^14+1/15*b^3*e*x^15+1/16*f*(b*x^4+a)^4/b
```

Rubi [A]
time = 0.07, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1596, 1671}

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f(a+bx^4)^4}{16b}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)
/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e
*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a + b
*x^4)^4)/(16*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx=\frac{f(a+bx^4)^4}{16b}+\int \left(c+dx+ex^2\right)\left(a+bx^4\right)^3\,dx$$
$$=\frac{f(a+bx^4)^4}{16b}+\int \left(a^3c+a^3dx+a^3ex^2+3a^2bcx^4+3a^2bdx^5+3a^2$$
$$=a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab$$

Mathematica [A]
time = 0.00, size = 180, normalized size = 1.19

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{4}a^3fx^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}a^2bfx^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{4}ab^2fx^{12}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{1}{16}b^3fx^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/5
 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9)/
3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*c*x
^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16
```

Maple [A]
time = 0.37, size = 151, normalized size = 1.00

method	result
gosper	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}a^3f\,x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x^9+$
default	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}a^3f\,x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x^9+$
norman	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}a^3f\,x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x^9+$
risch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}a^3f\,x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x^9+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+1/4*a^3*f*x^4+3/5*a^2*b*c*x^5+1/2*a^2*b
*d*x^6+3/7*a^2*b*e*x^7+3/8*f*a^2*b*x^8+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/
11*a*b^2*e*x^11+1/4*a*b^2*f*x^12+1/13*b^3*c*x^13+1/14*b^3*d*x^14+1/15*b^3*e
*x^15+1/16*b^3*f*x^16
```

Maxima [A]
time = 0.60, size = 154, normalized size = 1.02

$$\frac{1}{16}b^3fx^{16}+\frac{1}{15}b^3x^{15}e+\frac{1}{14}b^3dx^{14}+\frac{1}{13}b^3cx^{13}+\frac{1}{4}ab^2fx^{12}+\frac{3}{11}ab^2x^{11}e+\frac{3}{10}ab^2dx^{10}+\frac{1}{3}ab^2cx^9+\frac{3}{8}a^2bfx^8+\frac{3}{7}a^2bx^7e+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{4}a^3fx^4+\frac{1}{3}a^3x^3e+\frac{1}{2}a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

Fricas [A]
time = 0.37, size = 150, normalized size = 0.99

$$\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,b^3ex^{15}+\frac{1}{14}\,b^3dx^{14}+\frac{1}{13}\,b^3cx^{13}+\frac{1}{4}\,ab^2fx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bfx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3fx^4+\frac{1}{3}\,a^3ex^3+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Sympy [A]
time = 0.02, size = 180, normalized size = 1.19

$$a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{a^3fx^4}{4}+\frac{3a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{3a^2bfx^8}{8}+\frac{ab^2cx^9}{3}+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{ab^2fx^{12}}{4}+\frac{b^3cx^{13}}{13}+\frac{b^3dx^{14}}{14}+\frac{b^3ex^{15}}{15}+\frac{b^3fx^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5/
5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x**9
/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3*c*
x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16
```

Giac [A]
time = 0.53, size = 154, normalized size = 1.02

$$\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,b^3x^{15}e+\frac{1}{14}\,b^3dx^{14}+\frac{1}{13}\,b^3cx^{13}+\frac{1}{4}\,ab^2fx^{12}+\frac{3}{11}\,ab^2x^{11}e+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bfx^8+\frac{3}{7}\,a^2bx^7e+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3fx^4+\frac{1}{3}\,a^3x^3e+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B]
time = 4.86, size = 150, normalized size = 0.99

$$\frac{f\,a^3\,x^4}{4}+\frac{e\,a^3\,x^3}{3}+\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{3\,f\,a^2\,b\,x^8}{8}+\frac{3\,e\,a^2\,b\,x^7}{7}+\frac{d\,a^2\,b\,x^6}{2}+\frac{3\,c\,a^2\,b\,x^5}{5}+\frac{f\,a\,b^2\,x^{12}}{4}+\frac{3\,e\,a\,b^2\,x^{11}}{11}+\frac{3\,d\,a\,b^2\,x^{10}}{10}+\frac{c\,a\,b^2\,x^9}{3}+\frac{f\,b^3\,x^{16}}{16}+\frac{e\,b^3\,x^{15}}{15}+\frac{d\,b^3\,x^{14}}{14}+\frac{c\,b^3\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^3*d*x^2)/2 + (b^3*c*x^13)/13 + (a^3*e*x^3)/3 + (b^3*d*x^14)/14 + (a^3*f*
x^4)/4 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16 + a^3*c*x + (3*a^2*b*c*x^5)/5 +
(a*b^2*c*x^9)/3 + (a^2*b*d*x^6)/2 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^7)/7
 + (3*a*b^2*e*x^11)/11 + (3*a^2*b*f*x^8)/8 + (a*b^2*f*x^12)/4
```

3.149 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2} dx$

Optimal. Leaf size=155

$$\frac{af+bx(c+dx+ex^2)}{4ab(a-bx^4)}+\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{4a^{3/2}\sqrt{b}}$$

```
[Out] 1/4*(a*f+b*x*(e*x^2+d*x+c))/a/b/(-b*x^4+a)+1/4*d*arctanh(x^2*b^(1/2)/a^(1/2
))/a^(3/2)/b^(1/2)+1/8*arctan(b^(1/4)*x/a^(1/4))*(-e*a^(1/2)+3*c*b^(1/2))/a
^(7/4)/b^(3/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^(7/
4)/b^(3/4)
```

Rubi [A]
time = 0.08, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$,
Rules used = {1868, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{af+bx(c+dx+ex^2)}{4ab(a-bx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

```
[Out] (a*f + b*x*(c + d*x + e*x^2))/(4*a*b*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]
*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt
[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^2}\,dx &= \frac{af+bx(c+dx+ex^2)}{4ab\left(a-bx^4\right)}-\frac{\int \frac{-3c-2dx-ex^2}{a-bx^4}\,dx}{4a}\\
&= \frac{af+bx(c+dx+ex^2)}{4ab\left(a-bx^4\right)}-\frac{\int \left(-\frac{2dx}{a-bx^4}+\frac{-3c-ex^2}{a-bx^4}\right)dx}{4a}\\
&= \frac{af+bx(c+dx+ex^2)}{4ab\left(a-bx^4\right)}-\frac{\int \frac{-3c-ex^2}{a-bx^4}\,dx}{4a}+\frac{d\int \frac{x}{a-bx^4}\,dx}{2a}\\
&= \frac{af+bx(c+dx+ex^2)}{4ab\left(a-bx^4\right)}+\frac{d\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{4a}-\frac{\left(\frac{3\sqrt{b}\,c}{\sqrt{a}}-e\right)\int \frac{1}{-\sqrt{a}\,\sqrt{\ }}}{8a}\\
&= \frac{af+bx(c+dx+ex^2)}{4ab\left(a-bx^4\right)}+\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}\,c+\sqrt{a}\,e\right.}{8a^{7/}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 220, normalized size = 1.42

$$
\frac{\frac{4a(af+bx(c+x(d+ex)))}{a-bx^4}-2\sqrt[4]{a}\,\sqrt[4]{b}\left(-3\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\sqrt[4]{b}\left(3\sqrt[4]{a}\,\sqrt{b}\,c+2\sqrt{a}\,\sqrt[4]{b}\,d+a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+\sqrt[4]{b}\left(3\sqrt[4]{a}\,\sqrt{b}\,c-2\sqrt{a}\,\sqrt[4]{b}\,d+a^{3/4}e\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+2\sqrt{a}\,\sqrt{b}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{16a^2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*(a*f + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2*a^(1/4)*b^(1/4)*(-3*Sq
rt[b]*c + Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - b^(1/4)*(3*a^(1/4)*Sqrt[
b]*c + 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x] + b^(1/4)*
(3*a^(1/4)*Sqrt[b]*c - 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b^(1/
4)*x] + 2*Sqrt[a]*Sqrt[b]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^2*b)
```

Maple [A]
time = 0.33, size = 222, normalized size = 1.43

method	result
risch	$\frac{\frac{e\,x^{3}}{4a}+\frac{d\,x^{2}}{4a}+\frac{cx}{4a}+\frac{f}{4b}}{-b\,x^{4}+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{4}-a\right)}\frac{\left(\_R^{2}e+2\_Rd+3c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{16ba}$
default	$c\left(\frac{x}{4a(-b\,x^{4}+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{16a^{2}}\right)+d\left(\frac{x^{2}}{4a(-b\,x^{4}+a)}+\frac{\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)+e\left(\frac{}{4a(-}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/4*x/a/(-b*x^4+a)+3/16/a^2*(a/b)^(1/4)*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/
4)))+2*arctan(x/(a/b)^(1/4))))+d*(1/4*x^2/a/(-b*x^4+a)+1/8/a/(a*b)^(1/2)*ln
((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2))))+e*(1/4*x^3/a/(-b*x^4+a)-1/16/a/b
/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))))
+1/4*f*x^4/a/(-b*x^4+a)
```

Maxima [A]
time = 0.69, size = 203, normalized size = 1.31

$$-\frac{bx^{3}e+bdx^{2}+bcx+af}{4\left(ab^{2}x^{4}-a^{2}b\right)}+\frac{\frac{2\,d\log\left(\sqrt{b}\,x^{2}+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{2\,d\log\left(\sqrt{b}\,x^{2}-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x^3*e + b*d*x^2 + b*c*x + a*f)/(a*b^2*x^4 - a^2*b) + 1/16*(2*d*log(
sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))/(
sqrt(a)*sqrt(b)) + 2*(3*sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*sqrt(b)*c + sqrt(a
```

```
)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt
(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a
```

Fricas [C] Result contains complex when optimal does not.
time = 3.34, size = 117016, normalized size = 754.94

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] -1/9216*(2304*b*e*x^3 + 2304*b*d*x^2 + 2304*b*c*x + 2*(a*b^2*x^4 - a^2*b)*(
(-I*sqrt(3) + 1)*((a^2*b*sqrt(1/(a*b))*sqrt((6*a*b*c*e*sqrt(1/(a*b)) + 9*b*
c^2 + a*e^2)/(a^4*b^2*sqrt(1/(a*b)))) + 2*d)^2/(a^3*b) - 3*(4*a^2*b*d*sqrt(
(6*a*b*c*e*sqrt(1/(a*b)) + 9*b*c^2 + a*e^2)/(a^4*b^2*sqrt(1/(a*b)))) + 9*b*
c^2 - (2*(2*d^2 + 3*c*e)*b*sqrt(1/(a*b)) - e^2)*a)/(a^4*b^2*sqrt(1/(a*b))))
/(-1/24576*(4*a^2*b*d*sqrt((6*a*b*c*e*sqrt(1/(a*b)) + 9*b*c^2 + a*e^2)/(a^4
*b^2*sqrt(1/(a*b)))) + 9*b*c^2 - (2*(2*d^2 + 3*c*e)*b*sqrt(1/(a*b)) - e^2)*
a)*(a^2*b*sqrt(1/(a*b))*sqrt((6*a*b*c*e*sqrt(1/(a*b)) + 9*b*c^2 + a*e^2)/(a
^4*b^2*sqrt(1/(a*b)))) + 2*d)/(a^5*b^2) + 1/8192*(a^5*b^2*sqrt(1/(a*b))*((6
*a*b*c*e*sqrt(1/(a*b)) + 9*b*c^2 + a*e^2)/(a^4*b^2*sqrt(1/(a*b))))^(3/2) +
4*(d^2*sqrt((6*a*b*c*e*sqrt(1/(a*b)) + 9*b*c^2 + a*e^2)/(a^4*b^2*sqrt(1/(a*
b)))) - 3*c*e*sqrt((6*a*b*c*e*sqrt(1/(a*b)) + 9*b*c^2 + a*e^2)/(a^4*b^2*sqr
t(1/(a*b)))))*a^2*b*sqrt(1/(a*b)) - 18*b*c^2*d*sqrt(1/(a*b)) - 2*a*d*e^2*sq
rt(1/(a*b)) - 8*d^3 + 12* ...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(139) = 278$.
time = 45.64, size = 520, normalized size = 3.35

$$\operatorname{RootSum}\left(65536t^4a^7b^3 + t^2(-3072a^4b^2ce - 2048a^4b^2d^2) + t(128a^3bde^2 + 1152a^2b^2c^2d) - a^2e^4 + 18abc^2e^2 - 48abcd^2e + 16abd^4 - 81b^2c^4, \left(t \mapsto t\log\left(x + \frac{4096t^3a^7b^2e^3 + 36864t^3a^6b^3c^2e - 98304t^3a^6b^3cd^2 + 4608t^2a^5b^2cde^2 - 4096t^2a^5b^2d^3e - 13824t^2a^4b^3c^3d - 144ta^4bce^4 - 192ta^4bd^2e^3 - 1728ta^3b^2c^3e^2 + 5184ta^3b^2c^2d^2e + 1536ta^3b^2cd^4 - 3888ta^2b^3c^5 + 6a^3de^5 - 120a^2bcd^3e^2 + 64a^2bd^5e + 810ab^2c^4de - 1080ab^2c^3d^3}{a^3e^6 + 9a^2bc^2e^4 - 96a^2bcd^2e^3 + 64a^2bd^4e^2 - 81ab^2c^4e^2 + 864ab^2c^3d^2e - 576ab^2c^2d^4 - 729b^3c^6}\right)\right)\right) + \frac{-af - bcx - bdx^2 - bex^3}{-4a^2b + 4ab^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**2
*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*a*
b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _t*l
og(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 98304*_t
**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2
*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_t*a**4
*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e
+ 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 - 120*a
**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c*
*3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3 + 64*a**2*
b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**
```

```
2*d**4 - 729*b**3*c**6))) + (-a*f - b*c*x - b*d*x**2 - b*e*x**3)/(-4*a**2*
b + 4*a*b**2*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(119) = 238$.
time = 0.66, size = 320, normalized size = 2.06

$$-\frac{\sqrt{2}\left(3\,b^2c-2\,\sqrt{2}\,(-ab^3)^{\frac{1}{4}}\,bd+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a}-\frac{\sqrt{2}\left(3\,b^2c+2\,\sqrt{2}\,(-ab^3)^{\frac{1}{4}}\,bd-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a}-\frac{\sqrt{2}\left(3\,b^2c-\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a}+\frac{\sqrt{2}\left(3\,b^2c-\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a}-\frac{bx^3e+bdx^2+bcx+af}{4\,(bx^4-a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*
a) - 1/16*sqrt(2)*(3*b^2*c + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3
/4)*a) - 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)
^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b
)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) -
1/4*(b*x^3*e + b*d*x^2 + b*c*x + a*f)/((b*x^4 - a)*a*b)
```

Mupad [B]
time = 0.41, size = 483, normalized size = 3.12

$$\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(65536\,a^7\,b^3\,z^4-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4-81\,b^2\,c^4-a^2\,e^4,z,k\right)\left(\mathrm{root}\left(65536\,a^7\,b^3\,z^4-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4-81\,b^2\,c^4-a^2\,e^4,z,k\right)\left(12\,b^3\,c-8\,b^3\,d\,x\right)+\frac{x\,\left(4\,a^2\,b^2\,e^2+36\,a\,b^3\,c^2\right)}{16\,a^3}-\frac{b^2\,d\,e}{a}\right)-\frac{-9\,b^2\,c^2\,e+12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3}-\frac{x\,\left(2\,b^2\,d^3-3\,b^2\,c\,d\,e\right)}{16\,a^3}\right)\mathrm{root}\left(65536\,a^7\,b^3\,z^4-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4-81\,b^2\,c^4-a^2\,e^4,z,k\right)\right)+\frac{\frac{f}{4\,b}+\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{a-b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log(- root(65536*a^7*b^3*z^4 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d
^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b
*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k)*(root(65536*a^7*b^3*z^4
 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128
*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4
- a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 + 4*a^2*b^2*e^2)
)/(16*a^3) - (b^2*d*e)/a) - (12*b^2*c*d^2 - 9*b^2*c^2*e + a*b*e^3)/(64*a^3)
 - (x*(2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3))*root(65536*a^7*b^3*z^4 - 3072*a^
4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e
^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4,
 z, k), k, 1, 4) + (f/(4*b) + (d*x^2)/(4*a) + (e*x^3)/(4*a) + (c*x)/(4*a))/
(a - b*x^4)
```

3.150 $\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^3}\,dx$

Optimal. Leaf size=188

$$\frac{x(7c+6dx+5ex^2)}{32a^2\left(a-bx^4\right)}+\frac{af+bx(c+dx+ex^2)}{8ab\left(a-bx^4\right)^2}+\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}\,c+5\sqrt{a}\,e\right)\tan}{64a^{11/4}b^{3/4}}$$

```
[Out] 1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(-b*x^4+a)+1/8*(a*f+b*x*(e*x^2+d*x+c))/a/b/(
-b*x^4+a)^2+3/16*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)+1/64*arctan
(b^(1/4)*x/a^(1/4))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)+1/64*arcta
nh(b^(1/4)*x/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)
```

Rubi [A]
time = 0.10, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1868, 1869, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(7c+6dx+5ex^2)}{32a^2\left(a-bx^4\right)}+\frac{af+bx(c+dx+ex^2)}{8ab\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*
x^2))/(8*a*b*(a - b*x^4)^2) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)
*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh
[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx = \frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2} - \frac{\int \frac{-7c-6dx-5ex^2}{(a-bx^4)^2}\,dx}{8a}$$

$$= \frac{x(7c+6dx+5ex^2)}{32a^2\,(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2} + \frac{\int \frac{21c+12dx+5ex^2}{a-bx^4}\,dx}{32a^2}$$

$$= \frac{x(7c+6dx+5ex^2)}{32a^2\,(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2} + \frac{\int \left(\frac{12dx}{a-bx^4} + \frac{21c+5ex^2}{a-bx^4}\right)dx}{32a^2}$$

$$= \frac{x(7c+6dx+5ex^2)}{32a^2\,(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2} + \frac{\int \frac{21c+5ex^2}{a-bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a-bx^4}\,d}{8a^2}$$

$$= \frac{x(7c+6dx+5ex^2)}{32a^2\,(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2} + \frac{(3d)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{16a^2}$$

$$= \frac{x(7c+6dx+5ex^2)}{32a^2\,(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2} + \frac{\left(\frac{21\sqrt{b}\,c}{\sqrt{a}} - 5e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{3/4}}$$

Mathematica [A]
time = 0.14, size = 253, normalized size = 1.35

$$\frac{\frac{4ax(7c+x(6d+5ex))}{a-bx^4} + \frac{16a^2(af+bx(c+x(d+ex)))}{b(a-bx^4)^2} + \frac{2\sqrt[4]{a}\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}} - \frac{\left(21\sqrt[4]{a}\sqrt{b}\,c+12\sqrt{a}\sqrt[4]{b}\,d+5a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)}{b^{3/4}} + \frac{\left(21\sqrt[4]{a}\sqrt{b}\,c-12\sqrt{a}\sqrt[4]{b}\,d+5a^{3/4}e\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{b^{3/4}} + \frac{12\sqrt{a}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt{b}}}{128a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a*x*(7*c + x*(6*d + 5*e*x)))/(a - b*x^4) + (16*a^2*(a*f + b*x*(c + x*(d
 + e*x))))/(b*(a - b*x^4)^2) + (2*a^(1/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcT
an[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*Sqrt[b]*c + 12*Sqrt[a]*b^(1
/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*Sqrt[
b]*c - 12*Sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4
) + (12*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(146) = 292$.
time = 0.36, size = 312, normalized size = 1.66

method	result

risch	$\frac{-\frac{5be\,x^{7}}{32a^{2}}-\frac{3bd\,x^{6}}{16a^{2}}-\frac{7bc\,x^{5}}{32a^{2}}+\frac{9e\,x^{3}}{32a}+\frac{5d\,x^{2}}{16a}+\frac{11cx}{32a}+\frac{f}{8b}}{\left(-b\,x^{4}+a\right)^{2}}-\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{4}-a\right)}\frac{\left(5\_R^{2}e+12\_Rd+21c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{128a^{2}b}$
default	$c\left(\frac{x}{8a\left(-b\,x^{4}+a\right)^{2}}+\frac{\frac{7x}{32a\left(-b\,x^{4}+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{128a^{2}}}{a}\right)+d\left(\frac{x^{2}}{8a\left(-b\,x^{4}+a\right)^{2}}+\frac{\frac{3x^{2}}{16a\left(-b\,x^{4}+a\right)}+\frac{3\ln}{}}{a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/8*x/a/(-b*x^4+a)^2+7/8/a*(1/4*x/a/(-b*x^4+a)+3/16/a^2*(a/b)^(1/4)*(ln(
(x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))))+d*(1/8*x^2/a/(-
b*x^4+a)^2+3/4/a*(1/4*x^2/a/(-b*x^4+a)+1/8/a/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1
/2))/(a-x^2*(a*b)^(1/2)))))+e*(1/8*x^3/a/(-b*x^4+a)^2+5/8/a*(1/4*x^3/a/(-b*
x^4+a)-1/16/a/b/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-
(a/b)^(1/4))))))+f*(1/8*x^4/a/(-b*x^4+a)^2+1/8/a^2*x^4/(-b*x^4+a))
```

Maxima [A]
time = 0.79, size = 253, normalized size = 1.35

$$-\frac{5\,b^{2}x^{7}e+6\,b^{2}dx^{6}+7\,b^{2}cx^{5}-9\,abx^{3}e-10\,abdx^{2}-11\,abcx-4\,a^{2}f}{32\,\left(a^{2}b^{3}x^{8}-2\,a^{3}b^{2}x^{4}+a^{4}b\right)}+\frac{\frac{12\,d\log\left(\sqrt{b}x^{2}+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{12\,d\log\left(\sqrt{b}x^{2}-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(21\sqrt{b}c-5\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 - 9*a*b*x^3*e - 10*a*b*d*x^2
 - 11*a*b*c*x - 4*a^2*f)/(a^2*b^3*x^8 - 2*a^3*b^2*x^4 + a^4*b) + 1/128*(12*
d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 12*d*log(sqrt(b)*x^2 - sqr
t(a))/(sqrt(a)*sqrt(b)) + 2*(21*sqrt(b)*c - 5*sqrt(a)*e)*arctan(sqrt(b)*x/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*sqrt(b)
*c + 5*sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt
(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^2
```

Fricas [C] Result contains complex when optimal does not.
time = 5.92, size = 118761, normalized size = 631.71

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] -1/589824*(92160*b^2*e*x^7 + 110592*b^2*d*x^6 + 129024*b^2*c*x^5 - 165888*a
*b*e*x^3 - 184320*a*b*d*x^2 - 202752*a*b*c*x - 73728*a^2*f + 2*(a^2*b^3*x^8
 - 2*a^3*b^2*x^4 + a^4*b)*((-I*sqrt(3) + 1)*((a^3*b*sqrt(1/(a*b))*sqrt((210
*a*b*c*e*sqrt(1/(a*b)) + 441*b*c^2 + 25*a*e^2)/(a^6*b^2*sqrt(1/(a*b)))) + 1
2*d)^2/(a^5*b) - 3*(24*a^3*b*d*sqrt((210*a*b*c*e*sqrt(1/(a*b)) + 441*b*c^2
+ 25*a*e^2)/(a^6*b^2*sqrt(1/(a*b)))) + 441*b*c^2 - (6*(24*d^2 + 35*c*e)*b*s
qrt(1/(a*b)) - 25*e^2)*a)/(a^6*b^2*sqrt(1/(a*b)))))/(-1/12582912*(24*a^3*b*d
*sqrt((210*a*b*c*e*sqrt(1/(a*b)) + 441*b*c^2 + 25*a*e^2)/(a^6*b^2*sqrt(1/(a
*b)))) + 441*b*c^2 - (6*(24*d^2 + 35*c*e)*b*sqrt(1/(a*b)) - 25*e^2)*a)*(a^3
*b*sqrt(1/(a*b))*sqrt((210*a*b*c*e*sqrt(1/(a*b)) + 441*b*c^2 + 25*a*e^2)/(a
^6*b^2*sqrt(1/(a*b)))) + 12*d)/(a^8*b^2) + 1/4194304*(a^8*b^2*sqrt(1/(a*b))
*((210*a*b*c*e*sqrt(1/(a*b)) + 441*b*c^2 + 25*a*e^2)/(a^6*b^2*sqrt(1/(a*b))
))^(3/2) + 12*(12*d^2*sqrt((210*a*b*c*e*sqrt(1/(a*b)) + 441*b*c^2 + 25*a*e^
2)/(a^6*b^2*sqrt(1/(a*b)) ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(152) = 304.
time = 0.60, size = 358, normalized size = 1.90

$$-\frac{\sqrt{2}\left(21\,b^2c-12\,\sqrt{2}\,(-ab^3)^{\frac{1}{4}}\,bd+5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\,(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2}-\frac{\sqrt{2}\left(21\,b^2c+12\,\sqrt{2}\,(-ab^3)^{\frac{1}{4}}\,bd-5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\,(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2}-\frac{\sqrt{2}\left(21\,b^2c-5\,\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2}+\frac{\sqrt{2}\left(21\,b^2c-5\,\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2}-\frac{5\,b^2x^7e+6\,b^2dx^6+7\,b^2cx^5-9\,abx^3e-10\,abdx^2-11\,abcx-4\,a^2f}{32\,(bx^4-a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^2*c - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqrt(-a*b)*b*e
)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(
3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 5*sqr
t(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/
((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2
+ sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) + 1/256*sqrt(2)
*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b
))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 - 9
*a*b*x^3*e - 10*a*b*d*x^2 - 11*a*b*c*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

Mupad [B]
time = 5.18, size = 832, normalized size = 4.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x)
```

```
[Out] symsum(log(-(b*(125*a*e^3 + 3024*b*c*d^2 - 2205*b*c^2*e + 1728*b*d^3*x + 34
4064*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^
2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^
2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z,
k)^2*a^5*b^2*c + 3200*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2
 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z
 - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 19
4481*b^2*c^4, z, k)*a^3*b*e^2*x - 2520*b*c*d*e*x + 56448*root(268435456*a^1
1*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3
*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2
 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^2*b^2*c^2*x - 1966
08*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*
d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*
e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)
^2*a^5*b^2*d*x - 15360*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^
2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*
z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 1
94481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435456*a^11*b^3*z^4 -
 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*
z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*
b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k), k, 1, 4) + (f/(8*b) + (5*d*x^2
)/(16*a) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) - (7*b*c*x^5)/(32*a^2) - (3*b
*d*x^6)/(16*a^2) - (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4)
```

3.151 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4} \, dx$

Optimal. Leaf size=220

$$\frac{x(11c+10dx+9ex^2)}{96a^2(a-bx^4)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3(a-bx^4)}+\frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3}+\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{256a^{15/4}b^{3/4}}$$

```
[Out] 1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(-b*x^4+a)^2+1/384*x*(45*e*x^2+60*d*x+77*c
)/a^3/(-b*x^4+a)+1/12*(a*f+b*x*(e*x^2+d*x+c))/a/b/(-b*x^4+a)^3+5/32*d*arcta
nh(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)+1/256*arctan(b^(1/4)*x/a^(1/4))*(-1
5*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)+1/256*arctanh(b^(1/4)*x/a^(1/4))
*(15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)
```

Rubi [A]
time = 0.13, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1868, 1869, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(77c+60dx+45ex^2)}{384a^3(a-bx^4)}+\frac{x(11c+10dx+9ex^2)}{96a^2(a-bx^4)^2}+\frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x +
45*e*x^2))/(384*a^3*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*x^2))/(12*a*b*(a
 - b*x^4)^3) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/
(256*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)
/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(3
2*a^(7/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx &= \frac{af+bx(c+dx+ex^2)}{12ab\,(a-bx^4)^3} - \frac{\int \frac{-11c-10dx-9ex^2}{(a-bx^4)^3}\,dx}{12a} \\
&= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a-bx^4)^2} + \frac{af+bx(c+dx+ex^2)}{12ab\,(a-bx^4)^3} + \frac{\int \frac{77c+60dx+45ex^2}{(a-bx^4)^2}\,dx}{96a^2} \\
&= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a-bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{12ab\,(a-bx^4)^3} - \\
&= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a-bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{12ab\,(a-bx^4)^3} - \\
&= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a-bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{12ab\,(a-bx^4)^3} - \\
&= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a-bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{12ab\,(a-bx^4)^3} + \\
&= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a-bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{12ab\,(a-bx^4)^3} +
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 286, normalized size = 1.30

$$
\frac{\frac{4ax(77c+15x(4d+3ex))}{a-bx^4}+\frac{16a^2x(11c+x(10d+9ex))}{(a-bx^4)^2}-\frac{128a^3(af+bx(c+x(d+ex)))}{b(-a+bx^4)^3}+\frac{6\sqrt[4]{a}\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{3\left(77\sqrt[4]{a}\sqrt{b}\,c+40\sqrt{a}\sqrt[4]{b}\,d+15a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)}{b^{3/4}}+\frac{3\left(77\sqrt[4]{a}\sqrt{b}\,c-40\sqrt{a}\sqrt[4]{b}\,d+15a^{3/4}e\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{b^{3/4}}+\frac{120\sqrt{a}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt{b}}}{1536a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10*
d + 9*e*x)))/(a - b*x^4)^2 - (128*a^3*(a*f + b*x*(c + x*(d + e*x))))/(b*(-a
 + b*x^4)^3) + (6*a^(1/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/
a^(1/4)])/b^(3/4) - (3*(77*a^(1/4)*Sqrt[b]*c + 40*Sqrt[a]*b^(1/4)*d + 15*a^
(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*Sqrt[b]*c - 40*
Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (120*
Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

Maple [A]
time = 0.34, size = 254, normalized size = 1.15

method	result

risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}+\frac{f}{12b}}{(-b\,x^4+a)^3}-\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^4-a)}\frac{(15\_R^2e+40\_Rd\cdots}{\_R\cdots}}{512a^3b}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}+\frac{f}{12b}}{(-b\,x^4+a)^3}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\cdots\right.}{4a}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32/a^3*d*b^2*x^10+77/384*c/a^3*b^2*x^9-21/64*b*e/a
^2*x^7-5/12/a^2*b*d*x^6-33/64*b*c/a^2*x^5+113/384/a*e*x^3+11/32*d/a*x^2+51/
128/a*c*x+1/12*f/b)/(-b*x^4+a)^3+1/128/a^3*(77/4*c*(a/b)^(1/4)/a*(ln((x+(a/
b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))+10*d/(a*b)^(1/2)*ln((a+
x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2)))-15/4*e/b/(a/b)^(1/4)*(2*arctan(x/(a/b
)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))))
```

Maxima [A]
time = 0.50, size = 302, normalized size = 1.37

$$-\frac{45\,b^3x^{11}e+60\,b^3dx^{10}+77\,b^3cx^9-126\,ab^2x^7e-160\,ab^2dx^6-198\,ab^2cx^5+113\,a^2bx^3e+132\,a^2bdx^2+153\,a^2bcx+32\,a^3f}{384\,(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2x^4-a^6b)}+\frac{\frac{40\,d\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\,\left(77\,\sqrt{b}\,c-15\,\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(77\,\sqrt{b}\,c+15\,\sqrt{a}\,e\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{512\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*b^3*c*x^9 - 126*a*b^2*x^7*e - 16
0*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 + 153*a
^2*b*c*x + 32*a^3*f)/(a^3*b^4*x^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b)
 + 1/512*(40*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d*log(sqrt
(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(77*sqrt(b)*c - 15*sqrt(a)*e)*arct
an(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
 - (77*sqrt(b)*c + 15*sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(s
qrt(b)*x + sqrt(sqrt(a)*sqrt(b)))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))
/a^3
```

Fricas [C] Result contains complex when optimal does not.
time = 10.79, size = 118945, normalized size = 540.66

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] -1/9437184*(1105920*b^3*e*x^11 + 1474560*b^3*d*x^10 + 1892352*b^3*c*x^9 - 3
096576*a*b^2*e*x^7 - 3932160*a*b^2*d*x^6 - 4866048*a*b^2*c*x^5 + 2777088*a^
2*b*e*x^3 + 3244032*a^2*b*d*x^2 + 3760128*a^2*b*c*x + 786432*a^3*f + 2*(a^3
*b^4*x^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b)*((-I*sqrt(3) + 1)*((a^4*
b*sqrt(1/(a*b))*sqrt((2310*a*b*c*e*sqrt(1/(a*b)) + 5929*b*c^2 + 225*a*e^2)/
(a^8*b^2*sqrt(1/(a*b)))) + 40*d)^2/(a^7*b) - 3*(80*a^4*b*d*sqrt((2310*a*b*c
*e*sqrt(1/(a*b)) + 5929*b*c^2 + 225*a*e^2)/(a^8*b^2*sqrt(1/(a*b)))) + 5929*
b*c^2 - 5*(2*(160*d^2 + 231*c*e)*b*sqrt(1/(a*b)) - 45*e^2)*a)/(a^8*b^2*sqrt
(1/(a*b))))/(-1/805306368*(80*a^4*b*d*sqrt((2310*a*b*c*e*sqrt(1/(a*b)) + 59
29*b*c^2 + 225*a*e^2)/(a^8*b^2*sqrt(1/(a*b)))) + 5929*b*c^2 - 5*(2*(160*d^2
 + 231*c*e)*b*sqrt(1/(a*b)) - 45*e^2)*a)*(a^4*b*sqrt(1/(a*b))*sqrt((2310*a*
b*c*e*sqrt(1/(a*b)) + 5929*b*c^2 + 225*a*e^2)/(a^8*b^2*sqrt(1/(a*b)))) + 40
*d)/(a^11*b^2) + 1/268435456*(a^11*b^2*sqrt(1/(a*b))*((2310*a*b*c*e*sqrt(1/
(a*b)) + 5929*b*c^2 + 225 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(184) = 368.
time = 0.55, size = 395, normalized size = 1.80

$$-\frac{\sqrt{2}\left(77\,b^2c-40\,\sqrt{2}\,(-ab^3)^{\frac{1}{4}}\,bd+15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\,(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}-\frac{\sqrt{2}\left(77\,b^2c+40\,\sqrt{2}\,(-ab^3)^{\frac{1}{4}}\,bd-15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\,(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}-\frac{\sqrt{2}\left(77\,b^2c-15\,\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}+\frac{\sqrt{2}\left(77\,b^2c-15\,\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}-\frac{45\,b^3x^{11}e+60\,b^3dx^{10}+77\,b^3cx^9-126\,ab^2x^7e-160\,ab^2dx^6-198\,ab^2cx^5+113\,a^2bx^3e+132\,a^2bdx^2+153\,a^2bcx+32\,a^3f}{384\,(bx^4-a)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sqrt(-a*b)*b*
e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^
(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 15*s
qrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4)
)/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(
x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) + 1/1024*sq
rt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqr
t(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*b
^3*c*x^9 - 126*a*b^2*x^7*e - 160*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 113*a^2*b*
x^3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

Mupad [B]
time = 5.25, size = 880, normalized size = 4.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x)
```

```
[Out] symsum(log(-(b*(3375*a*e^3 + 123200*b*c*d^2 - 88935*b*c^2*e + 64000*b*d^3*x
 + 20185088*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 83
8860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*
z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*
b^2*c^4 - 50625*a^2*e^4, z, k)^2*a^7*b^2*c + 115200*root(68719476736*a^15*b
^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680
*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a
*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)*a^4*
b*e^2*x - 92400*b*c*d*e*x + 3035648*root(68719476736*a^15*b^3*z^4 - 1211105
280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z
 + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 256
0000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)*a^3*b^2*c^2*x - 1048
5760*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800
*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 739
2000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4
 - 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x - 614400*root(68719476736*a^15*b^3*z^
4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*
b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)*a^4*b*d*e
))/(2097152*a^9))*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^
2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*
d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 351
53041*b^2*c^4 - 50625*a^2*e^4, z, k), k, 1, 4) + (f/(12*b) + (11*d*x^2)/(32
*a) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9)/(384*a^3) + (
5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) - (33*b*c*x^5)/(64*a^2)
- (5*b*d*x^6)/(12*a^2) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^3*x^12 - 3*a^2*b*x
^4 + 3*a*b^2*x^8)
```

3.152 $\int \frac{a}{2+3x^4}\,dx$

Optimal. Leaf size=101

$$-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}-\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}$$

```
[Out] 1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)-1/48
*a*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/48*a*ln(6^(3/4)*x+3*x^2+6^(1/2))*
6^(3/4)
```

Rubi [A]
time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {12, 217, 1179, 642, 1176, 631, 210}

$$-\frac{a\text{ArcTan}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}+\frac{a\text{ArcTan}\left(\sqrt[4]{6}\,x+1\right)}{4\sqrt[4]{6}}-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Int[a/(2 + 3*x^4),x]
```

```
[Out] -1/4*(a*ArcTan[1 - 6^(1/4)*x])/6^(1/4) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/
4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(8*6^(1/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a}{2+3x^4}\,dx &= a\int \frac{1}{2+3x^4}\,dx\\
&= \frac{a\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}\,dx}{2\sqrt{2}}+\frac{a\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}\,dx}{2\sqrt{2}}\\
&= \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}}+\frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}}-\frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}}-\frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-}}{8\sqrt[4]{6}}\\
&= -\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt[4]{6}\right.}{4\sqrt[4]{6}}\\
&= -\frac{a\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}-\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}+\right.}{8}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 78, normalized size = 0.77

$$\frac{a\left(-2\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)+2\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)-\log\left(2-2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)+\log\left(2+2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)\right)}{8\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a/(2 + 3*x^4),x]
```

```
[Out] (a*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] - Log[2 - 2*6^(1/4)*
x + Sqrt[6]*x^2] + Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(1/4))
```

Maple [A]
time = 0.32, size = 94, normalized size = 0.93

method	result
risch	$\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}$
meijerg	$\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{3}}{2}\right.}{2\left(x^4\right)^{\frac{1}{4}}}}{96}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*(ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6
^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1/
2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))
```

Maxima [A]
time = 0.51, size = 123, normalized size = 1.22

$$\frac{1}{48}\left(2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+3^{\frac{3}{4}}2^{\frac{3}{4}}\log\left(\sqrt{3}\,x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)-3^{\frac{3}{4}}2^{\frac{3}{4}}\log\left(\sqrt{3}\,x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/48*(2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1
```

```
/4)*2^(3/4))) + 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(
2)) - 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(72) = 144$.
time = 0.39, size = 279, normalized size = 2.76

$$-\frac{1}{48}\cdot 24^{\frac{3}{4}}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}\arctan\left(-\frac{12\,a^{4}+6\cdot 24^{\frac{1}{4}}\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}ax-24^{\frac{1}{4}}\sqrt{3}\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}\sqrt{12\,a^{2}x^{2}+24^{\frac{3}{4}}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}ax+4\sqrt{6}\sqrt{a^{4}}}}{12\,a^{4}}\right)-\frac{1}{48}\cdot 24^{\frac{3}{4}}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}\arctan\left(\frac{12\,a^{4}-6\cdot 24^{\frac{1}{4}}\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}ax+24^{\frac{1}{4}}\sqrt{3}\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}\sqrt{12\,a^{2}x^{2}-24^{\frac{3}{4}}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}ax+4\sqrt{6}\sqrt{a^{4}}}}{12\,a^{4}}\right)+\frac{1}{192}\cdot 24^{\frac{3}{4}}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}\log\left(144\,a^{2}x^{2}+12\cdot 24^{\frac{3}{4}}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}ax+48\sqrt{6}\sqrt{a^{4}}\right)-\frac{1}{192}\cdot 24^{\frac{3}{4}}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}\log\left(144\,a^{2}x^{2}-12\cdot 24^{\frac{3}{4}}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}ax+48\sqrt{6}\sqrt{a^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/48*24^(3/4)*sqrt(2)*(a^4)^(1/4)*arctan(-1/12*(12*a^4 + 6*24^(1/4)*sqrt(2
)*(a^4)^(3/4)*a*x - 24^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(3/4)*sqrt(12*a^2*x^2 +
24^(3/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(6)*sqrt(a^4)))/a^4) - 1/48*24^(3/
4)*sqrt(2)*(a^4)^(1/4)*arctan(1/12*(12*a^4 - 6*24^(1/4)*sqrt(2)*(a^4)^(3/4)
*a*x + 24^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(3/4)*sqrt(12*a^2*x^2 - 24^(3/4)*sqrt
(2)*(a^4)^(1/4)*a*x + 4*sqrt(6)*sqrt(a^4)))/a^4) + 1/192*24^(3/4)*sqrt(2)*(
a^4)^(1/4)*log(144*a^2*x^2 + 12*24^(3/4)*sqrt(2)*(a^4)^(1/4)*a*x + 48*sqrt(
6)*sqrt(a^4)) - 1/192*24^(3/4)*sqrt(2)*(a^4)^(1/4)*log(144*a^2*x^2 - 12*24^
(3/4)*sqrt(2)*(a^4)^(1/4)*a*x + 48*sqrt(6)*sqrt(a^4))
```

Sympy [A]
time = 0.21, size = 88, normalized size = 0.87

$$a\left(-\frac{6^{\frac{3}{4}}\log\left(x^{2}-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{6^{\frac{3}{4}}\log\left(x^{2}+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}\,x-1\right)}{24}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}\,x+1\right)}{24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a/(3*x**4+2),x)
```

```
[Out] a*(-6**(3/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*log(x**2 +
6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*atan(6**(1/4)*x - 1)/24 + 6**(3/4)*
atan(6**(1/4)*x + 1)/24)
```

Giac [A]
time = 0.71, size = 97, normalized size = 0.96

$$\frac{1}{48}\left(2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+6^{\frac{3}{4}}\log\left(x^{2}+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-6^{\frac{3}{4}}\log\left(x^{2}-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/48*(2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) +
```

```
6^(3/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 6^(3/4)*log(x^2 - sq
rt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*a
```

Mupad [B]
time = 0.12, size = 36, normalized size = 0.36

$$-\frac{(-1)^{1/4}\,6144^{3/4}\,a\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\,6144^{1/4}\,x}{8}\right)\,1\mathrm{i}+\operatorname{atanh}\left(\frac{(-1)^{1/4}\,6144^{1/4}\,x}{8}\right)\,1\mathrm{i}\right)}{3072}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a/(3*x^4 + 2),x)
```

```
[Out] -((-1)^(1/4)*6144^(3/4)*a*(atan(((-1)^(1/4)*6144^(1/4)*x)/8)*1i + atanh(((-
1)^(1/4)*6144^(1/4)*x)/8)*1i))/3072
```

3.153 $\int \frac{bx}{2+3x^4} \, dx$

Optimal. Leaf size=22

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}$$

[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {12, 281, 209}

$$\frac{b\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(b*x)/(2 + 3*x^4),x]

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{bx}{2+3x^4}\,dx = b\int \frac{x}{2+3x^4}\,dx$$
$$= \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right)$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)/(2 + 3*x^4),x]
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

Maple [A]
time = 0.35, size = 16, normalized size = 0.73

method	result	size
default	$\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$	16
risch	$\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$	16
meijerg	$\frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,x^2}{2}\right)}{12}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x/(3*x^4+2),x,method=_RETURNVERBOSE)
[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Maxima [A]
time = 0.51, size = 15, normalized size = 0.68

$$\frac{1}{12}\,\sqrt{6}\,b\arctan\left(\frac{1}{2}\,\sqrt{6}\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x/(3*x^4+2),x, algorithm="maxima")
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

Fricas [A]
time = 0.36, size = 15, normalized size = 0.68

$$\frac{1}{12}\sqrt{6}\,b\arctan\left(\frac{1}{2}\sqrt{6}\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x/(3*x^4+2),x, algorithm="fricas")
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

Sympy [A]
time = 0.03, size = 19, normalized size = 0.86

$$\frac{\sqrt{6}\,b\,\mathrm{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x/(3*x**4+2),x)
[Out] sqrt(6)*b*atan(sqrt(6)*x**2/2)/12
```

Giac [A]
time = 0.64, size = 15, normalized size = 0.68

$$\frac{1}{12}\sqrt{6}\,b\arctan\left(\frac{1}{2}\sqrt{6}\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x/(3*x^4+2),x, algorithm="giac")
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

Mupad [B]
time = 4.77, size = 15, normalized size = 0.68

$$\frac{\sqrt{6}\,b\,\mathrm{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)/(3*x^4 + 2),x)
[Out] (6^(1/2)*b*atan((6^(1/2)*x^2)/2))/12
```

3.154 $\int \frac{a+bx}{2+3x^4} \, dx$

Optimal. Leaf size=123

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}-\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}\right.}{8}$$

```
[Out] 1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)-1/48
*a*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/48*a*ln(6^(3/4)*x+3*x^2+6^(1/2))*
6^(3/4)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1890, 217, 1179, 642, 1176, 631, 210, 281, 209}

$$-\frac{a\text{ArcTan}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}+\frac{a\text{ArcTan}\left(\sqrt[4]{6}\,x+1\right)}{4\sqrt[4]{6}}-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{b\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4
)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{a+bx}{2+3x^4}\,dx = \int \left(\frac{a}{2+3x^4}+\frac{bx}{2+3x^4}\right)dx$$
$$= a\int \frac{1}{2+3x^4}\,dx + b\int \frac{x}{2+3x^4}\,dx$$
$$= \frac{a\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}dx}{2\sqrt{2}} + \frac{a\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}dx}{2\sqrt{2}} + \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right)$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}dx}{4\sqrt{6}} + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}dx}{4\sqrt{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}}{8\sqrt[4]{6}}$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\text{Subst}\left(\right.}{}$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+\right.}{8\sqrt[4]{6}}$$

Mathematica [A]
time = 0.03, size = 107, normalized size = 0.87

$$\frac{-2\left(\sqrt[4]{6}a+2b\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt[4]{6}a-2b\right)\tan^{-1}\left(1+\sqrt[4]{6}x\right)+\sqrt[4]{6}a\left(-\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)+\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)\right)}{8\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(2 + 3*x^4),x]
```

```
[Out] (-2*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*(6^(1/4)*a - 2*b)*ArcTan[1
+ 6^(1/4)*x] + 6^(1/4)*a*(-Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + Log[2 + 2*6
^(1/4)*x + Sqrt[6]*x^2]))/(8*Sqrt[6])
```

Maple [A]
time = 0.39, size = 110, normalized size = 0.89

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R b+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$

default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+ b\arctan$
meijerg	$\frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{2}\sqrt{3}\,x^2}{2}\right)}{12}+\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+ x\sqrt{2}\right.}{96}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*(ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6
^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1/
2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))+1/12*b*a
rctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Maxima [A]
time = 0.49, size = 147, normalized size = 1.20

$$\frac{1}{48}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\log\left(\sqrt{3}\,x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)-\frac{1}{48}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\log\left(\sqrt{3}\,x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a-2\sqrt{2}\,b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\sqrt{2}\,b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/48*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) - 1/4
8*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*s
qrt(3)*(3^(1/4)*2^(3/4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt
(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)*b)*
arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.23, size = 12348, normalized size = 100.39

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/576*(216*(1/18)^(2/3)*(2*(2*sqrt(6)*sqrt(1/48*I*sqrt(6)*a^2) + I*b)^2 - s
qrt(6)*(sqrt(6)*b^2 + 3*I*a^2 + 24*I*sqrt(1/48*I*sqrt(6)*a^2)*b))*(-I*sqrt(
3) + 1)/(864*sqrt(6)*(2*sqrt(6)*sqrt(1/48*I*sqrt(6)*a^2) + I*b)^3 - 3888*(s
qrt(6)*b^2 + 3*I*a^2 + 24*I*sqrt(1/48*I*sqrt(6)*a^2)*b)*(2*sqrt(6)*sqrt(1/4
8*I*sqrt(6)*a^2) + I*b) + 5832*sqrt(6)*(sqrt(6)*a^2*b - 4*sqrt(6)*sqrt(1/48
*I*sqrt(6)*a^2)*b^2 + 2*I*b^3 + 96*sqrt(6)*(1/48*I*sqrt(6)*a^2)^(3/2)) + sq
```

```
rt(-362797056*I*sqrt(6)*a^6 + 967458816*I*sqrt(6)*a^2*b^4 + 5804752896*a^4*
b^2 - 5804752896*(3*sqrt(1/48*I*sqrt(6)*a^2)*a^2 - 8*I*sqrt(6)*(1/48*I*sqrt
(6)*a^2)^(3/2))*b^3 - 725594112*(5*I*sqrt(6)*sqrt(1/48*I*sqrt(6)*a^2)*a^4 -
 480*(1/48*I*sqrt(6)*a^2)^(3/2)*a^2 + 1152*I*sqrt(6)*(1/48*I*sqrt(6)*a^2)^(
5/2))*b))^(1/3) + (1/18)^(1/3)*(864*sqrt(6)*(2*sqrt(6)*sqrt(1/48*I*sqrt(6)*
a^2) + I*b)^3 - 3888*(sqrt(6)*b^2 + 3*I*a^2 + 24*I*sqrt(1/48*I*sqrt(6)*a^2)
*b)*(2*sqrt(6)*sqrt(1/48*I*sqrt(6)*a^2) + I*b) + 5832*sqrt(6)*(sqrt(6)*a^2*
b - 4*sqrt(6)*sqrt(1/48*I ...
```

Sympy [A]

time = 0.30, size = 88, normalized size = 0.72

$$\operatorname{RootSum}\left(18432t^4 + 384t^2b^2 - 96ta^2b + 3a^4 + 2b^4, \left(t \mapsto t\log\left(x + \frac{3072t^3b^2 + 192t^2a^2b + 24ta^4 + 32tb^4 - 10a^2b^3}{3a^5 - 8ab^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(18432*_t**4 + 384*_t**2*b**2 - 96*_t*a**2*b + 3*a**4 + 2*b**4, Lamb
da(_t, _t*log(x + (3072*_t**3*b**2 + 192*_t**2*a**2*b + 24*_t*a**4 + 32*_t*
b**4 - 10*a**2*b**3)/(3*a**5 - 8*a*b**4))))
```

Giac [A]

time = 0.58, size = 115, normalized size = 0.93

$$\frac{1}{48}\cdot 6^{\frac{3}{4}}a\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) - \frac{1}{48}\cdot 6^{\frac{3}{4}}a\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a - 2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/48*6^(3/4)*a*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*6^(3/4)*
a*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(3/4)*a - 2*sqrt(6
)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^
(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^
(1/4)))
```

Mupad [B]

time = 0.20, size = 119, normalized size = 0.97

$$\frac{2^{3/4}\,3^{3/4}\,a\,\ln\left(x^2 + \frac{6^{3/4}\,x}{3} + \frac{\sqrt{6}}{3}\right)}{48} - \frac{2^{3/4}\,3^{3/4}\,a\,\ln\left(x^2 - \frac{6^{3/4}\,x}{3} + \frac{\sqrt{6}}{3}\right)}{48} + \frac{2^{3/4}\,3^{3/4}\,a\,\mathrm{atan}\left(6^{1/4}\,x - 1\right)}{24} + \frac{2^{3/4}\,3^{3/4}\,a\,\mathrm{atan}\left(6^{1/4}\,x + 1\right)}{24} + \frac{\sqrt{2}\,\sqrt{3}\,b\,\mathrm{atan}\left(6^{1/4}\,x - 1\right)}{12} - \frac{\sqrt{2}\,\sqrt{3}\,b\,\mathrm{atan}\left(6^{1/4}\,x + 1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(3*x^4 + 2),x)
```

```
[Out] (2^(3/4)*3^(3/4)*a*log((6^(3/4)*x)/3 + 6^(1/2)/3 + x^2))/48 - (2^(3/4)*3^(3
/4)*a*log(6^(1/2)/3 - (6^(3/4)*x)/3 + x^2))/48 + (2^(3/4)*3^(3/4)*a*atan(6^
(1/4)*x - 1))/24 + (2^(3/4)*3^(3/4)*a*atan(6^(1/4)*x + 1))/24 + (2^(1/2)*3^
(1/2)*b*atan(6^(1/4)*x - 1))/12 - (2^(1/2)*3^(1/2)*b*atan(6^(1/4)*x + 1))/1
2
```

3.155 $\int \frac{cx^2}{2+3x^4} dx$

Optimal. Leaf size=101

$$-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{2\,6^{3/4}}+\frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}}-\frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\,6^{3/4}}$$

```
[Out] 1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)+1/24
*c*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/24*c*ln(6^(3/4)*x+3*x^2+6^(1/2))*
6^(1/4)
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {12, 303, 1176, 631, 210, 1179, 642}

$$-\frac{c\text{ArcTan}\left(1-\sqrt[4]{6}\,x\right)}{2\,6^{3/4}}+\frac{c\text{ArcTan}\left(\sqrt[4]{6}\,x+1\right)}{2\,6^{3/4}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)/(2 + 3*x^4),x]
```

```
[Out] -1/2*(c*ArcTan[1 - 6^(1/4)*x])/6^(3/4) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/
4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(4*6^(3/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{cx^2}{2+3x^4}\,dx &= c\int \frac{x^2}{2+3x^4}\,dx \\
&= -\frac{c\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{c\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}\,dx}{2\sqrt{3}} \\
&= \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\ 6^{3/4}} + \frac{c\int
\\
&= \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} + \frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt[4]{6}
\\
&= -\frac{c\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\ 6^{3/4}} + \frac{c\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{2\ 6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 78, normalized size = 0.77

$$\frac{c\left(-2\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)+2\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)+\log\left(2-2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)-\log\left(2+2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)\right)}{4\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (c*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] + Log[2 - 2*6^(1/4)*
x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(4*6^(3/4))
```

Maple [A]
time = 0.34, size = 94, normalized size = 0.93

method	result
risch	$\frac{c\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{12}$
default	$\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{144}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x}}{2}\right.}{2\left(x^4\right)^{\frac{3}{4}}}\right.}{216}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^2/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*(ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*
6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1
/2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))
```

Maxima [A]
time = 0.51, size = 123, normalized size = 1.22

$$\frac{1}{24}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-3^{\frac{1}{4}}2^{\frac{1}{4}}\log\left(\sqrt{3}\,x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+3^{\frac{1}{4}}2^{\frac{1}{4}}\log\left(\sqrt{3}\,x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*(2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1
```

```
/4)*2^(3/4))) - 3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(
2)) + 3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(72) = 144$.
time = 0.38, size = 299, normalized size = 2.96

$$-\frac{1}{108}\cdot 54^{\frac{3}{4}}\sqrt{2}\left(c^{4}\right)^{\frac{1}{4}}\arctan\left(-\frac{3\cdot 54^{\frac{3}{4}}\sqrt{2}\left(c^{4}\right)^{\frac{1}{4}}c^{3}x+54\,c^{4}-54^{\frac{3}{4}}\sqrt{2}\sqrt{9\,c^{6}x^{2}+3\cdot 54^{\frac{1}{4}}\sqrt{2}\left(c^{4}\right)^{\frac{3}{4}}c^{3}x+3\sqrt{6}\sqrt{c^{4}}\,c^{4}}\left(c^{4}\right)^{\frac{1}{4}}}{54\,c^{4}}\right)-\frac{1}{108}\cdot 54^{\frac{3}{4}}\sqrt{2}\left(c^{4}\right)^{\frac{1}{4}}\arctan\left(-\frac{3\cdot 54^{\frac{3}{4}}\sqrt{2}\left(c^{4}\right)^{\frac{1}{4}}c^{3}x-54\,c^{4}-54^{\frac{3}{4}}\sqrt{2}\sqrt{9\,c^{6}x^{2}-3\cdot 54^{\frac{1}{4}}\sqrt{2}\left(c^{4}\right)^{\frac{3}{4}}c^{3}x+3\sqrt{6}\sqrt{c^{4}}\,c^{4}}\left(c^{4}\right)^{\frac{1}{4}}}{54\,c^{4}}\right)-\frac{1}{432}\cdot 54^{\frac{3}{4}}\sqrt{2}\left(c^{4}\right)^{\frac{1}{4}}\log\left(9\,c^{6}x^{2}+3\cdot 54^{\frac{1}{4}}\sqrt{2}\left(c^{4}\right)^{\frac{3}{4}}c^{3}x+3\sqrt{6}\sqrt{c^{4}}\,c^{4}\right)+\frac{1}{432}\cdot 54^{\frac{3}{4}}\sqrt{2}\left(c^{4}\right)^{\frac{1}{4}}\log\left(9\,c^{6}x^{2}-3\cdot 54^{\frac{1}{4}}\sqrt{2}\left(c^{4}\right)^{\frac{3}{4}}c^{3}x+3\sqrt{6}\sqrt{c^{4}}\,c^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/108*54^(3/4)*sqrt(2)*(c^4)^(1/4)*arctan(-1/54*(3*54^(3/4)*sqrt(2)*(c^4)^
(1/4)*c^3*x + 54*c^4 - 54^(3/4)*sqrt(2)*sqrt(9*c^6*x^2 + 3*54^(1/4)*sqrt(2)
*(c^4)^(3/4)*c^3*x + 3*sqrt(6)*sqrt(c^4)*c^4)*(c^4)^(1/4))/c^4) - 1/108*54^
(3/4)*sqrt(2)*(c^4)^(1/4)*arctan(-1/54*(3*54^(3/4)*sqrt(2)*(c^4)^(1/4)*c^3*
x - 54*c^4 - 54^(3/4)*sqrt(2)*sqrt(9*c^6*x^2 - 3*54^(1/4)*sqrt(2)*(c^4)^(3/
4)*c^3*x + 3*sqrt(6)*sqrt(c^4)*c^4)*(c^4)^(1/4))/c^4) - 1/432*54^(3/4)*sqrt
(2)*(c^4)^(1/4)*log(9*c^6*x^2 + 3*54^(1/4)*sqrt(2)*(c^4)^(3/4)*c^3*x + 3*sq
rt(6)*sqrt(c^4)*c^4) + 1/432*54^(3/4)*sqrt(2)*(c^4)^(1/4)*log(9*c^6*x^2 - 3
*54^(1/4)*sqrt(2)*(c^4)^(3/4)*c^3*x + 3*sqrt(6)*sqrt(c^4)*c^4)
```

Sympy [A]
time = 0.17, size = 88, normalized size = 0.87

$$c\left(\frac{\sqrt[4]{6}\log\left(x^{2}-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}-\frac{\sqrt[4]{6}\log\left(x^{2}+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}+\frac{\sqrt[4]{6}\operatorname{atan}\left(\sqrt[4]{6}\,x-1\right)}{12}+\frac{\sqrt[4]{6}\operatorname{atan}\left(\sqrt[4]{6}\,x+1\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**2/(3*x**4+2),x)
```

```
[Out] c*(6**(1/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/24 - 6**(1/4)*log(x**2 + 6
**(3/4)*x/3 + sqrt(6)/3)/24 + 6**(1/4)*atan(6**(1/4)*x - 1)/12 + 6**(1/4)*a
tan(6**(1/4)*x + 1)/12)
```

Giac [A]
time = 0.66, size = 97, normalized size = 0.96

$$\frac{1}{24}\left(2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)-6^{\frac{1}{4}}\log\left(x^{2}+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)+6^{\frac{1}{4}}\log\left(x^{2}-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) -
```

```
6^(1/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 6^(1/4)*log(x^2 - sq
rt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*c
```

Mupad [B]
time = 4.97, size = 32, normalized size = 0.32

$$\frac{(-1)^{1/4}\,24^{1/4}\,c\left(\mathrm{atan}\left(\frac{(-1)^{1/4}\,24^{1/4}\,x}{2}\right)-\mathrm{atanh}\left(\frac{(-1)^{1/4}\,24^{1/4}\,x}{2}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)/(3*x^4 + 2),x)
```

```
[Out] ((-1)^(1/4)*24^(1/4)*c*(atan(((-1)^(1/4)*24^(1/4)*x)/2) - atanh(((-1)^(1/4)
*24^(1/4)*x)/2)))/12
```

3.156 $\int \frac{a+cx^2}{2+3x^4} dx$

Optimal. Leaf size=141

$$-\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\;6^{3/4}}+\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\;6^{3/4}}-\frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}\,-6^{3/4}x+3x\right.}{8\;6^{3/4}}$$

```
[Out] -1/48*ln(-6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/4)+1/48*ln(6^(3/4)
*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(-1+6^(1/4)*x)*(2*c+a
*6^(1/2))*6^(1/4)+1/24*arctan(1+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)
```

Rubi [A]
time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\sqrt{6}\,a+2c\right)\text{ArcTan}\left(1-\sqrt[4]{6}\,x\right)}{4\;6^{3/4}}+\frac{\left(\sqrt{6}\,a+2c\right)\text{ArcTan}\left(\sqrt[4]{6}\,x+1\right)}{4\;6^{3/4}}-\frac{\left(\sqrt{6}\,a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\;6^{3/4}}+\frac{\left(\sqrt{6}\,a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\;6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] -1/4*((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/6^(3/4) + ((Sqrt[6]*a + 2*c)
*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3
/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x +
3*x^2])/(8*6^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rubi steps

$$\int \frac{a+cx^2}{2+3x^4}\,dx = \frac{1}{12}\left(\sqrt{6}\,a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx + \frac{1}{12}\left(\sqrt{6}\,a+2c\right)\int \frac{\sqrt{6}+3x^2}{2+3x^4}\,dx$$

$$= -\frac{\left(\sqrt{6}\,a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}} - \frac{\left(\sqrt{6}\,a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}} + \frac{1}{24}\left(\sqrt{6}\,a+2c\right.$$

$$= -\frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}\right.}{}$$

$$= -\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\ 6^{3/4}} - \frac{\left(\sqrt{6}\,a-2c\right)\log\left(\right.}{8}$$

Mathematica [A]
time = 0.03, size = 113, normalized size = 0.80

$$\frac{-2\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)+2\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)-\left(\sqrt{6}\,a-2c\right)\left(\log\left(2-2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)-\log\left(2+2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)\right)}{8\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a + 2*c)*ArcTan[1
+ 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - Log[
2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))
```

Maple [A]
time = 0.34, size = 188, normalized size = 1.33

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^{4}+2\right)}\frac{\left(\_R^{2}c+a\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{c\sqrt{3}\cdots}{}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^{3}\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\cdots}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}\cdots\right)}{216}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*(ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6
^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1/
2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))+1/144*c*
3^(1/2)*6^(3/4)*2^(1/2)*(ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))
/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1/2)*3^(1
/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))
```

Maxima [A]
time = 0.51, size = 167, normalized size = 1.18

$$\frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\,a+\sqrt{2}\,c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\,a+\sqrt{2}\,c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\,a-\sqrt{2}\,c\right)\log\left(\sqrt{3}\,x^{2}+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)-\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\,a-\sqrt{2}\,c\right)\log\left(\sqrt{3}\,x^{2}-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*3^(1/4)*2^(3/4)*(sqrt(3)*a + sqrt(2)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*
sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*3^(1/4)*2^(3/4)*(sqrt(3)*a + sqrt(2)*c
)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/48*3^(1/4
```

```
)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqr
t(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 - 3^(1
/4)*2^(3/4)*x + sqrt(2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2556 vs. $2(104) = 208$.
time = 0.46, size = 2556, normalized size = 18.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/144*(2*sqrt(6)*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*a^4 -
12*a^2*c^2 + 4*c^4)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a
^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*arctan(1/36*(sqrt(2)*(5
4*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*(81*a^8 - 72*a^4*c^4 + 16*c^8)*x^
2 + 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*((9*a^4*c - 12*a^2*c^3 +
 4*c^5)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*x - 3*(27*a^7 - 18*a^5*c^2 - 12*
a^3*c^4 + 8*a*c^6)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72
*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 3*(27*a^6 - 18*a^4*
c^2 - 12*a^2*c^4 + 8*c^6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4))*(sqrt(6)*sqrt
(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*a - 2*sqrt(
6)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*c^3))*sqrt((9*a^4 + 12*a^2
*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^
2 + 4*c^4)) + 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*(sqrt(6)*(9*a^
5 - 4*a*c^4)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4
*c^4)*x - 2*sqrt(6)*(27*a^6*c + 18*a^4*c^3 - 12*a^2*c^5 - 8*c^7)*sqrt(9*a^4
 - 12*a^2*c^2 + 4*c^4)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4
+ 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) - 6*sqrt(6)*(81*a
^8 + 108*a^6*c^2 - 48*a^2*c^6 - 16*c^8)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*
sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4))/(729*a^12 + 972*a^10*c^2 - 324*a^8*c^4 -
864*a^6*c^6 - 144*a^4*c^8 + 192*a^2*c^10 + 64*c^12)) + 2*sqrt(6)*sqrt(2)*(5
4*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*sqrt((9
*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^
4 - 12*a^2*c^2 + 4*c^4))*arctan(1/36*(sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4
)^(3/4)*sqrt(9*(81*a^8 - 72*a^4*c^4 + 16*c^8)*x^2 - 3*sqrt(2)*(54*a^4 + 72*
a^2*c^2 + 24*c^4)^(1/4)*((9*a^4*c - 12*a^2*c^3 + 4*c^5)*sqrt(54*a^4 + 72*a^
2*c^2 + 24*c^4)*x - 3*(27*a^7 - 18*a^5*c^2 - 12*a^3*c^4 + 8*a*c^6)*x)*sqrt(
(9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*
a^4 - 12*a^2*c^2 + 4*c^4)) + 3*(27*a^6 - 18*a^4*c^2 - 12*a^2*c^4 + 8*c^6)*s
qrt(54*a^4 + 72*a^2*c^2 + 24*c^4))*(sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c
^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*a - 2*sqrt(6)*sqrt(9*a^4 - 12*a^2*c^2
+ 4*c^4)*(3*a^2*c + 2*c^3))*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^
4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 3*sqrt(2)*(54
```

```
*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*(sqrt(6)*(9*a^5 - 4*a*c^4)*sqrt(54*a^4 +
72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*x - 2*sqrt(6)*(27*a^6
*c + 18*a^4*c^3 - 12*a^2*c^5 - 8*c^7)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*x)*s
qrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)
/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 6*sqrt(6)*(81*a^8 + 108*a^6*c^2 - 48*a^2*c
^6 - 16*c^8)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4
*c^4))/(729*a^12 + 972*a^10*c^2 - 324*a^8*c^4 - 864*a^6*c^6 - 144*a^4*c^8 +
 192*a^2*c^10 + 64*c^12)) - 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*
(9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)*sqr
t((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(
9*a^4 - 12*a^2*c^2 + 4*c^4))*log(9*(81*a^8 - 72*a^4*c^4 + 16*c^8)*x^2 + 3*s
qrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*((9*a^4*c - 12*a^2*c^3 + 4*c^5)
*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*x - 3*(27*a^7 - 18*a^5*c^2 - 12*a^3*c^4
 + 8*a*c^6)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^
2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 3*(27*a^6 - 18*a^4*c^2 - 1
2*a^2*c^4 + 8*c^6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)) + 3*sqrt(2)*(54*a^4
+ 72*a^2*c^2 + 24*c^4)^(1/4)*(9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*sqrt(54*a^4 +
72*a^2*c^2 + 24*c^4)*a*c)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4
+ 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*log(9*(81*a^8 - 7
2*a^4*c^4 + 16*c^8)*x^2 - 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*((
9*a^4*c - 12*a^2*c^3 + 4*c^5)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*x - 3*(27*
a^7 - 18*a^5*c^2 - 12*a^3*c^4 + 8*a*c^6)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^
4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))
 + 3*(27*a^6 - 18*a^4*c^2 - 12*a^2*c^4 + 8*c^6)*sqrt(54*a^4 + 72*a^2*c^2 +
24*c^4)))/(9*a^4 + 12*a^2*c^2 + 4*c^4)
```

Sympy [A]
time = 0.22, size = 68, normalized size = 0.48

$$\operatorname{RootSum}\left(55296t^4 + 2304t^2ac + 9a^4 + 12a^2c^2 + 4c^4, \left(t \mapsto t\log\left(x + \frac{-4608t^3c + 72ta^3 - 144tac^2}{9a^4 - 4c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(3*x**4+2),x)
```

```
[Out] RootSum(55296*_t**4 + 2304*_t**2*a*c + 9*a**4 + 12*a**2*c**2 + 4*c**4, Lamb
da(_t, _t*log(x + (-4608*_t**3*c + 72*_t*a**3 - 144*_t*a*c**2)/(9*a**4 - 4*
c**4))))
```

Giac [A]
time = 0.58, size = 131, normalized size = 0.93

$$\frac{1}{24}\left(6^{\frac{3}{4}}a + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(
3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*log(x^2
+ sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*log(x
^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B]
time = 5.11, size = 315, normalized size = 2.23

$$-2\,\mathrm{atanh}\left(\frac{216\,a^2\,x\,\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3+18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2-12\,c^3}-\frac{144\,c^2\,x\,\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3+18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2-12\,c^3}\right)\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}+2\,\mathrm{atanh}\left(\frac{216\,a^2\,x\,\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3-18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2+12\,c^3}-\frac{144\,c^2\,x\,\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3-18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2+12\,c^3}\right)\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/(3*x^4 + 2),x)
```

```
[Out] 2*atanh((216*a^2*x*((6^(1/2)*a^2*1i)/192 - (a*c)/48 - (6^(1/2)*c^2*1i)/288)
^(1/2))/(6^(1/2)*a^3*9i - 18*a^2*c + 12*c^3 - 6^(1/2)*a*c^2*6i) - (144*c^2*
x*((6^(1/2)*a^2*1i)/192 - (a*c)/48 - (6^(1/2)*c^2*1i)/288)^(1/2))/(6^(1/2)*
a^3*9i - 18*a^2*c + 12*c^3 - 6^(1/2)*a*c^2*6i))*((6^(1/2)*a^2*1i)/192 - (a*
c)/48 - (6^(1/2)*c^2*1i)/288)^(1/2) - 2*atanh((216*a^2*x*((6^(1/2)*c^2*1i)/
288 - (6^(1/2)*a^2*1i)/192 - (a*c)/48)^(1/2))/(6^(1/2)*a^3*9i + 18*a^2*c -
12*c^3 - 6^(1/2)*a*c^2*6i) - (144*c^2*x*((6^(1/2)*c^2*1i)/288 - (6^(1/2)*a^
2*1i)/192 - (a*c)/48)^(1/2))/(6^(1/2)*a^3*9i + 18*a^2*c - 12*c^3 - 6^(1/2)*
a*c^2*6i))*((6^(1/2)*c^2*1i)/288 - (6^(1/2)*a^2*1i)/192 - (a*c)/48)^(1/2)
```

3.157 $\int \frac{bx+cx^2}{2+3x^4}\,dx$

Optimal. Leaf size=123

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{2\,6^{3/4}}+\frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}}-\frac{c\log\left(\sqrt{6}+\right.}{4\,6}$$

[Out] 1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)+1/24*c*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/24*c*ln(6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1607, 1845, 281, 209, 303, 1176, 631, 210, 1179, 642}

$$\frac{b\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}-\frac{c\text{ArcTan}\left(1-\sqrt[4]{6}\,x\right)}{2\,6^{3/4}}+\frac{c\text{ArcTan}\left(\sqrt[4]{6}\,x+1\right)}{2\,6^{3/4}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)/(2 + 3*x^4),x]

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4))

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
```

```
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = \int \frac{x(b+cx)}{2+3x^4}\,dx$$
$$= \int \left(\frac{bx}{2+3x^4}+\frac{cx^2}{2+3x^4}\right)dx$$
$$= b\int \frac{x}{2+3x^4}\,dx + c\int \frac{x^2}{2+3x^4}\,dx$$
$$= \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right) - \frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}}$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{c\int -}{}$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\,6^{3/4}} + \frac{c\text{Subst}(}{}$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}} + \frac{c\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2\,6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+\right.}{4\,6^{3/4}}$$

Mathematica [A]
time = 0.03, size = 99, normalized size = 0.80

$$\frac{-2\left(\sqrt[4]{6}b+c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\left(-\sqrt[4]{6}b+c\right)\tan^{-1}\left(1+\sqrt[4]{6}x\right)+c\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)-c\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)}{4\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(2 + 3*x^4),x]
[Out] (-2*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*(-(6^(1/4)*b) + c)*ArcTan[1 +
 6^(1/4)*x] + c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - c*Log[2 + 2*6^(1/4)*x
+ Sqrt[6]*x^2])/(4*6^(3/4))
```

Maple [A]
time = 0.33, size = 110, normalized size = 0.89

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^{4}+2\right)}\frac{\left(\_R^{2}c+\_Rb\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{12}$
default	$\frac{b\arctan\left(\frac{x^{2}\sqrt{6}}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\right.}{144}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^{3}\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x\right.}{2}}{2\left(x^{4}\right)^{\frac{3}{4}}}\right.}{216}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*(ln(
(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*
2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/
6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))
```

Maxima [A]
time = 0.51, size = 147, normalized size = 1.20

$$\frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}c-2\sqrt{3}\,b\right)\arctan\left(\frac{1}{6}\cdot3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}c+2\sqrt{3}\,b\right)\arctan\left(\frac{1}{6}\cdot3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-\frac{1}{24}\cdot3^{\frac{1}{4}}2^{\frac{1}{4}}c\log\left(\sqrt{3}\,x^{2}+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{24}\cdot3^{\frac{1}{4}}2^{\frac{1}{4}}c\log\left(\sqrt{3}\,x^{2}-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(2)*(3^(1/4)*2^(3/4)*c - 2*sqrt(3)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(
2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(2)*(3^(1/4)*2^(3/4)*c + 2*sqrt(
3)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) - 1/24*3^
(1/4)*2^(1/4)*c*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*3^(1/
4)*2^(1/4)*c*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.25, size = 12741, normalized size = 103.59

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/34828517376*((10077696*I*sqrt(6)*c^2 + (1296*I*sqrt(6)*b + 15552*sqrt(-1/
72*I*sqrt(6)*c^2))^2 - 120932352*I*sqrt(6)*sqrt(-1/72*I*sqrt(6)*c^2)*b - 30
233088*b^2)*(-I*sqrt(3) + 1)/(1/4608*I*sqrt(6)*b^3 + 1/812479653347328*(129
6*I*sqrt(6)*b + 15552*sqrt(-1/72*I*sqrt(6)*c^2))^3 - 1/2304*b*c^2 - 1/384*s
qrt(-1/72*I*sqrt(6)*c^2)*b^2 - 1/41472*(I*sqrt(6)*c^2 - 12*I*sqrt(6)*sqrt(-
1/72*I*sqrt(6)*c^2)*b - 3*b^2)*(-I*sqrt(6)*b - 12*sqrt(-1/72*I*sqrt(6)*c^2)
) + 1/16*(-1/72*I*sqrt(6)*c^2)^(3/2) + 1/11609505792*sqrt(10970982973440*I*
sqrt(6)*c^6 + 45137758519296*b^2*c^4 + 2166612408926208*I*sqrt(6)*(-1/72*I*
sqrt(6)*c^2)^(3/2)*b^3 + 313456656384*(-I*sqrt(6)*c^2 - 240*I*sqrt(6)*sqrt(
-1/72*I*sqrt(6)*c^2)*b + 96*b^2)*c^4 - 38999023360671744*I*sqrt(6)*(-1/72*I
*sqrt(6)*c^2)^(5/2)*b + 1880739938304*(-16*I*sqrt(6)*b^4 - 3*I*sqrt(6)*c^4
+ 24*b^2*c^2 + 288*sqrt(-1/72*I*sqrt(6)*c^2)*b^3 - 5760*(-1/72*I*sqrt(6)*c^
2)^(3/2)*b)*c^2))^(1/3) + 483729408*I*sqrt(6)*b + 8707129344*(1/4608*I*sqrt
(6)*b^3 + 1/8124796533473 ...
```

Sympy [A]
time = 0.31, size = 85, normalized size = 0.69

$$\operatorname{RootSum}\left(27648t^4+576t^2b^2+96tbc^2+3b^4+2c^4,\ \left(t\mapsto t\log\left(x+\frac{-1152t^3c^2+288t^2b^3-36tb^2c^2+3b^5-3bc^4}{6b^4c-c^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(3*x**4+2),x)
```

```
[Out] RootSum(27648*_t**4 + 576*_t**2*b**2 + 96*_t*b*c**2 + 3*b**4 + 2*c**4, Lamb
da(_t, _t*log(x + (-1152*_t**3*c**2 + 288*_t**2*b**3 - 36*_t*b**2*c**2 + 3*
b**5 - 3*b*c**4)/(6*b**4*c - c**5))))
```

Giac [A]
time = 0.62, size = 114, normalized size = 0.93

$$-\frac{1}{24}\cdot 6^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)+\frac{1}{24}\cdot 6^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-\frac{1}{12}\left(\sqrt{6}\,b-6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\left(\sqrt{6}\,b+6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/24*6^(1/4)*c*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*6^(1/4)
*c*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/12*(sqrt(6)*b - 6^(1/4)
*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/12*(sqr
t(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/
4)))
```

Mupad [B]
time = 0.22, size = 162, normalized size = 1.32

$$\sum_{k=1}^{4}\ln\left(9b^3x-6c^3-\mathrm{root}\left(z^4+\frac{b^2z^2}{48}+\frac{bc^2z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)bc\,144+\mathrm{root}\left(z^4+\frac{b^2z^2}{48}+\frac{bc^2z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)^2bx\,864+\mathrm{root}\left(z^4+\frac{b^2z^2}{48}+\frac{bc^2z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)c^2x\,72\right)\mathrm{root}\left(z^4+\frac{b^2z^2}{48}+\frac{bc^2z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(9*b^3*x - 6*c^3 - 144*root(z^4 + (b^2*z^2)/48 + (b*c^2*z)/288 +
c^4/13824 + b^4/9216, z, k)*b*c + 864*root(z^4 + (b^2*z^2)/48 + (b*c^2*z)/2
88 + c^4/13824 + b^4/9216, z, k)^2*b*x + 72*root(z^4 + (b^2*z^2)/48 + (b*c^
2*z)/288 + c^4/13824 + b^4/9216, z, k)*c^2*x)*root(z^4 + (b^2*z^2)/48 + (b*
c^2*z)/288 + c^4/13824 + b^4/9216, z, k), k, 1, 4)
```

3.158 $\int \frac{a+bx+cx^2}{2+3x^4}\,dx$

Optimal. Leaf size=163

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}-\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}\,a-2c\right)\log}{}$$

```
[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)-1/48*ln(-6^(3/4)*x+3*x^2+6^(1/2))*(-
2*c+a*6^(1/2))*6^(1/4)+1/48*ln(6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^
(1/4)+1/24*arctan(-1+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(1+6^(1/
4)*x)*(2*c+a*6^(1/2))*6^(1/4)
```

Rubi [A]
time = 0.08, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1890, 281, 209, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\sqrt{6}\,a+2c\right)\text{ArcTan}\left(1-\sqrt[4]{6}\,x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}\,a+2c\right)\text{ArcTan}\left(\sqrt[4]{6}\,x+1\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}\,a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}\,a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{b\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/
4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4))
- ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt
[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx = \int \left(\frac{bx}{2+3x^4}+\frac{a+cx^2}{2+3x^4}\right)dx$$

$$= b\int \frac{x}{2+3x^4}\,dx + \int \frac{a+cx^2}{2+3x^4}\,dx$$

$$= \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right) + \frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx + \frac{1}{12}\left(\sqrt{6}a+2c\right.$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}} - \frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}}{-\sqrt{\frac{2}{3}}+\ldots}}{8\ 6^{3/4}}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{\ldots}\right.}{8\ 6^{3\ldots}}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1+\sqrt[4]{\ldots}\right.}{4\ 6^{3/4}}$$

Mathematica [A]
time = 0.05, size = 129, normalized size = 0.79

$$\frac{-2\left(\sqrt{6}a+2\left(\sqrt[4]{6}b+c\right)\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt{6}a-2\sqrt[4]{6}b+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}x\right)-\left(\sqrt{6}a-2c\right)\left(\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)-\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)\right)}{8\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a -
2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6^(
1/4)*x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))
```

Maple [A]
time = 0.34, size = 203, normalized size = 1.25

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^2c+\_Rb+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$

default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan}{}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^{3}\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}\right.}{216}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*(ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6
^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1/
2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))+1/12*b*a
rctan(1/2*x^2*6^(1/2))*6^(1/2)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*(ln((x^2-1/3
*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+
1/3*6^(1/2)))+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2
)*3^(1/2)*6^(3/4)*x-1))
```

Maxima [A]
time = 0.49, size = 187, normalized size = 1.15

$$\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\,a-\sqrt{2}\,c\right)\log\left(\sqrt{3}\,x^{2}+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)-\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\,a-\sqrt{2}\,c\right)\log\left(\sqrt{3}\,x^{2}-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{24}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}a-2\sqrt{3}\sqrt{2}\,b+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}a+2\sqrt{3}\sqrt{2}\,b+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3
/4)*x + sqrt(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)
*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*(3^(3/4)*2^(3/4)*a - 2*sqrt(3)*s
qrt(2)*b + 2*3^(1/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3
^(1/4)*2^(3/4))) + 1/24*(3^(3/4)*2^(3/4)*a + 2*sqrt(3)*sqrt(2)*b + 2*3^(1/4
)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.63, size = 46651, normalized size = 286.20

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/626913312768*(((7776*I*sqrt(6)*b + 7776*sqrt(-sqrt(6)*(2*sqrt(6)*a*c - 3
*I*a^2 + 2*I*c^2)))^2 - 362797056*I*sqrt(6)*sqrt(-sqrt(6)*(2*sqrt(6)*a*c -
```

```
3*I*a^2 + 2*I*c^2))*b - 1088391168*b^2 - 2176782336*a*c - 181398528*I*sqrt(
6)*(3*a^2 - 2*c^2))*(-I*sqrt(3) + 1)/(1/4608*I*sqrt(6)*b^3 - 1/2304*I*sqrt(
6)*a*b*c + 1/175495605123022848*(7776*I*sqrt(6)*b + 7776*sqrt(-sqrt(6)*(2*s
qrt(6)*a*c - 3*I*a^2 + 2*I*c^2)))^3 + 1/1536*a^2*b - 1/2304*b*c^2 - 1/82944
*(-2*I*sqrt(6)*sqrt(-sqrt(6)*(2*sqrt(6)*a*c - 3*I*a^2 + 2*I*c^2))*b - 6*b^2
 - 12*a*c - I*sqrt(6)*(3*a^2 - 2*c^2))*(-I*sqrt(6)*b - sqrt(-sqrt(6)*(2*sqr
t(6)*a*c - 3*I*a^2 + 2*I*c^2))) - 1/4608*(b^2 - 4*a*c)*sqrt(-sqrt(6)*(2*sqr
t(6)*a*c - 3*I*a^2 + 2*I*c^2)) + 1/27648*(-sqrt(6)*(2*sqrt(6)*a*c - 3*I*a^2
 + 2*I*c^2))^(3/2) + 1/835884417024*sqrt(-21936950640377856*I*sqrt(6)*a^6 +
 233994140164030464*I*sqrt(6)*a^2*b^4 + 6499837226778624*I*sqrt(6)*c^6 - 11
6997070082015232*a*c^5 - 1828079220031488*sqrt(6)*(2*sqrt(6)*a*c - 3*I*a^2
+ 2*I*c^2)*a^4 + 81247965 ...
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(144) = 288$.
time = 2.76, size = 292, normalized size = 1.79

$$\operatorname{RootSum}\left(55296t^4 + t^2\cdot\left(2304ac + 1152b^2\right) + t\left(-288a^2b + 192bc^2\right) + 9a^4 + 12a^2c^2 - 24ab^2c + 6b^4 + 4c^4, \left(t \mapsto t\log\left(x + \frac{-13824t^3a^2c + 27648t^3ab^2 + 9216t^3c^3 + 1728t^2a^3b + 3456t^2abc^2 - 2304t^2b^3c + 216ta^5 - 576ta^3c^2 + 1296ta^2b^2c + 288tab^4 + 288tac^4 + 288tb^2c^3 + 90a^4bc - 90a^3b^3 + 60ab^3c^2 - 24b^5c + 24bc^5}{27a^6 - 18a^4c^2 + 144a^3b^2c - 72a^2b^4 - 12a^2c^4 + 96ab^2c^3 - 48b^4c^2 + 8c^6}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(55296*_t**4 + _t**2*(2304*a*c + 1152*b**2) + _t*(-288*a**2*b + 192*
b*c**2) + 9*a**4 + 12*a**2*c**2 - 24*a*b**2*c + 6*b**4 + 4*c**4, Lambda(_t,
 _t*log(x + (-13824*_t**3*a**2*c + 27648*_t**3*a*b**2 + 9216*_t**3*c**3 + 1
728*_t**2*a**3*b + 3456*_t**2*a*b*c**2 - 2304*_t**2*b**3*c + 216*_t*a**5 -
576*_t*a**3*c**2 + 1296*_t*a**2*b**2*c + 288*_t*a*b**4 + 288*_t*a*c**4 + 28
8*_t*b**2*c**3 + 90*a**4*b*c - 90*a**3*b**3 + 60*a*b**3*c**2 - 24*b**5*c +
24*b*c**5)/(27*a**6 - 18*a**4*c**2 + 144*a**3*b**2*c - 72*a**2*b**4 - 12*a*
*2*c**4 + 96*a*b**2*c**3 - 48*b**4*c**2 + 8*c**6))))
```

Giac [**A**]
time = 0.76, size = 143, normalized size = 0.88

$$\frac{1}{24}\left(6^{\frac{3}{4}}a - 2\sqrt{6}\,b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\sqrt{6}\,b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)
*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b + 2*6^(1/4)*c
)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/
4)*a - 2*6^(1/4)*c)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^
(3/4)*a - 2*6^(1/4)*c)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B]
time = 5.52, size = 270, normalized size = 1.66

$$\sum_{k=1}^{4}\ln\left(9\,a\,b^2-9\,a^2\,c-\mathrm{root}\left(z^4+\frac{z^2\,\left(2304\,a\,c+1152\,b^2\right)}{55296}-\frac{z\,\left(288\,a^2\,b-192\,b\,c^2\right)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\right)\,\left(\mathrm{root}\left(z^4+\frac{z^2\,\left(2304\,a\,c+1152\,b^2\right)}{55296}-\frac{z\,\left(288\,a^2\,b-192\,b\,c^2\right)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\right)\,\left(864\,a-864\,b\,x\right)+144\,b\,c+x\,\left(108\,a^2-72\,c^2\right)\right)-6\,c^3+x\,\left(9\,b^3-18\,a\,b\,c\right)\right)\,\mathrm{root}\left(z^4+\frac{z^2\,\left(2304\,a\,c+1152\,b^2\right)}{55296}-\frac{z\,\left(288\,a^2\,b-192\,b\,c^2\right)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(9*a*b^2 - 9*a^2*c - root(z^4 + (z^2*(2304*a*c + 1152*b^2))/55296
 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*c)/2304 + (a^2*c^2)/4608 + c^
4/13824 + b^4/9216 + a^4/6144, z, k)*(root(z^4 + (z^2*(2304*a*c + 1152*b^2)
)/55296 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*c)/2304 + (a^2*c^2)/46
08 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*(864*a - 864*b*x) + 144*b*c + x
*(108*a^2 - 72*c^2)) - 6*c^3 + x*(9*b^3 - 18*a*b*c))*root(z^4 + (z^2*(2304*
a*c + 1152*b^2))/55296 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*c)/2304
 + (a^2*c^2)/4608 + c^4/13824 + b^4/9216 + a^4/6144, z, k), k, 1, 4)
```

3.159 $\int \frac{dx^3}{2+3x^4} dx$

Optimal. Leaf size=13

$$\frac{1}{12}d\log\left(2+3x^4\right)$$

[Out] 1/12*d*ln(3*x^4+2)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {12, 266}

$$\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

[In] Int[(d*x^3)/(2 + 3*x^4),x]

[Out] (d*Log[2 + 3*x^4])/12

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned} \int \frac{dx^3}{2+3x^4}\,dx &= d\int \frac{x^3}{2+3x^4}\,dx \\ &= \frac{1}{12}d\log\left(2+3x^4\right) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{12}d\log\left(2+3x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x^3)/(2 + 3*x^4),x]
[Out] (d*Log[2 + 3*x^4])/12
```

Maple [A]
time = 0.32, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{d \ln(3x^4+2)}{12}$	12
default	$\frac{d \ln(3x^4+2)}{12}$	12
norman	$\frac{d \ln(3x^4+2)}{12}$	12
meijerg	$\frac{d \ln\left(1+\frac{3x^4}{2}\right)}{12}$	12
risch	$\frac{d \ln(3x^4+2)}{12}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x^3/(3*x^4+2),x,method=_RETURNVERBOSE)
[Out] 1/12*d*ln(3*x^4+2)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.85

$$\frac{1}{12} \, d \log \left(3 \, x^4 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3/(3*x^4+2),x, algorithm="maxima")
[Out] 1/12*d*log(3*x^4 + 2)
```

Fricas [A]
time = 0.40, size = 11, normalized size = 0.85

$$\frac{1}{12} \, d \log \left(3 \, x^4 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3/(3*x^4+2),x, algorithm="fricas")
[Out] 1/12*d*log(3*x^4 + 2)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.77

$$\frac{d \log \left(3x^4 + 2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x**3/(3*x**4+2),x)
```

```
[Out] d*log(3*x**4 + 2)/12
```

Giac [A]
time = 0.87, size = 11, normalized size = 0.85

$$\frac{1}{12}\,d\log\left(3\,x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/12*d*log(3*x^4 + 2)
```

Mupad [B]
time = 0.03, size = 9, normalized size = 0.69

$$\frac{d\,\ln\left(x^4+\frac{2}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3)/(3*x^4 + 2),x)
```

```
[Out] (d*log(x^4 + 2/3))/12
```

3.160 $\int \frac{a+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=114

$$-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}-\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{1}{12}d$$

```
[Out] 1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)+1/12
*d*ln(3*x^4+2)-1/48*a*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/48*a*ln(6^(3/4
)*x+3*x^2+6^(1/2))*6^(3/4)
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1890, 217, 1179, 642, 1176, 631, 210, 266}

$$-\frac{a\text{ArcTan}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}+\frac{a\text{ArcTan}\left(\sqrt[4]{6}\,x+1\right)}{4\sqrt[4]{6}}-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -1/4*(a*ArcTan[1 - 6^(1/4)*x])/6^(1/4) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/
4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*Log[2 + 3*x^4])/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \int \left(\frac{a}{2+3x^4}+\frac{dx^3}{2+3x^4}\right)dx$$
$$= a\int \frac{1}{2+3x^4}\,dx + d\int \frac{x^3}{2+3x^4}\,dx$$
$$= \frac{1}{12}d\log\left(2+3x^4\right) + \frac{a\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}dx}{2\sqrt{2}} + \frac{a\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}dx}{2\sqrt{2}}$$
$$= \frac{1}{12}d\log\left(2+3x^4\right) + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}dx}{4\sqrt{6}} + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}dx}{4\sqrt{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}dx}{8\sqrt[4]{6}}$$
$$= -\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{1}{12}d\log\left(2+3x^4\right) + \frac{a\text{Subst}\big(}{}$$
$$= -\frac{a\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}} + \frac{a\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+\right.}{8}$$

Mathematica [A]
time = 0.02, size = 108, normalized size = 0.95

$$\frac{1}{48}\left(-26^{3/4}a\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)+2\,6^{3/4}a\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)-6^{3/4}a\log\left(2-2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)+6^{3/4}a\log\left(2+2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)+4d\log\left(2+3x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + d*x^3)/(2 + 3*x^4),x]
[Out] (-2*6^(3/4)*a*ArcTan[1 - 6^(1/4)*x] + 2*6^(3/4)*a*ArcTan[1 + 6^(1/4)*x] - 6
^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(3/4)*a*Log[2 + 2*6^(1/4)*x
 + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [A]
time = 0.34, size = 106, normalized size = 0.93

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3 d+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{d\ln(3x^4}{12}$

meijerg	$\frac{d\ln\left(1+\frac{3x^4}{2}\right)}{12}+\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}} \right.}{2\left(x\right.}\right.}{96}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*(ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6
^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1/
2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))+1/12*d*l
n(3*x^4+2)
```

Maxima [A]
time = 0.52, size = 149, normalized size = 1.31

$$\frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d+3\,a\right)\log\left(\sqrt{3}\,x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}\,x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^
(3/4))) + 1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x -
3^(1/4)*2^(3/4))) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*log(s
qrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4
)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(83) = 166$.
time = 0.39, size = 353, normalized size = 3.10

$$-\frac{4\cdot 6^{\frac{1}{4}}\sqrt{3}\,\sqrt{2}\,\left(a^4\right)^{\frac{1}{4}}a^4\arctan\left(-\frac{3\cdot 6^{\frac{3}{4}}\sqrt{3}\,\sqrt{2}\,\left(a^4\right)^{\frac{3}{4}}a^5x+18\,a^8-6^{\frac{3}{4}}\sqrt{3}\,\sqrt{2}\,\left(a^4\right)^{\frac{3}{4}}\sqrt{9\,a^2x^2+3\cdot 6^{\frac{1}{4}}\sqrt{3}\,\sqrt{2}\,\left(a^4\right)^{\frac{1}{4}}ax+3\sqrt{6}\,\sqrt{a^4}}\,a^4}{18\,a^8}\right)+4\cdot 6^{\frac{1}{4}}\sqrt{3}\,\sqrt{2}\,\left(a^4\right)^{\frac{1}{4}}a^4\arctan\left(-\frac{3\cdot 6^{\frac{3}{4}}\sqrt{3}\,\sqrt{2}\,\left(a^4\right)^{\frac{3}{4}}a^5x-18\,a^8-6^{\frac{3}{4}}\sqrt{3}\,\sqrt{2}\,\left(a^4\right)^{\frac{3}{4}}\sqrt{9\,a^2x^2-3\cdot 6^{\frac{1}{4}}\sqrt{3}\,\sqrt{2}\,\left(a^4\right)^{\frac{1}{4}}ax+3\sqrt{6}\,\sqrt{a^4}}\,a^4}{18\,a^8}\right)-\left(6^{\frac{1}{4}}\sqrt{3}\,\sqrt{2}\,\left(a^4\right)^{\frac{1}{4}}a^4+4\,a^4d\right)\log\left(9\,a^2x^2+3\cdot 6^{\frac{1}{4}}\sqrt{3}\,\sqrt{2}\,\left(a^4\right)^{\frac{1}{4}}ax+3\sqrt{6}\,\sqrt{a^4}\right)+\left(6^{\frac{1}{4}}\sqrt{3}\,\sqrt{2}\,\left(a^4\right)^{\frac{1}{4}}a^4-4\,a^4d\right)\log\left(9\,a^2x^2-3\cdot 6^{\frac{1}{4}}\sqrt{3}\,\sqrt{2}\,\left(a^4\right)^{\frac{1}{4}}ax+3\sqrt{6}\,\sqrt{a^4}\right)}{48\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/48*(4*6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a^4*arctan(-1/18*(3*6^(3/4)*sq
rt(3)*sqrt(2)*(a^4)^(3/4)*a^5*x + 18*a^8 - 6^(3/4)*sqrt(3)*sqrt(2)*(a^4)^(3
/4)*sqrt(9*a^2*x^2 + 3*6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a*x + 3*sqrt(6)*
sqrt(a^4))*a^4)/a^8) + 4*6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a^4*arctan(-1/
18*(3*6^(3/4)*sqrt(3)*sqrt(2)*(a^4)^(3/4)*a^5*x - 18*a^8 - 6^(3/4)*sqrt(3)*
sqrt(2)*(a^4)^(3/4)*sqrt(9*a^2*x^2 - 3*6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*
a*x + 3*sqrt(6)*sqrt(a^4))*a^4)/a^8) - (6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)
*a^4 + 4*a^4*d)*log(9*a^2*x^2 + 3*6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a*x +
```

```
 3*sqrt(6)*sqrt(a^4)) + (6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a^4 - 4*a^4*d)
*log(9*a^2*x^2 - 3*6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a*x + 3*sqrt(6)*sqrt
(a^4)))/a^4
```

Sympy [A]
time = 0.18, size = 51, normalized size = 0.45

$$\text{RootSum}\left(165888t^4 - 55296t^3d + 6912t^2d^2 - 384td^3 + 27a^4 + 8d^4, \left(t \mapsto t\log\left(x + \frac{24t - 2d}{3a}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + 6912*_t**2*d**2 - 384*_t*d**3 + 27*a
**4 + 8*d**4, Lambda(_t, _t*log(x + (24*_t - 2*d)/(3*a))))
```

Giac [A]
time = 0.86, size = 109, normalized size = 0.96

$$\frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a + 4d\right)\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a - 4d\right)\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
+ 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))
) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1
/48*(6^(3/4)*a - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B]
time = 0.28, size = 117, normalized size = 1.03

$$\ln\left(x - \frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12} - \frac{6^{1/4}\,\sqrt{\frac{3}{4}\mathrm{i}}\;a}{12}\right) + \ln\left(x + \frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12} + \frac{6^{1/4}\,\sqrt{\frac{3}{4}\mathrm{i}}\;a}{12}\right) + \ln\left(x - \frac{(-1)^{3/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12} + \frac{6^{1/4}\,\sqrt{-\frac{3}{4}\mathrm{i}}\;a}{12}\right) + \ln\left(x + \frac{(-1)^{3/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12} - \frac{6^{1/4}\,\sqrt{-\frac{3}{4}\mathrm{i}}\;a}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] log(x - ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(3i/4)^(1/2)*a)/12
) + log(x + ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(3i/4)^(1/2)*a
)/12) + log(x - ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(-3i/4)^(1
/2)*a)/12) + log(x + ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(-3i/
4)^(1/2)*a)/12)
```

3.161 $\int \frac{bx+dx^3}{2+3x^4} \, dx$

Optimal. Leaf size=36

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(2+3x^4\right)$$

```
[Out] 1/12*d*ln(3*x^4+2)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1607, 1262, 649, 209, 266}

$$\frac{b\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) + (d*Log[2 + 3*x^4])/12
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, c, d, e, p, q}, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{bx+dx^3}{2+3x^4}\,dx &= \int \frac{x(b+dx^2)}{2+3x^4}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx,x,x^2\right)+\frac{1}{2}d\text{Subst}\left(\int \frac{x}{2+3x^2}\,dx,x,x^2\right)\\ &= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(2+3x^4\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.02, size = 65, normalized size = 1.81

$$\frac{1}{24}\left(i\sqrt{6}\,b+2d\right)\log\left(\sqrt{6}-3ix^2\right)+\frac{1}{24}\left(-i\sqrt{6}\,b+2d\right)\log\left(\sqrt{6}+3ix^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)/(2 + 3*x^4),x]
[Out] ((I*Sqrt[6]*b + 2*d)*Log[Sqrt[6] - (3*I)*x^2])/24 + (((-I)*Sqrt[6]*b + 2*d)
*Log[Sqrt[6] + (3*I)*x^2])/24
```

Maple [A]
time = 0.34, size = 28, normalized size = 0.78

method	result	size
default	$\frac{d\ln(3x^4+2)}{12}+\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$	28
risch	$\frac{d\ln(9x^4+6)}{12}+\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$	28

meijerg	$\frac{d\ln\left(1+\frac{3x^4}{2}\right)}{12}+\frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,x^2}{2}\right)}{12}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d*ln(3*x^4+2)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(27) = 54$.
time = 0.51, size = 113, normalized size = 3.14

$$-\frac{1}{12}\,\sqrt{3}\,\sqrt{2}\,b\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\,\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{12}\,\sqrt{3}\,\sqrt{2}\,b\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\,\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{12}\,d\log\left(\sqrt{3}\,x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{12}\,d\log\left(\sqrt{3}\,x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x -
 3^(1/4)*2^(3/4))) + 1/12*d*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2))
+ 1/12*d*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Fricas [A]
time = 0.39, size = 27, normalized size = 0.75

$$\frac{1}{12}\,\sqrt{6}\,b\arctan\left(\frac{1}{2}\,\sqrt{6}\,x^2\right)+\frac{1}{12}\,d\log\left(3\,x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2) + 1/12*d*log(3*x^4 + 2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.19, size = 53, normalized size = 1.47

$$\left(-\frac{\sqrt{6}\,ib}{24}+\frac{d}{12}\right)\log\left(x^2-\frac{\sqrt{6}\,i}{3}\right)+\left(\frac{\sqrt{6}\,ib}{24}+\frac{d}{12}\right)\log\left(x^2+\frac{\sqrt{6}\,i}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x)/(3*x**4+2),x)
```

```
[Out] (-sqrt(6)*I*b/24 + d/12)*log(x**2 - sqrt(6)*I/3) + (sqrt(6)*I*b/24 + d/12)*
log(x**2 + sqrt(6)*I/3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(27) = 54$.
time = 0.80, size = 93, normalized size = 2.58

$$-\frac{1}{12}\sqrt{6}\,b\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\sqrt{6}\,b\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\,d\log\left(x^{2}+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)+\frac{1}{12}\,d\log\left(x^{2}-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)
)) + 1/12*d*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/12*d*log(x^2 -
 sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B]
time = 0.06, size = 25, normalized size = 0.69

$$\frac{d\,\ln\left(x^4+\frac{2}{3}\right)}{12}+\frac{\sqrt{6}\,b\,\mathrm{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] (d*log(x^4 + 2/3))/12 + (6^(1/2)*b*atan((6^(1/2)*x^2)/2))/12
```

3.162 $\int \frac{a+bx+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=136

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}-\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}\right.}{8}$$

```
[Out] 1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)+1/12
*d*ln(3*x^4+2)-1/48*a*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/48*a*ln(6^(3/4
)*x+3*x^2+6^(1/2))*6^(3/4)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {1890, 217, 1179, 642, 1176, 631, 210, 1262, 649, 209, 266}

$$-\frac{a\text{ArcTan}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}+\frac{a\text{ArcTan}\left(\sqrt[4]{6}\,x+1\right)}{4\sqrt[4]{6}}-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{b\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4
)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*
Log[2 + 3*x^4])/12
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx = \int \left(\frac{a}{2+3x^4}+\frac{x(b+dx^2)}{2+3x^4}\right)dx$$

$$= a\int \frac{1}{2+3x^4}\,dx+\int \frac{x(b+dx^2)}{2+3x^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx,x,x^2\right)+\frac{a\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}\,dx}{2\sqrt{2}}+\frac{a\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}\,dx}{2\sqrt{2}}$$

$$= \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}}+\frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}}-\frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}}-\frac{a\int \frac{\frac{2^{3/}}{\sqrt[4]{3}}\cdots}{-\sqrt{\frac{2}{3}}\cdots}}{8\cdots}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}-\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{1}{12}\cdots$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}-\frac{a\log\left(\sqrt{6}-6^{3}\cdots\right.}{8\sqrt[4]{6}}\cdots$$

Mathematica [A]
time = 0.04, size = 128, normalized size = 0.94

$$\frac{1}{48}\left(-2\sqrt{6}\left(\sqrt[4]{6}\,a+2b\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)+2\sqrt{6}\left(\sqrt[4]{6}\,a-2b\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)-6^{3/4}a\log\left(2-2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)+6^{3/4}a\log\left(2+2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)+4d\log\left(2+3x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*Sqrt[6]*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*Sqrt[6]*(6^(1/4)*a
- 2*b)*ArcTan[1 + 6^(1/4)*x] - 6^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2]
 + 6^(3/4)*a*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [A]
time = 0.34, size = 121, normalized size = 0.89

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3 d+\_R b+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan}{}$
meijerg	$\frac{d\ln\left(1+\frac{3x^4}{2}\right)}{12}+\frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{2}\sqrt{3}\,x^2}{2}\right)}{12}+\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{}}}{8-3^{\frac{1}{4}}8}\right)}{\left(x^4\right)^{\frac{1}{4}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*(ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6
^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1/
2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))+1/12*b*a
rctan(1/2*x^2*6^(1/2))*6^(1/2)+1/12*d*ln(3*x^4+2)
```

Maxima [A]
time = 0.50, size = 171, normalized size = 1.26

$$\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d+3a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a-2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*log(sqrt(3)*x^2 + 3^(1/4)
*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d - 3*a)*l
og(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*sqrt(3)*(3^(1/4)*2^(3/
4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/
4))) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^
(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.34, size = 17085, normalized size = 125.62

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/1119744*((-725594112*I*sqrt(6)*b*d + (7776*I*sqrt(6)*b + 46656*d + 93312
*sqrt(1/48*I*sqrt(6)*a^2))^2 - 1088391168*b^2 - 2176782336*d^2 - 544195584*
sqrt(6)*(I*a^2 + 8*I*sqrt(1/48*I*sqrt(6)*a^2)*b) - 8707129344*sqrt(1/48*I*s
qrt(6)*a^2)*d)*(-I*sqrt(3) + 1)/((7776*I*sqrt(6)*b + 46656*d + 93312*sqrt(1
/48*I*sqrt(6)*a^2))^3 + 114254951251968*a^2*b + 304679870005248*I*sqrt(6)*s
qrt(1/48*I*sqrt(6)*a^2)*b*d + 76169967501312*b^2*d + 50779978334208*d^3 - 2
115832430592*(-4*I*sqrt(6)*b*d - 6*b^2 - 12*d^2 - 3*sqrt(6)*(I*a^2 + 8*I*sq
rt(1/48*I*sqrt(6)*a^2)*b) - 48*sqrt(1/48*I*sqrt(6)*a^2)*d)*(-I*sqrt(6)*b -
6*d - 12*sqrt(1/48*I*sqrt(6)*a^2)) + 12694994583552*I*sqrt(6)*(3*b^3 + 3*a^
2*d + 2*b*d^2) + 10968475320188928*(1/48*I*sqrt(6)*a^2)^(3/2) - 15233993500
2624*sqrt(1/48*I*sqrt(6)*a^2)*(3*b^2 - 2*d^2) + 104976*sqrt(-35099121024604
5696*I*sqrt(6)*a^6 + 935976560656121856*I*sqrt(6)*a^2*b^4 + 561585936393673
1136*a^4*b^2 - 5615859363936731136*(3*sqrt(1/48*I*sqrt(6)*a^2)*a^2 - 8*I*sq
rt(6)*(1/48*I*sqrt(6)*a^2 ...
```

Sympy [A]

time = 0.89, size = 199, normalized size = 1.46

$$\operatorname{RootSum}\left(165888t^4 - 55296t^3d + t^2\cdot\left(3456b^2 + 6912d^2\right) + t\left(-864a^2b - 576b^2d - 384d^3\right) + 27a^4 + 72a^2bd + 18b^4 + 24b^2d^2 + 8d^4, \left(t\mapsto t\log\left(x + \frac{27648t^3b^2 + 1728t^2a^2b - 6912t^2b^2d + 216ta^4 - 288ta^2bd + 288tb^4 + 576tb^2d^2 - 18a^4d - 90a^2b^3 + 12a^2bd^2 - 24b^4d - 16b^2d^3}{27a^5 - 72ab^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(3456*b**2 + 6912*d**2) + _t*(
-864*a**2*b - 576*b**2*d - 384*d**3) + 27*a**4 + 72*a**2*b*d + 18*b**4 + 24
*b**2*d**2 + 8*d**4, Lambda(_t, _t*log(x + (27648*_t**3*b**2 + 1728*_t**2*a
**2*b - 6912*_t**2*b**2*d + 216*_t*a**4 - 288*_t*a**2*b*d + 288*_t*b**4 + 5
76*_t*b**2*d**2 - 18*a**4*d - 90*a**2*b**3 + 12*a**2*b*d**2 - 24*b**4*d - 1
6*b**2*d**3)/(27*a**5 - 72*a*b**4))))
```

Giac [A]

time = 1.35, size = 125, normalized size = 0.92

$$\frac{1}{24}\left(6^{\frac{3}{4}}a - 2\sqrt{6}\,b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\sqrt{6}\,b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a + 4\,d\right)\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a - 4\,d\right)\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(
3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqrt(2
)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 4*d)*log(x^2 - sqrt(2)*(2/
3)^(1/4)*x + sqrt(2/3))
```

Mupad [B]

time = 5.50, size = 307, normalized size = 2.26

$$\sum_{k=1}^{4}\ln\left(x\left(9\,a^2\,d + 9\,b^3 + 6\,b\,d^2\right) + 9\,a\,b^2 - 6\,a\,d^2 - \operatorname{root}\left(z^4 - \frac{d\,z^3}{3} + \frac{z^2\,(3456\,b^2 + 6912\,d^2)}{165888} - \frac{z\,(864\,a^2\,b + 576\,b^2\,d + 384\,d^3)}{165888} + \frac{a^2\,b\,d}{2304} + \frac{b^2\,d^2}{6912} + \frac{d^4}{20736} + \frac{b^4}{9216} + \frac{a^4}{6144}, z, k\right)\left(\operatorname{root}\left(z^4 - \frac{d\,z^3}{3} + \frac{z^2\,(3456\,b^2 + 6912\,d^2)}{165888} - \frac{z\,(864\,a^2\,b + 576\,b^2\,d + 384\,d^3)}{165888} + \frac{a^2\,b\,d}{2304} + \frac{b^2\,d^2}{6912} + \frac{d^4}{20736} + \frac{b^4}{9216} + \frac{a^4}{6144}, z, k\right)(864\,a - 864\,b\,x) - 144\,a\,d + x\,(108\,a^2 + 144\,b\,d)\right)\right)\operatorname{root}\left(z^4 - \frac{d\,z^3}{3} + \frac{z^2\,(3456\,b^2 + 6912\,d^2)}{165888} - \frac{z\,(864\,a^2\,b + 576\,b^2\,d + 384\,d^3)}{165888} + \frac{a^2\,b\,d}{2304} + \frac{b^2\,d^2}{6912} + \frac{d^4}{20736} + \frac{b^4}{9216} + \frac{a^4}{6144}, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(x*(9*a^2*d + 6*b*d^2 + 9*b^3) + 9*a*b^2 - 6*a*d^2 - root(z^4 - (
d*z^3)/3 + (z^2*(3456*b^2 + 6912*d^2))/165888 - (z*(864*a^2*b + 576*b^2*d +
 384*d^3))/165888 + (a^2*b*d)/2304 + (b^2*d^2)/6912 + d^4/20736 + b^4/9216
+ a^4/6144, z, k)*(root(z^4 - (d*z^3)/3 + (z^2*(3456*b^2 + 6912*d^2))/16588
8 - (z*(864*a^2*b + 576*b^2*d + 384*d^3))/165888 + (a^2*b*d)/2304 + (b^2*d^
2)/6912 + d^4/20736 + b^4/9216 + a^4/6144, z, k)*(864*a - 864*b*x) - 144*a*
d + x*(144*b*d + 108*a^2)))*root(z^4 - (d*z^3)/3 + (z^2*(3456*b^2 + 6912*d^
2))/165888 - (z*(864*a^2*b + 576*b^2*d + 384*d^3))/165888 + (a^2*b*d)/2304
+ (b^2*d^2)/6912 + d^4/20736 + b^4/9216 + a^4/6144, z, k), k, 1, 4)
```

3.163 $\int \frac{cx^2+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=114

$$-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{2\ 6^{3/4}}+\frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}-\frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}+\frac{1}{12}c$$

```
[Out] 1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)+1/12
*d*ln(3*x^4+2)+1/24*c*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/24*c*ln(6^(3/4
)*x+3*x^2+6^(1/2))*6^(1/4)
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1607, 1845, 303, 1176, 631, 210, 1179, 642, 266}

$$-\frac{c\text{ArcTan}\left(1-\sqrt[4]{6}\,x\right)}{2\ 6^{3/4}}+\frac{c\text{ArcTan}\left(\sqrt[4]{6}\,x+1\right)}{2\ 6^{3/4}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -1/2*(c*ArcTan[1 - 6^(1/4)*x])/6^(3/4) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/
4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx = \int \frac{x^2(c+dx)}{2+3x^4}\,dx$$
$$= \int \left(\frac{cx^2}{2+3x^4}+\frac{dx^3}{2+3x^4}\right)dx$$
$$= c\int \frac{x^2}{2+3x^4}\,dx + d\int \frac{x^3}{2+3x^4}\,dx$$
$$= \frac{1}{12}d\log\left(2+3x^4\right) - \frac{c\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{c\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}\,dx}{2\sqrt{3}}$$

$$c\int -$$

$$= \frac{1}{12}d\log\left(2+3x^4\right) + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{-}{}$$
$$= \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} + \frac{1}{12}d\log\left(2+3x^4\right) + \frac{c\text{Subst}\big(}{}$$
$$= -\frac{c\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\ 6^{3/4}} + \frac{c\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{2\ 6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\big(\sqrt{6}}{}$$

Mathematica [A]
time = 0.02, size = 108, normalized size = 0.95

$$\frac{1}{24}\left(-2\sqrt[4]{6}\,c\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)+2\sqrt[4]{6}\,c\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)+\sqrt[4]{6}\,c\log\left(2-2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)-\sqrt[4]{6}\,c\log\left(2+2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)+2d\log\left(2+3x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*c*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*c*ArcTan[1 + 6^(1/4)*x] + 6
^(1/4)*c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - 6^(1/4)*c*Log[2 + 2*6^(1/4)*x
 + Sqrt[6]*x^2] + 2*d*Log[2 + 3*x^4])/24
```

Maple [A]
time = 0.34, size = 106, normalized size = 0.93

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3 d+\_R^2 c\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$

default	$\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{144}+\frac{d\ln(3x^4}{12}$
meijerg	$\frac{d\ln\left(1+\frac{3x^4}{2}\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}\right.}{2\left(x^4\right.}\right.}{216}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*(ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*
6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1
/2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))+1/12*d*
ln(3*x^4+2)
```

Maxima [A]
time = 0.51, size = 152, normalized size = 1.33

$$\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d-\sqrt{3}\,c\right)\log\left(\sqrt{3}\,x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d+\sqrt{3}\,c\right)\log\left(\sqrt{3}\,x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{12}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{12}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d - sqrt(3)*c)*log(sqrt(3)*x^2 + 3^(1
/4)*2^(3/4)*x + sqrt(2)) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d + sqrt(3
)*c)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/12*3^(1/4)*2^(1/4)*
c*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/12*3^(1/4
)*2^(1/4)*c*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(83) = 166.
time = 0.41, size = 299, normalized size = 2.62

$$-\frac{4\cdot 6^{\frac{1}{4}}\left(c^4\right)^{\frac{1}{4}}c^4\arctan\left(-\frac{3c^8+3\cdot 6^{\frac{1}{4}}\left(c^4\right)^{\frac{5}{4}}c^3x-6^{\frac{1}{4}}\sqrt{3}\,\sqrt{3c^6x^2+6^{\frac{3}{4}}\left(c^4\right)^{\frac{3}{4}}c^3x+\sqrt{6}\,\sqrt{c^4}\,c^4}\,\left(c^4\right)^{\frac{5}{4}}}{3c^8}\right)+4\cdot 6^{\frac{1}{4}}\left(c^4\right)^{\frac{1}{4}}c^4\arctan\left(\frac{3c^8-3\cdot 6^{\frac{1}{4}}\left(c^4\right)^{\frac{5}{4}}c^3x+6^{\frac{1}{4}}\sqrt{3}\,\sqrt{3c^6x^2-6^{\frac{3}{4}}\left(c^4\right)^{\frac{3}{4}}c^3x+\sqrt{6}\,\sqrt{c^4}\,c^4}\,\left(c^4\right)^{\frac{5}{4}}}{3c^8}\right)-\left(2c^4d-6^{\frac{1}{4}}\left(c^4\right)^{\frac{1}{4}}c^4\right)\log\left(36c^6x^2+12\cdot 6^{\frac{3}{4}}\left(c^4\right)^{\frac{3}{4}}c^3x+12\sqrt{6}\,\sqrt{c^4}\,c^4\right)-\left(2c^4d+6^{\frac{1}{4}}\left(c^4\right)^{\frac{1}{4}}c^4\right)\log\left(36c^6x^2-12\cdot 6^{\frac{3}{4}}\left(c^4\right)^{\frac{3}{4}}c^3x+12\sqrt{6}\,\sqrt{c^4}\,c^4\right)}{24c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/24*(4*6^(1/4)*(c^4)^(1/4)*c^4*arctan(-1/3*(3*c^8 + 3*6^(1/4)*(c^4)^(5/4)
*c^3*x - 6^(1/4)*sqrt(3)*sqrt(3*c^6*x^2 + 6^(3/4)*(c^4)^(3/4)*c^3*x + sqrt(
6)*sqrt(c^4)*c^4)*(c^4)^(5/4))/c^8) + 4*6^(1/4)*(c^4)^(1/4)*c^4*arctan(1/3*
(3*c^8 - 3*6^(1/4)*(c^4)^(5/4)*c^3*x + 6^(1/4)*sqrt(3)*sqrt(3*c^6*x^2 - 6^(
```

```
3/4)*(c^4)^(3/4)*c^3*x + sqrt(6)*sqrt(c^4)*c^4)*(c^4)^(5/4))/c^8) - (2*c^4*
d - 6^(1/4)*(c^4)^(1/4)*c^4)*log(36*c^6*x^2 + 12*6^(3/4)*(c^4)^(3/4)*c^3*x
+ 12*sqrt(6)*sqrt(c^4)*c^4) - (2*c^4*d + 6^(1/4)*(c^4)^(1/4)*c^4)*log(36*c^
6*x^2 - 12*6^(3/4)*(c^4)^(3/4)*c^3*x + 12*sqrt(6)*sqrt(c^4)*c^4))/c^4
```

Sympy [A]
time = 0.15, size = 70, normalized size = 0.61

$$\operatorname{RootSum}\left(41472t^4 - 13824t^3d + 1728t^2d^2 - 96td^3 + 3c^4 + 2d^4, \left(t \mapsto t\log\left(x + \frac{3456t^3 - 864t^2d + 72td^2 - 2d^3}{3c^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2)/(3*x**4+2),x)
```

```
[Out] RootSum(41472*_t**4 - 13824*_t**3*d + 1728*_t**2*d**2 - 96*_t*d**3 + 3*c**4
 + 2*d**4, Lambda(_t, _t*log(x + (3456*_t**3 - 864*_t**2*d + 72*_t*d**2 - 2
*d**3)/(3*c**3))))
```

Giac [A]
time = 0.92, size = 109, normalized size = 0.96

$$\frac{1}{12}\cdot 6^{\frac{1}{4}}c\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\cdot 6^{\frac{1}{4}}c\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24}\left(6^{\frac{1}{4}}c - 2d\right)\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) + \frac{1}{24}\left(6^{\frac{1}{4}}c + 2d\right)\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
+ 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))
) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1
/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B]
time = 0.37, size = 117, normalized size = 1.03

$$\ln\left(x - \frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12} + \frac{6^{1/4}\sqrt{-\frac{1}{2}\mathrm{i}}\;c}{12}\right) + \ln\left(x + \frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12} - \frac{6^{1/4}\sqrt{-\frac{1}{2}\mathrm{i}}\;c}{12}\right) + \ln\left(x - \frac{(-1)^{3/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12} - \frac{6^{1/4}\sqrt{\frac{1}{2}\mathrm{i}}\;c}{12}\right) + \ln\left(x + \frac{(-1)^{3/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12} + \frac{6^{1/4}\sqrt{\frac{1}{2}\mathrm{i}}\;c}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] log(x - ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(-1i/2)^(1/2)*c)/1
2) + log(x + ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(-1i/2)^(1/2)
*c)/12) + log(x - ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(1i/2)^(
1/2)*c)/12) + log(x + ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(1i/
2)^(1/2)*c)/12)
```

3.164 $\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=154

$$-\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}\,-6^{3/4}x+3x^2\right.}{8\ 6^{3/4}}$$

```
[Out] 1/12*d*ln(3*x^4+2)-1/48*ln(-6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/
4)+1/48*ln(6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(-1
+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(1+6^(1/4)*x)*(2*c+a*6^(1/2)
)*6^(1/4)
```

Rubi [A]
time = 0.08, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1890, 266, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\sqrt{6}\,a+2c\right)\text{ArcTan}\left(1-\sqrt[4]{6}\,x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}\,a+2c\right)\text{ArcTan}\left(\sqrt[4]{6}\,x+1\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}\,a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}\,a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -1/4*((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/6^(3/4) + ((Sqrt[6]*a + 2*c)
*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3
/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x +
3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx = \int \left(\frac{dx^3}{2+3x^4}+\frac{a+cx^2}{2+3x^4}\right)dx$$

$$= d\int \frac{x^3}{2+3x^4}\,dx + \int \frac{a+cx^2}{2+3x^4}\,dx$$

$$= \frac{1}{12}d\log\left(2+3x^4\right)+\frac{1}{12}\left(\sqrt{6}\,a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx+\frac{1}{12}\left(\sqrt{6}\,a+2c\right)\int \frac{\sqrt{6}+3x^2}{2+3x^4}$$

$$= \frac{1}{12}d\log\left(2+3x^4\right)-\frac{\left(\sqrt{6}\,a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}\,a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}}{\sqrt[4]{3}}}}{8\ 6^{3/4}}$$

$$= -\frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+$$

$$= -\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}\,a-2c\right.}{}$$

Mathematica [A]
time = 0.08, size = 148, normalized size = 0.96

$$\frac{1}{48}\left(-2\sqrt[4]{6}\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)+2\sqrt[4]{6}\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)-\sqrt[4]{6}\left(\sqrt{6}\,a-2c\right)\log\left(2-2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)+\sqrt[4]{6}\left(\sqrt{6}\,a-2c\right)\log\left(2+2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)+4d\log\left(2+3x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(Sqrt[6]*a
+ 2*c)*ArcTan[1 + 6^(1/4)*x] - 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 - 2*6^(1/4)*
x + Sqrt[6]*x^2] + 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 + 2*6^(1/4)*x + Sqrt[6]*
x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [A]
time = 0.33, size = 199, normalized size = 1.29

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3 d+\_R^2 c+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$

default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{c\sqrt{3}\cdots}{\cdots}$
meijerg	$\frac{d\ln\left(1+\frac{3x^4}{2}\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}(x^4)^{\frac{1}{4}}+\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{x^4}}{2}\right)}{2(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}(x^4)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}(x^4)^{\frac{1}{4}}}\right)}{(x^4)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}(x^4\cdots\right.}{2(\cdots}\cdots\right.}{216}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*(ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6
^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1/
2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))+1/144*c*
3^(1/2)*6^(3/4)*2^(1/2)*(ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))
/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1/2)*3^(1
/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))+1/12*d*ln(3*x^4
+2)
```

Maxima [A]
time = 0.50, size = 195, normalized size = 1.27

$$-\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\,\sqrt{2}\,c-2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}\,x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\,\sqrt{2}\,c+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}\,x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d - 3*a)*log(
sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(sqrt(3)
*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x
 + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c)*arct
an(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)*(3*3
^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(
3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2604 vs. $2(115) = 230$.
time = 0.43, size = 2604, normalized size = 16.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="fricas")
```

[Out] 1/144*(2*sqrt(6)*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*arctan(1/36*(sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*(81*a^8 - 72*a^4*c^4 + 16*c^8)*x^2 + 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*((9*a^4*c - 12*a^2*c^3 + 4*c^5)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*x - 3*(27*a^7 - 18*a^5*c^2 - 12*a^3*c^4 + 8*a*c^6)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 3*(27*a^6 - 18*a^4*c^2 - 12*a^2*c^4 + 8*c^6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4))*(sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*a - 2*sqrt(6)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*c^3))*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*(sqrt(6)*(9*a^5 - 4*a*c^4)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*x - 2*sqrt(6)*(27*a^6*c + 18*a^4*c^3 - 12*a^2*c^5 - 8*c^7)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) - 6*sqrt(6)*(81*a^8 + 108*a^6*c^2 - 48*a^2*c^6 - 16*c^8)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4))/(729*a^12 + 972*a^10*c^2 - 324*a^8*c^4 - 864*a^6*c^6 - 144*a^4*c^8 + 192*a^2*c^10 + 64*c^12)) + 2*sqrt(6)*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*arctan(1/36*(sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*(81*a^8 - 72*a^4*c^4 + 16*c^8)*x^2 - 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*((9*a^4*c - 12*a^2*c^3 + 4*c^5)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*x - 3*(27*a^7 - 18*a^5*c^2 - 12*a^3*c^4 + 8*a*c^6)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 3*(27*a^6 - 18*a^4*c^2 - 12*a^2*c^4 + 8*c^6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4))*(sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*a - 2*sqrt(6)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*c^3))*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*(sqrt(6)*(9*a^5 - 4*a*c^4)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*x - 2*sqrt(6)*(27*a^6*c + 18*a^4*c^3 - 12*a^2*c^5 - 8*c^7)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 6*sqrt(6)*(81*a^8 + 108*a^6*c^2 - 48*a^2*c^6 - 16*c^8)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4))/(729*a^12 + 972*a^10*c^2 - 324*a^8*c^4 - 864*a^6*c^6 - 144*a^4*c^8 + 192*a^2*c^10 + 64*c^12)) - 3*(sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) - 4*(9*a^4 + 12*a^2*c^2 + 4*c^4)*d)*log(9*(81*a^8 - 72*a^4*c^4 + 16*c^8)*x^2 + 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*((9*a^4*c - 12*a^2*c^3 + 4*c^5)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*x

```
- 3*(27*a^7 - 18*a^5*c^2 - 12*a^3*c^4 + 8*a*c^6)*x)*sqrt((9*a^4 + 12*a^2*c^
2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 +
 4*c^4)) + 3*(27*a^6 - 18*a^4*c^2 - 12*a^2*c^4 + 8*c^6)*sqrt(54*a^4 + 72*a^
2*c^2 + 24*c^4)) + 3*(sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(9*a^4 +
 12*a^2*c^2 + 4*c^4 - 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)*sqrt((9*a^4
 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 -
12*a^2*c^2 + 4*c^4)) + 4*(9*a^4 + 12*a^2*c^2 + 4*c^4)*d)*log(9*(81*a^8 - 72
*a^4*c^4 + 16*c^8)*x^2 - 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*((9
*a^4*c - 12*a^2*c^3 + 4*c^5)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*x - 3*(27*a
^7 - 18*a^5*c^2 - 12*a^3*c^4 + 8*a*c^6)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4
 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))
+ 3*(27*a^6 - 18*a^4*c^2 - 12*a^2*c^4 + 8*c^6)*sqrt(54*a^4 + 72*a^2*c^2 + 2
4*c^4)))/(9*a^4 + 12*a^2*c^2 + 4*c^4)
```

Sympy [**A**]
time = 0.86, size = 148, normalized size = 0.96

$$\operatorname{RootSum}\left(165888t^4 - 55296t^3d + t^2\cdot\left(6912ac + 6912d^2\right) + t\left(-1152acd - 384d^3\right) + 27a^4 + 36a^2c^2 + 48acd^2 + 12c^4 + 8d^4, \left(t \mapsto t\log\left(x + \frac{-13824t^3c + 3456t^2cd + 216ta^3 - 432tac^2 - 288tcd^2 - 18a^3d + 36ac^2d + 8cd^3}{27a^4 - 12c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 6912*d**2) + _t*(-
1152*a*c*d - 384*d**3) + 27*a**4 + 36*a**2*c**2 + 48*a*c*d**2 + 12*c**4 + 8
*d**4, Lambda(_t, _t*log(x + (-13824*_t**3*c + 3456*_t**2*c*d + 216*_t*a**3
 - 432*_t*a*c**2 - 288*_t*c*d**2 - 18*a**3*d + 36*a*c**2*d + 8*c*d**3)/(27*
a**4 - 12*c**4))))
```

Giac [**A**]
time = 0.80, size = 137, normalized size = 0.89

$$\frac{1}{24}\left(6^{\frac{3}{4}}a + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c + 4d\right)\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c - 4d\right)\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(
3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c + 4*d)*lo
g(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c
- 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [**B**]
time = 5.81, size = 286, normalized size = 1.86

$$\ln\left(-2c + \sqrt{6}\,a\,1\mathrm{i} + x\sqrt{3\mathrm{i}\sqrt{6}\,a^2 - 12ac - 2\mathrm{i}\sqrt{6}\,c^2}\right)\left(\frac{d}{12} + \frac{\sqrt{\frac{3\mathrm{i}\sqrt{6}\,a^2}{4} - 3ac - \frac{1\mathrm{i}\sqrt{6}\,c^2}{2}}}{12}\right) + \ln\left(2c - \sqrt{6}\,a\,1\mathrm{i} + x\sqrt{3\mathrm{i}\sqrt{6}\,a^2 - 12ac - 2\mathrm{i}\sqrt{6}\,c^2}\right)\left(\frac{d}{12} - \frac{\sqrt{\frac{3\mathrm{i}\sqrt{6}\,a^2}{4} - 3ac - \frac{1\mathrm{i}\sqrt{6}\,c^2}{2}}}{12}\right) + \ln\left(2c + \sqrt{6}\,a\,1\mathrm{i} + x\sqrt{-3\mathrm{i}\sqrt{6}\,a^2 - 12ac + 2\mathrm{i}\sqrt{6}\,c^2}\right)\left(\frac{d}{12} - \frac{\sqrt{-\frac{3\mathrm{i}\sqrt{6}\,a^2}{4} - 3ac + \frac{1\mathrm{i}\sqrt{6}\,c^2}{2}}}{12}\right) + \ln\left(2c + \sqrt{6}\,a\,1\mathrm{i} - x\sqrt{-3\mathrm{i}\sqrt{6}\,a^2 - 12ac + 2\mathrm{i}\sqrt{6}\,c^2}\right)\left(\frac{d}{12} + \frac{\sqrt{-\frac{3\mathrm{i}\sqrt{6}\,a^2}{4} - 3ac + \frac{1\mathrm{i}\sqrt{6}\,c^2}{2}}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] log(6^(1/2)*a*1i - 2*c + x*(6^(1/2)*a^2*3i - 12*a*c - 6^(1/2)*c^2*2i)^(1/2)
)*(d/12 + ((6^(1/2)*a^2*3i)/4 - 3*a*c - (6^(1/2)*c^2*1i)/2)^(1/2)/12) + log
(2*c - 6^(1/2)*a*1i + x*(6^(1/2)*a^2*3i - 12*a*c - 6^(1/2)*c^2*2i)^(1/2))*(
d/12 - ((6^(1/2)*a^2*3i)/4 - 3*a*c - (6^(1/2)*c^2*1i)/2)^(1/2)/12) + log(2*
c + 6^(1/2)*a*1i + x*(6^(1/2)*c^2*2i - 6^(1/2)*a^2*3i - 12*a*c)^(1/2))*(d/1
2 - ((6^(1/2)*c^2*1i)/2 - (6^(1/2)*a^2*3i)/4 - 3*a*c)^(1/2)/12) + log(2*c +
 6^(1/2)*a*1i - x*(6^(1/2)*c^2*2i - 6^(1/2)*a^2*3i - 12*a*c)^(1/2))*(d/12 +
 ((6^(1/2)*c^2*1i)/2 - (6^(1/2)*a^2*3i)/4 - 3*a*c)^(1/2)/12)
```

3.165 $\int \frac{bx+cx^2+dx^3}{2+3x^4} \, dx$

Optimal. Leaf size=136

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{2\,6^{3/4}}+\frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}}-\frac{c\log\left(\sqrt{6}+\right.}{4\,6}$$

```
[Out] 1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)+1/12
*d*ln(3*x^4+2)+1/24*c*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/24*c*ln(6^(3/4
)*x+3*x^2+6^(1/2))*6^(1/4)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1608, 1845, 303, 1176, 631, 210, 1179, 642, 1262, 649, 209, 266}

$$\frac{b\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}-\frac{c\text{ArcTan}\left(1-\sqrt[4]{6}\,x\right)}{2\,6^{3/4}}+\frac{c\text{ArcTan}\left(\sqrt[4]{6}\,x+1\right)}{2\,6^{3/4}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4
)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*
Log[2 + 3*x^4])/12
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx = \int \frac{x(b+cx+dx^2)}{2+3x^4}\,dx$$

$$= \int \left(\frac{cx^2}{2+3x^4}+\frac{x(b+dx^2)}{2+3x^4}\right)\,dx$$

$$= c\int \frac{x^2}{2+3x^4}\,dx + \int \frac{x(b+dx^2)}{2+3x^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx, x, x^2\right) - \frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right) + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3}}{\sqrt[4]{}}}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\,6^{3/4}} + \frac{}{1}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}} - \frac{c\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\,6^{3/4}} + \frac{c\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{2\,6^{3/4}} + \frac{c\log\left(\sqrt{6}-6\right.}{4\,6^{3}}$$

Mathematica [A]
time = 0.04, size = 125, normalized size = 0.92

$$\frac{1}{24}\left(-2\sqrt[4]{6}\left(\sqrt[4]{6}\,b+c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)+2\sqrt[4]{6}\left(-\sqrt[4]{6}\,b+c\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)+\sqrt[4]{6}\,c\log\left(2-2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)-\sqrt[4]{6}\,c\log\left(2+2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)+2d\log\left(2+3x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(-(6^(1/4)*b)
 + c)*ArcTan[1 + 6^(1/4)*x] + 6^(1/4)*c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2]
- 6^(1/4)*c*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 2*d*Log[2 + 3*x^4])/24
```

Maple [A]
time = 0.34, size = 121, normalized size = 0.89

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3 d+\_R^2 c+\_R b\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\sqrt{2}\right.\right.}{144}$
meijerg	$\frac{d\ln\left(1+\frac{3x^4}{2}\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}\right.}{2\left(x^4\right.}\right.}{216}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*(ln(
(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*
2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/
6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))+1/12*d*ln(3*x^4+2)
```

Maxima [A]
time = 0.51, size = 174, normalized size = 1.28

$$\frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c-6b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c+6b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d-\sqrt{3}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d+\sqrt{3}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/72*sqrt(3)*sqrt(2)*(3^(3/4)*2^(3/4)*c - 6*b)*arctan(1/6*3^(3/4)*2^(1/4)*(
2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)*sqrt(2)*(3^(3/4)*2^(3/4)*c +
 6*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/72*3^
(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d - sqrt(3)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(
3/4)*x + sqrt(2)) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d + sqrt(3)*c)*lo
g(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.36, size = 18086, normalized size = 132.99

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/1253826625536*(((7776*I*sqrt(6)*b + 46656*d + 93312*sqrt(-1/72*I*sqrt(6)*
c^2))^2 - 4353564672*I*sqrt(6)*sqrt(-1/72*I*sqrt(6)*c^2)*b - 1088391168*b^2
 - 2176782336*d^2 + 362797056*I*sqrt(6)*(c^2 - 2*b*d) - 8707129344*sqrt(-1/
72*I*sqrt(6)*c^2)*d)*(-I*sqrt(3) + 1)/(1/4608*I*sqrt(6)*b^3 + 1/17549560512
3022848*(7776*I*sqrt(6)*b + 46656*d + 93312*sqrt(-1/72*I*sqrt(6)*c^2))^3 -
1/2304*b*c^2 + 1/576*I*sqrt(6)*sqrt(-1/72*I*sqrt(6)*c^2)*b*d + 1/2304*b^2*d
 + 1/3456*d^3 - 1/41472*(-12*I*sqrt(6)*sqrt(-1/72*I*sqrt(6)*c^2)*b - 3*b^2
- 6*d^2 + I*sqrt(6)*(c^2 - 2*b*d) - 24*sqrt(-1/72*I*sqrt(6)*c^2)*d)*(-I*sqr
t(6)*b - 6*d - 12*sqrt(-1/72*I*sqrt(6)*c^2)) - 1/6912*I*sqrt(6)*(c^2*d - b*
d^2) + 1/16*(-1/72*I*sqrt(6)*c^2)^(3/2) - 1/1152*sqrt(-1/72*I*sqrt(6)*c^2)*
(3*b^2 - 2*d^2) + 1/417942208512*sqrt(14218393933578240*I*sqrt(6)*c^6 + 584
98535041007616*b^2*c^4 + 2807929681968365568*I*sqrt(6)*(-1/72*I*sqrt(6)*c^2
)^(3/2)*b^3 - 406239826673664*(I*sqrt(6)*c^2 + 240*I*sqrt(6)*sqrt(-1/72*I*s
qrt(6)*c^2)*b - 96*b^2)*c ...
```

Sympy [A]
time = 0.87, size = 189, normalized size = 1.39

$$\operatorname{RootSum}\left(82944t^4 - 27648t^3d + t^2\cdot\left(1728b^2 + 3456d^2\right) + t\left(-288b^2d + 288bc^2 - 192d^3\right) + 9b^4 + 12b^2d^2 - 24bc^2d + 6c^4 + 4d^4, \left(t \mapsto t\log\left(x + \frac{-3456t^3c^2 + 864t^2b^3 + 864t^2c^2d - 144tb^3d - 108tb^2c^2 - 72tc^2d^2 + 9b^5 + 6b^3d^2 + 9b^2c^2d - 9bc^4 + 2c^2d^3}{18b^4c - 3c^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x)/(3*x**4+2),x)
```

```
[Out] RootSum(82944*_t**4 - 27648*_t**3*d + _t**2*(1728*b**2 + 3456*d**2) + _t*(-
288*b**2*d + 288*b*c**2 - 192*d**3) + 9*b**4 + 12*b**2*d**2 - 24*b*c**2*d +
 6*c**4 + 4*d**4, Lambda(_t, _t*log(x + (-3456*_t**3*c**2 + 864*_t**2*b**3
+ 864*_t**2*c**2*d - 144*_t*b**3*d - 108*_t*b**2*c**2 - 72*_t*c**2*d**2 + 9
*b**5 + 6*b**3*d**2 + 9*b**2*c**2*d - 9*b*c**4 + 2*c**2*d**3)/(18*b**4*c -
3*c**5))))
```

Giac [A]
time = 1.21, size = 124, normalized size = 0.91

$$-\frac{1}{12}\left(\sqrt{6}\,b - 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\left(\sqrt{6}\,b + 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24}\left(6^{\frac{1}{4}}c - 2d\right)\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) + \frac{1}{24}\left(6^{\frac{1}{4}}c + 2d\right)\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(6)*b - 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)
*(2/3)^(1/4))) + 1/12*(sqrt(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4
)*(2*x - sqrt(2)*(2/3)^(1/4))) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2)*(
2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2/3)^
(1/4)*x + sqrt(2/3))
```

Mupad [B]
time = 5.39, size = 300, normalized size = 2.21

$$\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\,\left(1728\,b^2+3456\,d^2\right)}{82944}-\frac{z\,\left(-288\,b\,c^2+288\,b^2\,d+192\,d^3\right)}{82944}-\frac{b\,c^2\,d}{3456}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)\,\left(144\,b\,c+x\,\left(144\,b\,d-72\,c^2\right)-\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\,\left(1728\,b^2+3456\,d^2\right)}{82944}-\frac{z\,\left(-288\,b\,c^2+288\,b^2\,d+192\,d^3\right)}{82944}-\frac{b\,c^2\,d}{3456}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)\,b\,x\,864\right)+x\,\left(9\,b^3+6\,b\,d^2-6\,c^2\,d\right)-6\,c^3+12\,b\,c\,d\right)\,\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\,\left(1728\,b^2+3456\,d^2\right)}{82944}-\frac{z\,\left(-288\,b\,c^2+288\,b^2\,d+192\,d^3\right)}{82944}-\frac{b\,c^2\,d}{3456}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(x*(6*b*d^2 - 6*c^2*d + 9*b^3) - root(z^4 - (d*z^3)/3 + (z^2*(172
8*b^2 + 3456*d^2))/82944 - (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944 -
(b*c^2*d)/3456 + (b^2*d^2)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k)*(
144*b*c + x*(144*b*d - 72*c^2) - 864*root(z^4 - (d*z^3)/3 + (z^2*(1728*b^2
+ 3456*d^2))/82944 - (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944 - (b*c^2
*d)/3456 + (b^2*d^2)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k)*b*x) -
6*c^3 + 12*b*c*d)*root(z^4 - (d*z^3)/3 + (z^2*(1728*b^2 + 3456*d^2))/82944
- (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944 - (b*c^2*d)/3456 + (b^2*d^2
)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k), k, 1, 4)
```

3.166 $\int \frac{a+bx+cx^2+dx^3}{2+3x^4} \, dx$

Optimal. Leaf size=176

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}-\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\;6^{3/4}}+\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\;6^{3/4}}-\frac{\left(\sqrt{6}\,a-2c\right)\log}{}$$

[Out] 1/12*d*ln(3*x^4+2)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)-1/48*ln(-6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/4)+1/48*ln(6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(-1+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(1+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)

Rubi [A]
time = 0.10, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 209, 266}

$$-\frac{\left(\sqrt{6}\,a+2c\right)\text{ArcTan}\left(1-\sqrt[4]{6}\,x\right)}{4\;6^{3/4}}+\frac{\left(\sqrt{6}\,a+2c\right)\text{ArcTan}\left(\sqrt[4]{6}\,x+1\right)}{4\;6^{3/4}}-\frac{\left(\sqrt{6}\,a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\;6^{3/4}}+\frac{\left(\sqrt{6}\,a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\;6^{3/4}}+\frac{b\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = \int \left(\frac{a+cx^2}{2+3x^4}+\frac{x(b+dx^2)}{2+3x^4}\right)dx$$

$$= \int \frac{a+cx^2}{2+3x^4}\,dx + \int \frac{x(b+dx^2)}{2+3x^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx, x, x^2\right) + \frac{1}{12}\left(\sqrt{6}\,a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx + \frac{1}{12}\left(\sqrt{6}\,\right.$$

$$= \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right) - \frac{\left(\sqrt{6}\,a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}} - \frac{\left(\sqrt{6}\,\right.}{}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}\,a-2c\right)}{}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(\right.}{4\ 6^{3/4}}$$

Mathematica [A]
time = 0.07, size = 164, normalized size = 0.93

$$\frac{1}{48}\left(-2\sqrt[4]{6}\left(\sqrt{6}\,a+2\left(\sqrt[4]{6}\,b+c\right)\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)+2\sqrt[4]{6}\left(\sqrt{6}\,a-2\sqrt[4]{6}\,b+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)-\sqrt[4]{6}\left(\sqrt{6}\,a-2c\right)\log\left(2-2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)+\sqrt[4]{6}\left(\sqrt{6}\,a-2c\right)\log\left(2+2\sqrt[4]{6}\,x+\sqrt{6}\,x^2\right)+4d\log\left(2+3x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/
4)*(Sqrt[6]*a - 2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - 6^(1/4)*(Sqrt[6]
*a - 2*c)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(1/4)*(Sqrt[6]*a - 2*c)*Lo
g[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [A]
time = 0.34, size = 214, normalized size = 1.22

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3 d+\_R^2 c+\_R b+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan}{}$
meijerg	$\frac{d\ln\left(1+\frac{3x^4}{2}\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}\right.}{2\left(x^4\right.}\right.}{216}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*(ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6
^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+2*arctan(1/6*2^(1/
2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1))+1/12*b*a
rctan(1/2*x^2*6^(1/2))*6^(1/2)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*(ln((x^2-1/3
*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+
1/3*6^(1/2)))+2*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+2*arctan(1/6*2^(1/2
)*3^(1/2)*6^(3/4)*x-1))+1/12*d*ln(3*x^4+2)
```

Maxima [A]
time = 0.51, size = 207, normalized size = 1.18

$$-\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c-2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c-6\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c+6\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d - 3*a)*log(
sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(sqrt(3)
*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x
 + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c - 6*s
qrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/7
2*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c + 6*sqrt(2)*b)*arctan(
1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.85, size = 54479, normalized size = 309.54

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/6718464*(((46656*I*sqrt(6)*b + 279936*d + 46656*sqrt(-sqrt(6)*(2*sqrt(6)
*a*c - 3*I*a^2 + 2*I*c^2)))^2 - 13060694016*I*sqrt(6)*sqrt(-sqrt(6)*(2*sqrt
(6)*a*c - 3*I*a^2 + 2*I*c^2))*b - 39182082048*b^2 - 78364164096*a*c - 78364
164096*d^2 - 6530347008*I*sqrt(6)*(3*a^2 - 2*c^2 + 4*b*d) - 26121388032*sqr
t(-sqrt(6)*(2*sqrt(6)*a*c - 3*I*a^2 + 2*I*c^2))*d)*(-I*sqrt(3) + 1)/((46656
*I*sqrt(6)*b + 279936*d + 46656*sqrt(-sqrt(6)*(2*sqrt(6)*a*c - 3*I*a^2 + 2*
I*c^2)))^3 + 24679069470425088*a^2*b - 16452712980283392*b*c^2 + 5484237660
094464*I*sqrt(6)*sqrt(-sqrt(6)*(2*sqrt(6)*a*c - 3*I*a^2 + 2*I*c^2))*b*d + 1
6452712980283392*b^2*d + 32905425960566784*a*c*d + 10968475320188928*d^3 -
457019805007872*(-2*I*sqrt(6)*sqrt(-sqrt(6)*(2*sqrt(6)*a*c - 3*I*a^2 + 2*I*
c^2))*b - 6*b^2 - 12*a*c - 12*d^2 - I*sqrt(6)*(3*a^2 - 2*c^2 + 4*b*d) - 4*s
qrt(-sqrt(6)*(2*sqrt(6)*a*c - 3*I*a^2 + 2*I*c^2))*d)*(-I*sqrt(6)*b - 6*d -
sqrt(-sqrt(6)*(2*sqrt(6)*a*c - 3*I*a^2 + 2*I*c^2))) + 8226356490141696*I*sq
rt(6)*(b^3 + a^2*d) - 548 ...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(156) = 312$.
time = 5.19, size = 580, normalized size = 3.30

$$\operatorname{RootSum}\left(165888t^4 - 55296t^3d + t^2\cdot(6912ac + 3456b^2 + 6912d^2) + t(-864a^2b - 1152acd - 576b^2d + 576bc^2 - 384d^3) + 27a^4 + 72a^2bd + 36a^2c^2 - 72ab^2c + 48acd^2 + 18b^4 + 24b^2d^2 - 48bc^2d + 12c^4 + 8d^4, \left(t \mapsto t\log\left(x + \frac{-41472t^3a^2c + 82944t^3ab^2 + 27648t^3c^3 + 5184t^2a^3b + 10368t^2a^2cd - 20736t^2ab^2d + 10368t^2abc^2 - 6912t^2b^3c - 6912t^2c^3d + 648ta^5 - 864ta^3bd - 1728ta^3c^2 + 3888ta^2b^2c - 864ta^2cd^2 + 864tab^4 + 1728tab^2d^2 - 1728tabc^2d + 864tac^4 + 1152tb^3cd + 864tb^2c^3 + 576tc^3d^2 - 54a^5d + 270a^4bc - 270a^3b^3 + 36a^3bd^2 + 144a^3c^2d - 324a^2b^2cd + 24a^2cd^3 - 72ab^4d + 180ab^3c^2 - 48ab^2d^3 + 72abc^2d^2 - 72ac^4d - 72b^5c - 48b^3cd^2 - 72b^2c^3d + 72bc^5 - 16c^3d^3}{81a^6 - 54a^4c^2 + 432a^3b^2c - 216a^2b^4 - 36a^2c^4 + 288ab^2c^3 - 144b^4c^2 + 24c^6}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 3456*b**2 + 6912*d
**2) + _t*(-864*a**2*b - 1152*a*c*d - 576*b**2*d + 576*b*c**2 - 384*d**3) +
 27*a**4 + 72*a**2*b*d + 36*a**2*c**2 - 72*a*b**2*c + 48*a*c*d**2 + 18*b**4
 + 24*b**2*d**2 - 48*b*c**2*d + 12*c**4 + 8*d**4, Lambda(_t, _t*log(x + (-4
1472*_t**3*a**2*c + 82944*_t**3*a*b**2 + 27648*_t**3*c**3 + 5184*_t**2*a**3
*b + 10368*_t**2*a**2*c*d - 20736*_t**2*a*b**2*d + 10368*_t**2*a*b*c**2 - 6
912*_t**2*b**3*c - 6912*_t**2*c**3*d + 648*_t*a**5 - 864*_t*a**3*b*d - 1728
*_t*a**3*c**2 + 3888*_t*a**2*b**2*c - 864*_t*a**2*c*d**2 + 864*_t*a*b**4 +
1728*_t*a*b**2*d**2 - 1728*_t*a*b*c**2*d + 864*_t*a*c**4 + 1152*_t*b**3*c*d
 + 864*_t*b**2*c**3 + 576*_t*c**3*d**2 - 54*a**5*d + 270*a**4*b*c - 270*a**
3*b**3 + 36*a**3*b*d**2 + 144*a**3*c**2*d - 324*a**2*b**2*c*d + 24*a**2*c*d
**3 - 72*a*b**4*d + 180*a*b**3*c**2 - 48*a*b**2*d**3 + 72*a*b*c**2*d**2 - 7
2*a*c**4*d - 72*b**5*c - 48*b**3*c*d**2 - 72*b**2*c**3*d + 72*b*c**5 - 16*c
**3*d**3)/(81*a**6 - 54*a**4*c**2 + 432*a**3*b**2*c - 216*a**2*b**4 - 36*a*
*2*c**4 + 288*a*b**2*c**3 - 144*b**4*c**2 + 24*c**6))))
```

Giac [A]
time = 1.05, size = 149, normalized size = 0.85

$$\frac{1}{24}\left(6^{\frac{3}{4}}a - 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c + 4d\right)\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c - 4d\right)\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")

[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))

Mupad [B]
time = 5.64, size = 1168, normalized size = 6.64

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2 + d*x^3)/(3*x^4 + 2),x)

[Out] symsum(log(9*a*b^2 - 864*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216 + a^4/6144, z, k)^2*a - 9*a^2*c - 6*a*d^2 + 9*b^3*x - 6*c^3 + 144*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*a*d - 144*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*b*c + 12*b*c*d - 108*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*a^2*x + 864*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216 + a^4/6144, z, k)^2*b*x + 72*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*c^2*x + 9*a^2*d*x + 6*b*d^2*x - 6*c^2*d*x - 144*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^

```
2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 -
 (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)
/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216
+ a^4/6144, z, k)*b*d*x - 18*a*b*c*x)*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 +
 (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/28
8 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*
d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^
4/13824 + b^4/9216 + a^4/6144, z, k), k, 1, 4)
```

3.167 $\int \frac{1+x+x^2+x^3}{1-x^4}\,dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

```
[Out] -ln(1-x)
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1600, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

```
[Out] -Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2+x^3}{1-x^4}\,dx &= \int \frac{1}{1-x}\,dx\\ &= -\log(1-x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

[Out] -Log[1 - x]

Maple [A]
time = 0.32, size = 7, normalized size = 0.88

method	result
default	$-\ln \left(x -1\right)$
norman	$-\ln \left(x -1\right)$
risch	$-\ln \left(x -1\right)$
meijerg	$-\frac{\ln \left(-x^{4}+1\right)}{4}-\frac{x^{3} \left(\ln \left(1-\left(x^{4}\right)^{\frac{1}{4}}\right)-\ln \left(1+\left(x^{4}\right)^{\frac{1}{4}}\right)+2 \arctan \left(\left(x^{4}\right)^{\frac{1}{4}}\right)\right)}{4 \left(x^{4}\right)^{\frac{3}{4}}}+\frac{\operatorname{arctanh}\left(x^{2}\right)}{2}-\frac{x \left(\ln \left(1-\left(x^{4}\right)^{\frac{1}{4}}\right)-\ln \left(1+\left(x^{4}\right)^{\frac{1}{4}}\right)\right.}{4 \left(x^{4}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+x+1)/(-x^4+1),x,method=_RETURNVERBOSE)

[Out] -ln(x-1)

Maxima [A]
time = 0.29, size = 6, normalized size = 0.75

$$-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="maxima")

[Out] -log(x - 1)

Fricas [A]
time = 0.39, size = 6, normalized size = 0.75

$$-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="fricas")

[Out] -log(x - 1)

Sympy [A]
time = 0.01, size = 5, normalized size = 0.62

$$-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2+x+1)/(-x**4+1),x)

```
[Out] -log(x - 1)
```

Giac [**A**]
time = 1.11, size = 7, normalized size = 0.88

$$-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")
[Out] -log(abs(x - 1))
```

Mupad [**B**]
time = 0.02, size = 6, normalized size = 0.75

$$-\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2 + x^3 + 1)/(x^4 - 1),x)
[Out] -log(x - 1)
```

3.168 $\int \frac{1+x+x^2+x^3}{1+x^4} \, dx$

Optimal. Leaf size=53

$$\frac{1}{2}\tan^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{\sqrt{2}}+\frac{1}{4}\log\left(1+x^4\right)$$

```
[Out] 1/2*arctan(x^2)+1/4*ln(x^4+1)+1/2*arctan(-1+x*2^(1/2))*2^(1/2)+1/2*arctan(1
+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1890, 1176, 631, 210, 1262, 649, 209, 266}

$$\frac{\text{ArcTan}(x^2)}{2}-\frac{\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{\sqrt{2}}+\frac{1}{4}\log\left(x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

```
[Out] ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/Sqrt[
2] + Log[1 + x^4]/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2+x^3}{1+x^4}\,dx &= \int \left(\frac{1+x^2}{1+x^4}+\frac{x(1+x^2)}{1+x^4}\right)dx \\
&= \int \frac{1+x^2}{1+x^4}\,dx+\int \frac{x(1+x^2)}{1+x^4}\,dx \\
&= \frac{1}{2}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx+\frac{1}{2}\text{Subst}\left(\int \frac{1+x}{1+x^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,x^2\right)+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,\right.}{\sqrt{2}} \\
&= \frac{1}{2}\tan^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{\sqrt{2}}+\frac{1}{4}\log\left(1+x^4\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.94

$$\frac{1}{4}\left(-2\left(1+\sqrt{2}\right)\tan^{-1}\left(1-\sqrt{2}\,x\right)+2\left(-1+\sqrt{2}\right)\tan^{-1}\left(1+\sqrt{2}\,x\right)+\log\left(1+x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

```
[Out] (-2*(1 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(-1 + Sqrt[2])*ArcTan[1 + Sqrt[
2]*x] + Log[1 + x^4])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(41) = 82.
time = 0.36, size = 118, normalized size = 2.23

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+1)}\frac{\left(\_R^3+\_R^2+\_R+1\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}+\frac{\arctan(x^2)}{2}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\,\mathrm{arcta}\right.}{}$
meijerg	$\frac{\ln(x^4+1)}{4}+\frac{x^3\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x}\right.}{8(x^4)^{\frac{3}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*2^(1/2)*(ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))+2*arctan(x*2^(1/2)+1)+
2*arctan(x*2^(1/2)-1))+1/2*arctan(x^2)+1/8*2^(1/2)*(ln((1+x^2-x*2^(1/2))/(1
+x^2+x*2^(1/2)))+2*arctan(x*2^(1/2)+1)+2*arctan(x*2^(1/2)-1))+1/4*ln(x^4+1)
```

Maxima [A]
time = 0.49, size = 76, normalized size = 1.43

$$-\frac{1}{4}\sqrt{2}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\left(\sqrt{2}+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{4}\log\left(x^2+\sqrt{2}\,x+1\right)+\frac{1}{4}\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2
)*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(2)
*x + 1) + 1/4*log(x^2 - sqrt(2)*x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(41) = 82$.
time = 0.37, size = 150, normalized size = 2.83

$$-\sqrt{-2\sqrt{2}+3}\arctan\left(\sqrt{x^2+\sqrt{2}x+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+3}-\left(\sqrt{2}(x+1)+2x+1\right)\sqrt{-2\sqrt{2}+3}\right)+\sqrt{2\sqrt{2}+3}\arctan\left(-\left(\sqrt{2}(x+1)-\sqrt{x^2-\sqrt{2}x+1}\left(\sqrt{2}-2\right)-2x-1\right)\sqrt{2\sqrt{2}+3}\right)+\frac{1}{4}\log\left(4x^2+4\sqrt{2}x+4\right)+\frac{1}{4}\log\left(4x^2-4\sqrt{2}x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -sqrt(-2*sqrt(2) + 3)*arctan(sqrt(x^2 + sqrt(2)*x + 1)*(sqrt(2) + 2)*sqrt(-
2*sqrt(2) + 3) - (sqrt(2)*(x + 1) + 2*x + 1)*sqrt(-2*sqrt(2) + 3)) + sqrt(2
*sqrt(2) + 3)*arctan(-(sqrt(2)*(x + 1) - sqrt(x^2 - sqrt(2)*x + 1)*(sqrt(2)
 - 2) - 2*x - 1)*sqrt(2*sqrt(2) + 3)) + 1/4*log(4*x^2 + 4*sqrt(2)*x + 4) +
1/4*log(4*x^2 - 4*sqrt(2)*x + 4)
```

Sympy [A]
time = 0.17, size = 73, normalized size = 1.38

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4}+2\cdot\left(\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x-1\right)+2\left(-\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(x**4+1),x)
```

```
[Out] log(x**2 - sqrt(2)*x + 1)/4 + log(x**2 + sqrt(2)*x + 1)/4 + 2*(1/4 + sqrt(2
)/4)*atan(sqrt(2)*x - 1) + 2*(-1/4 + sqrt(2)/4)*atan(sqrt(2)*x + 1)
```

Giac [A]
time = 1.06, size = 70, normalized size = 1.32

$$\frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{4}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*(sqrt(2) + 1)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(2)*x + 1) + 1/4*log
(x^2 - sqrt(2)*x + 1)
```

Mupad [B]
time = 0.40, size = 156, normalized size = 2.94

$$\ln\left((16x-16)\left(\frac{\sqrt{-2\sqrt{2}-3}}{4}+\frac{1}{4}\right)-8x\right)\left(\frac{\sqrt{-2\sqrt{2}-3}}{4}+\frac{1}{4}\right)-\ln\left(8x+(16x-16)\left(\frac{\sqrt{-2\sqrt{2}-3}}{4}-\frac{1}{4}\right)\right)\left(\frac{\sqrt{-2\sqrt{2}-3}}{4}-\frac{1}{4}\right)-\ln\left(8x+(16x-16)\left(\frac{\sqrt{2\sqrt{2}-3}}{4}-\frac{1}{4}\right)\right)\left(\frac{\sqrt{2\sqrt{2}-3}}{4}-\frac{1}{4}\right)+\ln\left(8x-(16x-16)\left(\frac{\sqrt{2\sqrt{2}-3}}{4}+\frac{1}{4}\right)\right)\left(\frac{\sqrt{2\sqrt{2}-3}}{4}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 1)/(x^4 + 1),x)
```

```
[Out] log((16*x - 16)*((- 2*2^(1/2) - 3)^(1/2)/4 + 1/4) - 8*x)*((- 2*2^(1/2) - 3)
^(1/2)/4 + 1/4) - log(8*x + (16*x - 16)*((- 2*2^(1/2) - 3)^(1/2)/4 - 1/4))*
((- 2*2^(1/2) - 3)^(1/2)/4 - 1/4) - log(8*x + (16*x - 16)*((2*2^(1/2) - 3)^
(1/2)/4 - 1/4))*((2*2^(1/2) - 3)^(1/2)/4 - 1/4) + log(8*x - (16*x - 16)*((2
*2^(1/2) - 3)^(1/2)/4 + 1/4))*((2*2^(1/2) - 3)^(1/2)/4 + 1/4)
```

3.169 $\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx$

Optimal. Leaf size=124

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{\log\left(a-bx^4\right)}{4b}$$

```
[Out] -1/4*ln(-b*x^4+a)/b-1/2*arctan(b^(1/4)*x/a^(1/4))*(a^(1/2)-b^(1/2))/a^(3/4)
/b^(3/4)+1/2*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)+1/2*arctanh(b^(1/
4)*x/a^(1/4))*(a^(1/2)+b^(1/2))/a^(3/4)/b^(3/4)
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1890, 1181, 211, 214, 1262, 649, 266}

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}-\frac{\log\left(a-bx^4\right)}{4b}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

```
[Out] -1/2*((Sqrt[a] - Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(a^(3/4)*b^(3/4)) +
((Sqrt[a] + Sqrt[b])*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - Log[a - b*x^4]/(4*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx &= \int \left(\frac{1+x^2}{a-bx^4}+\frac{x(1+x^2)}{a-bx^4}\right)dx\\
&= \int \frac{1+x^2}{a-bx^4}\,dx+\int \frac{x(1+x^2)}{a-bx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{-\sqrt{a}\,\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(1+\cdots\right.\\
&= -\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{1}{2}\text{Subst}\left(\int \cdots\right.\\
&= -\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt{a}\cdots}\right.}{2\sqrt{a}\,\sqrt{b}\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 203, normalized size = 1.64

$$\frac{\left(-a^{3/4}+\sqrt[4]{a}\,\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2ab^{3/4}}-\frac{\left(a^{3/4}+\sqrt{a}\,\sqrt[4]{b}+\sqrt[4]{a}\,\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)}{4ab^{3/4}}-\frac{\left(-a^{3/4}+\sqrt{a}\,\sqrt[4]{b}-\sqrt[4]{a}\,\sqrt{b}\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{4ab^{3/4}}+\frac{\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{a}\,\sqrt{b}}-\frac{\log\left(a-bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

```
[Out] ((-a^(3/4) + a^(1/4)*Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a*b^(3/4)) -
((a^(3/4) + Sqrt[a]*b^(1/4) + a^(1/4)*Sqrt[b])*Log[a^(1/4) - b^(1/4)*x])/(4
*a*b^(3/4)) - ((-a^(3/4) + Sqrt[a]*b^(1/4) - a^(1/4)*Sqrt[b])*Log[a^(1/4) +
 b^(1/4)*x])/(4*a*b^(3/4)) + Log[Sqrt[a] + Sqrt[b]*x^2]/(4*Sqrt[a]*Sqrt[b])
 - Log[a - b*x^4]/(4*b)
```

Maple [A]
time = 0.36, size = 150, normalized size = 1.21

method	result	si
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(\_R^3+\_R^2+\_R+1\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	38
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{\ln\left(-b\,x^4+a\right)}{4b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/
4)))+1/4/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2)))-1/4/b/(a/b
)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4))))-1/4*l
n(-b*x^4+a)/b
```

Maxima [A]
time = 0.51, size = 160, normalized size = 1.29

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{4\sqrt{a}\,b}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{4\sqrt{a}\,b}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(a) - sqrt(b))*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(a) - sqrt(b))*log(sqrt(b)*x^2 + s
qrt(a))/(sqrt(a)*b) - 1/4*(sqrt(a) + sqrt(b))*log(sqrt(b)*x^2 - sqrt(a))/(s
qrt(a)*b) - 1/4*(sqrt(a) + sqrt(b))*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))
/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b
))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.44, size = 91748, normalized size = 739.90

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/48*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(((b*sqrt(1/(a*b))*sqrt(((2*a*sqrt
(1/(a*b)) + 1)*b + a)/(a^2*b^2*sqrt(1/(a*b)))) - 3*sqrt(1/(a*b)))*a + 1)^2/
(a*b) + 3*((2*b*sqrt(1/(a*b))*sqrt(((2*a*sqrt(1/(a*b)) + 1)*b + a)/(a^2*b^2
*sqrt(1/(a*b)))) - 3*sqrt(1/(a*b)))*a^2 + (b*(3*sqrt(1/(a*b)) - 2*sqrt(((2*
a*sqrt(1/(a*b)) + 1)*b + a)/(a^2*b^2*sqrt(1/(a*b))))) + 1)*a - b)/(a^2*b^2*
sqrt(1/(a*b))))/(9*((2*b*sqrt(1/(a*b))*sqrt(((2*a*sqrt(1/(a*b)) + 1)*b + a)
/(a^2*b^2*sqrt(1/(a*b)))) - 3*sqrt(1/(a*b)))*a^2 + (b*(3*sqrt(1/(a*b)) - 2*
sqrt(((2*a*sqrt(1/(a*b)) + 1)*b + a)/(a^2*b^2*sqrt(1/(a*b))))) + 1)*a - b)*
((b*sqrt(1/(a*b))*sqrt(((2*a*sqrt(1/(a*b)) + 1)*b + a)/(a^2*b^2*sqrt(1/(a*b
)))) - 3*sqrt(1/(a*b)))*a + 1)/(a^2*b^2) + 2*((b*sqrt(1/(a*b))*sqrt(((2*a*s
qrt(1/(a*b)) + 1)*b + a)/(a^2*b^2*sqrt(1/(a*b)))) - 3*sqrt(1/(a*b)))*a + 1)
^3/(a^3*b^3*(1/(a*b))^(3/2)) + 27*((b^3*sqrt(1/(a*b))*(((2*a*sqrt(1/(a*b))
+ 1)*b + a)/(a^2*b^2*sqrt(1/(a*b))))^(3/2) + b*sqrt(1/(a*b))*sqrt(((2*a*sqr
t(1/(a*b)) + 1)*b + a)/(a ...
```

Sympy [A]
time = 0.95, size = 187, normalized size = 1.51

$$-\operatorname{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3 + t^2\cdot\left(96a^3b^2 - 96a^2b^3\right) + t\left(-16a^3b + 32a^2b^2 - 16ab^3\right) + a^3 - 3a^2b + 3ab^2 - b^3, \left(t \mapsto t\log\left(x + \frac{-64t^3a^3b^3 + 48t^2a^3b^2 + 16t^2a^2b^3 - 12ta^3b + 16ta^2b^2 - 4tab^3 + a^3 - 2a^2b + ab^2}{a^2b - 2ab^2 + b^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**3*b**2 -
96*a**2*b**3) + _t*(-16*a**3*b + 32*a**2*b**2 - 16*a*b**3) + a**3 - 3*a**2*
b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**3 + 48*_t**2*
a**3*b**2 + 16*_t**2*a**2*b**3 - 12*_t*a**3*b + 16*_t*a**2*b**2 - 4*_t*a*b*
*3 + a**3 - 2*a**2*b + a*b**2)/(a**2*b - 2*a*b**2 + b**3))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(84) = 168.
time = 1.28, size = 290, normalized size = 2.34

$$-\frac{\log\left(|bx^4 - a|\right)}{4\,b} + \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2 - \sqrt{2}\sqrt{-ab^3}\,b + \left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2 + \sqrt{2}\sqrt{-ab^3}\,b + \left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2 - \left(-ab^3\right)^{\frac{3}{4}}\right)\log\left(x^2 + \sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2 - \left(-ab^3\right)^{\frac{3}{4}}\right)\log\left(x^2 - \sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(b*x^4 - a))/b + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 - sqrt(2)*sqrt
(-a*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)
)/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt(-a
*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(
-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*lo
g(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3
)^(1/4)*b^2 - (-a*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b)
)/(a*b^3)
```

Mupad [B]
time = 5.03, size = 312, normalized size = 2.52

$$\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2+16\,a^3\,b\,z+16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b+3\,a\,b^2-b^3+a^3,z,k\right)\,\left(\mathrm{root}\left(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2+16\,a^3\,b\,z+16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b+3\,a\,b^2-b^3+a^3,z,k\right)\,\left(16\,a\,b^3-16\,a\,b^3\,x\right)-x\,\left(4\,a\,b^2-4\,b^3\right)\right)\right)\,\mathrm{root}\left(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2+16\,a^3\,b\,z+16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b+3\,a\,b^2-b^3+a^3,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 1)/(a - b*x^4),x)
```

```
[Out] symsum(log(-root(256*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 - 96*a^
2*b^3*z^2 + 16*a^3*b*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*b^2 - b^
3 + a^3, z, k)*(root(256*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 - 9
6*a^2*b^3*z^2 + 16*a^3*b*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*b^2
- b^3 + a^3, z, k)*(16*a*b^3 - 16*a*b^3*x) - x*(4*a*b^2 - 4*b^3)))*root(256
*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 - 96*a^2*b^3*z^2 + 16*a^3*b
*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*b^2 - b^3 + a^3, z, k), k, 1
, 4)
```

3.170 $\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx$

Optimal. Leaf size=277

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}-\right.}{}$$

```
[Out] 1/4*ln(b*x^4+a)/b+1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(a
^(1/2)-b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)-1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^
(1/2)+x^2*b^(1/2))*(a^(1/2)-b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/2*arctan(x^2
*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)+1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*
(a^(1/2)+b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^
(1/4))*(a^(1/2)+b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$-\frac{\left(\sqrt{a}+\sqrt{b}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}+\frac{\log\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

```
[Out] ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt
[a] + Sqrt[b])*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
b^(3/4)) + ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) +
Log[a + b*x^4]/(4*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx &= \int \left(\frac{1+x^2}{a+bx^4}+\frac{x(1+x^2)}{a+bx^4}\right)dx \\
&= \int \frac{1+x^2}{a+bx^4}\,dx+\int \frac{x(1+x^2)}{a+bx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{a+bx^2}\,dx,x,x^2\right)-\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\int \cdots}{2\cdots} \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx,x,x^2\right)+\frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\int \cdots}{\cdots} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}-\cdots\right.}{\cdots} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\cdots\right.}{2\sqrt{2}\,a^{3/4}\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 283, normalized size = 1.02

$$\frac{-2\sqrt[4]{a}\left(\sqrt{2}\sqrt{a}+2\sqrt[4]{a}\sqrt[4]{b}+\sqrt{2}\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\left(\sqrt{2}\sqrt{a}-2\sqrt[4]{a}\sqrt[4]{b}+\sqrt{2}\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\sqrt{2}\left(a^{3/4}-\sqrt[4]{a}\sqrt{b}\right)\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+\sqrt{2}\left(-a^{3/4}+\sqrt[4]{a}\sqrt{b}\right)\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+2a\log\left(a+bx^4\right)}{8ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

```
[Out] (-2*a^(1/4)*(Sqrt[2]*Sqrt[a] + 2*a^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*(Sqrt[2]*Sqrt[a] - 2*a
^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^
(1/4)] + Sqrt[2]*(a^(3/4) - a^(1/4)*Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(-a^(3/4) + a^(1/4)*Sqrt[b])*b^(
1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a*Log[a + b
*x^4])/(8*a*b)
```

Maple [A]
time = 0.35, size = 236, normalized size = 0.85

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(\_R^3+\_R^2+\_R+1\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{\sqrt{2}\left(\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-
(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arc
tan(2^(1/2)/(a/b)^(1/4)*x-1))+1/2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8/b
/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)
^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2
^(1/2)/(a/b)^(1/4)*x-1))+1/4*ln(b*x^4+a)/b
```

Maxima [A]
time = 0.51, size = 296, normalized size = 1.07

$$\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}-\sqrt{a}\sqrt{b}+b\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}+\sqrt{a}\sqrt{b}-b\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\left(\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{a}\right)b+\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}+2a\right)\sqrt{b}-2a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}+\frac{\left(\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{a}\right)b+\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}-2a\right)\sqrt{b}+2a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*b^(1/4) - sqrt(a)*sqrt(b) + b)*log(sqrt(b)*x^2
 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/8*sqrt(2)*(sq
rt(2)*a^(3/4)*b^(1/4) + sqrt(a)*sqrt(b) - b)*log(sqrt(b)*x^2 - sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/4*((sqrt(2)*a^(1/4)*b^(1/4)
- 2*sqrt(a))*b + (sqrt(2)*a^(3/4)*b^(1/4) + 2*a)*sqrt(b) - 2*a*sqrt(b))*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 1/4*((sqrt(2)*a^(1/4)*b^(1/4)
```

```
 + 2*sqrt(a))*b + (sqrt(2)*a^(3/4)*b^(1/4) - 2*a)*sqrt(b) + 2*a*sqrt(b))*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.49, size = 96349, normalized size = 347.83

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/48*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(((b*sqrt(-1/(a*b))*sqrt(-((2*a*sqr
t(-1/(a*b)) + 1)*b - a)/(a^2*b^2*sqrt(-1/(a*b)))) + 3*sqrt(-1/(a*b)))*a - 1
)^2/(a*b) + 3*((2*b*sqrt(-1/(a*b))*sqrt(-((2*a*sqrt(-1/(a*b)) + 1)*b - a)/(
a^2*b^2*sqrt(-1/(a*b)))) + 3*sqrt(-1/(a*b)))*a^2 + (b*(3*sqrt(-1/(a*b)) - 2
*sqrt(-((2*a*sqrt(-1/(a*b)) + 1)*b - a)/(a^2*b^2*sqrt(-1/(a*b))))) - 1)*a -
 b)/(a^2*b^2*sqrt(-1/(a*b)))))/(9*((2*b*sqrt(-1/(a*b))*sqrt(-((2*a*sqrt(-1/(
a*b)) + 1)*b - a)/(a^2*b^2*sqrt(-1/(a*b)))) + 3*sqrt(-1/(a*b)))*a^2 + (b*(3
*sqrt(-1/(a*b)) - 2*sqrt(-((2*a*sqrt(-1/(a*b)) + 1)*b - a)/(a^2*b^2*sqrt(-1
/(a*b))))) - 1)*a - b)*((b*sqrt(-1/(a*b))*sqrt(-((2*a*sqrt(-1/(a*b)) + 1)*b
 - a)/(a^2*b^2*sqrt(-1/(a*b)))) + 3*sqrt(-1/(a*b)))*a - 1)/(a^2*b^2) + 2*((
b*sqrt(-1/(a*b))*sqrt(-((2*a*sqrt(-1/(a*b)) + 1)*b - a)/(a^2*b^2*sqrt(-1/(a
*b)))) + 3*sqrt(-1/(a*b)))*a - 1)^3/(a^3*b^3*(-1/(a*b))^(3/2)) + 27*((b^3*s
qrt(-1/(a*b))*(-((2*a*sqrt(-1/(a*b)) + 1)*b - a)/(a^2*b^2*sqrt(-1/(a*b))))^
(3/2) + b*sqrt(-1/(a*b))* ...
```

Sympy [A]
time = 1.12, size = 187, normalized size = 0.68

$$\operatorname{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3 + t^2\cdot\left(96a^3b^2 + 96a^2b^3\right) + t\left(-16a^3b - 32a^2b^2 - 16ab^3\right) + a^3 + 3a^2b + 3ab^2 + b^3, \left(t \mapsto t\log\left(x + \frac{64t^3a^3b^3 - 48t^2a^3b^2 + 16t^2a^2b^3 + 12ta^3b + 16ta^2b^2 + 4tab^3 - a^3 - 2a^2b - ab^2}{a^2b + 2ab^2 + b^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**3*b**2 + 9
6*a**2*b**3) + _t*(-16*a**3*b - 32*a**2*b**2 - 16*a*b**3) + a**3 + 3*a**2*b
 + 3*a*b**2 + b**3, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**3 - 48*_t**2*a*
*3*b**2 + 16*_t**2*a**2*b**3 + 12*_t*a**3*b + 16*_t*a**2*b**2 + 4*_t*a*b**3
 - a**3 - 2*a**2*b - a*b**2)/(a**2*b + 2*a*b**2 + b**3))))
```

Giac [A]
time = 1.80, size = 270, normalized size = 0.97

$$\frac{\log\left(|bx^4 + a|\right)}{4\,b} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2 - \sqrt{2}\sqrt{ab^3}\,b + \left(ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2 + \sqrt{2}\sqrt{ab^3}\,b + \left(ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2 - \left(ab^3\right)^{\frac{3}{4}}\right)\log\left(x^2 + \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2 - \left(ab^3\right)^{\frac{3}{4}}\right)\log\left(x^2 - \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 - sqrt(2)*sqrt(a
*b^3)*b + (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt(a*b^3)*b
+ (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*log(x^2 + sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2 - (a
*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

Mupad [B]
time = 5.04, size = 305, normalized size = 1.10

$$\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(256\,a^3\,b^4\,z^4-256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2+96\,a^2\,b^3\,z^2-16\,a^3\,b\,z-16\,a\,b^3\,z-32\,a^2\,b^2\,z+3\,a^2\,b+3\,a\,b^2+b^3+a^3,z,k\right)\,\left(\mathrm{root}\left(256\,a^3\,b^4\,z^4-256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2+96\,a^2\,b^3\,z^2-16\,a^3\,b\,z-16\,a\,b^3\,z-32\,a^2\,b^2\,z+3\,a^2\,b+3\,a\,b^2+b^3+a^3,z,k\right)\,\left(16\,a\,b^3-16\,a\,b^3\,x\right)+x\,\left(4\,b^3+4\,a\,b^2\right)\right)\right)\,\mathrm{root}\left(256\,a^3\,b^4\,z^4-256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2+96\,a^2\,b^3\,z^2-16\,a^3\,b\,z-16\,a\,b^3\,z-32\,a^2\,b^2\,z+3\,a^2\,b+3\,a\,b^2+b^3+a^3,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 1)/(a + b*x^4),x)
```

```
[Out] symsum(log(-root(256*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 + 96*a^
2*b^3*z^2 - 16*a^3*b*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*b^2 + b^
3 + a^3, z, k)*(root(256*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 + 9
6*a^2*b^3*z^2 - 16*a^3*b*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*b^2
+ b^3 + a^3, z, k)*(16*a*b^3 - 16*a*b^3*x) + x*(4*a*b^2 + 4*b^3)))*root(256
*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 + 96*a^2*b^3*z^2 - 16*a^3*b
*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*b^2 + b^3 + a^3, z, k), k, 1
, 4)
```

3.171 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx$

Optimal. Leaf size=148

$$-\frac{gx}{b}+\frac{\left(bc-\sqrt{a}\,\sqrt{b}\,e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\,\sqrt{b}\,e+ag\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

```
[Out] -g*x/b-1/4*f*ln(-b*x^4+a)/b+1/2*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1
/2)+1/2*arctan(b^(1/4)*x/a^(1/4))*(b*c+a*g-e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/
4)+1/2*arctanh(b^(1/4)*x/a^(1/4))*(b*c+a*g+e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/
4)
```

Rubi [A]
time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1899, 1262, 649, 214, 266, 1901, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\,\sqrt{b}\,e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,\sqrt{b}\,e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

```
[Out] -((g*x)/b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/
(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTanh[(b^(1/4)*x)/
a^(1/4)])/(2*a^(3/4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[
a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx &= \int \left(\frac{x(d+fx^2)}{a-bx^4}+\frac{c+ex^2+gx^4}{a-bx^4}\right)dx \\
&= \int \frac{x(d+fx^2)}{a-bx^4}\,dx + \int \frac{c+ex^2+gx^4}{a-bx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{a-bx^2}\,dx, x, x^2\right) + \int \left(-\frac{g}{b}+\frac{bc+ag+bex^2}{b\,(a-bx^4)}\right)dx \\
&= -\frac{gx}{b} + \frac{\int \frac{bc+ag+bex^2}{a-bx^4}\,dx}{b} + \frac{1}{2}d\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right) + \frac{1}{2}f\text{Subst}\Bigg(\\
&= -\frac{gx}{b} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}} - \frac{f\log\left(a-bx^4\right)}{4b} + \frac{1}{2}\left(e-\frac{bc+ag}{\sqrt{a}\,\sqrt{b}}\right)\int \frac{\ }{-\ } \\
&= -\frac{gx}{b} + \frac{\left(bc-\sqrt{a}\,\sqrt{b}\,e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}} + \frac{\left(bc+\sqrt{a}\,\sqrt{b}\,e+ag\right)}{2a^{3/4}b^{5/}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 249, normalized size = 1.68

$$
\frac{-4a^{3/4}\sqrt[4]{b}\,gx+2\left(bc-\sqrt{a}\,\sqrt{b}\,e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\left(bc+\sqrt[4]{a}\,b^{3/4}d+\sqrt{a}\,\sqrt{b}\,e+ag\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+bc\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)-\sqrt[4]{a}\,b^{3/4}d\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+\sqrt{a}\,\sqrt{b}\,e\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+ag\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+\sqrt[4]{a}\,b^{3/4}d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)-a^{3/4}\sqrt[4]{b}\,f\log\left(a-bx^4\right)}{4a^{3/4}b^{5/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

```
[Out] (-4*a^(3/4)*b^(1/4)*g*x + 2*(b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)
*x)/a^(1/4)] - (b*c + a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e + a*g)*Log[a^(1
/4) - b^(1/4)*x] + b*c*Log[a^(1/4) + b^(1/4)*x] - a^(1/4)*b^(3/4)*d*Log[a^(
1/4) + b^(1/4)*x] + Sqrt[a]*Sqrt[b]*e*Log[a^(1/4) + b^(1/4)*x] + a*g*Log[a^
(1/4) + b^(1/4)*x] + a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2] - a^(3/4)
*b^(1/4)*f*Log[a - b*x^4])/(4*a^(3/4)*b^(5/4))
```

Maple [A]
time = 0.34, size = 167, normalized size = 1.13

method	result
risch	$-\frac{gx}{b}+\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^4-a\right)}\frac{\left(-\_R^3bf-\_R^2be-\_Rbd-ag-bc\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$

default	$-\frac{gx}{b}+\frac{\frac{(ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{bd\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{f\ln\left(-b\,x^4\right.}{4}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -g*x/b+1/b*(1/4*(a*g+b*c)*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4))
)+2*arctan(x/(a/b)^(1/4)))+1/4*b*d/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^
2*(a*b)^(1/2)))-1/4*e/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4
))/(x-(a/b)^(1/4))))-1/4*f*ln(-b*x^4+a))
```

Maxima [A]
time = 0.51, size = 204, normalized size = 1.38

$$-\frac{gx}{b}+\frac{\frac{2\left(b^{\frac{3}{2}}c+a\sqrt{b}\,g-\sqrt{a}\,be\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\left(b^{\frac{3}{2}}d-\sqrt{a}\,bf\right)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}d+\sqrt{a}\,bf\right)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}c+a\sqrt{b}\,g+\sqrt{a}\,be\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] -g*x/b + 1/4*(2*(b^(3/2)*c + a*sqrt(b)*g - sqrt(a)*b*e)*arctan(sqrt(b)*x/sq
rt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b^(3/2)*d -
 sqrt(a)*b*f)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (b^(3/2)*d + sqrt(a)
*b*f)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (b^(3/2)*c + a*sqrt(b)*g + s
qrt(a)*b*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(
a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

Fricas [C] Result contains complex when optimal does not.
time = 29.57, size = 592528, normalized size = 4003.57

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/48*(12*(d*sqrt(1/(a*b)) + sqrt(1/2)*sqrt(4*f^2/b^2 + 2*(2*f^3/b^3 - ((3*
f^2 - 2*e*g)*a - (d^2 + 2*c*e)*b)*f/(a*b^3) + (b^2*c^2*d + (f^3 - 2*e*f*g +
 d*g^2)*a^2 + (d*e^2 - d^2*f - 2*(e*f - d*g)*c)*a*b)/(a^2*b^3))/(d*sqrt(1/(
a*b))) - ((3*f^2 - 2*e*g)*a - (d^2 + 2*c*e)*b)/(a*b^2) - ((f^2 - 2*e*g)*a +
 (d^2 - 2*c*e)*b)/(a*b^2)) + f/b)*b*log(5*a*b^5*c^3*d^3 + 5*a^2*b^4*c*d^3*e
^2 - 3*a^3*b^3*d*e^5 + a^6*f*g^5 - 5*(a^5*b*d*e - a^5*b*c*f)*g^4 - (2*a^3*b
^6*c*d^2 - a^3*b^6*c^2*e - a^4*b^5*e^3 - a^5*b^4*e*g^2 + 2*(a^4*b^5*d^2 - a
```

```
^4*b^5*c*e)*g)*(d*sqrt(1/(a*b)) + sqrt(1/2)*sqrt(4*f^2/b^2 + 2*(2*f^3/b^3 -
 ((3*f^2 - 2*e*g)*a - (d^2 + 2*c*e)*b)*f/(a*b^3) + (b^2*c^2*d + (f^3 - 2*e*
f*g + d*g^2)*a^2 + (d*e^2 - d^2*f - 2*(e*f - d*g)*c)*a*b)/(a^2*b^3))/(d*sqr
t(1/(a*b))) - ((3*f^2 - 2*e*g)*a - (d^2 + 2*c*e)*b)/(a*b^2) - ((f^2 - 2*e*g
)*a + (d^2 - 2*c*e)*b)/(a*b^2)) + f/b)^3 + (2*a^3*b^3*c*d^2 - a^3*b^3*c^2*e
 - a^4*b^2*e^3)*f^3 + (5*a^4*b^2*d^3 - 20*a^4*b^2*c*d*e + a^5*b*d*f^2 + 2*(
5*a^4*b^2*c^2 + 2*a^5*b*e ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(110) = 220.
time = 1.14, size = 303, normalized size = 2.05

$$-\frac{\sqrt{2}\left(b^{2}c+abg-\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c+abg+\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c+abg-\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b^{2}c+abg-\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{gx}{b}-\frac{f\log\left(|bx^{4}-a|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c + a*b*g - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*
arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4
) - 1/4*sqrt(2)*(b^2*c + a*b*g + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*
e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(
3/4) - 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a
/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(
-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) -
g*x/b - 1/4*f*log(abs(b*x^4 - a))/b
```

Mupad [B]
time = 5.51, size = 2500, normalized size = 16.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x)
```

```
[Out] symsum(log(b^2*c^2*e - b^2*c*d^2 + a^2*e*g^2 - a^2*f^2*g - b^2*d^3*x - a*b*
e^3 - a*b*c*f^2 - a*b*d^2*g - 16*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 -
```

64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)^2*a*b^3*c - 4*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*b^3*c^2*x - b^2*c^2*f*x - a^2*f*g^2*x - 16*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)^2*a^2*b^2*g + 16*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)^2*a*b^3*d*x - 4*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*a*b^2*e^2*x - 4*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f

```
*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^
2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z +
8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2
*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*
f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2
*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^
4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*a^2*b*g^2*x + 2*a*b*c*e*g + 2*a*
b*d*e*f - 8*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 -
 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*
e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16
*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z
 + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*
e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2
*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2
- 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3
*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*a*b^2*c*f + 8*root(256*a^3*b^
5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^
3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*
z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b
^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*...
```

3.172 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$

Optimal. Leaf size=172

$$\frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab(a-bx^4)}+\frac{\left(3bc-\sqrt{a}\sqrt{b}\,e-ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}}+\frac{\left(3bc+\sqrt{a}\sqrt{b}\,e-ag\right)\tanh^{-1}\left(\right.}{8a^{7/4}b^{5/4}}$$

```
[Out] 1/4*x*(b*f*x^3+b*e*x^2+b*d*x+a*g+b*c)/a/b/(-b*x^4+a)+1/4*d*arctanh(x^2*b^(1
/2)/a^(1/2))/a^(3/2)/b^(1/2)+1/8*arctan(b^(1/4)*x/a^(1/4))*(3*b*c-a*g-e*a^(
1/2)*b^(1/2))/a^(7/4)/b^(5/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))*(3*b*c-a*g+e*a
^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)
```

Rubi [A]
time = 0.11, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1872, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}\,e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}\,e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x(ag+bc+bdx+bex^2+bfx^3)}{4ab(a-bx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + ((3*b*c -
 Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4))
+ ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/
4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a-bx^4\right)^2}\,dx &= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab\left(a-bx^4\right)}+\frac{\int \frac{3bc-ag+2bdx+bex^2}{a-bx^4}\,dx}{4ab}\\
&= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab\left(a-bx^4\right)}+\frac{\int \left(\frac{2bdx}{a-bx^4}+\frac{3bc-ag+bex^2}{a-bx^4}\right)dx}{4ab}\\
&= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab\left(a-bx^4\right)}+\frac{\int \frac{3bc-ag+bex^2}{a-bx^4}\,dx}{4ab}+\frac{d\int \frac{x}{a-bx^4}\,dx}{2a}\\
&= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab\left(a-bx^4\right)}+\frac{d\,\mathrm{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{4a}-\frac{\left(3bc-\ldots\right.}{\ldots}\\
&= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab\left(a-bx^4\right)}+\frac{\left(3bc-\sqrt{a}\,\sqrt{b}\,e-ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\ldots}{\sqrt[4]{a}}\ldots\right.}{8a^{7/4}b^{5/4}}
\end{aligned}
$$

Mathematica [A]

time = 0.22, size = 221, normalized size = 1.28

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}\,(a(f+gx)+bx(c+x(d+ex)))}{a-bx^4}-2\left(-3bc+\sqrt{a}\,\sqrt{b}\,e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\left(3bc+2\sqrt[4]{a}\,b^{3/4}d+\sqrt{a}\,\sqrt{b}\,e-ag\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+\left(3bc-2\sqrt[4]{a}\,b^{3/4}d+\sqrt{a}\,\sqrt{b}\,e-ag\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+2\sqrt[4]{a}\,b^{3/4}d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{16a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2*
(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (3*b*c + 2
*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) - b^(1/4)*x] + (3
*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) + b^(1/4)
*x] + 2*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(5/4))
```

Maple [A]

time = 0.34, size = 206, normalized size = 1.20

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(\_R^2 e+2\_R d-\frac{ag-3bc}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^4+a}+\frac{\frac{(-ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{bd\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{2\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x}{x}\right.\right.}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{4ba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/a*e*x^3+1/4*d/a*x^2+1/4*(a*g+b*c)/a/b*x+1/4*f/b)/(-b*x^4+a)+1/4/b/a*(1
/4*(-a*g+3*b*c)*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan
(x/(a/b)^(1/4)))+1/2*b*d/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1
/2)))-1/4*e/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b
)^(1/4)))))
```

Maxima [A]

time = 0.52, size = 227, normalized size = 1.32

$$-\frac{bx^3e+bdx^2+af+(bc+ag)x}{4\left(ab^2x^4-a^2b\right)}+\frac{\frac{2\sqrt{b}\,d\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}}-\frac{2\sqrt{b}\,d\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}}+\frac{2\left(3b^{\frac{3}{2}}c-a\sqrt{b}\,g-\sqrt{a}\,be\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(3b^{\frac{3}{2}}c-a\sqrt{b}\,g+\sqrt{a}\,be\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{16\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x^3*e + b*d*x^2 + a*f + (b*c + a*g)*x)/(a*b^2*x^4 - a^2*b) + 1/16*(
2*sqrt(b)*d*log(sqrt(b)*x^2 + sqrt(a))/sqrt(a) - 2*sqrt(b)*d*log(sqrt(b)*x^
2 - sqrt(a))/sqrt(a) + 2*(3*b^(3/2)*c - a*sqrt(b)*g - sqrt(a)*b*e)*arctan(s
qrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (
3*b^(3/2)*c - a*sqrt(b)*g + sqrt(a)*b*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt
(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)))/(a*b)
```

Fricas [**C**] Result contains complex when optimal does not.
time = 21.21, size = 334837, normalized size = 1946.73

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] -1/9216*(2304*b*e*x^3 + 2304*b*d*x^2 + 1152*(a*b^2*x^4 - a^2*b)*(d*sqrt(1/(
a*b))/a + 4*sqrt(-1/64*(a*e*g - (2*d^2 + 3*c*e)*b)/(a^3*b^2) - 1/64*(a*e*g
+ (2*d^2 - 3*c*e)*b)/(a^3*b^2) + 1/64*(9*b^2*c^2*d + a^2*d*g^2 + (d*e^2 - 6
*c*d*g)*a*b)/(a^4*b^3*d*sqrt(1/(a*b)))))*log(1080*a*b^5*c^3*d^3 + 120*a^2*b
^4*c*d^3*e^2 - 6*a^3*b^3*d*e^5 - 10*a^5*b*d*e*g^4 - 40*(a^4*b^2*d^3 - 3*a^4
*b^2*c*d*e)*g^3 - 8*(24*a^6*b^6*c*d^2 - 9*a^6*b^6*c^2*e - a^7*b^5*e^3 - a^8
*b^4*e*g^2 - 2*(4*a^7*b^5*d^2 - 3*a^7*b^5*c*e)*g)*(d*sqrt(1/(a*b))/a + 4*sq
rt(-1/64*(a*e*g - (2*d^2 + 3*c*e)*b)/(a^3*b^2) - 1/64*(a*e*g + (2*d^2 - 3*c
*e)*b)/(a^3*b^2) + 1/64*(9*b^2*c^2*d + a^2*d*g^2 + (d*e^2 - 6*c*d*g)*a*b)/(
a^4*b^3*d*sqrt(1/(a*b)))))^3 + 180*(2*a^3*b^3*c*d^3 - 3*a^3*b^3*c^2*d*e)*g^
2 + 8*(27*a^4*b^6*c^3*d + 8*a^5*b^5*d^3*e - 9*a^5*b^5*c*d*e^2 + 9*a^6*b^4*c
*d*g^2 - a^7*b^3*d*g^3 - 3*(9*a^5*b^5*c^2*d - a^6*b^4*d*e^2)*g)*(d*sqrt(1/(
a*b))/a + 4*sqrt(-1/64*(a*e*g - (2*d^2 + 3*c*e)*b)/(a^3*b^2) - 1/64*(a*e*g
+ (2*d^2 - 3*c*e)*b)/(a^3 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(136) = 272.
time = 1.32, size = 344, normalized size = 2.00

$$-\frac{\sqrt{2}\left(3\,b^{2}c-abg-2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c-abg+2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c-abg-\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,\left(-ab^{3}\right)^{\frac{3}{4}}a}+\frac{\sqrt{2}\left(3\,b^{2}c-abg-\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{bx^{3}e+bdx^{2}+bcx+agx+af}{4\,(bx^{4}-a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - a*b*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*
b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
)^(3/4)*a) - 1/16*sqrt(2)*(3*b^2*c - a*b*g + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d -
 sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/((-a*b^3)^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*lo
g(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt
(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + s
qrt(-a/b))/((-a*b^3)^(3/4)*a) - 1/4*(b*x^3*e + b*d*x^2 + b*c*x + a*g*x + a*
f)/((b*x^4 - a)*a*b)
```

Mupad [B]
time = 5.56, size = 1393, normalized size = 8.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log(- (12*b^2*c*d^2 - 9*b^2*c^2*e - a^2*e*g^2 + a*b*e^3 - 4*a*b*d^2*
g + 6*a*b*c*e*g)/(64*a^3) - (root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2
- 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z + 128*a
^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2*d*z + 16*a^2*b^2*d^
2*e*g - 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*
c*g^3 - 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*
d^4 - 81*b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k)*b*(9*b^2*c^2*x + a^2*g^2*x
- 16*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 - 3072*a^4*b^4*c*e*z^2 -
 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z + 128*a^4*b^2*d*g^2*z + 128*a^3
*b^3*d*e^2*z + 1152*a^2*b^4*c^2*d*z + 16*a^2*b^2*d^2*e*g - 12*a^2*b^2*c*e^2
*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 - 54*a^2*b^2*c^2*g
^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 - 81*b^4*c^4 - a^2*b
^2*e^4 - a^4*g^4, z, k)*a^3*b*g + a*b*e^2*x + 48*root(65536*a^7*b^5*z^4 + 1
024*a^5*b^3*e*g*z^2 - 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3
*b^3*c*d*g*z + 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2
*d*z + 16*a^2*b^2*d^2*e*g - 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b
^3*c^3*g + 12*a^3*b*c*g^3 - 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3
*c^2*e^2 + 16*a*b^3*d^4 - 81*b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k)*a^2*b^2
*c - 4*a*b*d*e - 32*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 - 3072*a^
4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z + 128*a^4*b^2*d*
g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2*d*z + 16*a^2*b^2*d^2*e*g - 1
2*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 - 5
4*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 - 81*
b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k)*a^2*b^2*d*x - 6*a*b*c*g*x))/(4*a^2)
```

```
- (b*d*x*(2*b*d^2 - 3*b*c*e + a*e*g))/(16*a^3))*root(65536*a^7*b^5*z^4 + 10
24*a^5*b^3*e*g*z^2 - 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*
b^3*c*d*g*z + 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2*
d*z + 16*a^2*b^2*d^2*e*g - 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^
3*c^3*g + 12*a^3*b*c*g^3 - 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*
c^2*e^2 + 16*a*b^3*d^4 - 81*b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k), k, 1, 4
) + (f/(4*b) + (d*x^2)/(4*a) + (e*x^3)/(4*a) + (x*(b*c + a*g))/(4*a*b))/(a
- b*x^4)
```

3.173 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3} \, dx$

Optimal. Leaf size=221

$$\frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2}+\frac{4af+x(7bc-ag+6bdx+5bex^2)}{32a^2b\left(a-bx^4\right)}+\frac{\left(21bc-5\sqrt{a}\,\sqrt{b}\,e-3ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}}$$

```
[Out] 1/8*x*(b*f*x^3+b*e*x^2+b*d*x+a*g+b*c)/a/b/(-b*x^4+a)^2+1/32*(4*a*f+x*(5*b*e
*x^2+6*b*d*x-a*g+7*b*c))/a^2/b/(-b*x^4+a)+3/16*d*arctanh(x^2*b^(1/2)/a^(1/2
))/a^(5/2)/b^(1/2)+1/64*arctan(b^(1/4)*x/a^(1/4))*(21*b*c-3*a*g-5*e*a^(1/2)
*b^(1/2))/a^(11/4)/b^(5/4)+1/64*arctanh(b^(1/4)*x/a^(1/4))*(21*b*c-3*a*g+5*
e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)
```

Rubi [A]
time = 0.18, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1872, 1868, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\,\sqrt{b}\,e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\,\sqrt{b}\,e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(-ag+7bc+6bdx+5bex^2)+4af}{32a^2b\left(a-bx^4\right)}+\frac{x(ag+bc+bdx+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) + (4*a*f
+ x*(7*b*c - a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a - b*x^4)) + ((21*b*c
- 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^
(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4
)])/(64*a^(11/4)*b^(5/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2
)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx = \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{\int \frac{7bc-ag+6bdx+5bex^2+4bfx^3}{(a-bx^4)^2}\,dx}{8ab}$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af+x(7bc-ag+6bdx+5bex^2)}{32a^2b\,(a-bx^4)} -$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af+x(7bc-ag+6bdx+5bex^2)}{32a^2b\,(a-bx^4)} -$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af+x(7bc-ag+6bdx+5bex^2)}{32a^2b\,(a-bx^4)} -$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af+x(7bc-ag+6bdx+5bex^2)}{32a^2b\,(a-bx^4)} +$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af+x(7bc-ag+6bdx+5bex^2)}{32a^2b\,(a-bx^4)} +$$

Mathematica [A]
time = 0.33, size = 263, normalized size = 1.19

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}\,(a^2(4f+3gx)-b^2x^5(7c+x(6d+5ex))+abx(11c+x(10d+9ex+gx^3)))}{(a-bx^4)^2}+2\left(21bc-5\sqrt{a}\sqrt{b}\,e-3ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\left(21bc+12\sqrt[4]{a}\,b^{3/4}d+5\sqrt{a}\sqrt{b}\,e-3ag\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+\left(21bc-12\sqrt[4]{a}\,b^{3/4}d+5\sqrt{a}\sqrt{b}\,e-3ag\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+12\sqrt[4]{a}\,b^{3/4}d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{128a^{11/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(a^2*(4*f + 3*g*x) - b^2*x^5*(7*c + x*(6*d + 5*e*x)) +
a*b*x*(11*c + x*(10*d + 9*e*x + g*x^3))))/(a - b*x^4)^2 + 2*(21*b*c - 5*Sqr
t[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (21*b*c + 12*a^(1/4)*
b^(3/4)*d + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*Log[a^(1/4) - b^(1/4)*x] + (21*b*c
 - 12*a^(1/4)*b^(3/4)*d + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*Log[a^(1/4) + b^(1/4
)*x] + 12*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11/4)*b^(5/
4))
```

Maple [A]
time = 0.34, size = 244, normalized size = 1.10

method	result
risch	$\frac{-\frac{5be\,x^7}{32a^2}-\frac{3bd\,x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-b\,x^4+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(5\_R^2e+12\_Rd-\frac{3(ag-7bc)}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2b}$

default	$\frac{-\frac{5be\,x^{7}}{32a^{2}}-\frac{3bd\,x^{6}}{16a^{2}}+\frac{(ag-7bc)x^{5}}{32a^{2}}+\frac{9e\,x^{3}}{32a}+\frac{5d\,x^{2}}{16a}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-b\,x^{4}+a)^{2}}+\frac{\frac{(-3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{3bd\ln\left(\frac{a\cdots}{a\cdots}\right)}{\sqrt{\cdots}}}{32a^{2}b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-5/32*b*e/a^2*x^7-3/16/a^2*b*d*x^6+1/32*(a*g-7*b*c)/a^2*x^5+9/32/a*e*x^3+5
/16*d/a*x^2+1/32*(3*a*g+11*b*c)/a/b*x+1/8*f/b)/(-b*x^4+a)^2+1/32/a^2/b*(1/4
*(-3*a*g+21*b*c)*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arcta
n(x/(a/b)^(1/4)))+3*b*d/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/
2)))-5/4*e/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)
^(1/4)))))
```

Maxima [A]
time = 0.50, size = 288, normalized size = 1.30

$$-\frac{5\,b^{2}x^{7}e+6\,b^{2}dx^{6}+(7\,b^{2}c-abg)x^{5}-9\,abx^{3}e-10\,abdx^{2}-4\,a^{2}f-(11\,abc+3\,a^{2}g)x}{32\,(a^{2}b^{3}x^{8}-2\,a^{3}b^{2}x^{4}+a^{4}b)}+\frac{\frac{12\,\sqrt{b}\,d\log\left(\sqrt{b}\,x^{2}+\sqrt{a}\right)}{\sqrt{a}}-\frac{12\,\sqrt{b}\,d\log\left(\sqrt{b}\,x^{2}-\sqrt{a}\right)}{\sqrt{a}}+\frac{2\left(21\,b^{\frac{3}{2}}c-3\,a\sqrt{b}\,g-5\,\sqrt{a}\,be\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c-3\,a\sqrt{b}\,g+5\,\sqrt{a}\,be\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{128\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + (7*b^2*c - a*b*g)*x^5 - 9*a*b*x^3*e - 10
*a*b*d*x^2 - 4*a^2*f - (11*a*b*c + 3*a^2*g)*x)/(a^2*b^3*x^8 - 2*a^3*b^2*x^4
 + a^4*b) + 1/128*(12*sqrt(b)*d*log(sqrt(b)*x^2 + sqrt(a))/sqrt(a) - 12*sqr
t(b)*d*log(sqrt(b)*x^2 - sqrt(a))/sqrt(a) + 2*(21*b^(3/2)*c - 3*a*sqrt(b)*g
 - 5*sqrt(a)*b*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))*sqrt(b)) - (21*b^(3/2)*c - 3*a*sqrt(b)*g + 5*sqrt(a)*b*e)*log
((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(
sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^2*b)
```

Fricas [C] Result contains complex when optimal does not.
time = 30.95, size = 343626, normalized size = 1554.87

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] -1/589824*(92160*b^2*e*x^7 + 110592*b^2*d*x^6 - 165888*a*b*e*x^3 + 18432*(7
*b^2*c - a*b*g)*x^5 - 184320*a*b*d*x^2 - 73728*a^2*f + 18432*(a^2*b^3*x^8 -
 2*a^3*b^2*x^4 + a^4*b)*(16*sqrt(-3/4096*(5*a*e*g - (24*d^2 + 35*c*e)*b)/(a
```

```
^5*b^2) - 3/4096*(5*a*e*g + (24*d^2 - 35*c*e)*b)/(a^5*b^2) + 1/4096*(441*b^
2*c^2*d + 9*a^2*d*g^2 + (25*d*e^2 - 126*c*d*g)*a*b)/(a^6*b^3*d*sqrt(1/(a*b)
))) + 3*d*sqrt(1/(a*b))/a^2)*log(80015040*a*b^5*c^3*d^3 + 4536000*a^2*b^4*c
*d^3*e^2 - 112500*a^3*b^3*d*e^5 - 24300*a^5*b*d*e*g^4 - 19440*(12*a^4*b^2*d
^3 - 35*a^4*b^2*c*d*e)*g^3 - 64*(6048*a^9*b^6*c*d^2 - 2205*a^9*b^6*c^2*e -
125*a^10*b^5*e^3 - 45*a^11*b^4*e*g^2 - 18*(48*a^10*b^5*d^2 - 35*a^10*b^5*c*
e)*g)*(16*sqrt(-3/4096*(5*a*e*g - (24*d^2 + 35*c*e)*b)/(a^5*b^2) - 3/4096*(
5*a*e*g + (24*d^2 - 35*c*e)*b)/(a^5*b^2) + 1/4096*(441*b^2*c^2*d + 9*a^2*d*
g^2 + (25*d*e^2 - 126*c*d*g)*a*b)/(a^6*b^3*d*sqrt(1/(a*b)))) + 3*d*sqrt(1/(
a*b))/a^2)^3 + 204120*(24*a^3*b^3*c*d^3 - 35*a^3*b^3*c^2*d*e)*g^2 + 1728*(1
029*a^6*b^6*c^3*d + 160*a ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(185) = 370.
time = 1.06, size = 393, normalized size = 1.78

$$-\frac{\sqrt{2}\left(21\,b^{2}c-3\,abg-12\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{\sqrt{2}\left(21\,b^{2}c-3\,abg+12\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{\sqrt{2}\left(21\,b^{2}c-3\,abg-5\,\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}+\frac{\sqrt{2}\left(21\,b^{2}c-3\,abg-5\,\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{5\,b^{2}x^{7}e+6\,b^{2}dx^{6}+7\,b^{2}cx^{5}-abgx^{5}-9\,abx^{3}e-10\,abdx^{2}-11\,abcx-3\,a^{2}gx-4\,a^{2}f}{32\left(bx^{4}-a\right)^{2}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^2*c - 3*a*b*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqrt
(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(
(-a*b^3)^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c - 3*a*b*g + 12*sqrt(2)*(-a*b^
3)^(1/4)*b*d - 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(
1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 3*a*b*
g - 5*sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b
^3)^(3/4)*a^2) + 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a*b)*b*e)*log(
x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b
^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*x^3*e - 10*a*b*d*x
^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

Mupad [B]
time = 5.44, size = 1002, normalized size = 4.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x)
```

```
[Out] (f/(8*b) + (5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) - (x^5*(7*b*c - a*g))/(32*a^
2) + (x*(11*b*c + 3*a*g))/(32*a*b) - (3*b*d*x^6)/(16*a^2) - (5*b*e*x^7)/(32
*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(- root(268435456*a^11*b^5*z
^4 + 983040*a^7*b^3*e*g*z^2 - 6881280*a^6*b^4*c*e*z^2 - 4718592*a^6*b^4*d^2
*z^2 - 774144*a^4*b^3*c*d*g*z + 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d*e^
2*z + 2709504*a^3*b^4*c^2*d*z + 8640*a^2*b^2*d^2*e*g - 6300*a^2*b^2*c*e^2*g
 - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 - 23814*a^2*
b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 - 625*a^2*b^2*e^4 + 2
0736*a*b^3*d^4 - 81*a^4*g^4 - 194481*b^4*c^4, z, k)*(root(268435456*a^11*b^
5*z^4 + 983040*a^7*b^3*e*g*z^2 - 6881280*a^6*b^4*c*e*z^2 - 4718592*a^6*b^4*
d^2*z^2 - 774144*a^4*b^3*c*d*g*z + 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d
*e^2*z + 2709504*a^3*b^4*c^2*d*z + 8640*a^2*b^2*d^2*e*g - 6300*a^2*b^2*c*e^
2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 - 23814*a
^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 - 625*a^2*b^2*e^4
+ 20736*a*b^3*d^4 - 81*a^4*g^4 - 194481*b^4*c^4, z, k)*((344064*a^5*b^3*c -
 49152*a^6*b^2*g)/(32768*a^6) - (6*b^3*d*x)/a) + (x*(144*a^4*b*g^2 + 7056*a
^2*b^3*c^2 + 400*a^3*b^2*e^2 - 2016*a^3*b^2*c*g))/(4096*a^6) - (15*b^2*d*e)
/(32*a^3)) - (3024*b^2*c*d^2 - 2205*b^2*c^2*e - 45*a^2*e*g^2 + 125*a*b*e^3
- 432*a*b*d^2*g + 630*a*b*c*e*g)/(32768*a^6) - (x*(216*b^2*d^3 - 315*b^2*c*
d*e + 45*a*b*d*e*g))/(4096*a^6))*root(268435456*a^11*b^5*z^4 + 983040*a^7*b
^3*e*g*z^2 - 6881280*a^6*b^4*c*e*z^2 - 4718592*a^6*b^4*d^2*z^2 - 774144*a^4
*b^3*c*d*g*z + 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d*e^2*z + 2709504*a^3
*b^4*c^2*d*z + 8640*a^2*b^2*d^2*e*g - 6300*a^2*b^2*c*e^2*g - 60480*a*b^3*c*
d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 - 23814*a^2*b^2*c^2*g^2 + 450
*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 - 625*a^2*b^2*e^4 + 20736*a*b^3*d^4 -
81*a^4*g^4 - 194481*b^4*c^4, z, k), k, 1, 4)
```

3.174 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a-bx^4\right)^4}\,dx$

Optimal. Leaf size=266

$$\frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\left(a-bx^4\right)^3}+\frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b\left(a-bx^4\right)}+\frac{8af+x(11bc-ag+10bdx+9bex^2)}{96a^2b\left(a-bx^4\right)^2}$$

```
[Out] 1/12*x*(b*f*x^3+b*e*x^2+b*d*x+a*g+b*c)/a/b/(-b*x^4+a)^3+1/384*x*(45*b*e*x^2
+60*b*d*x-7*a*g+77*b*c)/a^3/b/(-b*x^4+a)+1/96*(8*a*f+x*(9*b*e*x^2+10*b*d*x-
a*g+11*b*c))/a^2/b/(-b*x^4+a)^2+5/32*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(7/2)
/b^(1/2)+1/256*arctan(b^(1/4)*x/a^(1/4))*(77*b*c-7*a*g-15*e*a^(1/2)*b^(1/2)
)/a^(15/4)/b^(5/4)+1/256*arctanh(b^(1/4)*x/a^(1/4))*(77*b*c-7*a*g+15*e*a^(1
/2)*b^(1/2))/a^(15/4)/b^(5/4)
```

Rubi [A]
time = 0.22, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1872, 1868, 1869, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\,\sqrt{b}\,e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\,\sqrt{b}\,e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b\left(a-bx^4\right)}+\frac{x(-ag+11bc+10bdx+9bex^2)+8af}{96a^2b\left(a-bx^4\right)^2}+\frac{x(ag+bc+bdx+bex^2+bfx^3)}{12ab\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3) + (x*(7*
(11*b*c - a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a - b*x^4)) + (8*a*f +
 x*(11*b*c - a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a - b*x^4)^2) + ((77*b
*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/
4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/
a^(1/4)])/(256*a^(15/4)*b^(5/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32
*a^(7/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{\int \frac{11bc-ag+10bdx+9bex^2+8bfx^3}{(a-bx^4)^3}\,dx}{12ab}$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{8af+x(11bc-ag+10bdx+9bex^2)}{96a^2b\,(a-bx^4)^2}$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b\,(a-bx^4)} +$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b\,(a-bx^4)} +$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b\,(a-bx^4)} +$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b\,(a-bx^4)} +$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b\,(a-bx^4)} +$$

Mathematica [A]
time = 0.21, size = 313, normalized size = 1.18

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}\,x(77bc-7ag+15bx(4d+3ex))}{a-bx^4}+\frac{16a^{7/4}\sqrt[4]{b}\,x(11bc-ag+bx(10d+9ex))}{(a-bx^4)^2}+\frac{128a^{11/4}\sqrt[4]{b}\,(a(f+gx)+bx(c+x(d+ex)))}{(a-bx^4)^3}+6\left(77bc-15\sqrt{a}\,\sqrt{b}\,e-7ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-3\left(77bc+40\sqrt[4]{a}\,b^{3/4}d+15\sqrt{a}\,\sqrt{b}\,e-7ag\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+3\left(77bc-40\sqrt[4]{a}\,b^{3/4}d+15\sqrt{a}\,\sqrt{b}\,e-7ag\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+120\sqrt[4]{a}\,b^{3/4}d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{1536a^{15/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*x*(77*b*c - 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a - b*x^4)
+ (16*a^(7/4)*b^(1/4)*x*(11*b*c - a*g + b*x*(10*d + 9*e*x)))/(a - b*x^4)^2
+ (128*a^(11/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^4)^
3 + 6*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] -
 3*(77*b*c + 40*a^(1/4)*b^(3/4)*d + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*Log[a^(1/
4) - b^(1/4)*x] + 3*(77*b*c - 40*a^(1/4)*b^(3/4)*d + 15*Sqrt[a]*Sqrt[b]*e -
 7*a*g)*Log[a^(1/4) + b^(1/4)*x] + 120*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt
[b]*x^2])/(1536*a^(15/4)*b^(5/4))
```

Maple [A]
time = 0.35, size = 285, normalized size = 1.07

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{(7ag+51bc)x}{128ab}+\frac{f}{12b}}{(-b\,x^4+a)^3} - \frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^4-a)}}{}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{(7ag+51bc)x}{128ab}+\frac{f}{12b}}{(-b\,x^4+a)^3} + \frac{(-7ag+77bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32/a^3*d*b^2*x^10-7/384*(a*g-11*b*c)/a^3*b*x^9-21/
64*b*e/a^2*x^7-5/12/a^2*b*d*x^6+3/64/a^2*(a*g-11*b*c)*x^5+113/384/a*e*x^3+1
1/32*d/a*x^2+1/128*(7*a*g+51*b*c)/a/b*x+1/12*f/b)/(-b*x^4+a)^3+1/128/a^3/b*
(1/4*(-7*a*g+77*b*c)*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*a
rctan(x/(a/b)^(1/4)))+10*b*d/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b
)^(1/2)))-15/4*e/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x
-(a/b)^(1/4)))))
```

Maxima [A]
time = 0.52, size = 350, normalized size = 1.32

$$-\frac{45\,b^3x^{11}e+60\,b^3dx^{10}-126\,ab^2x^7e-160\,ab^2dx^6+7\,(11\,b^3c-ab^2g)x^9+113\,a^2bx^3e+132\,a^2bdx^2-18\,(11\,ab^2c-a^2bg)x^5+32\,a^3f+3\,(51\,a^2bc+7\,a^3g)x}{384\,(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2x^4-a^6b)}+\frac{\frac{40\,\sqrt{b}\,d\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}}-\frac{40\,\sqrt{b}\,d\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}}+\frac{2\left(77\,b^{\frac{3}{2}}c-7\,a\sqrt{b}\,g-15\,\sqrt{a}\,be\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c-7\,a\sqrt{b}\,g+15\,\sqrt{a}\,be\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{512\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 - 126*a*b^2*x^7*e - 160*a*b^2*d*x^6 +
 7*(11*b^3*c - a*b^2*g)*x^9 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 - 18*(11*a*
b^2*c - a^2*b*g)*x^5 + 32*a^3*f + 3*(51*a^2*b*c + 7*a^3*g)*x)/(a^3*b^4*x^12
 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b) + 1/512*(40*sqrt(b)*d*log(sqrt(b)
*x^2 + sqrt(a))/sqrt(a) - 40*sqrt(b)*d*log(sqrt(b)*x^2 - sqrt(a))/sqrt(a) +
 2*(77*b^(3/2)*c - 7*a*sqrt(b)*g - 15*sqrt(a)*b*e)*arctan(sqrt(b)*x/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*b^(3/2)*c - 7
*a*sqrt(b)*g + 15*sqrt(a)*b*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqr
t(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(
a^3*b)
```

Fricas [C] Result contains complex when optimal does not.
time = 45.60, size = 343822, normalized size = 1292.56

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] -1/9437184*(1105920*b^3*e*x^11 + 1474560*b^3*d*x^10 - 3096576*a*b^2*e*x^7 -
 3932160*a*b^2*d*x^6 + 172032*(11*b^3*c - a*b^2*g)*x^9 + 2777088*a^2*b*e*x^
3 + 3244032*a^2*b*d*x^2 - 442368*(11*a*b^2*c - a^2*b*g)*x^5 + 786432*a^3*f
+ 147456*(a^3*b^4*x^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b)*(32*sqrt(-5
/65536*(21*a*e*g - (160*d^2 + 231*c*e)*b)/(a^7*b^2) - 5/65536*(21*a*e*g + (
160*d^2 - 231*c*e)*b)/(a^7*b^2) + 1/65536*(5929*b^2*c^2*d + 49*a^2*d*g^2 +
(225*d*e^2 - 1078*c*d*g)*a*b)/(a^8*b^3*d*sqrt(1/(a*b)))) + 5*d*sqrt(1/(a*b)
)/a^3)*log(146090560000*a*b^5*c^3*d^3 + 5544000000*a^2*b^4*c*d^3*e^2 - 9112
5000*a^3*b^3*d*e^5 - 7203000*a^5*b*d*e*g^4 - 1372000*(80*a^4*b^2*d^3 - 231*
a^4*b^2*c*d*e)*g^3 - 2560*(49280*a^12*b^6*c*d^2 - 17787*a^12*b^6*c^2*e - 67
5*a^13*b^5*e^3 - 147*a^14*b^4*e*g^2 - 14*(320*a^13*b^5*d^2 - 231*a^13*b^5*c
*e)*g)*(32*sqrt(-5/65536*(21*a*e*g - (160*d^2 + 231*c*e)*b)/(a^7*b^2) - 5/6
5536*(21*a*e*g + (160*d^2 - 231*c*e)*b)/(a^7*b^2) + 1/65536*(5929*b^2*c^2*d
 + 49*a^2*d*g^2 + (225*d* ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.30, size = 442, normalized size = 1.66

$$-\frac{\sqrt{2}\left(77\,b^2c-7\,abg-40\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd+15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{\sqrt{2}\left(77\,b^2c-7\,abg+40\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd-15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{\sqrt{2}\left(77\,b^2c-7\,abg-15\,\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^3\right)^{\frac{3}{4}}a^3}+\frac{\sqrt{2}\left(77\,b^2c-7\,abg-15\,\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{45\,b^3x^{11}e+60\,b^3dx^{10}+77\,b^3cx^9-7\,ab^2gx^9-126\,ab^2x^7e-160\,ab^2dx^6-198\,ab^2cx^5+18\,a^2bgx^5+113\,a^2bx^3e+132\,a^2bdx^2+153\,a^2bcx+21\,a^3gx+32\,a^3f}{384\left(bx^4-a\right)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 7*a*b*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sqr
t(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/
((-a*b^3)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c - 7*a*b*g + 40*sqrt(2)*(-a*b
^3)^(1/4)*b*d - 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)
^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 7*a
*b*g - 15*sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((
-a*b^3)^(3/4)*a^3) + 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqrt(-a*b)*b*e
)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/3
84*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 - 126*a*b^
```

```
2*x^7*e - 160*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^
3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x + 21*a^3*g*x + 32*a^3*f)/((b*x^4 - a)
^3*a^3*b)
```

Mupad [B]
time = 5.66, size = 1056, normalized size = 3.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x)
```

```
[Out] symsum(log(- root(68719476736*a^15*b^5*z^4 - 1211105280*a^8*b^4*c*e*z^2 + 1
10100480*a^9*b^3*e*g*z^2 - 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d
*g*z + 485703680*a^4*b^4*c^2*d*z + 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b
^3*d*e^2*z + 672000*a^2*b^2*d^2*e*g - 485100*a^2*b^2*c*e^2*g - 7392000*a*b^
3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g^3 - 1743126*a^2*b^2*c^2
*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2 - 50625*a^2*b^2*e^4 + 25
60000*a*b^3*d^4 - 2401*a^4*g^4 - 35153041*b^4*c^4, z, k)*(root(68719476736*
a^15*b^5*z^4 - 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 - 838
860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d*g*z + 485703680*a^4*b^4*c^2*d
*z + 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b^3*d*e^2*z + 672000*a^2*b^2*d^
2*e*g - 485100*a^2*b^2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 12782924*a*b^3*c^3
*g + 105644*a^3*b*c*g^3 - 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2
668050*a*b^3*c^2*e^2 - 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 - 2401*a^4*g^4
 - 35153041*b^4*c^4, z, k)*((20185088*a^7*b^3*c - 1835008*a^8*b^2*g)/(20971
52*a^9) - (5*b^3*d*x)/a^2) + (x*(1568*a^5*b*g^2 + 189728*a^3*b^3*c^2 + 7200
*a^4*b^2*e^2 - 34496*a^4*b^2*c*g))/(131072*a^9) - (75*b^2*d*e)/(256*a^5)) -
 (123200*b^2*c*d^2 - 88935*b^2*c^2*e - 735*a^2*e*g^2 + 3375*a*b*e^3 - 11200
*a*b*d^2*g + 16170*a*b*c*e*g)/(2097152*a^9) - (x*(4000*b^2*d^3 - 5775*b^2*c
*d*e + 525*a*b*d*e*g))/(131072*a^9))*root(68719476736*a^15*b^5*z^4 - 121110
5280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 - 838860800*a^8*b^4*d^2*z^
2 - 88309760*a^5*b^3*c*d*g*z + 485703680*a^4*b^4*c^2*d*z + 4014080*a^6*b^2*
d*g^2*z + 18432000*a^5*b^3*d*e^2*z + 672000*a^2*b^2*d^2*e*g - 485100*a^2*b^
2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g
^3 - 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2
- 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 - 2401*a^4*g^4 - 35153041*b^4*c^4,
z, k), k, 1, 4) + (f/(12*b) + (11*d*x^2)/(32*a) + (113*e*x^3)/(384*a) - (3*
x^5*(11*b*c - a*g))/(64*a^2) + (7*b*x^9*(11*b*c - a*g))/(384*a^3) + (x*(51*
b*c + 7*a*g))/(128*a*b) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^
3) - (5*b*d*x^6)/(12*a^2) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^3*x^12 - 3*a^2*
b*x^4 + 3*a*b^2*x^8)
```

3.175 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4} \, dx$

Optimal. Leaf size=319

$$\frac{gx}{b}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{\left(bc+\sqrt{a}\,\sqrt{b}\,e-ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\,\sqrt{b}\,e-ag\right)\tan^{-1}\left(1+\right.}{2\sqrt{2}\,a^{3/4}b^{5/4}}$$

```
[Out] g*x/b+1/4*f*ln(b*x^4+a)/b+1/2*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(b*c-a*g-e*a^(1/2)*
b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x
^2*b^(1/2))*(b*c-a*g-e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1/4*arctan(
-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(b*c-a*g+e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/4)*2
^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(b*c-a*g+e*a^(1/2)*b^(1/2))/
a^(3/4)/b^(5/4)*2^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1899, 1262, 649, 211, 266, 1901, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,\sqrt{b}\,e-ag+bc\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,\sqrt{b}\,e-ag+bc\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-\sqrt{a}\,\sqrt{b}\,e-ag+bc\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-\sqrt{a}\,\sqrt{b}\,e-ag+bc\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

```
[Out] (g*x)/b + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((b*c + S
qrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b]*e
- a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (f*Log[a + b*
x^4])/(4*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \int \left(\frac{x(d+fx^2)}{a+bx^4}+\frac{c+ex^2+gx^4}{a+bx^4}\right)dx$$
$$= \int \frac{x(d+fx^2)}{a+bx^4}\,dx + \int \frac{c+ex^2+gx^4}{a+bx^4}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{a+bx^2}\,dx,x,x^2\right) + \int \left(\frac{g}{b}+\frac{bc-ag+bex^2}{b\,(a+bx^4)}\right)dx$$
$$= \frac{gx}{b} + \frac{\int \frac{bc-ag+bex^2}{a+bx^4}\,dx}{b} + \frac{1}{2}d\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right) + \frac{1}{2}f\text{Subst}\left(\int\right.$$
$$= \frac{gx}{b} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}} + \frac{f\log\left(a+bx^4\right)}{4b} + \frac{\left(bc-\sqrt{a}\,\sqrt{b}\,e-ag\right)\int \sqrt{}}{2\sqrt{a}\,b^{3/2}}$$
$$= \frac{gx}{b} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}} + \frac{f\log\left(a+bx^4\right)}{4b} - \frac{\left(bc-\sqrt{a}\,\sqrt{b}\,e-ag\right)\int}{4\sqrt{2}\,a^{3/4}}$$
$$= \frac{gx}{b} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}} - \frac{\left(bc-\sqrt{a}\,\sqrt{b}\,e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{}\right.}{4\sqrt{2}\,a^{3/4}b^{5/4}}$$
$$= \frac{gx}{b} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}} - \frac{\left(bc+\sqrt{a}\,\sqrt{b}\,e-ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}$$

<u>Mathematica</u> [A]
time = 0.22, size = 311, normalized size = 0.97

$$\frac{8a^{3/4}\sqrt[4]{b}\,gx-2\left(\sqrt{2}\,bc+2\sqrt[4]{a}\,b^{3/4}d+\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,e-\sqrt{2}\,ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\,bc-2\sqrt[4]{a}\,b^{3/4}d+\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,e-\sqrt{2}\,ag\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\sqrt{2}\left(-bc+\sqrt{a}\,\sqrt{b}\,e+ag\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+\sqrt{2}\left(bc-\sqrt{a}\,\sqrt{b}\,e-ag\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+2a^{3/4}\sqrt[4]{b}\,f\log\left(a+bx^4\right)}{8a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

```
[Out] (8*a^(3/4)*b^(1/4)*g*x - 2*(Sqrt[2]*b*c + 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqr
t[a]*Sqrt[b]*e - Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(
Sqrt[2]*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e - Sqrt[2]*a*g
)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-(b*c) + Sqrt[a]*Sqrt[
b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2
]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*f*Log[a + b*x^4])/(8*a^(3/4)*b^(5/4))
```

Maple [A]
time = 0.34, size = 253, normalized size = 0.79

method	result
risch	$\frac{gx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(\_R^3bf+\_R^2be+\_Rbd-ag+bc\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$\frac{gx}{b}+\frac{\frac{(-ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{bd\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{e\sqrt{}}{}\cdots}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] g*x/b+1/b*(1/8*(-a*g+b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1
/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/
(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/2*b*d/(a*b)^(1/2)*arc
tan(x^2*(b/a)^(1/2))+1/8*e/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/
2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(
a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/4*f*ln(b*x^4+a))
```

Maxima [A]
time = 0.51, size = 332, normalized size = 1.04

$$\frac{gx}{b}+\frac{\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}f+b^2c-abg-\sqrt{a}\,b^{\frac{3}{2}}e\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}f-b^2c+abg+\sqrt{a}\,b^{\frac{3}{2}}e\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{9}{4}}c-\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}g+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}e-2\sqrt{a}\,b^2d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{9}{4}}c-\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}g+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}e+2\sqrt{a}\,b^2d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] g*x/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f + b^2*c - a*b*g - sqrt(a)*b
^(3/2)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b
^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f - b^2*c + a*b*g + sqrt(a)*b^(3
/2)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5
/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c - sqrt(2)*a^(5/4)*b^(5/4)*g + sqrt(2)*a
^(3/4)*b^(7/4)*e - 2*sqrt(a)*b^2*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b
^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c - sqrt(2)*a^(5/4)*b^(5/4)*g + sqrt(2
)*a^(3/4)*b^(7/4)*e + 2*sqrt(a)*b^2*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sq
rt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b)
)*b^(5/4)))/b
```

Fricas [C] Result contains complex when optimal does not.
time = 30.94, size = 622377, normalized size = 1951.03

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/48*(2*(2*(1/4)^(2/3)*(-I*sqrt(3) + 1)*((sqrt(2)*a*b*sqrt(-1/(a*b))*sqrt(-
(2*a*b^2*c*e*sqrt(-1/(a*b)) + b^2*c^2 - a*b*e^2 + a^2*g^2 - 2*(a^2*b*e*sqrt
(-1/(a*b)) + a*b*c)*g)/(a^2*b^3*sqrt(-1/(a*b)))) + sqrt(2)*(3*a*f*sqrt(-1/(
a*b)) - d))^2/(a*b) + 3*sqrt(2)*(2*sqrt(2)*a^2*b^2*f*sqrt(-1/(a*b))*sqrt(-(
2*a*b^2*c*e*sqrt(-1/(a*b)) + b^2*c^2 - a*b*e^2 + a^2*g^2 - 2*(a^2*b*e*sqrt(
-1/(a*b)) + a*b*c)*g)/(a^2*b^3*sqrt(-1/(a*b)))) - 2*sqrt(2)*a*b^2*d*sqrt(-(
2*a*b^2*c*e*sqrt(-1/(a*b)) + b^2*c^2 - a*b*e^2 + a^2*g^2 - 2*(a^2*b*e*sqrt(
-1/(a*b)) + a*b*c)*g)/(a^2*b^3*sqrt(-1/(a*b)))) - 2*sqrt(2)*(a^2*b*e*g*sqrt
(-1/(a*b)) - (b^2*c*e*sqrt(-1/(a*b)) - (d*f - c*g)*b)*a) - sqrt(2)*(b^2*c^2
 - (3*b*f^2*sqrt(-1/(a*b)) - g^2)*a^2 - (b^2*d^2*sqrt(-1/(a*b)) + b*e^2)*a)
)/(a^2*b^3*sqrt(-1/(a*b)))))/(9*(2*sqrt(2)*a^2*b^2*f*sqrt(-1/(a*b))*sqrt(-(2
*a*b^2*c*e*sqrt(-1/(a*b)) + b^2*c^2 - a*b*e^2 + a^2*g^2 - 2*(a^2*b*e*sqrt(-
1/(a*b)) + a*b*c)*g)/(a^2*b^3*sqrt(-1/(a*b)))) - 2*sqrt(2)*a*b^2*d*sqrt(-(2
*a*b^2*c*e*sqrt(-1/(a*b)) ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.07, size = 340, normalized size = 1.07

$$\frac{gx}{b}+\frac{f\log\left(|bx^4+a|\right)}{4\,b}+\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{ab}\,b^2d+(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{ab}\,b^2d+(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] g*x/b + 1/4*f*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d
+ (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*s
qrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt
(2)*sqrt(a*b)*b^2*d + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(
3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3)
 + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e
)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^
3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

Mupad [B]
time = 5.59, size = 2500, normalized size = 7.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x)
```

```
[Out] symsum(log(b^2*c*d^2 - b^2*c^2*e - a^2*e*g^2 + a^2*f^2*g + b^2*d^3*x - a*b*
e^3 - a*b*c*f^2 - a*b*d^2*g - 16*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 -
 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*
d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16
*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d
*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d
*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*
d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g
 + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^
2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)^2*a*b^3*
c - 4*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^
2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*
z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b
^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a
^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g
+ 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f -
 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^
2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4
 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*b^3*c^2*x + b^2*c^2*f*x + a^2*f*g^2
*x + 16*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*
a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*
g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2
*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8
*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*
g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f
 - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*
a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f
^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)^2*a^2*b^2*g + 16*root(256*a^3*b^5
*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3
*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z
 + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^
3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2
*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 -
4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a
^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*
b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4
 + b^4*c^4, z, k)^2*a*b^3*d*x + 4*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3
```

- 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4
*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 1
6*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*
d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*
d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b
*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*
g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e
^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a*b^2*e
^2*x - 4*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64
*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f
*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^
2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z -
8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2
*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*
f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2
*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*
f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a^2*b*g^2*x + 2*a*b*c*e*g + 2*a*
b*d*e*f + 8*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 +
64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*
e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16
*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z
- 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*
e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2
*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2
+ 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3
*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a*b^2*c*f - 8*root(256*a^3*b^
5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^
3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*
z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b
^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*...

3.176 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^4\right)^2}\,dx$

Optimal. Leaf size=341

$$\frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\left(a+bx^4\right)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{\left(3bc+\sqrt{a}\,\sqrt{b}\,e+ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(3bc}{}$$

```
[Out] 1/4*x*(b*f*x^3+b*e*x^2+b*d*x-a*g+b*c)/a/b/(b*x^4+a)+1/4*d*arctan(x^2*b^(1/2
)/a^(1/2))/a^(3/2)/b^(1/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b
^(1/2))*(3*b*c+a*g-e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/32*ln(a^(1/
4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(3*b*c+a*g-e*a^(1/2)*b^(1/2))/a^(
7/4)/b^(5/4)*2^(1/2)+1/16*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(3*b*c+a*g+e
*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a
^(1/4))*(3*b*c+a*g+e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1872, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,\sqrt{b}\,e+ag+3bc\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,\sqrt{b}\,e+ag+3bc\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-\sqrt{a}\,\sqrt{b}\,e+ag+3bc\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-\sqrt{a}\,\sqrt{b}\,e+ag+3bc\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (d*ArcTan
[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*b*c + Sqrt[a]*Sqrt[b]*e
+ a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4))
 + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/
4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
```

```
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx = \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \frac{-3bc-ag-2bdx-bex^2}{a+bx^4}\,dx}{4ab}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \left(-\frac{2bdx}{a+bx^4}+\frac{-3bc-ag-bex^2}{a+bx^4}\right)dx}{4ab}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \frac{-3bc-ag-bex^2}{a+bx^4}\,dx}{4ab} + \frac{d\int \frac{x}{a+bx^4}\,dx}{2a}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\,(a+bx^4)} + \frac{d\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4a} + \frac{\left(3bc-\sqrt{}\right.}{}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\,(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3bc-\sqrt{a}\,\sqrt{b}\,e\right.}{}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\,(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3bc-\sqrt{a}\,\sqrt{b}\,e\right.}{}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\,(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3bc+\sqrt{a}\,\sqrt{b}\,e\right.}{}$$

Mathematica [A]
time = 0.14, size = 319, normalized size = 0.94

$$\frac{-\frac{8a^{3/4}\sqrt[4]{b}\,(a(f+gx)-bx(c+x(d+ex)))}{a+bx^4}-2\left(3\sqrt{2}\,bc+4\sqrt[4]{a}\,b^{3/4}d+\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,e+\sqrt{2}\,ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\left(3\sqrt{2}\,bc-4\sqrt[4]{a}\,b^{3/4}d+\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,e+\sqrt{2}\,ag\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\sqrt{2}\left(-3bc+\sqrt{a}\,\sqrt{b}\,e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+\sqrt{2}\left(3bc-\sqrt{a}\,\sqrt{b}\,e+ag\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{32a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4) - 2
*(3*Sqrt[2]*b*c + 4*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[2]
*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b*c - 4*a^(1/4
)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[2]*a*g)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c - Sqrt[a]*Sqr
t[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a
^(7/4)*b^(5/4))
```

Maple [A]
time = 0.34, size = 291, normalized size = 0.85

method	result
risch	$\frac{\frac{e x^3}{4a}+\frac{d x^2}{4a}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(\_R^2 e+2\_R d+\frac{ag+3bc}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$\frac{\frac{e x^3}{4a}+\frac{d x^2}{4a}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b x^4+a}+\frac{\frac{(ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/a*e*x^3+1/4*d/a*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(b*x^4+a)+1/4/b/a*(1/
8*(a*g+3*b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1
/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*
x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+b*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1
/2))+1/8*e/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/
(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+
2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.51, size = 355, normalized size = 1.04

$$\frac{bx^3e+bdx^2-af+(bc-ag)x}{4\,(ab^2x^4+a^2b)}+\frac{\frac{\sqrt{2}\left(3b^{\frac{3}{2}}c+a\sqrt{b}\,g-\sqrt{a}\,be\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3b^{\frac{3}{2}}c+a\sqrt{b}\,g-\sqrt{a}\,be\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e-4\sqrt{a}\,b^{\frac{3}{2}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e+4\sqrt{a}\,b^{\frac{3}{2}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{32\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 - a*f + (b*c - a*g)*x)/(a*b^2*x^4 + a^2*b) + 1/32*(s
qrt(2)*(3*b^(3/2)*c + a*sqrt(b)*g - sqrt(a)*b*e)*log(sqrt(b)*x^2 + sqrt(2)*
```

```
a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*b^(3/2)*c + a*s
qrt(b)*g - sqrt(a)*b*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(
a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(5/4)*b^
(3/4)*g + sqrt(2)*a^(3/4)*b^(5/4)*e - 4*sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(
2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*
sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a
^(5/4)*b^(3/4)*g + sqrt(2)*a^(3/4)*b^(5/4)*e + 4*sqrt(a)*b^(3/2)*d)*arctan(
1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a*b)
```

Fricas [C] Result contains complex when optimal does not.
time = 24.57, size = 352423, normalized size = 1033.50

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9216*(2304*b*e*x^3 + 2304*b*d*x^2 + 2*(a*b^2*x^4 + a^2*b)*((-I*sqrt(3) +
1)*((a^2*b*sqrt(-1/(a*b))*sqrt(-(6*a*b^2*c*e*sqrt(-1/(a*b)) + 9*b^2*c^2 - a
*b*e^2 + a^2*g^2 + 2*(a^2*b*e*sqrt(-1/(a*b)) + 3*a*b*c)*g)/(a^4*b^3*sqrt(-1
/(a*b)))) - 2*d)^2/(a^3*b) - 3*(9*b^2*c^2 - (2*b*e*g*sqrt(-1/(a*b)) - 4*b^2
*d*sqrt(-(6*a*b^2*c*e*sqrt(-1/(a*b)) + 9*b^2*c^2 - a*b*e^2 + a^2*g^2 + 2*(a
^2*b*e*sqrt(-1/(a*b)) + 3*a*b*c)*g)/(a^4*b^3*sqrt(-1/(a*b)))) - g^2)*a^2 -
(2*(2*d^2 + 3*c*e)*b^2*sqrt(-1/(a*b)) + (e^2 - 6*c*g)*b)*a)/(a^4*b^3*sqrt(-
1/(a*b))))/(-1/24576*(9*b^2*c^2 - (2*b*e*g*sqrt(-1/(a*b)) - 4*b^2*d*sqrt(-(
6*a*b^2*c*e*sqrt(-1/(a*b)) + 9*b^2*c^2 - a*b*e^2 + a^2*g^2 + 2*(a^2*b*e*sqr
t(-1/(a*b)) + 3*a*b*c)*g)/(a^4*b^3*sqrt(-1/(a*b)))) - g^2)*a^2 - (2*(2*d^2
+ 3*c*e)*b^2*sqrt(-1/(a*b)) + (e^2 - 6*c*g)*b)*a)*(a^2*b*sqrt(-1/(a*b))*sqr
t(-(6*a*b^2*c*e*sqrt(-1/(a*b)) + 9*b^2*c^2 - a*b*e^2 + a^2*g^2 + 2*(a^2*b*e
*sqrt(-1/(a*b)) + 3*a*b*c)*g)/(a^4*b^3*sqrt(-1/(a*b)))) - 2*d)/(a^5*b^3) +
1/110592*(a^2*b*sqrt(-1/( ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.46, size = 365, normalized size = 1.07

$$\frac{bx^3e + bdx^2 + bcx - agx - af}{4\,(bx^4 + a)ab} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2d + 3\,(ab^3)^{\frac{1}{4}}\,b^2c + (ab^3)^{\frac{1}{4}}\,abg + (ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2d + 3\,(ab^3)^{\frac{1}{4}}\,b^2c + (ab^3)^{\frac{1}{4}}\,abg + (ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c + (ab^3)^{\frac{1}{4}}\,abg - (ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2 + \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3} - \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c + (ab^3)^{\frac{1}{4}}\,abg - (ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2 - \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt
(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*
g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)
*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - s
qrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*
b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^
(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(a^2*b^3)
```

Mupad [B]
time = 5.59, size = 1383, normalized size = 4.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log(- (9*b^2*c^2*e - 12*b^2*c*d^2 + a^2*e*g^2 + a*b*e^3 - 4*a*b*d^2*
g + 6*a*b*c*e*g)/(64*a^3) - (root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2
+ 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z - 128*a
^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2*d*z - 16*a^2*b^2*d^
2*e*g + 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*
c*g^3 + 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*
d^4 + 81*b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k)*b*(9*b^2*c^2*x + a^2*g^2*x
+ 16*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 + 3072*a^4*b^4*c*e*z^2 +
 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z - 128*a^4*b^2*d*g^2*z + 128*a^3
*b^3*d*e^2*z - 1152*a^2*b^4*c^2*d*z - 16*a^2*b^2*d^2*e*g + 12*a^2*b^2*c*e^2
*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 + 54*a^2*b^2*c^2*g
^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 + 81*b^4*c^4 + a^2*b
^2*e^4 + a^4*g^4, z, k)*a^3*b*g - a*b*e^2*x + 48*root(65536*a^7*b^5*z^4 + 1
024*a^5*b^3*e*g*z^2 + 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3
*b^3*c*d*g*z - 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2
*d*z - 16*a^2*b^2*d^2*e*g + 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b
^3*c^3*g + 12*a^3*b*c*g^3 + 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3
*c^2*e^2 + 16*a*b^3*d^4 + 81*b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k)*a^2*b^2
*c + 4*a*b*d*e - 32*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 + 3072*a^
4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z - 128*a^4*b^2*d*
g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2*d*z - 16*a^2*b^2*d^2*e*g + 1
2*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 + 5
4*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 + 81*
```

```
b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k)*a^2*b^2*d*x + 6*a*b*c*g*x))/(4*a^2)
- (b*d*x*(3*b*c*e - 2*b*d^2 + a*e*g))/(16*a^3))*root(65536*a^7*b^5*z^4 + 10
24*a^5*b^3*e*g*z^2 + 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*
b^3*c*d*g*z - 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2*
d*z - 16*a^2*b^2*d^2*e*g + 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^
3*c^3*g + 12*a^3*b*c*g^3 + 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*
c^2*e^2 + 16*a*b^3*d^4 + 81*b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k), k, 1, 4
) + ((d*x^2)/(4*a) - f/(4*b) + (e*x^3)/(4*a) + (x*(b*c - a*g))/(4*a*b))/(a
+ b*x^4)
```

3.177 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=394

$$\frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2}-\frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\,(a+bx^4)}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{\left(21bc+5\sqrt{a}\right.}{}$$

```
[Out] 1/8*x*(b*f*x^3+b*e*x^2+b*d*x-a*g+b*c)/a/b/(b*x^4+a)^2+1/32*(-4*a*f+x*(5*b*e
*x^2+6*b*d*x+a*g+7*b*c))/a^2/b/(b*x^4+a)+3/16*d*arctan(x^2*b^(1/2)/a^(1/2))
/a^(5/2)/b^(1/2)-1/256*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(
21*b*c+3*a*g-5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+1/256*ln(a^(1/4)
*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(21*b*c+3*a*g-5*e*a^(1/2)*b^(1/2))/
a^(11/4)/b^(5/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(21*b*c
+3*a*g+5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+1/128*arctan(1+b^(1/4)
*x*2^(1/2)/a^(1/4))*(21*b*c+3*a*g+5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(
1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1872, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}\,e+3ag+21bc\right)}{64\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\sqrt{b}\,e+3ag+21bc\right)}{64\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{3d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-5\sqrt{a}\sqrt{b}\,e+3ag+21bc\right)}{128\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-5\sqrt{a}\sqrt{b}\,e+3ag+21bc\right)}{128\sqrt{2}\,a^{11/4}b^{5/4}}-\frac{4af-x(ag+7bc+6bdx+5bex^2)}{32a^2b\,(a+bx^4)}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) - (4*a*f
- x*(7*b*c + a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a + b*x^4)) + (3*d*ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - ((21*b*c + 5*Sqrt[a]*Sqrt
[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4
)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) - ((21*b*c - 5*Sqrt[a]*Sqr
t[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(12
8*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^
(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx = \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{\int \frac{-7bc-ag-6bdx-5bex^2-4bfx^3}{(a+bx^4)^2}\,dx}{8ab}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\,(a+bx^4)} +$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\,(a+bx^4)} +$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\,(a+bx^4)} +$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\,(a+bx^4)} +$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\,(a+bx^4)} +$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\,(a+bx^4)} +$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\,(a+bx^4)} +$$

Mathematica [A]
time = 0.19, size = 366, normalized size = 0.93

$$\frac{\frac{8a^{3/4}\sqrt[4]{b}\,x(7bc+ag+bx(6d+5ex))}{a+bx^4} - \frac{32a^{7/4}\sqrt[4]{b}\,(a(f+gx)-bx(c+x(d+ex)))}{(a+bx^4)^2} - 2\left(21\sqrt{2}\,bc+24\sqrt[4]{a}\,b^{3/4}d+5\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,e+3\sqrt{2}\,ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) + 2\left(21\sqrt{2}\,bc-24\sqrt[4]{a}\,b^{3/4}d+5\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,e+3\sqrt{2}\,ag\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) + \sqrt{2}\left(-21bc+5\sqrt{a}\,\sqrt{b}\,e-3ag\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right) + \sqrt{2}\left(21bc-5\sqrt{a}\,\sqrt{b}\,e+3ag\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{256a^{11/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*b^(1/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x)))/(a + b*x^4) - (32*
a^(7/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4)^2 - 2*(2
1*Sqrt[2]*b*c + 24*a^(1/4)*b^(3/4)*d + 5*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 3*Sqrt
[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(21*Sqrt[2]*b*c - 24*a
^(1/4)*b^(3/4)*d + 5*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 3*Sqrt[2]*a*g)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a
```

```
*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(21*b*
c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(256*a^(11/4)*b^(5/4))
```

Maple [A]
time = 0.33, size = 330, normalized size = 0.84

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(5\_R^2e+12\_Rd+\frac{3ag+21bc}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2b}$
default	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\frac{(3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\right.\right.}{8a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (5/32*b*e/a^2*x^7+3/16/a^2*b*d*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32/a*e*x^3+5/
16*d/a*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*f/b)/(b*x^4+a)^2+1/32/a^2/b*(1/8*(
3*a*g+21*b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1
/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*
x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+6*b*d/(a*b)^(1/2)*arctan(x^2*(b/a)^
(1/2))+5/8*e/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)
)/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1
)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.51, size = 418, normalized size = 1.06

$$\frac{5\,b^2x^7e+6\,b^2dx^6+(7\,b^2c+abg)x^5+9\,abx^3e+10\,abdx^2-4\,a^2f+(11\,abc-3\,a^2g)x}{32\,(a^2b^3x^8+2\,a^3b^2x^4+a^4b)}+\frac{\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c+3\,a\sqrt{b}\,g-5\,\sqrt{a}\,be\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c+3\,a\sqrt{b}\,g-5\,\sqrt{a}\,be\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+3\,\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+5\,\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e-24\,\sqrt{a}\,b^{\frac{3}{2}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+3\,\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+5\,\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e+24\,\sqrt{a}\,b^{\frac{3}{2}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{3}{4}}}}{256\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + (7*b^2*c + a*b*g)*x^5 + 9*a*b*x^3*e + 10*
a*b*d*x^2 - 4*a^2*f + (11*a*b*c - 3*a^2*g)*x)/(a^2*b^3*x^8 + 2*a^3*b^2*x^4
+ a^4*b) + 1/256*(sqrt(2)*(21*b^(3/2)*c + 3*a*sqrt(b)*g - 5*sqrt(a)*b*e)*lo
g(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sq
rt(2)*(21*b^(3/2)*c + 3*a*sqrt(b)*g - 5*sqrt(a)*b*e)*log(sqrt(b)*x^2 - sqrt
(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*
b^(7/4)*c + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a^(3/4)*b^(5/4)*e - 24*
sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4
```

```
))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*s
qrt(2)*a^(1/4)*b^(7/4)*c + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a^(3/4)*
b^(5/4)*e + 24*sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)
*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(
3/4)))/(a^2*b)
```

Fricas [C] Result contains complex when optimal does not.
time = 44.57, size = 358509, normalized size = 909.92

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/589824*(92160*b^2*e*x^7 + 110592*b^2*d*x^6 + 165888*a*b*e*x^3 + 18432*(7*
b^2*c + a*b*g)*x^5 + 184320*a*b*d*x^2 - 73728*a^2*f + 2*(a^2*b^3*x^8 + 2*a^
3*b^2*x^4 + a^4*b)*((-I*sqrt(3) + 1)*((a^3*b*sqrt(-1/(a*b))*sqrt(-(210*a*b^
2*c*e*sqrt(-1/(a*b)) + 441*b^2*c^2 - 25*a*b*e^2 + 9*a^2*g^2 + 6*(5*a^2*b*e*
sqrt(-1/(a*b)) + 21*a*b*c)*g)/(a^6*b^3*sqrt(-1/(a*b)))) - 12*d)^2/(a^5*b) -
 3*(24*a^3*b^2*d*sqrt(-(210*a*b^2*c*e*sqrt(-1/(a*b)) + 441*b^2*c^2 - 25*a*b
*e^2 + 9*a^2*g^2 + 6*(5*a^2*b*e*sqrt(-1/(a*b)) + 21*a*b*c)*g)/(a^6*b^3*sqrt
(-1/(a*b)))) + 441*b^2*c^2 - 3*(10*b*e*g*sqrt(-1/(a*b)) - 3*g^2)*a^2 - (6*(
24*d^2 + 35*c*e)*b^2*sqrt(-1/(a*b)) + (25*e^2 - 126*c*g)*b)*a)/(a^6*b^3*sqr
t(-1/(a*b))))/(-1/12582912*(24*a^3*b^2*d*sqrt(-(210*a*b^2*c*e*sqrt(-1/(a*b)
) + 441*b^2*c^2 - 25*a*b*e^2 + 9*a^2*g^2 + 6*(5*a^2*b*e*sqrt(-1/(a*b)) + 21
*a*b*c)*g)/(a^6*b^3*sqrt(-1/(a*b)))) + 441*b^2*c^2 - 3*(10*b*e*g*sqrt(-1/(a
*b)) - 3*g^2)*a^2 - (6*(24*d^2 + 35*c*e)*b^2*sqrt(-1/(a*b)) + (25*e^2 - 126
*c*g)*b)*a)*(a^3*b*sqrt(- ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.17, size = 416, normalized size = 1.06

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^2d+21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^2d+21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}-\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}+\frac{5\,b^2x^7e+6\,b^2dx^6+7\,b^2cx^5+abgx^5+9\,abx^3e+10\,abdx^2+11\,abcx-3\,a^2gx-4\,a^2f}{32\left(bx^4+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b
^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d
 + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arct
an(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*s
qrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)
*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21
*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 -
 sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*x^7*e + 6*b^2*d
*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*x^3*e + 10*a*b*d*x^2 + 11*a*b*c*x -
3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

Mupad [B]
time = 0.71, size = 1001, normalized size = 2.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) - f/(8*b) + (9*e*x^3)/(32*a) + (x^5*(7*b*c + a*g))/(32*a^
2) + (x*(11*b*c - 3*a*g))/(32*a*b) + (3*b*d*x^6)/(16*a^2) + (5*b*e*x^7)/(32
*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log(- root(268435456*a^11*b^5*z
^4 + 983040*a^7*b^3*e*g*z^2 + 6881280*a^6*b^4*c*e*z^2 + 4718592*a^6*b^4*d^2
*z^2 - 774144*a^4*b^3*c*d*g*z - 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d*e^
2*z - 2709504*a^3*b^4*c^2*d*z - 8640*a^2*b^2*d^2*e*g + 6300*a^2*b^2*c*e^2*g
 - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 + 23814*a^2*
b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 + 625*a^2*b^2*e^4 + 2
0736*a*b^3*d^4 + 81*a^4*g^4 + 194481*b^4*c^4, z, k)*(root(268435456*a^11*b^
5*z^4 + 983040*a^7*b^3*e*g*z^2 + 6881280*a^6*b^4*c*e*z^2 + 4718592*a^6*b^4*
d^2*z^2 - 774144*a^4*b^3*c*d*g*z - 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d
*e^2*z - 2709504*a^3*b^4*c^2*d*z - 8640*a^2*b^2*d^2*e*g + 6300*a^2*b^2*c*e^
2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 + 23814*a
^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 + 625*a^2*b^2*e^4
+ 20736*a*b^3*d^4 + 81*a^4*g^4 + 194481*b^4*c^4, z, k)*((344064*a^5*b^3*c +
 49152*a^6*b^2*g)/(32768*a^6) - (6*b^3*d*x)/a) + (x*(144*a^4*b*g^2 + 7056*a
^2*b^3*c^2 - 400*a^3*b^2*e^2 + 2016*a^3*b^2*c*g))/(4096*a^6) + (15*b^2*d*e)
/(32*a^3)) - (2205*b^2*c^2*e - 3024*b^2*c*d^2 + 45*a^2*e*g^2 + 125*a*b*e^3
- 432*a*b*d^2*g + 630*a*b*c*e*g)/(32768*a^6) - (x*(315*b^2*c*d*e - 216*b^2*
d^3 + 45*a*b*d*e*g))/(4096*a^6))*root(268435456*a^11*b^5*z^4 + 983040*a^7*b
^3*e*g*z^2 + 6881280*a^6*b^4*c*e*z^2 + 4718592*a^6*b^4*d^2*z^2 - 774144*a^4
*b^3*c*d*g*z - 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d*e^2*z - 2709504*a^3
*b^4*c^2*d*z - 8640*a^2*b^2*d^2*e*g + 6300*a^2*b^2*c*e^2*g - 60480*a*b^3*c*
d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 + 23814*a^2*b^2*c^2*g^2 + 450
*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 + 625*a^2*b^2*e^4 + 20736*a*b^3*d^4 +
81*a^4*g^4 + 194481*b^4*c^4, z, k), k, 1, 4)
```

3.178 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4} \, dx$

Optimal. Leaf size=437

$$\frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3}+\frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b\left(a+bx^4\right)}-\frac{8af-x(11bc+ag+10bdx+9bex^2)}{96a^2b\left(a+bx^4\right)^2}$$

```
[Out] 1/12*x*(b*f*x^3+b*e*x^2+b*d*x-a*g+b*c)/a/b/(b*x^4+a)^3+1/384*x*(45*b*e*x^2+
60*b*d*x+7*a*g+77*b*c)/a^3/b/(b*x^4+a)+1/96*(-8*a*f+x*(9*b*e*x^2+10*b*d*x+a
*g+11*b*c))/a^2/b/(b*x^4+a)^2+5/32*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^
(1/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(77*b*c+7*a
*g-15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)+1/1024*ln(a^(1/4)*b^(1/4)
*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(77*b*c+7*a*g-15*e*a^(1/2)*b^(1/2))/a^(15/4
)/b^(5/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(77*b*c+7*a*g+
15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)+1/512*arctan(1+b^(1/4)*x*2^(
1/2)/a^(1/4))*(77*b*c+7*a*g+15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1872, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{5d\text{ArcTan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{x(7(ag+11bc)+60bdx+45bex^2)}{384a^3b\left(a+bx^4\right)}-\frac{8af-x(ag+11bc+10bdx+9bex^2)}{96a^2b\left(a+bx^4\right)^2}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3) + (x*(7*
(11*b*c + a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a + b*x^4)) - (8*a*f -
 x*(11*b*c + a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a + b*x^4)^2) + (5*d*A
rcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*b*c + 15*Sqrt[a]*
Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^
(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqr
t[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a
*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^
(15/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^4\right)^4}\,dx = \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-11bc-ag-10bdx-9bex^2-8bfx^3}{\left(a+bx^4\right)^3}\,dx}{12ab}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3} - \frac{8af-x(11bc+ag+10bdx+9bex^2)}{96a^2b\left(a+bx^4\right)^2}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b\left(a+bx^4\right)} -$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b\left(a+bx^4\right)} -$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b\left(a+bx^4\right)} -$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b\left(a+bx^4\right)} -$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b\left(a+bx^4\right)} -$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b\left(a+bx^4\right)} -$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b\left(a+bx^4\right)} -$$

Mathematica [A]
time = 0.24, size = 411, normalized size = 0.94

$$\frac{\frac{8a^{3/4}\sqrt[4]{b}\,x(77bc+7ag+15bx(4d+3ex))}{a+bx^4}+\frac{32a^{7/4}\sqrt[4]{b}\,x(11bc+ag+bx(10d+9ex))}{\left(a+bx^4\right)^2}-\frac{256a^{11/4}\sqrt[4]{b}\,(a(f+gx)-bx(c+x(d+ex)))}{\left(a+bx^4\right)^3}-6\left(77\sqrt{2}\,bc+80\sqrt[4]{a}\,b^{3/4}d+15\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,e+7\sqrt{2}\,ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+6\left(77\sqrt{2}\,bc-80\sqrt[4]{a}\,b^{3/4}d+15\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,e+7\sqrt{2}\,ag\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-3\sqrt{2}\left(77bc-15\sqrt{a}\,\sqrt{b}\,e+7ag\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+3\sqrt{2}\left(77bc-15\sqrt{a}\,\sqrt{b}\,e+7ag\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{3072a^{15/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*b^(1/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a + b*x^4)
+ (32*a^(7/4)*b^(1/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x)))/(a + b*x^4)^2
- (256*a^(11/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4)^
```

```
3 - 6*(77*Sqrt[2]*b*c + 80*a^(1/4)*b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[b]*e
 + 7*Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b
*c - 80*a^(1/4)*b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 7*Sqrt[2]*a*g)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(77*b*c - 15*Sqrt[a]*Sqr
t[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*
Sqrt[2]*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^(5/4))
```

Maple [A]
time = 0.36, size = 371, normalized size = 0.85

method	result
risch	$\frac{\frac{15 e\, b^{2} x^{11}}{128 a^{3}}+\frac{5 d\, b^{2} x^{10}}{32 a^{3}}+\frac{7 \left(a g +11 b c \right) b\, x^{9}}{384 a^{3}}+\frac{21 b e\, x^{7}}{64 a^{2}}+\frac{5 b d\, x^{6}}{12 a^{2}}+\frac{3 \left(a g +11 b c \right) x^{5}}{64 a^{2}}+\frac{113 e\, x^{3}}{384 a}+\frac{11 d\, x^{2}}{32 a}-\frac{\left(7 a g -51 b c \right) x}{128 a b}-\frac{f}{12 b}}{\left(b\, x^{4}+a \right)^{3}}+\frac{\sum_{\textit{\_R} =\operatorname{RootOf}\left(b\,\textit{\_Z}^{4}+a \right)} \ldots}{\ldots}$
default	$\frac{\frac{15 e\, b^{2} x^{11}}{128 a^{3}}+\frac{5 d\, b^{2} x^{10}}{32 a^{3}}+\frac{7 \left(a g +11 b c \right) b\, x^{9}}{384 a^{3}}+\frac{21 b e\, x^{7}}{64 a^{2}}+\frac{5 b d\, x^{6}}{12 a^{2}}+\frac{3 \left(a g +11 b c \right) x^{5}}{64 a^{2}}+\frac{113 e\, x^{3}}{384 a}+\frac{11 d\, x^{2}}{32 a}-\frac{\left(7 a g -51 b c \right) x}{128 a b}-\frac{f}{12 b}}{\left(b\, x^{4}+a \right)^{3}}+\frac{\left(7 a g +77 b c \right) \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ldots \right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32/a^3*d*b^2*x^10+7/384*(a*g+11*b*c)/a^3*b*x^9+21/
64*b*e/a^2*x^7+5/12/a^2*b*d*x^6+3/64/a^2*(a*g+11*b*c)*x^5+113/384/a*e*x^3+1
1/32*d/a*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x^4+a)^3+1/128/a^3/b*(
1/8*(7*a*g+77*b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/
b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(
1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+20*b*d/(a*b)^(1/2)*arctan(x^2*
(b/a)^(1/2))+15/8*e/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.50, size = 479, normalized size = 1.10

$$\frac{45\, b^{3} x^{11} e + 60\, b^{3} d x^{10} + 126\, a b^{2} x^{7} e + 160\, a b^{2} d x^{6} + 7\, {\left(11\, b^{3} c + a b^{2} g\right)} x^{9} + 113\, a^{2} b x^{3} e + 132\, a^{2} b d x^{2} + 18\, {\left(11\, a b^{2} c + a^{2} b g\right)} x^{5} - 32\, a^{3} f + 3\, {\left(51\, a^{2} b c - 7\, a^{3} g\right)} x}{384\, {\left(a^{3} b^{4} x^{12} + 3\, a^{4} b^{3} x^{8} + 3\, a^{5} b^{2} x^{4} + a^{6} b\right)}} + \frac{\frac{\sqrt{2} {\left(77\, b^{\frac{3}{2}} c + 7\, a \sqrt{b} g - 15\, \sqrt{a} b e\right)} \log\left(\sqrt{b} x^{2} + \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{3}{4}}} - \frac{\sqrt{2} {\left(77\, b^{\frac{3}{2}} c + 7\, a \sqrt{b} g - 15\, \sqrt{a} b e\right)} \log\left(\sqrt{b} x^{2} - \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{3}{4}}} + \frac{2 \, {\left(77 \, \sqrt{2} a^{\frac{1}{4}} b^{\frac{7}{4}} c + 7 \, \sqrt{2} a^{\frac{5}{4}} b^{\frac{3}{4}} g + 15 \, \sqrt{2} a^{\frac{3}{4}} b^{\frac{5}{4}} e - 80 \, \sqrt{a} b^{\frac{3}{2}} d\right)} \arctan\left(\frac{\sqrt{2} {\left(2 \, \sqrt{b} x + \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}\right)}}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{a^{\frac{3}{4}} \sqrt{\sqrt{a} \sqrt{b}} b^{\frac{3}{4}}} + \frac{2 \, {\left(77 \, \sqrt{2} a^{\frac{1}{4}} b^{\frac{7}{4}} c + 7 \, \sqrt{2} a^{\frac{5}{4}} b^{\frac{3}{4}} g + 15 \, \sqrt{2} a^{\frac{3}{4}} b^{\frac{5}{4}} e + 80 \, \sqrt{a} b^{\frac{3}{2}} d\right)} \arctan\left(\frac{\sqrt{2} {\left(2 \, \sqrt{b} x - \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}\right)}}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{a^{\frac{3}{4}} \sqrt{\sqrt{a} \sqrt{b}} b^{\frac{3}{4}}}}{1024 \, a^{3} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 +
7*(11*b^3*c + a*b^2*g)*x^9 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 + 18*(11*a*b
^2*c + a^2*b*g)*x^5 - 32*a^3*f + 3*(51*a^2*b*c - 7*a^3*g)*x)/(a^3*b^4*x^12
```

```
+ 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b) + 1/1024*(sqrt(2)*(77*b^(3/2)*c +
7*a*sqrt(b)*g - 15*sqrt(a)*b*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*b^(3/2)*c + 7*a*sqrt(b)*g - 15*
sqrt(a)*b*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4
)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 7*sqrt(2)*a^(5/4)*b^(3/4)*g
+ 15*sqrt(2)*a^(3/4)*b^(5/4)*e - 80*sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(2)*(
2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt
(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 7*sqrt(2)*a^
(5/4)*b^(3/4)*g + 15*sqrt(2)*a^(3/4)*b^(5/4)*e + 80*sqrt(a)*b^(3/2)*d)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^3*b)
```

Fricas [**C**] Result contains complex when optimal does not.
time = 83.46, size = 358702, normalized size = 820.83

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] 1/9437184*(1105920*b^3*e*x^11 + 1474560*b^3*d*x^10 + 3096576*a*b^2*e*x^7 +
3932160*a*b^2*d*x^6 + 172032*(11*b^3*c + a*b^2*g)*x^9 + 2777088*a^2*b*e*x^3
 + 3244032*a^2*b*d*x^2 + 442368*(11*a*b^2*c + a^2*b*g)*x^5 - 786432*a^3*f +
 2*(a^3*b^4*x^12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b)*((-I*sqrt(3) + 1)
*((a^4*b*sqrt(-1/(a*b))*sqrt(-(2310*a*b^2*c*e*sqrt(-1/(a*b)) + 5929*b^2*c^2
 - 225*a*b*e^2 + 49*a^2*g^2 + 14*(15*a^2*b*e*sqrt(-1/(a*b)) + 77*a*b*c)*g)/
(a^8*b^3*sqrt(-1/(a*b)))) - 40*d)^2/(a^7*b) - 3*(80*a^4*b^2*d*sqrt(-(2310*a
*b^2*c*e*sqrt(-1/(a*b)) + 5929*b^2*c^2 - 225*a*b*e^2 + 49*a^2*g^2 + 14*(15*
a^2*b*e*sqrt(-1/(a*b)) + 77*a*b*c)*g)/(a^8*b^3*sqrt(-1/(a*b)))) + 5929*b^2*
c^2 - 7*(30*b*e*g*sqrt(-1/(a*b)) - 7*g^2)*a^2 - (10*(160*d^2 + 231*c*e)*b^2
*sqrt(-1/(a*b)) + (225*e^2 - 1078*c*g)*b)*a)/(a^8*b^3*sqrt(-1/(a*b))))/(-1/
805306368*(80*a^4*b^2*d*sqrt(-(2310*a*b^2*c*e*sqrt(-1/(a*b)) + 5929*b^2*c^2
 - 225*a*b*e^2 + 49*a^2*g^2 + 14*(15*a^2*b*e*sqrt(-1/(a*b)) + 77*a*b*c)*g)/
(a^8*b^3*sqrt(-1/(a*b)))) ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.31, size = 466, normalized size = 1.07

$$\frac{\sqrt{2}\left(40\,\sqrt{2}\,\sqrt{ab}\,b^{2}d+77\,(ab^{3})^{\frac{1}{4}}\,b^{2}c+7\,(ab^{3})^{\frac{1}{4}}\,abg+15\,(ab^{3})^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^{4}b^{3}}+\frac{\sqrt{2}\left(40\,\sqrt{2}\,\sqrt{ab}\,b^{2}d+77\,(ab^{3})^{\frac{1}{4}}\,b^{2}c+7\,(ab^{3})^{\frac{1}{4}}\,abg+15\,(ab^{3})^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^{4}b^{3}}+\frac{\sqrt{2}\left(77\,(ab^{3})^{\frac{1}{4}}\,b^{2}c+7\,(ab^{3})^{\frac{1}{4}}\,abg-15\,(ab^{3})^{\frac{3}{4}}\,e\right)\log\left(x^{2}+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^{4}b^{3}}-\frac{\sqrt{2}\left(77\,(ab^{3})^{\frac{1}{4}}\,b^{2}c+7\,(ab^{3})^{\frac{1}{4}}\,abg-15\,(ab^{3})^{\frac{3}{4}}\,e\right)\log\left(x^{2}-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^{4}b^{3}}+\frac{45\,b^{3}x^{11}e+60\,b^{3}dx^{10}+77\,b^{3}cx^{9}+7\,ab^{2}gx^{9}+126\,ab^{2}x^{7}e+160\,ab^{2}dx^{6}+198\,ab^{2}cx^{5}+18\,a^{2}bgx^{5}+113\,a^{2}bx^{3}e+132\,a^{2}bdx^{2}+153\,a^{2}bcx-21\,a^{3}gx-32\,a^{3}f}{384\,(bx^{4}+a)^{3}a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b
^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/
b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*
d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/102
4*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4
)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2
)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log
(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*x^11*e
+ 60*b^3*d*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*x^7*e + 160*a*b^
2*d*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*
x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

Mupad [B]
time = 5.56, size = 1053, normalized size = 2.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x)
```

```
[Out] symsum(log(- root(68719476736*a^15*b^5*z^4 + 1211105280*a^8*b^4*c*e*z^2 + 1
10100480*a^9*b^3*e*g*z^2 + 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d
*g*z - 485703680*a^4*b^4*c^2*d*z - 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b
^3*d*e^2*z - 672000*a^2*b^2*d^2*e*g + 485100*a^2*b^2*c*e^2*g - 7392000*a*b^
3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g^3 + 1743126*a^2*b^2*c^2
*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2 + 50625*a^2*b^2*e^4 + 25
60000*a*b^3*d^4 + 2401*a^4*g^4 + 35153041*b^4*c^4, z, k)*(root(68719476736*
a^15*b^5*z^4 + 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 + 838
860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d*g*z - 485703680*a^4*b^4*c^2*d
*z - 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b^3*d*e^2*z - 672000*a^2*b^2*d^
2*e*g + 485100*a^2*b^2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 12782924*a*b^3*c^3
*g + 105644*a^3*b*c*g^3 + 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2
668050*a*b^3*c^2*e^2 + 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 + 2401*a^4*g^4
 + 35153041*b^4*c^4, z, k)*((20185088*a^7*b^3*c + 1835008*a^8*b^2*g)/(20971
52*a^9) - (5*b^3*d*x)/a^2) + (x*(1568*a^5*b*g^2 + 189728*a^3*b^3*c^2 - 7200
*a^4*b^2*e^2 + 34496*a^4*b^2*c*g))/(131072*a^9) + (75*b^2*d*e)/(256*a^5)) -
```

(88935*b^2*c^2*e - 123200*b^2*c*d^2 + 735*a^2*e*g^2 + 3375*a*b*e^3 - 11200*a*b*d^2*g + 16170*a*b*c*e*g)/(2097152*a^9) - (x*(5775*b^2*c*d*e - 4000*b^2*d^3 + 525*a*b*d*e*g))/(131072*a^9))*root(68719476736*a^15*b^5*z^4 + 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 + 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d*g*z - 485703680*a^4*b^4*c^2*d*z - 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b^3*d*e^2*z - 672000*a^2*b^2*d^2*e*g + 485100*a^2*b^2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g^3 + 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2 + 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 + 2401*a^4*g^4 + 35153041*b^4*c^4, z, k), k, 1, 4) + ((11*d*x^2)/(32*a) - f/(12*b) + (113*e*x^3)/(384*a) + (3*x^5*(11*b*c + a*g))/(64*a^2) + (7*b*x^9*(11*b*c + a*g))/(384*a^3) + (x*(51*b*c - 7*a*g))/(128*a*b) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (5*b*d*x^6)/(12*a^2) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8)

3.179 $\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{4}(1-x)^4$$

```
[Out] -1/4*(1-x)^4
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1600, 32}

$$-\frac{1}{4}(1-x)^4$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
```

```
[Out] -1/4*(1 - x)^4
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx &= \int (1-x)^3\,dx\\ &= -\frac{1}{4}(1-x)^4\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 0.82

$$-\frac{1}{4}(-1+x)^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
[Out] -1/4*(-1 + x)^4
```

Maple [A]
time = 0.31, size = 8, normalized size = 0.73

method	result	size
default	$-\frac{(x-1)^4}{4}$	8
gosper	$-\frac{x(x^3-4x^2+6x-4)}{4}$	17
risch	$-\frac{1}{4}x^4+x^3-\frac{3}{2}x^2+x-\frac{1}{4}$	17
norman	$\frac{-2x^5-2x^3-x^4-\frac{7}{4}x^2-\frac{1}{2}x-\frac{1}{4}x^8+\frac{1}{2}x^9-\frac{1}{4}x^{10}-\frac{3}{4}}{(x^3+x^2+x+1)^2}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^3/(x^3+x^2+x+1)^3,x,method=_RETURNVERBOSE)
[Out] -1/4*(x-1)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.
time = 0.28, size = 15, normalized size = 1.36

$$-\frac{1}{4}x^4+x^3-\frac{3}{2}x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="maxima")
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.
time = 0.37, size = 15, normalized size = 1.36

$$-\frac{1}{4}x^4+x^3-\frac{3}{2}x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="fricas")
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.
time = 0.02, size = 15, normalized size = 1.36

$$-\frac{x^4}{4}+x^3-\frac{3x^2}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**3/(x**3+x**2+x+1)**3,x)
```

```
[Out] -x**4/4 + x**3 - 3*x**2/2 + x
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.
time = 0.88, size = 15, normalized size = 1.36

$$-\frac{1}{4}x^4 + x^3 - \frac{3}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="giac")
```

```
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

Mupad [B]
time = 0.03, size = 15, normalized size = 1.36

$$-\frac{x^4}{4} + x^3 - \frac{3x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)^3/(x + x^2 + x^3 + 1)^3,x)
```

```
[Out] x - (3*x^2)/2 + x^3 - x^4/4
```

3.180 $\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} dx$

Optimal. Leaf size=11

$$-\frac{1}{3}(1-x)^3$$

[Out] -1/3*(1-x)^3

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1600, 32}

$$-\frac{1}{3}(1-x)^3$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]

[Out] -1/3*(1 - x)^3

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} dx &= \int (1-x)^2 dx \\ &= -\frac{1}{3}(1-x)^3\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.27

$$x-x^2+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]
```

```
[Out] x - x^2 + x^3/3
```

Maple [A]
time = 0.31, size = 8, normalized size = 0.73

method	result	size
default	$\frac{(x-1)^3}{3}$	8
gosper	$\frac{x(x^2-3x+3)}{3}$	12
risch	$\frac{1}{3}x^3-x^2+x-\frac{1}{3}$	14
norman	$\frac{-\frac{1}{3}x^2+\frac{2}{3}x+\frac{1}{3}x^4-\frac{2}{3}x^5+\frac{1}{3}x^6-\frac{1}{3}}{x^3+x^2+x+1}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^2/(x^3+x^2+x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(x-1)^3
```

Maxima [A]
time = 0.27, size = 12, normalized size = 1.09

$$\frac{1}{3}x^3-x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - x^2 + x
```

Fricas [A]
time = 0.39, size = 12, normalized size = 1.09

$$\frac{1}{3}x^3-x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 - x^2 + x
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.73

$$\frac{x^3}{3}-x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**2/(x**3+x**2+x+1)**2,x)
```

```
[Out] x**3/3 - x**2 + x
```

Giac [A]
time = 0.55, size = 12, normalized size = 1.09

$$\frac{1}{3}x^3 - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - x^2 + x
```

Mupad [B]
time = 0.02, size = 11, normalized size = 1.00

$$\frac{x\,(x^2 - 3\,x + 3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)^2/(x + x^2 + x^3 + 1)^2,x)
```

```
[Out] (x*(x^2 - 3*x + 3))/3
```

3.181 $\int \frac{1-x^4}{1+x+x^2+x^3} \, dx$

Optimal. Leaf size=9

$$x-\frac{x^2}{2}$$

[Out] x-1/2*x^2

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1600}

$$x-\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)/(1 + x + x^2 + x^3),x]

[Out] x - x^2/2

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1-x^4}{1+x+x^2+x^3} \, dx &= \int (1-x) \, dx \\ &= x-\frac{x^2}{2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$x-\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)/(1 + x + x^2 + x^3),x]

[Out] x - x^2/2

Maple [A]
time = 0.02, size = 8, normalized size = 0.89

method	result	size
gosper	$-\frac{x(x-2)}{2}$	7
default	$x-\frac{1}{2}x^2$	8
norman	$x-\frac{1}{2}x^2$	8
risch	$x-\frac{1}{2}x^2$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^3+x^2+x+1),x,method=_RETURNVERBOSE)
[Out] x-1/2*x^2
```

Maxima [A]
time = 0.29, size = 7, normalized size = 0.78

$$-\frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="maxima")
[Out] -1/2*x^2 + x
```

Fricas [A]
time = 0.37, size = 7, normalized size = 0.78

$$-\frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="fricas")
[Out] -1/2*x^2 + x
```

Sympy [A]
time = 0.01, size = 5, normalized size = 0.56

$$-\frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**3+x**2+x+1),x)
```

```
[Out] -x**2/2 + x
```

Giac [A]
time = 0.80, size = 7, normalized size = 0.78

$$-\frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 + x
```

Mupad [B]
time = 0.02, size = 6, normalized size = 0.67

$$-\frac{x(x-2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x + x^2 + x^3 + 1),x)
```

```
[Out] -(x*(x - 2))/2
```

3.182 $\int \frac{1+x+x^2+x^3}{1-x^4} dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

```
[Out] -ln(1-x)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1600, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

```
[Out] -Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x+x^2+x^3}{1-x^4}\, dx &= \int \frac{1}{1-x}\, dx \\ &= -\log(1-x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

[Out] -Log[1 - x]

Maple [A]
time = 0.33, size = 7, normalized size = 0.88

method	result
default	$-\ln \left(x -1\right)$
norman	$-\ln \left(x -1\right)$
risch	$-\ln \left(x -1\right)$
meijerg	$-\frac{\ln \left(-x^{4}+1\right)}{4}-\frac{x^{3} \left(\ln \left(1-\left(x^{4}\right)^{\frac{1}{4}}\right)-\ln \left(1+\left(x^{4}\right)^{\frac{1}{4}}\right)+2 \arctan \left(\left(x^{4}\right)^{\frac{1}{4}}\right)\right)}{4 \left(x^{4}\right)^{\frac{3}{4}}}+\frac{\operatorname{arctanh}\left(x^{2}\right)}{2}-\frac{x \left(\ln \left(1-\left(x^{4}\right)^{\frac{1}{4}}\right)-\ln \left(1+\left(x^{4}\right)^{\frac{1}{4}}\right)-\right.}{4 \left(x^{4}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x-1)
```

Maxima [A]
time = 0.28, size = 6, normalized size = 0.75

$$-\log \left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -log(x - 1)
```

Fricas [A]
time = 0.40, size = 6, normalized size = 0.75

$$-\log \left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] -log(x - 1)
```

Sympy [A]
time = 0.01, size = 5, normalized size = 0.62

$$-\log \left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(-x**4+1),x)
```

```
[Out] -log(x - 1)
```

Giac [A]
time = 0.94, size = 7, normalized size = 0.88

$$-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")
```

```
[Out] -log(abs(x - 1))
```

Mupad [B]
time = 0.00, size = 6, normalized size = 0.75

$$-\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2 + x^3 + 1)/(x^4 - 1),x)
```

```
[Out] -log(x - 1)
```

3.183 $\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} \, dx$

Optimal. Leaf size=7

$$\frac{1}{1-x}$$

```
[Out] 1/(1-x)
```

Rubi [A]
time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1600, 32}

$$\frac{1}{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]
```

```
[Out] (1 - x)^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} \, dx &= \int \frac{1}{(1-x)^2} \, dx \\ &= \frac{1}{1-x} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{-1+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]
```

```
[Out] -(-1 + x)^(-1)
```

Maple [A]
time = 0.35, size = 8, normalized size = 1.14

method	result
gosper	$-\frac{1}{x-1}$
default	$-\frac{1}{x-1}$
risch	$-\frac{1}{x-1}$
norman	$\frac{-x^{3}-x^{2}-x-1}{x^{4}-1}$
meijerg	$\frac{(-1)^{\frac{1}{4}}\left(-\frac{x^{3}(-1)^{\frac{3}{4}}}{-x^{4}+1}-\frac{3x^{3}(-1)^{\frac{3}{4}}\left(\ln\left(1-\left(x^{4}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{4}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^{4}\right)^{\frac{1}{4}}\right)\right)}{4\left(x^{4}\right)^{\frac{3}{4}}}\right)}{4}+\frac{i\left(-\frac{ix^{2}}{-x^{4}+1}+i\,\mathrm{arctanh}\left(x^{2}\right)\right)}{2}+\frac{3(-1)^{\frac{3}{4}}\left(-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)^2/(-x^4+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(x-1)
```

Maxima [A]
time = 0.28, size = 7, normalized size = 1.00

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="maxima")
```

```
[Out] -1/(x - 1)
```

Fricas [A]
time = 0.37, size = 7, normalized size = 1.00

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="fricas")
```

```
[Out] -1/(x - 1)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.71

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)**2/(-x**4+1)**2,x)
[Out] -1/(x - 1)
```

Giac [A]
time = 0.73, size = 7, normalized size = 1.00

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="giac")
[Out] -1/(x - 1)
```

Mupad [B]
time = 0.03, size = 7, normalized size = 1.00

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 1)^2/(x^4 - 1)^2,x)
[Out] -1/(x - 1)
```

3.184 $\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3} dx$

Optimal. Leaf size=11

$$\frac{1}{2(1-x)^2}$$

```
[Out] 1/2/(1-x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1600, 32}

$$\frac{1}{2(1-x)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]
```

```
[Out] 1/(2*(1 - x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3} dx &= \int \frac{1}{(1-x)^3} dx \\ &= \frac{1}{2(1-x)^2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 0.82

$$\frac{1}{2(-1+x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]
```

```
[Out] 1/(2*(-1 + x)^2)
```

Maple [A]
time = 0.42, size = 8, normalized size = 0.73

method	result
gosper	$\frac{1}{2(x-1)^2}$
default	$\frac{1}{2(x-1)^2}$
risch	$\frac{1}{2(x-1)^2}$
norman	$\frac{x+x^5+\frac{3}{2}x^4+\frac{3}{2}x^2+2x^3+\frac{1}{2}x^6+\frac{1}{2}}{(x^4-1)^2}$
meijerg	$-\frac{(-1)^{\frac{3}{4}}\left(\frac{(-1)^{\frac{1}{4}}x(-7x^4+11)}{4(-x^4+1)^2}-\frac{21x(-1)^{\frac{1}{4}}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{16(x^4)^{\frac{1}{4}}}\right)}{8}+\frac{5(-1)^{\frac{1}{4}}\left(-\frac{x^3(-1)^{\frac{3}{4}}(21x^4+7)}{28(-x^4+1)^2}-\frac{3x^3}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)^3/(-x^4+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(x-1)^2
```

Maxima [A]
time = 0.28, size = 12, normalized size = 1.09

$$\frac{1}{2\,(x^2-2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="maxima")
```

```
[Out] 1/2/(x^2 - 2*x + 1)
```

Fricas [A]
time = 0.42, size = 12, normalized size = 1.09

$$\frac{1}{2\,(x^2-2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="fricas")
```

```
[Out] 1/2/(x^2 - 2*x + 1)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.91

$$\frac{1}{2x^2 - 4x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)**3/(-x**4+1)**3,x)
[Out] 1/(2*x**2 - 4*x + 2)
```

Giac [A]
time = 0.63, size = 7, normalized size = 0.64

$$\frac{1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="giac")
[Out] 1/2/(x - 1)^2
```

Mupad [B]
time = 4.84, size = 7, normalized size = 0.64

$$\frac{1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2 + x^3 + 1)^3/(x^4 - 1)^3,x)
[Out] 1/(2*(x - 1)^2)
```

3.185 $\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} \, dx$

Optimal. Leaf size=11

$$\frac{1}{3(1-x)^3}$$

[Out] 1/3/(1-x)^3

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1600, 32}

$$\frac{1}{3(1-x)^3}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]

[Out] 1/(3*(1 - x)^3)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} \, dx &= \int \frac{1}{(1-x)^4} \, dx \\ &= \frac{1}{3(1-x)^3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 0.82

$$-\frac{1}{3(-1+x)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]
```

```
[Out] -1/3*1/(-1 + x)^3
```

Maple [A]
time = 0.43, size = 8, normalized size = 0.73

method	result	size
gosper	$-\frac{1}{3(x-1)^{3}}$	8
default	$-\frac{1}{3(x-1)^{3}}$	8
risch	$-\frac{1}{3(x-1)^{3}}$	8
norman	$\frac{-5x^{4}-\frac{1}{3}x^{12}-x-2x^{2}-\frac{10}{3}x^{3}-4x^{5}-\frac{10}{3}x^{6}-2x^{7}-\frac{1}{3}x^{9}}{(x^{4}-1)^{3}}$	53
meijerg	Expression too large to display	698

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)^4/(-x^4+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(x-1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.
time = 0.27, size = 17, normalized size = 1.55

$$-\frac{1}{3\left(x^{3}-3x^{2}+3x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="maxima")
```

```
[Out] -1/3/(x^3 - 3*x^2 + 3*x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.
time = 0.40, size = 17, normalized size = 1.55

$$-\frac{1}{3\left(x^{3}-3x^{2}+3x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="fricas")
```

```
[Out] -1/3/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.
time = 0.04, size = 17, normalized size = 1.55

$$-\frac{1}{3x^3 - 9x^2 + 9x - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)**4/(-x**4+1)**4,x)
```

```
[Out] -1/(3*x**3 - 9*x**2 + 9*x - 3)
```

Giac [**A**]
time = 0.62, size = 7, normalized size = 0.64

$$-\frac{1}{3(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="giac")
```

```
[Out] -1/3/(x - 1)^3
```

Mupad [**B**]
time = 4.81, size = 7, normalized size = 0.64

$$-\frac{1}{3(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 1)^4/(x^4 - 1)^4,x)
```

```
[Out] -1/(3*(x - 1)^3)
```

3.186 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx$

Optimal. Leaf size=165

$$-\frac{gx}{b}-\frac{hx^2}{2b}+\frac{\left(bc-\sqrt{a}\,\sqrt{b}\,e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\,\sqrt{b}\,e+ag\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{(bd+ah)\tan}{2\sqrt{}}$$

```
[Out] -g*x/b-1/2*h*x^2/b-1/4*f*ln(-b*x^4+a)/b+1/2*(a*h+b*d)*arctanh(x^2*b^(1/2)/a
^(1/2))/b^(3/2)/a^(1/2)+1/2*arctan(b^(1/4)*x/a^(1/4))*(b*c+a*g-e*a^(1/2)*b^
(1/2))/a^(3/4)/b^(5/4)+1/2*arctanh(b^(1/4)*x/a^(1/4))*(b*c+a*g+e*a^(1/2)*b^
(1/2))/a^(3/4)/b^(5/4)
```

Rubi [A]

time = 0.18, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1899, 1901, 1181, 211, 214, 1833, 1824, 649, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\,\sqrt{b}\,e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,\sqrt{b}\,e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}-\frac{hx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]
```

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/
4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcT
anh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - (f*Log[a - b*x^4])/(4*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx &= \int \left(\frac{c+ex^2+gx^4}{a-bx^4}+\frac{x(d+fx^2+hx^4)}{a-bx^4}\right)dx \\
&= \int \frac{c+ex^2+gx^4}{a-bx^4}\,dx+\int \frac{x(d+fx^2+hx^4)}{a-bx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{a-bx^2}\,dx,x,x^2\right)+\int \left(-\frac{g}{b}+\frac{bc+ag+bex^2}{b\left(a-bx^4\right)} \right. \\
&= -\frac{gx}{b}+\frac{1}{2}\text{Subst}\left(\int \left(-\frac{h}{b}+\frac{bd+ah+bfx}{b\left(a-bx^2\right)}\right)dx,x,x^2\right)+\frac{\int \frac{bc+ag}{a-}}{} \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}+\frac{\text{Subst}\left(\int \frac{bd+ah+bfx}{a-bx^2}\,dx,x,x^2\right)}{2b}+\frac{1}{2}\left(e-\frac{bc+ag}{\sqrt{a}\sqrt{b}}\right)\int \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}+\frac{\left(bc-\sqrt{a}\sqrt{b}\,e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\right.}{} \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}+\frac{\left(bc-\sqrt{a}\sqrt{b}\,e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\right.}{}
\end{aligned}
$$

Mathematica [A]

time = 0.17, size = 256, normalized size = 1.55

$$
\frac{-4a^{3/4}\sqrt{b}\,gx-2a^{3/4}\sqrt{b}\,hx^2+2\sqrt[4]{b}\left(bc-\sqrt{a}\sqrt{b}\,e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\left(b^{5/4}c+\sqrt[4]{a}\,bd+\sqrt{a}\,b^{3/4}e+a\sqrt[4]{b}\,g+a^{5/4}h\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+\left(b^{5/4}c-\sqrt[4]{a}\,bd+\sqrt{a}\,b^{3/4}e+a\sqrt[4]{b}\,g-a^{5/4}h\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+\sqrt[4]{a}\,(bd+ah)\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)-a^{3/4}\sqrt{b}\,f\log\left(a-bx^4\right)}{4a^{3/4}b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]
```

```
[Out] (-4*a^(3/4)*Sqrt[b]*g*x - 2*a^(3/4)*Sqrt[b]*h*x^2 + 2*b^(1/4)*(b*c - Sqrt[a
]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(5/4)*c + a^(1/4)*b*d +
 Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g + a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (b
^(5/4)*c - a^(1/4)*b*d + Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g - a^(5/4)*h)*Log[a
^(1/4) + b^(1/4)*x] + a^(1/4)*(b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2] - a^(3
/4)*Sqrt[b]*f*Log[a - b*x^4])/(4*a^(3/4)*b^(3/2))
```

Maple [A]

time = 0.34, size = 180, normalized size = 1.09

method	result
risch	$-\frac{h\,x^{2}}{2b}-\frac{gx}{b}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(bc+ag+(ah+bd)\_R+be\,\_R^{2}+\_R^{3}bf\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$

default	$-\frac{\frac{1}{2}h\,x^2+gx}{b}+\frac{\frac{(ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(ah+bd)\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(1/2*h*x^2+g*x)+1/b*(1/4*(a*g+b*c)*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(
x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))+1/4*(a*h+b*d)/(a*b)^(1/2)*ln((a+x^
2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2)))-1/4*e/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/
4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4))))-1/4*f*ln(-b*x^4+a))
```

Maxima [A]
time = 0.51, size = 224, normalized size = 1.36

$$-\frac{hx^2+2\,gx}{2\,b}+\frac{\frac{2\left(b^{\frac{3}{2}}c+a\sqrt{b}\,g-\sqrt{a}\,be\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\left(b^{\frac{3}{2}}d-\sqrt{a}\,bf+a\sqrt{b}\,h\right)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}d+\sqrt{a}\,bf+a\sqrt{b}\,h\right)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}c+a\sqrt{b}\,g+\sqrt{a}\,be\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*(h*x^2 + 2*g*x)/b + 1/4*(2*(b^(3/2)*c + a*sqrt(b)*g - sqrt(a)*b*e)*arc
tan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)
) + (b^(3/2)*d - sqrt(a)*b*f + a*sqrt(b)*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqr
t(a)*b) - (b^(3/2)*d + sqrt(a)*b*f + a*sqrt(b)*h)*log(sqrt(b)*x^2 - sqrt(a)
)/(sqrt(a)*b) - (b^(3/2)*c + a*sqrt(b)*g + sqrt(a)*b*e)*log((sqrt(b)*x - sq
rt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))*sqrt(b)))/b
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(125) = 250.
time = 0.72, size = 342, normalized size = 2.07

$$-\frac{\sqrt{2}\left(b^{2}c+abg-\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c+abg+\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c+abg-\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b^{2}c+abg-\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{f\log\left(|bx^{4}-a|\right)}{4\,b}-\frac{bhx^{2}+2\,bgx}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c + a*b*g - sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(2)*(-a*b^3)
^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)
)/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c + a*b*g + sqrt(2)*(-a*b
^3)^(1/4)*b*d + sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b*e)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*sqrt(2
)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(
-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^
2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) - 1/4*f*log(abs(b*x
^4 - a))/b - 1/2*(b*h*x^2 + 2*b*g*x)/b^2
```

Mupad [B]
time = 5.54, size = 2478, normalized size = 15.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x)
```

```
[Out] symsum(log(- root(256*a^3*b^6*z^4 + 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*g*z^2
- 64*a^3*b^4*d*h*z^2 - 64*a^2*b^5*c*e*z^2 - 32*a^4*b^3*h^2*z^2 + 96*a^3*b^4
*f^2*z^2 - 32*a^2*b^5*d^2*z^2 - 32*a^3*b^3*e*f*g*z - 32*a^3*b^3*d*f*h*z + 3
2*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*b^2*g^
2*h*z - 16*a^4*b^2*f*h^2*z + 16*a^3*b^3*e^2*h*z + 16*a^3*b^3*d*g^2*z + 16*a
^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z + 16*a*b^5*c^2*d*z
 + 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h + 8*a^2*b^3*c*d
*f*g - 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*a^3*b^
2*d*f^2*h + 4*a^3*b^2*d*f*g^2 + 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^2 - 4*a
^2*b^3*d^2*e*g + 4*a^2*b^3*d*e^2*f + 4*a^2*b^3*c*e^2*g - 4*a^2*b^3*c*e*f^2
+ 4*a^4*b*f*g^2*h - 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*d^2*e + 4
*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 - 6*a^
2*b^3*c^2*g^2 - 2*a^2*b^3*d^2*f^2 - 2*a^4*b*f^2*h^2 + 4*a^2*b^3*d^3*h - 4*a
```

```
^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a*b^4*d^4 + a^5*h^4 - a^2*b^
3*e^4 - a^4*b*g^4 - b^5*c^4, z, k)*((8*a*b^3*c*f - 8*a*b^3*d*e - 8*a^2*b^2*
e*h + 8*a^2*b^2*f*g)/b + root(256*a^3*b^6*z^4 + 256*a^3*b^5*f*z^3 - 64*a^3*
b^4*e*g*z^2 - 64*a^3*b^4*d*h*z^2 - 64*a^2*b^5*c*e*z^2 - 32*a^4*b^3*h^2*z^2
+ 96*a^3*b^4*f^2*z^2 - 32*a^2*b^5*d^2*z^2 - 32*a^3*b^3*e*f*g*z - 32*a^3*b^3
*d*f*h*z + 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 1
6*a^4*b^2*g^2*h*z - 16*a^4*b^2*f*h^2*z + 16*a^3*b^3*e^2*h*z + 16*a^3*b^3*d*
g^2*z + 16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z + 16*a
*b^5*c^2*d*z + 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h + 8
*a^2*b^3*c*d*f*g - 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*
g - 4*a^3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 + 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c
*e*h^2 - 4*a^2*b^3*d^2*e*g + 4*a^2*b^3*d*e^2*f + 4*a^2*b^3*c*e^2*g - 4*a^2*
b^3*c*e*f^2 + 4*a^4*b*f*g^2*h - 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4
*c*d^2*e + 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^
2*g^2 - 6*a^2*b^3*c^2*g^2 - 2*a^2*b^3*d^2*f^2 - 2*a^4*b*f^2*h^2 + 4*a^2*b^3
*d^3*h - 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a*b^4*d^4 + a^5*
h^4 - a^2*b^3*e^4 - a^4*b*g^4 - b^5*c^4, z, k)*((16*a^2*b^3*g + 16*a*b^4*c)
/b - (x*(16*a^2*b^3*h + 16*a*b^4*d))/b) + (x*(4*b^4*c^2 + 4*a*b^3*e^2 + 4*a
^2*b^2*g^2 + 8*a*b^3*c*g - 8*a*b^3*d*f - 8*a^2*b^2*f*h))/b) - (a*b^2*e^3 +
b^3*c*d^2 - b^3*c^2*e + a^3*g*h^2 + a*b^2*c*f^2 + a*b^2*d^2*g + a^2*b*c*h^2
 - a^2*b*e*g^2 + a^2*b*f^2*g + 2*a*b^2*c*d*h - 2*a*b^2*c*e*g - 2*a*b^2*d*e*
f + 2*a^2*b*d*g*h - 2*a^2*b*e*f*h)/b - (x*(b^3*d^3 + a^3*h^3 + b^3*c^2*f -
2*b^3*c*d*e - a*b^2*d*f^2 + a*b^2*e^2*f + 3*a*b^2*d^2*h + 3*a^2*b*d*h^2 + a
^2*b*f*g^2 - a^2*b*f^2*h - 2*a*b^2*c*e*h + 2*a*b^2*c*f*g - 2*a*b^2*d*e*g -
2*a^2*b*e*g*h))/b)*root(256*a^3*b^6*z^4 + 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*
g*z^2 - 64*a^3*b^4*d*h*z^2 - 64*a^2*b^5*c*e*z^2 - 32*a^4*b^3*h^2*z^2 + 96*a
^3*b^4*f^2*z^2 - 32*a^2*b^5*d^2*z^2 - 32*a^3*b^3*e*f*g*z - 32*a^3*b^3*d*f*h
*z + 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*
b^2*g^2*h*z - 16*a^4*b^2*f*h^2*z + 16*a^3*b^3*e^2*h*z + 16*a^3*b^3*d*g^2*z
+ 16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z + 16*a*b^5*c
^2*d*z + 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h + 8*a^2*b
^3*c*d*f*g - 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*
a^3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 + 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^2
 - 4*a^2*b^3*d^2*e*g + 4*a^2*b^3*d*e^2*f + 4*a^2*b^3*c*e^2*g - 4*a^2*b^3*c*
e*f^2 + 4*a^4*b*f*g^2*h - 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*d^2
*e + 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2
- 6*a^2*b^3*c^2*g^2 - 2*a^2*b^3*d^2*f^2 - 2*a^4*b*f^2*h^2 + 4*a^2*b^3*d^3*h
 - 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a*b^4*d^4 + a^5*h^4 -
a^2*b^3*e^4 - a^4*b*g^4 - b^5*c^4, z, k), k, 1, 4) - (h*x^2)/(2*b) - (g*x)/
b
```

3.187 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx$

Optimal. Leaf size=188

$$-\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}-\frac{\left(be-\frac{\sqrt{b}\,(bc+ag)}{\sqrt{a}}+ai\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\,b^{7/4}}+\frac{\left(be+\frac{\sqrt{b}\,(bc+ag)}{\sqrt{a}}+ai\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\,b^{7/4}}+\frac{(bd+\cdots}{\cdots}$$

```
[Out] -g*x/b-1/2*h*x^2/b-1/3*i*x^3/b-1/4*f*ln(-b*x^4+a)/b+1/2*(a*h+b*d)*arctanh(x
^2*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-1/2*arctan(b^(1/4)*x/a^(1/4))*(b*e+a*i-
(a*g+b*c)*b^(1/2)/a^(1/2))/a^(1/4)/b^(7/4)+1/2*arctanh(b^(1/4)*x/a^(1/4))*(
b*e+a*i+(a*g+b*c)*b^(1/2)/a^(1/2))/a^(1/4)/b^(7/4)
```

Rubi [A]
time = 0.22, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.220$, Rules used = {1899, 1833, 1824, 649, 214, 266, 1901, 1181, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}\,(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}\,b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}\,(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}\,b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x]
```

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - ((b*e - (Sqrt[b]*(b*c + a*g))/
Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*e + (
Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4
)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2
)) - (f*Log[a - b*x^4])/(4*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+187x^6}{a-bx^4}\,dx &= \int \left(\frac{x(d+fx^2+hx^4)}{a-bx^4}+\frac{c+ex^2+gx^4+187x^6}{a-bx^4}\right)\,dx \\
&= \int \frac{x(d+fx^2+hx^4)}{a-bx^4}\,dx+\int \frac{c+ex^2+gx^4+187x^6}{a-bx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{a-bx^2}\,dx,x,x^2\right)+\int \left(-\frac{g}{b}-\frac{187x^2}{b}\right. \\
&= -\frac{gx}{b}-\frac{187x^3}{3b}+\frac{1}{2}\text{Subst}\left(\int \left(-\frac{h}{b}+\frac{bd+ah+bfx}{b\,(a-bx^2)}\right)\,dx\right. \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{187x^3}{3b}+\frac{\text{Subst}\left(\int \frac{bd+ah+bfx}{a-bx^2}\,dx,x,x^2\right)}{2b}+ \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{187x^3}{3b}-\frac{\left(187a+be-\frac{\sqrt{b}\,(bc+ag)}{\sqrt{a}}\right)\tan^{-1}}{2\sqrt[4]{a}\,b^{7/4}} \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{187x^3}{3b}-\frac{\left(187a+be-\frac{\sqrt{b}\,(bc+ag)}{\sqrt{a}}\right)\tan^{-1}}{2\sqrt[4]{a}\,b^{7/4}}
\end{aligned}
$$

Mathematica [A]
time = 0.24, size = 301, normalized size = 1.60

$$
\frac{-12b^{3/4}gx-6b^{3/4}hx^2-4b^{3/4}ix^3+\frac{6\left(b^{3/2}c-\sqrt{a}\,be+a\sqrt{b}\,g-a^{3/2}i\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}}-\frac{3\left(b^{3/2}c+\sqrt[4]{a}\,b^{5/4}d+\sqrt{a}\,be+a\sqrt{b}\,g+a^{5/4}\sqrt[4]{b}\,h+a^{3/2}i\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)}{a^{3/4}}+\frac{3\left(b^{3/2}c-\sqrt[4]{a}\,b^{5/4}d+\sqrt{a}\,be+a\sqrt{b}\,g-a^{5/4}\sqrt[4]{b}\,h+a^{3/2}i\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{a^{3/4}}+\frac{3\sqrt[4]{b}\,(bd+ah)\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt{a}}-3b^{3/4}f\log\left(a-bx^4\right)}{12b^{7/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x]
```

```
[Out] (-12*b^(3/4)*g*x - 6*b^(3/4)*h*x^2 - 4*b^(3/4)*i*x^3 + (6*(b^(3/2)*c - Sqrt
[a]*b*e + a*Sqrt[b]*g - a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(3/4) - (
3*(b^(3/2)*c + a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g + a^(5/4)*b^(1
/4)*h + a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(3/4) + (3*(b^(3/2)*c - a^(1
/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g - a^(5/4)*b^(1/4)*h + a^(3/2)*i)*
Log[a^(1/4) + b^(1/4)*x])/a^(3/4) + (3*b^(1/4)*(b*d + a*h)*Log[Sqrt[a] + Sq
rt[b]*x^2])/Sqrt[a] - 3*b^(3/4)*f*Log[a - b*x^4])/(12*b^(7/4))
```

Maple [A]
time = 0.35, size = 195, normalized size = 1.04

method	result

risch	$-\frac{i x^3}{3b} - \frac{h x^2}{2b} - \frac{g x}{b} + \frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b-a\right)} \frac{\left(-\_R^3 bf+(-ai-be)\_R^2+(-ah-bd)\_R-ag-bc\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$-\frac{\frac{1}{3}i x^3+\frac{1}{2}h x^2+gx}{b} + \frac{\frac{(ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(ah+bd)\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{(ai+be)\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln \cdots\right.}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}} \cdots}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/b*(1/3*i*x^3+1/2*h*x^2+g*x)+1/b*(1/4*(a*g+b*c)*(a/b)^(1/4)/a*(ln((x+(a/b
)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))+1/4*(a*h+b*d)/(a*b)^(1/2
)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2)))-1/4*(a*i+b*e)/b/(a/b)^(1/4)*(
2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4))))-1/4*f*ln(-b*x^
4+a))
```

Maxima [A]
time = 0.51, size = 240, normalized size = 1.28

$$-\frac{3hx^2+2ix^3+6gx}{6b}+\frac{\frac{2\left(b^{\frac{3}{2}}c+a\sqrt{b}\,g-\sqrt{a}\,be-ia^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b^{\frac{3}{2}}d-\sqrt{a}\,bf+a\sqrt{b}\,h\right)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}d+\sqrt{a}\,bf+a\sqrt{b}\,h\right)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}c+a\sqrt{b}\,g+\sqrt{a}\,be+ia^{\frac{3}{2}}\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="ma
xima")
```

```
[Out] -1/6*(3*h*x^2 + 2*I*x^3 + 6*g*x)/b + 1/4*(2*(b^(3/2)*c + a*sqrt(b)*g - sqrt
(a)*b*e - I*a^(3/2))*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))*sqrt(b)) + (b^(3/2)*d - sqrt(a)*b*f + a*sqrt(b)*h)*log(sqr
t(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (b^(3/2)*d + sqrt(a)*b*f + a*sqrt(b)*h)*l
og(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (b^(3/2)*c + a*sqrt(b)*g + sqrt(a)*
b*e + I*a^(3/2))*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(
sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(145) = 290.
time = 0.66, size = 533, normalized size = 2.84

$$-\frac{\sqrt{2}\left(b^2c+abg-\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd-\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c+abg+\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd+\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c+abg-\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b^2c+abg-\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}-\frac{f\log\left(|bx^4-a|\right)}{4b}+\frac{i\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b^4}+\frac{i\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b^4}-\frac{i\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8b^4}+\frac{i\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8b^4}-\frac{3b^2hx^2+2ib^2x^3+6b^2gx}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="gi
ac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c + a*b*g - sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(2)*(-a*b^3)
^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)
)/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c + a*b*g + sqrt(2)*(-a*b
^3)^(1/4)*b*d + sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b*e)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*sqrt(2
)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(
-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^
2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) - 1/4*f*log(abs(b*x
^4 - a))/b + 1/4*I*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4 + 1/4*I*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4 - 1/8*I*sqrt(2)*(-a*
b^3)^(3/4)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4 + 1/8*I*sqrt(
2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4 - 1/6*
(3*b^2*h*x^2 + 2*I*b^2*x^3 + 6*b^2*g*x)/b^3
```

Mupad [B]
time = 5.07, size = 2500, normalized size = 13.30

$$\text{Too large to display}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x)
```

```
[Out] symsum(log(- (a^4*i^3 + a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^2*b^2*c*h^2 -
 a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i + a*b^3*c*f^2 + a*b^3*d^2*
```

g - a*b^3*c^2*i + 3*a^3*b*e*i^2 + a^3*b*g*h^2 - a^3*b*g^2*i - 2*a^2*b^2*c*g
*i - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h + 2*a*b^3*c*d*h -
2*a*b^3*c*e*g - 2*a*b^3*d*e*f - 2*a^3*b*f*h*i)/b^2 - root(256*a^3*b^7*z^4 +
 256*a^3*b^6*f*z^3 - 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d
*h*z^2 - 64*a^3*b^5*c*i*z^2 - 64*a^2*b^6*c*e*z^2 - 32*a^4*b^4*h^2*z^2 + 96*
a^3*b^5*f^2*z^2 - 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*i*z + 32*a^4*b^3*e*h*
i*z - 32*a^3*b^4*e*f*g*z - 32*a^3*b^4*d*f*h*z + 32*a^3*b^4*d*e*i*z + 32*a^3
*b^4*c*g*h*z - 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z + 32*a^2*b^5*c*d*g*z
 + 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^3*f*h^2*z + 16*a^4*b^
3*d*i^2*z + 16*a^3*b^4*e^2*h*z + 16*a^3*b^4*d*g^2*z + 16*a^2*b^5*c^2*h*z -
16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z + 16*a*b^6*c^2*d*z + 16*a^3*b^4*f^3
*z + 8*a^4*b^2*e*f*h*i - 8*a^4*b^2*d*g*h*i - 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*
d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^3*b^3*c*d*h*i + 8*a^2
*b^4*c*d*f*g - 8*a^2*b^4*c*d*e*h - 4*a^4*b^2*f^2*g*i + 4*a^4*b^2*f*g^2*h +
4*a^4*b^2*e*g^2*i - 4*a^4*b^2*e*g*h^2 - 4*a^4*b^2*c*h^2*i - 4*a^3*b^3*d^2*g
*i + 4*a^4*b^2*d*f*i^2 + 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3*e^2*f*h - 4*a^3*b^3*
e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^3*b^3*d*f*g^2 + 4*a^2
*b^4*c^2*f*h + 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2 - 4*a^2*b^4*d^2*e*g -
4*a^2*b^4*c*d^2*i + 4*a^2*b^4*d*e^2*f + 4*a^2*b^4*c*e^2*g - 4*a^2*b^4*c*e*f
^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a*b^5*c^2*d*f - 4*a*b^5*c*d^2*e
- 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g - 6*a^4*b^2*e^2*i^2 - 2*a^4*b^2*f^2*h^2 + 6
*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3*b^3*c^2*i^2 - 6*a^2*b^4*c^2*g^
2 - 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*a^3*b^3*e^3*i + 4*a^4*b^2*d*h^3
 + 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b^5*c^2*e^2 + a^3*b^3*f^4 + a^5*
b*h^4 + a*b^5*d^4 - a^4*b^2*g^4 - a^2*b^4*e^4 - a^6*i^4 - b^6*c^4, z, l)*(r
oot(256*a^3*b^7*z^4 + 256*a^3*b^6*f*z^3 - 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e
*g*z^2 - 64*a^3*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 - 64*a^2*b^6*c*e*z^2 - 32*
a^4*b^4*h^2*z^2 + 96*a^3*b^5*f^2*z^2 - 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*
i*z + 32*a^4*b^3*e*h*i*z - 32*a^3*b^4*e*f*g*z - 32*a^3*b^4*d*f*h*z + 32*a^3
*b^4*d*e*i*z + 32*a^3*b^4*c*g*h*z - 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z
 + 32*a^2*b^5*c*d*g*z + 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^
3*f*h^2*z + 16*a^4*b^3*d*i^2*z + 16*a^3*b^4*e^2*h*z + 16*a^3*b^4*d*g^2*z +
16*a^2*b^5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z + 16*a*b^6*c^2
*d*z + 16*a^3*b^4*f^3*z + 8*a^4*b^2*e*f*h*i - 8*a^4*b^2*d*g*h*i - 8*a^3*b^3
*d*e*g*h + 8*a^3*b^3*d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^
3*b^3*c*d*h*i + 8*a^2*b^4*c*d*f*g - 8*a^2*b^4*c*d*e*h - 4*a^4*b^2*f^2*g*i +
 4*a^4*b^2*f*g^2*h + 4*a^4*b^2*e*g^2*i - 4*a^4*b^2*e*g*h^2 - 4*a^4*b^2*c*h^
2*i - 4*a^3*b^3*d^2*g*i + 4*a^4*b^2*d*f*i^2 + 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3
*e^2*f*h - 4*a^3*b^3*e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^
3*b^3*d*f*g^2 + 4*a^2*b^4*c^2*f*h + 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2 -
 4*a^2*b^4*d^2*e*g - 4*a^2*b^4*c*d^2*i + 4*a^2*b^4*d*e^2*f + 4*a^2*b^4*c*e^
2*g - 4*a^2*b^4*c*e*f^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a*b^5*c^2*d
*f - 4*a*b^5*c*d^2*e - 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g - 6*a^4*b^2*e^2*i^2 -
2*a^4*b^2*f^2*h^2 + 6*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3*b^3*c^2*i
^2 - 6*a^2*b^4*c^2*g^2 - 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*a^3*b^3*e^

```
3*i + 4*a^4*b^2*d*h^3 + 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b^5*c^2*e^2
 + a^3*b^3*f^4 + a^5*b*h^4 + a*b^5*d^4 - a^4*b^2*g^4 - a^2*b^4*e^4 - a^6*i^
4 - b^6*c^4, z, l)*((16*a^2*b^4*g + 16*a*b^5*c)/b^2 - (x*(16*a^2*b^3*h + 16
*a*b^4*d))/b) - (8*a*b^4*d*e - 8*a*b^4*c*f + 8*a^2*b^3*d*i + 8*a^2*b^3*e*h
- 8*a^2*b^3*f*g + 8*a^3*b^2*h*i)/b^2 + (x*(4*b^4*c^2 + 4*a*b^3*e^2 + 4*a^3*
b*i^2 + 4*a^2*b^2*g^2 + 8*a*b^3*c*g - 8*a*b^3*d*f + 8*a^2*b^2*e*i - 8*a^2*b
^2*f*h))/b) - (x*(b^3*d^3 + a^3*h^3 + b^3*c^2*f + a^3*f*i^2 - 2*b^3*c*d*e -
 2*a^3*g*h*i - a*b^2*d*f^2 + a*b^2*e^2*f + 3*a*b^2*d^2*h + 3*a^2*b*d*h^2 +
a^2*b*f*g^2 - a^2*b*f^2*h - 2*a*b^2*c*d*i - 2*a*b^2*c*e*h + 2*a*b^2*c*f*g -
 2*a*b^2*d*e*g - 2*a^2*b*c*h*i - 2*a^2*b*d*g*i + 2*a^2*b*e*f*i - 2*a^2*b*e*
g*h))/b)*root(256*a^3*b^7*z^4 + 256*a^3*b^6*f*z^3 - 64*a^4*b^4*g*i*z^2 - 64
*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 - 64*a^2*b^6*c*e
*z^2 - 32*a^4*b^4*h^2*z^2 + 96*a^3*b^5*f^2*z^2 - 32*a^2*b^6*d^2*z^2 - 32*a^
4*b^3*f*g*i*z + 32*a^4*b^3*e*h*i*z - 32*a^3*b^4*e*f*g*z - 32*a^3*b^4*d*f*h*
z + 32*a^3*b^4*d*e*i*z + 32*a^3*b^4*c*g*h*z - 32*a^3*b^4*c*f*i*z - 32*a^2*b
^5*c*e*f*z + 32*a^2*b^5*c*d*g*z + 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z -
 16*a^4*b^3*f*h^2*z + 16*a^4*b^3*d*i^2*z + 16*a^3*b^4*e^2*h*z + 16*a^3*b^4*
d*g^2*z + 16*a^2*b^5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z + 16
*a*b^6*c^2*d*z + 16*a^3*b^4*f^3*z + 8*a^4*b^2*e*f*h*i - 8*a^4*b^2*d*g*h*i -
 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*d*e*f*i + 8*a^3*...
```

3.188 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx$

Optimal. Leaf size=205

$$-\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}-\frac{jx^4}{4b}-\frac{\left(be-\frac{\sqrt{b}\,(bc+ag)}{\sqrt{a}}+ai\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\,b^{7/4}}+\frac{\left(be+\frac{\sqrt{b}\,(bc+ag)}{\sqrt{a}}+ai\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\,b^{7/4}}+$$

```
[Out] -g*x/b-1/2*h*x^2/b-1/3*i*x^3/b-1/4*j*x^4/b-1/4*(a*j+b*f)*ln(-b*x^4+a)/b^2+1
/2*(a*h+b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-1/2*arctan(b^(1/4
)*x/a^(1/4))*(b*e+a*i-(a*g+b*c)*b^(1/2)/a^(1/2))/a^(1/4)/b^(7/4)+1/2*arctan
h(b^(1/4)*x/a^(1/4))*(b*e+a*i+(a*g+b*c)*b^(1/2)/a^(1/2))/a^(1/4)/b^(7/4)
```

Rubi [A]

time = 0.21, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.196$, Rules used = {1899, 1901, 1181, 211, 214, 1833, 1824, 649, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}\,(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}\,b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}\,(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}\,b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{(aj+bf)\log\left(a-bx^4\right)}{4b^2}-\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}-\frac{jx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4),x
]
```

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - (j*x^4)/(4*b) - ((b*e - (Sqrt[
b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7
/4)) + ((b*e + (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/a^(
1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(
2*Sqrt[a]*b^(3/2)) - ((b*f + a*j)*Log[a - b*x^4])/(4*b^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+188x^6+jx^7}{a-bx^4}\,dx = \int \left(\frac{c+ex^2+gx^4+188x^6}{a-bx^4}+\frac{x(d+fx^2+hx^4+jx\right.$$

$$= \int \frac{c+ex^2+gx^4+188x^6}{a-bx^4}\,dx + \int \frac{x(d+fx^2+hx^4}{a-bx^4}$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2+jx^3}{a-bx^2}\,dx, x, x^2\right) + \int \left(-\right.$$

$$= -\frac{gx}{b} - \frac{188x^3}{3b} + \frac{1}{2}\text{Subst}\left(\int \left(-\frac{h}{b} - \frac{jx}{b} + \frac{bd+ah}{b\,(a\right.\right.$$

$$= -\frac{gx}{b} - \frac{hx^2}{2b} - \frac{188x^3}{3b} - \frac{jx^4}{4b} + \frac{\text{Subst}\left(\int \frac{bd+ah+(bf+aj}{a-bx^2}\right.}{2b}$$

$$= -\frac{gx}{b} - \frac{hx^2}{2b} - \frac{188x^3}{3b} - \frac{jx^4}{4b} - \frac{\left(188a+be-\frac{\sqrt{b}\,(bc}{\sqrt{a}}\right.}{2\sqrt[4]{a}}$$

$$= -\frac{gx}{b} - \frac{hx^2}{2b} - \frac{188x^3}{3b} - \frac{jx^4}{4b} - \frac{\left(188a+be-\frac{\sqrt{b}\,(bc}{\sqrt{a}}\right.}{2\sqrt[4]{a}}$$

Mathematica [A]
time = 0.24, size = 318, normalized size = 1.55

$$\frac{-12b^{3/4}gx-6b^{3/4}hx^2-4b^{3/4}ix^3-3b^{3/4}jx^4+\frac{6\left(b^{3/2}c-\sqrt{a}\,be+a\sqrt{b}\,g-a^{3/2}i\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}}-\frac{3\left(b^{3/2}c+\sqrt[4]{a}\,b^{5/4}d+\sqrt{a}\,be+a\sqrt{b}\,g+a^{5/4}\sqrt[4]{b}\,h+a^{3/2}i\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)}{a^{3/4}}+\frac{3\left(b^{3/2}c-\sqrt[4]{a}\,b^{5/4}d+\sqrt{a}\,be+a\sqrt{b}\,g-a^{5/4}\sqrt[4]{b}\,h+a^{3/2}i\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{a^{3/4}}+\frac{3\sqrt[4]{b}\,(bd+ah)\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt{a}}-\frac{3(bf+aj)\log(a-bx^4)}{\sqrt[4]{b}}}{12b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4),x]
```

```
[Out] (-12*b^(3/4)*g*x - 6*b^(3/4)*h*x^2 - 4*b^(3/4)*i*x^3 - 3*b^(3/4)*j*x^4 + (6
*(b^(3/2)*c - Sqrt[a]*b*e + a*Sqrt[b]*g - a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(
1/4)])/a^(3/4) - (3*(b^(3/2)*c + a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b
]*g + a^(5/4)*b^(1/4)*h + a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(3/4) + (3
*(b^(3/2)*c - a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g - a^(5/4)*b^(1/
4)*h + a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x])/a^(3/4) + (3*b^(1/4)*(b*d + a*h
)*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[a] - (3*(b*f + a*j)*Log[a - b*x^4])/b^(1
/4))/(12*b^(7/4))
```

Maple [A]
time = 0.37, size = 210, normalized size = 1.02

method	result
risch	$-\frac{j\,x^{4}}{4b}-\frac{i\,x^{3}}{3b}-\frac{h\,x^{2}}{2b}-\frac{gx}{b}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(bc+ag+(ah+bd)\_R+(ai+be)\_R^{2}+(aj+bf)\_R^{3}\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$
default	$-\frac{\frac{1}{4}j\,x^{4}+\frac{1}{3}i\,x^{3}+\frac{1}{2}h\,x^{2}+gx}{b}+\frac{\frac{(ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(ah+bd)\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{(ai+be)\left(2\arctan\left(\cdots\right.\right.}{4b\cdots}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/b*(1/4*j*x^4+1/3*i*x^3+1/2*h*x^2+g*x)+1/b*(1/4*(a*g+b*c)*(a/b)^(1/4)/a*(
ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))+1/4*(a*h+b*d)/
(a*b)^(1/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2)))-1/4*(a*i+b*e)/b/(a/
b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4))))-1/4*
(a*j+b*f)*ln(-b*x^4+a)/b)
```

Maxima [A]
time = 0.51, size = 257, normalized size = 1.25

$$-\frac{3\,jx^4+6\,hx^2+4\,ix^3+12\,gx}{12\,b}+\frac{\frac{2\left(b^{\frac{3}{2}}c+a\sqrt{b}\,g-\sqrt{a}\,be-ia^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\left(b^{\frac{3}{2}}d-\sqrt{a}\,bf+a\sqrt{b}\,h-a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}d+\sqrt{a}\,bf+a\sqrt{b}\,h+a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}c+a\sqrt{b}\,g+\sqrt{a}\,be+ia^{\frac{3}{2}}\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorit
hm="maxima")
```

```
[Out] -1/12*(3*j*x^4 + 6*h*x^2 + 4*I*x^3 + 12*g*x)/b + 1/4*(2*(b^(3/2)*c + a*sqrt
(b)*g - sqrt(a)*b*e - I*a^(3/2))*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b^(3/2)*d - sqrt(a)*b*f + a*sqrt(b
)*h - a^(3/2)*j)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (b^(3/2)*d + sqrt
(a)*b*f + a*sqrt(b)*h + a^(3/2)*j)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) -
 (b^(3/2)*c + a*sqrt(b)*g + sqrt(a)*b*e + I*a^(3/2))*log((sqrt(b)*x - sqrt(
sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a
)*sqrt(b))*sqrt(b)))/b
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(160) = 320$.
time = 0.60, size = 548, normalized size = 2.67

$$-\frac{\sqrt{2}\left(b^{2}c+abg-\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c+abg+\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c+abg-\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b^{2}c+abg-\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{(bf+aj)\log\left(|bx^{4}-a|\right)}{4\,b^{2}}+\frac{i\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^{4}}+\frac{i\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^{4}}-\frac{i\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,b^{4}}+\frac{i\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,b^{4}}-\frac{3\,b^{3}jx^{4}+6\,b^{3}hx^{2}+4i\,b^{3}x^{3}+12\,b^{3}gx}{12\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorit
hm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c + a*b*g - sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(2)*(-a*b^3)
^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)
)/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c + a*b*g + sqrt(2)*(-a*b
^3)^(1/4)*b*d + sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b*e)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*sqrt(2
)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(
-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^
2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) - 1/4*(b*f + a*j)*l
og(abs(b*x^4 - a))/b^2 + 1/4*I*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4 + 1/4*I*sqrt(2)*(-a*b^3)^(3/4)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4 - 1/8*I*
sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4 +
 1/8*I*sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b)
)/b^4 - 1/12*(3*b^3*j*x^4 + 6*b^3*h*x^2 + 4*I*b^3*x^3 + 12*b^3*g*x)/b^4
```

Mupad [B]
time = 5.16, size = 2500, normalized size = 12.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4),x
)

[Out] symsum(log(- (a^4*i^3 + a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^4*g*j^2 + a^2
*b^2*c*h^2 - a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i - 2*a^4*h*i*j
+ a*b^3*c*f^2 + a*b^3*d^2*g - a*b^3*c^2*i + a^3*b*c*j^2 + 3*a^3*b*e*i^2 + a
^3*b*g*h^2 - a^3*b*g^2*i + 2*a^2*b^2*c*f*j - 2*a^2*b^2*c*g*i - 2*a^2*b^2*d*
e*j - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h + 2*a*b^3*c*d*h -
 2*a*b^3*c*e*g - 2*a*b^3*d*e*f - 2*a^3*b*d*i*j - 2*a^3*b*e*h*j + 2*a^3*b*f*
g*j - 2*a^3*b*f*h*i)/b^2 - root(256*a^3*b^8*z^4 + 256*a^4*b^6*j*z^3 + 256*a
^3*b^7*f*z^3 + 192*a^4*b^5*f*j*z^2 - 64*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^
2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*i*z^2 - 64*a^2*b^7*c*e*z^2 + 96*a^5*b
^4*j^2*z^2 - 32*a^4*b^5*h^2*z^2 + 96*a^3*b^6*f^2*z^2 - 32*a^2*b^7*d^2*z^2 -
 32*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i*z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*
e*g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*b^4*c*i*j*z - 32*a^3*b^5*e*f*g*z - 32
*a^3*b^5*d*f*h*z + 32*a^3*b^5*d*e*i*z + 32*a^3*b^5*c*g*h*z - 32*a^3*b^5*c*f
*i*z - 32*a^3*b^5*c*e*j*z - 32*a^2*b^6*c*e*f*z + 32*a^2*b^6*c*d*g*z - 16*a^
5*b^3*h^2*j*z + 16*a^5*b^3*h*i^2*z + 48*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*
z + 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*h^2*z - 16*a^3*b^5*d^2*j*z + 16*a^4*b
^4*d*i^2*z + 16*a^3*b^5*e^2*h*z + 16*a^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z -
 16*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2*z + 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^
3*z + 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g*i*j + 8*a^5*b^2*e*h*i*j + 8*a^4*b^3*
e*f*h*i - 8*a^4*b^3*e*f*g*j - 8*a^4*b^3*d*g*h*i - 8*a^4*b^3*d*f*h*j + 8*a^4
*b^3*d*e*i*j + 8*a^4*b^3*c*g*h*j - 8*a^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h +
8*a^3*b^4*d*e*f*i + 8*a^3*b^4*c*f*g*h + 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f
*j - 8*a^3*b^4*c*d*h*i + 8*a^3*b^4*c*d*g*j + 8*a^2*b^5*c*d*f*g - 8*a^2*b^5*
c*d*e*h + 4*a^5*b^2*g^2*h*j - 4*a^5*b^2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5
*b^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j + 4*a^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 -
4*a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*i*j^2 - 4*a^4*b^3*f^2*g*i + 4*a^4*b^3*f*g^2
*h + 4*a^4*b^3*e*g^2*i + 4*a^4*b^3*d*g^2*j + 4*a^3*b^4*c^2*h*j - 4*a^4*b^3*
e*g*h^2 - 4*a^4*b^3*c*h^2*i - 4*a^3*b^4*d^2*g*i - 4*a^3*b^4*d^2*f*j + 4*a^4
*b^3*d*f*i^2 + 4*a^4*b^3*c*g*i^2 + 4*a^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j -
4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*f^2*g - 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2
*i + 4*a^3*b^4*d*f*g^2 + 4*a^2*b^5*c^2*f*h + 4*a^2*b^5*c^2*e*i + 4*a^2*b^5*
c^2*d*j - 4*a^3*b^4*c*e*h^2 - 4*a^2*b^5*d^2*e*g - 4*a^2*b^5*c*d^2*i + 4*a^2
*b^5*d*e^2*f + 4*a^2*b^5*c*e^2*g - 4*a^2*b^5*c*e*f^2 + 4*a^6*b*h*i^2*j - 4*
a^6*b*g*i*j^2 + 4*a*b^6*c^2*d*f - 4*a*b^6*c*d^2*e + 4*a^6*b*f*j^3 - 4*a*b^6
*c^3*g + 6*a^5*b^2*f^2*j^2 + 2*a^5*b^2*g^2*i^2 - 6*a^4*b^3*e^2*i^2 - 2*a^4*
b^3*f^2*h^2 - 2*a^4*b^3*d^2*j^2 + 6*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2
*a^3*b^4*c^2*i^2 - 6*a^2*b^5*c^2*g^2 - 2*a^2*b^5*d^2*f^2 - 2*a^6*b*h^2*j^2
+ 4*a^4*b^3*f^3*j - 4*a^5*b^2*e*i^3 - 4*a^3*b^4*e^3*i + 4*a^4*b^3*d*h^3 + 4
*a^2*b^5*d^3*h - 4*a^3*b^4*c*g^3 + 2*a*b^6*c^2*e^2 + a^5*b^2*h^4 + a^3*b^4*
f^4 + a*b^6*d^4 + a^7*j^4 - a^4*b^3*g^4 - a^2*b^5*e^4 - a^6*b*i^4 - b^7*c^4
, z, m)*((8*a*b^4*c*f - 8*a*b^4*d*e + 8*a^2*b^3*c*j - 8*a^2*b^3*d*i - 8*a^2
*b^3*e*h + 8*a^2*b^3*f*g + 8*a^3*b^2*g*j - 8*a^3*b^2*h*i)/b^2 + root(256*a^
```

```
3*b^8*z^4 + 256*a^4*b^6*j*z^3 + 256*a^3*b^7*f*z^3 + 192*a^4*b^5*f*j*z^2 - 6
4*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*
i*z^2 - 64*a^2*b^7*c*e*z^2 + 96*a^5*b^4*j^2*z^2 - 32*a^4*b^5*h^2*z^2 + 96*a
^3*b^6*f^2*z^2 - 32*a^2*b^7*d^2*z^2 - 32*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i
*z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*e*g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*
b^4*c*i*j*z - 32*a^3*b^5*e*f*g*z - 32*a^3*b^5*d*f*h*z + 32*a^3*b^5*d*e*i*z
+ 32*a^3*b^5*c*g*h*z - 32*a^3*b^5*c*f*i*z - 32*a^3*b^5*c*e*j*z - 32*a^2*b^6
*c*e*f*z + 32*a^2*b^6*c*d*g*z - 16*a^5*b^3*h^2*j*z + 16*a^5*b^3*h*i^2*z + 4
8*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*z + 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*
h^2*z - 16*a^3*b^5*d^2*j*z + 16*a^4*b^4*d*i^2*z + 16*a^3*b^5*e^2*h*z + 16*a
^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z - 16*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2
*z + 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^3*z + 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g
*i*j + 8*a^5*b^2*e*h*i*j + 8*a^4*b^3*e*f*h*i - 8*a^4*b^3*e*f*g*j - 8*a^4*b^
3*d*g*h*i - 8*a^4*b^3*d*f*h*j + 8*a^4*b^3*d*e*i*j + 8*a^4*b^3*c*g*h*j - 8*a
^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h + 8*a^3*b^4*d*e*f*i + 8*a^3*b^4*c*f*g*h
+ 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f*j - 8*a^3*b^4*c*d*h*i + 8*a^3*b^4*c*d
*g*j + 8*a^2*b^5*c*d*f*g - 8*a^2*b^5*c*d*e*h + 4*a^5*b^2*g^2*h*j - 4*a^5*b^
2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5*b^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j + 4*a
^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 - 4*a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*i*j^2
- 4*a^4*b^3*f^2*g*i + 4*a^4*b^3*f*g^2*h + 4*a^4*b^3*e*g^2*i + 4*a^4*b^3*d*g
^2*j + 4*a^3*b^4*c^2*h*j - 4*a^4*b^3*e*g*h^2 - 4*a^4*b^3*c*h^2*i - 4*a^3*b^
4*d^2*g*i - 4*a^3*b^4*d^2*f*j + 4*a^4*b^3*d*f*i^2 + 4*a^4*b^3*c*g*i^2 + 4*a
^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j - 4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*f^2*g
- 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2*i + 4*a^3*b^4*d*f*g^2 + 4*a^2*b^5*c^2
*f*h + 4*a^2*b^5*c^2*e*i + 4*a^2*b^5*c^2*d*j - ...
```

3.189 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx$

Optimal. Leaf size=337

$$\frac{gx}{b}+\frac{hx^2}{2b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{\left(bc+\sqrt{a}\,\sqrt{b}\,e-ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\,\sqrt{b}\,e-a\right.}{2\sqrt{}}$$

```
[Out] g*x/b+1/2*h*x^2/b+1/4*f*ln(b*x^4+a)/b+1/2*(-a*h+b*d)*arctan(x^2*b^(1/2)/a^(
1/2))/b^(3/2)/a^(1/2)-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2)
)*(b*c-a*g-e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4
)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(b*c-a*g-e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/4
)*2^(1/2)+1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(b*c-a*g+e*a^(1/2)*b^(1/
2))/a^(3/4)/b^(5/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(b*c-a*
g+e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {1899, 1901, 1182, 1176, 631, 210, 1179, 642, 1833, 1824, 649, 211, 266}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,\sqrt{b}\,e-ag+bc\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,\sqrt{b}\,e-ag+bc\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-\sqrt{a}\,\sqrt{b}\,e-ag+bc\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-\sqrt{a}\,\sqrt{b}\,e-ag+bc\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)(bd-ah)}{2\sqrt{a}\,b^{3/2}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}+\frac{hx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sq
rt[a]*b^(3/2)) - ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a
*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) -
((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4
)*b^(5/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx = \int \left(\frac{c+ex^2+gx^4}{a+bx^4}+\frac{x(d+fx^2+hx^4)}{a+bx^4}\right)dx$$

$$= \int \frac{c+ex^2+gx^4}{a+bx^4}\,dx + \int \frac{x(d+fx^2+hx^4)}{a+bx^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{a+bx^2}\,dx, x, x^2\right) + \int \left(\frac{g}{b}+\frac{bc-ag+bex^2}{b\,(a+bx^4)}\right) d$$

$$= \frac{gx}{b} + \frac{1}{2}\text{Subst}\left(\int \left(\frac{h}{b}+\frac{bd-ah+bfx}{b\,(a+bx^2)}\right) dx, x, x^2\right) + \frac{\int \frac{bc-ag+bex^2}{a+bx^4}}{b}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{\text{Subst}\left(\int \frac{bd-ah+bfx}{a+bx^2}\,dx, x, x^2\right)}{2b} + \frac{\left(bc-\sqrt{a}\sqrt{b}\,e-ag\right)}{2\sqrt{a}\,b}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{1}{2}f\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, x^2\right) - \frac{\left(bc-\sqrt{a}\sqrt{b}\,e-\right.}{4}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}} - \frac{\left(bc-\sqrt{a}\sqrt{b}\,e-ag\right)\text{lo}}{4}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}} - \frac{\left(bc+\sqrt{a}\sqrt{b}\,e-ag\right)\text{ta}}{2\sqrt{2}\,a^{3/}}$$

Mathematica [A]
time = 0.23, size = 342, normalized size = 1.01

$$\frac{-2\left(\sqrt{2}\,b^{5/4}c+2\sqrt[4]{a}\,bd+\sqrt{2}\sqrt{a}\,b^{3/4}e-\sqrt{2}\,a\sqrt[4]{b}\,g-2a^{5/4}h\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\,b^{5/4}c-2\sqrt[4]{a}\,bd+\sqrt{2}\sqrt{a}\,b^{3/4}e-\sqrt{2}\,a\sqrt[4]{b}\,g+2a^{5/4}h\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\sqrt[4]{b}\left(\sqrt{2}\left(-bc+\sqrt{a}\sqrt{b}\,e+ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+\sqrt{2}\left(bc-\sqrt{a}\sqrt{b}\,e-ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+2a^{3/4}\sqrt[4]{b}\,(2x(2g+hx)+f\log(a+bx^4))\right)}{8a^{3/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*b^(5/4)*c + 2*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e - Sqrt[2
]*a*b^(1/4)*g - 2*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(S
qrt[2]*b^(5/4)*c - 2*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e - Sqrt[2]*a*b^
(1/4)*g + 2*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + b^(1/4)*(S
qrt[2]*(-(b*c) + Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*(2*x*(2*g +
 h*x) + f*Log[a + b*x^4])))/(8*a^(3/4)*b^(3/2))
```

Maple [A]

time = 0.35, size = 267, normalized size = 0.79

method	result
risch	$\frac{h\,x^{2}}{2b}+\frac{gx}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}b+a\right)}\frac{\left(bc-ag+\left(-ah+bd\right)\_R+be\,\_R^{2}+\_R^{3}bf\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$
default	$\frac{\frac{1}{2}h\,x^{2}+gx}{b}+\frac{\frac{\left(-ag+bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\left(-ah+bd\right)\arctan}{2\sqrt{a}}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*h*x^2+g*x)+1/b*(1/8*(-a*g+b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b
)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/2*(-a*h
+b*d)/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*e/(a/b)^(1/4)*2^(1/2)*(ln((x^
2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)
))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/4
*f*ln(b*x^4+a))
```

Maxima [A]

time = 0.51, size = 355, normalized size = 1.05

$$\frac{hx^{2}+2\,gx}{2\,b}+\frac{\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}f+b^{2}c-abg-\sqrt{a}\,b^{\frac{3}{2}}e\right)\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}f-b^{2}c+abg+\sqrt{a}\,b^{\frac{3}{2}}e\right)\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{9}{4}}c-\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}g+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}e-2\sqrt{a}\,b^{2}d+2\,a^{\frac{3}{2}}bh\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{9}{4}}c-\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}g+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}e+2\sqrt{a}\,b^{2}d-2\,a^{\frac{3}{2}}bh\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*(h*x^2 + 2*g*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f + b^2*c - a
*b*g - sqrt(a)*b^(3/2)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqr
t(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f - b^2*c + a*b*
g + sqrt(a)*b^(3/2)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a
))/(a^(3/4)*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c - sqrt(2)*a^(5/4)*b^(5/
4)*g + sqrt(2)*a^(3/4)*b^(7/4)*e - 2*sqrt(a)*b^2*d + 2*a^(3/2)*b*h)*arctan(
1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c - sq
rt(2)*a^(5/4)*b^(5/4)*g + sqrt(2)*a^(3/4)*b^(7/4)*e + 2*sqrt(a)*b^2*d - 2*a
^(3/2)*b*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt
(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))/b
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.62, size = 375, normalized size = 1.11

$$\frac{f\log\left(|bx^4+a|\right)}{4\,b}+\frac{bhx^2+2\,bgx}{2\,b^2}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d+\sqrt{2}\sqrt{ab}\,abh+(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d+\sqrt{2}\sqrt{ab}\,abh+(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*f*log(abs(b*x^4 + a))/b + 1/2*(b*h*x^2 + 2*b*g*x)/b^2 + 1/4*sqrt(2)*(sq
rt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c - (a*
b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)
^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(
2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3
/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3)
+ 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)
*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3
)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*
(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

Mupad [B]
time = 5.54, size = 2469, normalized size = 7.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x)
```

```
[Out] symsum(log(root(256*a^3*b^6*z^4 - 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*g*z^2 -
64*a^3*b^4*d*h*z^2 + 64*a^2*b^5*c*e*z^2 + 32*a^4*b^3*h^2*z^2 + 96*a^3*b^4*f
^2*z^2 + 32*a^2*b^5*d^2*z^2 + 32*a^3*b^3*e*f*g*z + 32*a^3*b^3*d*f*h*z - 32*
a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*b^2*g^2*
h*z - 16*a^4*b^2*f*h^2*z - 16*a^3*b^3*e^2*h*z - 16*a^3*b^3*d*g^2*z + 16*a^2
*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z - 16*a*b^5*c^2*d*z -
 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h - 8*a^2*b^3*c*d*f
*g + 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*a^3*b^2*
d*f^2*h + 4*a^3*b^2*d*f*g^2 - 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^2 + 4*a^2
*b^3*d^2*e*g - 4*a^2*b^3*d*e^2*f - 4*a^2*b^3*c*e^2*g + 4*a^2*b^3*c*e*f^2 -
4*a^4*b*f*g^2*h + 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*d^2*e - 4*a
^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 6*a^2*
b^3*c^2*g^2 + 2*a^2*b^3*d^2*f^2 + 2*a^4*b*f^2*h^2 - 4*a^2*b^3*d^3*h - 4*a^3
*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a^2*b^3*e^4 + a^4*b*g^4 + a*b^
4*d^4 + a^5*h^4 + b^5*c^4, z, k)*((8*a*b^3*c*f - 8*a*b^3*d*e + 8*a^2*b^2*e*
h - 8*a^2*b^2*f*g)/b + root(256*a^3*b^6*z^4 - 256*a^3*b^5*f*z^3 - 64*a^3*b^
4*e*g*z^2 - 64*a^3*b^4*d*h*z^2 + 64*a^2*b^5*c*e*z^2 + 32*a^4*b^3*h^2*z^2 +
96*a^3*b^4*f^2*z^2 + 32*a^2*b^5*d^2*z^2 + 32*a^3*b^3*e*f*g*z + 32*a^3*b^3*d
*f*h*z - 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*
a^4*b^2*g^2*h*z - 16*a^4*b^2*f*h^2*z - 16*a^3*b^3*e^2*h*z - 16*a^3*b^3*d*g^
2*z + 16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z - 16*a*b
^5*c^2*d*z - 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h - 8*a
^2*b^3*c*d*f*g + 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g
- 4*a^3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 - 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e
*h^2 + 4*a^2*b^3*d^2*e*g - 4*a^2*b^3*d*e^2*f - 4*a^2*b^3*c*e^2*g + 4*a^2*b^
3*c*e*f^2 - 4*a^4*b*f*g^2*h + 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c
*d^2*e - 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*
g^2 + 6*a^2*b^3*c^2*g^2 + 2*a^2*b^3*d^2*f^2 + 2*a^4*b*f^2*h^2 - 4*a^2*b^3*d
^3*h - 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a^2*b^3*e^4 + a^4*
b*g^4 + a*b^4*d^4 + a^5*h^4 + b^5*c^4, z, k)*((16*a^2*b^3*g - 16*a*b^4*c)/b
 - (x*(16*a^2*b^3*h - 16*a*b^4*d))/b) - (x*(4*b^4*c^2 - 4*a*b^3*e^2 + 4*a^2
*b^2*g^2 - 8*a*b^3*c*g + 8*a*b^3*d*f - 8*a^2*b^2*f*h))/b) - (a*b^2*e^3 - b^
3*c*d^2 + b^3*c^2*e + a^3*g*h^2 + a*b^2*c*f^2 + a*b^2*d^2*g - a^2*b*c*h^2 +
 a^2*b*e*g^2 - a^2*b*f^2*g + 2*a*b^2*c*d*h - 2*a*b^2*c*e*g - 2*a*b^2*d*e*f
- 2*a^2*b*d*g*h + 2*a^2*b*e*f*h)/b + (x*(b^3*d^3 - a^3*h^3 + b^3*c^2*f - 2*
b^3*c*d*e + a*b^2*d*f^2 - a*b^2*e^2*f - 3*a*b^2*d^2*h + 3*a^2*b*d*h^2 + a^2
*b*f*g^2 - a^2*b*f^2*h + 2*a*b^2*c*e*h - 2*a*b^2*c*f*g + 2*a*b^2*d*e*g - 2*
a^2*b*e*g*h))/b)*root(256*a^3*b^6*z^4 - 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*g*
z^2 - 64*a^3*b^4*d*h*z^2 + 64*a^2*b^5*c*e*z^2 + 32*a^4*b^3*h^2*z^2 + 96*a^3
*b^4*f^2*z^2 + 32*a^2*b^5*d^2*z^2 + 32*a^3*b^3*e*f*g*z + 32*a^3*b^3*d*f*h*z
 - 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*b^
2*g^2*h*z - 16*a^4*b^2*f*h^2*z - 16*a^3*b^3*e^2*h*z - 16*a^3*b^3*d*g^2*z +
16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z - 16*a*b^5*c^2
```

```
*d*z - 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h - 8*a^2*b^3
*c*d*f*g + 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*a^
3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 - 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^2 +
 4*a^2*b^3*d^2*e*g - 4*a^2*b^3*d*e^2*f - 4*a^2*b^3*c*e^2*g + 4*a^2*b^3*c*e*
f^2 - 4*a^4*b*f*g^2*h + 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*d^2*e
 - 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 +
6*a^2*b^3*c^2*g^2 + 2*a^2*b^3*d^2*f^2 + 2*a^4*b*f^2*h^2 - 4*a^2*b^3*d^3*h -
 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a^2*b^3*e^4 + a^4*b*g^4
+ a*b^4*d^4 + a^5*h^4 + b^5*c^4, z, k), k, 1, 4) + (h*x^2)/(2*b) + (g*x)/b
```

3.190 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx$

Optimal. Leaf size=384

$$\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{\left(\sqrt{b}\,(bc-ag)+\sqrt{a}\,(be-ai)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\left(\sqrt{b}\right.}{}$$

```
[Out] g*x/b+1/2*h*x^2/b+1/3*i*x^3/b+1/4*f*ln(b*x^4+a)/b+1/2*(-a*h+b*d)*arctan(x^2
*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*b^(1/2))*(-(-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1
/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(-a*i+b*e)*a^(1
/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)+1/4*arctan(-1+b^(1/4)*x*2^(
1/2)/a^(1/4))*((-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/
2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*((-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b
^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {1899, 1833, 1824, 649, 211, 266, 1901, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{b}\,(bc-ag)+\sqrt{a}\,(be-ai)\right)}{2\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}\,(bc-ag)+\sqrt{a}\,(be-ai)\right)}{2\sqrt{2}\,a^{3/4}b^{7/4}}-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(\sqrt{b}\,(bc-ag)-\sqrt{a}\,(be-ai)\right)}{4\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(\sqrt{b}\,(bc-ag)-\sqrt{a}\,(be-ai)\right)}{4\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)(bd-ah)}{2\sqrt{a}\,b^{3/2}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)
/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i
))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) + (
(Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*
e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2
]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4))
+ (f*Log[a + b*x^4])/(4*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+190x^6}{a+bx^4}\,dx &= \int \left(\frac{x(d+fx^2+hx^4)}{a+bx^4}+\frac{c+ex^2+gx^4+190x^6}{a+bx^4}\right)dx \\ &= \int \frac{x(d+fx^2+hx^4)}{a+bx^4}\,dx+\int \frac{c+ex^2+gx^4+190x^6}{a+bx^4}\,dx \\ &= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{a+bx^2}\,dx,x,x^2\right)+\int \left(\frac{g}{b}+\frac{190x^2}{b}+ \right. \\ &= \frac{gx}{b}+\frac{190x^3}{3b}+\frac{1}{2}\text{Subst}\left(\int \left(\frac{h}{b}+\frac{bd-ah+bfx}{b(a+bx^2)}\right)dx,x,x \right. \\ &= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{190x^3}{3b}+\frac{\text{Subst}\left(\int \frac{bd-ah+bfx}{a+bx^2}\,dx,x,x^2\right)}{2b}-\frac{(190}{} \\ &= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{190x^3}{3b}+\frac{1}{2}f\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx,x,x^2\right)- \\ &= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{190x^3}{3b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{(190a}{} \\ &= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{190x^3}{3b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}+\frac{(190a}{}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 427, normalized size = 1.11

$$\frac{24b^{3/4}gx+12b^{3/4}hx^2+8b^{3/4}ix^3+\frac{6\left(-\sqrt{2}b^{3/2}c-2\sqrt[4]{a}b^{5/4}d-\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+2a^{5/4}\sqrt[4]{b}h+\sqrt{2}a^{3/2}i\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{6\left(\sqrt{2}b^{3/2}c-2\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be-\sqrt{2}a\sqrt{b}g+2a^{5/4}\sqrt[4]{b}h-\sqrt{2}a^{3/2}i\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}}-\frac{3\sqrt{2}\left(b^{3/2}c-\sqrt{a}be-a\sqrt{b}g+a^{3/2}i\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{a^{3/4}}+\frac{3\sqrt{2}\left(b^{3/2}c-\sqrt{a}be-a\sqrt{b}g+a^{3/2}i\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{a^{3/4}}+6b^{3/4}f\log\left(a+bx^4\right)}{24b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x]
```

```
[Out] (24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + (6*(-(Sqrt[2]*b^(3/2
)*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 2*
a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(3/4) + (6*(Sqrt[2]*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a
]*b*e - Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - Sqrt[2]*a^(3/2)*i)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (3*Sqrt[2]*(b^(3/2)*c - Sqrt
[a]*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2])/a^(3/4) + (3*Sqrt[2]*(b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g
```

```
 + a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/
4) + 6*b^(3/4)*f*Log[a + b*x^4])/(24*b^(7/4))
```

Maple [A]
time = 0.35, size = 283, normalized size = 0.74

method	result
risch	$\frac{i x^{3}}{3 b}+\frac{h x^{2}}{2 b}+\frac{g x}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4} b+a\right)} \frac{\left(b c-a g+(-a h+b d) \_R+(-a i+b e) \_R^{2}+\_R^{3} b f\right) \ln \left(x-\_R\right)}{\_R^{3}}}{4 b^{2}}$
default	$\frac{\frac{1}{3} i x^{3}+\frac{1}{2} h x^{2}+g x}{b}+\frac{\frac{(-a g+b c)\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\left(\ln \left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8 a}+(-a h+b d}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/b*(1/3*i*x^3+1/2*h*x^2+g*x)+1/b*(1/8*(-a*g+b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln
((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
+1/2*(-a*h+b*d)/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*(-a*i+b*e)/b/(a/b)^
(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*
x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/
(a/b)^(1/4)*x-1))+1/4*f*ln(b*x^4+a))
```

Maxima [A]
time = 0.52, size = 399, normalized size = 1.04

$$\frac{3\,hx^2+2i\,x^3+6\,gx}{6\,b}+\frac{\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac34}b^{\frac54}f+b^2c-abg-\sqrt{a}\,b^{\frac32}e+i\,a^{\frac32}\sqrt{b}\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac54}}+\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac34}b^{\frac54}f-b^2c+abg+\sqrt{a}\,b^{\frac32}e-i\,a^{\frac32}\sqrt{b}\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac54}}+\frac{2\left(\sqrt{2}\,a^{\frac14}b^{\frac94}c-\sqrt{2}\,a^{\frac54}b^{\frac54}g+\sqrt{2}\,a^{\frac34}b^{\frac74}e-2\,\sqrt{a}\,b^2d+2\,a^{\frac32}bh-i\,\sqrt{2}\,a^{\frac74}b^{\frac34}\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac34}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac54}}+\frac{2\left(\sqrt{2}\,a^{\frac14}b^{\frac94}c-\sqrt{2}\,a^{\frac54}b^{\frac54}g+\sqrt{2}\,a^{\frac34}b^{\frac74}e+2\,\sqrt{a}\,b^2d-2\,a^{\frac32}bh-i\,\sqrt{2}\,a^{\frac74}b^{\frac34}\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac34}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac54}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="max
ima")
```

```
[Out] 1/6*(3*h*x^2 + 2*I*x^3 + 6*g*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f
 + b^2*c - a*b*g - sqrt(a)*b^(3/2)*e + I*a^(3/2)*sqrt(b))*log(sqrt(b)*x^2 +
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*
a^(3/4)*b^(5/4)*f - b^2*c + a*b*g + sqrt(a)*b^(3/2)*e - I*a^(3/2)*sqrt(b))*
log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) +
2*(sqrt(2)*a^(1/4)*b^(9/4)*c - sqrt(2)*a^(5/4)*b^(5/4)*g + sqrt(2)*a^(3/4)*
b^(7/4)*e - 2*sqrt(a)*b^2*d + 2*a^(3/2)*b*h - I*sqrt(2)*a^(7/4)*b^(3/4))*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
```

```
b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c
 - sqrt(2)*a^(5/4)*b^(5/4)*g + sqrt(2)*a^(3/4)*b^(7/4)*e + 2*sqrt(a)*b^2*d
- 2*a^(3/2)*b*h - I*sqrt(2)*a^(7/4)*b^(3/4))*arctan(1/2*sqrt(2)*(2*sqrt(b)*
x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*s
qrt(b))*b^(5/4)))/b
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 554, normalized size = 1.44

$$\frac{f\log\left(|bx^4+a|\right)}{4b}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d+\sqrt{2}\sqrt{ab}\,abh+(ab^3)^{\frac14}b^2c-(ab^3)^{\frac14}abg+(ab^3)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d+\sqrt{2}\sqrt{ab}\,abh+(ab^3)^{\frac14}b^2c-(ab^3)^{\frac14}abg+(ab^3)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4ab^3}+\frac{\sqrt{2}\left((ab^3)^{\frac14}b^2c-(ab^3)^{\frac14}abg-(ab^3)^{\frac34}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{8ab^3}-\frac{\sqrt{2}\left((ab^3)^{\frac14}b^2c-(ab^3)^{\frac14}abg-(ab^3)^{\frac34}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{8ab^3}-\frac{i\sqrt{2}(ab^3)^{\frac34}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4b^4}-\frac{i\sqrt{2}(ab^3)^{\frac34}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4b^4}+\frac{i\sqrt{2}(ab^3)^{\frac34}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{8b^4}-\frac{i\sqrt{2}(ab^3)^{\frac34}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{8b^4}+\frac{3b^2hx^2+2ib^2x^3+6b^2gx}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="gia
c")
```

```
[Out] 1/4*f*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(2
)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/
4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) +
 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^(
1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b
^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/
4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*
a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^
3) - 1/4*I*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
```

```
/4))/(a/b)^(1/4))/b^4 - 1/4*I*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/b^4 + 1/8*I*sqrt(2)*(a*b^3)^(3/4)*log(
x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 - 1/8*I*sqrt(2)*(a*b^3)^(3/4)*
log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 + 1/6*(3*b^2*h*x^2 + 2*I*b
^2*x^3 + 6*b^2*g*x)/b^3
```

Mupad [B]
time = 5.05, size = 2500, normalized size = 6.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x)
```

```
[Out] symsum(log((a^4*i^3 - a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^2*b^2*c*h^2 - a
^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i - a*b^3*c*f^2 - a*b^3*d^2*g
+ a*b^3*c^2*i - 3*a^3*b*e*i^2 - a^3*b*g*h^2 + a^3*b*g^2*i - 2*a^2*b^2*c*g*i
 - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h - 2*a*b^3*c*d*h + 2*
a*b^3*c*e*g + 2*a*b^3*d*e*f + 2*a^3*b*f*h*i)/b^2 + root(256*a^3*b^7*z^4 - 2
56*a^3*b^6*f*z^3 + 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d*h
*z^2 - 64*a^3*b^5*c*i*z^2 + 64*a^2*b^6*c*e*z^2 + 32*a^4*b^4*h^2*z^2 + 96*a^
3*b^5*f^2*z^2 + 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*i*z + 32*a^4*b^3*e*h*i*
z + 32*a^3*b^4*e*f*g*z + 32*a^3*b^4*d*f*h*z - 32*a^3*b^4*d*e*i*z - 32*a^3*b
^4*c*g*h*z + 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z + 32*a^2*b^5*c*d*g*z -
 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^3*f*h^2*z + 16*a^4*b^3*
d*i^2*z - 16*a^3*b^4*e^2*h*z - 16*a^3*b^4*d*g^2*z + 16*a^2*b^5*c^2*h*z - 16
*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z - 16*a*b^6*c^2*d*z - 16*a^3*b^4*f^3*z
 - 8*a^4*b^2*e*f*h*i + 8*a^4*b^2*d*g*h*i - 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*d*
e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^3*b^3*c*d*h*i - 8*a^2*b
^4*c*d*f*g + 8*a^2*b^4*c*d*e*h + 4*a^4*b^2*f^2*g*i - 4*a^4*b^2*f*g^2*h - 4*
a^4*b^2*e*g^2*i + 4*a^4*b^2*e*g*h^2 + 4*a^4*b^2*c*h^2*i - 4*a^3*b^3*d^2*g*i
 - 4*a^4*b^2*d*f*i^2 - 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3*e^2*f*h - 4*a^3*b^3*e*
f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^3*b^3*d*f*g^2 - 4*a^2*b
^4*c^2*f*h - 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2 + 4*a^2*b^4*d^2*e*g + 4*
a^2*b^4*c*d^2*i - 4*a^2*b^4*d*e^2*f - 4*a^2*b^4*c*e^2*g + 4*a^2*b^4*c*e*f^2
 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a*b^5*c^2*d*f - 4*a*b^5*c*d^2*e -
4*a^5*b*e*i^3 - 4*a*b^5*c^3*g + 6*a^4*b^2*e^2*i^2 + 2*a^4*b^2*f^2*h^2 + 6*a
^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3*b^3*c^2*i^2 + 6*a^2*b^4*c^2*g^2
+ 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*a^3*b^3*e^3*i - 4*a^4*b^2*d*h^3 -
 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b^5*c^2*e^2 + a^4*b^2*g^4 + a^3*b^
3*f^4 + a^2*b^4*e^4 + a^5*b*h^4 + a*b^5*d^4 + a^6*i^4 + b^6*c^4, z, l)*((8*
a*b^4*c*f - 8*a*b^4*d*e + 8*a^2*b^3*d*i + 8*a^2*b^3*e*h - 8*a^2*b^3*f*g - 8
*a^3*b^2*h*i)/b^2 + root(256*a^3*b^7*z^4 - 256*a^3*b^6*f*z^3 + 64*a^4*b^4*g
*i*z^2 - 64*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 + 64*
a^2*b^6*c*e*z^2 + 32*a^4*b^4*h^2*z^2 + 96*a^3*b^5*f^2*z^2 + 32*a^2*b^6*d^2*
```

z^2 - 32*a^4*b^3*f*g*i*z + 32*a^4*b^3*e*h*i*z + 32*a^3*b^4*e*f*g*z + 32*a^3*b^4*d*f*h*z - 32*a^3*b^4*d*e*i*z - 32*a^3*b^4*c*g*h*z + 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z + 32*a^2*b^5*c*d*g*z - 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^3*f*h^2*z + 16*a^4*b^3*d*i^2*z - 16*a^3*b^4*e^2*h*z - 16*a^3*b^4*d*g^2*z + 16*a^2*b^5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z - 16*a*b^6*c^2*d*z - 16*a^3*b^4*f^3*z - 8*a^4*b^2*e*f*h*i + 8*a^4*b^2*d*g*h*i - 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^3*b^3*c*d*h*i - 8*a^2*b^4*c*d*f*g + 8*a^2*b^4*c*d*e*h + 4*a^4*b^2*f^2*g*i - 4*a^4*b^2*f*g^2*h - 4*a^4*b^2*e*g^2*i + 4*a^4*b^2*e*g*h^2 + 4*a^4*b^2*c*h^2*i - 4*a^3*b^3*d^2*g*i - 4*a^4*b^2*d*f*i^2 - 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3*e^2*f*h - 4*a^3*b^3*e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^3*b^3*d*f*g^2 - 4*a^2*b^4*c^2*f*h - 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2 + 4*a^2*b^4*d^2*e*g + 4*a^2*b^4*c*d^2*i - 4*a^2*b^4*d*e^2*f - 4*a^2*b^4*c*e^2*g + 4*a^2*b^4*c*e*f^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a*b^5*c^2*d*f - 4*a*b^5*c*d^2*e - 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g + 6*a^4*b^2*e^2*i^2 + 2*a^4*b^2*f^2*h^2 + 6*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3*b^3*c^2*i^2 + 6*a^2*b^4*c^2*g^2 + 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*a^3*b^3*e^3*i - 4*a^4*b^2*d*h^3 - 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b^5*c^2*e^2 + a^4*b^2*g^4 + a^3*b^3*f^4 + a^2*b^4*e^4 + a^5*b*h^4 + a*b^5*d^4 + a^6*i^4 + b^6*c^4, z, l)*((16*a^2*b^4*g - 16*a*b^5*c)/b^2 - (x*(16*a^2*b^3*h - 16*a*b^4*d))/b) - (x*(4*b^4*c^2 - 4*a*b^3*e^2 - 4*a^3*b*i^2 + 4*a^2*b^2*g^2 - 8*a*b^3*c*g + 8*a*b^3*d*f + 8*a^2*b^2*e*i - 8*a^2*b^2*f*h))/b) + (x*(b^3*d^3 - a^3*h^3 + b^3*c^2*f - a^3*f*i^2 - 2*b^3*c*d*e + 2*a^3*g*h*i + a*b^2*d*f^2 - a*b^2*e^2*f - 3*a*b^2*d^2*h + 3*a^2*b*d*h^2 + a^2*b*f*g^2 - a^2*b*f^2*h + 2*a*b^2*c*d*i + 2*a*b^2*c*e*h - 2*a*b^2*c*f*g + 2*a*b^2*d*e*g - 2*a^2*b*c*h*i - 2*a^2*b*d*g*i + 2*a^2*b*e*f*i - 2*a^2*b*e*g*h))/b)*root(256*a^3*b^7*z^4 - 256*a^3*b^6*f*z^3 + 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 + 64*a^2*b^6*c*e*z^2 + 32*a^4*b^4*h^2*z^2 + 96*a^3*b^5*f^2*z^2 + 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*i*z + 32*a^4*b^3*e*h*i*z + 32*a^3*b^4*e*f*g*z + 32*a^3*b^4*d*f*h*z - 32*a^3*b^4*d*e*i*z - 32*a^3*b^4*c*g*h*z + 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z + 32*a^2*b^5*c*d*g*z - 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^3*f*h^2*z + 16*a^4*b^3*d*i^2*z - 16*a^3*b^4*e^2*h*z - 16*a^3*b^4*d*g^2*z + 16*a^2*b^5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z - 16*a*b^6*c^2*d*z - 16*a^3*b^4*f^3*z - 8*a^4*b^2*e*f*h*i + 8*a^4*b^2*d*g*h*i - 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*d*e*f*i + 8*a^3*b^...

3.191 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4} dx$

Optimal. Leaf size=402

$$\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{jx^4}{4b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{\left(\sqrt{b}\,(bc-ag)+\sqrt{a}\,(be-ai)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{7/4}}+$$

```
[Out] g*x/b+1/2*h*x^2/b+1/3*i*x^3/b+1/4*j*x^4/b+1/4*(-a*j+b*f)*ln(b*x^4+a)/b^2+1/
2*(-a*h+b*d)*arctan(x^2*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-1/8*ln(-a^(1/4)*b^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2
))/a^(3/4)/b^(7/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(
1/2))*(-(-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)+1/4*
arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*((-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2)
)/a^(3/4)/b^(7/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*((-a*i+b*
e)*a^(1/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {1899, 1901, 1182, 1176, 631, 210, 1179, 642, 1833, 1824, 649, 211, 266}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}\,(bc-ag)+\sqrt{a}\,(be-ai)\right)}{2\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}\,(bc-ag)+\sqrt{a}\,(be-ai)\right)}{2\sqrt{2}\,a^{3/4}b^{7/4}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}\,(bc-ag)-\sqrt{a}\,(be-ai)\right)}{4\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}\,(bc-ag)-\sqrt{a}\,(be-ai)\right)}{4\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(bd-ah)}{2\sqrt{a}\,b^{3/2}}+\frac{(bf-aj)\log\left(a+bx^4\right)}{4b^2}+\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{jx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4),x
]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + (j*x^4)/(4*b) + ((b*d - a*h)*ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sq
rt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*
g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e -
 a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(7/4)) + ((b*f - a*j)*Log[a + b*x^4])/(4*b^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+191x^6+jx^7}{a+bx^4}\,dx = \int \left(\frac{c+ex^2+gx^4+191x^6}{a+bx^4} + \frac{x(d+fx^2+hx^4+jx\cdots}{a+bx^4}\right.$$

$$= \int \frac{c+ex^2+gx^4+191x^6}{a+bx^4}\,dx + \int \frac{x(d+fx^2+hx^4\cdots}{a+bx^4}$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2+jx^3}{a+bx^2}\,dx, x, x^2\right) + \int \left(\frac{g}{b}\cdots\right.$$

$$= \frac{gx}{b} + \frac{191x^3}{3b} + \frac{1}{2}\text{Subst}\left(\int \left(\frac{h}{b} + \frac{jx}{b} + \frac{bd-ah+(b\cdots}{b(a+\cdots}\right.\right.$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{191x^3}{3b} + \frac{jx^4}{4b} + \frac{\text{Subst}\left(\int \frac{bd-ah+(bf-aj)x}{a+bx^2}\cdots\right.}{2b}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{191x^3}{3b} + \frac{jx^4}{4b} - \frac{\left(191a - be - \frac{\sqrt{b}\,(bc-a\cdots}{\sqrt{a}}\right.}{\cdots}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{191x^3}{3b} + \frac{jx^4}{4b} + \frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}\,x\cdots}{\sqrt{a}}\right.}{2\sqrt{a}\,b^{3/2}}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{191x^3}{3b} + \frac{jx^4}{4b} + \frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}\,x\cdots}{\sqrt{a}}\right.}{2\sqrt{a}\,b^{3/2}}$$

<u>Mathematica</u> [A]
time = 0.19, size = 445, normalized size = 1.11

$$\frac{24b^{3/4}gx+12b^{3/4}hx^2+8b^{3/4}ix^3+6b^{3/4}jx^4+\frac{6\left(-\sqrt{2}\,b^{3/2}c-2\sqrt[4]{a}\,b^{5/4}d-\sqrt{2}\,\sqrt{a}\,be+\sqrt{2}\,a\sqrt{b}\,g+2a^{5/4}\sqrt[4]{b}\,h+\sqrt{2}\,a^{3/2}i\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{6\left(\sqrt{2}\,b^{3/2}c-2\sqrt[4]{a}\,b^{5/4}d+\sqrt{2}\,\sqrt{a}\,be-\sqrt{2}\,a\sqrt{b}\,g+2a^{5/4}\sqrt[4]{b}\,h-\sqrt{2}\,a^{3/2}i\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}}-\frac{3\sqrt{2}\left(b^{3/2}c-\sqrt{a}\,be-a\sqrt{b}\,g+a^{3/2}i\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{3/4}}+\frac{3\sqrt{2}\left(b^{3/2}c-\sqrt{a}\,be-a\sqrt{b}\,g+a^{3/2}i\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{3/4}}+\frac{6(bf-aj)\log\left(a+bx^4\right)}{\sqrt[4]{b}}}{24b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4),x]
```

```
[Out] (24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + 6*b^(3/4)*j*x^4 + (6
*(-(Sqrt[2]*b^(3/2)*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*Sqrt[a]*b*e + Sqrt[2
]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (6*(Sqrt[2]*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*
d + Sqrt[2]*Sqrt[a]*b*e - Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - Sqrt[
2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (3*Sqrt[2]
*(b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*
```

```
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (3*Sqrt[2]*(b^(3/2)*c - Sqrt[a]
*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/a^(3/4) + (6*(b*f - a*j)*Log[a + b*x^4])/b^(1/4))/(24*b^(7/4))
```

Maple [A]

time = 0.36, size = 299, normalized size = 0.74

method	result
risch	$\frac{j\,x^4}{4b}+\frac{i\,x^3}{3b}+\frac{h\,x^2}{2b}+\frac{gx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(bc-ag+(-ah+bd)\_R+(-ai+be)\_R^2+(-aj+bf)\_R^3\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$\frac{\frac{1}{4}j\,x^4+\frac{1}{3}i\,x^3+\frac{1}{2}h\,x^2+gx}{b}+\frac{\frac{(-ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/b*(1/4*j*x^4+1/3*i*x^3+1/2*h*x^2+g*x)+1/b*(1/8*(-a*g+b*c)*(a/b)^(1/4)/a*2
^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/
2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(
1/4)*x-1))+1/2*(-a*h+b*d)/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*(-a*i+b*e
)/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a
/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arcta
n(2^(1/2)/(a/b)^(1/4)*x-1))+1/4*(-a*j+b*f)*ln(b*x^4+a)/b)
```

Maxima [A]

time = 0.51, size = 429, normalized size = 1.07

$$\frac{3jx^4+6hx^2+4ix^3+12gx}{12b}+\frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f-\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}j+b^2c-abg-\sqrt{a}b^{\frac{3}{2}}e+ia^{\frac{3}{2}}\sqrt{b}\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f-\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}j-b^2c+abg+\sqrt{a}b^{\frac{3}{2}}e-ia^{\frac{3}{2}}\sqrt{b}\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{9}{4}}c-\sqrt{2}a^{\frac{5}{4}}b^{\frac{5}{4}}g+\sqrt{2}a^{\frac{3}{4}}b^{\frac{7}{4}}e-2\sqrt{a}b^2d+2a^{\frac{3}{2}}bh-i\sqrt{2}a^{\frac{7}{4}}b^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{9}{4}}c-\sqrt{2}a^{\frac{5}{4}}b^{\frac{5}{4}}g+\sqrt{2}a^{\frac{3}{4}}b^{\frac{7}{4}}e+2\sqrt{a}b^2d-2a^{\frac{3}{2}}bh-i\sqrt{2}a^{\frac{7}{4}}b^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorith
m="maxima")
```

```
[Out] 1/12*(3*j*x^4 + 6*h*x^2 + 4*I*x^3 + 12*g*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/
4)*b^(5/4)*f - sqrt(2)*a^(7/4)*b^(1/4)*j + b^2*c - a*b*g - sqrt(a)*b^(3/2)*
e + I*a^(3/2)*sqrt(b))*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a
))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f - sqrt(2)*a^(7/4)
*b^(1/4)*j - b^2*c + a*b*g + sqrt(a)*b^(3/2)*e - I*a^(3/2)*sqrt(b))*log(sqr
t(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 2*(sqrt
(2)*a^(1/4)*b^(9/4)*c - sqrt(2)*a^(5/4)*b^(5/4)*g + sqrt(2)*a^(3/4)*b^(7/4)
```

```
*e - 2*sqrt(a)*b^2*d + 2*a^(3/2)*b*h - I*sqrt(2)*a^(7/4)*b^(3/4))*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c - sqrt
(2)*a^(5/4)*b^(5/4)*g + sqrt(2)*a^(3/4)*b^(7/4)*e + 2*sqrt(a)*b^2*d - 2*a^(
3/2)*b*h - I*sqrt(2)*a^(7/4)*b^(3/4))*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqr
t(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))
*b^(5/4)))/b
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.56, size = 570, normalized size = 1.42

$$\frac{(bf-aj)\log(|bx^4+a|)}{4\,b^2}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d+\sqrt{2}\sqrt{ab}\,abh+(ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{1}{4}}abg+(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d+\sqrt{2}\sqrt{ab}\,abh+(ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{1}{4}}abg+(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{1}{4}}abg-(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{1}{4}}abg-(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}-\frac{i\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}-\frac{i\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}+\frac{i\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4}-\frac{i\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4}+\frac{3\,b^3jx^4+6\,b^3hx^2+4i\,b^3x^3+12\,b^3gx}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorith
m="giac")
```

```
[Out] 1/4*(b*f - a*j)*log(abs(b*x^4 + a))/b^2 + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^
2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g +
 (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(a*b^3) + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h
 + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*
b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)
*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a
```

```
*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt
(a/b))/(a*b^3) - 1/4*I*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/b^4 - 1/4*I*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/b^4 + 1/8*I*sqrt(2)*(a*b^3
)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 - 1/8*I*sqrt(2)*(a
*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 + 1/12*(3*b^3*
j*x^4 + 6*b^3*h*x^2 + 4*I*b^3*x^3 + 12*b^3*g*x)/b^4
```

Mupad [B]
time = 5.20, size = 2500, normalized size = 6.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4),x
)
```

```
[Out] symsum(log((a^4*i^3 - a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^4*g*j^2 + a^2*b
^2*c*h^2 - a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i - 2*a^4*h*i*j -
a*b^3*c*f^2 - a*b^3*d^2*g + a*b^3*c^2*i - a^3*b*c*j^2 - 3*a^3*b*e*i^2 - a^3
*b*g*h^2 + a^3*b*g^2*i + 2*a^2*b^2*c*f*j - 2*a^2*b^2*c*g*i - 2*a^2*b^2*d*e*
j - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h - 2*a*b^3*c*d*h + 2
*a*b^3*c*e*g + 2*a*b^3*d*e*f + 2*a^3*b*d*i*j + 2*a^3*b*e*h*j - 2*a^3*b*f*g*
j + 2*a^3*b*f*h*i)/b^2 + root(256*a^3*b^8*z^4 + 256*a^4*b^6*j*z^3 - 256*a^3
*b^7*f*z^3 - 192*a^4*b^5*f*j*z^2 + 64*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^2
- 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*i*z^2 + 64*a^2*b^7*c*e*z^2 + 96*a^5*b^4
*j^2*z^2 + 32*a^4*b^5*h^2*z^2 + 96*a^3*b^6*f^2*z^2 + 32*a^2*b^7*d^2*z^2 + 3
2*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i*z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*e*
g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*b^4*c*i*j*z + 32*a^3*b^5*e*f*g*z + 32*a
^3*b^5*d*f*h*z - 32*a^3*b^5*d*e*i*z - 32*a^3*b^5*c*g*h*z + 32*a^3*b^5*c*f*i
*z + 32*a^3*b^5*c*e*j*z - 32*a^2*b^6*c*e*f*z + 32*a^2*b^6*c*d*g*z + 16*a^5*
b^3*h^2*j*z - 16*a^5*b^3*h*i^2*z - 48*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*z
+ 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*h^2*z + 16*a^3*b^5*d^2*j*z + 16*a^4*b^4
*d*i^2*z - 16*a^3*b^5*e^2*h*z - 16*a^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z - 1
6*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2*z - 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^3*
z - 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g*i*j + 8*a^5*b^2*e*h*i*j - 8*a^4*b^3*e*
f*h*i + 8*a^4*b^3*e*f*g*j + 8*a^4*b^3*d*g*h*i + 8*a^4*b^3*d*f*h*j - 8*a^4*b
^3*d*e*i*j - 8*a^4*b^3*c*g*h*j + 8*a^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h + 8*
a^3*b^4*d*e*f*i + 8*a^3*b^4*c*f*g*h + 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f*j
 - 8*a^3*b^4*c*d*h*i + 8*a^3*b^4*c*d*g*j - 8*a^2*b^5*c*d*f*g + 8*a^2*b^5*c*
d*e*h + 4*a^5*b^2*g^2*h*j - 4*a^5*b^2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5*b
^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j - 4*a^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 - 4*
a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*i*j^2 + 4*a^4*b^3*f^2*g*i - 4*a^4*b^3*f*g^2*h
 - 4*a^4*b^3*e*g^2*i - 4*a^4*b^3*d*g^2*j + 4*a^3*b^4*c^2*h*j + 4*a^4*b^3*e*
g*h^2 + 4*a^4*b^3*c*h^2*i - 4*a^3*b^4*d^2*g*i - 4*a^3*b^4*d^2*f*j - 4*a^4*b
```

^3*d*f*i^2 - 4*a^4*b^3*c*g*i^2 + 4*a^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j + 4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*f^2*g - 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2*i + 4*a^3*b^4*d*f*g^2 - 4*a^2*b^5*c^2*f*h - 4*a^2*b^5*c^2*e*i - 4*a^2*b^5*c^2*d*j - 4*a^3*b^4*c*e*h^2 + 4*a^2*b^5*d^2*e*g + 4*a^2*b^5*c*d^2*i - 4*a^2*b^5*d*e^2*f - 4*a^2*b^5*c*e^2*g + 4*a^2*b^5*c*e*f^2 - 4*a^6*b*h*i^2*j + 4*a^6*b*g*i*j^2 + 4*a*b^6*c^2*d*f - 4*a*b^6*c*d^2*e - 4*a^6*b*f*j^3 - 4*a*b^6*c^3*g + 6*a^5*b^2*f^2*j^2 + 2*a^5*b^2*g^2*i^2 + 6*a^4*b^3*e^2*i^2 + 2*a^4*b^3*f^2*h^2 + 2*a^4*b^3*d^2*j^2 + 6*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2*a^3*b^4*c^2*i^2 + 6*a^2*b^5*c^2*g^2 + 2*a^2*b^5*d^2*f^2 + 2*a^6*b*h^2*j^2 - 4*a^4*b^3*f^3*j - 4*a^5*b^2*e*i^3 - 4*a^3*b^4*e^3*i - 4*a^4*b^3*d*h^3 - 4*a^2*b^5*d^3*h - 4*a^3*b^4*c*g^3 + 2*a*b^6*c^2*e^2 + a^5*b^2*h^4 + a^4*b^3*g^4 + a^3*b^4*f^4 + a^2*b^5*e^4 + a^6*b*i^4 + a*b^6*d^4 + a^7*j^4 + b^7*c^4, z, m)*((8*a*b^4*c*f - 8*a*b^4*d*e - 8*a^2*b^3*c*j + 8*a^2*b^3*d*i + 8*a^2*b^3*e*h - 8*a^2*b^3*f*g + 8*a^3*b^2*g*j - 8*a^3*b^2*h*i)/b^2 + root(256*a^3*b^8*z^4 + 256*a^4*b^6*j*z^3 - 256*a^3*b^7*f*z^3 - 192*a^4*b^5*f*j*z^2 + 64*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*i*z^2 + 64*a^2*b^7*c*e*z^2 + 96*a^5*b^4*j^2*z^2 + 32*a^4*b^5*h^2*z^2 + 96*a^3*b^6*f^2*z^2 + 32*a^2*b^7*d^2*z^2 + 32*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i*z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*e*g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*b^4*c*i*j*z + 32*a^3*b^5*e*f*g*z + 32*a^3*b^5*d*f*h*z - 32*a^3*b^5*d*e*i*z - 32*a^3*b^5*c*g*h*z + 32*a^3*b^5*c*f*i*z + 32*a^3*b^5*c*e*j*z - 32*a^2*b^6*c*e*f*z + 32*a^2*b^6*c*d*g*z + 16*a^5*b^3*h^2*j*z - 16*a^5*b^3*h*i^2*z - 48*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*z + 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*h^2*z + 16*a^3*b^5*d^2*j*z + 16*a^4*b^4*d*i^2*z - 16*a^3*b^5*e^2*h*z - 16*a^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z - 16*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2*z - 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^3*z - 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g*i*j + 8*a^5*b^2*e*h*i*j - 8*a^4*b^3*e*f*h*i + 8*a^4*b^3*e*f*g*j + 8*a^4*b^3*d*g*h*i + 8*a^4*b^3*d*f*h*j - 8*a^4*b^3*d*e*i*j - 8*a^4*b^3*c*g*h*j + 8*a^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h + 8*a^3*b^4*d*e*f*i + 8*a^3*b^4*c*f*g*h + 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f*j - 8*a^3*b^4*c*d*h*i + 8*a^3*b^4*c*d*g*j - 8*a^2*b^5*c*d*f*g + 8*a^2*b^5*c*d*e*h + 4*a^5*b^2*g^2*h*j - 4*a^5*b^2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5*b^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j - 4*a^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 - 4*a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*i*j^2 + 4*a^4*b^3*f^2*g*i - 4*a^4*b^3*f*g^2*h - 4*a^4*b^3*e*g^2*i - 4*a^4*b^3*d*g^2*j + 4*a^3*b^4*c^2*h*j + 4*a^4*b^3*e*g*h^2 + 4*a^4*b^3*c*h^2*i - 4*a^3*b^4*d^2*g*i - 4*a^3*b^4*d^2*f*j - 4*a^4*b^3*d*f*i^2 - 4*a^4*b^3*c*g*i^2 + 4*a^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j + 4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*f^2*g - 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2*i + 4*a^3*b^4*d*f*g^2 - 4*a^2*b^5*c^2*f*h - 4*a^2*b^5*c^2*e*i - 4*a^2*b^5*c^2*d*j - 4*...

3.192 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx$

Optimal. Leaf size=184

$$\frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{4ab\,(a-bx^4)}+\frac{\left(3bc-\sqrt{a}\,\sqrt{b}\,e-ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}}+\frac{\left(3bc+\sqrt{a}\,\sqrt{b}\,e-ag\right)}{8a^{7/4}b^{5/4}}$$

```
[Out] 1/4*x*(b*c+a*g+(a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(-b*x^4+a)+1/4*(-a*h+b*d)*a
rctanh(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)+1/8*arctan(b^(1/4)*x/a^(1/4))*(
3*b*c-a*g-e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))
*(3*b*c-a*g+e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)
```

Rubi [A]
time = 0.14, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1872, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\,\sqrt{b}\,e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,\sqrt{b}\,e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x(x(ah+bd)+ag+bc+bex^2+bfx^3)}{4ab\,(a-bx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + (
(3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b
^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/
(8*a^(7/4)*b^(5/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/
2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^2}\,dx = \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-b(3bc-ag)-2b(bd-ah)x-b^2\cdots}{a-bx^4}}{4ab^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{4ab\left(a-bx^4\right)} - \frac{\int \left(-\frac{2b(bd-ah)x}{a-bx^4}+\frac{-b(3bc-\cdots}{a-\cdots}\right.}{4ab^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-b(3bc-ag)-b^2ex^2}{a-bx^4}\,dx}{4ab^2} + \cdots$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{4ab\left(a-bx^4\right)} - \frac{\left(3bc-\sqrt{a}\,\sqrt{b}\,e-ag\right)\cdots}{8a^{3/2}\sqrt{\cdots}}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{4ab\left(a-bx^4\right)} + \frac{\left(3bc-\sqrt{a}\,\sqrt{b}\,e-ag\right)\cdots}{8a^{7/4}b^{5/4}\cdots}$$

Mathematica [A]

time = 0.11, size = 257, normalized size = 1.40

$$\frac{\frac{4a^{3/4}\sqrt{b}\,(bx(c+x(d+ex))+a(f+x(g+hx)))}{a-bx^4}-2\sqrt[4]{b}\left(-3bc+\sqrt{a}\,\sqrt{b}\,e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\left(-3b^{5/4}c-2\sqrt[4]{a}\,bd-\sqrt{a}\,b^{3/4}e+a\sqrt[4]{b}\,g+2a^{5/4}h\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+\left(3b^{5/4}c-2\sqrt[4]{a}\,bd+\sqrt{a}\,b^{3/4}e-a\sqrt[4]{b}\,g+2a^{5/4}h\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)-2\sqrt[4]{a}\,(-bd+ah)\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{16a^{7/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x))))/(a - b*x
^4) - 2*b^(1/4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/
4)] + (-3*b^(5/4)*c - 2*a^(1/4)*b*d - Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g + 2*a
^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (3*b^(5/4)*c - 2*a^(1/4)*b*d + Sqrt[a]
*b^(3/4)*e - a*b^(1/4)*g + 2*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 2*a^(1/4
)*(-(b*d) + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(3/2))
```

Maple [A]

time = 0.34, size = 222, normalized size = 1.21

method	result
risch	$\frac{\frac{e\,x^{3}}{4a}+\frac{(ah+bd)x^{2}}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^{4}+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(\_R^{2}e-\frac{2(ah-bd)\_R}{b}-\frac{ag-3bc}{b}\right)\ln\left(x-\_R\right)}{\_R^{3}}}{16ba}$
default	$\frac{\frac{e\,x^{3}}{4a}+\frac{(ah+bd)x^{2}}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^{4}+a}+\frac{\frac{(-ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(-2ah+2bd)\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{4\sqrt{ab}}-e\left(2\arctan\cdots\right.}{4ba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/a*e*x^3+1/4*(a*h+b*d)/a/b*x^2+1/4*(a*g+b*c)/a/b*x+1/4*f/b)/(-b*x^4+a)+
1/4/b/a*(1/4*(-a*g+3*b*c)*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4))
)+2*arctan(x/(a/b)^(1/4)))+1/4*(-2*a*h+2*b*d)/(a*b)^(1/2)*ln((a+x^2*(a*b)^(
1/2))/(a-x^2*(a*b)^(1/2)))-1/4*e/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x
+(a/b)^(1/4))/(x-(a/b)^(1/4)))))
```

Maxima [A]

time = 0.51, size = 246, normalized size = 1.34

$$-\frac{bx^{3}e+(bd+ah)x^{2}+af+(bc+ag)x}{4\left(ab^{2}x^{4}-a^{2}b\right)}+\frac{\frac{2(bd-ah)\log\left(\sqrt{b}\,x^{2}+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{2(bd-ah)\log\left(\sqrt{b}\,x^{2}-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(3\,b^{\frac{3}{2}}c-a\sqrt{b}\,g-\sqrt{a}\,be\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(3\,b^{\frac{3}{2}}c-a\sqrt{b}\,g+\sqrt{a}\,be\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{16\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima
")
```

```
[Out] -1/4*(b*x^3*e + (b*d + a*h)*x^2 + a*f + (b*c + a*g)*x)/(a*b^2*x^4 - a^2*b)
+ 1/16*(2*(b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 2*(b*d
```

```
 - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(3*b^(3/2)*c - a*s
qrt(b)*g - sqrt(a)*b*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sq
rt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*b^(3/2)*c - a*sqrt(b)*g + sqrt(a)*b*e)*lo
g((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a*b)
```

Fricas [**C**] Result contains complex when optimal does not.
time = 194.96, size = 710521, normalized size = 3861.53

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas
")
```

```
[Out] -1/192*(48*b*e*x^3 + 48*(b*d + a*h)*x^2 + 12*(a*b^2*x^4 - a^2*b)*(sqrt((2*a
^2*h^2 - (e*g + 4*d*h)*a*b + (2*d^2 + 3*c*e)*b^2)/(a^3*b^3) - (2*a^2*h^2 +
(e*g - 4*d*h)*a*b + (2*d^2 - 3*c*e)*b^2)/(a^3*b^3) - (9*b^3*c^2*d - a^3*g^2
*h + (d*g^2 - e^2*h + 6*c*g*h)*a^2*b + (d*e^2 - 6*c*d*g - 9*c^2*h)*a*b^2)/(
(b*d - a*h)*a^4*b^3*sqrt(1/(a*b)))) - 2*(b*d - a*h)*sqrt(1/(a*b))/(a*b))*lo
g(1080*a*b^6*c^3*d^3 + 120*a^2*b^5*c*d^3*e^2 - 6*a^3*b^4*d*e^5 - 10*a^5*b^2
*d*e*g^4 - 320*a^6*b*d*e*h^4 + 64*a^7*e*h^5 - 40*(a^4*b^3*d^3 - 3*a^4*b^3*c
*d*e)*g^3 - 40*(27*a^4*b^3*c^3 - 16*a^5*b^2*d^2*e + 3*a^5*b^2*c*e^2 + 9*a^6
*b*c*g^2 - a^7*g^3 - (27*a^5*b^2*c^2 + a^6*b*e^2)*g)*h^3 - (24*a^6*b^7*c*d^
2 - 9*a^6*b^7*c^2*e - a^7*b^6*e^3 - a^8*b^5*e*g^2 + 8*(3*a^8*b^5*c - a^9*b^
4*g)*h^2 - 2*(4*a^7*b^6*d^2 - 3*a^7*b^6*c*e)*g - 16*(3*a^7*b^6*c*d - a^8*b^
5*d*g)*h)*(sqrt((2*a^2*h^2 - (e*g + 4*d*h)*a*b + (2*d^2 + 3*c*e)*b^2)/(a^3*
b^3) - (2*a^2*h^2 + (e*g - 4*d*h)*a*b + (2*d^2 - 3*c*e)*b^2)/(a^3*b^3) - (9
*b^3*c^2*d - a^3*g^2*h +  ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(148) = 296.
time = 0.54, size = 380, normalized size = 2.07

$$-\frac{\sqrt{2}\left(3\,b^{2}c-abg-2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c-abg+2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c-abg-\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^{3}\right)^{\frac{3}{4}}a}+\frac{\sqrt{2}\left(3\,b^{2}c-abg-\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{bx^{3}e+bdx^{2}+ahx^{2}+bcx+agx+af}{4\left(bx^{4}-a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - a*b*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + 2*sqrt(2)*(
-a*b^3)^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a) - 1/16*sqrt(2)*(3*b^2*c - a*b*g +
 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - 2*sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b
*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)
^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(
2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c
- a*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((
-a*b^3)^(3/4)*a) - 1/4*(b*x^3*e + b*d*x^2 + a*h*x^2 + b*c*x + a*g*x + a*f)/
((b*x^4 - a)*a*b)
```

Mupad [B]
time = 5.61, size = 1626, normalized size = 8.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log(- root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2 + 1024*a^5*b^4*e
*g*z^2 - 3072*a^4*b^5*c*e*z^2 - 2048*a^6*b^3*h^2*z^2 - 2048*a^4*b^5*d^2*z^2
 + 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2*g^2*h*z - 128*a^
4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z + 128*a^4*b^3*d*g^2*z + 128*a^3*b^4*d*
e^2*z + 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h + 96*a^2*b^3*c*d*e*h - 48
*a^3*b^2*c*e*h^2 + 16*a^2*b^3*d^2*e*g - 12*a^2*b^3*c*e^2*g + 16*a^4*b*e*g*h
^2 - 48*a*b^4*c*d^2*e - 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g + 96*a^3*b^2*d^2*h
^2 + 2*a^3*b^2*e^2*g^2 - 54*a^2*b^3*c^2*g^2 - 64*a^2*b^3*d^3*h + 12*a^3*b^2
*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 - 81*b^5*c^4 - a^2*b^
3*e^4 - a^4*b*g^4, z, k)*(root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2 + 1
024*a^5*b^4*e*g*z^2 - 3072*a^4*b^5*c*e*z^2 - 2048*a^6*b^3*h^2*z^2 - 2048*a^
4*b^5*d^2*z^2 + 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2*g^2
*h*z - 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z + 128*a^4*b^3*d*g^2*z + 1
28*a^3*b^4*d*e^2*z + 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h + 96*a^2*b^3
*c*d*e*h - 48*a^3*b^2*c*e*h^2 + 16*a^2*b^3*d^2*e*g - 12*a^2*b^3*c*e^2*g + 1
6*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e - 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g + 96*
a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 - 54*a^2*b^3*c^2*g^2 - 64*a^2*b^3*d^3*h
 + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 - 81*b^5
*c^4 - a^2*b^3*e^4 - a^4*b*g^4, z, k)*((768*a^3*b^4*c - 256*a^4*b^3*g)/(64*
a^3*b) - (x*(128*a^3*b^4*d - 128*a^4*b^3*h))/(16*a^3*b)) - (64*a^2*b^3*d*e
- 64*a^3*b^2*e*h)/(64*a^3*b) + (x*(36*a*b^4*c^2 + 4*a^2*b^3*e^2 + 4*a^3*b^2
*g^2 - 24*a^2*b^3*c*g))/(16*a^3*b)) - (a*b^2*e^3 + 12*b^3*c*d^2 - 9*b^3*c^2
*e - 4*a^3*g*h^2 - 4*a*b^2*d^2*g + 12*a^2*b*c*h^2 - a^2*b*e*g^2 - 24*a*b^2*
c*d*h + 6*a*b^2*c*e*g + 8*a^2*b*d*g*h)/(64*a^3*b) - (x*(2*b^3*d^3 - 2*a^3*h
^3 - 3*b^3*c*d*e - 6*a*b^2*d^2*h + 6*a^2*b*d*h^2 + 3*a*b^2*c*e*h + a*b^2*d*
```

```
e*g - a^2*b*e*g*h))/(16*a^3*b))*root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z
^2 + 1024*a^5*b^4*e*g*z^2 - 3072*a^4*b^5*c*e*z^2 - 2048*a^6*b^3*h^2*z^2 - 2
048*a^4*b^5*d^2*z^2 + 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b
^2*g^2*h*z - 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z + 128*a^4*b^3*d*g^2
*z + 128*a^3*b^4*d*e^2*z + 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h + 96*a
^2*b^3*c*d*e*h - 48*a^3*b^2*c*e*h^2 + 16*a^2*b^3*d^2*e*g - 12*a^2*b^3*c*e^2
*g + 16*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e - 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g
 + 96*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 - 54*a^2*b^3*c^2*g^2 - 64*a^2*b^3
*d^3*h + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 -
81*b^5*c^4 - a^2*b^3*e^4 - a^4*b*g^4, z, k), k, 1, 4) + (f/(4*b) + (e*x^3)/
(4*a) + (x*(b*c + a*g))/(4*a*b) + (x^2*(b*d + a*h))/(4*a*b))/(a - b*x^4)
```

3.193 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx$

Optimal. Leaf size=203

$$\frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{4ab(a-bx^4)}-\frac{\left(be-\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}}+\frac{\left(be+\frac{\sqrt{b}\,(3bc-$$

```
[Out] 1/4*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+b*f*x^3)/a/b/(-b*x^4+a)+1/4*(-a*h+
b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-1/8*arctan(b^(1/4)*x/a^(1
/4))*(b*e-3*a*i-(-a*g+3*b*c)*b^(1/2)/a^(1/2))/a^(5/4)/b^(7/4)+1/8*arctanh(b
^(1/4)*x/a^(1/4))*(b*e-3*a*i+(-a*g+3*b*c)*b^(1/2)/a^(1/2))/a^(5/4)/b^(7/4)
```

Rubi [A]
time = 0.18, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {1872, 1890, 281, 214, 1181, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3)}{4ab(a-bx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(4*a*b*(a - b*x
^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/
a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3
*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+193x^6}{\left(a-bx^4\right)^2}\,dx = \frac{x(bc+ag+(bd+ah)x+(193a+be)x^2+bfx^3)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-b(3}{}}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(193a+be)x^2+bfx^3)}{4ab\left(a-bx^4\right)} - \frac{\int \left(-\right.}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(193a+be)x^2+bfx^3)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-b(3}{}}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(193a+be)x^2+bfx^3)}{4ab\left(a-bx^4\right)} - \frac{\left(579a\right.}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(193a+be)x^2+bfx^3)}{4ab\left(a-bx^4\right)} + \frac{\left(579a\right.}{}$$

Mathematica [A]

time = 0.14, size = 302, normalized size = 1.49

$$\frac{\frac{4a^{3/4}b^{3/4}(bx(c+x(d+ex))+a(f+x(g+x(h+ix))))}{a-bx^4}+2\left(3b^{3/2}c-\sqrt{a}\,be-a\sqrt{b}\,g+3a^{3/2}i\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\left(-3b^{3/2}c-2\sqrt[4]{a}\,b^{5/4}d-\sqrt{a}\,be+a\sqrt{b}\,g+2a^{5/4}\sqrt[4]{b}\,h+3a^{3/2}i\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+\left(3b^{3/2}c-2\sqrt[4]{a}\,b^{5/4}d+\sqrt{a}\,be-a\sqrt{b}\,g+2a^{5/4}\sqrt[4]{b}\,h-3a^{3/2}i\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)-2\sqrt[4]{a}\,\sqrt[4]{b}\,(-bd+ah)\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{16a^{7/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x
]
```

```
[Out] ((4*a^(3/4)*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + x*(h + i*x)))))/
(a - b*x^4) + 2*(3*b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*Arc
Tan[(b^(1/4)*x)/a^(1/4)] + (-3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d - Sqrt[a]*b*
e + a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*
x] + (3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e - a*Sqrt[b]*g + 2*a^(
5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 2*a^(1/4)*b^(1/4)*
(-(b*d) + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(7/4))
```

Maple [A]
time = 0.36, size = 241, normalized size = 1.19

method	result
risch	$\frac{\frac{(ai+be)x^3}{4ab}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(-(3ai-be)\_R^2-2(ah-bd)\_R-ag+3bc\right)\ln\left(x-\_R\right)}{\_R^3}}{16a\,b^2}$
default	$\frac{\frac{(ai+be)x^3}{4ab}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^4+a}+\frac{\frac{(-ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(-2ah+2bd)\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{(-3 \cdots}{\cdots}}{4ba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERB
OSE)
```

```
[Out] (1/4*(a*i+b*e)/a/b*x^3+1/4*(a*h+b*d)/a/b*x^2+1/4*(a*g+b*c)/a/b*x+1/4*f/b)/(
-b*x^4+a)+1/4/b/a*(1/4*(-a*g+3*b*c)*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a
/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))+1/4*(-2*a*h+2*b*d)/(a*b)^(1/2)*ln((a+x
^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2)))-1/4*(-3*a*i+b*e)/b/(a/b)^(1/4)*(2*arct
an(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))))
```

Maxima [A]
time = 0.50, size = 261, normalized size = 1.29

$$-\frac{(be+i\,a)x^3+(bd+ah)x^2+af+(bc+ag)x}{4\left(ab^2x^4-a^2b\right)}+\frac{\frac{2\,(bd-ah)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{2\,(bd-ah)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(3\,b^{\frac{3}{2}}c-a\sqrt{b}\,g-\sqrt{a}\,be+3i\,a^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(3\,b^{\frac{3}{2}}c-a\sqrt{b}\,g+\sqrt{a}\,be-3i\,a^{\frac{3}{2}}\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{16\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="
maxima")
```

```
[Out] -1/4*((b*e + I*a)*x^3 + (b*d + a*h)*x^2 + a*f + (b*c + a*g)*x)/(a*b^2*x^4 -
 a^2*b) + 1/16*(2*(b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b))
- 2*(b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(3*b^(3/2)
*c - a*sqrt(b)*g - sqrt(a)*b*e + 3*I*a^(3/2))*arctan(sqrt(b)*x/sqrt(sqrt(a)
*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*b^(3/2)*c - a*sqrt(
b)*g + sqrt(a)*b*e - 3*I*a^(3/2))*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(
sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
)/(a*b)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(165) = 330.
time = 0.50, size = 576, normalized size = 2.84

$$-\frac{\sqrt{2}\left(3b^2c-abg-2\sqrt{2}(-ab^3)^{\frac14}bd+2\sqrt{2}(-ab^3)^{\frac14}ah+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}\right)}{2\left(-\frac{a}{b}\right)^{\frac14}}\right)}{16(-ab^3)^{\frac34}a}-\frac{\sqrt{2}\left(3b^2c-abg+2\sqrt{2}(-ab^3)^{\frac14}bd-2\sqrt{2}(-ab^3)^{\frac14}ah-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}\right)}{2\left(-\frac{a}{b}\right)^{\frac14}}\right)}{16(-ab^3)^{\frac34}a}-\frac{\sqrt{2}\left(3b^2c-abg-\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{32(-ab^3)^{\frac34}a}+\frac{\sqrt{2}\left(3b^2c-abg-\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{32(-ab^3)^{\frac34}a}-\frac{bx^3e+bdx^2+ahx^2+iax^3+bcx+agx+af}{4(bx^4-a)ab}-\frac{3i\sqrt{2}(-ab^3)^{\frac34}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}\right)}{2\left(-\frac{a}{b}\right)^{\frac14}}\right)}{16ab^4}-\frac{3i\sqrt{2}(-ab^3)^{\frac34}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}\right)}{2\left(-\frac{a}{b}\right)^{\frac14}}\right)}{16ab^4}+\frac{3i\sqrt{2}(-ab^3)^{\frac34}\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{32ab^4}-\frac{3i\sqrt{2}(-ab^3)^{\frac34}\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{32ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="
giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - a*b*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + 2*sqrt(2)*(
-a*b^3)^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a) - 1/16*sqrt(2)*(3*b^2*c - a*b*g +
 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - 2*sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b
```

```
*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)
^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(
2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c
- a*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((
-a*b^3)^(3/4)*a) - 1/4*(b*x^3*e + b*d*x^2 + a*h*x^2 + I*a*x^3 + b*c*x + a*g
*x + a*f)/((b*x^4 - a)*a*b) - 3/16*I*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^4) - 3/16*I*sqrt(2)*(-a
*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(
a*b^4) + 3/32*I*sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + s
qrt(-a/b))/(a*b^4) - 3/32*I*sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/
b)^(1/4) + sqrt(-a/b))/(a*b^4)
```

Mupad [B]
time = 5.67, size = 2611, normalized size = 12.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log((27*a^4*i^3 - a*b^3*e^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e - 12*a^2*b^
2*c*h^2 + a^2*b^2*e*g^2 + 9*a^2*b^2*e^2*i + 4*a*b^3*d^2*g - 27*a*b^3*c^2*i
- 27*a^3*b*e*i^2 + 4*a^3*b*g*h^2 - 3*a^3*b*g^2*i + 18*a^2*b^2*c*g*i - 8*a^2
*b^2*d*g*h + 24*a*b^3*c*d*h - 6*a*b^3*c*e*g)/(64*a^3*b^2) - root(65536*a^7*
b^7*z^4 - 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c*i*z^2 + 4096*a^5*b^5*d*h*z^
2 + 1024*a^5*b^5*e*g*z^2 - 3072*a^4*b^6*c*e*z^2 - 2048*a^6*b^4*h^2*z^2 - 20
48*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z - 768*a^4*b^4*d*e*i*z + 768*a^4*b^
4*c*g*h*z - 768*a^3*b^5*c*d*g*z - 1152*a^6*b^2*h*i^2*z - 128*a^5*b^3*g^2*h*
z + 1152*a^5*b^3*d*i^2*z - 128*a^4*b^4*e^2*h*z - 1152*a^3*b^5*c^2*h*z + 128
*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z + 1152*a^2*b^6*c^2*d*z + 96*a^4*b^2*
d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*c*e*g*i - 32*a^3*b^3*d*e*g*h + 9
6*a^2*b^4*c*d*e*h - 12*a^4*b^2*e*g^2*i + 144*a^4*b^2*c*h^2*i - 48*a^3*b^3*d
^2*g*i + 16*a^4*b^2*e*g*h^2 - 108*a^4*b^2*c*g*i^2 - 108*a^2*b^4*c^2*e*i + 1
44*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 + 16*a^2*b^4*d^2*e*g - 12*a^2*b^4*c
*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*e + 108*a^5*b*e*i^3 + 108*a*b^5*
c^3*g - 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^2*i^2 + 96*a^3*b^3*d^2*h^2 + 2*a
^3*b^3*e^2*g^2 - 54*a^2*b^4*c^2*g^2 + 18*a^5*b*g^2*i^2 + 12*a^3*b^3*e^3*i -
 64*a^4*b^2*d*h^3 - 64*a^2*b^4*d^3*h + 12*a^3*b^3*c*g^3 + 18*a*b^5*c^2*e^2
+ 16*a^5*b*h^4 + 16*a*b^5*d^4 - 81*a^6*i^4 - 81*b^6*c^4 - a^4*b^2*g^4 - a^2
*b^4*e^4, z, l)*(root(65536*a^7*b^7*z^4 - 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b
^5*c*i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^5*e*g*z^2 - 3072*a^4*b^6*c*e
*z^2 - 2048*a^6*b^4*h^2*z^2 - 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z -
768*a^4*b^4*d*e*i*z + 768*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z - 1152*a^6*
b^2*h*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^3*d*i^2*z - 128*a^4*b^4*e^2*
h*z - 1152*a^3*b^5*c^2*h*z + 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z + 11
```

52*a^2*b^6*c^2*d*z + 96*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*
c*e*g*i - 32*a^3*b^3*d*e*g*h + 96*a^2*b^4*c*d*e*h - 12*a^4*b^2*e*g^2*i + 14
4*a^4*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i + 16*a^4*b^2*e*g*h^2 - 108*a^4*b^2*c
*g*i^2 - 108*a^2*b^4*c^2*e*i + 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 + 1
6*a^2*b^4*d^2*e*g - 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*
e + 108*a^5*b*e*i^3 + 108*a*b^5*c^3*g - 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^
2*i^2 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 - 54*a^2*b^4*c^2*g^2 + 18*a^
5*b*g^2*i^2 + 12*a^3*b^3*e^3*i - 64*a^4*b^2*d*h^3 - 64*a^2*b^4*d^3*h + 12*a
^3*b^3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 - 81*a^6*i^4
- 81*b^6*c^4 - a^4*b^2*g^4 - a^2*b^4*e^4, z, l)*((768*a^3*b^5*c - 256*a^4*b
^4*g)/(64*a^3*b^2) - (x*(128*a^3*b^4*d - 128*a^4*b^3*h))/(16*a^3*b)) - (64*
a^2*b^4*d*e - 192*a^3*b^3*d*i - 64*a^3*b^3*e*h + 192*a^4*b^2*h*i)/(64*a^3*b
^2) + (x*(36*a*b^4*c^2 + 36*a^4*b*i^2 + 4*a^2*b^3*e^2 + 4*a^3*b^2*g^2 - 24*
a^2*b^3*c*g - 24*a^3*b^2*e*i))/(16*a^3*b)) - (x*(2*b^3*d^3 - 2*a^3*h^3 - 3*
b^3*c*d*e + 3*a^3*g*h*i - 6*a*b^2*d^2*h + 6*a^2*b*d*h^2 + 9*a*b^2*c*d*i + 3
*a*b^2*c*e*h + a*b^2*d*e*g - 9*a^2*b*c*h*i - 3*a^2*b*d*g*i - a^2*b*e*g*h))/
(16*a^3*b))*root(65536*a^7*b^7*z^4 - 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c*
i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^5*e*g*z^2 - 3072*a^4*b^6*c*e*z^2
- 2048*a^6*b^4*h^2*z^2 - 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z - 768*a
^4*b^4*d*e*i*z + 768*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z - 1152*a^6*b^2*h
*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^3*d*i^2*z - 128*a^4*b^4*e^2*h*z -
1152*a^3*b^5*c^2*h*z + 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z + 1152*a^
2*b^6*c^2*d*z + 96*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*c*e*g
*i - 32*a^3*b^3*d*e*g*h + 96*a^2*b^4*c*d*e*h - 12*a^4*b^2*e*g^2*i + 144*a^4
*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i + 16*a^4*b^2*e*g*h^2 - 108*a^4*b^2*c*g*i^
2 - 108*a^2*b^4*c^2*e*i + 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 + 16*a^2
*b^4*d^2*e*g - 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*e + 1
08*a^5*b*e*i^3 + 108*a*b^5*c^3*g - 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^2*i^2
+ 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 - 54*a^2*b^4*c^2*g^2 + 18*a^5*b*g
^2*i^2 + 12*a^3*b^3*e^3*i - 64*a^4*b^2*d*h^3 - 64*a^2*b^4*d^3*h + 12*a^3*b^
3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 - 81*a^6*i^4 - 81*
b^6*c^4 - a^4*b^2*g^4 - a^2*b^4*e^4, z, l), l, 1, 4) + (f/(4*b) + (x*(b*c +
a*g))/(4*a*b) + (x^2*(b*d + a*h))/(4*a*b) + (x^3*(b*e + a*i))/(4*a*b))/(a
- b*x^4)

3.194 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx$

Optimal. Leaf size=225

$$\frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{4ab\left(a-bx^4\right)}-\frac{\left(be-\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}}+\frac{\left(be+\right.}{}$$

[Out] 1/4*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+(a*j+b*f)*x^3)/a/b/(-b*x^4+a)+1/4*(-a*h+b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)+1/4*j*ln(-b*x^4+a)/b^2-1/8*arctan(b^(1/4)*x/a^(1/4))*(b*e-3*a*i-(-a*g+3*b*c)*b^(1/2)/a^(1/2))/a^(5/4)/b^(7/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))*(b*e-3*a*i+(-a*g+3*b*c)*b^(1/2)/a^(1/2))/a^(5/4)/b^(7/4)

Rubi [A]
time = 0.21, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$,
Rules used = {1872, 1890, 1181, 211, 214, 1262, 649, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{j\log\left(a-bx^4\right)}{4b^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{4ab\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^2,x]

[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(4*a*b*(a - b*x^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) + (j*Log[a - b*x^4])/(4*b^2)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+194x^6+jx^7}{(a-bx^4)^2}\,dx = \frac{x(bc+ag+(bd+ah)x+(194a+be)x^2+(bf+a}{4ab\,(a-bx^4)}$$

$$= \frac{x(bc+ag+(bd+ah)x+(194a+be)x^2+(bf+a}{4ab\,(a-bx^4)}$$

$$= \frac{x(bc+ag+(bd+ah)x+(194a+be)x^2+(bf+a}{4ab\,(a-bx^4)}$$

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$$= \frac{x(bc+ag+(bd+ah)x+(194a+be)x^2+(bf+a}{4ab\,(a-bx^4)}$$

Mathematica [A]
time = 0.14, size = 338, normalized size = 1.50

$$\frac{\frac{4(a^2j+b^2x(c+x(d+ex))+ab(f+x(g+x(h+ix))))}{a(a-bx^4)}+\frac{2\sqrt[4]{b}\left(3b^{3/2}c-\sqrt{a}\,be-a\sqrt{b}\,g+3a^{3/2}i\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{\sqrt[4]{b}\left(-3b^{3/2}c-2\sqrt[4]{a}\,b^{5/4}d-\sqrt{a}\,be+a\sqrt{b}\,g+2a^{5/4}\sqrt[4]{b}\,h+3a^{3/2}i\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)}{a^{7/4}}+\frac{\sqrt[4]{b}\left(3b^{3/2}c-2\sqrt[4]{a}\,b^{5/4}d+\sqrt{a}\,be-a\sqrt{b}\,g+2a^{5/4}\sqrt[4]{b}\,h-3a^{3/2}i\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{a^{7/4}}+\frac{2\sqrt{b}\,(bd-ah)\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{a^{3/2}}+4j\log\left(a-bx^4\right)}{16b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4)^2,x]
```

```
[Out] ((4*(a^2*j + b^2*x*(c + x*(d + e*x)) + a*b*(f + x*(g + x*(h + i*x)))))/(a*(
a - b*x^4)) + (2*b^(1/4)*(3*b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/
2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(7/4) + (b^(1/4)*(-3*b^(3/2)*c - 2*a^(
1/4)*b^(5/4)*d - Sqrt[a]*b*e + a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + 3*a^(3/2
)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(7/4) + (b^(1/4)*(3*b^(3/2)*c - 2*a^(1/4)*
b^(5/4)*d + Sqrt[a]*b*e - a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*
Log[a^(1/4) + b^(1/4)*x])/a^(7/4) + (2*Sqrt[b]*(b*d - a*h)*Log[Sqrt[a] + Sq
rt[b]*x^2])/a^(3/2) + 4*j*Log[a - b*x^4])/(16*b^2)
```

Maple [A]
time = 0.36, size = 262, normalized size = 1.16

method	result

risch	$\frac{\frac{(ai+be)x^3}{4ab}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{aj+bf}{4b^2}}{-b\,x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4b-a\right)}\frac{\left(-4j\_R^3-\frac{(3ai-be)\_R^2}{a}-\frac{2(ah-bd)\_R}{a}-\frac{ag-3bc}{a}\right)\ln\left(x-\_R\right)}{\_R^3}}{16b^2}$
default	$\frac{\frac{(ai+be)x^3}{4ab}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{aj+bf}{4b^2}}{-b\,x^4+a}+\frac{\frac{(-ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(-2ah+2bd)\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\cdots}{4ba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETU
RNVERBOSE)
```

```
[Out] (1/4*(a*i+b*e)/a/b*x^3+1/4*(a*h+b*d)/a/b*x^2+1/4*(a*g+b*c)/a/b*x+1/4*(a*j+b
*f)/b^2)/(-b*x^4+a)+1/4/b/a*(1/4*(-a*g+3*b*c)*(a/b)^(1/4)/a*(ln((x+(a/b)^(1
/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))+1/4*(-2*a*h+2*b*d)/(a*b)^(1/
2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2)))-1/4*(-3*a*i+b*e)/b/(a/b)^(1/
4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4))))+a*j*ln(-b*
x^4+a)/b)
```

Maxima [A]
time = 0.50, size = 300, normalized size = 1.33

$$-\frac{(b^2e+i\,ab)x^3+abf+a^2j+(b^2d+abh)x^2+(b^2c+abg)x}{4\,(ab^3x^4-a^2b^2)}+\frac{\frac{2\left(3b^{\frac{3}{2}}c-a\sqrt{b}\,g-\sqrt{a}\,be+3i\,a^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\left(b^{\frac{3}{2}}d-a\sqrt{b}\,h+2a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{2\left(b^{\frac{3}{2}}d-a\sqrt{b}\,h-2a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(3b^{\frac{3}{2}}c-a\sqrt{b}\,g+\sqrt{a}\,be-3i\,a^{\frac{3}{2}}\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algor
ithm="maxima")
```

```
[Out] -1/4*((b^2*e + I*a*b)*x^3 + a*b*f + a^2*j + (b^2*d + a*b*h)*x^2 + (b^2*c +
a*b*g)*x)/(a*b^3*x^4 - a^2*b^2) + 1/16*(2*(3*b^(3/2)*c - a*sqrt(b)*g - sqrt
(a)*b*e + 3*I*a^(3/2))*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqr
t(sqrt(a)*sqrt(b))*sqrt(b)) + 2*(b^(3/2)*d - a*sqrt(b)*h + 2*a^(3/2)*j)*log
(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - 2*(b^(3/2)*d - a*sqrt(b)*h - 2*a^(3/2
)*j)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (3*b^(3/2)*c - a*sqrt(b)*g +
sqrt(a)*b*e - 3*I*a^(3/2))*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)
*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a*b)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,
x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(185) = 370.
time = 0.60, size = 603, normalized size = 2.68

$$-\frac{\sqrt{2}\left(3\,b^{2}c-abg-2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c-abg+2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c-abg-\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^{3}\right)^{\frac{3}{4}}a}+\frac{\sqrt{2}\left(3\,b^{2}c-abg-\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^{3}\right)^{\frac{3}{4}}a}+\frac{j\log\left(\left|bx^{4}-a\right|\right)}{4\,b^{2}}-\frac{(be+i\,a)x^{3}+(bd+ah)x^{2}+(bc+ag)x+\frac{abf+a^{2}j}{b}}{4\left(bx^{4}-a\right)ab}-\frac{3i\,\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^{4}}-\frac{3i\,\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^{4}}+\frac{3i\,\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,ab^{4}}-\frac{3i\,\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,ab^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algor
ithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - a*b*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + 2*sqrt(2)*(
-a*b^3)^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a) - 1/16*sqrt(2)*(3*b^2*c - a*b*g +
 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - 2*sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b
*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)
^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(
2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c
- a*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((
-a*b^3)^(3/4)*a) + 1/4*j*log(abs(b*x^4 - a))/b^2 - 1/4*((b*e + I*a)*x^3 + (
b*d + a*h)*x^2 + (b*c + a*g)*x + (a*b*f + a^2*j)/b)/((b*x^4 - a)*a*b) - 3/1
6*I*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/
(-a/b)^(1/4))/(a*b^4) - 3/16*I*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^4) + 3/32*I*sqrt(2)*(-a*b^3)^
(3/4)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^4) - 3/32*I*sqrt(
2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^4)
```

Mupad [**B**]
time = 5.91, size = 2500, normalized size = 11.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^2
,x)
```

```
[Out] ((b*f + a*j)/(4*b^2) + (x*(b*c + a*g))/(4*a*b) + (x^2*(b*d + a*h))/(4*a*b)
+ (x^3*(b*e + a*i))/(4*a*b))/(a - b*x^4) + symsum(log((27*a^4*i^3 - a*b^3*e
^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e + 16*a^4*g*j^2 - 12*a^2*b^2*c*h^2 + a^2*b^2
*e*g^2 + 9*a^2*b^2*e^2*i - 48*a^4*h*i*j + 4*a*b^3*d^2*g - 27*a*b^3*c^2*i -
48*a^3*b*c*j^2 - 27*a^3*b*e*i^2 + 4*a^3*b*g*h^2 - 3*a^3*b*g^2*i + 18*a^2*b^
2*c*g*i - 16*a^2*b^2*d*e*j - 8*a^2*b^2*d*g*h + 24*a*b^3*c*d*h - 6*a*b^3*c*e
*g + 48*a^3*b*d*i*j + 16*a^3*b*e*h*j)/(64*a^3*b^2) - root(65536*a^7*b^8*z^4
 - 65536*a^7*b^6*j*z^3 - 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2 + 4096
*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 - 3072*a^4*b^7*c*e*z^2 + 24576*a^7*
b^4*j^2*z^2 - 2048*a^6*b^5*h^2*z^2 - 2048*a^4*b^7*d^2*z^2 + 1536*a^6*b^3*g*
i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^5*b^4*e*h*i*z -
 512*a^5*b^4*e*g*j*z + 1536*a^4*b^5*c*e*j*z - 768*a^4*b^5*d*e*i*z + 768*a^4
*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z + 1024*a^6*b^3*h^2*j*z - 1152*a^6*b^3*h*
i^2*z - 128*a^5*b^4*g^2*h*z + 1024*a^4*b^5*d^2*j*z + 1152*a^5*b^4*d*i^2*z -
 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z + 128*a^4*b^5*d*g^2*z + 128*a^3
*b^6*d*e^2*z + 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 192*a^5*b^2*e*h*
i*j + 192*a^4*b^3*d*e*i*j - 192*a^4*b^3*c*g*h*j + 96*a^4*b^3*d*g*h*i - 288*
a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i - 32*a^3*b^4*d*e
*g*h + 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*g*h^2*i - 288*a
^5*b^2*d*i^2*j + 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 + 256*a^5*b^2*d*h
*j^2 + 64*a^5*b^2*e*g*j^2 + 288*a^3*b^4*c^2*h*j - 32*a^4*b^3*d*g^2*j - 12*a
^4*b^3*e*g^2*i + 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i + 16*a^4*b^3*e*g*
h^2 - 108*a^4*b^3*c*g*i^2 - 32*a^3*b^4*d*e^2*j - 192*a^4*b^3*c*e*j^2 - 288*
a^2*b^5*c^2*d*j - 108*a^2*b^5*c^2*e*i + 144*a^2*b^5*c*d^2*i - 48*a^3*b^4*c*
e*h^2 + 16*a^2*b^5*d^2*e*g - 12*a^2*b^5*c*e^2*g + 288*a^6*b*h*i^2*j - 192*a
^6*b*g*i*j^2 - 48*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b^2*g^2*i^2 - 12
8*a^4*b^3*d^2*j^2 - 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2 + 96*a^3*b^4*d
^2*h^2 + 2*a^3*b^4*e^2*g^2 - 54*a^2*b^5*c^2*g^2 - 128*a^6*b*h^2*j^2 + 108*a
^5*b^2*e*i^3 + 12*a^3*b^4*e^3*i - 64*a^4*b^3*d*h^3 - 64*a^2*b^5*d^3*h + 12*
a^3*b^4*c*g^3 + 18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 - 81*a^6*b*i^4 + 16*a*b^6
*d^4 + 256*a^7*j^4 - 81*b^7*c^4 - a^4*b^3*g^4 - a^2*b^5*e^4, z, m)*(root(65
536*a^7*b^8*z^4 - 65536*a^7*b^6*j*z^3 - 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6
*c*i*z^2 + 4096*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 - 3072*a^4*b^7*c*e*z
^2 + 24576*a^7*b^4*j^2*z^2 - 2048*a^6*b^5*h^2*z^2 - 2048*a^4*b^7*d^2*z^2 +
1536*a^6*b^3*g*i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^
5*b^4*e*h*i*z - 512*a^5*b^4*e*g*j*z + 1536*a^4*b^5*c*e*j*z - 768*a^4*b^5*d*
e*i*z + 768*a^4*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z + 1024*a^6*b^3*h^2*j*z -
1152*a^6*b^3*h*i^2*z - 128*a^5*b^4*g^2*h*z + 1024*a^4*b^5*d^2*j*z + 1152*a^
5*b^4*d*i^2*z - 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z + 128*a^4*b^5*d*
g^2*z + 128*a^3*b^6*d*e^2*z + 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 1
```

92*a^5*b^2*e*h*i*j + 192*a^4*b^3*d*e*i*j - 192*a^4*b^3*c*g*h*j + 96*a^4*b^3
*d*g*h*i - 288*a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i -
32*a^3*b^4*d*e*g*h + 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*
g*h^2*i - 288*a^5*b^2*d*i^2*j + 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 +
256*a^5*b^2*d*h*j^2 + 64*a^5*b^2*e*g*j^2 + 288*a^3*b^4*c^2*h*j - 32*a^4*b^3
*d*g^2*j - 12*a^4*b^3*e*g^2*i + 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i +
16*a^4*b^3*e*g*h^2 - 108*a^4*b^3*c*g*i^2 - 32*a^3*b^4*d*e^2*j - 192*a^4*b^3
*c*e*j^2 - 288*a^2*b^5*c^2*d*j - 108*a^2*b^5*c^2*e*i + 144*a^2*b^5*c*d^2*i
- 48*a^3*b^4*c*e*h^2 + 16*a^2*b^5*d^2*e*g - 12*a^2*b^5*c*e^2*g + 288*a^6*b*
h*i^2*j - 192*a^6*b*g*i*j^2 - 48*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b
^2*g^2*i^2 - 128*a^4*b^3*d^2*j^2 - 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2
+ 96*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 - 54*a^2*b^5*c^2*g^2 - 128*a^6*b*
h^2*j^2 + 108*a^5*b^2*e*i^3 + 12*a^3*b^4*e^3*i - 64*a^4*b^3*d*h^3 - 64*a^2*
b^5*d^3*h + 12*a^3*b^4*c*g^3 + 18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 - 81*a^6*b
*i^4 + 16*a*b^6*d^4 + 256*a^7*j^4 - 81*b^7*c^4 - a^4*b^3*g^4 - a^2*b^5*e^4,
z, m)*((768*a^3*b^5*c - 256*a^4*b^4*g)/(64*a^3*b^2) - (x*(128*a^3*b^5*d -
128*a^4*b^4*h))/(16*a^3*b^2)) - (64*a^2*b^4*d*e + 384*a^3*b^3*c*j - 192*a^3
*b^3*d*i - 64*a^3*b^3*e*h - 128*a^4*b^2*g*j + 192*a^4*b^2*h*i)/(64*a^3*b^2)
+ (x*(36*a*b^5*c^2 + 4*a^2*b^4*e^2 + 4*a^3*b^3*g^2 + 36*a^4*b^2*i^2 - 24*a
^2*b^4*c*g + 64*a^3*b^3*d*j - 24*a^3*b^3*e*i - 64*a^4*b^2*h*j))/(16*a^3*b^2
)) + (x*(2*a^3*b*h^3 - 2*b^4*d^3 - 8*a^4*h*j^2 + 9*a^4*i^2*j - 6*a^2*b^2*d*
h^2 + a^2*b^2*e^2*j + 3*b^4*c*d*e + 6*a*b^3*d^2*h + 9*a*b^3*c^2*j + 8*a^3*b
*d*j^2 + a^3*b*g^2*j - 6*a^2*b^2*c*g*j + 9*a^2*b^2*c*h*i + 3*a^2*b^2*d*g*i
+ a^2*b^2*e*g*h - 9*a*b^3*c*d*i - 3*a*b^3*c*e*h - a*b^3*d*e*g - 6*a^3*b*e*i
*j - 3*a^3*b*g*h*i))/(16*a^3*b^2))*root(65536*a^7*b^8*z^4 - 65536*a^7*b^6*j
*z^3 - 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2 + 4096*a^5*b^6*d*h*z^2 +
1024*a^5*b^6*e*g*z^2 - 3072*a^4*b^7*c*e*z^2 + ...

3.195 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^2}\,dx$

Optimal. Leaf size=353

$$\frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}+\frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}-\frac{\left(3bc+\sqrt{a}\,\sqrt{b}\,e+ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\right.}{8\sqrt{2}\,a^{7/4}b^{5/4}}$$

```
[Out] 1/4*x*(b*c-a*g+(-a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(b*x^4+a)+1/4*(a*h+b*d)*ar
ctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*b^(1/2))*(3*b*c+a*g-e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2
)+1/32*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(3*b*c+a*g-e*a^(1/
2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4
))*(3*b*c+a*g+e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*arctan(1+b^(1
/4)*x*2^(1/2)/a^(1/4))*(3*b*c+a*g+e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2
)
```

Rubi [A]
time = 0.23, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1872, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,\sqrt{b}\,e+ag+3bc\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,\sqrt{b}\,e+ag+3bc\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)(ah+bd)}{4a^{3/2}b^{3/2}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-\sqrt{a}\,\sqrt{b}\,e+ag+3bc\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-\sqrt{a}\,\sqrt{b}\,e+ag+3bc\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (
(b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((3*b*c +
Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[
b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqr
t[2]*a^(7/4)*b^(5/4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
```

```
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^2}\,dx = \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-b(3bc+ag)-2b(bd+ah)x-b^2 \cdots}{a+bx^4} \cdots}{4ab^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\left(a+bx^4\right)} - \frac{\int \left(-\frac{2b(bd+ah)x}{a+bx^4} + \frac{-b(3bc+ \cdots}{a+ \cdots} \cdots\right.}{4ab^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-b(3bc+ag)-b^2ex^2}{a+bx^4}\,dx}{4ab^2} + \cdots$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\left(a+bx^4\right)} + \frac{\left(3bc-\sqrt{a}\,\sqrt{b}\,e+ag\right) \cdots}{8a^{3/2}b^{3/ \cdots}}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}} \cdots\right.}{4a^{3/2}b^{3/2}}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}} \cdots\right.}{4a^{3/2}b^{3/2}}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}} \cdots\right.}{4a^{3/2}b^{3/2}}$$

Mathematica [A]
time = 0.17, size = 359, normalized size = 1.02

$$\frac{-\frac{8a^{3/4}\sqrt{b}\,(-bx(c+x(d+ex))+a(f+x(g+hx)))}{a+bx^4} - 2\left(3\sqrt{2}\,b^{5/4}c+4\sqrt[4]{a}\,bd+\sqrt{2}\,\sqrt{a}\,b^{3/4}e+\sqrt{2}\,a\sqrt[4]{b}\,g+4a^{5/4}h\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) + 2\left(3\sqrt{2}\,b^{5/4}c-4\sqrt[4]{a}\,bd+\sqrt{2}\,\sqrt{a}\,b^{3/4}e+\sqrt{2}\,a\sqrt[4]{b}\,g-4a^{5/4}h\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) + \sqrt{2}\,\sqrt[4]{b}\left(-3bc+\sqrt{a}\,\sqrt{b}\,e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right) + \sqrt{2}\,\sqrt[4]{b}\left(3bc-\sqrt{a}\,\sqrt{b}\,e+ag\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{32a^{7/4}b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]

[Out] ((-8*a^(3/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(a + b*x^4) - 2*(3*Sqrt[2]*b^(5/4)*c + 4*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e + Sqrt[2]*a*b^(1/4)*g + 4*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b^(5/4)*c - 4*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e + Sqrt[2]*a*b^(1/4)*g - 4*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(7/4)*b^(3/2))

Maple [A]
time = 0.35, size = 309, normalized size = 0.88

method	result
risch	$\frac{\frac{e\,x^3}{4a}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(\_R^2 e+\frac{2(ah+bd)\_R}{b}+\frac{ag+3bc}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$\frac{\frac{e\,x^3}{4a}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\frac{(ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right.}{8a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)

[Out] (1/4/a*e*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(b*x^4+a)+1/4/b/a*(1/8*(a*g+3*b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/2*(2*a*h+2*b*d)/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*e/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))

Maxima [A]
time = 0.51, size = 379, normalized size = 1.07

$$\frac{bx^3e+(bd-ah)x^2-af+(bc-ag)x}{4\,(ab^2x^4+a^2b)}+\frac{\frac{\sqrt{2}\left(3b^{\frac{3}{2}}c+a\sqrt{b}\,g-\sqrt{a}\,be\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3b^{\frac{3}{2}}c+a\sqrt{b}\,g-\sqrt{a}\,be\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e-4\sqrt{a}\,b^{\frac{3}{2}}d-4a^{\frac{3}{2}}\sqrt{b}\,h\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e+4\sqrt{a}\,b^{\frac{3}{2}}d+4a^{\frac{3}{2}}\sqrt{b}\,h\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{32\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")

```
[Out] 1/4*(b*x^3*e + (b*d - a*h)*x^2 - a*f + (b*c - a*g)*x)/(a*b^2*x^4 + a^2*b) +
 1/32*(sqrt(2)*(3*b^(3/2)*c + a*sqrt(b)*g - sqrt(a)*b*e)*log(sqrt(b)*x^2 +
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*b^(3/2)
*c + a*sqrt(b)*g - sqrt(a)*b*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a^
(5/4)*b^(3/4)*g + sqrt(2)*a^(3/4)*b^(5/4)*e - 4*sqrt(a)*b^(3/2)*d - 4*a^(3/
2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sq
rt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)
*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(5/4)*b^(3/4)*g + sqrt(2)*a^(3/4)*b^(5/4)*e
+ 4*sqrt(a)*b^(3/2)*d + 4*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*
x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*s
qrt(b))*b^(3/4)))/(a*b)
```

Fricas [C] Result contains complex when optimal does not.
time = 207.37, size = 736847, normalized size = 2087.39

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas"
)
```

```
[Out] 1/192*(48*b*e*x^3 + 48*(b*d - a*h)*x^2 - 24*(a*b^2*x^4 + a^2*b)*(4*sqrt(-1/
64*(2*a^2*h^2 + (e*g + 4*d*h)*a*b + (2*d^2 + 3*c*e)*b^2)/(a^3*b^3) + 1/64*(
2*a^2*h^2 - (e*g - 4*d*h)*a*b + (2*d^2 - 3*c*e)*b^2)/(a^3*b^3) - 1/64*(9*b^
3*c^2*d + a^3*g^2*h + (d*g^2 - e^2*h + 6*c*g*h)*a^2*b - (d*e^2 - 6*c*d*g -
9*c^2*h)*a*b^2)/((b*d + a*h)*a^4*b^3*sqrt(-1/(a*b)))) + (b*d + a*h)*sqrt(-1
/(a*b))/(a*b))*log(1080*a*b^6*c^3*d^3 - 120*a^2*b^5*c*d^3*e^2 - 6*a^3*b^4*d
*e^5 - 10*a^5*b^2*d*e*g^4 + 320*a^6*b*d*e*h^4 + 64*a^7*e*h^5 + 40*(a^4*b^3*
d^3 - 3*a^4*b^3*c*d*e)*g^3 + 40*(27*a^4*b^3*c^3 + 16*a^5*b^2*d^2*e - 3*a^5*
b^2*c*e^2 + 9*a^6*b*c*g^2 + a^7*g^3 + (27*a^5*b^2*c^2 - a^6*b*e^2)*g)*h^3 +
 8*(24*a^6*b^7*c*d^2 - 9*a^6*b^7*c^2*e + a^7*b^6*e^3 - a^8*b^5*e*g^2 + 8*(3
*a^8*b^5*c + a^9*b^4*g)*h^2 + 2*(4*a^7*b^6*d^2 - 3*a^7*b^6*c*e)*g + 16*(3*a
^7*b^6*c*d + a^8*b^5*d*g)*h)*(4*sqrt(-1/64*(2*a^2*h^2 + (e*g + 4*d*h)*a*b +
 (2*d^2 + 3*c*e)*b^2)/(a^3*b^3) + 1/64*(2*a^2*h^2 - (e*g - 4*d*h)*a*b + (2*
d^2 - 3*c*e)*b^2)/(a^3*b^ ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.67, size = 398, normalized size = 1.13

$$\frac{bx^3e+bdx^2-ahx^2+bcx-agx-af}{4\,(bx^4+a)ab}+\frac{\sqrt{2}\left(2\,\sqrt{2}\,\sqrt{ab}\,b^2d+2\,\sqrt{2}\,\sqrt{ab}\,abh+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\left(2\,\sqrt{2}\,\sqrt{ab}\,b^2d+2\,\sqrt{2}\,\sqrt{ab}\,abh+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}-\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 - a*h*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b) +
 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a
*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(
2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqr
t(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c +
(a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a
/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (
a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b
*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3
)
```

Mupad [B]
time = 5.58, size = 1623, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log((12*b^3*c*d^2 - a*b^2*e^3 - 9*b^3*c^2*e + 4*a^3*g*h^2 + 4*a*b^2*
d^2*g + 12*a^2*b*c*h^2 - a^2*b*e*g^2 + 24*a*b^2*c*d*h - 6*a*b^2*c*e*g + 8*a
^2*b*d*g*h)/(64*a^3*b) - root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2 + 10
24*a^5*b^4*e*g*z^2 + 3072*a^4*b^5*c*e*z^2 + 2048*a^6*b^3*h^2*z^2 + 2048*a^4
*b^5*d^2*z^2 - 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2*g^2*
h*z + 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z - 128*a^4*b^3*d*g^2*z + 12
8*a^3*b^4*d*e^2*z - 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h - 96*a^2*b^3*
c*d*e*h - 48*a^3*b^2*c*e*h^2 - 16*a^2*b^3*d^2*e*g + 12*a^2*b^3*c*e^2*g - 16
*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e + 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g + 96*a
^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 54*a^2*b^3*c^2*g^2 + 64*a^2*b^3*d^3*h
+ 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 + 81*b^5*
c^4 + a^2*b^3*e^4 + a^4*b*g^4, z, k)*(root(65536*a^7*b^6*z^4 + 4096*a^5*b^4
*d*h*z^2 + 1024*a^5*b^4*e*g*z^2 + 3072*a^4*b^5*c*e*z^2 + 2048*a^6*b^3*h^2*z
^2 + 2048*a^4*b^5*d^2*z^2 - 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128
*a^5*b^2*g^2*h*z + 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z - 128*a^4*b^3
*d*g^2*z + 128*a^3*b^4*d*e^2*z - 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h
- 96*a^2*b^3*c*d*e*h - 48*a^3*b^2*c*e*h^2 - 16*a^2*b^3*d^2*e*g + 12*a^2*b^3
```

```
*c*e^2*g - 16*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e + 64*a^4*b*d*h^3 + 108*a*b^4
*c^3*g + 96*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 54*a^2*b^3*c^2*g^2 + 64*a
^2*b^3*d^3*h + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*
h^4 + 81*b^5*c^4 + a^2*b^3*e^4 + a^4*b*g^4, z, k)*((768*a^3*b^4*c + 256*a^4
*b^3*g)/(64*a^3*b) - (x*(128*a^3*b^4*d + 128*a^4*b^3*h))/(16*a^3*b)) + (64*
a^2*b^3*d*e + 64*a^3*b^2*e*h)/(64*a^3*b) + (x*(36*a*b^4*c^2 - 4*a^2*b^3*e^2
 + 4*a^3*b^2*g^2 + 24*a^2*b^3*c*g))/(16*a^3*b)) + (x*(2*b^3*d^3 + 2*a^3*h^3
 - 3*b^3*c*d*e + 6*a*b^2*d^2*h + 6*a^2*b*d*h^2 - 3*a*b^2*c*e*h - a*b^2*d*e*
g - a^2*b*e*g*h))/(16*a^3*b))*root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2
 + 1024*a^5*b^4*e*g*z^2 + 3072*a^4*b^5*c*e*z^2 + 2048*a^6*b^3*h^2*z^2 + 204
8*a^4*b^5*d^2*z^2 - 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2
*g^2*h*z + 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z - 128*a^4*b^3*d*g^2*z
 + 128*a^3*b^4*d*e^2*z - 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h - 96*a^2
*b^3*c*d*e*h - 48*a^3*b^2*c*e*h^2 - 16*a^2*b^3*d^2*e*g + 12*a^2*b^3*c*e^2*g
 - 16*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e + 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g +
 96*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 54*a^2*b^3*c^2*g^2 + 64*a^2*b^3*d
^3*h + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 + 81
*b^5*c^4 + a^2*b^3*e^4 + a^4*b*g^4, z, k), k, 1, 4) + ((e*x^3)/(4*a) - f/(4
*b) + (x*(b*c - a*g))/(4*a*b) + (x^2*(b*d - a*h))/(4*a*b))/(a + b*x^4)
```

3.196 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2} \, dx$

Optimal. Leaf size=395

$$\frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{4ab\,(a+bx^4)}+\frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}-\frac{\left(\sqrt{b}\,(3bc+ag)+\sqrt{a}\,(be+3a\right.}{8\sqrt{2}\,a^{7/}}$$

```
[Out] 1/4*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+b*f*x^3)/a/b/(b*x^4+a)+1/4*(a*h+
b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-1/32*ln(-a^(1/4)*b^(1/4)*x
*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^
(7/4)/b^(7/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2)
)*(-(3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+1/16*a
rctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2
))/a^(7/4)/b^(7/4)*2^(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i
+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1872, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{b}\,(ag+3bc)+\sqrt{a}\,(3ai+be)\right)}{8\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}\,(ag+3bc)+\sqrt{a}\,(3ai+be)\right)}{8\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)(ah+bd)}{4a^{3/2}b^{3/2}}-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(\sqrt{b}\,(ag+3bc)-\sqrt{a}\,(3ai+be)\right)}{16\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(\sqrt{b}\,(ag+3bc)-\sqrt{a}\,(3ai+be)\right)}{16\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{x(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3)}{4ab\,(a+bx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(4*a*b*(a + b*x
^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((
Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a
]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4
)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + (
(Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+196x^6}{(a+bx^4)^2}\,dx = \frac{x(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \frac{-b}{}}{}$$

$$= \frac{x(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \Big(-}{}$$

$$= \frac{x(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \frac{-b}{}}{}$$

$$= \frac{x(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\Big(58}{}$$

$$= \frac{x(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3)}{4ab\,(a+bx^4)} + \frac{(bd-}{}$$

$$= \frac{x(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3)}{4ab\,(a+bx^4)} + \frac{(bd-}{}$$

$$= \frac{x(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3)}{4ab\,(a+bx^4)} + \frac{(bd-}{}$$

Mathematica [A]

time = 0.23, size = 415, normalized size = 1.05

$$\frac{-\frac{8a^{3/4}b^{3/4}(-bx(c+x(d+ex))+a(f+x(g+x(h+ix))))}{a+bx^4}-2\left(3\sqrt{2}\,b^{3/2}c+4\sqrt[4]{a}\,b^{5/4}d+\sqrt{2}\,\sqrt{a}\,be+\sqrt{2}\,a\sqrt{b}\,g+4a^{5/4}\sqrt[4]{b}\,h+3\sqrt{2}\,a^{3/2}i\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\left(3\sqrt{2}\,b^{3/2}c-4\sqrt[4]{a}\,b^{5/4}d+\sqrt{2}\,\sqrt{a}\,be+\sqrt{2}\,a\sqrt{b}\,g-4a^{5/4}\sqrt[4]{b}\,h+3\sqrt{2}\,a^{3/2}i\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\sqrt{2}\left(-3b^{3/2}c+\sqrt{a}\,be-a\sqrt{b}\,g+3a^{3/2}i\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+\sqrt{2}\left(3b^{3/2}c-\sqrt{a}\,be+a\sqrt{b}\,g-3a^{3/2}i\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{32a^{7/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x
]
```

```
[Out] ((-8*a^(3/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + x*(h + i*x))
)))/(a + b*x^4) - 2*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sq
rt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b^(3/2)*c - 4*a^(1
/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g - 4*a^(5/4)*b^(1/
4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[
2]*(-3*b^(3/2)*c + Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b^(3/2)*c - Sqrt[a]*b*
e + a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/(32*a^(7/4)*b^(7/4))
```

Maple [A]
time = 0.35, size = 329, normalized size = 0.83

method	result
risch	$\frac{-\frac{(ai-be)x^3}{4ab}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\sum_{\_R=\text{RootOf}(\_Z^4 b+a)}\frac{\left((3ai+be)\_R^2+2(ah+bd)\_R+ag+3bc\right)\ln(x-\_R)}{\_R^3}}{16a\,b^2}$
default	$\frac{-\frac{(ai-be)x^3}{4ab}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\frac{(ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{b}\right)}\right.\right.}{8a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] (-1/4*(a*i-b*e)/a/b*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/
(b*x^4+a)+1/4/b/a*(1/8*(a*g+3*b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/
4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan
(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/2*(2*a*h+2*b
*d)/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*(3*a*i+b*e)/b/(a/b)^(1/4)*2^(1/
2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(
a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)
*x-1)))
```

Maxima [A]
time = 0.51, size = 416, normalized size = 1.05

$$\frac{(be-i\,a)x^3+(bd-ah)x^2-af+(bc-ag)x}{4\,(ab^2x^4+a^2b)}+\frac{\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c+a\sqrt{b}\,g-\sqrt{a}\,be-3i\,a^{\frac{3}{2}}\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c+a\sqrt{b}\,g-\sqrt{a}\,be-3i\,a^{\frac{3}{2}}\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e-4\sqrt{a}\,b^{\frac{3}{2}}d-4a^{\frac{3}{2}}\sqrt{b}\,h+3i\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{1}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e+4\sqrt{a}\,b^{\frac{3}{2}}d+4a^{\frac{3}{2}}\sqrt{b}\,h+3i\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{1}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{32\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="m
axima")
```

```
[Out] 1/4*((b*e - I*a)*x^3 + (b*d - a*h)*x^2 - a*f + (b*c - a*g)*x)/(a*b^2*x^4 +
a^2*b) + 1/32*(sqrt(2)*(3*b^(3/2)*c + a*sqrt(b)*g - sqrt(a)*b*e - 3*I*a^(3/
2))*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)
) - sqrt(2)*(3*b^(3/2)*c + a*sqrt(b)*g - sqrt(a)*b*e - 3*I*a^(3/2))*log(sqr
t(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sq
rt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(5/4)*b^(3/4)*g + sqrt(2)*a^(3/4)*b^(5/
4)*e - 4*sqrt(a)*b^(3/2)*d - 4*a^(3/2)*sqrt(b)*h + 3*I*sqrt(2)*a^(7/4)*b^(1
/4))*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a
)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*
b^(7/4)*c + sqrt(2)*a^(5/4)*b^(3/4)*g + sqrt(2)*a^(3/4)*b^(5/4)*e + 4*sqrt(
a)*b^(3/2)*d + 4*a^(3/2)*sqrt(b)*h + 3*I*sqrt(2)*a^(7/4)*b^(1/4))*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a*b)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.72, size = 582, normalized size = 1.47

$$\frac{bx^3e+bdx^2-ahx^2-iax^3+bcx-agx-af}{4\,(bx^4+a)ab}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2d+2\sqrt{2}\sqrt{ab}\,abh+3\,(ab^3)^{\frac{1}{4}}b^2c+(ab^3)^{\frac{1}{4}}abg+(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2d+2\sqrt{2}\sqrt{ab}\,abh+3\,(ab^3)^{\frac{1}{4}}b^2c+(ab^3)^{\frac{1}{4}}abg+(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}b^2c+(ab^3)^{\frac{1}{4}}abg-(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}-\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}b^2c+(ab^3)^{\frac{1}{4}}abg-(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}+\frac{3i\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4}+\frac{3i\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4}-\frac{3i\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^4}+\frac{3i\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="g
iac")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 - a*h*x^2 - I*a*x^3 + b*c*x - a*g*x - a*f)/((b*x^4 +
 a)*a*b) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*
b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan
(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt
(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4
)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)
*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)
^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(a^2*b^3) + 3/16*I*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2
)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 3/16*I*sqrt(2)*(a*b^3)^(3/4)*arctan(1
/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) - 3/32*I*sqrt(2
)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) + 3/32
*I*sqrt(2)*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^
4)
```

Mupad [B]
time = 5.70, size = 2605, normalized size = 6.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log(- root(65536*a^7*b^7*z^4 + 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c
*i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^5*e*g*z^2 + 3072*a^4*b^6*c*e*z^2
 + 2048*a^6*b^4*h^2*z^2 + 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z + 768*
a^4*b^4*d*e*i*z - 768*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z + 1152*a^6*b^2*
h*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^3*d*i^2*z + 128*a^4*b^4*e^2*h*z
- 1152*a^3*b^5*c^2*h*z - 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z - 1152*a
^2*b^6*c^2*d*z - 96*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*c*e*
g*i - 32*a^3*b^3*d*e*g*h - 96*a^2*b^4*c*d*e*h + 12*a^4*b^2*e*g^2*i - 144*a^
4*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i - 16*a^4*b^2*e*g*h^2 + 108*a^4*b^2*c*g*i
^2 + 108*a^2*b^4*c^2*e*i - 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 - 16*a^
2*b^4*d^2*e*g + 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*e +
```

108*a^5*b*e*i^3 + 108*a*b^5*c^3*g + 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^2*i^
2 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 54*a^2*b^4*c^2*g^2 + 18*a^5*b*
g^2*i^2 + 12*a^3*b^3*e^3*i + 64*a^4*b^2*d*h^3 + 64*a^2*b^4*d^3*h + 12*a^3*b
^3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 + 81*a^6*i^4 + 81
*b^6*c^4 + a^4*b^2*g^4 + a^2*b^4*e^4, z, l)*(root(65536*a^7*b^7*z^4 + 3072*
a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c*i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^
5*e*g*z^2 + 3072*a^4*b^6*c*e*z^2 + 2048*a^6*b^4*h^2*z^2 + 2048*a^4*b^6*d^2*
z^2 + 768*a^5*b^3*e*h*i*z + 768*a^4*b^4*d*e*i*z - 768*a^4*b^4*c*g*h*z - 768
*a^3*b^5*c*d*g*z + 1152*a^6*b^2*h*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^
3*d*i^2*z + 128*a^4*b^4*e^2*h*z - 1152*a^3*b^5*c^2*h*z - 128*a^4*b^4*d*g^2*
z + 128*a^3*b^5*d*e^2*z - 1152*a^2*b^6*c^2*d*z - 96*a^4*b^2*d*g*h*i - 288*a
^3*b^3*c*d*h*i + 72*a^3*b^3*c*e*g*i - 32*a^3*b^3*d*e*g*h - 96*a^2*b^4*c*d*e
*h + 12*a^4*b^2*e*g^2*i - 144*a^4*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i - 16*a^4
*b^2*e*g*h^2 + 108*a^4*b^2*c*g*i^2 + 108*a^2*b^4*c^2*e*i - 144*a^2*b^4*c*d^
2*i - 48*a^3*b^3*c*e*h^2 - 16*a^2*b^4*d^2*e*g + 12*a^2*b^4*c*e^2*g - 48*a^5
*b*g*h^2*i - 48*a*b^5*c*d^2*e + 108*a^5*b*e*i^3 + 108*a*b^5*c^3*g + 54*a^4*
b^2*e^2*i^2 + 162*a^3*b^3*c^2*i^2 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2
+ 54*a^2*b^4*c^2*g^2 + 18*a^5*b*g^2*i^2 + 12*a^3*b^3*e^3*i + 64*a^4*b^2*d*h
^3 + 64*a^2*b^4*d^3*h + 12*a^3*b^3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4
+ 16*a*b^5*d^4 + 81*a^6*i^4 + 81*b^6*c^4 + a^4*b^2*g^4 + a^2*b^4*e^4, z, l)
*((768*a^3*b^5*c + 256*a^4*b^4*g)/(64*a^3*b^2) - (x*(128*a^3*b^4*d + 128*a^
4*b^3*h))/(16*a^3*b)) + (64*a^2*b^4*d*e + 192*a^3*b^3*d*i + 64*a^3*b^3*e*h
+ 192*a^4*b^2*h*i)/(64*a^3*b^2) + (x*(36*a*b^4*c^2 - 36*a^4*b*i^2 - 4*a^2*b
^3*e^2 + 4*a^3*b^2*g^2 + 24*a^2*b^3*c*g - 24*a^3*b^2*e*i))/(16*a^3*b)) - (2
7*a^4*i^3 + a*b^3*e^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e - 12*a^2*b^2*c*h^2 + a^2
*b^2*e*g^2 + 9*a^2*b^2*e^2*i - 4*a*b^3*d^2*g + 27*a*b^3*c^2*i + 27*a^3*b*e*
i^2 - 4*a^3*b*g*h^2 + 3*a^3*b*g^2*i + 18*a^2*b^2*c*g*i - 8*a^2*b^2*d*g*h -
24*a*b^3*c*d*h + 6*a*b^3*c*e*g)/(64*a^3*b^2) - (x*(3*b^3*c*d*e - 2*a^3*h^3
- 2*b^3*d^3 + 3*a^3*g*h*i - 6*a*b^2*d^2*h - 6*a^2*b*d*h^2 + 9*a*b^2*c*d*i +
3*a*b^2*c*e*h + a*b^2*d*e*g + 9*a^2*b*c*h*i + 3*a^2*b*d*g*i + a^2*b*e*g*h)
)/(16*a^3*b))*root(65536*a^7*b^7*z^4 + 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*
c*i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^5*e*g*z^2 + 3072*a^4*b^6*c*e*z^
2 + 2048*a^6*b^4*h^2*z^2 + 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z + 768
*a^4*b^4*d*e*i*z - 768*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z + 1152*a^6*b^2
*h*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^3*d*i^2*z + 128*a^4*b^4*e^2*h*z
- 1152*a^3*b^5*c^2*h*z - 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z - 1152*
a^2*b^6*c^2*d*z - 96*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*c*e
*g*i - 32*a^3*b^3*d*e*g*h - 96*a^2*b^4*c*d*e*h + 12*a^4*b^2*e*g^2*i - 144*a
^4*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i - 16*a^4*b^2*e*g*h^2 + 108*a^4*b^2*c*g*
i^2 + 108*a^2*b^4*c^2*e*i - 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 - 16*a
^2*b^4*d^2*e*g + 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*e +
108*a^5*b*e*i^3 + 108*a*b^5*c^3*g + 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^2*i
^2 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 54*a^2*b^4*c^2*g^2 + 18*a^5*b
*g^2*i^2 + 12*a^3*b^3*e^3*i + 64*a^4*b^2*d*h^3 + 64*a^2*b^4*d^3*h + 12*a^3*
b^3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 + 81*a^6*i^4 + 8

```
1*b^6*c^4 + a^4*b^2*g^4 + a^2*b^4*e^4, z, l), l, 1, 4) + ((x*(b*c - a*g))/(
4*a*b) - f/(4*b) + (x^2*(b*d - a*h))/(4*a*b) + (x^3*(b*e - a*i))/(4*a*b))/(
a + b*x^4)
```

3.197 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a+bx^4\right)^2} \, dx$

Optimal. Leaf size=417

$$\frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{4ab\left(a+bx^4\right)}+\frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}-\frac{\left(\sqrt{b}\,(3bc+ag)+\sqrt{a}\,(\cdots\right.}{8\cdots}$$

```
[Out] 1/4*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+(-a*j+b*f)*x^3)/a/b/(b*x^4+a)+1/
4*(a*h+b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)+1/4*j*ln(b*x^4+a)/b
^2-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+b*e)*a^
(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*
x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a
^(7/4)/b^(7/4)*2^(1/2)+1/16*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i+b*
e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+1/16*arctan(1+b^(1/
4)*x*2^(1/2)/a^(1/4))*((3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(
7/4)*2^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1872, 1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{b}\,(ag+3bc)+\sqrt{a}\,(3ai+be)\right)}{8\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}\,(ag+3bc)+\sqrt{a}\,(3ai+be)\right)}{8\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)(ah+bd)}{4a^{3/2}b^{3/2}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(\sqrt{b}\,(ag+3bc)-\sqrt{a}\,(3ai+be)\right)}{16\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(\sqrt{b}\,(ag+3bc)-\sqrt{a}\,(3ai+be)\right)}{16\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{j\log\left(a+bx^4\right)}{4b^2}+\frac{x(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^2
,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(4*a*b*
(a + b*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/
2)) - ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g)
+ Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7
/4)) + ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + (j*Log[
a + b*x^4])/(4*b^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+197x^6+jx^7}{(a+bx^4)^2}\,dx = \frac{x(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj}{4ab\,(a+bx^4)}$$

$$= \frac{x(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj}{4ab\,(a+bx^4)}$$

$$= \frac{x(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj}{4ab\,(a+bx^4)}$$

$$= \frac{x(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj}{4ab\,(a+bx^4)}$$

$$= \frac{x(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj}{4ab\,(a+bx^4)}$$

$$= \frac{x(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj}{4ab\,(a+bx^4)}$$

$$= \frac{x(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj}{4ab\,(a+bx^4)}$$

<u>Mathematica</u> [A]

time = 0.21, size = 460, normalized size = 1.10

$$\frac{\frac{8(a^2j+b^2x(c+x(d+ex))-ab(f+x(g+x(h+ix))))}{a(a+bx^4)} - \frac{2\sqrt[4]{b}\left(3\sqrt{2}\,b^{3/2}c+4\sqrt[4]{a}\,b^{5/4}d+\sqrt{2}\,\sqrt{a}\,be+\sqrt{2}\,a\sqrt{b}\,g+4a^{5/4}\sqrt[4]{b}\,h+3\sqrt{2}\,a^{3/2}i\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{2\sqrt[4]{b}\left(3\sqrt{2}\,b^{3/2}c-4\sqrt[4]{a}\,b^{5/4}d+\sqrt{2}\,\sqrt{a}\,be+\sqrt{2}\,a\sqrt{b}\,g-4a^{5/4}\sqrt[4]{b}\,h+3\sqrt{2}\,a^{3/2}i\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{\sqrt{2}\sqrt[4]{b}\left(-3b^{3/2}c+\sqrt{a}\,be-a\sqrt{b}\,g+3a^{3/2}i\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}} + \frac{\sqrt{2}\sqrt[4]{b}\left(3b^{3/2}c-\sqrt{a}\,be+a\sqrt{b}\,g-3a^{3/2}i\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}} + 8j\log\left(a+bx^4\right)}{32b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4)^2,x]
```

```
[Out] ((8*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x)))))/(a*(
a + b*x^4)) - (2*b^(1/4)*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/4)*b^(5/4)*d + Sqrt[
2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3
/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*b^(1/4)*(3*Sqr
t[2]*b^(3/2)*c - 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt
[b]*g - 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]*b^(1/4)*(-3*b^(3/2)*c + Sqrt[a]*b*e - a
*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
```

```
*x^2])/a^(7/4) + (Sqrt[2]*b^(1/4)*(3*b^(3/2)*c - Sqrt[a]*b*e + a*Sqrt[b]*g
- 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7
/4) + 8*j*Log[a + b*x^4])/(32*b^2)
```

Maple [A]
time = 0.36, size = 350, normalized size = 0.84

method	result
risch	$\frac{-\frac{(ai-be)x^3}{4ab}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}+\frac{aj-bf}{4b^2}}{b\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{\left(4j\_R^3+\frac{(3ai+be)\_R^2}{a}+\frac{2(ah+bd)\_R}{a}+\frac{ag+3bc}{a}\right)\ln(x-\_}{\_R^3}}{16b^2}$
default	$\frac{-\frac{(ai-be)x^3}{4ab}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}+\frac{aj-bf}{4b^2}}{b\,x^4+a}+\frac{\frac{(ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{8a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETUR
NVERBOSE)
```

```
[Out] (-1/4*(a*i-b*e)/a/b*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x+1/4*(a*j-
b*f)/b^2)/(b*x^4+a)+1/4/b/a*(1/8*(a*g+3*b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2
+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))
)+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/2*
(2*a*h+2*b*d)/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*(3*a*i+b*e)/b/(a/b)^(
1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x
*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(
a/b)^(1/4)*x-1))+a*j*ln(b*x^4+a)/b)
```

Maxima [A]
time = 0.53, size = 458, normalized size = 1.10

$$\frac{(b^2e-i\,ab)x^3-abf+a^2j+(b^2d-abh)x^2+(b^2c-abg)x}{4\,(ab^3x^4+a^2b^2)}+\frac{\frac{\sqrt{2}\left(4\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{1}{4}}j+3b^2c+abg-\sqrt{a}\,b^{\frac{3}{2}}e-3i\,a^{\frac{3}{2}}\sqrt{b}\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(4\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{1}{4}}j-3b^2c-abg+\sqrt{a}\,b^{\frac{3}{2}}e+3i\,a^{\frac{3}{2}}\sqrt{b}\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{9}{4}}c+\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}g+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}e-4\sqrt{a}\,b^2d-4a^{\frac{3}{2}}bh+3i\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{9}{4}}c+\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}g+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}e+4\sqrt{a}\,b^2d+4a^{\frac{3}{2}}bh+3i\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}}{32\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algori
thm="maxima")
```

```
[Out] 1/4*((b^2*e - I*a*b)*x^3 - a*b*f + a^2*j + (b^2*d - a*b*h)*x^2 + (b^2*c - a
*b*g)*x)/(a*b^3*x^4 + a^2*b^2) + 1/32*(sqrt(2)*(4*sqrt(2)*a^(7/4)*b^(1/4)*j
 + 3*b^2*c + a*b*g - sqrt(a)*b^(3/2)*e - 3*I*a^(3/2)*sqrt(b))*log(sqrt(b)*x
^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(4*sq
rt(2)*a^(7/4)*b^(1/4)*j - 3*b^2*c - a*b*g + sqrt(a)*b^(3/2)*e + 3*I*a^(3/2)
*sqrt(b))*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b
```

```
^(5/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(5/4)*b^(5/4)*g + sqrt
(2)*a^(3/4)*b^(7/4)*e - 4*sqrt(a)*b^2*d - 4*a^(3/2)*b*h + 3*I*sqrt(2)*a^(7/
4)*b^(3/4))*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt
(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(3*sqrt(2)*a
^(1/4)*b^(9/4)*c + sqrt(2)*a^(5/4)*b^(5/4)*g + sqrt(2)*a^(3/4)*b^(7/4)*e +
4*sqrt(a)*b^2*d + 4*a^(3/2)*b*h + 3*I*sqrt(2)*a^(7/4)*b^(3/4))*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3
/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))/(a*b)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x
)
```

```
[Out] Timed out
```

Giac [A]
time = 0.64, size = 608, normalized size = 1.46

$$\frac{j\log\left(|bx^4+a|\right)}{4\,b^2}+\frac{(be-i\,a)x^3+(bd-ah)x^2+(bc-ag)x-\frac{abf-a^2j}{b}}{4\,(bx^4+a)ab}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2d+2\sqrt{2}\sqrt{ab}\,abh+3\,(ab^3)^{\frac{1}{4}}b^2c+(ab^3)^{\frac{1}{4}}abg+(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2d+2\sqrt{2}\sqrt{ab}\,abh+3\,(ab^3)^{\frac{1}{4}}b^2c+(ab^3)^{\frac{1}{4}}abg+(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}b^2c+(ab^3)^{\frac{1}{4}}abg-(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}-\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}b^2c+(ab^3)^{\frac{1}{4}}abg-(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}+\frac{3i\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4}+\frac{3i\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4}-\frac{3i\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^4}+\frac{3i\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algori
thm="giac")
```

```
[Out] 1/4*j*log(abs(b*x^4 + a))/b^2 + 1/4*((b*e - I*a)*x^3 + (b*d - a*h)*x^2 + (b
*c - a*g)*x - (a*b*f - a^2*j)/b)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*sqrt(2
)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*
b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)
^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d +
2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (
```

```
a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))
/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a
*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/3
2*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*l
og(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) + 3/16*I*sqrt(2)*(a*b
^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^
4) + 3/16*I*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(
1/4))/(a/b)^(1/4))/(a*b^4) - 3/32*I*sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)
*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) + 3/32*I*sqrt(2)*(a*b^3)^(3/4)*log(x^2
- sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)
```

Mupad [B]
time = 5.84, size = 2500, normalized size = 6.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^2
,x)
```

```
[Out] ((x*(b*c - a*g))/(4*a*b) - (b*f - a*j)/(4*b^2) + (x^2*(b*d - a*h))/(4*a*b)
+ (x^3*(b*e - a*i))/(4*a*b))/(a + b*x^4) + symsum(log(- root(65536*a^7*b^8*
z^4 - 65536*a^7*b^6*j*z^3 + 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2 + 4
096*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 + 3072*a^4*b^7*c*e*z^2 + 24576*a
^7*b^4*j^2*z^2 + 2048*a^6*b^5*h^2*z^2 + 2048*a^4*b^7*d^2*z^2 - 1536*a^6*b^3
*g*i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^5*b^4*e*h*i*
z - 512*a^5*b^4*e*g*j*z - 1536*a^4*b^5*c*e*j*z + 768*a^4*b^5*d*e*i*z - 768*
a^4*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z - 1024*a^6*b^3*h^2*j*z + 1152*a^6*b^3
*h*i^2*z - 128*a^5*b^4*g^2*h*z - 1024*a^4*b^5*d^2*j*z + 1152*a^5*b^4*d*i^2*
z + 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z - 128*a^4*b^5*d*g^2*z + 128*
a^3*b^6*d*e^2*z - 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 192*a^5*b^2*e
*h*i*j - 192*a^4*b^3*d*e*i*j + 192*a^4*b^3*c*g*h*j - 96*a^4*b^3*d*g*h*i - 2
88*a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i - 32*a^3*b^4*
d*e*g*h - 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*g*h^2*i - 28
8*a^5*b^2*d*i^2*j - 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 + 256*a^5*b^2*
d*h*j^2 + 64*a^5*b^2*e*g*j^2 + 288*a^3*b^4*c^2*h*j + 32*a^4*b^3*d*g^2*j + 1
2*a^4*b^3*e*g^2*i - 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i - 16*a^4*b^3*e
*g*h^2 + 108*a^4*b^3*c*g*i^2 - 32*a^3*b^4*d*e^2*j + 192*a^4*b^3*c*e*j^2 + 2
88*a^2*b^5*c^2*d*j + 108*a^2*b^5*c^2*e*i - 144*a^2*b^5*c*d^2*i - 48*a^3*b^4
*c*e*h^2 - 16*a^2*b^5*d^2*e*g + 12*a^2*b^5*c*e^2*g - 288*a^6*b*h*i^2*j + 19
2*a^6*b*g*i*j^2 - 48*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b^2*g^2*i^2 +
 128*a^4*b^3*d^2*j^2 + 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2 + 96*a^3*b^
4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 54*a^2*b^5*c^2*g^2 + 128*a^6*b*h^2*j^2 + 10
8*a^5*b^2*e*i^3 + 12*a^3*b^4*e^3*i + 64*a^4*b^3*d*h^3 + 64*a^2*b^5*d^3*h +
12*a^3*b^4*c*g^3 + 18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 + 81*a^6*b*i^4 + 16*a*
```

b^6*d^4 + 256*a^7*j^4 + 81*b^7*c^4 + a^4*b^3*g^4 + a^2*b^5*e^4, z, m)*(root (65536*a^7*b^8*z^4 - 65536*a^7*b^6*j*z^3 + 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2 + 4096*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 + 3072*a^4*b^7*c*e*z^2 + 24576*a^7*b^4*j^2*z^2 + 2048*a^6*b^5*h^2*z^2 + 2048*a^4*b^7*d^2*z^2 - 1536*a^6*b^3*g*i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^5*b^4*e*h*i*z - 512*a^5*b^4*e*g*j*z - 1536*a^4*b^5*c*e*j*z + 768*a^4*b^5*d*e*i*z - 768*a^4*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z - 1024*a^6*b^3*h^2*j*z + 1152*a^6*b^3*h*i^2*z - 128*a^5*b^4*g^2*h*z - 1024*a^4*b^5*d^2*j*z + 1152*a^5*b^4*d*i^2*z + 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z - 128*a^4*b^5*d*g^2*z + 128*a^3*b^6*d*e^2*z - 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 192*a^5*b^2*e*h*i*j - 192*a^4*b^3*d*e*i*j + 192*a^4*b^3*c*g*h*j - 96*a^4*b^3*d*g*h*i - 288*a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i - 32*a^3*b^4*d*e*g*h - 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*g*h^2*i - 288*a^5*b^2*d*i^2*j - 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 + 256*a^5*b^2*d*h*j^2 + 64*a^5*b^2*e*g*j^2 + 288*a^3*b^4*c^2*h*j + 32*a^4*b^3*d*g^2*j + 12*a^4*b^3*e*g^2*i - 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i - 16*a^4*b^3*e*g*h^2 + 108*a^4*b^3*c*g*i^2 - 32*a^3*b^4*d*e^2*j + 192*a^4*b^3*c*e*j^2 + 288*a^2*b^5*c^2*d*j + 108*a^2*b^5*c^2*e*i - 144*a^2*b^5*c*d^2*i - 48*a^3*b^4*c*e*h^2 - 16*a^2*b^5*d^2*e*g + 12*a^2*b^5*c*e^2*g - 288*a^6*b*h*i^2*j + 192*a^6*b*g*i*j^2 - 48*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b^2*g^2*i^2 + 128*a^4*b^3*d^2*j^2 + 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2 + 96*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 54*a^2*b^5*c^2*g^2 + 128*a^6*b*h^2*j^2 + 108*a^5*b^2*e*i^3 + 12*a^3*b^4*e^3*i + 64*a^4*b^3*d*h^3 + 64*a^2*b^5*d^3*h + 12*a^3*b^4*c*g^3 + 18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 + 81*a^6*b*i^4 + 16*a*b^6*d^4 + 256*a^7*j^4 + 81*b^7*c^4 + a^4*b^3*g^4 + a^2*b^5*e^4, z, m)*((768*a^3*b^5*c + 256*a^4*b^4*g)/(64*a^3*b^2) - (x*(128*a^3*b^5*d + 128*a^4*b^4*h))/(16*a^3*b^2)) + (64*a^2*b^4*d*e - 384*a^3*b^3*c*j + 192*a^3*b^3*d*i + 64*a^3*b^3*e*h - 128*a^4*b^2*g*j + 192*a^4*b^2*h*i)/(64*a^3*b^2) + (x*(36*a*b^5*c^2 - 4*a^2*b^4*e^2 + 4*a^3*b^3*g^2 - 36*a^4*b^2*i^2 + 24*a^2*b^4*c*g + 64*a^3*b^3*d*j - 24*a^3*b^3*e*i + 64*a^4*b^2*h*j))/(16*a^3*b^2)) - (27*a^4*i^3 + a*b^3*e^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e + 16*a^4*g*j^2 - 12*a^2*b^2*c*h^2 + a^2*b^2*e*g^2 + 9*a^2*b^2*e^2*i - 48*a^4*h*i*j - 4*a*b^3*d^2*g + 27*a*b^3*c^2*i + 48*a^3*b*c*j^2 + 27*a^3*b*e*i^2 - 4*a^3*b*g*h^2 + 3*a^3*b*g^2*i + 18*a^2*b^2*c*g*i - 16*a^2*b^2*d*e*j - 8*a^2*b^2*d*g*h - 24*a*b^3*c*d*h + 6*a*b^3*c*e*g - 48*a^3*b*d*i*j - 16*a^3*b*e*h*j)/(64*a^3*b^2) - (x*(9*a^4*i^2*j - 2*a^3*b*h^3 - 8*a^4*h*j^2 - 2*b^4*d^3 - 6*a^2*b^2*d*h^2 + a^2*b^2*e^2*j + 3*b^4*c*d*e - 6*a*b^3*d^2*h - 9*a*b^3*c^2*j - 8*a^3*b*d*j^2 - a^3*b*g^2*j - 6*a^2*b^2*c*g*j + 9*a^2*b^2*c*h*i + 3*a^2*b^2*d*g*i + a^2*b^2*e*g*h + 9*a*b^3*c*d*i + 3*a*b^3*c*e*h + a*b^3*d*e*g + 6*a^3*b*e*i*j + 3*a^3*b*g*h*i))/(16*a^3*b^2))*root(65536*a^7*b^8*z^4 - 65536*a^7*b^6*j*z^3 + 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2 + 4096*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 + 3072*a^4*b^7*c*e*z^2 ...

3.198 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=241

$$\frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab\,(a-bx^4)^2}+\frac{4af+x(7bc-ag+2(3bd-ah)x+5bex^2)}{32a^2b\,(a-bx^4)}+\frac{\left(21bc-5\sqrt{a}\,\sqrt{b}\,e\right.}{64a^1}$$

```
[Out] 1/8*x*(b*c+a*g+(a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(-b*x^4+a)^2+1/32*(4*a*f+x*
(7*b*c-a*g+2*(-a*h+3*b*d)*x+5*b*e*x^2))/a^2/b/(-b*x^4+a)+1/16*(-a*h+3*b*d)*
arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)+1/64*arctan(b^(1/4)*x/a^(1/4))
*(21*b*c-3*a*g-5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)+1/64*arctanh(b^(1/4)*x
/a^(1/4))*(21*b*c-3*a*g+5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)
```

Rubi [A]

time = 0.22, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1872, 1868, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\,\sqrt{b}\,e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\,\sqrt{b}\,e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x(2x(3bd-ah)-ag+7bc+5bex^2)+4af}{32a^2b\,(a-bx^4)}+\frac{x(x(ah+bd)+ag+bc+bex^2+bfx^3)}{8ab\,(a-bx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) +
 (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a - b
*x^4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)
])/(64*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx = \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab\,(a-bx^4)^2} - \frac{\int \frac{-b(7bc-ag)-2b(3bd-ah)x-}{(a-bx^4)^2}}{8ab^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af+x(7bc-ag+2(}{32a^2b\,(a}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af+x(7bc-ag+2(}{32a^2b\,(a}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af+x(7bc-ag+2(}{32a^2b\,(a}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af+x(7bc-ag+2(}{32a^2b\,(a}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af+x(7bc-ag+2(}{32a^2b\,(a}$$

Mathematica [A]
time = 0.18, size = 309, normalized size = 1.28

$$\frac{\frac{4a^{3/4}\sqrt{b}\,x(7bc+bx(6d+5ex)-a(g+2hx))}{a-bx^4}+\frac{16a^{7/4}\sqrt{b}\,(bx(c+x(d+ex))+a(f+x(g+hx)))}{(a-bx^4)^2}+2\sqrt[4]{b}\left(21bc-5\sqrt{a}\sqrt{b}\,e-3ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\left(-21b^{5/4}c-12\sqrt[4]{a}\,bd-5\sqrt{a}\,b^{3/4}e+3a\sqrt[4]{b}\,g+4a^{5/4}h\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+\left(21b^{5/4}c-12\sqrt[4]{a}\,bd+5\sqrt{a}\,b^{3/4}e-3a\sqrt[4]{b}\,g+4a^{5/4}h\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)-4\sqrt[4]{a}\,(-3bd+ah)\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{128a^{11/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x) - a*(g + 2*h*x)))/(a - b*x
^4) + (16*a^(7/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x))))/(a
 - b*x^4)^2 + 2*b^(1/4)*(21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1
/4)*x)/a^(1/4)] + (-21*b^(5/4)*c - 12*a^(1/4)*b*d - 5*Sqrt[a]*b^(3/4)*e + 3
*a*b^(1/4)*g + 4*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (21*b^(5/4)*c - 12*a
^(1/4)*b*d + 5*Sqrt[a]*b^(3/4)*e - 3*a*b^(1/4)*g + 4*a^(5/4)*h)*Log[a^(1/4)
 + b^(1/4)*x] - 4*a^(1/4)*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a
^(11/4)*b^(3/2))
```

Maple [A]
time = 0.35, size = 267, normalized size = 1.11

method	result

risch	$\frac{-\frac{5be\,x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-b\,x^4+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\frac{\left(5\_R^2e-\frac{4(ah-3bd)\_R}{b}-\frac{3(ag-}{b}\right.}{\_R^3}}{128a^2b}$
default	$\frac{-\frac{5be\,x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-b\,x^4+a)^2}+\frac{\frac{(-3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-5/32*b*e/a^2*x^7+1/16*(a*h-3*b*d)/a^2*x^6+1/32*(a*g-7*b*c)/a^2*x^5+9/32/a
*e*x^3+1/16*(a*h+5*b*d)/a/b*x^2+1/32*(3*a*g+11*b*c)/a/b*x+1/8*f/b)/(-b*x^4+
a)^2+1/32/a^2/b*(1/4*(-3*a*g+21*b*c)*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(
a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))+1/4*(-4*a*h+12*b*d)/(a*b)^(1/2)*ln((a
+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2)))-5/4*e/(a/b)^(1/4)*(2*arctan(x/(a/b)^
(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))))
```

Maxima [A]
time = 0.53, size = 320, normalized size = 1.33

$$-\frac{5\,b^2x^7e+2\,(3\,b^2d-abh)x^6+(7\,b^2c-abg)x^5-9\,abx^3e-4\,a^2f-2\,(5\,abd+a^2h)x^2-(11\,abc+3\,a^2g)x}{32\,(a^2b^3x^8-2\,a^3b^2x^4+a^4b)}+\frac{\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(21\,b^{\frac{3}{2}}c-3\,a\sqrt{b}\,g-5\,\sqrt{a}\,be\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c-3\,a\sqrt{b}\,g+5\,\sqrt{a}\,be\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{128\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima
")
```

```
[Out] -1/32*(5*b^2*x^7*e + 2*(3*b^2*d - a*b*h)*x^6 + (7*b^2*c - a*b*g)*x^5 - 9*a*
b*x^3*e - 4*a^2*f - 2*(5*a*b*d + a^2*h)*x^2 - (11*a*b*c + 3*a^2*g)*x)/(a^2*
b^3*x^8 - 2*a^3*b^2*x^4 + a^4*b) + 1/128*(4*(3*b*d - a*h)*log(sqrt(b)*x^2 +
 sqrt(a))/(sqrt(a)*sqrt(b)) - 4*(3*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(s
qrt(a)*sqrt(b)) + 2*(21*b^(3/2)*c - 3*a*sqrt(b)*g - 5*sqrt(a)*b*e)*arctan(s
qrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (
21*b^(3/2)*c - 3*a*sqrt(b)*g + 5*sqrt(a)*b*e)*log((sqrt(b)*x - sqrt(sqrt(a)
*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b))*sqrt(b)))/(a^2*b)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(205) = 410.
time = 1.10, size = 440, normalized size = 1.83

$$-\frac{\sqrt{2}\left(21\,b^{2}c-3\,abg-12\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+4\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah+5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{\sqrt{2}\left(21\,b^{2}c-3\,abg+12\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-4\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah-5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{\sqrt{2}\left(21\,b^{2}c-3\,abg-5\,\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}+\frac{\sqrt{2}\left(21\,b^{2}c-3\,abg-5\,\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{5\,b^{2}x^{7}e+6\,b^{2}dx^{6}-2\,abhx^{6}+7\,b^{2}cx^{5}-abgx^{5}-9\,abx^{3}e-10\,abdx^{2}-2\,a^{2}hx^{2}-11\,abcx-3\,a^{2}gx-4\,a^{2}f}{32\left(bx^{4}-a\right)^{2}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^2*c - 3*a*b*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 4*sqrt
(2)*(-a*b^3)^(1/4)*a*h + 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2
)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*
c - 3*a*b*g + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 4*sqrt(2)*(-a*b^3)^(1/4)*a*h
- 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^
(1/4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a
*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2
) + 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)
*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b^2*x^7*e + 6*
b^2*d*x^6 - 2*a*b*h*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*x^3*e - 10*a*b*d*
x^2 - 2*a^2*h*x^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

Mupad [**B**]
time = 5.73, size = 1687, normalized size = 7.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x)
```

```
[Out] (f/(8*b) + (9*e*x^3)/(32*a) - (x^5*(7*b*c - a*g))/(32*a^2) - (x^6*(3*b*d -
a*h))/(16*a^2) + (x*(11*b*c + 3*a*g))/(32*a*b) + (x^2*(5*b*d + a*h))/(16*a*
b) - (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(- root(
```

268435456*a^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 - 6881280*a^6*b^5*c*e*z^2 - 524288*a^8*b^3*h^2*z^2 - 4718592*a^6*b^5*d^2*z^2 + 258048*a^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z - 51200*a^5*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z + 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z + 2709504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h + 40320*a^2*b^3*c*d*e*h + 8640*a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 - 6300*a^2*b^3*c*e^2*g + 960*a^4*b*e*g*h^2 - 60480*a*b^4*c*d^2*e - 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 - 23814*a^2*b^3*c^2*g^2 - 27648*a^2*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 - 625*a^2*b^3*e^4 - 81*a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 - 194481*b^5*c^4, z, k)*(root(268435456*a^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 - 6881280*a^6*b^5*c*e*z^2 - 524288*a^8*b^3*h^2*z^2 - 4718592*a^6*b^5*d^2*z^2 + 258048*a^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z - 51200*a^5*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z + 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z + 2709504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h + 40320*a^2*b^3*c*d*e*h + 8640*a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 - 6300*a^2*b^3*c*e^2*g + 960*a^4*b*e*g*h^2 - 60480*a*b^4*c*d^2*e - 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 - 23814*a^2*b^3*c^2*g^2 - 27648*a^2*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 - 625*a^2*b^3*e^4 - 81*a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 - 194481*b^5*c^4, z, k)*((344064*a^5*b^4*c - 49152*a^6*b^3*g)/(32768*a^6*b) - (x*(24576*a^5*b^4*d - 8192*a^6*b^3*h))/(4096*a^6*b)) - (15360*a^3*b^3*d*e - 5120*a^4*b^2*e*h)/(32768*a^6*b) + (x*(7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 + 144*a^4*b^2*g^2 - 2016*a^3*b^3*c*g))/(4096*a^6*b)) - (125*a*b^2*e^3 + 3024*b^3*c*d^2 - 2205*b^3*c^2*e - 48*a^3*g*h^2 - 432*a*b^2*d^2*g + 336*a^2*b*c*h^2 - 45*a^2*b*e*g^2 - 2016*a*b^2*c*d*h + 630*a*b^2*c*e*g + 288*a^2*b*d*g*h)/(32768*a^6*b) - (x*(216*b^3*d^3 - 8*a^3*h^3 - 315*b^3*c*d*e - 216*a*b^2*d^2*h + 72*a^2*b*d*h^2 + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g - 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 - 6881280*a^6*b^5*c*e*z^2 - 524288*a^8*b^3*h^2*z^2 - 4718592*a^6*b^5*d^2*z^2 + 258048*a^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z - 51200*a^5*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z + 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z + 2709504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h + 40320*a^2*b^3*c*d*e*h + 8640*a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 - 6300*a^2*b^3*c*e^2*g + 960*a^4*b*e*g*h^2 - 60480*a*b^4*c*d^2*e - 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 - 23814*a^2*b^3*c^2*g^2 - 27648*a^2*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 - 625*a^2*b^3*e^4 - 81*a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 - 194481*b^5*c^4, z, k), k, 1, 4)

3.199 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3} \, dx$

Optimal. Leaf size=268

$$\frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{8ab\,(a-bx^4)^2}+\frac{4af+x(7bc-ag+2(3bd-ah)x+(5be-3ai)x^2)}{32a^2b\,(a-bx^4)}-\frac{\Big(5be}{}$$

```
[Out] 1/8*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+b*f*x^3)/a/b/(-b*x^4+a)^2+1/32*(4*
a*f+x*(7*b*c-a*g+2*(-a*h+3*b*d)*x+(-3*a*i+5*b*e)*x^2))/a^2/b/(-b*x^4+a)+1/1
6*(-a*h+3*b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)-1/64*arctan(b^(
1/4)*x/a^(1/4))*(5*b*e-3*a*i-3*(-a*g+7*b*c)*b^(1/2)/a^(1/2))/a^(9/4)/b^(7/4
)+1/64*arctanh(b^(1/4)*x/a^(1/4))*(5*b*e-3*a*i+3*(-a*g+7*b*c)*b^(1/2)/a^(1/
2))/a^(9/4)/b^(7/4)
```

Rubi [A]
time = 0.28, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1872, 1868, 1890, 281, 214, 1181, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}\,(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}\,(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x(2x(3bd-ah)+x^2(5be-3ai)-ag+7bc)+4af}{32a^2b\,(a-bx^4)}+\frac{x(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3)}{8ab\,(a-bx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(8*a*b*(a - b*x
^4)^2) + (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + (5*b*e - 3*a*i)*x^2)
)/(32*a^2*b*(a - b*x^4)) - ((5*b*e - (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*
a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((5*b*e + (3*Sqrt[
b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4
)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(
3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+199x^6}{(a-bx^4)^3}\,dx = \frac{x(bc+ag+(bd+ah)x+(199a+be)x^2+bfx^3)}{8ab\,(a-bx^4)^2} - \frac{\int -b}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(199a+be)x^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(199a+be)x^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(199a+be)x^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(199a+be)x^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(199a+be)x^2+bfx^3)}{8ab\,(a-bx^4)^2} + \frac{4af}{}$$

Mathematica [A]
time = 0.22, size = 359, normalized size = 1.34

$$\frac{-\frac{4a^{3/4}b^{3/4}x(-b(7c+x(6d+5ex))+a(g+x(2h+3ix)))}{a-bx^4}+\frac{16a^{7/4}b^{3/4}(bx(c+x(d+ex))+a(f+x(g+x(h+ix))))}{(a-bx^4)^2}+2\left(21b^{3/2}c-5\sqrt{a}\,be-3a\sqrt{b}\,g+3a^{3/2}i\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\left(-21b^{3/2}c-12\sqrt[4]{a}\,b^{5/4}d-5\sqrt{a}\,be+3a\sqrt{b}\,g+4a^{5/4}\sqrt[4]{b}\,h+3a^{3/2}i\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+\left(21b^{3/2}c-12\sqrt[4]{a}\,b^{5/4}d+5\sqrt{a}\,be-3a\sqrt{b}\,g+4a^{5/4}\sqrt[4]{b}\,h-3a^{3/2}i\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)-4\sqrt[4]{a}\,\sqrt[4]{b}\,(-3bd+ah)\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{128a^{11/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x
]
```

```
[Out] ((-4*a^(3/4)*b^(3/4)*x*(-(b*(7*c + x*(6*d + 5*e*x))) + a*(g + x*(2*h + 3*i*
x))))/(a - b*x^4) + (16*a^(7/4)*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x*(
g + x*(h + i*x)))))/(a - b*x^4)^2 + 2*(21*b^(3/2)*c - 5*Sqrt[a]*b*e - 3*a*S
qrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + (-21*b^(3/2)*c - 12*a
^(1/4)*b^(5/4)*d - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*
a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + (21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d
+ 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^
(1/4) + b^(1/4)*x] - 4*a^(1/4)*b^(1/4)*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]
*x^2])/(128*a^(11/4)*b^(7/4))
```

Maple [A]
time = 0.43, size = 295, normalized size = 1.10

method	result

risch	$\frac{\frac{(3ai-5be)x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{(ai+9be)x^3}{32ab}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-bx^4+a)^2} - \frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)} \frac{\left(-(3ai-5be)\_R^2-4(ah\right.}{}}{128a^2b^2}$
default	$\frac{\frac{(3ai-5be)x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{(ai+9be)x^3}{32ab}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-bx^4+a)^2} + \frac{\frac{(-3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\right.\right.}{4a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERB
OSE)
```

```
[Out] (1/32*(3*a*i-5*b*e)/a^2*x^7+1/16*(a*h-3*b*d)/a^2*x^6+1/32*(a*g-7*b*c)/a^2*x
^5+1/32*(a*i+9*b*e)/a/b*x^3+1/16*(a*h+5*b*d)/a/b*x^2+1/32*(3*a*g+11*b*c)/a/
b*x+1/8*f/b)/(-b*x^4+a)^2+1/32/a^2/b*(1/4*(-3*a*g+21*b*c)*(a/b)^(1/4)/a*(ln
((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))+1/4*(-4*a*h+12*b
*d)/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2)))-1/4*(-3*a*i+5*b
*e)/b/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4
)))))
```

Maxima [A]
time = 0.50, size = 344, normalized size = 1.28

$$-\frac{(5\,b^2e-3i\,ab)x^7+2\,(3\,b^2d-abh)x^6+(7\,b^2c-abg)x^5-(9\,abe+i\,a^2)x^3-4\,a^2f-2\,(5\,abd+a^2h)x^2-(11\,abc+3\,a^2g)x}{32\,(a^2b^3x^8-2\,a^3b^2x^4+a^4b)}+\frac{\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(21\,b^{\frac{3}{2}}c-3\,a\sqrt{b}\,g-5\,\sqrt{a}\,be+3i\,a^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c-3\,a\sqrt{b}\,g+5\,\sqrt{a}\,be-3i\,a^{\frac{3}{2}}\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{128\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
maxima")
```

```
[Out] -1/32*((5*b^2*e - 3*I*a*b)*x^7 + 2*(3*b^2*d - a*b*h)*x^6 + (7*b^2*c - a*b*g
)*x^5 - (9*a*b*e + I*a^2)*x^3 - 4*a^2*f - 2*(5*a*b*d + a^2*h)*x^2 - (11*a*b
*c + 3*a^2*g)*x)/(a^2*b^3*x^8 - 2*a^3*b^2*x^4 + a^4*b) + 1/128*(4*(3*b*d -
a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 4*(3*b*d - a*h)*log(sqr
t(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(21*b^(3/2)*c - 3*a*sqrt(b)*g - 5
*sqrt(a)*b*e + 3*I*a^(3/2))*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*b^(3/2)*c - 3*a*sqrt(b)*g + 5*sqrt(a
)*b*e - 3*I*a^(3/2))*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + s
qrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^2*b)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(229) = 458.
time = 1.07, size = 645, normalized size = 2.41

$$-\frac{\sqrt{2}\left(21\,b^2c-3\,abg-12\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd+4\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah+5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^3\right)^{\frac{3}{4}}a^2}-\frac{\sqrt{2}\left(21\,b^2c-3\,abg+12\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd-4\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah-5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^3\right)^{\frac{3}{4}}a^2}-\frac{\sqrt{2}\left(21\,b^2c-3\,abg-5\,\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^3\right)^{\frac{3}{4}}a^2}+\frac{\sqrt{2}\left(21\,b^2c-3\,abg-5\,\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^3\right)^{\frac{3}{4}}a^2}-\frac{3i\,\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4}-\frac{3i\,\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4}+\frac{3i\,\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^2b^4}-\frac{3i\,\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^2b^4}-\frac{5\,b^2x^7e+6\,b^2dx^6-2\,abhx^6-3i\,abx^7+7\,b^2cx^5-abgx^5-9\,abx^3e-10\,abdx^2-2\,a^2hx^2-i\,a^2x^3-11\,abcx-3\,a^2gx-4\,a^2f}{32\left(bx^4-a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^2*c - 3*a*b*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 4*sqrt
(2)*(-a*b^3)^(1/4)*a*h + 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2
)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*
c - 3*a*b*g + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 4*sqrt(2)*(-a*b^3)^(1/4)*a*h
- 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^
(1/4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a
*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2
) + 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)
*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) - 3/128*I*sqrt(2)*(-a*b^
3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2
*b^4) - 3/128*I*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^4) + 3/256*I*sqrt(2)*(-a*b^3)^(3/4)*log(x^
2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^4) - 3/256*I*sqrt(2)*(-a*b^
3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^4) - 1/32*(5
*b^2*x^7*e + 6*b^2*d*x^6 - 2*a*b*h*x^6 - 3*I*a*b*x^7 + 7*b^2*c*x^5 - a*b*g*
x^5 - 9*a*b*x^3*e - 10*a*b*d*x^2 - 2*a^2*h*x^2 - I*a^2*x^3 - 11*a*b*c*x - 3
*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

Mupad [**B**]
time = 5.80, size = 2680, normalized size = 10.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x)
```

```
[Out] symsum(log((27*a^4*i^3 - 125*a*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e -
336*a^2*b^2*c*h^2 + 45*a^2*b^2*e*g^2 + 225*a^2*b^2*e^2*i + 432*a*b^3*d^2*g
- 1323*a*b^3*c^2*i - 135*a^3*b*e*i^2 + 48*a^3*b*g*h^2 - 27*a^3*b*g^2*i + 37
8*a^2*b^2*c*g*i - 288*a^2*b^2*d*g*h + 2016*a*b^3*c*d*h - 630*a*b^3*c*e*g)/(
32768*a^6*b^2) - root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2 + 412
8768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 - 6
881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*z^2 +
 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i*z -
774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55
296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z + 5529
6*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z + 3456
*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*
b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^
2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*i^2 -
 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g - 672
0*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*
c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2 + 138
24*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 23814*a^2
*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*d^3*h
 - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*a^4*b
^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^4 - 1
94481*b^6*c^4, z, l)*(root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2
+ 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^
2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*
z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i
*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z
 + 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z +
 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z +
 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760
*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b
^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*
i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g
- 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a
*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2
+ 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 2381
4*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*
d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*
a^4*b^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^
4 - 194481*b^6*c^4, z, l)*((344064*a^5*b^5*c - 49152*a^6*b^4*g)/(32768*a^6*
b^2) - (x*(24576*a^5*b^4*d - 8192*a^6*b^3*h))/(4096*a^6*b)) - (15360*a^3*b^
4*d*e - 9216*a^4*b^3*d*i - 5120*a^4*b^3*e*h + 3072*a^5*b^2*h*i)/(32768*a^6*
```

b^2) + (x*(144*a^5*b*i^2 + 7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 + 144*a^4*b^2*g^2 - 2016*a^3*b^3*c*g - 480*a^4*b^2*e*i))/(4096*a^6*b)) - (x*(216*b^3*d^3 - 8*a^3*h^3 - 315*b^3*c*d*e + 9*a^3*g*h*i - 216*a*b^2*d^2*h + 72*a^2*b*d*h^2 + 189*a*b^2*c*d*i + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g - 63*a^2*b*c*h*i - 27*a^2*b*d*g*i - 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z + 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z + 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*a^4*b^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^4 - 194481*b^6*c^4, z, l), l, 1, 4) + (f/(8*b) - (x^5*(7*b*c - a*g))/(32*a^2) - (x^6*(3*b*d - a*h))/(16*a^2) - (x^7*(5*b*e - 3*a*i))/(32*a^2) + (x*(11*b*c + 3*a*g))/(32*a*b) + (x^2*(5*b*d + a*h))/(16*a*b) + (x^3*(9*b*e + a*i))/(32*a*b))/(a^2 + b^2*x^8 - 2*a*b*x^4)

3.200 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=285

$$\frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{8ab\,(a-bx^4)^2}+\frac{4a(bf-aj)+x(b(7bc-ag)+2b(3bd-ah)x+b(5b}{32a^2b^2\,(a-bx^4)}$$

```
[Out] 1/8*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+(a*j+b*f)*x^3)/a/b/(-b*x^4+a)^2+1/
32*(4*a*(-a*j+b*f)+x*(b*(-a*g+7*b*c)+2*b*(-a*h+3*b*d)*x+b*(-3*a*i+5*b*e)*x^
2))/a^2/b^2/(-b*x^4+a)+1/16*(-a*h+3*b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/
2)/b^(3/2)-1/64*arctan(b^(1/4)*x/a^(1/4))*(5*b*e-3*a*i-3*(-a*g+7*b*c)*b^(1/
2)/a^(1/2))/a^(9/4)/b^(7/4)+1/64*arctanh(b^(1/4)*x/a^(1/4))*(5*b*e-3*a*i+3*
(-a*g+7*b*c)*b^(1/2)/a^(1/2))/a^(9/4)/b^(7/4)
```

Rubi [A]
time = 0.26, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1872, 1868, 1890, 281, 214, 1181, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}\,(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}\,(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x(b(7bc-ag)+2bx(3bd-ah)+bx^2(5be-3ai))+4a(bf-aj)}{32a^2b^2\,(a-bx^4)}+\frac{x(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc)}{8ab\,(a-bx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^3
,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(8*a*b*
(a - b*x^4)^2) + (4*a*(b*f - a*j) + x*(b*(7*b*c - a*g) + 2*b*(3*b*d - a*h)*
x + b*(5*b*e - 3*a*i)*x^2))/(32*a^2*b^2*(a - b*x^4)) - ((5*b*e - (3*Sqrt[b]
*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b
^(7/4)) + ((5*b*e + (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(
1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^
2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+200x^6+jx^7}{(a-bx^4)^3}\,dx = \frac{x(bc+ag+(bd+ah)x+(200a+be)x^2+(bf+aj}{8ab\,(a-bx^4)^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(200a+be)x^2+(bf+aj}{8ab\,(a-bx^4)^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(200a+be)x^2+(bf+aj}{8ab\,(a-bx^4)^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(200a+be)x^2+(bf+aj}{8ab\,(a-bx^4)^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(200a+be)x^2+(bf+aj}{8ab\,(a-bx^4)^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(200a+be)x^2+(bf+aj}{8ab\,(a-bx^4)^2}$$

Mathematica [A]
time = 0.17, size = 380, normalized size = 1.33

$$\frac{-\frac{4a^{3/4}(8a^2j-b^2x(7c+x(6d+5ex))+abx(g+x(2h+3ix)))}{a-bx^4}+\frac{16a^{7/4}(a^2j+b^2x(c+x(d+ex))+ab(f+x(g+x(h+ix))))}{(a-bx^4)^2}+2\sqrt[4]{b}\left(21b^{3/2}c-5\sqrt{a}\,be-3a\sqrt{b}\,g+3a^{3/2}i\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\sqrt[4]{b}\left(-21b^{3/2}c-12\sqrt[4]{a}\,b^{5/4}d-5\sqrt{a}\,be+3a\sqrt{b}\,g+4a^{5/4}\sqrt[4]{b}\,h+3a^{3/2}i\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+\sqrt[4]{b}\left(21b^{3/2}c-12\sqrt[4]{a}\,b^{5/4}d+5\sqrt{a}\,be-3a\sqrt{b}\,g+4a^{5/4}\sqrt[4]{b}\,h-3a^{3/2}i\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)-4\sqrt[4]{a}\,\sqrt{b}\,(-3bd+ah)\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{128a^{11/4}b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4)^3,x]
```

```
[Out] ((-4*a^(3/4)*(8*a^2*j - b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g + x*(2*h +
 3*i*x))))/(a - b*x^4) + (16*a^(7/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) + a*b
*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^2 + 2*b^(1/4)*(21*b^(3/2)*c - 5*Sq
rt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + b^(1
/4)*(-21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g +
 4*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + b^(1/4)*(21*
b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 4*a^(5/4
)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 4*a^(1/4)*Sqrt[b]*(-3
*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11/4)*b^2)
```

Maple [A]
time = 0.40, size = 311, normalized size = 1.09

method	result

risch	$\frac{\frac{(3ai-5be)x^{7}}{32a^{2}}+\frac{(ah-3bd)x^{6}}{16a^{2}}+\frac{(ag-7bc)x^{5}}{32a^{2}}+\frac{j\,x^{4}}{4b}+\frac{(ai+9be)x^{3}}{32ab}+\frac{(ah+5bd)x^{2}}{16ab}+\frac{(3ag+11bc)x}{32ab}-\frac{aj-bf}{8b^{2}}}{\left(-b\,x^{4}+a\right)^{2}}-\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b-a\right)}\left(-(3ai-5be)\right.$
default	$\frac{\frac{(3ai-5be)x^{7}}{32a^{2}}+\frac{(ah-3bd)x^{6}}{16a^{2}}+\frac{(ag-7bc)x^{5}}{32a^{2}}+\frac{j\,x^{4}}{4b}+\frac{(ai+9be)x^{3}}{32ab}+\frac{(ah+5bd)x^{2}}{16ab}+\frac{(3ag+11bc)x}{32ab}-\frac{aj-bf}{8b^{2}}}{\left(-b\,x^{4}+a\right)^{2}}+\frac{\frac{(-3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right.}{4a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETU
RNVERBOSE)
```

```
[Out] (1/32*(3*a*i-5*b*e)/a^2*x^7+1/16*(a*h-3*b*d)/a^2*x^6+1/32*(a*g-7*b*c)/a^2*x
^5+1/4*j*x^4/b+1/32*(a*i+9*b*e)/a/b*x^3+1/16*(a*h+5*b*d)/a/b*x^2+1/32*(3*a*
g+11*b*c)/a/b*x-1/8*(a*j-b*f)/b^2)/(-b*x^4+a)^2+1/32/a^2/b*(1/4*(-3*a*g+21*
b*c)*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1
/4)))+1/4*(-4*a*h+12*b*d)/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(
1/2)))-1/4*(-3*a*i+5*b*e)/b/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b
)^(1/4))/(x-(a/b)^(1/4)))))
```

Maxima [A]
time = 0.51, size = 378, normalized size = 1.33

$$\frac{8\,a^{2}bjx^{4}-(5\,b^{3}e-3i\,ab^{2})x^{7}-2\,(3\,b^{3}d-ab^{2}h)x^{6}-(7\,b^{3}c-ab^{2}g)x^{5}+4\,a^{2}bf-4\,a^{3}j+(9\,ab^{2}e+i\,a^{2}b)x^{3}+2\,(5\,ab^{2}d+a^{2}bh)x^{2}+(11\,ab^{2}c+3\,a^{2}bg)x}{32\,(a^{2}b^{4}x^{8}-2\,a^{3}b^{3}x^{4}+a^{4}b^{2})}+\frac{\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}\,x^{2}+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}\,x^{2}-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(21\,b^{\frac{3}{2}}c-3\,a\sqrt{b}\,g-5\,\sqrt{a}\,be+3i\,a^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c-3\,a\sqrt{b}\,g+5\,\sqrt{a}\,be-3i\,a^{\frac{3}{2}}\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{128\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*a^2*b*j*x^4 - (5*b^3*e - 3*I*a*b^2)*x^7 - 2*(3*b^3*d - a*b^2*h)*x^6
 - (7*b^3*c - a*b^2*g)*x^5 + 4*a^2*b*f - 4*a^3*j + (9*a*b^2*e + I*a^2*b)*x^
3 + 2*(5*a*b^2*d + a^2*b*h)*x^2 + (11*a*b^2*c + 3*a^2*b*g)*x)/(a^2*b^4*x^8
- 2*a^3*b^3*x^4 + a^4*b^2) + 1/128*(4*(3*b*d - a*h)*log(sqrt(b)*x^2 + sqrt(
a))/(sqrt(a)*sqrt(b)) - 4*(3*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)
*sqrt(b)) + 2*(21*b^(3/2)*c - 3*a*sqrt(b)*g - 5*sqrt(a)*b*e + 3*I*a^(3/2))*
arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt
(b)) - (21*b^(3/2)*c - 3*a*sqrt(b)*g + 5*sqrt(a)*b*e - 3*I*a^(3/2))*log((sq
rt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^2*b)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,
x)
```

```
[Out] Timed out
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(246) = 492$.
time = 1.91, size = 677, normalized size = 2.38

$$-\frac{\sqrt{2}\left(21\,b^{2}c-3\,abg-12\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+4\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah+5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{\sqrt{2}\left(21\,b^{2}c-3\,abg+12\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-4\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah-5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{\sqrt{2}\left(21\,b^{2}c-3\,abg-5\,\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}+\frac{\sqrt{2}\left(21\,b^{2}c-3\,abg-5\,\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{3i\,\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^{2}b^{4}}-\frac{3i\,\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^{2}b^{4}}+\frac{3i\,\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^{2}b^{4}}-\frac{3i\,\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^{2}b^{4}}-\frac{5\,b^{3}x^{7}e+6\,b^{3}dx^{6}-2\,ab^{2}hx^{6}-3i\,ab^{2}x^{7}+7\,b^{3}cx^{5}-ab^{2}gx^{5}-8\,a^{2}bjx^{4}-9\,ab^{2}x^{3}e-10\,ab^{2}dx^{2}-2\,a^{2}bhx^{2}-i\,a^{2}bx^{3}-11\,ab^{2}cx-3\,a^{2}bgx-4\,a^{2}bf+4\,a^{3}j}{32\left(bx^{4}-a\right)^{2}a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algor
ithm="giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^2*c - 3*a*b*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 4*sqrt
(2)*(-a*b^3)^(1/4)*a*h + 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2
)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*
c - 3*a*b*g + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 4*sqrt(2)*(-a*b^3)^(1/4)*a*h
- 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^
(1/4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a
*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2
) + 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)
*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) - 3/128*I*sqrt(2)*(-a*b^
3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2
*b^4) - 3/128*I*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^4) + 3/256*I*sqrt(2)*(-a*b^3)^(3/4)*log(x^
2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^4) - 3/256*I*sqrt(2)*(-a*b^
3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^4) - 1/32*(5
*b^3*x^7*e + 6*b^3*d*x^6 - 2*a*b^2*h*x^6 - 3*I*a*b^2*x^7 + 7*b^3*c*x^5 - a*
b^2*g*x^5 - 8*a^2*b*j*x^4 - 9*a*b^2*x^3*e - 10*a*b^2*d*x^2 - 2*a^2*b*h*x^2
- I*a^2*b*x^3 - 11*a*b^2*c*x - 3*a^2*b*g*x - 4*a^2*b*f + 4*a^3*j)/((b*x^4 -
 a)^2*a^2*b^2)
```

Mupad [B]
time = 5.91, size = 2696, normalized size = 9.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^3
,x)
```

```
[Out] symsum(log((27*a^4*i^3 - 125*a*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e -
336*a^2*b^2*c*h^2 + 45*a^2*b^2*e*g^2 + 225*a^2*b^2*e^2*i + 432*a*b^3*d^2*g
- 1323*a*b^3*c^2*i - 135*a^3*b*e*i^2 + 48*a^3*b*g*h^2 - 27*a^3*b*g^2*i + 37
8*a^2*b^2*c*g*i - 288*a^2*b^2*d*g*h + 2016*a*b^3*c*d*h - 630*a*b^3*c*e*g)/(
32768*a^6*b^2) - root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2 + 412
8768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 - 6
881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*z^2 +
 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i*z -
774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55
296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z + 5529
6*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z + 3456
*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*
b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^
2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*i^2 -
 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g - 672
0*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*
c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2 + 138
24*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 23814*a^2
*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*d^3*h
 - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*a^4*b
^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^4 - 1
94481*b^6*c^4, z, m)*(root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2
+ 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^
2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*
z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i
*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z
 + 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z +
 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z +
 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760
*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b
^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*
i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g
- 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a
*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2
+ 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 2381
4*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*
```

d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*a^4*b^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^4 - 194481*b^6*c^4, z, m)*((344064*a^5*b^5*c - 49152*a^6*b^4*g)/(32768*a^6*b^2) - (x*(24576*a^5*b^4*d - 8192*a^6*b^3*h))/(4096*a^6*b)) - (15360*a^3*b^4*d*e - 9216*a^4*b^3*d*i - 5120*a^4*b^3*e*h + 3072*a^5*b^2*h*i)/(32768*a^6*b^2) + (x*(144*a^5*b*i^2 + 7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 + 144*a^4*b^2*g^2 - 2016*a^3*b^3*c*g - 480*a^4*b^2*e*i))/(4096*a^6*b)) - (x*(216*b^3*d^3 - 8*a^3*h^3 - 315*b^3*c*d*e + 9*a^3*g*h*i - 216*a*b^2*d^2*h + 72*a^2*b*d*h^2 + 189*a*b^2*c*d*i + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g - 63*a^2*b*c*h*i - 27*a^2*b*d*g*i - 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z + 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z + 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*a^4*b^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^4 - 194481*b^6*c^4, z, m), m, 1, 4) + ((b*f - a*j)/(8*b^2) + (j*x^4)/(4*b) - (x^5*(7*b*c - a*g))/(32*a^2) - (x^6*(3*b*d - a*h))/(16*a^2) - (x^7*(5*b*e - 3*a*i))/(32*a^2) + (x*(11*b*c + 3*a*g))/(32*a*b) + (x^2*(5*b*d + a*h))/(16*a*b) + (x^3*(9*b*e + a*i))/(32*a*b))/(a^2 + b^2*x^8 - 2*a*b*x^4)

3.201 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^3}\,dx$

Optimal. Leaf size=413

$$\frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2}-\frac{4af-x\left(7bc+ag+2(3bd+ah)x+5bex^2\right)}{32a^2b\left(a+bx^4\right)}+\frac{(3bd+ah)\tan^{-1}\left(\right.}{16a^{5/2}b^{3/2}}$$

```
[Out] 1/8*x*(b*c-a*g+(-a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(b*x^4+a)^2+1/32*(-4*a*f+x
*(7*b*c+a*g+2*(a*h+3*b*d)*x+5*b*e*x^2))/a^2/b/(b*x^4+a)+1/16*(a*h+3*b*d)*ar
ctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)-1/256*ln(-a^(1/4)*b^(1/4)*x*2^(1/
2)+a^(1/2)+x^2*b^(1/2))*(21*b*c+3*a*g-5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)
*2^(1/2)+1/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(21*b*c+3*
a*g-5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x
*2^(1/2)/a^(1/4))*(21*b*c+3*a*g+5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(1/
2)+1/128*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(21*b*c+3*a*g+5*e*a^(1/2)*b^(1
/2))/a^(11/4)/b^(5/4)*2^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {1872, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+3bd)}{16a^{5/2}b^{3/2}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}-\frac{4af-x\left(2x(ah+3bd)+ag+7bc+5bex^2\right)}{32a^2b\left(a+bx^4\right)}+\frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{8ab\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) -
 (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a + b
*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
 - ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3
*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)
) - ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt
[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(128*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx = \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{\int \frac{-b(7bc+ag)-2b(3bd+ah)x-5}{(a+bx^4)^2}}{8ab^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+2(3}{32a^2b\,(a+}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+2(3}{32a^2b\,(a+}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+2(3}{32a^2b\,(a+}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+2(3}{32a^2b\,(a+}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+2(3}{32a^2b\,(a+}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+2(3}{32a^2b\,(a+}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+2(3}{32a^2b\,(a+}$$

Mathematica [A]
time = 0.23, size = 411, normalized size = 1.00

$$\frac{\frac{8a^{3/4}\sqrt{b}\,x(7bc+bx(6d+5ex)+a(g+2hx))}{a+bx^4} - \frac{32a^{7/4}\sqrt{b}\,(-bx(c+x(d+ex))+a(f+x(g+hx)))}{(a+bx^4)^2} - 2\left(21\sqrt{2}\,b^{5/4}c+24\sqrt[4]{a}\,bd+5\sqrt{2}\,\sqrt{a}\,b^{3/4}e+3\sqrt{2}\,a\sqrt[4]{b}\,g+8a^{5/4}h\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\left(21\sqrt{2}\,b^{5/4}c-24\sqrt[4]{a}\,bd+5\sqrt{2}\,\sqrt{a}\,b^{3/4}e+3\sqrt{2}\,a\sqrt[4]{b}\,g-8a^{5/4}h\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\sqrt{2}\sqrt[4]{b}\left(-21bc+5\sqrt{a}\sqrt{b}\,e-3ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+\sqrt{2}\sqrt[4]{b}\left(21bc-5\sqrt{a}\sqrt{b}\,e+3ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{256a^{11/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x) + a*(g + 2*h*x)))/(a + b*x
^4) - (32*a^(7/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))
/(a + b*x^4)^2 - 2*(21*Sqrt[2]*b^(5/4)*c + 24*a^(1/4)*b*d + 5*Sqrt[2]*Sqrt[
a]*b^(3/4)*e + 3*Sqrt[2]*a*b^(1/4)*g + 8*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)] + 2*(21*Sqrt[2]*b^(5/4)*c - 24*a^(1/4)*b*d + 5*Sqrt[2]*Sqr
t[a]*b^(3/4)*e + 3*Sqrt[2]*a*b^(1/4)*g - 8*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b
```

```
^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*
(21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(3/2))
```

Maple [A]
time = 0.36, size = 353, normalized size = 0.85

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{\left(5\_R^2e+\frac{4(ah+3bd)}{b}\_R+\frac{3ag+ \ldots}{b} \ldots\right.}{\_R^3}}{128a^2b}$ …
default	$\frac{\frac{5be\,x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\frac{(3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+ \ldots\right.}{8a}}{\ldots}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (5/32*b*e/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32/a*
e*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*f/b)/(b*x^4+a)
^2+1/32/a^2/b*(1/8*(3*a*g+21*b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4
)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(
2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/2*(4*a*h+12*b
*d)/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+5/8*e/(a/b)^(1/4)*2^(1/2)*(ln((x^2-
(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))
+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.51, size = 452, normalized size = 1.09

$$\frac{5\,b^2x^7e+2\,(3\,b^2d+abh)x^6+(7\,b^2c+abg)x^5+9\,abx^3e-4\,a^2f+2\,(5\,abd-a^2h)x^2+(11\,abc-3\,a^2g)x}{32\,(a^2b^3x^8+2\,a^3b^2x^4+a^4b)}+\frac{\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c+3\,a\sqrt{b}\,g-5\,\sqrt{a}\,be\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c+3\,a\sqrt{b}\,g-5\,\sqrt{a}\,be\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+3\,\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+5\,\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e-24\,\sqrt{a}\,b^{\frac{3}{2}}d-8\,a^{\frac{3}{2}}\sqrt{b}\,h\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+3\,\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+5\,\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e+24\,\sqrt{a}\,b^{\frac{3}{2}}d+8\,a^{\frac{3}{2}}\sqrt{b}\,h\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{3}{4}}}}{256\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima"
)
```

```
[Out] 1/32*(5*b^2*x^7*e + 2*(3*b^2*d + a*b*h)*x^6 + (7*b^2*c + a*b*g)*x^5 + 9*a*b
*x^3*e - 4*a^2*f + 2*(5*a*b*d - a^2*h)*x^2 + (11*a*b*c - 3*a^2*g)*x)/(a^2*b
^3*x^8 + 2*a^3*b^2*x^4 + a^4*b) + 1/256*(sqrt(2)*(21*b^(3/2)*c + 3*a*sqrt(b
)*g - 5*sqrt(a)*b*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))
/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*b^(3/2)*c + 3*a*sqrt(b)*g - 5*sqrt(a)*b*e)
*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) +
```

```
2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*
a^(3/4)*b^(5/4)*e - 24*sqrt(a)*b^(3/2)*d - 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*
sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 3*s
qrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 24*sqrt(a)*b^(3/2)
*d + 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(
a^2*b)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.59, size = 459, normalized size = 1.11

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^2d + 4\sqrt{2}\sqrt{ab}\,abh + 21\left(ab^3\right)^{\frac{1}{4}}b^2c + 3\left(ab^3\right)^{\frac{1}{4}}abg + 5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3} + \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^2d + 4\sqrt{2}\sqrt{ab}\,abh + 21\left(ab^3\right)^{\frac{1}{4}}b^2c + 3\left(ab^3\right)^{\frac{1}{4}}abg + 5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3} + \frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c + 3\left(ab^3\right)^{\frac{1}{4}}abg - 5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2 + \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{256\,a^3b^3} - \frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c + 3\left(ab^3\right)^{\frac{1}{4}}abg - 5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2 - \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{256\,a^3b^3} + \frac{5\,b^2x^7e + 6\,b^2dx^6 + 2\,abhx^6 + 7\,b^2cx^5 + abgx^5 + 9\,abx^3e + 10\,abdx^2 - 2\,a^2hx^2 + 11\,abcx - 3\,a^2gx - 4\,a^2f}{32\left(bx^4+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*
(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)
*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)
*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^
(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*
```

```
c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 2*a*b*h*x^6
+ 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*x^3*e + 10*a*b*d*x^2 - 2*a^2*h*x^2 + 11*a
*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

Mupad [B]
time = 5.69, size = 1686, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x)
```

```
[Out] ((9*e*x^3)/(32*a) - f/(8*b) + (x^5*(7*b*c + a*g))/(32*a^2) + (x^6*(3*b*d +
a*h))/(16*a^2) + (x*(11*b*c - 3*a*g))/(32*a*b) + (x^2*(5*b*d - a*h))/(16*a*
b) + (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log((3024*b
^3*c*d^2 - 125*a*b^2*e^3 - 2205*b^3*c^2*e + 48*a^3*g*h^2 + 432*a*b^2*d^2*g
+ 336*a^2*b*c*h^2 - 45*a^2*b*e*g^2 + 2016*a*b^2*c*d*h - 630*a*b^2*c*e*g + 2
88*a^2*b*d*g*h)/(32768*a^6*b) - root(268435456*a^11*b^6*z^4 + 3145728*a^7*b
^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 + 6881280*a^6*b^5*c*e*z^2 + 524288*a^8*
b^3*h^2*z^2 + 4718592*a^6*b^5*d^2*z^2 - 258048*a^5*b^3*c*g*h*z - 774144*a^4
*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z + 51200*a^5*b^3*e^2*h*z - 903168*a^4*b
^4*c^2*h*z - 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z - 2709504*a^3*b
^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h - 40320*a^2*b^3*c*d*e*h - 8640*a^2*b^3*d^
2*e*g - 6720*a^3*b^2*c*e*h^2 + 6300*a^2*b^3*c*e^2*g - 960*a^4*b*e*g*h^2 - 6
0480*a*b^4*c*d^2*e + 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*a^3*b^2*
d^2*h^2 + 450*a^3*b^2*e^2*g^2 + 23814*a^2*b^3*c^2*g^2 + 27648*a^2*b^3*d^3*h
 + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 + 625*a^2*b^3*e^4 + 81*a^4*b*g^
4 + 20736*a*b^4*d^4 + 256*a^5*h^4 + 194481*b^5*c^4, z, k)*(root(268435456*a
^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 + 6881280*a^
6*b^5*c*e*z^2 + 524288*a^8*b^3*h^2*z^2 + 4718592*a^6*b^5*d^2*z^2 - 258048*a
^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z + 51200*a^5
*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z - 55296*a^5*b^3*d*g^2*z + 153600*a^4*
b^4*d*e^2*z - 2709504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h - 40320*a^2*b^
3*c*d*e*h - 8640*a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 + 6300*a^2*b^3*c*e^
2*g - 960*a^4*b*e*g*h^2 - 60480*a*b^4*c*d^2*e + 3072*a^4*b*d*h^3 + 111132*a
*b^4*c^3*g + 13824*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 + 23814*a^2*b^3*c^
2*g^2 + 27648*a^2*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 + 62
5*a^2*b^3*e^4 + 81*a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 + 194481*b^5*c
^4, z, k)*((344064*a^5*b^4*c + 49152*a^6*b^3*g)/(32768*a^6*b) - (x*(24576*a
^5*b^4*d + 8192*a^6*b^3*h))/(4096*a^6*b)) + (15360*a^3*b^3*d*e + 5120*a^4*b
^2*e*h)/(32768*a^6*b) + (x*(7056*a^2*b^4*c^2 - 400*a^3*b^3*e^2 + 144*a^4*b^
2*g^2 + 2016*a^3*b^3*c*g))/(4096*a^6*b)) + (x*(216*b^3*d^3 + 8*a^3*h^3 - 31
5*b^3*c*d*e + 216*a*b^2*d^2*h + 72*a^2*b*d*h^2 - 105*a*b^2*c*e*h - 45*a*b^2
*d*e*g - 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^6*z^4 + 31457
```

```
28*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 + 6881280*a^6*b^5*c*e*z^2 + 524
288*a^8*b^3*h^2*z^2 + 4718592*a^6*b^5*d^2*z^2 - 258048*a^5*b^3*c*g*h*z - 77
4144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z + 51200*a^5*b^3*e^2*h*z - 9031
68*a^4*b^4*c^2*h*z - 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z - 27095
04*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h - 40320*a^2*b^3*c*d*e*h - 8640*a^
2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 + 6300*a^2*b^3*c*e^2*g - 960*a^4*b*e*g
*h^2 - 60480*a*b^4*c*d^2*e + 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*
a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 + 23814*a^2*b^3*c^2*g^2 + 27648*a^2*b
^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 + 625*a^2*b^3*e^4 + 81*
a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 + 194481*b^5*c^4, z, k), k, 1, 4)
```

3.202 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=463

$$\frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{8ab\,(a+bx^4)^2}-\frac{4af-x(7bc+ag+2(3bd+ah)x+(5be+3ai)x^2)}{32a^2b\,(a+bx^4)}+\frac{(3bd\ \cdots}{\cdots}$$

```
[Out] 1/8*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+b*f*x^3)/a/b/(b*x^4+a)^2+1/32*(-
4*a*f+x*(7*b*c+a*g+2*(a*h+3*b*d)*x+(3*a*i+5*b*e)*x^2))/a^2/b/(b*x^4+a)+1/16
*(a*h+3*b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)-1/256*ln(-a^(1/4)*
b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+5*b*e)*a^(1/2)+3*(a*g+7*b*c
)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*b^(1/2))*(-(3*a*i+5*b*e)*a^(1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/b^
(7/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i+5*b*e)*a^(
1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/128*arctan(1+b^(1/4)
*x*2^(1/2)/a^(1/4))*((3*a*i+5*b*e)*a^(1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/
b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {1872, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+3bd)}{16a^{5/2}b^{3/2}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}-\frac{4af-x(2x(ah+3bd)+x^2(3ai+5be)+ag+7bc)}{32a^2b(a+bx^4)}+\frac{x(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3)}{8ab(a+bx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(8*a*b*(a + b*x
^4)^2) - (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + (5*b*e + 3*a*i)*x^2)
)/(32*a^2*b*(a + b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(1
6*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3
*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) - ((3*Sqrt[b]*(7*b*c + a*g) -
 Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]
*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(
128*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+202x^6}{(a+bx^4)^3}\,dx = \frac{x(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{\int \frac{-b(7}{}}{}$$

$$= \frac{x(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-}{}$$

$$= \frac{x(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-}{}$$

$$= \frac{x(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-}{}$$

$$= \frac{x(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-}{}$$

$$= \frac{x(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-}{}$$

$$= \frac{x(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-}{}$$

$$= \frac{x(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-}{}$$

<u>Mathematica</u> [A]
time = 0.35, size = 473, normalized size = 1.02

$$\frac{\frac{8a^{3/4}b^{3/4}x(7bc+ag+bx(6d+5ex)+ax(2h+3ix))}{a+bx^4} - \frac{32a^{7/4}b^{3/4}(-bx(c+x(d+ex))+a(f+x(g+x(h+ix))))}{(a+bx^4)^2} - 2\left(21\sqrt{2}\,b^{3/2}c+24\sqrt[4]{a}\,b^{5/4}d+5\sqrt{2}\sqrt{a}\,be+3\sqrt{2}\,a\sqrt{b}\,g+8a^{5/4}\sqrt[4]{b}\,h+3\sqrt{2}\,a^{3/2}i\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) + 2\left(21\sqrt{2}\,b^{3/2}c-24\sqrt[4]{a}\,b^{5/4}d+5\sqrt{2}\sqrt{a}\,be+3\sqrt{2}\,a\sqrt{b}\,g-8a^{5/4}\sqrt[4]{b}\,h+3\sqrt{2}\,a^{3/2}i\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) + \sqrt{2}\left(-21b^{3/2}c+5\sqrt{a}\,be-3a\sqrt{b}\,g+3a^{3/2}i\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right) + \sqrt{2}\left(21b^{3/2}c-5\sqrt{a}\,be+3a\sqrt{b}\,g-3a^{3/2}i\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{256a^{11/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x
]
```

```
[Out] ((8*a^(3/4)*b^(3/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x) + a*x*(2*h + 3*i*x))
)/(a + b*x^4) - (32*a^(7/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g
 + x*(h + i*x)))))/(a + b*x^4)^2 - 2*(21*Sqrt[2]*b^(3/2)*c + 24*a^(1/4)*b^(
5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*
h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(21*Sq
```

```
rt[2]*b^(3/2)*c - 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*
a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-21*b^(3/2)*c + 5*Sqrt[a]*b*e - 3*a*Sqrt[b
]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] +
 Sqrt[2]*(21*b^(3/2)*c - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(7/4))
```

Maple [A]
time = 0.35, size = 381, normalized size = 0.82

method	result
risch	$\frac{\frac{(3ai+5be)x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}-\frac{(ai-9be)x^3}{32ab}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b+a)}\frac{\left((3ai+5be)\_R^2+4(ah\right.}{}}{128a^2b^2}$
default	$\frac{\frac{(3ai+5be)x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}-\frac{(ai-9be)x^3}{32ab}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{(3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBO
SE)
```

```
[Out] (1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x
^5-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/
b*x-1/8*f/b)/(b*x^4+a)^2+1/32/a^2/b*(1/8*(3*a*g+21*b*c)*(a/b)^(1/4)/a*2^(1/
2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(
a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)
*x-1))+1/2*(4*a*h+12*b*d)/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*(3*a*i+5*
b*e)/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2
+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*ar
ctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.52, size = 497, normalized size = 1.07

$$\frac{(5\,b^2e+3i\,ab)x^7+2\,(3\,b^2d+abh)x^6+(7\,b^2c+abg)x^5+(9\,abe-i\,a^2)x^3-4\,a^2f+2\,(5\,abd-a^2h)x^2+(11\,abc-3\,a^2g)x}{32\,(a^2b^3x^8+2\,a^3b^2x^4+a^4b)}+\frac{\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c+3\,a\sqrt{b}\,g-5\,\sqrt{a}\,be-3i\,a^{\frac{3}{2}}\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c+3\,a\sqrt{b}\,g-5\,\sqrt{a}\,be-3i\,a^{\frac{3}{2}}\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(21\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+3\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+5\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e-24\sqrt{a}\,b^{\frac{3}{2}}d-8\,a^{\frac{3}{2}}\sqrt{b}\,h+3i\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{1}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(21\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+3\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+5\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e+24\sqrt{a}\,b^{\frac{3}{2}}d+8\,a^{\frac{3}{2}}\sqrt{b}\,h+3i\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{1}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{256\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="m
axima")
```

```
[Out] 1/32*((5*b^2*e + 3*I*a*b)*x^7 + 2*(3*b^2*d + a*b*h)*x^6 + (7*b^2*c + a*b*g)
*x^5 + (9*a*b*e - I*a^2)*x^3 - 4*a^2*f + 2*(5*a*b*d - a^2*h)*x^2 + (11*a*b*
```

```
c - 3*a^2*g)*x)/(a^2*b^3*x^8 + 2*a^3*b^2*x^4 + a^4*b) + 1/256*(sqrt(2)*(21*
b^(3/2)*c + 3*a*sqrt(b)*g - 5*sqrt(a)*b*e - 3*I*a^(3/2))*log(sqrt(b)*x^2 +
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*b^(3/2
)*c + 3*a*sqrt(b)*g - 5*sqrt(a)*b*e - 3*I*a^(3/2))*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^
(7/4)*c + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a^(3/4)*b^(5/4)*e - 24*sq
rt(a)*b^(3/2)*d - 8*a^(3/2)*sqrt(b)*h + 3*I*sqrt(2)*a^(7/4)*b^(1/4))*arctan
(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))
/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c
+ 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 24*sqrt(a)*b^
(3/2)*d + 8*a^(3/2)*sqrt(b)*h + 3*I*sqrt(2)*a^(7/4)*b^(1/4))*arctan(1/2*sqr
t(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4
)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^2*b)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.17, size = 652, normalized size = 1.41

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^2d+4\sqrt{2}\sqrt{ab}\,abh+21\left(ab^3\right)^{\frac14}b^2c+3\left(ab^3\right)^{\frac14}abg+5\left(ab^3\right)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^2d+4\sqrt{2}\sqrt{ab}\,abh+21\left(ab^3\right)^{\frac14}b^2c+3\left(ab^3\right)^{\frac14}abg+5\left(ab^3\right)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac14}b^2c+3\left(ab^3\right)^{\frac14}abg-5\left(ab^3\right)^{\frac34}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}-\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac14}b^2c+3\left(ab^3\right)^{\frac14}abg-5\left(ab^3\right)^{\frac34}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}+\frac{3i\sqrt{2}\left(ab^3\right)^{\frac14}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{128\,a^2b^4}+\frac{3i\sqrt{2}\left(ab^3\right)^{\frac14}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{128\,a^2b^4}-\frac{3i\sqrt{2}\left(ab^3\right)^{\frac34}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}+\frac{3i\sqrt{2}\left(ab^3\right)^{\frac34}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}+\frac{5\,b^2x^7e+6\,b^2dx^6+2\,abhx^6+3i\,abx^7+7\,b^2cx^5+abgx^5+9\,abx^3e+10\,abdx^2-2\,a^2hx^2-i\,a^2x^3+11\,abcx-3\,a^2gx-4\,a^2f}{32\left(bx^4+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="g
iac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*
(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2
```

```
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)
*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)
*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^
(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*
c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(a^3*b^3) + 3/128*I*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(
2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/128*I*sqrt(2)*(a*
b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2
*b^4) - 3/256*I*sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(a^2*b^4) + 3/256*I*sqrt(2)*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)
^(1/4) + sqrt(a/b))/(a^2*b^4) + 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 2*a*b*h*x
^6 + 3*I*a*b*x^7 + 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*x^3*e + 10*a*b*d*x^2 - 2
*a^2*h*x^2 - I*a^2*x^3 + 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a
^2*b)
```

Mupad [B]
time = 5.75, size = 2680, normalized size = 5.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x)
```

```
[Out] symsum(log(- root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 4128768
*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 + 68812
80*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2 + 614
40*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z - 7741
44*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55296*
a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 55296*a^
5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 3456*a^4
*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*
d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*
i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2 + 264
60*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 6720*a^
3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^
2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13824*a
^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^2*b^4
*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*h + 3
072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*b^2*g
^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 + 19448
1*b^6*c^4, z, l)*(root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 41
28768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 +
6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2
```

+ 61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z -
774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 5
5296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 552
96*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 345
6*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3
*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d
^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2
+ 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 67
20*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5
*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13
824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^
2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*
h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*
b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 +
194481*b^6*c^4, z, l)*((344064*a^5*b^5*c + 49152*a^6*b^4*g)/(32768*a^6*b^2)
- (x*(24576*a^5*b^4*d + 8192*a^6*b^3*h))/(4096*a^6*b)) + (15360*a^3*b^4*d*
e + 9216*a^4*b^3*d*i + 5120*a^4*b^3*e*h + 3072*a^5*b^2*h*i)/(32768*a^6*b^2)
- (x*(144*a^5*b*i^2 - 7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 - 144*a^4*b^2*g^2
- 2016*a^3*b^3*c*g + 480*a^4*b^2*e*i))/(4096*a^6*b)) - (27*a^4*i^3 + 125*a
*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e - 336*a^2*b^2*c*h^2 + 45*a^2*b^2
*e*g^2 + 225*a^2*b^2*e^2*i - 432*a*b^3*d^2*g + 1323*a*b^3*c^2*i + 135*a^3*b
*e*i^2 - 48*a^3*b*g*h^2 + 27*a^3*b*g^2*i + 378*a^2*b^2*c*g*i - 288*a^2*b^2*
d*g*h - 2016*a*b^3*c*d*h + 630*a*b^3*c*e*g)/(32768*a^6*b^2) - (x*(315*b^3*c
*d*e - 8*a^3*h^3 - 216*b^3*d^3 + 9*a^3*g*h*i - 216*a*b^2*d^2*h - 72*a^2*b*d
*h^2 + 189*a*b^2*c*d*i + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g + 63*a^2*b*c*h*i
+ 27*a^2*b*d*g*i + 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^7*z
^4 + 589824*a^8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h
*z^2 + 983040*a^7*b^5*e*g*z^2 + 6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^
2*z^2 + 4718592*a^6*b^6*d^2*z^2 + 61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*
g*h*z + 184320*a^5*b^4*d*e*i*z - 774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i
^2*z - 18432*a^6*b^3*g^2*h*z + 55296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*
z - 903168*a^4*b^5*c^2*h*z - 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z
- 2709504*a^3*b^6*c^2*d*z - 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i +
7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*
a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2
*e*g*h^2 + 2268*a^4*b^2*c*g*i^2 + 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d
^2*i - 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g -
576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c
^3*g + 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2
+ 450*a^3*b^3*e^2*g^2 + 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^
3*b^3*e^3*i + 27648*a^2*b^4*d^3*h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3
+ 22050*a*b^5*c^2*e^2 + 81*a^4*b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 +
20736*a*b^5*d^4 + 81*a^6*i^4 + 194481*b^6*c^4, z, l), l, 1, 4) + ((x^5*(7*
b*c + a*g))/(32*a^2) - f/(8*b) + (x^6*(3*b*d + a*h))/(16*a^2) + (x^7*(5*b*e
+ 3*a*i))/(32*a^2) + (x*(11*b*c - 3*a*g))/(32*a*b) + (x^2*(5*b*d - a*h))/(

```
16*a*b) + (x^3*(9*b*e - a*i))/(32*a*b))/(a^2 + b^2*x^8 + 2*a*b*x^4)
```

3.203 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=480

$$\frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{8ab\,(a+bx^4)^2}-\frac{4a(bf+aj)-x(b(7bc+ag)+2b(3bd+ah)x+b(5b}{32a^2b^2\,(a+bx^4)}$$

```
[Out] 1/8*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+(-a*j+b*f)*x^3)/a/b/(b*x^4+a)^2+
1/32*(-4*a*(a*j+b*f)+x*(b*(a*g+7*b*c)+2*b*(a*h+3*b*d)*x+b*(3*a*i+5*b*e)*x^2
))/a^2/b^2/(b*x^4+a)+1/16*(a*h+3*b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b
^(3/2)-1/256*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+5*
b*e)*a^(1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/256*ln(a^(1/
4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+5*b*e)*a^(1/2)+3*(a*g+7*
b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x*2^(1/2)/a^
(1/4))*((3*a*i+5*b*e)*a^(1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/
2)+1/128*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i+5*b*e)*a^(1/2)+3*(a*g+
7*b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.244$, Rules used = {1872, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+3bd)}{16a^{5/2}b^{3/2}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}-\frac{4a(aj+bf)-x(b(ag+7bc)+2bx(ah+3bd)+bx^2(3ai+5be))}{32a^2b^2(a+bx^4)}+\frac{x(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc)}{8ab(a+bx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^3
,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(8*a*b*
(a + b*x^4)^2) - (4*a*(b*f + a*j) - x*(b*(7*b*c + a*g) + 2*b*(3*b*d + a*h)*
x + b*(5*b*e + 3*a*i)*x^2))/(32*a^2*b^2*(a + b*x^4)) + ((3*b*d + a*h)*ArcTa
n[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g)
+ Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqr
t[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i
))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) -
 ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqr
t[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+203x^6+jx^7}{(a+bx^4)^3}\,dx = \frac{x(bc-ag+(bd-ah)x-(203a-be)x^2+(bf-a}{8ab\,(a+bx^4)^2}$$

$$= \frac{x(bc-ag+(bd-ah)x-(203a-be)x^2+(bf-a}{8ab\,(a+bx^4)^2}$$

$$= \frac{x(bc-ag+(bd-ah)x-(203a-be)x^2+(bf-a}{8ab\,(a+bx^4)^2}$$

$$= \frac{x(bc-ag+(bd-ah)x-(203a-be)x^2+(bf-a}{8ab\,(a+bx^4)^2}$$

$$= \frac{x(bc-ag+(bd-ah)x-(203a-be)x^2+(bf-a}{8ab\,(a+bx^4)^2}$$

$$= \frac{x(bc-ag+(bd-ah)x-(203a-be)x^2+(bf-a}{8ab\,(a+bx^4)^2}$$

$$= \frac{x(bc-ag+(bd-ah)x-(203a-be)x^2+(bf-a}{8ab\,(a+bx^4)^2}$$

$$= \frac{x(bc-ag+(bd-ah)x-(203a-be)x^2+(bf-a}{8ab\,(a+bx^4)^2}$$

Mathematica [A]
time = 0.27, size = 500, normalized size = 1.04

$$\frac{\frac{8a^{3/4}(-8a^2j+b^2x(7c+x(6d+5ex))+abx(g+x(2h+3ix)))}{a+bx^4}+\frac{32a^{7/4}(a^2j+b^2x(c+x(d+ex))-ab(f+x(g+x(h+ix))))}{(a+bx^4)^2}-2\sqrt[4]{b}\left(21\sqrt{2}\,b^{3/2}c+24\sqrt[4]{a}\,b^{5/4}d+5\sqrt{2}\,\sqrt{a}\,be+3\sqrt{2}\,a\sqrt{b}\,g+8a^{5/4}\sqrt[4]{b}\,h+3\sqrt{2}\,a^{3/2}i\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\sqrt[4]{b}\left(21\sqrt{2}\,b^{3/2}c-24\sqrt[4]{a}\,b^{5/4}d+5\sqrt{2}\,\sqrt{a}\,be+3\sqrt{2}\,a\sqrt{b}\,g-8a^{5/4}\sqrt[4]{b}\,h+3\sqrt{2}\,a^{3/2}i\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\sqrt{2}\,\sqrt[4]{b}\left(-21b^{3/2}c+5\sqrt{a}\,be-3a\sqrt{b}\,g+3a^{3/2}i\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+\sqrt{2}\,\sqrt[4]{b}\left(21b^{3/2}c-5\sqrt{a}\,be+3a\sqrt{b}\,g-3a^{3/2}i\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{256a^{11/4}b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*(-8*a^2*j + b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g + x*(2*h +
 3*i*x))))/(a + b*x^4) + (32*a^(7/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b
*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^2 - 2*b^(1/4)*(21*Sqrt[2]*b^(3/2)*
c + 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g +
8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a
```

```
^(1/4)] + 2*b^(1/4)*(21*Sqrt[2]*b^(3/2)*c - 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2
]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(
3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-21*b^(3
/2)*c + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(21*b^(3/2)*c - 5*Sqrt[a
]*b*e + 3*a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/(256*a^(11/4)*b^2)
```

Maple [A]
time = 0.37, size = 396, normalized size = 0.82

method	result
risch	$\frac{\frac{(3ai+5be)x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}-\frac{j\,x^4}{4b}-\frac{(ai-9be)x^3}{32ab}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{aj+bf}{8b^2}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b+a)} \left((3ai+5be)\_R \ldots\right.}{1\ldots}$
default	$\frac{\frac{(3ai+5be)x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}-\frac{j\,x^4}{4b}-\frac{(ai-9be)x^3}{32ab}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{aj+bf}{8b^2}}{(b\,x^4+a)^2}+\frac{(3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\ldots}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\ldots}\right)\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETUR
NVERBOSE)
```

```
[Out] (1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x
^5-1/4*j*x^4/b-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*
g-11*b*c)/a/b*x-1/8*(a*j+b*f)/b^2)/(b*x^4+a)^2+1/32/a^2/b*(1/8*(3*a*g+21*b*
c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(
a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arct
an(2^(1/2)/(a/b)^(1/4)*x-1))+1/2*(4*a*h+12*b*d)/(a*b)^(1/2)*arctan(x^2*(b/a
)^(1/2))+1/8*(3*a*i+5*b*e)/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(
1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)
/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.50, size = 535, normalized size = 1.11

$$-\frac{8a^2bjx^4-(5b^3e+3i\,ab^2)x^7-2(3b^3d+ab^2h)x^6-(7b^3c+ab^2g)x^5+4a^2bf+4a^3j-(9ab^2e-i\,a^2b)x^3-2(5ab^2d-a^2bh)x^2-(11ab^2c-3a^2bg)x}{32(a^2b^4x^8+2a^3b^3x^4+a^4b^2)}+\frac{\frac{\sqrt{2}\left(21b^{\frac{3}{2}}c+3a\sqrt{b}g-5\sqrt{a}be-3ia^{\frac{3}{2}}\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21b^{\frac{3}{2}}c+3a\sqrt{b}g-5\sqrt{a}be-3ia^{\frac{3}{2}}\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(21\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+3\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+5\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e-24\sqrt{a}b^{\frac{3}{2}}d-8a^{\frac{3}{2}}\sqrt{b}h+3i\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(21\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+3\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+5\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+24\sqrt{a}b^{\frac{3}{2}}d+8a^{\frac{3}{2}}\sqrt{b}h+3i\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{256a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algori
thm="maxima")
```

```
[Out] -1/32*(8*a^2*b*j*x^4 - (5*b^3*e + 3*I*a*b^2)*x^7 - 2*(3*b^3*d + a*b^2*h)*x^
6 - (7*b^3*c + a*b^2*g)*x^5 + 4*a^2*b*f + 4*a^3*j - (9*a*b^2*e - I*a^2*b)*x
```

```
^3 - 2*(5*a*b^2*d - a^2*b*h)*x^2 - (11*a*b^2*c - 3*a^2*b*g)*x)/(a^2*b^4*x^8
 + 2*a^3*b^3*x^4 + a^4*b^2) + 1/256*(sqrt(2)*(21*b^(3/2)*c + 3*a*sqrt(b)*g
- 5*sqrt(a)*b*e - 3*I*a^(3/2))*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x
+ sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*b^(3/2)*c + 3*a*sqrt(b)*g - 5*sq
rt(a)*b*e - 3*I*a^(3/2))*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt
(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 3*sqrt(2)*a^(5/4
)*b^(3/4)*g + 5*sqrt(2)*a^(3/4)*b^(5/4)*e - 24*sqrt(a)*b^(3/2)*d - 8*a^(3/2
)*sqrt(b)*h + 3*I*sqrt(2)*a^(7/4)*b^(1/4))*arctan(1/2*sqrt(2)*(2*sqrt(b)*x
+ sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqr
t(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 3*sqrt(2)*a^(5/4)*b^(3/4
)*g + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 24*sqrt(a)*b^(3/2)*d + 8*a^(3/2)*sqrt(b
)*h + 3*I*sqrt(2)*a^(7/4)*b^(1/4))*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2
)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^
(3/4)))/(a^2*b)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x
)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.08, size = 684, normalized size = 1.42

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^2d+4\sqrt{2}\sqrt{ab}\,abh+21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^2d+4\sqrt{2}\sqrt{ab}\,abh+21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}-\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}+\frac{3i\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4}+\frac{3i\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4}-\frac{3i\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}+\frac{3i\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}+\frac{5b^3x^7e+6b^3dx^6+2ab^2hx^6+3iab^2x^7+7b^3cx^5+ab^2gx^5-8a^2bjx^4+9ab^2x^3e+10ab^2dx^2-2a^2bhx^2-ia^2bx^3+11ab^2cx-3a^2bgx-4a^2bf-4a^3j}{32\left(bx^4+a\right)^2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algori
thm="giac")
```

[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 3/128*I*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/128*I*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) - 3/256*I*sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) + 3/256*I*sqrt(2)*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) + 1/32*(5*b^3*x^7*e + 6*b^3*d*x^6 + 2*a*b^2*h*x^6 + 3*I*a*b^2*x^7 + 7*b^3*c*x^5 + a*b^2*g*x^5 - 8*a^2*b*j*x^4 + 9*a*b^2*x^3*e + 10*a*b^2*d*x^2 - 2*a^2*b*h*x^2 - I*a^2*b*x^3 + 11*a*b^2*c*x - 3*a^2*b*g*x - 4*a^2*b*f - 4*a^3*j)/((b*x^4 + a)^2*a^2*b^2)

Mupad [B]
time = 5.79, size = 2695, normalized size = 5.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^3,x)

[Out] symsum(log(- root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 + 6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2 + 61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z - 774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2 + 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 + 19448

1*b^6*c^4, z, m)*(root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 41
28768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 +
6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2
+ 61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z -
774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 5
5296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 552
96*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 345
6*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3
*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d
^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2
+ 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 67
20*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5
*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13
824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^
2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*
h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*
b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 +
194481*b^6*c^4, z, m)*((344064*a^5*b^5*c + 49152*a^6*b^4*g)/(32768*a^6*b^2)
- (x*(24576*a^5*b^4*d + 8192*a^6*b^3*h))/(4096*a^6*b)) + (15360*a^3*b^4*d*
e + 9216*a^4*b^3*d*i + 5120*a^4*b^3*e*h + 3072*a^5*b^2*h*i)/(32768*a^6*b^2)
- (x*(144*a^5*b*i^2 - 7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 - 144*a^4*b^2*g^2
- 2016*a^3*b^3*c*g + 480*a^4*b^2*e*i))/(4096*a^6*b)) - (27*a^4*i^3 + 125*a
*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e - 336*a^2*b^2*c*h^2 + 45*a^2*b^2
*e*g^2 + 225*a^2*b^2*e^2*i - 432*a*b^3*d^2*g + 1323*a*b^3*c^2*i + 135*a^3*b
*e*i^2 - 48*a^3*b*g*h^2 + 27*a^3*b*g^2*i + 378*a^2*b^2*c*g*i - 288*a^2*b^2*
d*g*h - 2016*a*b^3*c*d*h + 630*a*b^3*c*e*g)/(32768*a^6*b^2) - (x*(315*b^3*c
*d*e - 8*a^3*h^3 - 216*b^3*d^3 + 9*a^3*g*h*i - 216*a*b^2*d^2*h - 72*a^2*b*d
*h^2 + 189*a*b^2*c*d*i + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g + 63*a^2*b*c*h*i
+ 27*a^2*b*d*g*i + 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^7*z
^4 + 589824*a^8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h
*z^2 + 983040*a^7*b^5*e*g*z^2 + 6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^
2*z^2 + 4718592*a^6*b^6*d^2*z^2 + 61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*
g*h*z + 184320*a^5*b^4*d*e*i*z - 774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i
^2*z - 18432*a^6*b^3*g^2*h*z + 55296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*
z - 903168*a^4*b^5*c^2*h*z - 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z
- 2709504*a^3*b^6*c^2*d*z - 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i +
7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*
a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2
*e*g*h^2 + 2268*a^4*b^2*c*g*i^2 + 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d
^2*i - 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g -
576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c
^3*g + 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2
+ 450*a^3*b^3*e^2*g^2 + 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^
3*b^3*e^3*i + 27648*a^2*b^4*d^3*h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3
+ 22050*a*b^5*c^2*e^2 + 81*a^4*b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 +

```
 20736*a*b^5*d^4 + 81*a^6*i^4 + 194481*b^6*c^4, z, m), m, 1, 4) + ((x^5*(7*
b*c + a*g))/(32*a^2) - (j*x^4)/(4*b) - (b*f + a*j)/(8*b^2) + (x^6*(3*b*d +
a*h))/(16*a^2) + (x^7*(5*b*e + 3*a*i))/(32*a^2) + (x*(11*b*c - 3*a*g))/(32*
a*b) + (x^2*(5*b*d - a*h))/(16*a*b) + (x^3*(9*b*e - a*i))/(32*a*b))/(a^2 +
b^2*x^8 + 2*a*b*x^4)
```

3.204 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^4}\,dx$

Optimal. Leaf size=293

$$\frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}+\frac{x\left(7(11bc-ag)+12(5bd-ah)x+45bex^2\right)}{384a^3b\left(a-bx^4\right)}+\frac{8af+x(11bc-ag+}{96a^2b}$$

```
[Out] 1/12*x*(b*c+a*g+(a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(-b*x^4+a)^3+1/384*x*(-7*a
*g+77*b*c+12*(-a*h+5*b*d)*x+45*b*e*x^2)/a^3/b/(-b*x^4+a)+1/96*(8*a*f+x*(11*
b*c-a*g+2*(-a*h+5*b*d)*x+9*b*e*x^2))/a^2/b/(-b*x^4+a)^2+1/32*(-a*h+5*b*d)*a
rctanh(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)+1/256*arctan(b^(1/4)*x/a^(1/4))
*(77*b*c-7*a*g-15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)+1/256*arctanh(b^(1/4)
*x/a^(1/4))*(77*b*c-7*a*g+15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)
```

Rubi [A]
time = 0.28, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1872, 1868, 1869, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\,\sqrt{b}\,e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\,\sqrt{b}\,e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(11bc-ag)+12x(5bd-ah)+45bex^2\right)}{384a^3b\left(a-bx^4\right)}+\frac{x\left(2x(5bd-ah)-ag+11bc+9bex^2\right)+8af}{96a^2b\left(a-bx^4\right)^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3)
 + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a -
b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x + 9*b*e*x^2))/(96*a^
2*b*(a - b*x^4)^2) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/
4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e -
7*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((5*b*d - a*h
)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx = \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} - \frac{\int \frac{-b(11bc-ag)-2b(5bd-ah)x}{(a-bx^4)^3}}{12ab^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{8af+x(11bc-ag+2}{96a^2b\,(a}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+12(5}{384a^3b\,(a}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+12(5}{384a^3b\,(a}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+12(5}{384a^3b\,(a}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+12(5}{384a^3b\,(a}$$

$$= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+12(5}{384a^3b\,(a}$$

Mathematica [A]
time = 0.26, size = 360, normalized size = 1.23

$$\frac{\frac{4a^{3/4}\sqrt{b}\,x(77bc-7ag+60bdx-12ahx+45bex^2)}{a-bx^4}+\frac{16a^{7/4}\sqrt{b}\,x(11bc+bx(10d+9ex)-a(g+2hx))}{(a-bx^4)^2}+\frac{128a^{11/4}\sqrt{b}\,(bx(c+x(d+ex))+a(f+x(g+hx)))}{(a-bx^4)^3}+6\sqrt[4]{b}\left(77bc-15\sqrt{a}\sqrt{b}\,e-7ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-3\left(77b^{5/4}c+40\sqrt[4]{a}\,bd+15\sqrt{a}\,b^{3/4}e-7a\sqrt[4]{b}\,g-8a^{5/4}h\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+3\left(77b^{5/4}c-40\sqrt[4]{a}\,bd+15\sqrt{a}\,b^{3/4}e-7a\sqrt[4]{b}\,g+8a^{5/4}h\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)-24\sqrt[4]{a}\,(-5bd+ah)\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{1536a^{15/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*x*(77*b*c - 7*a*g + 60*b*d*x - 12*a*h*x + 45*b*e*x^2))/
(a - b*x^4) + (16*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d + 9*e*x) - a*(g + 2
*h*x)))/(a - b*x^4)^2 + (128*a^(11/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f
 + x*(g + h*x))))/(a - b*x^4)^3 + 6*b^(1/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e
- 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(77*b^(5/4)*c + 40*a^(1/4)*b*d + 1
5*Sqrt[a]*b^(3/4)*e - 7*a*b^(1/4)*g - 8*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x]
 + 3*(77*b^(5/4)*c - 40*a^(1/4)*b*d + 15*Sqrt[a]*b^(3/4)*e - 7*a*b^(1/4)*g
+ 8*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 24*a^(1/4)*(-5*b*d + a*h)*Log[Sqr
t[a] + Sqrt[b]*x^2])/(1536*a^(15/4)*b^(3/2))
```

Maple [A]
time = 0.35, size = 313, normalized size = 1.07

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}-\frac{21be\,x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7ag+51bc)x}{128ab}+\frac{f}{12b}}{(-b\,x^4+a)^3}-\frac{\_R\!=\!\mathrm{Ro}}{}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}-\frac{21be\,x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7ag+51bc)x}{128ab}+\frac{f}{12b}}{(-b\,x^4+a)^3}+\frac{(-7ag+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/128*e/a^3*b^2*x^11-1/32*(a*h-5*b*d)/a^3*b*x^10-7/384*(a*g-11*b*c)/a^3*b
*x^9-21/64*b*e/a^2*x^7+1/12/a^2*(a*h-5*b*d)*x^6+3/64/a^2*(a*g-11*b*c)*x^5+1
13/384/a*e*x^3+1/32*(a*h+11*b*d)/a/b*x^2+1/128*(7*a*g+51*b*c)/a/b*x+1/12*f/
b)/(-b*x^4+a)^3+1/128/a^3/b*(1/4*(-7*a*g+77*b*c)*(a/b)^(1/4)/a*(ln((x+(a/b)
^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))+1/4*(-8*a*h+40*b*d)/(a*b)
^(1/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2)))-15/4*e/(a/b)^(1/4)*(2*ar
ctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))))
```

Maxima [A]
time = 0.52, size = 394, normalized size = 1.34

$$-\frac{45\,b^3x^{11}e+12\,(5\,b^3d-ab^2h)x^{10}-126\,ab^2x^7e+7\,(11\,b^3c-ab^2g)x^9-32\,(5\,ab^2d-a^2bh)x^6+113\,a^2bx^3e-18\,(11\,ab^2c-a^2bg)x^5+32\,a^3f+12\,(11\,a^2bd+a^3h)x^2+3\,(51\,a^2bc+7\,a^3g)x}{384\,(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2x^4-a^6b)}+\frac{\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(77\,b^{\frac{3}{2}}c-7\,a\sqrt{b}\,g-15\,\sqrt{a}\,be\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c-7\,a\sqrt{b}\,g+15\,\sqrt{a}\,be\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{512\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima
")
```

```
[Out] -1/384*(45*b^3*x^11*e + 12*(5*b^3*d - a*b^2*h)*x^10 - 126*a*b^2*x^7*e + 7*(
11*b^3*c - a*b^2*g)*x^9 - 32*(5*a*b^2*d - a^2*b*h)*x^6 + 113*a^2*b*x^3*e -
18*(11*a*b^2*c - a^2*b*g)*x^5 + 32*a^3*f + 12*(11*a^2*b*d + a^3*h)*x^2 + 3*
(51*a^2*b*c + 7*a^3*g)*x)/(a^3*b^4*x^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a
^6*b) + 1/512*(8*(5*b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b))
 - 8*(5*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(77*b^(
3/2)*c - 7*a*sqrt(b)*g - 15*sqrt(a)*b*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*b^(3/2)*c - 7*a*sqrt(b)
*g + 15*sqrt(a)*b*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + s
qrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3*b)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.82, size = 501, normalized size = 1.71

$$-\frac{\sqrt{2}\left(77\,b^{2}c-7\,abg-40\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+8\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah+15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^{3}\right)^{\frac{3}{4}}a^{3}}-\frac{\sqrt{2}\left(77\,b^{2}c-7\,abg+40\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-8\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah-15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^{3}\right)^{\frac{3}{4}}a^{3}}-\frac{\sqrt{2}\left(77\,b^{2}c-7\,abg-15\,\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^{3}\right)^{\frac{3}{4}}a^{3}}+\frac{\sqrt{2}\left(77\,b^{2}c-7\,abg-15\,\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^{3}\right)^{\frac{3}{4}}a^{3}}-\frac{45\,b^{3}x^{11}e+60\,b^{3}dx^{10}-12\,ab^{2}hx^{10}+77\,b^{3}cx^{9}-7\,ab^{2}gx^{9}-126\,ab^{2}x^{7}e-160\,ab^{2}dx^{6}+32\,a^{2}bhx^{6}-198\,ab^{2}cx^{5}+18\,a^{2}bgx^{5}+113\,a^{2}bx^{3}e+132\,a^{2}bdx^{2}+12\,a^{3}hx^{2}+153\,a^{2}bcx+21\,a^{3}gx+32\,a^{3}f}{384\left(bx^{4}-a\right)^{3}a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 7*a*b*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 8*sqrt
(2)*(-a*b^3)^(1/4)*a*h + 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2
*c - 7*a*b*g + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 8*sqrt(2)*(-a*b^3)^(1/4)*a*h
 - 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b
)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqr
t(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)
*a^3) + 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqrt(-a*b)*b*e)*log(x^2 - s
qrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(45*b^3*x^
11*e + 60*b^3*d*x^10 - 12*a*b^2*h*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 - 126
*a*b^2*x^7*e - 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 - 198*a*b^2*c*x^5 + 18*a^2*
b*g*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 + 12*a^3*h*x^2 + 153*a^2*b*c*x
+ 21*a^3*g*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

Mupad [**B**]
time = 5.99, size = 1747, normalized size = 5.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x)
```

```
[Out] symsum(log(- root(68719476736*a^15*b^6*z^4 - 1211105280*a^8*b^5*c*e*z^2 + 3
35544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 - 838860800*a^8*b^5*d^
2*z^2 - 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z + 17661952*a^6
*b^3*c*g*h*z + 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 80281
6*a^7*b^2*g^2*h*z - 3686400*a^6*b^3*e^2*h*z + 4014080*a^6*b^3*d*g^2*z + 184
32000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h + 2956800*a^2*b^3*c*d*e*h +
672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 - 485100*a^2*b^3*c*e^2*g +
26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e - 81920*a^4*b*d*h^3 + 12782924*
a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 - 1743126*a^2*
b^3*c^2*g^2 - 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*b^4*
c^2*e^2 - 50625*a^2*b^3*e^4 - 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096*a^5
*h^4 - 35153041*b^5*c^4, z, k)*(root(68719476736*a^15*b^6*z^4 - 1211105280*
a^8*b^5*c*e*z^2 + 335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 - 8
38860800*a^8*b^5*d^2*z^2 - 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d
*g*z + 17661952*a^6*b^3*c*g*h*z + 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*
b^4*c^2*h*z - 802816*a^7*b^2*g^2*h*z - 3686400*a^6*b^3*e^2*h*z + 4014080*a^
6*b^3*d*g^2*z + 18432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h + 2956800
*a^2*b^3*c*d*e*h + 672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 - 485100
*a^2*b^3*c*e^2*g + 26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e - 81920*a^4*
b*d*h^3 + 12782924*a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2
*g^2 - 1743126*a^2*b^3*c^2*g^2 - 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g
^3 + 2668050*a*b^4*c^2*e^2 - 50625*a^2*b^3*e^4 - 2401*a^4*b*g^4 + 2560000*a
*b^4*d^4 + 4096*a^5*h^4 - 35153041*b^5*c^4, z, k)*((20185088*a^7*b^4*c - 18
35008*a^8*b^3*g)/(2097152*a^9*b) - (x*(655360*a^7*b^4*d - 131072*a^8*b^3*h)
)/(131072*a^9*b)) - (614400*a^4*b^3*d*e - 122880*a^5*b^2*e*h)/(2097152*a^9*
b) + (x*(189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 + 1568*a^5*b^2*g^2 - 34496*a
^4*b^3*c*g))/(131072*a^9*b)) - (3375*a*b^2*e^3 + 123200*b^3*c*d^2 - 88935*b
^3*c^2*e - 448*a^3*g*h^2 - 11200*a*b^2*d^2*g + 4928*a^2*b*c*h^2 - 735*a^2*b
*e*g^2 - 49280*a*b^2*c*d*h + 16170*a*b^2*c*e*g + 4480*a^2*b*d*g*h)/(2097152
*a^9*b) - (x*(4000*b^3*d^3 - 32*a^3*h^3 - 5775*b^3*c*d*e - 2400*a*b^2*d^2*h
 + 480*a^2*b*d*h^2 + 1155*a*b^2*c*e*h + 525*a*b^2*d*e*g - 105*a^2*b*e*g*h))
/(131072*a^9*b))*root(68719476736*a^15*b^6*z^4 - 1211105280*a^8*b^5*c*e*z^2
 + 335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 - 838860800*a^8*b^
5*d^2*z^2 - 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z + 17661952
*a^6*b^3*c*g*h*z + 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 8
02816*a^7*b^2*g^2*h*z - 3686400*a^6*b^3*e^2*h*z + 4014080*a^6*b^3*d*g^2*z +
 18432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h + 2956800*a^2*b^3*c*d*e*
h + 672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 - 485100*a^2*b^3*c*e^2*
g + 26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e - 81920*a^4*b*d*h^3 + 12782
924*a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 - 1743126*
a^2*b^3*c^2*g^2 - 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*
b^4*c^2*e^2 - 50625*a^2*b^3*e^4 - 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096
*a^5*h^4 - 35153041*b^5*c^4, z, k), k, 1, 4) + (f/(12*b) + (113*e*x^3)/(384
*a) - (3*x^5*(11*b*c - a*g))/(64*a^2) - (x^6*(5*b*d - a*h))/(12*a^2) + (7*b
*x^9*(11*b*c - a*g))/(384*a^3) + (x*(51*b*c + 7*a*g))/(128*a*b) + (b*x^10*(
```

```
5*b*d - a*h))/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (x^2*(11*b*d + a*h))/(
32*a*b) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^
8)
```

3.205 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a-bx^4\right)^4} \, dx$

Optimal. Leaf size=331

$$\frac{x\left(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}+\frac{x\left(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2\right)}{384a^3b\left(a-bx^4\right)}+\frac{8af+\ldots}{}$$

```
[Out] 1/12*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+b*f*x^3)/a/b/(-b*x^4+a)^3+1/384*x
*(-7*a*g+77*b*c+12*(-a*h+5*b*d)*x+15*(-a*i+3*b*e)*x^2)/a^3/b/(-b*x^4+a)+1/9
6*(8*a*f+x*(11*b*c-a*g+2*(-a*h+5*b*d)*x+3*(-a*i+3*b*e)*x^2))/a^2/b/(-b*x^4+
a)^2+1/32*(-a*h+5*b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)+1/256*a
rctanh(b^(1/4)*x/a^(1/4))*(15*b*e-5*a*i+7*(-a*g+11*b*c)*b^(1/2)/a^(1/2))/a^
(13/4)/b^(7/4)+1/256*arctan(b^(1/4)*x/a^(1/4))*(5*a*i-15*b*e+7*(-a*g+11*b*c
)*b^(1/2)/a^(1/2))/a^(13/4)/b^(7/4)
```

Rubi [A]
time = 0.38, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {1872, 1868, 1869, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(11bc-ag)+12x(5bd-ah)+15x^2(3be-ai)\right)}{384a^3b\left(a-bx^4\right)}+\frac{x\left(2x(5bd-ah)+3x^2(3be-ai)-ag+11bc\right)+8af}{96a^2b\left(a-bx^4\right)^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(12*a*b*(a - b*
x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a*i)*x^2)
)/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x +
3*(3*b*e - a*i)*x^2))/(96*a^2*b*(a - b*x^4)^2) + (((7*Sqrt[b]*(11*b*c - a*g
))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^
(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*a*i)*ArcTanh[(b^
(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((5*b*d - a*h)*ArcTanh[(Sqrt[b]
*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+205x^6}{\left(a-bx^4\right)^4}\,dx = \frac{x(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3)}{12ab\left(a-bx^4\right)^3} - \int \frac{-b(1}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3)}{12ab\left(a-bx^4\right)^3} + \frac{8af+}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3)}{12ab\left(a-bx^4\right)^3} + \frac{x(7(1}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3)}{12ab\left(a-bx^4\right)^3} + \frac{x(7(1}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3)}{12ab\left(a-bx^4\right)^3} + \frac{x(7(1}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3)}{12ab\left(a-bx^4\right)^3} + \frac{x(7(1}{}$$

$$= \frac{x(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3)}{12ab\left(a-bx^4\right)^3} + \frac{x(7(1}{}$$

Mathematica [A]
time = 0.23, size = 422, normalized size = 1.27

$$\frac{-\frac{4ab^{3/4}x(-77bc+7ag-15bx(4d+3ex)+3ax(4h+5ix))}{a-bx^4}-\frac{16a^2b^{3/4}x(-b(11c+x(10d+9ex))+a(g+x(2h+3ix)))}{(a-bx^4)^2}+\frac{128a^3b^{3/4}(bx(c+x(d+ex))+a(f+x(g+x(h+ix))))}{(a-bx^4)^3}+6\sqrt[4]{a}\left(77b^{3/2}c-15\sqrt{a}\,be-7a\sqrt{b}\,g+5a^{3/2}i\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+3\sqrt[4]{a}\left(-77b^{3/2}c-40\sqrt[4]{a}\,b^{5/4}d-15\sqrt{a}\,be+7a\sqrt{b}\,g+8a^{5/4}\sqrt[4]{b}\,h+5a^{3/2}i\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)-3\sqrt[4]{a}\left(-77b^{3/2}c+40\sqrt[4]{a}\,b^{5/4}d-15\sqrt{a}\,be+7a\sqrt{b}\,g-8a^{5/4}\sqrt[4]{b}\,h+5a^{3/2}i\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)-24\sqrt{a}\,\sqrt[4]{b}\,(-5bd+ah)\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{1536a^4b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x
]
```

```
[Out] ((-4*a*b^(3/4)*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i
*x)))/(a - b*x^4) - (16*a^2*b^(3/4)*x*(-(b*(11*c + x*(10*d + 9*e*x))) + a*(
g + x*(2*h + 3*i*x))))/(a - b*x^4)^2 + (128*a^3*b^(3/4)*(b*x*(c + x*(d + e*
x)) + a*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^3 + 6*a^(1/4)*(77*b^(3/2)*c
 - 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)
] + 3*a^(1/4)*(-77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*b*e + 7*a*
Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] - 3
*a^(1/4)*(-77*b^(3/2)*c + 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*b*e + 7*a*Sqrt[
b]*g - 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 24*Sqr
t[a]*b^(1/4)*(-5*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(1536*a^4*b^(7/4))
```

Maple [A]
time = 0.37, size = 346, normalized size = 1.05

method	result
risch	$\frac{-\frac{5(ai-3be)b\,x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}+\frac{7(ai-3be)x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{(5ai+113be)x^3}{384ab}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7ag+51bc}{128ab}}{(-b\,x^4+a)^3}$
default	$\frac{-\frac{5(ai-3be)b\,x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}+\frac{7(ai-3be)x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{(5ai+113be)x^3}{384ab}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7ag+51bc}{128ab}}{(-b\,x^4+a)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERB
OSE)
```

```
[Out] (-5/128*(a*i-3*b*e)/a^3*b*x^11-1/32*(a*h-5*b*d)/a^3*b*x^10-7/384*(a*g-11*b*
c)/a^3*b*x^9+7/64*(a*i-3*b*e)/a^2*x^7+1/12/a^2*(a*h-5*b*d)*x^6+3/64/a^2*(a*
g-11*b*c)*x^5+1/384*(5*a*i+113*b*e)/a/b*x^3+1/32*(a*h+11*b*d)/a/b*x^2+1/128
*(7*a*g+51*b*c)/a/b*x+1/12*f/b)/(-b*x^4+a)^3+1/128/a^3/b*(1/4*(-7*a*g+77*b*
c)*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4
)))+1/4*(-8*a*h+40*b*d)/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/
2)))-1/4*(-5*a*i+15*b*e)/b/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)
^(1/4))/(x-(a/b)^(1/4)))))
```

Maxima [A]
time = 0.53, size = 429, normalized size = 1.30

$$-\frac{15\,(3\,b^3e-i\,ab^2)x^{11}+12\,(5\,b^3d-ab^2h)x^{10}+7\,(11\,b^3c-ab^2g)x^9-42\,(3\,ab^2e-i\,a^2b)x^7-32\,(5\,ab^2d-a^2bh)x^6-18\,(11\,ab^2c-a^2bg)x^5+32\,a^3f+(113\,a^2be+5i\,a^3)x^3+12\,(11\,a^2bd+a^3h)x^2+3\,(51\,a^2bc+7\,a^3g)x}{384\,(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2x^4-a^6b)}+\frac{\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77\,b^{\frac{3}{2}}c-7\,a\sqrt{b}\,g-15\,\sqrt{a}\,be+5i\,a^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c-7\,a\sqrt{b}\,g+15\,\sqrt{a}\,be-5i\,a^{\frac{3}{2}}\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{512\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="
maxima")
```

```
[Out] -1/384*(15*(3*b^3*e - I*a*b^2)*x^11 + 12*(5*b^3*d - a*b^2*h)*x^10 + 7*(11*b
^3*c - a*b^2*g)*x^9 - 42*(3*a*b^2*e - I*a^2*b)*x^7 - 32*(5*a*b^2*d - a^2*b*
h)*x^6 - 18*(11*a*b^2*c - a^2*b*g)*x^5 + 32*a^3*f + (113*a^2*b*e + 5*I*a^3)
*x^3 + 12*(11*a^2*b*d + a^3*h)*x^2 + 3*(51*a^2*b*c + 7*a^3*g)*x)/(a^3*b^4*x
^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b) + 1/512*(8*(5*b*d - a*h)*log(s
qrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 8*(5*b*d - a*h)*log(sqrt(b)*x^2 -
 sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(77*b^(3/2)*c - 7*a*sqrt(b)*g - 15*sqrt(a)*
b*e + 5*I*a^(3/2))*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) - (77*b^(3/2)*c - 7*a*sqrt(b)*g + 15*sqrt(a)*b*e -
```

```
5*I*a^(3/2))*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt
(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3*b)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(285) = 570.
time = 0.57, size = 717, normalized size = 2.17

$$-\frac{\sqrt{2}\left(77\,b^2c-7\,abg-40\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd+8\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah+15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{\sqrt{2}\left(77\,b^2c-7\,abg+40\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd-8\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah-15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{\sqrt{2}\left(77\,b^2c-7\,abg-15\,\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^3\right)^{\frac{3}{4}}a^3}+\frac{\sqrt{2}\left(77\,b^2c-7\,abg-15\,\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{5i\,\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3b^4}-\frac{5i\,\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3b^4}+\frac{5i\,\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^3b^4}-\frac{5i\,\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^3b^4}-\frac{45\,b^3x^{11}e+60\,b^3dx^{10}-12\,ab^2hx^{10}-15i\,ab^2x^{11}+77\,b^3cx^9-7\,ab^2gx^9-126\,ab^2x^7e-160\,ab^2dx^6+32\,a^2bhx^6+42i\,a^2bx^7-198\,ab^2cx^5+18\,a^2bgx^5+113\,a^2bx^3e+132\,a^2bdx^2+12\,a^3hx^2+5i\,a^3x^3+153\,a^2bcx+21\,a^3gx+32\,a^3f}{384\left(bx^4-a\right)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="
giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 7*a*b*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 8*sqrt
(2)*(-a*b^3)^(1/4)*a*h + 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2
*c - 7*a*b*g + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 8*sqrt(2)*(-a*b^3)^(1/4)*a*h
 - 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b
)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqr
t(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)
*a^3) + 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqrt(-a*b)*b*e)*log(x^2 - s
qrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) - 5/512*I*sqrt(2)*
(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4)
)/(a^3*b^4) - 5/512*I*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^4) + 5/1024*I*sqrt(2)*(-a*b^3)^(3/4)
```

```
*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^4) - 5/1024*I*sqrt(2
)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^4) -
 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 - 12*a*b^2*h*x^10 - 15*I*a*b^2*x^11 +
 77*b^3*c*x^9 - 7*a*b^2*g*x^9 - 126*a*b^2*x^7*e - 160*a*b^2*d*x^6 + 32*a^2*
b*h*x^6 + 42*I*a^2*b*x^7 - 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^3
*e + 132*a^2*b*d*x^2 + 12*a^3*h*x^2 + 5*I*a^3*x^3 + 153*a^2*b*c*x + 21*a^3*
g*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

Mupad [B]
time = 6.14, size = 2500, normalized size = 7.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x)
```

```
[Out] (f/(12*b) - (3*x^5*(11*b*c - a*g))/(64*a^2) - (x^6*(5*b*d - a*h))/(12*a^2)
- (7*x^7*(3*b*e - a*i))/(64*a^2) + (7*b*x^9*(11*b*c - a*g))/(384*a^3) + (x*
(51*b*c + 7*a*g))/(128*a*b) + (b*x^10*(5*b*d - a*h))/(32*a^3) + (5*b*x^11*(
3*b*e - a*i))/(128*a^3) + (x^2*(11*b*d + a*h))/(32*a*b) + (x^3*(113*b*e + 5
*a*i))/(384*a*b))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log
((125*a^4*i^3 - 3375*a*b^3*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^2*e - 4928*
a^2*b^2*c*h^2 + 735*a^2*b^2*e*g^2 + 3375*a^2*b^2*e^2*i + 11200*a*b^3*d^2*g
- 29645*a*b^3*c^2*i - 1125*a^3*b*e*i^2 + 448*a^3*b*g*h^2 - 245*a^3*b*g^2*i
+ 5390*a^2*b^2*c*g*i - 4480*a^2*b^2*d*g*h + 49280*a*b^3*c*d*h - 16170*a*b^3
*c*e*g)/(2097152*a^9*b^2) - root(68719476736*a^15*b^7*z^4 - 1211105280*a^8*
b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 11010
0480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i*z^2 - 838860800*a^8*b^6*d^2*z^
2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*
c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 12288000*a^6*b^4*d*e*i*z + 485703680*a
^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816
*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z + 4014
080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z + 89600*a^4*b^2*d*g*h*i - 98
5600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h + 29
56800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i + 98
560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2 - 53900*a^4*b^2*c*g*i^2 - 17787
00*a^2*b^4*c^2*e*i + 2464000*a^2*b^4*c*d^2*i + 672000*a^2*b^4*d^2*e*g - 295
680*a^3*b^3*c*e*h^2 - 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000
*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g - 33750*a^4*b^2*e^
2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2
*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i -
 2048000*a^2*b^4*d^3*h - 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 26680
50*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^4 - 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 +
2560000*a*b^5*d^4 - 625*a^6*i^4 - 35153041*b^6*c^4, z, l)*(root(68719476736
*a^15*b^7*z^4 - 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 33
```

5544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i
*z^2 - 838860800*a^8*b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*
b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 1228800
0*a^6*b^4*d*e*i*z + 485703680*a^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97
140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z +
 2048000*a^7*b^3*d*i^2*z + 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2
*z + 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*
i - 268800*a^3*b^3*d*e*g*h + 2956800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*
i - 224000*a^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2
- 53900*a^4*b^2*c*g*i^2 - 1778700*a^2*b^4*c^2*e*i + 2464000*a^2*b^4*c*d^2*i
 + 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 - 485100*a^2*b^4*c*e^2*g
 - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924
*a*b^5*c^3*g - 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*
b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*
g^2*i^2 + 67500*a^3*b^3*e^3*i - 2048000*a^2*b^4*d^3*h - 81920*a^4*b^2*d*h^3
 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^4 - 50625*
a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 - 625*a^6*i^4 - 35153041*b
^6*c^4, z, l)*((20185088*a^7*b^5*c - 1835008*a^8*b^4*g)/(2097152*a^9*b^2) -
 (x*(655360*a^7*b^4*d - 131072*a^8*b^3*h))/(131072*a^9*b)) - (614400*a^4*b^
4*d*e - 204800*a^5*b^3*d*i - 122880*a^5*b^3*e*h + 40960*a^6*b^2*h*i)/(20971
52*a^9*b^2) + (x*(800*a^6*b*i^2 + 189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 + 1
568*a^5*b^2*g^2 - 34496*a^4*b^3*c*g - 4800*a^5*b^2*e*i))/(131072*a^9*b)) -
(x*(4000*b^3*d^3 - 32*a^3*h^3 - 5775*b^3*c*d*e + 35*a^3*g*h*i - 2400*a*b^2*
d^2*h + 480*a^2*b*d*h^2 + 1925*a*b^2*c*d*i + 1155*a*b^2*c*e*h + 525*a*b^2*d
*e*g - 385*a^2*b*c*h*i - 175*a^2*b*d*g*i - 105*a^2*b*e*g*h))/(131072*a^9*b)
)*root(68719476736*a^15*b^7*z^4 - 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^
9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 - 367
00160*a^10*b^4*g*i*z^2 - 838860800*a^8*b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*
z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z + 17661952*a^6*b^4
*c*g*h*z - 12288000*a^6*b^4*d*e*i*z + 485703680*a^4*b^6*c^2*d*z - 409600*a^
8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z - 3686400
*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z + 4014080*a^6*b^4*d*g^2*z + 1843
2000*a^5*b^5*d*e^2*z + 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323
400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h + 2956800*a^2*b^4*c*d*e*h - 14
700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^2*i + 2688
0*a^4*b^2*e*g*h^2 - 53900*a^4*b^2*c*g*i^2 - 1778700*a^2*b^4*c^2*e*i + 24640
00*a^2*b^4*c*d^2*i + 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 - 4851
00*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 73920...

3.206 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx$

Optimal. Leaf size=349

$$\frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{12ab\,(a-bx^4)^3}+\frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b\,(a-bx^4)}$$

```
[Out] 1/12*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+(a*j+b*f)*x^3)/a/b/(-b*x^4+a)^3+1
/384*x*(-7*a*g+77*b*c+12*(-a*h+5*b*d)*x+15*(-a*i+3*b*e)*x^2)/a^3/b/(-b*x^4+
a)+1/96*(4*a*(-a*j+2*b*f)+x*(b*(-a*g+11*b*c)+2*b*(-a*h+5*b*d)*x+3*b*(-a*i+3
*b*e)*x^2))/a^2/b^2/(-b*x^4+a)^2+1/32*(-a*h+5*b*d)*arctanh(x^2*b^(1/2)/a^(1
/2))/a^(7/2)/b^(3/2)+1/256*arctanh(b^(1/4)*x/a^(1/4))*(15*b*e-5*a*i+7*(-a*g
+11*b*c)*b^(1/2)/a^(1/2))/a^(13/4)/b^(7/4)+1/256*arctan(b^(1/4)*x/a^(1/4))*
(5*a*i-15*b*e+7*(-a*g+11*b*c)*b^(1/2)/a^(1/2))/a^(13/4)/b^(7/4)
```

Rubi [A]
time = 0.39, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1872, 1868, 1869, 1890, 281, 214, 1181, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}\,(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}\,(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x(7(11bc-ag)+12x(5bd-ah)+15x^2(3be-ai))}{384a^3b\,(a-bx^4)}+\frac{x(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai))+4a(2bf-aj)}{96a^2b^2\,(a-bx^4)^2}+\frac{x(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc)}{12ab\,(a-bx^4)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^4
,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(12*a*b
*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a
*i)*x^2))/(384*a^3*b*(a - b*x^4)) + (4*a*(2*b*f - a*j) + x*(b*(11*b*c - a*g
) + 2*b*(5*b*d - a*h)*x + 3*b*(3*b*e - a*i)*x^2))/(96*a^2*b^2*(a - b*x^4)^2
) + (((7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)
*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g)
)/Sqrt[a] - 5*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + (
(5*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+206x^6+jx^7}{\left(a-bx^4\right)^4}\,dx = \frac{x(bc+ag+(bd+ah)x+(206a+be)x^2+(bf+a}{12ab\left(a-bx^4\right)^3}$$

$$= \frac{x(bc+ag+(bd+ah)x+(206a+be)x^2+(bf+a}{12ab\left(a-bx^4\right)^3}$$

$$= \frac{x(bc+ag+(bd+ah)x+(206a+be)x^2+(bf+a}{12ab\left(a-bx^4\right)^3}$$

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$$= \frac{x(bc+ag+(bd+ah)x+(206a+be)x^2+(bf+a}{12ab\left(a-bx^4\right)^3}$$

Mathematica [A]
time = 0.23, size = 439, normalized size = 1.26

$$\frac{-\frac{4abx(-77bc+7ag-15bx(4d+3ex)+3ax(4h+5ix))}{a-bx^4}-\frac{16a^2(12a^2j-b^2x(11c+x(10d+9ex))+abx(g+x(2h+3ix)))}{(a-bx^4)^2}+\frac{128a^3(a^2j+b^2x(c+x(d+ex))+ab(f+x(g+x(h+ix))))}{(a-bx^4)^3}+6\sqrt[4]{a}\sqrt[4]{b}\left(77b^{3/2}c-15\sqrt{a}\,be-7a\sqrt{b}\,g+5a^{3/2}i\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+3\sqrt[4]{a}\sqrt[4]{b}\left(-77b^{3/2}c-40\sqrt[4]{a}\,b^{5/4}d-15\sqrt{a}\,be+7a\sqrt{b}\,g+8a^{5/4}\sqrt[4]{b}\,h+5a^{3/2}i\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+3\sqrt[4]{a}\sqrt[4]{b}\left(77b^{3/2}c-40\sqrt[4]{a}\,b^{5/4}d+15\sqrt{a}\,be-7a\sqrt{b}\,g+8a^{5/4}\sqrt[4]{b}\,h-5a^{3/2}i\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)-24\sqrt{a}\sqrt{b}\,(-5bd+ah)\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{1536a^4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4)^4,x]
```

```
[Out] ((-4*a*b*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*x)))/
(a - b*x^4) - (16*a^2*(12*a^2*j - b^2*x*(11*c + x*(10*d + 9*e*x)) + a*b*x*(
g + x*(2*h + 3*i*x))))/(a - b*x^4)^2 + (128*a^3*(a^2*j + b^2*x*(c + x*(d +
e*x)) + a*b*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^3 + 6*a^(1/4)*b^(1/4)*(
77*b^(3/2)*c - 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*ArcTan[(b^(1/4
)*x)/a^(1/4)] + 3*a^(1/4)*b^(1/4)*(-77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d - 1
5*Sqrt[a]*b*e + 7*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1
/4) - b^(1/4)*x] + 3*a^(1/4)*b^(1/4)*(77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d +
 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h - 5*a^(3/2)*i)*Log[a^
```

```
(1/4) + b^(1/4)*x] - 24*Sqrt[a]*Sqrt[b]*(-5*b*d + a*h)*Log[Sqrt[a] + Sqrt[b
]*x^2])/(1536*a^4*b^2)
```

Maple [A]

time = 0.37, size = 362, normalized size = 1.04

method	result
risch	$\frac{-\frac{5(ai-3be)b\,x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}+\frac{7(ai-3be)x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{j\,x^4}{8b}+\frac{(5ai+113be)x^3}{384ab}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7ag+}{12}}{\left(-b\,x^4+a\right)^3}$
default	$\frac{-\frac{5(ai-3be)b\,x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}+\frac{7(ai-3be)x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{j\,x^4}{8b}+\frac{(5ai+113be)x^3}{384ab}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7ag+}{12}}{\left(-b\,x^4+a\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETU
RNVERBOSE)
```

```
[Out] (-5/128*(a*i-3*b*e)/a^3*b*x^11-1/32*(a*h-5*b*d)/a^3*b*x^10-7/384*(a*g-11*b*
c)/a^3*b*x^9+7/64*(a*i-3*b*e)/a^2*x^7+1/12/a^2*(a*h-5*b*d)*x^6+3/64/a^2*(a*
g-11*b*c)*x^5+1/8*j*x^4/b+1/384*(5*a*i+113*b*e)/a/b*x^3+1/32*(a*h+11*b*d)/a
/b*x^2+1/128*(7*a*g+51*b*c)/a/b*x-1/24*(a*j-2*b*f)/b^2)/(-b*x^4+a)^3+1/128/
a^3/b*(1/4*(-7*a*g+77*b*c)*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)
))+2*arctan(x/(a/b)^(1/4)))+1/4*(-8*a*h+40*b*d)/(a*b)^(1/2)*ln((a+x^2*(a*b)
^(1/2))/(a-x^2*(a*b)^(1/2)))-1/4*(-5*a*i+15*b*e)/b/(a/b)^(1/4)*(2*arctan(x/
(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))))
```

Maxima [A]

time = 0.53, size = 463, normalized size = 1.33

$$-\frac{15\,(3\,b^4e-i\,ab^3)x^{11}+12\,(5\,b^4d-ab^3h)x^{10}+7\,(11\,b^4c-ab^3g)x^9+48\,a^3bjx^4-42\,(3\,ab^3e-i\,a^2b^2)x^7-32\,(5\,ab^3d-a^2b^2h)x^6-18\,(11\,ab^3c-a^2b^2g)x^5+32\,a^3bf-16\,a^4j+(113\,a^2b^2e+5i\,a^3b)x^3+12\,(11\,a^2b^2d+a^3bh)x^2+3\,(51\,a^2b^2c+7\,a^3bg)x}{384\,(a^3b^5x^{12}-3\,a^4b^4x^8+3\,a^5b^3x^4-a^6b^2)}+\frac{\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(77\,b^{\frac{3}{2}}c-7\,a\sqrt{b}\,g-15\,\sqrt{a}\,be+5i\,a^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c-7\,a\sqrt{b}\,g+15\,\sqrt{a}\,be-5i\,a^{\frac{3}{2}}\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{512\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algor
ithm="maxima")
```

```
[Out] -1/384*(15*(3*b^4*e - I*a*b^3)*x^11 + 12*(5*b^4*d - a*b^3*h)*x^10 + 7*(11*b
^4*c - a*b^3*g)*x^9 + 48*a^3*b*j*x^4 - 42*(3*a*b^3*e - I*a^2*b^2)*x^7 - 32*
(5*a*b^3*d - a^2*b^2*h)*x^6 - 18*(11*a*b^3*c - a^2*b^2*g)*x^5 + 32*a^3*b*f
- 16*a^4*j + (113*a^2*b^2*e + 5*I*a^3*b)*x^3 + 12*(11*a^2*b^2*d + a^3*b*h)*
x^2 + 3*(51*a^2*b^2*c + 7*a^3*b*g)*x)/(a^3*b^5*x^12 - 3*a^4*b^4*x^8 + 3*a^5
*b^3*x^4 - a^6*b^2) + 1/512*(8*(5*b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sq
rt(a)*sqrt(b)) - 8*(5*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b
```

```
)) + 2*(77*b^(3/2)*c - 7*a*sqrt(b)*g - 15*sqrt(a)*b*e + 5*I*a^(3/2))*arctan
(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) -
 (77*b^(3/2)*c - 7*a*sqrt(b)*g + 15*sqrt(a)*b*e - 5*I*a^(3/2))*log((sqrt(b)
*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3*b)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,
x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 749 vs. 2(303) = 606.
time = 0.56, size = 749, normalized size = 2.15

$$-\frac{\sqrt{2}\left(77\,b^2c-7\,abg-40\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd+8\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah+15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{\sqrt{2}\left(77\,b^2c-7\,abg+40\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd-8\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah-15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{\sqrt{2}\left(77\,b^2c-7\,abg-15\,\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^3\right)^{\frac{3}{4}}a^3}+\frac{\sqrt{2}\left(77\,b^2c-7\,abg-15\,\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{5i\,\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3b^4}-\frac{5i\,\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3b^4}+\frac{5i\,\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^3b^4}-\frac{5i\,\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^3b^4}-\frac{45\,b^4x^{11}e+60\,b^4dx^{10}-12\,ab^3hx^{10}-15i\,ab^3x^{11}+77\,b^4cx^9-7\,ab^3gx^9-126\,ab^3x^7e-160\,ab^3dx^6+32\,a^2b^2hx^6+42i\,a^2b^2x^7-198\,ab^3cx^5+18\,a^2b^2gx^5+48\,a^3bjx^4+113\,a^2b^2x^3e+132\,a^2b^2dx^2+12\,a^3bhx^2+5i\,a^3bx^3+153\,a^2b^2cx+21\,a^3bgx+32\,a^3bf-16\,a^4j}{384\left(bx^4-a\right)^3a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algor
ithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 7*a*b*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 8*sqrt
(2)*(-a*b^3)^(1/4)*a*h + 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2
*c - 7*a*b*g + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 8*sqrt(2)*(-a*b^3)^(1/4)*a*h
 - 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b
)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqr
t(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)
*a^3) + 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqrt(-a*b)*b*e)*log(x^2 - s
```

```
qrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) - 5/512*I*sqrt(2)*
(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4)
)/(a^3*b^4) - 5/512*I*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2sqrt(2)*(2*x - sqrt
(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^4) + 5/1024*I*sqrt(2)*(-a*b^3)^(3/4)
*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^4) - 5/1024*I*sqrt(2
)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^4) -
 1/384*(45*b^4*x^11*e + 60*b^4*d*x^10 - 12*a*b^3*h*x^10 - 15*I*a*b^3*x^11 +
 77*b^4*c*x^9 - 7*a*b^3*g*x^9 - 126*a*b^3*x^7*e - 160*a*b^3*d*x^6 + 32*a^2*
b^2*h*x^6 + 42*I*a^2*b^2*x^7 - 198*a*b^3*c*x^5 + 18*a^2*b^2*g*x^5 + 48*a^3*
b*j*x^4 + 113*a^2*b^2*x^3*e + 132*a^2*b^2*d*x^2 + 12*a^3*b*h*x^2 + 5*I*a^3*
b*x^3 + 153*a^2*b^2*c*x + 21*a^3*b*g*x + 32*a^3*b*f - 16*a^4*j)/((b*x^4 - a
)^3*a^3*b^2)
```

Mupad [B]
time = 6.40, size = 2500, normalized size = 7.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^4
,x)
```

```
[Out] symsum(log((125*a^4*i^3 - 3375*a*b^3*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^2
*e - 4928*a^2*b^2*c*h^2 + 735*a^2*b^2*e*g^2 + 3375*a^2*b^2*e^2*i + 11200*a*
b^3*d^2*g - 29645*a*b^3*c^2*i - 1125*a^3*b*e*i^2 + 448*a^3*b*g*h^2 - 245*a^
3*b*g^2*i + 5390*a^2*b^2*c*g*i - 4480*a^2*b^2*d*g*h + 49280*a*b^3*c*d*h - 1
6170*a*b^3*c*e*g)/(2097152*a^9*b^2) - root(68719476736*a^15*b^7*z^4 - 12111
05280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z
^2 + 110100480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i*z^2 - 838860800*a^8*
b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 8830976
0*a^5*b^5*c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 12288000*a^6*b^4*d*e*i*z + 4
85703680*a^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*
z - 802816*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^
2*z + 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z + 89600*a^4*b^2*d*
g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*
e*g*h + 2956800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^
2*g*i + 98560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2 - 53900*a^4*b^2*c*g*i
^2 - 1778700*a^2*b^4*c^2*e*i + 2464000*a^2*b^4*c*d^2*i + 672000*a^2*b^4*d^2
*e*g - 295680*a^3*b^3*c*e*h^2 - 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i
 - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g - 33750*
a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a
^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b
^3*e^3*i - 2048000*a^2*b^4*d^3*h - 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g
^3 + 2668050*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^4 - 50625*a^2*b^4*e^4 + 4096*a^
5*b*h^4 + 2560000*a*b^5*d^4 - 625*a^6*i^4 - 35153041*b^6*c^4, z, m)*(root(6
```

8719476736*a^15*b^7*z^4 - 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i*z^2 - 838860800*a^8*b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 12288000*a^6*b^4*d*e*i*z + 485703680*a^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z + 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z + 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h + 2956800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2 - 53900*a^4*b^2*c*g*i^2 - 1778700*a^2*b^4*c^2*e*i + 2464000*a^2*b^4*c*d^2*i + 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 - 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g - 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i - 2048000*a^2*b^4*d^3*h - 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^4 - 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 - 625*a^6*i^4 - 35153041*b^6*c^4, z, m)*((20185088*a^7*b^5*c - 1835008*a^8*b^4*g)/(2097152*a^9*b^2) - (x*(655360*a^7*b^4*d - 131072*a^8*b^3*h))/(131072*a^9*b)) - (614400*a^4*b^4*d*e - 204800*a^5*b^3*d*i - 122880*a^5*b^3*e*h + 40960*a^6*b^2*h*i)/(2097152*a^9*b^2) + (x*(800*a^6*b*i^2 + 189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 + 1568*a^5*b^2*g^2 - 34496*a^4*b^3*c*g - 4800*a^5*b^2*e*i))/(131072*a^9*b)) - (x*(4000*b^3*d^3 - 32*a^3*h^3 - 5775*b^3*c*d*e + 35*a^3*g*h*i - 2400*a*b^2*d^2*h + 480*a^2*b*d*h^2 + 1925*a*b^2*c*d*i + 1155*a*b^2*c*e*h + 525*a*b^2*d*e*g - 385*a^2*b*c*h*i - 175*a^2*b*d*g*i - 105*a^2*b*e*g*h))/(131072*a^9*b))*root(68719476736*a^15*b^7*z^4 - 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i*z^2 - 838860800*a^8*b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 12288000*a^6*b^4*d*e*i*z + 485703680*a^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z + 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z + 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h + 2956800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2 - 53900*a^4*b^2*c*g*i^2 - 1778700*a^2*b^4*c^2*e*i + 2464000*a^2*b^4*c*d^2*i + 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 - 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g - 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i - 2048000*a^2*b^4*d^3*h - 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^4 - 50625*a^2*b^4*e^4 + 4096*...

3.207 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=462

$$\frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\,(a+bx^4)^3}+\frac{x(7(11bc+ag)+12(5bd+ah)x+45bex^2)}{384a^3b\,(a+bx^4)}-\frac{8af-x(11bc+ag+}{96a^2b\,(a}$$

```
[Out] 1/12*x*(b*c-a*g+(-a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(b*x^4+a)^3+1/384*x*(7*a*
g+77*b*c+12*(a*h+5*b*d)*x+45*b*e*x^2)/a^3/b/(b*x^4+a)+1/96*(-8*a*f+x*(11*b*
c+a*g+2*(a*h+5*b*d)*x+9*b*e*x^2))/a^2/b/(b*x^4+a)^2+1/32*(a*h+5*b*d)*arctan
(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*b^(1/2))*(77*b*c+7*a*g-15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2
^(1/2)+1/1024*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(77*b*c+7*a
*g-15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x
*2^(1/2)/a^(1/4))*(77*b*c+7*a*g+15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1
/2)+1/512*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(77*b*c+7*a*g+15*e*a^(1/2)*b^
(1/2))/a^(15/4)/b^(5/4)*2^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {1872, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+5bd)}{32a^{7/2}b^{3/2}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{x(7(ag+11bc)+12x(ah+5bd)+45bex^2)}{384a^3b\,(a+bx^4)}-\frac{8af-x(2x(ah+5bd)+ag+11bc+9bex^2)}{96a^2b\,(a+bx^4)^2}+\frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{12ab\,(a+bx^4)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3)
+ (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a +
b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x + 9*b*e*x^2))/(96*a^
2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7
/2)*b^(3/2)) - ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a
]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*
a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) +
((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx = \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\,(a+bx^4)^3} - \frac{\int \frac{-b(11bc+ag)-2b(5bd+ah)x}{(a+bx^4)^3}}{12ab^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\,(a+bx^4)^3} - \frac{8af-x(11bc+ag+2}{96a^2b\,(a}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5}{384a^3b\,(a}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5}{384a^3b\,(a}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5}{384a^3b\,(a}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5}{384a^3b\,(a}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5}{384a^3b\,(a}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5}{384a^3b\,(a}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5}{384a^3b\,(a}$$

Mathematica [A]
time = 0.30, size = 461, normalized size = 1.00

$$\frac{\frac{8a^{3/4}\sqrt{b}\,x(77bc+7ag+60bdx+12ahx+45bex^2)}{a+bx^4}+\frac{32a^{7/4}\sqrt{b}\,x(11bc+bx(10d+9ex)+a(g+2hx))}{(a+bx^4)^2}-\frac{256a^{11/4}\sqrt{b}\,(-bx(c+x(d+ex))+a(f+x(g+hx)))}{(a+bx^4)^3}-6\left(77\sqrt{2}\,b^{5/4}c+80\sqrt[4]{a}\,bd+15\sqrt{2}\,\sqrt{a}\,b^{3/4}e+7\sqrt{2}\,a\sqrt[4]{b}\,g+16a^{5/4}h\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+6\left(77\sqrt{2}\,b^{5/4}c-80\sqrt[4]{a}\,bd+15\sqrt{2}\,\sqrt{a}\,b^{3/4}e+7\sqrt{2}\,a\sqrt[4]{b}\,g-16a^{5/4}h\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-3\sqrt{2}\,\sqrt[4]{b}\left(77bc-15\sqrt{a}\,\sqrt{b}\,e+7ag\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+3\sqrt{2}\,\sqrt[4]{b}\left(77bc-15\sqrt{a}\,\sqrt{b}\,e+7ag\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{3072a^{15/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*Sqrt[b]*x*(77*b*c + 7*a*g + 60*b*d*x + 12*a*h*x + 45*b*e*x^2))/
(a + b*x^4) + (32*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d + 9*e*x) + a*(g + 2
*h*x)))/(a + b*x^4)^2 - (256*a^(11/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a
```

```
*(f + x*(g + h*x))))/(a + b*x^4)^3 - 6*(77*Sqrt[2]*b^(5/4)*c + 80*a^(1/4)*b
*d + 15*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g + 16*a^(5/4)*h)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b^(5/4)*c - 80*a^(1/
4)*b*d + 15*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g - 16*a^(5/4)*
h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*b^(1/4)*(77*b*c - 15
*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + 3*Sqrt[2]*b^(1/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^(3/2))
```

Maple [A]

time = 0.34, size = 399, normalized size = 0.86

method	result
risch	$\frac{\frac{15 e\, b^{2} x^{11}}{128 a^{3}}+\frac{(a h+5 b d) b\, x^{10}}{32 a^{3}}+\frac{7(a g+11 b c) b\, x^{9}}{384 a^{3}}+\frac{21 b e\, x^{7}}{64 a^{2}}+\frac{(a h+5 b d) x^{6}}{12 a^{2}}+\frac{3(a g+11 b c) x^{5}}{64 a^{2}}+\frac{113 e\, x^{3}}{384 a}-\frac{(a h-11 b d) x^{2}}{32 a b}-\frac{(7 a g-51 b c) x}{128 a b}-\frac{f}{12 b}}{(b\, x^{4}+a)^{3}}+\frac{\_R\text{=Ro}}{}$
default	$\frac{\frac{15 e\, b^{2} x^{11}}{128 a^{3}}+\frac{(a h+5 b d) b\, x^{10}}{32 a^{3}}+\frac{7(a g+11 b c) b\, x^{9}}{384 a^{3}}+\frac{21 b e\, x^{7}}{64 a^{2}}+\frac{(a h+5 b d) x^{6}}{12 a^{2}}+\frac{3(a g+11 b c) x^{5}}{64 a^{2}}+\frac{113 e\, x^{3}}{384 a}-\frac{(a h-11 b d) x^{2}}{32 a b}-\frac{(7 a g-51 b c) x}{128 a b}-\frac{f}{12 b}}{(b\, x^{4}+a)^{3}}+\frac{(7ag+77}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/128*e/a^3*b^2*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c)/a^3*b
*x^9+21/64*b*e/a^2*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g+11*b*c)*x^5+1
13/384/a*e*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12*f/
b)/(b*x^4+a)^3+1/128/a^3/b*(1/8*(7*a*g+77*b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x
^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2
)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/
2*(8*a*h+40*b*d)/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+15/8*e/(a/b)^(1/4)*2^(
1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/
4)*x-1)))
```

Maxima [A]

time = 0.50, size = 524, normalized size = 1.13

$$\frac{45\, b^{3} x^{11} e + 12\,(5\, b^{3} d + a b^{2} h) x^{10} + 126\, a b^{2} x^{7} e + 7\,(11\, b^{3} c + a b^{2} g) x^{9} + 32\,(5\, a b^{2} d + a^{2} b h) x^{6} + 113\, a^{2} b x^{3} e + 18\,(11\, a b^{2} c + a^{2} b g) x^{5} - 32\, a^{3} f + 12\,(11\, a^{2} b d - a^{3} h) x^{2} + 3\,(51\, a^{2} b c - 7\, a^{3} g) x}{384\,(a^{3} b^{4} x^{12} + 3\, a^{4} b^{3} x^{8} + 3\, a^{5} b^{2} x^{4} + a^{6} b)} + \frac{\frac{\sqrt{2}\,(77\, b^{\frac{3}{2}} c + 7\, a \sqrt{b}\, g - 15\,\sqrt{a}\, b e) \log\left(\sqrt{b}\, x^{2} + \sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{3}{4}}} - \frac{\sqrt{2}\,(77\, b^{\frac{3}{2}} c + 7\, a \sqrt{b}\, g - 15\,\sqrt{a}\, b e) \log\left(\sqrt{b}\, x^{2} - \sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{3}{4}}} + \frac{2\,(77\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{7}{4}} c + 7\sqrt{2}\, a^{\frac{5}{4}} b^{\frac{3}{4}} g + 15\sqrt{2}\, a^{\frac{3}{4}} b^{\frac{5}{4}} e - 80\,\sqrt{a}\, b^{\frac{3}{2}} d - 16\, a^{\frac{3}{2}} \sqrt{b}\, h) \arctan\left(\frac{\sqrt{2}\,(2\sqrt{b}\, x + \sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}})}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}} \sqrt{\sqrt{a}\sqrt{b}}\, b^{\frac{3}{4}}} + \frac{2\,(77\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{7}{4}} c + 7\sqrt{2}\, a^{\frac{5}{4}} b^{\frac{3}{4}} g + 15\sqrt{2}\, a^{\frac{3}{4}} b^{\frac{5}{4}} e + 80\,\sqrt{a}\, b^{\frac{3}{2}} d + 16\, a^{\frac{3}{2}} \sqrt{b}\, h) \arctan\left(\frac{\sqrt{2}\,(2\sqrt{b}\, x - \sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}})}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}} \sqrt{\sqrt{a}\sqrt{b}}\, b^{\frac{3}{4}}}}{1024\, a^{3} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima"
)
```

```
[Out] 1/384*(45*b^3*x^11*e + 12*(5*b^3*d + a*b^2*h)*x^10 + 126*a*b^2*x^7*e + 7*(1
1*b^3*c + a*b^2*g)*x^9 + 32*(5*a*b^2*d + a^2*b*h)*x^6 + 113*a^2*b*x^3*e + 1
8*(11*a*b^2*c + a^2*b*g)*x^5 - 32*a^3*f + 12*(11*a^2*b*d - a^3*h)*x^2 + 3*(
51*a^2*b*c - 7*a^3*g)*x)/(a^3*b^4*x^12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^
6*b) + 1/1024*(sqrt(2)*(77*b^(3/2)*c + 7*a*sqrt(b)*g - 15*sqrt(a)*b*e)*log(
sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt
(2)*(77*b^(3/2)*c + 7*a*sqrt(b)*g - 15*sqrt(a)*b*e)*log(sqrt(b)*x^2 - sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b
^(7/4)*c + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 15*sqrt(2)*a^(3/4)*b^(5/4)*e - 80*
sqrt(a)*b^(3/2)*d - 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x +
 sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt
(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 7*sqrt(2)*a^(5/4)*b^(3/4)
*g + 15*sqrt(2)*a^(3/4)*b^(5/4)*e + 80*sqrt(a)*b^(3/2)*d + 16*a^(3/2)*sqrt(
b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(
a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^3*b)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.52, size = 521, normalized size = 1.13

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d+8\sqrt{2}\sqrt{ab}\,abh+77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d+8\sqrt{2}\sqrt{ab}\,abh+77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}-\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}+\frac{45\,b^3x^{11}e+60\,b^3dx^{10}+12\,ab^2hx^{10}+77\,b^3cx^9+7\,ab^2gx^9+126\,ab^2x^7e+160\,ab^2dx^6+32\,a^2bhx^6+198\,ab^2cx^5+18\,a^2bgx^5+113\,a^2bx^3e+132\,a^2bdx^2-12\,a^3hx^2+153\,a^2bcx-21\,a^3gx-32\,a^3f}{384\,(bx^4+a)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*
(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/
```

```
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2
)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4
)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^
3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)
*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a
/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 1
2*a*b^2*h*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*x^7*e + 160*a*b^2
*d*x^6 + 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^3*
e + 132*a^2*b*d*x^2 - 12*a^3*h*x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)
/((b*x^4 + a)^3*a^3*b)
```

Mupad [B]
time = 6.08, size = 1743, normalized size = 3.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x)
```

```
[Out] symsum(log((123200*b^3*c*d^2 - 3375*a*b^2*e^3 - 88935*b^3*c^2*e + 448*a^3*g
*h^2 + 11200*a*b^2*d^2*g + 4928*a^2*b*c*h^2 - 735*a^2*b*e*g^2 + 49280*a*b^2
*c*d*h - 16170*a*b^2*c*e*g + 4480*a^2*b*d*g*h)/(2097152*a^9*b) - root(68719
476736*a^15*b^6*z^4 + 1211105280*a^8*b^5*c*e*z^2 + 335544320*a^9*b^4*d*h*z^
2 + 110100480*a^9*b^4*e*g*z^2 + 838860800*a^8*b^5*d^2*z^2 + 33554432*a^10*b
^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z - 17661952*a^6*b^3*c*g*h*z - 48570368
0*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 802816*a^7*b^2*g^2*h*z + 368
6400*a^6*b^3*e^2*h*z - 4014080*a^6*b^3*d*g^2*z + 18432000*a^5*b^4*d*e^2*z -
 268800*a^3*b^2*d*e*g*h - 2956800*a^2*b^3*c*d*e*h - 672000*a^2*b^3*d^2*e*g
- 295680*a^3*b^2*c*e*h^2 + 485100*a^2*b^3*c*e^2*g - 26880*a^4*b*e*g*h^2 - 7
392000*a*b^4*c*d^2*e + 81920*a^4*b*d*h^3 + 12782924*a*b^4*c^3*g + 614400*a^
3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 + 1743126*a^2*b^3*c^2*g^2 + 2048000*a
^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*b^4*c^2*e^2 + 50625*a^2*b^3
*e^4 + 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096*a^5*h^4 + 35153041*b^5*c^4
, z, k)*(root(68719476736*a^15*b^6*z^4 + 1211105280*a^8*b^5*c*e*z^2 + 33554
4320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 + 838860800*a^8*b^5*d^2*z^
2 + 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z - 17661952*a^6*b^3
*c*g*h*z - 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 802816*a^
7*b^2*g^2*h*z + 3686400*a^6*b^3*e^2*h*z - 4014080*a^6*b^3*d*g^2*z + 1843200
0*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h - 2956800*a^2*b^3*c*d*e*h - 6720
00*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 + 485100*a^2*b^3*c*e^2*g - 2688
0*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e + 81920*a^4*b*d*h^3 + 12782924*a*b^
4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 + 1743126*a^2*b^3*
c^2*g^2 + 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*b^4*c^2*
```

```
e^2 + 50625*a^2*b^3*e^4 + 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096*a^5*h^4
 + 35153041*b^5*c^4, z, k)*((20185088*a^7*b^4*c + 1835008*a^8*b^3*g)/(20971
52*a^9*b) - (x*(655360*a^7*b^4*d + 131072*a^8*b^3*h))/(131072*a^9*b)) + (61
4400*a^4*b^3*d*e + 122880*a^5*b^2*e*h)/(2097152*a^9*b) + (x*(189728*a^3*b^4
*c^2 - 7200*a^4*b^3*e^2 + 1568*a^5*b^2*g^2 + 34496*a^4*b^3*c*g))/(131072*a^
9*b)) + (x*(4000*b^3*d^3 + 32*a^3*h^3 - 5775*b^3*c*d*e + 2400*a*b^2*d^2*h +
 480*a^2*b*d*h^2 - 1155*a*b^2*c*e*h - 525*a*b^2*d*e*g - 105*a^2*b*e*g*h))/(
131072*a^9*b))*root(68719476736*a^15*b^6*z^4 + 1211105280*a^8*b^5*c*e*z^2 +
 335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 + 838860800*a^8*b^5*
d^2*z^2 + 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z - 17661952*a
^6*b^3*c*g*h*z - 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 802
816*a^7*b^2*g^2*h*z + 3686400*a^6*b^3*e^2*h*z - 4014080*a^6*b^3*d*g^2*z + 1
8432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h - 2956800*a^2*b^3*c*d*e*h
- 672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 + 485100*a^2*b^3*c*e^2*g
- 26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e + 81920*a^4*b*d*h^3 + 1278292
4*a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 + 1743126*a^
2*b^3*c^2*g^2 + 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*b^
4*c^2*e^2 + 50625*a^2*b^3*e^4 + 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096*a
^5*h^4 + 35153041*b^5*c^4, z, k), k, 1, 4) + ((113*e*x^3)/(384*a) - f/(12*b
) + (3*x^5*(11*b*c + a*g))/(64*a^2) + (x^6*(5*b*d + a*h))/(12*a^2) + (7*b*x
^9*(11*b*c + a*g))/(384*a^3) + (x*(51*b*c - 7*a*g))/(128*a*b) + (b*x^10*(5*
b*d + a*h))/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (x^2*(11*b*d - a*h))/(32
*a*b) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

3.208 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=516

$$\frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{12ab\,(a+bx^4)^3}+\frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)}-\frac{8af-}{}$$

```
[Out] 1/12*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+b*f*x^3)/a/b/(b*x^4+a)^3+1/384*
x*(7*a*g+77*b*c+12*(a*h+5*b*d)*x+15*(a*i+3*b*e)*x^2)/a^3/b/(b*x^4+a)+1/96*(
-8*a*f+x*(11*b*c+a*g+2*(a*h+5*b*d)*x+3*(a*i+3*b*e)*x^2))/a^2/b/(b*x^4+a)^2+
1/32*(a*h+5*b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)-1/1024*ln(-a^(
1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*(a*i+3*b*e)*a^(1/2)+7*(a*g+
11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)+1/1024*ln(a^(1/4)*b^(1/4)*x*2^(1/
2)+a^(1/2)+x^2*b^(1/2))*(-5*(a*i+3*b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))/a^(
15/4)/b^(7/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*(a*i+3*
b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)+1/512*arctan(
1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*(a*i+3*b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))
/a^(15/4)/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 516, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1872, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+5bd)}{32a^{7/2}b^{3/2}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{x(7(ag+11bc)+12x(ah+5bd)+15x^2(ai+3be))}{384a^3b\,(a+bx^4)}-\frac{8af-x(2x(ah+5bd)+3x^2(ai+3be)+ag+11bc)}{96a^2b\,(a+bx^4)^2}+\frac{x(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3)}{12ab\,(a+bx^4)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(12*a*b*(a + b*
x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a*i)*x^2)
)/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x +
3*(3*b*e + a*i)*x^2))/(96*a^2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqr
t[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((7*Sqrt[b]*(11*b*c + a*g) + 5*S
qrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]
*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) - (
(7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt
[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+208x^6}{(a+bx^4)^4}\,dx = \frac{x(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3)}{12ab\,(a+bx^4)^3} - \frac{\int -b \cdots}{\cdots}$$

$$= \frac{x(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3)}{12ab\,(a+bx^4)^3} - \frac{8af \cdots}{\cdots}$$

$$= \frac{x(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(\cdots}{\cdots}$$

$$= \frac{x(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(\cdots}{\cdots}$$

$$= \frac{x(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(\cdots}{\cdots}$$

$$= \frac{x(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(\cdots}{\cdots}$$

$$= \frac{x(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(\cdots}{\cdots}$$

$$= \frac{x(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(\cdots}{\cdots}$$

$$= \frac{x(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(\cdots}{\cdots}$$

Mathematica [A]
time = 0.44, size = 530, normalized size = 1.03

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x
]
```

```
[Out] ((32*a^(7/4)*b^(3/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x) + a*x*(2*h + 3*i*
x)))/(a + b*x^4)^2 + (8*a^(3/4)*b^(3/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3
```

```
*e*x) + 3*a*x*(4*h + 5*i*x)))/(a + b*x^4) - (256*a^(11/4)*b^(3/4)*(-(b*x*(c
 + x*(d + e*x))) + a*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^3 - 6*(77*Sqrt
[2]*b^(3/2)*c + 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a
*Sqrt[b]*g + 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b^(3/2)*c - 80*a^(1/4)*b^(5/4)*d + 15
*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g - 16*a^(5/4)*b^(1/4)*h + 5*Sqr
t[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*(-77*b^
(3/2)*c + 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]*(77*b^(3/2)*c - 15*Sqrt[a]*
b*e + 7*a*Sqrt[b]*g - 5*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2])/(3072*a^(15/4)*b^(7/4))
```

Maple [A]
time = 0.36, size = 432, normalized size = 0.84

method	result
risch	$\frac{\frac{5(ai+3be)b\,x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{7(ai+3be)x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}-\frac{(5ai-113be)x^3}{384ab}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-51bc)x}{128ab}-}{(b\,x^4+a)^3}$
default	$\frac{\frac{5(ai+3be)b\,x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{7(ai+3be)x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}-\frac{(5ai-113be)x^3}{384ab}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-51bc)x}{128ab}-}{(b\,x^4+a)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBO
SE)
```

```
[Out] (5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c
)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g
+11*b*c)*x^5-1/384*(5*a*i-113*b*e)/a/b*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/128*
(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x^4+a)^3+1/128/a^3/b*(1/8*(7*a*g+77*b*c)*
(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b
)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(
2^(1/2)/(a/b)^(1/4)*x-1))+1/2*(8*a*h+40*b*d)/(a*b)^(1/2)*arctan(x^2*(b/a)^(
1/2))+1/8*(5*a*i+15*b*e)/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/
2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(
a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.52, size = 581, normalized size = 1.13

$$\frac{15\,(3\,b^3e+i\,ab^2)x^{11}+12\,(5\,b^3d+ab^2h)x^{10}+7\,(11\,b^3c+ab^2g)x^9+42\,(3\,ab^2e+i\,a^2b)x^7+32\,(5\,ab^2d+a^2bh)x^6+18\,(11\,ab^2c+a^2bg)x^5-32\,a^3f+(113\,a^2be-5i\,a^3)x^3+12\,(11\,a^2bd-a^3h)x^2+3\,(51\,a^2bc-7\,a^3g)x}{384\,(a^3b^4x^{12}+3\,a^4b^3x^8+3\,a^5b^2x^4+a^6b)}+\frac{\frac{\sqrt{2}\left(77b^{\frac{3}{2}}c+7a\sqrt{b}\,g-15\sqrt{a}\,be-5ia^{\frac{3}{2}}\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77b^{\frac{3}{2}}c+7a\sqrt{b}\,g-15\sqrt{a}\,be-5ia^{\frac{3}{2}}\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+7\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+15\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e-80\sqrt{a}\,b^{\frac{3}{2}}d-16a^{\frac{3}{2}}\sqrt{b}\,h+5i\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{1}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(77\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+7\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+15\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e+80\sqrt{a}\,b^{\frac{3}{2}}d+16a^{\frac{3}{2}}\sqrt{b}\,h+5i\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{1}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{1024\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="m
axima")
```

```
[Out] 1/384*(15*(3*b^3*e + I*a*b^2)*x^11 + 12*(5*b^3*d + a*b^2*h)*x^10 + 7*(11*b^
3*c + a*b^2*g)*x^9 + 42*(3*a*b^2*e + I*a^2*b)*x^7 + 32*(5*a*b^2*d + a^2*b*h
)*x^6 + 18*(11*a*b^2*c + a^2*b*g)*x^5 - 32*a^3*f + (113*a^2*b*e - 5*I*a^3)*
x^3 + 12*(11*a^2*b*d - a^3*h)*x^2 + 3*(51*a^2*b*c - 7*a^3*g)*x)/(a^3*b^4*x^
12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b) + 1/1024*(sqrt(2)*(77*b^(3/2)*c
 + 7*a*sqrt(b)*g - 15*sqrt(a)*b*e - 5*I*a^(3/2))*log(sqrt(b)*x^2 + sqrt(2)*
a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*b^(3/2)*c + 7*
a*sqrt(b)*g - 15*sqrt(a)*b*e - 5*I*a^(3/2))*log(sqrt(b)*x^2 - sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c
 + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 15*sqrt(2)*a^(3/4)*b^(5/4)*e - 80*sqrt(a)*
b^(3/2)*d - 16*a^(3/2)*sqrt(b)*h + 5*I*sqrt(2)*a^(7/4)*b^(1/4))*arctan(1/2*
sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 7*s
qrt(2)*a^(5/4)*b^(3/4)*g + 15*sqrt(2)*a^(3/4)*b^(5/4)*e + 80*sqrt(a)*b^(3/2
)*d + 16*a^(3/2)*sqrt(b)*h + 5*I*sqrt(2)*a^(7/4)*b^(1/4))*arctan(1/2*sqrt(2
)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*s
qrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^3*b)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.59, size = 725, normalized size = 1.41

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2 d + 8\sqrt{2}\sqrt{ab}\,abh + 77\left(ab^3\right)^{\frac{1}{4}} b^2 c + 7\left(ab^3\right)^{\frac{1}{4}} abg + 15\left(ab^3\right)^{\frac{3}{4}} e\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4 b^3} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2 d + 8\sqrt{2}\sqrt{ab}\,abh + 77\left(ab^3\right)^{\frac{1}{4}} b^2 c + 7\left(ab^3\right)^{\frac{1}{4}} abg + 15\left(ab^3\right)^{\frac{3}{4}} e\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4 b^3} + \frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac{1}{4}} b^2 c + 7\left(ab^3\right)^{\frac{1}{4}} abg - 15\left(ab^3\right)^{\frac{3}{4}} e\right)\log\left(x^2 + \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4 b^3} - \frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac{1}{4}} b^2 c + 7\left(ab^3\right)^{\frac{1}{4}} abg - 15\left(ab^3\right)^{\frac{3}{4}} e\right)\log\left(x^2 - \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4 b^3} + \frac{5i\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3 b^4} + \frac{5i\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3 b^4} - \frac{5i\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(x^2 + \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^3 b^4} + \frac{5i\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(x^2 - \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^3 b^4} + \frac{45\,b^3 x^{11} e + 60\,b^3 d x^{10} + 12\,ab^2 h x^{10} + 15i\,ab^2 x^{11} + 77\,b^3 c x^9 + 7\,ab^2 g x^9 + 126\,ab^2 x^7 e + 160\,ab^2 d x^6 + 32\,a^2 bh x^6 + 42i\,a^2 b x^7 + 198\,ab^2 c x^5 + 18\,a^2 bg x^5 + 113\,a^2 b x^3 e + 132\,a^2 bd x^2 - 12\,a^3 h x^2 - 5i\,a^3 x^3 + 153\,a^2 bc x - 21\,a^3 g x - 32\,a^3 f}{384\left(bx^4 + a\right)^3 a^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="g
iac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*
(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2
)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4
)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^
3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)
*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a
/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 5/512*I*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) + 5/512*I*sqrt(
2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a^3*b^4) - 5/1024*I*sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/(a^3*b^4) + 5/1024*I*sqrt(2)*(a*b^3)^(3/4)*log(x^2 - sqrt(2)
*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) + 1/384*(45*b^3*x^11*e + 60*b^3*d*x^1
0 + 12*a*b^2*h*x^10 + 15*I*a*b^2*x^11 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*
a*b^2*x^7*e + 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 + 42*I*a^2*b*x^7 + 198*a*b^2
*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 - 12*a^3*h*x^2
- 5*I*a^3*x^3 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b
)
```

Mupad [B]
time = 6.08, size = 2500, normalized size = 4.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x)
```

```
[Out] ((3*x^5*(11*b*c + a*g))/(64*a^2) - f/(12*b) + (x^6*(5*b*d + a*h))/(12*a^2)
+ (7*x^7*(3*b*e + a*i))/(64*a^2) + (7*b*x^9*(11*b*c + a*g))/(384*a^3) + (x*
(51*b*c - 7*a*g))/(128*a*b) + (b*x^10*(5*b*d + a*h))/(32*a^3) + (5*b*x^11*(
3*b*e + a*i))/(128*a^3) + (x^2*(11*b*d - a*h))/(32*a*b) + (x^3*(113*b*e - 5
*a*i))/(384*a*b))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log
(- root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 403701760*a
^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 + 36
700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a^10*b^4*h^2
*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 17661952*a^6*b^
4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z + 409600*a
^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z + 368640
0*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*g^2*z + 184
```

32000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 32
3400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*c*d*e*h + 1
4700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c*h^2*i - 268
80*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2*e*i - 2464
000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 + 485
100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5
*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^
2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 + 1743126*a^2*b^4*c^
2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i + 2048000*a^2*b^4*d^3*h +
81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 + 2401*a
^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 + 625*a
^6*i^4 + 35153041*b^6*c^4, z, l)*(root(68719476736*a^15*b^7*z^4 + 121110528
0*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 +
110100480*a^9*b^5*e*g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*
d^2*z^2 + 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^
5*b^5*c*d*g*z - 17661952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 48570
3680*a^4*b^6*c^2*d*z + 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z -
802816*a^7*b^3*g^2*h*z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z
- 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*
i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*
h - 2956800*a^2*b^4*c*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*
i - 98560*a^4*b^2*c*h^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 +
1778700*a^2*b^4*c^2*e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g
- 295680*a^3*b^3*c*e*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7
392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*
b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b
^3*e^2*g^2 + 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e
^3*i + 2048000*a^2*b^4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 +
2668050*a*b^5*c^2*e^2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*
h^4 + 2560000*a*b^5*d^4 + 625*a^6*i^4 + 35153041*b^6*c^4, z, l)*((20185088*
a^7*b^5*c + 1835008*a^8*b^4*g)/(2097152*a^9*b^2) - (x*(655360*a^7*b^4*d + 1
31072*a^8*b^3*h))/(131072*a^9*b)) + (614400*a^4*b^4*d*e + 204800*a^5*b^3*d*
i + 122880*a^5*b^3*e*h + 40960*a^6*b^2*h*i)/(2097152*a^9*b^2) - (x*(800*a^6
*b*i^2 - 189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 - 1568*a^5*b^2*g^2 - 34496*a
^4*b^3*c*g + 4800*a^5*b^2*e*i))/(131072*a^9*b)) - (125*a^4*i^3 + 3375*a*b^3
*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^2*e - 4928*a^2*b^2*c*h^2 + 735*a^2*b^
2*e*g^2 + 3375*a^2*b^2*e^2*i - 11200*a*b^3*d^2*g + 29645*a*b^3*c^2*i + 1125
*a^3*b*e*i^2 - 448*a^3*b*g*h^2 + 245*a^3*b*g^2*i + 5390*a^2*b^2*c*g*i - 448
0*a^2*b^2*d*g*h - 49280*a*b^3*c*d*h + 16170*a*b^3*c*e*g)/(2097152*a^9*b^2)
- (x*(5775*b^3*c*d*e - 32*a^3*h^3 - 4000*b^3*d^3 + 35*a^3*g*h*i - 2400*a*b^
2*d^2*h - 480*a^2*b*d*h^2 + 1925*a*b^2*c*d*i + 1155*a*b^2*c*e*h + 525*a*b^2
*d*e*g + 385*a^2*b*c*h*i + 175*a^2*b*d*g*i + 105*a^2*b*e*g*h))/(131072*a^9*
b))*root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 403701760*
a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 + 3
6700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a^10*b^4*h^

```
2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 17661952*a^6*b
^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z + 409600*
a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z + 36864
00*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*g^2*z + 18
432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 3
23400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*c*d*e*h +
14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c*h^2*i - 26
880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2*e*i - 246
4000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 + 48
5100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 739...
```

3.209 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4} dx$

Optimal. Leaf size=534

$$\frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{12ab\,(a+bx^4)^3}+\frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)}$$

```
[Out] 1/12*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+(-a*j+b*f)*x^3)/a/b/(b*x^4+a)^3
+1/384*x*(7*a*g+77*b*c+12*(a*h+5*b*d)*x+15*(a*i+3*b*e)*x^2)/a^3/b/(b*x^4+a)
+1/96*(-4*a*(a*j+2*b*f)+x*(b*(a*g+11*b*c)+2*b*(a*h+5*b*d)*x+3*b*(a*i+3*b*e)
*x^2))/a^2/b^2/(b*x^4+a)^2+1/32*(a*h+5*b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(
7/2)/b^(3/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*
(a*i+3*b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)+1/1024
*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*(a*i+3*b*e)*a^(1/2)+
7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*
2^(1/2)/a^(1/4))*(5*(a*i+3*b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^
(7/4)*2^(1/2)+1/512*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*(a*i+3*b*e)*a^(1
/2)+7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1872, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+5bd)}{32a^{7/2}b^{3/2}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{x(7(ag+11bc)+12x(ah+5bd)+15x^2(ai+3be))}{384a^3b\,(a+bx^4)}-\frac{4a(aj+2bf)-x(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3be))}{96a^2b^2\,(a+bx^4)^2}+\frac{x(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc)}{12ab\,(a+bx^4)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^4
,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(12*a*b
*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a
*i)*x^2))/(384*a^3*b*(a + b*x^4)) - (4*a*(2*b*f + a*j) - x*(b*(11*b*c + a*g
) + 2*b*(5*b*d + a*h)*x + 3*b*(3*b*e + a*i)*x^2))/(96*a^2*b^2*(a + b*x^4)^2
) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((
7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*
g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256
*Sqrt[2]*a^(15/4)*b^(7/4)) - ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e
+ a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2
]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15
/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+209x^6+jx^7}{\left(a+bx^4\right)^4}\,dx = \frac{x(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj}{12ab\left(a+bx^4\right)^3}$$

$$= \frac{x(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj}{12ab\left(a+bx^4\right)^3}$$

$$= \frac{x(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj}{12ab\left(a+bx^4\right)^3}$$

$$= \frac{x(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj}{12ab\left(a+bx^4\right)^3}$$

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$$= \frac{x(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj}{12ab\left(a+bx^4\right)^3}$$

$$= \frac{x(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj}{12ab\left(a+bx^4\right)^3}$$

Mathematica [A]
time = 0.35, size = 555, normalized size = 1.04

$$\frac{\frac{8a^{3/4}bx(77bc+7ag+15bx(4d+3ex)+3ax(4h+5ix))}{a+bx^4}-\frac{32a^{7/4}\left(12a^2j-b^2x(11c+x(10d+9ex))-abx(g+x(2h+3ix))\right)}{\left(a+bx^4\right)^2}+\frac{256a^{11/4}\left(a^2j+b^2x(c+x(d+ex))-ab(f+x(g+x(h+ix)))\right)}{\left(a+bx^4\right)^3}-6\sqrt[4]{b}\left(77\sqrt{2}b^{3/2}c+80\sqrt[4]{a}b^{5/4}d+15\sqrt{2}\sqrt{a}be+7\sqrt{2}a\sqrt{b}g+16a^{5/4}\sqrt[4]{b}h+5\sqrt{2}a^{3/2}i\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+6\sqrt[4]{b}\left(77\sqrt{2}b^{3/2}c-80\sqrt[4]{a}b^{5/4}d+15\sqrt{2}\sqrt{a}be+7\sqrt{2}a\sqrt{b}g-16a^{5/4}\sqrt[4]{b}h+5\sqrt{2}a^{3/2}i\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+3\sqrt{2}\sqrt[4]{b}\left(-77b^{3/2}c+15\sqrt{a}be-7a\sqrt{b}g+5a^{3/2}i\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)+3\sqrt{2}\sqrt[4]{b}\left(77b^{3/2}c-15\sqrt{a}be+7a\sqrt{b}g-5a^{3/2}i\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{3072a^{15/4}b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*b*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*x
)))/(a + b*x^4) - (32*a^(7/4)*(12*a^2*j - b^2*x*(11*c + x*(10*d + 9*e*x)) -
```

```
a*b*x*(g + x*(2*h + 3*i*x))))/(a + b*x^4)^2 + (256*a^(11/4)*(a^2*j + b^2*x
*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^3 - 6*b^(1
/4)*(77*Sqrt[2]*b^(3/2)*c + 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e +
 7*Sqrt[2]*a*Sqrt[b]*g + 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*b^(1/4)*(77*Sqrt[2]*b^(3/2)*c - 80*a^
(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g - 16*a^(5/
4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
 + 3*Sqrt[2]*b^(1/4)*(-77*b^(3/2)*c + 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^
(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]
*b^(1/4)*(77*b^(3/2)*c - 15*Sqrt[a]*b*e + 7*a*Sqrt[b]*g - 5*a^(3/2)*i)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^2)
```

Maple [A]
time = 0.35, size = 448, normalized size = 0.84

method	result
risch	$\frac{\frac{5(ai+3be)b\,x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{7(ai+3be)x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}-\frac{j\,x^4}{8b}-\frac{(5ai-113be)x^3}{384ab}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-\ldots}{12\ldots}}{(b\,x^4+a)^3}$ …
default	$\frac{\frac{5(ai+3be)b\,x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{7(ai+3be)x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}-\frac{j\,x^4}{8b}-\frac{(5ai-113be)x^3}{384ab}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-\ldots}{12\ldots}}{(b\,x^4+a)^3}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETUR
NVERBOSE)
```

```
[Out] (5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c
)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g
+11*b*c)*x^5-1/8*j*x^4/b-1/384*(5*a*i-113*b*e)/a/b*x^3-1/32*(a*h-11*b*d)/a/
b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/24*(a*j+2*b*f)/b^2)/(b*x^4+a)^3+1/128/a^
3/b*(1/8*(7*a*g+77*b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2
)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a
/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/2*(8*a*h+40*b*d)/(a*b)^
(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*(5*a*i+15*b*e)/b/(a/b)^(1/4)*2^(1/2)*(ln(
(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1
/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.54, size = 615, normalized size = 1.15

$$\frac{15\,(3\,b^4e+i\,ab^3)x^{11}+12\,(5\,b^4d+ab^3h)x^{10}+7\,(11\,b^4c+ab^3g)x^9-48\,a^3bjx^4+42\,(3\,ab^3e+i\,a^2b^2)x^7+32\,(5\,ab^3d+a^2b^2h)x^6+18\,(11\,ab^3c+a^2b^2g)x^5-32\,a^3bf-16\,a^4j+(113\,a^2b^2e-5i\,a^3b)x^3+12\,(11\,a^2b^2d-a^3bh)x^2+3\,(51\,a^2b^2c-7\,a^3bg)x}{384\,(a^3b^5x^{12}+3\,a^4b^4x^8+3\,a^5b^3x^4+a^6b^2)}+\frac{\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c+7\,a\sqrt{b}\,g-15\,\sqrt{a}\,be-5i\,a^{\frac{3}{2}}\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{7}{4}}}-\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c+7\,a\sqrt{b}\,g-15\,\sqrt{a}\,be-5i\,a^{\frac{3}{2}}\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{7}{4}}}+\frac{2\left(77\,\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+7\,\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+15\,\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e-80\,\sqrt{a}\,b^{\frac{3}{2}}d-16\,a^{\frac{3}{2}}\sqrt{b}\,h+5i\,\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{1}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{2}}}+\frac{2\left(77\,\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{7}{4}}c+7\,\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{3}{4}}g+15\,\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}e+80\,\sqrt{a}\,b^{\frac{3}{2}}d+16\,a^{\frac{3}{2}}\sqrt{b}\,h+5i\,\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{1}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{2}}}}{1024\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algori
thm="maxima")
```

```
[Out] 1/384*(15*(3*b^4*e + I*a*b^3)*x^11 + 12*(5*b^4*d + a*b^3*h)*x^10 + 7*(11*b^
4*c + a*b^3*g)*x^9 - 48*a^3*b*j*x^4 + 42*(3*a*b^3*e + I*a^2*b^2)*x^7 + 32*(
5*a*b^3*d + a^2*b^2*h)*x^6 + 18*(11*a*b^3*c + a^2*b^2*g)*x^5 - 32*a^3*b*f -
 16*a^4*j + (113*a^2*b^2*e - 5*I*a^3*b)*x^3 + 12*(11*a^2*b^2*d - a^3*b*h)*x
^2 + 3*(51*a^2*b^2*c - 7*a^3*b*g)*x)/(a^3*b^5*x^12 + 3*a^4*b^4*x^8 + 3*a^5*
b^3*x^4 + a^6*b^2) + 1/1024*(sqrt(2)*(77*b^(3/2)*c + 7*a*sqrt(b)*g - 15*sqr
t(a)*b*e - 5*I*a^(3/2))*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(
a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*b^(3/2)*c + 7*a*sqrt(b)*g - 15*sqrt(a)*
b*e - 5*I*a^(3/2))*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(
a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 7*sqrt(2)*a^(5/4)*b^(3
/4)*g + 15*sqrt(2)*a^(3/4)*b^(5/4)*e - 80*sqrt(a)*b^(3/2)*d - 16*a^(3/2)*sq
rt(b)*h + 5*I*sqrt(2)*a^(7/4)*b^(1/4))*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sq
rt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b)
)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 7*sqrt(2)*a^(5/4)*b^(3/4)*g
+ 15*sqrt(2)*a^(3/4)*b^(5/4)*e + 80*sqrt(a)*b^(3/2)*d + 16*a^(3/2)*sqrt(b)*
h + 5*I*sqrt(2)*a^(7/4)*b^(1/4))*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*
a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3
/4)))/(a^3*b)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x
)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.64, size = 757, normalized size = 1.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algori
thm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*
(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2
)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4
)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^
3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)
*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a
/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 5/512*I*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) + 5/512*I*sqrt(
2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a^3*b^4) - 5/1024*I*sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/(a^3*b^4) + 5/1024*I*sqrt(2)*(a*b^3)^(3/4)*log(x^2 - sqrt(2)
*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) + 1/384*(45*b^4*x^11*e + 60*b^4*d*x^1
0 + 12*a*b^3*h*x^10 + 15*I*a*b^3*x^11 + 77*b^4*c*x^9 + 7*a*b^3*g*x^9 + 126*
a*b^3*x^7*e + 160*a*b^3*d*x^6 + 32*a^2*b^2*h*x^6 + 42*I*a^2*b^2*x^7 + 198*a
*b^3*c*x^5 + 18*a^2*b^2*g*x^5 - 48*a^3*b*j*x^4 + 113*a^2*b^2*x^3*e + 132*a^
2*b^2*d*x^2 - 12*a^3*b*h*x^2 - 5*I*a^3*b*x^3 + 153*a^2*b^2*c*x - 21*a^3*b*g
*x - 32*a^3*b*f - 16*a^4*j)/((b*x^4 + a)^3*a^3*b^2)
```

Mupad [B]
time = 6.48, size = 2500, normalized size = 4.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^4
,x)
```

```
[Out] symsum(log(- root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 4
03701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*
g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a^
10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 17661
952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z
+ 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*
z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*g
^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*
d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*c
*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c*h
^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2*
```

e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e
*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e
+ 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2 + 614400*
a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 + 1743126*
a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i + 2048000*a^2*b^
4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^
2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d
^4 + 625*a^6*i^4 + 35153041*b^6*c^4, z, m)*(root(68719476736*a^15*b^7*z^4 +
 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5
*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 83886080
0*a^8*b^6*d^2*z^2 + 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 8
8309760*a^5*b^5*c*d*g*z - 17661952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i
*z - 485703680*a^4*b^6*c^2*d*z + 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*
c^2*h*z - 802816*a^7*b^3*g^2*h*z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^
3*d*i^2*z - 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*
b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*
b^3*d*e*g*h - 2956800*a^2*b^4*c*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*
b^3*d^2*g*i - 98560*a^4*b^2*c*h^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2
*c*g*i^2 + 1778700*a^2*b^4*c^2*e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b
^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g
*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g +
33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 2
2050*a^3*b^3*e^2*g^2 + 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500
*a^3*b^3*e^3*i + 2048000*a^2*b^4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b
^3*c*g^3 + 2668050*a*b^5*c^2*e^2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4
096*a^5*b*h^4 + 2560000*a*b^5*d^4 + 625*a^6*i^4 + 35153041*b^6*c^4, z, m)*(
(20185088*a^7*b^5*c + 1835008*a^8*b^4*g)/(2097152*a^9*b^2) - (x*(655360*a^7
*b^4*d + 131072*a^8*b^3*h))/(131072*a^9*b)) + (614400*a^4*b^4*d*e + 204800*
a^5*b^3*d*i + 122880*a^5*b^3*e*h + 40960*a^6*b^2*h*i)/(2097152*a^9*b^2) - (
x*(800*a^6*b*i^2 - 189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 - 1568*a^5*b^2*g^2
 - 34496*a^4*b^3*c*g + 4800*a^5*b^2*e*i))/(131072*a^9*b)) - (125*a^4*i^3 +
3375*a*b^3*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^2*e - 4928*a^2*b^2*c*h^2 +
735*a^2*b^2*e*g^2 + 3375*a^2*b^2*e^2*i - 11200*a*b^3*d^2*g + 29645*a*b^3*c^
2*i + 1125*a^3*b*e*i^2 - 448*a^3*b*g*h^2 + 245*a^3*b*g^2*i + 5390*a^2*b^2*c
*g*i - 4480*a^2*b^2*d*g*h - 49280*a*b^3*c*d*h + 16170*a*b^3*c*e*g)/(2097152
*a^9*b^2) - (x*(5775*b^3*c*d*e - 32*a^3*h^3 - 4000*b^3*d^3 + 35*a^3*g*h*i -
 2400*a*b^2*d^2*h - 480*a^2*b*d*h^2 + 1925*a*b^2*c*d*i + 1155*a*b^2*c*e*h +
 525*a*b^2*d*e*g + 385*a^2*b*c*h*i + 175*a^2*b*d*g*i + 105*a^2*b*e*g*h))/(1
31072*a^9*b))*root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 +
403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e
*g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a
^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 1766
1952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z
 + 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h
*z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*

```
g^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c
*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*
c*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c*
h^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2
*e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*
e*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e
 + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2 + 614400
*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 + 1743126
*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i + 2048000*a^2*b
^4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e
^2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 409...
```

3.210 $\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=121

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{a+bx^4}}$$

```
[Out] 1/2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+1/2*c*(cos(2*arctan(b^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*a
rctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^
(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1899, 226, 281, 223, 212}

$$\frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx &= \int \left(\frac{c}{\sqrt{a+bx^4}}+\frac{dx}{\sqrt{a+bx^4}}\right)dx\\ &= c\int \frac{1}{\sqrt{a+bx^4}}\,dx+d\int \frac{x}{\sqrt{a+bx^4}}\,dx\\ &= \frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{\sqrt{a+b}}\right.\\ &= \frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{1-bx^2}\right.\\ &= \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\right.}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 79, normalized size = 0.65

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{cx\sqrt{1+\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.32, size = 96, normalized size = 0.79

method	result	size
default	$\frac{d \ln\left(x^{2} \sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{2 \sqrt{b}}+\frac{c \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}$	96
elliptic	$\frac{c \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}+\frac{d \ln\left(2 x^{2} \sqrt{b}+2 \sqrt{b\,x^{4}+a}\right)}{2 \sqrt{b}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+c/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^4 + a), x)
```

Fricas [A]
time = 0.11, size = 72, normalized size = 0.60

$$\frac{4\,b^{\frac{3}{2}} c\left(-\frac{a}{b}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right) \mid -1) + a\sqrt{b}\, d \log\left(-2\,bx^{4} - 2\,\sqrt{bx^{4}+a}\,\sqrt{b}\,x^{2} - a\right)}{4\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^(3/2)*c*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + a*sq
rt(b)*d*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a))/(a*b)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.26, size = 61, normalized size = 0.50

$$\frac{d\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2)
, (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^4)^(1/2),x)
```

```
[Out] int((c + d*x)/(a + b*x^4)^(1/2), x)
```

3.211 $\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=87

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}+\frac{\sqrt[4]{a}\,c\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\,\sqrt{a-bx^4}}$$

```
[Out] 1/2*d*arctan(x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(1/2)+a^(1/4)*c*EllipticF(b^(1
/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/(-b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1899, 230, 227, 281, 223, 209}

$$\frac{\sqrt[4]{a}\,c\sqrt{1-\frac{bx^4}{a}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\,\sqrt{a-bx^4}}+\frac{d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1 -
 (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b
*x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx &= \int \left(\frac{c}{\sqrt{a-bx^4}}+\frac{dx}{\sqrt{a-bx^4}}\right)dx\\
&= c\int \frac{1}{\sqrt{a-bx^4}}\,dx+d\int \frac{x}{\sqrt{a-bx^4}}\,dx\\
&= \frac{1}{2}d\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,x^2\right)+\frac{\left(c\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{\sqrt{a-bx^4}}\\
&= \frac{\sqrt[4]{a}\,c\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\,\sqrt{a-bx^4}}+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{x^2}{\sqrt{a-bx^4}}\right)\\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}+\frac{\sqrt[4]{a}\,c\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\,\sqrt{a-bx^4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 81, normalized size = 0.93

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}+\frac{cx\sqrt{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{bx^4}{a}\right)}{\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 - (b*x^
4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/Sqrt[a - b*x^4]
```

Maple [A]
time = 0.37, size = 90, normalized size = 1.03

method	result	size
default	$\frac{d \arctan\left(\frac{x^{2} \sqrt{b}}{\sqrt{-b\,x^{4}+a}}\right)}{2 \sqrt{b}}+\frac{c \sqrt{1-\frac{x^{2} \sqrt{b}}{\sqrt{a}}}\, \sqrt{1+\frac{x^{2} \sqrt{b}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b\,x^{4}+a}}$	90
elliptic	$\frac{c \sqrt{1-\frac{x^{2} \sqrt{b}}{\sqrt{a}}}\, \sqrt{1+\frac{x^{2} \sqrt{b}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b\,x^{4}+a}}+\frac{d \ln\left(-\frac{2 b\,x^{2}}{\sqrt{-b}}+2 \sqrt{-b\,x^{4}+a}\right)}{2 \sqrt{-b}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d*arctan(x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(1/2)+c/(1/a^(1/2)*b^(1/2))^(1
/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^
(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^4 + a), x)
```

Fricas [A]
time = 0.12, size = 79, normalized size = 0.91

$$\frac{4 \sqrt{-b}\, bc\left(\frac{a}{b}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right) \mid -1) - a\sqrt{-b}\, d \log\left(2\, bx^{4} - 2\, \sqrt{-bx^{4}+a}\, \sqrt{-b}\, x^{2} - a\right)}{4\, ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(-b)*b*c*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) - a*s
qrt(-b)*d*log(2*b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(-b)*x^2 - a))/(a*b)
```

Sympy [A]
time = 1.36, size = 95, normalized size = 1.09

$$d\left(\begin{cases} -\frac{i\operatorname{acosh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for}\ \left|\frac{bx^4}{a}\right| > 1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a)**(1/2),x)
```

```
[Out] d*Piecewise((-I*acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4/a) > 1)
, (asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) + c*x*gamma(1/4)*hyper((1
/4, 1/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{\sqrt{a-b\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a - b*x^4)^(1/2),x)
```

```
[Out] int((c + d*x)/(a - b*x^4)^(1/2), x)
```

3.212 $\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx$

Optimal. Leaf size=89

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{-a+bx^4}}\right)}{2\sqrt{b}}+\frac{\sqrt[4]{a}\,c\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\,\sqrt{-a+bx^4}}$$

```
[Out] 1/2*d*arctanh(x^2*b^(1/2)/(b*x^4-a)^(1/2))/b^(1/2)+a^(1/4)*c*EllipticF(b^(1
/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/(b*x^4-a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1899, 230, 227, 281, 223, 212}

$$\frac{\sqrt[4]{a}\,c\sqrt{1-\frac{bx^4}{a}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\,\sqrt{bx^4-a}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{bx^4-a}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[-a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1
 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[-a
+ b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx &= \int \left(\frac{c}{\sqrt{-a+bx^4}}+\frac{dx}{\sqrt{-a+bx^4}}\right) dx \\
&= c\int \frac{1}{\sqrt{-a+bx^4}}\,dx + d\int \frac{x}{\sqrt{-a+bx^4}}\,dx \\
&= \frac{1}{2} d\,\text{Subst}\left(\int \frac{1}{\sqrt{-a+bx^2}}\,dx, x, x^2\right) + \frac{\left(c\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{\sqrt{-a+bx^4}} \\
&= \frac{\sqrt[4]{a}\,c\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\,\sqrt{-a+bx^4}} + \frac{1}{2} d\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{-a+bx^4}}\right) \\
&= \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{-a+bx^4}}\right)}{2\sqrt{b}} + \frac{\sqrt[4]{a}\,c\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\,\sqrt{-a+bx^4}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 83, normalized size = 0.93

$$
\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{-a+bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{bx^4}{a}\right)}{\sqrt{-a+bx^4}}
$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/Sqrt[-a + b*x^4],x]

[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/Sqrt[-a + b*x^4]

Maple [A]
time = 0.37, size = 95, normalized size = 1.07

method	result	size
default	$\frac{d \ln\left(x^{2} \sqrt{b}+\sqrt{b\,x^{4}-a}\right)}{2 \sqrt{b}}+\frac{c \sqrt{1+\frac{x^{2} \sqrt{b}}{\sqrt{a}}}\, \sqrt{1-\frac{x^{2} \sqrt{b}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}-a}}$	95
elliptic	$\frac{c \sqrt{1+\frac{x^{2} \sqrt{b}}{\sqrt{a}}}\, \sqrt{1-\frac{x^{2} \sqrt{b}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}-a}}+\frac{d \ln\left(2 x^{2} \sqrt{b}+2 \sqrt{b\,x^{4}-a}\right)}{2 \sqrt{b}}$	98

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(b*x^4-a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*d*ln(x^2*b^(1/2)+(b*x^4-a)^(1/2))/b^(1/2)+c/(-1/a^(1/2)*b^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)/(b*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*b^(1/2))^(1/2),I)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(b*x^4 - a), x)

Fricas [A]
time = 0.11, size = 73, normalized size = 0.82

$$-\frac{4\, b^{\frac{3}{2}} c \left(\frac{a}{b}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right) \mid -1) - a \sqrt{b}\, d \log\left(2\, b x^{4} + 2\, \sqrt{b x^{4} - a}\, \sqrt{b}\, x^{2} - a\right)}{4\, a b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^(3/2)*c*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) - a*sqr
t(b)*d*log(2*b*x^4 + 2*sqrt(b*x^4 - a)*sqrt(b)*x^2 - a))/(a*b)
```

Sympy [A]
time = 1.41, size = 90, normalized size = 1.01

$$d\left(\begin{cases}\frac{\operatorname{acosh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for}\ \left|\frac{bx^4}{a}\right|>1\\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise}\end{cases}\right)-\frac{icx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4-a)**(1/2),x)
```

```
[Out] d*Piecewise((acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4/a) > 1), (
-I*asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) - I*c*x*gamma(1/4)*hyper(
(1/4, 1/2), (5/4,), b*x**4/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^4 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{\sqrt{b\,x^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(b*x^4 - a)^(1/2),x)
```

```
[Out] int((c + d*x)/(b*x^4 - a)^(1/2), x)
```

3.213 $\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx$

Optimal. Leaf size=127

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}+\frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{-a-bx^4}}$$

```
[Out] 1/2*d*arctan(x^2*b^(1/2)/(-b*x^4-a)^(1/2))/b^(1/2)+1/2*c*(cos(2*arctan(b^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*a
rctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^
(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(-b*x^4-a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1899, 226, 281, 223, 209}

$$\frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{-a-bx^4}}+\frac{d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[-a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[-a - b*x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx &= \int \left(\frac{c}{\sqrt{-a-bx^4}}+\frac{dx}{\sqrt{-a-bx^4}}\right)dx\\
&= c\int \frac{1}{\sqrt{-a-bx^4}}\,dx + d\int \frac{x}{\sqrt{-a-bx^4}}\,dx\\
&= \frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{-a-bx^4}} + \frac{1}{2}d\text{Subst}\left(\int \frac{}{\sqrt{-a}}\right.\\
&= \frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{-a-bx^4}} + \frac{1}{2}d\text{Subst}\left(\int \frac{1}{1+b}\right.\\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}} + \frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right.\right.}{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{-a-bx^4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 85, normalized size = 0.67

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}+\frac{cx\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{-a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[-a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[-a - b*x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.32, size = 101, normalized size = 0.80

method	result	size
default	$\frac{d \arctan\left(\frac{x^{2} \sqrt{b}}{\sqrt{-b\,x^{4}-a}}\right)}{2 \sqrt{b}}+\frac{c \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{-\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{-\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{-b\,x^{4}-a}}$	101
elliptic	$\frac{c \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{-\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{-\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{-b\,x^{4}-a}}+\frac{d \ln\left(-\frac{2 b\,x^{2}}{\sqrt{-b}}+2 \sqrt{-b\,x^{4}-a}\right)}{2 \sqrt{-b}}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d*arctan(x^2*b^(1/2)/(-b*x^4-a)^(1/2))/b^(1/2)+c/(-I/a^(1/2)*b^(1/2))^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(-b*x^
4-a)^(1/2)*EllipticF(x*(-I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^4 - a), x)
```

Fricas [A]
time = 0.11, size = 82, normalized size = 0.65

$$-\frac{4 \sqrt{-b}\, bc\left(-\frac{a}{b}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right) \mid -1) + a\sqrt{-b}\, d \log\left(-2\, bx^{4} + 2\, \sqrt{-bx^{4} - a}\, \sqrt{-b}\, x^{2} - a\right)}{4\, ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(-b)*b*c*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) +
a*sqrt(-b)*d*log(-2*b*x^4 + 2*sqrt(-b*x^4 - a)*sqrt(-b)*x^2 - a))/(a*b)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.32, size = 66, normalized size = 0.52

$$-\frac{id\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}}-\frac{icx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4-a)**(1/2),x)
```

```
[Out] -I*d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - I*c*x*gamma(1/4)*hyper((1/4,
 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^4 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{\sqrt{-b\,x^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(- a - b*x^4)^(1/2),x)
```

```
[Out] int((c + d*x)/(- a - b*x^4)^(1/2), x)
```

3.214 $\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=257

$$\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right.\right.}{b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] 1/2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+e*x*(b*x^4+a)^(1/2)/b^(1
/2)/(a^(1/2)+x^2*b^(1/2))-a^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)
^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(e+c*b^(1/2)/a^(1/2))*((b*x^
4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1899, 281, 223, 212, 1212, 226, 1210}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/Sqrt[a + b*x^4],x]
```

```
[Out] (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[a] +
 e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \int \left(\frac{dx}{\sqrt{a+bx^4}}+\frac{c+ex^2}{\sqrt{a+bx^4}}\right)dx$$

$$= d\int \frac{x}{\sqrt{a+bx^4}}\,dx + \int \frac{c+ex^2}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{1}{2}d\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right) - \frac{\left(\sqrt{a}\,e\right)\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}} + \left(c+\frac{\sqrt{a}\,e}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a+ \dots}}$$

$$= \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\dots\right.}{b^{3/4}\sqrt{a+bx^4}}$$

$$= \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b \dots}{\left(\sqrt{a}+\sqrt{\dots}\right.}}}{b^{3/4}\sqrt{a+\dots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.09, size = 131, normalized size = 0.51

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1+\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}} + \frac{ex^3\sqrt{1+\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4] + (e
*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(3
*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.32, size = 193, normalized size = 0.75

method	result

default	$\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}+\frac{d\ln\left(x^2\sqrt{b}+\sqrt{\right.}}{2\sqrt{b}}$
elliptic	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*d*ln(x^2*b^
(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d*x + c)/sqrt(b*x^4 + a), x)
```

Fricas [A]
time = 0.12, size = 128, normalized size = 0.50

$$\frac{4\,a\sqrt{b}\,ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+a\sqrt{b}\,dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\,\sqrt{b}\,x^2-a\right)+4\,(bc-ae)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+4\,\sqrt{bx^4+a}\,ae}{4\,abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) +
a*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 4*(b*c -
a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 4*sqrt
(b*x^4 + a)*a*e)/(a*b*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.58, size = 102, normalized size = 0.40

$$\frac{d\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2)
, (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(3
/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/
4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)/sqrt(b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d\,x+c}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^4)^(1/2),x)
```

```
[Out] int((c + d*x + e*x^2)/(a + b*x^4)^(1/2), x)
```

3.215 $\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=14

$$\frac{gx}{\sqrt{a+bx^4}}$$

[Out] g*x/(b*x^4+a)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{391\}$

$$\frac{gx}{\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (g*x)/Sqrt[a + b*x^4]

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{gx}{\sqrt{a+bx^4}}$$

Mathematica [A]
time = 1.06, size = 14, normalized size = 1.00

$$\frac{gx}{\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (g*x)/Sqrt[a + b*x^4]

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 2.
time = 0.34, size = 195, normalized size = 13.93

method	result
gosper	$\frac{gx}{\sqrt{b\,x^{4}+a}}$
trager	$\frac{gx}{\sqrt{b\,x^{4}+a}}$
elliptic	$\frac{gx}{\sqrt{b\,x^{4}+a}}$
default	$g\left(-b\left(-\frac{x}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+a\left(\frac{x}{2a\sqrt{\left(x^{4}+\right.}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(-b*(-1/2/b*x/((x^4+a/b)*b)^(1/2)+1/2/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*El
lipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+a*(1/2/a*x/((x^4+a/b)*b)^(1/2)+1/2/
a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Maxima [A]
time = 0.32, size = 12, normalized size = 0.86

$$\frac{gx}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] g*x/sqrt(b*x^4 + a)
```

Fricas [A]
time = 0.40, size = 12, normalized size = 0.86

$$\frac{gx}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] g*x/sqrt(b*x^4 + a)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.35, size = 80, normalized size = 5.71

$$\frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}-\frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x**4+a*g)/(b*x**4+a)**(3/2),x)
```

```
[Out] g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Giac [A]
time = 0.62, size = 12, normalized size = 0.86

$$\frac{gx}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] g*x/sqrt(b*x^4 + a)
```

Mupad [B]
time = 5.04, size = 12, normalized size = 0.86

$$\frac{g\,x}{\sqrt{b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g - b*g*x^4)/(a + b*x^4)^(3/2),x)
```

```
[Out] (g*x)/(a + b*x^4)^(1/2)
```

3.216 $\int \frac{ag+ex-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

```
[Out] 1/2*(2*a*g*x+e*x^2)/a/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{1870\}$

$$\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (2*a*g*x + e*x^2)/(2*a*Sqrt[a + b*x^4])
```

Rule 1870

```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, Simp[-(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Rubi steps

$$\int \frac{ag+ex-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx = \frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

Mathematica [A]
time = 9.78, size = 27, normalized size = 0.93

$$\frac{x(2ag+ex)}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (x*(2*a*g + e*x))/(2*a*Sqrt[a + b*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 2.
time = 0.33, size = 213, normalized size = 7.34

method	result
gosper	$\frac{x(2ag+ex)}{2\sqrt{b\,x^4+a}\,a}$
trager	$\frac{x(2ag+ex)}{2\sqrt{b\,x^4+a}\,a}$
elliptic	$\frac{e\,x^2}{2a\sqrt{b\,x^4+a}}+\frac{gx}{\sqrt{b\,x^4+a}}$
default	$-bg\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+ag\left(\frac{x}{2a\sqrt{\left(x^4+\right.}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b*g*(-1/2/b*x/((x^4+a/b)*b)^(1/2)+1/2/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(
1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+a*g*(1/2/a*x/((x^4+a/b)*b)^(1/2)+1/2
/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
+1/2*e*x^2/a/(b*x^4+a)^(1/2)
```

Maxima [A]
time = 0.32, size = 26, normalized size = 0.90

$$\frac{2\,agx+x^2e}{2\,\sqrt{bx^4+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*g*x + x^2*e)/(sqrt(b*x^4 + a)*a)
```

Fricas [A]
time = 0.40, size = 34, normalized size = 1.17

$$\frac{\sqrt{bx^4+a}\,(2\,agx+ex^2)}{2\,(abx^4+a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*g*x + e*x^2)/(a*b*x^4 + a^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.50, size = 104, normalized size = 3.59

$$\frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}-\frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x**4+a*g+e*x)/(b*x**4+a)**(3/2),x)
```

```
[Out] g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/
a))
```

Giac [A]
time = 0.61, size = 23, normalized size = 0.79

$$\frac{\left(2\,g+\frac{xe}{a}\right)x}{2\,\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*g + x*e/a)*x/sqrt(b*x^4 + a)
```

Mupad [B]
time = 4.91, size = 23, normalized size = 0.79

$$\frac{g\,x+\frac{e\,x^2}{2\,a}}{\sqrt{b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x)
```

```
[Out] (g*x + (e*x^2)/(2*a))/(a + b*x^4)^(1/2)
```

3.217 $\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}} \, dx$

Optimal. Leaf size=25

$$-\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

[Out] 1/2*(2*b*g*x-f)/b/(b*x^4+a)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1870}

$$-\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] -1/2*(f - 2*b*g*x)/(b*Sqrt[a + b*x^4])

Rule 1870

```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, Simp[-(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Rubi steps

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}} \, dx = -\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

Mathematica [A]
time = 10.03, size = 27, normalized size = 1.08

$$\frac{-f+2bgx}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (-f + 2*b*g*x)/(2*b*Sqrt[a + b*x^4])

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 2.
time = 0.36, size = 210, normalized size = 8.40

method	result
gosper	$\frac{2bgx-f}{2b\sqrt{b\,x^4+a}}$
trager	$\frac{2bgx-f}{2b\sqrt{b\,x^4+a}}$
elliptic	$-\frac{f}{2b\sqrt{b\,x^4+a}}+\frac{gx}{\sqrt{b\,x^4+a}}$
default	$-bg\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)-\frac{f}{2b\sqrt{b\,x^4+a}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b*g*(-1/2/b*x/((x^4+a/b)*b)^(1/2)+1/2/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(
1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*f/b/(b*x^4+a)^(1/2)+a*g*(1/2/a*x
/((x^4+a/b)*b)^(1/2)+1/2/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*b^(1/2))^(1/2),I))
```

Maxima [A]
time = 0.32, size = 23, normalized size = 0.92

$$\frac{2\,bgx-f}{2\,\sqrt{bx^4+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*g*x - f)/(sqrt(b*x^4 + a)*b)
```

Fricas [A]
time = 0.40, size = 33, normalized size = 1.32

$$\frac{\sqrt{bx^4+a}\,(2\,bgx-f)}{2\,(b^2x^4+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*b*g*x - f)/(b^2*x^4 + a*b)
```

Sympy [A]
time = 5.94, size = 109, normalized size = 4.36

$$f\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{gx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x**4+f*x**3+a*g)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Giac [A]
time = 0.69, size = 22, normalized size = 0.88

$$\frac{2\,gx - \frac{f}{b}}{2\,\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*g*x - f/b)/sqrt(b*x^4 + a)
```

Mupad [B]
time = 4.90, size = 20, normalized size = 0.80

$$\frac{g\,x - \frac{f}{2\,b}}{\sqrt{b\,x^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x)
```

```
[Out] (g*x - f/(2*b))/(a + b*x^4)^(1/2)
```

3.218 $\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}} dx$

Optimal. Leaf size=38

$$-\frac{af-2abgx-bex^2}{2ab\sqrt{a+bx^4}}$$

[Out] 1/2*(2*a*b*g*x+b*e*x^2-a*f)/a/b/(b*x^4+a)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = $\{1870\}$

$$-\frac{-2abgx+af-bex^2}{2ab\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] -1/2*(a*f - 2*a*b*g*x - b*e*x^2)/(a*b*Sqrt[a + b*x^4])

Rule 1870

```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, Simp[-(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Rubi steps

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}} dx = -\frac{af-2abgx-bex^2}{2ab\sqrt{a+bx^4}}$$

Mathematica [A]
time = 10.05, size = 38, normalized size = 1.00

$$\frac{-af+2abgx+bex^2}{2ab\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]

```
[Out] (-(a*f) + 2*a*b*g*x + b*e*x^2)/(2*a*b*Sqrt[a + b*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 2.
time = 0.33, size = 228, normalized size = 6.00

method	result
gosper	$\frac{2abgx+be\,x^{2}-af}{2ab\sqrt{b\,x^{4}+a}}$
trager	$\frac{2abgx+be\,x^{2}-af}{2ab\sqrt{b\,x^{4}+a}}$
elliptic	$-\frac{-be\,x^{2}+af}{2\sqrt{b\,x^{4}+a}\,ab}+\frac{gx}{\sqrt{b\,x^{4}+a}}$
default	$-bg\left(-\frac{x}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)-\frac{f}{2b\sqrt{b\,x^{4}+a}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b*g*(-1/2/b*x/((x^4+a/b)*b)^(1/2)+1/2/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(
1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*f/b/(b*x^4+a)^(1/2)+a*g*(1/2/a*x
/((x^4+a/b)*b)^(1/2)+1/2/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*b^(1/2))^(1/2),I))+1/2*e*x^2/a/(b*x^4+a)^(1/2)
```

Maxima [A]
time = 0.32, size = 45, normalized size = 1.18

$$\frac{\sqrt{bx^4+a}\,(2\,abgx+bx^2e-af)}{2\,(ab^2x^4+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*x^2*e - a*f)/(a*b^2*x^4 + a^2*b)
```

Fricas [A]
time = 0.41, size = 44, normalized size = 1.16

$$\frac{\sqrt{bx^4+a}\,(2\,abgx+bex^2-af)}{2\,(ab^2x^4+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*b)
```

Sympy [A]
time = 7.33, size = 133, normalized size = 3.50

$$f\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{gx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x**4+f*x**3+a*g+e*x)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*
x**4/a))
```

Giac [A]
time = 0.63, size = 31, normalized size = 0.82

$$\frac{\left(2\,g + \frac{xe}{a}\right)x - \frac{f}{b}}{2\,\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*g + x*e/a)*x - f/b)/sqrt(b*x^4 + a)
```

Mupad [B]
time = 4.84, size = 29, normalized size = 0.76

$$\frac{g\,x - \frac{f}{2\,b} + \frac{e\,x^2}{2\,a}}{\sqrt{b\,x^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x)
```

```
[Out] (g*x - f/(2*b) + (e*x^2)/(2*a))/(a + b*x^4)^(1/2)
```

3.219 $\int \frac{-1+x^4}{(1+x^4)^{3/2}} dx$

Optimal. Leaf size=12

$$-\frac{x}{\sqrt{1+x^4}}$$

```
[Out] -x/(x^4+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {391}

$$-\frac{x}{\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/(1 + x^4)^(3/2),x]
```

```
[Out] -(x/Sqrt[1 + x^4])
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}} dx = -\frac{x}{\sqrt{1+x^4}}$$

Mathematica [A]
time = 0.13, size = 12, normalized size = 1.00

$$-\frac{x}{\sqrt{1+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(1 + x^4)^(3/2),x]
```

```
[Out] -(x/Sqrt[1 + x^4])
```

Maple [A]
time = 0.33, size = 11, normalized size = 0.92

method	result	size
gosper	$-\frac{x}{\sqrt{x^4+1}}$	11
default	$-\frac{x}{\sqrt{x^4+1}}$	11
trager	$-\frac{x}{\sqrt{x^4+1}}$	11
risch	$-\frac{x}{\sqrt{x^4+1}}$	11
elliptic	$-\frac{x}{\sqrt{x^4+1}}$	11
meijerg	$-x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],-x^4\right)+\frac{x^5\,\mathrm{hypergeom}\left(\left[\frac{5}{4},\frac{3}{2}\right],\left[\frac{9}{4}\right],-x^4\right)}{5}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -x/(x^4+1)^(1/2)
```

Maxima [A]
time = 0.57, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="maxima")
[Out] -x/sqrt(x^4 + 1)
```

Fricas [A]
time = 0.40, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="fricas")
[Out] -x/sqrt(x^4 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.86, size = 58, normalized size = 4.83

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/(x**4+1)**(3/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi))/(4*gamma(9/
4)) - x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi))/(4*gamma
(5/4))
```

Giac [A]
time = 0.58, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] -x/sqrt(x^4 + 1)
```

Mupad [B]
time = 4.85, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/(x^4 + 1)^(3/2),x)
```

```
[Out] -x/(x^4 + 1)^(1/2)
```

3.220 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=385

$$\frac{f\sqrt{a+bx^4}}{2b}+\frac{gx\sqrt{a+bx^4}}{3b}+\frac{hx^2\sqrt{a+bx^4}}{4b}+\frac{ix^3\sqrt{a+bx^4}}{5b}+\frac{(5be-3ai)x\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{(2bd-ah)\tanh^{-1}\left(\right.}{4b^{3/2}}$$

```
[Out] 1/4*(-a*h+2*b*d)*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/2*f*(b*x^4+
a)^(1/2)/b+1/3*g*x*(b*x^4+a)^(1/2)/b+1/4*h*x^2*(b*x^4+a)^(1/2)/b+1/5*i*x^3*
(b*x^4+a)^(1/2)/b+1/5*(-3*a*i+5*b*e)*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2
*b^(1/2))-1/5*a^(1/4)*(-3*a*i+5*b*e)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(
1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2
)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)+1/30*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4
)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(15*b*e-9*a*i+5*(-a*g+3*b*
c)*b^(1/2)/a^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^
4+a)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1899, 1833, 1829, 655, 223, 212, 1902, 1212, 226, 1210}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\frac{5\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-9ai+15be\right)}{30b^{7/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,(5be-3ai)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}+\frac{(2bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}+\frac{x\sqrt{a+bx^4}\,(5be-3ai)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{f\sqrt{a+bx^4}}{2b}+\frac{gx\sqrt{a+bx^4}}{3b}+\frac{hx^2\sqrt{a+bx^4}}{4b}+\frac{ix^3\sqrt{a+bx^4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4],x]
```

```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (g*x*Sqrt[a + b*x^4])/(3*b) + (h*x^2*Sqrt[a + b
*x^4])/(4*b) + (i*x^3*Sqrt[a + b*x^4])/(5*b) + ((5*b*e - 3*a*i)*x*Sqrt[a +
b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*b*d - a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*(5*b*e - 3*a*i)*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(15*b*
e + (5*Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 9*a*i)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
```

```
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+220x^6}{\sqrt{a+bx^4}}\,dx = \int \left(\frac{x(d+fx^2+hx^4)}{\sqrt{a+bx^4}}+\frac{c+ex^2+gx^4+220x^6}{\sqrt{a+bx^4}}\right)dx$$

$$= \int \frac{x(d+fx^2+hx^4)}{\sqrt{a+bx^4}}\,dx + \int \frac{c+ex^2+gx^4+220x^6}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{44x^3\sqrt{a+bx^4}}{b} + \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{\sqrt{a+bx^2}}\,dx, x, x^2\right) +$$

$$= \frac{gx\sqrt{a+bx^4}}{3b} + \frac{hx^2\sqrt{a+bx^4}}{4b} + \frac{44x^3\sqrt{a+bx^4}}{b} + \int \frac{5b(}{}$$

$$= \frac{f\sqrt{a+bx^4}}{2b} + \frac{gx\sqrt{a+bx^4}}{3b} + \frac{hx^2\sqrt{a+bx^4}}{4b} + \frac{44x^3\sqrt{a}}{b}$$

$$= \frac{f\sqrt{a+bx^4}}{2b} + \frac{gx\sqrt{a+bx^4}}{3b} + \frac{hx^2\sqrt{a+bx^4}}{4b} + \frac{44x^3\sqrt{a}}{b}$$

$$= \frac{f\sqrt{a+bx^4}}{2b} + \frac{gx\sqrt{a+bx^4}}{3b} + \frac{hx^2\sqrt{a+bx^4}}{4b} + \frac{44x^3\sqrt{a}}{b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.15, size = 281, normalized size = 0.73

$$\frac{30a\sqrt{b}\,f+20a\sqrt{b}\,gx+15a\sqrt{b}\,hx^2+12a\sqrt{b}\,ix^3+30b^{3/2}fx^4+20b^{3/2}gx^5+15b^{3/2}hx^6+12b^{3/2}ix^7+30bd\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-15ah\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-20\sqrt{b}\,(-3bc+ag)x\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+4\sqrt{b}\,(5be-3ai)x^3\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{60b^{3/2}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4]
,x]
```

```
[Out] (30*a*Sqrt[b]*f + 20*a*Sqrt[b]*g*x + 15*a*Sqrt[b]*h*x^2 + 12*a*Sqrt[b]*i*x^
3 + 30*b^(3/2)*f*x^4 + 20*b^(3/2)*g*x^5 + 15*b^(3/2)*h*x^6 + 12*b^(3/2)*i*x
^7 + 30*b*d*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 15*a*h
*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*Sqrt[b]*(-3*b*
c + a*g)*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a
)] + 4*Sqrt[b]*(5*b*e - 3*a*i)*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/
2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^(3/2)*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.35, size = 460, normalized size = 1.19

method	result
elliptic	$\frac{i x^3\sqrt{b x^4+a}}{5b}+\frac{h x^2\sqrt{b x^4+a}}{4b}+\frac{g x\sqrt{b x^4+a}}{3b}+\frac{f\sqrt{b x^4+a}}{2b}+\frac{\left(c-\frac{a g}{3b}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4}}$
default	$i\left(\frac{x^3\sqrt{b x^4+a}}{5b}-\frac{3 i a^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5 b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}\right.$
risch	$\frac{\left(12 i x^3+15 h x^2+20 g x+30 f\right)\sqrt{b x^4+a}}{60 b}-\frac{3 i a^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right) i}{5 b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}+\frac{i\sqrt{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] i*(1/5*x^3*(b*x^4+a)^(1/2)/b-3/5*I*a^(3/2)/b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2
)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)
^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)))+h*(1/4*x^2*(b*x^4+a)^(1/2)/b-1/4*a/b^(3/2)*ln(x^2*b^(1/2)+
(b*x^4+a)^(1/2)))+g*(1/3*x*(b*x^4+a)^(1/2)/b-1/3*a/b/(I/a^(1/2)*b^(1/2))^(1
/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*f*(b*x^4+a)^(1/2)/b+
I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*d*ln(x^2*b^
(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((h*x^5 + I*x^6 + g*x^4 + f*x^3 + x^2*e + d*x + c)/sqrt(b*x^4 + a)
, x)
```

Fricas [A]
time = 0.16, size = 204, normalized size = 0.53

$$\frac{24\,(5\,abe-3\,a^2i)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+8\,(15\,b^2c-15\,abe-5\,abg+9\,a^2i)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-15\,(2\,abd-a^2h)\sqrt{b}\,x\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\,\sqrt{b}\,x^2-a\right)+2\,(12\,abix^4+15\,abhx^3+20\,abgx^2+30\,abfx+60\,abe-36\,a^2i)\sqrt{bx^4+a}}{120\,ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorith
m="fricas")
```

```
[Out] 1/120*(24*(5*a*b*e - 3*a^2*i)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/
b)^(1/4)/x), -1) + 8*(15*b^2*c - 15*a*b*e - 5*a*b*g + 9*a^2*i)*sqrt(b)*x*(-
a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) - 15*(2*a*b*d - a^2*h)*sq
rt(b)*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(12*a*b*i*x^4
 + 15*a*b*h*x^3 + 20*a*b*g*x^2 + 30*a*b*f*x + 60*a*b*e - 36*a^2*i)*sqrt(b*x
^4 + a))/(a*b^2*x)
```

Sympy [A]
time = 3.50, size = 260, normalized size = 0.68

$$\frac{\sqrt{a}\,hx^2\sqrt{1+\frac{bx^4}{a}}}{4b}-\frac{ah\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+f\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for}\ b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{d\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{gx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{ix^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*h*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*h*asinh(sqrt(b)*x**2/sqrt(a))/(
4*b**(3/2)) + f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(
2*b), True)) + d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*h
yper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) +
 e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(7/4)) + g*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp
_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + i*x**7*gamma(7/4)*hyper((1/2, 7/4)
, (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((h*x^5 + I*x^6 + g*x^4 + f*x^3 + x^2*e + d*x + c)/sqrt(b*x^4 + a)
, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{i\,x^6+h\,x^5+g\,x^4+f\,x^3+e\,x^2+d\,x+c}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^(1/2),x)
[Out] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^(1/2), x)
```

3.221 $\int \frac{1+x}{1+x^5} \, dx$

Optimal. Leaf size=109

$$-\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(\sqrt[5]{-1}-x\right)+\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\log\left(-(-1)^{4/5}-x\right)+\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right.$$

```
[Out] -1/5*(-1)^(1/5)*(1+(-1)^(1/5))*ln((-1)^(1/5)-x)+1/5*(-1)^(4/5)*(1-(-1)^(4/5
))*ln(-(-1)^(4/5)-x)+1/5*(-1)^(2/5)*(1-(-1)^(2/5))*ln((-1)^(2/5)+x)-1/5*(-1
)^(3/5)*(1+(-1)^(3/5))*ln(-(-1)^(3/5)+x)
```

Rubi [A]

time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1600, 2093}

$$-\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(\sqrt[5]{-1}-x\right)+\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\log\left(-x-(-1)^{4/5}\right)+\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left(x+(-1)^{2/5}\right)-\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(x-(-1)^{3/5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(1 + x^5),x]
```

```
[Out] -1/5*((-1)^(1/5)*(1 + (-1)^(1/5))*Log[(-1)^(1/5) - x]) + ((-1)^(4/5)*(1 - (
-1)^(4/5))*Log[-(-1)^(4/5) - x])/5 + ((-1)^(2/5)*(1 - (-1)^(2/5))*Log[(-1)^
(2/5) + x])/5 - ((-1)^(3/5)*(1 + (-1)^(3/5))*Log[-(-1)^(3/5) + x])/5
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2093

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[1/a^
(3*p), Int[ExpandIntegrand[1/((a - b*x)^p/(a^5 - b^5*x^5)^p), x], x], x] /;
 NeQ[a, 0] && EqQ[c, b^2/a] && EqQ[d, b^3/a^2] && EqQ[e, b^4/a^3]] /; FreeQ
[p, x] && PolyQ[P4, x, 4] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x}{1+x^5} \, dx &= \int \frac{1}{1-x+x^2-x^3+x^4} \, dx \\
&= \int \left(\frac{-1+(-1)^{4/5}}{5\left(-1+\sqrt[5]{-1}\,x\right)}+\frac{-1-(-1)^{3/5}}{5\left(-1-(-1)^{2/5}x\right)}+\frac{-1+(-1)^{2/5}}{5\left(-1+(-1)^{3/5}x\right)}+\frac{-1-\sqrt[5]{-1}}{5\left(-1-(-1)^{4/5}x\right)}\right) d \\
&= -\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(\sqrt[5]{-1}-x\right)+\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\log\left(-(-1)^{4/5}-x\right)+\frac{1}{5}(-
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 51, normalized size = 0.47

$$\text{RootSum}\left[1-\#1+\#1^2-\#1^3+\#1^4\&,\frac{\log(x-\#1)}{-1+2\#1-3\#1^2+4\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(1 + x^5),x]
[Out] RootSum[1 - #1 + #1^2 - #1^3 + #1^4 & , Log[x - #1]/(-1 + 2*#1 - 3*#1^2 + 4
*#1^3) & ]
```

Maple [A]
time = 0.32, size = 143, normalized size = 1.31

method	result
risch	$\sum_{\_R=\text{RootOf}(\_Z^4-\_Z^3+\_Z^2-\_Z+1)}\frac{\ln(x-\_R)}{4\_R^3-3\_R^2+2\_R-1}$
default	$-\frac{\sqrt{5}\,\ln\left(-x\sqrt{5}+2x^2-x+2\right)}{10}-\frac{2\left(-\frac{\sqrt{5}\,\left(-\sqrt{5}-1\right)}{2}-\sqrt{5}-5\right)\arctan\left(\frac{-\sqrt{5}+4x-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{\sqrt{5}\,\ln\left(x\sqrt{5}+2x^2\right.}{10}$
meijerg	$-\frac{x^2\ln\left(1+(x^5)^{\frac{1}{5}}\right)}{5(x^5)^{\frac{2}{5}}}-\frac{x^2\cos\left(\frac{2\pi}{5}\right)\ln\left(1-2\cos\left(\frac{\pi}{5}\right)(x^5)^{\frac{1}{5}}+(x^5)^{\frac{2}{5}}\right)}{5(x^5)^{\frac{2}{5}}}+\frac{2x^2\sin\left(\frac{2\pi}{5}\right)\arctan\left(\frac{\sin\left(\frac{\pi}{5}\right)(x^5)^{\frac{1}{5}}}{1-\cos\left(\frac{\pi}{5}\right)(x^5)^{\frac{1}{5}}}\right)}{5(x^5)^{\frac{2}{5}}}+\frac{x^2\cos\left(\frac{\pi}{5}\right)\ln\left(1-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^5+1),x,method=_RETURNVERBOSE)
[Out] -1/10*5^(1/2)*ln(-x*5^(1/2)+2*x^2-x+2)-2/5*(-1/2*5^(1/2)*(-5^(1/2)-1)-5^(1/
2)-5)/(10-2*5^(1/2))^(1/2)*arctan((-5^(1/2)+4*x-1)/(10-2*5^(1/2))^(1/2))+1/
10*5^(1/2)*ln(x*5^(1/2)+2*x^2-x+2)+2/5*(-1/2*5^(1/2)*(5^(1/2)-1)+5-5^(1/2))
/(10+2*5^(1/2))^(1/2)*arctan((5^(1/2)+4*x-1)/(10+2*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^5+1),x, algorithm="maxima")
[Out] integrate((x + 1)/(x^5 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(73) = 146$.

time = 1.15, size = 835, normalized size = 7.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^5+1),x, algorithm="fricas")
```

```
[Out] -1/10*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))*log(3/8*(sqrt(5) + 5*sqrt(-2/
25*sqrt(5) - 1/5))^3 + 1/8*(3*sqrt(5) + 15*sqrt(-2/25*sqrt(5) - 1/5) + 8)*(
sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2 + 3/8*((sqrt(5) + 5*sqrt(-2/25*sqr
t(5) - 1/5))^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 11*x + 1) -
1/10*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))*log(-3/8*(sqrt(5) + 5*sqrt(-2/
25*sqrt(5) - 1/5))^3 + (sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 + 11*x - 9
/2*sqrt(5) - 45/2*sqrt(-2/25*sqrt(5) - 1/5) - 14) + 1/10*(sqrt(5) + 5*sqrt(
-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) + 5*sqrt(-
2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/100*(sqrt(
5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2))*log(-1/8*(3*sqrt(5) + 15*sqrt(-2/25*s
qrt(5) - 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - (sqrt(5) + 5
*sqrt(-2/25*sqrt(5) - 1/5))^2 - 3/8*((sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5)
)^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 5/4*sqrt(-3/100*(sqrt(5
) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) -
 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/100*(sqrt(5) - 5*sqrt(-2
/25*sqrt(5) - 1/5))^2)*((3*sqrt(5) + 15*sqrt(-2/25*sqrt(5) - 1/5) + 8)*(sqr
t(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 8*sqrt(5) + 40*sqrt(-2/25*sqrt(5) - 1
/5) + 36) + 22*x + 9/2*sqrt(5) + 45/2*sqrt(-2/25*sqrt(5) - 1/5) + 2) + 1/10
*(sqrt(5) - 5*sqrt(-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*
(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1
/5)) - 3/100*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2))*log(-1/8*(3*sqrt(5
) + 15*sqrt(-2/25*sqrt(5) - 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5
))^2 - (sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 3/8*((sqrt(5) + 5*sqrt(-
2/25*sqrt(5) - 1/5))^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 5/4*
sqrt(-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) + 5*s
qrt(-2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/100*(
sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2)*((3*sqrt(5) + 15*sqrt(-2/25*sqrt(
5) - 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 8*sqrt(5) + 40*sqr
t(-2/25*sqrt(5) - 1/5) + 36) + 22*x + 9/2*sqrt(5) + 45/2*sqrt(-2/25*sqrt(5)
 - 1/5) + 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. $2(109) = 218$.

time = 0.55, size = 1287, normalized size = 11.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**5+1),x)
```

```
[Out] sqrt(5)*log(x**2 + x*(-48/11 - 21*sqrt(5)/11 + 4*sqrt(10)*sqrt(sqrt(5) + 3)
/11 + 45*sqrt(2)*sqrt(sqrt(5) + 3)/22) - 1381*sqrt(10)*sqrt(sqrt(5) + 3)/48
4 - 3045*sqrt(2)*sqrt(sqrt(5) + 3)/484 + 2213*sqrt(5)/242 + 5217/242)/10 -
sqrt(5)*log(x**2 + x*(-48/11 - 45*sqrt(2)*sqrt(3 - sqrt(5))/22 + 4*sqrt(10)
*sqrt(3 - sqrt(5))/11 + 21*sqrt(5)/11) - 2213*sqrt(5)/242 - 1381*sqrt(10)*s
qrt(3 - sqrt(5))/484 + 3045*sqrt(2)*sqrt(3 - sqrt(5))/484 + 5217/242)/10 +
2*sqrt(-sqrt(10)*sqrt(3 - sqrt(5))/50 + 3/20)*atan(44*x/(-8*sqrt(5)*sqrt(-2
*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sq
rt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15
)) - 96/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*s
qrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt
(10)*sqrt(3 - sqrt(5)) + 15)) - 45*sqrt(2)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(
-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5))
 + 15)) + 8*sqrt(10)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3
- sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - s
qrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) + 42*sqrt(5)/(
-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - s
qrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt
(3 - sqrt(5)) + 15))) + 2*sqrt(-sqrt(10)*sqrt(sqrt(5) + 3)/50 + 3/20)*atan(
44*x/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(
10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)
*sqrt(sqrt(5) + 3) + 15)) - 96/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3
) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqr
t(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)) - 42*sqrt(5)/(8*sqrt(5)
*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5
) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) +
 3) + 15)) + 8*sqrt(10)*sqrt(sqrt(5) + 3)/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(
sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(1
0)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)) + 45*sqrt(2)
*sqrt(sqrt(5) + 3)/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18
*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sq
rt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)))
```

Giac [A]
time = 0.54, size = 101, normalized size = 0.93

$$\frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)+\frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)-\frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)+\frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^5+1),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) +
 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))
- 1/10*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/10*sqrt(5)*log(x^2 +
1/2*x*(sqrt(5) - 1) + 1)
```

Mupad [B]
time = 4.92, size = 64, normalized size = 0.59

$$\sum_{k=1}^{4} \ln\left(\mathrm{root}\left(z^4 - \frac{z}{25} + \frac{1}{125}, z, k\right)\left(-4\,x + \mathrm{root}\left(z^4 - \frac{z}{25} + \frac{1}{125}, z, k\right)\left(25\,\mathrm{root}\left(z^4 - \frac{z}{25} + \frac{1}{125}, z, k\right) + 15\,x - 15\right) + 1\right)\right)\mathrm{root}\left(z^4 - \frac{z}{25} + \frac{1}{125}, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x^5 + 1),x)
```

```
[Out] symsum(log(root(z^4 - z/25 + 1/125, z, k)*(root(z^4 - z/25 + 1/125, z, k)*(
25*root(z^4 - z/25 + 1/125, z, k) + 15*x - 15) - 4*x + 1))*root(z^4 - z/25
+ 1/125, z, k), k, 1, 4)
```

3.222 $\int \frac{1-x}{1-x^5} \, dx$

Optimal. Leaf size=109

$$-\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(-(-1)^{3/5}-x\right)+\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right.$$

```
[Out] -1/5*(-1)^(2/5)*(1-(-1)^(2/5))*ln((-1)^(2/5)-x)+1/5*(-1)^(3/5)*(1+(-1)^(3/5
))*ln(-(-1)^(3/5)-x)+1/5*(-1)^(1/5)*(1+(-1)^(1/5))*ln((-1)^(1/5)+x)-1/5*(-1
)^(4/5)*(1-(-1)^(4/5))*ln(-(-1)^(4/5)+x)
```

Rubi [A]

time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1600, 2093}

$$-\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(-x-(-1)^{3/5}\right)+\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(x+\sqrt[5]{-1}\right)-\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\log\left(x-(-1)^{4/5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 - x^5),x]
```

```
[Out] -1/5*((-1)^(2/5)*(1 - (-1)^(2/5))*Log[(-1)^(2/5) - x]) + ((-1)^(3/5)*(1 + (
-1)^(3/5))*Log[-(-1)^(3/5) - x])/5 + ((-1)^(1/5)*(1 + (-1)^(1/5))*Log[(-1)^
(1/5) + x])/5 - ((-1)^(4/5)*(1 - (-1)^(4/5))*Log[-(-1)^(4/5) + x])/5
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2093

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[1/a^
(3*p), Int[ExpandIntegrand[1/((a - b*x)^p/(a^5 - b^5*x^5)^p), x], x], x] /;
 NeQ[a, 0] && EqQ[c, b^2/a] && EqQ[d, b^3/a^2] && EqQ[e, b^4/a^3]] /; FreeQ
[p, x] && PolyQ[P4, x, 4] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x}{1-x^5} \, dx &= \int \frac{1}{1+x+x^2+x^3+x^4} \, dx \\
&= \int \left(\frac{1-(-1)^{4/5}}{5\left(1+\sqrt[5]{-1}\,x\right)}+\frac{1+(-1)^{3/5}}{5\left(1-(-1)^{2/5}x\right)}+\frac{1-(-1)^{2/5}}{5\left(1+(-1)^{3/5}x\right)}+\frac{1+\sqrt[5]{-1}}{5\left(1-(-1)^{4/5}x\right)}\right) dx \\
&= -\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(-(-1)^{3/5}-x\right)+
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 47, normalized size = 0.43

$$\text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{\log(x-\#1)}{1+2\#1+3\#1^2+4\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(1 - x^5),x]
```

```
[Out] RootSum[1 + #1 + #1^2 + #1^3 + #1^4 & , Log[x - #1]/(1 + 2*#1 + 3*#1^2 + 4*
#1^3) & ]
```

Maple [A]
time = 0.33, size = 135, normalized size = 1.24

method	result
risch	$\sum_{\_R=\text{RootOf}(\_Z^4+\_Z^3+\_Z^2+\_Z+1)}\frac{\ln(x-\_R)}{4\_R^3+3\_R^2+2\_R+1}$
default	$\frac{\sqrt{5}\,\ln\left(x\sqrt{5}+2x^2+x+2\right)}{10}+\frac{2\left(-\frac{\sqrt{5}\,(\sqrt{5}+1)}{2}+5+\sqrt{5}\right)\arctan\left(\frac{1+4x+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{\sqrt{5}\,\ln\left(-x\sqrt{5}+2x^2+\right.}{10}$
meijerg	$-\frac{x\left(\ln\left(1-(x^5)^{\frac{1}{5}}\right)+\cos(\frac{2\pi}{5})\ln\left(1-2\cos(\frac{2\pi}{5})(x^5)^{\frac{1}{5}}+(x^5)^{\frac{2}{5}}\right)-2\sin(\frac{2\pi}{5})\arctan\left(\frac{\sin(\frac{2\pi}{5})(x^5)^{\frac{1}{5}}}{1-\cos(\frac{2\pi}{5})(x^5)^{\frac{1}{5}}}\right)-\cos(\frac{\pi}{5})\ln\left(1+2\cos(\frac{\pi}{5})(x\right.\right.}{5(x^5)^{\frac{1}{5}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(-x^5+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*5^(1/2)*ln(x*5^(1/2)+2*x^2+x+2)+2/5*(-1/2*5^(1/2)*(5^(1/2)+1)+5+5^(1/2
))/(10-2*5^(1/2))^(1/2)*arctan((1+4*x+5^(1/2))/(10-2*5^(1/2))^(1/2))-1/10*5
^(1/2)*ln(-x*5^(1/2)+2*x^2+x+2)-2/5*(-1/2*5^(1/2)*(-5^(1/2)+1)+5^(1/2)-5)/(
10+2*5^(1/2))^(1/2)*arctan((1+4*x-5^(1/2))/(10+2*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^5+1),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/(x^5 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(73) = 146$.
time = 1.17, size = 799, normalized size = 7.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^5+1),x, algorithm="fricas")
```

```
[Out] -1/10*(sqrt(5) - sqrt(2*sqrt(5) - 5))*log(3/8*(sqrt(5) + sqrt(2*sqrt(5) - 5
))^3 + 1/8*(3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5
) - 5))^2 + 3/8*((sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 12)*(sqrt(5) - sqrt(2*
sqrt(5) - 5)) + 11*x - 1) - 1/10*(sqrt(5) + sqrt(2*sqrt(5) - 5))*log(-3/8*(
sqrt(5) + sqrt(2*sqrt(5) - 5))^3 - (sqrt(5) + sqrt(2*sqrt(5) - 5))^2 + 11*x
 - 9/2*sqrt(5) - 9/2*sqrt(2*sqrt(5) - 5) + 14) + 1/10*(sqrt(5) + 5*sqrt(-3/
100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sqrt(5) + sqrt(2*sqrt(5) - 5)
)*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2
))*log(-1/8*(3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(
5) - 5))^2 + (sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 3/8*((sqrt(5) + sqrt(2*sqr
t(5) - 5))^2 - 12)*(sqrt(5) - sqrt(2*sqrt(5) - 5)) + 5/4*sqrt(-3/100*(sqrt(
5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sqrt(5) + sqrt(2*sqrt(5) - 5))*(sqrt(5)
 - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2)*((3*sqrt
(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 8*sqrt(5
) - 8*sqrt(2*sqrt(5) - 5) + 36) + 22*x + 9/2*sqrt(5) + 9/2*sqrt(2*sqrt(5) -
 5) - 2) + 1/10*(sqrt(5) - 5*sqrt(-3/100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2
- 1/50*(sqrt(5) + sqrt(2*sqrt(5) - 5))*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/
100*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2))*log(-1/8*(3*sqrt(5) + 3*sqrt(2*sqrt
(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2 + (sqrt(5) + sqrt(2*sqrt(5)
 - 5))^2 - 3/8*((sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 12)*(sqrt(5) - sqrt(2*s
qrt(5) - 5)) - 5/4*sqrt(-3/100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sq
rt(5) + sqrt(2*sqrt(5) - 5))*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(
5) - sqrt(2*sqrt(5) - 5))^2)*((3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt
(5) - sqrt(2*sqrt(5) - 5)) - 8*sqrt(5) - 8*sqrt(2*sqrt(5) - 5) + 36) + 22*x
 + 9/2*sqrt(5) + 9/2*sqrt(2*sqrt(5) - 5) - 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. $2(109) = 218$.
time = 0.54, size = 1287, normalized size = 11.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x**5+1),x)
```

```
[Out] sqrt(5)*log(x**2 + x*(-21*sqrt(5)/11 - 4*sqrt(10)*sqrt(3 - sqrt(5))/11 + 45
*sqrt(2)*sqrt(3 - sqrt(5))/22 + 48/11) - 2213*sqrt(5)/242 - 1381*sqrt(10)*s
```

```
qrt(3 - sqrt(5))/484 + 3045*sqrt(2)*sqrt(3 - sqrt(5))/484 + 5217/242)/10 -
sqrt(5)*log(x**2 + x*(-45*sqrt(2)*sqrt(sqrt(5) + 3)/22 - 4*sqrt(10)*sqrt(sq
rt(5) + 3)/11 + 21*sqrt(5)/11 + 48/11) - 1381*sqrt(10)*sqrt(sqrt(5) + 3)/48
4 - 3045*sqrt(2)*sqrt(sqrt(5) + 3)/484 + 2213*sqrt(5)/242 + 5217/242)/10 +
2*sqrt(-sqrt(10)*sqrt(3 - sqrt(5))/50 + 3/20)*atan(44*x/(-8*sqrt(5)*sqrt(-2
*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sq
rt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15
)) - 42*sqrt(5)/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sq
rt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt
(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) - 8*sqrt(10)*sqrt(3 - sqrt(5))/(-8*sq
rt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5
))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 -
sqrt(5)) + 15)) + 45*sqrt(2)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*sqrt(-2*sqrt(10)
*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sq
rt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) + 96/(
-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - s
qrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt
(3 - sqrt(5)) + 15))) + 2*sqrt(-sqrt(10)*sqrt(sqrt(5) + 3)/50 + 3/20)*atan(
44*x/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(
10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)
*sqrt(sqrt(5) + 3) + 15)) - 45*sqrt(2)*sqrt(sqrt(5) + 3)/(8*sqrt(5)*sqrt(-2
*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) +
 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15
)) - 8*sqrt(10)*sqrt(sqrt(5) + 3)/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5)
+ 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(
sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)) + 42*sqrt(5)/(8*sqrt
(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqr
t(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5
) + 3) + 15)) + 96/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18
*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sq
rt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)))
```

Giac [A]
time = 0.61, size = 101, normalized size = 0.93

$$\frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\sqrt{5}+10}}\right)+\frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)-\frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^5+1),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x - sqrt(5) + 1)/sqrt(2*sqrt(5) + 10)) +
 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x + sqrt(5) + 1)/sqrt(-2*sqrt(5) + 10))
+ 1/10*sqrt(5)*log(x^2 + 1/2*x*(sqrt(5) + 1) + 1) - 1/10*sqrt(5)*log(x^2 -
1/2*x*(sqrt(5) - 1) + 1)
```

Mupad [B]
time = 4.98, size = 65, normalized size = 0.60

$$\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)\,\left(4\,x+\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)\,\left(25\,\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)+15\,x+15\right)+1\right)\right)\,\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x^5 - 1),x)
```

```
[Out] symsum(log(-root(z^4 + z/25 + 1/125, z, k)*(4*x + root(z^4 + z/25 + 1/125,
z, k)*(25*root(z^4 + z/25 + 1/125, z, k) + 15*x + 15) + 1))*root(z^4 + z/25
 + 1/125, z, k), k, 1, 4)
```

3.223 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=208

$$\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^6}-\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^6}{6b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^9}{9b^4}+\frac{(b^2d-abe}{12}$$

```
[Out] 1/3*a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^3/b^6-1/6*a*(-a^3*f+a^2*b*e-a*b^2*
d+b^3*c)*x^6/b^5+1/9*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^9/b^4+1/12*(a^2*f-a*b
*e+b^2*d)*x^12/b^3+1/15*(-a*f+b*e)*x^15/b^2+1/18*f*x^18/b-1/3*a^3*(-a^3*f+a
^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/b^7
```

Rubi [A]
time = 0.21, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{x^{12}(a^2f-abe+b^2d)}{12b^3}-\frac{a^3\log(a+bx^3)\,(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^7}+\frac{a^2x^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^6}-\frac{ax^6(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5}+\frac{x^9(a^3(-f)+a^2be-ab^2d+b^3c)}{9b^4}+\frac{x^{15}(be-af)}{15b^2}+\frac{fx^{18}}{18b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^6) - (a*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^6)/(6*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9
)/(9*b^4) + ((b^2*d - a*b*e + a^2*f)*x^12)/(12*b^3) + ((b*e - a*f)*x^15)/(1
5*b^2) + (f*x^18)/(18*b) - (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a +
 b*x^3])/(3*b^7)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)}{b^6}+\frac{a(-b^3c+ab^2d-a^2be+a$$
$$= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^6}-\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^6}{6b^5}+\frac{(b^3c-$$

Mathematica [A]
time = 0.06, size = 187, normalized size = 0.90

$$\frac{bx^3(-60a^5f+30a^4b(2e+fx^3)-10a^3b^2(6d+3ex^3+2fx^6)+5a^2b^3(12c+6dx^3+4ex^6+3fx^9)+b^5x^6(20c+15dx^3+12ex^6+10fx^9)-ab^4x^3(30c+20dx^3+15ex^6+12fx^9))+60a^3(-b^3c+ab^2d-a^2be+a^3f)\log(a+bx^3)}{180b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(-60*a^5*f + 30*a^4*b*(2*e + f*x^3) - 10*a^3*b^2*(6*d + 3*e*x^3 + 2*
f*x^6) + 5*a^2*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9) + b^5*x^6*(20*c + 1
5*d*x^3 + 12*e*x^6 + 10*f*x^9) - a*b^4*x^3*(30*c + 20*d*x^3 + 15*e*x^6 + 12
*f*x^9)) + 60*a^3*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a + b*x^3])/(1
80*b^7)
```

Maple [A]
time = 0.34, size = 236, normalized size = 1.13

method	result
norman	$-\frac{(a^3f-a^2be+ab^2d-b^3c)x^9}{9b^4}-\frac{(af-be)x^{15}}{15b^2}+\frac{fx^{18}}{18b}+\frac{(a^2f-abe+b^2d)x^{12}}{12b^3}+\frac{a(a^3f-a^2be+ab^2d-b^3c)x^6}{6b^5}-\frac{a^2(a^3f-a^2be+a}{3b^6}$
default	$-\frac{-\frac{1}{6}fx^{18}b^5+\frac{1}{5}ab^4fx^{15}-\frac{1}{5}b^5ex^{15}-\frac{1}{4}a^2b^3fx^{12}+\frac{1}{4}ab^4ex^{12}-\frac{1}{4}b^5dx^{12}+\frac{1}{3}a^3b^2fx^9-\frac{1}{3}a^2b^3ex^9+\frac{1}{3}ab^4dx^9-\frac{1}{3}b^5cx^9-\frac{1}{2}a^4bfx^6+\frac{1}{2}a}{3b^6}$
risch	$\frac{fx^{18}}{18b}-\frac{afx^{15}}{15b^2}+\frac{ex^{15}}{15b}+\frac{a^2fx^{12}}{12b^3}-\frac{aex^{12}}{12b^2}+\frac{dx^{12}}{12b}-\frac{a^3fx^9}{9b^4}+\frac{a^2ex^9}{9b^3}-\frac{adx^9}{9b^2}+\frac{cx^9}{9b}+\frac{a^4fx^6}{6b^5}-\frac{a^3ex^6}{6b^4}+\frac{a^2dx^6}{6b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b^6*(-1/6*f*x^18*b^5+1/5*a*b^4*f*x^15-1/5*b^5*e*x^15-1/4*a^2*b^3*f*x^1
2+1/4*a*b^4*e*x^12-1/4*b^5*d*x^12+1/3*a^3*b^2*f*x^9-1/3*a^2*b^3*e*x^9+1/3*a
*b^4*d*x^9-1/3*b^5*c*x^9-1/2*a^4*b*f*x^6+1/2*a^3*b^2*e*x^6-1/2*a^2*b^3*d*x^
6+1/2*a*b^4*c*x^6+a^5*f*x^3-a^4*b*e*x^3+a^3*b^2*d*x^3-a^2*b^3*c*x^3)+1/3*a^
3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^7*ln(b*x^3+a)
```

Maxima [A]
time = 0.28, size = 215, normalized size = 1.03

$$\frac{10\,b^5fx^{18}-12\,(ab^4f-b^5e)x^{15}+15\,(b^5d+a^2b^3f-ab^4e)x^{12}+20\,(b^5c-ab^4d-a^3b^2f+a^2b^3e)x^9-30\,(ab^4c-a^2b^3d-a^4bf+a^3b^2e)x^6+60\,(a^2b^3c-a^3b^2d-a^5f+a^4be)x^3}{180\,b^6}-\frac{(a^3b^3c-a^4b^2d-a^6f+a^5be)\log(bx^3+a)}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/180*(10*b^5*f*x^18 - 12*(a*b^4*f - b^5*e)*x^15 + 15*(b^5*d + a^2*b^3*f -
a*b^4*e)*x^12 + 20*(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*x^9 - 30*(a*b^
4*c - a^2*b^3*d - a^4*b*f + a^3*b^2*e)*x^6 + 60*(a^2*b^3*c - a^3*b^2*d - a^
5*f + a^4*b*e)*x^3)/b^6 - 1/3*(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)*log
(b*x^3 + a)/b^7
```

Fricas [A]
time = 0.41, size = 210, normalized size = 1.01

$$\frac{10\,b^6fx^{18}+12\,(b^6e-ab^5f)x^{15}+15\,(b^6d-ab^5e+a^2b^4f)x^{12}+20\,(b^6c-ab^5d+a^2b^4e-a^3b^3f)x^9-30\,(ab^5c-a^2b^4d+a^3b^3e-a^4b^2f)x^6+60\,(a^2b^4c-a^3b^3d+a^4b^2e-a^5bf)x^3-60\,(a^3b^3c-a^4b^2d+a^5be-a^6f)\log\left(bx^3+a\right)}{180\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/180*(10*b^6*f*x^18 + 12*(b^6*e - a*b^5*f)*x^15 + 15*(b^6*d - a*b^5*e + a^
2*b^4*f)*x^12 + 20*(b^6*c - a*b^5*d + a^2*b^4*e - a^3*b^3*f)*x^9 - 30*(a*b^
5*c - a^2*b^4*d + a^3*b^3*e - a^4*b^2*f)*x^6 + 60*(a^2*b^4*c - a^3*b^3*d +
a^4*b^2*e - a^5*b*f)*x^3 - 60*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*log
(b*x^3 + a))/b^7
```

Sympy [A]
time = 0.62, size = 216, normalized size = 1.04

$$\frac{a^3(a^3f-a^2be+ab^2d-b^3c)\log\left(a+bx^3\right)}{3b^7}+x^{15}\left(-\frac{af}{15b^2}+\frac{e}{15b}\right)+x^{12}\left(\frac{a^2f}{12b^3}-\frac{ae}{12b^2}+\frac{d}{12b}\right)+x^9\left(-\frac{a^3f}{9b^4}+\frac{a^2e}{9b^3}-\frac{ad}{9b^2}+\frac{c}{9b}\right)+x^6\left(\frac{a^4f}{6b^5}-\frac{a^3e}{6b^4}+\frac{a^2d}{6b^3}-\frac{ac}{6b^2}\right)+x^3\left(-\frac{a^5f}{3b^6}+\frac{a^4e}{3b^5}-\frac{a^3d}{3b^4}+\frac{a^2c}{3b^3}\right)+\frac{fx^{18}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a**3*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**7) + x**
15*(-a*f/(15*b**2) + e/(15*b)) + x**12*(a**2*f/(12*b**3) - a*e/(12*b**2) +
d/(12*b)) + x**9*(-a**3*f/(9*b**4) + a**2*e/(9*b**3) - a*d/(9*b**2) + c/(9*
b)) + x**6*(a**4*f/(6*b**5) - a**3*e/(6*b**4) + a**2*d/(6*b**3) - a*c/(6*b*
*2)) + x**3*(-a**5*f/(3*b**6) + a**4*e/(3*b**5) - a**3*d/(3*b**4) + a**2*c/
(3*b**3)) + f*x**18/(18*b)
```

Giac [A]
time = 0.61, size = 246, normalized size = 1.18

$$\frac{10\,b^5fx^{18}-12\,ab^4fx^{15}+12\,b^5x^{15}e+15\,b^5dx^{12}+15\,a^2b^3fx^{12}-15\,ab^4x^{12}e+20\,b^5cx^9-20\,ab^4dx^9-20\,a^3b^2fx^9+20\,a^2b^3x^9e-30\,ab^4cx^6+30\,a^2b^3dx^6+30\,a^4bfx^6-30\,a^3b^2x^6e+60\,a^2b^3cx^3-60\,a^3b^2dx^3-60\,a^5fx^3+60\,a^4bx^3e}{180\,b^6}-\frac{(a^3b^3c-a^4b^2d-a^6f+a^5be)\log\left(|bx^3+a|\right)}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/180*(10*b^5*f*x^18 - 12*a*b^4*f*x^15 + 12*b^5*x^15*e + 15*b^5*d*x^12 + 15
*a^2*b^3*f*x^12 - 15*a*b^4*x^12*e + 20*b^5*c*x^9 - 20*a*b^4*d*x^9 - 20*a^3*
b^2*f*x^9 + 20*a^2*b^3*x^9*e - 30*a*b^4*c*x^6 + 30*a^2*b^3*d*x^6 + 30*a^4*b
*f*x^6 - 30*a^3*b^2*x^6*e + 60*a^2*b^3*c*x^3 - 60*a^3*b^2*d*x^3 - 60*a^5*f*
x^3 + 60*a^4*b*x^3*e)/b^6 - 1/3*(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)*l
og(abs(b*x^3 + a))/b^7
```

Mupad [B]
time = 4.92, size = 237, normalized size = 1.14

$$x^{15}\left(\frac{e}{15\,b}-\frac{a\,f}{15\,b^2}\right)+x^{12}\left(\frac{d}{12\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{12\,b}\right)+x^9\left(\frac{c}{9\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{9\,b}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^6-e\,a^5\,b+d\,a^4\,b^2-c\,a^3\,b^3\right)}{3\,b^7}+\frac{f\,x^{18}}{18\,b}+\frac{a^2\,x^3\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{3\,b^2}-\frac{a\,x^6\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^15*(e/(15*b) - (a*f)/(15*b^2)) + x^12*(d/(12*b) - (a*(e/b - (a*f)/b^2))/(
12*b)) + x^9*(c/(9*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(9*b)) + (log(a
 + b*x^3)*(a^6*f - a^3*b^3*c + a^4*b^2*d - a^5*b*e))/(3*b^7) + (f*x^18)/(18
*b) + (a^2*x^3*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(3*b^2) - (a*
x^6*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(6*b)
```

3.224 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=170

$$-\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^6}{6b^4}+\frac{(b^2d-abe+a^2f)\,x^9}{9b^3}+\frac{(be-af)x^{12}}{12b^2}+\frac{fx^{15}}{15b}$$

```
[Out] -1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^3/b^5+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b
^3*c)*x^6/b^4+1/9*(a^2*f-a*b*e+b^2*d)*x^9/b^3+1/12*(-a*f+b*e)*x^12/b^2+1/15
*f*x^15/b+1/3*a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/b^6
```

Rubi [A]
time = 0.16, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{x^9(a^2f-abe+b^2d)}{9b^3}+\frac{a^2\log(a+bx^3)\,(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^6}-\frac{ax^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5}+\frac{x^6(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^4}+\frac{x^{12}(be-af)}{12b^2}+\frac{fx^{15}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -1/3*(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/b^5 + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x^6)/(6*b^4) + ((b^2*d - a*b*e + a^2*f)*x^9)/(9*b^3) + ((b
*e - a*f)*x^12)/(12*b^2) + (f*x^15)/(15*b) + (a^2*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx+ex^2+fx^3)}{a+bx}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-b^3c+ab^2d-a^2be+a^3f)}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4}\right.\right.$$
$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^6}{6b^4} + \frac{(b^2d-\cdots}{}$$

Mathematica [A]
time = 0.05, size = 154, normalized size = 0.91

$$\frac{bx^3(60a^4f-30a^3b(2e+fx^3)+10a^2b^2(6d+3ex^3+2fx^6)-5ab^3(12c+6dx^3+4ex^6+3fx^9)+b^4x^3(30c+20dx^3+15ex^6+12fx^9))-60a^2(-b^3c+ab^2d-a^2be+a^3f)\log(a+bx^3)}{180b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(60*a^4*f - 30*a^3*b*(2*e + f*x^3) + 10*a^2*b^2*(6*d + 3*e*x^3 + 2*f
*x^6) - 5*a*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9) + b^4*x^3*(30*c + 20*d
*x^3 + 15*e*x^6 + 12*f*x^9)) - 60*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*Log[a + b*x^3])/(180*b^6)
```

Maple [A]
time = 0.37, size = 188, normalized size = 1.11

method	result
norman	$-\frac{(a^3f-a^2be+a\,b^2d-b^3c)x^6}{6b^4}-\frac{(af-be)x^{12}}{12b^2}+\frac{f\,x^{15}}{15b}+\frac{(a^2f-abe+b^2d)x^9}{9b^3}+\frac{a(a^3f-a^2be+a\,b^2d-b^3c)x^3}{3b^5}-\frac{a^2(a^3f-a^2be+a\cdots}{}$
default	$\frac{\frac{1}{5}f\,x^{15}b^4-\frac{1}{4}a\,b^3f\,x^{12}+\frac{1}{4}b^4e\,x^{12}+\frac{1}{3}a^2b^2f\,x^9-\frac{1}{3}a\,b^3e\,x^9+\frac{1}{3}b^4d\,x^9-\frac{1}{2}a^3bf\,x^6+\frac{1}{2}a^2b^2e\,x^6-\frac{1}{2}a\,b^3d\,x^6+\frac{1}{2}b^4c\,x^6+a^4f\,x^3-a^3be\,x^3+a^2b^2\cdots}{3b^5}$
risch	$\frac{f\,x^{15}}{15b}-\frac{af\,x^{12}}{12b^2}+\frac{e\,x^{12}}{12b}+\frac{a^2f\,x^9}{9b^3}-\frac{ae\,x^9}{9b^2}+\frac{d\,x^9}{9b}-\frac{a^3f\,x^6}{6b^4}+\frac{a^2e\,x^6}{6b^3}-\frac{ad\,x^6}{6b^2}+\frac{c\,x^6}{6b}+\frac{a^4f\,x^3}{3b^5}-\frac{a^3e\,x^3}{3b^4}+\frac{a^2d\,x^3}{3b^3}-\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b^5*(1/5*f*x^15*b^4-1/4*a*b^3*f*x^12+1/4*b^4*e*x^12+1/3*a^2*b^2*f*x^9-1
/3*a*b^3*e*x^9+1/3*b^4*d*x^9-1/2*a^3*b*f*x^6+1/2*a^2*b^2*e*x^6-1/2*a*b^3*d*
x^6+1/2*b^4*c*x^6+a^4*f*x^3-a^3*b*e*x^3+a^2*b^2*d*x^3-a*b^3*c*x^3)-1/3*a^2*
(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^6*ln(b*x^3+a)
```

Maxima [A]
time = 0.27, size = 174, normalized size = 1.02

$$\frac{12\,b^4fx^{15}-15\,(ab^3f-b^4e)x^{12}+20\,(b^4d+a^2b^2f-ab^3e)x^9+30\,(b^4c-ab^3d-a^3bf+a^2b^2e)x^6-60\,(ab^3c-a^2b^2d-a^4f+a^3be)x^3}{180\,b^5}+\frac{(a^2b^3c-a^3b^2d-a^5f+a^4be)\log(bx^3+a)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/180*(12*b^4*f*x^15 - 15*(a*b^3*f - b^4*e)*x^12 + 20*(b^4*d + a^2*b^2*f -
a*b^3*e)*x^9 + 30*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*x^6 - 60*(a*b^3*c
 - a^2*b^2*d - a^4*f + a^3*b*e)*x^3)/b^5 + 1/3*(a^2*b^3*c - a^3*b^2*d - a^5
*f + a^4*b*e)*log(b*x^3 + a)/b^6
```

Fricas [A]
time = 0.37, size = 170, normalized size = 1.00

$$\frac{12\,b^5fx^{15}+15\,(b^5e-ab^4f)x^{12}+20\,(b^5d-ab^4e+a^2b^3f)x^9+30\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^6-60\,(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^3+60\,(a^2b^3c-a^3b^2d+a^4be-a^5f)\log\left(bx^3+a\right)}{180\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^5*f*x^15 + 15*(b^5*e - a*b^4*f)*x^12 + 20*(b^5*d - a*b^4*e + a^
2*b^3*f)*x^9 + 30*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^6 - 60*(a*b^4
*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 + 60*(a^2*b^3*c - a^3*b^2*d + a^4
*b*e - a^5*f)*log(b*x^3 + a))/b^6
```

Sympy [A]
time = 0.58, size = 172, normalized size = 1.01

$$-\frac{a^2(a^3f-a^2be+ab^2d-b^3c)\log\left(a+bx^3\right)}{3b^6}+x^{12}\left(-\frac{af}{12b^2}+\frac{e}{12b}\right)+x^9\left(\frac{a^2f}{9b^3}-\frac{ae}{9b^2}+\frac{d}{9b}\right)+x^6\left(-\frac{a^3f}{6b^4}+\frac{a^2e}{6b^3}-\frac{ad}{6b^2}+\frac{c}{6b}\right)+x^3\left(\frac{a^4f}{3b^5}-\frac{a^3e}{3b^4}+\frac{a^2d}{3b^3}-\frac{ac}{3b^2}\right)+\frac{fx^{15}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] -a**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**6) + x*
*12*(-a*f/(12*b**2) + e/(12*b)) + x**9*(a**2*f/(9*b**3) - a*e/(9*b**2) + d/
(9*b)) + x**6*(-a**3*f/(6*b**4) + a**2*e/(6*b**3) - a*d/(6*b**2) + c/(6*b))
 + x**3*(a**4*f/(3*b**5) - a**3*e/(3*b**4) + a**2*d/(3*b**3) - a*c/(3*b**2)
) + f*x**15/(15*b)
```

Giac [A]
time = 0.56, size = 197, normalized size = 1.16

$$\frac{12\,b^4fx^{15}-15\,ab^3fx^{12}+15\,b^4x^{12}e+20\,b^4dx^9+20\,a^2b^2fx^9-20\,ab^3x^9e+30\,b^4cx^6-30\,ab^3dx^6-30\,a^3bfx^6+30\,a^2b^2x^6e-60\,ab^3cx^3+60\,a^2b^2dx^3+60\,a^4fx^3-60\,a^3bx^3e}{180\,b^5}+\frac{(a^2b^3c-a^3b^2d-a^5f+a^4be)\log\left(|bx^3+a|\right)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/180*(12*b^4*f*x^15 - 15*a*b^3*f*x^12 + 15*b^4*x^12*e + 20*b^4*d*x^9 + 20*
a^2*b^2*f*x^9 - 20*a*b^3*x^9*e + 30*b^4*c*x^6 - 30*a*b^3*d*x^6 - 30*a^3*b*f
*x^6 + 30*a^2*b^2*x^6*e - 60*a*b^3*c*x^3 + 60*a^2*b^2*d*x^3 + 60*a^4*f*x^3
```

```
- 60*a^3*b*x^3*e)/b^5 + 1/3*(a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*log(a
bs(b*x^3 + a))/b^6
```

Mupad [B]
time = 4.96, size = 189, normalized size = 1.11

$$x^{12}\left(\frac{e}{12\,b}-\frac{a\,f}{12\,b^2}\right)+x^9\left(\frac{d}{9\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{9\,b}\right)+x^6\left(\frac{c}{6\,b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{6\,b}\right)-\frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^5-e\,a^4\,b+d\,a^3\,b^2-c\,a^2\,b^3\right)}{3\,b^6}+\frac{f\,x^{15}}{15\,b}-\frac{a\,x^3\,\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^12*(e/(12*b) - (a*f)/(12*b^2)) + x^9*(d/(9*b) - (a*(e/b - (a*f)/b^2))/(9*
b)) + x^6*(c/(6*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(6*b)) - (log(a +
b*x^3)*(a^5*f - a^2*b^3*c + a^3*b^2*d - a^4*b*e))/(3*b^6) + (f*x^15)/(15*b)
 - (a*x^3*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(3*b)
```

3.225 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=132

$$\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^4}+\frac{(b^2d-abe+a^2f)\,x^6}{6b^3}+\frac{(be-af)x^9}{9b^2}+\frac{fx^{12}}{12b}-\frac{a(b^3c-ab^2d+a^2be-a^3f)\log(a$$

```
[Out] 1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^3/b^4+1/6*(a^2*f-a*b*e+b^2*d)*x^6/b^3+
1/9*(-a*f+b*e)*x^9/b^2+1/12*f*x^12/b-1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*l
n(b*x^3+a)/b^5
```

Rubi [A]

time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{x^6(a^2f-abe+b^2d)}{6b^3}-\frac{a\log(a+bx^3)\,(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5}+\frac{x^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^4}+\frac{x^9(be-af)}{9b^2}+\frac{fx^{12}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^6)/(6*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (f*x^12)/(12*b) - (a*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx+ex^2+fx^3)}{a+bx}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^4} + \frac{(b^2d-abe+a^2f)\,x}{b^3} + \frac{(be-af)x}{b^2}\right.\right.$$
$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^4} + \frac{(b^2d-abe+a^2f)\,x^6}{6b^3} + \frac{(be-af)x^9}{9b^2} + \frac{fx^1}{12b}$$

Mathematica [A]
time = 0.04, size = 119, normalized size = 0.90

$$\frac{bx^3(-12a^3f+6a^2b(2e+fx^3)-2ab^2(6d+3ex^3+2fx^6)+b^3(12c+6dx^3+4ex^6+3fx^9))+12a(-b^3c+ab^2d-a^2be+a^3f)\log(a+bx^3)}{36b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(-12*a^3*f + 6*a^2*b*(2*e + f*x^3) - 2*a*b^2*(6*d + 3*e*x^3 + 2*f*x^
6) + b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9)) + 12*a*(-(b^3*c) + a*b^2*d -
 a^2*b*e + a^3*f)*Log[a + b*x^3])/(36*b^5)
```

Maple [A]
time = 0.36, size = 140, normalized size = 1.06

method	result
norman	$-\frac{(a^3f-a^2be+a\,b^2d-b^3c)x^3}{3b^4}-\frac{(af-be)x^9}{9b^2}+\frac{f\,x^{12}}{12b}+\frac{(a^2f-abe+b^2d)x^6}{6b^3}+\frac{a(a^3f-a^2be+a\,b^2d-b^3c)\ln(b\,x^3+a)}{3b^5}$
default	$-\frac{-\frac{1}{4}f\,x^{12}b^3+\frac{1}{3}a\,b^2f\,x^9-\frac{1}{3}b^3e\,x^9-\frac{1}{2}a^2bf\,x^6+\frac{1}{2}a\,b^2e\,x^6-\frac{1}{2}b^3d\,x^6+a^3f\,x^3-a^2be\,x^3+a\,b^2d\,x^3-b^3c\,x^3}{3b^4}+\frac{a(a^3f-a^2be+a\,b^2d-b^3c)\ln}{3b^5}$
risch	$\frac{f\,x^{12}}{12b}-\frac{af\,x^9}{9b^2}+\frac{e\,x^9}{9b}+\frac{a^2f\,x^6}{6b^3}-\frac{ae\,x^6}{6b^2}+\frac{d\,x^6}{6b}-\frac{a^3f\,x^3}{3b^4}+\frac{a^2e\,x^3}{3b^3}-\frac{ad\,x^3}{3b^2}+\frac{c\,x^3}{3b}+\frac{a^4\ln(b\,x^3+a)f}{3b^5}-\frac{a^3\ln(b\,x^3+a)}{3b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b^4*(-1/4*f*x^12*b^3+1/3*a*b^2*f*x^9-1/3*b^3*e*x^9-1/2*a^2*b*f*x^6+1/2
*a*b^2*e*x^6-1/2*b^3*d*x^6+a^3*f*x^3-a^2*b*e*x^3+a*b^2*d*x^3-b^3*c*x^3)+1/3
*a/b^5*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)*ln(b*x^3+a)
```

Maxima [A]
time = 0.28, size = 133, normalized size = 1.01

$$\frac{3\,b^3fx^{12}-4\,(ab^2f-b^3e)x^9+6\,(b^3d+a^2bf-ab^2e)x^6+12\,(b^3c-ab^2d-a^3f+a^2be)x^3}{36\,b^4}-\frac{(ab^3c-a^2b^2d-a^4f+a^3be)\log(bx^3+a)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/36*(3*b^3*f*x^12 - 4*(a*b^2*f - b^3*e)*x^9 + 6*(b^3*d + a^2*b*f - a*b^2*e
)*x^6 + 12*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x^3)/b^4 - 1/3*(a*b^3*c - a^
2*b^2*d - a^4*f + a^3*b*e)*log(b*x^3 + a)/b^5
```

Fricas [A]
time = 0.39, size = 130, normalized size = 0.98

$$\frac{3\,b^4fx^{12}+4\,(b^4e-ab^3f)x^9+6\,(b^4d-ab^3e+a^2b^2f)x^6+12\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^3-12\,(ab^3c-a^2b^2d+a^3be-a^4f)\log{(bx^3+a)}}{36\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^4*f*x^12 + 4*(b^4*e - a*b^3*f)*x^9 + 6*(b^4*d - a*b^3*e + a^2*b^2
*f)*x^6 + 12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 12*(a*b^3*c - a^
2*b^2*d + a^3*b*e - a^4*f)*log(b*x^3 + a))/b^5
```

Sympy [A]
time = 0.59, size = 128, normalized size = 0.97

$$\frac{a(a^3f-a^2be+ab^2d-b^3c)\log{(a+bx^3)}}{3b^5}+x^9\left(-\frac{af}{9b^2}+\frac{e}{9b}\right)+x^6\left(\frac{a^2f}{6b^3}-\frac{ae}{6b^2}+\frac{d}{6b}\right)+x^3\left(-\frac{a^3f}{3b^4}+\frac{a^2e}{3b^3}-\frac{ad}{3b^2}+\frac{c}{3b}\right)+\frac{fx^{12}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**5) + x**9*(
-a*f/(9*b**2) + e/(9*b)) + x**6*(a**2*f/(6*b**3) - a*e/(6*b**2) + d/(6*b))
+ x**3*(-a**3*f/(3*b**4) + a**2*e/(3*b**3) - a*d/(3*b**2) + c/(3*b)) + f*x*
*12/(12*b)
```

Giac [A]
time = 0.59, size = 148, normalized size = 1.12

$$\frac{3\,b^3fx^{12}-4\,ab^2fx^9+4\,b^3x^9e+6\,b^3dx^6+6\,a^2bfx^6-6\,ab^2x^6e+12\,b^3cx^3-12\,ab^2dx^3-12\,a^3fx^3+12\,a^2bx^3e}{36\,b^4}-\frac{(ab^3c-a^2b^2d-a^4f+a^3be)\log{(|bx^3+a|)}}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/36*(3*b^3*f*x^12 - 4*a*b^2*f*x^9 + 4*b^3*x^9*e + 6*b^3*d*x^6 + 6*a^2*b*f*
x^6 - 6*a*b^2*x^6*e + 12*b^3*c*x^3 - 12*a*b^2*d*x^3 - 12*a^3*f*x^3 + 12*a^2
*b*x^3*e)/b^4 - 1/3*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*log(abs(b*x^3 +
 a))/b^5
```

Mupad [B]
time = 4.93, size = 141, normalized size = 1.07

$$x^9\left(\frac{e}{9\,b}-\frac{a\,f}{9\,b^2}\right)+x^6\left(\frac{d}{6\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{6\,b}\right)+x^3\left(\frac{c}{3\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{3\,b}\right)+\frac{f\,x^{12}}{12\,b}+\frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^4-e\,a^3\,b+d\,a^2\,b^2-c\,a\,b^3\right)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^9*(e/(9*b) - (a*f)/(9*b^2)) + x^6*(d/(6*b) - (a*(e/b - (a*f)/b^2))/(6*b))
 + x^3*(c/(3*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(3*b)) + (f*x^12)/(12
*b) + (log(a + b*x^3)*(a^4*f + a^2*b^2*d - a*b^3*c - a^3*b*e))/(3*b^5)
```

3.226 $\int \frac{x^2\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3} \, dx$

Optimal. Leaf size=96

$$\frac{\left(b^2d-abe+a^2f\right)x^3}{3b^3}+\frac{(be-af)x^6}{6b^2}+\frac{fx^9}{9b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a+bx^3\right)}{3b^4}$$

```
[Out] 1/3*(a^2*f-a*b*e+b^2*d)*x^3/b^3+1/6*(-a*f+b*e)*x^6/b^2+1/9*f*x^9/b+1/3*(-a^
3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/b^4
```

Rubi [A]
time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1833, 1864}

$$\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4}+\frac{x^6(be-af)}{6b^2}+\frac{fx^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x^3)/(3*b^3) + ((b*e - a*f)*x^6)/(6*b^2) + (f*x^9)
/(9*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^4)
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{a+bx}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2d-abe+a^2f}{b^3}+\frac{(be-af)x}{b^2}+\frac{fx^2}{b}+\frac{b^3c-ab^2d+a^2be-}{b^3(a+bx)}\right.\right.\\ &= \frac{\left(b^2d-abe+a^2f\right)x^3}{3b^3}+\frac{(be-af)x^6}{6b^2}+\frac{fx^9}{9b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\mathrm{l}}{3b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 0.92

$$\frac{bx^3(6a^2f-3ab(2e+fx^3)+b^2(6d+3ex^3+2fx^6))+6(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{18b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(6*a^2*f - 3*a*b*(2*e + f*x^3) + b^2*(6*d + 3*e*x^3 + 2*f*x^6)) + 6*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*b^4)
```

Maple [A]
time = 0.36, size = 96, normalized size = 1.00

method	result	si
norman	$-\frac{(af-be)x^6}{6b^2}+\frac{f x^9}{9b}+\frac{(a^2f-abe+b^2d)x^3}{3b^3}-\frac{(a^3f-a^2be+a b^2d-b^3c)\ln(b x^3+a)}{3b^4}$	89
default	$\frac{\frac{1}{3}f x^9b^2-\frac{1}{2}abf x^6+\frac{1}{2}b^2e x^6+a^2f x^3-abe x^3+b^2d x^3}{3b^3}+\frac{(-a^3f+a^2be-a b^2d+b^3c)\ln(b x^3+a)}{3b^4}$	96
risch	$\frac{f x^9}{9b}-\frac{af x^6}{6b^2}+\frac{e x^6}{6b}+\frac{a^2f x^3}{3b^3}-\frac{ae x^3}{3b^2}+\frac{d x^3}{3b}-\frac{\ln(b x^3+a)a^3f}{3b^4}+\frac{\ln(b x^3+a)a^2e}{3b^3}-\frac{\ln(b x^3+a)ad}{3b^2}+\frac{c\ln(b x^3+a)}{3b}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b^3*(1/3*f*x^9*b^2-1/2*a*b*f*x^6+1/2*b^2*e*x^6+a^2*f*x^3-a*b*e*x^3+b^2*
d*x^3)+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/b^4
```

Maxima [A]
time = 0.29, size = 94, normalized size = 0.98

$$\frac{2\,b^2fx^9-3\,(abf-b^2e)x^6+6\,(b^2d+a^2f-abe)x^3}{18\,b^3}+\frac{(b^3c-ab^2d-a^3f+a^2be)\log(bx^3+a)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/18*(2*b^2*f*x^9 - 3*(a*b*f - b^2*e)*x^6 + 6*(b^2*d + a^2*f - a*b*e)*x^3)/
b^3 + 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(b*x^3 + a)/b^4
```

Fricas [A]
time = 0.40, size = 92, normalized size = 0.96

$$\frac{2\,b^3fx^9+3\,(b^3e-ab^2f)x^6+6\,(b^3d-ab^2e+a^2bf)x^3+6\,(b^3c-ab^2d+a^2be-a^3f)\log(bx^3+a)}{18\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^3*f*x^9 + 3*(b^3*e - a*b^2*f)*x^6 + 6*(b^3*d - a*b^2*e + a^2*b*f)
*x^3 + 6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a))/b^4
```

Sympy [A]
time = 0.57, size = 88, normalized size = 0.92

$$x^6\left(-\frac{af}{6b^2}+\frac{e}{6b}\right)+x^3\left(\frac{a^2f}{3b^3}-\frac{ae}{3b^2}+\frac{d}{3b}\right)+\frac{fx^9}{9b}-\frac{(a^3f-a^2be+ab^2d-b^3c)\log\left(a+bx^3\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**6*(-a*f/(6*b**2) + e/(6*b)) + x**3*(a**2*f/(3*b**3) - a*e/(3*b**2) + d/(
3*b)) + f*x**9/(9*b) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x*
*3)/(3*b**4)
```

Giac [A]
time = 0.53, size = 101, normalized size = 1.05

$$\frac{2\,b^2fx^9-3\,abfx^6+3\,b^2x^6e+6\,b^2dx^3+6\,a^2fx^3-6\,abx^3e}{18\,b^3}+\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(|bx^3+a|\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/18*(2*b^2*f*x^9 - 3*a*b*f*x^6 + 3*b^2*x^6*e + 6*b^2*d*x^3 + 6*a^2*f*x^3 -
 6*a*b*x^3*e)/b^3 + 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(abs(b*x^3 +
 a))/b^4
```

Mupad [B]
time = 4.83, size = 96, normalized size = 1.00

$$x^6\left(\frac{e}{6\,b}-\frac{a\,f}{6\,b^2}\right)+x^3\left(\frac{d}{3\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{3\,b}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^4}+\frac{f\,x^9}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^6*(e/(6*b) - (a*f)/(6*b^2)) + x^3*(d/(3*b) - (a*(e/b - (a*f)/b^2))/(3*b))
 + (log(a + b*x^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^4) + (f*x^9)/(
9*b)
```

3.227 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)} \, dx$

Optimal. Leaf size=80

$$\frac{(be-af)x^3}{3b^2}+\frac{fx^6}{6b}+\frac{c\log(x)}{a}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3ab^3}$$

```
[Out] 1/3*(-a*f+b*e)*x^3/b^2+1/6*f*x^6/b+c*ln(x)/a-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^
3*c)*ln(b*x^3+a)/a/b^3
```

Rubi [A]
time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$-\frac{\log(a+bx^3)(a^3(-f)+a^2be-ab^2d+b^3c)}{3ab^3}+\frac{x^3(be-af)}{3b^2}+\frac{c\log(x)}{a}+\frac{fx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]
```

```
[Out] ((b*e - a*f)*x^3)/(3*b^2) + (f*x^6)/(6*b) + (c*Log[x])/a - ((b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a*b^3)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x(a+bx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{be-af}{b^2}+\frac{c}{ax}+\frac{fx}{b}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2(a+bx)}\right)dx,x,x^3\right)\\ &= \frac{(be-af)x^3}{3b^2}+\frac{fx^6}{6b}+\frac{c\log(x)}{a}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3ab^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 75, normalized size = 0.94

$$\frac{abx^3(2be-2af+bfx^3)+6b^3c\log(x)-2(b^3c-ab^2d+a^2be-a^3f)\log\left(a+bx^3\right)}{6ab^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]
```

```
[Out] (a*b*x^3*(2*b*e - 2*a*f + b*f*x^3) + 6*b^3*c*Log[x] - 2*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*Log[a + b*x^3])/(6*a*b^3)
```

Maple [A]
time = 0.36, size = 75, normalized size = 0.94

method	result
default	$\frac{\left(-f\,x^3 b+af-be\right)^2}{6b^3 f}+\frac{\left(a^3 f-a^2 be+a\,b^2 d-b^3 c\right)\ln\left(b\,x^3+a\right)}{3a\,b^3}+\frac{c\ln(x)}{a}$
norman	$-\frac{(af-be)x^3}{3b^2}+\frac{f\,x^6}{6b}+\frac{c\ln(x)}{a}+\frac{\left(a^3 f-a^2 be+a\,b^2 d-b^3 c\right)\ln\left(b\,x^3+a\right)}{3a\,b^3}$
risch	$\frac{f\,x^6}{6b}-\frac{fa\,x^3}{3b^2}+\frac{e\,x^3}{3b}+\frac{f\,a^2}{6b^3}-\frac{ae}{3b^2}+\frac{e^2}{6bf}+\frac{c\ln(x)}{a}+\frac{a^2\ln\left(-b\,x^3-a\right)f}{3b^3}-\frac{a\ln\left(-b\,x^3-a\right)e}{3b^2}+\frac{\ln\left(-b\,x^3-a\right)d}{3b}-\frac{\ln(-b}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-b*f*x^3+a*f-b*e)^2/b^3/f+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a/b^3*ln(b
*x^3+a)+c*ln(x)/a
```

Maxima [A]
time = 0.30, size = 79, normalized size = 0.99

$$\frac{c\log\left(x^3\right)}{3\,a}+\frac{bfx^6-2\,(af-be)x^3}{6\,b^2}-\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(bx^3+a\right)}{3\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*c*log(x^3)/a + 1/6*(b*f*x^6 - 2*(a*f - b*e)*x^3)/b^2 - 1/3*(b^3*c - a*b
^2*d - a^3*f + a^2*b*e)*log(b*x^3 + a)/(a*b^3)
```

Fricas [A]
time = 0.42, size = 80, normalized size = 1.00

$$\frac{ab^2fx^6+6\,b^3c\log\left(x\right)+2\,(ab^2e-a^2bf)x^3-2\,(b^3c-ab^2d+a^2be-a^3f)\log\left(bx^3+a\right)}{6\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(a*b^2*f*x^6 + 6*b^3*c*log(x) + 2*(a*b^2*e - a^2*b*f)*x^3 - 2*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a))/(a*b^3)
```

Sympy [A]
time = 3.03, size = 70, normalized size = 0.88

$$x^3\left(-\frac{af}{3b^2}+\frac{e}{3b}\right)+\frac{fx^6}{6b}+\frac{c\log{(x)}}{a}+\frac{(a^3f-a^2be+ab^2d-b^3c)\log{\left(\frac{a}{b}+x^3\right)}}{3ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a),x)
```

```
[Out] x**3*(-a*f/(3*b**2) + e/(3*b)) + f*x**6/(6*b) + c*log(x)/a + (a**3*f - a**2
*b*e + a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a*b**3)
```

Giac [A]
time = 0.54, size = 79, normalized size = 0.99

$$\frac{c\log{(|x|)}}{a}+\frac{bfx^6-2\,afx^3+2\,bx^3e}{6\,b^2}-\frac{(b^3c-ab^2d-a^3f+a^2be)\log{(|bx^3+a|)}}{3\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] c*log(abs(x))/a + 1/6*(b*f*x^6 - 2*a*f*x^3 + 2*b*x^3*e)/b^2 - 1/3*(b^3*c -
a*b^2*d - a^3*f + a^2*b*e)*log(abs(b*x^3 + a))/(a*b^3)
```

Mupad [B]
time = 4.93, size = 76, normalized size = 0.95

$$x^3\left(\frac{e}{3\,b}-\frac{a\,f}{3\,b^2}\right)+\frac{f\,x^6}{6\,b}+\frac{c\,\ln{(x)}}{a}-\frac{\ln{(b\,x^3+a)}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x)
```

```
[Out] x^3*(e/(3*b) - (a*f)/(3*b^2)) + (f*x^6)/(6*b) + (c*log(x))/a - (log(a + b*x
^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a*b^3)
```

3.228 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx$

Optimal. Leaf size=81

$$-\frac{c}{3ax^3}+\frac{fx^3}{3b}-\frac{(bc-ad)\log(x)}{a^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^2b^2}$$

```
[Out] -1/3*c/a/x^3+1/3*f*x^3/b-(-a*d+b*c)*ln(x)/a^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b
^3*c)*ln(b*x^3+a)/a^2/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$-\frac{\log(x)(bc-ad)}{a^2}+\frac{\log(a+bx^3)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^2b^2}-\frac{c}{3ax^3}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*c/(a*x^3) + (f*x^3)/(3*b) - ((b*c - a*d)*Log[x])/a^2 + ((b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^2*b^2)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{f}{b}+\frac{c}{ax^2}+\frac{-bc+ad}{a^2x}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx)}\right)\,dx,x,x^3\right)\\ &= -\frac{c}{3ax^3}+\frac{fx^3}{3b}-\frac{(bc-ad)\log(x)}{a^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a+bx^3\right)}{3a^2b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 0.95

$$\frac{1}{3}\left(-\frac{c}{ax^3}+\frac{fx^3}{b}+\frac{3(-bc+ad)\log(x)}{a^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a+bx^3\right)}{a^2b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]
```

```
[Out] (-(c/(a*x^3)) + (f*x^3)/b + (3*(-(b*c) + a*d)*Log[x])/a^2 + ((b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(a^2*b^2))/3
```

Maple [A]
time = 0.34, size = 75, normalized size = 0.93

method	result	size
default	$\frac{f\,x^3}{3b}-\frac{\left(a^3 f-a^2 b e+a\,b^2 d-b^3 c\right)\ln\left(b\,x^3+a\right)}{3a^2b^2}-\frac{c}{3a\,x^3}+\frac{(ad-bc)\ln(x)}{a^2}$	75
norman	$\frac{-\frac{c}{3a}+\frac{f\,x^6}{3b}}{x^3}+\frac{(ad-bc)\ln(x)}{a^2}-\frac{\left(a^3 f-a^2 b e+a\,b^2 d-b^3 c\right)\ln\left(b\,x^3+a\right)}{3a^2b^2}$	77
risch	$\frac{f\,x^3}{3b}-\frac{a\ln\left(b\,x^3+a\right)f}{3b^2}+\frac{e\ln\left(b\,x^3+a\right)}{3b}-\frac{d\ln\left(b\,x^3+a\right)}{3a}+\frac{b\ln\left(b\,x^3+a\right)c}{3a^2}-\frac{c}{3a\,x^3}+\frac{d\ln(x)}{a}-\frac{bc\ln(x)}{a^2}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*f*x^3/b-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^2/b^2*ln(b*x^3+a)-1/3*c/a/x
^3+(a*d-b*c)/a^2*ln(x)
```

Maxima [A]
time = 0.31, size = 78, normalized size = 0.96

$$\frac{fx^3}{3\,b}-\frac{(bc-ad)\log\left(x^3\right)}{3\,a^2}+\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(bx^3+a\right)}{3\,a^2b^2}-\frac{c}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3/b - 1/3*(b*c - a*d)*log(x^3)/a^2 + 1/3*(b^3*c - a*b^2*d - a^3*f +
 a^2*b*e)*log(b*x^3 + a)/(a^2*b^2) - 1/3*c/(a*x^3)
```

Fricas [A]
time = 0.41, size = 85, normalized size = 1.05

$$\frac{a^2bfx^6+(b^3c-ab^2d+a^2be-a^3f)x^3\log\left(bx^3+a\right)-3\left(b^3c-ab^2d\right)x^3\log\left(x\right)-ab^2c}{3\,a^2b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*b*f*x^6 + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3*log(b*x^3 + a) -
 3*(b^3*c - a*b^2*d)*x^3*log(x) - a*b^2*c)/(a^2*b^2*x^3)
```

Sympy [A]
time = 50.85, size = 70, normalized size = 0.86

$$\frac{fx^3}{3b}-\frac{c}{3ax^3}+\frac{(ad-bc)\log{(x)}}{a^2}-\frac{(a^3f-a^2be+ab^2d-b^3c)\log{\left(\frac{a}{b}+x^3\right)}}{3a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a),x)
```

```
[Out] f*x**3/(3*b) - c/(3*a*x**3) + (a*d - b*c)*log(x)/a**2 - (a**3*f - a**2*b*e
+ a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a**2*b**2)
```

Giac [A]
time = 0.69, size = 95, normalized size = 1.17

$$\frac{fx^3}{3\,b}-\frac{(bc-ad)\log{(|x|)}}{a^2}+\frac{(b^3c-ab^2d-a^3f+a^2be)\log{(|bx^3+a|)}}{3\,a^2b^2}+\frac{bcx^3-adx^3-ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*f*x^3/b - (b*c - a*d)*log(abs(x))/a^2 + 1/3*(b^3*c - a*b^2*d - a^3*f +
a^2*b*e)*log(abs(b*x^3 + a))/(a^2*b^2) + 1/3*(b*c*x^3 - a*d*x^3 - a*c)/(a^2
*x^3)
```

Mupad [B]
time = 4.97, size = 74, normalized size = 0.91

$$\frac{f\,x^3}{3\,b}-\frac{c}{3\,a\,x^3}+\frac{\ln{(x)}\,\left(a\,d-b\,c\right)}{a^2}+\frac{\ln{\left(b\,x^3+a\right)}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x)
```

```
[Out] (f*x^3)/(3*b) - c/(3*a*x^3) + (log(x)*(a*d - b*c))/a^2 + (log(a + b*x^3)*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^2*b^2)
```

3.229 $\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=95

$$-\frac{c}{6ax^6}+\frac{bc-ad}{3a^2x^3}+\frac{\left(b^2c-abd+a^2e\right)\log(x)}{a^3}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a+bx^3\right)}{3a^3b}$$

```
[Out] -1/6*c/a/x^6+1/3*(-a*d+b*c)/a^2/x^3+(a^2*e-a*b*d+b^2*c)*ln(x)/a^3-1/3*(-a^3
*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/a^3/b
```

Rubi [A]

time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\frac{bc-ad}{3a^2x^3}+\frac{\log(x)\left(a^2e-abd+b^2c\right)}{a^3}-\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b}-\frac{c}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x]
```

```
[Out] -1/6*c/(a*x^6) + (b*c - a*d)/(3*a^2*x^3) + ((b^2*c - a*b*d + a^2*e)*Log[x])
/a^3 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^3*b)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^3}+\frac{-bc+ad}{a^2x^2}+\frac{b^2c-abd+a^2e}{a^3x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3(a+bx)}\right.\right.\\ &= -\frac{c}{6ax^6}+\frac{bc-ad}{3a^2x^3}+\frac{\left(b^2c-abd+a^2e\right)\log(x)}{a^3}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log(}{3a^3b}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 88, normalized size = 0.93

$$\frac{-\frac{a(ac-2bcx^3+2adx^3)}{x^6}+6(b^2c-abd+a^2e)\log(x)+\left(-2b^2c+2abd-2a^2e+\frac{2a^3f}{b}\right)\log\left(a+bx^3\right)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x]
```

```
[Out] (-((a*(a*c - 2*b*c*x^3 + 2*a*d*x^3))/x^6) + 6*(b^2*c - a*b*d + a^2*e)*Log[x
] + (-2*b^2*c + 2*a*b*d - 2*a^2*e + (2*a^3*f)/b)*Log[a + b*x^3])/(6*a^3)
```

Maple [A]
time = 0.35, size = 90, normalized size = 0.95

method	result
default	$\frac{\left(a^{3} f-a^{2} b e+a\,b^{2} d-b^{3} c\right) \ln\left(b\,x^{3}+a\right)}{3 a^{3} b}-\frac{c}{6 x^{6} a}-\frac{a d-b c}{3 a^{2} x^{3}}+\frac{\left(a^{2} e-a b d+b^{2} c\right) \ln\left(x\right)}{a^{3}}$
norman	$\frac{-\frac{c}{6 a}-\frac{\left(a d-b c\right) x^{3}}{3 a^{2}}}{x^{6}}+\frac{\left(a^{2} e-a b d+b^{2} c\right) \ln\left(x\right)}{a^{3}}+\frac{\left(a^{3} f-a^{2} b e+a\,b^{2} d-b^{3} c\right) \ln\left(b\,x^{3}+a\right)}{3 a^{3} b}$
risch	$\frac{-\frac{c}{6 a}-\frac{\left(a d-b c\right) x^{3}}{3 a^{2}}}{x^{6}}+\frac{e \ln\left(x\right)}{a}-\frac{\ln\left(x\right) b d}{a^{2}}+\frac{\ln\left(x\right) b^{2} c}{a^{3}}+\frac{\ln\left(-b\,x^{3}-a\right) f}{3 b}-\frac{\ln\left(-b\,x^{3}-a\right) e}{3 a}+\frac{b \ln\left(-b\,x^{3}-a\right) d}{3 a^{2}}-\frac{b^{2} \ln\left(-b\,x^{3}-a\right) c}{3 a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^3/b*ln(b*x^3+a)-1/6*c/x^6/a-1/3*(a*d-b*
c)/a^2/x^3+(a^2*e-a*b*d+b^2*c)*ln(x)/a^3
```

Maxima [A]
time = 0.30, size = 95, normalized size = 1.00

$$\frac{(b^2c-abd+a^2e)\log\left(x^3\right)}{3\,a^3}-\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(bx^3+a\right)}{3\,a^3b}+\frac{2\,(bc-ad)x^3-ac}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*c - a*b*d + a^2*e)*log(x^3)/a^3 - 1/3*(b^3*c - a*b^2*d - a^3*f + a
^2*b*e)*log(b*x^3 + a)/(a^3*b) + 1/6*(2*(b*c - a*d)*x^3 - a*c)/(a^2*x^6)
```

Fricas [A]
time = 0.44, size = 101, normalized size = 1.06

$$-\frac{2\,(b^3c-ab^2d+a^2be-a^3f)x^6\log\left(bx^3+a\right)-6\,(b^3c-ab^2d+a^2be)x^6\log\left(x\right)+a^2bc-2\,(ab^2c-a^2bd)x^3}{6\,a^3bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6*log(b*x^3 + a) - 6*(b^3*c -
 a*b^2*d + a^2*b*e)*x^6*log(x) + a^2*b*c - 2*(a*b^2*c - a^2*b*d)*x^3)/(a^3*
b*x^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.64, size = 126, normalized size = 1.33

$$\frac{(b^2c - abd + a^2e)\log(|x|)}{a^3} - \frac{(b^3c - ab^2d - a^3f + a^2be)\log(|bx^3 + a|)}{3a^3b} - \frac{3b^2cx^6 - 3abdx^6 + 3a^2x^6e - 2abcx^3 + 2a^2dx^3 + a^2c}{6a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="giac")
```

```
[Out] (b^2*c - a*b*d + a^2*e)*log(abs(x))/a^3 - 1/3*(b^3*c - a*b^2*d - a^3*f + a^
2*b*e)*log(abs(b*x^3 + a))/(a^3*b) - 1/6*(3*b^2*c*x^6 - 3*a*b*d*x^6 + 3*a^2
*x^6*e - 2*a*b*c*x^3 + 2*a^2*d*x^3 + a^2*c)/(a^3*x^6)
```

Mupad [B]
time = 4.99, size = 92, normalized size = 0.97

$$\frac{\ln(x)\,\left(e\,a^2 - d\,a\,b + c\,b^2\right)}{a^3} - \frac{\frac{c}{6\,a} + \frac{x^3\,(a\,d - b\,c)}{3\,a^2}}{x^6} - \frac{\ln(b\,x^3 + a)\,\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{3\,a^3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x)
```

```
[Out] (log(x)*(b^2*c + a^2*e - a*b*d))/a^3 - (c/(6*a) + (x^3*(a*d - b*c))/(3*a^2)
)/x^6 - (log(a + b*x^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^3*b)
```

3.230 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)}\,dx$

Optimal. Leaf size=128

$$-\frac{c}{9ax^9}+\frac{bc-ad}{6a^2x^6}-\frac{b^2c-abd+a^2e}{3a^3x^3}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log(x)}{a^4}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+b$$

```
[Out] -1/9*c/a/x^9+1/6*(-a*d+b*c)/a^2/x^6+1/3*(-a^2*e+a*b*d-b^2*c)/a^3/x^3-(-a^3*
f+a^2*b*e-a*b^2*d+b^3*c)*ln(x)/a^4+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*
x^3+a)/a^4
```

Rubi [A]
time = 0.11, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{bc-ad}{6a^2x^6}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{\log(a+bx^3)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4}-\frac{\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^4}-\frac{c}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x]
```

```
[Out] -1/9*c/(a*x^9) + (b*c - a*d)/(6*a^2*x^6) - (b^2*c - a*b*d + a^2*e)/(3*a^3*x
^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x])/a^4 + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^4}+\frac{-bc+ad}{a^2x^3}+\frac{b^2c-abd+a^2e}{a^3x^2}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x}\right.\right.$$
$$= -\frac{c}{9ax^9}+\frac{bc-ad}{6a^2x^6}-\frac{b^2c-abd+a^2e}{3a^3x^3}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log(x)}{a^4}+\frac{(b^3}{}$$

Mathematica [A]
time = 0.05, size = 128, normalized size = 1.00

$$-\frac{c}{9ax^9}+\frac{bc-ad}{6a^2x^6}+\frac{-b^2c+abd-a^2e}{3a^3x^3}+\frac{(-b^3c+ab^2d-a^2be+a^3f)\log(x)}{a^4}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x]
```

```
[Out] -1/9*c/(a*x^9) + (b*c - a*d)/(6*a^2*x^6) + (-(b^2*c) + a*b*d - a^2*e)/(3*a^
3*x^3) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[x])/a^4 + ((b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4)
```

Maple [A]
time = 0.35, size = 120, normalized size = 0.94

method	result
default	$-\frac{(a^3f-a^2be+ab^2d-b^3c)\ln(bx^3+a)}{3a^4}-\frac{c}{9ax^9}-\frac{ad-bc}{6a^2x^6}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{(a^3f-a^2be+ab^2d-b^3c)\ln(x)}{a^4}$
norman	$\frac{-\frac{c}{9a}-\frac{(ad-bc)x^3}{6a^2}-\frac{(a^2e-abd+b^2c)x^6}{3a^3}}{x^9}+\frac{(a^3f-a^2be+ab^2d-b^3c)\ln(x)}{a^4}-\frac{(a^3f-a^2be+ab^2d-b^3c)\ln(bx^3+a)}{3a^4}$
risch	$\frac{-\frac{c}{9a}-\frac{(ad-bc)x^3}{6a^2}-\frac{(a^2e-abd+b^2c)x^6}{3a^3}}{x^9}+\frac{\ln(x)f}{a}-\frac{\ln(x)be}{a^2}+\frac{\ln(x)b^2d}{a^3}-\frac{\ln(x)b^3c}{a^4}-\frac{\ln(bx^3+a)f}{3a}+\frac{\ln(bx^3+a)be}{3a^2}-\frac{\ln(bx^3+}{3a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4*ln(b*x^3+a)-1/9*c/a/x^9-1/6*(a*d-b*c
)/a^2/x^6-1/3*(a^2*e-a*b*d+b^2*c)/a^3/x^3+(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4
*ln(x)
```

Maxima [A]
time = 0.29, size = 128, normalized size = 1.00

$$\frac{(b^3c-ab^2d-a^3f+a^2be)\log(bx^3+a)}{3a^4}-\frac{(b^3c-ab^2d-a^3f+a^2be)\log(x^3)}{3a^4}-\frac{6(b^2c-abd+a^2e)x^6-3(abc-a^2d)x^3+2a^2c}{18a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(b*x^3 + a)/a^4 - 1/3*(b^3*c - a
*b^2*d - a^3*f + a^2*b*e)*log(x^3)/a^4 - 1/18*(6*(b^2*c - a*b*d + a^2*e)*x^
6 - 3*(a*b*c - a^2*d)*x^3 + 2*a^2*c)/(a^3*x^9)
```

Fricas [A]
time = 0.42, size = 127, normalized size = 0.99

$$\frac{6\,(b^3c-ab^2d+a^2be-a^3f)x^9\log\left(bx^3+a\right)-18\,(b^3c-ab^2d+a^2be-a^3f)x^9\log\left(x\right)-6\,(ab^2c-a^2bd+a^3e)x^6-2\,a^3c+3\,(a^2bc-a^3d)x^3}{18\,a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9*log(b*x^3 + a) - 18*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^9*log(x) - 6*(a*b^2*c - a^2*b*d + a^3*e)*x^6
 - 2*a^3*c + 3*(a^2*b*c - a^3*d)*x^3)/(a^4*x^9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.62, size = 184, normalized size = 1.44

$$-\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(|x|\right)}{a^4}+\frac{(b^4c-ab^3d-a^3bf+a^2b^2e)\log\left(|bx^3+a|\right)}{3\,a^4b}+\frac{11\,b^3cx^9-11\,ab^2dx^9-11\,a^3fx^9+11\,a^2bx^9e-6\,ab^2cx^6+6\,a^2bdx^6-6\,a^3x^6e+3\,a^2bcx^3-3\,a^3dx^3-2\,a^3c}{18\,a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(abs(x))/a^4 + 1/3*(b^4*c - a*b^3*d
 - a^3*b*f + a^2*b^2*e)*log(abs(b*x^3 + a))/(a^4*b) + 1/18*(11*b^3*c*x^9 -
11*a*b^2*d*x^9 - 11*a^3*f*x^9 + 11*a^2*b*x^9*e - 6*a*b^2*c*x^6 + 6*a^2*b*d*
x^6 - 6*a^3*x^6*e + 3*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^4*x^9)
```

Mupad [B]
time = 5.02, size = 123, normalized size = 0.96

$$\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^4}-\frac{\frac{c}{9\,a}+\frac{x^3\,\left(a\,d-b\,c\right)}{6\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{3\,a^3}}{x^9}-\frac{\ln\left(x\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x)
```

```
[Out] (log(a + b*x^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^4) - (c/(9*a) + (
x^3*(a*d - b*c))/(6*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(3*a^3))/x^9 - (lo
g(x)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^4
```

3.231 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx$

Optimal. Leaf size=164

$$-\frac{c}{12ax^{12}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{6a^3x^6}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\log(x)}{a^5}-\frac{b(b^3c-$$

```
[Out] -1/12*c/a/x^12+1/9*(-a*d+b*c)/a^2/x^9+1/6*(-a^2*e+a*b*d-b^2*c)/a^3/x^6+1/3*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^3+b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(
x)/a^5-1/3*b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/a^5
```

Rubi [A]
time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\frac{bc-ad}{9a^2x^9}-\frac{a^2e-abd+b^2c}{6a^3x^6}-\frac{b\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5}+\frac{b\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^5}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4x^3}-\frac{c}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x]
```

```
[Out] -1/12*c/(a*x^12) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(6*a^3
*x^6) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) + (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*Log[x])/a^5 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a + b*x^3])/(3*a^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5(a+bx)}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^5}+\frac{-bc+ad}{a^2x^4}+\frac{b^2c-abd+a^2e}{a^3x^3}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^2}\right.\right.$$
$$= -\frac{c}{12ax^{12}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{6a^3x^6}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}+\frac{b(b^3c-ab\ldots}{}$$

Mathematica [A]

time = 0.04, size = 164, normalized size = 1.00

$$\frac{12ab^3cx^9-6a^2b^2x^6(c+2dx^3)+2a^3bx^3(2c+3dx^3+6ex^6)-a^4(3c+4dx^3+6ex^6+12fx^9)+36b(b^3c-ab^2d+a^2be-a^3f)\,x^{12}\log(x)-12b(b^3c-ab^2d+a^2be-a^3f)\,x^{12}\log\left(a+bx^3\right)}{36a^5x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x]
```

```
[Out] (12*a*b^3*c*x^9 - 6*a^2*b^2*x^6*(c + 2*d*x^3) + 2*a^3*b*x^3*(2*c + 3*d*x^3
+ 6*e*x^6) - a^4*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + 36*b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*x^12*Log[x] - 12*b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f
)*x^12*Log[a + b*x^3])/(36*a^5*x^12)
```

Maple [A]

time = 0.35, size = 156, normalized size = 0.95

method	result
default	$\frac{b\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{3a^{5}}-\frac{c}{12a\,x^{12}}-\frac{ad-bc}{9a^{2}x^{9}}-\frac{a^{2}e-abd+b^{2}c}{6a^{3}x^{6}}-\frac{a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c}{3a^{4}x^{3}}-\frac{\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)\ldots}{a^{5}}$
norman	$\frac{-\frac{c}{12a}-\frac{(ad-bc)x^{3}}{9a^{2}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{6a^{3}}-\frac{\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{9}}{3a^{4}}}{x^{12}}-\frac{\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)b\ln(x)}{a^{5}}+\frac{b\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln(b\ldots}{3a^{5}}$
risch	$\frac{-\frac{c}{12a}-\frac{(ad-bc)x^{3}}{9a^{2}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{6a^{3}}-\frac{\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{9}}{3a^{4}}}{x^{12}}-\frac{b\ln(x)f}{a^{2}}+\frac{b^{2}\ln(x)e}{a^{3}}-\frac{b^{3}\ln(x)d}{a^{4}}+\frac{b^{4}\ln(x)c}{a^{5}}+\frac{b\ln\left(-b\,x^{3}-\ldots\right.}{3a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*ln(b*x^3+a)-1/12*c/a/x^12-1/9*(a*d-
b*c)/a^2/x^9-1/6*(a^2*e-a*b*d+b^2*c)/a^3/x^6-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3
*c)/a^4/x^3-(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b*ln(x)
```

Maxima [A]

time = 0.31, size = 170, normalized size = 1.04

$$-\frac{(b^4c-ab^3d-a^3bf+a^2b^2e)\log\left(bx^3+a\right)}{3\,a^5}+\frac{(b^4c-ab^3d-a^3bf+a^2b^2e)\log\left(x^3\right)}{3\,a^5}+\frac{12\,(b^3c-ab^2d-a^3f+a^2be)x^9-6\,(ab^2c-a^2bd+a^3e)x^6-3\,a^3c+4\,(a^2bc-a^3d)x^3}{36\,a^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*log(b*x^3 + a)/a^5 + 1/3*(b^4*
c - a*b^3*d - a^3*b*f + a^2*b^2*e)*log(x^3)/a^5 + 1/36*(12*(b^3*c - a*b^2*d
 - a^3*f + a^2*b*e)*x^9 - 6*(a*b^2*c - a^2*b*d + a^3*e)*x^6 - 3*a^3*c + 4*(
a^2*b*c - a^3*d)*x^3)/(a^4*x^12)
```

Fricas [A]
time = 0.42, size = 168, normalized size = 1.02

$$-\frac{12\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{12}\log\left(bx^3+a\right)-36\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{12}\log\left(x\right)-12\,(ab^3c-a^2b^2d+a^3be-a^4f)x^9+6\,(a^2b^2c-a^3bd+a^4e)x^6+3\,a^4c-4\,(a^3bc-a^4d)x^3}{36\,a^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12*log(b*x^3 + a) - 36*
(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12*log(x) - 12*(a*b^3*c - a^2*b^2
*d + a^3*b*e - a^4*f)*x^9 + 6*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 3*a^4*c -
 4*(a^3*b*c - a^4*d)*x^3)/(a^5*x^12)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 235, normalized size = 1.43

$$\frac{(b^4c-ab^3d-a^3bf+a^2b^2e)\log\left(|x|\right)}{a^5}-\frac{(b^5c-ab^4d-a^3b^2f+a^2b^3e)\log\left(|bx^3+a|\right)}{3\,a^5b}-\frac{25\,b^4cx^{12}-25\,ab^3dx^{12}-25\,a^3bfx^{12}+25\,a^2b^2x^{12}e-12\,ab^3cx^9+12\,a^2b^2dx^9+12\,a^4fx^9-12\,a^3bx^9e+6\,a^2b^2cx^6-6\,a^3bdx^6+6\,a^4x^6e-4\,a^3bcx^3+4\,a^4dx^3+3\,a^4c}{36\,a^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="giac")
```

```
[Out] (b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*log(abs(x))/a^5 - 1/3*(b^5*c - a*b^
4*d - a^3*b^2*f + a^2*b^3*e)*log(abs(b*x^3 + a))/(a^5*b) - 1/36*(25*b^4*c*x
^12 - 25*a*b^3*d*x^12 - 25*a^3*b*f*x^12 + 25*a^2*b^2*x^12*e - 12*a*b^3*c*x^
9 + 12*a^2*b^2*d*x^9 + 12*a^4*f*x^9 - 12*a^3*b*x^9*e + 6*a^2*b^2*c*x^6 - 6*
a^3*b*d*x^6 + 6*a^4*x^6*e - 4*a^3*b*c*x^3 + 4*a^4*d*x^3 + 3*a^4*c)/(a^5*x^1
2)
```

Mupad [B]
time = 5.07, size = 161, normalized size = 0.98

$$\frac{\ln(x)\left(-f\,a^3\,b+e\,a^2\,b^2-d\,a\,b^3+c\,b^4\right)}{a^5}-\frac{\ln\left(b\,x^3+a\right)\left(-f\,a^3\,b+e\,a^2\,b^2-d\,a\,b^3+c\,b^4\right)}{3\,a^5}-\frac{\frac{c}{12\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^4}+\frac{x^3\,\left(a\,d-b\,c\right)}{9\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{6\,a^3}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x)
```

```
[Out] (log(x)*(b^4*c + a^2*b^2*e - a*b^3*d - a^3*b*f))/a^5 - (log(a + b*x^3)*(b^4
*c + a^2*b^2*e - a*b^3*d - a^3*b*f))/(3*a^5) - (c/(12*a) - (x^9*(b^3*c - a^
3*f - a*b^2*d + a^2*b*e))/(3*a^4) + (x^3*(a*d - b*c))/(9*a^2) + (x^6*(b^2*c
 + a^2*e - a*b*d))/(6*a^3))/x^12
```

3.232 $\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx$

Optimal. Leaf size=205

$$-\frac{c}{15ax^{15}}+\frac{bc-ad}{12a^2x^{12}}-\frac{b^2c-abd+a^2e}{9a^3x^9}+\frac{b^3c-ab^2d+a^2be-a^3f}{6a^4x^6}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{3a^5x^3}-\frac{b^2(b^3c-ab^2d}{}$$

```
[Out] -1/15*c/a/x^15+1/12*(-a*d+b*c)/a^2/x^12+1/9*(-a^2*e+a*b*d-b^2*c)/a^3/x^9+1/
6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^6-1/3*b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)/a^5/x^3-b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(x)/a^6+1/3*b^2*(-a^3*f+a^2
*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/a^6
```

Rubi [A]
time = 0.14, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{bc-ad}{12a^2x^{12}}-\frac{a^2e-abd+b^2c}{9a^3x^9}+\frac{b^2\log(a+bx^3)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^6}-\frac{b^2\log(x)(a^3(-f)+a^2be-ab^2d+b^3c)}{a^6}-\frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^5x^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^4x^6}-\frac{c}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]
```

```
[Out] -1/15*c/(a*x^15) + (b*c - a*d)/(12*a^2*x^12) - (b^2*c - a*b*d + a^2*e)/(9*a
^3*x^9) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*x^6) - (b*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f))/(3*a^5*x^3) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*Log[x])/a^6 + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])
/(3*a^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}\left(a+bx^3\right)}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^6(a+bx)}\,dx,x,x^3\right)$$
$$=\frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^6}+\frac{-bc+ad}{a^2x^5}+\frac{b^2c-abd+a^2e}{a^3x^4}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^3}\right.\right.$$
$$=-\frac{c}{15ax^{15}}+\frac{bc-ad}{12a^2x^{12}}-\frac{b^2c-abd+a^2e}{9a^3x^9}+\frac{b^3c-ab^2d+a^2be-a^3f}{6a^4x^6}-\frac{b(b^3c-a}{}$$

Mathematica [A]

time = 0.10, size = 194, normalized size = 0.95

$$-\frac{\frac{a\left(60b^4cx^{12}-30ab^3x^9\left(c+2dx^3\right)+10a^2b^2x^6\left(2c+3dx^3+6ex^6\right)-5a^3bx^3\left(3c+4dx^3+6ex^6+12fx^9\right)+a^4\left(12c+15dx^3+20ex^6+30fx^9\right)\right)}{x^{15}}+180b^2(b^3c-ab^2d+a^2be-a^3f)\log(x)-60b^2(b^3c-ab^2d+a^2be-a^3f)\log\left(a+bx^3\right)}{180a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]
```

```
[Out] -1/180*((a*(60*b^4*c*x^12 - 30*a*b^3*x^9*(c + 2*d*x^3) + 10*a^2*b^2*x^6*(2*
c + 3*d*x^3 + 6*e*x^6) - 5*a^3*b*x^3*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) +
 a^4*(12*c + 15*d*x^3 + 20*e*x^6 + 30*f*x^9)))/x^15 + 180*b^2*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*Log[x] - 60*b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a + b*x^3])/a^6
```

Maple [A]

time = 0.34, size = 193, normalized size = 0.94

method	result
default	$-\frac{b^2\left(a^3f-a^2be+a\,b^2d-b^3c\right)\ln\left(b\,x^3+a\right)}{3a^6}-\frac{c}{15a\,x^{15}}-\frac{ad-bc}{12a^2x^{12}}-\frac{a^2e-abd+b^2c}{9a^3x^9}-\frac{a^3f-a^2be+a\,b^2d-b^3c}{6a^4x^6}+\frac{(a^3f-a^2be+a\,b^2d}{a^6}$
norman	$\frac{-\frac{c}{15a}-\frac{(ad-bc)x^3}{12a^2}-\frac{\left(a^2e-abd+b^2c\right)x^6}{9a^3}-\frac{\left(a^3f-a^2be+a\,b^2d-b^3c\right)x^9}{6a^4}+\frac{\left(a^3f-a^2be+a\,b^2d-b^3c\right)b\,x^{12}}{3a^5}}{x^{15}}+\frac{(a^3f-a^2be+a\,b^2d-b^3c)b^2\ln(x)}{a^6}-$
risch	$\frac{-\frac{c}{15a}-\frac{(ad-bc)x^3}{12a^2}-\frac{\left(a^2e-abd+b^2c\right)x^6}{9a^3}-\frac{\left(a^3f-a^2be+a\,b^2d-b^3c\right)x^9}{6a^4}+\frac{\left(a^3f-a^2be+a\,b^2d-b^3c\right)b\,x^{12}}{3a^5}}{x^{15}}+\frac{b^2\ln(x)f}{a^3}-\frac{b^3\ln(x)e}{a^4}+\frac{b^4\ln(x)}{a^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^6*ln(b*x^3+a)-1/15*c/a/x^15-1/12*(
a*d-b*c)/a^2/x^12-1/9*(a^2*e-a*b*d+b^2*c)/a^3/x^9-1/6*(a^3*f-a^2*b*e+a*b^2*
d-b^3*c)/a^4/x^6+(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^6*b^2*ln(x)+1/3*(a^3*f-a^2
*b*e+a*b^2*d-b^3*c)/a^5*b/x^3
```

Maxima [A]

time = 0.34, size = 213, normalized size = 1.04

$$\frac{\left(b^5c-ab^4d-a^3b^2f+a^2b^3e\right)\log\left(bx^3+a\right)}{3\,a^6}-\frac{\left(b^5c-ab^4d-a^3b^2f+a^2b^3e\right)\log\left(x^3\right)}{3\,a^6}-\frac{60\,\left(b^4c-ab^3d-a^3bf+a^2b^2e\right)x^{12}-30\,\left(ab^3c-a^2b^2d-a^4f+a^3be\right)x^9+20\,\left(a^2b^2c-a^3bd+a^4e\right)x^6+12\,a^4c-15\,\left(a^3bc-a^4d\right)x^3}{180\,a^5x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*log(b*x^3 + a)/a^6 - 1/3*(b^5
*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*log(x^3)/a^6 - 1/180*(60*(b^4*c - a*b
^3*d - a^3*b*f + a^2*b^2*e)*x^12 - 30*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*
e)*x^9 + 20*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 12*a^4*c - 15*(a^3*b*c - a^
4*d)*x^3)/(a^5*x^15)
```

Fricas [A]
time = 0.43, size = 210, normalized size = 1.02

$$\frac{60\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{15}\log\left(bx^3+a\right)-180\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{15}\log\left(x\right)-60\,(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^{12}+30\,(a^2b^3c-a^3b^2d+a^4be-a^5f)x^9-20\,(a^3b^2c-a^4bd+a^5e)x^6-12\,a^5c+15\,(a^4bc-a^5d)x^3}{180\,a^6x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/180*(60*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^15*log(b*x^3 + a) - 1
80*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^15*log(x) - 60*(a*b^4*c - a^
2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 30*(a^2*b^3*c - a^3*b^2*d + a^4*b*e -
 a^5*f)*x^9 - 20*(a^3*b^2*c - a^4*b*d + a^5*e)*x^6 - 12*a^5*c + 15*(a^4*b*c
 - a^5*d)*x^3)/(a^6*x^15)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**16/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.63, size = 287, normalized size = 1.40

$$-\frac{(b^5c-ab^4d-a^3b^2f+a^2b^3e)\log\left(|x|\right)}{a^6}+\frac{(b^6c-ab^5d-a^3b^3f+a^2b^4e)\log\left(|bx^3+a|\right)}{3\,a^6b}+\frac{137\,b^5cx^{15}-137\,ab^4dx^{15}-137\,a^3b^2fx^{15}+137\,a^2b^3x^{15}e-60\,ab^4cx^{12}+60\,a^2b^3dx^{12}+60\,a^4bfx^{12}-60\,a^3b^2x^{12}e+30\,a^2b^3cx^9-30\,a^3b^2dx^9-30\,a^5fx^9+30\,a^4bx^9e-20\,a^3b^2cx^6+20\,a^4bdx^6-20\,a^5x^6e+15\,a^4bcx^3-15\,a^5dx^3-12\,a^5c}{180\,a^6x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*log(abs(x))/a^6 + 1/3*(b^6*c - a
*b^5*d - a^3*b^3*f + a^2*b^4*e)*log(abs(b*x^3 + a))/(a^6*b) + 1/180*(137*b^
5*c*x^15 - 137*a*b^4*d*x^15 - 137*a^3*b^2*f*x^15 + 137*a^2*b^3*x^15*e - 60*
a*b^4*c*x^12 + 60*a^2*b^3*d*x^12 + 60*a^4*b*f*x^12 - 60*a^3*b^2*x^12*e + 30
*a^2*b^3*c*x^9 - 30*a^3*b^2*d*x^9 - 30*a^5*f*x^9 + 30*a^4*b*x^9*e - 20*a^3*
```

```
b^2*c*x^6 + 20*a^4*b*d*x^6 - 20*a^5*x^6*e + 15*a^4*b*c*x^3 - 15*a^5*d*x^3 -
 12*a^5*c)/(a^6*x^15)
```

Mupad [B]
time = 0.26, size = 200, normalized size = 0.98

$$\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3\,b^2+e\,a^2\,b^3-d\,a\,b^4+c\,b^5\right)}{3\,a^6}-\frac{\frac{c}{15\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{6\,a^4}+\frac{x^3\,\left(a\,d-b\,c\right)}{12\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{9\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^5}}{x^{15}}-\frac{\ln\left(x\right)\,\left(-f\,a^3\,b^2+e\,a^2\,b^3-d\,a\,b^4+c\,b^5\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x)
```

```
[Out] (log(a + b*x^3)*(b^5*c + a^2*b^3*e - a^3*b^2*f - a*b^4*d))/(3*a^6) - (c/(15
*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(6*a^4) + (x^3*(a*d - b*c))
/(12*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(9*a^3) + (b*x^12*(b^3*c - a^3*f
- a*b^2*d + a^2*b*e))/(3*a^5))/x^15 - (log(x)*(b^5*c + a^2*b^3*e - a^3*b^2*
f - a*b^4*d))/a^6
```

3.233 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=348

$$\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{b^6}-\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^7}{7b^4}+\frac{(b^2d-abe+}{10b}$$

```
[Out] a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^6-1/4*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)*x^4/b^5+1/7*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^7/b^4+1/10*(a^2*f-a*b*e+b^2
*d)*x^10/b^3+1/13*(-a*f+b*e)*x^13/b^2+1/16*f*x^16/b-1/3*a^(7/3)*(-a^3*f+a^2
*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(19/3)+1/6*a^(7/3)*(-a^3*f+a^2*
b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(19/3)+1/3*a
^(7/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1
/3)*3^(1/2))/b^(19/3)*3^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1850, 1502, 206, 31, 648, 631, 210, 642}

$$\frac{x^{10}(a^2f-abe+b^2d)}{10b^3}+\frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{b^6}-\frac{ax^4(a^3(-f)+a^2be-ab^2d+b^3c)}{4b^5}+\frac{x^7(a^3(-f)+a^2be-ab^2d+b^3c)}{7b^4}+\frac{a^{7/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}b^{19/3}}+\frac{a^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^{19/3}}-\frac{a^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^{19/3}}+\frac{x^{13}(be-af)}{13b^2}+\frac{fx^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7)/(7*b
^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3) + ((b*e - a*f)*x^13)/(13*b^2)
 + (f*x^16)/(16*b) + (a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) - (a^(7/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(19/3)) + (
a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(6*b^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{fx^{16}}{16b} + \frac{\int \frac{x^9\left(16bc+16bdx^3+16(be-af)x^6\right)}{a+bx^3}\,dx}{16b}$$

$$= \frac{fx^{16}}{16b} + \frac{\int \left(\frac{16a^2\left(b^3c-ab^2d+a^2be-a^3f\right)}{b^5} - \frac{16a\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{b^4} + \frac{16\left(b^3c-ab^2d+a^2\right.}{b^3}\right.}{}$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6} - \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^5} + \frac{\left(b^3c-\right.}{}$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6} - \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^5} + \frac{\left(b^3c-\right.}{}$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6} - \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^5} + \frac{\left(b^3c-\right.}{}$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6} - \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^5} + \frac{\left(b^3c-\right.}{}$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6} - \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^5} + \frac{\left(b^3c-\right.}{}$$

Mathematica [A]
time = 0.05, size = 351, normalized size = 1.01

$$-\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)x}{b^6} + \frac{a(-b^3c+ab^2d-a^2be+a^3f)x^4}{4b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^7}{7b^4} + \frac{(b^2d-abe+a^2f)x^{10}}{10b^3} + \frac{(be-af)x^{13}}{13b^2} + \frac{fx^{16}}{16b} + \frac{a^{7/3}(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{19/3}} + \frac{a^{7/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{19/3}} - \frac{a^{7/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3*c) + a*b^
2*d - a^2*b*e + a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x^7)/(7*b^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3) + ((b*e - a*f)*x^13)
/(13*b^2) + (f*x^16)/(16*b) + (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) +
 (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(3*b^(19/3)) - (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(19/3))
```

Maple [A]
time = 0.35, size = 309, normalized size = 0.89

method	result
risch	$\frac{f\,x^{16}}{16b}-\frac{af\,x^{13}}{13b^{2}}+\frac{e\,x^{13}}{13b}+\frac{a^{2}f\,x^{10}}{10b^{3}}-\frac{ae\,x^{10}}{10b^{2}}+\frac{d\,x^{10}}{10b}-\frac{a^{3}f\,x^{7}}{7b^{4}}+\frac{a^{2}e\,x^{7}}{7b^{3}}-\frac{ad\,x^{7}}{7b^{2}}+\frac{c\,x^{7}}{7b}+\frac{a^{4}f\,x^{4}}{4b^{5}}-\frac{a^{3}e\,x^{4}}{4b^{4}}+\frac{a^{2}d\,x^{4}}{4b^{3}}$
default	$-\frac{-\frac{1}{16}f\,x^{16}b^{5}+\frac{1}{13}a\,b^{4}f\,x^{13}-\frac{1}{13}b^{5}e\,x^{13}-\frac{1}{10}a^{2}b^{3}f\,x^{10}+\frac{1}{10}a\,b^{4}e\,x^{10}-\frac{1}{10}b^{5}d\,x^{10}+\frac{1}{7}a^{3}b^{2}f\,x^{7}-\frac{1}{7}a^{2}b^{3}e\,x^{7}+\frac{1}{7}a\,b^{4}d\,x^{7}-\frac{1}{7}b^{5}c\,x^{7}-\frac{1}{4}a^{4}bf\,x}{b^{6}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6*(-1/16*f*x^16*b^5+1/13*a*b^4*f*x^13-1/13*b^5*e*x^13-1/10*a^2*b^3*f*x
^10+1/10*a*b^4*e*x^10-1/10*b^5*d*x^10+1/7*a^3*b^2*f*x^7-1/7*a^2*b^3*e*x^7+1
/7*a*b^4*d*x^7-1/7*b^5*c*x^7-1/4*a^4*b*f*x^4+1/4*a^3*b^2*e*x^4-1/4*a^2*b^3*
d*x^4+1/4*a*b^4*c*x^4+a^5*f*x-a^4*b*e*x+a^3*b^2*d*x-a^2*b^3*c*x)+(1/3/b/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a^3*
(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^6
```

Maxima [A]
time = 0.53, size = 359, normalized size = 1.03

$$\frac{455\,b^{5}fx^{16}-560\,(ab^{4}f-b^{5}e)x^{13}+728\,(b^{5}d+a^{2}b^{3}f-ab^{4}e)x^{10}+1040\,(b^{5}c-ab^{4}d-a^{3}b^{2}f+a^{2}b^{3}e)x^{7}-1820\,(ab^{4}c-a^{2}b^{3}d-a^{4}bf+a^{3}b^{2}e)x^{4}+7280\,(a^{2}b^{3}c-a^{3}b^{2}d-a^{5}f+a^{4}be)x}{7280\,b^{6}}-\frac{\sqrt{3}\,(a^{3}b^{3}c-a^{4}b^{2}d-a^{6}f+a^{5}be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{7}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(a^{3}b^{3}c-a^{4}b^{2}d-a^{6}f+a^{5}be)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{7}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(a^{3}b^{3}c-a^{4}b^{2}d-a^{6}f+a^{5}be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{7}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/7280*(455*b^5*f*x^16 - 560*(a*b^4*f - b^5*e)*x^13 + 728*(b^5*d + a^2*b^3*
f - a*b^4*e)*x^10 + 1040*(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*x^7 - 18
20*(a*b^4*c - a^2*b^3*d - a^4*b*f + a^3*b^2*e)*x^4 + 7280*(a^2*b^3*c - a^3*
b^2*d - a^5*f + a^4*b*e)*x)/b^6 - 1/3*sqrt(3)*(a^3*b^3*c - a^4*b^2*d - a^6*
f + a^5*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)
^(2/3)) + 1/6*(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(b^7*(a/b)^(2/3)) - 1/3*(a^3*b^3*c - a^4*b^2*d - a^6*f
+ a^5*b*e)*log(x + (a/b)^(1/3))/(b^7*(a/b)^(2/3))
```

Fricas [A]
time = 0.41, size = 342, normalized size = 0.98

$$\frac{1365\,b^{5}fx^{16}+1680\,(b^{5}e-ab^{4}f)x^{13}+2184\,(b^{5}d-ab^{4}e+a^{2}b^{3}f)x^{10}+3120\,(b^{5}c-ab^{4}d+a^{2}b^{3}e-a^{3}b^{2}f)x^{7}-5460\,(ab^{4}c-a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf)x^{4}-7280\,\sqrt{3}\,(a^{2}b^{3}c-a^{3}b^{2}d+a^{4}be-a^{5}f)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+3640\,(a^{2}b^{3}c-a^{3}b^{2}d+a^{4}be-a^{5}f)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-7280\,(a^{2}b^{3}c-a^{3}b^{2}d+a^{4}be-a^{5}f)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+21840\,(a^{2}b^{3}c-a^{3}b^{2}d+a^{4}be-a^{5}f)x}{21840\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/21840*(1365*b^5*f*x^16 + 1680*(b^5*e - a*b^4*f)*x^13 + 2184*(b^5*d - a*b^
4*e + a^2*b^3*f)*x^10 + 3120*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^7
- 5460*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4 - 7280*sqrt(3)*(a^2*
b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*
(a/b)^(2/3) - sqrt(3)*a)/a) + 3640*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f
)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 7280*(a^2*b^3*c - a^
3*b^2*d + a^4*b*e - a^5*f)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 21840*(a^2*b^
3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6
```

Sympy [A]
time = 1.15, size = 469, normalized size = 1.35

$$x^{13}\left(-\frac{af}{13b^2}+\frac{e}{13b}\right)+x^{10}\left(\frac{a^2f}{10b^3}-\frac{ae}{10b^2}+\frac{d}{10b}\right)+x^7\left(-\frac{a^3f}{7b^4}+\frac{a^2e}{7b^3}-\frac{ad}{7b^2}+\frac{c}{7b}\right)+x^4\left(\frac{a^4f}{4b^5}-\frac{a^3e}{4b^4}+\frac{a^2d}{4b^3}-\frac{ac}{4b^2}\right)+x\left(-\frac{a^5f}{b^6}+\frac{a^4e}{b^5}-\frac{a^3d}{b^4}+\frac{a^2c}{b^3}\right)+\operatorname{RootSum}\left(27t^3b^{19}-a^{16}f^3+3a^{15}bef^2-3a^{14}b^2df^2-3a^{14}b^2e^2f+3a^{13}b^3cf^2+6a^{13}b^3def+a^{13}b^3e^3-6a^{12}b^4cef-3a^{12}b^4d^2f-3a^{12}b^4de^2+6a^{11}b^5cdf+3a^{11}b^5ce^2+3a^{11}b^5d^2e-3a^{10}b^6c^2f-6a^{10}b^6cde-a^{10}b^6d^3+3a^9b^7c^2e+3a^9b^7cd^2-3a^8b^8c^2d+a^7b^9c^3,\left(t\mapsto t\log\left(\frac{3tb^6}{a^5f-a^4be+a^3b^2d-a^2b^3c}+x\right)\right)\right)+\frac{fx^{16}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**13*(-a*f/(13*b**2) + e/(13*b)) + x**10*(a**2*f/(10*b**3) - a*e/(10*b**2)
 + d/(10*b)) + x**7*(-a**3*f/(7*b**4) + a**2*e/(7*b**3) - a*d/(7*b**2) + c/
(7*b)) + x**4*(a**4*f/(4*b**5) - a**3*e/(4*b**4) + a**2*d/(4*b**3) - a*c/(4
*b**2)) + x*(-a**5*f/b**6 + a**4*e/b**5 - a**3*d/b**4 + a**2*c/b**3) + Root
Sum(27*_t**3*b**19 - a**16*f**3 + 3*a**15*b*e*f**2 - 3*a**14*b**2*d*f**2 -
3*a**14*b**2*e**2*f + 3*a**13*b**3*c*f**2 + 6*a**13*b**3*d*e*f + a**13*b**3
*e**3 - 6*a**12*b**4*c*e*f - 3*a**12*b**4*d**2*f - 3*a**12*b**4*d*e**2 + 6*
a**11*b**5*c*d*f + 3*a**11*b**5*c*e**2 + 3*a**11*b**5*d**2*e - 3*a**10*b**6
*c**2*f - 6*a**10*b**6*c*d*e - a**10*b**6*d**3 + 3*a**9*b**7*c**2*e + 3*a**
9*b**7*c*d**2 - 3*a**8*b**8*c**2*d + a**7*b**9*c**3, Lambda(_t, _t*log(3*_t
*b**6/(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c) + x))) + f*x**16/(16*
b)
```

Giac [A]
time = 0.64, size = 454, normalized size = 1.30

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}a^2b^3c-\left(-ab^2\right)^{\frac{1}{3}}a^3b^2d-\left(-ab^2\right)^{\frac{1}{3}}a^5f+\left(-ab^2\right)^{\frac{1}{3}}a^4be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^7}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}a^2b^3c-\left(-ab^2\right)^{\frac{1}{3}}a^3b^2d-\left(-ab^2\right)^{\frac{1}{3}}a^5f+\left(-ab^2\right)^{\frac{1}{3}}a^4be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^7}+\frac{\left(a^3b^{13}c-a^4b^{12}d-a^6b^{10}f+a^5b^{11}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{16}}+\frac{455b^{15}fx^{16}-560ab^{14}fx^{13}+560b^{15}x^{13}e+728b^{15}dx^{10}+728a^2b^{13}fx^{10}-728ab^{14}x^{10}e+1040b^{15}cx^7-1040ab^{14}dx^7-1040a^3b^{12}fx^7+1040a^2b^{13}x^7e-1820ab^{14}cx^4+1820a^2b^{13}dx^4+1820a^4b^{11}fx^4-1820a^3b^{12}x^4e+7280a^2b^{13}cx-7280a^3b^{12}dx-7280a^5b^{10}fx+7280a^4b^{11}xe}{7280b^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*a^2*b^3*c - (-a*b^2)^(1/3)*a^3*b^2*d - (-a*b^2
)^(1/3)*a^5*f + (-a*b^2)^(1/3)*a^4*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/b^7 - 1/6*((-a*b^2)^(1/3)*a^2*b^3*c - (-a*b^2)^(1/3)*a^3
*b^2*d - (-a*b^2)^(1/3)*a^5*f + (-a*b^2)^(1/3)*a^4*b*e)*log(x^2 + x*(-a/b)^
```

```
(1/3) + (-a/b)^(2/3))/b^7 + 1/3*(a^3*b^13*c - a^4*b^12*d - a^6*b^10*f + a^5
*b^11*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^16) + 1/7280*(455*b^1
5*f*x^16 - 560*a*b^14*f*x^13 + 560*b^15*x^13*e + 728*b^15*d*x^10 + 728*a^2*
b^13*f*x^10 - 728*a*b^14*x^10*e + 1040*b^15*c*x^7 - 1040*a*b^14*d*x^7 - 104
0*a^3*b^12*f*x^7 + 1040*a^2*b^13*x^7*e - 1820*a*b^14*c*x^4 + 1820*a^2*b^13*
d*x^4 + 1820*a^4*b^11*f*x^4 - 1820*a^3*b^12*x^4*e + 7280*a^2*b^13*c*x - 728
0*a^3*b^12*d*x - 7280*a^5*b^10*f*x + 7280*a^4*b^11*x*e)/b^16
```

Mupad [B]
time = 0.31, size = 358, normalized size = 1.03

$$x^{13}\left(\frac{e}{13\,b}-\frac{a\,f}{13\,b^2}\right)+x^{10}\left(\frac{d}{10\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{10\,b}\right)+x^7\left(\frac{c}{7\,b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{7\,b}\right)+\frac{f\,x^{16}}{16\,b}-\frac{a^{7/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{19/3}}+\frac{a^2\,x\,\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b^2}-\frac{a\,x^4\,\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{4\,b}-\frac{a^{7/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{19/3}}+\frac{a^{7/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{19/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^13*(e/(13*b) - (a*f)/(13*b^2)) + x^10*(d/(10*b) - (a*(e/b - (a*f)/b^2))/(
10*b)) + x^7*(c/(7*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(7*b)) + (f*x^1
6)/(16*b) - (a^(7/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^
2*b*e))/(3*b^(19/3)) + (a^2*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b)
)/b^2 - (a*x^4*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(4*b) - (a^(7
/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*
(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(19/3)) + (a^(7/3)*log(3^(1/2)*a^
(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a
*b^2*d + a^2*b*e))/(3*b^(19/3))
```

3.234 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3} \, dx$

Optimal. Leaf size=316

$$-\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4}+\frac{(b^2d-abe+a^2f)\,x^8}{8b^3}+\frac{(be-af)x^{11}}{11b^2}+\frac{fx^{14}}{14b}$$

```
[Out] -1/2*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/b^5+1/5*(-a^3*f+a^2*b*e-a*b^2*d+b
^3*c)*x^5/b^4+1/8*(a^2*f-a*b*e+b^2*d)*x^8/b^3+1/11*(-a*f+b*e)*x^11/b^2+1/14
*f*x^14/b-1/3*a^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/
b^(17/3)+1/6*a^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/b^(17/3)-1/3*a^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arct
an(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(17/3)*3^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1850, 1502, 298, 31, 648, 631, 210, 642}

$$\frac{x^8(a^2f-abe+b^2d)}{8b^3}-\frac{ax^2(a^3(-f)+a^2be-ab^2d+b^3c)}{2b^5}+\frac{x^5(a^3(-f)+a^2be-ab^2d+b^3c)}{5b^4}-\frac{a^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}b^{17/3}}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^{17/3}}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^{17/3}}+\frac{x^{11}(be-af)}{11b^2}+\frac{fx^{14}}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -1/2*(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/b^5 + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^8)/(8*b^3) + ((b
*e - a*f)*x^11)/(11*b^2) + (f*x^14)/(14*b) - (a^(5/3)*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
b^(17/3)) - (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(17/3)) + (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{fx^{14}}{14b} + \frac{\int \frac{x^7(14bc+14bdx^3+14(be-af)x^6)}{a+bx^3}\,dx}{14b}$$

$$= \frac{fx^{14}}{14b} + \frac{\int \left(-\frac{14a(b^3c-ab^2d+a^2be-a^3f)x}{b^4} + \frac{14(b^3c-ab^2d+a^2be-a^3f)x^4}{b^3} + \frac{14(b^2d-abe+a^2\cdots}{b^2}\right.}{14b}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4} + \frac{(b^2d-\cdots}{}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4} + \frac{(b^2d-\cdots}{}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4} + \frac{(b^2d-\cdots}{}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4} + \frac{(b^2d-\cdots}{}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4} + \frac{(b^2d-\cdots}{}$$

Mathematica [A]
time = 0.06, size = 311, normalized size = 0.98

$$\frac{a(-b^3c+ab^2d-a^2be+a^3f)\,x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4} + \frac{(b^2d-abe+a^2f)\,x^8}{8b^3} + \frac{(be-af)x^{11}}{11b^2} + \frac{fx^{14}}{14b} + \frac{a^{5/3}(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{17/3}} + \frac{a^{5/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{17/3}} - \frac{a^{5/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(2*b^5) + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^8)/(8*b^3) + (
(b*e - a*f)*x^11)/(11*b^2) + (f*x^14)/(14*b) + (a^(5/3)*(-(b^3*c) + a*b^2*d
 - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b
^(17/3)) + (a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^
(1/3)*x])/(3*b^(17/3)) - (a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(17/3))
```

Maple [A]
time = 0.36, size = 249, normalized size = 0.79

method	result

risch	$\frac{f x^{14}}{14b} - \frac{x^{11}af}{11b^2} + \frac{x^{11}e}{11b} + \frac{x^8a^2f}{8b^3} - \frac{x^8ae}{8b^2} + \frac{x^8d}{8b} - \frac{x^5a^3f}{5b^4} + \frac{x^5a^2e}{5b^3} - \frac{x^5ad}{5b^2} + \frac{x^5c}{5b} + \frac{x^2a^4f}{2b^5} - \frac{x^2a^3e}{2b^4} + \frac{x^2a^2d}{2b^3} - \frac{x^2c}{2b}$
default	$\frac{\frac{f x^{14}b^4}{14} + \frac{(-a b^3 f + b^4 e)x^{11}}{11} + \frac{(a^2b^2f - a b^3 e + d b^4)x^8}{8} + \frac{(-a^3bf + a^2eb^2 - ad b^3 + c b^4)x^5}{5} + \frac{(a^4f - a^3be + a^2b^2d - a b^3 c)x^2}{2}}{b^5} - \left(-\frac{\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/14*f*x^14*b^4+1/11*(-a*b^3*f+b^4*e)*x^11+1/8*(a^2*b^2*f-a*b^3*e+b^
4*d)*x^8+1/5*(-a^3*b*f+a^2*b^2*e-a*b^3*d+b^4*c)*x^5+1/2*(a^4*f-a^3*b*e+a^2*
b^2*d-a*b^3*c)*x^2)-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1)))*a^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^5
```

Maxima [A]
time = 0.50, size = 320, normalized size = 1.01

$$\frac{\sqrt{3}\,(a^2b^3c - a^3b^2d - a^5f + a^4be)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{220\,b^4fx^{14} - 280\,(ab^3f - b^4e)x^{11} + 385\,(b^4d + a^2b^2f - ab^3e)x^8 + 616\,(b^4c - ab^3d - a^3bf + a^2b^2e)x^5 - 1540\,(ab^3c - a^2b^2d - a^4f + a^3be)x^2}{3080\,b^5} + \frac{(a^2b^3c - a^3b^2d - a^5f + a^4be)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(a^2b^3c - a^3b^2d - a^5f + a^4be)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(1/3)) + 1/3080*(220*b^4*f*x^14 -
 280*(a*b^3*f - b^4*e)*x^11 + 385*(b^4*d + a^2*b^2*f - a*b^3*e)*x^8 + 616*(
b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*x^5 - 1540*(a*b^3*c - a^2*b^2*d - a^
4*f + a^3*b*e)*x^2)/b^5 + 1/6*(a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*log
(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6*(a/b)^(1/3)) - 1/3*(a^2*b^3*c - a^
3*b^2*d - a^5*f + a^4*b*e)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(1/3))
```

Fricas [A]
time = 0.41, size = 321, normalized size = 1.02

$$\frac{660\,b^4fx^{14} + 840\,(b^4e - ab^3f)x^{11} + 1155\,(b^4d - ab^3e + a^2b^2f)x^8 + 1848\,(b^4c - ab^3d + a^2b^2e - a^3bf)x^5 - 4620\,(ab^3c - a^2b^2d + a^3be - a^4f)x^2 + 3080\,\sqrt{3}\,(ab^3c - a^2b^2d + a^3be - a^4f)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} - \sqrt{3}\,a}{3\,a}\right) + 1540\,(ab^3c - a^2b^2d + a^3be - a^4f)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2 - bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}} + a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right) - 3080\,(ab^3c - a^2b^2d + a^3be - a^4f)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax + b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{9240\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/9240*(660*b^4*f*x^14 + 840*(b^4*e - a*b^3*f)*x^11 + 1155*(b^4*d - a*b^3*e
 + a^2*b^2*f)*x^8 + 1848*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^5 - 4620
*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^2 + 3080*sqrt(3)*(a*b^3*c - a^2*
b^2*d + a^3*b*e - a^4*f)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2
)^(1/3) - sqrt(3)*a)/a) + 1540*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(a^2
/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 3080*(a*
b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^
(2/3)))/b^5
```

Sympy [**A**]
time = 0.98, size = 513, normalized size = 1.62

$$x^{11}\left(-\frac{af}{11b^2}+\frac{e}{11b}\right)+x^8\left(\frac{a^2f}{8b^3}-\frac{ae}{8b^2}+\frac{d}{8b}\right)+x^5\left(-\frac{a^3f}{5b^4}+\frac{a^2e}{5b^3}-\frac{ad}{5b^2}+\frac{c}{5b}\right)+x^2\left(\frac{a^4f}{2b^5}-\frac{a^3e}{2b^4}+\frac{a^2d}{2b^3}-\frac{ac}{2b^2}\right)+\operatorname{RootSum}\left(27t^3b^{17}-a^{14}f^3+3a^{13}bef^2-3a^{12}b^2df^2-3a^{12}b^2e^2f+3a^{11}b^3cf^2+6a^{11}b^3def+a^{11}b^3e^3-6a^{10}b^4cef-3a^{10}b^4d^2f-3a^{10}b^4de^2+6a^9b^5cdf+3a^9b^5ce^2+3a^9b^5d^2e-3a^8b^6c^2f-6a^8b^6cde-a^8b^6d^3+3a^7b^7c^2e+3a^7b^7cd^2-3a^6b^8c^2d+a^5b^9c^3,\left(t\mapsto t\log\left(\frac{9t^2b^{11}}{a^9f^2-2a^8bef+2a^7b^2df+a^7b^2e^2-2a^6b^3cf-2a^6b^3de+2a^5b^4ce+a^5b^4d^2-2a^4b^5cd+a^3b^6c^2}+x\right)\right)\right)+\frac{fx^{14}}{14b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**11*(-a*f/(11*b**2) + e/(11*b)) + x**8*(a**2*f/(8*b**3) - a*e/(8*b**2) +
d/(8*b)) + x**5*(-a**3*f/(5*b**4) + a**2*e/(5*b**3) - a*d/(5*b**2) + c/(5*b
)) + x**2*(a**4*f/(2*b**5) - a**3*e/(2*b**4) + a**2*d/(2*b**3) - a*c/(2*b**
2)) + RootSum(27*_t**3*b**17 - a**14*f**3 + 3*a**13*b*e*f**2 - 3*a**12*b**2
*d*f**2 - 3*a**12*b**2*e**2*f + 3*a**11*b**3*c*f**2 + 6*a**11*b**3*d*e*f +
a**11*b**3*e**3 - 6*a**10*b**4*c*e*f - 3*a**10*b**4*d**2*f - 3*a**10*b**4*d
*e**2 + 6*a**9*b**5*c*d*f + 3*a**9*b**5*c*e**2 + 3*a**9*b**5*d**2*e - 3*a**
8*b**6*c**2*f - 6*a**8*b**6*c*d*e - a**8*b**6*d**3 + 3*a**7*b**7*c**2*e + 3
*a**7*b**7*c*d**2 - 3*a**6*b**8*c**2*d + a**5*b**9*c**3, Lambda(_t, _t*log(
9*_t**2*b**11/(a**9*f**2 - 2*a**8*b*e*f + 2*a**7*b**2*d*f + a**7*b**2*e**2
- 2*a**6*b**3*c*f - 2*a**6*b**3*d*e + 2*a**5*b**4*c*e + a**5*b**4*d**2 - 2*
a**4*b**5*c*d + a**3*b**6*c**2) + x))) + f*x**14/(14*b)
```

Giac [**A**]
time = 0.63, size = 441, normalized size = 1.40

$$-\frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}ab^3c-(-ab^2)^{\frac{2}{3}}a^2b^2d-(-ab^2)^{\frac{2}{3}}a^4f+(-ab^2)^{\frac{2}{3}}a^3be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^7}+\frac{\left((-ab^2)^{\frac{2}{3}}ab^3c-(-ab^2)^{\frac{2}{3}}a^2b^2d-(-ab^2)^{\frac{2}{3}}a^4f+(-ab^2)^{\frac{2}{3}}a^3be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^7}-\frac{\left(a^2b^{12}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^{11}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^5b^9f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^4b^{10}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{14}}+\frac{220b^{13}fx^{14}-280ab^{12}fx^{11}+280b^{13}x^{11}e+385b^{13}dx^8+385a^2b^{11}fx^8-385ab^{12}x^8e+616b^{13}cx^5-616ab^{12}dx^5-616a^3b^{10}fx^5+616a^2b^{11}x^5e-1540ab^{12}cx^2+1540a^2b^{11}dx^2+1540a^4b^9fx^2-1540a^3b^{10}x^2e}{3080b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*a*b^3*c - (-a*b^2)^(2/3)*a^2*b^2*d - (-a*b^2)^
(2/3)*a^4*f + (-a*b^2)^(2/3)*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/b^7 + 1/6*((-a*b^2)^(2/3)*a*b^3*c - (-a*b^2)^(2/3)*a^2*b^2
*d - (-a*b^2)^(2/3)*a^4*f + (-a*b^2)^(2/3)*a^3*b*e)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/b^7 - 1/3*(a^2*b^12*c*(-a/b)^(1/3) - a^3*b^11*d*(-a/b)^(1
/3) - a^5*b^9*f*(-a/b)^(1/3) + a^4*b^10*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(ab
```

```
s(x - (-a/b)^(1/3)))/(a*b^14) + 1/3080*(220*b^13*f*x^14 - 280*a*b^12*f*x^11
 + 280*b^13*x^11*e + 385*b^13*d*x^8 + 385*a^2*b^11*f*x^8 - 385*a*b^12*x^8*e
 + 616*b^13*c*x^5 - 616*a*b^12*d*x^5 - 616*a^3*b^10*f*x^5 + 616*a^2*b^11*x^
5*e - 1540*a*b^12*c*x^2 + 1540*a^2*b^11*d*x^2 + 1540*a^4*b^9*f*x^2 - 1540*a
^3*b^10*x^2*e)/b^14
```

Mupad [B]
time = 5.16, size = 313, normalized size = 0.99

$$x^{11}\left(\frac{e}{11\,b}-\frac{a\,f}{11\,b^2}\right)+x^8\left(\frac{d}{8\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{8\,b}\right)+x^5\left(\frac{c}{5\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{5\,b}\right)+\frac{f\,x^{14}}{14\,b}-\frac{a^{5/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{17/3}}-\frac{a\,x^2\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{2\,b}+\frac{a^{5/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{17/3}}-\frac{a^{5/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{17/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^11*(e/(11*b) - (a*f)/(11*b^2)) + x^8*(d/(8*b) - (a*(e/b - (a*f)/b^2))/(8*
b)) + x^5*(c/(5*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(5*b)) + (f*x^14)/
(14*b) - (a^(5/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b
*e))/(3*b^(17/3)) - (a*x^2*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(
2*b) + (a^(5/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1
i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(17/3)) - (a^(5/3)*lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c
 - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(17/3))
```

3.235 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=312

$$-\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4}+\frac{(b^2d-abe+a^2f)\,x^7}{7b^3}+\frac{(be-af)x^{10}}{10b^2}+\frac{fx^{13}}{13b}$$

```
[Out] -a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^5+1/4*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
x^4/b^4+1/7*(a^2*f-a*b*e+b^2*d)*x^7/b^3+1/10*(-a*f+b*e)*x^10/b^2+1/13*f*x^1
3/b+1/3*a^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(16/
3)-1/6*a^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/b^(16/3)-1/3*a^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3
*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(16/3)*3^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1850, 1502, 206, 31, 648, 631, 210, 642}

$$\frac{x^7(a^2f-abe+b^2d)}{7b^3}-\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5}+\frac{x^4(a^3(-f)+a^2be-ab^2d+b^3c)}{4b^4}-\frac{a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}b^{16/3}}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^{16/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^{16/3}}+\frac{x^{10}(be-af)}{10b^2}+\frac{fx^{13}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e -
 a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1
6/3)) + (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(16/3)) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{fx^{13}}{13b} + \frac{\int \frac{x^6(13bc+13bdx^3+13(be-af)x^6)}{a+bx^3}\,dx}{13b}$$

$$= \frac{fx^{13}}{13b} + \frac{\int \left(-\frac{13a(b^3c-ab^2d+a^2be-a^3f)}{b^4} + \frac{13(b^3c-ab^2d+a^2be-a^3f)x^3}{b^3} + \frac{13(b^2d-abe+a^2f}{b^2}\right.}{13b}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4} + \frac{(b^2d-}{}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4} + \frac{(b^2d-}{}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4} + \frac{(b^2d-}{}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4} + \frac{(b^2d-}{}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4} + \frac{(b^2d-}{}$$

Mathematica [A]
time = 0.06, size = 306, normalized size = 0.98

$$\frac{a(-b^3c+ab^2d-a^2be+a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4} + \frac{(b^2d-abe+a^2f)\,x^7}{7b^3} + \frac{(be-af)x^{10}}{10b^2} + \frac{fx^{13}}{13b} + \frac{a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{16/3}} - \frac{a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{16/3}} + \frac{a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^5 + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e -
 a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) + (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2
*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(16/3
)) - (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(16/3)) + (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))
```

Maple [A]
time = 0.34, size = 261, normalized size = 0.84

method	result

risch	$\frac{f\,x^{13}}{13b}-\frac{af\,x^{10}}{10b^2}+\frac{e\,x^{10}}{10b}+\frac{a^2f\,x^7}{7b^3}-\frac{ae\,x^7}{7b^2}+\frac{d\,x^7}{7b}-\frac{a^3f\,x^4}{4b^4}+\frac{a^2e\,x^4}{4b^3}-\frac{ad\,x^4}{4b^2}+\frac{c\,x^4}{4b}+\frac{a^4fx}{b^5}-\frac{a^3ex}{b^4}+\frac{a^2dx}{b^3}-\frac{acx}{b^2}$
default	$\frac{\frac{1}{13}f\,x^{13}b^4-\frac{1}{10}a\,b^3f\,x^{10}+\frac{1}{10}b^4e\,x^{10}+\frac{1}{7}a^2b^2f\,x^7-\frac{1}{7}a\,b^3e\,x^7+\frac{1}{7}b^4d\,x^7-\frac{1}{4}a^3bf\,x^4+\frac{1}{4}a^2b^2e\,x^4-\frac{1}{4}a\,b^3d\,x^4+\frac{1}{4}b^4c\,x^4+a^4fx-a^3bex+a^2b^2}{b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/13*f*x^13*b^4-1/10*a*b^3*f*x^10+1/10*b^4*e*x^10+1/7*a^2*b^2*f*x^7-
1/7*a*b^3*e*x^7+1/7*b^4*d*x^7-1/4*a^3*b*f*x^4+1/4*a^2*b^2*e*x^4-1/4*a*b^3*d
*x^4+1/4*b^4*c*x^4+a^4*f*x-a^3*b*e*x+a^2*b^2*d*x-a*b^3*c*x)-(1/3/b/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1
/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a^2*(a^3*
f-a^2*b*e+a*b^2*d-b^3*c)/b^5
```

Maxima [A]
time = 0.50, size = 318, normalized size = 1.02

$$\frac{140\,b^4fx^{13}-182\,(ab^3f-b^4e)x^{10}+260\,(b^4d+a^2b^2f-ab^3e)x^7+455\,(b^4c-ab^3d-a^3bf+a^2b^2e)x^4-1820\,(ab^3c-a^2b^2d-a^4f+a^3be)x}{1820\,b^5}+\frac{\sqrt{3}\,(a^2b^3c-a^3b^2d-a^5f+a^4be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(a^2b^3c-a^3b^2d-a^5f+a^4be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(a^2b^3c-a^3b^2d-a^5f+a^4be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/1820*(140*b^4*f*x^13 - 182*(a*b^3*f - b^4*e)*x^10 + 260*(b^4*d + a^2*b^2*
f - a*b^3*e)*x^7 + 455*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*x^4 - 1820*(
a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*x)/b^5 + 1/3*sqrt(3)*(a^2*b^3*c - a^
3*b^2*d - a^5*f + a^4*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(b^6*(a/b)^(2/3)) - 1/6*(a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6*(a/b)^(2/3)) + 1/3*(a^2*b^3*c - a^3*
b^2*d - a^5*f + a^4*b*e)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(2/3))
```

Fricas [A]
time = 0.41, size = 304, normalized size = 0.97

$$\frac{420\,b^4fx^{13}+546\,(b^4e-ab^3f)x^{10}+780\,(b^4d-ab^3e+a^2b^2f)x^7+1365\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^4-1820\,\sqrt{3}\,(ab^3c-a^2b^2d+a^3be-a^4f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+910\,(ab^3c-a^2b^2d+a^3be-a^4f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)-1820\,(ab^3c-a^2b^2d+a^3be-a^4f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)-5460\,(ab^3c-a^2b^2d+a^3be-a^4f)x}{5460\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/5460*(420*b^4*f*x^13 + 546*(b^4*e - a*b^3*f)*x^10 + 780*(b^4*d - a*b^3*e
+ a^2*b^2*f)*x^7 + 1365*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^4 - 1820*
sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*arctan(1/3*(2*
sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*(a*b^3*c - a^2*b^2*d + a^3*b
*e - a^4*f)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 1820*(a
*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) -
5460*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5
```

Sympy [**A**]
time = 1.07, size = 423, normalized size = 1.36

$$x^{10}\left(-\frac{af}{10b^2}+\frac{e}{10b}\right)+x^7\left(\frac{a^2f}{7b^3}-\frac{ae}{7b^2}+\frac{d}{7b}\right)+x^4\left(-\frac{a^3f}{4b^4}+\frac{a^2e}{4b^3}-\frac{ad}{4b^2}+\frac{c}{4b}\right)+x\left(\frac{a^4f}{b^5}-\frac{a^3e}{b^4}+\frac{a^2d}{b^3}-\frac{ac}{b^2}\right)+\operatorname{RootSum}\left(27t^3b^{16}+a^{13}f^3-3a^{12}bef^2+3a^{11}b^2df^2+3a^{11}b^2e^2f-3a^{10}b^3cf^2-6a^{10}b^3def-a^{10}b^3e^3+6a^9b^4cef+3a^9b^4d^2f+3a^9b^4de^2-6a^8b^5cdf-3a^8b^5ce^2-3a^8b^5d^2e+3a^7b^6c^2f+6a^7b^6cde+a^7b^6d^3-3a^6b^7c^2e-3a^6b^7cd^2+3a^5b^8c^2d-a^4b^9c^3,\left(t\mapsto t\log\left(-\frac{3tb^5}{a^4f-a^3be+a^2b^2d-ab^3c}+x\right)\right)\right)+\frac{fx^{13}}{13b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**10*(-a*f/(10*b**2) + e/(10*b)) + x**7*(a**2*f/(7*b**3) - a*e/(7*b**2) +
d/(7*b)) + x**4*(-a**3*f/(4*b**4) + a**2*e/(4*b**3) - a*d/(4*b**2) + c/(4*b
)) + x*(a**4*f/b**5 - a**3*e/b**4 + a**2*d/b**3 - a*c/b**2) + RootSum(27*_t
**3*b**16 + a**13*f**3 - 3*a**12*b*e*f**2 + 3*a**11*b**2*d*f**2 + 3*a**11*b
**2*e**2*f - 3*a**10*b**3*c*f**2 - 6*a**10*b**3*d*e*f - a**10*b**3*e**3 + 6
*a**9*b**4*c*e*f + 3*a**9*b**4*d**2*f + 3*a**9*b**4*d*e**2 - 6*a**8*b**5*c*
d*f - 3*a**8*b**5*c*e**2 - 3*a**8*b**5*d**2*e + 3*a**7*b**6*c**2*f + 6*a**7
*b**6*c*d*e + a**7*b**6*d**3 - 3*a**6*b**7*c**2*e - 3*a**6*b**7*c*d**2 + 3*
a**5*b**8*c**2*d - a**4*b**9*c**3, Lambda(_t, _t*log(-3*_t*b**5/(a**4*f - a
**3*b*e + a**2*b**2*d - a*b**3*c) + x))) + f*x**13/(13*b)
```

Giac [**A**]
time = 0.62, size = 401, normalized size = 1.29

$$\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}ab^3c-(-ab^2)^{\frac{1}{3}}a^2b^2d-(-ab^2)^{\frac{1}{3}}a^4f+(-ab^2)^{\frac{1}{3}}a^3be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^6}+\frac{\left((-ab^2)^{\frac{1}{3}}ab^3c-(-ab^2)^{\frac{1}{3}}a^2b^2d-(-ab^2)^{\frac{1}{3}}a^4f+(-ab^2)^{\frac{1}{3}}a^3be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^6}-\frac{(a^2b^{11}c-a^3b^{10}d-a^5b^8f+a^4b^9e)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{13}}+\frac{140b^{12}fx^{13}-182ab^{11}fx^{10}+182b^{12}x^{10}e+260b^{12}dx^7+260a^2b^{10}fx^7-260ab^{11}x^7e+455b^{12}cx^4-455ab^{11}dx^4-455a^3b^9fx^4+455a^2b^{10}x^4e-1820ab^{11}cx+1820a^2b^{10}dx+1820a^4b^8fx-1820a^3b^9xe}{1820b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*a*b^3*c - (-a*b^2)^(1/3)*a^2*b^2*d - (-a*b^2)^(
1/3)*a^4*f + (-a*b^2)^(1/3)*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/b^6 + 1/6*((-a*b^2)^(1/3)*a*b^3*c - (-a*b^2)^(1/3)*a^2*b^2*
d - (-a*b^2)^(1/3)*a^4*f + (-a*b^2)^(1/3)*a^3*b*e)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/b^6 - 1/3*(a^2*b^11*c - a^3*b^10*d - a^5*b^8*f + a^4*b^9*e
)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^13) + 1/1820*(140*b^12*f*x^1
3 - 182*a*b^11*f*x^10 + 182*b^12*x^10*e + 260*b^12*d*x^7 + 260*a^2*b^10*f*x
^7 - 260*a*b^11*x^7*e + 455*b^12*c*x^4 - 455*a*b^11*d*x^4 - 455*a^3*b^9*f*x
^4 + 455*a^2*b^10*x^4*e - 1820*a*b^11*c*x + 1820*a^2*b^10*d*x + 1820*a^4*b^
8*f*x - 1820*a^3*b^9*x*e)/b^13
```

Mupad [B]
time = 5.19, size = 311, normalized size = 1.00

$$x^{10}\left(\frac{e}{10\,b}-\frac{a\,f}{10\,b^2}\right)+x^7\left(\frac{d}{7\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{7\,b}\right)+x^4\left(\frac{c}{4\,b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{4\,b}\right)+\frac{f\,x^{13}}{13\,b}+\frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{16/3}}-\frac{a\,x\,\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b}+\frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{16/3}}-\frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^10*(e/(10*b) - (a*f)/(10*b^2)) + x^7*(d/(7*b) - (a*(e/b - (a*f)/b^2))/(7*
b)) + x^4*(c/(4*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(4*b)) + (f*x^13)/
(13*b) + (a^(4/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b
*e))/(3*b^(16/3)) - (a*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/b +
 (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 -
 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(16/3)) - (a^(4/3)*log(3^(1
/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3
*f - a*b^2*d + a^2*b*e))/(3*b^(16/3))
```

3.236 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=279

$$\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^4}+\frac{(b^2d-abe+a^2f)\,x^5}{5b^3}+\frac{(be-af)x^8}{8b^2}+\frac{fx^{11}}{11b}+\frac{a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\,\mathrm{ta}}{\sqrt{3}\,b^{14/3}}$$

```
[Out] 1/2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/b^4+1/5*(a^2*f-a*b*e+b^2*d)*x^5/b^3+
1/8*(-a*f+b*e)*x^8/b^2+1/11*f*x^11/b+1/3*a^(2/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^
3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(14/3)-1/6*a^(2/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3
*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(14/3)+1/3*a^(2/3)*(-a^3*f+
a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^
(14/3)*3^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1850, 1502, 298, 31, 648, 631, 210, 642}

$$\frac{x^5(a^2f-abe+b^2d)}{5b^3}+\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{2b^4}+\frac{a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}\,b^{14/3}}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^{14/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^{14/3}}+\frac{x^8(be-af)}{8b^2}+\frac{fx^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^5)/(5*b^3) + ((b*e - a*f)*x^8)/(8*b^2) + (f*x^11)/(11*b) + (a^(2/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*b^(14/3)) + (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a^(1/3) + b^(1/3)*x])/(3*b^(14/3)) - (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{fx^{11}}{11b} + \frac{\int \frac{x^4(11bc+11bdx^3+11(be-af)x^6)}{a+bx^3}\,dx}{11b}$$

$$= \frac{fx^{11}}{11b} + \frac{\int \left(\frac{11(b^3c-ab^2d+a^2be-a^3f)x}{b^3} + \frac{11(b^2d-abe+a^2f)x^4}{b^2} + \frac{11(be-af)x^7}{b} + \frac{11(-ab^3c}{}\right.}{11b}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^4} + \frac{(b^2d-abe+a^2f)\,x^5}{5b^3} + \frac{(be-af)x^8}{8b^2} + \frac{fx}{11}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^4} + \frac{(b^2d-abe+a^2f)\,x^5}{5b^3} + \frac{(be-af)x^8}{8b^2} + \frac{fx}{11}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^4} + \frac{(b^2d-abe+a^2f)\,x^5}{5b^3} + \frac{(be-af)x^8}{8b^2} + \frac{fx}{11}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^4} + \frac{(b^2d-abe+a^2f)\,x^5}{5b^3} + \frac{(be-af)x^8}{8b^2} + \frac{fx}{11}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^4} + \frac{(b^2d-abe+a^2f)\,x^5}{5b^3} + \frac{(be-af)x^8}{8b^2} + \frac{fx}{11}$$

Mathematica [A]
time = 0.06, size = 266, normalized size = 0.95

$$\frac{660b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\,x^2+264b^{5/3}(b^2d-abe+a^2f)\,x^5+165b^{8/3}(be-af)x^8+120b^{11/3}fx^{11}-440\sqrt{3}\,a^{2/3}(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-440a^{2/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+220a^{2/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{1320b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (660*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2 + 264*b^(5/3)*(b^2*d -
 a*b*e + a^2*f)*x^5 + 165*b^(8/3)*(b*e - a*f)*x^8 + 120*b^(11/3)*f*x^11 - 4
40*Sqrt[3]*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]] - 440*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^
3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
 a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1320*b^(14/3))
```

Maple [A]
time = 0.34, size = 210, normalized size = 0.75

method	result

risch	$\frac{f\,x^{11}}{11b}-\frac{x^8 fa}{8b^2}+\frac{x^8 e}{8b}+\frac{x^5 f\,a^2}{5b^3}-\frac{x^5 ae}{5b^2}+\frac{d\,x^5}{5b}-\frac{x^2 a^3 f}{2b^4}+\frac{x^2 a^2 e}{2b^3}-\frac{x^2 ad}{2b^2}+\frac{c\,x^2}{2b}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(a^3 f-a^2 be \cdots\right.}{\cdots}\right.}{3b^5}$
default	$-\frac{-\frac{b^3 f\,x^{11}}{11}+\frac{\left(fa\,b^2-e\,b^3\right)x^8}{8}+\frac{\left(-f\,a^2 b+a\,b^2 e-b^3 d\right)x^5}{5}+\frac{\left(a^3 f-a^2 be+a\,b^2 d-b^3 c\right)x^2}{2}}{b^4}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\cdots}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/11*b^3*f*x^11+1/8*(a*b^2*f-b^3*e)*x^8+1/5*(-a^2*b*f+a*b^2*e-b^3*
d)*x^5+1/2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)*x^2)+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a*(a^3*f-a^2*b*e+a*b^2
*d-b^3*c)/b^4
```

Maxima [A]

time = 0.48, size = 275, normalized size = 0.99

$$-\frac{\sqrt{3}\left(ab^3c-a^2b^2d-a^4f+a^3be\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{40\,b^3fx^{11}-55\left(ab^2f-b^3e\right)x^8+88\left(b^3d+a^2bf-ab^2e\right)x^5+220\left(b^3c-ab^2d-a^3f+a^2be\right)x^2}{440\,b^4}-\frac{\left(ab^3c-a^2b^2d-a^4f+a^3be\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(ab^3c-a^2b^2d-a^4f+a^3be\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/440*(40*b^3*f*x^11 - 55
*(a*b^2*f - b^3*e)*x^8 + 88*(b^3*d + a^2*b*f - a*b^2*e)*x^5 + 220*(b^3*c -
a*b^2*d - a^3*f + a^2*b*e)*x^2)/b^4 - 1/6*(a*b^3*c - a^2*b^2*d - a^4*f + a^
3*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(1/3)) + 1/3*(a*b^
3*c - a^2*b^2*d - a^4*f + a^3*b*e)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(1/3))
```

Fricas [A]

time = 0.41, size = 281, normalized size = 1.01

$$\frac{120\,b^3fx^{11}+165\left(b^3e-ab^2f\right)x^8+264\left(b^3d-ab^2e+a^2bf\right)x^5+660\left(b^3c-ab^2d+a^2be-a^3f\right)x^2-440\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}\,a}{3a}\right)+220\left(b^3c-ab^2d+a^2be-a^3f\right)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-440\left(b^3c-ab^2d+a^2be-a^3f\right)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{1320\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/1320*(120*b^3*f*x^11 + 165*(b^3*e - a*b^2*f)*x^8 + 264*(b^3*d - a*b^2*e +
 a^2*b*f)*x^5 + 660*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2 - 440*sqrt(3)*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b
*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/3))
- 440*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^
2/b^2)^(2/3)))/b^4
```

Sympy [**A**]
time = 0.90, size = 469, normalized size = 1.68

$$x^{8}\left(-\frac{af}{8b^{2}}+\frac{e}{8b}\right)+x^{5}\left(\frac{a^{2}f}{5b^{3}}-\frac{ae}{5b^{2}}+\frac{d}{5b}\right)+x^{2}\left(-\frac{a^{3}f}{2b^{4}}+\frac{a^{2}e}{2b^{3}}-\frac{ad}{2b^{2}}+\frac{c}{2b}\right)+\operatorname{RootSum}\left(27t^{3}b^{14}+a^{11}f^{3}-3a^{10}bef^{2}+3a^{9}b^{2}df^{2}+3a^{9}b^{2}e^{2}f-3a^{8}b^{3}cf^{2}-6a^{8}b^{3}def-a^{8}b^{3}e^{3}+6a^{7}b^{4}cef+3a^{7}b^{4}d^{2}f+3a^{7}b^{4}de^{2}-6a^{6}b^{5}cdf-3a^{6}b^{5}ce^{2}-3a^{6}b^{5}d^{2}e+3a^{5}b^{6}c^{2}f+6a^{5}b^{6}cde+a^{5}b^{6}d^{3}-3a^{4}b^{7}c^{2}e-3a^{4}b^{7}cd^{2}+3a^{3}b^{8}c^{2}d-a^{2}b^{9}c^{3},\left(t\mapsto t\log\left(\frac{9t^{2}b^{9}}{a^{7}f^{2}-2a^{6}bef+2a^{5}b^{2}df+a^{5}b^{2}e^{2}-2a^{4}b^{3}cf-2a^{4}b^{3}de+2a^{3}b^{4}ce+a^{3}b^{4}d^{2}-2a^{2}b^{5}cd+ab^{6}c^{2}}+x\right)\right)\right)+\frac{fx^{11}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**8*(-a*f/(8*b**2) + e/(8*b)) + x**5*(a**2*f/(5*b**3) - a*e/(5*b**2) + d/(
5*b)) + x**2*(-a**3*f/(2*b**4) + a**2*e/(2*b**3) - a*d/(2*b**2) + c/(2*b))
+ RootSum(27*_t**3*b**14 + a**11*f**3 - 3*a**10*b*e*f**2 + 3*a**9*b**2*d*f*
*2 + 3*a**9*b**2*e**2*f - 3*a**8*b**3*c*f**2 - 6*a**8*b**3*d*e*f - a**8*b**
3*e**3 + 6*a**7*b**4*c*e*f + 3*a**7*b**4*d**2*f + 3*a**7*b**4*d*e**2 - 6*a*
*6*b**5*c*d*f - 3*a**6*b**5*c*e**2 - 3*a**6*b**5*d**2*e + 3*a**5*b**6*c**2*
f + 6*a**5*b**6*c*d*e + a**5*b**6*d**3 - 3*a**4*b**7*c**2*e - 3*a**4*b**7*c
*d**2 + 3*a**3*b**8*c**2*d - a**2*b**9*c**3, Lambda(_t, _t*log(9*_t**2*b**9
/(a**7*f**2 - 2*a**6*b*e*f + 2*a**5*b**2*d*f + a**5*b**2*e**2 - 2*a**4*b**3
*c*f - 2*a**4*b**3*d*e + 2*a**3*b**4*c*e + a**3*b**4*d**2 - 2*a**2*b**5*c*d
 + a*b**6*c**2) + x))) + f*x**11/(11*b)
```

Giac [**A**]
time = 0.59, size = 386, normalized size = 1.38

$$\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^{6}}-\frac{\left(\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^{6}}+\frac{\left(ab^{10}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}b^{9}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{4}b^{7}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{3}b^{8}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{11}}+\frac{40b^{10}fx^{11}-55ab^{9}fx^{8}+55b^{10}x^{8}e+88b^{10}dx^{5}+88a^{2}b^{8}fx^{5}-88ab^{9}x^{5}e+220b^{10}cx^{2}-220ab^{9}dx^{2}-220a^{3}b^{7}fx^{2}+220a^{2}b^{8}x^{2}e}{440b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)
*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/b^6 - 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*
b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/b^6 + 1/3*(a*b^10*c*(-a/b)^(1/3) - a^2*b^9*d*(-a/b)^(1/3) - a^4*b^
7*f*(-a/b)^(1/3) + a^3*b^8*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a*b^11) + 1/440*(40*b^10*f*x^11 - 55*a*b^9*f*x^8 + 55*b^10*x^8*e +
 88*b^10*d*x^5 + 88*a^2*b^8*f*x^5 - 88*a*b^9*x^5*e + 220*b^10*c*x^2 - 220*a
*b^9*d*x^2 - 220*a^3*b^7*f*x^2 + 220*a^2*b^8*x^2*e)/b^11
```

Mupad [B]
time = 5.15, size = 267, normalized size = 0.96

$$x^8\left(\frac{e}{8b}-\frac{af}{8b^2}\right)+x^5\left(\frac{d}{5b}-\frac{a\left(\frac{e}{b}-\frac{af}{b^2}\right)}{5b}\right)+x^2\left(\frac{c}{2b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{af}{b^2}\right)}{b}\right)}{2b}\right)+\frac{fx^{11}}{11b}+\frac{a^{2/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3b^{14/3}}-\frac{a^{2/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3b^{14/3}}+\frac{a^{2/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3b^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^8*(e/(8*b) - (a*f)/(8*b^2)) + x^5*(d/(5*b) - (a*(e/b - (a*f)/b^2))/(5*b))
 + x^2*(c/(2*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(2*b)) + (f*x^11)/(11
*b) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e)
)/(3*b^(14/3)) - (a^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*(
(3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(14/3)) + (
a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1
/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(14/3))
```

3.237 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=274

$$\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4}+\frac{(b^2d-abe+a^2f)\,x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}+\frac{\sqrt[3]{a}\,(b^3c-ab^2d+a^2be-a^3f)\tan}{\sqrt{3}\,b^{13/3}}$$

```
[Out] (-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^4+1/4*(a^2*f-a*b*e+b^2*d)*x^4/b^3+1/7*(-
a*f+b*e)*x^7/b^2+1/10*f*x^10/b-1/3*a^(1/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*l
n(a^(1/3)+b^(1/3)*x)/b^(13/3)+1/6*a^(1/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(13/3)+1/3*a^(1/3)*(-a^3*f+a^2*b*
e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(13/3)
*3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1850, 1502, 206, 31, 648, 631, 210, 642}

$$\frac{x^4(a^2f-abe+b^2d)}{4b^3}+\frac{\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}\,b^{13/3}}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^{13/3}}+\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4}+\frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^{13/3}}+\frac{x^7(be-af)}{7b^2}+\frac{fx^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^4)
/(4*b^3) + ((b*e - a*f)*x^7)/(7*b^2) + (f*x^10)/(10*b) + (a^(1/3)*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*b^(13/3)) - (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^
(1/3) + b^(1/3)*x])/(3*b^(13/3)) + (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{fx^{10}}{10b} + \frac{\int \frac{x^3(10bc+10bdx^3+10(be-af)x^6)}{a+bx^3}\,dx}{10b}$$

$$= \frac{fx^{10}}{10b} + \frac{\int \left(\frac{10(b^3c-ab^2d+a^2be-a^3f)}{b^3} + \frac{10(b^2d-abe+a^2f)x^3}{b^2} + \frac{10(be-af)x^6}{b} + \frac{10(-ab^3c-}{b}\right.}{10b}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4} + \frac{(b^2d-abe+a^2f)\,x^4}{4b^3} + \frac{(be-af)x^7}{7b^2} + \frac{fx}{10}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4} + \frac{(b^2d-abe+a^2f)\,x^4}{4b^3} + \frac{(be-af)x^7}{7b^2} + \frac{fx}{10}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4} + \frac{(b^2d-abe+a^2f)\,x^4}{4b^3} + \frac{(be-af)x^7}{7b^2} + \frac{fx}{10}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4} + \frac{(b^2d-abe+a^2f)\,x^4}{4b^3} + \frac{(be-af)x^7}{7b^2} + \frac{fx}{10}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4} + \frac{(b^2d-abe+a^2f)\,x^4}{4b^3} + \frac{(be-af)x^7}{7b^2} + \frac{fx}{10}$$

Mathematica [A]
time = 0.06, size = 264, normalized size = 0.96

$$\frac{420\sqrt[3]{b}\,(b^3c-ab^2d+a^2be-a^3f)\,x+105b^{4/3}(b^2d-abe+a^2f)\,x^4+60b^{7/3}(be-af)x^7+42b^{10/3}fx^{10}-140\sqrt{3}\,\sqrt[3]{a}\,(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+140\sqrt[3]{a}\,(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-70\sqrt[3]{a}\,(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{420b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (420*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x + 105*b^(4/3)*(b^2*d - a
*b*e + a^2*f)*x^4 + 60*b^(7/3)*(b*e - a*f)*x^7 + 42*b^(10/3)*f*x^10 - 140*S
qrt[3]*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)
*Log[a^(1/3) + b^(1/3)*x] - 70*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(420*b^(13/3))
```

Maple [A]
time = 0.34, size = 213, normalized size = 0.78

method	result

risch	$\frac{f\,x^{10}}{10b}-\frac{af\,x^{7}}{7b^{2}}+\frac{e\,x^{7}}{7b}+\frac{a^{2}f\,x^{4}}{4b^{3}}-\frac{ae\,x^{4}}{4b^{2}}+\frac{d\,x^{4}}{4b}-\frac{a^{3}fx}{b^{4}}+\frac{a^{2}ex}{b^{3}}-\frac{adx}{b^{2}}+\frac{cx}{b}+\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(a^{3}f-a^{2}be+a\,b\cdots\right.}{\cdots}\right.}{3b^{5}}$
default	$-\frac{-\frac{1}{10}b^{3}f\,x^{10}+\frac{1}{7}a\,b^{2}f\,x^{7}-\frac{1}{7}b^{3}e\,x^{7}-\frac{1}{4}a^{2}bf\,x^{4}+\frac{1}{4}a\,b^{2}e\,x^{4}-\frac{1}{4}b^{3}d\,x^{4}+a^{3}fx-a^{2}bex+a\,b^{2}dx-b^{3}cx}{b^{4}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x\cdots\right.}{6b\left(\frac{a}{b}\right)\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/10*b^3*f*x^10+1/7*a*b^2*f*x^7-1/7*b^3*e*x^7-1/4*a^2*b*f*x^4+1/4*
a*b^2*e*x^4-1/4*b^3*d*x^4+a^3*f*x-a^2*b*e*x+a*b^2*d*x-b^3*c*x)+(1/3/b/(a/b)
^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a*(a^3
*f-a^2*b*e+a*b^2*d-b^3*c)/b^4
```

Maxima [A]
time = 0.48, size = 273, normalized size = 1.00

$$\frac{14\,b^{3}fx^{10}-20\,(ab^{2}f-b^{3}e)x^{7}+35\,(b^{3}d+a^{2}bf-ab^{2}e)x^{4}+140\,(b^{3}c-ab^{2}d-a^{3}f+a^{2}be)x}{140\,b^{4}}-\frac{\sqrt{3}\,(ab^{3}c-a^{2}b^{2}d-a^{4}f+a^{3}be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(ab^{3}c-a^{2}b^{2}d-a^{4}f+a^{3}be)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(ab^{3}c-a^{2}b^{2}d-a^{4}f+a^{3}be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/140*(14*b^3*f*x^10 - 20*(a*b^2*f - b^3*e)*x^7 + 35*(b^3*d + a^2*b*f - a*b
^2*e)*x^4 + 140*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x)/b^4 - 1/3*sqrt(3)*(a
*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) + 1/6*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b
*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(2/3)) - 1/3*(a*b^3*c
 - a^2*b^2*d - a^4*f + a^3*b*e)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(2/3))
```

Fricas [A]
time = 0.40, size = 249, normalized size = 0.91

$$\frac{42\,b^{3}fx^{10}+60\,(b^{3}e-ab^{2}f)x^{7}+105\,(b^{3}d-ab^{2}e+a^{2}bf)x^{4}-140\,\sqrt{3}\,(b^{3}c-ab^{2}d+a^{2}be-a^{3}f)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+70\,(b^{3}c-ab^{2}d+a^{2}be-a^{3}f)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-140\,(b^{3}c-ab^{2}d+a^{2}be-a^{3}f)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+420\,(b^{3}c-ab^{2}d+a^{2}be-a^{3}f)x}{420\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/420*(42*b^3*f*x^10 + 60*(b^3*e - a*b^2*f)*x^7 + 105*(b^3*d - a*b^2*e + a^
2*b*f)*x^4 - 140*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*ar
ctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4
```

Sympy [A]
time = 1.07, size = 376, normalized size = 1.37

$$x^{7}\left(-\frac{af}{7b^{2}}+\frac{e}{7b}\right)+x^{4}\left(\frac{a^{2}f}{4b^{3}}-\frac{ae}{4b^{2}}+\frac{d}{4b}\right)+x\left(-\frac{a^{3}f}{b^{4}}+\frac{a^{2}e}{b^{3}}-\frac{ad}{b^{2}}+\frac{c}{b}\right)+\operatorname{RootSum}\left(27t^{3}b^{13}-a^{10}f^{3}+3a^{9}bef^{2}-3a^{8}b^{2}df^{2}-3a^{8}b^{2}e^{2}f+3a^{7}b^{3}cf^{2}+6a^{7}b^{3}def+a^{7}b^{3}e^{3}-6a^{6}b^{4}cef-3a^{6}b^{4}d^{2}f-3a^{6}b^{4}de^{2}+6a^{5}b^{5}cdf+3a^{5}b^{5}ce^{2}+3a^{5}b^{5}d^{2}e-3a^{4}b^{6}c^{2}f-6a^{4}b^{6}cde-a^{4}b^{6}d^{3}+3a^{3}b^{7}c^{2}e+3a^{3}b^{7}cd^{2}-3a^{2}b^{8}c^{2}d+ab^{9}c^{3},\left(t\mapsto t\log\left(\frac{3tb^{4}}{a^{3}f-a^{2}be+ab^{2}d-b^{3}c}+x\right)\right)\right)+\frac{fx^{10}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**7*(-a*f/(7*b**2) + e/(7*b)) + x**4*(a**2*f/(4*b**3) - a*e/(4*b**2) + d/(
4*b)) + x*(-a**3*f/b**4 + a**2*e/b**3 - a*d/b**2 + c/b) + RootSum(27*_t**3*
b**13 - a**10*f**3 + 3*a**9*b*e*f**2 - 3*a**8*b**2*d*f**2 - 3*a**8*b**2*e**
2*f + 3*a**7*b**3*c*f**2 + 6*a**7*b**3*d*e*f + a**7*b**3*e**3 - 6*a**6*b**4
*c*e*f - 3*a**6*b**4*d**2*f - 3*a**6*b**4*d*e**2 + 6*a**5*b**5*c*d*f + 3*a*
*5*b**5*c*e**2 + 3*a**5*b**5*d**2*e - 3*a**4*b**6*c**2*f - 6*a**4*b**6*c*d*
e - a**4*b**6*d**3 + 3*a**3*b**7*c**2*e + 3*a**3*b**7*c*d**2 - 3*a**2*b**8*
c**2*d + a*b**9*c**3, Lambda(_t, _t*log(3*_t*b**4/(a**3*f - a**2*b*e + a*b*
*2*d - b**3*c) + x))) + f*x**10/(10*b)
```

Giac [A]
time = 0.60, size = 346, normalized size = 1.26

$$-\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^{5}}-\frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^{5}}+\frac{\left(ab^{9}c-a^{2}b^{8}d-a^{4}b^{6}f+a^{3}b^{7}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{10}}+\frac{14b^{9}fx^{10}-20ab^{8}fx^{7}+20b^{9}x^{7}e+35b^{9}dx^{4}+35a^{2}b^{7}fx^{4}-35ab^{8}x^{4}e+140b^{9}cx-140ab^{8}dx-140a^{3}b^{6}fx+140a^{2}b^{7}xe}{140b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3
)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/b^5 - 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a
*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3))/b^5 + 1/3*(a*b^9*c - a^2*b^8*d - a^4*b^6*f + a^3*b^7*e)*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(14*b^9*f*x^10 - 20*a*b^8*f
*x^7 + 20*b^9*x^7*e + 35*b^9*d*x^4 + 35*a^2*b^7*f*x^4 - 35*a*b^8*x^4*e + 14
0*b^9*c*x - 140*a*b^8*d*x - 140*a^3*b^6*f*x + 140*a^2*b^7*x*e)/b^10
```

Mupad [B]
time = 5.10, size = 264, normalized size = 0.96

$$x^{7}\left(\frac{e}{7b}-\frac{af}{7b^{2}}\right)+x^{4}\left(\frac{d}{4b}-\frac{a\left(\frac{e}{b}-\frac{af}{b^{2}}\right)}{4b}\right)+x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{af}{b^{2}}\right)}{b}\right)}{b}\right)+\frac{fx^{10}}{10b}-\frac{a^{1/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(-fa^{3}+ea^{2}b-dab^{2}+cb^{3}\right)}{3b^{13/3}}-\frac{a^{1/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-fa^{3}+ea^{2}b-dab^{2}+cb^{3}\right)}{3b^{13/3}}+\frac{a^{1/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-fa^{3}+ea^{2}b-dab^{2}+cb^{3}\right)}{3b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^7*(e/(7*b) - (a*f)/(7*b^2)) + x^4*(d/(4*b) - (a*(e/b - (a*f)/b^2))/(4*b))
 + x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b) + (f*x^10)/(10*b) - (a^(
1/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(13
/3)) - (a^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1
i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(13/3)) + (a^(1/3)*lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c
 - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(13/3))
```

3.238 $\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=245

$$\frac{(b^2d-abe+a^2f)\,x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{11/3}}-\frac{(b^3c-ab^2d+}{}$$

```
[Out] 1/2*(a^2*f-a*b*e+b^2*d)*x^2/b^3+1/5*(-a*f+b*e)*x^5/b^2+1/8*f*x^8/b-1/3*(-a^
3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(11/3)+1/6*(-a^3
*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)
/b^(11/3)-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(11/3)*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1850, 1502, 298, 31, 648, 631, 210, 642}

$$\frac{x^2(a^2f-abe+b^2d)}{2b^3}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}\,\sqrt[3]{a}\,b^{11/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3\sqrt[3]{a}\,b^{11/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6\sqrt[3]{a}\,b^{11/3}}+\frac{x^5(be-af)}{5b^2}+\frac{fx^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x^2)/(2*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^8)
/(8*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(11/3)) - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(11/3)) + ((b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*
a^(1/3)*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx &= \frac{fx^8}{8b} + \frac{\int \frac{x(8bc+8bdx^3+8(be-af)x^6)}{a+bx^3}\,dx}{8b} \\
&= \frac{fx^8}{8b} + \frac{\int \left(\frac{8(b^2d-abe+a^2f)x}{b^2} + \frac{8(be-af)x^4}{b} + \frac{8(b^3c-ab^2d+a^2be-a^3f)x}{b^2(a+bx^3)}\right)\,dx}{8b} \\
&= \frac{(b^2d-abe+a^2f)\,x^2}{2b^3} + \frac{(be-af)x^5}{5b^2} + \frac{fx^8}{8b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\int}{b^3} \\
&= \frac{(b^2d-abe+a^2f)\,x^2}{2b^3} + \frac{(be-af)x^5}{5b^2} + \frac{fx^8}{8b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\int}{3\sqrt[3]{a}\,b^{10/3}} \\
&= \frac{(b^2d-abe+a^2f)\,x^2}{2b^3} + \frac{(be-af)x^5}{5b^2} + \frac{fx^8}{8b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,\mathrm{lo}}{3\sqrt[3]{a}\,b^{11/3}} \\
&= \frac{(b^2d-abe+a^2f)\,x^2}{2b^3} + \frac{(be-af)x^5}{5b^2} + \frac{fx^8}{8b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,\mathrm{lo}}{3\sqrt[3]{a}\,b^{11/3}} \\
&= \frac{(b^2d-abe+a^2f)\,x^2}{2b^3} + \frac{(be-af)x^5}{5b^2} + \frac{fx^8}{8b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,\mathrm{ta}}{\sqrt{3}\,\sqrt[3]{a}\,b^{1}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 231, normalized size = 0.94

$$
\frac{60b^{2/3}(b^2d-abe+a^2f)\,x^2+24b^{5/3}(be-af)x^5+15b^{8/3}fx^8+\frac{40\sqrt{3}\,(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{40(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}}+\frac{20(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{a}}}{120b^{11/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (60*b^(2/3)*(b^2*d - a*b*e + a^2*f)*x^2 + 24*b^(5/3)*(b*e - a*f)*x^5 + 15*b
^(8/3)*f*x^8 + (40*Sqrt[3]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-(b^3*c) + a*b^2*d - a^2
*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (20*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(12
0*b^(11/3))
```

Maple [A]
time = 0.33, size = 173, normalized size = 0.71

method	result
risch	$\frac{f x^8}{8b} - \frac{x^5 f a}{5b^2} + \frac{x^5 e}{5b} + \frac{x^2 a^2 f}{2b^3} - \frac{x^2 a e}{2b^2} + \frac{d x^2}{2b} + \frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)} \frac{(-a^3 f + a^2 b e - a b^2 d + b^3 c)\ln(x-\_R)}{\_R}}{3b^4}$
default	$\frac{\frac{b^2 f x^8}{8} + \frac{(-fab+b^2e)x^5}{5} + \frac{(a^2f-abe+b^2d)x^2}{2}}{b^3} - \frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)(a^3}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/8*b^2*f*x^8+1/5*(-a*b*f+b^2*e)*x^5+1/2*(a^2*f-a*b*e+b^2*d)*x^2)-(-
1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^3
```

Maxima [A]

time = 0.48, size = 230, normalized size = 0.94

$$\frac{\sqrt{3}\,(b^3c - ab^2d - a^3f + a^2be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{5\,b^2fx^8 - 8\,(abf - b^2e)x^5 + 20\,(b^2d + a^2f - abe)x^2}{40\,b^3} + \frac{(b^3c - ab^2d - a^3f + a^2be)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^3c - ab^2d - a^3f + a^2be)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) + 1/40*(5*b^2*f*x^8 - 8*(a*b*f -
 b^2*e)*x^5 + 20*(b^2*d + a^2*f - a*b*e)*x^2)/b^3 + 1/6*(b^3*c - a*b^2*d -
a^3*f + a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) -
 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1
/3))
```

Fricas [A]

time = 0.41, size = 568, normalized size = 2.32

$$\left[\frac{15\,ab^4fx^8 + 24\,(ab^4e - a^2b^3f)x^5 + 60\,(ab^4d - a^2b^3e + a^3b^2f)x^2 - 60\sqrt{\frac{1}{3}}\,(ab^4c - a^2b^3d + a^3b^2e - a^4bf)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3 - ab - 3\sqrt{\frac{1}{3}}\left(abx + 2\,(ab^2)^{\frac{2}{3}}x^2 - (ab^2)^{\frac{1}{3}}a\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}} - 3\,(ab^2)^{\frac{2}{3}}x}{bx^3 + a}\right) + 20\,(b^3c - ab^2d + a^2be - a^3f)(ab^2)^{\frac{2}{3}}\log\left(b^2x^2 - (ab^2)^{\frac{1}{3}}bx + (ab^2)^{\frac{2}{3}}\right) - 40\,(b^3c - ab^2d + a^2be - a^3f)(ab^2)^{\frac{2}{3}}\log\left(bx + (ab^2)^{\frac{1}{3}}\right)}{120\,ab^5}, \frac{15\,ab^4fx^8 + 24\,(ab^4e - a^2b^3f)x^5 + 60\,(ab^4d - a^2b^3e + a^3b^2f)x^2 - 120\sqrt{\frac{1}{3}}\,(ab^4c - a^2b^3d + a^3b^2e - a^4bf)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2\,bx - (ab^2)^{\frac{1}{3}}\right)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}}{b}\right) + 20\,(b^3c - ab^2d + a^2be - a^3f)(ab^2)^{\frac{2}{3}}\log\left(b^2x^2 - (ab^2)^{\frac{1}{3}}bx + (ab^2)^{\frac{2}{3}}\right) - 40\,(b^3c - ab^2d + a^2be - a^3f)(ab^2)^{\frac{2}{3}}\log\left(bx + (ab^2)^{\frac{1}{3}}\right)}{120\,ab^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/120*(15*a*b^4*f*x^8 + 24*(a*b^4*e - a^2*b^3*f)*x^5 + 60*(a*b^4*d - a^2*b
^3*e + a^3*b^2*f)*x^2 - 60*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4
*b*f)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*
(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/
3)*x)/(b*x^3 + a)) + 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*l
og(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^5), 1/120*(15*a*
b^4*f*x^8 + 24*(a*b^4*e - a^2*b^3*f)*x^5 + 60*(a*b^4*d - a^2*b^3*e + a^3*b^
2*f)*x^2 - 120*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt((
a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3
)/a)/b) + 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*log(b^2*x^2
- (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^5)]
```

Sympy [**A**]
time = 0.85, size = 427, normalized size = 1.74

$$x^{5}\left(-\frac{af}{5b^{2}}+\frac{e}{5b}\right)+x^{2}\left(\frac{a^{2}f}{2b^{3}}-\frac{ae}{2b^{2}}+\frac{d}{2b}\right)+\operatorname{RootSum}\left(27t^{3}ab^{11}-a^{9}f^{3}+3a^{8}bef^{2}-3a^{7}b^{2}df^{2}-3a^{7}b^{2}e^{2}f+3a^{6}b^{3}cf^{2}+6a^{6}b^{3}def+a^{6}b^{3}e^{3}-6a^{5}b^{4}cef-3a^{5}b^{4}d^{2}f-3a^{5}b^{4}de^{2}+6a^{4}b^{5}cdf+3a^{4}b^{5}ce^{2}+3a^{4}b^{5}d^{2}e-3a^{3}b^{6}c^{2}f-6a^{3}b^{6}cde-a^{3}b^{6}d^{3}+3a^{2}b^{7}c^{2}e+3a^{2}b^{7}cd^{2}-3ab^{8}c^{2}d+b^{9}c^{3},\left(t\mapsto t\log\left(\frac{9t^{2}ab^{7}}{a^{6}f^{2}-2a^{5}bef+2a^{4}b^{2}df+a^{4}b^{2}e^{2}-2a^{3}b^{3}cf-2a^{3}b^{3}de+2a^{2}b^{4}ce+a^{2}b^{4}d^{2}-2ab^{5}cd+b^{6}c^{2}}+x\right)\right)\right)+\frac{fx^{8}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**5*(-a*f/(5*b**2) + e/(5*b)) + x**2*(a**2*f/(2*b**3) - a*e/(2*b**2) + d/(
2*b)) + RootSum(27*_t**3*a*b**11 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**
2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a*
*6*b**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2
+ 6*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6
*c**2*f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*
b**7*c*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(9*_t**2*a*b**7
/(a**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3
*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d +
b**6*c**2) + x))) + f*x**8/(8*b)
```

Giac [**A**]
time = 0.61, size = 291, normalized size = 1.19

$$\frac{\sqrt{3}\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}}-\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}}-\frac{\left(b^{8}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^{7}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}b^{5}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}b^{6}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{8}}+\frac{5b^{7}fx^{8}-8ab^{6}fx^{5}+8b^{7}x^{5}e+20b^{7}dx^{2}+20a^{2}b^{5}fx^{2}-20ab^{6}x^{2}e}{40b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^3) - 1/6*(b^3*c - a*b^2*d - a^
3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^3
```

```
) - 1/3*(b^8*c*(-a/b)^(1/3) - a*b^7*d*(-a/b)^(1/3) - a^3*b^5*f*(-a/b)^(1/3)
 + a^2*b^6*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^8)
+ 1/40*(5*b^7*f*x^8 - 8*a*b^6*f*x^5 + 8*b^7*x^5*e + 20*b^7*d*x^2 + 20*a^2*b
^5*f*x^2 - 20*a*b^6*x^2*e)/b^8
```

Mupad [B]
time = 5.14, size = 225, normalized size = 0.92

$$x^5\left(\frac{e}{5\,b}-\frac{a\,f}{5\,b^2}\right)+x^2\left(\frac{d}{2\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{2\,b}\right)+\frac{f\,x^8}{8\,b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{1/3}\,b^{11/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{1/3}\,b^{11/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{1/3}\,b^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^5*(e/(5*b) - (a*f)/(5*b^2)) + x^2*(d/(2*b) - (a*(e/b - (a*f)/b^2))/(2*b))
 + (f*x^8)/(8*b) - (log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2
*b*e))/(3*a^(1/3)*b^(11/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/
3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(1/3)*
b^(11/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/
2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(1/3)*b^(11/3))
```

3.239 $\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx$

Optimal. Leaf size=240

$$\frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{10/3}}+\frac{(b^3c-ab^2d+a}{}$$

```
[Out] (a^2*f-a*b*e+b^2*d)*x/b^3+1/4*(-a*f+b*e)*x^4/b^2+1/7*f*x^7/b+1/3*(-a^3*f+a^
2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(10/3)-1/6*(-a^3*f+a^2
*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(10
/3)-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/a^(2/3)/b^(10/3)*3^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1901, 206, 31, 648, 631, 210, 642}

$$\frac{x(a^2f-abe+b^2d)}{b^3}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}\,a^{2/3}b^{10/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{2/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{2/3}b^{10/3}}+\frac{x^4(be-af)}{4b^2}+\frac{fx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^4)/(4*b^2) + (f*x^7)/(7*b)
 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx = \int \left(\frac{b^2d-abe+a^2f}{b^3}+\frac{(be-af)x^3}{b^2}+\frac{fx^6}{b}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\,(a+bx^3)}\right)dx$$

$$= \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{a+bx}}{b^3}$$

$$= \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{}{\sqrt[3]{a}}}{3a^{2/3}b^3}$$

$$= \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt{}\right.}{3a^{2/3}b^{10/3}}$$

$$= \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt{}\right.}{3a^{2/3}b^{10/3}}$$

$$= \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}}{\sqrt{3}\,a^{2/3}b^{10/3}}$$

Mathematica [A]
time = 0.10, size = 229, normalized size = 0.95

$$\frac{84\sqrt[3]{b}\,(b^2d-abe+a^2f)\,x+21b^{4/3}(be-af)x^4+12b^{7/3}fx^7+\frac{28\sqrt{3}\,(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{28(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}}+\frac{14(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}}}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x]
```

```
[Out] (84*b^(1/3)*(b^2*d - a*b*e + a^2*f)*x + 21*b^(4/3)*(b*e - a*f)*x^4 + 12*b^(
7/3)*f*x^7 + (28*Sqrt[3]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b^3*c) + a*b^2*d - a^2*
b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(84*b
^(10/3))
```

Maple [A]
time = 0.33, size = 170, normalized size = 0.71

method	result
risch	$\frac{f\,x^7}{7b}-\frac{af\,x^4}{4b^2}+\frac{e\,x^4}{4b}+\frac{a^2fx}{b^3}-\frac{aex}{b^2}+\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-a^3f+a^2be-a\,b^2d+b^3c\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^4}$

default	$\frac{\frac{1}{7}b^{2}f\,x^{7}-\frac{1}{4}abf\,x^{4}+\frac{1}{4}b^{2}e\,x^{4}+a^{2}fx-abex+b^{2}dx}{b^{3}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(}{b^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/7*b^2*f*x^7-1/4*a*b*f*x^4+1/4*b^2*e*x^4+a^2*f*x-a*b*e*x+b^2*d*x)+(
1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1)))*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b^3
```

Maxima [A]
time = 0.51, size = 228, normalized size = 0.95

$$\frac{4\,b^{2}fx^{7}-7\,(abf-b^{2}e)x^{4}+28\,(b^{2}d+a^{2}f-abe)x}{28\,b^{3}}+\frac{\sqrt{3}\,(b^{3}c-ab^{2}d-a^{3}f+a^{2}be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(b^{3}c-ab^{2}d-a^{3}f+a^{2}be)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(b^{3}c-ab^{2}d-a^{3}f+a^{2}be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/28*(4*b^2*f*x^7 - 7*(a*b*f - b^2*e)*x^4 + 28*(b^2*d + a^2*f - a*b*e)*x)/b
^3 + 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) - 1/6*(b^3*c - a*b^2*d - a^
3*f + a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1
/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3
))
```

Fricas [A]
time = 0.41, size = 600, normalized size = 2.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/84*(12*a^2*b^3*f*x^7 + 21*(a^2*b^3*e - a^3*b^2*f)*x^4 - 42*sqrt(1/3)*(a*
b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*
x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*
```

```
x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-
a^2*b)^(1/3)*a) + 28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log
(a*b*x + (-a^2*b)^(2/3)) + 84*(a^2*b^3*d - a^3*b^2*e + a^4*b*f)*x)/(a^2*b^4
), 1/84*(12*a^2*b^3*f*x^7 + 21*(a^2*b^3*e - a^3*b^2*f)*x^4 + 84*sqrt(1/3)*(
a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(s
qrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^
2) - 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-
a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 84*(a^2*b^3*d - a^3*b^2*e + a
^4*b*f)*x)/(a^2*b^4)]
```

Sympy [A]
time = 1.00, size = 342, normalized size = 1.42

$$x^4\left(-\frac{af}{4b^2}+\frac{e}{4b}\right)+x\left(\frac{a^2f}{b^3}-\frac{ae}{b^2}+\frac{d}{b}\right)+\operatorname{RootSum}\left(27t^3a^2b^{10}+a^9f^3-3a^8bef^2+3a^7b^2df^2+3a^7b^2e^2f-3a^6b^3cf^2-6a^6b^3def-a^6b^3e^3+6a^5b^4cef+3a^5b^4d^2f+3a^5b^4de^2-6a^4b^5cdf-3a^4b^5ce^2-3a^4b^5d^2e+3a^3b^6c^2f+6a^3b^6cde+a^3b^6d^3-3a^2b^7c^2e-3a^2b^7cd^2+3ab^8c^2d-b^9c^3,\left(t\mapsto t\log\left(-\frac{3tab^3}{a^3f-a^2be+ab^2d-b^3c}+x\right)\right)\right)+\frac{fx^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**4*(-a*f/(4*b**2) + e/(4*b)) + x*(a**2*f/b**3 - a*e/b**2 + d/b) + RootSum
(27*_t**3*a**2*b**10 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f**2 + 3
*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b**3*e**3
 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a**4*b**
5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*f + 6*
a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c*d**2
+ 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a*b**3/(a**3*f - a**
2*b*e + a*b**2*d - b**3*c) + x))) + f*x**7/(7*b)
```

Giac [A]
time = 0.63, size = 253, normalized size = 1.05

$$-\frac{\sqrt{3}\,(b^3c-ab^2d-a^3f+a^2be)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}\,b^2}-\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}\,b^2}-\frac{(b^7c-ab^6d-a^3b^4f+a^2b^5e)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7}+\frac{4\,b^6fx^7-7\,ab^5fx^4+7\,b^6x^4e+28\,b^6dx+28\,a^2b^4fx-28\,ab^5xe}{28\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/6*(b^3*c - a*b^2*d - a
^3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^
2) - 1/3*(b^7*c - a*b^6*d - a^3*b^4*f + a^2*b^5*e)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a*b^7) + 1/28*(4*b^6*f*x^7 - 7*a*b^5*f*x^4 + 7*b^6*x^4*e +
 28*b^6*d*x + 28*a^2*b^4*f*x - 28*a*b^5*x*e)/b^7
```

Mupad [B]
time = 5.17, size = 222, normalized size = 0.92

$$x^4\left(\frac{e}{4\,b}-\frac{a\,f}{4\,b^2}\right)+x\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)+\frac{f\,x^7}{7\,b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{2/3}\,b^{10/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{2/3}\,b^{10/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{2/3}\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x)
```

```
[Out] x^4*(e/(4*b) - (a*f)/(4*b^2)) + x*(d/b - (a*(e/b - (a*f)/b^2))/b) + (f*x^7)
/(7*b) + (log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*
a^(2/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1
/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(2/3)*b^(10/3))
- (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(2/3)*b^(10/3))
```

3.240 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=227

$$-\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{8/3}}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log(}{3a^{4/3}b^{8/3}}$$

```
[Out] -c/a/x+1/2*(-a*f+b*e)*x^2/b^2+1/5*f*x^5/b+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(8/3)-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(8/3)+1/3*(-a^3*f+a^2*b
*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)
/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 298, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}\,a^{4/3}b^{8/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{4/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{4/3}b^{8/3}}+\frac{x^2(be-af)}{2b^2}-\frac{c}{ax}+\frac{fx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x^2)/(2*b^2) + (f*x^5)/(5*b) + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*a^(4/3)*b^(8/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b
^(1/3)*x])/(3*a^(4/3)*b^(8/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx = \int \left(\frac{c}{ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{b}+\frac{(-b^3c+ab^2d-a^2be+a^3f)\,x}{ab^2(a+bx^3)}\right)dx$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{(-b^3c+ab^2d-a^2be+a^3f)\int \frac{x}{a+bx^3}\,dx}{ab^2}$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{4/3}b^{7/3}} - \frac{(b}{}$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}b^{8/3}} -$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}b^{8/3}} -$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{8/3}}$$

Mathematica [A]
time = 0.09, size = 224, normalized size = 0.99

$$\frac{-30\sqrt[3]{a}\,b^{8/3}c+15a^{4/3}b^{2/3}(be-af)x^3+6a^{4/3}b^{5/3}fx^6+10\sqrt{3}\,(b^3c-ab^2d+a^2be-a^3f)\,x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10(b^3c-ab^2d+a^2be-a^3f)\,x\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-5(b^3c-ab^2d+a^2be-a^3f)\,x\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{30a^{4/3}b^{8/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-30*a^(1/3)*b^(8/3)*c + 15*a^(4/3)*b^(2/3)*(b*e - a*f)*x^3 + 6*a^(4/3)*b^(
5/3)*f*x^6 + 10*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*L
og[a^(1/3) + b^(1/3)*x] - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(30*a^(4/3)*b^(8/3)*x)
```

Maple [A]
time = 0.41, size = 159, normalized size = 0.70

method	result

default	$-\frac{-\frac{bf\,x^{5}}{5}+\frac{(af-be)x^{2}}{2}}{b^{2}}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c)}{a\,b^{2}}$
risch	$\frac{f\,x^{5}}{5b}-\frac{x^{2}af}{2b^{2}}+\frac{e\,x^{2}}{2b}-\frac{c}{ax}+\frac{\_R=\mathrm{RootOf}\left(a^{9}f^{3}-3a^{8}be\,f^{2}+3a^{7}b^{2}d\,f^{2}+3a^{7}b^{2}e^{2}f-3a^{6}b^{3}c\,f^{2}-6a^{6}b^{3}def-a^{6}b^{3}e^{3}+6a^{5}b^{4}cef+3a^{5}b^{4}d^{2}f+3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*(-1/5*b*f*x^5+1/2*(a*f-b*e)*x^2)+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3
))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^
(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c
)/a/b^2-c/a/x
```

Maxima [A]

time = 0.49, size = 221, normalized size = 0.97

$$\frac{2\,bfx^{5}-5\,(af-be)x^{2}}{10\,b^{2}}-\frac{c}{ax}-\frac{\sqrt{3}\,(b^{3}c-ab^{2}d-a^{3}f+a^{2}be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(b^{3}c-ab^{2}d-a^{3}f+a^{2}be)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(b^{3}c-ab^{2}d-a^{3}f+a^{2}be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/10*(2*b*f*x^5 - 5*(a*f - b*e)*x^2)/b^2 - c/(a*x) - 1/3*sqrt(3)*(b^3*c - a
*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a*b^3*(a/b)^(1/3)) - 1/6*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(1/3)) + 1/3*(b^3*c - a*b^2*d - a
^3*f + a^2*b*e)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(1/3))
```

Fricas [A]

time = 0.41, size = 560, normalized size = 2.47

$$\left[\frac{6\,a^{2}b^{3}fx^{6}-30\,ab^{4}c+15\,(a^{2}b^{3}e-a^{3}b^{2}f)x^{3}-15\,\sqrt{\frac{1}{3}}\,(ab^{4}c-a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf)x\sqrt{\frac{(-ab^{2})^{\frac{1}{3}}}{a}}\log\left(\frac{2b^{2}x^{3}-ab+3\sqrt{\frac{1}{3}}\left(abx+2(-ab^{2})^{\frac{2}{3}}x^{2}+(-ab^{2})^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^{2})^{\frac{1}{3}}}{a}}-3(-ab^{2})^{\frac{2}{3}}x}{bx^{3}+a}\right)-5\,(b^{3}c-ab^{2}d+a^{2}be-a^{3}f)(-ab^{2})^{\frac{2}{3}}x\log\left(b^{2}x^{2}+(-ab^{2})^{\frac{1}{3}}bx+(-ab^{2})^{\frac{2}{3}}\right)+10\,(b^{3}c-ab^{2}d+a^{2}be-a^{3}f)(-ab^{2})^{\frac{2}{3}}x\log\left(bx-(-ab^{2})^{\frac{1}{3}}\right)}{30\,a^{2}b^{4}x},\ \frac{6\,a^{2}b^{3}fx^{6}-30\,ab^{4}c+15\,(a^{2}b^{3}e-a^{3}b^{2}f)x^{3}-30\,\sqrt{\frac{1}{3}}\,(ab^{4}c-a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf)x\sqrt{-\frac{(-ab^{2})^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx+(-ab^{2})^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^{2})^{\frac{1}{3}}}{a}}}{b}\right)-5\,(b^{3}c-ab^{2}d+a^{2}be-a^{3}f)(-ab^{2})^{\frac{2}{3}}x\log\left(b^{2}x^{2}+(-ab^{2})^{\frac{1}{3}}bx+(-ab^{2})^{\frac{2}{3}}\right)+10\,(b^{3}c-ab^{2}d+a^{2}be-a^{3}f)(-ab^{2})^{\frac{2}{3}}x\log\left(bx-(-ab^{2})^{\frac{1}{3}}\right)}{30\,a^{2}b^{4}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a^2*b^3*f*x^6 - 30*a*b^4*c + 15*(a^2*b^3*e - a^3*b^2*f)*x^3 - 15*s
qrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x*sqrt((-a*b^2)^(1/3)/
a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b
^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b^2*x^2 + (-a*b^2)
^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b
^2)^(2/3)*x*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x), 1/30*(6*a^2*b^3*f*x^6 -
 30*a*b^4*c + 15*(a^2*b^3*e - a^3*b^2*f)*x^3 - 30*sqrt(1/3)*(a*b^4*c - a^2*
b^3*d + a^3*b^2*e - a^4*b*f)*x*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*
b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^
(2/3)) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b*x -
(-a*b^2)^(1/3)))/(a^2*b^4*x)]
```

Sympy [A]
time = 1.44, size = 408, normalized size = 1.80

$$x^{2}\left(-\frac{af}{2b^{2}}+\frac{e}{2b}\right)+\operatorname{RootSum}\left(27t^{3}a^{4}b^{8}+a^{9}f^{3}-3a^{8}bef^{2}+3a^{7}b^{2}df^{2}+3a^{7}b^{2}e^{2}f-3a^{6}b^{3}cf^{2}-6a^{6}b^{3}def-a^{6}b^{3}e^{3}+6a^{5}b^{4}cef+3a^{5}b^{4}d^{2}f+3a^{5}b^{4}de^{2}-6a^{4}b^{5}cdf-3a^{4}b^{5}ce^{2}-3a^{4}b^{5}d^{2}e+3a^{3}b^{6}c^{2}f+6a^{3}b^{6}cde+a^{3}b^{6}d^{3}-3a^{2}b^{7}c^{2}e-3a^{2}b^{7}cd^{2}+3ab^{8}c^{2}d-b^{9}c^{3},\left(t\mapsto t\log\left(\frac{9t^{2}a^{3}b^{5}}{a^{6}f^{2}-2a^{5}bef+2a^{4}b^{2}df+a^{4}b^{2}e^{2}-2a^{3}b^{3}cf-2a^{3}b^{3}de+2a^{2}b^{4}ce+a^{2}b^{4}d^{2}-2ab^{5}cd+b^{6}c^{2}}+x\right)\right)\right)+\frac{fx^{5}}{5b}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a),x)
```

```
[Out] x**2*(-a*f/(2*b**2) + e/(2*b)) + RootSum(27*_t**3*a**4*b**8 + a**9*f**3 - 3
*a**8*b*e*f**2 + 3*a**7*b**2*d*f**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f*
*2 - 6*a**6*b**3*d*e*f - a**6*b**3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d
**2*f + 3*a**5*b**4*d*e**2 - 6*a**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**
4*b**5*d**2*e + 3*a**3*b**6*c**2*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3
*a**2*b**7*c**2*e - 3*a**2*b**7*c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambd
a(_t, _t*log(9*_t**2*a**3*b**5/(a**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f
+ a**4*b**2*e**2 - 2*a**3*b**3*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a*
*2*b**4*d**2 - 2*a*b**5*c*d + b**6*c**2) + x))) + f*x**5/(5*b) - c/(a*x)
```

Giac [A]
time = 1.40, size = 269, normalized size = 1.19

$$-\frac{\sqrt{3}\,(b^{3}c-ab^{2}d-a^{3}f+a^{2}be)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^{2})^{\frac{1}{3}}ab^{2}}-\frac{c}{ax}+\frac{(b^{3}c-ab^{2}d-a^{3}f+a^{2}be)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^{2})^{\frac{1}{3}}ab^{2}}+\frac{\left(b^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{2}b^{2}}+\frac{2\,b^{4}fx^{5}-5\,ab^{3}fx^{2}+5\,b^{4}x^{2}e}{10\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a*b^2) - c/(a*x) + 1/6*(b^3*c -
 a*b^2*d - a^3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b
^2)^(1/3)*a*b^2) + 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) - a^3*f*(
-a/b)^(1/3) + a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))
/(a^2*b^2) + 1/10*(2*b^4*f*x^5 - 5*a*b^3*f*x^2 + 5*b^4*x^2*e)/b^5
```

Mupad [B]
time = 5.37, size = 204, normalized size = 0.90

$$x^2\left(\frac{e}{2\,b}-\frac{a\,f}{2\,b^2}\right)-\frac{c}{a\,x}+\frac{f\,x^5}{5\,b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{4/3}\,b^{8/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{4/3}\,b^{8/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{4/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x)
```

```
[Out] x^2*(e/(2*b) - (a*f)/(2*b^2)) - c/(a*x) + (f*x^5)/(5*b) + (log(b^(1/3)*x +
a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(4/3)*b^(8/3)) - (log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a
^3*f - a*b^2*d + a^2*b*e))/(3*a^(4/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1i -
2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^
2*b*e))/(3*a^(4/3)*b^(8/3))
```

3.241 $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=224

$$-\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}b^{7/3}}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log}{3a^{5/3}b^{7/3}}$$

```
[Out] -1/2*c/a/x^2+(-a*f+b*e)*x/b^2+1/4*f*x^4/b-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(7/3)+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(7/3)+1/3*(-a^3*f+a^2*b
*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)
/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 206, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}\,a^{5/3}b^{7/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{5/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{5/3}b^{7/3}}+\frac{x(be-af)}{b^2}-\frac{c}{2ax^2}+\frac{fx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*c/(a*x^2) + ((b*e - a*f)*x)/b^2 + (f*x^4)/(4*b) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*a^(5/3)*b^(7/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(5/3)*b^(7/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx = \int \left(\frac{be-af}{b^2}+\frac{c}{ax^3}+\frac{fx^3}{b}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2(a+bx^3)}\right)dx$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}+\frac{(-b^3c+ab^2d-a^2be+a^3f)\int\frac{1}{a+bx^3}\,dx}{ab^2}$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{5/3}b^2}-$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}b^{7/3}}$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}b^{7/3}}$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}b^{7/3}}$$

Mathematica [A]
time = 0.08, size = 218, normalized size = 0.97

$$\frac{1}{12}\left(-\frac{6c}{ax^2}+\frac{12(be-af)x}{b^2}+\frac{3fx^4}{b}+\frac{4\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}b^{7/3}}+\frac{4(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}b^{7/3}}+\frac{2(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}b^{7/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-6*c)/(a*x^2) + (12*(b*e - a*f)*x)/b^2 + (3*f*x^4)/b + (4*Sqrt[3]*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(
a^(5/3)*b^(7/3)) + (4*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) +
b^(1/3)*x])/(a^(5/3)*b^(7/3)) + (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(5/3)*b^(7/3)))/12
```

Maple [A]
time = 0.40, size = 155, normalized size = 0.69

method	result

default	$-\frac{-\frac{1}{4}bf\,x^4+afx-bex}{b^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(a^3f-a^2be+a\,b^2d-b^3c)}{a\,b^2}$ -
risch	$\frac{f\,x^4}{4b}-\frac{afx}{b^2}+\frac{ex}{b}-\frac{c}{2a\,x^2}+\frac{\_R=\mathrm{RootOf}\left(-a^9f^3+3a^8be\,f^2-3a^7b^2d\,f^2-3a^7b^2e^2f+3a^6b^3c\,f^2+6a^6b^3def+a^6b^3e^3-6a^5b^4cef-3a^5b^4d^2f\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*(-1/4*b*f*x^4+a*f*x-b*e*x)+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/
b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a/b^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)
-1/2*c/a/x^2
```

Maxima [A]
time = 0.48, size = 218, normalized size = 0.97

$$\frac{bfx^4-4\,(af-be)x}{4\,b^2}-\frac{c}{2\,ax^2}-\frac{\sqrt{3}\,(b^3c-ab^2d-a^3f+a^2be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*f*x^4 - 4*(a*f - b*e)*x)/b^2 - 1/2*c/(a*x^2) - 1/3*sqrt(3)*(b^3*c -
a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(a*b^3*(a/b)^(2/3)) + 1/6*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) - 1/3*(b^3*c - a*b^2*d -
a^3*f + a^2*b*e)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

Fricas [A]
time = 0.40, size = 565, normalized size = 2.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a^3*b^2*f*x^6 - 6*a^2*b^3*c - 6*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a
^3*b^2*e - a^4*b*f)*x^2*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(
1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*(a^2*b)^(2/3)*x^2*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x + (a^2*b)^(
2/3)) + 12*(a^3*b^2*e - a^4*b*f)*x^3)/(a^3*b^3*x^2), 1/12*(3*a^3*b^2*f*x^6
- 6*a^2*b^3*c - 12*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^
2*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)
*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2
*b)^(2/3)*x^2*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x + (a^2*b)^(2/3)) +
12*(a^3*b^2*e - a^4*b*f)*x^3)/(a^3*b^3*x^2)]
```

Sympy [A]
time = 1.71, size = 326, normalized size = 1.46

$$x\left(-\frac{af}{b^2}+\frac{e}{b}\right)+\operatorname{RootSum}\left(27t^3a^5b^7-a^9f^3+3a^8bef^2-3a^7b^2df^2-3a^7b^2e^2f+3a^6b^3cf^2+6a^6b^3def+a^6b^3e^3-6a^5b^4cef-3a^5b^4d^2f-3a^5b^4de^2+6a^4b^5cdf+3a^4b^5ce^2+3a^4b^5d^2e-3a^3b^6c^2f-6a^3b^6cde-a^3b^6d^3+3a^2b^7c^2e+3a^2b^7cd^2-3ab^8c^2d+b^9c^3,\left(t\mapsto t\log\left(\frac{3ta^2b^2}{a^3f-a^2be+ab^2d-b^3c}+x\right)\right)\right)+\frac{fx^4}{4b}-\frac{c}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a),x)
```

```
[Out] x*(-a*f/b**2 + e/b) + RootSum(27*_t**3*a**5*b**7 - a**9*f**3 + 3*a**8*b*e*f
**2 - 3*a**7*b**2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6
*b**3*d*e*f + a**6*b**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a
**5*b**4*d*e**2 + 6*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2
*e - 3*a**3*b**6*c**2*f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*
c**2*e + 3*a**2*b**7*c*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*lo
g(3*_t*a**2*b**2/(a**3*f - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x**4/(4
*b) - c/(2*a*x**2)
```

Giac [A]
time = 1.23, size = 232, normalized size = 1.04

$$\frac{\sqrt{3}\,(b^3c-ab^2d-a^3f+a^2be)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}\,ab}+\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}\,ab}+\frac{(b^3c-ab^2d-a^3f+a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2b^2}-\frac{c}{2\,ax^2}+\frac{b^3fx^4-4\,ab^2fx+4\,b^3xe}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) + 1/6*(b^3*c - a*b^2*d - a^
3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b
) + 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^2) - 1/2*c/(a*x^2) + 1/4*(b^3*f*x^4 - 4*a*b^2*f*x + 4*b^3*x
*e)/b^4
```

Mupad [B]
time = 0.28, size = 201, normalized size = 0.90

$$x\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)-\frac{c}{2\,a\,x^2}+\frac{f\,x^4}{4\,b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{5/3}\,b^{7/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{5/3}\,b^{7/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{5/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x)
```

```
[Out] x*(e/b - (a*f)/b^2) - c/(2*a*x^2) + (f*x^4)/(4*b) - (log(b^(1/3)*x + a^(1/3
))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(5/3)*b^(7/3)) - (log(3^(1/2)*
a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f -
 a*b^2*d + a^2*b*e))/(3*a^(5/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1
/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e)
)/(3*a^(5/3)*b^(7/3))
```

3.242 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}\,dx$

Optimal. Leaf size=227

$$-\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}b^{5/3}}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{\right.}}{3a^{7/3}b^{5/3}}$$

```
[Out] -1/4*c/a/x^4+(-a*d+b*c)/a^2/x+1/2*f*x^2/b-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(5/3)+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(5/3)-1/3*(-a^3*f+a^2*b
*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)
/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 298, 31, 648, 631, 210, 642}

$$\frac{bc-ad}{a^2x}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}\,a^{7/3}b^{5/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{7/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{7/3}b^{5/3}}-\frac{c}{4ax^4}+\frac{fx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]
```

```
[Out] -1/4*c/(a*x^4) + (b*c - a*d)/(a^2*x) + (f*x^2)/(2*b) - ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*a^(7/3)*b^(5/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(7/3)*b^(5/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}\,dx = \int \left(\frac{c}{ax^5}+\frac{-bc+ad}{a^2x^2}+\frac{fx}{b}-\frac{(-b^3c+ab^2d-a^2be+a^3f)\,x}{a^2b\,(a+bx^3)}\right)dx$$

$$= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{(-b^3c+ab^2d-a^2be+a^3f)\int \frac{x}{a+bx^3}\,dx}{a^2b}$$

$$= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{7/3}b^{4/3}}+\frac{(b^3c}{}$$

$$= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}b^{5/3}}+\frac{(}{}$$

$$= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}b^{5/3}}+\frac{(}{}$$

$$= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}b^{5/3}}-$$

Mathematica [A]
time = 0.08, size = 220, normalized size = 0.97

$$\frac{1}{12}\left(-\frac{3c}{ax^4}+\frac{12(bc-ad)}{a^2x}+\frac{6fx^2}{b}+\frac{4\sqrt{3}\,(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}b^{5/3}}+\frac{4(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{7/3}b^{5/3}}+\frac{2(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{7/3}b^{5/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]
```

```
[Out] ((-3*c)/(a*x^4) + (12*(b*c - a*d))/(a^2*x) + (6*f*x^2)/b + (4*Sqrt[3]*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]])/(a^(7/3)*b^(5/3)) + (4*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/
3) + b^(1/3)*x])/(a^(7/3)*b^(5/3)) + (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(7/3)*b^(5/3)))/12
```

Maple [A]
time = 0.38, size = 159, normalized size = 0.70

method	result

default	$\frac{f x^2}{2b}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{a^2b}-\frac{c}{4a\,x^4}-\frac{ad-}{a^2}$
risch	$\frac{f x^2}{2b}+\frac{-\frac{b(ad-bc)x^3}{a^2}-\frac{bc}{4a}}{b\,x^4}+\frac{\_R=\mathrm{RootOf}\left(a^7b^2\_Z^3-a^9f^3+3a^8be\,f^2-3a^7b^2d\,f^2-3a^7b^2e^2f+3a^6b^3c\,f^2+6a^6b^3def+a^6b^3e^3-6a^5b^4cef-3a^5b\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2/b-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^2/b-1/4*c/a/x^4-(a*d-b*c
)/a^2/x
```

Maxima [A]

time = 0.50, size = 220, normalized size = 0.97

$$\frac{fx^2}{2b}+\frac{\sqrt{3}\,(b^3c-ab^2d-a^3f+a^2be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4\,(bc-ad)x^3-ac}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*f*x^2/b + 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sq
rt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3)) + 1/6*(b^3*c -
 a*b^2*d - a^3*f + a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2
*(a/b)^(1/3)) - 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(x + (a/b)^(1/3)
)/(a^2*b^2*(a/b)^(1/3)) + 1/4*(4*(b*c - a*d)*x^3 - a*c)/(a^2*x^4)
```

Fricas [A]

time = 0.40, size = 556, normalized size = 2.45

$$\left[\frac{6a^3b^2fx^6-6\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^4\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab-3\sqrt{\frac{1}{3}}\left(abx+2(ab^2)^{\frac{2}{3}}x^2-(ab^2)^{\frac{1}{3}}a\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}-3(ab^2)^{\frac{2}{3}}x}{bx^3+a}\right)+2(b^3c-ab^2d+a^2be-a^3f)(ab^2)^{\frac{2}{3}}x^4\log\left(b^2x^2-(ab^2)^{\frac{1}{3}}bx+(ab^2)^{\frac{2}{3}}\right)-4(b^3c-ab^2d+a^2be-a^3f)(ab^2)^{\frac{2}{3}}x^4\log\left(bx+(ab^2)^{\frac{1}{3}}\right)-3a^2b^3c+12(ab^4c-a^2b^3d)x^3}{12a^3b^3x^4},\ \frac{6a^3b^2fx^6-12\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^4\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2bx-(ab^2)^{\frac{1}{3}}\right)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}}{b}\right)+2(b^3c-ab^2d+a^2be-a^3f)(ab^2)^{\frac{2}{3}}x^4\log\left(b^2x^2-(ab^2)^{\frac{1}{3}}bx+(ab^2)^{\frac{2}{3}}\right)-4(b^3c-ab^2d+a^2be-a^3f)(ab^2)^{\frac{2}{3}}x^4\log\left(bx+(ab^2)^{\frac{1}{3}}\right)-3a^2b^3c+12(ab^4c-a^2b^3d)x^3}{12a^3b^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*a^3*b^2*f*x^6 - 6*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4
*b*f)*x^4*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x
 + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)
^(2/3)*x)/(b*x^3 + a)) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3
)*x^4*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b*x + (a*b^2)^(1/3)) - 3*a^2*b^3*
c + 12*(a*b^4*c - a^2*b^3*d)*x^3)/(a^3*b^3*x^4), 1/12*(6*a^3*b^2*f*x^6 - 12
*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4*sqrt((a*b^2)^(1/
3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) +
2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b^2*x^2 - (a*b^
2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^
2)^(2/3)*x^4*log(b*x + (a*b^2)^(1/3)) - 3*a^2*b^3*c + 12*(a*b^4*c - a^2*b^3
*d)*x^3)/(a^3*b^3*x^4)]
```

Sympy [A]
time = 4.56, size = 411, normalized size = 1.81

$$\operatorname{RootSum}\left(27t^3a^7b^5 - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4cef - 3a^5b^4d^2f - 3a^5b^4de^2 + 6a^4b^5cdf + 3a^4b^5ce^2 + 3a^4b^5d^2e - 3a^3b^6c^2f - 6a^3b^6cde - a^3b^6d^3 + 3a^2b^7c^2e + 3a^2b^7cd^2 - 3ab^8c^2d + b^9c^3, \left(t \mapsto t\log\left(\frac{9t^2a^5b^3}{a^6f^2 - 2a^5bef + 2a^4b^2df + a^4b^2e^2 - 2a^3b^3cf - 2a^3b^3de + 2a^2b^4ce + a^2b^4d^2 - 2ab^5cd + b^6c^2} + x\right)\right)\right) + \frac{fx^2}{2b} + \frac{-ac + x^3(-4ad + 4bc)}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**7*b**5 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f*
*2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b**
3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2 + 6*a*
*4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*
f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c
*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**5*b**3/(a
**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*
f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**
6*c**2) + x))) + f*x**2/(2*b) + (-a*c + x**3*(-4*a*d + 4*b*c))/(4*a**2*x**4
)
```

Giac [A]
time = 1.52, size = 261, normalized size = 1.15

$$\frac{fx^2}{2\,b} + \frac{\sqrt{3}\,(b^3c - ab^2d - a^3f + a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{1}{3}}\,a^2b} - \frac{(b^3c - ab^2d - a^3f + a^2be)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{1}{3}}\,a^2b} - \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b} + \frac{4\,bcx^3 - 4\,adx^3 - ac}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b + 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sq
rt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^2*b) - 1/6*(b^3*
c - a*b^2*d - a^3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-
a*b^2)^(1/3)*a^2*b) - 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) - a^3*
```

```
f*(-a/b)^(1/3) + a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3
)))/(a^3*b) + 1/4*(4*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^2*x^4)
```

Mupad [B]
time = 5.16, size = 209, normalized size = 0.92

$$\frac{f\,x^2}{2\,b}-\frac{\frac{b\,c}{4\,a}+\frac{b\,x^3\,\left(a\,d-b\,c\right)}{a^2}}{b\,x^4}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{7/3}\,b^{5/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{7/3}\,b^{5/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{7/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x)
```

```
[Out] (f*x^2)/(2*b) - ((b*c)/(4*a) + (b*x^3*(a*d - b*c))/a^2)/(b*x^4) - (log(b^(1
/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(7/3)*b^(5/3)) +
 (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(7/3)*b^(5/3)) - (log(3^(1/2)*a^(1/
3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^
2*d + a^2*b*e))/(3*a^(7/3)*b^(5/3))
```

3.243 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx$

Optimal. Leaf size=225

$$-\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}b^{4/3}}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{\cdots}\right.}{3a^{8/3}b^{4/3}}$$

```
[Out] -1/5*c/a/x^5+1/2*(-a*d+b*c)/a^2/x^2+f*x/b+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(4/3)-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(4/3)-1/3*(-a^3*f+a^2*b
*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)
/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 206, 31, 648, 631, 210, 642}

$$\frac{bc-ad}{2a^2x^2}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}\,a^{8/3}b^{4/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{8/3}b^{4/3}}-\frac{c}{5ax^5}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]
```

```
[Out] -1/5*c/(a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b - ((b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/
3)*x])/(3*a^(8/3)*b^(4/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx = \int \left(\frac{f}{b}+\frac{c}{ax^6}+\frac{-bc+ad}{a^2x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx^3)}\right)dx$$

$$= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{a+bx^3}\,dx}{a^2b}$$

$$= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{8/3}b}+\frac{(b^3c-\cdots}{\cdots}$$

$$= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}b^{4/3}}-\frac{(b\cdots}{\cdots}$$

$$= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}b^{4/3}}-\frac{(b\cdots}{\cdots}$$

$$= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}b^{4/3}}+\cdots$$

Mathematica [A]
time = 0.06, size = 220, normalized size = 0.98

$$-\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{8/3}b^{4/3}}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}b^{4/3}}+\frac{(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{8/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]
```

```
[Out] -1/5*c/(a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b + ((-(b^3*c) + a*b^2*d -
 a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(
8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(8/3)*b^(4/3)) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*b^(4/3))
```

Maple [A]
time = 0.37, size = 155, normalized size = 0.69

method	result

default	$\frac{fx}{b}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(-a^3f+a^2be-a\,b^2d+b^3c)}{a^2b}-\frac{c}{5a\,x^5}-\frac{ad-b}{2a^2x}$
risch	$\frac{fx}{b}+\frac{-\frac{b(ad-bc)x^3}{2a^2}-\frac{bc}{5a}}{b\,x^5}+\frac{\_R=\mathrm{RootOf}\left(a^8b\_Z^3+a^9f^3-3a^8be\,f^2+3a^7b^2d\,f^2+3a^7b^2e^2f-3a^6b^3c\,f^2-6a^6b^3def-a^6b^3e^3+6a^5b^4cef+3a^5b^4d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] f*x/b+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1)))/a^2/b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)-1/5*c/a/x^5-1/2*(a*d-b*c)/
a^2/x^2
```

Maxima [A]
time = 0.50, size = 217, normalized size = 0.96

$$\frac{fx}{b}+\frac{\sqrt{3}\,(b^3c-ab^2d-a^3f+a^2be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,(bc-ad)x^3-2\,ac}{10\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] f*x/b + 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) - 1/6*(b^3*c - a*b^2
*d - a^3*f + a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)
^(2/3)) + 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(x + (a/b)^(1/3))/(a^2
*b^2*(a/b)^(2/3)) + 1/10*(5*(b*c - a*d)*x^3 - 2*a*c)/(a^2*x^5)
```

Fricas [A]
time = 0.44, size = 584, normalized size = 2.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/30*(30*a^4*b*f*x^6 - 15*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4
*b*f)*x^5*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^
2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^
2*b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b
)^(2/3)*x^5*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x + (-a^2*b)^(2/3))
 - 6*a^3*b^2*c + 15*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^5), 1/30*(30*a^
4*b*f*x^6 + 30*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^5*sq
rt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)
*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-
a^2*b)^(2/3)*x^5*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x + (-a^2*b)^(
2/3)) - 6*a^3*b^2*c + 15*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^5)]
```

Sympy [**A**]
time = 8.25, size = 328, normalized size = 1.46

$$\operatorname{RootSum}\left(27t^3a^8b^4 + a^9f^3 - 3a^8bef^2 + 3a^7b^2df^2 + 3a^7b^2e^2f - 3a^6b^3cf^2 - 6a^6b^3def - a^6b^3e^3 + 6a^5b^4cef + 3a^5b^4d^2f + 3a^5b^4de^2 - 6a^4b^5cdf - 3a^4b^5ce^2 - 3a^4b^5d^2e + 3a^3b^6c^2f + 6a^3b^6cde + a^3b^6d^3 - 3a^2b^7c^2e - 3a^2b^7cd^2 + 3ab^8c^2d - b^9c^3, \left(t \mapsto t\log\left(-\frac{3ta^3b}{a^3f - a^2be + ab^2d - b^3c} + x\right)\right)\right) + \frac{fx}{b} + \frac{-2ac + x^3(-5ad + 5bc)}{10a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**8*b**4 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f*
*2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b**
3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a*
*4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*
f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c
*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a**3*b/(a**3*f
 - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x/b + (-2*a*c + x**3*(-5*a*d +
5*b*c))/(10*a**2*x**5)
```

Giac [**A**]
time = 0.82, size = 220, normalized size = 0.98

$$\frac{fx}{b} - \frac{\sqrt{3}\,(b^3c - ab^2d - a^3f + a^2be)\arctan\left(\frac{\sqrt{3}\,\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{(b^3c - ab^2d - a^3f + a^2be)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{(b^3c - ab^2d - a^3f + a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b} + \frac{5\,bcx^3 - 5\,adx^3 - 2\,ac}{10\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="giac")
```

```
[Out] f*x/b - 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/6*(b^3*c - a*b^
2*d - a^3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(
2/3)*a^2) - 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^3*b) + 1/10*(5*b*c*x^3 - 5*a*d*x^3 - 2*a*c)/(a^2*x^5)
```

Mupad [B]
time = 5.09, size = 207, normalized size = 0.92

$$\frac{f\,x}{b}-\frac{\frac{b\,c}{5\,a}+\frac{b\,x^3\,\left(a\,d-b\,c\right)}{2\,a^2}}{b\,x^5}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{8/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{8/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{8/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x)
```

```
[Out] (f*x)/b - ((b*c)/(5*a) + (b*x^3*(a*d - b*c))/(2*a^2))/(b*x^5) + (log(b^(1/3
)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(8/3)*b^(4/3)) + (
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3
*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(8/3)*b^(4/3)) - (log(3^(1/2)*a^(1/3)
*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*
d + a^2*b*e))/(3*a^(8/3)*b^(4/3))
```

3.244 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx$

Optimal. Leaf size=242

$$-\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}b^{2/3}}+\frac{(b^3c-ab^2d+a^2be-}{3a^1}$$

```
[Out] -1/7*c/a/x^7+1/4*(-a*d+b*c)/a^2/x^4+(-a^2*e+a*b*d-b^2*c)/a^3/x+1/3*(-a^3*f+
a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(2/3)-1/6*(-a^3*f+a
^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^
(2/3)+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a
^(1/3)*3^(1/2))/a^(10/3)/b^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 298, 31, 648, 631, 210, 642}

$$\frac{bc-ad}{4a^2x^4}-\frac{a^2e-abd+b^2c}{a^3x}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}\,a^{10/3}b^{2/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{10/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{10/3}b^{2/3}}-\frac{c}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]
```

```
[Out] -1/7*c/(a*x^7) + (b*c - a*d)/(4*a^2*x^4) - (b^2*c - a*b*d + a^2*e)/(a^3*x)
+ ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(Sqrt[3]*a^(10/3)*b^(2/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a
^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(10/3)*b^(2/3)) - ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(10/3
)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^8}+\frac{-bc+ad}{a^2x^5}+\frac{b^2c-abd+a^2e}{a^3x^2}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{a^3\left(a+bx^3\right)}\right)\,dx$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int\frac{x}{a+bx^3}\,dx}{a^3}$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}}}{3a^{10/3}\sqrt[3]{b}}$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\right.}{3a^{10/3}b^{2/3}}$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\right.}{3a^{10/3}b^{2/3}}$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}}{\sqrt{}}\right.}{\sqrt{3}\,a^{10/3}b^{2/3}}$$

Mathematica [A]
time = 0.08, size = 231, normalized size = 0.95

$$\frac{-\frac{12a^{7/3}c}{x^7}+\frac{21a^{4/3}(bc-ad)}{x^4}-\frac{84\sqrt[3]{a}\left(b^2c-abd+a^2e\right)}{x}+\frac{28\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{28\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{14\left(-b^3c+ab^2d-a^2be+a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}}{84a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]
```

```
[Out] ((-12*a^(7/3)*c)/x^7 + (21*a^(4/3)*(b*c - a*d))/x^4 - (84*a^(1/3)*(b^2*c -
a*b*d + a^2*e))/x + (28*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (14*(-(b^3*c) + a*b^2*d -
a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(
84*a^(10/3))
```

Maple [A]
time = 0.36, size = 170, normalized size = 0.70

method	result

default	$\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)}{a^{3}}-\frac{c}{7a\,x^{7}}-\frac{ad-bc}{4a^{2}x^{4}}-\frac{a}{}$
risch	$\frac{-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{a^{3}}-\frac{(ad-bc)x^{3}}{4a^{2}}-\frac{c}{7a}}{x^{7}}+\frac{\left(\_R=\text{RootOf}\left(a^{10}b^{2}\_Z^{3}+a^{9}f^{3}-3a^{8}be\,f^{2}+3a^{7}b^{2}d\,f^{2}+3a^{7}b^{2}e^{2}f-3a^{6}b^{3}c\,f^{2}-6a^{6}b^{3}def-a^{6}b^{3}e^{3}+6\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1)))/a^3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)-1/7*c/a/x^7-1/4*(a*d-b*c)/a^2/x^4-
(a^2*e-a*b*d+b^2*c)/a^3/x
```

Maxima [A]
time = 0.49, size = 238, normalized size = 0.98

$$-\frac{\sqrt{3}\,(b^3c-ab^2d-a^3f+a^2be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{28\,(b^2c-abd+a^2e)x^6-7\,(abc-a^2d)x^3+4\,a^2c}{28\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(1/3)) - 1/6*(b^3*c - a*b^2*d - a^3*
f + a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(1/3)) + 1
/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(1
/3)) - 1/28*(28*(b^2*c - a*b*d + a^2*e)*x^6 - 7*(a*b*c - a^2*d)*x^3 + 4*a^2
*c)/(a^3*x^7)
```

Fricas [A]
time = 0.40, size = 610, normalized size = 2.52

$$\left[-\frac{42\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^7\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\text{[illegible]}\right)+14(b^3c-ab^2d+a^2be-a^3f)(-ab^2)^{\frac{2}{3}}x^7\log\left(b^2x^2+(-ab^2)^{\frac{1}{3}}bx+(-ab^2)^{\frac{2}{3}}\right)-28(b^3c-ab^2d+a^2be-a^3f)(-ab^2)^{\frac{2}{3}}x^7\log\left(bx-(-ab^2)^{\frac{1}{3}}\right)+84(ab^4c-a^2b^3d+a^3b^2e)x^6+12a^3b^2c-21(a^2b^3c-a^3b^2d)x^3}{84a^4b^2x^7},\ -\frac{84\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^7\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right)+14(b^3c-ab^2d+a^2be-a^3f)(-ab^2)^{\frac{2}{3}}x^7\log\left(b^2x^2+(-ab^2)^{\frac{1}{3}}bx+(-ab^2)^{\frac{2}{3}}\right)-28(b^3c-ab^2d+a^2be-a^3f)(-ab^2)^{\frac{2}{3}}x^7\log\left(bx-(-ab^2)^{\frac{1}{3}}\right)+84(ab^4c-a^2b^3d+a^3b^2e)x^6+12a^3b^2c-21(a^2b^3c-a^3b^2d)x^3}{84a^4b^2x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^7*sqrt((
-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/
3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*
x^3 + a)) + 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b
^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b*x - (-a*b^2)^(1/3)) + 84*(a*b^4*c - a^
2*b^3*d + a^3*b^2*e)*x^6 + 12*a^3*b^2*c - 21*(a^2*b^3*c - a^3*b^2*d)*x^3)/(
a^4*b^2*x^7), -1/84*(84*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*
f)*x^7*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sq
rt(-(-a*b^2)^(1/3)/a)/b) + 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^
(2/3)*x^7*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b*x - (-a*b^2)^(1/3)) + 8
4*(a*b^4*c - a^2*b^3*d + a^3*b^2*e)*x^6 + 12*a^3*b^2*c - 21*(a^2*b^3*c - a^
3*b^2*d)*x^3)/(a^4*b^2*x^7)]
```

Sympy [**A**]
time = 45.53, size = 432, normalized size = 1.79

$$\operatorname{RootSum}\left(27t^3a^{10}b^2+a^9f^3-3a^8bef^2+3a^7b^2df^2+3a^7b^2e^2f-3a^6b^3cf^2-6a^6b^3def-a^6b^3e^3+6a^5b^4cef+3a^5b^4d^2f+3a^5b^4de^2-6a^4b^5cdf-3a^4b^5ce^2-3a^4b^5d^2e+3a^3b^6c^2f+6a^3b^6cde+a^3b^6d^3-3a^2b^7c^2e-3a^2b^7cd^2+3ab^8c^2d-b^9c^3,\left(t\mapsto t\log\left(\frac{9t^2a^7b}{a^6f^2-2a^5bef+2a^4b^2df+a^4b^2e^2-2a^3b^3cf-2a^3b^3de+2a^2b^4ce+a^2b^4d^2-2ab^5cd+b^6c^2}+x\right)\right)\right)+\frac{-4a^2c+x^6(-28a^2e+28abd-28b^2c)+x^3(-7a^2d+7abc)}{28a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**10*b**2 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f
**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b*
*3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a
**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2
*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*
c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**7*b/(a**
6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*f
- 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**6*
c**2) + x))) + (-4*a**2*c + x**6*(-28*a**2*e + 28*a*b*d - 28*b**2*c) + x**3
*(-7*a**2*d + 7*a*b*c))/(28*a**3*x**7)
```

Giac [**A**]
time = 1.02, size = 275, normalized size = 1.14

$$-\frac{\sqrt{3}\,(b^3c-ab^2d-a^3f+a^2be)\arctan\left(\frac{\sqrt{3}\,\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{1}{3}}\,a^3}+\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{1}{3}}\,a^3}+\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^4}-\frac{28\,b^2cx^6-28\,abdx^6+28\,a^2x^6e-7\,abcx^3+7\,a^2dx^3+4\,a^2c}{28\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^3) + 1/6*(b^3*c - a*b^2*d - a
^3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^
```

```
3) + 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) - a^3*f*(-a/b)^(1/3) +
a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - 1/28*(2
8*b^2*c*x^6 - 28*a*b*d*x^6 + 28*a^2*x^6*e - 7*a*b*c*x^3 + 7*a^2*d*x^3 + 4*a
^2*c)/(a^3*x^7)
```

Mupad [B]
time = 5.20, size = 219, normalized size = 0.90

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{10/3}\,b^{2/3}}-\frac{\frac{c}{7\,a}+\frac{x^3\,\left(a\,d-b\,c\right)}{4\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{a^3}}{x^7}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{10/3}\,b^{2/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{10/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(10/3)*
b^(2/3)) - (c/(7*a) + (x^3*(a*d - b*c))/(4*a^2) + (x^6*(b^2*c + a^2*e - a*b
*d))/a^3)/x^7 - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*
1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(10/3)*b^(2/3)) + (l
og(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*
c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(10/3)*b^(2/3))
```

3.245 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx$

Optimal. Leaf size=244

$$-\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{11/3}\sqrt[3]{b}}-\frac{(b^3c-ab^2d+a^2be-}{3a^1}$$

```
[Out] -1/8*c/a/x^8+1/5*(-a*d+b*c)/a^2/x^5+1/2*(-a^2*e+a*b*d-b^2*c)/a^3/x^2-1/3*(-
a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(1/3)+1/6*(-a
^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11
/3)/b^(1/3)+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3
)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 206, 31, 648, 631, 210, 642}

$$\frac{bc-ad}{5a^2x^5}-\frac{a^2e-abd+b^2c}{2a^3x^2}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}\,a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{11/3}\sqrt[3]{b}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{11/3}\sqrt[3]{b}}-\frac{c}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```

```
[Out] -1/8*c/(a*x^8) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(2*a^3*x
^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(11/3)*b^(1/3)) - ((b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(11/3)*b^(1/3)) + ((b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(
11/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^9}+\frac{-bc+ad}{a^2x^6}+\frac{b^2c-abd+a^2e}{a^3x^3}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3\left(a+bx^3\right)}\right)\,dx$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{1}{a+bx^3}\,dx}{a^3}$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}}}{3a^{11/3}}$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\right.}{3a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\right.}{3a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}}{\sqrt{}}\right.}{\sqrt{3}\,a^{11/3}\sqrt[3]{b}}$$

Mathematica [A]
time = 0.08, size = 231, normalized size = 0.95

$$\frac{-\frac{15a^{8/3}c}{x^8}+\frac{24a^{5/3}(bc-ad)}{x^5}-\frac{60a^{2/3}\left(b^2c-abd+a^2e\right)}{x^2}+\frac{40\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{40\left(-b^3c+ab^2d-a^2be+a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+\frac{20\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}}}{120a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```

```
[Out] ((-15*a^(8/3)*c)/x^8 + (24*a^(5/3)*(b*c - a*d))/x^5 - (60*a^(2/3)*(b^2*c -
a*b*d + a^2*e))/x^2 + (40*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-(b^3*c) + a*b^2*d -
 a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (20*(b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))
/(120*a^(11/3))
```

Maple [A]
time = 0.37, size = 170, normalized size = 0.70

method	result

default	$\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(a^3f-a^2be+ab^2d-b^3c)}{a^3}-\frac{c}{8a\,x^8}-\frac{ad-bc}{5a^2x^5}-\frac{a^2e}{}$
risch	$\frac{-\frac{(a^2e-abd+b^2c)x^6}{2a^3}-\frac{(ad-bc)x^3}{5a^2}-\frac{c}{8a}}{x^8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}b\_Z^3-a^9f^3+3a^8bef^2-3a^7b^2df^2-3a^7b^2e^2f+3a^6b^3cf^2+6a^6b^3def+a^6b^3e^3-6a\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1)))/a^3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)-1/8*c/a/x^8-1/5*(a*d-b*c)/a^2/x^5-1
/2*(a^2*e-a*b*d+b^2*c)/a^3/x^2
```

Maxima [A]
time = 0.50, size = 238, normalized size = 0.98

$$-\frac{\sqrt{3}\,(b^3c-ab^2d-a^3f+a^2be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(b^3c-ab^2d-a^3f+a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{20\,(b^2c-abd+a^2e)x^6-8\,(abc-a^2d)x^3+5\,a^2c}{40\,a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) + 1/6*(b^3*c - a*b^2*d - a^3*
f + a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) - 1
/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2
/3)) - 1/40*(20*(b^2*c - a*b*d + a^2*e)*x^6 - 8*(a*b*c - a^2*d)*x^3 + 5*a^2
*c)/(a^3*x^8)
```

Fricas [A]
time = 0.40, size = 595, normalized size = 2.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/120*(60*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8*sqrt(
-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*
(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*
x^3 + a)) - 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^8*log(a*
b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*(a^2*b)^(2/3)*x^8*log(a*b*x + (a^2*b)^(2/3)) + 60*(a^2*b^3*c - a^3
*b^2*d + a^4*b*e)*x^6 + 15*a^4*b*c - 24*(a^3*b^2*c - a^4*b*d)*x^3)/(a^5*b*x
^8), -1/120*(120*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8*
sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*
b)^(2/3)*x^8*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^8*log(a*b*x + (a^2*b)^(2/3)) +
60*(a^2*b^3*c - a^3*b^2*d + a^4*b*e)*x^6 + 15*a^4*b*c - 24*(a^3*b^2*c - a^4
*b*d)*x^3)/(a^5*b*x^8)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.28, size = 297, normalized size = 1.22

$$\frac{(b^3c-ab^2d-a^3f+a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^4}-\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}\,b^3c-(-ab^2)^{\frac{1}{3}}\,ab^2d-(-ab^2)^{\frac{1}{3}}\,a^3f+(-ab^2)^{\frac{1}{3}}\,a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^4b}-\frac{\left((-ab^2)^{\frac{1}{3}}\,b^3c-(-ab^2)^{\frac{1}{3}}\,ab^2d-(-ab^2)^{\frac{1}{3}}\,a^3f+(-ab^2)^{\frac{1}{3}}\,a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^4b}-\frac{20\,b^2cx^6-20\,abdx^6+20\,a^2x^6e-8\,abcx^3+8\,a^2dx^3+5\,a^2c}{40\,a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/
3)))/a^4 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a
*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)
*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/40*(20*b^2*c*x^6 - 20*a*b*d*x^6 + 20*a^
2*x^6*e - 8*a*b*c*x^3 + 8*a^2*d*x^3 + 5*a^2*c)/(a^3*x^8)
```

Mupad [**B**]
time = 5.13, size = 220, normalized size = 0.90

$$-\frac{\frac{c}{8\,a}+\frac{x^3\,(a\,d-b\,c)}{5\,a^2}+\frac{x^6\,(e\,a^2-d\,a\,b+c\,b^2)}{2\,a^3}}{x^8}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{11/3}\,b^{1/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{11/3}\,b^{1/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{11/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^
3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(11/3)*b^(1/3)) - (log(b^(1/3)*x + a
^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(11/3)*b^(1/3)) - (log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a
^3*f - a*b^2*d + a^2*b*e))/(3*a^(11/3)*b^(1/3)) - (c/(8*a) + (x^3*(a*d - b*
c))/(5*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(2*a^3))/x^8
```

3.246 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx$

Optimal. Leaf size=277

$$-\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}-\frac{\sqrt[3]{b}\,(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}}{\sqrt{}}\right.}{\sqrt{3}\,a^{13/3}}$$

```
[Out] -1/10*c/a/x^10+1/7*(-a*d+b*c)/a^2/x^7+1/4*(-a^2*e+a*b*d-b^2*c)/a^3/x^4+(-a^
3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x-1/3*b^(1/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)+1/6*b^(1/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(13/3)-1/3*b^(1/3)*(-a^3*f+a^2*
b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/
3)*3^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 298, 31, 648, 631, 210, 642}

$$\frac{bc-ad}{7a^2x^7}-\frac{a^2e-abd+b^2c}{4a^3x^4}-\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}\,a^{13/3}}-\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{13/3}}+\frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{13/3}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]
```

```
[Out] -1/10*c/(a*x^10) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(4*a^3
*x^4) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (b^(1/3)*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*a^(13/3)) - (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/
3) + b^(1/3)*x])/(3*a^(13/3)) + (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)}\,dx &= \int \left(\frac{c}{ax^{11}}+\frac{-bc+ad}{a^2x^8}+\frac{b^2c-abd+a^2e}{a^3x^5}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^2}-\frac{b(-\cdots}{\cdots}\right.\\
&= -\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}+\frac{(b(b^3c-\cdots}{\cdots}\\
&= -\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}-\frac{(b^{2/3}(b^3\cdots}{\cdots}\\
&= -\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}-\frac{\sqrt[3]{b}\,(b^3c\cdots}{\cdots}\\
&= -\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}-\frac{\sqrt[3]{b}\,(b^3c\cdots}{\cdots}\\
&= -\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}-\frac{\sqrt[3]{b}\,(b^3c\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 266, normalized size = 0.96

$$
\frac{-\frac{42a^{10/3}c}{x^{10}}+\frac{60a^{7/3}(bc-ad)}{x^7}-\frac{105a^{4/3}(b^2c-abd+a^2e)}{x^4}+\frac{420\sqrt[3]{a}\,(b^3c-ab^2d+a^2be-a^3f)}{x}-140\sqrt{3}\,\sqrt[3]{b}\,(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+140\sqrt[3]{b}\,(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+70\sqrt[3]{b}\,(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{420a^{13/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]
```

```
[Out] ((-42*a^(10/3)*c)/x^10 + (60*a^(7/3)*(b*c - a*d))/x^7 - (105*a^(4/3)*(b^2*c
 - a*b*d + a^2*e))/x^4 + (420*a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/
x - 140*Sqrt[3]*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 70*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(420*a^(13/3))
```

Maple [A]
time = 0.36, size = 205, normalized size = 0.74

method	result

default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)b}{a^{4}}-\frac{c}{10a\,x^{10}}-\frac{ad-bc}{7a^{2}x^{7}}$
risch	$\frac{-\frac{\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{9}}{a^{4}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{4a^{3}}-\frac{(ad-bc)x^{3}}{7a^{2}}-\frac{c}{10a}}{x^{10}}+\Big(\_R=\mathrm{RootOf}\big(a^{13}\_Z^{3}-a^{9}b\,f^{3}+3a^{8}b^{2}e\,f^{2}-3a^{7}b^{3}d\,f^{2}-3a^{7}b^{3}e^{2}f+3a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4*b-1/10*c/a/x^10-1/7*(a*d-b*c)/a^2
/x^7-1/4*(a^2*e-a*b*d+b^2*c)/a^3/x^4-(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x
```

Maxima [A]
time = 0.50, size = 265, normalized size = 0.96

$$\frac{\sqrt{3}\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{4}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{4}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^{4}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{140\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)x^{9}-35\left(ab^{2}c-a^{2}bd+a^{3}e\right)x^{6}-14\,a^{3}c+20\left(a^{2}bc-a^{3}d\right)x^{3}}{140\,a^{4}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a^4*(a/b)^(1/3)) + 1/6*(b^3*c - a*b^2*d - a^3*f +
 a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(1/3)) - 1/3*(b
^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(1/3)) +
1/140*(140*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x^9 - 35*(a*b^2*c - a^2*b*d
+ a^3*e)*x^6 - 14*a^3*c + 20*(a^2*b*c - a^3*d)*x^3)/(a^4*x^10)
```

Fricas [A]
time = 0.38, size = 262, normalized size = 0.95

$$\frac{140\,\sqrt{3}\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{10}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+70\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{10}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-140\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{10}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)+420\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{9}-105\left(ab^{2}c-a^{2}bd+a^{3}e\right)x^{6}-42\,a^{3}c+60\left(a^{2}bc-a^{3}d\right)x^{3}}{420\,a^{4}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/420*(140*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^10*(b/a)^(1/3)*arc
tan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*x^10*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) -
140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^10*(b/a)^(1/3)*log(b*x + a*(b/a)^
(2/3)) + 420*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 105*(a*b^2*c - a^2*b
*d + a^3*e)*x^6 - 42*a^3*c + 60*(a^2*b*c - a^3*d)*x^3)/(a^4*x^10)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.42, size = 376, normalized size = 1.36

$$-\frac{\left(b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^5} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}a^3f + \left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^5b} + \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}a^3f + \left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^5b} + \frac{140\,b^3cx^9 - 140\,ab^2dx^9 - 140\,a^3fx^9 + 140\,a^2bx^9e - 35\,ab^2cx^6 + 35\,a^2bdx^6 - 35\,a^3x^6e + 20\,a^2bcx^3 - 20\,a^3dx^3 - 14\,a^3c}{140\,a^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(b^4*c*(-a/b)^(1/3) - a*b^3*d*(-a/b)^(1/3) - a^3*b*f*(-a/b)^(1/3) + a^
2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 1/3*sqr
t(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f
+ (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(a^5*b) + 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2
)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/(a^5*b) + 1/140*(140*b^3*c*x^9 - 140*a*b^2*d*x^9 - 140*a^3*f*x^9 + 14
0*a^2*b*x^9*e - 35*a*b^2*c*x^6 + 35*a^2*b*d*x^6 - 35*a^3*x^6*e + 20*a^2*b*c
*x^3 - 20*a^3*d*x^3 - 14*a^3*c)/(a^4*x^10)
```

Mupad [B]
time = 5.33, size = 253, normalized size = 0.91

$$-\frac{\frac{c}{10\,a} - \frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^4} + \frac{x^3\,\left(a\,d-b\,c\right)}{7\,a^2} + \frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{4\,a^3}}{x^{10}} - \frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{13/3}} + \frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{13/3}} - \frac{b^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x)
```

```
[Out] (b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(13/3)) - (b^(1/3)*log(b^(1/
3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(13/3)) - (c/(10*
a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^4 + (x^3*(a*d - b*c))/(7*a
^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(4*a^3))/x^10 - (b^(1/3)*log(3^(1/2)*a^
(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a
*b^2*d + a^2*b*e))/(3*a^(13/3))
```

3.247 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx$

Optimal. Leaf size=280

$$-\frac{c}{11ax^{11}}+\frac{bc-ad}{8a^2x^8}-\frac{b^2c-abd+a^2e}{5a^3x^5}+\frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2}-\frac{b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{\sqrt{3}\,a^{14/3}}$$

```
[Out] -1/11*c/a/x^11+1/8*(-a*d+b*c)/a^2/x^8+1/5*(-a^2*e+a*b*d-b^2*c)/a^3/x^5+1/2*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^2+1/3*b^(2/3)*(-a^3*f+a^2*b*e-a*b^2*d+
b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)-1/6*b^(2/3)*(-a^3*f+a^2*b*e-a*b^2*d+b
^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)-1/3*b^(2/3)*(-a^3*
f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
a^(14/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 206, 31, 648, 631, 210, 642}

$$\frac{bc-ad}{8a^2x^8}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}\,a^{14/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{14/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{14/3}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{2a^4x^2}-\frac{c}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]
```

```
[Out] -1/11*c/(a*x^11) + (b*c - a*d)/(8*a^2*x^8) - (b^2*c - a*b*d + a^2*e)/(5*a^3
*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a^4*x^2) - (b^(2/3)*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(14/3)) + (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a
^(1/3) + b^(1/3)*x])/(3*a^(14/3)) - (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^{12}} + \frac{-bc+ad}{a^2x^9} + \frac{b^2c-abd+a^2e}{a^3x^6} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^3} - \frac{b(-b^3\right.$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{(b(b^3c-a}{}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{(b(b^3c-a}{}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{b^{2/3}(b^3c-}{}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{b^{2/3}(b^3c-}{}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} - \frac{b^{2/3}(b^3c-}{}$$

Mathematica [A]
time = 0.08, size = 266, normalized size = 0.95

$$\frac{-\frac{120a^{11/3}c}{x^{11}} + \frac{165a^{8/3}(bc-ad)}{x^8} - \frac{264a^{5/3}(b^2c-abd+a^2e)}{x^5} + \frac{660a^{2/3}(b^3c-ab^2d+a^2be-a^3f)}{x^2} - 440\sqrt{3}\,b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 440b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) + 220b^{2/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{1320a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]
```

```
[Out] ((-120*a^(11/3)*c)/x^11 + (165*a^(8/3)*(b*c - a*d))/x^8 - (264*a^(5/3)*(b^2
*c - a*b*d + a^2*e))/x^5 + (660*a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
)/x^2 - 440*Sqrt[3]*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 440*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*b^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*
e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1320*a^(14/3))
```

Maple [A]
time = 0.37, size = 205, normalized size = 0.73

method	result

default	$-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)b}{a^{4}}-\frac{c}{11a\,x^{11}}-\frac{ad-bc}{8a^{2}x^{8}}$
risch	$\frac{-\frac{\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{9}}{2a^{4}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{5a^{3}}-\frac{(ad-bc)x^{3}}{8a^{2}}-\frac{c}{11a}}{x^{11}}+\frac{\left(\_R=\mathrm{RootOf}\left(a^{14}\_Z^{3}+a^{9}b^{2}f^{3}-3a^{8}b^{3}e\,f^{2}+3a^{7}b^{4}d\,f^{2}+3a^{7}b^{4}e^{2}f\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4*b-1/11*c/a/x^11-1/8*(a*d-b*c)/a^2/
x^8-1/5*(a^2*e-a*b*d+b^2*c)/a^3/x^5-1/2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x
^2
```

Maxima [A]
time = 0.52, size = 265, normalized size = 0.95

$$\frac{\sqrt{3}\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{220\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)x^{9}-88\left(ab^{2}c-a^{2}bd+a^{3}e\right)x^{6}-40\,a^{3}c+55\left(a^{2}bc-a^{3}d\right)x^{3}}{440\,a^{4}x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a^4*(a/b)^(2/3)) - 1/6*(b^3*c - a*b^2*d - a^3*f +
 a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) + 1/3*(b
^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(2/3)) +
1/440*(220*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x^9 - 88*(a*b^2*c - a^2*b*d
+ a^3*e)*x^6 - 40*a^3*c + 55*(a^2*b*c - a^3*d)*x^3)/(a^4*x^11)
```

Fricas [A]
time = 0.38, size = 295, normalized size = 1.05

$$-\frac{440\sqrt{3}\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{11}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-220\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{11}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}+abx\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)+440\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{11}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\right)-660\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{9}+264\left(ab^{2}c-a^{2}bd+a^{3}e\right)x^{6}+120\,a^{3}c-165\left(a^{2}bc-a^{3}d\right)x^{3}}{1320\,a^{4}x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(1
/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 220*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b
^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 440*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*x^11*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 660*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^9 + 264*(a*b^2*c - a^2*b*d + a^3*e)*x^6 + 120*a^3
*c - 165*(a^2*b*c - a^3*d)*x^3)/(a^4*x^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.57, size = 338, normalized size = 1.21

$$\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}b^3c-(-ab^2)^{\frac{1}{3}}ab^2d-(-ab^2)^{\frac{1}{3}}a^3f+(-ab^2)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^5}-\frac{(b^4c-ab^3d-a^3bf+a^2b^2e)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^5}+\frac{\left((-ab^2)^{\frac{1}{3}}b^3c-(-ab^2)^{\frac{1}{3}}ab^2d-(-ab^2)^{\frac{1}{3}}a^3f+(-ab^2)^{\frac{1}{3}}a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^5}+\frac{220b^3cx^9-220ab^2dx^9-220a^3fx^9+220a^2bx^9e-88ab^2cx^6+88a^2bdx^6-88a^3x^6e+55a^2bcx^3-55a^3dx^3-40a^3c}{440a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)
*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/a^5 - 1/3*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*(-a/b)^(1/3)*
log(abs(x - (-a/b)^(1/3)))/a^5 + 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)
*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/a^5 + 1/440*(220*b^3*c*x^9 - 220*a*b^2*d*x^9 - 220*
a^3*f*x^9 + 220*a^2*b*x^9*e - 88*a*b^2*c*x^6 + 88*a^2*b*d*x^6 - 88*a^3*x^6*
e + 55*a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^4*x^11)
```

Mupad [**B**]
time = 5.15, size = 253, normalized size = 0.90

$$\frac{b^{2/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{14/3}}-\frac{\frac{c}{11\,a}-\frac{x^9\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{2\,a^4}+\frac{x^3\left(a\,d-b\,c\right)}{8\,a^2}+\frac{x^6\left(e\,a^2-d\,a\,b+c\,b^2\right)}{5\,a^3}}{x^{11}}+\frac{b^{2/3}\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{14/3}}-\frac{b^{2/3}\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x)
```

```
[Out] (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a
^(14/3)) - (c/(11*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(2*a^4) +
```

```
(x^3*(a*d - b*c))/(8*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(5*a^3))/x^11 + (
b^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1
/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(14/3)) - (b^(2/3)*log(3^(1/2
)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f
 - a*b^2*d + a^2*b*e))/(3*a^(14/3))
```

3.248 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx$

Optimal. Leaf size=313

$$-\frac{c}{13ax^{13}}+\frac{bc-ad}{10a^2x^{10}}-\frac{b^2c-abd+a^2e}{7a^3x^7}+\frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{a^5x}+\frac{b^{4/3}(b^3c-ab^2}{}$$

```
[Out] -1/13*c/a/x^13+1/10*(-a*d+b*c)/a^2/x^10+1/7*(-a^2*e+a*b*d-b^2*c)/a^3/x^7+1/
4*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^4-b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a
^5/x+1/3*b^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(16
/3)-1/6*b^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/a^(16/3)+1/3*b^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(16/3)*3^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 298, 31, 648, 631, 210, 642}

$$\frac{bc-ad}{10a^2x^{10}}-\frac{a^2e-abd+b^2c}{7a^3x^7}+\frac{b^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{16/3}}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{16/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{16/3}}-\frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{a^5x}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{4a^4x^4}-\frac{c}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]
```

```
[Out] -1/13*c/(a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2*e)/(7*a
^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4) - (b*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f))/(a^5*x) + (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(16/3))
+ (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3
*a^(16/3)) - (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx = \int \left(\frac{c}{ax^{14}} + \frac{-bc+ad}{a^2x^{11}} + \frac{b^2c-abd+a^2e}{a^3x^8} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^5} - \frac{b(-b^3 \right.$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b(b^3c-a}{}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b(b^3c-a}{}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b(b^3c-a}{}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b(b^3c-a}{}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b(b^3c-a}{}$$

Mathematica [A]
time = 0.07, size = 308, normalized size = 0.98

$$-\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} + \frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^5x} + \frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{16/3}} + \frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{16/3}} + \frac{b^{4/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]
```

```
[Out] -1/13*c/(a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2*e)/(7*a
^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4) + (b*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f))/(a^5*x) + (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(16/3)) + (
b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^
(16/3)) + (b^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

Maple [A]
time = 0.38, size = 239, normalized size = 0.76

method	result

default	$\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)b^{2}}{a^{5}}-\frac{c}{13a\,x^{13}}-\frac{ad-b}{10a^{2}x}$
risch	$\frac{\frac{\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)b\,x^{12}}{a^{5}}-\frac{\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{9}}{4a^{4}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{7a^{3}}-\frac{(ad-bc)x^{3}}{10a^{2}}-\frac{c}{13a}}{x^{13}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{16}\_Z^{3}+a^{9}b^{4}f^{3}-3a^{8}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b^2-1/13*c/a/x^13-1/10*(a*d-b*c)/a
^2/x^10-1/7*(a^2*e-a*b*d+b^2*c)/a^3/x^7-1/4*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a
^4/x^4+(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b/x
```

Maxima [A]
time = 0.52, size = 313, normalized size = 1.00

$$-\frac{\sqrt{3}\,(b^4c-ab^3d-a^3bf+a^2b^2e)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(b^4c-ab^3d-a^3bf+a^2b^2e)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(b^4c-ab^3d-a^3bf+a^2b^2e)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{1820\,(b^4c-ab^3d-a^3bf+a^2b^2e)x^{12}-455\,(ab^3c-a^2b^2d-a^4f+a^3be)x^9+260\,(a^2b^2c-a^3bd+a^4e)x^6+140\,a^4c-182\,(a^3bc-a^4d)x^3}{1820\,a^5x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(1/3)) - 1/6*(b^4*c - a*b^3*d - a^
3*b*f + a^2*b^2*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(1/3))
 + 1/3*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*log(x + (a/b)^(1/3))/(a^5*(a
/b)^(1/3)) - 1/1820*(1820*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*x^12 - 45
5*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*x^9 + 260*(a^2*b^2*c - a^3*b*d +
a^4*e)*x^6 + 140*a^4*c - 182*(a^3*b*c - a^4*d)*x^3)/(a^5*x^13)
```

Fricas [A]
time = 0.40, size = 317, normalized size = 1.01

$$-\frac{1820\,\sqrt{3}\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{13}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)-910\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{13}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+1820\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{13}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)+5460\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{12}-1365\,(ab^3c-a^2b^2d+a^3be-a^4f)x^9+780\,(a^2b^2c-a^3bd+a^4e)x^6+420\,a^4c-546\,(a^3bc-a^4d)x^3}{5460\,a^5x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/5460*(1820*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(
1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*(b^4*c - a*b^3*
d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a
*(-b/a)^(1/3)) + 1820*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(
1/3)*log(b*x + a*(-b/a)^(2/3)) + 5460*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*
f)*x^12 - 1365*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 780*(a^2*b^2*c
 - a^3*b*d + a^4*e)*x^6 + 420*a^4*c - 546*(a^3*b*c - a^4*d)*x^3)/(a^5*x^13)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.38, size = 419, normalized size = 1.34

$$\frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}b^3c-(-ab^2)^{\frac{2}{3}}ab^2d-(-ab^2)^{\frac{2}{3}}a^3f+(-ab^2)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^6}+\frac{\left(b^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^2f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^6}-\frac{\left((-ab^2)^{\frac{2}{3}}b^3c-(-ab^2)^{\frac{2}{3}}ab^2d-(-ab^2)^{\frac{2}{3}}a^3f+(-ab^2)^{\frac{2}{3}}a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^6}-\frac{1820\,b^4cx^{12}-1820\,ab^3dx^{12}-1820\,a^3bfx^{12}+1820\,a^2b^2x^{12}e-455\,ab^3cx^9+455\,a^2b^2dx^9+455\,a^4fx^9-455\,a^3bx^9e+260\,a^2b^2cx^6-260\,a^3bdx^6+260\,a^4x^6e-182\,a^3bcx^3+182\,a^4dx^3+140\,a^4c}{1820\,a^5x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)
*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/a^6 + 1/3*(b^5*c*(-a/b)^(1/3) - a*b^4*d*(-a/b)^(1/3) - a^3*b^2*
f*(-a/b)^(1/3) + a^2*b^3*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1
/3)))/a^6 - 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(
2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/a^6 - 1/1820*(1820*b^4*c*x^12 - 1820*a*b^3*d*x^12 - 1820*a^3*b*f*x^12 +
1820*a^2*b^2*x^12*e - 455*a*b^3*c*x^9 + 455*a^2*b^2*d*x^9 + 455*a^4*f*x^9 -
 455*a^3*b*x^9*e + 260*a^2*b^2*c*x^6 - 260*a^3*b*d*x^6 + 260*a^4*x^6*e - 18
2*a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)/(a^5*x^13)
```

Mupad [B]
time = 5.23, size = 286, normalized size = 0.91

$$\frac{b^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{16/3}}-\frac{\frac{c}{13\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{4\,a^4}+\frac{x^3\,\left(a\,d-b\,c\right)}{10\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{7\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^5}}{x^{13}}-\frac{b^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{16/3}}+\frac{b^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x)
```

```
[Out] (b^(4/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a
^(16/3)) - (c/(13*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(4*a^4) +
(x^3*(a*d - b*c))/(10*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(7*a^3) + (b*x^1
2*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^5)/x^13 - (b^(4/3)*log(3^(1/2)*a^(
1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*
b^2*d + a^2*b*e))/(3*a^(16/3)) + (b^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3
)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/
(3*a^(16/3))
```

3.249 $\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx$

Optimal. Leaf size=315

$$-\frac{c}{14ax^{14}}+\frac{bc-ad}{11a^2x^{11}}-\frac{b^2c-abd+a^2e}{8a^3x^8}+\frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{2a^5x^2}+\frac{b^{5/3}(b^3c-ab^2c}{}$$

```
[Out] -1/14*c/a/x^14+1/11*(-a*d+b*c)/a^2/x^11+1/8*(-a^2*e+a*b*d-b^2*c)/a^3/x^8+1/
5*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^5-1/2*b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)/a^5/x^2-1/3*b^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)
/a^(17/3)+1/6*b^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(17/3)+1/3*b^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(17/3)*3^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 206, 31, 648, 631, 210, 642}

$$\frac{bc-ad}{11a^2x^{11}}-\frac{a^2e-abd+b^2c}{8a^3x^8}+\frac{b^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}\,a^{17/3}}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{17/3}}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{17/3}}-\frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{2a^5x^2}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5}-\frac{c}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]
```

```
[Out] -1/14*c/(a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2*e)/(8*a
^3*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f))/(2*a^5*x^2) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(17/
3)) - (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x]
)/(3*a^(17/3)) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^{15}} + \frac{-bc+ad}{a^2x^{12}} + \frac{b^2c-abd+a^2e}{a^3x^9} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^6} - \frac{b(-b^3}{}\right.$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b(b^3c-a}{}$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b(b^3c-a}{}$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b(b^3c-a}{}$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b(b^3c-a}{}$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b(b^3c-a}{}$$

Mathematica [A]
time = 0.07, size = 311, normalized size = 0.99

$$-\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} + \frac{b(-b^3c+ab^2d-a^2be+a^3f)}{2a^5x^2} + \frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{17/3}} + \frac{b^{5/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{17/3}} + \frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]
```

```
[Out] -1/14*c/(a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2*e)/(8*a
^3*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) + (b*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f))/(2*a^5*x^2) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(17/3))
 + (b^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x]
)/(3*a^(17/3)) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(17/3))
```

Maple [A]
time = 0.38, size = 240, normalized size = 0.76

method	result

default	$\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)b^{2}}{a^{5}}-\frac{c}{14a\,x^{14}}-\frac{ad-bc}{11a^{2}x^{11}}$
risch	$\frac{\frac{\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)b\,x^{12}}{2a^{5}}-\frac{\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{9}}{5a^{4}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{8a^{3}}-\frac{(ad-bc)x^{3}}{11a^{2}}-\frac{c}{14a}}{x^{14}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{17}\_Z^{3}-a^{9}b^{5}f^{3}+3a^{8}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b^2-1/14*c/a/x^14-1/11*(a*d-b*c)/a^
2/x^11-1/8*(a^2*e-a*b*d+b^2*c)/a^3/x^8-1/5*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^
4/x^5+1/2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b/x^2
```

Maxima [A]

time = 0.51, size = 313, normalized size = 0.99

$$-\frac{\sqrt{3}\left(b^{4}c-ab^{3}d-a^{3}bf+a^{2}b^{2}e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^{4}c-ab^{3}d-a^{3}bf+a^{2}b^{2}e\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^{4}c-ab^{3}d-a^{3}bf+a^{2}b^{2}e\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{1540\left(b^{4}c-ab^{3}d-a^{3}bf+a^{2}b^{2}e\right)x^{12}-616\left(ab^{3}c-a^{2}b^{2}d-a^{4}f+a^{3}be\right)x^{9}+385\left(a^{2}b^{2}c-a^{3}bd+a^{4}e\right)x^{6}+220\,a^{4}c-280\left(a^{3}bc-a^{4}d\right)x^{3}}{3080\,a^{5}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(2/3)) + 1/6*(b^4*c - a*b^3*d - a^
3*b*f + a^2*b^2*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(2/3))
 - 1/3*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*log(x + (a/b)^(1/3))/(a^5*(a
/b)^(2/3)) - 1/3080*(1540*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*x^12 - 61
6*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*x^9 + 385*(a^2*b^2*c - a^3*b*d +
a^4*e)*x^6 + 220*a^4*c - 280*(a^3*b*c - a^4*d)*x^3)/(a^5*x^14)
```

Fricas [A]

time = 0.40, size = 335, normalized size = 1.06

$$-\frac{3080\sqrt{3}\left(b^{4}c-ab^{3}d+a^{2}b^{2}e-a^{3}bf\right)x^{14}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-1540\left(b^{4}c-ab^{3}d+a^{2}b^{2}e-a^{3}bf\right)x^{14}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}-abx\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)+3080\left(b^{4}c-ab^{3}d+a^{2}b^{2}e-a^{3}bf\right)x^{14}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\right)+4620\left(b^{4}c-ab^{3}d+a^{2}b^{2}e-a^{3}bf\right)x^{12}-1848\left(ab^{3}c-a^{2}b^{2}d+a^{3}be-a^{4}f\right)x^{9}+1155\left(a^{2}b^{2}c-a^{3}bd+a^{4}e\right)x^{6}+660\,a^{4}c-840\left(a^{3}bc-a^{4}d\right)x^{3}}{9240\,a^{5}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/9240*(3080*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2
)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 1540*(b
^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*
b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 3080*(b^4*c - a*b^3*d + a^2*b^
2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 4620*(b^
4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 1848*(a*b^3*c - a^2*b^2*d + a^3
*b*e - a^4*f)*x^9 + 1155*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 660*a^4*c - 84
0*(a^3*b*c - a^4*d)*x^3)/(a^5*x^14)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**15/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.91, size = 393, normalized size = 1.25

$$-\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{4}c-\left(-ab^{2}\right)^{\frac{1}{3}}ab^{3}d-\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}bf+\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}b^{2}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^{6}}+\frac{\left(b^{5}c-ab^{4}d-a^{3}b^{2}f+a^{2}b^{3}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^{6}}-\frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{4}c-\left(-ab^{2}\right)^{\frac{1}{3}}ab^{3}d-\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}bf+\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}b^{2}e\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^{6}}-\frac{1540\,b^{4}cx^{12}-1540\,ab^{3}dx^{12}-1540\,a^{3}bfx^{12}+1540\,a^{2}b^{2}x^{12}e-616\,ab^{3}cx^{9}+616\,a^{2}b^{2}dx^{9}+616\,a^{4}fx^{9}-616\,a^{3}bx^{9}e+385\,a^{2}b^{2}cx^{6}-385\,a^{3}bdx^{6}+385\,a^{4}x^{6}e-280\,a^{3}bcx^{3}+280\,a^{4}dx^{3}+220\,a^{4}c}{3080\,a^{5}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*b^4*c - (-a*b^2)^(1/3)*a*b^3*d - (-a*b^2)^(1/3
)*a^3*b*f + (-a*b^2)^(1/3)*a^2*b^2*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/a^6 + 1/3*(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 - 1/6*((-a*b^2)^(1/3)*b^4*c - (-a*b^2
)^(1/3)*a*b^3*d - (-a*b^2)^(1/3)*a^3*b*f + (-a*b^2)^(1/3)*a^2*b^2*e)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6 - 1/3080*(1540*b^4*c*x^12 - 1540*a*b
^3*d*x^12 - 1540*a^3*b*f*x^12 + 1540*a^2*b^2*x^12*e - 616*a*b^3*c*x^9 + 616
*a^2*b^2*d*x^9 + 616*a^4*f*x^9 - 616*a^3*b*x^9*e + 385*a^2*b^2*c*x^6 - 385*
a^3*b*d*x^6 + 385*a^4*x^6*e - 280*a^3*b*c*x^3 + 280*a^4*d*x^3 + 220*a^4*c)/
(a^5*x^14)
```

Mupad [**B**]
time = 5.17, size = 287, normalized size = 0.91

$$-\frac{\frac{c}{14\,a}-\frac{x^{9}\,\left(-f\,a^{3}+e\,a^{2}\,b-d\,a\,b^{2}+c\,b^{3}\right)}{5\,a^{4}}+\frac{x^{3}\,\left(a\,d-b\,c\right)}{11\,a^{2}}+\frac{x^{6}\,\left(e\,a^{2}-d\,a\,b+c\,b^{2}\right)}{8\,a^{3}}+\frac{b\,x^{12}\,\left(-f\,a^{3}+e\,a^{2}\,b-d\,a\,b^{2}+c\,b^{3}\right)}{2\,a^{5}}}{x^{14}}-\frac{b^{5/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^{3}+e\,a^{2}\,b-d\,a\,b^{2}+c\,b^{3}\right)}{3\,a^{17/3}}-\frac{b^{5/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^{3}+e\,a^{2}\,b-d\,a\,b^{2}+c\,b^{3}\right)}{3\,a^{17/3}}+\frac{b^{5/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^{3}+e\,a^{2}\,b-d\,a\,b^{2}+c\,b^{3}\right)}{3\,a^{17/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x)
```

```
[Out] (b^(5/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(17/3)) - (b^(5/3)*log(b^(1/
3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(17/3)) - (b^(5/3
)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(17/3)) - (c/(14*a) - (x^9*(b^3*c -
 a^3*f - a*b^2*d + a^2*b*e))/(5*a^4) + (x^3*(a*d - b*c))/(11*a^2) + (x^6*(b
^2*c + a^2*e - a*b*d))/(8*a^3) + (b*x^12*(b^3*c - a^3*f - a*b^2*d + a^2*b*e
))/(2*a^5))/x^14
```

3.250 $\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx$

Optimal. Leaf size=351

$$-\frac{c}{16ax^{16}}+\frac{bc-ad}{13a^2x^{13}}-\frac{b^2c-abd+a^2e}{10a^3x^{10}}+\frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{4a^5x^4}+\frac{b^2(b^3c-ab^2d}{a^6}$$

```
[Out] -1/16*c/a/x^16+1/13*(-a*d+b*c)/a^2/x^13+1/10*(-a^2*e+a*b*d-b^2*c)/a^3/x^10+
1/7*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^7-1/4*b*(-a^3*f+a^2*b*e-a*b^2*d+b^
3*c)/a^5/x^4+b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^6/x-1/3*b^(7/3)*(-a^3*f+a
^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(19/3)+1/6*b^(7/3)*(-a^3*f+a^
2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(19/3)-1/3
*b^(7/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/a^(19/3)*3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 298, 31, 648, 631, 210, 642}

$$\frac{bc-ad}{13a^2x^{13}}-\frac{a^2e-abd+b^2c}{10a^3x^{10}}-\frac{b^{7/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{19/3}}+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{19/3}}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{19/3}}+\frac{b^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6x}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^5x^4}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{7a^4x^7}-\frac{c}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]
```

```
[Out] -1/16*c/(a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2*e)/(10*
a^3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^7) - (b*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f))/(4*a^5*x^4) + (b^2*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f))/(a^6*x) - (b^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(19/3)) - (b^(7/3)*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3)) + (b^(
7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(6*a^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^{17}} + \frac{-bc+ad}{a^2x^{14}} + \frac{b^2c-abd+a^2e}{a^3x^{11}} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^8} - \frac{b(-b^3}{}\right.$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b(b^3c-a}{}$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b(b^3c-a}{}$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b(b^3c-a}{}$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b(b^3c-a}{}$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b(b^3c-a}{}$$

Mathematica [A]
time = 0.08, size = 346, normalized size = 0.99

$$-\frac{c}{16ax^{16}}+\frac{bc-ad}{13a^2x^{13}}-\frac{b^2c-abd+a^2e}{10a^3x^{10}}+\frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7}+\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{4a^5x^4}+\frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x}+\frac{b^{7/3}(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{19/3}}+\frac{b^{7/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{19/3}}+\frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]
```

```
[Out] -1/16*c/(a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2*e)/(10*
a^3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^7) + (b*(-(b^3*c)
+ a*b^2*d - a^2*b*e + a^3*f))/(4*a^5*x^4) + (b^2*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f))/(a^6*x) + (b^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan
[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(19/3)) + (b^(7/3)*(-(b^3
*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3)) + (
b^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(6*a^(19/3))
```

Maple [A]
time = 0.38, size = 277, normalized size = 0.79

method	result

default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{3}\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)}{a^{6}}-\frac{c}{16a\,x^{16}}-\frac{ad}{13a}$
risch	$\frac{-\frac{c}{16a}-\frac{(ad-bc)x^{3}}{13a^{2}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{10a^{3}}-\frac{\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{9}}{7a^{4}}+\frac{\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)b\,x^{12}}{4a^{5}}-\frac{\left(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c\right)b^{2}x^{15}}{a^{6}}}{x^{16}}+\frac{(}{-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))*b^3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^6-1/16*c/a/x^16-1/13*(a*d-b*c)/
a^2/x^13-1/10*(a^2*e-a*b*d+b^2*c)/a^3/x^10-1/7*(a^3*f-a^2*b*e+a*b^2*d-b^3*c
)/a^4/x^7-(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^6*b^2/x+1/4*(a^3*f-a^2*b*e+a*b^2*
d-b^3*c)/a^5*b/x^4
```

<u>**Maxima**</u> [A]

time = 0.50, size = 360, normalized size = 1.03

$$\frac{\sqrt{3}\,(b^5c-ab^4d-a^3b^2f+a^2b^3e)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(b^5c-ab^4d-a^3b^2f+a^2b^3e)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(b^5c-ab^4d-a^3b^2f+a^2b^3e)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7280\,(b^5c-ab^4d-a^3b^2f+a^2b^3e)x^{15}-1820\,(ab^4c-a^2b^3d-a^4bf+a^3b^2e)x^{12}+1040\,(a^2b^3c-a^3b^2d-a^5f+a^4be)x^9-728\,(a^3b^2c-a^4bd+a^5e)x^6-455\,a^5c+560\,(a^4bc-a^5d)x^3}{7280\,a^6x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^6*(a/b)^(1/3)) + 1/6*(b^5*c - a*b^4*d - a
^3*b^2*f + a^2*b^3*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(1/
3)) - 1/3*(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*log(x + (a/b)^(1/3))/(a
^6*(a/b)^(1/3)) + 1/7280*(7280*(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*x^
15 - 1820*(a*b^4*c - a^2*b^3*d - a^4*b*f + a^3*b^2*e)*x^12 + 1040*(a^2*b^3*
c - a^3*b^2*d - a^5*f + a^4*b*e)*x^9 - 728*(a^3*b^2*c - a^4*b*d + a^5*e)*x^
6 - 455*a^5*c + 560*(a^4*b*c - a^5*d)*x^3)/(a^6*x^16)
```

<u>**Fricas**</u> [A]

time = 0.38, size = 355, normalized size = 1.01

$$\frac{7280\,\sqrt{3}\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{16}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+3640\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{16}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-7280\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{16}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)+21840\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{15}-5460\,(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^{12}+3120\,(a^2b^3c-a^3b^2d+a^4be-a^5f)x^9-2184\,(a^3b^2c-a^4bd+a^5e)x^6-1365\,a^5c+1680\,(a^4bc-a^5d)x^3}{21840\,a^6x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/21840*(7280*sqrt(3)*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^
(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 3640*(b^5*c - a*b^4
*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) +
a*(b/a)^(1/3)) - 7280*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^
(1/3)*log(b*x + a*(b/a)^(2/3)) + 21840*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b
^2*f)*x^15 - 5460*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 3120*(
a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^9 - 2184*(a^3*b^2*c - a^4*b*d +
a^5*e)*x^6 - 1365*a^5*c + 1680*(a^4*b*c - a^5*d)*x^3)/(a^6*x^16)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**17/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.99, size = 474, normalized size = 1.35

$$-\frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}b^4c-(-ab^2)^{\frac{2}{3}}ab^3d-(-ab^2)^{\frac{2}{3}}a^3bf+(-ab^2)^{\frac{2}{3}}a^2b^2e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^7}-\frac{\left(b^6c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^5d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^7}+\frac{\left((-ab^2)^{\frac{2}{3}}b^4c-(-ab^2)^{\frac{2}{3}}ab^3d-(-ab^2)^{\frac{2}{3}}a^3bf+(-ab^2)^{\frac{2}{3}}a^2b^2e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^7}+\frac{7280\,b^5cx^{15}-7280\,ab^4dx^{15}-7280\,a^3b^2fx^{15}+7280\,a^2b^3x^{15}e-1820\,ab^4cx^{12}+1820\,a^2b^3dx^{12}+1820\,a^4bfx^{12}-1820\,a^3b^2x^{12}e+1040\,a^2b^3cx^9-1040\,a^3b^2dx^9-1040\,a^5fx^9+1040\,a^4bx^9e-728\,a^3b^2cx^6+728\,a^4bdx^6-728\,a^5x^6e+560\,a^4bcx^3-560\,a^5dx^3-455\,a^5c}{7280\,a^6x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*b^4*c - (-a*b^2)^(2/3)*a*b^3*d - (-a*b^2)^(2/3
)*a^3*b*f + (-a*b^2)^(2/3)*a^2*b^2*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/a^7 - 1/3*(b^6*c*(-a/b)^(1/3) - a*b^5*d*(-a/b)^(1/3) - a^3
*b^3*f*(-a/b)^(1/3) + a^2*b^4*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/a^7 + 1/6*((-a*b^2)^(2/3)*b^4*c - (-a*b^2)^(2/3)*a*b^3*d - (-a*b
^2)^(2/3)*a^3*b*f + (-a*b^2)^(2/3)*a^2*b^2*e)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/a^7 + 1/7280*(7280*b^5*c*x^15 - 7280*a*b^4*d*x^15 - 7280*a^3*b^
2*f*x^15 + 7280*a^2*b^3*x^15*e - 1820*a*b^4*c*x^12 + 1820*a^2*b^3*d*x^12 +
1820*a^4*b*f*x^12 - 1820*a^3*b^2*x^12*e + 1040*a^2*b^3*c*x^9 - 1040*a^3*b^2
*d*x^9 - 1040*a^5*f*x^9 + 1040*a^4*b*x^9*e - 728*a^3*b^2*c*x^6 + 728*a^4*b*
d*x^6 - 728*a^5*x^6*e + 560*a^4*b*c*x^3 - 560*a^5*d*x^3 - 455*a^5*c)/(a^6*x
^16)
```

Mupad [B]
time = 5.16, size = 323, normalized size = 0.92

$$-\frac{\frac{c}{16\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{7\,a^4}+\frac{x^3\,\left(a\,d-b\,c\right)}{13\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{10\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{4\,a^5}-\frac{b^2\,x^{15}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^6}}{x^{16}}-\frac{b^{7/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{19/3}}+\frac{b^{7/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{19/3}}-\frac{b^{7/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{19/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x)
```

```
[Out] (b^(7/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(19/3)) - (b^(7/3)*log(b^(1/
3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(19/3)) - (c/(16*
a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(7*a^4) + (x^3*(a*d - b*c))/
(13*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(10*a^3) + (b*x^12*(b^3*c - a^3*f
- a*b^2*d + a^2*b*e))/(4*a^5) - (b^2*x^15*(b^3*c - a^3*f - a*b^2*d + a^2*b*
e))/a^6)/x^16 - (b^(7/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(19/3))
```

3.251 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=220

$$-\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x^3}{3b^6}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^6}{6b^5}+\frac{(b^2d-2abe+3a^2f)\,x^9}{9b^4}+\frac{(be-2af)}{12b^3}$$

```
[Out] -1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*x^3/b^6+1/6*(-4*a^3*f+3*a^2*b
*e-2*a*b^2*d+b^3*c)*x^6/b^5+1/9*(3*a^2*f-2*a*b*e+b^2*d)*x^9/b^4+1/12*(-2*a*
f+b*e)*x^12/b^3+1/15*f*x^15/b^2+1/3*a^3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b^7/
(b*x^3+a)+1/3*a^2*(-6*a^3*f+5*a^2*b*e-4*a*b^2*d+3*b^3*c)*ln(b*x^3+a)/b^7
```

Rubi [A]
time = 0.23, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{x^9(3a^2f-2abe+b^2d)}{9b^4}+\frac{a^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^7(a+bx^3)}+\frac{a^2\log(a+bx^3)(-6a^3f+5a^2be-4ab^2d+3b^3c)}{3b^7}-\frac{ax^3(-5a^3f+4a^2be-3ab^2d+2b^3c)}{3b^6}+\frac{x^6(-4a^3f+3a^2be-2ab^2d+b^3c)}{6b^5}+\frac{x^{12}(be-2af)}{12b^3}+\frac{fx^{15}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x^3)/b^6 + ((b^3*c - 2*
a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6)/(6*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*
x^9)/(9*b^4) + ((b*e - 2*a*f)*x^12)/(12*b^3) + (f*x^15)/(15*b^2) + (a^3*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^7*(a + b*x^3)) + (a^2*(3*b^3*c - 4*a
*b^2*d + 5*a^2*b*e - 6*a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-2b^3c+3ab^2d-4a^2be+5a^3f)}{b^6} + \frac{(b^3c-2ab^2d+3a^2be}{b^5}\right.\right.$$
$$= -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x^3}{3b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^6}{6b^5}$$

Mathematica [A]
time = 0.09, size = 205, normalized size = 0.93

$$\frac{60ab(-2b^3c+3ab^2d-4a^2be+5a^3f)\,x^3+30b^2(b^3c-2ab^2d+3a^2be-4a^3f)\,x^6+20b^3(b^2d-2abe+3a^2f)\,x^9+15b^4(be-2af)x^{12}+12b^5fx^{15}-\frac{60a^3(-b^3c+ab^2d-a^2be+a^3f)}{a+bx^3}+60a^2(3b^3c-4ab^2d+5a^2be-6a^3f)\log(a+bx^3)}{180b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (60*a*b*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x^3 + 30*b^2*(b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6 + 20*b^3*(b^2*d - 2*a*b*e + 3*a^2*f)*x
^9 + 15*b^4*(b*e - 2*a*f)*x^12 + 12*b^5*f*x^15 - (60*a^3*(-(b^3*c) + a*b^2*
d - a^2*b*e + a^3*f))/(a + b*x^3) + 60*a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e
 - 6*a^3*f)*Log[a + b*x^3])/(180*b^7)
```

Maple [A]
time = 0.36, size = 217, normalized size = 0.99

method	result
default	$\frac{\frac{f\,x^{15}b^4}{15}+\frac{\left(-2a\,b^3f+b^4e\right)x^{12}}{12}+\frac{\left(3a^2b^2f-2a\,b^3e+d\,b^4\right)x^9}{9}+\frac{\left(-4a^3bf+3a^2e\,b^2-2ad\,b^3+c\,b^4\right)x^6}{6}+\frac{\left(5a^4f-4a^3be+3a^2b^2d-2a\,b^3c\right)x^3}{3}}{b^6} - \frac{a^2(\ldots}{}$
norman	$\frac{-\frac{a\left(6f\,a^5-5e\,a^4b+4d\,a^3b^2-3a^2c\,b^3\right)}{3b^7}+\frac{f\,x^{18}}{15b}-\frac{(6af-5be)x^{15}}{60b^2}+\frac{\left(6a^2f-5abe+4b^2d\right)x^{12}}{36b^3}-\frac{\left(6a^3f-5a^2be+4a\,b^2d-3b^3c\right)x^9}{18b^4}+\frac{a\left(6a^3f-5a^2be+4a\ldots\right.}{6b^5}}{b\,x^3+a}$
risch	$\frac{f\,x^{15}}{15b^2}-\frac{af\,x^{12}}{6b^3}+\frac{e\,x^{12}}{12b^2}+\frac{a^2f\,x^9}{3b^4}-\frac{2ae\,x^9}{9b^3}+\frac{d\,x^9}{9b^2}-\frac{2a^3f\,x^6}{3b^5}+\frac{a^2e\,x^6}{2b^4}-\frac{ad\,x^6}{3b^3}+\frac{c\,x^6}{6b^2}+\frac{5a^4f\,x^3}{3b^6}-\frac{4a^3e\,x^3}{3b^5}+\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/15*f*x^15*b^4+1/12*(-2*a*b^3*f+b^4*e)*x^12+1/9*(3*a^2*b^2*f-2*a*b^
3*e+b^4*d)*x^9+1/6*(-4*a^3*b*f+3*a^2*b^2*e-2*a*b^3*d+b^4*c)*x^6+1/3*(5*a^4*
f-4*a^3*b*e+3*a^2*b^2*d-2*a*b^3*c)*x^3)-1/3*a^2/b^6*(a*(a^3*f-a^2*b*e+a*b^2
*d-b^3*c)/b/(b*x^3+a)+(6*a^3*f-5*a^2*b*e+4*a*b^2*d-3*b^3*c)/b*ln(b*x^3+a))
```

Maxima [A]
time = 0.27, size = 229, normalized size = 1.04

$$\frac{a^{3} b^{3} c - a^{4} b^{2} d - a^{6} f + a^{5} b e}{3 \left(b^{8} x^{3} + a b^{7}\right)} + \frac{12 \, b^{4} f x^{15} - 15 \left(2 \, a b^{3} f - b^{4} e\right) x^{12} + 20 \left(b^{4} d + 3 \, a^{2} b^{2} f - 2 \, a b^{3} e\right) x^{9} + 30 \left(b^{4} c - 2 \, a b^{3} d - 4 \, a^{3} b f + 3 \, a^{2} b^{2} e\right) x^{6} - 60 \left(2 \, a b^{3} c - 3 \, a^{2} b^{2} d - 5 \, a^{4} f + 4 \, a^{3} b e\right) x^{3}}{180 \, b^{6}} + \frac{\left(3 \, a^{2} b^{3} c - 4 \, a^{3} b^{2} d - 6 \, a^{5} f + 5 \, a^{4} b e\right) \log\left(b x^{3} + a\right)}{3 \, b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)/(b^8*x^3 + a*b^7) + 1/180*(12
*b^4*f*x^15 - 15*(2*a*b^3*f - b^4*e)*x^12 + 20*(b^4*d + 3*a^2*b^2*f - 2*a*b
^3*e)*x^9 + 30*(b^4*c - 2*a*b^3*d - 4*a^3*b*f + 3*a^2*b^2*e)*x^6 - 60*(2*a*
b^3*c - 3*a^2*b^2*d - 5*a^4*f + 4*a^3*b*e)*x^3)/b^6 + 1/3*(3*a^2*b^3*c - 4*
a^3*b^2*d - 6*a^5*f + 5*a^4*b*e)*log(b*x^3 + a)/b^7
```

Fricas [A]
time = 0.38, size = 303, normalized size = 1.38

$$\frac{12 \, b^{6} f x^{18} + 3 \left(5 \, b^{6} e - 6 \, a b^{5} f\right) x^{15} + 5 \left(4 \, b^{6} d - 5 \, a b^{5} e + 6 \, a^{2} b^{4} f\right) x^{12} + 10 \left(3 \, b^{6} c - 4 \, a b^{5} d + 5 \, a^{2} b^{4} e - 6 \, a^{3} b^{3} f\right) x^{9} + 60 \, a^{3} b^{3} c - 60 \, a^{4} b^{2} d + 60 \, a^{5} b e - 60 \, a^{6} f - 30 \left(3 \, a b^{5} c - 4 \, a^{2} b^{4} d + 5 \, a^{3} b^{3} e - 6 \, a^{4} b^{2} f\right) x^{6} - 60 \left(2 \, a^{2} b^{4} c - 3 \, a^{3} b^{3} d + 4 \, a^{4} b^{2} e - 5 \, a^{5} b f\right) x^{3} + 60 \left(3 \, a^{3} b^{3} c - 4 \, a^{4} b^{2} d + 5 \, a^{5} b e - 6 \, a^{6} f + \left(3 \, a^{2} b^{4} c - 4 \, a^{3} b^{3} d + 5 \, a^{4} b^{2} e - 6 \, a^{5} b f\right) x^{3}\right) \log\left(b x^{3} + a\right)}{180 \left(b^{8} x^{3} + a b^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^6*f*x^18 + 3*(5*b^6*e - 6*a*b^5*f)*x^15 + 5*(4*b^6*d - 5*a*b^5*
e + 6*a^2*b^4*f)*x^12 + 10*(3*b^6*c - 4*a*b^5*d + 5*a^2*b^4*e - 6*a^3*b^3*f
)*x^9 + 60*a^3*b^3*c - 60*a^4*b^2*d + 60*a^5*b*e - 60*a^6*f - 30*(3*a*b^5*c
 - 4*a^2*b^4*d + 5*a^3*b^3*e - 6*a^4*b^2*f)*x^6 - 60*(2*a^2*b^4*c - 3*a^3*b
^3*d + 4*a^4*b^2*e - 5*a^5*b*f)*x^3 + 60*(3*a^3*b^3*c - 4*a^4*b^2*d + 5*a^5
*b*e - 6*a^6*f + (3*a^2*b^4*c - 4*a^3*b^3*d + 5*a^4*b^2*e - 6*a^5*b*f)*x^3)
*log(b*x^3 + a))/(b^8*x^3 + a*b^7)
```

Sympy [A]
time = 15.28, size = 236, normalized size = 1.07

$$- \frac{a^{2} \cdot \left(6 a^{3} f - 5 a^{2} b e + 4 a b^{2} d - 3 b^{3} c\right) \log{\left(a + b x^{3} \right)}}{3 b^{7}} + x^{12} \left(- \frac{a f}{6 b^{3}} + \frac{e}{12 b^{2}}\right) + x^{9} \left(\frac{a^{2} f}{3 b^{4}} - \frac{2 a e}{9 b^{3}} + \frac{d}{9 b^{2}}\right) + x^{6} \left(- \frac{2 a^{3} f}{3 b^{5}} + \frac{a^{2} e}{2 b^{4}} - \frac{a d}{3 b^{3}} + \frac{c}{6 b^{2}}\right) + x^{3} \cdot \left(\frac{5 a^{4} f}{3 b^{6}} - \frac{4 a^{3} e}{3 b^{5}} + \frac{a^{2} d}{b^{4}} - \frac{2 a c}{3 b^{3}}\right) + \frac{- a^{6} f + a^{5} b e - a^{4} b^{2} d + a^{3} b^{3} c}{3 a b^{7} + 3 b^{8} x^{3}} + \frac{f x^{15}}{15 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -a**2*(6*a**3*f - 5*a**2*b*e + 4*a*b**2*d - 3*b**3*c)*log(a + b*x**3)/(3*b*
*7) + x**12*(-a*f/(6*b**3) + e/(12*b**2)) + x**9*(a**2*f/(3*b**4) - 2*a*e/(
9*b**3) + d/(9*b**2)) + x**6*(-2*a**3*f/(3*b**5) + a**2*e/(2*b**4) - a*d/(3
*b**3) + c/(6*b**2)) + x**3*(5*a**4*f/(3*b**6) - 4*a**3*e/(3*b**5) + a**2*d
/b**4 - 2*a*c/(3*b**3)) + (-a**6*f + a**5*b*e - a**4*b**2*d + a**3*b**3*c)/
(3*a*b**7 + 3*b**8*x**3) + f*x**15/(15*b**2)
```

Giac [A]
time = 0.96, size = 300, normalized size = 1.36

$$\frac{(3a^2b^3c - 4a^3b^2d - 6a^5f + 5a^4be)\log(|bx^3 + a|)}{3b^7} - \frac{3a^2b^4cx^3 - 4a^3b^3dx^3 - 6a^5bfx^3 + 5a^4b^2x^3e + 2a^3b^3c - 3a^4b^2d - 5a^6f + 4a^5be}{3(bx^3 + a)b^7} + \frac{12b^8fx^{15} - 30ab^7fx^{12} + 15b^8x^{12}e + 20b^8dx^9 + 60a^2b^6fx^9 - 40ab^7x^9e + 30b^8cx^6 - 60ab^7dx^6 - 120a^3b^5fx^6 + 90a^2b^6x^6e - 120ab^7cx^3 + 180a^2b^6dx^3 + 300a^4b^4fx^3 - 240a^3b^5x^3e}{180b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*b^3*c - 4*a^3*b^2*d - 6*a^5*f + 5*a^4*b*e)*log(abs(b*x^3 + a))/b
^7 - 1/3*(3*a^2*b^4*c*x^3 - 4*a^3*b^3*d*x^3 - 6*a^5*b*f*x^3 + 5*a^4*b^2*x^3
*e + 2*a^3*b^3*c - 3*a^4*b^2*d - 5*a^6*f + 4*a^5*b*e)/((b*x^3 + a)*b^7) + 1
/180*(12*b^8*f*x^15 - 30*a*b^7*f*x^12 + 15*b^8*x^12*e + 20*b^8*d*x^9 + 60*a
^2*b^6*f*x^9 - 40*a*b^7*x^9*e + 30*b^8*c*x^6 - 60*a*b^7*d*x^6 - 120*a^3*b^5
*f*x^6 + 90*a^2*b^6*x^6*e - 120*a*b^7*c*x^3 + 180*a^2*b^6*d*x^3 + 300*a^4*b
^4*f*x^3 - 240*a^3*b^5*x^3*e)/b^10
```

Mupad [B]
time = 4.99, size = 356, normalized size = 1.62

$$x^{12}\left(\frac{e}{12b^2} - \frac{af}{6b^3}\right) - x^3\left(\frac{2a\left(\frac{c}{b^2} - \frac{a^2\left(\frac{e}{b^2} - \frac{2af}{b^3}\right)}{b^2} + \frac{2a\left(\frac{a^2f}{b^4} - \frac{d}{b^2} + \frac{2a\left(\frac{e}{b^2} - \frac{2af}{b^3}\right)}{b}\right)}{b}\right)}{3b} - \frac{a^2\left(\frac{a^2f}{b^4} - \frac{d}{b^2} + \frac{2a\left(\frac{e}{b^2} - \frac{2af}{b^3}\right)}{b}\right)}{3b^2}\right) - x^9\left(\frac{a^2f}{9b^4} - \frac{d}{9b^2} + \frac{2a\left(\frac{e}{b^2} - \frac{2af}{b^3}\right)}{9b}\right) + x^6\left(\frac{c}{6b^2} - \frac{a^2\left(\frac{e}{b^2} - \frac{2af}{b^3}\right)}{6b^2} + \frac{a\left(\frac{a^2f}{b^4} - \frac{d}{b^2} + \frac{2a\left(\frac{e}{b^2} - \frac{2af}{b^3}\right)}{b}\right)}{3b}\right) - \frac{\ln(bx^3 + a)\,(6fa^5 - 5ea^4b + 4da^3b^2 - 3ca^2b^3)}{3b^7} + \frac{fx^{15}}{15b^2} - \frac{fa^6 - ea^5b + da^4b^2 - ca^3b^3}{3b\,(b^7x^3 + ab^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^12*(e/(12*b^2) - (a*f)/(6*b^3)) - x^3*((2*a*(c/b^2 - (a^2*(e/b^2 - (2*a*f
)/b^3))/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b
))/(3*b) - (a^2*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b
^2)) - x^9*((a^2*f)/(9*b^4) - d/(9*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(9*b)
) + x^6*(c/(6*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))/(6*b^2) + (a*((a^2*f)/b^4
- d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b)) - (log(a + b*x^3)*(6*a^5*f
 - 3*a^2*b^3*c + 4*a^3*b^2*d - 5*a^4*b*e))/(3*b^7) + (f*x^15)/(15*b^2) - (a
^6*f - a^3*b^3*c + a^4*b^2*d - a^5*b*e)/(3*b*(a*b^6 + b^7*x^3))
```

3.252 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=180

$$\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^3}{3b^5}+\frac{(b^2d-2abe+3a^2f)\,x^6}{6b^4}+\frac{(be-2af)x^9}{9b^3}+\frac{fx^{12}}{12b^2}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f}{3b^6\,(a+bx^3)}$$

```
[Out] 1/3*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*x^3/b^5+1/6*(3*a^2*f-2*a*b*e+b^2*d
)*x^6/b^4+1/9*(-2*a*f+b*e)*x^9/b^3+1/12*f*x^12/b^2-1/3*a^2*(-a^3*f+a^2*b*e-
a*b^2*d+b^3*c)/b^6/(b*x^3+a)-1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*l
n(b*x^3+a)/b^6
```

Rubi [A]
time = 0.18, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{x^6(3a^2f-2abe+b^2d)}{6b^4}-\frac{a^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^6\,(a+bx^3)}-\frac{a\log\left(a+bx^3\right)\left(-5a^3f+4a^2be-3ab^2d+2b^3c\right)}{3b^6}+\frac{x^3(-4a^3f+3a^2be-2ab^2d+b^3c)}{3b^5}+\frac{x^9(be-2af)}{9b^3}+\frac{fx^{12}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d - 2*a*b*e
 + 3*a^2*f)*x^6)/(6*b^4) + ((b*e - 2*a*f)*x^9)/(9*b^3) + (f*x^12)/(12*b^2)
- (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^6*(a + b*x^3)) - (a*(2*b^3
*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^3c-2ab^2d+3a^2be-4a^3f}{b^5} + \frac{(b^2d-2abe+3a^2f)\,x}{b^4} + \frac{(be\right.\right.$$
$$= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^3}{3b^5} + \frac{(b^2d-2abe+3a^2f)\,x^6}{6b^4} + \frac{(be-2af)x}{9b^3}$$

Mathematica [A]
time = 0.08, size = 167, normalized size = 0.93

$$\frac{12b(b^3c-2ab^2d+3a^2be-4a^3f)\,x^3+6b^2(b^2d-2abe+3a^2f)\,x^6+4b^3(be-2af)x^9+3b^4fx^{12}+\frac{12a^2(-b^3c+ab^2d-a^2be+a^3f)}{a+bx^3}+12a(-2b^3c+3ab^2d-4a^2be+5a^3f)\log(a+bx^3)}{36b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (12*b*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3 + 6*b^2*(b^2*d - 2*a*b*
e + 3*a^2*f)*x^6 + 4*b^3*(b*e - 2*a*f)*x^9 + 3*b^4*f*x^12 + (12*a^2*(-(b^3*
c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 12*a*(-2*b^3*c + 3*a*b^2*d -
 4*a^2*b*e + 5*a^3*f)*Log[a + b*x^3])/(36*b^6)
```

Maple [A]
time = 0.36, size = 179, normalized size = 0.99

method	result
default	$-\frac{-\frac{f\,x^{12}b^3}{12}+\frac{\left(2fa\,b^2-e\,b^3\right)x^9}{9}+\frac{\left(-3f\,a^2b+2a\,b^2e-b^3d\right)x^6}{6}+\frac{\left(4a^3f-3a^2be+2a\,b^2d-b^3c\right)x^3}{3}}{b^5}+\frac{a\left(\frac{a\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{b\left(b\,x^3+a\right)}+\frac{(5a^3f-4a^2b}{}\right.}{3b^5}$
norman	$\frac{-\frac{\left(5a^3f-4a^2be+3a\,b^2d-2b^3c\right)x^6}{6b^4}+\frac{f\,x^{15}}{12b}-\frac{(5af-4be)x^{12}}{36b^2}+\frac{\left(5a^2f-4abe+3b^2d\right)x^9}{18b^3}-\frac{\left(5f\,a^5-4e\,a^4b+3d\,a^3b^2-2a^2c\,b^3\right)x^3}{3a\,b^5}}{b\,x^3+a}+\frac{(5a^3f-4a^2be+}{}$
risch	$\frac{f\,x^{12}}{12b^2}-\frac{2x^9fa}{9b^3}+\frac{e\,x^9}{9b^2}+\frac{x^6f\,a^2}{2b^4}-\frac{ae\,x^6}{3b^3}+\frac{x^6d}{6b^2}-\frac{4x^3a^3f}{3b^5}+\frac{a^2e\,x^3}{b^4}-\frac{2x^3ad}{3b^3}+\frac{x^3c}{3b^2}+\frac{a^5f}{3b^6(b\,x^3+a)}-\frac{a^4e}{3b^5(b\,x^3+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*(-1/12*f*x^12*b^3+1/9*(2*a*b^2*f-b^3*e)*x^9+1/6*(-3*a^2*b*f+2*a*b^2*
e-b^3*d)*x^6+1/3*(4*a^3*f-3*a^2*b*e+2*a*b^2*d-b^3*c)*x^3)+1/3*a/b^5*(a*(a^3
*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)+(5*a^3*f-4*a^2*b*e+3*a*b^2*d-2*b^3*c)
/b*ln(b*x^3+a))
```

Maxima [A]
time = 0.28, size = 186, normalized size = 1.03

$$-\frac{a^2b^3c-a^3b^2d-a^5f+a^4be}{3\,(b^7x^3+ab^6)}+\frac{3\,b^3fx^{12}-4\,(2\,ab^2f-b^3e)x^9+6\,(b^3d+3\,a^2bf-2\,ab^2e)x^6+12\,(b^3c-2\,ab^2d-4\,a^3f+3\,a^2be)x^3}{36\,b^5}-\frac{(2\,ab^3c-3\,a^2b^2d-5\,a^4f+4\,a^3be)\log(bx^3+a)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)/(b^7*x^3 + a*b^6) + 1/36*(3*
b^3*f*x^12 - 4*(2*a*b^2*f - b^3*e)*x^9 + 6*(b^3*d + 3*a^2*b*f - 2*a*b^2*e)*
x^6 + 12*(b^3*c - 2*a*b^2*d - 4*a^3*f + 3*a^2*b*e)*x^3)/b^5 - 1/3*(2*a*b^3*
c - 3*a^2*b^2*d - 5*a^4*f + 4*a^3*b*e)*log(b*x^3 + a)/b^6
```

Fricas [A]
time = 0.38, size = 257, normalized size = 1.43

$$\frac{3\,b^5fx^{15}+(4\,b^5e-5\,ab^4f)x^{12}+2\,(3\,b^5d-4\,ab^4e+5\,a^2b^3f)x^9+6\,(2\,b^5c-3\,ab^4d+4\,a^2b^3e-5\,a^3b^2f)x^6-12\,a^2b^3c+12\,a^3b^2d-12\,a^4be+12\,a^5f+12\,(ab^4c-2\,a^2b^3d+3\,a^3b^2e-4\,a^4bf)x^3-12\,(2\,a^2b^3c-3\,a^3b^2d+4\,a^4be-5\,a^5f+(2\,ab^4c-3\,a^2b^3d+4\,a^3b^2e-5\,a^4bf)x^3)\log\left(bx^3+a\right)}{36\,(b^7x^3+ab^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^5*f*x^15 + (4*b^5*e - 5*a*b^4*f)*x^12 + 2*(3*b^5*d - 4*a*b^4*e +
5*a^2*b^3*f)*x^9 + 6*(2*b^5*c - 3*a*b^4*d + 4*a^2*b^3*e - 5*a^3*b^2*f)*x^6
- 12*a^2*b^3*c + 12*a^3*b^2*d - 12*a^4*b*e + 12*a^5*f + 12*(a*b^4*c - 2*a^2
*b^3*d + 3*a^3*b^2*e - 4*a^4*b*f)*x^3 - 12*(2*a^2*b^3*c - 3*a^3*b^2*d + 4*a
^4*b*e - 5*a^5*f + (2*a*b^4*c - 3*a^2*b^3*d + 4*a^3*b^2*e - 5*a^4*b*f)*x^3)
*log(b*x^3 + a))/(b^7*x^3 + a*b^6)
```

Sympy [A]
time = 14.50, size = 189, normalized size = 1.05

$$\frac{a(5a^3f-4a^2be+3ab^2d-2b^3c)\log\left(a+bx^3\right)}{3b^6}+x^9\left(-\frac{2af}{9b^3}+\frac{e}{9b^2}\right)+x^6\left(\frac{a^2f}{2b^4}-\frac{ae}{3b^3}+\frac{d}{6b^2}\right)+x^3\left(-\frac{4a^3f}{3b^5}+\frac{a^2e}{b^4}-\frac{2ad}{3b^3}+\frac{c}{3b^2}\right)+\frac{a^5f-a^4be+a^3b^2d-a^2b^3c}{3ab^6+3b^7x^3}+\frac{fx^{12}}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] a*(5*a**3*f - 4*a**2*b*e + 3*a*b**2*d - 2*b**3*c)*log(a + b*x**3)/(3*b**6)
+ x**9*(-2*a*f/(9*b**3) + e/(9*b**2)) + x**6*(a**2*f/(2*b**4) - a*e/(3*b**3
) + d/(6*b**2)) + x**3*(-4*a**3*f/(3*b**5) + a**2*e/b**4 - 2*a*d/(3*b**3) +
 c/(3*b**2)) + (a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*b**6 +
3*b**7*x**3) + f*x**12/(12*b**2)
```

Giac [A]
time = 0.98, size = 248, normalized size = 1.38

$$-\frac{(2\,ab^3c-3\,a^2b^2d-5\,a^4f+4\,a^3be)\log\left(|bx^3+a|\right)}{3\,b^6}+\frac{2\,ab^4cx^3-3\,a^2b^3dx^3-5\,a^4bfx^3+4\,a^3b^2x^3e+a^2b^3c-2\,a^3b^2d-4\,a^5f+3\,a^4be}{3\,(bx^3+a)b^6}+\frac{3\,b^6fx^{12}-8\,ab^5fx^9+4\,b^6x^9e+6\,b^6dx^6+18\,a^2b^4fx^6-12\,ab^5x^6e+12\,b^6cx^3-24\,ab^5dx^3-48\,a^3b^3fx^3+36\,a^2b^4x^3e}{36\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(2*a*b^3*c - 3*a^2*b^2*d - 5*a^4*f + 4*a^3*b*e)*log(abs(b*x^3 + a))/b^
6 + 1/3*(2*a*b^4*c*x^3 - 3*a^2*b^3*d*x^3 - 5*a^4*b*f*x^3 + 4*a^3*b^2*x^3*e
+ a^2*b^3*c - 2*a^3*b^2*d - 4*a^5*f + 3*a^4*b*e)/((b*x^3 + a)*b^6) + 1/36*(
3*b^6*f*x^12 - 8*a*b^5*f*x^9 + 4*b^6*x^9*e + 6*b^6*d*x^6 + 18*a^2*b^4*f*x^6
 - 12*a*b^5*x^6*e + 12*b^6*c*x^3 - 24*a*b^5*d*x^3 - 48*a^3*b^3*f*x^3 + 36*a
^2*b^4*x^3*e)/b^8
```

Mupad [B]
time = 5.00, size = 233, normalized size = 1.29

$$x^9\left(\frac{e}{9\,b^2}-\frac{2\,a\,f}{9\,b^3}\right)-x^6\left(\frac{a^2\,f}{6\,b^4}-\frac{d}{6\,b^2}+\frac{a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b}\right)+x^3\left(\frac{c}{3\,b^2}-\frac{a^2\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b^2}+\frac{2\,a\,\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b}\right)+\frac{f\,x^{12}}{12\,b^2}+\frac{f\,a^5-e\,a^4\,b+d\,a^3\,b^2-c\,a^2\,b^3}{3\,b\,\left(b^6\,x^3+a\,b^5\right)}+\frac{\ln\left(b\,x^3+a\right)\,\left(5\,f\,a^4-4\,e\,a^3\,b+3\,d\,a^2\,b^2-2\,c\,a\,b^3\right)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^9*(e/(9*b^2) - (2*a*f)/(9*b^3)) - x^6*((a^2*f)/(6*b^4) - d/(6*b^2) + (a*(
e/b^2 - (2*a*f)/b^3))/(3*b)) + x^3*(c/(3*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))
/(3*b^2) + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b
)) + (f*x^12)/(12*b^2) + (a^5*f - a^2*b^3*c + a^3*b^2*d - a^4*b*e)/(3*b*(a*
b^5 + b^6*x^3)) + (log(a + b*x^3)*(5*a^4*f + 3*a^2*b^2*d - 2*a*b^3*c - 4*a^
3*b*e))/(3*b^6)
```

3.253 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=140

$$\frac{(b^2d-2abe+3a^2f)\,x^3}{3b^4}+\frac{(be-2af)x^6}{6b^3}+\frac{fx^9}{9b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)}{3b^5\,(a+bx^3)}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\log}{3b^5}$$

```
[Out] 1/3*(3*a^2*f-2*a*b*e+b^2*d)*x^3/b^4+1/6*(-2*a*f+b*e)*x^6/b^3+1/9*f*x^9/b^2+
1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b^5/(b*x^3+a)+1/3*(-4*a^3*f+3*a^2*b*e-
2*a*b^2*d+b^3*c)*ln(b*x^3+a)/b^5
```

Rubi [A]
time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{x^3(3a^2f-2abe+b^2d)}{3b^4}+\frac{a(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5\,(a+bx^3)}+\frac{\log\left(a+bx^3\right)\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}+\frac{x^6(be-2af)}{6b^3}+\frac{fx^9}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^6)/(6*b^3) + (
f*x^9)/(9*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^5*(a + b*x^3)
) + ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*Log[a + b*x^3])/(3*b^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2d-2abe+3a^2f}{b^4}+\frac{(be-2af)x}{b^3}+\frac{fx^2}{b^2}+\frac{a(-b^3c+ab^2d}{b^4(a+}\right.\right.$$
$$= \frac{(b^2d-2abe+3a^2f)\,x^3}{3b^4}+\frac{(be-2af)x^6}{6b^3}+\frac{fx^9}{9b^2}+\frac{a(b^3c-ab^2d+a^2be-}{3b^5\,(a+bx^3)}$$

Mathematica [A]
time = 0.06, size = 129, normalized size = 0.92

$$\frac{6b(b^2d-2abe+3a^2f)\,x^3+3b^2(be-2af)x^6+2b^3fx^9+\frac{6a(b^3c-ab^2d+a^2be-a^3f)}{a+bx^3}+6(b^3c-2ab^2d+3a^2be-4a^3f)\log(a+bx^3)}{18b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (6*b*(b^2*d - 2*a*b*e + 3*a^2*f)*x^3 + 3*b^2*(b*e - 2*a*f)*x^6 + 2*b^3*f*x^
9 + (6*a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^3) + 6*(b^3*c - 2*a*
b^2*d + 3*a^2*b*e - 4*a^3*f)*Log[a + b*x^3])/(18*b^5)
```

Maple [A]
time = 0.34, size = 138, normalized size = 0.99

method	result
default	$\frac{\frac{f\,x^9b^2}{9}+\frac{\left(-2fab+b^2e\right)x^6}{6}+\frac{\left(3a^2f-2abe+b^2d\right)x^3}{3}}{b^4}-\frac{\frac{a\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{b\left(b\,x^3+a\right)}+\frac{\left(4a^3f-3a^2be+2a\,b^2d-b^3c\right)\ln\left(b\,x^3+a\right)}{b}}{3b^4}$
norman	$\frac{\frac{f\,x^{12}}{9b}-\frac{(4af-3be)x^9}{18b^2}+\frac{\left(4a^2f-3abe+2b^2d\right)x^6}{6b^3}+\frac{\left(4a^4f-3a^3be+2a^2b^2d-a\,b^3c\right)x^3}{3a\,b^4}}{b\,x^3+a}-\frac{(4a^3f-3a^2be+2a\,b^2d-b^3c)\ln(b\,x^3+a)}{3b^5}$
risch	$\frac{f\,x^9}{9b^2}-\frac{af\,x^6}{3b^3}+\frac{e\,x^6}{6b^2}+\frac{a^2f\,x^3}{b^4}-\frac{2ae\,x^3}{3b^3}+\frac{d\,x^3}{3b^2}-\frac{a^4f}{3b^5(b\,x^3+a)}+\frac{a^3e}{3b^4(b\,x^3+a)}-\frac{a^2d}{3b^3(b\,x^3+a)}+\frac{ac}{3b^2(b\,x^3+a)}-\frac{4\ln(b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/9*f*x^9*b^2+1/6*(-2*a*b*f+b^2*e)*x^6+1/3*(3*a^2*f-2*a*b*e+b^2*d)*x
^3)-1/3/b^4*(a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)+(4*a^3*f-3*a^2*b*e
+2*a*b^2*d-b^3*c)/b*ln(b*x^3+a))
```

Maxima [A]
time = 0.28, size = 143, normalized size = 1.02

$$\frac{ab^3c-a^2b^2d-a^4f+a^3be}{3\,(b^6x^3+ab^5)}+\frac{2\,b^2fx^9-3\,(2\,abf-b^2e)x^6+6\,(b^2d+3\,a^2f-2\,abe)x^3}{18\,b^4}+\frac{(b^3c-2\,ab^2d-4\,a^3f+3\,a^2be)\log(bx^3+a)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)/(b^6*x^3 + a*b^5) + 1/18*(2*b^2
*f*x^9 - 3*(2*a*b*f - b^2*e)*x^6 + 6*(b^2*d + 3*a^2*f - 2*a*b*e)*x^3)/b^4 +
 1/3*(b^3*c - 2*a*b^2*d - 4*a^3*f + 3*a^2*b*e)*log(b*x^3 + a)/b^5
```

Fricas [A]
time = 0.40, size = 202, normalized size = 1.44

$$\frac{2\,b^4fx^{12}+(3\,b^4e-4\,ab^3f)x^9+3\,(2\,b^4d-3\,ab^3e+4\,a^2b^2f)x^6+6\,ab^3c-6\,a^2b^2d+6\,a^3be-6\,a^4f+6\,(ab^3d-2\,a^2b^2e+3\,a^3bf)x^3+6\,(ab^3c-2\,a^2b^2d+3\,a^3be-4\,a^4f+(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a^3bf)x^3)\log\left(bx^3+a\right)}{18\,(b^6x^3+ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^4*f*x^12 + (3*b^4*e - 4*a*b^3*f)*x^9 + 3*(2*b^4*d - 3*a*b^3*e + 4
*a^2*b^2*f)*x^6 + 6*a*b^3*c - 6*a^2*b^2*d + 6*a^3*b*e - 6*a^4*f + 6*(a*b^3*
d - 2*a^2*b^2*e + 3*a^3*b*f)*x^3 + 6*(a*b^3*c - 2*a^2*b^2*d + 3*a^3*b*e - 4
*a^4*f + (b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^3)*log(b*x^3 + a))
/(b^6*x^3 + a*b^5)
```

Sympy [A]
time = 12.94, size = 141, normalized size = 1.01

$$x^6\left(-\frac{af}{3b^3}+\frac{e}{6b^2}\right)+x^3\left(\frac{a^2f}{b^4}-\frac{2ae}{3b^3}+\frac{d}{3b^2}\right)+\frac{-a^4f+a^3be-a^2b^2d+ab^3c}{3ab^5+3b^6x^3}+\frac{fx^9}{9b^2}-\frac{(4a^3f-3a^2be+2ab^2d-b^3c)\log\left(a+bx^3\right)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**6*(-a*f/(3*b**3) + e/(6*b**2)) + x**3*(a**2*f/b**4 - 2*a*e/(3*b**3) + d/
(3*b**2)) + (-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(3*a*b**5 + 3*b**
6*x**3) + f*x**9/(9*b**2) - (4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)*l
og(a + b*x**3)/(3*b**5)
```

Giac [A]
time = 1.00, size = 217, normalized size = 1.55

$$\frac{\frac{(bx^3+a)^3\left(2\,f-\frac{3\,(4\,abf-b^2e)}{(bx^3+a)b}+\frac{6\,(b^4d+6\,a^2b^2f-3\,ab^3e)}{(bx^3+a)^2b^2}\right)}{b^4}-\frac{6\,(b^3c-2\,ab^2d-4\,a^3f+3\,a^2be)\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^4}+\frac{6\left(\frac{ab^6c}{bx^3+a}-\frac{a^2b^5d}{bx^3+a}-\frac{a^4b^3f}{bx^3+a}+\frac{a^3b^4e}{bx^3+a}\right)}{b^7}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/18*((b*x^3 + a)^3*(2*f - 3*(4*a*b*f - b^2*e)/((b*x^3 + a)*b) + 6*(b^4*d +
 6*a^2*b^2*f - 3*a*b^3*e)/((b*x^3 + a)^2*b^2))/b^4 - 6*(b^3*c - 2*a*b^2*d -
```

```
 4*a^3*f + 3*a^2*b*e)*log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^4 + 6*(a
*b^6*c/(b*x^3 + a) - a^2*b^5*d/(b*x^3 + a) - a^4*b^3*f/(b*x^3 + a) + a^3*b^
4*e/(b*x^3 + a))/b^7)/b
```

Mupad [B]
time = 4.93, size = 155, normalized size = 1.11

$$x^6\left(\frac{e}{6\,b^2}-\frac{a\,f}{3\,b^3}\right)-x^3\left(\frac{a^2\,f}{3\,b^4}-\frac{d}{3\,b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(-4\,f\,a^3+3\,e\,a^2\,b-2\,d\,a\,b^2+c\,b^3\right)}{3\,b^5}-\frac{f\,a^4-e\,a^3\,b+d\,a^2\,b^2-c\,a\,b^3}{3\,b\,\left(b^5\,x^3+a\,b^4\right)}+\frac{f\,x^9}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^6*(e/(6*b^2) - (a*f)/(3*b^3)) - x^3*((a^2*f)/(3*b^4) - d/(3*b^2) + (2*a*(
e/b^2 - (2*a*f)/b^3))/(3*b)) + (log(a + b*x^3)*(b^3*c - 4*a^3*f - 2*a*b^2*d
 + 3*a^2*b*e))/(3*b^5) - (a^4*f + a^2*b^2*d - a*b^3*c - a^3*b*e)/(3*b*(a*b^
4 + b^5*x^3)) + (f*x^9)/(9*b^2)
```

3.254 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$

Optimal. Leaf size=103

$$\frac{(be-2af)x^3}{3b^3}+\frac{fx^6}{6b^2}-\frac{b^3c-ab^2d+a^2be-a^3f}{3b^4(a+bx^3)}+\frac{(b^2d-2abe+3a^2f)\log(a+bx^3)}{3b^4}$$

```
[Out] 1/3*(-2*a*f+b*e)*x^3/b^3+1/6*f*x^6/b^2+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^
4/(b*x^3+a)+1/3*(3*a^2*f-2*a*b*e+b^2*d)*ln(b*x^3+a)/b^4
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1833, 1864}

$$\frac{\log(a+bx^3)(3a^2f-2abe+b^2d)}{3b^4}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3b^4(a+bx^3)}+\frac{x^3(be-2af)}{3b^3}+\frac{fx^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^6)/(6*b^2) - (b^3*c - a*b^2*d + a^2*b*e
- a^3*f)/(3*b^4*(a + b*x^3)) + ((b^2*d - 2*a*b*e + 3*a^2*f)*Log[a + b*x^3])
/(3*b^4)
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{be-2af}{b^3}+\frac{fx}{b^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3(a+bx)^2}+\frac{b^2d-2abe+}{b^3(a+bx}\right.\right.$$
$$= \frac{(be-2af)x^3}{3b^3}+\frac{fx^6}{6b^2}-\frac{b^3c-ab^2d+a^2be-a^3f}{3b^4\,(a+bx^3)}+\frac{(b^2d-2abe+3a^2f)\,\text{lo}}{3b^4}$$

Mathematica [A]
time = 0.04, size = 93, normalized size = 0.90

$$\frac{2b(be-2af)x^3+b^2fx^6+\frac{2(-b^3c+ab^2d-a^2be+a^3f)}{a+bx^3}+2(b^2d-2abe+3a^2f)\log\left(a+bx^3\right)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (2*b*(b*e - 2*a*f)*x^3 + b^2*f*x^6 + (2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3
*f))/(a + b*x^3) + 2*(b^2*d - 2*a*b*e + 3*a^2*f)*Log[a + b*x^3])/(6*b^4)
```

Maple [A]
time = 0.37, size = 102, normalized size = 0.99

method	result
norman	$\frac{\frac{3a^3f-2a^2be+a\,b^2d-b^3c}{3b^4}+\frac{f\,x^9}{6b}-\frac{(3af-2be)x^6}{6b^2}}{b\,x^3+a}+\frac{(3a^2f-2abe+b^2d)\ln(b\,x^3+a)}{3b^4}$
default	$\frac{(-f\,x^3b+2af-be)^2}{6b^4f}+\frac{-\frac{-a^3f+a^2be-a\,b^2d+b^3c}{b(b\,x^3+a)}+\frac{(3a^2f-2abe+b^2d)\ln(b\,x^3+a)}{b}}{3b^3}$
risch	$\frac{f\,x^6}{6b^2}-\frac{2fa\,x^3}{3b^3}+\frac{e\,x^3}{3b^2}+\frac{2f\,a^2}{3b^4}-\frac{2ae}{3b^3}+\frac{e^2}{6b^2f}+\frac{a^3f}{3b^4(b\,x^3+a)}-\frac{a^2e}{3b^3(b\,x^3+a)}+\frac{ad}{3b^2(b\,x^3+a)}-\frac{c}{3b(b\,x^3+a)}+\frac{\ln(b\,x^3+}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-b*f*x^3+2*a*f-b*e)^2/b^4/f+1/3/b^3*(-(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b
/(b*x^3+a)+(3*a^2*f-2*a*b*e+b^2*d)/b*ln(b*x^3+a))
```

Maxima [A]
time = 0.28, size = 102, normalized size = 0.99

$$-\frac{b^3c-ab^2d-a^3f+a^2be}{3\,(b^5x^3+ab^4)}+\frac{bfx^6-2\,(2\,af-be)x^3}{6\,b^3}+\frac{(b^2d+3\,a^2f-2\,abe)\log\left(bx^3+a\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)/(b^5*x^3 + a*b^4) + 1/6*(b*f*x^6 -
 2*(2*a*f - b*e)*x^3)/b^3 + 1/3*(b^2*d + 3*a^2*f - 2*a*b*e)*log(b*x^3 + a)/
b^4
```

Fricas [**A**]
time = 0.38, size = 143, normalized size = 1.39

$$\frac{b^3 f x^9 + (2\,b^3 e - 3\,a b^2 f)x^6 - 2\,b^3 c + 2\,a b^2 d - 2\,a^2 b e + 2\,a^3 f + 2\,(a b^2 e - 2\,a^2 b f)x^3 + 2\,(a b^2 d - 2\,a^2 b e + 3\,a^3 f + (b^3 d - 2\,a b^2 e + 3\,a^2 b f)x^3)\log\left(b x^3 + a\right)}{6\,(b^5 x^3 + a b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*f*x^9 + (2*b^3*e - 3*a*b^2*f)*x^6 - 2*b^3*c + 2*a*b^2*d - 2*a^2*b*
e + 2*a^3*f + 2*(a*b^2*e - 2*a^2*b*f)*x^3 + 2*(a*b^2*d - 2*a^2*b*e + 3*a^3*
f + (b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^3)*log(b*x^3 + a))/(b^5*x^3 + a*b^4)
```

Sympy [**A**]
time = 6.81, size = 100, normalized size = 0.97

$$x^3\left(-\frac{2af}{3b^3} + \frac{e}{3b^2}\right) + \frac{a^3 f - a^2 b e + a b^2 d - b^3 c}{3ab^4 + 3b^5 x^3} + \frac{f x^6}{6b^2} + \frac{(3a^2 f - 2abe + b^2 d)\log\left(a + bx^3\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**3*(-2*a*f/(3*b**3) + e/(3*b**2)) + (a**3*f - a**2*b*e + a*b**2*d - b**3*
c)/(3*a*b**4 + 3*b**5*x**3) + f*x**6/(6*b**2) + (3*a**2*f - 2*a*b*e + b**2*
d)*log(a + b*x**3)/(3*b**4)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(99) = 198.
time = 0.88, size = 206, normalized size = 2.00

$$-\frac{1}{6}\,f\left(\frac{(bx^3+a)^2\left(\frac{6\,a}{bx^3+a}-1\right)}{b^4} + \frac{6\,a^2\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^4} - \frac{2\,a^3}{(bx^3+a)b^4}\right) + \frac{1}{3}\left(\frac{2\,a\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^3} + \frac{bx^3+a}{b^3} - \frac{a^2}{(bx^3+a)b^3}\right)e - \frac{d\left(\frac{\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b} - \frac{a}{(bx^3+a)b}\right)}{3\,b} - \frac{c}{3\,(bx^3+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*f*((b*x^3 + a)^2*(6*a/(b*x^3 + a) - 1)/b^4 + 6*a^2*log(abs(b*x^3 + a)/
((b*x^3 + a)^2*abs(b)))/b^4 - 2*a^3/((b*x^3 + a)*b^4)) + 1/3*(2*a*log(abs(b
*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^3 + (b*x^3 + a)/b^3 - a^2/((b*x^3 + a)*
```

```
b^3))*e - 1/3*d*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 +
 a)*b))/b - 1/3*c/((b*x^3 + a)*b)
```

Mupad [B]
time = 0.09, size = 103, normalized size = 1.00

$$x^3\left(\frac{e}{3\,b^2}-\frac{2\,a\,f}{3\,b^3}\right)+\frac{f\,x^6}{6\,b^2}-\frac{-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3}{3\,b\,\left(b^4\,x^3+a\,b^3\right)}+\frac{\ln\left(b\,x^3+a\right)\,\left(3\,f\,a^2-2\,e\,a\,b+d\,b^2\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^3*(e/(3*b^2) - (2*a*f)/(3*b^3)) + (f*x^6)/(6*b^2) - (b^3*c - a^3*f - a*b^
2*d + a^2*b*e)/(3*b*(a*b^3 + b^4*x^3)) + (log(a + b*x^3)*(b^2*d + 3*a^2*f -
 2*a*b*e))/(3*b^4)
```

3.255 $\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=100

$$\frac{fx^3}{3b^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{3ab^3\left(a+bx^3\right)}+\frac{c\log(x)}{a^2}-\frac{\left(b^3c-a^2be+2a^3f\right)\log\left(a+bx^3\right)}{3a^2b^3}$$

```
[Out] 1/3*f*x^3/b^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a/b^3/(b*x^3+a)+c*ln(x)/a^
2-1/3*(2*a^3*f-a^2*b*e+b^3*c)*ln(b*x^3+a)/a^2/b^3
```

Rubi [A]
time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{c\log(x)}{a^2}-\frac{\log\left(a+bx^3\right)\left(2a^3f-a^2be+b^3c\right)}{3a^2b^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3ab^3\left(a+bx^3\right)}+\frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]
```

```
[Out] (f*x^3)/(3*b^2) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a*b^3*(a + b*x^3))
 + (c*Log[x])/a^2 - ((b^3*c - a^2*b*e + 2*a^3*f)*Log[a + b*x^3])/(3*a^2*b^3
)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{f}{b^2}+\frac{c}{a^2x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2(a+bx)^2}+\frac{-b^3c+a^2be-2a^3f}{a^2b^2(a+bx)}\right)\right.$$
$$= \frac{fx^3}{3b^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{3ab^3\left(a+bx^3\right)}+\frac{c\log(x)}{a^2}-\frac{\left(b^3c-a^2be+2a^3f\right)\log\left(a+bx^3\right.}{3a^2b^3}$$

Mathematica [A]
time = 0.08, size = 95, normalized size = 0.95

$$\frac{3c\log(x)+\frac{\frac{a\left(b^3c-a^3f+a^2b\left(e+fx^3\right)+ab^2\left(-d+fx^6\right)\right)}{a+bx^3}+\left(-b^3c+a^2be-2a^3f\right)\log\left(a+bx^3\right)}{b^3}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]
```

```
[Out] (3*c*Log[x] + ((a*(b^3*c - a^3*f + a^2*b*(e + f*x^3) + a*b^2*(-d + f*x^6)))
/(a + b*x^3) + (-(b^3*c) + a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/b^3)/(3*a^2)
```

Maple [A]
time = 0.40, size = 97, normalized size = 0.97

method	result
default	$\frac{f\,x^3}{3b^2}-\frac{\frac{a\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{b\left(b\,x^3+a\right)}+\frac{\left(2a^3f-a^2be+b^3c\right)\ln\left(b\,x^3+a\right)}{b}}{3a^2b^2}+\frac{c\ln(x)}{a^2}$
norman	$\frac{\frac{f\,x^6}{3b}-\frac{2a^3f-a^2be+a\,b^2d-b^3c}{3a\,b^3}}{b\,x^3+a}+\frac{c\ln(x)}{a^2}-\frac{\left(2a^3f-a^2be+b^3c\right)\ln\left(b\,x^3+a\right)}{3a^2b^3}$
risch	$\frac{f\,x^3}{3b^2}-\frac{a^2f}{3b^3(b\,x^3+a)}+\frac{ae}{3b^2(b\,x^3+a)}-\frac{d}{3b(b\,x^3+a)}+\frac{c}{3a(b\,x^3+a)}+\frac{c\ln(x)}{a^2}-\frac{2\ln(b\,x^3+a)af}{3b^3}+\frac{\ln(b\,x^3+a)e}{3b^2}-\frac{c\ln(b\,x^3+}{3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*f*x^3/b^2-1/3/a^2/b^2*(a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)+(2*a
^3*f-a^2*b*e+b^3*c)/b*ln(b*x^3+a))+c*ln(x)/a^2
```

Maxima [A]
time = 0.28, size = 102, normalized size = 1.02

$$\frac{fx^3}{3\,b^2}+\frac{b^3c-ab^2d-a^3f+a^2be}{3\left(ab^4x^3+a^2b^3\right)}+\frac{c\log\left(x^3\right)}{3\,a^2}-\frac{\left(b^3c+2\,a^3f-a^2be\right)\log\left(bx^3+a\right)}{3\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3/b^2 + 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)/(a*b^4*x^3 + a^2*b^
3) + 1/3*c*log(x^3)/a^2 - 1/3*(b^3*c + 2*a^3*f - a^2*b*e)*log(b*x^3 + a)/(a
^2*b^3)
```

Fricas [A]
time = 0.42, size = 145, normalized size = 1.45

$$\frac{a^2b^2fx^6 + a^3bfx^3 + ab^3c - a^2b^2d + a^3be - a^4f - (ab^3c - a^3be + 2\,a^4f + (b^4c - a^2b^2e + 2\,a^3bf)x^3)\log\left(bx^3 + a\right) + 3\left(b^4cx^3 + ab^3c\right)\log\left(x\right)}{3\left(a^2b^4x^3 + a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*b^2*f*x^6 + a^3*b*f*x^3 + a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f -
(a*b^3*c - a^3*b*e + 2*a^4*f + (b^4*c - a^2*b^2*e + 2*a^3*b*f)*x^3)*log(b*x
^3 + a) + 3*(b^4*c*x^3 + a*b^3*c)*log(x))/(a^2*b^4*x^3 + a^3*b^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.76, size = 125, normalized size = 1.25

$$\frac{fx^3}{3\,b^2} + \frac{c\log\left(|x|\right)}{a^2} - \frac{\left(b^3c + 2\,a^3f - a^2be\right)\log\left(|bx^3 + a|\right)}{3\,a^2b^3} + \frac{b^4cx^3 + 2\,a^3bfx^3 - a^2b^2x^3e + 2\,ab^3c - a^2b^2d + a^4f}{3\left(bx^3 + a\right)a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*f*x^3/b^2 + c*log(abs(x))/a^2 - 1/3*(b^3*c + 2*a^3*f - a^2*b*e)*log(abs
(b*x^3 + a))/(a^2*b^3) + 1/3*(b^4*c*x^3 + 2*a^3*b*f*x^3 - a^2*b^2*x^3*e + 2
*a*b^3*c - a^2*b^2*d + a^4*f)/((b*x^3 + a)*a^2*b^3)
```

Mupad [B]
time = 5.03, size = 100, normalized size = 1.00

$$\frac{f\,x^3}{3\,b^2} + \frac{c\,\ln\left(x\right)}{a^2} + \frac{-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3}{3\,a\,b\,\left(b^3\,x^3 + a\,b^2\right)} - \frac{\ln\left(b\,x^3 + a\right)\,\left(2\,f\,a^3 - e\,a^2\,b + c\,b^3\right)}{3\,a^2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x)
```

```
[Out] (f*x^3)/(3*b^2) + (c*log(x))/a^2 + (b^3*c - a^3*f - a*b^2*d + a^2*b*e)/(3*a
*b*(a*b^2 + b^3*x^3)) - (log(a + b*x^3)*(b^3*c + 2*a^3*f - a^2*b*e))/(3*a^2
*b^3)
```

3.256 $\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=109

$$-\frac{c}{3a^2x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{3a^2b^2\left(a+bx^3\right)}-\frac{(2bc-ad)\log(x)}{a^3}+\frac{\left(2b^3c-ab^2d+a^3f\right)\log\left(a+bx^3\right)}{3a^3b^2}$$

```
[Out] -1/3*c/a^2/x^3+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^2/b^2/(b*x^3+a)-(-a*d+2*
b*c)*ln(x)/a^3+1/3*(a^3*f-a*b^2*d+2*b^3*c)*ln(b*x^3+a)/a^3/b^2
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{\log\left(a+bx^3\right)\left(a^3f-ab^2d+2b^3c\right)}{3a^3b^2}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{c}{3a^2x^3}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^2b^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^2*b^2*(a + b*x^
3)) - ((2*b*c - a*d)*Log[x])/a^3 + ((2*b^3*c - a*b^2*d + a^3*f)*Log[a + b*x
^3])/(3*a^3*b^2)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^2x^2}+\frac{-2bc+ad}{a^3x}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx)^2}+\frac{2b^3c-ab^2d+}{a^3b(a+bx)}\right.\right.$$
$$= -\frac{c}{3a^2x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{3a^2b^2\left(a+bx^3\right)}-\frac{(2bc-ad)\log(x)}{a^3}+\frac{(2b^3c-ab^2d+a^3f}{3a^3b^2}$$

Mathematica [A]
time = 0.05, size = 97, normalized size = 0.89

$$\frac{-\frac{ac}{x^3}+\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^2(a+bx^3)}+3(-2bc+ad)\log(x)+\frac{\left(2b^3c-ab^2d+a^3f\right)\log\left(a+bx^3\right)}{b^2}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-((a*c)/x^3) + (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2*(a + b*x^3)
) + 3*(-2*b*c + a*d)*Log[x] + ((2*b^3*c - a*b^2*d + a^3*f)*Log[a + b*x^3])/
b^2)/(3*a^3)
```

Maple [A]
time = 0.36, size = 101, normalized size = 0.93

method	result	size
default	$\frac{\frac{a\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{b^2\left(b\,x^3+a\right)}+\frac{\left(a^3f-a\,b^2d+2b^3c\right)\ln\left(b\,x^3+a\right)}{b^2}}{3a^3}-\frac{c}{3a^2x^3}+\frac{(ad-2bc)\ln(x)}{a^3}$	101
norman	$\frac{-\frac{c}{3a}+\frac{\left(a^3f-a^2be+a\,b^2d-2b^3c\right)x^3}{3a^2b^2}}{x^3\left(b\,x^3+a\right)}+\frac{(ad-2bc)\ln(x)}{a^3}+\frac{\left(a^3f-a\,b^2d+2b^3c\right)\ln\left(b\,x^3+a\right)}{3a^3b^2}$	107
risch	$\frac{-\frac{c}{3a}+\frac{\left(a^3f-a^2be+a\,b^2d-2b^3c\right)x^3}{3a^2b^2}}{x^3\left(b\,x^3+a\right)}+\frac{d\ln(x)}{a^2}-\frac{2bc\ln(x)}{a^3}+\frac{\ln\left(-b\,x^3-a\right)f}{3b^2}-\frac{\ln\left(-b\,x^3-a\right)d}{3a^2}+\frac{2b\ln\left(-b\,x^3-a\right)c}{3a^3}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^3*(a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^2/(b*x^3+a)+(a^3*f-a*b^2*d+2*b^3
*c)/b^2*ln(b*x^3+a))-1/3*c/a^2/x^3+(a*d-2*b*c)/a^3*ln(x)
```

Maxima [A]
time = 0.28, size = 117, normalized size = 1.07

$$-\frac{ab^2c+(2\,b^3c-ab^2d-a^3f+a^2be)x^3}{3\left(a^2b^3x^6+a^3b^2x^3\right)}-\frac{(2\,bc-ad)\log\left(x^3\right)}{3\,a^3}+\frac{(2\,b^3c-ab^2d+a^3f)\log\left(bx^3+a\right)}{3\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a*b^2*c + (2*b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x^3)/(a^2*b^3*x^6 + a
^3*b^2*x^3) - 1/3*(2*b*c - a*d)*log(x^3)/a^3 + 1/3*(2*b^3*c - a*b^2*d + a^3
*f)*log(b*x^3 + a)/(a^3*b^2)
```

Fricas [A]
time = 0.41, size = 172, normalized size = 1.58

$$-\frac{a^{2}b^{2}c+\left(2\,ab^{3}c-a^{2}b^{2}d+a^{3}be-a^{4}f\right)x^{3}-\left(\left(2\,b^{4}c-ab^{3}d+a^{3}bf\right)x^{6}+\left(2\,ab^{3}c-a^{2}b^{2}d+a^{4}f\right)x^{3}\right)\log\left(bx^{3}+a\right)+3\left(\left(2\,b^{4}c-ab^{3}d\right)x^{6}+\left(2\,ab^{3}c-a^{2}b^{2}d\right)x^{3}\right)\log\left(x\right)}{3\left(a^{3}b^{3}x^{6}+a^{4}b^{2}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(a^2*b^2*c + (2*a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^3 - ((2*b^4*c
 - a*b^3*d + a^3*b*f)*x^6 + (2*a*b^3*c - a^2*b^2*d + a^4*f)*x^3)*log(b*x^3
+ a) + 3*((2*b^4*c - a*b^3*d)*x^6 + (2*a*b^3*c - a^2*b^2*d)*x^3)*log(x))/(a
^3*b^3*x^6 + a^4*b^2*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.88, size = 131, normalized size = 1.20

$$-\frac{(2\,bc-ad)\log(|x|)}{a^{3}}+\frac{(2\,b^{3}c-ab^{2}d+a^{3}f)\log(|bx^{3}+a|)}{3\,a^{3}b^{2}}-\frac{a^{2}bfx^{6}+4\,b^{3}cx^{3}-2\,ab^{2}dx^{3}-a^{3}fx^{3}+2\,a^{2}bx^{3}e+2\,ab^{2}c}{6\,(bx^{6}+ax^{3})a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -(2*b*c - a*d)*log(abs(x))/a^3 + 1/3*(2*b^3*c - a*b^2*d + a^3*f)*log(abs(b*
x^3 + a))/(a^3*b^2) - 1/6*(a^2*b*f*x^6 + 4*b^3*c*x^3 - 2*a*b^2*d*x^3 - a^3*
f*x^3 + 2*a^2*b*x^3*e + 2*a*b^2*c)/((b*x^6 + a*x^3)*a^2*b^2)
```

Mupad [B]
time = 5.05, size = 109, normalized size = 1.00

$$\frac{\ln\left(x\right)\,\left(a\,d-2\,b\,c\right)}{a^3}-\frac{\frac{c}{3\,a}+\frac{x^3\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+2\,c\,b^3\right)}{3\,a^2\,b^2}}{b\,x^6+a\,x^3}+\frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^3-d\,a\,b^2+2\,c\,b^3\right)}{3\,a^3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (log(x)*(a*d - 2*b*c))/a^3 - (c/(3*a) + (x^3*(2*b^3*c - a^3*f - a*b^2*d + a
^2*b*e))/(3*a^2*b^2))/(a*x^3 + b*x^6) + (log(a + b*x^3)*(2*b^3*c + a^3*f -
a*b^2*d))/(3*a^3*b^2)
```

3.257 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^2}\,dx$

Optimal. Leaf size=130

$$-\frac{c}{6a^2x^6}+\frac{2bc-ad}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^3b\,(a+bx^3)}+\frac{(3b^2c-2abd+a^2e)\log(x)}{a^4}-\frac{(3b^2c-2abd+a^2e)\log\left(a+bx^3\right)}{3a^4}$$

```
[Out] -1/6*c/a^2/x^6+1/3*(-a*d+2*b*c)/a^3/x^3+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/
a^3/b/(b*x^3+a)+(a^2*e-2*a*b*d+3*b^2*c)*ln(x)/a^4-1/3*(a^2*e-2*a*b*d+3*b^2*
c)*ln(b*x^3+a)/a^4
```

Rubi [A]
time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{2bc-ad}{3a^3x^3}-\frac{c}{6a^2x^6}-\frac{\log\left(a+bx^3\right)\left(a^2e-2abd+3b^2c\right)}{3a^4}+\frac{\log(x)\left(a^2e-2abd+3b^2c\right)}{a^4}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^3b\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -1/6*c/(a^2*x^6) + (2*b*c - a*d)/(3*a^3*x^3) + (b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)/(3*a^3*b*(a + b*x^3)) + ((3*b^2*c - 2*a*b*d + a^2*e)*Log[x])/a^4 -
((3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/(3*a^4)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^2x^3}+\frac{-2bc+ad}{a^3x^2}+\frac{3b^2c-2abd+a^2e}{a^4x}+\frac{-b^3c+ab^2d-a^2be+}{a^3(a+bx)^2}\right.\right.$$
$$= -\frac{c}{6a^2x^6}+\frac{2bc-ad}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^3b\left(a+bx^3\right)}+\frac{\left(3b^2c-2abd+a^2e\right)\log(x)}{a^4}$$

Mathematica [A]
time = 0.08, size = 118, normalized size = 0.91

$$-\frac{\frac{a^2c}{x^6}+\frac{2a(-2bc+ad)}{x^3}+\frac{2a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b(a+bx^3)}-6(3b^2c-2abd+a^2e)\log(x)+2(3b^2c-2abd+a^2e)\log\left(a+bx^3\right)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -1/6*((a^2*c)/x^6 + (2*a*(-2*b*c + a*d))/x^3 + (2*a*(-(b^3*c) + a*b^2*d - a
^2*b*e + a^3*f))/(b*(a + b*x^3)) - 6*(3*b^2*c - 2*a*b*d + a^2*e)*Log[x] + 2
*(3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/a^4
```

Maple [A]
time = 0.35, size = 123, normalized size = 0.95

method	result
default	$\frac{-\frac{a\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{b\left(b\,x^3+a\right)}+\left(-a^2e+2abd-3b^2c\right)\ln\left(b\,x^3+a\right)}{3a^4}-\frac{c}{6a^2x^6}-\frac{ad-2bc}{3a^3x^3}+\frac{\left(a^2e-2abd+3b^2c\right)\ln(x)}{a^4}$
norman	$\frac{-\frac{c}{6a}-\frac{(2ad-3bc)x^3}{6a^2}+\frac{\left(a^3f-a^2be+2a\,b^2d-3b^3c\right)x^9}{3a^4}}{x^6\left(b\,x^3+a\right)}+\frac{\left(a^2e-2abd+3b^2c\right)\ln(x)}{a^4}-\frac{\left(a^2e-2abd+3b^2c\right)\ln\left(b\,x^3+a\right)}{3a^4}$
risch	$\frac{-\frac{\left(a^3f-a^2be+2a\,b^2d-3b^3c\right)x^6}{3a^3b}-\frac{(2ad-3bc)x^3}{6a^2}-\frac{c}{6a}}{x^6\left(b\,x^3+a\right)}+\frac{e\ln(x)}{a^2}-\frac{2\ln(x)bd}{a^3}+\frac{3\ln(x)b^2c}{a^4}-\frac{e\ln\left(b\,x^3+a\right)}{3a^2}+\frac{2\ln\left(b\,x^3+a\right)bd}{3a^3}-\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^4*(-a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)+(-a^2*e+2*a*b*d-3*b^2
*c)*ln(b*x^3+a))-1/6*c/a^2/x^6-1/3*(a*d-2*b*c)/a^3/x^3+(a^2*e-2*a*b*d+3*b^2
*c)*ln(x)/a^4
```

Maxima [A]
time = 0.28, size = 141, normalized size = 1.08

$$\frac{2\left(3\,b^3c-2\,ab^2d-a^3f+a^2be\right)x^6-a^2bc+\left(3\,ab^2c-2\,a^2bd\right)x^3}{6\left(a^3b^2x^9+a^4bx^6\right)}-\frac{\left(3\,b^2c-2\,abd+a^2e\right)\log\left(bx^3+a\right)}{3\,a^4}+\frac{\left(3\,b^2c-2\,abd+a^2e\right)\log\left(x^3\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(3*b^3*c - 2*a*b^2*d - a^3*f + a^2*b*e)*x^6 - a^2*b*c + (3*a*b^2*c -
 2*a^2*b*d)*x^3)/(a^3*b^2*x^9 + a^4*b*x^6) - 1/3*(3*b^2*c - 2*a*b*d + a^2*e
)*log(b*x^3 + a)/a^4 + 1/3*(3*b^2*c - 2*a*b*d + a^2*e)*log(x^3)/a^4
```

Fricas [A]
time = 0.42, size = 208, normalized size = 1.60

$$\frac{2\,(3\,ab^3c-2\,a^2b^2d+a^3be-a^4f)x^6-a^3bc+(3\,a^2b^2c-2\,a^3bd)x^3-2\,((3\,b^4c-2\,ab^3d+a^2b^2e)x^9+(3\,ab^3c-2\,a^2b^2d+a^3be)x^6)\log\left(bx^3+a\right)+6\,((3\,b^4c-2\,ab^3d+a^2b^2e)x^9+(3\,ab^3c-2\,a^2b^2d+a^3be)x^6)\log\left(x\right)}{6\,(a^4b^2x^9+a^5bx^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e - a^4*f)*x^6 - a^3*b*c + (3*a^2*b
^2*c - 2*a^3*b*d)*x^3 - 2*((3*b^4*c - 2*a*b^3*d + a^2*b^2*e)*x^9 + (3*a*b^3
*c - 2*a^2*b^2*d + a^3*b*e)*x^6)*log(b*x^3 + a) + 6*((3*b^4*c - 2*a*b^3*d +
 a^2*b^2*e)*x^9 + (3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e)*x^6)*log(x))/(a^4*b^2
*x^9 + a^5*b*x^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.88, size = 201, normalized size = 1.55

$$\frac{(3\,b^2c-2\,abd+a^2e)\log\left(|x|\right)}{a^4}-\frac{(3\,b^3c-2\,ab^2d+a^2be)\log\left(|bx^3+a|\right)}{3\,a^4b}+\frac{3\,b^4cx^3-2\,ab^3dx^3+a^2b^2x^3e+4\,ab^3c-3\,a^2b^2d-a^4f+2\,a^3be}{3\,(bx^3+a)a^4b}-\frac{9\,b^2cx^6-6\,abdx^6+3\,a^2x^6e-4\,abcx^3+2\,a^2dx^3+a^2c}{6\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (3*b^2*c - 2*a*b*d + a^2*e)*log(abs(x))/a^4 - 1/3*(3*b^3*c - 2*a*b^2*d + a^
2*b*e)*log(abs(b*x^3 + a))/(a^4*b) + 1/3*(3*b^4*c*x^3 - 2*a*b^3*d*x^3 + a^2
*b^2*x^3*e + 4*a*b^3*c - 3*a^2*b^2*d - a^4*f + 2*a^3*b*e)/((b*x^3 + a)*a^4*
b) - 1/6*(9*b^2*c*x^6 - 6*a*b*d*x^6 + 3*a^2*x^6*e - 4*a*b*c*x^3 + 2*a^2*d*x
^3 + a^2*c)/(a^4*x^6)
```

Mupad [B]
time = 5.01, size = 130, normalized size = 1.00

$$\frac{\ln(x)\,\left(e\,a^2-2\,d\,a\,b+3\,c\,b^2\right)}{a^4}-\frac{\ln\left(b\,x^3+a\right)\,\left(e\,a^2-2\,d\,a\,b+3\,c\,b^2\right)}{3\,a^4}-\frac{\frac{c}{6\,a}+\frac{x^3\,(2\,a\,d-3\,b\,c)}{6\,a^2}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-2\,d\,a\,b^2+3\,c\,b^3\right)}{3\,a^3\,b}}{b\,x^9+a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x)
```

```
[Out] (log(x)*(3*b^2*c + a^2*e - 2*a*b*d))/a^4 - (log(a + b*x^3)*(3*b^2*c + a^2*e
 - 2*a*b*d))/(3*a^4) - (c/(6*a) + (x^3*(2*a*d - 3*b*c))/(6*a^2) - (x^6*(3*b
^3*c - a^3*f - 2*a*b^2*d + a^2*b*e))/(3*a^3*b))/(a*x^6 + b*x^9)
```

3.258 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2} \, dx$

Optimal. Leaf size=175

$$-\frac{c}{9a^2x^9}+\frac{2bc-ad}{6a^3x^6}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4\left(a+bx^3\right)}-\frac{\left(4b^3c-3ab^2d+2a^2be-a^3f\right)\log(x)}{a^5}+\frac{(4b$$

```
[Out] -1/9*c/a^2/x^9+1/6*(-a*d+2*b*c)/a^3/x^6+1/3*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x^
3+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/(b*x^3+a)-(-a^3*f+2*a^2*b*e-3*a*b^2
*d+4*b^3*c)*ln(x)/a^5+1/3*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)*ln(b*x^3+a)/
a^5
```

Rubi [A]
time = 0.14, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{2bc-ad}{6a^3x^6}-\frac{c}{9a^2x^9}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+2a^2be-3ab^2d+4b^3c\right)}{3a^5}-\frac{\log(x)\left(a^3(-f)+2a^2be-3ab^2d+4b^3c\right)}{a^5}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x]
```

```
[Out] -1/9*c/(a^2*x^9) + (2*b*c - a*d)/(6*a^3*x^6) - (3*b^2*c - 2*a*b*d + a^2*e)/
(3*a^4*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*(a + b*x^3)) - ((4
*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*Log[x])/a^5 + ((4*b^3*c - 3*a*b^2*d
 + 2*a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^2x^4} + \frac{-2bc+ad}{a^3x^3} + \frac{3b^2c-2abd+a^2e}{a^4x^2} + \frac{-4b^3c+3ab^2d-2a^2}{a^5x}\right.\right.$$
$$= -\frac{c}{9a^2x^9} + \frac{2bc-ad}{6a^3x^6} - \frac{3b^2c-2abd+a^2e}{3a^4x^3} - \frac{b^3c-ab^2d+a^2be-a^3f}{3a^4\left(a+bx^3\right)} - \frac{(4b^3c}{}$$

Mathematica [A]
time = 0.07, size = 160, normalized size = 0.91

$$\frac{-\frac{2a^3c}{x^9} - \frac{3a^2(-2bc+ad)}{x^6} - \frac{6a(3b^2c-2abd+a^2e)}{x^3} + \frac{6a(-b^3c+ab^2d-a^2be+a^3f)}{a+bx^3} + 18(-4b^3c+3ab^2d-2a^2be+a^3f)\log(x) + 6(4b^3c-3ab^2d+2a^2be-a^3f)\log\left(a+bx^3\right)}{18a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x]
```

```
[Out] ((-2*a^3*c)/x^9 - (3*a^2*(-2*b*c + a*d))/x^6 - (6*a*(3*b^2*c - 2*a*b*d + a^
2*e))/x^3 + (6*a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 18*(
-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f)*Log[x] + 6*(4*b^3*c - 3*a*b^2*d +
 2*a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*a^5)
```

Maple [A]
time = 0.37, size = 169, normalized size = 0.97

method	result
default	$-\frac{b\left(-\frac{a\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{b\left(b\,x^3+a\right)}+\frac{\left(a^3f-2a^2be+3a\,b^2d-4b^3c\right)\ln\left(b\,x^3+a\right)}{b}\right)}{3a^5}-\frac{c}{9a^2x^9}-\frac{ad-2bc}{6a^3x^6}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{(a^3f-2a^2b}{}$
norman	$\frac{-\frac{c}{9a}-\frac{(3ad-4bc)x^3}{18a^2}-\frac{\left(2a^2e-3abd+4b^2c\right)x^6}{6a^3}+\frac{b\left(-a^3f+2a^2be-3a\,b^2d+4b^3c\right)x^{12}}{3a^5}}{x^9\left(b\,x^3+a\right)}+\frac{\left(a^3f-2a^2be+3a\,b^2d-4b^3c\right)\ln(x)}{a^5}-\frac{(a^3f-2a^2be+}{}$
risch	$\frac{\frac{\left(a^3f-2a^2be+3a\,b^2d-4b^3c\right)x^9}{3a^4}-\frac{\left(2a^2e-3abd+4b^2c\right)x^6}{6a^3}-\frac{(3ad-4bc)x^3}{18a^2}-\frac{c}{9a}}{x^9\left(b\,x^3+a\right)}+\frac{\ln(x)f}{a^2}-\frac{2\ln(x)be}{a^3}+\frac{3\ln(x)b^2d}{a^4}-\frac{4\ln(x)b^3c}{a^5}-\frac{\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^5*b*(-a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)+(a^3*f-2*a^2*b*e+3
*a*b^2*d-4*b^3*c)/b*ln(b*x^3+a))-1/9*c/a^2/x^9-1/6*(a*d-2*b*c)/a^3/x^6-1/3*
(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^3+(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5*ln(
x)
```

Maxima [A]
time = 0.28, size = 185, normalized size = 1.06

$$-\frac{6\,(4\,b^{3}c-3\,ab^{2}d-a^{3}f+2\,a^{2}be)x^{9}+3\,(4\,ab^{2}c-3\,a^{2}bd+2\,a^{3}e)x^{6}+2\,a^{3}c-(4\,a^{2}bc-3\,a^{3}d)x^{3}}{18\,(a^{4}bx^{12}+a^{5}x^{9})}+\frac{(4\,b^{3}c-3\,ab^{2}d-a^{3}f+2\,a^{2}be)\log\left(bx^{3}+a\right)}{3\,a^{5}}-\frac{(4\,b^{3}c-3\,ab^{2}d-a^{3}f+2\,a^{2}be)\log\left(x^{3}\right)}{3\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/18*(6*(4*b^3*c - 3*a*b^2*d - a^3*f + 2*a^2*b*e)*x^9 + 3*(4*a*b^2*c - 3*a
^2*b*d + 2*a^3*e)*x^6 + 2*a^3*c - (4*a^2*b*c - 3*a^3*d)*x^3)/(a^4*b*x^12 +
a^5*x^9) + 1/3*(4*b^3*c - 3*a*b^2*d - a^3*f + 2*a^2*b*e)*log(b*x^3 + a)/a^5
 - 1/3*(4*b^3*c - 3*a*b^2*d - a^3*f + 2*a^2*b*e)*log(x^3)/a^5
```

Fricas [A]
time = 0.42, size = 261, normalized size = 1.49

$$-\frac{6\,(4\,ab^{3}c-3\,a^{2}b^{2}d+2\,a^{3}be-a^{4}f)x^{9}+3\,(4\,a^{2}b^{2}c-3\,a^{3}bd+2\,a^{4}e)x^{6}+2\,a^{4}c-(4\,a^{3}bc-3\,a^{4}d)x^{3}-6\,((4\,b^{4}c-3\,ab^{3}d+2\,a^{2}b^{2}e-a^{3}bf)x^{12}+(4\,ab^{3}c-3\,a^{2}b^{2}d+2\,a^{3}be-a^{4}f)x^{9})\log\left(bx^{3}+a\right)+18\,((4\,b^{4}c-3\,ab^{3}d+2\,a^{2}b^{2}e-a^{3}bf)x^{12}+(4\,ab^{3}c-3\,a^{2}b^{2}d+2\,a^{3}be-a^{4}f)x^{9})\log\left(x\right)}{18\,(a^{5}bx^{12}+a^{6}x^{9})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(6*(4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9 + 3*(4*a^2*b^2*c
 - 3*a^3*b*d + 2*a^4*e)*x^6 + 2*a^4*c - (4*a^3*b*c - 3*a^4*d)*x^3 - 6*((4*b
^4*c - 3*a*b^3*d + 2*a^2*b^2*e - a^3*b*f)*x^12 + (4*a*b^3*c - 3*a^2*b^2*d +
 2*a^3*b*e - a^4*f)*x^9)*log(b*x^3 + a) + 18*((4*b^4*c - 3*a*b^3*d + 2*a^2*
b^2*e - a^3*b*f)*x^12 + (4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9)*
log(x))/(a^5*b*x^12 + a^6*x^9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.21, size = 275, normalized size = 1.57

$$-\frac{(4\,b^{3}c-3\,ab^{2}d-a^{3}f+2\,a^{2}be)\log\left(|x|\right)}{a^{5}}+\frac{(4\,b^{4}c-3\,ab^{3}d-a^{3}bf+2\,a^{2}b^{2}e)\log\left(|bx^{3}+a|\right)}{3\,a^{5}b}-\frac{4\,b^{4}cx^{3}-3\,ab^{3}dx^{3}-a^{3}bfx^{3}+2\,a^{2}b^{2}x^{3}e+5\,ab^{3}c-4\,a^{2}b^{2}d-2\,a^{4}f+3\,a^{3}be}{3\,(bx^{3}+a)a^{5}}+\frac{44\,b^{3}cx^{9}-33\,ab^{2}dx^{9}-11\,a^{3}fx^{9}+22\,a^{2}bx^{9}e-18\,ab^{2}cx^{6}+12\,a^{2}bdx^{6}-6\,a^{3}x^{6}e+6\,a^{2}bcx^{3}-3\,a^{3}dx^{3}-2\,a^{3}c}{18\,a^{5}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -(4*b^3*c - 3*a*b^2*d - a^3*f + 2*a^2*b*e)*log(abs(x))/a^5 + 1/3*(4*b^4*c -
 3*a*b^3*d - a^3*b*f + 2*a^2*b^2*e)*log(abs(b*x^3 + a))/(a^5*b) - 1/3*(4*b^
4*c*x^3 - 3*a*b^3*d*x^3 - a^3*b*f*x^3 + 2*a^2*b^2*x^3*e + 5*a*b^3*c - 4*a^2
*b^2*d - 2*a^4*f + 3*a^3*b*e)/((b*x^3 + a)*a^5) + 1/18*(44*b^3*c*x^9 - 33*a
*b^2*d*x^9 - 11*a^3*f*x^9 + 22*a^2*b*x^9*e - 18*a*b^2*c*x^6 + 12*a^2*b*d*x^
6 - 6*a^3*x^6*e + 6*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^5*x^9)
```

Mupad [B]
time = 5.08, size = 175, normalized size = 1.00

$$\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+2\,e\,a^2\,b-3\,d\,a\,b^2+4\,c\,b^3\right)}{3\,a^5}-\frac{\frac{c}{9\,a}+\frac{x^9\,\left(-f\,a^3+2\,e\,a^2\,b-3\,d\,a\,b^2+4\,c\,b^3\right)}{3\,a^4}+\frac{x^3\,\left(3\,a\,d-4\,b\,c\right)}{18\,a^2}+\frac{x^6\,\left(2\,e\,a^2-3\,d\,a\,b+4\,c\,b^2\right)}{6\,a^3}}{b\,x^{12}+a\,x^9}-\frac{\ln\left(x\right)\,\left(-f\,a^3+2\,e\,a^2\,b-3\,d\,a\,b^2+4\,c\,b^3\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x)
```

```
[Out] (log(a + b*x^3)*(4*b^3*c - a^3*f - 3*a*b^2*d + 2*a^2*b*e))/(3*a^5) - (c/(9*
a) + (x^9*(4*b^3*c - a^3*f - 3*a*b^2*d + 2*a^2*b*e))/(3*a^4) + (x^3*(3*a*d
- 4*b*c))/(18*a^2) + (x^6*(4*b^2*c + 2*a^2*e - 3*a*b*d))/(6*a^3))/(a*x^9 +
b*x^12) - (log(x)*(4*b^3*c - a^3*f - 3*a*b^2*d + 2*a^2*b*e))/a^5
```

3.259 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=214

$$-\frac{c}{12a^2x^{12}}+\frac{2bc-ad}{9a^3x^9}-\frac{3b^2c-2abd+a^2e}{6a^4x^6}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}+\frac{b(b^3c-ab^2d+a^2be-a^3f)}{3a^5\left(a+bx^3\right)}+\frac{b(5b^3c$$

```
[Out] -1/12*c/a^2/x^12+1/9*(-a*d+2*b*c)/a^3/x^9+1/6*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/
x^6+1/3*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x^3+1/3*b*(-a^3*f+a^2*b*e-
a*b^2*d+b^3*c)/a^5/(b*x^3+a)+b*(-2*a^3*f+3*a^2*b*e-4*a*b^2*d+5*b^3*c)*ln(x)
/a^6-1/3*b*(-2*a^3*f+3*a^2*b*e-4*a*b^2*d+5*b^3*c)*ln(b*x^3+a)/a^6
```

Rubi [A]
time = 0.16, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{2bc-ad}{9a^3x^9}-\frac{c}{12a^2x^{12}}-\frac{a^2e-2abd+3b^2c}{6a^4x^6}-\frac{b\log\left(a+bx^3\right)\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{3a^6}+\frac{b\log(x)\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{a^6}+\frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^5\left(a+bx^3\right)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x]
```

```
[Out] -1/12*c/(a^2*x^12) + (2*b*c - a*d)/(9*a^3*x^9) - (3*b^2*c - 2*a*b*d + a^2*e
)/(6*a^4*x^6) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) + (b*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*(a + b*x^3)) + (b*(5*b^3*c - 4*
a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x])/a^6 - (b*(5*b^3*c - 4*a*b^2*d + 3*a^
2*b*e - 2*a^3*f)*Log[a + b*x^3])/(3*a^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5(a+bx)^2}\,dx,x,x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^2x^5}+\frac{-2bc+ad}{a^3x^4}+\frac{3b^2c-2abd+a^2e}{a^4x^3}+\frac{-4b^3c+3ab^2d-2a^2\cdots}{a^5x^2}\cdots\right.\right.$$

$$= -\frac{c}{12a^2x^{12}}+\frac{2bc-ad}{9a^3x^9}-\frac{3b^2c-2abd+a^2e}{6a^4x^6}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}+\cdots$$

Mathematica [A]

time = 0.14, size = 198, normalized size = 0.93

$$-\frac{\frac{3a^4c}{x^{12}}+\frac{4a^3(-2bc+ad)}{x^9}+\frac{6a^2(3b^2c-2abd+a^2e)}{x^6}+\frac{12a(-4b^3c+3ab^2d-2a^2be+a^3f)}{x^3}+\frac{12ab(-b^3c+ab^2d-a^2be+a^3f)}{a+bx^3}-36b(5b^3c-4ab^2d+3a^2be-2a^3f)\log(x)+12b(5b^3c-4ab^2d+3a^2be-2a^3f)\log\left(a+bx^3\right)}{36a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x]
```

```
[Out] -1/36*((3*a^4*c)/x^12 + (4*a^3*(-2*b*c + a*d))/x^9 + (6*a^2*(3*b^2*c - 2*a*
b*d + a^2*e))/x^6 + (12*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/x^3 +
 (12*a*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) - 36*b*(5*b^3*
c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x] + 12*b*(5*b^3*c - 4*a*b^2*d + 3
*a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/a^6
```

Maple [A]

time = 0.38, size = 209, normalized size = 0.98

method	result
default	$\frac{b^2\left(-\frac{a\left(a^3f-a^2be+ab^2d-b^3c\right)}{b\left(bx^3+a\right)}+\frac{\left(2a^3f-3a^2be+4ab^2d-5b^3c\right)\ln\left(bx^3+a\right)}{b}\right)}{3a^6}-\frac{c}{12a^2x^{12}}-\frac{ad-2bc}{9a^3x^9}-\frac{a^2e-2abd+3b^2c}{6a^4x^6}-\frac{a^3f-2a^2b\cdots}{\cdots}$
norman	$\frac{-\frac{c}{12a}-\frac{\left(2a^3f-3a^2be+4ab^2d-5b^3c\right)x^9}{6a^4}-\frac{(4ad-5bc)x^3}{36a^2}-\frac{\left(3a^2e-4abd+5b^2c\right)x^6}{18a^3}+\frac{b\left(2a^3bf-3a^2eb^2+4adb^3-5cb^4\right)x^{15}}{3a^6}}{x^{12}\left(bx^3+a\right)}-\frac{b\left(2a^3f-3a^2be+4\cdots\right)}{a\cdots}$
risch	$\frac{-\frac{c}{12a}-\frac{(4ad-5bc)x^3}{36a^2}-\frac{\left(3a^2e-4abd+5b^2c\right)x^6}{18a^3}-\frac{\left(2a^3f-3a^2be+4ab^2d-5b^3c\right)x^9}{6a^4}-\frac{b\left(2a^3f-3a^2be+4ab^2d-5b^3c\right)x^{12}}{3a^5}}{x^{12}\left(bx^3+a\right)}-\frac{2b\ln(x)f}{a^3}+\frac{3b^2\ln(\cdots}{a^4}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2/a^6*(-a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)+(2*a^3*f-3*a^2*b*
e+4*a*b^2*d-5*b^3*c)/b*ln(b*x^3+a))-1/12*c/a^2/x^12-1/9*(a*d-2*b*c)/a^3/x^9
-1/6*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^6-1/3*(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c
)/a^5/x^3-b*(2*a^3*f-3*a^2*b*e+4*a*b^2*d-5*b^3*c)/a^6*ln(x)
```

Maxima [A]
time = 0.28, size = 231, normalized size = 1.08

$$\frac{12\,(5\,b^4c-4\,ab^3d-2\,a^3bf+3\,a^2b^2e)x^{12}+6\,(5\,ab^3c-4\,a^2b^2d-2\,a^4f+3\,a^3be)x^9-2\,(5\,a^2b^2c-4\,a^3bd+3\,a^4e)x^6-3\,a^4c+(5\,a^3bc-4\,a^4d)x^3}{36\,(a^5bx^{15}+a^6x^{12})}-\frac{(5\,b^4c-4\,ab^3d-2\,a^3bf+3\,a^2b^2e)\log\left(bx^3+a\right)}{3\,a^6}+\frac{(5\,b^4c-4\,ab^3d-2\,a^3bf+3\,a^2b^2e)\log\left(x^3\right)}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/36*(12*(5*b^4*c - 4*a*b^3*d - 2*a^3*b*f + 3*a^2*b^2*e)*x^12 + 6*(5*a*b^3*
c - 4*a^2*b^2*d - 2*a^4*f + 3*a^3*b*e)*x^9 - 2*(5*a^2*b^2*c - 4*a^3*b*d + 3
*a^4*e)*x^6 - 3*a^4*c + (5*a^3*b*c - 4*a^4*d)*x^3)/(a^5*b*x^15 + a^6*x^12)
- 1/3*(5*b^4*c - 4*a*b^3*d - 2*a^3*b*f + 3*a^2*b^2*e)*log(b*x^3 + a)/a^6 +
1/3*(5*b^4*c - 4*a*b^3*d - 2*a^3*b*f + 3*a^2*b^2*e)*log(x^3)/a^6
```

Fricas [A]
time = 0.45, size = 310, normalized size = 1.45

$$\frac{12\,(5\,ab^4c-4\,a^2b^3d+3\,a^3b^2e-2\,a^4bf)x^{12}+6\,(5\,a^2b^3c-4\,a^3b^2d+3\,a^4be-2\,a^5f)x^9-2\,(5\,a^3b^2c-4\,a^4bd+3\,a^5e)x^6-3\,a^5c+(5\,a^4bc-4\,a^5d)x^3-12\,((5\,b^5c-4\,ab^4d+3\,a^2b^3e-2\,a^3b^2f)x^{15}+(5\,ab^4c-4\,a^2b^3d+3\,a^3b^2e-2\,a^4bf)x^{12})\log\left(bx^3+a\right)+36\,((5\,b^5c-4\,ab^4d+3\,a^2b^3e-2\,a^3b^2f)x^{15}+(5\,ab^4c-4\,a^2b^3d+3\,a^3b^2e-2\,a^4bf)x^{12})\log\left(x\right)}{36\,(a^6bx^{15}+a^7x^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*(5*a*b^4*c - 4*a^2*b^3*d + 3*a^3*b^2*e - 2*a^4*b*f)*x^12 + 6*(5*a^
2*b^3*c - 4*a^3*b^2*d + 3*a^4*b*e - 2*a^5*f)*x^9 - 2*(5*a^3*b^2*c - 4*a^4*b
*d + 3*a^5*e)*x^6 - 3*a^5*c + (5*a^4*b*c - 4*a^5*d)*x^3 - 12*((5*b^5*c - 4*
a*b^4*d + 3*a^2*b^3*e - 2*a^3*b^2*f)*x^15 + (5*a*b^4*c - 4*a^2*b^3*d + 3*a^
3*b^2*e - 2*a^4*b*f)*x^12)*log(b*x^3 + a) + 36*((5*b^5*c - 4*a*b^4*d + 3*a^
2*b^3*e - 2*a^3*b^2*f)*x^15 + (5*a*b^4*c - 4*a^2*b^3*d + 3*a^3*b^2*e - 2*a^
4*b*f)*x^12)*log(x))/(a^6*b*x^15 + a^7*x^12)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.02, size = 331, normalized size = 1.55

$$\frac{(5\,b^4c-4\,ab^3d-2\,a^3bf+3\,a^2b^2e)\log\left(|x|\right)}{a^6}-\frac{(5\,b^5c-4\,ab^4d-2\,a^3b^2f+3\,a^2b^3e)\log\left(|bx^3+a|\right)}{3\,a^6b}+\frac{5\,b^5cx^3-4\,ab^4dx^3-2\,a^3b^2fx^3+3\,a^2b^3x^3e+6\,ab^4c-5\,a^2b^3d-3\,a^4bf+4\,a^3b^2e}{3\,(bx^3+a)a^6}-\frac{125\,b^4cx^{12}-100\,ab^3dx^{12}-50\,a^3bfx^{12}+75\,a^2b^2x^{12}e-48\,ab^3cx^9+36\,a^2b^2dx^9+12\,a^4fx^9-24\,a^3bx^9e+18\,a^2b^2cx^6-12\,a^3bdx^6+6\,a^4x^6e-8\,a^3bcx^3+4\,a^4dx^3+3\,a^4c}{36\,a^6x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (5*b^4*c - 4*a*b^3*d - 2*a^3*b*f + 3*a^2*b^2*e)*log(abs(x))/a^6 - 1/3*(5*b^
5*c - 4*a*b^4*d - 2*a^3*b^2*f + 3*a^2*b^3*e)*log(abs(b*x^3 + a))/(a^6*b) +
1/3*(5*b^5*c*x^3 - 4*a*b^4*d*x^3 - 2*a^3*b^2*f*x^3 + 3*a^2*b^3*x^3*e + 6*a*
b^4*c - 5*a^2*b^3*d - 3*a^4*b*f + 4*a^3*b^2*e)/((b*x^3 + a)*a^6) - 1/36*(12
5*b^4*c*x^12 - 100*a*b^3*d*x^12 - 50*a^3*b*f*x^12 + 75*a^2*b^2*x^12*e - 48*
a*b^3*c*x^9 + 36*a^2*b^2*d*x^9 + 12*a^4*f*x^9 - 24*a^3*b*x^9*e + 18*a^2*b^2
*c*x^6 - 12*a^3*b*d*x^6 + 6*a^4*x^6*e - 8*a^3*b*c*x^3 + 4*a^4*d*x^3 + 3*a^4
*c)/(a^6*x^12)
```

Mupad [B]
time = 5.09, size = 216, normalized size = 1.01

$$\frac{\ln (x)\,\left(-2\,f\,a^3\,b+3\,e\,a^2\,b^2-4\,d\,a\,b^3+5\,c\,b^4\right)}{a^6}-\frac{\ln \left(b\,x^3+a\right)\,\left(-2\,f\,a^3\,b+3\,e\,a^2\,b^2-4\,d\,a\,b^3+5\,c\,b^4\right)}{3\,a^6}-\frac{\frac{c}{12\,a}-\frac{x^9\,\left(-2\,f\,a^3+3\,e\,a^2\,b-4\,d\,a\,b^2+5\,c\,b^3\right)}{6\,a^4}+\frac{x^3\,\left(4\,a\,d-5\,b\,c\right)}{36\,a^2}+\frac{x^6\,\left(3\,e\,a^2-4\,d\,a\,b+5\,c\,b^2\right)}{18\,a^3}-\frac{b\,x^{12}\,\left(-2\,f\,a^3+3\,e\,a^2\,b-4\,d\,a\,b^2+5\,c\,b^3\right)}{3\,a^5}}{b\,x^{15}+a\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x)
```

```
[Out] (log(x)*(5*b^4*c + 3*a^2*b^2*e - 4*a*b^3*d - 2*a^3*b*f))/a^6 - (log(a + b*x
^3)*(5*b^4*c + 3*a^2*b^2*e - 4*a*b^3*d - 2*a^3*b*f))/(3*a^6) - (c/(12*a) -
(x^9*(5*b^3*c - 2*a^3*f - 4*a*b^2*d + 3*a^2*b*e))/(6*a^4) + (x^3*(4*a*d - 5
*b*c))/(36*a^2) + (x^6*(5*b^2*c + 3*a^2*e - 4*a*b*d))/(18*a^3) - (b*x^12*(5
*b^3*c - 2*a^3*f - 4*a*b^2*d + 3*a^2*b*e))/(3*a^5))/(a*x^12 + b*x^15)
```

3.260 $\int \frac{x^9\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=369

$$-\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)x}{b^6}+\frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^4}{4b^5}+\frac{\left(b^2d-2abe+3a^2f\right)x^7}{7b^4}+\frac{(be-2af)x^{10}}{10b^3}$$

```
[Out] -a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*x/b^6+1/4*(-4*a^3*f+3*a^2*b*e-2*a
*b^2*d+b^3*c)*x^4/b^5+1/7*(3*a^2*f-2*a*b*e+b^2*d)*x^7/b^4+1/10*(-2*a*f+b*e)
*x^10/b^3+1/13*f*x^13/b^2-1/3*a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^6/(b*x
^3+a)+1/9*a^(4/3)*(-16*a^3*f+13*a^2*b*e-10*a*b^2*d+7*b^3*c)*ln(a^(1/3)+b^(1
/3)*x)/b^(19/3)-1/18*a^(4/3)*(-16*a^3*f+13*a^2*b*e-10*a*b^2*d+7*b^3*c)*ln(a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(19/3)-1/9*a^(4/3)*(-16*a^3*f+13*a^
2*b*e-10*a*b^2*d+7*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))
/b^(19/3)*3^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1842, 1901, 206, 31, 648, 631, 210, 642}

$$\frac{x^7\left(3a^2f-2abe+b^2d\right)}{7b^4}-\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^6\left(a+bx^3\right)}-\frac{ax\left(-5a^3f+4a^2be-3ab^2d+2b^3c\right)}{b^6}+\frac{x^4\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{4b^5}-\frac{a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-16a^3f+13a^2be-10ab^2d+7b^3c\right)}{3\sqrt{3}b^{19/3}}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-16a^3f+13a^2be-10ab^2d+7b^3c\right)}{18b^{19/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-16a^3f+13a^2be-10ab^2d+7b^3c\right)}{9b^{19/3}}+\frac{x^{10}(be-2af)}{10b^3}+\frac{fx^{13}}{13b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] -((a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x)/b^6) + ((b^3*c - 2*a*b^
2*d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^7)
/(7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f*x^13)/(13*b^2) - (a^2*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^6*(a + b*x^3)) - (a^(4/3)*(7*b^3*c - 1
0*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(3*Sqrt[3]*b^(19/3)) + (a^(4/3)*(7*b^3*c - 10*a*b^2*d + 13*a^2*b
*e - 16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) - (a^(4/3)*(7*b^3*c -
 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(18*b^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^6\,(a+bx^3)} - \frac{\int \frac{-a^3(b^3c-ab^2d+a^2be-a^3f)+3a^2b(b^3c-ab^2d+a^2be}{}}{}$$

$$= -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^6\,(a+bx^3)} - \frac{\int \Big(3a^2(2b^3c-3ab^2d+4a^2be-5a^3f) -}{}$$

$$= -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x}{b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5} + $$

$$= -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x}{b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5} + $$

$$= -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x}{b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5} + $$

$$= -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x}{b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5} + $$

$$= -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x}{b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5} + $$

Mathematica [A]
time = 0.23, size = 364, normalized size = 0.99

$$\frac{a(-2b^3c+3ab^2d-4a^2be+5a^3f)\,x}{b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5} + \frac{(b^2d-2abe+3a^2f)\,x^7}{7b^4} + \frac{(be-2af)x^{10}}{10b^3} + \frac{fx^{13}}{13b^2} + \frac{a^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{3b^6\,(a+bx^3)} + \frac{a^{4/3}(-7b^3c+10ab^2d-13a^2be+16a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{19/3}} - \frac{a^{4/3}(-7b^3c+10ab^2d-13a^2be+16a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{19/3}} + \frac{a^{4/3}(-7b^3c+10ab^2d-13a^2be+16a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18b^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c - 2*a*b^2*
d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^7)/(
7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f*x^13)/(13*b^2) + (a^2*(-(b^3*c)
 + a*b^2*d - a^2*b*e + a^3*f)*x)/(3*b^6*(a + b*x^3)) + (a^(4/3)*(-7*b^3*c +
 10*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/(3*Sqrt[3]*b^(19/3)) - (a^(4/3)*(-7*b^3*c + 10*a*b^2*d - 13*a^2*b*e
+ 16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) + (a^(4/3)*(-7*b^3*c + 1
0*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*b^(19/3))
```

Maple [A]
time = 0.36, size = 306, normalized size = 0.83

method	result
risch	$\frac{f\,x^{13}}{13b^2}-\frac{af\,x^{10}}{5b^3}+\frac{e\,x^{10}}{10b^2}+\frac{3a^2f\,x^7}{7b^4}-\frac{2ae\,x^7}{7b^3}+\frac{d\,x^7}{7b^2}-\frac{a^3f\,x^4}{b^5}+\frac{3a^2e\,x^4}{4b^4}-\frac{ad\,x^4}{2b^3}+\frac{c\,x^4}{4b^2}+\frac{5a^4fx}{b^6}-\frac{4a^3ex}{b^5}+\frac{3a^2d}{b^4}$
default	$\frac{\frac{1}{13}f\,x^{13}b^4-\frac{1}{5}a\,b^3f\,x^{10}+\frac{1}{10}b^4e\,x^{10}+\frac{3}{7}a^2b^2f\,x^7-\frac{2}{7}a\,b^3e\,x^7+\frac{1}{7}b^4d\,x^7-a^3bf\,x^4+\frac{3}{4}a^2b^2e\,x^4-\frac{1}{2}a\,b^3d\,x^4+\frac{1}{4}b^4c\,x^4+5a^4fx-4a^3bex+3a^2}{b^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/13*f*x^13*b^4-1/5*a*b^3*f*x^10+1/10*b^4*e*x^10+3/7*a^2*b^2*f*x^7-2
/7*a*b^3*e*x^7+1/7*b^4*d*x^7-a^3*b*f*x^4+3/4*a^2*b^2*e*x^4-1/2*a*b^3*d*x^4+
1/4*b^4*c*x^4+5*a^4*f*x-4*a^3*b*e*x+3*a^2*b^2*d*x-2*a*b^3*c*x)-a^2/b^6*((-1
/3*a^3*f+1/3*a^2*b*e-1/3*a*b^2*d+1/3*b^3*c)*x/(b*x^3+a)+1/3*(16*a^3*f-13*a^
2*b*e+10*a*b^2*d-7*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^
(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.51, size = 378, normalized size = 1.02

$$-\frac{(a^2b^3c-a^3b^2d-a^5f+a^4be)x}{3\,(b^7x^3+ab^6)}+\frac{140\,b^4fx^{13}-182\,(2\,ab^3f-b^4e)x^{10}+260\,(b^4d+3\,a^2b^2f-2\,ab^3e)x^7+455\,(b^4c-2\,ab^3d-4\,a^3bf+3\,a^2b^2e)x^4-1820\,(2\,ab^3c-3\,a^2b^2d-5\,a^4f+4\,a^3be)x}{1820\,b^6}+\frac{\sqrt{3}\,(7\,a^2b^3c-10\,a^3b^2d-16\,a^5f+13\,a^4be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(7\,a^2b^3c-10\,a^3b^2d-16\,a^5f+13\,a^4be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(7\,a^2b^3c-10\,a^3b^2d-16\,a^5f+13\,a^4be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*x/(b^7*x^3 + a*b^6) + 1/1820
*(140*b^4*f*x^13 - 182*(2*a*b^3*f - b^4*e)*x^10 + 260*(b^4*d + 3*a^2*b^2*f
```

```
- 2*a*b^3*e)*x^7 + 455*(b^4*c - 2*a*b^3*d - 4*a^3*b*f + 3*a^2*b^2*e)*x^4 -
1820*(2*a*b^3*c - 3*a^2*b^2*d - 5*a^4*f + 4*a^3*b*e)*x)/b^6 + 1/9*sqrt(3)*(
7*a^2*b^3*c - 10*a^3*b^2*d - 16*a^5*f + 13*a^4*b*e)*arctan(1/3*sqrt(3)*(2*x
 - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)^(2/3)) - 1/18*(7*a^2*b^3*c - 10*a^3
*b^2*d - 16*a^5*f + 13*a^4*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^7
*(a/b)^(2/3)) + 1/9*(7*a^2*b^3*c - 10*a^3*b^2*d - 16*a^5*f + 13*a^4*b*e)*lo
g(x + (a/b)^(1/3))/(b^7*(a/b)^(2/3))
```

Fricas [**A**]
time = 0.41, size = 488, normalized size = 1.32

$$\frac{1260 b^{5} f x^{16} + 126 \left(13 b^{5} e - 16 a b^{4} f\right) x^{13} + 234 \left(10 b^{5} d - 13 a b^{4} e + 16 a^{2} b^{3} f\right) x^{10} + 585 \left(7 b^{5} c - 10 a b^{4} d + 13 a^{2} b^{3} e - 16 a^{3} b^{2} f\right) x^{7} - 4095 \left(7 a b^{4} c - 10 a^{2} b^{3} d + 13 a^{3} b^{2} e - 16 a^{4} b f\right) x^{4} - 1820 \sqrt{3} \left(7 a^{2} b^{3} c - 10 a^{3} b^{2} d + 13 a^{4} b e - 16 a^{5} f + \left(7 a b^{4} c - 10 a^{2} b^{3} d + 13 a^{3} b^{2} e - 16 a^{4} b f\right) x^{3}\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{2 \sqrt{3} b x \left(-\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3} a}{3 a}\right) + 910 \left(7 a^{2} b^{3} c - 10 a^{3} b^{2} d + 13 a^{4} b e - 16 a^{5} f + \left(7 a b^{4} c - 10 a^{2} b^{3} d + 13 a^{3} b^{2} e - 16 a^{4} b f\right) x^{3}\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - 1820 \left(7 a^{2} b^{3} c - 10 a^{3} b^{2} d + 13 a^{4} b e - 16 a^{5} f + \left(7 a b^{4} c - 10 a^{2} b^{3} d + 13 a^{3} b^{2} e - 16 a^{4} b f\right) x^{3}\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) - 5460 \left(7 a^{2} b^{3} c - 10 a^{3} b^{2} d + 13 a^{4} b e - 16 a^{5} f\right) x}{16380 \left(b^{7} x^{3} + a b^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16380*(1260*b^5*f*x^16 + 126*(13*b^5*e - 16*a*b^4*f)*x^13 + 234*(10*b^5*d
 - 13*a*b^4*e + 16*a^2*b^3*f)*x^10 + 585*(7*b^5*c - 10*a*b^4*d + 13*a^2*b^3
*e - 16*a^3*b^2*f)*x^7 - 4095*(7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16
*a^4*b*f)*x^4 - 1820*sqrt(3)*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*
a^5*f + (7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^
(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*(7*a^2*b
^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f + (7*a*b^4*c - 10*a^2*b^3*d + 1
3*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3)) - 1820*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f + (7*a
*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^(1/3)*log(x
- (-a/b)^(1/3)) - 5460*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f)
*x)/(b^7*x^3 + a*b^6)
```

Sympy [**A**]
time = 85.92, size = 500, normalized size = 1.36

$$x^{10} \left(- \frac{a f}{5 b^{3}} + \frac{e}{10 b^{2}}\right) + x^{7} \cdot \left(\frac{3 a^{2} f}{7 b^{4}} - \frac{2 a e}{7 b^{3}} + \frac{d}{7 b^{2}}\right) + x^{4} \left(- \frac{a^{3} f}{b^{5}} + \frac{3 a^{2} e}{4 b^{4}} - \frac{a d}{2 b^{3}} + \frac{c}{4 b^{2}}\right) + x \left(\frac{5 a^{4} f}{b^{6}} - \frac{4 a^{3} e}{b^{5}} + \frac{3 a^{2} d}{b^{4}} - \frac{2 a c}{b^{3}}\right) + \frac{x \left(a^{5} f - a^{4} b e + a^{3} b^{2} d - a^{2} b^{3} c\right)}{3 a b^{6} + 3 b^{7} x^{3}} + \operatorname{RootSum}{\left(729 t^{3} b^{19} + 4096 a^{13} f^{3} - 9984 a^{12} b e f^{2} + 7680 a^{11} b^{2} d f^{2} + 8112 a^{11} b^{2} e^{2} f - 5376 a^{10} b^{3} c f^{2} - 12480 a^{10} b^{3} d e f - 2197 a^{10} b^{3} e^{3} + 8736 a^{9} b^{4} c e f + 4800 a^{9} b^{4} d^{2} f + 5070 a^{9} b^{4} d e^{2} - 6720 a^{8} b^{5} c d f - 3549 a^{8} b^{5} c e^{2} - 3900 a^{8} b^{5} d^{2} e + 2352 a^{7} b^{6} c^{2} f + 5460 a^{7} b^{6} c d e + 1000 a^{7} b^{6} d^{3} - 1911 a^{6} b^{7} c^{2} e - 2100 a^{6} b^{7} c d^{2} + 1470 a^{5} b^{8} c^{2} d - 343 a^{4} b^{9} c^{3}, \left(t \mapsto t \log{\left(- \frac{9 t b^{6}}{16 a^{4} f - 13 a^{3} b e + 10 a^{2} b^{2} d - 7 a b^{3} c} + x \right)} \right)\right)} + \frac{f x^{13}}{13 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**10*(-a*f/(5*b**3) + e/(10*b**2)) + x**7*(3*a**2*f/(7*b**4) - 2*a*e/(7*b*
*3) + d/(7*b**2)) + x**4*(-a**3*f/b**5 + 3*a**2*e/(4*b**4) - a*d/(2*b**3) +
 c/(4*b**2)) + x*(5*a**4*f/b**6 - 4*a**3*e/b**5 + 3*a**2*d/b**4 - 2*a*c/b**
3) + x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*b**6 + 3*b**7*x
**3) + RootSum(729*_t**3*b**19 + 4096*a**13*f**3 - 9984*a**12*b*e*f**2 + 76
80*a**11*b**2*d*f**2 + 8112*a**11*b**2*e**2*f - 5376*a**10*b**3*c*f**2 - 12
480*a**10*b**3*d*e*f - 2197*a**10*b**3*e**3 + 8736*a**9*b**4*c*e*f + 4800*a
**9*b**4*d**2*f + 5070*a**9*b**4*d*e**2 - 6720*a**8*b**5*c*d*f - 3549*a**8*
b**5*c*e**2 - 3900*a**8*b**5*d**2*e + 2352*a**7*b**6*c**2*f + 5460*a**7*b**
```

```
6*c*d*e + 1000*a**7*b**6*d**3 - 1911*a**6*b**7*c**2*e - 2100*a**6*b**7*c*d*
*2 + 1470*a**5*b**8*c**2*d - 343*a**4*b**9*c**3, Lambda(_t, _t*log(-9*_t*b*
*6/(16*a**4*f - 13*a**3*b*e + 10*a**2*b**2*d - 7*a*b**3*c) + x))) + f*x**13
/(13*b**2)
```

Giac [A]
time = 0.86, size = 451, normalized size = 1.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*a*b^3*c - 10*(-a*b^2)^(1/3)*a^2*b^2*d - 16*(-
a*b^2)^(1/3)*a^4*f + 13*(-a*b^2)^(1/3)*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/b^7 - 1/9*(7*a^2*b^3*c - 10*a^3*b^2*d - 16*a^5*f
 + 13*a^4*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^6) + 1/18*(7*(-
a*b^2)^(1/3)*a*b^3*c - 10*(-a*b^2)^(1/3)*a^2*b^2*d - 16*(-a*b^2)^(1/3)*a^4*
f + 13*(-a*b^2)^(1/3)*a^3*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7
 - 1/3*(a^2*b^3*c*x - a^3*b^2*d*x - a^5*f*x + a^4*b*x*e)/((b*x^3 + a)*b^6)
+ 1/1820*(140*b^24*f*x^13 - 364*a*b^23*f*x^10 + 182*b^24*x^10*e + 260*b^24*
d*x^7 + 780*a^2*b^22*f*x^7 - 520*a*b^23*x^7*e + 455*b^24*c*x^4 - 910*a*b^23
*d*x^4 - 1820*a^3*b^21*f*x^4 + 1365*a^2*b^22*x^4*e - 3640*a*b^23*c*x + 5460
*a^2*b^22*d*x + 9100*a^4*b^20*f*x - 7280*a^3*b^21*x*e)/b^26
```

Mupad [B]
time = 0.35, size = 481, normalized size = 1.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^10*(e/(10*b^2) - (a*f)/(5*b^3)) - x*((2*a*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/
b^3))/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b))
/b - (a^2*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b^2) - x^7
*((a^2*f)/(7*b^4) - d/(7*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(7*b)) + x^4*(c
/(4*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))/(4*b^2) + (a*((a^2*f)/b^4 - d/b^2 +
(2*a*(e/b^2 - (2*a*f)/b^3))/b))/(2*b)) + (f*x^13)/(13*b^2) + (x*((a^5*f)/3
- (a^2*b^3*c)/3 + (a^3*b^2*d)/3 - (a^4*b*e)/3))/(a*b^6 + b^7*x^3) + (a^(4/3
)*log(b^(1/3)*x + a^(1/3))*(7*b^3*c - 16*a^3*f - 10*a*b^2*d + 13*a^2*b*e))/
(9*b^(19/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3
^(1/2)*1i)/2 - 1/2)*(7*b^3*c - 16*a^3*f - 10*a*b^2*d + 13*a^2*b*e))/(9*b^(1
9/3)) - (a^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*
1i)/2 + 1/2)*(7*b^3*c - 16*a^3*f - 10*a*b^2*d + 13*a^2*b*e))/(9*b^(19/3))
```

3.261 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=335

$$\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^2}{2b^5}+\frac{(b^2d-2abe+3a^2f)\,x^5}{5b^4}+\frac{(be-2af)x^8}{8b^3}+\frac{fx^{11}}{11b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)}{3b^5\,(a+bx^3)}$$

```
[Out] 1/2*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*x^2/b^5+1/5*(3*a^2*f-2*a*b*e+b^2*d
)*x^5/b^4+1/8*(-2*a*f+b*e)*x^8/b^3+1/11*f*x^11/b^2+1/3*a*(-a^3*f+a^2*b*e-a*
b^2*d+b^3*c)*x^2/b^5/(b*x^3+a)+1/9*a^(2/3)*(-14*a^3*f+11*a^2*b*e-8*a*b^2*d+
5*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(17/3)-1/18*a^(2/3)*(-14*a^3*f+11*a^2*b*e-
8*a*b^2*d+5*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(17/3)+1/9*a
^(2/3)*(-14*a^3*f+11*a^2*b*e-8*a*b^2*d+5*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/b^(17/3)*3^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1842, 1865, 1850, 1502, 298, 31, 648, 631, 210, 642}

$$\frac{x^5(3a^2f-2abe+b^2d)}{5b^4}+\frac{x^2(-4a^3f+3a^2be-2ab^2d+b^3c)}{2b^5}+\frac{ax^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5\,(a+bx^3)}+\frac{a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(-14a^3f+11a^2be-8ab^2d+5b^3c)}{3\sqrt{3}\,b^{17/3}}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(-14a^3f+11a^2be-8ab^2d+5b^3c)}{18b^{17/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(-14a^3f+11a^2be-8ab^2d+5b^3c)}{9b^{17/3}}+\frac{x^8(be-2af)}{8b^3}+\frac{fx^{11}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2)/(2*b^5) + ((b^2*d - 2*a*b*e
 + 3*a^2*f)*x^5)/(5*b^4) + ((b*e - 2*a*f)*x^8)/(8*b^3) + (f*x^11)/(11*b^2)
+ (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^5*(a + b*x^3)) + (a^(2/3
)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(17/3)) + (a^(2/3)*(5*b^3*c - 8*a*b^2*
d + 11*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(17/3)) - (a^(2/3
)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(18*b^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1865

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx &= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^5\,(a+bx^3)} - \frac{\int \frac{2a^2b(b^3c-ab^2d+a^2be-a^3f)x-3ab^2(b^3c-ab^2d+a^2b}{}}{} \\
&= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^5\,(a+bx^3)} - \frac{\int \frac{x(2a^2b(b^3c-ab^2d+a^2be-a^3f)-3ab^2(b^3c-ab^2d+a^2}{}}{} \\
&= \frac{fx^{11}}{11b^2} + \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^5\,(a+bx^3)} - \frac{\int \frac{x(22a^2b^2(b^3c-ab^2d+a^2be-a^3f)-33ab^3}{}}{} \\
&= \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^5\,(a+bx^3)} - \frac{\int \frac{x(176a^2b^3(b^3c-ab}{}}{} \\
&= \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^5\,(a+bx^3)} - \frac{\int \left(-264ab^3(b^3c\right.}{} \\
&= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^2}{2b^5} + \frac{(b^2d-2abe+3a^2f)\,x^5}{5b^4} + \frac{(be-2af)x}{8b^3} \\
&= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^2}{2b^5} + \frac{(b^2d-2abe+3a^2f)\,x^5}{5b^4} + \frac{(be-2af)x}{8b^3} \\
&= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^2}{2b^5} + \frac{(b^2d-2abe+3a^2f)\,x^5}{5b^4} + \frac{(be-2af)x}{8b^3} \\
&= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^2}{2b^5} + \frac{(b^2d-2abe+3a^2f)\,x^5}{5b^4} + \frac{(be-2af)x}{8b^3} \\
&= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^2}{2b^5} + \frac{(b^2d-2abe+3a^2f)\,x^5}{5b^4} + \frac{(be-2af)x}{8b^3}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 319, normalized size = 0.95

$$
\frac{1980b^{2/3}(b^3c-2ab^2d+3a^2be-4a^3f)\,x^2+792b^{5/3}(b^2d-2abe+3a^2f)\,x^5+495b^{8/3}(be-2af)x^8+360b^{11/3}fx^{11}+\frac{1320ab^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{a+bx^3}-440\sqrt{3}\,a^{2/3}(-5b^3c+8ab^2d-11a^2be+14a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-440a^{2/3}(-5b^3c+8ab^2d-11a^2be+14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+220a^{2/3}(-5b^3c+8ab^2d-11a^2be+14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{3960b^{17/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (1980*b^(2/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2 + 792*b^(5/3)*(
b^2*d - 2*a*b*e + 3*a^2*f)*x^5 + 495*b^(8/3)*(b*e - 2*a*f)*x^8 + 360*b^(11/
3)*f*x^11 + (1320*a*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a + b
```

```
*x^3) - 440*Sqrt[3]*a^(2/3)*(-5*b^3*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 440*a^(2/3)*(-5*b^3*c + 8*a*b
^2*d - 11*a^2*b*e + 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*a^(2/3)*(-5*b^
3*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(3960*b^(17/3))
```

Maple [A]
time = 0.36, size = 258, normalized size = 0.77

method	result
risch	$\frac{f x^{11}}{11 b^2} - \frac{x^8 f a}{4 b^3} + \frac{x^8 e}{8 b^2} + \frac{3 x^5 f a^2}{5 b^4} - \frac{2 x^5 a e}{5 b^3} + \frac{x^5 d}{5 b^2} - \frac{2 x^2 a^3 f}{b^5} + \frac{3 x^2 a^2 e}{2 b^4} - \frac{x^2 a d}{b^3} + \frac{x^2 c}{2 b^2} + \frac{\left(-\frac{1}{3} a^4 f + \frac{1}{3} a^3 b e - \frac{1}{3} a^2 b^2 d + \frac{1}{3} a b^3 \right.}{b^5 (b x^3 + a)}$
default	$-\frac{-\frac{b^3 f x^{11}}{11} + \frac{(2 f a b^2 - e b^3) x^8}{8} + \frac{(-3 f a^2 b + 2 a b^2 e - b^3 d) x^5}{5} + \frac{(4 a^3 f - 3 a^2 b e + 2 a b^2 d - b^3 c) x^2}{2}}{b^5} + \frac{a \left(\frac{(-\frac{1}{3} a^3 f + \frac{1}{3} a^2 b e - \frac{1}{3} a b^2 d + \frac{1}{3} b^3 c) x^2}{b x^3 + a} + (\frac{14}{3} \right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*(-1/11*b^3*f*x^11+1/8*(2*a*b^2*f-b^3*e)*x^8+1/5*(-3*a^2*b*f+2*a*b^2*
e-b^3*d)*x^5+1/2*(4*a^3*f-3*a^2*b*e+2*a*b^2*d-b^3*c)*x^2)+a/b^5*((-1/3*a^3*
f+1/3*a^2*b*e-1/3*a*b^2*d+1/3*b^3*c)*x^2/(b*x^3+a)+(14/3*a^3*f-11/3*a^2*b*e
+8/3*a*b^2*d-5/3*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(
1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.49, size = 333, normalized size = 0.99

$$\frac{(ab^3c - a^2b^2d - a^4f + a^3be)x^2}{3(b^6x^3 + ab^5)} - \frac{\sqrt{3}(5ab^3c - 8a^2b^2d - 14a^4f + 11a^3be)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{40b^3fx^{11} - 55(2ab^2f - b^3e)x^8 + 88(b^3d + 3a^2bf - 2ab^2e)x^5 + 220(b^3c - 2ab^2d - 4a^3f + 3a^2be)x^2}{440b^5} - \frac{(5ab^3c - 8a^2b^2d - 14a^4f + 11a^3be)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(5ab^3c - 8a^2b^2d - 14a^4f + 11a^3be)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*x^2/(b^6*x^3 + a*b^5) - 1/9*sqr
t(3)*(5*a*b^3*c - 8*a^2*b^2*d - 14*a^4*f + 11*a^3*b*e)*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(1/3)) + 1/440*(40*b^3*f*x^11 -
55*(2*a*b^2*f - b^3*e)*x^8 + 88*(b^3*d + 3*a^2*b*f - 2*a*b^2*e)*x^5 + 220*(
```

```
b^3*c - 2*a*b^2*d - 4*a^3*f + 3*a^2*b*e)*x^2)/b^5 - 1/18*(5*a*b^3*c - 8*a^2
*b^2*d - 14*a^4*f + 11*a^3*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6
*(a/b)^(1/3)) + 1/9*(5*a*b^3*c - 8*a^2*b^2*d - 14*a^4*f + 11*a^3*b*e)*log(x
 + (a/b)^(1/3))/(b^6*(a/b)^(1/3))
```

Fricas [**A**]
time = 0.39, size = 455, normalized size = 1.36

$$\frac{360\,b^4fx^{14}+45\,(11\,b^4e-14\,ab^3f)x^{11}+99\,(8\,b^4d-11\,ab^3e+14\,a^2b^2f)x^8+396\,(5\,b^4c-8\,ab^3d+11\,a^2b^2e-14\,a^3bf)x^5+660\,(5\,ab^3c-8\,a^2b^2d+11\,a^3be-14\,a^4f)x^2-440\,\sqrt{3}\,(5\,ab^3c-8\,a^2b^2d+11\,a^3be-14\,a^4f+(5\,b^4c-8\,ab^3d+11\,a^2b^2e-14\,a^3bf)x^3)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}\,a}{3\,a}\right)+220\,(5\,ab^3c-8\,a^2b^2d+11\,a^3be-14\,a^4f+(5\,b^4c-8\,ab^3d+11\,a^2b^2e-14\,a^3bf)x^3)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-440\,(5\,ab^3c-8\,a^2b^2d+11\,a^3be-14\,a^4f+(5\,b^4c-8\,ab^3d+11\,a^2b^2e-14\,a^3bf)x^3)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{3960\,(b^6x^3+ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3960*(360*b^4*f*x^14 + 45*(11*b^4*e - 14*a*b^3*f)*x^11 + 99*(8*b^4*d - 11
*a*b^3*e + 14*a^2*b^2*f)*x^8 + 396*(5*b^4*c - 8*a*b^3*d + 11*a^2*b^2*e - 14
*a^3*b*f)*x^5 + 660*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f)*x^2 -
 440*sqrt(3)*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f + (5*b^4*c -
8*a*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f)*x^3)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*
sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*(5*a*b^3*c - 8*a^2*b^2*d
 + 11*a^3*b*e - 14*a^4*f + (5*b^4*c - 8*a*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f
)*x^3)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/
3)) - 440*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f + (5*b^4*c - 8*a
*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2
/b^2)^(2/3)))/(b^6*x^3 + a*b^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.11, size = 442, normalized size = 1.32

$$\frac{\left(5\,ab^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-8\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,a^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+11\,a^3b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5}+\frac{\sqrt{3}\left(5\,(-ab^2)^{\frac{2}{3}}\,b^3c-8\,(-ab^2)^{\frac{2}{3}}\,ab^2d-14\,(-ab^2)^{\frac{2}{3}}\,a^3f+11\,(-ab^2)^{\frac{2}{3}}\,a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^7}+\frac{ab^3cx^2-a^2b^2dx^2-a^4fx^2+a^3bx^2e}{3\,(bx^3+a)b^5}-\frac{\left(5\,(-ab^2)^{\frac{2}{3}}\,b^3c-8\,(-ab^2)^{\frac{2}{3}}\,ab^2d-14\,(-ab^2)^{\frac{2}{3}}\,a^3f+11\,(-ab^2)^{\frac{2}{3}}\,a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^7}+\frac{40\,b^{20}fx^{11}-110\,ab^{19}fx^8+55\,b^{20}x^8e+88\,b^{20}dx^5+264\,a^2b^{18}fx^5-176\,ab^{19}x^5e+220\,b^{20}cx^2-440\,ab^{19}dx^2-880\,a^3b^{17}fx^2+660\,a^2b^{18}x^2e}{440\,b^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(5*a*b^3*c*(-a/b)^(1/3) - 8*a^2*b^2*d*(-a/b)^(1/3) - 14*a^4*f*(-a/b)^(1
/3) + 11*a^3*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b
^5) + 1/9*sqrt(3)*(5*(-a*b^2)^(2/3)*b^3*c - 8*(-a*b^2)^(2/3)*a*b^2*d - 14*(
```

```
-a*b^2)^(2/3)*a^3*f + 11*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/b^7 + 1/3*(a*b^3*c*x^2 - a^2*b^2*d*x^2 - a^4*f*
x^2 + a^3*b*x^2*e)/((b*x^3 + a)*b^5) - 1/18*(5*(-a*b^2)^(2/3)*b^3*c - 8*(-a
*b^2)^(2/3)*a*b^2*d - 14*(-a*b^2)^(2/3)*a^3*f + 11*(-a*b^2)^(2/3)*a^2*b*e)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7 + 1/440*(40*b^20*f*x^11 - 110*
a*b^19*f*x^8 + 55*b^20*x^8*e + 88*b^20*d*x^5 + 264*a^2*b^18*f*x^5 - 176*a*b
^19*x^5*e + 220*b^20*c*x^2 - 440*a*b^19*d*x^2 - 880*a^3*b^17*f*x^2 + 660*a^
2*b^18*x^2*e)/b^22
```

Mupad [B]

time = 5.28, size = 362, normalized size = 1.08

$$x^8\left(\frac{e}{8\,b^2}-\frac{a\,f}{4\,b^3}\right)-x^5\left(\frac{a^2\,f}{5\,b^4}-\frac{d}{5\,b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{5\,b}\right)+x^2\left(\frac{c}{2\,b^2}-\frac{a^2\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{2\,b^2}+\frac{a\,\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)+\frac{f\,x^{11}}{11\,b^2}-\frac{x^2\,\left(\frac{f\,a^4}{3}-\frac{e\,a^3\,b}{3}+\frac{d\,a^2\,b^2}{3}-\frac{c\,a\,b^3}{3}\right)}{b^6\,x^3+a\,b^5}+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-14\,f\,a^3+11\,e\,a^2\,b-8\,d\,a\,b^2+5\,c\,b^3\right)}{9\,b^{17/3}}-\frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+11\,e\,a^2\,b-8\,d\,a\,b^2+5\,c\,b^3\right)}{9\,b^{17/3}}+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+11\,e\,a^2\,b-8\,d\,a\,b^2+5\,c\,b^3\right)}{9\,b^{17/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^8*(e/(8*b^2) - (a*f)/(4*b^3)) - x^5*((a^2*f)/(5*b^4) - d/(5*b^2) + (2*a*(
e/b^2 - (2*a*f)/b^3))/(5*b)) + x^2*(c/(2*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))
/(2*b^2) + (a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b) + (
f*x^11)/(11*b^2) - (x^2*((a^4*f)/3 + (a^2*b^2*d)/3 - (a*b^3*c)/3 - (a^3*b*e
)/3))/(a*b^5 + b^6*x^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*b^3*c - 14*a
^3*f - 8*a*b^2*d + 11*a^2*b*e))/(9*b^(17/3)) - (a^(2/3)*log(3^(1/2)*a^(1/3)
*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*b^3*c - 14*a^3*f - 8
*a*b^2*d + 11*a^2*b*e))/(9*b^(17/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*
b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c - 14*a^3*f - 8*a*b^2*d
 + 11*a^2*b*e))/(9*b^(17/3))
```

3.262 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=328

$$\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5}+\frac{(b^2d-2abe+3a^2f)\,x^4}{4b^4}+\frac{(be-2af)x^7}{7b^3}+\frac{fx^{10}}{10b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f}{3b^5\,(a+bx^3)}$$

```
[Out] (-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*x/b^5+1/4*(3*a^2*f-2*a*b*e+b^2*d)*x^4/
b^4+1/7*(-2*a*f+b*e)*x^7/b^3+1/10*f*x^10/b^2+1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+
b^3*c)*x/b^5/(b*x^3+a)-1/9*a^(1/3)*(-13*a^3*f+10*a^2*b*e-7*a*b^2*d+4*b^3*c)
*ln(a^(1/3)+b^(1/3)*x)/b^(16/3)+1/18*a^(1/3)*(-13*a^3*f+10*a^2*b*e-7*a*b^2*
d+4*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(16/3)+1/9*a^(1/3)*(
-13*a^3*f+10*a^2*b*e-7*a*b^2*d+4*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/b^(16/3)*3^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1842, 1901, 206, 31, 648, 631, 210, 642}

$$\frac{x^4(3a^2f-2abe+b^2d)}{4b^4}+\frac{\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(-13a^3f+10a^2be-7ab^2d+4b^3c)}{3\sqrt{3}\,b^{16/3}}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(-13a^3f+10a^2be-7ab^2d+4b^3c)}{9b^{16/3}}+\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5\,(a+bx^3)}+\frac{x(-4a^3f+3a^2be-2ab^2d+b^3c)}{b^5}+\frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(-13a^3f+10a^2be-7ab^2d+4b^3c)}{18b^{16/3}}+\frac{x^7(be-2af)}{7b^3}+\frac{fx^{10}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a*b*e + 3*a
^2*f)*x^4)/(4*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^10)/(10*b^2) + (a*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^5*(a + b*x^3)) + (a^(1/3)*(4*b^3
*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(3*Sqrt[3]*b^(16/3)) - (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a
^2*b*e - 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(16/3)) + (a^(1/3)*(4*b^3
*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(18*b^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

reeQ[{a, b}, x]

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^5\,(a+bx^3)} - \frac{\int \frac{a^2(b^3c-ab^2d+a^2be-a^3f)-3ab(b^3c-ab^2d+a^2be-a^3}{a}}{}$$

$$= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^5\,(a+bx^3)} - \frac{\int \Big(-3a(b^3c-2ab^2d+3a^2be-4a^3f)-3}{}$$

$$= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5} + \frac{(b^2d-2abe+3a^2f)\,x^4}{4b^4} + \frac{(be-2af)x}{7b^3}$$

$$= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5} + \frac{(b^2d-2abe+3a^2f)\,x^4}{4b^4} + \frac{(be-2af)x}{7b^3}$$

$$= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5} + \frac{(b^2d-2abe+3a^2f)\,x^4}{4b^4} + \frac{(be-2af)x}{7b^3}$$

$$= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5} + \frac{(b^2d-2abe+3a^2f)\,x^4}{4b^4} + \frac{(be-2af)x}{7b^3}$$

$$= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5} + \frac{(b^2d-2abe+3a^2f)\,x^4}{4b^4} + \frac{(be-2af)x}{7b^3}$$

Mathematica [A]
time = 0.16, size = 315, normalized size = 0.96

$$\frac{1260\sqrt[3]{b}\,(b^3c-2ab^2d+3a^2be-4a^3f)\,x+315b^{4/3}(b^2d-2abe+3a^2f)\,x^4+180b^{7/3}(be-2af)x^7+126b^{10/3}fx^{10}+\frac{420a\sqrt[3]{b}\,(b^3c-ab^2d+a^2be-a^3f)x}{a+bx^3}-140\sqrt{3}\,\sqrt[3]{a}\,(-4b^3c+7ab^2d-10a^2be+13a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+140\sqrt[3]{a}\,(-4b^3c+7ab^2d-10a^2be+13a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-70\sqrt[3]{a}\,(-4b^3c+7ab^2d-10a^2be+13a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{1260b^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (1260*b^(1/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x + 315*b^(4/3)*(b^
2*d - 2*a*b*e + 3*a^2*f)*x^4 + 180*b^(7/3)*(b*e - 2*a*f)*x^7 + 126*b^(10/3)
*f*x^10 + (420*a*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3)
 - 140*Sqrt[3]*a^(1/3)*(-4*b^3*c + 7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-4*b^3*c + 7*a*b^2*d
- 10*a^2*b*e + 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x] - 70*a^(1/3)*(-4*b^3*c +
7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(1260*b^(16/3))
```

Maple [A]
time = 0.35, size = 258, normalized size = 0.79

method	result
risch	$\frac{fx^{10}}{10b^2} - \frac{2afx^7}{7b^3} + \frac{ex^7}{7b^2} + \frac{3a^2fx^4}{4b^4} - \frac{aex^4}{2b^3} + \frac{dx^4}{4b^2} - \frac{4a^3fx}{b^5} + \frac{3a^2ex}{b^4} - \frac{2adx}{b^3} + \frac{cx}{b^2} + \frac{\left(-\frac{1}{3}a^4f + \frac{1}{3}a^3be - \frac{1}{3}a^2b^2d + \frac{1}{3}ab^3c\right.}{b^5(bx^3+a)}$
default	$-\frac{-\frac{1}{10}b^3fx^{10} + \frac{2}{7}ab^2fx^7 - \frac{1}{7}b^3ex^7 - \frac{3}{4}a^2bfx^4 + \frac{1}{2}ab^2ex^4 - \frac{1}{4}b^3dx^4 + 4a^3fx - 3a^2bex + 2ab^2dx - b^3cx}{b^5} + \frac{a\left(\frac{\left(-\frac{1}{3}a^3f + \frac{1}{3}a^2be - \frac{1}{3}ab^2d + \right.}{bx^3+a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*(-1/10*b^3*f*x^10+2/7*a*b^2*f*x^7-1/7*b^3*e*x^7-3/4*a^2*b*f*x^4+1/2*
a*b^2*e*x^4-1/4*b^3*d*x^4+4*a^3*f*x-3*a^2*b*e*x+2*a*b^2*d*x-b^3*c*x)+a/b^5*
((-1/3*a^3*f+1/3*a^2*b*e-1/3*a*b^2*d+1/3*b^3*c)*x/(b*x^3+a)+1/3*(13*a^3*f-1
0*a^2*b*e+7*a*b^2*d-4*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/
b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]

time = 0.50, size = 329, normalized size = 1.00

$$\frac{(ab^3c - a^2b^2d - a^4f + a^3be)x}{3\,(b^6x^3 + ab^5)} + \frac{14\,b^3fx^{10} - 20\,(2\,ab^2f - b^3e)x^7 + 35\,(b^3d + 3\,a^2bf - 2\,ab^2e)x^4 + 140\,(b^3c - 2\,ab^2d - 4\,a^3f + 3\,a^2be)x}{140\,b^5} - \frac{\sqrt{3}\,(4\,ab^3c - 7\,a^2b^2d - 13\,a^4f + 10\,a^3be)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(4\,ab^3c - 7\,a^2b^2d - 13\,a^4f + 10\,a^3be)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(4\,ab^3c - 7\,a^2b^2d - 13\,a^4f + 10\,a^3be)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*x/(b^6*x^3 + a*b^5) + 1/140*(14
*b^3*f*x^10 - 20*(2*a*b^2*f - b^3*e)*x^7 + 35*(b^3*d + 3*a^2*b*f - 2*a*b^2*
e)*x^4 + 140*(b^3*c - 2*a*b^2*d - 4*a^3*f + 3*a^2*b*e)*x)/b^5 - 1/9*sqrt(3)
```

```
*(4*a*b^3*c - 7*a^2*b^2*d - 13*a^4*f + 10*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(2/3)) + 1/18*(4*a*b^3*c - 7*a^2*b^2
*d - 13*a^4*f + 10*a^3*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6*(a/
b)^(2/3)) - 1/9*(4*a*b^3*c - 7*a^2*b^2*d - 13*a^4*f + 10*a^3*b*e)*log(x + (
a/b)^(1/3))/(b^6*(a/b)^(2/3))
```

Fricas [A]
time = 0.39, size = 423, normalized size = 1.29

$$\frac{126\,b^4 f x^{13} + 18\,(10\,b^4 e - 13\,a b^3 f)x^{10} + 45\,(7\,b^4 d - 10\,a b^3 e + 13\,a^2 b^2 f)x^7 + 315\,(4\,b^4 c - 7\,a b^3 d + 10\,a^2 b^2 e - 13\,a^3 b f)x^4 - 140\,\sqrt{3}\,(4\,a b^3 c - 7\,a^2 b^2 d + 10\,a^3 b e - 13\,a^4 f + (4\,b^4 c - 7\,a b^3 d + 10\,a^2 b^2 e - 13\,a^3 b f)x^3)\left(\frac{a}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{2\,\sqrt{3}\,b x \left(\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}\,a}{3\,a}\right) + 70\,(4\,a b^3 c - 7\,a^2 b^2 d + 10\,a^3 b e - 13\,a^4 f + (4\,b^4 c - 7\,a b^3 d + 10\,a^2 b^2 e - 13\,a^3 b f)x^3)\left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 140\,(4\,a b^3 c - 7\,a^2 b^2 d + 10\,a^3 b e - 13\,a^4 f + (4\,b^4 c - 7\,a b^3 d + 10\,a^2 b^2 e - 13\,a^3 b f)x^3)\left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + 420\,(4\,a b^3 c - 7\,a^2 b^2 d + 10\,a^3 b e - 13\,a^4 f)x}{1260\,(b^6 x^3 + a b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(126*b^4*f*x^13 + 18*(10*b^4*e - 13*a*b^3*f)*x^10 + 45*(7*b^4*d - 10
*a*b^3*e + 13*a^2*b^2*f)*x^7 + 315*(4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13
*a^3*b*f)*x^4 - 140*sqrt(3)*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*
f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3*b*f)*x^3)*(a/b)^(1/3)*arct
an(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*(4*a*b^3*c - 7*a^2*b
^2*d + 10*a^3*b*e - 13*a^4*f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3
*b*f)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*(4*a*b^
3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b
^2*e - 13*a^3*b*f)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*(4*a*b^3*c -
 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f)*x)/(b^6*x^3 + a*b^5)
```

Sympy [A]
time = 76.64, size = 449, normalized size = 1.37

$$x^7\left(-\frac{2af}{7b^3} + \frac{e}{7b^2}\right) + x^4\cdot\left(\frac{3a^2f}{4b^4} - \frac{ae}{2b^3} + \frac{d}{4b^2}\right) + x\left(-\frac{4a^3f}{b^5} + \frac{3a^2e}{b^4} - \frac{2ad}{b^3} + \frac{c}{b^2}\right) + \frac{x(-a^4f + a^3be - a^2b^2d + ab^3c)}{3ab^5 + 3b^6x^3} + \operatorname{RootSum}\left(729t^3b^{16} - 2197a^{10}f^3 + 5070a^9bef^2 - 3549a^8b^2df^2 - 3900a^8b^2e^2f + 2028a^7b^3cf^2 + 5460a^7b^3def + 1000a^7b^3e^3 - 3120a^6b^4cef - 1911a^6b^4d^2f - 2100a^6b^4de^2 + 2184a^5b^5cdf + 1200a^5b^5ce^2 + 1470a^5b^5d^2e - 624a^4b^6c^2f - 1680a^4b^6cde - 343a^4b^6d^3 + 480a^3b^7c^2e + 588a^3b^7cd^2 - 336a^2b^8c^2d + 64ab^9c^3, \left(t \mapsto t\log\left(\frac{9tb^5}{13a^3f - 10a^2be + 7ab^2d - 4b^3c} + x\right)\right)\right) + \frac{fx^{10}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**7*(-2*a*f/(7*b**3) + e/(7*b**2)) + x**4*(3*a**2*f/(4*b**4) - a*e/(2*b**3
) + d/(4*b**2)) + x*(-4*a**3*f/b**5 + 3*a**2*e/b**4 - 2*a*d/b**3 + c/b**2)
+ x*(-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(3*a*b**5 + 3*b**6*x**3)
+ RootSum(729*_t**3*b**16 - 2197*a**10*f**3 + 5070*a**9*b*e*f**2 - 3549*a**
8*b**2*d*f**2 - 3900*a**8*b**2*e**2*f + 2028*a**7*b**3*c*f**2 + 5460*a**7*b
**3*d*e*f + 1000*a**7*b**3*e**3 - 3120*a**6*b**4*c*e*f - 1911*a**6*b**4*d**
2*f - 2100*a**6*b**4*d*e**2 + 2184*a**5*b**5*c*d*f + 1200*a**5*b**5*c*e**2
+ 1470*a**5*b**5*d**2*e - 624*a**4*b**6*c**2*f - 1680*a**4*b**6*c*d*e - 343
*a**4*b**6*d**3 + 480*a**3*b**7*c**2*e + 588*a**3*b**7*c*d**2 - 336*a**2*b*
*8*c**2*d + 64*a*b**9*c**3, Lambda(_t, _t*log(9*_t*b**5/(13*a**3*f - 10*a**
2*b*e + 7*a*b**2*d - 4*b**3*c) + x))) + f*x**10/(10*b**2)
```

Giac [A]
time = 1.01, size = 394, normalized size = 1.20

$$-\frac{\sqrt{3}\left(4\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-7\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-13\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+10\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^{6}}+\frac{\left(4ab^{3}c-7a^{2}b^{2}d-13a^{4}f+10a^{3}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9ab^{5}}-\frac{\left(4\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-7\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-13\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+10\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^{6}}+\frac{ab^{3}cx-a^{2}b^{2}dx-a^{4}fx+a^{3}bxe}{3\left(bx^{3}+a\right)b^{5}}+\frac{14b^{18}fx^{10}-40ab^{17}fx^{7}+20b^{18}x^{7}e+35b^{18}dx^{4}+105a^{2}b^{16}fx^{4}-70ab^{17}x^{4}e+140b^{18}cx-280ab^{17}dx-560a^{3}b^{15}fx+420a^{2}b^{16}xe}{140b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(4*(-a*b^2)^(1/3)*b^3*c - 7*(-a*b^2)^(1/3)*a*b^2*d - 13*(-a*b^
2)^(1/3)*a^3*f + 10*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/b^6 + 1/9*(4*a*b^3*c - 7*a^2*b^2*d - 13*a^4*f + 10*a
^3*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) - 1/18*(4*(-a*b^2)^
(1/3)*b^3*c - 7*(-a*b^2)^(1/3)*a*b^2*d - 13*(-a*b^2)^(1/3)*a^3*f + 10*(-a*b
^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^6 + 1/3*(a*b^
3*c*x - a^2*b^2*d*x - a^4*f*x + a^3*b*x*e)/((b*x^3 + a)*b^5) + 1/140*(14*b^
18*f*x^10 - 40*a*b^17*f*x^7 + 20*b^18*x^7*e + 35*b^18*d*x^4 + 105*a^2*b^16*
f*x^4 - 70*a*b^17*x^4*e + 140*b^18*c*x - 280*a*b^17*d*x - 560*a^3*b^15*f*x
+ 420*a^2*b^16*x*e)/b^20
```

Mupad [B]
time = 5.20, size = 358, normalized size = 1.09

$$x^{7}\left(\frac{e}{7b^{2}}-\frac{2af}{7b^{3}}\right)+x\left(\frac{c}{b^{2}}-\frac{a^{2}\left(\frac{e}{b^{2}}-\frac{2af}{b^{3}}\right)}{b^{2}}+\frac{2a\left(\frac{a^{2}f}{b^{4}}-\frac{d}{b^{2}}+\frac{2a\left(\frac{e}{b^{2}}-\frac{2af}{b^{3}}\right)}{b}\right)}{b}\right)-x^{4}\left(\frac{a^{2}f}{4b^{4}}-\frac{d}{4b^{2}}+\frac{a\left(\frac{e}{b^{2}}-\frac{2af}{b^{3}}\right)}{2b}\right)-\frac{x\left(\frac{fa^{4}}{3}-\frac{ea^{3}b}{3}+\frac{da^{2}b^{2}}{3}-\frac{cab^{3}}{3}\right)}{b^{6}x^{3}+ab^{5}}+\frac{fx^{10}}{10b^{2}}-\frac{a^{1/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(-13fa^{3}+10ea^{2}b-7dab^{2}+4cb^{3}\right)}{9b^{16/3}}-\frac{a^{1/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)\left(-13fa^{3}+10ea^{2}b-7dab^{2}+4cb^{3}\right)}{9b^{16/3}}+\frac{a^{1/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)\left(-13fa^{3}+10ea^{2}b-7dab^{2}+4cb^{3}\right)}{9b^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^7*(e/(7*b^2) - (2*a*f)/(7*b^3)) + x*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/b^3))/
b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b) - x^4*
((a^2*f)/(4*b^4) - d/(4*b^2) + (a*(e/b^2 - (2*a*f)/b^3))/(2*b)) - (x*((a^4*
f)/3 + (a^2*b^2*d)/3 - (a*b^3*c)/3 - (a^3*b*e)/3))/(a*b^5 + b^6*x^3) + (f*x
^10)/(10*b^2) - (a^(1/3)*log(b^(1/3)*x + a^(1/3))*(4*b^3*c - 13*a^3*f - 7*a
*b^2*d + 10*a^2*b*e))/(9*b^(16/3)) - (a^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^
(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*b^3*c - 13*a^3*f - 7*a*b^2*d +
 10*a^2*b*e))/(9*b^(16/3)) + (a^(1/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
+ a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(4*b^3*c - 13*a^3*f - 7*a*b^2*d + 10*a^2*
b*e))/(9*b^(16/3))
```

3.263 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$

Optimal. Leaf size=298

$$\frac{(b^2d-2abe+3a^2f)\,x^2}{2b^4}+\frac{(be-2af)x^5}{5b^3}+\frac{fx^8}{8b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)}-\frac{(2b^3c-5ab^2d+8a^2be-11a^3}{3\sqrt{3}\,\sqrt[3]{a}}$$

```
[Out] 1/2*(3*a^2*f-2*a*b*e+b^2*d)*x^2/b^4+1/5*(-2*a*f+b*e)*x^5/b^3+1/8*f*x^8/b^2-
1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/b^4/(b*x^3+a)-1/9*(-11*a^3*f+8*a^2*b
*e-5*a*b^2*d+2*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(14/3)+1/18*(-11*a^3*
f+8*a^2*b*e-5*a*b^2*d+2*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^
(1/3)/b^(14/3)-1/9*(-11*a^3*f+8*a^2*b*e-5*a*b^2*d+2*b^3*c)*arctan(1/3*(a^(1
/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(14/3)*3^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1842, 1865, 1850, 1502, 298, 31, 648, 631, 210, 642}

$$\frac{x^2(3a^2f-2abe+b^2d)}{2b^4}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(-11a^3f+8a^2be-5ab^2d+2b^3c)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{14/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(-11a^3f+8a^2be-5ab^2d+2b^3c)}{9\sqrt[3]{a}\,b^{14/3}}-\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^4\,(a+bx^3)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(-11a^3f+8a^2be-5ab^2d+2b^3c)}{18\sqrt[3]{a}\,b^{14/3}}+\frac{x^5(be-2af)}{5b^3}+\frac{fx^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^2)/(2*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (
f*x^8)/(8*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^4*(a + b*x^
3)) - ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(14/3)) - ((2*b^3*c - 5*a*
b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(14/3)
) + ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
```

```
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1865

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} - \frac{\int \frac{-2ab(b^3c-ab^2d+a^2be-a^3f)x-3ab^2(b^2d-abe+a^2\cdots}{a+bx^3}}{3ab^5}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} - \frac{\int \frac{x(-2ab(b^3c-ab^2d+a^2be-a^3f)-3ab^2(b^2d-abe+a^2\cdots}{a+bx^3}}{3ab^5}$$

$$= \frac{fx^8}{8b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} - \frac{\int \frac{x(-16ab^2(b^3c-ab^2d+a^2be-a^3f)-24ab^3(b^2\cdots}{a+bx^3}}{24ab^{\cdots}}$$

$$= \frac{fx^8}{8b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} - \frac{\int \Big(-24ab^2(b^2d-2abe+3a^2f)\,x \cdots}{\cdots}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{(b^3c-ab^2d+a^2be-a^3\cdots}{3b^4\,(a+bx^3)}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{(b^3c-ab^2d+a^2be-a^3\cdots}{3b^4\,(a+bx^3)}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{(b^3c-ab^2d+a^2be-a^3\cdots}{3b^4\,(a+bx^3)}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{(b^3c-ab^2d+a^2be-a^3\cdots}{3b^4\,(a+bx^3)}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{(b^3c-ab^2d+a^2be-a^3\cdots}{3b^4\,(a+bx^3)}$$

Mathematica [A]

time = 0.10, size = 282, normalized size = 0.95

$$\frac{180b^{2/3}(b^2d-2abe+3a^2f)\,x^2+72b^{5/3}(be-2af)x^5+45b^{8/3}fx^8-\frac{120b^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{a+bx^3}+\frac{40\sqrt{3}\,(-2b^3c+5ab^2d-8a^2be+11a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{40(-2b^3c+5ab^2d-8a^2be+11a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}}+\frac{20(2b^3c-5ab^2d+8a^2be-11a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{a}}}{360b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (180*b^(2/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x^2 + 72*b^(5/3)*(b*e - 2*a*f)*x^5
 + 45*b^(8/3)*f*x^8 - (120*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)
/(a + b*x^3) + (40*Sqrt[3]*(-2*b^3*c + 5*a*b^2*d - 8*a^2*b*e + 11*a^3*f)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-2*b^3*c + 5*a*b^
2*d - 8*a^2*b*e + 11*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (20*(2*b^3*
c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/a^(1/3))/(360*b^(14/3))
```

Maple [A]
time = 0.36, size = 218, normalized size = 0.73

method	result
risch	$\frac{f\,x^8}{8b^2}-\frac{2x^5fa}{5b^3}+\frac{x^5e}{5b^2}+\frac{3x^2a^2f}{2b^4}-\frac{x^2ae}{b^3}+\frac{d\,x^2}{2b^2}+\frac{\left(\frac{1}{3}a^3f-\frac{1}{3}a^2be+\frac{1}{3}a\,b^2d-\frac{1}{3}b^3c\right)x^2}{b^4(b\,x^3+a)}+\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-11a^3f+8a^2b\cdots\right.}{}}{9b^5}$
default	$\frac{\frac{b^2f\,x^8}{8}+\frac{\left(-2fab+b^2e\right)x^5}{5}+\frac{\left(3a^2f-2abe+b^2d\right)x^2}{2}}{b^4}-\frac{\frac{\left(-\frac{1}{3}a^3f+\frac{1}{3}a^2be-\frac{1}{3}a\,b^2d+\frac{1}{3}b^3c\right)x^2}{b\,x^3+a}+\left(\frac{11}{3}a^3f-\frac{8}{3}a^2be+\frac{5}{3}a\,b^2d-\frac{2}{3}b^3c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\cdots\right.}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/8*b^2*f*x^8+1/5*(-2*a*b*f+b^2*e)*x^5+1/2*(3*a^2*f-2*a*b*e+b^2*d)*x
^2)-1/b^4*((-1/3*a^3*f+1/3*a^2*b*e-1/3*a*b^2*d+1/3*b^3*c)*x^2/(b*x^3+a)+(11
/3*a^3*f-8/3*a^2*b*e+5/3*a*b^2*d-2/3*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^
(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a
/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.49, size = 284, normalized size = 0.95

$$-\frac{(b^3c-ab^2d-a^3f+a^2be)x^2}{3\,(b^5x^3+ab^4)}+\frac{\sqrt{3}\,(2\,b^3c-5\,ab^2d-11\,a^3f+8\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\,b^2fx^8-8\,(2\,abf-b^2e)x^5+20\,(b^2d+3\,a^2f-2\,abe)x^2}{40\,b^4}+\frac{(2\,b^3c-5\,ab^2d-11\,a^3f+8\,a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(2\,b^3c-5\,ab^2d-11\,a^3f+8\,a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x^2/(b^5*x^3 + a*b^4) + 1/9*sqrt(3
)*(2*b^3*c - 5*a*b^2*d - 11*a^3*f + 8*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a
/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/40*(5*b^2*f*x^8 - 8*(2*a*b*f
- b^2*e)*x^5 + 20*(b^2*d + 3*a^2*f - 2*a*b*e)*x^2)/b^4 + 1/18*(2*b^3*c - 5*
a*b^2*d - 11*a^3*f + 8*a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5
*(a/b)^(1/3)) - 1/9*(2*b^3*c - 5*a*b^2*d - 11*a^3*f + 8*a^2*b*e)*log(x + (a
/b)^(1/3))/(b^5*(a/b)^(1/3))
```

Fricas [A]
time = 0.43, size = 920, normalized size = 3.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/360*(45*a*b^5*f*x^11 + 9*(8*a*b^5*e - 11*a^2*b^4*f)*x^8 + 36*(5*a*b^5*d
- 8*a^2*b^4*e + 11*a^3*b^3*f)*x^5 - 60*(2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3
*e - 11*a^4*b^2*f)*x^2 - 60*sqrt(1/3)*(2*a^2*b^4*c - 5*a^3*b^3*d + 8*a^4*b^
2*e - 11*a^5*b*f + (2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*f)*x
^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a
*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)
*x)/(b*x^3 + a)) + 20*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*
b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^
2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^
3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a
*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^7*x^3 + a^2*b^6), 1/360*(45*a*b^
5*f*x^11 + 9*(8*a*b^5*e - 11*a^2*b^4*f)*x^8 + 36*(5*a*b^5*d - 8*a^2*b^4*e +
 11*a^3*b^3*f)*x^5 - 60*(2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2
*f)*x^2 - 120*sqrt(1/3)*(2*a^2*b^4*c - 5*a^3*b^3*d + 8*a^4*b^2*e - 11*a^5*b
*f + (2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*f)*x^3)*sqrt((a*b^
2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)
/b) + 20*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b
^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(
1/3)*b*x + (a*b^2)^(2/3)) - 40*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^
4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*l
og(b*x + (a*b^2)^(1/3)))/(a*b^7*x^3 + a^2*b^6)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.08, size = 344, normalized size = 1.15

$$\frac{\sqrt{3}\,(2\,b^3c-5\,ab^2d-11\,a^3f+8\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{1}{3}}\,b^4}-\frac{(2\,b^3c-5\,ab^2d-11\,a^3f+8\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{1}{3}}\,b^4}-\frac{\left(2\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-11\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+8\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4}-\frac{b^3cx^2-ab^2dx^2-a^3fx^2+a^2bx^2e}{3\,(bx^3+a)b^4}+\frac{5\,b^{14}fx^8-16\,ab^{13}fx^5+8\,b^{14}x^5e+20\,b^{14}dx^2+60\,a^2b^{12}fx^2-40\,ab^{13}x^2e}{40\,b^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(2*b^3*c - 5*a*b^2*d - 11*a^3*f + 8*a^2*b*e)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^4) - 1/18*(2*b^3*c -
5*a*b^2*d - 11*a^3*f + 8*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
((-a*b^2)^(1/3)*b^4) - 1/9*(2*b^3*c*(-a/b)^(1/3) - 5*a*b^2*d*(-a/b)^(1/3) -
 11*a^3*f*(-a/b)^(1/3) + 8*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/(a*b^4) - 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^
2*e)/((b*x^3 + a)*b^4) + 1/40*(5*b^14*f*x^8 - 16*a*b^13*f*x^5 + 8*b^14*x^5*
e + 20*b^14*d*x^2 + 60*a^2*b^12*f*x^2 - 40*a*b^13*x^2*e)/b^16
```

Mupad [B]
time = 5.22, size = 287, normalized size = 0.96

$$x^5\left(\frac{e}{5\,b^2}-\frac{2\,a\,f}{5\,b^3}\right)-x^2\left(\frac{a^2\,f}{2\,b^4}-\frac{d}{2\,b^2}+\frac{a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)+\frac{f\,x^8}{8\,b^2}-\frac{x^2\left(-\frac{f\,a^3}{3}+\frac{e\,a^2\,b}{3}-\frac{d\,a\,b^2}{3}+\frac{c\,b^3}{3}\right)}{b^5\,x^3+a\,b^4}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-11\,f\,a^3+8\,e\,a^2\,b-5\,d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{1/3}\,b^{14/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-11\,f\,a^3+8\,e\,a^2\,b-5\,d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{1/3}\,b^{14/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-11\,f\,a^3+8\,e\,a^2\,b-5\,d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{1/3}\,b^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^5*(e/(5*b^2) - (2*a*f)/(5*b^3)) - x^2*((a^2*f)/(2*b^4) - d/(2*b^2) + (a*(
e/b^2 - (2*a*f)/b^3))/b) + (f*x^8)/(8*b^2) - (x^2*((b^3*c)/3 - (a^3*f)/3 -
(a*b^2*d)/3 + (a^2*b*e)/3))/(a*b^4 + b^5*x^3) - (log(b^(1/3)*x + a^(1/3))*(
2*b^3*c - 11*a^3*f - 5*a*b^2*d + 8*a^2*b*e))/(9*a^(1/3)*b^(14/3)) + (log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*b^3*c -
 11*a^3*f - 5*a*b^2*d + 8*a^2*b*e))/(9*a^(1/3)*b^(14/3)) - (log(3^(1/2)*a^(
1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*b^3*c - 11*a^3*f
 - 5*a*b^2*d + 8*a^2*b*e))/(9*a^(1/3)*b^(14/3))
```

3.264 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=288

$$\frac{(b^2d-2abe+3a^2f)\,x}{b^4}+\frac{(be-2af)x^4}{4b^3}+\frac{fx^7}{7b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^4\,(a+bx^3)}-\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\,\mathrm{t}}{3\sqrt{3}\,a^{2/3}b^{13/}}$$

```
[Out] (3*a^2*f-2*a*b*e+b^2*d)*x/b^4+1/4*(-2*a*f+b*e)*x^4/b^3+1/7*f*x^7/b^2-1/3*(-
a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^4/(b*x^3+a)+1/9*(-10*a^3*f+7*a^2*b*e-4*a*b
^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(13/3)-1/18*(-10*a^3*f+7*a^2*b*
e-4*a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(13/
3)-1/9*(-10*a^3*f+7*a^2*b*e-4*a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(13/3)*3^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1842, 1901, 206, 31, 648, 631, 210, 642}

$$\frac{x(3a^2f-2abe+b^2d)}{b^4}-\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^4\,(a+bx^3)}-\frac{\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(-10a^3f+7a^2be-4ab^2d+b^3c)}{3\sqrt{3}\,a^{2/3}b^{13/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(-10a^3f+7a^2be-4ab^2d+b^3c)}{18a^{2/3}b^{13/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(-10a^3f+7a^2be-4ab^2d+b^3c)}{9a^{2/3}b^{13/3}}+\frac{x^4(be-2af)}{4b^3}+\frac{fx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^4)/(4*b^3) + (f*x^7)
/(7*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^4*(a + b*x^3)) - ((
b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(13/3)) + ((b^3*c - 4*a*b^2*d + 7*a^
2*b*e - 10*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(13/3)) - ((b^3*c
- 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(18*a^(2/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^4\,(a+bx^3)} - \frac{\int \frac{-a(b^3c-ab^2d+a^2be-a^3f)-3ab(b^2d-abe+a^2f)x^3-}{a+bx^3}}{3ab^4}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^4\,(a+bx^3)} - \frac{\int \Big(-3a(b^2d-2abe+3a^2f)-3ab(be-}{}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f}{3b^4\,(a+bx^3)}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f}{3b^4\,(a+bx^3)}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f}{3b^4\,(a+bx^3)}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f}{3b^4\,(a+bx^3)}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f}{3b^4\,(a+bx^3)}$$

Mathematica [A]
time = 0.10, size = 277, normalized size = 0.96

$$\frac{252\sqrt[3]{b}\,(b^2d-2abe+3a^2f)\,x+63b^{4/3}(be-2af)x^4+36b^{7/3}fx^7-\frac{84\sqrt[3]{b}\,(b^3c-ab^2d+a^2be-a^3f)x}{a+bx^3}+\frac{28\sqrt{3}\,(-b^3c+4ab^2d-7a^2be+10a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{28(b^3c-4ab^2d+7a^2be-10a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}}+\frac{14(-b^3c+4ab^2d-7a^2be+10a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}}}{252b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (252*b^(1/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x + 63*b^(4/3)*(b*e - 2*a*f)*x^4 +
 36*b^(7/3)*f*x^7 - (84*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a +
 b*x^3) + (28*Sqrt[3]*(-(b^3*c) + 4*a*b^2*d - 7*a^2*b*e + 10*a^3*f)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(b^3*c - 4*a*b^2*d + 7*
a^2*b*e - 10*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b^3*c) + 4*a
*b^2*d - 7*a^2*b*e + 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/a^(2/3))/(252*b^(13/3))
```

Maple [A]
time = 0.37, size = 215, normalized size = 0.75

method	result
risch	$\frac{f x^7}{7b^2} - \frac{af x^4}{2b^3} + \frac{e x^4}{4b^2} + \frac{3a^2 f x}{b^4} - \frac{2aex}{b^3} + \frac{dx}{b^2} + \frac{(\frac{1}{3}a^3 f - \frac{1}{3}a^2 be + \frac{1}{3}a b^2 d - \frac{1}{3}b^3 c)x}{b^4(b x^3+a)} + \frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)} \frac{(-10a^3 f + 7a^2 be - 4a \cdots}{-\cdots}}{9b^5}$
default	$\frac{\frac{1}{7}b^2 f x^7 - \frac{1}{2}abf x^4 + \frac{1}{4}b^2 e x^4 + 3a^2 f x - 2abex + b^2 dx}{b^4} - \frac{\frac{(-\frac{1}{3}a^3 f + \frac{1}{3}a^2 be - \frac{1}{3}a b^2 d + \frac{1}{3}b^3 c)x}{b x^3+a} + (10a^3 f - 7a^2 be + 4a b^2 d - b^3 c)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \cdots\right.}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/7*b^2*f*x^7-1/2*a*b*f*x^4+1/4*b^2*e*x^4+3*a^2*f*x-2*a*b*e*x+b^2*d*
x)-1/b^4*((-1/3*a^3*f+1/3*a^2*b*e-1/3*a*b^2*d+1/3*b^3*c)*x/(b*x^3+a)+1/3*(1
0*a^3*f-7*a^2*b*e+4*a*b^2*d-b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6
/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.49, size = 277, normalized size = 0.96

$$-\frac{(b^3c - ab^2d - a^3f + a^2be)x}{3\,(b^5x^3 + ab^4)} + \frac{4\,b^2fx^7 - 7\,(2\,abf - b^2e)x^4 + 28\,(b^2d + 3\,a^2f - 2\,abe)x}{28\,b^4} + \frac{\sqrt{3}\,(b^3c - 4\,ab^2d - 10\,a^3f + 7\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^3c - 4\,ab^2d - 10\,a^3f + 7\,a^2be)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^3c - 4\,ab^2d - 10\,a^3f + 7\,a^2be)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x/(b^5*x^3 + a*b^4) + 1/28*(4*b^2*
f*x^7 - 7*(2*a*b*f - b^2*e)*x^4 + 28*(b^2*d + 3*a^2*f - 2*a*b*e)*x)/b^4 + 1
/9*sqrt(3)*(b^3*c - 4*a*b^2*d - 10*a^3*f + 7*a^2*b*e)*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) - 1/18*(b^3*c - 4*a*b^2*d
- 10*a^3*f + 7*a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(
2/3)) + 1/9*(b^3*c - 4*a*b^2*d - 10*a^3*f + 7*a^2*b*e)*log(x + (a/b)^(1/3))
/(b^5*(a/b)^(2/3))
```

Fricas [A]
time = 0.41, size = 946, normalized size = 3.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/252*(36*a^2*b^4*f*x^10 + 9*(7*a^2*b^4*e - 10*a^3*b^3*f)*x^7 + 63*(4*a^2*
b^4*d - 7*a^3*b^3*e + 10*a^4*b^2*f)*x^4 - 42*sqrt(1/3)*(a^2*b^4*c - 4*a^3*b
^3*d + 7*a^4*b^2*e - 10*a^5*b*f + (a*b^5*c - 4*a^2*b^4*d + 7*a^3*b^3*e - 10
*a^4*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a
*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sq
rt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(a*b^3*c - 4*a^2*b^2*d + 7*a^3*b*e
- 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3)*(-a^2*b)^(
2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^3*c - 4*a
^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3
*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(a^2*b^4*c - 4*a
^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f)*x)/(a^2*b^6*x^3 + a^3*b^5), 1/252*(36*
a^2*b^4*f*x^10 + 9*(7*a^2*b^4*e - 10*a^3*b^3*f)*x^7 + 63*(4*a^2*b^4*d - 7*a
^3*b^3*e + 10*a^4*b^2*f)*x^4 + 84*sqrt(1/3)*(a^2*b^4*c - 4*a^3*b^3*d + 7*a^
4*b^2*e - 10*a^5*b*f + (a*b^5*c - 4*a^2*b^4*d + 7*a^3*b^3*e - 10*a^4*b^2*f)
*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*
b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 14*(a*b^3*c - 4*a^2*b^2*d + 7*a^
3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3)*(-a^
2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^3*c
 - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e -
10*a^3*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(a^2*b^4*c
 - 4*a^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f)*x)/(a^2*b^6*x^3 + a^3*b^5)]
```

Sympy [A]
time = 67.93, size = 401, normalized size = 1.39

$$x^4\left(-\frac{af}{2b^3}+\frac{e}{4b^2}\right)+x\left(\frac{3a^2f}{b^4}-\frac{2ae}{b^3}+\frac{d}{b^2}\right)+\frac{x(a^3f-a^2be+ab^2d-b^3c)}{3ab^4+3b^5x^3}+\operatorname{RootSum}\left(729t^3a^2b^{13}+1000a^9f^3-2100a^8bef^2+1200a^7b^2df^2+1470a^7b^2e^2f-300a^6b^3cf^2-1680a^6b^3def-343a^6b^3e^3+420a^5b^4cef+480a^5b^4d^2f+588a^5b^4de^2-240a^4b^5cdf-147a^4b^5ce^2-336a^4b^5d^2e+30a^3b^6c^2f+168a^3b^6cde+64a^3b^6d^3-21a^2b^7c^2e-48a^2b^7cd^2+12ab^8c^2d-b^9c^3,\left(t\mapsto t\log\left(-\frac{9tab^4}{10a^3f-7a^2be+4ab^2d-b^3c}+x\right)\right)\right)+\frac{fx^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**4*(-a*f/(2*b**3) + e/(4*b**2)) + x*(3*a**2*f/b**4 - 2*a*e/b**3 + d/b**2)
 + x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b**5*x**3) + Roo
tSum(729*_t**3*a**2*b**13 + 1000*a**9*f**3 - 2100*a**8*b*e*f**2 + 1200*a**7
*b**2*d*f**2 + 1470*a**7*b**2*e**2*f - 300*a**6*b**3*c*f**2 - 1680*a**6*b**
3*d*e*f - 343*a**6*b**3*e**3 + 420*a**5*b**4*c*e*f + 480*a**5*b**4*d**2*f +
 588*a**5*b**4*d*e**2 - 240*a**4*b**5*c*d*f - 147*a**4*b**5*c*e**2 - 336*a*
*4*b**5*d**2*e + 30*a**3*b**6*c**2*f + 168*a**3*b**6*c*d*e + 64*a**3*b**6*d
**3 - 21*a**2*b**7*c**2*e - 48*a**2*b**7*c*d**2 + 12*a*b**8*c**2*d - b**9*c
**3, Lambda(_t, _t*log(-9*_t*a*b**4/(10*a**3*f - 7*a**2*b*e + 4*a*b**2*d -
b**3*c) + x))) + f*x**7/(7*b**2)
```

Giac [A]

time = 1.29, size = 295, normalized size = 1.02

$$-\frac{\sqrt{3}\,(b^3c-4\,ab^2d-10\,a^3f+7\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,b^3}-\frac{(b^3c-4\,ab^2d-10\,a^3f+7\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,b^3}-\frac{(b^3c-4\,ab^2d-10\,a^3f+7\,a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4}-\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{3\,(bx^3+a)b^4}+\frac{4\,b^{12}fx^7-14\,ab^{11}fx^4+7\,b^{12}x^4e+28\,b^{12}dx+84\,a^2b^{10}fx-56\,ab^{11}xe}{28\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(b^3*c - 4*a*b^2*d - 10*a^3*f + 7*a^2*b*e)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^3) - 1/18*(b^3*c - 4*a
*b^2*d - 10*a^3*f + 7*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-
a*b^2)^(2/3)*b^3) - 1/9*(b^3*c - 4*a*b^2*d - 10*a^3*f + 7*a^2*b*e)*(-a/b)^(
1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 1/3*(b^3*c*x - a*b^2*d*x - a^3*f*
x + a^2*b*x*e)/((b*x^3 + a)*b^4) + 1/28*(4*b^12*f*x^7 - 14*a*b^11*f*x^4 + 7
*b^12*x^4*e + 28*b^12*d*x + 84*a^2*b^10*f*x - 56*a*b^11*x*e)/b^14
```

Mupad [B]
time = 0.31, size = 280, normalized size = 0.97

$$x^4\left(\frac{e}{4\,b^2}-\frac{a\,f}{2\,b^3}\right)-x\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)-\frac{x\left(-\frac{f\,a^3}{3}+\frac{e\,a^2\,b}{3}-\frac{d\,a\,b^2}{3}+\frac{c\,b^3}{3}\right)}{b^5\,x^3+a\,b^4}+\frac{f\,x^7}{7\,b^2}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-10\,f\,a^3+7\,e\,a^2\,b-4\,d\,a\,b^2+c\,b^3\right)}{9\,a^{2/3}\,b^{13/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-10\,f\,a^3+7\,e\,a^2\,b-4\,d\,a\,b^2+c\,b^3\right)}{9\,a^{2/3}\,b^{13/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-10\,f\,a^3+7\,e\,a^2\,b-4\,d\,a\,b^2+c\,b^3\right)}{9\,a^{2/3}\,b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^4*(e/(4*b^2) - (a*f)/(2*b^3)) - x*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2
*a*f)/b^3))/b) - (x*((b^3*c)/3 - (a^3*f)/3 - (a*b^2*d)/3 + (a^2*b*e)/3))/(a
*b^4 + b^5*x^3) + (f*x^7)/(7*b^2) + (log(b^(1/3)*x + a^(1/3))*(b^3*c - 10*a
^3*f - 4*a*b^2*d + 7*a^2*b*e))/(9*a^(2/3)*b^(13/3)) + (log(3^(1/2)*a^(1/3)*
1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - 10*a^3*f - 4*a*
b^2*d + 7*a^2*b*e))/(9*a^(2/3)*b^(13/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1
/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - 10*a^3*f - 4*a*b^2*d + 7*a
^2*b*e))/(9*a^(2/3)*b^(13/3))
```

3.265 $\int \frac{x\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=271

$$\frac{(be-2af)x^2}{2b^3}+\frac{fx^5}{5b^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)}-\frac{\left(b^3c+2ab^2d-5a^2be+8a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{11/3}}-\cdots$$

```
[Out] 1/2*(-2*a*f+b*e)*x^2/b^3+1/5*f*x^5/b^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x
^2/a/b^3/(b*x^3+a)-1/9*(8*a^3*f-5*a^2*b*e+2*a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/
3)*x)/a^(4/3)/b^(11/3)+1/18*(8*a^3*f-5*a^2*b*e+2*a*b^2*d+b^3*c)*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(11/3)-1/9*(8*a^3*f-5*a^2*b*e+2*a*
b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(1
1/3)*3^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1842, 1608, 1502, 298, 31, 648, 631, 210, 642}

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{3\sqrt{3}\,a^{4/3}b^{11/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{18a^{4/3}b^{11/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{9a^{4/3}b^{11/3}}+\frac{x^2(be-2af)}{2b^3}+\frac{fx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x^2)/(2*b^3) + (f*x^5)/(5*b^2) + ((b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*x^2)/(3*a*b^3*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3
)*b^(11/3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1
/3)*x])/(9*a^(4/3)*b^(11/3)) + ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
```

```
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx &= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3ab^3\,(a+bx^3)} - \frac{\int \frac{-b(b^3c+2ab^2d-2a^2be+2a^3f)x-3ab^2(be-af)x^4-3ab^3}{a+bx^3}}{3ab^4} \\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3ab^3\,(a+bx^3)} - \frac{\int \frac{x(-b(b^3c+2ab^2d-2a^2be+2a^3f)-3ab^2(be-af)x^3-3ab}{a+bx^3}}{3ab^4} \\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3ab^3\,(a+bx^3)} - \frac{\int \Big(-3ab(be-2af)x-3ab^2fx^4+\frac{(-b^4c-2a}{}}{3ab^4} \\
&= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3ab^3\,(a+bx^3)} + \frac{(b^3c+2ab^2d-5a^2}{3} \\
&= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3ab^3\,(a+bx^3)} - \frac{(b^3c+2ab^2d-5a^2}{9} \\
&= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3ab^3\,(a+bx^3)} - \frac{(b^3c+2ab^2d-5a^2}{} \\
&= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3ab^3\,(a+bx^3)} - \frac{(b^3c+2ab^2d-5a^2}{} \\
&= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3ab^3\,(a+bx^3)} - \frac{(b^3c+2ab^2d-5a^2}{}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 255, normalized size = 0.94

$$
\frac{45b^{2/3}(be-2af)x^2+18b^{5/3}fx^5+\frac{30b^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{a(a+bx^3)}-\frac{10\sqrt{3}\,(b^3c+2ab^2d-5a^2be+8a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}-\frac{10(b^3c+2ab^2d-5a^2be+8a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{4/3}}+\frac{5(b^3c+2ab^2d-5a^2be+8a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{4/3}}}{90b^{11/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (45*b^(2/3)*(b*e - 2*a*f)*x^2 + 18*b^(5/3)*f*x^5 + (30*b^(2/3)*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^2)/(a*(a + b*x^3)) - (10*Sqrt[3]*(b^3*c + 2*a*b^2
```

```
*d - 5*a^2*b*e + 8*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4
/3) - (10*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1/3)*x
])/a^(4/3) + (5*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(90*b^(11/3))
```

Maple [A]
time = 0.35, size = 197, normalized size = 0.73

method	result
risch	$\frac{f x^5}{5b^2} - \frac{x^2 a f}{b^3} + \frac{e x^2}{2b^2} - \frac{(a^3 f - a^2 b e + a b^2 d - b^3 c) x^2}{3 a b^3 (b x^3 + a)} + \frac{\sum_{\_R = \mathrm{RootOf}(b\_Z^3 + a)} \frac{(8a^3 f - 5a^2 b e + 2a b^2 d + b^3 c) \ln(x - \_R)}{\_R}}{9 b^4 a}$
default	$-\frac{-\frac{b f x^5}{5} + \frac{(2af - be)x^2}{2}}{b^3} + \frac{-\frac{(a^3 f - a^2 b e + a b^2 d - b^3 c) x^2}{3a(b x^3 + a)} + \frac{(8a^3 f - 5a^2 b e + 2a b^2 d + b^3 c)\left(-\frac{\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}\, a}{\ldots}\right.}{3a}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/5*b*f*x^5+1/2*(2*a*f-b*e)*x^2)+1/b^3*(-1/3*(a^3*f-a^2*b*e+a*b^2*
d-b^3*c)/a*x^2/(b*x^3+a)+1/3*(8*a^3*f-5*a^2*b*e+2*a*b^2*d+b^3*c)/a*(-1/3/b/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.51, size = 265, normalized size = 0.98

$$\frac{(b^3c - ab^2d - a^3f + a^2be)x^2}{3(ab^4x^3 + a^2b^3)} + \frac{2bfx^5 - 5(2af - be)x^2}{10b^3} + \frac{\sqrt{3}(b^3c + 2ab^2d + 8a^3f - 5a^2be)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(b^3c + 2ab^2d + 8a^3f - 5a^2be)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18ab^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^3c + 2ab^2d + 8a^3f - 5a^2be)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x^2/(a*b^4*x^3 + a^2*b^3) + 1/10*(2
*b*f*x^5 - 5*(2*a*f - b*e)*x^2)/b^3 + 1/9*sqrt(3)*(b^3*c + 2*a*b^2*d + 8*a^
3*f - 5*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^4
*(a/b)^(1/3)) + 1/18*(b^3*c + 2*a*b^2*d + 8*a^3*f - 5*a^2*b*e)*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(a*b^4*(a/b)^(1/3)) - 1/9*(b^3*c + 2*a*b^2*d + 8
*a^3*f - 5*a^2*b*e)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(1/3))
```

Fricas [A]
time = 0.42, size = 874, normalized size = 3.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/90*(18*a^2*b^4*f*x^8 + 9*(5*a^2*b^4*e - 8*a^3*b^3*f)*x^5 + 15*(2*a*b^5*c
 - 2*a^2*b^4*d + 5*a^3*b^3*e - 8*a^4*b^2*f)*x^2 + 15*sqrt(1/3)*(a^2*b^4*c +
 2*a^3*b^3*d - 5*a^4*b^2*e + 8*a^5*b*f + (a*b^5*c + 2*a^2*b^4*d - 5*a^3*b^3
*e + 8*a^4*b^2*f)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt
(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)
/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 5*(a*b^3*c + 2*a^2*b^2*d - 5*a^3*b
*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-a*b^2)^
(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*(a*b^3*c + 2*
a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*
b*f)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^3 + a^3*b^5)
, 1/90*(18*a^2*b^4*f*x^8 + 9*(5*a^2*b^4*e - 8*a^3*b^3*f)*x^5 + 15*(2*a*b^5*
c - 2*a^2*b^4*d + 5*a^3*b^3*e - 8*a^4*b^2*f)*x^2 + 30*sqrt(1/3)*(a^2*b^4*c
+ 2*a^3*b^3*d - 5*a^4*b^2*e + 8*a^5*b*f + (a*b^5*c + 2*a^2*b^4*d - 5*a^3*b^
3*e + 8*a^4*b^2*f)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 5*(a*b^3*c + 2*a^2*b^2*d - 5*a^
3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-a*b^
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*(a*b^3*c +
 2*a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a
^3*b*f)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^3 + a^3*b
^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.37, size = 318, normalized size = 1.17

$$\frac{\sqrt{3}\,(b^3c+2\,ab^2d+8\,a^3f-5\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{1}{3}}\,ab^3}-\frac{(b^3c+2\,ab^2d+8\,a^3f-5\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{1}{3}}\,ab^3}-\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+8\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3}+\frac{b^3cx^2-ab^2dx^2-a^3fx^2+a^2bx^2e}{3\,(bx^3+a)ab^3}+\frac{2\,b^8fx^5-10\,ab^7fx^2+5\,b^8x^2e}{10\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(b^3*c + 2*a*b^2*d + 8*a^3*f - 5*a^2*b*e)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a*b^3) - 1/18*(b^3*c + 2*a
*b^2*d + 8*a^3*f - 5*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a
*b^2)^(1/3)*a*b^3) - 1/9*(b^3*c*(-a/b)^(1/3) + 2*a*b^2*d*(-a/b)^(1/3) + 8*a
^3*f*(-a/b)^(1/3) - 5*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^3) + 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^2*e
)/((b*x^3 + a)*a*b^3) + 1/10*(2*b^8*f*x^5 - 10*a*b^7*f*x^2 + 5*b^8*x^2*e)/b
^10
```

Mupad [B]
time = 5.23, size = 246, normalized size = 0.91

$$x^2\left(\frac{e}{2\,b^2}-\frac{a\,f}{b^3}\right)+\frac{f\,x^5}{5\,b^2}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(8\,f\,a^3-5\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{9\,a^{4/3}\,b^{11/3}}+\frac{x^2\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a\,\left(b^4\,x^3+a\,b^3\right)}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(8\,f\,a^3-5\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{9\,a^{4/3}\,b^{11/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(8\,f\,a^3-5\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{9\,a^{4/3}\,b^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^2*(e/(2*b^2) - (a*f)/b^3) + (f*x^5)/(5*b^2) - (log(b^(1/3)*x + a^(1/3))*(
b^3*c + 8*a^3*f + 2*a*b^2*d - 5*a^2*b*e))/(9*a^(4/3)*b^(11/3)) + (x^2*(b^3*
c - a^3*f - a*b^2*d + a^2*b*e))/(3*a*(a*b^3 + b^4*x^3)) + (log(3^(1/2)*a^(1
/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c + 8*a^3*f + 2
*a*b^2*d - 5*a^2*b*e))/(9*a^(4/3)*b^(11/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b
^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c + 8*a^3*f + 2*a*b^2*d - 5
*a^2*b*e))/(9*a^(4/3)*b^(11/3))
```

3.266 $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=264

$$\frac{(be-2af)x}{b^3}+\frac{fx^4}{4b^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3ab^3\,(a+bx^3)}-\frac{(2b^3c+ab^2d-4a^2be+7a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{10/3}}+\frac{(2b^3 \cdots}{\cdots}$$

```
[Out] (-2*a*f+b*e)*x/b^3+1/4*f*x^4/b^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a/b^3
/(b*x^3+a)+1/9*(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^
(5/3)/b^(10/3)-1/18*(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(10/3)-1/9*(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^
3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(10/3)*3^(
1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1872, 1425, 396, 206, 31, 648, 631, 210, 642}

$$\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3ab^3\,(a+bx^3)}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(7a^3f-4a^2be+ab^2d+2b^3c)}{3\sqrt{3}\,a^{5/3}b^{10/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(7a^3f-4a^2be+ab^2d+2b^3c)}{18a^{5/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(7a^3f-4a^2be+ab^2d+2b^3c)}{9a^{5/3}b^{10/3}}+\frac{x(be-2af)}{b^3}+\frac{fx^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^4)/(4*b^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*x)/(3*a*b^3*(a + b*x^3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/
3)) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(9*a^(5/3)*b^(10/3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
```

```
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx &= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3ab^3\,(a+bx^3)} - \frac{\int \frac{-2b^3c-ab^2d+a^2be-a^3f-3ab(be-af)x^3-3ab^2fx^6}{a+bx^3}\,dx}{3ab^3} \\
&= \frac{fx^4}{4b^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3ab^3\,(a+bx^3)} - \frac{\int \frac{4b(-2b^3c-ab^2d+a^2be-a^3f)-(-12a^2b^2f+12ab^2(b \cdots}{a+bx^3} \cdots}{12ab^4 \cdots} \\
&= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3ab^3\,(a+bx^3)} + \frac{(2b^3c+ab^2d-4a^2be+ \cdots}{3ab^3 \cdots} \\
&= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3ab^3\,(a+bx^3)} + \frac{(2b^3c+ab^2d-4a^2be+ \cdots}{9a^{5/3} \cdots} \\
&= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3ab^3\,(a+bx^3)} + \frac{(2b^3c+ab^2d-4a^2be+ \cdots}{9a^{5/3} \cdots} \\
&= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3ab^3\,(a+bx^3)} + \frac{(2b^3c+ab^2d-4a^2be+ \cdots}{9a^{5/3} \cdots} \\
&= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3ab^3\,(a+bx^3)} - \frac{(2b^3c+ab^2d-4a^2be+ \cdots}{3\sqrt{3} \cdots}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 251, normalized size = 0.95

$$\frac{36\sqrt[3]{b}\,(be-2af)x+9b^{4/3}fx^4+\frac{12\sqrt[3]{b}\,(b^3c-ab^2d+a^2be-a^3f)x}{a(a+bx^3)}-\frac{4\sqrt{3}\,(2b^3c+ab^2d-4a^2be+7a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{4(2b^3c+ab^2d-4a^2be+7a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}}-\frac{2(2b^3c+ab^2d-4a^2be+7a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}}{36b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*(b*e - 2*a*f)*x + 9*b^(4/3)*f*x^4 + (12*b^(1/3)*(b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*x)/(a*(a + b*x^3)) - (4*Sqrt[3]*(2*b^3*c + a*b^2*d - 4
*a^2*b*e + 7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) +
(4*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(5
```

```
/3) - (2*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(36*b^(10/3))
```

Maple [A]

time = 0.36, size = 191, normalized size = 0.72

method	result
risch	$\frac{f\,x^4}{4b^2} - \frac{2afx}{b^3} + \frac{ex}{b^2} - \frac{(a^3f - a^2be + a\,b^2d - b^3c)x}{3a\,b^3(b\,x^3+a)} + \frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)} \frac{(7a^3f - 4a^2be + a\,b^2d + 2b^3c)\ln(x-\_R)}{\_R^2}}{9b^4a}$
default	$-\frac{-\frac{1}{4}bf\,x^4 + 2afx - bex}{b^3} + \frac{-\frac{(a^3f - a^2be + a\,b^2d - b^3c)x}{3a(b\,x^3+a)} + \frac{(7a^3f - 4a^2be + a\,b^2d + 2b^3c)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\,\mathrm{arc}}{}\right)}{3a}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/4*b*f*x^4+2*a*f*x-b*e*x)+1/b^3*(-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*
c)/a*x/(b*x^3+a)+1/3*(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)/a*(1/3/b/(a/b)^(2/
3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/
3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]

time = 0.52, size = 260, normalized size = 0.98

$$\frac{(b^3c - ab^2d - a^3f + a^2be)x}{3(ab^4x^3 + a^2b^3)} + \frac{bfx^4 - 4(2af - be)x}{4b^3} + \frac{\sqrt{3}(2b^3c + ab^2d + 7a^3f - 4a^2be)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2b^3c + ab^2d + 7a^3f - 4a^2be)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2b^3c + ab^2d + 7a^3f - 4a^2be)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x/(a*b^4*x^3 + a^2*b^3) + 1/4*(b*f*
x^4 - 4*(2*a*f - b*e)*x)/b^3 + 1/9*sqrt(3)*(2*b^3*c + a*b^2*d + 7*a^3*f - 4
*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^4*(a/b)^
(2/3)) - 1/18*(2*b^3*c + a*b^2*d + 7*a^3*f - 4*a^2*b*e)*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(a*b^4*(a/b)^(2/3)) + 1/9*(2*b^3*c + a*b^2*d + 7*a^3*f
- 4*a^2*b*e)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3))
```

Fricas [A]
time = 0.44, size = 861, normalized size = 3.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/36*(9*a^3*b^3*f*x^7 + 9*(4*a^3*b^3*e - 7*a^4*b^2*f)*x^4 + 6*sqrt(1/3)*(2
*a^2*b^4*c + a^3*b^3*d - 4*a^4*b^2*e + 7*a^5*b*f + (2*a*b^5*c + a^2*b^4*d -
 4*a^3*b^3*e + 7*a^4*b^2*f)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*
(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b
)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 2*(2*a*b^3*c + a^2*b^2*d
- 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^3)*
(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3
*c + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7
*a^3*b*f)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*b^4*c - a
^3*b^3*d + 4*a^4*b^2*e - 7*a^5*b*f)*x)/(a^3*b^5*x^3 + a^4*b^4), 1/36*(9*a^3
*b^3*f*x^7 + 9*(4*a^3*b^3*e - 7*a^4*b^2*f)*x^4 + 12*sqrt(1/3)*(2*a^2*b^4*c
+ a^3*b^3*d - 4*a^4*b^2*e + 7*a^5*b*f + (2*a*b^5*c + a^2*b^4*d - 4*a^3*b^3*
e + 7*a^4*b^2*f)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/
3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(2*a*b^3*c + a^2*b^2
*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^
3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*
b^3*c + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e
+ 7*a^3*b*f)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*b^4*c
- a^3*b^3*d + 4*a^4*b^2*e - 7*a^5*b*f)*x)/(a^3*b^5*x^3 + a^4*b^4)]
```

Sympy [A]
time = 3.67, size = 377, normalized size = 1.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x*(-2*a*f/b**3 + e/b**2) + x*(-a**3*f + a**2*b*e - a*b**2*d + b**3*c)/(3*a*
*2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**3*a**5*b**10 - 343*a**9*f**3 + 5
88*a**8*b*e*f**2 - 147*a**7*b**2*d*f**2 - 336*a**7*b**2*e**2*f - 294*a**6*b
**3*c*f**2 + 168*a**6*b**3*d*e*f + 64*a**6*b**3*e**3 + 336*a**5*b**4*c*e*f
- 21*a**5*b**4*d**2*f - 48*a**5*b**4*d*e**2 - 84*a**4*b**5*c*d*f - 96*a**4*
b**5*c*e**2 + 12*a**4*b**5*d**2*e - 84*a**3*b**6*c**2*f + 48*a**3*b**6*c*d*
e - a**3*b**6*d**3 + 48*a**2*b**7*c**2*e - 6*a**2*b**7*c*d**2 - 12*a*b**8*c
**2*d - 8*b**9*c**3, Lambda(_t, _t*log(9*_t*a**2*b**3/(7*a**3*f - 4*a**2*b*
e + a*b**2*d + 2*b**3*c) + x))) + f*x**4/(4*b**2)
```

Giac [A]
time = 1.65, size = 273, normalized size = 1.03

$$-\frac{\sqrt{3}\,(2\,b^3c+ab^2d+7\,a^3f-4\,a^2be)\arctan\left(\frac{\sqrt{3}\,\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}\,ab^2}-\frac{(2\,b^3c+ab^2d+7\,a^3f-4\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}\,ab^2}-\frac{(2\,b^3c+ab^2d+7\,a^3f-4\,a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3}+\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{3\,(bx^3+a)ab^3}+\frac{b^6fx^4-8\,ab^5fx+4\,b^6xe}{4\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b^3*c + a*b^2*d + 7*a^3*f - 4*a^2*b*e)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1/18*(2*b^3*c +
a*b^2*d + 7*a^3*f - 4*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-
a*b^2)^(2/3)*a*b^2) - 1/9*(2*b^3*c + a*b^2*d + 7*a^3*f - 4*a^2*b*e)*(-a/b)^
(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3) + 1/3*(b^3*c*x - a*b^2*d*x - a^3
*f*x + a^2*b*x*e)/((b*x^3 + a)*a*b^3) + 1/4*(b^6*f*x^4 - 8*a*b^5*f*x + 4*b^
6*x*e)/b^8
```

Mupad [B]
time = 5.18, size = 241, normalized size = 0.91

$$x\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)+\frac{f\,x^4}{4\,b^2}+\frac{x\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{3\,a\,\left(b^4\,x^3+a\,b^3\right)}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(7\,f\,a^3-4\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{5/3}\,b^{10/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,f\,a^3-4\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{5/3}\,b^{10/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,f\,a^3-4\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{5/3}\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x)
```

```
[Out] x*(e/b^2 - (2*a*f)/b^3) + (f*x^4)/(4*b^2) + (x*(b^3*c - a^3*f - a*b^2*d + a
^2*b*e))/(3*a*(a*b^3 + b^4*x^3)) + (log(b^(1/3)*x + a^(1/3))*(2*b^3*c + 7*a
^3*f + a*b^2*d - 4*a^2*b*e))/(9*a^(5/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i
 + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*b^3*c + 7*a^3*f + a*b^2
*d - 4*a^2*b*e))/(9*a^(5/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)
*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*b^3*c + 7*a^3*f + a*b^2*d - 4*a^2*b
*e))/(9*a^(5/3)*b^(10/3))
```

3.267 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2} dx$

Optimal. Leaf size=265

$$-\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^2b^2\,(a+bx^3)}+\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{8/3}}+\frac{(4b^3c-a$$

```
[Out] -c/a^2/x+1/2*f*x^2/b^2-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^2/b^2/(b*x^
3+a)+1/9*(5*a^3*f-2*a^2*b*e-a*b^2*d+4*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/
b^(8/3)-1/18*(5*a^3*f-2*a^2*b*e-a*b^2*d+4*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/a^(7/3)/b^(8/3)+1/9*(5*a^3*f-2*a^2*b*e-a*b^2*d+4*b^3*c)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(8/3)*3^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1502, 298, 31, 648, 631, 210, 642}

$$-\frac{c}{a^2x}-\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^2b^2\,(a+bx^3)}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(5a^3f-2a^2be-ab^2d+4b^3c)}{3\sqrt{3}\,a^{7/3}b^{8/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(5a^3f-2a^2be-ab^2d+4b^3c)}{18a^{7/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(5a^3f-2a^2be-ab^2d+4b^3c)}{9a^{7/3}b^{8/3}}+\frac{fx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x^2)/(2*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/
(3*a^2*b^2*(a + b*x^3)) + ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(8/3)) +
((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(
7/3)*b^(8/3)) - ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+b\left(\frac{b^3c}{a}-b^2d-2abe+2a^2f\right)x^3-3ab^2fx^6}{x^2(a+bx^3)}\,dx}{3ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^2}-3abfx+\frac{b(4b^3c-ab^2d-2a^2be+5a^3f)x}{a(a+bx^3)}\right.}{3ab^3}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)}{3a^2b^2}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)}{9a^{7/3}b^{7/3}}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)}{9a^{7/3}b^{8/3}}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)}{9a^{7/3}b^{8/3}}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)}{3\sqrt{3}\,a^{7/3}b}$$

Mathematica [A]
time = 0.10, size = 255, normalized size = 0.96

$$\frac{1}{18}\left(-\frac{18c}{a^2x}+\frac{9fx^2}{b^2}+\frac{6(-b^3c+ab^2d-a^2be+a^3f)x^2}{a^2b^2(a+bx^3)}+\frac{2\sqrt{3}\,(4b^3c-ab^2d-2a^2be+5a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}b^{8/3}}+\frac{2(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{7/3}b^{8/3}}-\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{7/3}b^{8/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] ((-18*c)/(a^2*x) + (9*f*x^2)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*x^2)/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(4*b^3*c - a*b^2*d - 2*a^2*b*e +
5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(7/3)*b^(8/3)) + (
2*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(7
/3)*b^(8/3)) - ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(7/3)*b^(8/3)))/18
```

Maple [A]
time = 0.37, size = 187, normalized size = 0.71

method	result
default	$\frac{f x^2}{2b^2} - \frac{\frac{\left(-\frac{1}{3}a^3 f+\frac{1}{3}a^2 b e-\frac{1}{3}a b^2 d+\frac{1}{3}b^3 c\right)x^2}{b x^3+a}+\left(\frac{5}{3}a^3 f-\frac{1}{3}a b^2 d+\frac{4}{3}b^3 c-\frac{2}{3}a^2 b e\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan(\ldots}{\ldots}\right.}{a^2 b^2}$
risch	$\frac{f x^2}{2b^2}+\frac{\frac{\left(a^3 f-a^2 b e+a b^2 d-4b^3 c\right)x^3}{3a^2}-\frac{b^2 c}{a}}{b^2 x\left(b x^3+a\right)}+\frac{\_R=\text{RootOf}\left(a^7 b^2 \_Z^3-125a^9 f^3+150a^8 b e f^2+75a^7 b^2 d f^2-60a^7 b^2 e^2 f-300a^6 b^3 c f^2-60a^6 b^3 d e \ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*f*x^2/b^2-1/a^2/b^2*((-1/3*a^3*f+1/3*a^2*b*e-1/3*a*b^2*d+1/3*b^3*c)*x^2 /(b*x^3+a)+(5/3*a^3*f-1/3*a*b^2*d+4/3*b^3*c-2/3*a^2*b*e)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3 *3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-c/a^2/x

Maxima [A]

time = 0.50, size = 262, normalized size = 0.99

$$\frac{fx^2}{2b^2}-\frac{3ab^2c+\left(4b^3c-ab^2d-a^3f+a^2be\right)x^3}{3\left(a^2b^3x^4+a^3b^2x\right)}-\frac{\sqrt{3}\left(4b^3c-ab^2d+5a^3f-2a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(4b^3c-ab^2d+5a^3f-2a^2be\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(4b^3c-ab^2d+5a^3f-2a^2be\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")

[Out] 1/2*f*x^2/b^2 - 1/3*(3*a*b^2*c + (4*b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x^3) /(a^2*b^3*x^4 + a^3*b^2*x) - 1/9*sqrt(3)*(4*b^3*c - a*b^2*d + 5*a^3*f - 2*a ^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^ (1/3)) - 1/18*(4*b^3*c - a*b^2*d + 5*a^3*f - 2*a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(1/3)) + 1/9*(4*b^3*c - a*b^2*d + 5*a^3* f - 2*a^2*b*e)*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(1/3))

Fricas [A]

time = 0.42, size = 860, normalized size = 3.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(9*a^3*b^3*f*x^6 - 18*a^2*b^4*c - 3*(8*a*b^5*c - 2*a^2*b^4*d + 2*a^3*
b^3*e - 5*a^4*b^2*f)*x^3 + 3*sqrt(1/3)*((4*a*b^5*c - a^2*b^4*d - 2*a^3*b^3*
e + 5*a^4*b^2*f)*x^4 + (4*a^2*b^4*c - a^3*b^3*d - 2*a^4*b^2*e + 5*a^5*b*f)*
x)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*
b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*
x)/(b*x^3 + a)) - ((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a
*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a
*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*
a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/
3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^5*x^4 + a^4*b^4*x), 1/18*(9*a^3*b^3*f*x
^6 - 18*a^2*b^4*c - 3*(8*a*b^5*c - 2*a^2*b^4*d + 2*a^3*b^3*e - 5*a^4*b^2*f)
*x^3 + 6*sqrt(1/3)*((4*a*b^5*c - a^2*b^4*d - 2*a^3*b^3*e + 5*a^4*b^2*f)*x^4
 + (4*a^2*b^4*c - a^3*b^3*d - 2*a^4*b^2*e + 5*a^5*b*f)*x)*sqrt((a*b^2)^(1/3
)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - (
(4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d
- 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (
a*b^2)^(2/3)) + 2*((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a
*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b*x + (a*b^2
)^(1/3)))/(a^3*b^5*x^4 + a^4*b^4*x)]
```

Sympy [A]
time = 67.21, size = 457, normalized size = 1.72

$$\frac{-3ab^2c + x^3(a^3f - a^2be + ab^2d - 4b^3c)}{3a^3b^2x + 3a^2b^3x^4} + \operatorname{RootSum}\left(729t^3a^7b^8 - 125a^9f^3 + 150a^8bef^2 + 75a^7b^2df^2 - 60a^7b^2e^2f - 300a^6b^3cf^2 - 60a^6b^3def + 8a^6b^3e^3 + 240a^5b^4cef - 15a^5b^4d^2f + 12a^5b^4de^2 + 120a^4b^5cdf - 48a^4b^5ce^2 + 6a^4b^5d^2e - 240a^3b^6c^2f - 48a^3b^6cde + a^3b^6d^3 + 96a^2b^7c^2e - 12a^2b^7cd^2 + 48ab^8c^2d - 64b^9c^3, \left(t \mapsto t\log\left(\frac{81t^2a^5b^5}{25a^6f^2 - 20a^5bef - 10a^4b^2df + 4a^4b^2e^2 + 40a^3b^3cf + 4a^3b^3de - 16a^2b^4ce + a^2b^4d^2 - 8ab^5cd + 16b^6c^2} + x\right)\right)\right) + \frac{fx^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] (-3*a*b**2*c + x**3*(a**3*f - a**2*b*e + a*b**2*d - 4*b**3*c))/(3*a**3*b**2
*x + 3*a**2*b**3*x**4) + RootSum(729*_t**3*a**7*b**8 - 125*a**9*f**3 + 150*
a**8*b*e*f**2 + 75*a**7*b**2*d*f**2 - 60*a**7*b**2*e**2*f - 300*a**6*b**3*c
*f**2 - 60*a**6*b**3*d*e*f + 8*a**6*b**3*e**3 + 240*a**5*b**4*c*e*f - 15*a*
*5*b**4*d**2*f + 12*a**5*b**4*d*e**2 + 120*a**4*b**5*c*d*f - 48*a**4*b**5*c
*e**2 + 6*a**4*b**5*d**2*e - 240*a**3*b**6*c**2*f - 48*a**3*b**6*c*d*e + a*
*3*b**6*d**3 + 96*a**2*b**7*c**2*e - 12*a**2*b**7*c*d**2 + 48*a*b**8*c**2*d
 - 64*b**9*c**3, Lambda(_t, _t*log(81*_t**2*a**5*b**5/(25*a**6*f**2 - 20*a*
*5*b*e*f - 10*a**4*b**2*d*f + 4*a**4*b**2*e**2 + 40*a**3*b**3*c*f + 4*a**3*
b**3*d*e - 16*a**2*b**4*c*e + a**2*b**4*d**2 - 8*a*b**5*c*d + 16*b**6*c**2)
 + x))) + f*x**2/(2*b**2)
```

Giac [A]
time = 1.41, size = 305, normalized size = 1.15

$$\frac{fx^2}{2\,b^2} - \frac{\sqrt{3}\,(4\,b^3c - ab^2d + 5\,a^3f - 2\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{1}{3}}\,a^2b^2} + \frac{(4\,b^3c - ab^2d + 5\,a^3f - 2\,a^2be)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{1}{3}}\,a^2b^2} + \frac{\left(4\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^2} - \frac{4\,b^3cx^3 - ab^2dx^3 - a^3fx^3 + a^2bx^3e + 3\,ab^2c}{3\,(bx^4 + ax)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b^2 - 1/9*sqrt(3)*(4*b^3*c - a*b^2*d + 5*a^3*f - 2*a^2*b*e)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^2*b^2) +
 1/18*(4*b^3*c - a*b^2*d + 5*a^3*f - 2*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b^2) + 1/9*(4*b^3*c*(-a/b)^(1/3) - a*b^2*
d*(-a/b)^(1/3) + 5*a^3*f*(-a/b)^(1/3) - 2*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3
)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^2) - 1/3*(4*b^3*c*x^3 - a*b^2*d*x^3 - a
^3*f*x^3 + a^2*b*x^3*e + 3*a*b^2*c)/((b*x^4 + a*x)*a^2*b^2)
```

Mupad [B]
time = 5.39, size = 244, normalized size = 0.92

$$\frac{f\,x^2}{2\,b^2}-\frac{\frac{x^3\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{3\,a^2}+\frac{b^2\,c}{a}}{b^3\,x^4+a\,b^2\,x}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a^{7/3}\,b^{8/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a^{7/3}\,b^{8/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a^{7/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] (f*x^2)/(2*b^2) - ((x^3*(4*b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^2) + (b
^2*c)/a)/(b^3*x^4 + a*b^2*x) + (log(b^(1/3)*x + a^(1/3))*(4*b^3*c + 5*a^3*f
 - a*b^2*d - 2*a^2*b*e))/(9*a^(7/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*
b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(4*b^3*c + 5*a^3*f - a*b^2*d -
2*a^2*b*e))/(9*a^(7/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a
^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*b^3*c + 5*a^3*f - a*b^2*d - 2*a^2*b*e))/(
9*a^(7/3)*b^(8/3))
```

3.268 $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^2}\,dx$

Optimal. Leaf size=260

$$-\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^2b^2\,(a+bx^3)}+\frac{(5b^3c-2ab^2d-a^2be+4a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}b^{7/3}}-\frac{(5b^3c-2}{}$$

```
[Out] -1/2*c/a^2/x^2+f*x/b^2-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^2/b^2/(b*x^3+
a)-1/9*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^
(7/3)+1/18*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/a^(8/3)/b^(7/3)+1/9*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(7/3)*3^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1502, 206, 31, 648, 631, 210, 642}

$$-\frac{c}{2a^2x^2}-\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^2b^2\,(a+bx^3)}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(4a^3f-a^2be-2ab^2d+5b^3c)}{3\sqrt{3}\,a^{8/3}b^{7/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(4a^3f-a^2be-2ab^2d+5b^3c)}{18a^{8/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(4a^3f-a^2be-2ab^2d+5b^3c)}{9a^{8/3}b^{7/3}}+\frac{fx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/2*c/(a^2*x^2) + (f*x)/b^2 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a
^2*b^2*(a + b*x^3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(7/3)) - ((5*
b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)
*b^(7/3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+b\left(\frac{2b^3c}{a}-2b^2d-abe+a^2f\right)x^3-3ab^2fx^6}{x^3(a+bx^3)}\,dx}{3ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)} - \frac{\int \left(-3abf-\frac{3b^3c}{ax^3}+\frac{b\left(5b^3c-2ab^2d-a^2be+4a^3f\right)}{a(a+bx^3)}\right)\,dx}{3ab^3}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)}{3a^2b^2}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)}{9a^{8/3}b^2}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)}{9a^{8/3}b^{7/3}}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)}{9a^{8/3}b^{7/3}}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)}{3\sqrt{3}\,a^{8/3}b}$$

Mathematica [A]
time = 0.11, size = 250, normalized size = 0.96

$$\frac{1}{18}\left(-\frac{9c}{a^2x^2}+\frac{18fx}{b^2}+\frac{6\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{a^2b^2\left(a+bx^3\right)}+\frac{2\sqrt{3}\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}b^{7/3}}-\frac{2\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{8/3}b^{7/3}}+\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{8/3}b^{7/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*c)/(a^2*x^2) + (18*f*x)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*x))/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*
a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(8/3)*b^(7/3)) - (2*
(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(8/3
)*b^(7/3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(8/3)*b^(7/3)))/18
```

Maple [A]
time = 0.37, size = 183, normalized size = 0.70

method	result
default	$\frac{fx}{b^2}-\frac{\frac{\left(-\frac{1}{3}a^3f+\frac{1}{3}a^2be-\frac{1}{3}a\,b^2d+\frac{1}{3}b^3c\right)x}{b\,x^3+a}+\frac{\left(4a^3f-a^2be-2a\,b^2d+5b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\cdots\right.}{3\cdots}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{3}}{a^2b^2}$
risch	$\frac{fx}{b^2}+\frac{\frac{\left(2a^3f-2a^2be+2a\,b^2d-5b^3c\right)x^3}{6a^2}-\frac{b^2c}{2a}}{b^2x^2\left(b\,x^3+a\right)}+\frac{\_R=\text{RootOf}\left(a^8b\_Z^3+64a^9f^3-48a^8be\,f^2-96a^7b^2d\,f^2+12a^7b^2e^2f+240a^6b^3c\,f^2+48a^6b^3def-\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f*x/b^2-1/a^2/b^2*((-1/3*a^3*f+1/3*a^2*b*e-1/3*a*b^2*d+1/3*b^3*c)*x/(b*x^3+
a)+1/3*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1
/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/2*c/a^2/x^2
```

Maxima [A]

time = 0.51, size = 262, normalized size = 1.01

$$-\frac{3\,ab^2c+\left(5\,b^3c-2\,ab^2d-2\,a^3f+2\,a^2be\right)x^3}{6\left(a^2b^3x^5+a^3b^2x^2\right)}+\frac{fx}{b^2}-\frac{\sqrt{3}\left(5\,b^3c-2\,ab^2d+4\,a^3f-a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(5\,b^3c-2\,ab^2d+4\,a^3f-a^2be\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(5\,b^3c-2\,ab^2d+4\,a^3f-a^2be\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a*b^2*c + (5*b^3*c - 2*a*b^2*d - 2*a^3*f + 2*a^2*b*e)*x^3)/(a^2*b^3
*x^5 + a^3*b^2*x^2) + f*x/b^2 - 1/9*sqrt(3)*(5*b^3*c - 2*a*b^2*d + 4*a^3*f
- a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/
b)^(2/3)) + 1/18*(5*b^3*c - 2*a*b^2*d + 4*a^3*f - a^2*b*e)*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(2/3)) - 1/9*(5*b^3*c - 2*a*b^2*d + 4
*a^3*f - a^2*b*e)*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))
```

Fricas [A]

time = 0.41, size = 902, normalized size = 3.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(18*a^4*b^2*f*x^6 - 9*a^3*b^3*c - 3*(5*a^2*b^4*c - 2*a^3*b^3*d + 2*a^
4*b^2*e - 8*a^5*b*f)*x^3 + 3*sqrt(1/3)*((5*a*b^5*c - 2*a^2*b^4*d - a^3*b^3*
e + 4*a^4*b^2*f)*x^5 + (5*a^2*b^4*c - 2*a^3*b^3*d - a^4*b^2*e + 4*a^5*b*f)*
x^2)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3
*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^
(1/3)/b))/(b*x^3 + a)) + ((5*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5
 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*
b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((5*b^4*c - 2*a*b^3*d - a^
2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^
2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^4*x^5 + a^5*b^3*x^2),
 1/18*(18*a^4*b^2*f*x^6 - 9*a^3*b^3*c - 3*(5*a^2*b^4*c - 2*a^3*b^3*d + 2*a^
4*b^2*e - 8*a^5*b*f)*x^3 - 6*sqrt(1/3)*((5*a*b^5*c - 2*a^2*b^4*d - a^3*b^3*
e + 4*a^4*b^2*f)*x^5 + (5*a^2*b^4*c - 2*a^3*b^3*d - a^4*b^2*e + 4*a^5*b*f)*
x^2)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b
)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + ((5*b^4*c - 2*a*b^3*d - a^2*b^2*e
 + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^
2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((5*b^4*c
 - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*
b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^4*x^
5 + a^5*b^3*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.49, size = 261, normalized size = 1.00

$$\frac{fx}{b^2}+\frac{\sqrt{3}\,(5\,b^3c-2\,ab^2d+4\,a^3f-a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a^2b}+\frac{(5\,b^3c-2\,ab^2d+4\,a^3f-a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a^2b}+\frac{(5\,b^3c-2\,ab^2d+4\,a^3f-a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^2}-\frac{c}{2\,a^2x^2}-\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{3\,(bx^3+a)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] f*x/b^2 + 1/9*sqrt(3)*(5*b^3*c - 2*a*b^2*d + 4*a^3*f - a^2*b*e)*arctan(1/3*
sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b) + 1/18*(5
*b^3*c - 2*a*b^2*d + 4*a^3*f - a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(2/3)*a^2*b) + 1/9*(5*b^3*c - 2*a*b^2*d + 4*a^3*f - a^2*b*e
```

```
)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^2) - 1/2*c/(a^2*x^2) - 1/3
*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*a^2*b^2)
```

Mupad [B]
time = 5.22, size = 245, normalized size = 0.94

$$\frac{f\,x}{b^2}-\frac{\frac{x^3\,\left(-2\,f\,a^3+2\,e\,a^2\,b-2\,d\,a\,b^2+5\,c\,b^3\right)}{6\,a^2}+\frac{b^2\,c}{2\,a}}{b^3\,x^5+a\,b^2\,x^2}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(4\,f\,a^3-e\,a^2\,b-2\,d\,a\,b^2+5\,c\,b^3\right)}{9\,a^{8/3}\,b^{7/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,f\,a^3-e\,a^2\,b-2\,d\,a\,b^2+5\,c\,b^3\right)}{9\,a^{8/3}\,b^{7/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,f\,a^3-e\,a^2\,b-2\,d\,a\,b^2+5\,c\,b^3\right)}{9\,a^{8/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] (f*x)/b^2 - ((x^3*(5*b^3*c - 2*a^3*f - 2*a*b^2*d + 2*a^2*b*e))/(6*a^2) + (b
^2*c)/(2*a))/(b^3*x^5 + a*b^2*x^2) - (log(b^(1/3)*x + a^(1/3))*(5*b^3*c + 4
*a^3*f - 2*a*b^2*d - a^2*b*e))/(9*a^(8/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1
i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c + 4*a^3*f - 2*a*
b^2*d - a^2*b*e))/(9*a^(8/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)
*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*b^3*c + 4*a^3*f - 2*a*b^2*d - a^2*b
*e))/(9*a^(8/3)*b^(7/3))
```

3.269 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2}\,dx$

Optimal. Leaf size=269

$$-\frac{c}{4a^2x^4}+\frac{2bc-ad}{a^3x}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^3b\,(a+bx^3)}-\frac{(7b^3c-4ab^2d+a^2be+2a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}b^{5/3}}-\frac{(7$$

```
[Out] -1/4*c/a^2/x^4+(-a*d+2*b*c)/a^3/x+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^
3/b/(b*x^3+a)-1/9*(2*a^3*f+a^2*b*e-4*a*b^2*d+7*b^3*c)*ln(a^(1/3)+b^(1/3)*x)
/a^(10/3)/b^(5/3)+1/18*(2*a^3*f+a^2*b*e-4*a*b^2*d+7*b^3*c)*ln(a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(5/3)-1/9*(2*a^3*f+a^2*b*e-4*a*b^2*d+7
*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(5/3)*
3^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1502, 298, 31, 648, 631, 210, 642}

$$\frac{2bc-ad}{a^3x}-\frac{c}{4a^2x^4}+\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^3b\,(a+bx^3)}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(2a^3f+a^2be-4ab^2d+7b^3c)}{3\sqrt{3}\,a^{10/3}b^{5/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(2a^3f+a^2be-4ab^2d+7b^3c)}{18a^{10/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(2a^3f+a^2be-4ab^2d+7b^3c)}{9a^{10/3}b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] -1/4*c/(a^2*x^4) + (2*b*c - a*d)/(a^3*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*x^2)/(3*a^3*b*(a + b*x^3)) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)*b^
(5/3)) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x
])/(9*a^(10/3)*b^(5/3)) + ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(10/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^2}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-b^2\left(\frac{b^3c}{a^2}-\frac{b^2d}{a}+be+2af\right)x^6}{x^5(a+bx^3)}\,dx}{3ab^3}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^5} - \frac{3b^3(-2bc+ad)}{a^2x^2} - \frac{b^2\left(7b^3c-4ab^2d+a^2be+2a^3\right.}{a^2(a+bx^3)}\right.}{3ab^3}$$

$$= -\frac{c}{4a^2x^4} + \frac{2bc-ad}{a^3x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)} + \frac{\left(7b^3c-4ab^2d+a^2be+\right.}{3a^3b}$$

$$= -\frac{c}{4a^2x^4} + \frac{2bc-ad}{a^3x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)} - \frac{\left(7b^3c-4ab^2d+a^2be+\right.}{9a^{10/}}$$

$$= -\frac{c}{4a^2x^4} + \frac{2bc-ad}{a^3x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)} - \frac{\left(7b^3c-4ab^2d+a^2be+\right.}{9a^1}$$

$$= -\frac{c}{4a^2x^4} + \frac{2bc-ad}{a^3x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)} - \frac{\left(7b^3c-4ab^2d+a^2be+\right.}{9a^1}$$

$$= -\frac{c}{4a^2x^4} + \frac{2bc-ad}{a^3x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)} - \frac{\left(7b^3c-4ab^2d+a^2be+\right.}{3\sqrt{3}}$$

Mathematica [A]
time = 0.11, size = 255, normalized size = 0.95

$$\frac{-\frac{9a^{4/3}c}{x^4} - \frac{36\sqrt[3]{a}\,(-2bc+ad)}{x} - \frac{12\sqrt[3]{a}\,\left(-b^3c+ab^2d-a^2be+a^3f\right)x^2}{b(a+bx^3)} - \frac{4\sqrt{3}\,\left(7b^3c-4ab^2d+a^2be+2a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} - \frac{4\left(7b^3c-4ab^2d+a^2be+2a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{5/3}} + \frac{2\left(7b^3c-4ab^2d+a^2be+2a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{5/3}}}{36a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(4/3)*c)/x^4 - (36*a^(1/3)*(-2*b*c + a*d))/x - (12*a^(1/3)*(-(b^3*c)
 + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)) - (4*Sqrt[3]*(7*b^3*c -
4*a*b^2*d + a^2*b*e + 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])
/b^(5/3) - (4*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/
3)*x])/b^(5/3) + (2*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3))/(36*a^(10/3))
```

Maple [A]
time = 0.36, size = 195, normalized size = 0.72

method	result
default	$\frac{-\frac{\left(a^3 f-a^2 b e+a\,b^2 d-b^3 c\right)x^2}{3b\left(b\,x^3+a\right)}+\frac{\left(2a^3 f+a^2 b e-4a\,b^2 d+7b^3 c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}}{a^3}$
risch	$\frac{-\frac{\left(a^3 f-a^2 b e+4a\,b^2 d-7b^3 c\right)x^6}{3a^3 b}-\frac{(4ad-7bc)x^3}{4a^2}-\frac{c}{4a}}{x^4\left(b\,x^3+a\right)}+\frac{\left(\left(\_R=\mathrm{RootOf}\left(a^{10}b^5\_Z^3+8a^9 f^3+12a^8 be\,f^2-48a^7 b^2 d\,f^2+6a^7 b^2 e^2 f+84a^6 b^3 c\,f^2-48a^6\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/a^3*(-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b*x^2/(b*x^3+a)+1/3*(2*a^3*f+a^2*b*e-4*a*b^2*d+7*b^3*c)/b*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/4*c/a^2/x^4-(a*d-2*b*c)/a^3/x

Maxima [A]

time = 0.50, size = 271, normalized size = 1.01

$$\frac{4\left(7b^3c-4ab^2d-a^3f+a^2be\right)x^6-3a^2bc+3\left(7ab^2c-4a^2bd\right)x^3}{12\left(a^3b^2x^7+a^4bx^4\right)}+\frac{\sqrt{3}\left(7b^3c-4ab^2d+2a^3f+a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(7b^3c-4ab^2d+2a^3f+a^2be\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(7b^3c-4ab^2d+2a^3f+a^2be\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="maxima")

[Out] 1/12*(4*(7*b^3*c - 4*a*b^2*d - a^3*f + a^2*b*e)*x^6 - 3*a^2*b*c + 3*(7*a*b^2*c - 4*a^2*b*d)*x^3)/(a^3*b^2*x^7 + a^4*b*x^4) + 1/9*sqrt(3)*(7*b^3*c - 4*a*b^2*d + 2*a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3)) + 1/18*(7*b^3*c - 4*a*b^2*d + 2*a^3*f + a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(1/3)) - 1/9*(7*b^3*c - 4*a*b^2*d + 2*a^3*f + a^2*b*e)*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3))

Fricas [A]

time = 0.43, size = 902, normalized size = 3.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/36*(9*a^3*b^3*c - 12*(7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e - a^4*b^2*f)*
x^6 - 9*(7*a^2*b^4*c - 4*a^3*b^3*d)*x^3 - 6*sqrt(1/3)*((7*a*b^5*c - 4*a^2*b
^4*d + a^3*b^3*e + 2*a^4*b^2*f)*x^7 + (7*a^2*b^4*c - 4*a^3*b^3*d + a^4*b^2*
e + 2*a^5*b*f)*x^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/
3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a)
 - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 2*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e +
 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 4*((7*b^4*c -
 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*
e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^4*x^7 +
a^5*b^3*x^4), -1/36*(9*a^3*b^3*c - 12*(7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e
- a^4*b^2*f)*x^6 - 9*(7*a^2*b^4*c - 4*a^3*b^3*d)*x^3 - 12*sqrt(1/3)*((7*a*b
^5*c - 4*a^2*b^4*d + a^3*b^3*e + 2*a^4*b^2*f)*x^7 + (7*a^2*b^4*c - 4*a^3*b^
3*d + a^4*b^2*e + 2*a^5*b*f)*x^4)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*
(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 2*((7*b^4*c - 4*a*b^3
*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^
4*f)*x^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3))
 + 4*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^
2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))
/(a^4*b^4*x^7 + a^5*b^3*x^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.39, size = 310, normalized size = 1.15

$$\frac{\sqrt{3}\,(7\,b^3c-4\,ab^2d+2\,a^3f+a^2be)\arctan\left(\frac{\sqrt{3}\,\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{1}{3}}\,a^3b}-\frac{(7\,b^3c-4\,ab^2d+2\,a^3f+a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{1}{3}}\,a^3b}-\frac{\left(7\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4b}+\frac{b^3cx^2-ab^2dx^2-a^3fx^2+a^2bx^2e}{3\,(bx^3+a)a^3b}+\frac{8\,bcx^3-4\,adx^3-ac}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(7*b^3*c - 4*a*b^2*d + 2*a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^3*b) - 1/18*(7*b^3*c - 4
*a*b^2*d + 2*a^3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a
```

```
*b^2)^(1/3)*a^3*b) - 1/9*(7*b^3*c*(-a/b)^(1/3) - 4*a*b^2*d*(-a/b)^(1/3) + 2
*a^3*f*(-a/b)^(1/3) + a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^4*b) + 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^2*e)/
((b*x^3 + a)*a^3*b) + 1/4*(8*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^3*x^4)
```

Mupad [B]
time = 5.18, size = 247, normalized size = 0.92

$$-\frac{\frac{c}{4\,a}+\frac{x^3\,\left(4\,a\,d-7\,b\,c\right)}{4\,a^2}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-4\,d\,a\,b^2+7\,c\,b^3\right)}{3\,a^3\,b}}{b\,x^7+a\,x^4}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(2\,f\,a^3+e\,a^2\,b-4\,d\,a\,b^2+7\,c\,b^3\right)}{9\,a^{10/3}\,b^{5/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,f\,a^3+e\,a^2\,b-4\,d\,a\,b^2+7\,c\,b^3\right)}{9\,a^{10/3}\,b^{5/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,f\,a^3+e\,a^2\,b-4\,d\,a\,b^2+7\,c\,b^3\right)}{9\,a^{10/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(7*
b^3*c + 2*a^3*f - 4*a*b^2*d + a^2*b*e))/(9*a^(10/3)*b^(5/3)) - (log(b^(1/3)
*x + a^(1/3))*(7*b^3*c + 2*a^3*f - 4*a*b^2*d + a^2*b*e))/(9*a^(10/3)*b^(5/3
)) - (c/(4*a) + (x^3*(4*a*d - 7*b*c))/(4*a^2) - (x^6*(7*b^3*c - a^3*f - 4*a
*b^2*d + a^2*b*e))/(3*a^3*b))/(a*x^4 + b*x^7) - (log(3^(1/2)*a^(1/3)*1i - 2
*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(7*b^3*c + 2*a^3*f - 4*a*b^2*d
 + a^2*b*e))/(9*a^(10/3)*b^(5/3))
```

3.270 $\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=270

$$-\frac{c}{5a^2x^5}+\frac{2bc-ad}{2a^3x^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)}-\frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}b^{4/3}}+\frac{(8b}{}$$

```
[Out] -1/5*c/a^2/x^5+1/2*(-a*d+2*b*c)/a^3/x^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
x/a^3/b/(b*x^3+a)+1/9*(a^3*f+2*a^2*b*e-5*a*b^2*d+8*b^3*c)*ln(a^(1/3)+b^(1/3
)*x)/a^(11/3)/b^(4/3)-1/18*(a^3*f+2*a^2*b*e-5*a*b^2*d+8*b^3*c)*ln(a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(4/3)-1/9*(a^3*f+2*a^2*b*e-5*a*b^2
*d+8*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(4
/3)*3^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1502, 206, 31, 648, 631, 210, 642}

$$\frac{2bc-ad}{2a^3x^2}-\frac{c}{5a^2x^5}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{3\sqrt{3}\,a^{11/3}b^{4/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{18a^{11/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{9a^{11/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] -1/5*c/(a^2*x^5) + (2*b*c - a*d)/(2*a^3*x^2) + ((b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*x)/(3*a^3*b*(a + b*x^3)) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3
*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(11/3)*
b^(4/3)) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)
*x])/(9*a^(11/3)*b^(4/3)) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(11/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^2}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-b^2\left(\frac{2b^3c}{a^2}-\frac{2b^2d}{a}+2be+af\right)x^6}{x^6\left(a+bx^3\right)}\,dx}{3ab^3}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^6} - \frac{3b^3(-2bc+ad)}{a^2x^3} - \frac{b^2\left(8b^3c-5ab^2d+2a^2be+a^3f\right)}{a^2\left(a+bx^3\right)}\right.}{3ab^3}$$

$$= -\frac{c}{5a^2x^5} + \frac{2bc-ad}{2a^3x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)} + \frac{\left(8b^3c-5ab^2d+2a^2be\right.}{3a^3b}$$

$$= -\frac{c}{5a^2x^5} + \frac{2bc-ad}{2a^3x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)} + \frac{\left(8b^3c-5ab^2d+2a^2be\right.}{9a^{11}}$$

$$= -\frac{c}{5a^2x^5} + \frac{2bc-ad}{2a^3x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)} + \frac{\left(8b^3c-5ab^2d+2a^2be\right.}{9a^{11}}$$

$$= -\frac{c}{5a^2x^5} + \frac{2bc-ad}{2a^3x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)} + \frac{\left(8b^3c-5ab^2d+2a^2be\right.}{9a^{11}}$$

$$= -\frac{c}{5a^2x^5} + \frac{2bc-ad}{2a^3x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)} - \frac{\left(8b^3c-5ab^2d+2a^2be\right.}{3\sqrt{3}}$$

Mathematica [A]
time = 0.11, size = 253, normalized size = 0.94

$$\frac{-\frac{18a^{5/3}c}{x^5} - \frac{45a^{2/3}(-2bc+ad)}{x^2} - \frac{30a^{2/3}\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{b\left(a+bx^3\right)} - \frac{10\sqrt{3}\left(8b^3c-5ab^2d+2a^2be+a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}} + \frac{10\left(8b^3c-5ab^2d+2a^2be+a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{4/3}} - \frac{5\left(8b^3c-5ab^2d+2a^2be+a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{4/3}}}{90a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(5/3)*c)/x^5 - (45*a^(2/3)*(-2*b*c + a*d))/x^2 - (30*a^(2/3)*(-(b^3
*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)) - (10*Sqrt[3]*(8*b^3*c
- 5*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/b^(4/3) + (10*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^
(1/3)*x])/b^(4/3) - (5*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3))/(90*a^(11/3))
```

Maple [A]
time = 0.44, size = 193, normalized size = 0.71

method	result
default	$\frac{-\frac{\left(a^{3} f-a^{2} b e+a \,b^{2} d-b^{3} c\right) x}{3 b \left(b \,x^{3}+a\right)}+\frac{\left(a^{3} f+2 a^{2} b e-5 a \,b^{2} d+8 b^{3} c\right) \left(\frac{\ln \left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3 b}}{a^{3}}$
risch	$\frac{-\frac{\left(2 a^{3} f-2 a^{2} b e+5 a \,b^{2} d-8 b^{3} c\right) x^{6}}{6 a^{3} b}-\frac{\left(5 a d-8 b c\right) x^{3}}{10 a^{2}}-\frac{c}{5 a}}{x^{5} \left(b \,x^{3}+a\right)}+\Big(\_R=\mathrm{RootOf}\big(a^{11} b^{4} \_Z^{3}-a^{9} f^{3}-6 a^{8} b e \,f^{2}+15 a^{7} b^{2} d \,f^{2}-12 a^{7} b^{2} e^{2} f-24 a^{6} b^{3} c \,f^{2}+60 a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b*x/(b*x^3+a)+1/3*(a^3*f+2*a^2*b*
e-5*a*b^2*d+8*b^3*c)/b*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/5*c/a^2/x^5-1/2*(a*d-2*b*c)/a^3/x^2
```

Maxima [A]
time = 0.51, size = 272, normalized size = 1.01

$$\frac{5\,(8\,b^{3}c-5\,ab^{2}d-2\,a^{3}f+2\,a^{2}be)x^{6}-6\,a^{2}bc+3\,(8\,ab^{2}c-5\,a^{2}bd)x^{3}}{30\,(a^{3}b^{2}x^{8}+a^{4}bx^{5})}+\frac{\sqrt{3}\,(8\,b^{3}c-5\,ab^{2}d+a^{3}f+2\,a^{2}be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(8\,b^{3}c-5\,ab^{2}d+a^{3}f+2\,a^{2}be)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{3}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(8\,b^{3}c-5\,ab^{2}d+a^{3}f+2\,a^{2}be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{3}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(5*(8*b^3*c - 5*a*b^2*d - 2*a^3*f + 2*a^2*b*e)*x^6 - 6*a^2*b*c + 3*(8*
a*b^2*c - 5*a^2*b*d)*x^3)/(a^3*b^2*x^8 + a^4*b*x^5) + 1/9*sqrt(3)*(8*b^3*c
- 5*a*b^2*d + a^3*f + 2*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(a^3*b^2*(a/b)^(2/3)) - 1/18*(8*b^3*c - 5*a*b^2*d + a^3*f + 2*a^2
*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) + 1/9*(8
*b^3*c - 5*a*b^2*d + a^3*f + 2*a^2*b*e)*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)
^(2/3))
```

Fricas [A]
time = 0.44, size = 897, normalized size = 3.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/90*(18*a^4*b^2*c - 15*(8*a^2*b^4*c - 5*a^3*b^3*d + 2*a^4*b^2*e - 2*a^5*
b*f)*x^6 - 9*(8*a^3*b^3*c - 5*a^4*b^2*d)*x^3 - 15*sqrt(1/3)*((8*a*b^5*c - 5
*a^2*b^4*d + 2*a^3*b^3*e + a^4*b^2*f)*x^8 + (8*a^2*b^4*c - 5*a^3*b^3*d + 2*
a^4*b^2*e + a^5*b*f)*x^5)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)
^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)
*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + 5*((8*b^4*c - 5*a*b^3*d + 2*a^2*
b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(
a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 10*((8*b^4*
c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a
^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^3*x^8
 + a^6*b^2*x^5), -1/90*(18*a^4*b^2*c - 15*(8*a^2*b^4*c - 5*a^3*b^3*d + 2*a^
4*b^2*e - 2*a^5*b*f)*x^6 - 9*(8*a^3*b^3*c - 5*a^4*b^2*d)*x^3 - 30*sqrt(1/3)
*((8*a*b^5*c - 5*a^2*b^4*d + 2*a^3*b^3*e + a^4*b^2*f)*x^8 + (8*a^2*b^4*c -
5*a^3*b^3*d + 2*a^4*b^2*e + a^5*b*f)*x^5)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt
(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 5*
((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2
*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x +
(a^2*b)^(1/3)*a) - 10*((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 +
(8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x
+ (a^2*b)^(2/3)))/(a^5*b^3*x^8 + a^6*b^2*x^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.74, size = 264, normalized size = 0.98

$$-\frac{\sqrt{3}\,(8\,b^3c-5\,ab^2d+a^3f+2\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a^3}-\frac{(8\,b^3c-5\,ab^2d+a^3f+2\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a^3}-\frac{(8\,b^3c-5\,ab^2d+a^3f+2\,a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4b}+\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{3\,(bx^3+a)a^3b}+\frac{10\,bcx^3-5\,adx^3-2\,ac}{10\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(8*b^3*c - 5*a*b^2*d + a^3*f + 2*a^2*b*e)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/18*(8*b^3*c - 5*
a*b^2*d + a^3*f + 2*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*
```

```
b^2)^(2/3)*a^3) - 1/9*(8*b^3*c - 5*a*b^2*d + a^3*f + 2*a^2*b*e)*(-a/b)^(1/3
)*log(abs(x - (-a/b)^(1/3)))/(a^4*b) + 1/3*(b^3*c*x - a*b^2*d*x - a^3*f*x +
 a^2*b*x*e)/((b*x^3 + a)*a^3*b) + 1/10*(10*b*c*x^3 - 5*a*d*x^3 - 2*a*c)/(a^
3*x^5)
```

Mupad [B]
time = 5.13, size = 248, normalized size = 0.92

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{9\,a^{11/3}\,b^{4/3}}-\frac{\frac{c}{5\,a}+\frac{x^3\,\left(5\,a\,d-8\,b\,c\right)}{10\,a^2}-\frac{x^6\,\left(-2\,f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{6\,a^3\,b}}{b\,x^8+a\,x^5}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{9\,a^{11/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{9\,a^{11/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(8*b^3*c + a^3*f - 5*a*b^2*d + 2*a^2*b*e))/(9*a^(
11/3)*b^(4/3)) - (c/(5*a) + (x^3*(5*a*d - 8*b*c))/(10*a^2) - (x^6*(8*b^3*c
- 2*a^3*f - 5*a*b^2*d + 2*a^2*b*e))/(6*a^3*b))/(a*x^5 + b*x^8) + (log(3^(1/
2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(8*b^3*c + a^
3*f - 5*a*b^2*d + 2*a^2*b*e))/(9*a^(11/3)*b^(4/3)) - (log(3^(1/2)*a^(1/3)*1
i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(8*b^3*c + a^3*f - 5*a*b^
2*d + 2*a^2*b*e))/(9*a^(11/3)*b^(4/3))
```

3.271 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx$

Optimal. Leaf size=297

$$-\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4\,(a+bx^3)}+\frac{(10b^3c-7ab^2d+4a^2be-a^3f)\tan^{-}}{3\sqrt{3}\,a^{13/3}b^{2/3}}$$

```
[Out] -1/7*c/a^2/x^7+1/4*(-a*d+2*b*c)/a^3/x^4+(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x-1/3*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^4/(b*x^3+a)+1/9*(-a^3*f+4*a^2*b*e-7*a*
b^2*d+10*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)/b^(2/3)-1/18*(-a^3*f+4*a^2*b
*e-7*a*b^2*d+10*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(13/3)/b
^(2/3)+1/9*(-a^3*f+4*a^2*b*e-7*a*b^2*d+10*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1848, 298, 31, 648, 631, 210, 642}

$$\frac{2bc-ad}{4a^3x^4}-\frac{c}{7a^2x^7}-\frac{a^2e-2abd+3b^2c}{a^4x}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+4a^2be-7ab^2d+10b^3c)}{3\sqrt{3}\,a^{13/3}b^{2/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)(a^3(-f)+4a^2be-7ab^2d+10b^3c)}{18a^{13/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(a^3(-f)+4a^2be-7ab^2d+10b^3c)}{9a^{13/3}b^{2/3}}-\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^4\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```

```
[Out] -1/7*c/(a^2*x^7) + (2*b*c - a*d)/(4*a^3*x^4) - (3*b^2*c - 2*a*b*d + a^2*e)/
(a^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^4*(a + b*x^3)) + (
(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(13/3)*b^(2/3)) + ((10*b^3*c - 7*a*b^2*d +
4*a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(13/3)*b^(2/3)) - ((10*b^
3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(18*a^(13/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^2}\,dx &= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{b^3\left(b^3c-ab\right.}{}}{x^8\left(a+bx^3\right)}}{3ab^3} \\
&= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^8}-\frac{3b^3(-2bc+ad)}{a^2x^5}-\frac{3b^3\left(3b^2c-2abd+a^2e\right)}{a^3x^2}-\right.}{3ab^3} \\
&= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)}-\frac{(}{} \\
&= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)}+-\frac{(}{} \\
&= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)}+-\frac{(}{} \\
&= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)}+-\frac{(}{} \\
&= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)}+-\frac{(}{}
\end{aligned}
$$

Mathematica [A]

time = 0.11, size = 281, normalized size = 0.95

$$
\frac{-\frac{36a^{7/3}c}{x^7}-\frac{63a^{4/3}(-2bc+ad)}{x^4}-\frac{252\sqrt[3]{a}\left(3b^2c-2abd+a^2e\right)}{x}+\frac{84\sqrt[3]{a}\left(-b^3c+ab^2d-a^2be+a^3f\right)x^2}{a+bx^3}+\frac{28\sqrt{3}\left(10b^3c-7ab^2d+4a^2be-a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{28\left(10b^3c-7ab^2d+4a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{14\left(-10b^3c+7ab^2d-4a^2be+a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}}{252a^{13/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```

```
[Out] ((-36*a^(7/3)*c)/x^7 - (63*a^(4/3)*(-2*b*c + a*d))/x^4 - (252*a^(1/3)*(3*b^
2*c - 2*a*b*d + a^2*e))/x + (84*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3
*f)*x^2)/(a + b*x^3) + (28*Sqrt[3]*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*
f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(10*b^3*c - 7
*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (14*(-10*
b^3*c + 7*a*b^2*d - 4*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/b^(2/3))/(252*a^(13/3))
```

Maple [A]

time = 0.42, size = 215, normalized size = 0.72

method	result
default	$\frac{\frac{\left(\frac{1}{3}a^3f-\frac{1}{3}a^2be+\frac{1}{3}a\,b^2d-\frac{1}{3}b^3c\right)x^2}{b\,x^3+a}+\left(-\frac{4}{3}a^2be+\frac{7}{3}a\,b^2d-\frac{10}{3}b^3c+\frac{1}{3}a^3f\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(}{}\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{a^4}$
risch	$\frac{\frac{\left(a^3f-4a^2be+7a\,b^2d-10b^3c\right)x^9}{3a^4}-\frac{\left(4a^2e-7abd+10b^2c\right)x^6}{4a^3}-\frac{(7ad-10bc)x^3}{28a^2}-\frac{c}{7a}}{x^7\left(b\,x^3+a\right)}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^{13}b^2\_Z^3+a^9f^3-12a^8be\,f^2+21a^7b^2d\,f^2+48\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*((1/3*a^3*f-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x^2/(b*x^3+a)+(-4/3*a^
2*b*e+7/3*a*b^2*d-10/3*b^3*c+1/3*a^3*f)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3
))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^
(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/7*c/a^2/x^7-1/4*(a*d-2*b*
c)/a^3/x^4-(a^2*e-2*a*b*d+3*b^2*c)/a^4/x
```

Maxima [A]

time = 0.50, size = 297, normalized size = 1.00

$$-\frac{28\left(10\,b^3c-7\,ab^2d-a^3f+4\,a^2be\right)x^9+21\left(10\,ab^2c-7\,a^2bd+4\,a^3e\right)x^6+12\,a^3c-3\left(10\,a^2bc-7\,a^3d\right)x^3}{84\left(a^4bx^{10}+a^5x^7\right)}-\frac{\sqrt{3}\left(10\,b^3c-7\,ab^2d-a^3f+4\,a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(10\,b^3c-7\,ab^2d-a^3f+4\,a^2be\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(10\,b^3c-7\,ab^2d-a^3f+4\,a^2be\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/84*(28*(10*b^3*c - 7*a*b^2*d - a^3*f + 4*a^2*b*e)*x^9 + 21*(10*a*b^2*c -
 7*a^2*b*d + 4*a^3*e)*x^6 + 12*a^3*c - 3*(10*a^2*b*c - 7*a^3*d)*x^3)/(a^4*b
*x^10 + a^5*x^7) - 1/9*sqrt(3)*(10*b^3*c - 7*a*b^2*d - a^3*f + 4*a^2*b*e)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(1/3)) - 1/
18*(10*b^3*c - 7*a*b^2*d - a^3*f + 4*a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/
b)^(2/3))/(a^4*b*(a/b)^(1/3)) + 1/9*(10*b^3*c - 7*a*b^2*d - a^3*f + 4*a^2*b
*e)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(1/3))
```

Fricas [A]

time = 0.41, size = 982, normalized size = 3.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/252*(84*(10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x^9 + 36*a
^4*b^2*c + 63*(10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e)*x^6 - 9*(10*a^3*b^
3*c - 7*a^4*b^2*d)*x^3 + 42*sqrt(1/3)*((10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^
3*e - a^4*b^2*f)*x^10 + (10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e - a^5*b*f
)*x^7)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2
*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)
^(2/3)*x)/(b*x^3 + a)) + 14*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)
*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7)*(-a*b^2)^(2/3)*
log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((10*b^4*c - 7*a*b^
3*d + 4*a^2*b^2*e - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e -
 a^4*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^3*x^10 + a^6*
b^2*x^7), -1/252*(84*(10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x
^9 + 36*a^4*b^2*c + 63*(10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e)*x^6 - 9*(
10*a^3*b^3*c - 7*a^4*b^2*d)*x^3 + 84*sqrt(1/3)*((10*a*b^5*c - 7*a^2*b^4*d +
 4*a^3*b^3*e - a^4*b^2*f)*x^10 + (10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e
- a^5*b*f)*x^7)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^
(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e
 - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((10*b^4
*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d +
4*a^3*b*e - a^4*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^3*
x^10 + a^6*b^2*x^7)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.01, size = 333, normalized size = 1.12

$$-\frac{\sqrt{3}\,(10\,b^3c-7\,ab^2d-a^3f+4\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}a^4}+\frac{(10\,b^3c-7\,ab^2d-a^3f+4\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}a^4}+\frac{\left(10\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5}-\frac{b^3cx^2-ab^2dx^2-a^3fx^2+a^2bx^2e}{3\,(bx^3+a)a^4}-\frac{84\,b^2cx^6-56\,abdx^6+28\,a^2x^6e-14\,abcx^3+7\,a^2dx^3+4\,a^2c}{28\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(10*b^3*c - 7*a*b^2*d - a^3*f + 4*a^2*b*e)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^4) + 1/18*(10*b^3*c -
```

```
7*a*b^2*d - a^3*f + 4*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-
a*b^2)^(1/3)*a^4) + 1/9*(10*b^3*c*(-a/b)^(1/3) - 7*a*b^2*d*(-a/b)^(1/3) - a
^3*f*(-a/b)^(1/3) + 4*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/a^5 - 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^2*e)/((b*
x^3 + a)*a^4) - 1/28*(84*b^2*c*x^6 - 56*a*b*d*x^6 + 28*a^2*x^6*e - 14*a*b*c
*x^3 + 7*a^2*d*x^3 + 4*a^2*c)/(a^4*x^7)
```

Mupad [B]
time = 5.18, size = 274, normalized size = 0.92

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+4\,e\,a^2\,b-7\,d\,a\,b^2+10\,c\,b^3\right)}{9\,a^{13/3}\,b^{2/3}}-\frac{\frac{c}{7\,a}+\frac{x^9\,\left(-f\,a^3+4\,e\,a^2\,b-7\,d\,a\,b^2+10\,c\,b^3\right)}{3\,a^4}+\frac{x^3\,\left(7\,a\,d-10\,b\,c\right)}{28\,a^2}+\frac{x^6\,\left(4\,e\,a^2-7\,d\,a\,b+10\,c\,b^2\right)}{4\,a^3}}{b\,x^{10}+a\,x^7}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+4\,e\,a^2\,b-7\,d\,a\,b^2+10\,c\,b^3\right)}{9\,a^{13/3}\,b^{2/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+4\,e\,a^2\,b-7\,d\,a\,b^2+10\,c\,b^3\right)}{9\,a^{13/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e))/(9*a^
(13/3)*b^(2/3)) - (c/(7*a) + (x^9*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e
))/(3*a^4) + (x^3*(7*a*d - 10*b*c))/(28*a^2) + (x^6*(10*b^2*c + 4*a^2*e - 7
*a*b*d))/(4*a^3))/(a*x^7 + b*x^10) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
- a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e
))/(9*a^(13/3)*b^(2/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*
((3^(1/2)*1i)/2 - 1/2)*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e))/(9*a^(13
/3)*b^(2/3))
```

3.272 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^2}\,dx$

Optimal. Leaf size=297

$$-\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^4\,(a+bx^3)}+\frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\tan^{}}{3\sqrt{3}\,a^{14/3}\sqrt[3]{b}}$$

```
[Out] -1/8*c/a^2/x^8+1/5*(-a*d+2*b*c)/a^3/x^5+1/2*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x^
2-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^3+a)-1/9*(-2*a^3*f+5*a^2*b*
e-8*a*b^2*d+11*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)/b^(1/3)+1/18*(-2*a^3*f
+5*a^2*b*e-8*a*b^2*d+11*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^
(14/3)/b^(1/3)+1/9*(-2*a^3*f+5*a^2*b*e-8*a*b^2*d+11*b^3*c)*arctan(1/3*(a^(1
/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1848, 206, 31, 648, 631, 210, 642}

$$\frac{2bc-ad}{5a^3x^5}-\frac{c}{8a^2x^8}-\frac{a^2e-2abd+3b^2c}{2a^4x^2}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(-2a^3f+5a^2be-8ab^2d+11b^3c)}{3\sqrt{3}\,a^{14/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(-2a^3f+5a^2be-8ab^2d+11b^3c)}{9a^{14/3}\sqrt[3]{b}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(-2a^3f+5a^2be-8ab^2d+11b^3c)}{18a^{14/3}\sqrt[3]{b}}-\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^4\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```

```
[Out] -1/8*c/(a^2*x^8) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d + a^2*e)/
(2*a^4*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^4*(a + b*x^3)) +
 ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(14/3)*b^(1/3)) - ((11*b^3*c - 8*a*b^2*
d + 5*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(14/3)*b^(1/3)) + (
(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(18*a^(14/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{2b^3\left(b^3c-ab\cdots\right.}{}}{x^9(a+bx^3)}}{3ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^9}-\frac{3b^3(-2bc+ad)}{a^2x^6}-\frac{3b^3\left(3b^2c-2abd+a^2e\right)}{a^3x^3}-\cdots\right.}{3ab^3}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}-\frac{(1\cdots}{}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}-\frac{(1\cdots}{\cdots}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}-\frac{(1\cdots}{\cdots}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}-\frac{(1\cdots}{\cdots}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}+\frac{(1\cdots}{\cdots}$$

Mathematica [A]
time = 0.11, size = 280, normalized size = 0.94

$$\frac{-\frac{45a^{8/3}c}{x^8}-\frac{72a^{5/3}(-2bc+ad)}{x^5}-\frac{180a^{2/3}\left(3b^2c-2abd+a^2e\right)}{x^2}+\frac{120a^{2/3}\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{a+bx^3}+\frac{40\sqrt{3}\left(11b^3c-8ab^2d+5a^2be-2a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{40\left(-11b^3c+8ab^2d-5a^2be+2a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{20\left(11b^3c-8ab^2d+5a^2be-2a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}}{360a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```

```
[Out] ((-45*a^(8/3)*c)/x^8 - (72*a^(5/3)*(-2*b*c + a*d))/x^5 - (180*a^(2/3)*(3*b^
2*c - 2*a*b*d + a^2*e))/x^2 + (120*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*x)/(a + b*x^3) + (40*Sqrt[3]*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a
^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-11*b^3*c
 + 8*a*b^2*d - 5*a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (20
*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/b^(1/3))/(360*a^(14/3))
```

Maple [A]
time = 0.38, size = 214, normalized size = 0.72

method	result
default	$\frac{\frac{\left(\frac{1}{3}a^3f-\frac{1}{3}a^2be+\frac{1}{3}a\,b^2d-\frac{1}{3}b^3c\right)x}{b\,x^3+a}+\frac{\left(2a^3f-5a^2be+8a\,b^2d-11b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{3}}{a^4}$
risch	$\frac{\frac{\left(2a^3f-5a^2be+8a\,b^2d-11b^3c\right)x^9}{6a^4}-\frac{\left(5a^2e-8abd+11b^2c\right)x^6}{10a^3}-\frac{(8ad-11bc)x^3}{40a^2}-\frac{c}{8a}}{x^8\left(b\,x^3+a\right)}+\frac{\left(\_R=\text{RootOf}\left(a^{14}b\_Z^3-8a^9f^3+60a^8be\,f^2-96a^7b^2d\,f^2-1\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*((1/3*a^3*f-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x/(b*x^3+a)+1/3*(2*a^3
*f-5*a^2*b*e+8*a*b^2*d-11*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b
/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/8*c/a^2/x^8-1/5*(a*d-2*b*c)/a^3/x
^5-1/2*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^2
```

Maxima [A]

time = 0.49, size = 297, normalized size = 1.00

$$-\frac{20\,(11\,b^3c-8\,ab^2d-2\,a^3f+5\,a^2be)x^9+12\,(11\,ab^2c-8\,a^2bd+5\,a^3e)x^6+15\,a^3c-3\,(11\,a^2bc-8\,a^3d)x^3}{120\,(a^4bx^{11}+a^5x^8)}-\frac{\sqrt{3}\,(11\,b^3c-8\,ab^2d-2\,a^3f+5\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(11\,b^3c-8\,ab^2d-2\,a^3f+5\,a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(11\,b^3c-8\,ab^2d-2\,a^3f+5\,a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/120*(20*(11*b^3*c - 8*a*b^2*d - 2*a^3*f + 5*a^2*b*e)*x^9 + 12*(11*a*b^2*
c - 8*a^2*b*d + 5*a^3*e)*x^6 + 15*a^3*c - 3*(11*a^2*b*c - 8*a^3*d)*x^3)/(a^
4*b*x^11 + a^5*x^8) - 1/9*sqrt(3)*(11*b^3*c - 8*a*b^2*d - 2*a^3*f + 5*a^2*b
*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
 + 1/18*(11*b^3*c - 8*a*b^2*d - 2*a^3*f + 5*a^2*b*e)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) - 1/9*(11*b^3*c - 8*a*b^2*d - 2*a^3*f
+ 5*a^2*b*e)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

Fricas [A]

time = 0.42, size = 959, normalized size = 3.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/360*(60*(11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*f)*x^9 + 45
*a^5*b*c + 36*(11*a^3*b^3*c - 8*a^4*b^2*d + 5*a^5*b*e)*x^6 - 9*(11*a^4*b^2*
c - 8*a^5*b*d)*x^3 + 60*sqrt(1/3)*((11*a*b^5*c - 8*a^2*b^4*d + 5*a^3*b^3*e
- 2*a^4*b^2*f)*x^11 + (11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*f
)*x^8)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 +
3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(
1/3)/b))/(b*x^3 + a)) - 20*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f
)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*(a^2*b)^(2/3
)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*((11*b^4*c - 8*a*b^
3*d + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e
 - 2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^2*x^11 +
a^7*b*x^8), -1/360*(60*(11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*
f)*x^9 + 45*a^5*b*c + 36*(11*a^3*b^3*c - 8*a^4*b^2*d + 5*a^5*b*e)*x^6 - 9*(
11*a^4*b^2*c - 8*a^5*b*d)*x^3 + 120*sqrt(1/3)*((11*a*b^5*c - 8*a^2*b^4*d +
5*a^3*b^3*e - 2*a^4*b^2*f)*x^11 + (11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e
 - 2*a^5*b*f)*x^8)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*
x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*((11*b^4*c - 8*a*b^3*d
 + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e -
2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a
) + 40*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c
 - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b
)^(2/3)))/(a^6*b^2*x^11 + a^7*b*x^8)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.05, size = 347, normalized size = 1.17

$$\frac{(11\,b^3c-8\,ab^2d-2\,a^3f+5\,a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5}-\frac{\sqrt{3}\left(11\,(-ab^2)^{\frac{1}{3}}\,b^3c-8\,(-ab^2)^{\frac{1}{3}}\,ab^2d-2\,(-ab^2)^{\frac{1}{3}}\,a^3f+5\,(-ab^2)^{\frac{1}{3}}\,a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^5b}-\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{3\,(bx^3+a)a^4}-\frac{\left(11\,(-ab^2)^{\frac{1}{3}}\,b^3c-8\,(-ab^2)^{\frac{1}{3}}\,ab^2d-2\,(-ab^2)^{\frac{1}{3}}\,a^3f+5\,(-ab^2)^{\frac{1}{3}}\,a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^5b}-\frac{60\,b^2cx^6-40\,abdx^6+20\,a^2x^6e-16\,abcx^3+8\,a^2dx^3+5\,a^2c}{40\,a^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(11*b^3*c - 8*a*b^2*d - 2*a^3*f + 5*a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/a^5 - 1/9*sqrt(3)*(11*(-a*b^2)^(1/3)*b^3*c - 8*(-a*b^2)^(1/3)
```

```
*a*b^2*d - 2*(-a*b^2)^(1/3)*a^3*f + 5*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sq
rt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) - 1/3*(b^3*c*x - a*b^2*d*x
 - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*a^4) - 1/18*(11*(-a*b^2)^(1/3)*b^3*c -
 8*(-a*b^2)^(1/3)*a*b^2*d - 2*(-a*b^2)^(1/3)*a^3*f + 5*(-a*b^2)^(1/3)*a^2*b
*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/40*(60*b^2*c*x^6 -
 40*a*b*d*x^6 + 20*a^2*x^6*e - 16*a*b*c*x^3 + 8*a^2*d*x^3 + 5*a^2*c)/(a^4*x
^8)
```

Mupad [B]
time = 5.20, size = 274, normalized size = 0.92

$$-\frac{\frac{c}{8\,a}+\frac{x^9\,\left(-2\,f\,a^3+5\,e\,a^2\,b-8\,d\,a\,b^2+11\,c\,b^3\right)}{6\,a^4}+\frac{x^3\,\left(8\,a\,d-11\,b\,c\right)}{40\,a^2}+\frac{x^6\,\left(5\,e\,a^2-8\,d\,a\,b+11\,c\,b^2\right)}{10\,a^3}}{b\,x^{11}+a\,x^8}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-2\,f\,a^3+5\,e\,a^2\,b-8\,d\,a\,b^2+11\,c\,b^3\right)}{9\,a^{14/3}\,b^{1/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-2\,f\,a^3+5\,e\,a^2\,b-8\,d\,a\,b^2+11\,c\,b^3\right)}{9\,a^{14/3}\,b^{1/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-2\,f\,a^3+5\,e\,a^2\,b-8\,d\,a\,b^2+11\,c\,b^3\right)}{9\,a^{14/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(11
*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(9*a^(14/3)*b^(1/3)) - (log(b^(1
/3)*x + a^(1/3))*(11*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(9*a^(14/3)*
b^(1/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2
 - 1/2)*(11*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(9*a^(14/3)*b^(1/3))
- (c/(8*a) + (x^9*(11*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(6*a^4) + (
x^3*(8*a*d - 11*b*c))/(40*a^2) + (x^6*(11*b^2*c + 5*a^2*e - 8*a*b*d))/(10*a
^3))/(a*x^8 + b*x^11)
```

3.273 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=334

$$-\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^5\,(a+bx^3)}-\frac{\sqrt[3]{}}{}$$

```
[Out] -1/10*c/a^2/x^10+1/7*(-a*d+2*b*c)/a^3/x^7+1/4*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/
x^4+(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x+1/3*b*(-a^3*f+a^2*b*e-a*b^2*
d+b^3*c)*x^2/a^5/(b*x^3+a)-1/9*b^(1/3)*(-4*a^3*f+7*a^2*b*e-10*a*b^2*d+13*b^
3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(16/3)+1/18*b^(1/3)*(-4*a^3*f+7*a^2*b*e-10*a*b
^2*d+13*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(16/3)-1/9*b^(1/
3)*(-4*a^3*f+7*a^2*b*e-10*a*b^2*d+13*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x
)/a^(1/3)*3^(1/2))/a^(16/3)*3^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1848, 298, 31, 648, 631, 210, 642}

$$\frac{2bc-ad}{7a^3x^7}-\frac{c}{10a^2x^{10}}-\frac{a^2e-2abd+3b^2c}{4a^4x^4}-\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(-4a^3f+7a^2be-10ab^2d+13b^3c)}{3\sqrt{3}\,a^{16/3}}-\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(-4a^3f+7a^2be-10ab^2d+13b^3c)}{9a^{16/3}}+\frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(-4a^3f+7a^2be-10ab^2d+13b^3c)}{18a^{16/3}}+\frac{bx^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^5\,(a+bx^3)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{a^5x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]
```

```
[Out] -1/10*c/(a^2*x^10) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d + a^2*e
)/(4*a^4*x^4) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) + (b*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^5*(a + b*x^3)) - (b^(1/3)*(13*b^3
*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(3*Sqrt[3]*a^(16/3)) - (b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a
^2*b*e - 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(16/3)) + (b^(1/3)*(13*b^3
*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(18*a^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx = \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^5\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{3b^3\left(b^3c-ab\right.}{}}{x^{11}(a+bx^3)}}{3ab^3}$$

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^5\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^{11}}-\frac{3b^3(-2bc+ad)}{a^2x^8}-\frac{3b^3(3b^2c-2abd+a^2e)}{a^3x^5}-\right.}{}$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+$$

Mathematica [A]
time = 0.12, size = 319, normalized size = 0.96

$$\frac{-\frac{126a^{10/3}c}{x^{10}}-\frac{180a^{7/3}(-2bc+ad)}{x^7}-\frac{315a^{4/3}(3b^2c-2abd+a^2e)}{x^4}-\frac{1260\sqrt[3]{a}\,(-4b^3c+3ab^2d-2a^2be+a^3f)}{x}-\frac{420\sqrt[3]{a}\,b(-b^3c+ab^2d-a^2be+a^3f)x^2}{a+bx^3}-140\sqrt{3}\,\sqrt[3]{b}\,(13b^3c-10ab^2d+7a^2be-4a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+140\sqrt[3]{b}\,(-13b^3c+10ab^2d-7a^2be+4a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+70\sqrt[3]{b}\,(13b^3c-10ab^2d+7a^2be-4a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{1260a^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]
```

```
[Out] ((-126*a^(10/3)*c)/x^10 - (180*a^(7/3)*(-2*b*c + a*d))/x^7 - (315*a^(4/3)*(
3*b^2*c - 2*a*b*d + a^2*e))/x^4 - (1260*a^(1/3)*(-4*b^3*c + 3*a*b^2*d - 2*a
^2*b*e + a^3*f))/x - (420*a^(1/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*
x^2)/(a + b*x^3) - 140*Sqrt[3]*b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e -
 4*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(1/3)*(-13*b^
3*c + 10*a*b^2*d - 7*a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 70*b^(1/
3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(1260*a^(16/3))
```

Maple [A]
time = 0.38, size = 251, normalized size = 0.75

method	result
default	$-\frac{b\left(\frac{\left(\frac{1}{3}a^3f-\frac{1}{3}a^2be+\frac{1}{3}a\,b^2d-\frac{1}{3}b^3c\right)x^2}{b\,x^3+a}+\left(\frac{4}{3}a^3f-\frac{7}{3}a^2be+\frac{10}{3}a\,b^2d-\frac{13}{3}b^3c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\sqrt{\ldots}\right.}{3b(\ldots}\right.\right.}{a^5}$
risch	$\frac{-\frac{b\left(4a^3f-7a^2be+10a\,b^2d-13b^3c\right)x^{12}}{3a^5}-\frac{\left(4a^3f-7a^2be+10a\,b^2d-13b^3c\right)x^9}{4a^4}-\frac{\left(7a^2e-10abd+13b^2c\right)x^6}{28a^3}-\frac{(10ad-13bc)x^3}{70a^2}-\frac{c}{10a}}{x^{10}(b\,x^3+a)}+\frac{\left(\_R=\text{RootOf}(\ldots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/a^5*b*((1/3*a^3*f-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x^2/(b*x^3+a)+(4/3*a^3*f-7/3*a^2*b*e+10/3*a*b^2*d-13/3*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/10*c/a^2/x^10-1/7*(a*d-2*b*c)/a^3/x^7-1/4*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^4-(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x

Maxima [A]
time = 0.50, size = 329, normalized size = 0.99

$$\frac{140\,(13\,b^4c-10\,ab^3d-4\,a^3bf+7\,a^2b^2e)x^{12}+105\,(13\,ab^3c-10\,a^2b^2d-4\,a^4f+7\,a^3be)x^9-15\,(13\,a^2b^2c-10\,a^3bd+7\,a^4e)x^6-42\,a^4c+6\,(13\,a^3bc-10\,a^4d)x^3}{420\,(a^5bx^{13}+a^6x^{10})}+\frac{\sqrt{3}\,(13\,b^3c-10\,ab^2d-4\,a^3f+7\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(13\,b^3c-10\,ab^2d-4\,a^3f+7\,a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(13\,b^3c-10\,ab^2d-4\,a^3f+7\,a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="maxima")

[Out] 1/420*(140*(13*b^4*c - 10*a*b^3*d - 4*a^3*b*f + 7*a^2*b^2*e)*x^12 + 105*(13*a*b^3*c - 10*a^2*b^2*d - 4*a^4*f + 7*a^3*b*e)*x^9 - 15*(13*a^2*b^2*c - 10*a^3*b*d + 7*a^4*e)*x^6 - 42*a^4*c + 6*(13*a^3*b*c - 10*a^4*d)*x^3)/(a^5*b*x^13 + a^6*x^10) + 1/9*sqrt(3)*(13*b^3*c - 10*a*b^2*d - 4*a^3*f + 7*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(1/3)) + 1/18*(13*b^3*c - 10*a*b^2*d - 4*a^3*f + 7*a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(1/3)) - 1/9*(13*b^3*c - 10*a*b^2*d - 4*a^3*f + 7*a^2*b*e)*log(x + (a/b)^(1/3))/(a^5*(a/b)^(1/3))

Fricas [A]
time = 0.42, size = 442, normalized size = 1.32

$$\frac{420\,(13\,b^4c-10\,ab^3d+7\,a^2b^2e-4\,a^3bf)x^{12}+315\,(13\,ab^3c-10\,a^2b^2d+7\,a^3be-4\,a^4f)x^9-45\,(13\,a^2b^2c-10\,a^3bd+7\,a^4e)x^6-126\,a^4c+18\,(13\,a^3bc-10\,a^4d)x^3+140\,\sqrt{3}\,((13\,b^4c-10\,ab^3d+7\,a^2b^2e-4\,a^3bf)x^{13}+(13\,ab^3c-10\,a^2b^2d+7\,a^3be-4\,a^4f)x^{10})\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+70\,((13\,b^4c-10\,ab^3d+7\,a^2b^2e-4\,a^3bf)x^{13}+(13\,ab^3c-10\,a^2b^2d+7\,a^3be-4\,a^4f)x^{10})\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-140\,((13\,b^4c-10\,ab^3d+7\,a^2b^2e-4\,a^3bf)x^{13}+(13\,ab^3c-10\,a^2b^2d+7\,a^3be-4\,a^4f)x^{10})\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{1260\,(a^5bx^{13}+a^6x^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(420*(13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^12 + 315*(1
3*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^9 - 45*(13*a^2*b^2*c - 10
*a^3*b*d + 7*a^4*e)*x^6 - 126*a^4*c + 18*(13*a^3*b*c - 10*a^4*d)*x^3 + 140*
sqrt(3)*((13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^13 + (13*a*b^3
*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^10)*(b/a)^(1/3)*arctan(2/3*sqrt(
3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*((13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e
- 4*a^3*b*f)*x^13 + (13*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^10)
*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 140*((13*b^4*c
- 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^13 + (13*a*b^3*c - 10*a^2*b^2*d +
 7*a^3*b*e - 4*a^4*f)*x^10)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)))/(a^5*b*x^
13 + a^6*x^10)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.11, size = 437, normalized size = 1.31

$$-\frac{\left(13\,b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-10\,ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}}+7\,a^2b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^6}-\frac{\sqrt{3}\left(13\left(-ab^2\right)^{\frac{2}{3}}b^3c-10\left(-ab^2\right)^{\frac{2}{3}}ab^2d-4\left(-ab^2\right)^{\frac{2}{3}}a^3f+7\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^6b}+\frac{b^4cx^2-ab^3dx^2-a^3bfx^2+a^2b^2x^2e}{3\left(bx^3+a\right)a^5}+\frac{\left(13\left(-ab^2\right)^{\frac{2}{3}}b^3c-10\left(-ab^2\right)^{\frac{2}{3}}ab^2d-4\left(-ab^2\right)^{\frac{2}{3}}a^3f+7\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^6b}+\frac{560\,b^3cx^9-420\,ab^2dx^9-140\,a^3fx^9+280\,a^2bx^9e-105\,ab^2cx^6+70\,a^2bdx^6-35\,a^3x^6e+40\,a^2bcx^3-20\,a^3dx^3-14\,a^3c}{140\,a^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(13*b^4*c*(-a/b)^(1/3) - 10*a*b^3*d*(-a/b)^(1/3) - 4*a^3*b*f*(-a/b)^(1
/3) + 7*a^2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6
 - 1/9*sqrt(3)*(13*(-a*b^2)^(2/3)*b^3*c - 10*(-a*b^2)^(2/3)*a*b^2*d - 4*(-a
*b^2)^(2/3)*a^3*f + 7*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/(a^6*b) + 1/3*(b^4*c*x^2 - a*b^3*d*x^2 - a^3*b*f*x
^2 + a^2*b^2*x^2*e)/((b*x^3 + a)*a^5) + 1/18*(13*(-a*b^2)^(2/3)*b^3*c - 10*
(-a*b^2)^(2/3)*a*b^2*d - 4*(-a*b^2)^(2/3)*a^3*f + 7*(-a*b^2)^(2/3)*a^2*b*e)
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b) + 1/140*(560*b^3*c*x^9 -
420*a*b^2*d*x^9 - 140*a^3*f*x^9 + 280*a^2*b*x^9*e - 105*a*b^2*c*x^6 + 70*a^
2*b*d*x^6 - 35*a^3*x^6*e + 40*a^2*b*c*x^3 - 20*a^3*d*x^3 - 14*a^3*c)/(a^5*x
^10)
```

Mupad [B]
time = 5.41, size = 310, normalized size = 0.93

$$-\frac{\frac{c}{10\,a}-\frac{x^9\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+13\,c\,b^3\right)}{4\,a^4}+\frac{x^3\,\left(10\,a\,d-13\,b\,c\right)}{70\,a^2}+\frac{x^6\,\left(7\,e\,a^2-10\,d\,a\,b+13\,c\,b^2\right)}{28\,a^3}-\frac{b\,x^{12}\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+13\,c\,b^3\right)}{3\,a^5}}{b\,x^{13}+a\,x^{10}}-\frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+13\,c\,b^3\right)}{9\,a^{16/3}}+\frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+13\,c\,b^3\right)}{9\,a^{16/3}}-\frac{b^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+13\,c\,b^3\right)}{9\,a^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x)
```

```
[Out] (b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e))/(9*a^(16/3)) - (b^(1/3)
*log(b^(1/3)*x + a^(1/3))*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e))/(9
*a^(16/3)) - (c/(10*a) - (x^9*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e)
)/(4*a^4) + (x^3*(10*a*d - 13*b*c))/(70*a^2) + (x^6*(13*b^2*c + 7*a^2*e - 1
0*a*b*d))/(28*a^3) - (b*x^12*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e))
/(3*a^5))/(a*x^10 + b*x^13) - (b^(1/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
 + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2
*b*e))/(9*a^(16/3))
```

3.274 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^2}\,dx$

Optimal. Leaf size=335

$$-\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^5\,(a+bx^3)}-\frac{b^{2/3}}{}$$

[Out] -1/11*c/a^2/x^11+1/8*(-a*d+2*b*c)/a^3/x^8+1/5*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x^5+1/2*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x^2+1/3*b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^5/(b*x^3+a)+1/9*b^(2/3)*(-5*a^3*f+8*a^2*b*e-11*a*b^2*d+14*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(17/3)-1/18*b^(2/3)*(-5*a^3*f+8*a^2*b*e-11*a*b^2*d+14*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(17/3)-1/9*b^(2/3)*(-5*a^3*f+8*a^2*b*e-11*a*b^2*d+14*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(17/3)*3^(1/2)

Rubi [A]

time = 0.29, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1848, 206, 31, 648, 631, 210, 642}

$$\frac{2bc-ad}{8a^3x^8}-\frac{c}{11a^2x^{11}}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-5a^3f+8a^2be-11ab^2d+14b^3c)}{3\sqrt{3}a^{17/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-5a^3f+8a^2be-11ab^2d+14b^3c)}{18a^{17/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-5a^3f+8a^2be-11ab^2d+14b^3c)}{9a^{17/3}}+\frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^5(a+bx^3)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{2a^5x^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]

[Out] -1/11*c/(a^2*x^11) + (2*b*c - a*d)/(8*a^3*x^8) - (3*b^2*c - 2*a*b*d + a^2*e)/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(2*a^5*x^2) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^5*(a + b*x^3)) - (b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(17/3)) + (b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(17/3)) - (b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(17/3))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx = \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^5\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{3b^3\left(b^3c-ab^2\right.}{}}{x^{12}(a+bx^3)}}{3ab^3}$$

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^5\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^{12}}-\frac{3b^3(-2bc+ad)}{a^2x^9}-\frac{3b^3\left(3b^2c-2abd+a^2e\right)}{a^3x^6}-\right.}{}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+$$

Mathematica [A]
time = 0.12, size = 317, normalized size = 0.95

$$\frac{-\frac{360a^{11/3}c}{x^{11}}-\frac{495a^{8/3}(-2bc+ad)}{x^8}-\frac{792a^{5/3}(3b^2c-2abd+a^2e)}{x^5}-\frac{1980a^{2/3}(-4b^3c+3ab^2d-2a^2be+a^3f)}{x^2}-\frac{1320a^{2/3}b(-b^3c+ab^2d-a^2be+a^3f)x}{a+bx^3}-440\sqrt{3}\,b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+440b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+220b^{2/3}(-14b^3c+11ab^2d-8a^2be+5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{3960a^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]
```

```
[Out] ((-360*a^(11/3)*c)/x^11 - (495*a^(8/3)*(-2*b*c + a*d))/x^8 - (792*a^(5/3)*(
3*b^2*c - 2*a*b*d + a^2*e))/x^5 - (1980*a^(2/3)*(-4*b^3*c + 3*a*b^2*d - 2*a
^2*b*e + a^3*f))/x^2 - (1320*a^(2/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*x)/(a + b*x^3) - 440*Sqrt[3]*b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e
- 5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 440*b^(2/3)*(14*b^
3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*b^(2
/3)*(-14*b^3*c + 11*a*b^2*d - 8*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(3960*a^(17/3))
```

Maple [A]
time = 0.37, size = 250, normalized size = 0.75

method	result
default	$-\frac{b\left(\frac{\left(\frac{1}{3}a^3f-\frac{1}{3}a^2be+\frac{1}{3}a\,b^2d-\frac{1}{3}b^3c\right)x}{b\,x^3+a}+\frac{\left(5a^3f-8a^2be+11a\,b^2d-14b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\cdots\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}\right)}{a^5}$
risch	$\frac{-\frac{b\left(5a^3f-8a^2be+11a\,b^2d-14b^3c\right)x^{12}}{6a^5}-\frac{\left(5a^3f-8a^2be+11a\,b^2d-14b^3c\right)x^9}{10a^4}-\frac{\left(8a^2e-11abd+14b^2c\right)x^6}{40a^3}-\frac{(11ad-14bc)x^3}{88a^2}-\frac{c}{11a}}{x^{11}(b\,x^3+a)}+\left(\sum_{\_R=\text{RootOf}(\cdots)}\cdots\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^5*b*((1/3*a^3*f-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x/(b*x^3+a)+1/3*(5*
a^3*f-8*a^2*b*e+11*a*b^2*d-14*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1
/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/11*c/a^2/x^11-1/8*(a*d-2*b*c)
/a^3/x^8-1/5*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^5-1/2*(a^3*f-2*a^2*b*e+3*a*b^2*d
-4*b^3*c)/a^5/x^2
```

Maxima [A]

time = 0.52, size = 329, normalized size = 0.98

$$\frac{220\,(14\,b^4c-11\,ab^3d-5\,a^3bf+8\,a^2b^2e)x^{12}+132\,(14\,ab^3c-11\,a^2b^2d-5\,a^4f+8\,a^3be)x^9-33\,(14\,a^2b^2c-11\,a^3bd+8\,a^4e)x^6-120\,a^4c+15\,(14\,a^3bc-11\,a^4d)x^3}{1320\,(a^5bx^{14}+a^6x^{11})}+\frac{\sqrt{3}\,(14\,b^3c-11\,ab^2d-5\,a^3f+8\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(14\,b^3c-11\,ab^2d-5\,a^3f+8\,a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(14\,b^3c-11\,ab^2d-5\,a^3f+8\,a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1320*(220*(14*b^4*c - 11*a*b^3*d - 5*a^3*b*f + 8*a^2*b^2*e)*x^12 + 132*(1
4*a*b^3*c - 11*a^2*b^2*d - 5*a^4*f + 8*a^3*b*e)*x^9 - 33*(14*a^2*b^2*c - 11
*a^3*b*d + 8*a^4*e)*x^6 - 120*a^4*c + 15*(14*a^3*b*c - 11*a^4*d)*x^3)/(a^5*
```

```
b*x^14 + a^6*x^11) + 1/9*sqrt(3)*(14*b^3*c - 11*a*b^2*d - 5*a^3*f + 8*a^2*b
*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(2/3)) -
 1/18*(14*b^3*c - 11*a*b^2*d - 5*a^3*f + 8*a^2*b*e)*log(x^2 - x*(a/b)^(1/3)
 + (a/b)^(2/3))/(a^5*(a/b)^(2/3)) + 1/9*(14*b^3*c - 11*a*b^2*d - 5*a^3*f +
8*a^2*b*e)*log(x + (a/b)^(1/3))/(a^5*(a/b)^(2/3))
```

Fricas [**A**]
time = 0.42, size = 475, normalized size = 1.42

$$\frac{660\,(14\,b^4c-11\,ab^3d+8\,a^2b^2e-5\,a^3bf)x^{12}+396\,(14\,ab^3c-11\,a^2b^2d+8\,a^3be-5\,a^4f)x^9-99\,(14\,a^2b^2c-11\,a^3bd+8\,a^4e)x^6-360\,a^4c+45\,(14\,a^3bc-11\,a^4d)x^3-440\,\sqrt{3}\,((14\,b^4c-11\,ab^3d+8\,a^2b^2e-5\,a^3bf)x^{14}+(14\,ab^3c-11\,a^2b^2d+8\,a^3be-5\,a^4f)x^{11})\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)+220\,((14\,b^4c-11\,ab^3d+8\,a^2b^2e-5\,a^3bf)x^{14}+(14\,ab^3c-11\,a^2b^2d+8\,a^3be-5\,a^4f)x^{11})\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-440\,((14\,b^4c-11\,ab^3d+8\,a^2b^2e-5\,a^3bf)x^{14}+(14\,ab^3c-11\,a^2b^2d+8\,a^3be-5\,a^4f)x^{11})\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)}{3960\,(a^5bx^{14}+a^6x^{11})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3960*(660*(14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^12 + 396*(1
4*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^9 - 99*(14*a^2*b^2*c - 11
*a^3*b*d + 8*a^4*e)*x^6 - 360*a^4*c + 45*(14*a^3*b*c - 11*a^4*d)*x^3 - 440*
sqrt(3)*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a*b^3
*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*arctan(1/3*
(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 220*((14*b^4*c - 11*a*b^3
*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e
 - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a
^2*(-b^2/a^2)^(2/3)) - 440*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*
f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2
)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^5*b*x^14 + a^6*x^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.86, size = 391, normalized size = 1.17

$$\frac{\sqrt{3}\left(14\,(-ab^2)^{\frac{1}{3}}\,b^3c-11\,(-ab^2)^{\frac{1}{3}}\,ab^2d-5\,(-ab^2)^{\frac{1}{3}}\,a^3f+8\,(-ab^2)^{\frac{1}{3}}\,a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^6}-\frac{(14\,b^4c-11\,ab^3d-5\,a^3bf+8\,a^2b^2e)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^6}+\frac{\left(14\,(-ab^2)^{\frac{1}{3}}\,b^3c-11\,(-ab^2)^{\frac{1}{3}}\,ab^2d-5\,(-ab^2)^{\frac{1}{3}}\,a^3f+8\,(-ab^2)^{\frac{1}{3}}\,a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^6}+\frac{b^4cx-ab^3dx-a^3bfx+a^2b^2xe}{3\,(bx^3+a)a^5}+\frac{880\,b^3cx^9-660\,ab^2dx^9-220\,a^3fx^9+440\,a^2bx^9e-264\,ab^2cx^6+176\,a^2bdx^6-88\,a^3x^6e+110\,a^2bcx^3-55\,a^3dx^3-40\,a^3c}{440\,a^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(14*(-a*b^2)^(1/3)*b^3*c - 11*(-a*b^2)^(1/3)*a*b^2*d - 5*(-a*b^
2)^(1/3)*a^3*f + 8*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
```

```
^(1/3))/(-a/b)^(1/3))/a^6 - 1/9*(14*b^4*c - 11*a*b^3*d - 5*a^3*b*f + 8*a^2*
b^2*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 + 1/18*(14*(-a*b^2)^(1/3
)*b^3*c - 11*(-a*b^2)^(1/3)*a*b^2*d - 5*(-a*b^2)^(1/3)*a^3*f + 8*(-a*b^2)^(
1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6 + 1/3*(b^4*c*x -
 a*b^3*d*x - a^3*b*f*x + a^2*b^2*x*e)/((b*x^3 + a)*a^5) + 1/440*(880*b^3*c*
x^9 - 660*a*b^2*d*x^9 - 220*a^3*f*x^9 + 440*a^2*b*x^9*e - 264*a*b^2*c*x^6 +
 176*a^2*b*d*x^6 - 88*a^3*x^6*e + 110*a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c
)/(a^5*x^11)
```

Mupad [B]
time = 5.12, size = 310, normalized size = 0.93

$$\frac{b^{2/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(-5fa^3+8ea^2b-11dab^2+14cb^3\right)}{9a^{17/3}}-\frac{\frac{c}{11a}-\frac{x^9\left(-5fa^3+8ea^2b-11dab^2+14cb^3\right)}{10a^4}+\frac{x^3\left(11ad-14bc\right)}{88a^2}+\frac{x^6\left(8ea^2-11dab+14cb^2\right)}{40a^3}-\frac{bx^{12}\left(-5fa^3+8ea^2b-11dab^2+14cb^3\right)}{6a^5}}{bx^{14}+ax^{11}}+\frac{b^{2/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-5fa^3+8ea^2b-11dab^2+14cb^3\right)}{9a^{17/3}}-\frac{b^{2/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-5fa^3+8ea^2b-11dab^2+14cb^3\right)}{9a^{17/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x)
```

```
[Out] (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(14*b^3*c - 5*a^3*f - 11*a*b^2*d + 8*a^2*
b*e))/(9*a^(17/3)) - (c/(11*a) - (x^9*(14*b^3*c - 5*a^3*f - 11*a*b^2*d + 8*
a^2*b*e))/(10*a^4) + (x^3*(11*a*d - 14*b*c))/(88*a^2) + (x^6*(14*b^2*c + 8*
a^2*e - 11*a*b*d))/(40*a^3) - (b*x^12*(14*b^3*c - 5*a^3*f - 11*a*b^2*d + 8*
a^2*b*e))/(6*a^5))/(a*x^11 + b*x^14) + (b^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*
b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(14*b^3*c - 5*a^3*f - 11*a*b^2*
d + 8*a^2*b*e))/(9*a^(17/3)) - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*
x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(14*b^3*c - 5*a^3*f - 11*a*b^2*d + 8*a^
2*b*e))/(9*a^(17/3))
```

3.275 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=375

$$-\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-\frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{a^6x}-$$

```
[Out] -1/13*c/a^2/x^13+1/10*(-a*d+2*b*c)/a^3/x^10+1/7*(-a^2*e+2*a*b*d-3*b^2*c)/a^
4/x^7+1/4*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x^4-b*(-2*a^3*f+3*a^2*b*
e-4*a*b^2*d+5*b^3*c)/a^6/x-1/3*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^6/(
b*x^3+a)+1/9*b^(4/3)*(-7*a^3*f+10*a^2*b*e-13*a*b^2*d+16*b^3*c)*ln(a^(1/3)+b
^(1/3)*x)/a^(19/3)-1/18*b^(4/3)*(-7*a^3*f+10*a^2*b*e-13*a*b^2*d+16*b^3*c)*l
n(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(19/3)+1/9*b^(4/3)*(-7*a^3*f+10*
a^2*b*e-13*a*b^2*d+16*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/
2))/a^(19/3)*3^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1848, 298, 31, 648, 631, 210, 642}

$$\frac{2bc-ad}{10a^3x^{10}}-\frac{c}{13a^2x^{13}}-\frac{a^2e-2abd+3b^2c}{7a^4x^7}+\frac{b^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-7a^3f+10a^2be-13ab^2d+16b^3c)}{3\sqrt{3}a^{19/3}}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-7a^3f+10a^2be-13ab^2d+16b^3c)}{18a^{19/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-7a^3f+10a^2be-13ab^2d+16b^3c)}{9a^{19/3}}-\frac{b^2x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^6(a+bx^3)}-\frac{b(-2a^3f+3a^2be-4ab^2d+5b^3c)}{a^6x}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{4a^5x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]
```

```
[Out] -1/13*c/(a^2*x^13) + (2*b*c - a*d)/(10*a^3*x^10) - (3*b^2*c - 2*a*b*d + a^2
*e)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(4*a^5*x^4) - (
b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6*x) - (b^2*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*x^2)/(3*a^6*(a + b*x^3)) + (b^(4/3)*(16*b^3*c - 13*a
*b^2*d + 10*a^2*b*e - 7*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(3*Sqrt[3]*a^(19/3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e
- 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(19/3)) - (b^(4/3)*(16*b^3*c - 13
*a*b^2*d + 10*a^2*b*e - 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(18*a^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx = -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^6\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{3b^3\left(b^3c\right.}{}}{}}{}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^6\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^{14}}-\frac{3b^3(-2bc+ad)}{a^2x^{11}}-\frac{3b^3\left(3b^2c-2abd+a^2e\right)}{a^3x^8}\right.}{}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-$$

Mathematica [A]

time = 0.23, size = 370, normalized size = 0.99

$$-\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}+\frac{b\left(-5b^3c+4ab^2d-3a^2be+2a^3f\right)}{a^6x}+\frac{b^2\left(-b^3c+ab^2d-a^2be+a^3f\right)x^2}{3a^6\left(a+bx^3\right)}+\frac{b^{4/3}\left(16b^3c-13ab^2d+10a^2be-7a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{19/3}}+\frac{b^{4/3}\left(16b^3c-13ab^2d+10a^2be-7a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{19/3}}+\frac{b^{4/3}\left(-16b^3c+13ab^2d-10a^2be+7a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]
```

```
[Out] -1/13*c/(a^2*x^13) + (2*b*c - a*d)/(10*a^3*x^10) - (3*b^2*c - 2*a*b*d + a^2
*e)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(4*a^5*x^4) + (
b*(-5*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))/(a^6*x) + (b^2*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f)*x^2)/(3*a^6*(a + b*x^3)) + (b^(4/3)*(16*b^3*c -
13*a*b^2*d + 10*a^2*b*e - 7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]])/(3*Sqrt[3]*a^(19/3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e -
7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(19/3)) + (b^(4/3)*(-16*b^3*c + 13*
a*b^2*d - 10*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(19/3))
```

Maple [A]
time = 0.38, size = 287, normalized size = 0.77

method	result
default	$\frac{b^{2}\left(\frac{\left(\frac{1}{3}a^{3}f-\frac{1}{3}a^{2}be+\frac{1}{3}a\,b^{2}d-\frac{1}{3}b^{3}c\right)x^{2}}{b\,x^{3}+a}+\left(\frac{7}{3}a^{3}f-\frac{10}{3}a^{2}be+\frac{13}{3}a\,b^{2}d-\frac{16}{3}b^{3}c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\cdots\right.}{3b\left(\cdots\right.}\right.\right.}{a^{6}}$
risch	$\frac{-\frac{c}{13a}-\frac{(13ad-16bc)x^{3}}{130a^{2}}-\frac{\left(10a^{2}e-13abd+16b^{2}c\right)x^{6}}{70a^{3}}-\frac{\left(7a^{3}f-10a^{2}be+13a\,b^{2}d-16b^{3}c\right)x^{9}}{28a^{4}}+\frac{b\left(7a^{3}f-10a^{2}be+13a\,b^{2}d-16b^{3}c\right)x^{12}}{4a^{5}}+\frac{b^{2}\left(7a^{3}f-10a^{2}\cdots\right.}{}}{x^{13}(b\,x^{3}+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/a^6*((1/3*a^3*f-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x^2/(b*x^3+a)+(7/3*a
^3*f-10/3*a^2*b*e+13/3*a*b^2*d-16/3*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/
b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/13*c/a^2/x^13-1/10*(a*
d-2*b*c)/a^3/x^10-1/7*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^7-1/4*(a^3*f-2*a^2*b*e+
3*a*b^2*d-4*b^3*c)/a^5/x^4+b*(2*a^3*f-3*a^2*b*e+4*a*b^2*d-5*b^3*c)/a^6/x
```

Maxima [A]
time = 0.52, size = 381, normalized size = 1.02

$$-\frac{1820\,(16\,b^5c-13\,ab^4d-7\,a^3b^2f+10\,a^2b^3e)x^{15}+1365\,(16\,ab^4c-13\,a^2b^3d-7\,a^4bf+10\,a^3b^2e)x^{12}-195\,(16\,a^2b^3c-13\,a^3b^2d-7\,a^5f+10\,a^4be)x^9+78\,(16\,a^3b^2c-13\,a^4bd+10\,a^5e)x^6+420\,a^5c-42\,(16\,a^4bc-13\,a^5d)x^3}{5460\,(a^6bx^{16}+a^7x^{13})}-\frac{\sqrt{3}\,(16\,b^4c-13\,ab^3d-7\,a^3bf+10\,a^2b^2e)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(16\,b^4c-13\,ab^3d-7\,a^3bf+10\,a^2b^2e)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(16\,b^4c-13\,ab^3d-7\,a^3bf+10\,a^2b^2e)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/5460*(1820*(16*b^5*c - 13*a*b^4*d - 7*a^3*b^2*f + 10*a^2*b^3*e)*x^15 + 1
365*(16*a*b^4*c - 13*a^2*b^3*d - 7*a^4*b*f + 10*a^3*b^2*e)*x^12 - 195*(16*a
^2*b^3*c - 13*a^3*b^2*d - 7*a^5*f + 10*a^4*b*e)*x^9 + 78*(16*a^3*b^2*c - 13
*a^4*b*d + 10*a^5*e)*x^6 + 420*a^5*c - 42*(16*a^4*b*c - 13*a^5*d)*x^3)/(a^6
*b*x^16 + a^7*x^13) - 1/9*sqrt(3)*(16*b^4*c - 13*a*b^3*d - 7*a^3*b*f + 10*a
^2*b^2*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^6*(a/b)^(1
/3)) - 1/18*(16*b^4*c - 13*a*b^3*d - 7*a^3*b*f + 10*a^2*b^2*e)*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(1/3)) + 1/9*(16*b^4*c - 13*a*b^3*d -
 7*a^3*b*f + 10*a^2*b^2*e)*log(x + (a/b)^(1/3))/(a^6*(a/b)^(1/3))
```

Fricas [A]
time = 0.39, size = 507, normalized size = 1.35

$$-\frac{5460\,(16\,b^5c-13\,ab^4d+10\,a^2b^3e-7\,a^3b^2f)x^{15}+4095\,(16\,ab^4c-13\,a^2b^3d+10\,a^3b^2e-7\,a^4bf)x^{12}-585\,(16\,a^2b^3c-13\,a^3b^2d+10\,a^4be-7\,a^5f)x^9+234\,(16\,a^3b^2c-13\,a^4bd+10\,a^5e)x^6+1260\,a^5c-126\,(16\,a^4bc-13\,a^5d)x^3+1820\,\sqrt{3}\,((16\,b^5c-13\,ab^4d+10\,a^2b^3e-7\,a^3b^2f)x^{16}+(16\,ab^4c-13\,a^2b^3d+10\,a^3b^2e-7\,a^4bf)x^{13})\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)-910\,((16\,b^5c-13\,ab^4d+10\,a^2b^3e-7\,a^3b^2f)x^{16}+(16\,ab^4c-13\,a^2b^3d+10\,a^3b^2e-7\,a^4bf)x^{13})\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+1820\,((16\,b^5c-13\,ab^4d+10\,a^2b^3e-7\,a^3b^2f)x^{16}+(16\,ab^4c-13\,a^2b^3d+10\,a^3b^2e-7\,a^4bf)x^{13})\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{16380\,(a^6bx^{16}+a^7x^{13})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16380*(5460*(16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^15 +
4095*(16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^12 - 585*(16*
a^2*b^3*c - 13*a^3*b^2*d + 10*a^4*b*e - 7*a^5*f)*x^9 + 234*(16*a^3*b^2*c -
13*a^4*b*d + 10*a^5*e)*x^6 + 1260*a^5*c - 126*(16*a^4*b*c - 13*a^5*d)*x^3 +
 1820*sqrt(3)*((16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^16 +
(16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/3)*a
rctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*((16*b^5*c - 13*a*b^4
*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^16 + (16*a*b^4*c - 13*a^2*b^3*d + 10*a^3
*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b
/a)^(1/3)) + 1820*((16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^1
6 + (16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/
3)*log(b*x + a*(-b/a)^(2/3)))/(a^6*b*x^16 + a^7*x^13)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.79, size = 482, normalized size = 1.29

$$\frac{\sqrt{3}\left(16\,(-ab^2)^{\frac{2}{3}}\,b^3c-13\,(-ab^2)^{\frac{2}{3}}\,ab^2d-7\,(-ab^2)^{\frac{2}{3}}\,a^3f+10\,(-ab^2)^{\frac{2}{3}}\,a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^7}+\frac{\left(16\,b^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-13\,ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,a^3b^2f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+10\,a^2b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^7}-\frac{\left(16\,(-ab^2)^{\frac{2}{3}}\,b^3c-13\,(-ab^2)^{\frac{2}{3}}\,ab^2d-7\,(-ab^2)^{\frac{2}{3}}\,a^3f+10\,(-ab^2)^{\frac{2}{3}}\,a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^7}-\frac{b^5cx^2-ab^4dx^2-a^3b^2fx^2+a^2b^3x^2e}{3\,(bx^3+a)a^6}-\frac{9100\,b^4cx^{12}-7280\,ab^3dx^{12}-3640\,a^3bfx^{12}+5460\,a^2b^2x^{12}e-1820\,ab^3cx^9+1365\,a^2b^2dx^9+455\,a^4fx^9-910\,a^3bx^9e+780\,a^2b^2cx^6-520\,a^3bdx^6+260\,a^4x^6e-364\,a^3bcx^3+182\,a^4dx^3+140\,a^4c}{1820\,a^6x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(16*(-a*b^2)^(2/3)*b^3*c - 13*(-a*b^2)^(2/3)*a*b^2*d - 7*(-a*b^
2)^(2/3)*a^3*f + 10*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/a^7 + 1/9*(16*b^5*c*(-a/b)^(1/3) - 13*a*b^4*d*(-a/b)
^(1/3) - 7*a^3*b^2*f*(-a/b)^(1/3) + 10*a^2*b^3*(-a/b)^(1/3)*e)*(-a/b)^(1/3)
*log(abs(x - (-a/b)^(1/3)))/a^7 - 1/18*(16*(-a*b^2)^(2/3)*b^3*c - 13*(-a*b^
2)^(2/3)*a*b^2*d - 7*(-a*b^2)^(2/3)*a^3*f + 10*(-a*b^2)^(2/3)*a^2*b*e)*log(
```

```
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7 - 1/3*(b^5*c*x^2 - a*b^4*d*x^2 - a
^3*b^2*f*x^2 + a^2*b^3*x^2*e)/((b*x^3 + a)*a^6) - 1/1820*(9100*b^4*c*x^12 -
 7280*a*b^3*d*x^12 - 3640*a^3*b*f*x^12 + 5460*a^2*b^2*x^12*e - 1820*a*b^3*c
*x^9 + 1365*a^2*b^2*d*x^9 + 455*a^4*f*x^9 - 910*a^3*b*x^9*e + 780*a^2*b^2*c
*x^6 - 520*a^3*b*d*x^6 + 260*a^4*x^6*e - 364*a^3*b*c*x^3 + 182*a^4*d*x^3 +
140*a^4*c)/(a^6*x^13)
```

Mupad [B]
time = 5.12, size = 348, normalized size = 0.93

$$\frac{b^{4/3}\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^{19/3}}-\frac{\frac{c}{13\,a}-\frac{x^9\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{28\,a^4}+\frac{x^3\,\left(13\,a\,d-16\,b\,c\right)}{130\,a^2}+\frac{x^6\,\left(10\,e\,a^2-13\,d\,a\,b+16\,c\,b^2\right)}{70\,a^3}+\frac{b\,x^{12}\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{4\,a^5}+\frac{b^2\,x^{15}\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{3\,a^6}}{b\,x^{16}+a\,x^{13}}-\frac{b^{4/3}\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^{19/3}}+\frac{b^{4/3}\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^{19/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x)
```

```
[Out] (b^(4/3)*log(b^(1/3)*x + a^(1/3))*(16*b^3*c - 7*a^3*f - 13*a*b^2*d + 10*a^2
*b*e))/(9*a^(19/3)) - (c/(13*a) - (x^9*(16*b^3*c - 7*a^3*f - 13*a*b^2*d + 1
0*a^2*b*e))/(28*a^4) + (x^3*(13*a*d - 16*b*c))/(130*a^2) + (x^6*(16*b^2*c +
 10*a^2*e - 13*a*b*d))/(70*a^3) + (b*x^12*(16*b^3*c - 7*a^3*f - 13*a*b^2*d
+ 10*a^2*b*e))/(4*a^5) + (b^2*x^15*(16*b^3*c - 7*a^3*f - 13*a*b^2*d + 10*a^
2*b*e))/(3*a^6))/(a*x^13 + b*x^16) - (b^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^
(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(16*b^3*c - 7*a^3*f - 13*a*b^2*d
+ 10*a^2*b*e))/(9*a^(19/3)) + (b^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
 + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(16*b^3*c - 7*a^3*f - 13*a*b^2*d + 10*a^
2*b*e))/(9*a^(19/3))
```

3.276 $\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=266

$$-\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x^3}{3b^7}+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^6}{6b^6}+\frac{(b^2d-3abe+6a^2f)\,x^9}{9b^5}+\frac{(be-$$

```
[Out] -1/3*a*(-15*a^3*f+10*a^2*b*e-6*a*b^2*d+3*b^3*c)*x^3/b^7+1/6*(-10*a^3*f+6*a^
2*b*e-3*a*b^2*d+b^3*c)*x^6/b^6+1/9*(6*a^2*f-3*a*b*e+b^2*d)*x^9/b^5+1/12*(-3
*a*f+b*e)*x^12/b^4+1/15*f*x^15/b^3-1/6*a^4*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b
^8/(b*x^3+a)^2+1/3*a^3*(-7*a^3*f+6*a^2*b*e-5*a*b^2*d+4*b^3*c)/b^8/(b*x^3+a)
+1/3*a^2*(-21*a^3*f+15*a^2*b*e-10*a*b^2*d+6*b^3*c)*ln(b*x^3+a)/b^8
```

Rubi [A]
time = 0.30, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{x^9(6a^2f-3abe+b^2d)}{9b^5}+\frac{a^3(-7a^3f+6a^2be-5ab^2d+4b^3c)}{3b^8(a+bx^3)}+\frac{a^2\log(a+bx^3)(-21a^3f+15a^2be-10ab^2d+6b^3c)}{3b^8}-\frac{ax^3(-15a^3f+10a^2be-6ab^2d+3b^3c)}{3b^7}+\frac{x^6(-10a^3f+6a^2be-3ab^2d+b^3c)}{6b^6}-\frac{a^4(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^8(a+bx^3)^2}+\frac{x^{12}(be-3af)}{12b^4}+\frac{fx^{15}}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] -1/3*(a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x^3)/b^7 + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6)/(6*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*
f)*x^9)/(9*b^5) + ((b*e - 3*a*f)*x^12)/(12*b^4) + (f*x^15)/(15*b^3) - (a^4*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^8*(a + b*x^3)^2) + (a^3*(4*b^3*c
- 5*a*b^2*d + 6*a^2*b*e - 7*a^3*f))/(3*b^8*(a + b*x^3)) + (a^2*(6*b^3*c - 1
0*a*b^2*d + 15*a^2*b*e - 21*a^3*f)*Log[a + b*x^3])/(3*b^8)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx)^3}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-3b^3c+6ab^2d-10a^2be+15a^3f)}{b^7} + \frac{(b^3c-3ab^2d+6a^2be}{b^6}\right.\right.$$

$$= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x^3}{3b^7} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{6b^6}$$

Mathematica [A]
time = 0.10, size = 246, normalized size = 0.92

$$\frac{60ab(-3b^3c+6ab^2d-10a^2be+15a^3f)\,x^3+30b^2(b^3c-3ab^2d+6a^2be-10a^3f)\,x^6+20b^3(b^2d-3abe+6a^2f)\,x^9+15b^4(be-3af)x^{12}+12b^5fx^{15}+\frac{30a^4(-b^3c+ab^2d-a^2be+a^3f)}{(a+bx^3)^2}-\frac{60a^3(-4b^3c+5ab^2d-6a^2be+7a^3f)}{a+bx^3}+60a^2(6b^3c-10ab^2d+15a^2be-21a^3f)\log(a+bx^3)}{180b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (60*a*b*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x^3 + 30*b^2*(b^3*c
- 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6 + 20*b^3*(b^2*d - 3*a*b*e + 6*a^2*f
)*x^9 + 15*b^4*(b*e - 3*a*f)*x^12 + 12*b^5*f*x^15 + (30*a^4*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 - (60*a^3*(-4*b^3*c + 5*a*b^2*d - 6*
a^2*b*e + 7*a^3*f))/(a + b*x^3) + 60*a^2*(6*b^3*c - 10*a*b^2*d + 15*a^2*b*e
 - 21*a^3*f)*Log[a + b*x^3])/(180*b^8)
```

Maple [A]
time = 0.36, size = 261, normalized size = 0.98

method	result
norman	$\frac{-\frac{a^2\left(21fa^5-15ea^4b+10da^3b^2-6a^2cb^3\right)}{2b^8}+\frac{fx^{21}}{15b}-\frac{(7af-5be)x^{18}}{60b^2}+\frac{\left(21a^2f-15abe+10b^2d\right)x^{15}}{90b^3}-\frac{\left(21a^3f-15a^2be+10ab^2d-6b^3c\right)x^{12}}{36b^4}+\frac{a(21a^3f}{}}{(bx^3+a)^2}$
default	$\frac{\frac{fx^{15}b^4}{15}+\frac{\left(-3ab^3f+b^4e\right)x^{12}}{12}+\frac{\left(6a^2b^2f-3ab^3e+b^4d\right)x^9}{9}+\frac{\left(-10a^3bf+6a^2eb^2-3adb^3+cb^4\right)x^6}{6}+\frac{\left(15a^4f-10a^3be+6a^2b^2d-3ab^3c\right)x^3}{3}}{b^7}-\frac{a^2(}{}$
risch	$\frac{fx^{15}}{15b^3}-\frac{afx^{12}}{4b^4}+\frac{ex^{12}}{12b^3}+\frac{2a^2fx^9}{3b^5}-\frac{aex^9}{3b^4}+\frac{dx^9}{9b^3}-\frac{5a^3fx^6}{3b^6}+\frac{a^2ex^6}{b^5}-\frac{adx^6}{2b^4}+\frac{cx^6}{6b^3}+\frac{5a^4fx^3}{b^7}-\frac{10a^3ex^3}{3b^6}+\frac{2a^2}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^7*(1/15*f*x^15*b^4+1/12*(-3*a*b^3*f+b^4*e)*x^12+1/9*(6*a^2*b^2*f-3*a*b^
3*e+b^4*d)*x^9+1/6*(-10*a^3*b*f+6*a^2*b^2*e-3*a*b^3*d+b^4*c)*x^6+1/3*(15*a^
4*f-10*a^3*b*e+6*a^2*b^2*d-3*a*b^3*c)*x^3)-1/3*a^2/b^7*(a*(7*a^3*f-6*a^2*b*
```

```
e+5*a*b^2*d-4*b^3*c)/b/(b*x^3+a)-1/2*a^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b
*x^3+a)^2+(21*a^3*f-15*a^2*b*e+10*a*b^2*d-6*b^3*c)/b*ln(b*x^3+a))
```

Maxima [A]
time = 0.28, size = 283, normalized size = 1.06

$$\frac{7\,a^4b^3c-9\,a^5b^2d-13\,a^7f+11\,a^6be+2\,(4\,a^3b^4c-5\,a^4b^3d-7\,a^6bf+6\,a^5b^2e)x^3}{6\,(b^{10}x^6+2\,ab^9x^3+a^2b^8)}+\frac{12\,b^4fx^{15}-15\,(3\,ab^3f-b^4e)x^{12}+20\,(b^4d+6\,a^2b^2f-3\,ab^3e)x^9+30\,(b^4c-3\,ab^3d-10\,a^3bf+6\,a^2b^2e)x^6-60\,(3\,ab^3c-6\,a^2b^2d-15\,a^4f+10\,a^3be)x^3}{180\,b^7}+\frac{(6\,a^2b^3c-10\,a^3b^2d-21\,a^5f+15\,a^4be)\log\left(bx^3+a\right)}{3\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(7*a^4*b^3*c - 9*a^5*b^2*d - 13*a^7*f + 11*a^6*b*e + 2*(4*a^3*b^4*c - 5
*a^4*b^3*d - 7*a^6*b*f + 6*a^5*b^2*e)*x^3)/(b^10*x^6 + 2*a*b^9*x^3 + a^2*b^
8) + 1/180*(12*b^4*f*x^15 - 15*(3*a*b^3*f - b^4*e)*x^12 + 20*(b^4*d + 6*a^2
*b^2*f - 3*a*b^3*e)*x^9 + 30*(b^4*c - 3*a*b^3*d - 10*a^3*b*f + 6*a^2*b^2*e)
*x^6 - 60*(3*a*b^3*c - 6*a^2*b^2*d - 15*a^4*f + 10*a^3*b*e)*x^3)/b^7 + 1/3*
(6*a^2*b^3*c - 10*a^3*b^2*d - 21*a^5*f + 15*a^4*b*e)*log(b*x^3 + a)/b^8
```

Fricas [A]
time = 0.37, size = 396, normalized size = 1.49

$$\frac{12\,b^7fx^{21}+3\,(5\,b^7e-7\,ab^6f)x^{18}+2\,(10\,b^7d-15\,ab^6e+21\,a^2b^5f)x^{15}+5\,(6\,b^7c-10\,ab^6d+15\,a^2b^5e-21\,a^3b^4f)x^{12}-20\,(6\,ab^6c-10\,a^2b^5d+15\,a^3b^4e-21\,a^4b^3f)x^9+210\,a^4b^3c-270\,a^5b^2d+330\,a^6be-390\,a^7f-30\,(11\,a^2b^5c-21\,a^3b^4d+34\,a^4b^3e-50\,a^5b^2f)x^6+60\,(a^3b^4c+a^4b^3d-4\,a^5b^2e+8\,a^6bf)x^3+60\,(6\,a^4b^3c-10\,a^5b^2d+15\,a^6be-21\,a^7f+(6\,a^2b^5c-10\,a^3b^4d+15\,a^4b^3e-21\,a^5b^2f)x^6+2\,(6\,a^3b^4c-10\,a^4b^3d+15\,a^5b^2e-21\,a^6bf)x^3)\log\left(bx^3+a\right)}{180\,(b^{10}x^6+2\,ab^9x^3+a^2b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^7*f*x^21 + 3*(5*b^7*e - 7*a*b^6*f)*x^18 + 2*(10*b^7*d - 15*a*b^
6*e + 21*a^2*b^5*f)*x^15 + 5*(6*b^7*c - 10*a*b^6*d + 15*a^2*b^5*e - 21*a^3*
b^4*f)*x^12 - 20*(6*a*b^6*c - 10*a^2*b^5*d + 15*a^3*b^4*e - 21*a^4*b^3*f)*x
^9 + 210*a^4*b^3*c - 270*a^5*b^2*d + 330*a^6*b*e - 390*a^7*f - 30*(11*a^2*b
^5*c - 21*a^3*b^4*d + 34*a^4*b^3*e - 50*a^5*b^2*f)*x^6 + 60*(a^3*b^4*c + a^
4*b^3*d - 4*a^5*b^2*e + 8*a^6*b*f)*x^3 + 60*(6*a^4*b^3*c - 10*a^5*b^2*d + 1
5*a^6*b*e - 21*a^7*f + (6*a^2*b^5*c - 10*a^3*b^4*d + 15*a^4*b^3*e - 21*a^5*
b^2*f)*x^6 + 2*(6*a^3*b^4*c - 10*a^4*b^3*d + 15*a^5*b^2*e - 21*a^6*b*f)*x^3
)*log(b*x^3 + a))/(b^10*x^6 + 2*a*b^9*x^3 + a^2*b^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.65, size = 349, normalized size = 1.31

$$\frac{\left(6\,a^{2}b^{3}c-10\,a^{3}b^{2}d-21\,a^{5}f+15\,a^{4}be\right)\log\left(\left|bx^{3}+a\right|\right)}{3\,b^{8}}-\frac{18\,a^{2}b^{5}cx^{6}-30\,a^{3}b^{4}dx^{6}-63\,a^{5}b^{2}fx^{6}+45\,a^{4}b^{3}x^{6}e+28\,a^{3}b^{4}cx^{3}-50\,a^{4}b^{3}dx^{3}-112\,a^{6}bfx^{3}+78\,a^{5}b^{2}x^{3}e+11\,a^{4}b^{3}c-21\,a^{5}b^{2}d-50\,a^{7}f+34\,a^{6}be}{6\,(bx^{3}+a)^{2}b^{8}}+\frac{12\,b^{12}fx^{15}-45\,ab^{11}fx^{12}+15\,b^{12}x^{12}e+20\,b^{12}dx^{9}+120\,a^{2}b^{10}fx^{9}-60\,ab^{11}x^{9}e+30\,b^{12}cx^{6}-90\,ab^{11}dx^{6}-300\,a^{3}b^{9}fx^{6}+180\,a^{2}b^{10}x^{6}e-180\,ab^{11}cx^{3}+360\,a^{2}b^{10}dx^{3}+900\,a^{4}b^{8}fx^{3}-600\,a^{3}b^{9}x^{3}e}{180\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^2*b^3*c - 10*a^3*b^2*d - 21*a^5*f + 15*a^4*b*e)*log(abs(b*x^3 + a)
)/b^8 - 1/6*(18*a^2*b^5*c*x^6 - 30*a^3*b^4*d*x^6 - 63*a^5*b^2*f*x^6 + 45*a^
4*b^3*x^6*e + 28*a^3*b^4*c*x^3 - 50*a^4*b^3*d*x^3 - 112*a^6*b*f*x^3 + 78*a^
5*b^2*x^3*e + 11*a^4*b^3*c - 21*a^5*b^2*d - 50*a^7*f + 34*a^6*b*e)/((b*x^3
+ a)^2*b^8) + 1/180*(12*b^12*f*x^15 - 45*a*b^11*f*x^12 + 15*b^12*x^12*e + 2
0*b^12*d*x^9 + 120*a^2*b^10*f*x^9 - 60*a*b^11*x^9*e + 30*b^12*c*x^6 - 90*a*
b^11*d*x^6 - 300*a^3*b^9*f*x^6 + 180*a^2*b^10*x^6*e - 180*a*b^11*c*x^3 + 36
0*a^2*b^10*d*x^3 + 900*a^4*b^8*f*x^3 - 600*a^3*b^9*x^3*e)/b^15
```

Mupad [B]
time = 4.96, size = 449, normalized size = 1.69

$$x^{12}\left(\frac{e}{12\,b^{3}}-\frac{a\,f}{4\,b^{4}}\right)+x^{6}\left(\frac{c}{6\,b^{3}}-\frac{a^{3}\,f}{6\,b^{6}}-\frac{a^{2}\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{2\,b^{2}}+\frac{a\left(\frac{3\,a^{2}\,f}{b^{5}}-\frac{d}{b^{3}}+\frac{3\,a\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{b}\right)}{2\,b}\right)-x^{9}\left(\frac{a^{2}\,f}{3\,b^{5}}-\frac{d}{9\,b^{3}}+\frac{a\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{3\,b}\right)-\frac{\frac{13\,f\,a^{7}-11\,e\,a^{6}\,b+9\,d\,a^{5}\,b^{2}-7\,c\,a^{4}\,b^{3}}{6\,b}+x^{3}\left(\frac{7\,f\,a^{6}}{3}-2\,e\,a^{5}\,b+\frac{5\,d\,a^{4}\,b^{2}}{3}-\frac{4\,c\,a^{3}\,b^{3}}{3}\right)}{a^{2}\,b^{7}+2\,a\,b^{8}\,x^{3}+b^{9}\,x^{6}}-x^{3}\left(\frac{a\left(\frac{c}{b^{3}}-\frac{a^{3}\,f}{b^{6}}-\frac{3\,a^{2}\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{b^{2}}+\frac{3\,a\left(\frac{3\,a^{2}\,f}{b^{5}}-\frac{d}{b^{3}}+\frac{3\,a\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{b}\right)}{b}\right)}{b}-\frac{a^{2}\left(\frac{3\,a^{2}\,f}{b^{5}}-\frac{d}{b^{3}}+\frac{3\,a\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{b}\right)}{b^{2}}+\frac{a^{3}\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{3\,b^{3}}\right)-\frac{\ln\left(b\,x^{3}+a\right)\left(21\,f\,a^{5}-15\,e\,a^{4}\,b+10\,d\,a^{3}\,b^{2}-6\,c\,a^{2}\,b^{3}\right)}{3\,b^{8}}+\frac{f\,x^{15}}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^12*(e/(12*b^3) - (a*f)/(4*b^4)) + x^6*(c/(6*b^3) - (a^3*f)/(6*b^6) - (a^2
*(e/b^3 - (3*a*f)/b^4))/(2*b^2) + (a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 -
 (3*a*f)/b^4))/b))/(2*b)) - x^9*((a^2*f)/(3*b^5) - d/(9*b^3) + (a*(e/b^3 -
(3*a*f)/b^4))/(3*b)) - ((13*a^7*f - 7*a^4*b^3*c + 9*a^5*b^2*d - 11*a^6*b*e)
/(6*b) + x^3*((7*a^6*f)/3 - (4*a^3*b^3*c)/3 + (5*a^4*b^2*d)/3 - 2*a^5*b*e))
/(a^2*b^7 + b^9*x^6 + 2*a*b^8*x^3) - x^3*((a*(c/b^3 - (a^3*f)/b^6 - (3*a^2*
(e/b^3 - (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3
*a*f)/b^4))/b))/b))/b - (a^2*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)
/b^4))/b))/b^2 + (a^3*(e/b^3 - (3*a*f)/b^4))/(3*b^3)) - (log(a + b*x^3)*(21
*a^5*f - 6*a^2*b^3*c + 10*a^3*b^2*d - 15*a^4*b*e))/(3*b^8) + (f*x^15)/(15*b
^3)
```

3.277 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=226

$$\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^3}{3b^6}+\frac{(b^2d-3abe+6a^2f)\,x^6}{6b^5}+\frac{(be-3af)x^9}{9b^4}+\frac{fx^{12}}{12b^3}+\frac{a^3(b^3c-ab^2d+a^2be-a}{6b^7\,(a+bx^3)^2}$$

```
[Out] 1/3*(-10*a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*x^3/b^6+1/6*(6*a^2*f-3*a*b*e+b^2*
d)*x^6/b^5+1/9*(-3*a*f+b*e)*x^9/b^4+1/12*f*x^12/b^3+1/6*a^3*(-a^3*f+a^2*b*e
-a*b^2*d+b^3*c)/b^7/(b*x^3+a)^2-1/3*a^2*(-6*a^3*f+5*a^2*b*e-4*a*b^2*d+3*b^3
*c)/b^7/(b*x^3+a)-1/3*a*(-15*a^3*f+10*a^2*b*e-6*a*b^2*d+3*b^3*c)*ln(b*x^3+a
)/b^7
```

Rubi [A]
time = 0.23, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{x^6(6a^2f-3abe+b^2d)}{6b^5}-\frac{a^2(-6a^3f+5a^2be-4ab^2d+3b^3c)}{3b^7\,(a+bx^3)}+\frac{a^3(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^7\,(a+bx^3)^2}-\frac{a\log(a+bx^3)\,(-15a^3f+10a^2be-6ab^2d+3b^3c)}{3b^7}+\frac{x^3(-10a^3f+6a^2be-3ab^2d+b^3c)}{3b^6}+\frac{x^9(be-3af)}{9b^4}+\frac{fx^{12}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^6)/(6*b^5) + ((b*e - 3*a*f)*x^9)/(9*b^4) + (f*x^12)/(12*b^3)
 + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^7*(a + b*x^3)^2) - (a^2*(
3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6*a^3*f))/(3*b^7*(a + b*x^3)) - (a*(3*b^3
*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx)^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^3c-3ab^2d+6a^2be-10a^3f}{b^6} + \frac{(b^2d-3abe+6a^2f)\,x}{b^5} + \frac{(be$$
$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^3}{3b^6} + \frac{(b^2d-3abe+6a^2f)\,x^6}{6b^5} + \frac{(be-3af)}{9b^4}$$

Mathematica [A]
time = 0.08, size = 208, normalized size = 0.92

$$\frac{12b(b^3c-3ab^2d+6a^2be-10a^3f)\,x^3+6b^2(b^2d-3abe+6a^2f)\,x^6+4b^3(be-3af)x^9+3b^4fx^{12}+\frac{6a^3(b^3c-ab^2d+a^2be-a^3f)}{(a+bx^3)^2}+\frac{12a^2(-3b^3c+4ab^2d-5a^2be+6a^3f)}{a+bx^3}+12a(-3b^3c+6ab^2d-10a^2be+15a^3f)\log(a+bx^3)}{36b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (12*b*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3 + 6*b^2*(b^2*d - 3*a*b
*e + 6*a^2*f)*x^6 + 4*b^3*(b*e - 3*a*f)*x^9 + 3*b^4*f*x^12 + (6*a^3*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^3)^2 + (12*a^2*(-3*b^3*c + 4*a*b^2*d
 - 5*a^2*b*e + 6*a^3*f))/(a + b*x^3) + 12*a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*
b*e + 15*a^3*f)*Log[a + b*x^3])/(36*b^7)
```

Maple [A]
time = 0.37, size = 223, normalized size = 0.99

method	result
norman	$\frac{\frac{a^2\left(15a^4f-10a^3be+6a^2b^2d-3a\,b^3c\right)}{2b^7}-\frac{\left(15a^3f-10a^2be+6a\,b^2d-3b^3c\right)x^9}{9b^4}+\frac{f\,x^{18}}{12b}-\frac{(3af-2be)x^{15}}{18b^2}+\frac{\left(15a^2f-10abe+6b^2d\right)x^{12}}{36b^3}+\frac{2a\left(15a^4f-10a^3be\right.}{}}{\left(b\,x^3+a\right)^2}$
default	$-\frac{-\frac{f\,x^{12}b^3}{12}+\frac{\left(3fa\,b^2-e\,b^3\right)x^9}{9}+\frac{\left(-6f\,a^2b+3a\,b^2e-b^3d\right)x^6}{6}+\frac{\left(10a^3f-6a^2be+3a\,b^2d-b^3c\right)x^3}{3}}{b^6}+\frac{a\left(\frac{a\left(6a^3f-5a^2be+4a\,b^2d-3b^3c\right)}{b\left(b\,x^3+a\right)}-\frac{a^2\left(a^3f-\right.}{2b}\right.}{}$
risch	$\frac{f\,x^{12}}{12b^3}-\frac{x^9fa}{3b^4}+\frac{e\,x^9}{9b^3}+\frac{x^6f\,a^2}{b^5}-\frac{ae\,x^6}{2b^4}+\frac{x^6d}{6b^3}-\frac{10x^3a^3f}{3b^6}+\frac{2a^2e\,x^3}{b^5}-\frac{x^3ad}{b^4}+\frac{x^3c}{3b^3}+\frac{(2f\,a^5-\frac{5}{3}e\,a^4b+\frac{4}{3}d\,a^3b^2-a^2c\,b^3}{b^6(b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6*(-1/12*f*x^12*b^3+1/9*(3*a*b^2*f-b^3*e)*x^9+1/6*(-6*a^2*b*f+3*a*b^2*
e-b^3*d)*x^6+1/3*(10*a^3*f-6*a^2*b*e+3*a*b^2*d-b^3*c)*x^3)+1/3*a/b^6*(a*(6*
a^3*f-5*a^2*b*e+4*a*b^2*d-3*b^3*c)/b/(b*x^3+a)-1/2*a^2*(a^3*f-a^2*b*e+a*b^2
*d-b^3*c)/b/(b*x^3+a)^2+(15*a^3*f-10*a^2*b*e+6*a*b^2*d-3*b^3*c)/b*ln(b*x^3+
a))
```

Maxima [A]
time = 0.28, size = 240, normalized size = 1.06

$$-\frac{5\,a^3b^3c - 7\,a^4b^2d - 11\,a^6f + 9\,a^5be + 2\,(3\,a^2b^4c - 4\,a^3b^3d - 6\,a^5bf + 5\,a^4b^2e)x^3}{6\,(b^9x^6 + 2\,ab^8x^3 + a^2b^7)} + \frac{3\,b^3fx^{12} - 4\,(3\,ab^2f - b^3e)x^9 + 6\,(b^3d + 6\,a^2bf - 3\,ab^2e)x^6 + 12\,(b^3c - 3\,ab^2d - 10\,a^3f + 6\,a^2be)x^3}{36\,b^6} - \frac{(3\,ab^3c - 6\,a^2b^2d - 15\,a^4f + 10\,a^3be)\log(bx^3 + a)}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(5*a^3*b^3*c - 7*a^4*b^2*d - 11*a^6*f + 9*a^5*b*e + 2*(3*a^2*b^4*c - 4
*a^3*b^3*d - 6*a^5*b*f + 5*a^4*b^2*e)*x^3)/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7
) + 1/36*(3*b^3*f*x^12 - 4*(3*a*b^2*f - b^3*e)*x^9 + 6*(b^3*d + 6*a^2*b*f -
 3*a*b^2*e)*x^6 + 12*(b^3*c - 3*a*b^2*d - 10*a^3*f + 6*a^2*b*e)*x^3)/b^6 -
1/3*(3*a*b^3*c - 6*a^2*b^2*d - 15*a^4*f + 10*a^3*b*e)*log(b*x^3 + a)/b^7
```

Fricas [A]
time = 0.38, size = 353, normalized size = 1.56

$$\frac{3\,b^6fx^{18} + 2\,(2\,b^6e - 3\,ab^5f)x^{15} + (6\,b^6d - 10\,ab^5e + 15\,a^2b^4f)x^{12} + 4\,(3\,b^6c - 6\,ab^5d + 10\,a^2b^4e - 15\,a^3b^3f)x^9 - 30\,a^3b^3c + 42\,a^4b^2d - 54\,a^5be + 66\,a^6f + 6\,(4\,ab^5c - 11\,a^2b^4d + 21\,a^3b^3e - 34\,a^4b^2f)x^6 - 12\,(2\,a^2b^4c - a^3b^3d - a^4b^2e + 4\,a^5bf)x^3 - 12\,(3\,a^3b^3c - 6\,a^4b^2d + 10\,a^5be - 15\,a^6f + (3\,ab^5c - 6\,a^2b^4d + 10\,a^3b^3e - 15\,a^4b^2f)x^6 + 2\,(3\,a^2b^4c - 6\,a^3b^3d + 10\,a^4b^2e - 15\,a^5bf)x^3)\log(bx^3 + a)}{36\,(b^9x^6 + 2\,ab^8x^3 + a^2b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^6*f*x^18 + 2*(2*b^6*e - 3*a*b^5*f)*x^15 + (6*b^6*d - 10*a*b^5*e +
 15*a^2*b^4*f)*x^12 + 4*(3*b^6*c - 6*a*b^5*d + 10*a^2*b^4*e - 15*a^3*b^3*f)
*x^9 - 30*a^3*b^3*c + 42*a^4*b^2*d - 54*a^5*b*e + 66*a^6*f + 6*(4*a*b^5*c -
 11*a^2*b^4*d + 21*a^3*b^3*e - 34*a^4*b^2*f)*x^6 - 12*(2*a^2*b^4*c - a^3*b^
3*d - a^4*b^2*e + 4*a^5*b*f)*x^3 - 12*(3*a^3*b^3*c - 6*a^4*b^2*d + 10*a^5*b
*e - 15*a^6*f + (3*a*b^5*c - 6*a^2*b^4*d + 10*a^3*b^3*e - 15*a^4*b^2*f)*x^6
 + 2*(3*a^2*b^4*c - 6*a^3*b^3*d + 10*a^4*b^2*e - 15*a^5*b*f)*x^3)*log(b*x^3
 + a))/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 298, normalized size = 1.32

$$-\frac{(3\,ab^3c - 6\,a^2b^2d - 15\,a^4f + 10\,a^3be)\log(|bx^3 + a|)}{3\,b^7} + \frac{9\,ab^5cx^6 - 18\,a^2b^4dx^6 - 45\,a^4b^2fx^6 + 30\,a^3b^3x^6e + 12\,a^2b^4cx^3 - 28\,a^3b^3dx^3 - 78\,a^5bfx^3 + 50\,a^4b^2x^3e + 4\,a^3b^3c - 11\,a^4b^2d - 34\,a^6f + 21\,a^5be}{6\,(bx^3 + a)^2b^7} + \frac{3\,b^9fx^{12} - 12\,ab^8fx^9 + 4\,b^9x^9e + 6\,b^9dx^6 + 36\,a^2b^7fx^6 - 18\,ab^8x^6e + 12\,b^9cx^3 - 36\,ab^8dx^3 - 120\,a^3b^6fx^3 + 72\,a^2b^7x^3e}{36\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] -1/3*(3*a*b^3*c - 6*a^2*b^2*d - 15*a^4*f + 10*a^3*b*e)*log(abs(b*x^3 + a))/b^7 + 1/6*(9*a*b^5*c*x^6 - 18*a^2*b^4*d*x^6 - 45*a^4*b^2*f*x^6 + 30*a^3*b^3*x^6*e + 12*a^2*b^4*c*x^3 - 28*a^3*b^3*d*x^3 - 78*a^5*b*f*x^3 + 50*a^4*b^2*x^3*e + 4*a^3*b^3*c - 11*a^4*b^2*d - 34*a^6*f + 21*a^5*b*e)/((b*x^3 + a)^2*b^7) + 1/36*(3*b^9*f*x^12 - 12*a*b^8*f*x^9 + 4*b^9*x^9*e + 6*b^9*d*x^6 + 36*a^2*b^7*f*x^6 - 18*a*b^8*x^6*e + 12*b^9*c*x^3 - 36*a*b^8*d*x^3 - 120*a^3*b^6*f*x^3 + 72*a^2*b^7*x^3*e)/b^12

Mupad [B]
time = 4.97, size = 293, normalized size = 1.30

$$x^9\,\left(\frac{e}{9\,b^3}-\frac{a\,f}{3\,b^4}\right)+x^3\,\left(\frac{c}{3\,b^3}-\frac{a^3\,f}{3\,b^6}-\frac{a^2\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{a\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)-x^6\,\left(\frac{a^2\,f}{2\,b^5}-\frac{d}{6\,b^3}+\frac{a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{2\,b}\right)+\frac{\frac{11\,f\,a^6-9\,e\,a^5\,b+7\,d\,a^4\,b^2-5\,c\,a^3\,b^3}{6\,b}+x^3\,\left(2\,f\,a^5-\frac{5\,e\,a^4\,b}{3}+\frac{4\,d\,a^3\,b^2}{3}-c\,a^2\,b^3\right)}{a^2\,b^6+2\,a\,b^7\,x^3+b^8\,x^6}+\frac{f\,x^{12}}{12\,b^3}+\frac{\ln\left(b\,x^3+a\right)\,\left(15\,f\,a^4-10\,e\,a^3\,b+6\,d\,a^2\,b^2-3\,c\,a\,b^3\right)}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)

[Out] x^9*(e/(9*b^3) - (a*f)/(3*b^4)) + x^3*(c/(3*b^3) - (a^3*f)/(3*b^6) - (a^2*(e/b^3 - (3*a*f)/b^4))/b^2 + (a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b) - x^6*((a^2*f)/(2*b^5) - d/(6*b^3) + (a*(e/b^3 - (3*a*f)/b^4))/(2*b)) + ((11*a^6*f - 5*a^3*b^3*c + 7*a^4*b^2*d - 9*a^5*b*e)/(6*b) + x^3*(2*a^5*f - a^2*b^3*c + (4*a^3*b^2*d)/3 - (5*a^4*b*e)/3))/(a^2*b^6 + b^8*x^6 + 2*a*b^7*x^3) + (f*x^12)/(12*b^3) + (log(a + b*x^3)*(15*a^4*f + 6*a^2*b^2*d - 3*a*b^3*c - 10*a^3*b*e))/(3*b^7)

3.278 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$

Optimal. Leaf size=186

$$\frac{(b^2d-3abe+6a^2f)\,x^3}{3b^5}+\frac{(be-3af)x^6}{6b^4}+\frac{fx^9}{9b^3}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)}{6b^6\,(a+bx^3)^2}+\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f}{3b^6\,(a+bx^3)}$$

```
[Out] 1/3*(6*a^2*f-3*a*b*e+b^2*d)*x^3/b^5+1/6*(-3*a*f+b*e)*x^6/b^4+1/9*f*x^9/b^3-
1/6*a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b^6/(b*x^3+a)^2+1/3*a*(-5*a^3*f+4*a^
2*b*e-3*a*b^2*d+2*b^3*c)/b^6/(b*x^3+a)+1/3*(-10*a^3*f+6*a^2*b*e-3*a*b^2*d+b
^3*c)*ln(b*x^3+a)/b^6
```

Rubi [A]
time = 0.18, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{x^3(6a^2f-3abe+b^2d)}{3b^5}+\frac{a(-5a^3f+4a^2be-3ab^2d+2b^3c)}{3b^6\,(a+bx^3)}-\frac{a^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6\,(a+bx^3)^2}+\frac{\log\left(a+bx^3\right)(-10a^3f+6a^2be-3ab^2d+b^3c)}{3b^6}+\frac{x^6(be-3af)}{6b^4}+\frac{fx^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^6)/(6*b^4) + (
f*x^9)/(9*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^6*(a + b*x^
3)^2) + (a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f))/(3*b^6*(a + b*x^3))
 + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx)^3}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2d-3abe+6a^2f}{b^5} + \frac{(be-3af)x}{b^4} + \frac{fx^2}{b^3} - \frac{a^2(-b^3c+ab^2d -}{b^5(a+}\right.\right.$$

$$= \frac{(b^2d-3abe+6a^2f)\,x^3}{3b^5} + \frac{(be-3af)x^6}{6b^4} + \frac{fx^9}{9b^3} - \frac{a^2(b^3c-ab^2d+a^2be-a}{6b^6\,(a+bx^3)^2}$$

Mathematica [A]
time = 0.07, size = 170, normalized size = 0.91

$$\frac{6b(b^2d-3abe+6a^2f)\,x^3+3b^2(be-3af)x^6+2b^3fx^9+\frac{3a^2(-b^3c+ab^2d-a^2be+a^3f)}{(a+bx^3)^2}-\frac{6a(-2b^3c+3ab^2d-4a^2be+5a^3f)}{a+bx^3}+6(b^3c-3ab^2d+6a^2be-10a^3f)\log(a+bx^3)}{18b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (6*b*(b^2*d - 3*a*b*e + 6*a^2*f)*x^3 + 3*b^2*(b*e - 3*a*f)*x^6 + 2*b^3*f*x^
9 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 - (6*a*(-2
*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f))/(a + b*x^3) + 6*(b^3*c - 3*a*b^2
*d + 6*a^2*b*e - 10*a^3*f)*Log[a + b*x^3])/(18*b^6)
```

Maple [A]
time = 0.35, size = 182, normalized size = 0.98

method	result
norman	$\frac{-\frac{a^2\left(10a^3f-6a^2be+3a\,b^2d-b^3c\right)}{2b^6}+\frac{f\,x^{15}}{9b}-\frac{(5af-3be)x^{12}}{18b^2}+\frac{\left(10a^2f-6abe+3b^2d\right)x^9}{9b^3}-\frac{2a\left(10a^3f-6a^2be+3a\,b^2d-b^3c\right)x^3}{3b^5}}{(b\,x^3+a)^2} - (10a^3f-6a^2be+3a$
default	$\frac{\frac{f\,x^9b^2}{9}+\frac{\left(-3fab+b^2e\right)x^6}{6}+\frac{\left(6a^2f-3abe+b^2d\right)x^3}{3}}{b^5} - \frac{\frac{\left(5a^3f-4a^2be+3a\,b^2d-2b^3c\right)a}{b\left(b\,x^3+a\right)}-\frac{a^2\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{2b\left(b\,x^3+a\right)^2}+\frac{\left(10a^3f-6a^2be+3a\,b^2d-b^3c\right)}{b}}{3b^5}$
risch	$\frac{f\,x^9}{9b^3}-\frac{af\,x^6}{2b^4}+\frac{e\,x^6}{6b^3}+\frac{2a^2f\,x^3}{b^5}-\frac{ae\,x^3}{b^4}+\frac{d\,x^3}{3b^3}+\frac{(-\frac{5}{3}a^4f+\frac{4}{3}a^3be-a^2b^2d+\frac{2}{3}a\,b^3c)x^3-\frac{a^2\left(9a^3f-7a^2be+5a\,b^2d-3b^3c\right)}{6b}}{b^5(b\,x^3+a)^2} - 10$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/9*f*x^9*b^2+1/6*(-3*a*b*f+b^2*e)*x^6+1/3*(6*a^2*f-3*a*b*e+b^2*d)*x
^3)-1/3/b^5*((5*a^3*f-4*a^2*b*e+3*a*b^2*d-2*b^3*c)*a/b/(b*x^3+a)-1/2*a^2*(a
^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)^2+(10*a^3*f-6*a^2*b*e+3*a*b^2*d-b^3
*c)/b*ln(b*x^3+a))
```

Maxima [A]
time = 0.28, size = 197, normalized size = 1.06

$$\frac{3\,a^{2}b^{3}c-5\,a^{3}b^{2}d-9\,a^{5}f+7\,a^{4}be+2\,(2\,ab^{4}c-3\,a^{2}b^{3}d-5\,a^{4}bf+4\,a^{3}b^{2}e)x^{3}}{6\,(b^{8}x^{6}+2\,ab^{7}x^{3}+a^{2}b^{6})}+\frac{2\,b^{2}fx^{9}-3\,(3\,abf-b^{2}e)x^{6}+6\,(b^{2}d+6\,a^{2}f-3\,abe)x^{3}}{18\,b^{5}}+\frac{(b^{3}c-3\,ab^{2}d-10\,a^{3}f+6\,a^{2}be)\log\left(bx^{3}+a\right)}{3\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*b^3*c - 5*a^3*b^2*d - 9*a^5*f + 7*a^4*b*e + 2*(2*a*b^4*c - 3*a^2
*b^3*d - 5*a^4*b*f + 4*a^3*b^2*e)*x^3)/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6) +
1/18*(2*b^2*f*x^9 - 3*(3*a*b*f - b^2*e)*x^6 + 6*(b^2*d + 6*a^2*f - 3*a*b*e)
*x^3)/b^5 + 1/3*(b^3*c - 3*a*b^2*d - 10*a^3*f + 6*a^2*b*e)*log(b*x^3 + a)/b
^6
```

Fricas [A]
time = 0.38, size = 295, normalized size = 1.59

$$\frac{2\,b^{5}fx^{15}+(3\,b^{5}e-5\,ab^{4}f)x^{12}+2\,(3\,b^{5}d-6\,ab^{4}e+10\,a^{2}b^{3}f)x^{9}+3\,(4\,ab^{4}d-11\,a^{2}b^{3}e+21\,a^{3}b^{2}f)x^{6}+9\,a^{2}b^{3}c-15\,a^{3}b^{2}d+21\,a^{4}be-27\,a^{5}f+6\,(2\,ab^{4}c-2\,a^{2}b^{3}d+a^{3}b^{2}e+a^{4}bf)x^{3}+6\,((b^{5}c-3\,ab^{4}d+6\,a^{2}b^{3}e-10\,a^{3}b^{2}f)x^{6}+a^{2}b^{3}c-3\,a^{3}b^{2}d+6\,a^{4}be-10\,a^{5}f+2\,(ab^{4}c-3\,a^{2}b^{3}d+6\,a^{3}b^{2}e-10\,a^{4}bf)x^{3})\log\left(bx^{3}+a\right)}{18\,(b^{8}x^{6}+2\,ab^{7}x^{3}+a^{2}b^{6})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^5*f*x^15 + (3*b^5*e - 5*a*b^4*f)*x^12 + 2*(3*b^5*d - 6*a*b^4*e +
10*a^2*b^3*f)*x^9 + 3*(4*a*b^4*d - 11*a^2*b^3*e + 21*a^3*b^2*f)*x^6 + 9*a^2
*b^3*c - 15*a^3*b^2*d + 21*a^4*b*e - 27*a^5*f + 6*(2*a*b^4*c - 2*a^2*b^3*d
+ a^3*b^2*e + a^4*b*f)*x^3 + 6*((b^5*c - 3*a*b^4*d + 6*a^2*b^3*e - 10*a^3*b
^2*f)*x^6 + a^2*b^3*c - 3*a^3*b^2*d + 6*a^4*b*e - 10*a^5*f + 2*(a*b^4*c - 3
*a^2*b^3*d + 6*a^3*b^2*e - 10*a^4*b*f)*x^3)*log(b*x^3 + a))/(b^8*x^6 + 2*a*
b^7*x^3 + a^2*b^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.66, size = 236, normalized size = 1.27

$$\frac{(b^{3}c-3\,ab^{2}d-10\,a^{3}f+6\,a^{2}be)\log\left(|bx^{3}+a|\right)}{3\,b^{6}}-\frac{3\,b^{5}cx^{6}-9\,ab^{4}dx^{6}-30\,a^{3}b^{2}fx^{6}+18\,a^{2}b^{3}x^{6}e+2\,ab^{4}cx^{3}-12\,a^{2}b^{3}dx^{3}-50\,a^{4}bfx^{3}+28\,a^{3}b^{2}x^{3}e-4\,a^{3}b^{2}d-21\,a^{5}f+11\,a^{4}be}{6\,(bx^{3}+a)^{2}b^{6}}+\frac{2\,b^{6}fx^{9}-9\,ab^{5}fx^{6}+3\,b^{6}x^{6}e+6\,b^{6}dx^{3}+36\,a^{2}b^{4}fx^{3}-18\,ab^{5}x^{3}e}{18\,b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c - 3*a*b^2*d - 10*a^3*f + 6*a^2*b*e)*log(abs(b*x^3 + a))/b^6 - 1/
6*(3*b^5*c*x^6 - 9*a*b^4*d*x^6 - 30*a^3*b^2*f*x^6 + 18*a^2*b^3*x^6*e + 2*a*
b^4*c*x^3 - 12*a^2*b^3*d*x^3 - 50*a^4*b*f*x^3 + 28*a^3*b^2*x^3*e - 4*a^3*b^
2*d - 21*a^5*f + 11*a^4*b*e)/((b*x^3 + a)^2*b^6) + 1/18*(2*b^6*f*x^9 - 9*a*
b^5*f*x^6 + 3*b^6*x^6*e + 6*b^6*d*x^3 + 36*a^2*b^4*f*x^3 - 18*a*b^5*x^3*e)/
b^9
```

Mupad [B]
time = 4.92, size = 204, normalized size = 1.10

$$x^6\left(\frac{e}{6\,b^3}-\frac{a\,f}{2\,b^4}\right)-\frac{x^3\,\left(\frac{5\,f\,a^4}{3}-\frac{4\,e\,a^3\,b}{3}+d\,a^2\,b^2-\frac{2\,c\,a\,b^3}{3}\right)+\frac{9\,f\,a^5-7\,e\,a^4\,b+5\,d\,a^3\,b^2-3\,c\,a^2\,b^3}{6\,b}}{a^2\,b^5+2\,a\,b^6\,x^3+b^7\,x^6}-x^3\,\left(\frac{a^2\,f}{b^5}-\frac{d}{3\,b^3}+\frac{a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(-10\,f\,a^3+6\,e\,a^2\,b-3\,d\,a\,b^2+c\,b^3\right)}{3\,b^6}+\frac{f\,x^9}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^6*(e/(6*b^3) - (a*f)/(2*b^4)) - (x^3*((5*a^4*f)/3 + a^2*b^2*d - (2*a*b^3*
c)/3 - (4*a^3*b*e)/3) + (9*a^5*f - 3*a^2*b^3*c + 5*a^3*b^2*d - 7*a^4*b*e)/(
6*b))/(a^2*b^5 + b^7*x^6 + 2*a*b^6*x^3) - x^3*((a^2*f)/b^5 - d/(3*b^3) + (a
*(e/b^3 - (3*a*f)/b^4))/b) + (log(a + b*x^3)*(b^3*c - 10*a^3*f - 3*a*b^2*d
+ 6*a^2*b*e))/(3*b^6) + (f*x^9)/(9*b^3)
```

3.279 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=146

$$\frac{(be-3af)x^3}{3b^4}+\frac{fx^6}{6b^3}+\frac{a(b^3c-ab^2d+a^2be-a^3f)}{6b^5(a+bx^3)^2}-\frac{b^3c-2ab^2d+3a^2be-4a^3f}{3b^5(a+bx^3)}+\frac{(b^2d-3abe+6a^2f)\log(a}{3b^5}$$

```
[Out] 1/3*(-3*a*f+b*e)*x^3/b^4+1/6*f*x^6/b^3+1/6*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
/b^5/(b*x^3+a)^2+1/3*(4*a^3*f-3*a^2*b*e+2*a*b^2*d-b^3*c)/b^5/(b*x^3+a)+1/3*
(6*a^2*f-3*a*b*e+b^2*d)*ln(b*x^3+a)/b^5
```

Rubi [A]

time = 0.14, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{\log(a+bx^3)(6a^2f-3abe+b^2d)}{3b^5}-\frac{-4a^3f+3a^2be-2ab^2d+b^3c}{3b^5(a+bx^3)}+\frac{a(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}+\frac{x^3(be-3af)}{3b^4}+\frac{fx^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x^3)/(3*b^4) + (f*x^6)/(6*b^3) + (a*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f))/(6*b^5*(a + b*x^3)^2) - (b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3
*f)/(3*b^5*(a + b*x^3)) + ((b^2*d - 3*a*b*e + 6*a^2*f)*Log[a + b*x^3])/(3*b
^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx)^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{be-3af}{b^4}+\frac{fx}{b^3}+\frac{a(-b^3c+ab^2d-a^2be+a^3f)}{b^4(a+bx)^3}+\frac{b^3c-2ab^2}{b^4}\right.\right.$$
$$= \frac{(be-3af)x^3}{3b^4}+\frac{fx^6}{6b^3}+\frac{a(b^3c-ab^2d+a^2be-a^3f)}{6b^5\left(a+bx^3\right)^2}-\frac{b^3c-2ab^2d+3a^2be}{3b^5\left(a+bx^3\right)}$$

Mathematica [A]
time = 0.05, size = 145, normalized size = 0.99

$$\frac{7a^4f+a^3b(-5e+2fx^3)+a^2b^2(3d-4ex^3-11fx^6)+b^4x^3(-2c+2ex^6+fx^9)-ab^3(c-4x^3(d+ex^3-fx^6))+2(b^2d-3abe+6a^2f)\left(a+bx^3\right)^2\log\left(a+bx^3\right)}{6b^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (7*a^4*f + a^3*b*(-5*e + 2*f*x^3) + a^2*b^2*(3*d - 4*e*x^3 - 11*f*x^6) + b^
4*x^3*(-2*c + 2*e*x^6 + f*x^9) - a*b^3*(c - 4*x^3*(d + e*x^3 - f*x^6)) + 2*
(b^2*d - 3*a*b*e + 6*a^2*f)*(a + b*x^3)^2*Log[a + b*x^3])/(6*b^5*(a + b*x^3
)^2)
```

Maple [A]
time = 0.37, size = 143, normalized size = 0.98

method	result
norman	$\frac{\frac{a\left(18a^3f-9a^2be+3a\,b^2d-b^3c\right)}{6b^5}+\frac{f\,x^{12}}{6b}-\frac{(2af-be)x^9}{3b^2}+\frac{\left(12a^3f-6a^2be+2a\,b^2d-b^3c\right)x^3}{3b^4}}{\left(b\,x^3+a\right)^2}+\frac{(6a^2f-3abe+b^2d)\ln\left(b\,x^3+a\right)}{3b^5}$
default	$\frac{\left(-f\,x^3b+3af-be\right)^2}{6b^5f}+\frac{-\frac{-4a^3f+3a^2be-2a\,b^2d+b^3c}{b\left(b\,x^3+a\right)}-\frac{a\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{2b\left(b\,x^3+a\right)^2}+\frac{\left(6a^2f-3abe+b^2d\right)\ln\left(b\,x^3+a\right)}{b}}{3b^4}$
risch	$\frac{f\,x^6}{6b^3}-\frac{fa\,x^3}{b^4}+\frac{e\,x^3}{3b^3}+\frac{3f\,a^2}{2b^5}-\frac{ae}{b^4}+\frac{e^2}{6b^3f}+\frac{(\frac{4}{3}a^3f-a^2be+\frac{2}{3}a\,b^2d-\frac{1}{3}b^3c)x^3+\frac{a\left(7a^3f-5a^2be+3a\,b^2d-b^3c\right)}{6b}}{b^4(b\,x^3+a)^2}+\frac{2\ln(b\,x^3+a)a^2}{b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-b*f*x^3+3*a*f-b*e)^2/b^5/f+1/3/b^4*(-(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^
3*c)/b/(b*x^3+a)-1/2*a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)^2+(6*a^2*f
-3*a*b*e+b^2*d)/b*ln(b*x^3+a))
```

Maxima [A]
time = 0.27, size = 152, normalized size = 1.04

$$-\frac{ab^3c-3\,a^2b^2d-7\,a^4f+5\,a^3be+2\left(b^4c-2\,ab^3d-4\,a^3bf+3\,a^2b^2e\right)x^3}{6\left(b^7x^6+2\,ab^6x^3+a^2b^5\right)}+\frac{bfx^6-2\left(3\,af-be\right)x^3}{6\,b^4}+\frac{\left(b^2d+6\,a^2f-3\,abe\right)\log\left(bx^3+a\right)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(a*b^3*c - 3*a^2*b^2*d - 7*a^4*f + 5*a^3*b*e + 2*(b^4*c - 2*a*b^3*d -
4*a^3*b*f + 3*a^2*b^2*e)*x^3)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5) + 1/6*(b*f*
x^6 - 2*(3*a*f - b*e)*x^3)/b^4 + 1/3*(b^2*d + 6*a^2*f - 3*a*b*e)*log(b*x^3
+ a)/b^5
```

Fricas [**A**]
time = 0.39, size = 225, normalized size = 1.54

$$\frac{b^4fx^{12}+2\,(b^4e-2\,ab^3f)x^9+(4\,ab^3e-11\,a^2b^2f)x^6-ab^3c+3\,a^2b^2d-5\,a^3be+7\,a^4f-2\,(b^4c-2\,ab^3d+2\,a^2b^2e-a^3bf)x^3+2\,((b^4d-3\,ab^3e+6\,a^2b^2f)x^6+a^2b^2d-3\,a^3be+6\,a^4f+2\,(ab^3d-3\,a^2b^2e+6\,a^3bf)x^3)\log\left(bx^3+a\right)}{6\,(b^7x^6+2\,ab^6x^3+a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*f*x^12 + 2*(b^4*e - 2*a*b^3*f)*x^9 + (4*a*b^3*e - 11*a^2*b^2*f)*x^
6 - a*b^3*c + 3*a^2*b^2*d - 5*a^3*b*e + 7*a^4*f - 2*(b^4*c - 2*a*b^3*d + 2*
a^2*b^2*e - a^3*b*f)*x^3 + 2*((b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^6 + a^2*b
^2*d - 3*a^3*b*e + 6*a^4*f + 2*(a*b^3*d - 3*a^2*b^2*e + 6*a^3*b*f)*x^3)*log
(b*x^3 + a))/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.70, size = 146, normalized size = 1.00

$$\frac{(b^2d+6\,a^2f-3\,abe)\log\left(|bx^3+a|\right)}{3\,b^5}+\frac{b^3fx^6-6\,ab^2fx^3+2\,b^3x^3e}{6\,b^6}-\frac{ab^3c-3\,a^2b^2d-7\,a^4f+5\,a^3be+2\,(b^4c-2\,ab^3d-4\,a^3bf+3\,a^2b^2e)x^3}{6\,(bx^3+a)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*d + 6*a^2*f - 3*a*b*e)*log(abs(b*x^3 + a))/b^5 + 1/6*(b^3*f*x^6 -
6*a*b^2*f*x^3 + 2*b^3*x^3*e)/b^6 - 1/6*(a*b^3*c - 3*a^2*b^2*d - 7*a^4*f + 5
*a^3*b*e + 2*(b^4*c - 2*a*b^3*d - 4*a^3*b*f + 3*a^2*b^2*e)*x^3)/((b*x^3 + a
)^2*b^5)
```

Mupad [B]
time = 0.10, size = 152, normalized size = 1.04

$$x^3\left(\frac{e}{3\,b^3}-\frac{a\,f}{b^4}\right)+\frac{\frac{7\,f\,a^4-5\,e\,a^3\,b+3\,d\,a^2\,b^2-c\,a\,b^3}{6\,b}-x^3\left(-\frac{4\,f\,a^3}{3}+e\,a^2\,b-\frac{2\,d\,a\,b^2}{3}+\frac{c\,b^3}{3}\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+\frac{f\,x^6}{6\,b^3}+\frac{\ln\left(b\,x^3+a\right)\,\left(6\,f\,a^2-3\,e\,a\,b+d\,b^2\right)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^3*(e/(3*b^3) - (a*f)/b^4) + ((7*a^4*f + 3*a^2*b^2*d - a*b^3*c - 5*a^3*b*e
)/(6*b) - x^3*((b^3*c)/3 - (4*a^3*f)/3 - (2*a*b^2*d)/3 + a^2*b*e))/(a^2*b^4
 + b^6*x^6 + 2*a*b^5*x^3) + (f*x^6)/(6*b^3) + (log(a + b*x^3)*(b^2*d + 6*a^
2*f - 3*a*b*e))/(3*b^5)
```

3.280 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=109

$$\frac{fx^3}{3b^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{6b^4(a+bx^3)^2}-\frac{b^2d-2abe+3a^2f}{3b^4(a+bx^3)}+\frac{(be-3af)\log(a+bx^3)}{3b^4}$$

```
[Out] 1/3*f*x^3/b^3+1/6*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^4/(b*x^3+a)^2+1/3*(-3*a^2
*f+2*a*b*e-b^2*d)/b^4/(b*x^3+a)+1/3*(-3*a*f+b*e)*ln(b*x^3+a)/b^4
```

Rubi [A]
time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1833, 1864}

$$-\frac{3a^2f-2abe+b^2d}{3b^4(a+bx^3)}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6b^4(a+bx^3)^2}+\frac{(be-3af)\log(a+bx^3)}{3b^4}+\frac{fx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (f*x^3)/(3*b^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*b^4*(a + b*x^3)^2)
 - (b^2*d - 2*a*b*e + 3*a^2*f)/(3*b^4*(a + b*x^3)) + ((b*e - 3*a*f)*Log[a +
 b*x^3])/(3*b^4)
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{(a+bx)^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{f}{b^3} + \frac{b^3c-ab^2d+a^2be-a^3f}{b^3(a+bx)^3} + \frac{b^2d-2abe+3a^2f}{b^3(a+bx)^2} + \frac{be-3}{b^3(a+}\right.\right.$$
$$= \frac{fx^3}{3b^3} - \frac{b^3c-ab^2d+a^2be-a^3f}{6b^4(a+bx^3)^2} - \frac{b^2d-2abe+3a^2f}{3b^4(a+bx^3)} + \frac{(be-3af)\log(a+}{3b^4}$$

Mathematica [A]
time = 0.04, size = 105, normalized size = 0.96

$$\frac{-5a^3f+a^2b(3e-4fx^3)+ab^2(-d+4ex^3+4fx^6)-b^3(c+2dx^3-2fx^9)+2(be-3af)\left(a+bx^3\right)^2\log\left(a+bx^3\right)}{6b^4\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (-5*a^3*f + a^2*b*(3*e - 4*f*x^3) + a*b^2*(-d + 4*e*x^3 + 4*f*x^6) - b^3*(c
 + 2*d*x^3 - 2*f*x^9) + 2*(b*e - 3*a*f)*(a + b*x^3)^2*Log[a + b*x^3])/(6*b^
4*(a + b*x^3)^2)
```

Maple [A]
time = 0.34, size = 109, normalized size = 1.00

method	result	size
norman	$\frac{-\frac{9a^3f-3a^2be+a\,b^2d+b^3c}{6b^4}+\frac{f\,x^9}{3b}-\frac{\left(6a^2f-2abe+b^2d\right)x^3}{3b^3}}{\left(b\,x^3+a\right)^2}-\frac{(3af-be)\ln\left(b\,x^3+a\right)}{3b^4}$	99
risch	$\frac{f\,x^3}{3b^3}+\frac{\left(-a^2f+\frac{2}{3}abe-\frac{1}{3}b^2d\right)x^3-\frac{5a^3f-3a^2be+a\,b^2d+b^3c}{6b}}{b^3\left(b\,x^3+a\right)^2}-\frac{\ln\left(b\,x^3+a\right)af}{b^4}+\frac{\ln\left(b\,x^3+a\right)e}{3b^3}$	106
default	$\frac{f\,x^3}{3b^3}-\frac{-\frac{-3a^2f+2abe-b^2d}{b\left(b\,x^3+a\right)}-\frac{a^3f-a^2be+a\,b^2d-b^3c}{2b\left(b\,x^3+a\right)^2}+\frac{(3af-be)\ln\left(b\,x^3+a\right)}{b}}{3b^3}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*f*x^3/b^3-1/3/b^3*(-(-3*a^2*f+2*a*b*e-b^2*d)/b/(b*x^3+a)-1/2*(a^3*f-a^2
*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)^2+1/b*(3*a*f-b*e)*ln(b*x^3+a))
```

Maxima [A]
time = 0.28, size = 113, normalized size = 1.04

$$\frac{fx^3}{3\,b^3} - \frac{b^3c+ab^2d+5\,a^3f+2\,(b^3d+3\,a^2bf-2\,ab^2e)x^3-3\,a^2be}{6\,(b^6x^6+2\,ab^5x^3+a^2b^4)} - \frac{(3\,af-be)\log\left(bx^3+a\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3/b^3 - 1/6*(b^3*c + a*b^2*d + 5*a^3*f + 2*(b^3*d + 3*a^2*b*f - 2*a
*b^2*e)*x^3 - 3*a^2*b*e)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) - 1/3*(3*a*f - b
*e)*log(b*x^3 + a)/b^4
```

Fricas [**A**]
time = 0.38, size = 158, normalized size = 1.45

$$\frac{2 b^3 f x^9 + 4 a b^2 f x^6 - b^3 c - a b^2 d + 3 a^2 b e - 5 a^3 f - 2 (b^3 d - 2 a b^2 e + 2 a^2 b f) x^3 + 2 ((b^3 e - 3 a b^2 f) x^6 + a^2 b e - 3 a^3 f + 2 (a b^2 e - 3 a^2 b f) x^3) \log (b x^3 + a)}{6 (b^6 x^6 + 2 a b^5 x^3 + a^2 b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*f*x^9 + 4*a*b^2*f*x^6 - b^3*c - a*b^2*d + 3*a^2*b*e - 5*a^3*f -
2*(b^3*d - 2*a*b^2*e + 2*a^2*b*f)*x^3 + 2*((b^3*e - 3*a*b^2*f)*x^6 + a^2*b*
e - 3*a^3*f + 2*(a*b^2*e - 3*a^2*b*f)*x^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b
^5*x^3 + a^2*b^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.62, size = 100, normalized size = 0.92

$$\frac{f x^3}{3 b^3} - \frac{(3 a f - b e) \log (|b x^3 + a|)}{3 b^4} - \frac{b^3 c + a b^2 d + 5 a^3 f + 2 (b^3 d + 3 a^2 b f - 2 a b^2 e) x^3 - 3 a^2 b e}{6 (b x^3 + a)^2 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*f*x^3/b^3 - 1/3*(3*a*f - b*e)*log(abs(b*x^3 + a))/b^4 - 1/6*(b^3*c + a*
b^2*d + 5*a^3*f + 2*(b^3*d + 3*a^2*b*f - 2*a*b^2*e)*x^3 - 3*a^2*b*e)/((b*x^
3 + a)^2*b^4)
```

Mupad [**B**]
time = 4.94, size = 112, normalized size = 1.03

$$\frac{f x^3}{3 b^3} - \frac{x^3 \left(f a^2 - \frac{2 e a b}{3} + \frac{d b^2}{3}\right) + \frac{5 f a^3 - 3 e a^2 b + d a b^2 + c b^3}{6 b}}{a^2 b^3 + 2 a b^4 x^3 + b^5 x^6} - \frac{\ln (b x^3 + a) (3 a f - b e)}{3 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] (f*x^3)/(3*b^3) - (x^3*((b^2*d)/3 + a^2*f - (2*a*b*e)/3) + (b^3*c + 5*a^3*f
 + a*b^2*d - 3*a^2*b*e)/(6*b))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) - (log(a +
 b*x^3)*(3*a*f - b*e))/(3*b^4)
```

3.281 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^3}\,dx$

Optimal. Leaf size=114

$$\frac{b^3c-ab^2d+a^2be-a^3f}{6ab^3(a+bx^3)^2}+\frac{b^3c-a^2be+2a^3f}{3a^2b^3(a+bx^3)}+\frac{c\log(x)}{a^3}-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)$$

```
[Out] 1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a/b^3/(b*x^3+a)^2+1/3*(2*a^3*f-a^2*b*e+b
^3*c)/a^2/b^3/(b*x^3+a)+c*ln(x)/a^3-1/3*(c/a^3-f/b^3)*ln(b*x^3+a)
```

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)+\frac{c\log(x)}{a^3}+\frac{2a^3f-a^2be+b^3c}{3a^2b^3(a+bx^3)}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6ab^3(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]
```

```
[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a*b^3*(a + b*x^3)^2) + (b^3*c - a^2*
b*e + 2*a^3*f)/(3*a^2*b^3*(a + b*x^3)) + (c*Log[x])/a^3 - ((c/a^3 - f/b^3)*
Log[a + b*x^3])/3
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x(a+bx)^3}\,dx,x,x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2(a+bx)^3}+\frac{-b^3c+a^2be-2a^3f}{a^2b^2(a+bx)^2}+\frac{-b^3c}{a^3b^2(c\right.\right. \\
&= \frac{b^3c-ab^2d+a^2be-a^3f}{6ab^3\left(a+bx^3\right)^2}+\frac{b^3c-a^2be+2a^3f}{3a^2b^3\left(a+bx^3\right)}+\frac{c\log(x)}{a^3}-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a\right.
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 104, normalized size = 0.91

$$
\frac{6c\log(x)+\frac{\frac{a\left(3ab^3c+3a^4f+2b^4cx^3-a^2b^2\left(d+2ex^3\right)-a^3b\left(e-4fx^3\right)\right)}{\left(a+bx^3\right)^2}+2\left(-b^3c+a^3f\right)\log\left(a+bx^3\right)}{b^3}}{6a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]
```

```
[Out] (6*c*Log[x] + ((a*(3*a*b^3*c + 3*a^4*f + 2*b^4*c*x^3 - a^2*b^2*(d + 2*e*x^3
) - a^3*b*(e - 4*f*x^3)))/(a + b*x^3)^2 + 2*(-(b^3*c) + a^3*f)*Log[a + b*x^
3])/b^3)/(6*a^3)
```

Maple [A]
time = 0.34, size = 114, normalized size = 1.00

method	result	size
norman	$\frac{\frac{3a^3f-a^2be-a\,b^2d+3b^3c}{6a\,b^3}+\frac{\left(2a^3f-a^2be+b^3c\right)x^3}{3a^2b^2}}{\left(b\,x^3+a\right)^2}+\frac{c\ln(x)}{a^3}+\frac{\left(a^3f-b^3c\right)\ln\left(b\,x^3+a\right)}{3a^3b^3}$	113
default	$\frac{\frac{a\left(2a^3f-a^2be+b^3c\right)}{b^3\left(b\,x^3+a\right)}-\frac{a^2\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{2b^3\left(b\,x^3+a\right)^2}+\frac{\left(a^3f-b^3c\right)\ln\left(b\,x^3+a\right)}{b^3}}{3a^3}+\frac{c\ln(x)}{a^3}$	114
risch	$\frac{\frac{3a^3f-a^2be-a\,b^2d+3b^3c}{6a\,b^3}+\frac{\left(2a^3f-a^2be+b^3c\right)x^3}{3a^2b^2}}{\left(b\,x^3+a\right)^2}+\frac{c\ln(x)}{a^3}+\frac{\ln\left(-b\,x^3-a\right)f}{3b^3}-\frac{\ln\left(-b\,x^3-a\right)c}{3a^3}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^3*(a/b^3*(2*a^3*f-a^2*b*e+b^3*c)/(b*x^3+a)-1/2*a^2*(a^3*f-a^2*b*e+a*b
^2*d-b^3*c)/b^3/(b*x^3+a)^2+(a^3*f-b^3*c)/b^3*ln(b*x^3+a))+c*ln(x)/a^3
```

Maxima [A]
time = 0.28, size = 131, normalized size = 1.15

$$
\frac{3\,ab^3c-a^2b^2d+3\,a^4f-a^3be+2\,(b^4c+2\,a^3bf-a^2b^2e)x^3}{6\,(a^2b^5x^6+2\,a^3b^4x^3+a^4b^3)}+\frac{c\log\left(x^3\right)}{3\,a^3}-\frac{(b^3c-a^3f)\log\left(bx^3+a\right)}{3\,a^3b^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b^3*c - a^2*b^2*d + 3*a^4*f - a^3*b*e + 2*(b^4*c + 2*a^3*b*f - a^2
*b^2*e)*x^3)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3) + 1/3*c*log(x^3)/a^3 -
 1/3*(b^3*c - a^3*f)*log(b*x^3 + a)/(a^3*b^3)
```

Fricas [A]
time = 0.41, size = 187, normalized size = 1.64

$$\frac{3\,a^{2}b^{3}c-a^{3}b^{2}d-a^{4}be+3\,a^{5}f+2\,(ab^{4}c-a^{3}b^{2}e+2\,a^{4}bf)x^{3}-2\,((b^{5}c-a^{3}b^{2}f)x^{6}+a^{2}b^{3}c-a^{5}f+2\,(ab^{4}c-a^{4}bf)x^{3})\log\left(bx^{3}+a\right)+6\,(b^{5}cx^{6}+2\,ab^{4}cx^{3}+a^{2}b^{3}c)\log\left(x\right)}{6\,(a^{3}b^{5}x^{6}+2\,a^{4}b^{4}x^{3}+a^{5}b^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*b^3*c - a^3*b^2*d - a^4*b*e + 3*a^5*f + 2*(a*b^4*c - a^3*b^2*e +
 2*a^4*b*f)*x^3 - 2*((b^5*c - a^3*b^2*f)*x^6 + a^2*b^3*c - a^5*f + 2*(a*b^4
*c - a^4*b*f)*x^3)*log(b*x^3 + a) + 6*(b^5*c*x^6 + 2*a*b^4*c*x^3 + a^2*b^3*
c)*log(x))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.68, size = 128, normalized size = 1.12

$$\frac{c\log\left(|x|\right)}{a^{3}}-\frac{(b^{3}c-a^{3}f)\log\left(|bx^{3}+a|\right)}{3\,a^{3}b^{3}}+\frac{3\,b^{4}cx^{6}-3\,a^{3}bfx^{6}+8\,ab^{3}cx^{3}-2\,a^{4}fx^{3}-2\,a^{3}bx^{3}e+6\,a^{2}b^{2}c-a^{3}bd-a^{4}e}{6\,(bx^{3}+a)^{2}a^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] c*log(abs(x))/a^3 - 1/3*(b^3*c - a^3*f)*log(abs(b*x^3 + a))/(a^3*b^3) + 1/6
*(3*b^4*c*x^6 - 3*a^3*b*f*x^6 + 8*a*b^3*c*x^3 - 2*a^4*f*x^3 - 2*a^3*b*x^3*e
 + 6*a^2*b^2*c - a^3*b*d - a^4*e)/((b*x^3 + a)^2*a^3*b^2)
```

Mupad [B]
time = 0.18, size = 123, normalized size = 1.08

$$\frac{\frac{3\,f\,a^{3}-e\,a^{2}\,b-d\,a\,b^{2}+3\,c\,b^{3}}{6\,a\,b^{3}}+\frac{x^{3}\,\left(2\,f\,a^{3}-e\,a^{2}\,b+c\,b^{3}\right)}{3\,a^{2}\,b^{2}}}{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}+\frac{c\,\ln\left(x\right)}{a^{3}}-\frac{\ln\left(b\,x^{3}+a\right)\,\left(b^{3}\,c-a^{3}\,f\right)}{3\,a^{3}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x)
```

```
[Out] ((3*b^3*c + 3*a^3*f - a*b^2*d - a^2*b*e)/(6*a*b^3) + (x^3*(b^3*c + 2*a^3*f
- a^2*b*e))/(3*a^2*b^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (c*log(x))/a^3 - (lo
g(a + b*x^3)*(b^3*c - a^3*f))/(3*a^3*b^3)
```

3.282 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3} dx$

Optimal. Leaf size=134

$$-\frac{c}{3a^3x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{6a^2b^2(a+bx^3)^2}-\frac{2b^3c-ab^2d+a^3f}{3a^3b^2(a+bx^3)}-\frac{(3bc-ad)\log(x)}{a^4}+\frac{(3bc-ad)\log(a+bx^3)}{3a^4}$$

```
[Out] -1/3*c/a^3/x^3+1/6*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^2/b^2/(b*x^3+a)^2+1/3*(-
a^3*f+a*b^2*d-2*b^3*c)/a^3/b^2/(b*x^3+a)-(-a*d+3*b*c)*ln(x)/a^4+1/3*(-a*d+3
*b*c)*ln(b*x^3+a)/a^4
```

Rubi [A]
time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{(3bc-ad)\log(a+bx^3)}{3a^4}-\frac{\log(x)(3bc-ad)}{a^4}-\frac{a^3f-ab^2d+2b^3c}{3a^3b^2(a+bx^3)}-\frac{c}{3a^3x^3}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^2b^2(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -1/3*c/(a^3*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^2*b^2*(a + b*x^
3)^2) - (2*b^3*c - a*b^2*d + a^3*f)/(3*a^3*b^2*(a + b*x^3)) - ((3*b*c - a*d
)*Log[x])/a^4 + ((3*b*c - a*d)*Log[a + b*x^3])/(3*a^4)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^3}dx = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx)^3}dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x^2}+\frac{-3bc+ad}{a^4x}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx)^3}+\frac{2b^3c-ab^2d+a}{a^3b(a+bx)^2}\right.\right.$$
$$= -\frac{c}{3a^3x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{6a^2b^2\left(a+bx^3\right)^2}-\frac{2b^3c-ab^2d+a^3f}{3a^3b^2\left(a+bx^3\right)}-\frac{(3bc-ad)\log(x)}{a^4}+$$

Mathematica [A]
time = 0.06, size = 121, normalized size = 0.90

$$\frac{-\frac{2ac}{x^3}+\frac{a^2\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^2(a+bx^3)^2}-\frac{2a(2b^3c-ab^2d+a^3f)}{b^2(a+bx^3)}+6(-3bc+ad)\log(x)+2(3bc-ad)\log\left(a+bx^3\right)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a*c)/x^3 + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2*(a + b*x^
3)^2) - (2*a*(2*b^3*c - a*b^2*d + a^3*f))/(b^2*(a + b*x^3)) + 6*(-3*b*c + a
*d)*Log[x] + 2*(3*b*c - a*d)*Log[a + b*x^3])/(6*a^4)
```

Maple [A]
time = 0.35, size = 125, normalized size = 0.93

method	result	size
norman	$\frac{-\frac{c}{3a}+\frac{\left(a^2e-2abd+6b^2c\right)x^6}{3a^3}+\frac{\left(a^3f+a^2be-3a\,b^2d+9b^3c\right)x^9}{6a^4}}{x^3\left(b\,x^3+a\right)^2}+\frac{(ad-3bc)\ln(x)}{a^4}-\frac{(ad-3bc)\ln\left(b\,x^3+a\right)}{3a^4}$	115
default	$\frac{-\frac{a\left(a^3f-a\,b^2d+2b^3c\right)}{b^2\left(b\,x^3+a\right)}+\frac{a^2\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{2b^2\left(b\,x^3+a\right)^2}+(-ad+3bc)\ln\left(b\,x^3+a\right)}{3a^4}-\frac{c}{3a^3x^3}+\frac{(ad-3bc)\ln(x)}{a^4}$	125
risch	$\frac{-\frac{\left(a^3f-a\,b^2d+3b^3c\right)x^6}{3a^3b}-\frac{\left(a^3f+a^2be-3a\,b^2d+9b^3c\right)x^3}{6a^2b^2}-\frac{c}{3a}}{x^3\left(b\,x^3+a\right)^2}+\frac{d\ln(x)}{a^3}-\frac{3bc\ln(x)}{a^4}-\frac{d\ln\left(b\,x^3+a\right)}{3a^3}+\frac{bc\ln\left(b\,x^3+a\right)}{a^4}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^4*(-a*(a^3*f-a*b^2*d+2*b^3*c)/b^2/(b*x^3+a)+1/2*a^2*(a^3*f-a^2*b*e+a*
b^2*d-b^3*c)/b^2/(b*x^3+a)^2+(-a*d+3*b*c)*ln(b*x^3+a))-1/3*c/a^3/x^3+(a*d-3
*b*c)/a^4*ln(x)
```

Maxima [A]
time = 0.27, size = 145, normalized size = 1.08

$$-\frac{2\left(3\,b^4c-ab^3d+a^3bf\right)x^6+2\,a^2b^2c+\left(9\,ab^3c-3\,a^2b^2d+a^4f+a^3be\right)x^3}{6\left(a^3b^4x^9+2\,a^4b^3x^6+a^5b^2x^3\right)}+\frac{(3\,bc-ad)\log\left(bx^3+a\right)}{3\,a^4}-\frac{(3\,bc-ad)\log\left(x^3\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(3*b^4*c - a*b^3*d + a^3*b*f)*x^6 + 2*a^2*b^2*c + (9*a*b^3*c - 3*a^
2*b^2*d + a^4*f + a^3*b*e)*x^3)/(a^3*b^4*x^9 + 2*a^4*b^3*x^6 + a^5*b^2*x^3)
 + 1/3*(3*b*c - a*d)*log(b*x^3 + a)/a^4 - 1/3*(3*b*c - a*d)*log(x^3)/a^4
```

Fricas [A]
time = 0.40, size = 250, normalized size = 1.87

$$-\frac{2\,(3\,ab^4c-a^2b^3d+a^4bf)x^6+2\,a^3b^2c+(9\,a^2b^3c-3\,a^3b^2d+a^4be+a^5f)x^3-2\,((3\,b^5c-ab^4d)x^9+2\,(3\,ab^4c-a^2b^3d)x^6+(3\,a^2b^3c-a^3b^2d)x^3)\log{(bx^3+a)}+6\,((3\,b^5c-ab^4d)x^9+2\,(3\,ab^4c-a^2b^3d)x^6+(3\,a^2b^3c-a^3b^2d)x^3)\log{(x)}}{6\,(a^4b^4x^9+2\,a^5b^3x^6+a^6b^2x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(3*a*b^4*c - a^2*b^3*d + a^4*b*f)*x^6 + 2*a^3*b^2*c + (9*a^2*b^3*c
- 3*a^3*b^2*d + a^4*b*e + a^5*f)*x^3 - 2*((3*b^5*c - a*b^4*d)*x^9 + 2*(3*a*
b^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c - a^3*b^2*d)*x^3)*log(b*x^3 + a) + 6*
((3*b^5*c - a*b^4*d)*x^9 + 2*(3*a*b^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c - a
^3*b^2*d)*x^3)*log(x))/(a^4*b^4*x^9 + 2*a^5*b^3*x^6 + a^6*b^2*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 173, normalized size = 1.29

$$-\frac{(3\,bc-ad)\log{(|x|)}}{a^4}+\frac{(3\,b^2c-abd)\log{(|bx^3+a|)}}{3\,a^4b}+\frac{3\,bcx^3-adx^3-ac}{3\,a^4x^3}-\frac{9\,b^5cx^6-3\,ab^4dx^6+22\,ab^4cx^3-8\,a^2b^3dx^3+2\,a^4bfx^3+14\,a^2b^3c-6\,a^3b^2d+a^5f+a^4be}{6\,(bx^3+a)^2a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -(3*b*c - a*d)*log(abs(x))/a^4 + 1/3*(3*b^2*c - a*b*d)*log(abs(b*x^3 + a))/
(a^4*b) + 1/3*(3*b*c*x^3 - a*d*x^3 - a*c)/(a^4*x^3) - 1/6*(9*b^5*c*x^6 - 3*
a*b^4*d*x^6 + 22*a*b^4*c*x^3 - 8*a^2*b^3*d*x^3 + 2*a^4*b*f*x^3 + 14*a^2*b^3
*c - 6*a^3*b^2*d + a^5*f + a^4*b*e)/((b*x^3 + a)^2*a^4*b^2)
```

Mupad [B]
time = 5.07, size = 135, normalized size = 1.01

$$\frac{\ln\left(x\right)\,\left(a\,d-3\,b\,c\right)}{a^4}-\frac{\ln\left(b\,x^3+a\right)\,\left(a\,d-3\,b\,c\right)}{3\,a^4}-\frac{\frac{c}{3\,a}+\frac{x^6\,\left(f\,a^3-d\,a\,b^2+3\,c\,b^3\right)}{3\,a^3\,b}+\frac{x^3\,\left(f\,a^3+e\,a^2\,b-3\,d\,a\,b^2+9\,c\,b^3\right)}{6\,a^2\,b^2}}{a^2\,x^3+2\,a\,b\,x^6+b^2\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x)
```

```
[Out] (log(x)*(a*d - 3*b*c))/a^4 - (log(a + b*x^3)*(a*d - 3*b*c))/(3*a^4) - (c/(3
*a) + (x^6*(3*b^3*c + a^3*f - a*b^2*d))/(3*a^3*b) + (x^3*(9*b^3*c + a^3*f -
 3*a*b^2*d + a^2*b*e))/(6*a^2*b^2))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6)
```

3.283 $\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=163

$$-\frac{c}{6a^3x^6}+\frac{3bc-ad}{3a^4x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{6a^3b\left(a+bx^3\right)^2}+\frac{3b^2c-2abd+a^2e}{3a^4\left(a+bx^3\right)}+\frac{\left(6b^2c-3abd+a^2e\right)\log(x)}{a^5}-\frac{(6b^2c-3ab}{}$$

```
[Out] -1/6*c/a^3/x^6+1/3*(-a*d+3*b*c)/a^4/x^3+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/
a^3/b/(b*x^3+a)^2+1/3*(a^2*e-2*a*b*d+3*b^2*c)/a^4/(b*x^3+a)+(a^2*e-3*a*b*d+
6*b^2*c)*ln(x)/a^5-1/3*(a^2*e-3*a*b*d+6*b^2*c)*ln(b*x^3+a)/a^5
```

Rubi [A]
time = 0.13, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\frac{3bc-ad}{3a^4x^3}-\frac{c}{6a^3x^6}-\frac{\log\left(a+bx^3\right)\left(a^2e-3abd+6b^2c\right)}{3a^5}+\frac{\log(x)\left(a^2e-3abd+6b^2c\right)}{a^5}+\frac{a^2e-2abd+3b^2c}{3a^4\left(a+bx^3\right)}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^3b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] -1/6*c/(a^3*x^6) + (3*b*c - a*d)/(3*a^4*x^3) + (b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)/(6*a^3*b*(a + b*x^3)^2) + (3*b^2*c - 2*a*b*d + a^2*e)/(3*a^4*(a + b
*x^3)) + ((6*b^2*c - 3*a*b*d + a^2*e)*Log[x])/a^5 - ((6*b^2*c - 3*a*b*d + a
^2*e)*Log[a + b*x^3])/(3*a^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx)^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x^3}+\frac{-3bc+ad}{a^4x^2}+\frac{6b^2c-3abd+a^2e}{a^5x}+\frac{-b^3c+ab^2d-a^2be+}{a^3(a+bx)^3}\right.\right.$$
$$= -\frac{c}{6a^3x^6}+\frac{3bc-ad}{3a^4x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{6a^3b\left(a+bx^3\right)^2}+\frac{3b^2c-2abd+a^2e}{3a^4\left(a+bx^3\right)}+\frac{(6b^2c-}{}$$

Mathematica [A]
time = 0.08, size = 149, normalized size = 0.91

$$\frac{-\frac{a^2c}{x^6}-\frac{2a(-3bc+ad)}{x^3}+\frac{a^2(b^3c-ab^2d+a^2be-a^3f)}{b(a+bx^3)^2}+\frac{2a(3b^2c-2abd+a^2e)}{a+bx^3}+6(6b^2c-3abd+a^2e)\log(x)-2(6b^2c-3abd+a^2e)\log\left(a+bx^3\right)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] (-((a^2*c)/x^6) - (2*a*(-3*b*c + a*d))/x^3 + (a^2*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f))/(b*(a + b*x^3)^2) + (2*a*(3*b^2*c - 2*a*b*d + a^2*e))/(a + b*x^
3) + 6*(6*b^2*c - 3*a*b*d + a^2*e)*Log[x] - 2*(6*b^2*c - 3*a*b*d + a^2*e)*L
og[a + b*x^3])/(6*a^5)
```

Maple [A]
time = 0.37, size = 153, normalized size = 0.94

method	result
default	$\frac{\frac{a\left(a^2e-2abd+3b^2c\right)}{b\,x^3+a}-\frac{a^2\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{2b\left(b\,x^3+a\right)^2}+\left(-a^2e+3abd-6b^2c\right)\ln\left(b\,x^3+a\right)}{3a^5}-\frac{c}{6a^3x^6}-\frac{ad-3bc}{3a^4x^3}+\frac{\left(a^2e-3abd+6b^2c\right)\ln(x)}{a^5}$
norman	$\frac{-\frac{c}{6a}-\frac{(ad-2bc)x^3}{3a^2}+\frac{\left(a^3f-2a^2be+6a\,b^2d-12b^3c\right)x^9}{3a^4}+\frac{b\left(a^3f-3a^2be+9a\,b^2d-18b^3c\right)x^{12}}{6a^5}}{x^6\left(b\,x^3+a\right)^2}+\frac{\left(a^2e-3abd+6b^2c\right)\ln(x)}{a^5}-\frac{\left(a^2e-3abd+6b^2c\right)}{3a^5}$
risch	$\frac{\frac{b\left(a^2e-3abd+6b^2c\right)x^9}{3a^4}-\frac{\left(a^3f-3a^2be+9a\,b^2d-18b^3c\right)x^6}{6a^3b}-\frac{(ad-2bc)x^3}{3a^2}-\frac{c}{6a}}{x^6\left(b\,x^3+a\right)^2}+\frac{e\ln(x)}{a^3}-\frac{3\ln(x)bd}{a^4}+\frac{6\ln(x)b^2c}{a^5}-\frac{e\ln\left(b\,x^3+a\right)}{3a^3}+\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^5*(a*(a^2*e-2*a*b*d+3*b^2*c)/(b*x^3+a)-1/2*a^2*(a^3*f-a^2*b*e+a*b^2*d
-b^3*c)/b/(b*x^3+a)^2+(-a^2*e+3*a*b*d-6*b^2*c)*ln(b*x^3+a))-1/6*c/a^3/x^6-1
/3*(a*d-3*b*c)/a^4/x^3+(a^2*e-3*a*b*d+6*b^2*c)*ln(x)/a^5
```

Maxima [A]
time = 0.28, size = 186, normalized size = 1.14

$$\frac{2\,(6\,b^4c-3\,ab^3d+a^2b^2e)x^9+(18\,ab^3c-9\,a^2b^2d-a^4f+3\,a^3be)x^6-a^3bc+2\,(2\,a^2b^2c-a^3bd)x^3}{6\,(a^4b^3x^{12}+2\,a^5b^2x^9+a^6bx^6)}-\frac{(6\,b^2c-3\,abd+a^2e)\log\left(bx^3+a\right)}{3\,a^5}+\frac{(6\,b^2c-3\,abd+a^2e)\log\left(x^3\right)}{3\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(6*b^4*c - 3*a*b^3*d + a^2*b^2*e)*x^9 + (18*a*b^3*c - 9*a^2*b^2*d -
a^4*f + 3*a^3*b*e)*x^6 - a^3*b*c + 2*(2*a^2*b^2*c - a^3*b*d)*x^3)/(a^4*b^3*
x^12 + 2*a^5*b^2*x^9 + a^6*b*x^6) - 1/3*(6*b^2*c - 3*a*b*d + a^2*e)*log(b*x
^3 + a)/a^5 + 1/3*(6*b^2*c - 3*a*b*d + a^2*e)*log(x^3)/a^5
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(153) = 306$.
time = 0.40, size = 316, normalized size = 1.94

$$\frac{2\,(6\,ab^4c-3\,a^2b^3d+a^3b^2e)x^9+(18\,a^2b^3c-9\,a^3b^2d+3\,a^4be-a^5f)x^6-a^4bc+2\,(2\,a^3b^2c-a^4bd)x^3-2\,((6\,b^5c-3\,ab^4d+a^2b^3e)x^{12}+2\,(6\,ab^4c-3\,a^2b^3d+a^3b^2e)x^9+(6\,a^2b^3c-3\,a^3b^2d+a^4be)x^6)\log\left(bx^3+a\right)+6\,((6\,b^5c-3\,ab^4d+a^2b^3e)x^{12}+2\,(6\,ab^4c-3\,a^2b^3d+a^3b^2e)x^9+(6\,a^2b^3c-3\,a^3b^2d+a^4be)x^6)\log\left(x\right)}{6\,(a^5b^3x^{12}+2\,a^6b^2x^9+a^7bx^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e)*x^9 + (18*a^2*b^3*c - 9*a^3*b^
2*d + 3*a^4*b*e - a^5*f)*x^6 - a^4*b*c + 2*(2*a^3*b^2*c - a^4*b*d)*x^3 - 2*
((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^12 + 2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*
b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*log(b*x^3 + a) + 6*
((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^12 + 2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*
b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*log(x))/(a^5*b^3*x^
12 + 2*a^6*b^2*x^9 + a^7*b*x^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.56, size = 189, normalized size = 1.16

$$\frac{(6\,b^2c-3\,abd+a^2e)\log\left(|x|\right)}{a^5}-\frac{(6\,b^3c-3\,ab^2d+a^2be)\log\left(|bx^3+a|\right)}{3\,a^5b}+\frac{12\,b^4cx^9-6\,ab^3dx^9+2\,a^2b^2x^9e+18\,ab^3cx^6-9\,a^2b^2dx^6-a^4fx^6+3\,a^3bx^6e+4\,a^2b^2cx^3-2\,a^3bdx^3-a^3bc}{6\,(bx^6+ax^3)^2a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (6*b^2*c - 3*a*b*d + a^2*e)*log(abs(x))/a^5 - 1/3*(6*b^3*c - 3*a*b^2*d + a^
2*b*e)*log(abs(b*x^3 + a))/(a^5*b) + 1/6*(12*b^4*c*x^9 - 6*a*b^3*d*x^9 + 2*
```

```
a^2*b^2*x^9*e + 18*a*b^3*c*x^6 - 9*a^2*b^2*d*x^6 - a^4*f*x^6 + 3*a^3*b*x^6*
e + 4*a^2*b^2*c*x^3 - 2*a^3*b*d*x^3 - a^3*b*c)/((b*x^6 + a*x^3)^2*a^4*b)
```

Mupad [B]
time = 5.10, size = 167, normalized size = 1.02

$$\frac{\ln(x)\,\left(e\,a^2-3\,d\,a\,b+6\,c\,b^2\right)}{a^5}-\frac{\ln\left(b\,x^3+a\right)\,\left(e\,a^2-3\,d\,a\,b+6\,c\,b^2\right)}{3\,a^5}-\frac{\frac{c}{6\,a}+\frac{x^3\,(a\,d-2\,b\,c)}{3\,a^2}-\frac{b\,x^9\,\left(e\,a^2-3\,d\,a\,b+6\,c\,b^2\right)}{3\,a^4}-\frac{x^6\,\left(-f\,a^3+3\,e\,a^2\,b-9\,d\,a\,b^2+18\,c\,b^3\right)}{6\,a^3\,b}}{a^2\,x^6+2\,a\,b\,x^9+b^2\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x)
```

```
[Out] (log(x)*(6*b^2*c + a^2*e - 3*a*b*d))/a^5 - (log(a + b*x^3)*(6*b^2*c + a^2*e
 - 3*a*b*d))/(3*a^5) - (c/(6*a) + (x^3*(a*d - 2*b*c))/(3*a^2) - (b*x^9*(6*b
^2*c + a^2*e - 3*a*b*d))/(3*a^4) - (x^6*(18*b^3*c - a^3*f - 9*a*b^2*d + 3*a
^2*b*e))/(6*a^3*b))/(a^2*x^6 + b^2*x^12 + 2*a*b*x^9)
```

3.284 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^3}\,dx$

Optimal. Leaf size=218

$$-\frac{c}{9a^3x^9}+\frac{3bc-ad}{6a^4x^6}-\frac{6b^2c-3abd+a^2e}{3a^5x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{6a^4\left(a+bx^3\right)^2}-\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5\left(a+bx^3\right)}-\frac{(10b^3c-6$$

```
[Out] -1/9*c/a^3/x^9+1/6*(-a*d+3*b*c)/a^4/x^6+1/3*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x^
3+1/6*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/(b*x^3+a)^2+1/3*(a^3*f-2*a^2*b*e+3*
a*b^2*d-4*b^3*c)/a^5/(b*x^3+a)-(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)*ln(x)/
a^6+1/3*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)*ln(b*x^3+a)/a^6
```

Rubi [A]
time = 0.18, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{3bc-ad}{6a^4x^6}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+3a^2be-6ab^2d+10b^3c\right)}{3a^6}-\frac{\log(x)\left(a^3(-f)+3a^2be-6ab^2d+10b^3c\right)}{a^6}-\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5\left(a+bx^3\right)}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^4\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x]
```

```
[Out] -1/9*c/(a^3*x^9) + (3*b*c - a*d)/(6*a^4*x^6) - (6*b^2*c - 3*a*b*d + a^2*e)/
(3*a^5*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*(a + b*x^3)^2) - (
4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*(a + b*x^3)) - ((10*b^3*c -
 6*a*b^2*d + 3*a^2*b*e - a^3*f)*Log[x])/a^6 + ((10*b^3*c - 6*a*b^2*d + 3*a^
2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx)^3}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x^4} + \frac{-3bc+ad}{a^4x^3} + \frac{6b^2c-3abd+a^2e}{a^5x^2} + \frac{-10b^3c+6ab^2d-3a^2}{a^6x}\right.\right.$$

$$= -\frac{c}{9a^3x^9} + \frac{3bc-ad}{6a^4x^6} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} - \frac{b^3c-ab^2d+a^2be-a^3f}{6a^4\left(a+bx^3\right)^2} - \frac{4b^3c-}{}$$

Mathematica [A]
time = 0.10, size = 200, normalized size = 0.92

$$\frac{-\frac{2a^3c}{x^9} - \frac{3a^2(-3bc+ad)}{x^6} - \frac{6a(6b^2c-3abd+a^2e)}{x^3} + \frac{3a^2(-b^3c+ab^2d-a^2be+a^3f)}{(a+bx^3)^2} + \frac{6a(-4b^3c+3ab^2d-2a^2be+a^3f)}{a+bx^3} + 18(-10b^3c+6ab^2d-3a^2be+a^3f)\log(x) + 6(10b^3c-6ab^2d+3a^2be-a^3f)\log\left(a+bx^3\right)}{18a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a^3*c)/x^9 - (3*a^2*(-3*b*c + a*d))/x^6 - (6*a*(6*b^2*c - 3*a*b*d + a^
2*e))/x^3 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 +
(6*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/(a + b*x^3) + 18*(-10*b^3*
c + 6*a*b^2*d - 3*a^2*b*e + a^3*f)*Log[x] + 6*(10*b^3*c - 6*a*b^2*d + 3*a^2
*b*e - a^3*f)*Log[a + b*x^3])/(18*a^6)
```

Maple [A]
time = 0.37, size = 212, normalized size = 0.97

method	result
default	$-\frac{b\left(-\frac{a\left(a^3f-2a^2be+3a\,b^2d-4b^3c\right)}{b\left(b\,x^3+a\right)}-\frac{a^2\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{2b\left(b\,x^3+a\right)^2}+\frac{\left(a^3f-3a^2be+6a\,b^2d-10b^3c\right)\ln\left(b\,x^3+a\right)}{b}\right)}{3a^6}-\frac{c}{9a^3x^9}-\frac{ad-3bc}{6a^4x^6}-\frac{a^2e}{}$
norman	$\frac{-\frac{c}{9a}-\frac{(3ad-5bc)x^3}{18a^2}-\frac{\left(3a^2e-6abd+10b^2c\right)x^6}{9a^3}+\frac{\left(a^3b^2f-3a^2b^3e+6a\,b^4d-10b^5c\right)x^9}{2a^4b^2}+\frac{\left(a^3b^2f-3a^2b^3e+6a\,b^4d-10b^5c\right)x^{12}}{3a^5b}}{x^9\left(b\,x^3+a\right)^2}+\frac{\left(a^3f-3a^2be+6a\right.}{a}$
risch	$\frac{-\frac{c}{9a}-\frac{(3ad-5bc)x^3}{18a^2}-\frac{\left(3a^2e-6abd+10b^2c\right)x^6}{9a^3}+\frac{\left(a^3f-3a^2be+6a\,b^2d-10b^3c\right)x^9}{2a^4}+\frac{b\left(a^3f-3a^2be+6a\,b^2d-10b^3c\right)x^{12}}{3a^5}}{x^9\left(b\,x^3+a\right)^2}+\frac{\ln(x)f}{a^3}-\frac{3\ln(x)be}{a^4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b/a^6*(-a*(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/b/(b*x^3+a)-1/2*a^2*(a^3
*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)^2+(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c
)/b*ln(b*x^3+a))-1/9*c/a^3/x^9-1/6*(a*d-3*b*c)/a^4/x^6-1/3*(a^2*e-3*a*b*d+6
*b^2*c)/a^5/x^3+(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6*ln(x)
```

Maxima [A]
time = 0.28, size = 237, normalized size = 1.09

$$-\frac{6\,(10\,b^4c - 6\,ab^3d - a^3bf + 3\,a^2b^2e)x^{12} + 9\,(10\,ab^3c - 6\,a^2b^2d - a^4f + 3\,a^3be)x^9 + 2\,(10\,a^2b^2c - 6\,a^3bd + 3\,a^4e)x^6 + 2\,a^4c - (5\,a^3bc - 3\,a^4d)x^3}{18\,(a^5b^2x^{15} + 2\,a^6bx^{12} + a^7x^9)} + \frac{(10\,b^3c - 6\,ab^2d - a^3f + 3\,a^2be)\log\left(bx^3 + a\right)}{3\,a^6} - \frac{(10\,b^3c - 6\,ab^2d - a^3f + 3\,a^2be)\log\left(x^3\right)}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(6*(10*b^4*c - 6*a*b^3*d - a^3*b*f + 3*a^2*b^2*e)*x^12 + 9*(10*a*b^3*
c - 6*a^2*b^2*d - a^4*f + 3*a^3*b*e)*x^9 + 2*(10*a^2*b^2*c - 6*a^3*b*d + 3*
a^4*e)*x^6 + 2*a^4*c - (5*a^3*b*c - 3*a^4*d)*x^3)/(a^5*b^2*x^15 + 2*a^6*b*x
^12 + a^7*x^9) + 1/3*(10*b^3*c - 6*a*b^2*d - a^3*f + 3*a^2*b*e)*log(b*x^3 +
 a)/a^6 - 1/3*(10*b^3*c - 6*a*b^2*d - a^3*f + 3*a^2*b*e)*log(x^3)/a^6
```

Fricas [A]
time = 0.42, size = 396, normalized size = 1.82

$$-\frac{6\,(10\,ab^4c - 6\,a^2b^3d + 3\,a^3b^2e - a^4bf)x^{12} + 9\,(10\,a^2b^3c - 6\,a^3b^2d + 3\,a^4be - a^5f)x^9 + 2\,(10\,a^3b^2c - 6\,a^4bd + 3\,a^5e)x^6 + 2\,a^5c - (5\,a^4bc - 3\,a^5d)x^3 - 6\,((10\,b^5c - 6\,ab^4d + 3\,a^2b^3e - a^3b^2f)x^{15} + 2\,(10\,ab^4c - 6\,a^2b^3d + 3\,a^3b^2e - a^4bf)x^{12} + (10\,a^2b^3c - 6\,a^3b^2d + 3\,a^4be - a^5f)x^9)\log\left(bx^3 + a\right) + 18\,((10\,b^5c - 6\,ab^4d + 3\,a^2b^3e - a^3b^2f)x^{15} + 2\,(10\,ab^4c - 6\,a^2b^3d + 3\,a^3b^2e - a^4bf)x^{12} + (10\,a^2b^3c - 6\,a^3b^2d + 3\,a^4be - a^5f)x^9)\log\left(x\right)}{18\,(a^6b^2x^{15} + 2\,a^7bx^{12} + a^8x^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/18*(6*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^12 + 9*(10*a^
2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^9 + 2*(10*a^3*b^2*c - 6*a^4*b*
d + 3*a^5*e)*x^6 + 2*a^5*c - (5*a^4*b*c - 3*a^5*d)*x^3 - 6*((10*b^5*c - 6*a
*b^4*d + 3*a^2*b^3*e - a^3*b^2*f)*x^15 + 2*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^
3*b^2*e - a^4*b*f)*x^12 + (10*a^2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*
x^9)*log(b*x^3 + a) + 18*((10*b^5*c - 6*a*b^4*d + 3*a^2*b^3*e - a^3*b^2*f)*
x^15 + 2*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^12 + (10*a^2*
b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^9)*log(x))/(a^6*b^2*x^15 + 2*a^7
*b*x^12 + a^8*x^9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 324, normalized size = 1.49

$$-\frac{(10\,b^3c - 6\,ab^2d - a^3f + 3\,a^2be)\log\left(|x|\right)}{a^6} + \frac{(10\,b^4c - 6\,ab^3d - a^3bf + 3\,a^2b^2e)\log\left(|bx^3 + a|\right)}{3\,a^6b} - \frac{30\,b^5cx^6 - 18\,ab^4dx^6 - 3\,a^3b^2fx^6 + 9\,a^2b^3x^6e + 68\,ab^4cx^3 - 42\,a^2b^3dx^3 - 8\,a^4bfx^3 + 22\,a^3b^2x^3e + 39\,a^2b^3c - 25\,a^3b^2d - 6\,a^5f + 14\,a^4be}{6\,(bx^3 + a)^2a^6} + \frac{110\,b^3cx^9 - 66\,ab^2dx^9 - 11\,a^3fx^9 + 33\,a^2bx^9e - 36\,ab^2cx^6 + 18\,a^2bdx^6 - 6\,a^3x^6e + 9\,a^2bcx^3 - 3\,a^3dx^3 - 2\,a^3c}{18\,a^6x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -(10*b^3*c - 6*a*b^2*d - a^3*f + 3*a^2*b*e)*log(abs(x))/a^6 + 1/3*(10*b^4*c
 - 6*a*b^3*d - a^3*b*f + 3*a^2*b^2*e)*log(abs(b*x^3 + a))/(a^6*b) - 1/6*(30
*b^5*c*x^6 - 18*a*b^4*d*x^6 - 3*a^3*b^2*f*x^6 + 9*a^2*b^3*x^6*e + 68*a*b^4*
c*x^3 - 42*a^2*b^3*d*x^3 - 8*a^4*b*f*x^3 + 22*a^3*b^2*x^3*e + 39*a^2*b^3*c
- 25*a^3*b^2*d - 6*a^5*f + 14*a^4*b*e)/((b*x^3 + a)^2*a^6) + 1/18*(110*b^3*
c*x^9 - 66*a*b^2*d*x^9 - 11*a^3*f*x^9 + 33*a^2*b*x^9*e - 36*a*b^2*c*x^6 + 1
8*a^2*b*d*x^6 - 6*a^3*x^6*e + 9*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^6*x
^9)
```

Mupad [B]
time = 5.17, size = 222, normalized size = 1.02

$$\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{3\,a^6}-\frac{\frac{c}{9\,a}+\frac{x^9\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{2\,a^4}+\frac{x^3\,\left(3\,a\,d-5\,b\,c\right)}{18\,a^2}+\frac{x^6\,\left(3\,e\,a^2-6\,d\,a\,b+10\,c\,b^2\right)}{9\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{3\,a^5}}{a^2\,x^9+2\,a\,b\,x^{12}+b^2\,x^{15}}-\frac{\ln\left(x\right)\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x)
```

```
[Out] (log(a + b*x^3)*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/(3*a^6) - (c/(9
*a) + (x^9*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/(2*a^4) + (x^3*(3*a*
d - 5*b*c))/(18*a^2) + (x^6*(10*b^2*c + 3*a^2*e - 6*a*b*d))/(9*a^3) + (b*x^
12*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/(3*a^5))/(a^2*x^9 + b^2*x^15
 + 2*a*b*x^12) - (log(x)*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/a^6
```

3.285 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^3}\,dx$

Optimal. Leaf size=258

$$-\frac{c}{12a^3x^{12}}+\frac{3bc-ad}{9a^4x^9}-\frac{6b^2c-3abd+a^2e}{6a^5x^6}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3}+\frac{b(b^3c-ab^2d+a^2be-a^3f)}{6a^5\left(a+bx^3\right)^2}+\frac{b(5b$$

```
[Out] -1/12*c/a^3/x^12+1/9*(-a*d+3*b*c)/a^4/x^9+1/6*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/
x^6+1/3*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x^3+1/6*b*(-a^3*f+a^2*b*e
-a*b^2*d+b^3*c)/a^5/(b*x^3+a)^2+1/3*b*(-2*a^3*f+3*a^2*b*e-4*a*b^2*d+5*b^3*c
)/a^6/(b*x^3+a)+b*(-3*a^3*f+6*a^2*b*e-10*a*b^2*d+15*b^3*c)*ln(x)/a^7-1/3*b*
(-3*a^3*f+6*a^2*b*e-10*a*b^2*d+15*b^3*c)*ln(b*x^3+a)/a^7
```

Rubi [A]
time = 0.21, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\frac{3bc-ad}{9a^4x^9}-\frac{c}{12a^3x^{12}}-\frac{a^2e-3abd+6b^2c}{6a^5x^6}-\frac{b\log\left(a+bx^3\right)\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{3a^7}+\frac{b\log(x)\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{a^7}+\frac{b(-2a^3f+3a^2be-4ab^2d+5b^3c)}{3a^6\left(a+bx^3\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{3a^6x^3}+\frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]
```

```
[Out] -1/12*c/(a^3*x^12) + (3*b*c - a*d)/(9*a^4*x^9) - (6*b^2*c - 3*a*b*d + a^2*e
)/(6*a^5*x^6) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) + (b
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*a^5*(a + b*x^3)^2) + (b*(5*b^3*c -
 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(3*a^6*(a + b*x^3)) + (b*(15*b^3*c - 10*
a*b^2*d + 6*a^2*b*e - 3*a^3*f)*Log[x])/a^7 - (b*(15*b^3*c - 10*a*b^2*d + 6*
a^2*b*e - 3*a^3*f)*Log[a + b*x^3])/(3*a^7)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5(a+bx)^3}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x^5} + \frac{-3bc+ad}{a^4x^4} + \frac{6b^2c-3abd+a^2e}{a^5x^3} + \frac{-10b^3c+6ab^2d-3a^2}{a^6x^2}\right.\right.$$

$$= -\frac{c}{12a^3x^{12}} + \frac{3bc-ad}{9a^4x^9} - \frac{6b^2c-3abd+a^2e}{6a^5x^6} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3} +$$

Mathematica [A]
time = 0.13, size = 238, normalized size = 0.92

$$\frac{-\frac{a(-180b^5cx^{15}+30ab^4x^{12}(-9c+4dx^3)-12a^2b^3x^9(5c-15dx^3+6ex^6)-2a^4bx^3(3c+5dx^3+12ex^6-27fx^9)+a^5(3c+4dx^3+6ex^6+12fx^9)+a^3b^2x^6(15c+40dx^3-108ex^6+36fx^9))}{x^{12}(a+bx^3)^2}+36b(15b^3c-10ab^2d+6a^2be-3a^3f)\log(x)+12b(-15b^3c+10ab^2d-6a^2be+3a^3f)\log(a+bx^3)}{36a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]
```

```
[Out] (-((a*(-180*b^5*c*x^15 + 30*a*b^4*x^12*(-9*c + 4*d*x^3) - 12*a^2*b^3*x^9*(5
*c - 15*d*x^3 + 6*e*x^6) - 2*a^4*b*x^3*(3*c + 5*d*x^3 + 12*e*x^6 - 27*f*x^9
) + a^5*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + a^3*b^2*x^6*(15*c + 40*d*x^3
 - 108*e*x^6 + 36*f*x^9)))/(x^12*(a + b*x^3)^2)) + 36*b*(15*b^3*c - 10*a*b^
2*d + 6*a^2*b*e - 3*a^3*f)*Log[x] + 12*b*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*
e + 3*a^3*f)*Log[a + b*x^3])/(36*a^7)
```

Maple [A]
time = 0.40, size = 253, normalized size = 0.98

method	result
default	$\frac{b^2\left(-\frac{a\left(2a^3f-3a^2be+4a\,b^2d-5b^3c\right)}{b\left(b\,x^3+a\right)}-\frac{a^2\left(a^3f-a^2be+a\,b^2d-b^3c\right)}{2b\left(b\,x^3+a\right)^2}+\frac{\left(3a^3f-6a^2be+10a\,b^2d-15b^3c\right)\ln\left(b\,x^3+a\right)}{b}\right)}{3a^7}-\frac{c}{12a^3x^{12}}-\frac{ad-3bc}{9a^4x^9}-$
norman	$\frac{-\frac{c}{12a}-\frac{(2ad-3bc)x^3}{18a^2}-\frac{\left(6a^2e-10abd+15b^2c\right)x^6}{36a^3}-\frac{\left(3a^3f-6a^2be+10a\,b^2d-15b^3c\right)x^9}{9a^4}+\frac{\left(-3a^3b^3f+6a^2b^4e-10a\,b^5d+15b^6c\right)x^{12}}{2a^5b^2}+\frac{\left(-3a^3b^3f+6a^2b\right.}{}}{x^{12}\left(b\,x^3+a\right)^2}$
risch	$\frac{-\frac{c}{12a}-\frac{(2ad-3bc)x^3}{18a^2}-\frac{\left(6a^2e-10abd+15b^2c\right)x^6}{36a^3}-\frac{\left(3a^3f-6a^2be+10a\,b^2d-15b^3c\right)x^9}{9a^4}-\frac{b\left(3a^3f-6a^2be+10a\,b^2d-15b^3c\right)x^{12}}{2a^5}-\frac{b^2\left(3a^3f-6a^2be+10a\right.}{3a^6}}{x^{12}\left(b\,x^3+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2/a^7*(-a*(2*a^3*f-3*a^2*b*e+4*a*b^2*d-5*b^3*c)/b/(b*x^3+a)-1/2*a^2*(
a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)^2+(3*a^3*f-6*a^2*b*e+10*a*b^2*d-15
*b^3*c)/b*ln(b*x^3+a))-1/12*c/a^3/x^12-1/9*(a*d-3*b*c)/a^4/x^9-1/6*(a^2*e-3
```

```
*a*b*d+6*b^2*c)/a^5/x^6-1/3*(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x^3-b*
(3*a^3*f-6*a^2*b*e+10*a*b^2*d-15*b^3*c)/a^7*ln(x)
```

Maxima [A]
time = 0.28, size = 286, normalized size = 1.11

$$\frac{12\,(15\,b^5c-10\,ab^4d-3\,a^3b^2f+6\,a^2b^3e)x^{15}+18\,(15\,ab^4c-10\,a^2b^3d-3\,a^4bf+6\,a^3b^2e)x^{12}+4\,(15\,a^2b^3c-10\,a^3b^2d-3\,a^5f+6\,a^4be)x^9-(15\,a^3b^2c-10\,a^4bd+6\,a^5e)x^6-3\,a^5c+2\,(3\,a^4bc-2\,a^5d)x^3}{36\,(a^6b^2x^{18}+2\,a^7bx^{15}+a^8x^{12})}-\frac{(15\,b^4c-10\,ab^3d-3\,a^3bf+6\,a^2b^2e)\log\left(bx^3+a\right)}{3\,a^7}+\frac{(15\,b^4c-10\,ab^3d-3\,a^3bf+6\,a^2b^2e)\log\left(x^3\right)}{3\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(12*(15*b^5*c - 10*a*b^4*d - 3*a^3*b^2*f + 6*a^2*b^3*e)*x^15 + 18*(15*
a*b^4*c - 10*a^2*b^3*d - 3*a^4*b*f + 6*a^3*b^2*e)*x^12 + 4*(15*a^2*b^3*c -
10*a^3*b^2*d - 3*a^5*f + 6*a^4*b*e)*x^9 - (15*a^3*b^2*c - 10*a^4*b*d + 6*a^
5*e)*x^6 - 3*a^5*c + 2*(3*a^4*b*c - 2*a^5*d)*x^3)/(a^6*b^2*x^18 + 2*a^7*b*x
^15 + a^8*x^12) - 1/3*(15*b^4*c - 10*a*b^3*d - 3*a^3*b*f + 6*a^2*b^2*e)*log
(b*x^3 + a)/a^7 + 1/3*(15*b^4*c - 10*a*b^3*d - 3*a^3*b*f + 6*a^2*b^2*e)*log
(x^3)/a^7
```

Fricas [A]
time = 0.44, size = 448, normalized size = 1.74

$$\frac{12\,(15\,ab^5c-10\,a^2b^4d+6\,a^3b^3e-3\,a^4b^2f)x^{15}+18\,(15\,a^2b^4c-10\,a^3b^3d+6\,a^4b^2e-3\,a^5bf)x^{12}+4\,(15\,a^3b^3c-10\,a^4b^2d+6\,a^5be-3\,a^6f)x^9-3\,a^6c-(15\,a^4b^2c-10\,a^5bd+6\,a^6e)x^6+2\,(3\,a^5bc-2\,a^6d)x^3-12\,((15\,b^6c-10\,ab^5d+6\,a^2b^4e-3\,a^3b^3f)x^{18}+2\,(15\,ab^5c-10\,a^2b^4d+6\,a^3b^3e-3\,a^4b^2f)x^{15}+(15\,a^2b^4c-10\,a^3b^3d+6\,a^4b^2e-3\,a^5bf)x^{12})\log\left(bx^3+a\right)+36\,((15\,b^6c-10\,ab^5d+6\,a^2b^4e-3\,a^3b^3f)x^{18}+2\,(15\,ab^5c-10\,a^2b^4d+6\,a^3b^3e-3\,a^4b^2f)x^{15}+(15\,a^2b^4c-10\,a^3b^3d+6\,a^4b^2e-3\,a^5bf)x^{12})\log\left(x\right)}{36\,(a^7b^2x^{18}+2\,a^8bx^{15}+a^9x^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/36*(12*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3*e - 3*a^4*b^2*f)*x^15 + 18*
(15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e - 3*a^5*b*f)*x^12 + 4*(15*a^3*b^
3*c - 10*a^4*b^2*d + 6*a^5*b*e - 3*a^6*f)*x^9 - 3*a^6*c - (15*a^4*b^2*c - 1
0*a^5*b*d + 6*a^6*e)*x^6 + 2*(3*a^5*b*c - 2*a^6*d)*x^3 - 12*((15*b^6*c - 10
*a*b^5*d + 6*a^2*b^4*e - 3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^4*d +
 6*a^3*b^3*e - 3*a^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2
*e - 3*a^5*b*f)*x^12)*log(b*x^3 + a) + 36*((15*b^6*c - 10*a*b^5*d + 6*a^2*b
^4*e - 3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3*e - 3*a
^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e - 3*a^5*b*f)*x^
12)*log(x))/(a^7*b^2*x^18 + 2*a^8*b*x^15 + a^9*x^12)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.64, size = 380, normalized size = 1.47

$$\frac{(15\,b^4c-10\,ab^3d-3\,a^3bf+6\,a^2b^2e)\log(|x|)}{a^7}-\frac{(15\,b^5c-10\,ab^4d-3\,a^3b^2f+6\,a^2b^3e)\log(|bx^3+a|)}{3\,a^7b}+\frac{45\,b^6cx^6-30\,ab^5dx^6-9\,a^3b^3fx^6+18\,a^2b^4x^6e+100\,ab^5cx^3-68\,a^2b^4dx^3-22\,a^4b^2fx^3+42\,a^3b^3x^3e+56\,a^2b^4c-39\,a^3b^3d-14\,a^5bf+25\,a^4b^2e}{6\,(bx^3+a)^2a^7}-\frac{375\,b^4cx^{12}-250\,ab^3dx^{12}-75\,a^3bfx^{12}+150\,a^2b^2x^{12}e-120\,ab^3cx^9+72\,a^2b^2dx^9+12\,a^4fx^9-36\,a^3bx^9e+36\,a^2b^2cx^6-18\,a^3bdx^6+6\,a^4x^6e-12\,a^3bcx^3+4\,a^4dx^3+3\,a^4c}{36\,a^7x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (15*b^4*c - 10*a*b^3*d - 3*a^3*b*f + 6*a^2*b^2*e)*log(abs(x))/a^7 - 1/3*(15
*b^5*c - 10*a*b^4*d - 3*a^3*b^2*f + 6*a^2*b^3*e)*log(abs(b*x^3 + a))/(a^7*b
) + 1/6*(45*b^6*c*x^6 - 30*a*b^5*d*x^6 - 9*a^3*b^3*f*x^6 + 18*a^2*b^4*x^6*e
 + 100*a*b^5*c*x^3 - 68*a^2*b^4*d*x^3 - 22*a^4*b^2*f*x^3 + 42*a^3*b^3*x^3*e
 + 56*a^2*b^4*c - 39*a^3*b^3*d - 14*a^5*b*f + 25*a^4*b^2*e)/((b*x^3 + a)^2*
a^7) - 1/36*(375*b^4*c*x^12 - 250*a*b^3*d*x^12 - 75*a^3*b*f*x^12 + 150*a^2*
b^2*x^12*e - 120*a*b^3*c*x^9 + 72*a^2*b^2*d*x^9 + 12*a^4*f*x^9 - 36*a^3*b*x
^9*e + 36*a^2*b^2*c*x^6 - 18*a^3*b*d*x^6 + 6*a^4*x^6*e - 12*a^3*b*c*x^3 + 4
*a^4*d*x^3 + 3*a^4*c)/(a^7*x^12)
```

Mupad [B]
time = 0.31, size = 265, normalized size = 1.03

$$\frac{\ln(x)\,\left(-3\,f\,a^3\,b+6\,e\,a^2\,b^2-10\,d\,a\,b^3+15\,c\,b^4\right)}{a^7}-\frac{\ln(b\,x^3+a)\,\left(-3\,f\,a^3\,b+6\,e\,a^2\,b^2-10\,d\,a\,b^3+15\,c\,b^4\right)}{3\,a^7}-\frac{\frac{c}{12\,a}-\frac{x^9\,\left(-3\,f\,a^3+6\,e\,a^2\,b-10\,d\,a\,b^2+15\,c\,b^3\right)}{9\,a^4}+\frac{x^3\,\left(2\,a\,d-3\,b\,c\right)}{18\,a^2}+\frac{x^6\,\left(6\,e\,a^2-10\,d\,a\,b+15\,c\,b^2\right)}{36\,a^3}-\frac{b\,x^{12}\,\left(-3\,f\,a^3+6\,e\,a^2\,b-10\,d\,a\,b^2+15\,c\,b^3\right)}{2\,a^5}-\frac{b^2\,x^{15}\,\left(-3\,f\,a^3+6\,e\,a^2\,b-10\,d\,a\,b^2+15\,c\,b^3\right)}{3\,a^6}}{a^2\,x^{12}+2\,a\,b\,x^{15}+b^2\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x)
```

```
[Out] (log(x)*(15*b^4*c + 6*a^2*b^2*e - 10*a*b^3*d - 3*a^3*b*f))/a^7 - (log(a + b
*x^3)*(15*b^4*c + 6*a^2*b^2*e - 10*a*b^3*d - 3*a^3*b*f))/(3*a^7) - (c/(12*a
) - (x^9*(15*b^3*c - 3*a^3*f - 10*a*b^2*d + 6*a^2*b*e))/(9*a^4) + (x^3*(2*a
*d - 3*b*c))/(18*a^2) + (x^6*(15*b^2*c + 6*a^2*e - 10*a*b*d))/(36*a^3) - (b
*x^12*(15*b^3*c - 3*a^3*f - 10*a*b^2*d + 6*a^2*b*e))/(2*a^5) - (b^2*x^15*(1
5*b^3*c - 3*a^3*f - 10*a*b^2*d + 6*a^2*b*e))/(3*a^6))/(a^2*x^12 + b^2*x^18
+ 2*a*b*x^15)
```

3.286 $\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=416

$$-\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x}{b^7}+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^4}{4b^6}+\frac{(b^2d-3abe+6a^2f)\,x^7}{7b^5}+\frac{(be-$$

```
[Out] -a*(-15*a^3*f+10*a^2*b*e-6*a*b^2*d+3*b^3*c)*x/b^7+1/4*(-10*a^3*f+6*a^2*b*e-
3*a*b^2*d+b^3*c)*x^4/b^6+1/7*(6*a^2*f-3*a*b*e+b^2*d)*x^7/b^5+1/10*(-3*a*f+b
*e)*x^10/b^4+1/13*f*x^13/b^3+1/6*a^3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^7/(
b*x^3+a)^2-1/18*a^2*(-37*a^3*f+31*a^2*b*e-25*a*b^2*d+19*b^3*c)*x/b^7/(b*x^3
+a)+1/27*a^(4/3)*(-152*a^3*f+104*a^2*b*e-65*a*b^2*d+35*b^3*c)*ln(a^(1/3)+b^
(1/3)*x)/b^(22/3)-1/54*a^(4/3)*(-152*a^3*f+104*a^2*b*e-65*a*b^2*d+35*b^3*c)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(22/3)-1/27*a^(4/3)*(-152*a^3*
f+104*a^2*b*e-65*a*b^2*d+35*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)
*3^(1/2))/b^(22/3)*3^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1842, 1872, 1901, 206, 31, 648, 631, 210, 642}

$$\frac{x^7(6a^2f-3abe+b^2d)}{7b^5}-\frac{a^2x(-37a^3f+31a^2be-25ab^2d+19b^3c)}{18b^7(a+bx^3)}+\frac{a^3x(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^7(a+bx^3)^2}-\frac{ax(-15a^3f+10a^2be-6ab^2d+3b^3c)}{b^7}+\frac{x^4(-10a^3f+6a^2be-3ab^2d+b^3c)}{4b^6}-\frac{a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-152a^3f+104a^2be-65ab^2d+35b^3c)}{9\sqrt{3}b^{22/3}}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-152a^3f+104a^2be-65ab^2d+35b^3c)}{54b^{22/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-152a^3f+104a^2be-65ab^2d+35b^3c)}{27b^{22/3}}+\frac{x^{10}(be-3af)}{10b^4}+\frac{fx^{13}}{13b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] -((a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x)/b^7) + ((b^3*c - 3*a*
b^2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x
^7)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) + (f*x^13)/(13*b^3) + (a^3*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^7*(a + b*x^3)^2) - (a^2*(19*b^3*c -
 25*a*b^2*d + 31*a^2*b*e - 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) - (a^(4/3)*(35
*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(22/3)) + (a^(4/3)*(35*b^3*c - 65*a*b^2
*d + 104*a^2*b*e - 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(22/3)) - (a^
(4/3)*(35*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(22/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
```

```
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2} - \frac{\int \frac{a^4(b^3c-ab^2d+a^2be-a^3f)-6a^3b(b^3c-ab^2d+a^2be-}{}}{}$$

$$= \frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2} - \frac{a^2(19b^3c-25ab^2d+31a^2be-37a^3f)\,x}{18b^7\,(a+bx^3)}$$

$$= \frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2} - \frac{a^2(19b^3c-25ab^2d+31a^2be-37a^3f)\,x}{18b^7\,(a+bx^3)}$$

$$= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x}{b^7} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)}{4b^6}$$

$$= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x}{b^7} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)}{4b^6}$$

$$= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x}{b^7} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)}{4b^6}$$

$$= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x}{b^7} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)}{4b^6}$$

$$= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x}{b^7} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)}{4b^6}$$

Mathematica [A]
time = 0.30, size = 411, normalized size = 0.99

$$\frac{a(-3b^3c+6ab^2d-10a^2be+15a^3f)\,x}{b^7} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^4}{4b^6} + \frac{(b^2d-3abe+6a^2f)\,x^7}{7b^5} + \frac{(be-3af)x^{10}}{10b^4} + \frac{fx^{13}}{13b^3} + \frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2} + \frac{a^2(-19b^3c+25ab^2d-31a^2be+37a^3f)\,x}{18b^7\,(a+bx^3)} + \frac{a^{4/3}(-35b^3c+65ab^2d-104a^2be+152a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{22/3}} - \frac{a^{4/3}(-35b^3c+65ab^2d-104a^2be+152a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27b^{22/3}} + \frac{a^{4/3}(-35b^3c+65ab^2d-104a^2be+152a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54b^{22/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*c - 3*a*b^
2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^7
)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) + (f*x^13)/(13*b^3) + (a^3*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^7*(a + b*x^3)^2) + (a^2*(-19*b^3*c +
25*a*b^2*d - 31*a^2*b*e + 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) + (a^(4/3)*(-35
*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(22/3)) - (a^(4/3)*(-35*b^3*c + 65*a*b^2*d -
104*a^2*b*e + 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(22/3)) + (a^(4/3)
*(-35*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*b^(22/3))
```

Maple [A]
time = 0.40, size = 344, normalized size = 0.83

method	result
risch	$\frac{f\,x^{13}}{13b^{3}}-\frac{3af\,x^{10}}{10b^{4}}+\frac{e\,x^{10}}{10b^{3}}+\frac{6a^{2}f\,x^{7}}{7b^{5}}-\frac{3ae\,x^{7}}{7b^{4}}+\frac{d\,x^{7}}{7b^{3}}-\frac{5a^{3}f\,x^{4}}{2b^{6}}+\frac{3a^{2}e\,x^{4}}{2b^{5}}-\frac{3ad\,x^{4}}{4b^{4}}+\frac{c\,x^{4}}{4b^{3}}+\frac{15a^{4}fx}{b^{7}}-\frac{10a^{3}ex}{b^{6}}+$
default	$\frac{\frac{1}{13}f\,x^{13}b^{4}-\frac{3}{10}a\,b^{3}f\,x^{10}+\frac{1}{10}b^{4}e\,x^{10}+\frac{6}{7}a^{2}b^{2}f\,x^{7}-\frac{3}{7}a\,b^{3}e\,x^{7}+\frac{1}{7}b^{4}d\,x^{7}-\frac{5}{2}a^{3}bf\,x^{4}+\frac{3}{2}a^{2}b^{2}e\,x^{4}-\frac{3}{4}a\,b^{3}d\,x^{4}+\frac{1}{4}b^{4}c\,x^{4}+15a^{4}fx-10a^{3}bex+}{b^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^7*(1/13*f*x^13*b^4-3/10*a*b^3*f*x^10+1/10*b^4*e*x^10+6/7*a^2*b^2*f*x^7-
3/7*a*b^3*e*x^7+1/7*b^4*d*x^7-5/2*a^3*b*f*x^4+3/2*a^2*b^2*e*x^4-3/4*a*b^3*d
```

```
*x^4+1/4*b^4*c*x^4+15*a^4*f*x-10*a^3*b*e*x+6*a^2*b^2*d*x-3*a*b^3*c*x)-a^2/b
^7*(((-37/18*a^3*b*f+31/18*a^2*e*b^2-25/18*a*d*b^3+19/18*c*b^4)*x^4-1/9*a*(
17*a^3*f-14*a^2*b*e+11*a*b^2*d-8*b^3*c)*x)/(b*x^3+a)^2+1/9*(152*a^3*f-104*a
^2*b*e+65*a*b^2*d-35*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b
)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [**A**]
time = 0.50, size = 434, normalized size = 1.04

$$-\frac{(19\,a^2b^4c-25\,a^3b^3d-37\,a^5bf+31\,a^4b^2e)x^4+2\,(8\,a^3b^3c-11\,a^4b^2d-17\,a^6f+14\,a^5be)x}{18\,(b^9x^6+2\,ab^8x^3+a^2b^7)}+\frac{140\,b^4fx^{13}-182\,(3\,ab^3f-b^4e)x^{10}+260\,(b^4d+6\,a^2b^2f-3\,ab^3e)x^7+455\,(b^4c-3\,ab^3d-10\,a^3bf+6\,a^2b^2e)x^4-1820\,(3\,ab^3c-6\,a^2b^2d-15\,a^4f+10\,a^3be)x}{1820\,b^7}+\frac{\sqrt{3}\,(35\,a^2b^3c-65\,a^3b^2d-152\,a^5f+104\,a^4be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^8\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(35\,a^2b^3c-65\,a^3b^2d-152\,a^5f+104\,a^4be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^8\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(35\,a^2b^3c-65\,a^3b^2d-152\,a^5f+104\,a^4be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^8\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((19*a^2*b^4*c - 25*a^3*b^3*d - 37*a^5*b*f + 31*a^4*b^2*e)*x^4 + 2*(8
*a^3*b^3*c - 11*a^4*b^2*d - 17*a^6*f + 14*a^5*b*e)*x)/(b^9*x^6 + 2*a*b^8*x^
3 + a^2*b^7) + 1/1820*(140*b^4*f*x^13 - 182*(3*a*b^3*f - b^4*e)*x^10 + 260*
(b^4*d + 6*a^2*b^2*f - 3*a*b^3*e)*x^7 + 455*(b^4*c - 3*a*b^3*d - 10*a^3*b*f
 + 6*a^2*b^2*e)*x^4 - 1820*(3*a*b^3*c - 6*a^2*b^2*d - 15*a^4*f + 10*a^3*b*e
)*x)/b^7 + 1/27*sqrt(3)*(35*a^2*b^3*c - 65*a^3*b^2*d - 152*a^5*f + 104*a^4*
b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^8*(a/b)^(2/3))
- 1/54*(35*a^2*b^3*c - 65*a^3*b^2*d - 152*a^5*f + 104*a^4*b*e)*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(b^8*(a/b)^(2/3)) + 1/27*(35*a^2*b^3*c - 65*a^3*
b^2*d - 152*a^5*f + 104*a^4*b*e)*log(x + (a/b)^(1/3))/(b^8*(a/b)^(2/3))
```

Fricas [**A**]
time = 0.39, size = 667, normalized size = 1.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/49140*(3780*b^6*f*x^19 + 378*(13*b^6*e - 19*a*b^5*f)*x^16 + 108*(65*b^6*d
 - 104*a*b^5*e + 152*a^2*b^4*f)*x^13 + 351*(35*b^6*c - 65*a*b^5*d + 104*a^2
*b^4*e - 152*a^3*b^3*f)*x^10 - 3510*(35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^
3*e - 152*a^4*b^2*f)*x^7 - 9555*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*
e - 152*a^5*b*f)*x^4 - 1820*sqrt(3)*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*
b*e - 152*a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*
f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)
*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*
(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e - 152*a^6*f + (35*a*b^5*c - 65*a
^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^
3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3)) - 1820*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e - 152*
```

```
a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*f)*x^6 + 2
*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)*(-a/b)^(1
/3)*log(x - (-a/b)^(1/3)) - 5460*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e
 - 152*a^6*f)*x)/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.56, size = 500, normalized size = 1.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(35*(-a*b^2)^(1/3)*a*b^3*c - 65*(-a*b^2)^(1/3)*a^2*b^2*d - 152
*(-a*b^2)^(1/3)*a^4*f + 104*(-a*b^2)^(1/3)*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x
 + (-a/b)^(1/3))/(-a/b)^(1/3))/b^8 - 1/27*(35*a^2*b^3*c - 65*a^3*b^2*d - 15
2*a^5*f + 104*a^4*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/
54*(35*(-a*b^2)^(1/3)*a*b^3*c - 65*(-a*b^2)^(1/3)*a^2*b^2*d - 152*(-a*b^2)^
(1/3)*a^4*f + 104*(-a*b^2)^(1/3)*a^3*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)
^(2/3))/b^8 - 1/18*(19*a^2*b^4*c*x^4 - 25*a^3*b^3*d*x^4 - 37*a^5*b*f*x^4 +
31*a^4*b^2*x^4*e + 16*a^3*b^3*c*x - 22*a^4*b^2*d*x - 34*a^6*f*x + 28*a^5*b*
x*e)/((b*x^3 + a)^2*b^7) + 1/1820*(140*b^36*f*x^13 - 546*a*b^35*f*x^10 + 18
2*b^36*x^10*e + 260*b^36*d*x^7 + 1560*a^2*b^34*f*x^7 - 780*a*b^35*x^7*e + 4
55*b^36*c*x^4 - 1365*a*b^35*d*x^4 - 4550*a^3*b^33*f*x^4 + 2730*a^2*b^34*x^4
*e - 5460*a*b^35*c*x + 10920*a^2*b^34*d*x + 27300*a^4*b^32*f*x - 18200*a^3*
b^33*x*e)/b^39
```

Mupad [**B**]
time = 5.24, size = 575, normalized size = 1.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^10*(e/(10*b^3) - (3*a*f)/(10*b^4)) + x^4*(c/(4*b^3) - (a^3*f)/(4*b^6) - (
3*a^2*(e/b^3 - (3*a*f)/b^4))/(4*b^2) + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(
e/b^3 - (3*a*f)/b^4))/b))/(4*b)) + (x*((17*a^6*f)/9 - (8*a^3*b^3*c)/9 + (11
*a^4*b^2*d)/9 - (14*a^5*b*e)/9) - x^4*((19*a^2*b^4*c)/18 - (25*a^3*b^3*d)/1
8 + (31*a^4*b^2*e)/18 - (37*a^5*b*f)/18))/(a^2*b^7 + b^9*x^6 + 2*a*b^8*x^3)
 - x*((3*a*(c/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3 - (3*a*f)/b^4))/b^2 + (3*a*
((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b))/b - (3*a^2*((3
*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b^2 + (a^3*(e/b^3 - (
3*a*f)/b^4))/b^3) - x^7*((3*a^2*f)/(7*b^5) - d/(7*b^3) + (3*a*(e/b^3 - (3*a
*f)/b^4))/(7*b)) + (f*x^13)/(13*b^3) + (a^(4/3)*log(b^(1/3)*x + a^(1/3))*(3
5*b^3*c - 152*a^3*f - 65*a*b^2*d + 104*a^2*b*e))/(27*b^(22/3)) + (a^(4/3)*l
og(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(35*b
^3*c - 152*a^3*f - 65*a*b^2*d + 104*a^2*b*e))/(27*b^(22/3)) - (a^(4/3)*log(
3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(35*b^3*
c - 152*a^3*f - 65*a*b^2*d + 104*a^2*b*e))/(27*b^(22/3))
```

3.287 $\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=384

$$\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6}+\frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5}+\frac{(be-3af)x^8}{8b^4}+\frac{fx^{11}}{11b^3}-\frac{a^2(b^3c-ab^2d+a^2be-a^3}{6b^6\,(a+bx^3)^2}$$

```
[Out] 1/2*(-10*a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*x^2/b^6+1/5*(6*a^2*f-3*a*b*e+b^2*
d)*x^5/b^5+1/8*(-3*a*f+b*e)*x^8/b^4+1/11*f*x^11/b^3-1/6*a^2*(-a^3*f+a^2*b*e
-a*b^2*d+b^3*c)*x^2/b^6/(b*x^3+a)^2+1/9*a*(-16*a^3*f+13*a^2*b*e-10*a*b^2*d+
7*b^3*c)*x^2/b^6/(b*x^3+a)+1/27*a^(2/3)*(-119*a^3*f+77*a^2*b*e-44*a*b^2*d+2
0*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(20/3)-1/54*a^(2/3)*(-119*a^3*f+77*a^2*b*e
-44*a*b^2*d+20*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(20/3)+1/
27*a^(2/3)*(-119*a^3*f+77*a^2*b*e-44*a*b^2*d+20*b^3*c)*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(20/3)*3^(1/2)
```

Rubi [A]
time = 0.69, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1842, 1865, 1850, 1502, 298, 31, 648, 631, 210, 642}

$$\frac{x^5(6a^2f-3abe+b^2d)}{5b^5}+\frac{x^2(-10a^3f+6a^2be-3ab^2d+b^3c)}{2b^6}+\frac{ax^2(-16a^3f+13a^2be-10ab^2d+7b^3c)}{9b^6\,(a+bx^3)}-\frac{a^2x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6\,(a+bx^3)^2}+\frac{a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(-119a^3f+77a^2be-44ab^2d+20b^3c)}{9\sqrt{3}\,b^{20/3}}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(-119a^3f+77a^2be-44ab^2d+20b^3c)}{54b^{20/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(-119a^3f+77a^2be-44ab^2d+20b^3c)}{27b^{20/3}}+\frac{x^8(be-3af)}{8b^4}+\frac{fx^{11}}{11b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4) + (f*x^11)/(11*b^3)
 - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^6*(a + b*x^3)^2) + (a
*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3)) +
(a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(20/3)) + (a^(2/3)*(20*b^3*c
 - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(20
/3)) - (a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1865

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} - \frac{\int \frac{-2a^3b(b^3c-ab^2d+a^2be-a^3f)x+6a^2b^2(b^3c-ab\cdots}{\cdots}}{\cdots} \\
&= -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} - \frac{\int \frac{x(-2a^3b(b^3c-ab^2d+a^2be-a^3f)+6a^2b^2(b^3c-\cdots}{\cdots}}{\cdots} \\
&= -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} + \frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,\cdots}{9b^6\,(a+bx^3)} \\
&= -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} + \frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,\cdots}{9b^6\,(a+bx^3)} \\
&= \frac{fx^{11}}{11b^3} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} + \frac{a(7b^3c-10ab^2d+13a^2be-16\cdots}{9b^6\,(a+bx^3)} \\
&= \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} + \frac{a(7b^3c-10ab\cdots}{9\cdots} \\
&= \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} + \frac{a(7b^3c-10ab\cdots}{9\cdots} \\
&= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6} + \frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5} + \frac{(be-3af\cdots}{8b^4} \\
&= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6} + \frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5} + \frac{(be-3af\cdots}{8b^4} \\
&= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6} + \frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5} + \frac{(be-3af\cdots}{8b^4} \\
&= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6} + \frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5} + \frac{(be-3af\cdots}{8b^4} \\
&= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6} + \frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5} + \frac{(be-3af\cdots}{8b^4}
\end{aligned}
$$

Mathematica [A]

time = 0.26, size = 380, normalized size = 0.99

$$
\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6} + \frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} + \frac{a^2(-b^3c+ab^2d-a^2be+a^3f)\,x^2}{6b^6\,(a+bx^3)^2} + \frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)} - \frac{a^{2/3}(-20b^3c+44ab^2d-77a^2be+119a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{20/3}} - \frac{a^{2/3}(-20b^3c+44ab^2d-77a^2be+119a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27b^{20/3}} + \frac{a^{2/3}(-20b^3c+44ab^2d-77a^2be+119a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54b^{20/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4) + (f*x^11)/(11*b^3)
 + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(6*b^6*(a + b*x^3)^2) +
 (a*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3))
 - (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(20/3)) - (a^(2/3)*(-20*b^3*c
+ 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(20/
3)) + (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3))
```

Maple [A]
time = 0.36, size = 296, normalized size = 0.77

method	result
risch	$\frac{f\,x^{11}}{11 b^{3}}-\frac{3 x^{8} f a}{8 b^{4}}+\frac{x^{8} e}{8 b^{3}}+\frac{6 x^{5} a^{2} f}{5 b^{5}}-\frac{3 x^{5} a e}{5 b^{4}}+\frac{x^{5} d}{5 b^{3}}-\frac{5 x^{2} a^{3} f}{b^{6}}+\frac{3 x^{2} a^{2} e}{b^{5}}-\frac{3 x^{2} a d}{2 b^{4}}+\frac{x^{2} c}{2 b^{3}}+\frac{\left(-\frac{16}{9} a^{4} b f+\frac{13}{9} a^{3} b^{2} e-\frac{10}{9} a^{2} b^{3} c \ldots$
default	$-\frac{-\frac{b^{3} f\,x^{11}}{11}+\frac{\left(3 f a\,b^{2}-e\,b^{3}\right) x^{8}}{8}+\frac{\left(-6 a^{2} b f+3 a\,b^{2} e-b^{3} d\right) x^{5}}{5}+\frac{\left(10 a^{3} f-6 a^{2} b e+3 a\,b^{2} d-b^{3} c\right) x^{2}}{2}}{b^{6}}+\frac{a\left(\frac{\left(-\frac{16}{9} a^{3} b f+\frac{13}{9} a^{2} e\,b^{2}-\frac{10}{9} a d\,b^{3}+\frac{7}{9} c\,b^{4}\right) x^{5} \ldots}{\left(b\,x^{3}+\ldots\right.} \ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6*(-1/11*b^3*f*x^11+1/8*(3*a*b^2*f-b^3*e)*x^8+1/5*(-6*a^2*b*f+3*a*b^2*
e-b^3*d)*x^5+1/2*(10*a^3*f-6*a^2*b*e+3*a*b^2*d-b^3*c)*x^2)+a/b^6*(((-16/9*a
^3*b*f+13/9*a^2*e*b^2-10/9*a*d*b^3+7/9*c*b^4)*x^5-1/18*a*(29*a^3*f-23*a^2*b
*e+17*a*b^2*d-11*b^3*c)*x^2)/(b*x^3+a)^2+(119/9*a^3*f-77/9*a^2*b*e+44/9*a*b
^2*d-20/9*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.49, size = 389, normalized size = 1.01

$$\frac{2\,(7\,ab^4c-10\,a^2b^3d-16\,a^4bf+13\,a^3b^2e)x^5+(11\,a^2b^3c-17\,a^3b^2d-29\,a^5f+23\,a^4be)x^2}{18\,(b^8x^6+2\,ab^7x^3+a^2b^6)}-\frac{\sqrt{3}\,(20\,ab^3c-44\,a^2b^2d-119\,a^4f+77\,a^3be)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{40\,b^3fx^{11}-55\,(3\,ab^2f-b^3e)x^8+88\,(b^3d+6\,a^2bf-3\,ab^2e)x^5+220\,(b^3c-3\,ab^2d-10\,a^3f+6\,a^2be)x^2}{440\,b^6}-\frac{(20\,ab^3c-44\,a^2b^2d-119\,a^4f+77\,a^3be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(20\,ab^3c-44\,a^2b^2d-119\,a^4f+77\,a^3be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(7*a*b^4*c - 10*a^2*b^3*d - 16*a^4*b*f + 13*a^3*b^2*e)*x^5 + (11*a^
2*b^3*c - 17*a^3*b^2*d - 29*a^5*f + 23*a^4*b*e)*x^2)/(b^8*x^6 + 2*a*b^7*x^3
 + a^2*b^6) - 1/27*sqrt(3)*(20*a*b^3*c - 44*a^2*b^2*d - 119*a^4*f + 77*a^3*
b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)^(1/3))
+ 1/440*(40*b^3*f*x^11 - 55*(3*a*b^2*f - b^3*e)*x^8 + 88*(b^3*d + 6*a^2*b*f
 - 3*a*b^2*e)*x^5 + 220*(b^3*c - 3*a*b^2*d - 10*a^3*f + 6*a^2*b*e)*x^2)/b^6
 - 1/54*(20*a*b^3*c - 44*a^2*b^2*d - 119*a^4*f + 77*a^3*b*e)*log(x^2 - x*(a
/b)^(1/3) + (a/b)^(2/3))/(b^7*(a/b)^(1/3)) + 1/27*(20*a*b^3*c - 44*a^2*b^2*
d - 119*a^4*f + 77*a^3*b*e)*log(x + (a/b)^(1/3))/(b^7*(a/b)^(1/3))
```

Fricas [**A**]
time = 0.41, size = 634, normalized size = 1.65

$$\frac{1080\,b^5fx^{17}+135\,(11\,b^5e-17\,ab^4f)x^{14}+54\,(44\,b^5d-77\,ab^4e+119\,a^2b^3f)x^{11}+297\,(20\,b^5c-44\,ab^4d+77\,a^2b^3e-119\,a^3b^2f)x^8+1056\,(20\,ab^4c-44\,a^2b^3d+77\,a^3b^2e-119\,a^4bf)x^5+660\,(20\,a^2b^3c-44\,a^3b^2d+77\,a^4be-119\,a^5f)x^2-440\,\sqrt{3}\,((20\,b^5c-44\,ab^4d+77\,a^2b^3e-119\,a^3b^2f)x^6+20\,a^2b^3c-44\,a^3b^2d+77\,a^4be-119\,a^5f+2\,(20\,ab^4c-44\,a^2b^3d+77\,a^3b^2e-119\,a^4bf)x^3)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}\,a}{3\,a}\right)+220\,((20\,b^5c-44\,ab^4d+77\,a^2b^3e-119\,a^3b^2f)x^6+20\,a^2b^3c-44\,a^3b^2d+77\,a^4be-119\,a^5f+2\,(20\,ab^4c-44\,a^2b^3d+77\,a^3b^2e-119\,a^4bf)x^3)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-440\,((20\,b^5c-44\,ab^4d+77\,a^2b^3e-119\,a^3b^2f)x^6+20\,a^2b^3c-44\,a^3b^2d+77\,a^4be-119\,a^5f+2\,(20\,ab^4c-44\,a^2b^3d+77\,a^3b^2e-119\,a^4bf)x^3)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{11880\,(b^8x^6+2\,ab^7x^3+a^2b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/11880*(1080*b^5*f*x^17 + 135*(11*b^5*e - 17*a*b^4*f)*x^14 + 54*(44*b^5*d
- 77*a*b^4*e + 119*a^2*b^3*f)*x^11 + 297*(20*b^5*c - 44*a*b^4*d + 77*a^2*b^
3*e - 119*a^3*b^2*f)*x^8 + 1056*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*e -
 119*a^4*b*f)*x^5 + 660*(20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5
*f)*x^2 - 440*sqrt(3)*((20*b^5*c - 44*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*
f)*x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5*f + 2*(20*a*b^4
*c - 44*a^2*b^3*d + 77*a^3*b^2*e - 119*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*arcta
n(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*((20*b^5*c - 44
*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*f)*x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d
+ 77*a^4*b*e - 119*a^5*f + 2*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*e - 11
9*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2
/b^2)^(1/3)) - 440*((20*b^5*c - 44*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*f)*
x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5*f + 2*(20*a*b^4*c
- 44*a^2*b^3*d + 77*a^3*b^2*e - 119*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x
+ b*(-a^2/b^2)^(2/3)))/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 491, normalized size = 1.28

$$\frac{\left(20\,ab^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-44\,a^{2}b^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-119\,a^{4}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+77\,a^{3}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^{6}}+\frac{\sqrt{3}\left(20\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-44\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-119\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+77\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^{8}}-\frac{\left(20\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-44\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-119\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+77\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^{8}}+\frac{14\,ab^{4}cx^{5}-20\,a^{2}b^{3}dx^{5}-32\,a^{4}bfx^{5}+26\,a^{3}b^{2}x^{5}e+11\,a^{2}b^{3}cx^{2}-17\,a^{3}b^{2}dx^{2}-29\,a^{5}fx^{2}+23\,a^{4}bx^{2}e}{18\left(bx^{3}+a\right)^{2}b^{6}}+\frac{40\,b^{30}fx^{11}-165\,ab^{29}fx^{8}+55\,b^{30}x^{8}e+88\,b^{30}dx^{5}+528\,a^{2}b^{28}fx^{5}-264\,ab^{29}x^{5}e+220\,b^{30}cx^{2}-660\,ab^{29}dx^{2}-2200\,a^{3}b^{27}fx^{2}+1320\,a^{2}b^{28}x^{2}e}{440\,b^{33}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*(20*a*b^3*c*(-a/b)^(1/3) - 44*a^2*b^2*d*(-a/b)^(1/3) - 119*a^4*f*(-a/b
)^(1/3) + 77*a^3*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^6) + 1/27*sqrt(3)*(20*(-a*b^2)^(2/3)*b^3*c - 44*(-a*b^2)^(2/3)*a*b^2*d
 - 119*(-a*b^2)^(2/3)*a^3*f + 77*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^8 - 1/54*(20*(-a*b^2)^(2/3)*b^3*c - 4
4*(-a*b^2)^(2/3)*a*b^2*d - 119*(-a*b^2)^(2/3)*a^3*f + 77*(-a*b^2)^(2/3)*a^2
*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^8 + 1/18*(14*a*b^4*c*x^5 -
 20*a^2*b^3*d*x^5 - 32*a^4*b*f*x^5 + 26*a^3*b^2*x^5*e + 11*a^2*b^3*c*x^2 -
17*a^3*b^2*d*x^2 - 29*a^5*f*x^2 + 23*a^4*b*x^2*e)/((b*x^3 + a)^2*b^6) + 1/4
40*(40*b^30*f*x^11 - 165*a*b^29*f*x^8 + 55*b^30*x^8*e + 88*b^30*d*x^5 + 528
*a^2*b^28*f*x^5 - 264*a*b^29*x^5*e + 220*b^30*c*x^2 - 660*a*b^29*d*x^2 - 22
00*a^3*b^27*f*x^2 + 1320*a^2*b^28*x^2*e)/b^33
```

Mupad [B]
time = 5.34, size = 425, normalized size = 1.11

$$x^{8}\left(\frac{e}{8\,b^{3}}-\frac{3\,a\,f}{8\,b^{4}}\right)+x^{2}\left(\frac{c}{2\,b^{3}}-\frac{a^{3}\,f}{2\,b^{6}}-\frac{3\,a^{2}\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{2\,b^{2}}+\frac{3\,a\left(\frac{3\,a^{2}\,f}{b^{5}}-\frac{d}{b^{3}}+\frac{3\,a\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{b}\right)}{2\,b}\right)-\frac{\left(\frac{16\,f\,a^{4}\,b}{9}-\frac{13\,e\,a^{3}\,b^{2}}{9}+\frac{10\,d\,a^{2}\,b^{3}}{9}-\frac{7\,c\,a\,b^{4}}{9}\right)x^{5}+\left(\frac{29\,f\,a^{5}}{18}-\frac{23\,e\,a^{4}\,b}{18}+\frac{17\,d\,a^{3}\,b^{2}}{18}-\frac{11\,c\,a^{2}\,b^{3}}{18}\right)x^{2}}{a^{2}\,b^{6}+2\,a\,b^{7}\,x^{3}+b^{8}\,x^{6}}-x^{5}\left(\frac{3\,a^{2}\,f}{5\,b^{5}}-\frac{d}{5\,b^{3}}+\frac{3\,a\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{5\,b}\right)+\frac{f\,x^{11}}{11\,b^{3}}+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-119\,f\,a^{3}+77\,e\,a^{2}\,b-44\,d\,a\,b^{2}+20\,c\,b^{3}\right)}{27\,b^{20/3}}-\frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-119\,f\,a^{3}+77\,e\,a^{2}\,b-44\,d\,a\,b^{2}+20\,c\,b^{3}\right)}{27\,b^{20/3}}+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-119\,f\,a^{3}+77\,e\,a^{2}\,b-44\,d\,a\,b^{2}+20\,c\,b^{3}\right)}{27\,b^{20/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^8*(e/(8*b^3) - (3*a*f)/(8*b^4)) + x^2*(c/(2*b^3) - (a^3*f)/(2*b^6) - (3*a
^2*(e/b^3 - (3*a*f)/b^4))/(2*b^2) + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b
^3 - (3*a*f)/b^4))/b))/(2*b)) - (x^2*((29*a^5*f)/18 - (11*a^2*b^3*c)/18 + (
17*a^3*b^2*d)/18 - (23*a^4*b*e)/18) + x^5*((10*a^2*b^3*d)/9 - (13*a^3*b^2*e
)/9 - (7*a*b^4*c)/9 + (16*a^4*b*f)/9))/(a^2*b^6 + b^8*x^6 + 2*a*b^7*x^3) -
x^5*((3*a^2*f)/(5*b^5) - d/(5*b^3) + (3*a*(e/b^3 - (3*a*f)/b^4))/(5*b)) + (
f*x^11)/(11*b^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(20*b^3*c - 119*a^3*f
- 44*a*b^2*d + 77*a^2*b*e))/(27*b^(20/3)) - (a^(2/3)*log(3^(1/2)*a^(1/3)*1i
 + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(20*b^3*c - 119*a^3*f - 44
*a*b^2*d + 77*a^2*b*e))/(27*b^(20/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2
*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(20*b^3*c - 119*a^3*f - 44*a*b
^2*d + 77*a^2*b*e))/(27*b^(20/3))
```

3.288 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=375

$$\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6}+\frac{(b^2d-3abe+6a^2f)\,x^4}{4b^5}+\frac{(be-3af)x^7}{7b^4}+\frac{fx^{10}}{10b^3}-\frac{a^2(b^3c-ab^2d+a^2be-a}{6b^6\,(a+bx^3)^2}$$

```
[Out] (-10*a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*x/b^6+1/4*(6*a^2*f-3*a*b*e+b^2*d)*x^4
/b^5+1/7*(-3*a*f+b*e)*x^7/b^4+1/10*f*x^10/b^3-1/6*a^2*(-a^3*f+a^2*b*e-a*b^2
*d+b^3*c)*x/b^6/(b*x^3+a)^2+1/18*a*(-31*a^3*f+25*a^2*b*e-19*a*b^2*d+13*b^3*
c)*x/b^6/(b*x^3+a)-1/27*a^(1/3)*(-104*a^3*f+65*a^2*b*e-35*a*b^2*d+14*b^3*c)
*ln(a^(1/3)+b^(1/3)*x)/b^(19/3)+1/54*a^(1/3)*(-104*a^3*f+65*a^2*b*e-35*a*b^
2*d+14*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(19/3)+1/27*a^(1/
3)*(-104*a^3*f+65*a^2*b*e-35*a*b^2*d+14*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3
)*x)/a^(1/3)*3^(1/2))/b^(19/3)*3^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1842, 1872, 1901, 206, 31, 648, 631, 210, 642}

$$\frac{x^4(6a^2f-3abe+b^2d)}{4b^5}+\frac{\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(-104a^3f+65a^2be-35ab^2d+14b^3c)}{9\sqrt{3}\,b^{19/3}}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(-104a^3f+65a^2be-35ab^2d+14b^3c)}{27b^{19/3}}+\frac{ax(-31a^3f+25a^2be-19ab^2d+13b^3c)}{18b^6\,(a+bx^3)}-\frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6\,(a+bx^3)^2}+\frac{x(-10a^3f+6a^2be-3ab^2d+b^3c)}{b^6}+\frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(-104a^3f+65a^2be-35ab^2d+14b^3c)}{54b^{19/3}}+\frac{x^7(be-3af)}{7b^4}+\frac{fx^{10}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*a*b*e + 6*
a^2*f)*x^4)/(4*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*x^10)/(10*b^3) - (a^
2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^6*(a + b*x^3)^2) + (a*(13*b^3
*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x)/(18*b^6*(a + b*x^3)) + (a^(1/3)
*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(19/3)) - (a^(1/3)*(14*b^3*c - 35*a*
b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(19/3)) + (
a^(1/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
```

```
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^6\,(a+bx^3)^2} - \frac{\int \frac{-a^3(b^3c-ab^2d+a^2be-a^3f)+6a^2b(b^3c-ab^2d+a^2}{}}{}$$

$$= -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^6\,(a+bx^3)^2} + \frac{a(13b^3c-19ab^2d+25a^2be-31a^3f)\,x}{18b^6\,(a+bx^3)}$$

$$= -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^6\,(a+bx^3)^2} + \frac{a(13b^3c-19ab^2d+25a^2be-31a^3f)\,x}{18b^6\,(a+bx^3)}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6} + \frac{(b^2d-3abe+6a^2f)\,x^4}{4b^5} + \frac{(be-3af)x}{7b^4}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6} + \frac{(b^2d-3abe+6a^2f)\,x^4}{4b^5} + \frac{(be-3af)x}{7b^4}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6} + \frac{(b^2d-3abe+6a^2f)\,x^4}{4b^5} + \frac{(be-3af)x}{7b^4}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6} + \frac{(b^2d-3abe+6a^2f)\,x^4}{4b^5} + \frac{(be-3af)x}{7b^4}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6} + \frac{(b^2d-3abe+6a^2f)\,x^4}{4b^5} + \frac{(be-3af)x}{7b^4}$$

Mathematica [A]
time = 0.24, size = 362, normalized size = 0.97

$$\frac{3780\sqrt[3]{b}\,(b^3c-3ab^2d+6a^2be-10a^3f)\,x+945b^{4/3}(b^2d-3abe+6a^2f)\,x^4+540b^{7/3}(be-3af)x^7+378b^{10/3}fx^{10}+\frac{630a^2\sqrt[3]{b}\,(-b^3c+ab^2d-a^2be+a^3f)x}{(a+bx^3)^2}+\frac{210a\sqrt[3]{b}\,(13b^3c-19ab^2d+25a^2be-31a^3f)x}{a+bx^3}-140\sqrt{3}\,\sqrt[3]{a}\,(-14b^3c+35ab^2d-65a^2be+104a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+140\sqrt[3]{a}\,(-14b^3c+35ab^2d-65a^2be+104a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-70\sqrt[3]{a}\,(-14b^3c+35ab^2d-65a^2be+104a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{3780b^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (3780*b^(1/3)*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x + 945*b^(4/3)*(b
^2*d - 3*a*b*e + 6*a^2*f)*x^4 + 540*b^(7/3)*(b*e - 3*a*f)*x^7 + 378*b^(10/3
)*f*x^10 + (630*a^2*b^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(a +
b*x^3)^2 + (210*a*b^(1/3)*(13*b^3*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x
)/(a + b*x^3) - 140*Sqrt[3]*a^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e +
104*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-14*b
^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x] - 70*a
^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(3780*b^(19/3))
```

Maple [A]
time = 0.35, size = 296, normalized size = 0.79

method	result
risch	$\frac{f\,x^{10}}{10b^3}-\frac{3af\,x^7}{7b^4}+\frac{e\,x^7}{7b^3}+\frac{3a^2f\,x^4}{2b^5}-\frac{3ae\,x^4}{4b^4}+\frac{d\,x^4}{4b^3}-\frac{10a^3fx}{b^6}+\frac{6a^2ex}{b^5}-\frac{3adx}{b^4}+\frac{cx}{b^3}+\frac{(-\frac{31}{18}a^4bf+\frac{25}{18}a^3b^2e-\frac{19}{18}a^2b^3d+}{}$
default	$-\frac{-\frac{1}{10}b^3f\,x^{10}+\frac{3}{7}a\,b^2f\,x^7-\frac{1}{7}b^3e\,x^7-\frac{3}{2}a^2bf\,x^4+\frac{3}{4}a\,b^2e\,x^4-\frac{1}{4}b^3d\,x^4+10a^3fx-6a^2bex+3a\,b^2dx-b^3cx}{b^6}+\frac{a\left(\frac{(-\frac{31}{18}a^3bf+\frac{25}{18}a^2e\,b^2-\frac{19}{18}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6*(-1/10*b^3*f*x^10+3/7*a*b^2*f*x^7-1/7*b^3*e*x^7-3/2*a^2*b*f*x^4+3/4*
a*b^2*e*x^4-1/4*b^3*d*x^4+10*a^3*f*x-6*a^2*b*e*x+3*a*b^2*d*x-b^3*c*x)+a/b^6
*(((-31/18*a^3*b*f+25/18*a^2*e*b^2-19/18*a*d*b^3+13/18*c*b^4)*x^4-1/9*a*(14
*a^3*f-11*a^2*b*e+8*a*b^2*d-5*b^3*c)*x)/(b*x^3+a)^2+1/9*(104*a^3*f-65*a^2*b
*e+35*a*b^2*d-14*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2
```

```
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.49, size = 385, normalized size = 1.03

$$\frac{(13\,ab^4c-19\,a^2b^3d-31\,a^4bf+25\,a^3b^2e)x^4+2\,(5\,a^2b^3c-8\,a^3b^2d-14\,a^5f+11\,a^4be)x}{18\,(b^8x^6+2\,ab^7x^3+a^2b^6)}+\frac{14\,b^3fx^{10}-20\,(3\,ab^2f-b^3e)x^7+35\,(b^3d+6\,a^2bf-3\,ab^2e)x^4+140\,(b^3c-3\,ab^2d-10\,a^3f+6\,a^2be)x}{140\,b^6}-\frac{\sqrt{3}\,(14\,ab^3c-35\,a^2b^2d-104\,a^4f+65\,a^3be)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(14\,ab^3c-35\,a^2b^2d-104\,a^4f+65\,a^3be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(14\,ab^3c-35\,a^2b^2d-104\,a^4f+65\,a^3be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((13*a*b^4*c - 19*a^2*b^3*d - 31*a^4*b*f + 25*a^3*b^2*e)*x^4 + 2*(5*a^
2*b^3*c - 8*a^3*b^2*d - 14*a^5*f + 11*a^4*b*e)*x)/(b^8*x^6 + 2*a*b^7*x^3 +
a^2*b^6) + 1/140*(14*b^3*f*x^10 - 20*(3*a*b^2*f - b^3*e)*x^7 + 35*(b^3*d +
6*a^2*b*f - 3*a*b^2*e)*x^4 + 140*(b^3*c - 3*a*b^2*d - 10*a^3*f + 6*a^2*b*e)
*x)/b^6 - 1/27*sqrt(3)*(14*a*b^3*c - 35*a^2*b^2*d - 104*a^4*f + 65*a^3*b*e)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)^(2/3)) + 1/
54*(14*a*b^3*c - 35*a^2*b^2*d - 104*a^4*f + 65*a^3*b*e)*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(b^7*(a/b)^(2/3)) - 1/27*(14*a*b^3*c - 35*a^2*b^2*d - 1
04*a^4*f + 65*a^3*b*e)*log(x + (a/b)^(1/3))/(b^7*(a/b)^(2/3))
```

Fricas [A]
time = 0.42, size = 602, normalized size = 1.61

$$\frac{378\,b^5fx^{16}+108\,(5\,b^5e-8\,ab^4f)x^{13}+27\,(35\,b^5d-65\,ab^4e+104\,a^2b^3f)x^{10}+270\,(14\,b^5c-35\,ab^4d+65\,a^2b^3e-104\,a^3b^2f)x^7+735\,(14\,ab^4c-35\,a^2b^3d+65\,a^3b^2e-104\,a^4bf)x^4-140\,\sqrt{3}\,((14\,b^5c-35\,ab^4d+65\,a^2b^3e-104\,a^3b^2f)x^6+14\,a^2b^3c-35\,a^3b^2d+65\,a^4be-104\,a^5f+2\,(14\,ab^4c-35\,a^2b^3d+65\,a^3b^2e-104\,a^4bf)x^3)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+70\,((14\,b^5c-35\,ab^4d+65\,a^2b^3e-104\,a^3b^2f)x^6+14\,a^2b^3c-35\,a^3b^2d+65\,a^4be-104\,a^5f+2\,(14\,ab^4c-35\,a^2b^3d+65\,a^3b^2e-104\,a^4bf)x^3)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-140\,((14\,b^5c-35\,ab^4d+65\,a^2b^3e-104\,a^3b^2f)x^6+14\,a^2b^3c-35\,a^3b^2d+65\,a^4be-104\,a^5f+2\,(14\,ab^4c-35\,a^2b^3d+65\,a^3b^2e-104\,a^4bf)x^3)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+420\,(14\,a^2b^3c-35\,a^3b^2d+65\,a^4be-104\,a^5f)x}{3780\,(b^8x^6+2\,ab^7x^3+a^2b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3780*(378*b^5*f*x^16 + 108*(5*b^5*e - 8*a*b^4*f)*x^13 + 27*(35*b^5*d - 65
*a*b^4*e + 104*a^2*b^3*f)*x^10 + 270*(14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e
- 104*a^3*b^2*f)*x^7 + 735*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*
a^4*b*f)*x^4 - 140*sqrt(3)*((14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3
*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a^4*b*e - 104*a^5*f + 2*(14*
a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*b*f)*x^3)*(a/b)^(1/3)*arcta
n(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*((14*b^5*c - 35*a*b^4
*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a
^4*b*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*
b*f)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*((14*b^5
*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3
*b^2*d + 65*a^4*b*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2
*e - 104*a^4*b*f)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*(14*a^2*b^3*c
 - 35*a^3*b^2*d + 65*a^4*b*e - 104*a^5*f)*x)/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b
^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.59, size = 443, normalized size = 1.18

$$-\frac{\sqrt{3}\left(14\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-35\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-104\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+65\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^{7}}+\frac{\left(14ab^{3}c-35a^{2}b^{2}d-104a^{4}f+65a^{3}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27ab^{6}}-\frac{\left(14\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-35\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-104\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+65\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54b^{7}}+\frac{13ab^{4}cx^{4}-19a^{2}b^{3}dx^{4}-31a^{4}bfx^{4}+25a^{3}b^{2}x^{4}e+10a^{2}b^{3}cx-16a^{3}b^{2}dx-28a^{5}fx+22a^{4}bxe}{18\left(bx^{3}+a\right)^{2}b^{6}}+\frac{14b^{27}fx^{10}-60ab^{26}fx^{7}+20b^{27}x^{7}e+35b^{27}dx^{4}+210a^{2}b^{25}fx^{4}-105ab^{26}x^{4}e+140b^{27}cx-420ab^{26}dx-1400a^{3}b^{24}fx+840a^{2}b^{25}xe}{140b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(14*(-a*b^2)^(1/3)*b^3*c - 35*(-a*b^2)^(1/3)*a*b^2*d - 104*(-
a*b^2)^(1/3)*a^3*f + 65*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/b^7 + 1/27*(14*a*b^3*c - 35*a^2*b^2*d - 104*a^4*
f + 65*a^3*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^6) - 1/54*(14*
(-a*b^2)^(1/3)*b^3*c - 35*(-a*b^2)^(1/3)*a*b^2*d - 104*(-a*b^2)^(1/3)*a^3*f
 + 65*(-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7
+ 1/18*(13*a*b^4*c*x^4 - 19*a^2*b^3*d*x^4 - 31*a^4*b*f*x^4 + 25*a^3*b^2*x^4
*e + 10*a^2*b^3*c*x - 16*a^3*b^2*d*x - 28*a^5*f*x + 22*a^4*b*x*e)/((b*x^3 +
 a)^2*b^6) + 1/140*(14*b^27*f*x^10 - 60*a*b^26*f*x^7 + 20*b^27*x^7*e + 35*b
^27*d*x^4 + 210*a^2*b^25*f*x^4 - 105*a*b^26*x^4*e + 140*b^27*c*x - 420*a*b^
26*d*x - 1400*a^3*b^24*f*x + 840*a^2*b^25*x*e)/b^30
```

Mupad [**B**]
time = 5.35, size = 420, normalized size = 1.12

$$x^{7}\left(\frac{e}{7b^{3}}-\frac{3af}{7b^{4}}\right)+x\left(\frac{c}{b^{3}}-\frac{a^{3}f}{b^{6}}-\frac{3a^{2}\left(\frac{e}{b^{3}}-\frac{3af}{b^{4}}\right)}{b^{2}}+\frac{3a\left(\frac{3a^{2}f}{b^{5}}-\frac{d}{b^{3}}+\frac{3a\left(\frac{e}{b^{3}}-\frac{3af}{b^{4}}\right)}{b}\right)}{b}\right)-x^{4}\left(\frac{3a^{2}f}{4b^{5}}-\frac{d}{4b^{3}}+\frac{3a\left(\frac{e}{b^{3}}-\frac{3af}{b^{4}}\right)}{4b}\right)-\frac{\left(\frac{31fa^{4}b}{18}-\frac{25ea^{3}b^{2}}{18}+\frac{19da^{2}b^{3}}{18}-\frac{13cab^{4}}{18}\right)x^{4}+\left(\frac{14fa^{5}}{9}-\frac{11ea^{4}b}{9}+\frac{8da^{3}b^{2}}{9}-\frac{5ca^{2}b^{3}}{9}\right)x}{a^{2}b^{6}+2ab^{7}x^{3}+b^{8}x^{6}}+\frac{fx^{10}}{10b^{3}}-\frac{a^{1/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(-104fa^{3}+65ea^{2}b-35dab^{2}+14cb^{3}\right)}{27b^{19/3}}-\frac{a^{1/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}1i\right)\left(-\frac{1}{2}+\frac{\sqrt{3}1i}{2}\right)\left(-104fa^{3}+65ea^{2}b-35dab^{2}+14cb^{3}\right)}{27b^{19/3}}+\frac{a^{1/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}1i\right)\left(\frac{1}{2}+\frac{\sqrt{3}1i}{2}\right)\left(-104fa^{3}+65ea^{2}b-35dab^{2}+14cb^{3}\right)}{27b^{19/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^7*(e/(7*b^3) - (3*a*f)/(7*b^4)) + x*(c/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3
- (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b
^4))/b))/b) - x^4*((3*a^2*f)/(4*b^5) - d/(4*b^3) + (3*a*(e/b^3 - (3*a*f)/b^
4))/(4*b)) - (x*((14*a^5*f)/9 - (5*a^2*b^3*c)/9 + (8*a^3*b^2*d)/9 - (11*a^4
*b*e)/9) + x^4*((19*a^2*b^3*d)/18 - (25*a^3*b^2*e)/18 - (13*a*b^4*c)/18 + (
31*a^4*b*f)/18))/(a^2*b^6 + b^8*x^6 + 2*a*b^7*x^3) + (f*x^10)/(10*b^3) - (a
^(1/3)*log(b^(1/3)*x + a^(1/3))*(14*b^3*c - 104*a^3*f - 35*a*b^2*d + 65*a^2
```

```
*b*e))/(27*b^(19/3)) - (a^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(14*b^3*c - 104*a^3*f - 35*a*b^2*d + 65*a^2*b*e
))/(27*b^(19/3)) + (a^(1/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))
*((3^(1/2)*1i)/2 + 1/2)*(14*b^3*c - 104*a^3*f - 35*a*b^2*d + 65*a^2*b*e))/(
27*b^(19/3))
```

3.289 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=345

$$\frac{(b^2d-3abe+6a^2f)\,x^2}{2b^5}+\frac{(be-3af)x^5}{5b^4}+\frac{fx^8}{8b^3}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2}-\frac{(4b^3c-7ab^2d+10a^2be-13a$$

```
[Out] 1/2*(6*a^2*f-3*a*b*e+b^2*d)*x^2/b^5+1/5*(-3*a*f+b*e)*x^5/b^4+1/8*f*x^8/b^3+
1/6*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/b^5/(b*x^3+a)^2-1/9*(-13*a^3*f+10*
a^2*b*e-7*a*b^2*d+4*b^3*c)*x^2/b^5/(b*x^3+a)-1/27*(-77*a^3*f+44*a^2*b*e-20*
a*b^2*d+5*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(17/3)+1/54*(-77*a^3*f+44*
a^2*b*e-20*a*b^2*d+5*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/
3)/b^(17/3)-1/27*(-77*a^3*f+44*a^2*b*e-20*a*b^2*d+5*b^3*c)*arctan(1/3*(a^(1
/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(17/3)*3^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1842, 1865, 1850, 1502, 298, 31, 648, 631, 210, 642}

$$\frac{x^2(6a^2f-3abe+b^2d)}{2b^5}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-77a^3f+44a^2be-20ab^2d+5b^3c)}{9\sqrt{3}\sqrt[3]{a}\,b^{17/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-77a^3f+44a^2be-20ab^2d+5b^3c)}{27\sqrt[3]{a}\,b^{17/3}}-\frac{x^2(-13a^3f+10a^2be-7ab^2d+4b^3c)}{9b^5(a+bx^3)}+\frac{ax^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-77a^3f+44a^2be-20ab^2d+5b^3c)}{54\sqrt[3]{a}\,b^{17/3}}+\frac{x^5(be-3af)}{5b^4}+\frac{fx^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^2)/(2*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (
f*x^8)/(8*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^5*(a + b*
x^3)^2) - ((4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*x^2)/(9*b^5*(a + b
*x^3)) - ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(1/3)*b^(17/3)) - ((5*b^3*c -
 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*
b^(17/3)) + ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1865

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2} - \frac{\int \frac{2a^2b(b^3c-ab^2d+a^2be-a^3f)x-6ab^2(b^3c-ab^2d+a^2b}{}}{}$$

$$= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2} - \frac{\int \frac{x(2a^2b(b^3c-ab^2d+a^2be-a^3f)-6ab^2(b^3c-ab^2d+a^2}{}}{}$$

$$= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2} - \frac{(4b^3c-7ab^2d+10a^2be-13a^3f)\,x^2}{9b^5\,(a+bx^3)} + \frac{\int}{}$$

$$= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2} - \frac{(4b^3c-7ab^2d+10a^2be-13a^3f)\,x^2}{9b^5\,(a+bx^3)} + \frac{\int}{}$$

$$= \frac{fx^8}{8b^3} + \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2} - \frac{(4b^3c-7ab^2d+10a^2be-13a^3f)}{9b^5\,(a+bx^3)}$$

$$= \frac{fx^8}{8b^3} + \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2} - \frac{(4b^3c-7ab^2d+10a^2be-13a^3f)}{9b^5\,(a+bx^3)}$$

$$= \frac{(b^2d-3abe+6a^2f)\,x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c-ab^2d+a^2be-a}{6b^5\,(a+bx^3)^2}$$

$$= \frac{(b^2d-3abe+6a^2f)\,x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c-ab^2d+a^2be-a}{6b^5\,(a+bx^3)^2}$$

$$= \frac{(b^2d-3abe+6a^2f)\,x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c-ab^2d+a^2be-a}{6b^5\,(a+bx^3)^2}$$

$$= \frac{(b^2d-3abe+6a^2f)\,x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c-ab^2d+a^2be-a}{6b^5\,(a+bx^3)^2}$$

$$= \frac{(b^2d-3abe+6a^2f)\,x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c-ab^2d+a^2be-a}{6b^5\,(a+bx^3)^2}$$

Mathematica [A]
time = 0.15, size = 329, normalized size = 0.95

$$\frac{540b^{2/3}(b^2d-3abe+6a^2f)\,x^2+216b^{5/3}(be-3af)x^5+135b^{8/3}fx^8+\frac{180ab^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{(a+bx^3)^2}-\frac{120b^{2/3}(4b^3c-7ab^2d+10a^2be-13a^3f)x^2}{a+bx^3}+\frac{40\sqrt{3}\,(-5b^3c+20ab^2d-44a^2be+77a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{40(-5b^3c+20ab^2d-44a^2be+77a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}}+\frac{20(5b^3c-20ab^2d+44a^2be-77a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{a}}}{1080b^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (540*b^(2/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x^2 + 216*b^(5/3)*(b*e - 3*a*f)*x^
5 + 135*b^(8/3)*f*x^8 + (180*a*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x^2)/(a + b*x^3)^2 - (120*b^(2/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^
3*f)*x^2)/(a + b*x^3) + (40*Sqrt[3]*(-5*b^3*c + 20*a*b^2*d - 44*a^2*b*e + 7
7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-5*b^3
*c + 20*a*b^2*d - 44*a^2*b*e + 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3)
+ (20*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(1080*b^(17/3))
```

Maple [A]
time = 0.36, size = 256, normalized size = 0.74

method	result
risch	$\frac{f x^8}{8b^3} - \frac{3x^5 fa}{5b^4} + \frac{x^5 e}{5b^3} + \frac{3x^2a^2f}{b^5} - \frac{3x^2ae}{2b^4} + \frac{x^2d}{2b^3} + \frac{(\frac{13}{9}a^3bf - \frac{10}{9}a^2eb^2 + \frac{7}{9}adb^3 - \frac{4}{9}cb^4)x^5 + \frac{a(23a^3f - 17a^2be + 11ab^2d - 5b^3c)x^2}{18}}{b^5(bx^3+a)^2}$
default	$\frac{\frac{b^2fx^8}{8} + \frac{(-3fab+b^2e)x^5}{5} + \frac{(6a^2f-3abe+b^2d)x^2}{2}}{b^5} - \frac{\frac{(-\frac{13}{9}a^3bf + \frac{10}{9}a^2eb^2 - \frac{7}{9}adb^3 + \frac{4}{9}cb^4)x^5 - \frac{a(23a^3f - 17a^2be + 11ab^2d - 5b^3c)x^2}{18}}{(bx^3+a)^2} + (\frac{77}{9}a^3f}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/8*b^2*f*x^8+1/5*(-3*a*b*f+b^2*e)*x^5+1/2*(6*a^2*f-3*a*b*e+b^2*d)*x
^2)-1/b^5*(((-13/9*a^3*b*f+10/9*a^2*e*b^2-7/9*a*d*b^3+4/9*c*b^4)*x^5-1/18*a
*(23*a^3*f-17*a^2*b*e+11*a*b^2*d-5*b^3*c)*x^2)/(b*x^3+a)^2+(77/9*a^3*f-44/9
*a^2*b*e+20/9*a*b^2*d-5/9*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/
b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.50, size = 338, normalized size = 0.98

$$-\frac{2(4b^4c - 7ab^3d - 13a^3bf + 10a^2b^2e)x^5 + (5ab^3c - 11a^2b^2d - 23a^4f + 17a^3be)x^2}{18(b^7x^6 + 2ab^6x^3 + a^2b^5)} + \frac{\sqrt{3}(5b^3c - 20ab^2d - 77a^3f + 44a^2be)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{5b^2fx^8 - 8(3abf - b^2e)x^5 + 20(b^2d + 6a^2f - 3abe)x^2}{40b^5} + \frac{(5b^3c - 20ab^2d - 77a^3f + 44a^2be)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(5b^3c - 20ab^2d - 77a^3f + 44a^2be)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(4*b^4*c - 7*a*b^3*d - 13*a^3*b*f + 10*a^2*b^2*e)*x^5 + (5*a*b^3*c
 - 11*a^2*b^2*d - 23*a^4*f + 17*a^3*b*e)*x^2)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*
```

```
b^5) + 1/27*sqrt(3)*(5*b^3*c - 20*a*b^2*d - 77*a^3*f + 44*a^2*b*e)*arctan(1
/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(1/3)) + 1/40*(5*b^2
*f*x^8 - 8*(3*a*b*f - b^2*e)*x^5 + 20*(b^2*d + 6*a^2*f - 3*a*b*e)*x^2)/b^5
+ 1/54*(5*b^3*c - 20*a*b^2*d - 77*a^3*f + 44*a^2*b*e)*log(x^2 - x*(a/b)^(1/
3) + (a/b)^(2/3))/(b^6*(a/b)^(1/3)) - 1/27*(5*b^3*c - 20*a*b^2*d - 77*a^3*f
 + 44*a^2*b*e)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(298) = 596$.
time = 0.43, size = 1278, normalized size = 3.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1080*(135*a*b^6*f*x^14 + 54*(4*a*b^6*e - 7*a^2*b^5*f)*x^11 + 27*(20*a*b^
6*d - 44*a^2*b^5*e + 77*a^3*b^4*f)*x^8 - 96*(5*a*b^6*c - 20*a^2*b^5*d + 44*
a^3*b^4*e - 77*a^4*b^3*f)*x^5 - 60*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3
*e - 77*a^5*b^2*f)*x^2 - 60*sqrt(1/3)*(5*a^3*b^4*c - 20*a^4*b^3*d + 44*a^5*
b^2*e - 77*a^6*b*f + (5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e - 77*a^4*b^3*
f)*x^6 + 2*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5*b^2*f)*x^3)*
sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2
)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/
(b*x^3 + a)) + 20*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b^2*f)*x^6
 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4*c - 20*a
^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b
^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e -
 77*a^3*b^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2
*(5*a*b^4*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*
log(b*x + (a*b^2)^(1/3)))/(a*b^9*x^6 + 2*a^2*b^8*x^3 + a^3*b^7), 1/1080*(13
5*a*b^6*f*x^14 + 54*(4*a*b^6*e - 7*a^2*b^5*f)*x^11 + 27*(20*a*b^6*d - 44*a^
2*b^5*e + 77*a^3*b^4*f)*x^8 - 96*(5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e -
 77*a^4*b^3*f)*x^5 - 60*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5
*b^2*f)*x^2 - 120*sqrt(1/3)*(5*a^3*b^4*c - 20*a^4*b^3*d + 44*a^5*b^2*e - 77
*a^6*b*f + (5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e - 77*a^4*b^3*f)*x^6 + 2
*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5*b^2*f)*x^3)*sqrt((a*b^
2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)
/b) + 20*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b^2*f)*x^6 + 5*a^2*
b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4*c - 20*a^2*b^3*d
+ 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)
*b*x + (a*b^2)^(2/3)) - 40*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b
^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4
*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b*x +
 (a*b^2)^(1/3)))/(a*b^9*x^6 + 2*a^2*b^8*x^3 + a^3*b^7)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.64, size = 391, normalized size = 1.13

$$\frac{\sqrt{3}\,(5\,b^3c-20\,ab^2d-77\,a^3f+44\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}b^5}-\frac{(5\,b^3c-20\,ab^2d-77\,a^3f+44\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}b^5}-\frac{\left(5\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-77\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+44\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^5}-\frac{8\,b^4cx^5-14\,ab^3dx^5-26\,a^3bfx^5+20\,a^2b^2x^5e+5\,ab^3cx^2-11\,a^2b^2dx^2-23\,a^4fx^2+17\,a^3bx^2e}{18\,(bx^3+a)^2b^5}+\frac{5\,b^{21}fx^8-24\,ab^{20}fx^5+8\,b^{21}x^5e+20\,b^{21}dx^2+120\,a^2b^{19}fx^2-60\,ab^{20}x^2e}{40\,b^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(5*b^3*c - 20*a*b^2*d - 77*a^3*f + 44*a^2*b*e)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^5) - 1/54*(5*b^3*c
 - 20*a*b^2*d - 77*a^3*f + 44*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))/((-a*b^2)^(1/3)*b^5) - 1/27*(5*b^3*c*(-a/b)^(1/3) - 20*a*b^2*d*(-a/b)^
(1/3) - 77*a^3*f*(-a/b)^(1/3) + 44*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a*b^5) - 1/18*(8*b^4*c*x^5 - 14*a*b^3*d*x^5 - 26*a^3
*b*f*x^5 + 20*a^2*b^2*x^5*e + 5*a*b^3*c*x^2 - 11*a^2*b^2*d*x^2 - 23*a^4*f*x
^2 + 17*a^3*b*x^2*e)/((b*x^3 + a)^2*b^5) + 1/40*(5*b^21*f*x^8 - 24*a*b^20*f
*x^5 + 8*b^21*x^5*e + 20*b^21*d*x^2 + 120*a^2*b^19*f*x^2 - 60*a*b^20*x^2*e)
/b^24
```

Mupad [**B**]
time = 5.53, size = 338, normalized size = 0.98

$$x^5\left(\frac{e}{5\,b^3}-\frac{3\,a\,f}{5\,b^4}\right)+\frac{x^2\left(\frac{23\,f\,a^4}{18}-\frac{17\,e\,a^3\,b}{18}+\frac{11\,d\,a^2\,b^2}{18}-\frac{5\,c\,a\,b^3}{18}\right)-x^5\left(-\frac{13\,f\,a^3\,b}{9}+\frac{10\,e\,a^2\,b^2}{9}-\frac{7\,d\,a\,b^3}{9}+\frac{4\,c\,b^4}{9}\right)}{a^2\,b^5+2\,a\,b^6\,x^3+b^7\,x^6}-x^2\left(\frac{3\,a^2\,f}{2\,b^5}-\frac{d}{2\,b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{2\,b}\right)+\frac{f\,x^8}{8\,b^3}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-77\,f\,a^3+44\,e\,a^2\,b-20\,d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{1/3}\,b^{17/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-77\,f\,a^3+44\,e\,a^2\,b-20\,d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{1/3}\,b^{17/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-77\,f\,a^3+44\,e\,a^2\,b-20\,d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{1/3}\,b^{17/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^5*(e/(5*b^3) - (3*a*f)/(5*b^4)) + (x^2*((23*a^4*f)/18 + (11*a^2*b^2*d)/18
 - (5*a*b^3*c)/18 - (17*a^3*b*e)/18) - x^5*((4*b^4*c)/9 + (10*a^2*b^2*e)/9
- (7*a*b^3*d)/9 - (13*a^3*b*f)/9))/(a^2*b^5 + b^7*x^6 + 2*a*b^6*x^3) - x^2*
((3*a^2*f)/(2*b^5) - d/(2*b^3) + (3*a*(e/b^3 - (3*a*f)/b^4))/(2*b)) + (f*x^
8)/(8*b^3) - (log(b^(1/3)*x + a^(1/3))*(5*b^3*c - 77*a^3*f - 20*a*b^2*d + 4
4*a^2*b*e))/(27*a^(1/3)*b^(17/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x -
 a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*b^3*c - 77*a^3*f - 20*a*b^2*d + 44*a^2*
b*e))/(27*a^(1/3)*b^(17/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/
3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c - 77*a^3*f - 20*a*b^2*d + 44*a^2*b*e))/
(27*a^(1/3)*b^(17/3))
```

3.290 $\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=336

$$\frac{(b^2d-3abe+6a^2f)\,x}{b^5}+\frac{(be-3af)x^4}{4b^4}+\frac{fx^7}{7b^3}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2}-\frac{(7b^3c-13ab^2d+19a^2be-25a}{18b^5\,(a+bx^3)}$$

```
[Out] (6*a^2*f-3*a*b*e+b^2*d)*x/b^5+1/4*(-3*a*f+b*e)*x^4/b^4+1/7*f*x^7/b^3+1/6*a*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^5/(b*x^3+a)^2-1/18*(-25*a^3*f+19*a^2*b*e
-13*a*b^2*d+7*b^3*c)*x/b^5/(b*x^3+a)+1/27*(-65*a^3*f+35*a^2*b*e-14*a*b^2*d+
2*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(16/3)-1/54*(-65*a^3*f+35*a^2*b*e-
14*a*b^2*d+2*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(16
/3)-1/27*(-65*a^3*f+35*a^2*b*e-14*a*b^2*d+2*b^3*c)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(16/3)*3^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1842, 1872, 1901, 206, 31, 648, 631, 210, 642}

$$\frac{x(6a^2f-3abe+b^2d)}{b^5}-\frac{x(-25a^3f+19a^2be-13ab^2d+7b^3c)}{18b^5\,(a+bx^3)}+\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5\,(a+bx^3)^2}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(-65a^3f+35a^2be-14ab^2d+2b^3c)}{9\sqrt{3}\,a^{2/3}b^{16/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(-65a^3f+35a^2be-14ab^2d+2b^3c)}{54a^{2/3}b^{16/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(-65a^3f+35a^2be-14ab^2d+2b^3c)}{27a^{2/3}b^{16/3}}+\frac{x^4(be-3af)}{4b^4}+\frac{fx^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x)/b^5 + ((b*e - 3*a*f)*x^4)/(4*b^4) + (f*x^7)
/(7*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^5*(a + b*x^3)^2)
- ((7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3*f)*x)/(18*b^5*(a + b*x^3)) -
 ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3
)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(16/3)) + ((2*b^3*c - 14*a*b^
2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(16/3)
) - ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*a^(2/3)*b^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
```

```
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2} - \frac{\int \frac{a^2(b^3c-ab^2d+a^2be-a^3f)-6ab(b^3c-ab^2d+a^2be-a^3}{(a}}{}$$

$$= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2} - \frac{(7b^3c-13ab^2d+19a^2be-25a^3f)\,x}{18b^5\,(a+bx^3)} + \frac{\int}{}$$

$$= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2} - \frac{(7b^3c-13ab^2d+19a^2be-25a^3f)\,x}{18b^5\,(a+bx^3)} + \frac{\int}{}$$

$$= \frac{(b^2d-3abe+6a^2f)\,x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a(b^3c-ab^2d+a^2be-a^3}{6b^5\,(a+bx^3)^2}$$

$$= \frac{(b^2d-3abe+6a^2f)\,x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a(b^3c-ab^2d+a^2be-a^3}{6b^5\,(a+bx^3)^2}$$

$$= \frac{(b^2d-3abe+6a^2f)\,x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a(b^3c-ab^2d+a^2be-a^3}{6b^5\,(a+bx^3)^2}$$

$$= \frac{(b^2d-3abe+6a^2f)\,x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a(b^3c-ab^2d+a^2be-a^3}{6b^5\,(a+bx^3)^2}$$

$$= \frac{(b^2d-3abe+6a^2f)\,x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a(b^3c-ab^2d+a^2be-a^3}{6b^5\,(a+bx^3)^2}$$

Mathematica [A]
time = 0.21, size = 323, normalized size = 0.96

$$\frac{756\sqrt[3]{b}\,(b^2d-3abe+6a^2f)\,x+189b^{4/3}(be-3af)x^4+108b^{7/3}fx^7+\frac{126a\sqrt[3]{b}\,(b^3c-ab^2d+a^2be-a^3f)x}{(a+bx^3)^2}-\frac{42\sqrt[3]{b}\,(7b^3c-13ab^2d+19a^2be-25a^3f)x}{a+bx^3}+\frac{28\sqrt{3}\,(-2b^3c+14ab^2d-35a^2be+65a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{28(2b^3c-14ab^2d+35a^2be-65a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}}+\frac{14(-2b^3c+14ab^2d-35a^2be+65a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}}}{756b^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (756*b^(1/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x + 189*b^(4/3)*(b*e - 3*a*f)*x^4
+ 108*b^(7/3)*f*x^7 + (126*a*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)
/(a + b*x^3)^2 - (42*b^(1/3)*(7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3*f)
*x)/(a + b*x^3) + (28*Sqrt[3]*(-2*b^3*c + 14*a*b^2*d - 35*a^2*b*e + 65*a^3*
f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(2*b^3*c - 14
*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(
-2*b^3*c + 14*a*b^2*d - 35*a^2*b*e + 65*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/a^(2/3))/(756*b^(16/3))
```

Maple [A]
time = 0.37, size = 253, normalized size = 0.75

method	result
risch	$\frac{f\,x^{7}}{7b^{3}}-\frac{3af\,x^{4}}{4b^{4}}+\frac{e\,x^{4}}{4b^{3}}+\frac{6a^{2}fx}{b^{5}}-\frac{3aex}{b^{4}}+\frac{dx}{b^{3}}+\frac{\left(\frac{25}{18}a^{3}bf-\frac{19}{18}a^{2}e\,b^{2}+\frac{13}{18}ad\,b^{3}-\frac{7}{18}c\,b^{4}\right)x^{4}+\frac{a\left(11a^{3}f-8a^{2}be+5a\,b^{2}d-2b^{3}c\right)x}{9}}{b^{5}\left(b\,x^{3}+a\right)^{2}}+$ … $(65a$
default	$\frac{\frac{1}{7}b^{2}f\,x^{7}-\frac{3}{4}abf\,x^{4}+\frac{1}{4}b^{2}e\,x^{4}+6a^{2}fx-3abex+b^{2}dx}{b^{5}}-\frac{\frac{\left(-\frac{25}{18}a^{3}bf+\frac{19}{18}a^{2}e\,b^{2}-\frac{13}{18}ad\,b^{3}+\frac{7}{18}c\,b^{4}\right)x^{4}-\frac{a\left(11a^{3}f-8a^{2}be+5a\,b^{2}d-2b^{3}c\right)x}{9}}{\left(b\,x^{3}+a\right)^{2}}+}{}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/7*b^2*f*x^7-3/4*a*b*f*x^4+1/4*b^2*e*x^4+6*a^2*f*x-3*a*b*e*x+b^2*d*
x)-1/b^5*(((-25/18*a^3*b*f+19/18*a^2*e*b^2-13/18*a*d*b^3+7/18*c*b^4)*x^4-1/
9*a*(11*a^3*f-8*a^2*b*e+5*a*b^2*d-2*b^3*c)*x)/(b*x^3+a)^2+1/9*(65*a^3*f-35*
a^2*b*e+14*a*b^2*d-2*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b
)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.50, size = 334, normalized size = 0.99

$$-\frac{{\left(7\,b^{4}c-13\,ab^{3}d-25\,a^{3}bf+19\,a^{2}b^{2}e\right)}x^{4}+2\,{\left(2\,ab^{3}c-5\,a^{2}b^{2}d-11\,a^{4}f+8\,a^{3}be\right)}x}{18\,{\left(b^{7}x^{6}+2\,ab^{6}x^{3}+a^{2}b^{5}\right)}}+\frac{4\,b^{2}fx^{7}-7\,{\left(3\,abf-b^{2}e\right)}x^{4}+28\,{\left(b^{2}d+6\,a^{2}f-3\,abe\right)}x}{28\,b^{5}}+\frac{\sqrt{3}\,{\left(2\,b^{3}c-14\,ab^{2}d-65\,a^{3}f+35\,a^{2}be\right)}\arctan\left(\frac{\sqrt{3}\,{\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^{6}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{{\left(2\,b^{3}c-14\,ab^{2}d-65\,a^{3}f+35\,a^{2}be\right)}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^{6}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{{\left(2\,b^{3}c-14\,ab^{2}d-65\,a^{3}f+35\,a^{2}be\right)}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^{6}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((7*b^4*c - 13*a*b^3*d - 25*a^3*b*f + 19*a^2*b^2*e)*x^4 + 2*(2*a*b^3*
c - 5*a^2*b^2*d - 11*a^4*f + 8*a^3*b*e)*x)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5
) + 1/28*(4*b^2*f*x^7 - 7*(3*a*b*f - b^2*e)*x^4 + 28*(b^2*d + 6*a^2*f - 3*a
*b*e)*x)/b^5 + 1/27*sqrt(3)*(2*b^3*c - 14*a*b^2*d - 65*a^3*f + 35*a^2*b*e)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(2/3)) - 1/5
4*(2*b^3*c - 14*a*b^2*d - 65*a^3*f + 35*a^2*b*e)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(b^6*(a/b)^(2/3)) + 1/27*(2*b^3*c - 14*a*b^2*d - 65*a^3*f + 35
*a^2*b*e)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(291) = 582$.
time = 0.42, size = 1318, normalized size = 3.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/756*(108*a^2*b^5*f*x^13 + 27*(7*a^2*b^5*e - 13*a^3*b^4*f)*x^10 + 54*(14*
a^2*b^5*d - 35*a^3*b^4*e + 65*a^4*b^3*f)*x^7 - 147*(2*a^2*b^5*c - 14*a^3*b^
4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^4 - 42*sqrt(1/3)*(2*a^3*b^4*c - 14*a^4
*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f + (2*a*b^6*c - 14*a^2*b^5*d + 35*a^3*b^4
*e - 65*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3*e - 65*
a^5*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*
x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqr
t((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*
e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f
+ 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*(-a^2*b)^(2
/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*((2*b^5*c - 14*
a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 3
5*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*
b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(2*a^3*b^4*c - 14
*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f)*x)/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a
^4*b^6), 1/756*(108*a^2*b^5*f*x^13 + 27*(7*a^2*b^5*e - 13*a^3*b^4*f)*x^10 +
 54*(14*a^2*b^5*d - 35*a^3*b^4*e + 65*a^4*b^3*f)*x^7 - 147*(2*a^2*b^5*c - 1
4*a^3*b^4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^4 + 84*sqrt(1/3)*(2*a^3*b^4*c
- 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f + (2*a*b^6*c - 14*a^2*b^5*d + 35
*a^3*b^4*e - 65*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3
*e - 65*a^5*b^2*f)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b
)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 14*((2*b^5*c -
 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d
 + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*
a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3
)*a) + 28*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2
*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d
```

```
 + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3
)) - 84*(2*a^3*b^4*c - 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f)*x)/(a^2*b^
8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.83, size = 345, normalized size = 1.03

$$-\frac{\sqrt{3}\,(2\,b^3c-14\,ab^2d-65\,a^3f+35\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}b^4}-\frac{(2\,b^3c-14\,ab^2d-65\,a^3f+35\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{2}{3}}b^4}-\frac{(2\,b^3c-14\,ab^2d-65\,a^3f+35\,a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^5}-\frac{7\,b^4cx^4-13\,ab^3dx^4-25\,a^3bfx^4+19\,a^2b^2x^4e+4\,ab^3cx-10\,a^2b^2dx-22\,a^4fx+16\,a^3bxe}{18\,(bx^3+a)^2b^5}+\frac{4\,b^{18}fx^7-21\,ab^{17}fx^4+7\,b^{18}x^4e+28\,b^{18}dx+168\,a^2b^{16}fx-84\,ab^{17}xe}{28\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*b^3*c - 14*a*b^2*d - 65*a^3*f + 35*a^2*b*e)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^4) - 1/54*(2*b^3*
c - 14*a*b^2*d - 65*a^3*f + 35*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(2/3)*b^4) - 1/27*(2*b^3*c - 14*a*b^2*d - 65*a^3*f + 35*a^2
*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) - 1/18*(7*b^4*c*x^4 -
 13*a*b^3*d*x^4 - 25*a^3*b*f*x^4 + 19*a^2*b^2*x^4*e + 4*a*b^3*c*x - 10*a^2*
b^2*d*x - 22*a^4*f*x + 16*a^3*b*x*e)/((b*x^3 + a)^2*b^5) + 1/28*(4*b^18*f*x
^7 - 21*a*b^17*f*x^4 + 7*b^18*x^4*e + 28*b^18*d*x + 168*a^2*b^16*f*x - 84*a
*b^17*x*e)/b^21
```

Mupad [**B**]
time = 5.30, size = 335, normalized size = 1.00

$$x^4\left(\frac{e}{4\,b^3}-\frac{3\,a\,f}{4\,b^4}\right)-x\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)-\frac{x^4\left(-\frac{25\,f\,a^3\,b}{18}+\frac{19\,e\,a^2\,b^2}{18}-\frac{13\,d\,a\,b^3}{18}+\frac{7\,c\,b^4}{18}\right)-x\left(\frac{11\,f\,a^4}{9}-\frac{8\,e\,a^3\,b}{9}+\frac{5\,d\,a^2\,b^2}{9}-\frac{2\,c\,a\,b^3}{9}\right)}{a^2\,b^5+2\,a\,b^6\,x^3+b^7\,x^6}+\frac{f\,x^7}{7\,b^3}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-65\,f\,a^3+35\,e\,a^2\,b-14\,d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{2/3}\,b^{16/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-65\,f\,a^3+35\,e\,a^2\,b-14\,d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{2/3}\,b^{16/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-65\,f\,a^3+35\,e\,a^2\,b-14\,d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{2/3}\,b^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^4*(e/(4*b^3) - (3*a*f)/(4*b^4)) - x*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3
 - (3*a*f)/b^4))/b) - (x^4*((7*b^4*c)/18 + (19*a^2*b^2*e)/18 - (13*a*b^3*d)/
18 - (25*a^3*b*f)/18) - x*((11*a^4*f)/9 + (5*a^2*b^2*d)/9 - (2*a*b^3*c)/9 -
 (8*a^3*b*e)/9))/(a^2*b^5 + b^7*x^6 + 2*a*b^6*x^3) + (f*x^7)/(7*b^3) + (log
(b^(1/3)*x + a^(1/3))*(2*b^3*c - 65*a^3*f - 14*a*b^2*d + 35*a^2*b*e))/(27*a
^(2/3)*b^(16/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/
2)*1i)/2 - 1/2)*(2*b^3*c - 65*a^3*f - 14*a*b^2*d + 35*a^2*b*e))/(27*a^(2/3)
*b^(16/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)
/2 + 1/2)*(2*b^3*c - 65*a^3*f - 14*a*b^2*d + 35*a^2*b*e))/(27*a^(2/3)*b^(16
/3))
```

3.291 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=316

$$\frac{(be-3af)x^2}{2b^4}+\frac{fx^5}{5b^3}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2}+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\,x^2}{9ab^4\,(a+bx^3)}-\frac{(b^3c+5ab^2d-20a}{}$$

```
[Out] 1/2*(-3*a*f+b*e)*x^2/b^4+1/5*f*x^5/b^3-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x
^2/b^4/(b*x^3+a)^2+1/9*(-10*a^3*f+7*a^2*b*e-4*a*b^2*d+b^3*c)*x^2/a/b^4/(b*x
^3+a)-1/27*(44*a^3*f-20*a^2*b*e+5*a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(4
/3)/b^(14/3)+1/54*(44*a^3*f-20*a^2*b*e+5*a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(14/3)-1/27*(44*a^3*f-20*a^2*b*e+5*a*b^2*d
+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(14/3)*
3^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1842, 1865, 1608, 1502, 298, 31, 648, 631, 210, 642}

$$\frac{x^2(-10a^3f+7a^2be-4ab^2d+b^3c)}{9ab^4\,(a+bx^3)}-\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^4\,(a+bx^3)^2}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(44a^3f-20a^2be+5ab^2d+b^3c)}{9\sqrt{3}\,a^{4/3}b^{14/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(44a^3f-20a^2be+5ab^2d+b^3c)}{54a^{4/3}b^{14/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(44a^3f-20a^2be+5ab^2d+b^3c)}{27a^{4/3}b^{14/3}}+\frac{x^2(be-3af)}{2b^4}+\frac{fx^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x^2)/(2*b^4) + (f*x^5)/(5*b^3) - ((b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*x^2)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10
*a^3*f)*x^2)/(9*a*b^4*(a + b*x^3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3
)*b^(14/3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^
(1/3)*x])/(27*a^(4/3)*b^(14/3)) + ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3
*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
```

```
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1865

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} - \frac{\int \frac{-2ab(b^3c-ab^2d+a^2be-a^3f)x-6ab^2(b^2d-abe+a^2 \cdots}{(a+bx^3)^2}}{6ab^5}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} - \frac{\int \frac{x(-2ab(b^3c-ab^2d+a^2be-a^3f)-6ab^2(b^2d-abe+a^2 \cdots}{(a+bx^3)^2}}{6ab^5}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-4ab^2d+7a^2be-10a^3f)\,x^2}{9ab^4\,(a+bx^3)} + \int \cdots$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-4ab^2d+7a^2be-10a^3f)\,x^2}{9ab^4\,(a+bx^3)} + \int \cdots$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-4ab^2d+7a^2be-10a^3f)\,x^2}{9ab^4\,(a+bx^3)} + \int \cdots$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-4ab^2d+7a \cdots}{9ab^4\,(a \cdots}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-4ab^2d+7a \cdots}{9ab^4\,(a \cdots}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-4ab^2d+7a \cdots}{9ab^4\,(a \cdots}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-4ab^2d+7a \cdots}{9ab^4\,(a \cdots}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-4ab^2d+7a \cdots}{9ab^4\,(a \cdots}$$

Mathematica [A]
time = 0.14, size = 300, normalized size = 0.95

$$\frac{135b^{2/3}(be-3af)x^2+54b^{5/3}fx^5-\frac{45b^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{(a+bx^3)^2}+\frac{30b^{2/3}(b^3c-4ab^2d+7a^2be-10a^3f)x^2}{a(a+bx^3)}-\frac{10\sqrt{3}\,(b^3c+5ab^2d-20a^2be+44a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}-\frac{10(b^3c+5ab^2d-20a^2be+44a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{4/3}}+\frac{5(b^3c+5ab^2d-20a^2be+44a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{4/3}}}{270b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (135*b^(2/3)*(b*e - 3*a*f)*x^2 + 54*b^(5/3)*f*x^5 - (45*b^(2/3)*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*x^2)/(a + b*x^3)^2 + (30*b^(2/3)*(b^3*c - 4*a*b^2*
d + 7*a^2*b*e - 10*a^3*f)*x^2)/(a*(a + b*x^3)) - (10*Sqrt[3]*(b^3*c + 5*a*b
^2*d - 20*a^2*b*e + 44*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/
a^(4/3) - (10*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^(
1/3)*x])/a^(4/3) + (5*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(270*b^(14/3))
```

Maple [A]
time = 0.36, size = 233, normalized size = 0.74

method	result
risch	$\frac{f\,x^5}{5b^3}-\frac{3x^2af}{2b^4}+\frac{e\,x^2}{2b^3}+\frac{-\frac{b\left(10a^3f-7a^2be+4a\,b^2d-b^3c\right)x^5}{9a}+\left(-\frac{17}{18}a^3f+\frac{11}{18}a^2be-\frac{5}{18}a\,b^2d-\frac{1}{18}b^3c\right)x^2}{b^4\left(b\,x^3+a\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\left(44a\right.}{}$
default	$-\frac{-\frac{bf\,x^5}{5}+\frac{(3af-be)x^2}{2}}{b^4}+\frac{\frac{-\frac{b\left(10a^3f-7a^2be+4a\,b^2d-b^3c\right)x^5}{9a}+\left(-\frac{17}{18}a^3f+\frac{11}{18}a^2be-\frac{5}{18}a\,b^2d-\frac{1}{18}b^3c\right)x^2}{\left(b\,x^3+a\right)^2}+\frac{\left(44a^3f-20a^2be+5a\,b^2d+b^3c\right)\left(-\frac{\ln}{}\right.}{}}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/5*b*f*x^5+1/2*(3*a*f-b*e)*x^2)+1/b^4*((-1/9*b*(10*a^3*f-7*a^2*b*
e+4*a*b^2*d-b^3*c)/a*x^5+(-17/18*a^3*f+11/18*a^2*b*e-5/18*a*b^2*d-1/18*b^3*
c)*x^2)/(b*x^3+a)^2+1/9*(44*a^3*f-20*a^2*b*e+5*a*b^2*d+b^3*c)/a*(-1/3/b/(a/
b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.51, size = 318, normalized size = 1.01

$$\frac{2\,(b^4c-4\,ab^3d-10\,a^3bf+7\,a^2b^2e)x^5-(ab^3c+5\,a^2b^2d+17\,a^4f-11\,a^3be)x^2}{18\,(ab^6x^6+2\,a^2b^5x^3+a^3b^4)}+\frac{2\,bfx^5-5\,(3\,af-be)x^2}{10\,b^4}+\frac{\sqrt{3}\,(b^3c+5\,ab^2d+44\,a^3f-20\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(b^3c+5\,ab^2d+44\,a^3f-20\,a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(b^3c+5\,ab^2d+44\,a^3f-20\,a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(b^4*c - 4*a*b^3*d - 10*a^3*b*f + 7*a^2*b^2*e)*x^5 - (a*b^3*c + 5*a
^2*b^2*d + 17*a^4*f - 11*a^3*b*e)*x^2)/(a*b^6*x^6 + 2*a^2*b^5*x^3 + a^3*b^4
) + 1/10*(2*b*f*x^5 - 5*(3*a*f - b*e)*x^2)/b^4 + 1/27*sqrt(3)*(b^3*c + 5*a*
b^2*d + 44*a^3*f - 20*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/(a*b^5*(a/b)^(1/3)) + 1/54*(b^3*c + 5*a*b^2*d + 44*a^3*f - 20*a^2*b
*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(1/3)) - 1/27*(b^3*
c + 5*a*b^2*d + 44*a^3*f - 20*a^2*b*e)*log(x + (a/b)^(1/3))/(a*b^5*(a/b)^(1
/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. 2(271) = 542.
time = 0.43, size = 1224, normalized size = 3.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/270*(54*a^2*b^5*f*x^11 + 27*(5*a^2*b^5*e - 11*a^3*b^4*f)*x^8 + 6*(5*a*b^
6*c - 20*a^2*b^5*d + 80*a^3*b^4*e - 176*a^4*b^3*f)*x^5 - 15*(a^2*b^5*c + 5*
a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^2 + 15*sqrt(1/3)*(a^3*b^4*c + 5*
a^4*b^3*d - 20*a^5*b^2*e + 44*a^6*b*f + (a*b^6*c + 5*a^2*b^5*d - 20*a^3*b^4
*e + 44*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5
*b^2*f)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b
*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-
a*b^2)^(2/3)*x)/(b*x^3 + a)) + 5*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^
3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c
 + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2
 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*((b^5*c + 5*a*b^4*d - 20*a^2*b
^3*e + 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f
+ 2*(a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)
*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6), 1/270*
(54*a^2*b^5*f*x^11 + 27*(5*a^2*b^5*e - 11*a^3*b^4*f)*x^8 + 6*(5*a*b^6*c - 2
0*a^2*b^5*d + 80*a^3*b^4*e - 176*a^4*b^3*f)*x^5 - 15*(a^2*b^5*c + 5*a^3*b^4
*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^2 + 30*sqrt(1/3)*(a^3*b^4*c + 5*a^4*b^3
*d - 20*a^5*b^2*e + 44*a^6*b*f + (a*b^6*c + 5*a^2*b^5*d - 20*a^3*b^4*e + 44
*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)
```

```
*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqr
t(-(-a*b^2)^(1/3)/a)/b) + 5*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^3*b^2
*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c + 5*
a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-
a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e
+ 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(
a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)*log(
b*x - (-a*b^2)^(1/3)))/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.32, size = 365, normalized size = 1.16

$$\frac{\sqrt{3}\,(b^3c+5\,ab^2d+44\,a^3f-20\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{1}{3}}ab^4}-\frac{(b^3c+5\,ab^2d+44\,a^3f-20\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{1}{3}}ab^4}-\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+44\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^4}+\frac{2\,b^4cx^5-8\,ab^3dx^5-20\,a^3bfx^5+14\,a^2b^2x^5e-ab^3cx^2-5\,a^2b^2dx^2-17\,a^4fx^2+11\,a^3bx^2e}{18\,(bx^3+a)^2ab^4}+\frac{2\,b^{12}fx^5-15\,ab^{11}fx^2+5\,b^{12}x^2e}{10\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(b^3*c + 5*a*b^2*d + 44*a^3*f - 20*a^2*b*e)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a*b^4) - 1/54*(b^3*c +
5*a*b^2*d + 44*a^3*f - 20*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/((-a*b^2)^(1/3)*a*b^4) - 1/27*(b^3*c*(-a/b)^(1/3) + 5*a*b^2*d*(-a/b)^(1/3)
 + 44*a^3*f*(-a/b)^(1/3) - 20*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^2*b^4) + 1/18*(2*b^4*c*x^5 - 8*a*b^3*d*x^5 - 20*a^3*b*f
*x^5 + 14*a^2*b^2*x^5*e - a*b^3*c*x^2 - 5*a^2*b^2*d*x^2 - 17*a^4*f*x^2 + 11
*a^3*b*x^2*e)/((b*x^3 + a)^2*a*b^4) + 1/10*(2*b^12*f*x^5 - 15*a*b^11*f*x^2
+ 5*b^12*x^2*e)/b^15
```

Mupad [**B**]
time = 5.27, size = 295, normalized size = 0.93

$$x^2\left(\frac{e}{2\,b^3}-\frac{3\,a\,f}{2\,b^4}\right)-\frac{x^2\left(\frac{17\,f\,a^3}{18}-\frac{11\,e\,a^2\,b}{18}+\frac{5\,d\,a\,b^2}{18}+\frac{c\,b^3}{18}\right)-\frac{x^5\,\left(-10\,f\,a^3\,b+7\,e\,a^2\,b^2-4\,d\,a\,b^3+c\,b^4\right)}{9\,a}}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+\frac{f\,x^5}{5\,b^3}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(44\,f\,a^3-20\,e\,a^2\,b+5\,d\,a\,b^2+c\,b^3\right)}{27\,a^{4/3}\,b^{14/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(44\,f\,a^3-20\,e\,a^2\,b+5\,d\,a\,b^2+c\,b^3\right)}{27\,a^{4/3}\,b^{14/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(44\,f\,a^3-20\,e\,a^2\,b+5\,d\,a\,b^2+c\,b^3\right)}{27\,a^{4/3}\,b^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^2*(e/(2*b^3) - (3*a*f)/(2*b^4)) - (x^2*((b^3*c)/18 + (17*a^3*f)/18 + (5*a
*b^2*d)/18 - (11*a^2*b*e)/18) - (x^5*(b^4*c + 7*a^2*b^2*e - 4*a*b^3*d - 10*
```

```
a^3*b*f))/(9*a))/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + (f*x^5)/(5*b^3) - (log
(b^(1/3)*x + a^(1/3))*(b^3*c + 44*a^3*f + 5*a*b^2*d - 20*a^2*b*e))/(27*a^(4
/3)*b^(14/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*
1i)/2 + 1/2)*(b^3*c + 44*a^3*f + 5*a*b^2*d - 20*a^2*b*e))/(27*a^(4/3)*b^(14
/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1
/2)*(b^3*c + 44*a^3*f + 5*a*b^2*d - 20*a^2*b*e))/(27*a^(4/3)*b^(14/3))
```

3.292 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=307

$$\frac{(be-3af)x}{b^4}+\frac{fx^4}{4b^3}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2}+\frac{(b^3c-7ab^2d+13a^2be-19a^3f)\,x}{18ab^4\,(a+bx^3)}-\frac{(b^3c+2ab^2d-14a^2be}{9\surd}$$

```
[Out] (-3*a*f+b*e)*x/b^4+1/4*f*x^4/b^3-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^4/(
b*x^3+a)^2+1/18*(-19*a^3*f+13*a^2*b*e-7*a*b^2*d+b^3*c)*x/a/b^4/(b*x^3+a)+1/
27*(35*a^3*f-14*a^2*b*e+2*a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(1
3/3)-1/54*(35*a^3*f-14*a^2*b*e+2*a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/a^(5/3)/b^(13/3)-1/27*(35*a^3*f-14*a^2*b*e+2*a*b^2*d+b^3*c)*
arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(13/3)*3^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1842, 1872, 1425, 396, 206, 31, 648, 631, 210, 642}

$$\frac{x(-19a^3f+13a^2be-7ab^2d+b^3c)}{18ab^4\,(a+bx^3)}-\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^4\,(a+bx^3)^2}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(35a^3f-14a^2be+2ab^2d+b^3c)}{9\sqrt{3}\,a^{5/3}b^{13/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(35a^3f-14a^2be+2ab^2d+b^3c)}{54a^{5/3}b^{13/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(35a^3f-14a^2be+2ab^2d+b^3c)}{27a^{5/3}b^{13/3}}+\frac{x(be-3af)}{b^4}+\frac{fx^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^4)/(4*b^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*x)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 7*a*b^2*d + 13*a^2*b*e - 19*a^3*f)
*x)/(18*a*b^4*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(13/
3)) + ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x]
)/(27*a^(5/3)*b^(13/3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
```

```
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} - \frac{\int \frac{-a(b^3c-ab^2d+a^2be-a^3f)-6ab(b^2d-abe+a^2f)x^3-6a}{(a+bx^3)^2}}{6ab^4}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-7ab^2d+13a^2be-19a^3f)\,x}{18ab^4\,(a+bx^3)} + \frac{\int \frac{2ab}{}}{}$$

$$= \frac{fx^4}{4b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-7ab^2d+13a^2be-19a^3f)\,x}{18ab^4\,(a+bx^3)} +$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-7ab^2d+13a^2b}{18ab^4\,(a+b}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-7ab^2d+13a^2b}{18ab^4\,(a+b}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-7ab^2d+13a^2b}{18ab^4\,(a+b}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-7ab^2d+13a^2b}{18ab^4\,(a+b}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-7ab^2d+13a^2b}{18ab^4\,(a+b}$$

Mathematica [A]

time = 0.14, size = 294, normalized size = 0.96

$$\frac{108\sqrt[3]{b}\,(be-3af)x+27b^{4/3}fx^4-\frac{18\sqrt[3]{b}\,(b^3c-ab^2d+a^2be-a^3f)x}{(a+bx^3)^2}+\frac{6\sqrt[3]{b}\,(b^3c-7ab^2d+13a^2be-19a^3f)x}{a(a+bx^3)}-\frac{4\sqrt{3}\,(b^3c+2ab^2d-14a^2be+35a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{4(b^3c+2ab^2d-14a^2be+35a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}}-\frac{2(b^3c+2ab^2d-14a^2be+35a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}}{108b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (108*b^(1/3)*(b*e - 3*a*f)*x + 27*b^(4/3)*f*x^4 - (18*b^(1/3)*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3)^2 + (6*b^(1/3)*(b^3*c - 7*a*b^2*d + 1
3*a^2*b*e - 19*a^3*f)*x)/(a*(a + b*x^3)) - (4*Sqrt[3]*(b^3*c + 2*a*b^2*d -
14*a^2*b*e + 35*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3)
 + (4*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])
/a^(5/3) - (2*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(108*b^(13/3))
```

Maple [A]

time = 0.35, size = 227, normalized size = 0.74

method	result
risch	$\frac{f x^4}{4b^3}-\frac{3afx}{b^4}+\frac{ex}{b^3}+\frac{-\frac{b(19a^3f-13a^2be+7a\,b^2d-b^3c)x^4}{18a}+(-\frac{8}{9}a^3f+\frac{5}{9}a^2be-\frac{2}{9}a\,b^2d-\frac{1}{9}b^3c)x}{b^4(b\,x^3+a)^2}+\frac{\sum_{\_R=\text{RootOf}(b\_Z^3+a)}\frac{(35a^3f-1\cdots}{\cdots}}{27b^5\cdots}$
default	$-\frac{-\frac{1}{4}bf\,x^4+3afx-bex}{b^4}+\frac{\frac{-\frac{b(19a^3f-13a^2be+7a\,b^2d-b^3c)x^4}{18a}+\left(-\frac{8}{9}a^3f+\frac{5}{9}a^2be-\frac{2}{9}a\,b^2d-\frac{1}{9}b^3c\right)x}{(b\,x^3+a)^2}+\frac{(35a^3f-14a^2be+2a\,b^2d+b^3c)\left(\frac{\ln(x+\cdots}{3b(\cdots}\cdots\right.}{\cdots}}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/4*b*f*x^4+3*a*f*x-b*e*x)+1/b^4*((-1/18*b*(19*a^3*f-13*a^2*b*e+7*
a*b^2*d-b^3*c)/a*x^4+(-8/9*a^3*f+5/9*a^2*b*e-2/9*a*b^2*d-1/9*b^3*c)*x)/(b*x
^3+a)^2+1/9*(35*a^3*f-14*a^2*b*e+2*a*b^2*d+b^3*c)/a*(1/3/b/(a/b)^(2/3)*ln(x
+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.51, size = 312, normalized size = 1.02

$$\frac{(b^4c-7\,ab^3d-19\,a^3bf+13\,a^2b^2e)x^4-2\,(ab^3c+2\,a^2b^2d+8\,a^4f-5\,a^3be)x}{18\,(ab^6x^6+2\,a^2b^5x^3+a^3b^4)}+\frac{bfx^4-4\,(3\,af-be)x}{4\,b^4}+\frac{\sqrt{3}\,(b^3c+2\,ab^2d+35\,a^3f-14\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(b^3c+2\,ab^2d+35\,a^3f-14\,a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(b^3c+2\,ab^2d+35\,a^3f-14\,a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((b^4*c - 7*a*b^3*d - 19*a^3*b*f + 13*a^2*b^2*e)*x^4 - 2*(a*b^3*c + 2*
a^2*b^2*d + 8*a^4*f - 5*a^3*b*e)*x)/(a*b^6*x^6 + 2*a^2*b^5*x^3 + a^3*b^4) +
 1/4*(b*f*x^4 - 4*(3*a*f - b*e)*x)/b^4 + 1/27*sqrt(3)*(b^3*c + 2*a*b^2*d +
35*a^3*f - 14*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/
(a*b^5*(a/b)^(2/3)) - 1/54*(b^3*c + 2*a*b^2*d + 35*a^3*f - 14*a^2*b*e)*log(
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(2/3)) + 1/27*(b^3*c + 2*a*
b^2*d + 35*a^3*f - 14*a^2*b*e)*log(x + (a/b)^(1/3))/(a*b^5*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(264) = 528.
time = 0.42, size = 1213, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/108*(27*a^3*b^4*f*x^10 + 54*(2*a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 3*(2*a^2*b
^5*c - 14*a^3*b^4*d + 98*a^4*b^3*e - 245*a^5*b^2*f)*x^4 + 6*sqrt(1/3)*(a^3*
b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e + 35*a^6*b*f + (a*b^6*c + 2*a^2*b^5*d -
14*a^3*b^4*e + 35*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 2*a^3*b^4*d - 14*a^4*b^3*
e + 35*a^5*b^2*f)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1
/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)
*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 2*((b^5*c + 2*a*b^4*d - 14*a^2*b^3*
e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2
*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(2/3)*log
(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*((b^5*c + 2*a*b^4*d - 14*
a^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a
^5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(
2/3)*log(a*b*x + (a^2*b)^(2/3)) - 12*(a^3*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*
e + 35*a^6*b*f)*x)/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5), 1/108*(27*a^3*b
^4*f*x^10 + 54*(2*a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 3*(2*a^2*b^5*c - 14*a^3*b^
4*d + 98*a^4*b^3*e - 245*a^5*b^2*f)*x^4 + 12*sqrt(1/3)*(a^3*b^4*c + 2*a^4*b
^3*d - 14*a^5*b^2*e + 35*a^6*b*f + (a*b^6*c + 2*a^2*b^5*d - 14*a^3*b^4*e +
35*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 2*a^3*b^4*d - 14*a^4*b^3*e + 35*a^5*b^2*
f)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)
^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*((b^5*c + 2*a*b^4*d - 14*a^2*b^3*e
```

```
 + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2*
(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*((b^5*c + 2*a*b^4*d - 14*a
^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^
5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) - 12*(a^3*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e
 + 35*a^6*b*f)*x)/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.12, size = 319, normalized size = 1.04

$$-\frac{\sqrt{3}\,(b^3c+2\,ab^2d+35\,a^3f-14\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,ab^3}-\frac{(b^3c+2\,ab^2d+35\,a^3f-14\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,ab^3}-\frac{(b^3c+2\,ab^2d+35\,a^3f-14\,a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^4}+\frac{b^4cx^4-7\,ab^3dx^4-19\,a^3bfx^4+13\,a^2b^2x^4e-2\,ab^3cx-4\,a^2b^2dx-16\,a^4fx+10\,a^3bxe}{18\,(bx^3+a)^2ab^4}+\frac{b^9fx^4-12\,ab^8fx+4\,b^9xe}{4\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(b^3*c + 2*a*b^2*d + 35*a^3*f - 14*a^2*b*e)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^3) - 1/54*(b^3*c +
 2*a*b^2*d + 35*a^3*f - 14*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/((-a*b^2)^(2/3)*a*b^3) - 1/27*(b^3*c + 2*a*b^2*d + 35*a^3*f - 14*a^2*b*e)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^4) + 1/18*(b^4*c*x^4 - 7*a*
b^3*d*x^4 - 19*a^3*b*f*x^4 + 13*a^2*b^2*x^4*e - 2*a*b^3*c*x - 4*a^2*b^2*d*x
 - 16*a^4*f*x + 10*a^3*b*x*e)/((b*x^3 + a)^2*a*b^4) + 1/4*(b^9*f*x^4 - 12*a
*b^8*f*x + 4*b^9*x*e)/b^12
```

Mupad [**B**]
time = 5.14, size = 290, normalized size = 0.94

$$x\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)-\frac{x\left(\frac{8\,f\,a^3}{9}-\frac{5\,e\,a^2\,b}{9}+\frac{2\,d\,a\,b^2}{9}+\frac{c\,b^3}{9}\right)-\frac{x^4\left(-19\,f\,a^3\,b+13\,e\,a^2\,b^2-7\,d\,a\,b^3+c\,b^4\right)}{18\,a}}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+\frac{f\,x^4}{4\,b^3}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(35\,f\,a^3-14\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{27\,a^{5/3}\,b^{13/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(35\,f\,a^3-14\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{27\,a^{5/3}\,b^{13/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(35\,f\,a^3-14\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{27\,a^{5/3}\,b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x*(e/b^3 - (3*a*f)/b^4) - (x*((b^3*c)/9 + (8*a^3*f)/9 + (2*a*b^2*d)/9 - (5*
a^2*b*e)/9) - (x^4*(b^4*c + 13*a^2*b^2*e - 7*a*b^3*d - 19*a^3*b*f))/(18*a))
/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + (f*x^4)/(4*b^3) + (log(b^(1/3)*x + a^(
```

```
1/3))*(b^3*c + 35*a^3*f + 2*a*b^2*d - 14*a^2*b*e))/(27*a^(5/3)*b^(13/3)) +
(log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^
3*c + 35*a^3*f + 2*a*b^2*d - 14*a^2*b*e))/(27*a^(5/3)*b^(13/3)) - (log(3^(1
/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c + 35*
a^3*f + 2*a*b^2*d - 14*a^2*b*e))/(27*a^(5/3)*b^(13/3))
```

3.293 $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$

Optimal. Leaf size=301

$$\frac{fx^2}{2b^3}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2}+\frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)}-\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\tan^{-}}{9\sqrt{3}\,a^{7/3}b^{11/3}}$$

```
[Out] 1/2*f*x^2/b^3+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a/b^3/(b*x^3+a)^2+1/9*
(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)*x^2/a^2/b^3/(b*x^3+a)-1/27*(-20*a^3*f+5
*a^2*b*e+a*b^2*d+2*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(11/3)+1/54*(-20*
a^3*f+5*a^2*b*e+a*b^2*d+2*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
a^(7/3)/b^(11/3)-1/27*(-20*a^3*f+5*a^2*b*e+a*b^2*d+2*b^3*c)*arctan(1/3*(a^(
1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(11/3)*3^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1842, 1608, 1496, 470, 298, 31, 648, 631, 210, 642}

$$\frac{x^2(7a^3f-4a^2be+ab^2d+2b^3c)}{9a^2b^3\,(a+bx^3)}+\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6ab^3\,(a+bx^3)^2}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(-20a^3f+5a^2be+ab^2d+2b^3c)}{9\sqrt{3}\,a^{7/3}b^{11/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(-20a^3f+5a^2be+ab^2d+2b^3c)}{54a^{7/3}b^{11/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(-20a^3f+5a^2be+ab^2d+2b^3c)}{27a^{7/3}b^{11/3}}+\frac{fx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (f*x^2)/(2*b^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a*b^3*(a + b
*x^3)^2) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2)/(9*a^2*b^3*(a +
b*x^3)) - ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(11/3)) - ((2*b^3*c + a*
b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(11/3
)) + ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1496

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
 Mod[m, n])/n)*(q + 1))), x] + Dist[1/(n*e^(2*p + (m - Mod[m, n])/n)*(q + 1
)), Int[x^Mod[m, n]*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n)
)*(n*e^(2*p + (m - Mod[m, n])/n)*(q + 1)*x^(m - Mod[m, n])*(a + b*x^n + c*x
^(2*n))^p - (-d)^((m - Mod[m, n])/n - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d*(Mod[
m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGt
Q[p, 0] && ILtQ[q, -1] && IGtQ[m, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2} - \frac{\int \frac{-2b(2b^3c+ab^2d-a^2be+a^3f)x-6ab^2(be-af)x^4-6ab^3f}{(a+bx^3)^2}}{6ab^4}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2} - \frac{\int \frac{x(-2b(2b^3c+ab^2d-a^2be+a^3f)-6ab^2(be-af)x^3-6ab^3}{(a+bx^3)^2}}{6ab^4}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2} + \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)} + \int \frac{x(2b^3}{}$$

$$= \frac{fx^2}{2b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2} + \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)} +$$

$$= \frac{fx^2}{2b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2} + \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)} -$$

$$= \frac{fx^2}{2b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2} + \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)} -$$

$$= \frac{fx^2}{2b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2} + \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)} -$$

$$= \frac{fx^2}{2b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2} + \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)} -$$

Mathematica [A]
time = 0.13, size = 284, normalized size = 0.94

$$\frac{27b^{2/3}fx^2+\frac{9b^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{a(a+bx^3)^2}+\frac{6b^{2/3}(2b^3c+ab^2d-4a^2be+7a^3f)x^2}{a^2(a+bx^3)}-\frac{2\sqrt{3}\,(2b^3c+ab^2d+5a^2be-20a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}}-\frac{2(2b^3c+ab^2d+5a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{7/3}}+\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{7/3}}}{54b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (27*b^(2/3)*f*x^2 + (9*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a*
(a + b*x^3)^2) + (6*b^(2/3)*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2)/
(a^2*(a + b*x^3)) - (2*Sqrt[3]*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(7/3) - (2*(2*b^3*c + a*b^2*d
 + 5*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(7/3) + ((2*b^3*c + a*
b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/a^(7/3))/(54*b^(11/3))
```

Maple [A]
time = 0.36, size = 220, normalized size = 0.73

method	result
risch	$\frac{f x^2}{2b^3}+\frac{\frac{b\left(7a^3f-4a^2be+a\,b^2d+2b^3c\right)x^5}{9a^2}+\frac{\left(11a^3f-5a^2be-a\,b^2d+7b^3c\right)x^2}{18a}}{b^3(b\,x^3+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(20a^3f-5a^2be-a\,b^2d-2b^3c\right)\ln\left(x-\_R\right.}{\_R}}{27b^4a^2}$
default	$\frac{f x^2}{2b^3}-\frac{\frac{-\frac{b\left(7a^3f-4a^2be+a\,b^2d+2b^3c\right)x^5}{9a^2}-\frac{\left(11a^3f-5a^2be-a\,b^2d+7b^3c\right)x^2}{18a}}{\left(b\,x^3+a\right)^2}+\frac{\left(20a^3f-5a^2be-a\,b^2d-2b^3c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{9a^2}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2/b^3-1/b^3*((-1/9*b*(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)/a^2*x^5-1/
18*(11*a^3*f-5*a^2*b*e-a*b^2*d+7*b^3*c)/a*x^2)/(b*x^3+a)^2+1/9*(20*a^3*f-5*
a^2*b*e-a*b^2*d-2*b^3*c)/a^2*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a
/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.51, size = 301, normalized size = 1.00

$$\frac{2\left(2b^4c+ab^3d+7a^3bf-4a^2b^2e\right)x^5+\left(7ab^3c-a^2b^2d+11a^4f-5a^3be\right)x^2}{18\left(a^2b^5x^6+2a^3b^4x^3+a^4b^3\right)}+\frac{fx^2}{2b^3}+\frac{\sqrt{3}\left(2b^3c+ab^2d-20a^3f+5a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^2b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(2b^3c+ab^2d-20a^3f+5a^2be\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^2b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(2b^3c+ab^2d-20a^3f+5a^2be\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^2b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(2*b^4*c + a*b^3*d + 7*a^3*b*f - 4*a^2*b^2*e)*x^5 + (7*a*b^3*c - a^
2*b^2*d + 11*a^4*f - 5*a^3*b*e)*x^2)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3
) + 1/2*f*x^2/b^3 + 1/27*sqrt(3)*(2*b^3*c + a*b^2*d - 20*a^3*f + 5*a^2*b*e)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4*(a/b)^(1/3))
+ 1/54*(2*b^3*c + a*b^2*d - 20*a^3*f + 5*a^2*b*e)*log(x^2 - x*(a/b)^(1/3) +
 (a/b)^(2/3))/(a^2*b^4*(a/b)^(1/3)) - 1/27*(2*b^3*c + a*b^2*d - 20*a^3*f +
5*a^2*b*e)*log(x + (a/b)^(1/3))/(a^2*b^4*(a/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(258) = 516.
time = 0.42, size = 1158, normalized size = 3.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(27*a^3*b^4*f*x^8 + 6*(2*a*b^6*c + a^2*b^5*d - 4*a^3*b^4*e + 16*a^4*b
^3*f)*x^5 + 3*(7*a^2*b^5*c - a^3*b^4*d - 5*a^4*b^3*e + 20*a^5*b^2*f)*x^2 -
3*sqrt(1/3)*(2*a^3*b^4*c + a^4*b^3*d + 5*a^5*b^2*e - 20*a^6*b*f + (2*a*b^6*
c + a^2*b^5*d + 5*a^3*b^4*e - 20*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c + a^3*b^4*
d + 5*a^4*b^3*e - 20*a^5*b^2*f)*x^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3
- a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(
a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((2*b^5*c + a*b^4*d + 5
*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e - 20*a
^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^2)^(2
/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 2*((2*b^5*c + a*b^4*
d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e -
 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^
2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5),
 1/54*(27*a^3*b^4*f*x^8 + 6*(2*a*b^6*c + a^2*b^5*d - 4*a^3*b^4*e + 16*a^4*b
^3*f)*x^5 + 3*(7*a^2*b^5*c - a^3*b^4*d - 5*a^4*b^3*e + 20*a^5*b^2*f)*x^2 -
6*sqrt(1/3)*(2*a^3*b^4*c + a^4*b^3*d + 5*a^5*b^2*e - 20*a^6*b*f + (2*a*b^6*
c + a^2*b^5*d + 5*a^3*b^4*e - 20*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c + a^3*b^4*
d + 5*a^4*b^3*e - 20*a^5*b^2*f)*x^3)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(1/3
)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + ((2*b^5*c + a*b^4*d +
5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e - 20*
```

```
a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^2)^(
2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 2*((2*b^5*c + a*b^4
*d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e
- 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b
^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5)
]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.21, size = 339, normalized size = 1.13

$$\frac{fx^2}{2\,b^3}+\frac{\sqrt{3}\,(2\,b^3c+ab^2d-20\,a^3f+5\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{1}{3}}\,a^2b^3}-\frac{(2\,b^3c+ab^2d-20\,a^3f+5\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{1}{3}}\,a^2b^3}-\frac{\left(2\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^3}+\frac{4\,b^4cx^5+2\,ab^3dx^5+14\,a^3bfx^5-8\,a^2b^2x^5e+7\,ab^3cx^2-a^2b^2dx^2+11\,a^4fx^2-5\,a^3bx^2e}{18\,(bx^3+a)^2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b^3 + 1/27*sqrt(3)*(2*b^3*c + a*b^2*d - 20*a^3*f + 5*a^2*b*e)*arc
tan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^2*b^3)
 - 1/54*(2*b^3*c + a*b^2*d - 20*a^3*f + 5*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b^3) - 1/27*(2*b^3*c*(-a/b)^(1/3) + a*
b^2*d*(-a/b)^(1/3) - 20*a^3*f*(-a/b)^(1/3) + 5*a^2*b*(-a/b)^(1/3)*e)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3) + 1/18*(4*b^4*c*x^5 + 2*a*b^3*d
*x^5 + 14*a^3*b*f*x^5 - 8*a^2*b^2*x^5*e + 7*a*b^3*c*x^2 - a^2*b^2*d*x^2 + 1
1*a^4*f*x^2 - 5*a^3*b*x^2*e)/((b*x^3 + a)^2*a^2*b^3)
```

Mupad [**B**]
time = 5.27, size = 280, normalized size = 0.93

$$\frac{\frac{x^2\,(11\,f\,a^3-5\,e\,a^2\,b-d\,a\,b^2+7\,c\,b^3)}{18\,a}+\frac{x^5\,(7\,f\,a^3\,b-4\,e\,a^2\,b^2+d\,a\,b^3+2\,c\,b^4)}{9\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{f\,x^2}{2\,b^3}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-20\,f\,a^3+5\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{7/3}\,b^{11/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-20\,f\,a^3+5\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{7/3}\,b^{11/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-20\,f\,a^3+5\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{7/3}\,b^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] ((x^2*(7*b^3*c + 11*a^3*f - a*b^2*d - 5*a^2*b*e))/(18*a) + (x^5*(2*b^4*c -
4*a^2*b^2*e + a*b^3*d + 7*a^3*b*f))/(9*a^2))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x
^3) + (f*x^2)/(2*b^3) - (log(b^(1/3)*x + a^(1/3))*(2*b^3*c - 20*a^3*f + a*b
^2*d + 5*a^2*b*e))/(27*a^(7/3)*b^(11/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1
/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*b^3*c - 20*a^3*f + a*b^2*d + 5*a
^2*b*e))/(27*a^(7/3)*b^(11/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^
(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*b^3*c - 20*a^3*f + a*b^2*d + 5*a^2*b*e))/(
27*a^(7/3)*b^(11/3))
```

3.294 $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3} dx$

Optimal. Leaf size=292

$$\frac{fx}{b^3}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6ab^3\,(a+bx^3)^2}+\frac{(5b^3c+ab^2d-7a^2be+13a^3f)\,x}{18a^2b^3\,(a+bx^3)}-\frac{(5b^3c+ab^2d+2a^2be-14a^3f)\tan^{-1}}{9\sqrt{3}\,a^{8/3}b^{10/3}}$$

```
[Out] f*x/b^3+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a/b^3/(b*x^3+a)^2+1/18*(13*a^3
*f-7*a^2*b*e+a*b^2*d+5*b^3*c)*x/a^2/b^3/(b*x^3+a)+1/27*(-14*a^3*f+2*a^2*b*e
+a*b^2*d+5*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(10/3)-1/54*(-14*a^3*f+2*
a^2*b*e+a*b^2*d+5*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/
b^(10/3)-1/27*(-14*a^3*f+2*a^2*b*e+a*b^2*d+5*b^3*c)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1872, 1423, 396, 206, 31, 648, 631, 210, 642}

$$\frac{x(13a^3f-7a^2be+ab^2d+5b^3c)}{18a^2b^3\,(a+bx^3)}+\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6ab^3\,(a+bx^3)^2}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(-14a^3f+2a^2be+ab^2d+5b^3c)}{9\sqrt{3}\,a^{8/3}b^{10/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(-14a^3f+2a^2be+ab^2d+5b^3c)}{54a^{8/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(-14a^3f+2a^2be+ab^2d+5b^3c)}{27a^{8/3}b^{10/3}}+\frac{fx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]
```

```
[Out] (f*x)/b^3 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a*b^3*(a + b*x^3)^2)
 + ((5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(18*a^2*b^3*(a + b*x^3))
- ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(10/3)) + ((5*b^3*c + a*b^2*d +
2*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(10/3)) - ((5
*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(54*a^(8/3)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
```

```
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6ab^3\,(a+bx^3)^2} - \frac{\int \frac{-5b^3c-ab^2d+a^2be-a^3f-6ab(be-af)x^3-6ab^2fx^6}{(a+bx^3)^2}\,dx}{6ab^3}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6ab^3\,(a+bx^3)^2} + \frac{(5b^3c+ab^2d-7a^2be+13a^3f)\,x}{18a^2b^3\,(a+bx^3)} + \frac{\int \frac{2b^2(5b^3c+a}{}}{}$$

$$= \frac{fx}{b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6ab^3\,(a+bx^3)^2} + \frac{(5b^3c+ab^2d-7a^2be+13a^3f)\,x}{18a^2b^3\,(a+bx^3)} + \frac{(5b^3}{}$$

$$= \frac{fx}{b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6ab^3\,(a+bx^3)^2} + \frac{(5b^3c+ab^2d-7a^2be+13a^3f)\,x}{18a^2b^3\,(a+bx^3)} + \frac{(5b^3}{}$$

$$= \frac{fx}{b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6ab^3\,(a+bx^3)^2} + \frac{(5b^3c+ab^2d-7a^2be+13a^3f)\,x}{18a^2b^3\,(a+bx^3)} + \frac{(5b^3}{}$$

$$= \frac{fx}{b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6ab^3\,(a+bx^3)^2} + \frac{(5b^3c+ab^2d-7a^2be+13a^3f)\,x}{18a^2b^3\,(a+bx^3)} + \frac{(5b^3}{}$$

$$= \frac{fx}{b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6ab^3\,(a+bx^3)^2} + \frac{(5b^3c+ab^2d-7a^2be+13a^3f)\,x}{18a^2b^3\,(a+bx^3)} - \frac{(5b^3}{}$$

Mathematica [A]
time = 0.13, size = 279, normalized size = 0.96

$$\frac{54\sqrt[3]{b}\,fx + \frac{9\sqrt[3]{b}\,(b^3c-ab^2d+a^2be-a^3f)x}{a(a+bx^3)^2} + \frac{3\sqrt[3]{b}\,(5b^3c+ab^2d-7a^2be+13a^3f)x}{a^2(a+bx^3)} - \frac{2\sqrt{3}\,(5b^3c+ab^2d+2a^2be-14a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}} + \frac{2(5b^3c+ab^2d+2a^2be-14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{8/3}} - \frac{(5b^3c+ab^2d+2a^2be-14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{8/3}}}{54b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(1/3)*f*x + (9*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a*(a +
 b*x^3)^2) + (3*b^(1/3)*(5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(a^2*
(a + b*x^3)) - (2*Sqrt[3]*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*ArcTan
```

```
[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(8/3) + (2*(5*b^3*c + a*b^2*d + 2*
a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) - ((5*b^3*c + a*b^2*d
 + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^
(8/3))/(54*b^(10/3))
```

Maple [A]
time = 0.36, size = 215, normalized size = 0.74

method	result
risch	$\frac{fx}{b^3}+\frac{\frac{b\left(13a^3f-7a^2be+a\,b^2d+5b^3c\right)x^4}{18a^2}+\frac{\left(5a^3f-2a^2be-a\,b^2d+4b^3c\right)x}{9a}}{b^3\left(b\,x^3+a\right)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(14a^3f-2a^2be-a\,b^2d-5b^3c\right)\ln\left(x-\_R\right)}{\_R^2}}{27b^4a^2}$
default	$\frac{fx}{b^3}-\frac{\frac{-\frac{b\left(13a^3f-7a^2be+a\,b^2d+5b^3c\right)x^4}{18a^2}-\frac{\left(5a^3f-2a^2be-a\,b^2d+4b^3c\right)x}{9a}}{\left(b\,x^3+a\right)^2}+\frac{\left(14a^3f-2a^2be-a\,b^2d-5b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right.\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{9a^2}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f*x/b^3-1/b^3*((-1/18*b*(13*a^3*f-7*a^2*b*e+a*b^2*d+5*b^3*c)/a^2*x^4-1/9*(5
*a^3*f-2*a^2*b*e-a*b^2*d+4*b^3*c)/a*x)/(b*x^3+a)^2+1/9*(14*a^3*f-2*a^2*b*e-
a*b^2*d-5*b^3*c)/a^2*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.51, size = 296, normalized size = 1.01

$$\frac{(5\,b^4c+ab^3d+13\,a^3bf-7\,a^2b^2e)x^4+2\,(4\,ab^3c-a^2b^2d+5\,a^4f-2\,a^3be)x}{18\,(a^2b^5x^6+2\,a^3b^4x^3+a^4b^3)}+\frac{fx}{b^3}+\frac{\sqrt{3}\,(5\,b^3c+ab^2d-14\,a^3f+2\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(5\,b^3c+ab^2d-14\,a^3f+2\,a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(5\,b^3c+ab^2d-14\,a^3f+2\,a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((5*b^4*c + a*b^3*d + 13*a^3*b*f - 7*a^2*b^2*e)*x^4 + 2*(4*a*b^3*c - a
^2*b^2*d + 5*a^4*f - 2*a^3*b*e)*x)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)
+ f*x/b^3 + 1/27*sqrt(3)*(5*b^3*c + a*b^2*d - 14*a^3*f + 2*a^2*b*e)*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4*(a/b)^(2/3)) - 1/54*(
5*b^3*c + a*b^2*d - 14*a^3*f + 2*a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
```

```
2/3))/(a^2*b^4*(a/b)^(2/3)) + 1/27*(5*b^3*c + a*b^2*d - 14*a^3*f + 2*a^2*b*
e)*log(x + (a/b)^(1/3))/(a^2*b^4*(a/b)^(2/3))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(251) = 502$.
time = 0.40, size = 1184, normalized size = 4.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(54*a^4*b^3*f*x^7 + 3*(5*a^2*b^5*c + a^3*b^4*d - 7*a^4*b^3*e + 49*a^5
*b^2*f)*x^4 - 3*sqrt(1/3)*(5*a^3*b^4*c + a^4*b^3*d + 2*a^5*b^2*e - 14*a^6*b
*f + (5*a*b^6*c + a^2*b^5*d + 2*a^3*b^4*e - 14*a^4*b^3*f)*x^6 + 2*(5*a^2*b^
5*c + a^3*b^4*d + 2*a^4*b^3*e - 14*a^5*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*l
og((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2
*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - ((5*
b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d
 + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b
*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a)
+ 2*((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a
^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e -
14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 6*(4*a^3*b^4*
c - a^4*b^3*d - 2*a^5*b^2*e + 14*a^6*b*f)*x)/(a^4*b^6*x^6 + 2*a^5*b^5*x^3 +
 a^6*b^4), 1/54*(54*a^4*b^3*f*x^7 + 3*(5*a^2*b^5*c + a^3*b^4*d - 7*a^4*b^3*
e + 49*a^5*b^2*f)*x^4 + 6*sqrt(1/3)*(5*a^3*b^4*c + a^4*b^3*d + 2*a^5*b^2*e
- 14*a^6*b*f + (5*a*b^6*c + a^2*b^5*d + 2*a^3*b^4*e - 14*a^4*b^3*f)*x^6 + 2
*(5*a^2*b^5*c + a^3*b^4*d + 2*a^4*b^3*e - 14*a^5*b^2*f)*x^3)*sqrt(-(-a^2*b)
^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-
a^2*b)^(1/3)/b)/a^2) - ((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^
6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3
*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(
2/3)*x - (-a^2*b)^(1/3)*a) + 2*((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b
^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c +
 a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^
2*b)^(2/3)) + 6*(4*a^3*b^4*c - a^4*b^3*d - 2*a^5*b^2*e + 14*a^6*b*f)*x)/(a^
4*b^6*x^6 + 2*a^5*b^5*x^3 + a^6*b^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.06, size = 295, normalized size = 1.01

$$\frac{fx}{b^3}-\frac{\sqrt{3}\,(5\,b^3c+ab^2d-14\,a^3f+2\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^2b^2}-\frac{(5\,b^3c+ab^2d-14\,a^3f+2\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^2b^2}-\frac{(5\,b^3c+ab^2d-14\,a^3f+2\,a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^3}+\frac{5\,b^4cx^4+ab^3dx^4+13\,a^3bfx^4-7\,a^2b^2x^4e+8\,ab^3cx-2\,a^2b^2dx+10\,a^4fx-4\,a^3bxe}{18\,(bx^3+a)^2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] f*x/b^3 - 1/27*sqrt(3)*(5*b^3*c + a*b^2*d - 14*a^3*f + 2*a^2*b*e)*arctan(1/
3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b^2) - 1/5
4*(5*b^3*c + a*b^2*d - 14*a^3*f + 2*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a
/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b^2) - 1/27*(5*b^3*c + a*b^2*d - 14*a^3*f +
2*a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3) + 1/18*(5*b^4*
c*x^4 + a*b^3*d*x^4 + 13*a^3*b*f*x^4 - 7*a^2*b^2*x^4*e + 8*a*b^3*c*x - 2*a^
2*b^2*d*x + 10*a^4*f*x - 4*a^3*b*x*e)/((b*x^3 + a)^2*a^2*b^3)
```

Mupad [B]
time = 5.20, size = 275, normalized size = 0.94

$$\frac{\frac{x\,(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3)}{9\,a}+\frac{x^4\,(13\,f\,a^3\,b-7\,e\,a^2\,b^2+d\,a\,b^3+5\,c\,b^4)}{18\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{f\,x}{b^3}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(-14\,f\,a^3+2\,e\,a^2\,b+d\,a\,b^2+5\,c\,b^3)}{27\,a^{8/3}\,b^{10/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(-14\,f\,a^3+2\,e\,a^2\,b+d\,a\,b^2+5\,c\,b^3)}{27\,a^{8/3}\,b^{10/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(-14\,f\,a^3+2\,e\,a^2\,b+d\,a\,b^2+5\,c\,b^3)}{27\,a^{8/3}\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x)
```

```
[Out] ((x*(4*b^3*c + 5*a^3*f - a*b^2*d - 2*a^2*b*e))/(9*a) + (x^4*(5*b^4*c - 7*a^
2*b^2*e + a*b^3*d + 13*a^3*b*f))/(18*a^2))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3
) + (f*x)/b^3 + (log(b^(1/3)*x + a^(1/3))*(5*b^3*c - 14*a^3*f + a*b^2*d + 2
*a^2*b*e))/(27*a^(8/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x -
a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c - 14*a^3*f + a*b^2*d + 2*a^2*b*e))
/(27*a^(8/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*(
(3^(1/2)*1i)/2 + 1/2)*(5*b^3*c - 14*a^3*f + a*b^2*d + 2*a^2*b*e))/(27*a^(8/
3)*b^(10/3))
```

3.295 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx$

Optimal. Leaf size=303

$$-\frac{c}{a^3x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^2b^2\,(a+bx^3)^2}-\frac{(5b^3c-2ab^2d-a^2be+4a^3f)\,x^2}{9a^3b^2\,(a+bx^3)}+\frac{(14b^3c-2ab^2d-a^2be-5a^3f)\tan}{9\sqrt{3}\,a^{10/3}b^{8/3}}$$

```
[Out] -c/a^3/x-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^2/b^2/(b*x^3+a)^2-1/9*(4*
a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*x^2/a^3/b^2/(b*x^3+a)+1/27*(-5*a^3*f-a^2*b
*e-2*a*b^2*d+14*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(8/3)-1/54*(-5*a^3*
f-a^2*b*e-2*a*b^2*d+14*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(
10/3)/b^(8/3)+1/27*(-5*a^3*f-a^2*b*e-2*a*b^2*d+14*b^3*c)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1843, 1498, 464, 298, 31, 648, 631, 210, 642}

$$-\frac{c}{a^3x}-\frac{x^2(4a^3f-a^2be-2ab^2d+5b^3c)}{9a^3b^2\,(a+bx^3)}-\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^2b^2\,(a+bx^3)^2}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(-5a^3f-a^2be-2ab^2d+14b^3c)}{9\sqrt{3}\,a^{10/3}b^{8/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(-5a^3f-a^2be-2ab^2d+14b^3c)}{54a^{10/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(-5a^3f-a^2be-2ab^2d+14b^3c)}{27a^{10/3}b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -(c/(a^3*x)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^2*b^2*(a + b*
x^3)^2) - ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*x^2)/(9*a^3*b^2*(a + b
*x^3)) + ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(8/3)) + ((14*b^3*c - 2*
a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(8/3)
) - ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1498

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1))), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n))*
(n*(-d)^(-(m - Mod[m, n])/n + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p
- ((c*d^2 - b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*
(Mod[m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\int \frac{-6b^3c+2b\left(\frac{2b^3c}{a}-2b^2d-abe+a^2f\right)x^3-6ab^2fx^6}{x^2\left(a+bx^3\right)^2}\,dx}{6ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)}+\frac{\int \frac{18ab^5c-\cdots}{\cdots}}{\cdots}$$

$$= -\frac{c}{a^3x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)}-\cdots$$

$$= -\frac{c}{a^3x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)}+\cdots$$

$$= -\frac{c}{a^3x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)}+\cdots$$

$$= -\frac{c}{a^3x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)}+\cdots$$

$$= -\frac{c}{a^3x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)}+\cdots$$

Mathematica [A]
time = 0.14, size = 286, normalized size = 0.94

$$\frac{-\frac{54\sqrt[3]{a}\,c}{x}+\frac{9a^{4/3}\left(-b^3c+ab^2d-a^2be+a^3f\right)x^2}{b^2\left(a+bx^3\right)^2}-\frac{6\sqrt[3]{a}\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{b^2\left(a+bx^3\right)}+\frac{2\sqrt{3}\left(14b^3c-2ab^2d-a^2be-5a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{8/3}}-\frac{2\left(-14b^3c+2ab^2d+a^2be+5a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{8/3}}+\frac{\left(-14b^3c+2ab^2d+a^2be+5a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{8/3}}}{54a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(1/3)*c)/x + (9*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)
/(b^2*(a + b*x^3)^2) - (6*a^(1/3)*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)
*x^2)/(b^2*(a + b*x^3)) + (2*Sqrt[3]*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^
3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(8/3) - (2*(-14*b^3*c +
 2*a*b^2*d + a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(8/3) + ((-14*b
^3*c + 2*a*b^2*d + a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/b^(8/3))/(54*a^(10/3))
```

Maple [A]
time = 0.37, size = 218, normalized size = 0.72

method	result
default	$\frac{\frac{-\frac{\left(4a^3f-a^2be-2a\,b^2d+5b^3c\right)x^5}{9b}-\frac{a\left(5a^3f+a^2be-7a\,b^2d+13b^3c\right)x^2}{18b^2}}{\left(b\,x^3+a\right)^2}+\frac{\left(5a^3f+a^2be+2a\,b^2d-14b^3c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{9b^2}}{a^3}$
risch	$\frac{-\frac{\left(4a^3f-a^2be-2a\,b^2d+14b^3c\right)x^6}{9a^3b}-\frac{\left(5a^3f+a^2be-7a\,b^2d+49b^3c\right)x^3}{18a^2b^2}-\frac{c}{a}}{x\left(b\,x^3+a\right)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}b^8\_Z^3+125a^9f^3+75a^8be\,f^2+150a^7b^2d\,f^2+15a^7b\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*((-1/9*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)/b*x^5-1/18*a*(5*a^3*f+a^2*
b*e-7*a*b^2*d+13*b^3*c)/b^2*x^2)/(b*x^3+a)^2+1/9*(5*a^3*f+a^2*b*e+2*a*b^2*d
-14*b^3*c)/b^2*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))))-c/a^3/x
```

Maxima [A]
time = 0.50, size = 305, normalized size = 1.01

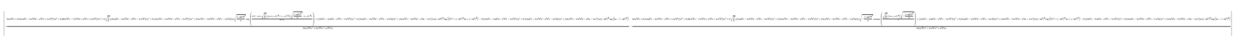
$$-\frac{2\,(14\,b^4c-2\,ab^3d+4\,a^3bf-a^2b^2e)x^6+18\,a^2b^2c+(49\,ab^3c-7\,a^2b^2d+5\,a^4f+a^3be)x^3}{18\,(a^3b^4x^7+2\,a^4b^3x^4+a^5b^2x)}-\frac{\sqrt{3}\,(14\,b^3c-2\,ab^2d-5\,a^3f-a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(14\,b^3c-2\,ab^2d-5\,a^3f-a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(14\,b^3c-2\,ab^2d-5\,a^3f-a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(14*b^4*c - 2*a*b^3*d + 4*a^3*b*f - a^2*b^2*e)*x^6 + 18*a^2*b^2*c
+ (49*a*b^3*c - 7*a^2*b^2*d + 5*a^4*f + a^3*b*e)*x^3)/(a^3*b^4*x^7 + 2*a^4*
b^3*x^4 + a^5*b^2*x) - 1/27*sqrt(3)*(14*b^3*c - 2*a*b^2*d - 5*a^3*f - a^2*b
*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^3*(a/b)^(1/3
)) - 1/54*(14*b^3*c - 2*a*b^2*d - 5*a^3*f - a^2*b*e)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(a^3*b^3*(a/b)^(1/3)) + 1/27*(14*b^3*c - 2*a*b^2*d - 5*a^3
*f - a^2*b*e)*log(x + (a/b)^(1/3))/(a^3*b^3*(a/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(262) = 524$.
time = 0.42, size = 1206, normalized size = 3.98



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(54*a^3*b^4*c + 6*(14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e + 4*a^4*b^3*
f)*x^6 + 3*(49*a^2*b^5*c - 7*a^3*b^4*d + a^4*b^3*e + 5*a^5*b^2*f)*x^3 + 3*s
qrt(1/3)*((14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 2*(14*
a^2*b^5*c - 2*a^3*b^4*d - a^4*b^3*e - 5*a^5*b^2*f)*x^4 + (14*a^3*b^4*c - 2*
a^4*b^3*d - a^5*b^2*e - 5*a^6*b*f)*x)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3
 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt
((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((14*b^5*c - 2*a*b^
4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2*a^2*b^3*d - a^3*b^2*
e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4*b*e - 5*a^5*f)*x)*(-
a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*((14*b^
5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2*a^2*b^3*
d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4*b*e - 5*
a^5*f)*x)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^6*x^7 + 2*a^5*b^
5*x^4 + a^6*b^4*x), -1/54*(54*a^3*b^4*c + 6*(14*a*b^6*c - 2*a^2*b^5*d - a^3
*b^4*e + 4*a^4*b^3*f)*x^6 + 3*(49*a^2*b^5*c - 7*a^3*b^4*d + a^4*b^3*e + 5*a
^5*b^2*f)*x^3 + 6*sqrt(1/3)*((14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e - 5*a^4*
b^3*f)*x^7 + 2*(14*a^2*b^5*c - 2*a^3*b^4*d - a^4*b^3*e - 5*a^5*b^2*f)*x^4 +
 (14*a^3*b^4*c - 2*a^4*b^3*d - a^5*b^2*e - 5*a^6*b*f)*x)*sqrt(-(-a*b^2)^(1/
3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b)
+ ((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2
*a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4
*b*e - 5*a^5*f)*x)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^
2)^(2/3)) - 2*((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14
*a*b^4*c - 2*a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3
*b^2*d - a^4*b*e - 5*a^5*f)*x)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a
^4*b^6*x^7 + 2*a^5*b^5*x^4 + a^6*b^4*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.83, size = 341, normalized size = 1.13

$$-\frac{\sqrt{3}\,(14\,b^3c-2\,ab^2d-5\,a^3f-a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{1}{3}}\,a^3b^2}-\frac{c}{a^3x}+\frac{(14\,b^3c-2\,ab^2d-5\,a^3f-a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{1}{3}}\,a^3b^2}+\frac{\left(14\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4b^2}-\frac{10\,b^4cx^5-4\,ab^3dx^5+8\,a^3bfx^5-2\,a^2b^2x^5e+13\,ab^3cx^2-7\,a^2b^2dx^2+5\,a^4fx^2+a^3bx^2e}{18\,(bx^3+a)^2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(14*b^3*c - 2*a*b^2*d - 5*a^3*f - a^2*b*e)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^3*b^2) - c/(a^3*x) +
1/54*(14*b^3*c - 2*a*b^2*d - 5*a^3*f - a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(1/3)*a^3*b^2) + 1/27*(14*b^3*c*(-a/b)^(1/3) - 2*a*
b^2*d*(-a/b)^(1/3) - 5*a^3*f*(-a/b)^(1/3) - a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b^2) - 1/18*(10*b^4*c*x^5 - 4*a*b^3*d*x
^5 + 8*a^3*b*f*x^5 - 2*a^2*b^2*x^5*e + 13*a*b^3*c*x^2 - 7*a^2*b^2*d*x^2 + 5
*a^4*f*x^2 + a^3*b*x^2*e)/((b*x^3 + a)^2*a^3*b^2)
```

Mupad [B]
time = 5.20, size = 276, normalized size = 0.91

$$-\frac{\frac{c}{a}+\frac{x^6\,(4\,f\,a^3-e\,a^2\,b-2\,d\,a\,b^2+14\,c\,b^3)}{9\,a^3\,b}+\frac{x^3\,(5\,f\,a^3+e\,a^2\,b-7\,d\,a\,b^2+49\,c\,b^3)}{18\,a^2\,b^2}}{a^2\,x+2\,a\,b\,x^4+b^2\,x^7}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,f\,a^3+e\,a^2\,b+2\,d\,a\,b^2-14\,c\,b^3)}{27\,a^{10/3}\,b^{8/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(5\,f\,a^3+e\,a^2\,b+2\,d\,a\,b^2-14\,c\,b^3)}{27\,a^{10/3}\,b^{8/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(5\,f\,a^3+e\,a^2\,b+2\,d\,a\,b^2-14\,c\,b^3)}{27\,a^{10/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*
a^3*f - 14*b^3*c + 2*a*b^2*d + a^2*b*e))/(27*a^(10/3)*b^(8/3)) - (log(b^(1/
3)*x + a^(1/3))*(5*a^3*f - 14*b^3*c + 2*a*b^2*d + a^2*b*e))/(27*a^(10/3)*b^
(8/3)) - (c/a + (x^6*(14*b^3*c + 4*a^3*f - 2*a*b^2*d - a^2*b*e))/(9*a^3*b)
+ (x^3*(49*b^3*c + 5*a^3*f - 7*a*b^2*d + a^2*b*e))/(18*a^2*b^2))/(a^2*x + b
^2*x^7 + 2*a*b*x^4) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(5*a^3*f - 14*b^3*c + 2*a*b^2*d + a^2*b*e))/(27*a^(10/3)
*b^(8/3))
```

3.296 $\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=301

$$-\frac{c}{2a^3x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)}+\frac{\left(20b^3c-5ab^2d-a^2be-2a^3f\right)\text{ta}}{9\sqrt{3}\,a^{11/3}b^{7/3}}$$

```
[Out] -1/2*c/a^3/x^2-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^2/b^2/(b*x^3+a)^2-1/1
8*(7*a^3*f-a^2*b*e-5*a*b^2*d+11*b^3*c)*x/a^3/b^2/(b*x^3+a)-1/27*(-2*a^3*f-a
^2*b*e-5*a*b^2*d+20*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(7/3)+1/54*(-2*
a^3*f-a^2*b*e-5*a*b^2*d+20*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/a^(11/3)/b^(7/3)+1/27*(-2*a^3*f-a^2*b*e-5*a*b^2*d+20*b^3*c)*arctan(1/3*(a^
(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1843, 1498, 464, 206, 31, 648, 631, 210, 642}

$$-\frac{c}{2a^3x^2}-\frac{x\left(7a^3f-a^2be-5ab^2d+11b^3c\right)}{18a^3b^2\left(a+bx^3\right)}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)}{9\sqrt{3}\,a^{11/3}b^{7/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)}{54a^{11/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)}{27a^{11/3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -1/2*c/(a^3*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^2*b^2*(a +
b*x^3)^2) - ((11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f)*x)/(18*a^3*b^2*(a +
 b*x^3)) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(7/3)) - ((20*b^3*c -
5*a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(7/
3)) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1498

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1))), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n))*
(n*(-d)^(-(m - Mod[m, n])/n + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p
- ((c*d^2 - b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*
(Mod[m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+b\left(\frac{5b^3c}{a}-5b^2d-abe+a^2f\right)x^3-6ab^2fx^6}{x^3\left(a+bx^3\right)^2}\,dx}{6ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} + \frac{\int \frac{18ab^5c-\cdots}{\cdots}}{\cdots}$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} - \cdots$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} - \cdots$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} - \cdots$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} - \cdots$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} + \cdots$$

Mathematica [A]
time = 0.14, size = 283, normalized size = 0.94

$$\frac{-\frac{27a^{2/3}c}{x^2}+\frac{9a^{5/3}\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{b^2\left(a+bx^3\right)^2}-\frac{3a^{2/3}\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{b^2\left(a+bx^3\right)}+\frac{2\sqrt{3}\left(20b^3c-5ab^2d-a^2be-2a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{7/3}}+\frac{2\left(-20b^3c+5ab^2d+a^2be+2a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{7/3}}-\frac{\left(-20b^3c+5ab^2d+a^2be+2a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{7/3}}}{54a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(2/3)*c)/x^2 + (9*a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)
/(b^2*(a + b*x^3)^2) - (3*a^(2/3)*(11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f
)*x)/(b^2*(a + b*x^3)) + (2*Sqrt[3]*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3
*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(7/3) + (2*(-20*b^3*c +
5*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(7/3) - ((-20*b^
3*c + 5*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/b^(7/3))/(54*a^(11/3))
```

Maple [A]
time = 0.39, size = 216, normalized size = 0.72

method	result
default	$\frac{\frac{-\frac{\left(7a^{3}f-a^{2}be-5a\,b^{2}d+11b^{3}c\right)x^{4}}{18b}-\frac{a\left(2a^{3}f+a^{2}be-4a\,b^{2}d+7b^{3}c\right)x}{9b^{2}}}{\left(b\,x^{3}+a\right)^{2}}+\frac{\left(2a^{3}f+a^{2}be+5a\,b^{2}d-20b^{3}c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\cdots\right.}{9b^{2}}}{a^{3}}$
risch	$\frac{-\frac{\left(7a^{3}f-a^{2}be-5a\,b^{2}d+20b^{3}c\right)x^{6}}{18a^{3}b}-\frac{\left(2a^{3}f+a^{2}be-4a\,b^{2}d+16b^{3}c\right)x^{3}}{9a^{2}b^{2}}-\frac{c}{2a}}{x^{2}\left(b\,x^{3}+a\right)^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}b^{7}\_Z^{3}-8a^{9}f^{3}-12a^{8}be\,f^{2}-60a^{7}b^{2}d\,f^{2}-6a^{7}b^{2}e^{\cdots}\right.}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*((-1/18*(7*a^3*f-a^2*b*e-5*a*b^2*d+11*b^3*c)/b*x^4-1/9*a*(2*a^3*f+a^2
*b*e-4*a*b^2*d+7*b^3*c)/b^2*x)/(b*x^3+a)^2+1/9*(2*a^3*f+a^2*b*e+5*a*b^2*d-2
0*b^3*c)/b^2*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))))-1/2*c/a^3/x^2
```

Maxima [A]
time = 0.51, size = 307, normalized size = 1.02

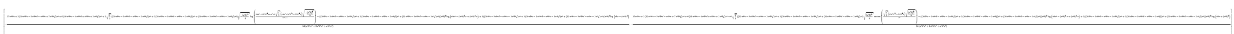
$$-\frac{(20\,b^4c-5\,ab^3d+7\,a^3bf-a^2b^2e)x^6+9\,a^2b^2c+2\,(16\,ab^3c-4\,a^2b^2d+2\,a^4f+a^3be)x^3}{18\,(a^3b^4x^8+2\,a^4b^3x^5+a^5b^2x^2)}-\frac{\sqrt{3}\,(20\,b^3c-5\,ab^2d-2\,a^3f-a^2be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(20\,b^3c-5\,ab^2d-2\,a^3f-a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(20\,b^3c-5\,ab^2d-2\,a^3f-a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((20*b^4*c - 5*a*b^3*d + 7*a^3*b*f - a^2*b^2*e)*x^6 + 9*a^2*b^2*c + 2
*(16*a*b^3*c - 4*a^2*b^2*d + 2*a^4*f + a^3*b*e)*x^3)/(a^3*b^4*x^8 + 2*a^4*b
^3*x^5 + a^5*b^2*x^2) - 1/27*sqrt(3)*(20*b^3*c - 5*a*b^2*d - 2*a^3*f - a^2*
b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^3*(a/b)^(2/
3)) + 1/54*(20*b^3*c - 5*a*b^2*d - 2*a^3*f - a^2*b*e)*log(x^2 - x*(a/b)^(1/
3) + (a/b)^(2/3))/(a^3*b^3*(a/b)^(2/3)) - 1/27*(20*b^3*c - 5*a*b^2*d - 2*a^
3*f - a^2*b*e)*log(x + (a/b)^(1/3))/(a^3*b^3*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(258) = 516$.
time = 0.41, size = 1217, normalized size = 4.04



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(27*a^4*b^3*c + 3*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e + 7*a^5*b^
2*f)*x^6 + 6*(16*a^3*b^4*c - 4*a^4*b^3*d + a^5*b^2*e + 2*a^6*b*f)*x^3 + 3*s
qrt(1/3)*((20*a*b^6*c - 5*a^2*b^5*d - a^3*b^4*e - 2*a^4*b^3*f)*x^8 + 2*(20*
a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e - 2*a^5*b^2*f)*x^5 + (20*a^3*b^4*c - 5*
a^4*b^3*d - a^5*b^2*e - 2*a^6*b*f)*x^2)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x
^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x -
 (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - ((20*b^5*c - 5*a*b
^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2
*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((20*b^
5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*
d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*
a^5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^5*x^8 + 2*a^6*
b^4*x^5 + a^7*b^3*x^2), -1/54*(27*a^4*b^3*c + 3*(20*a^2*b^5*c - 5*a^3*b^4*d
 - a^4*b^3*e + 7*a^5*b^2*f)*x^6 + 6*(16*a^3*b^4*c - 4*a^4*b^3*d + a^5*b^2*e
 + 2*a^6*b*f)*x^3 + 6*sqrt(1/3)*((20*a*b^6*c - 5*a^2*b^5*d - a^3*b^4*e - 2*
a^4*b^3*f)*x^8 + 2*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e - 2*a^5*b^2*f)*x
^5 + (20*a^3*b^4*c - 5*a^4*b^3*d - a^5*b^2*e - 2*a^6*b*f)*x^2)*sqrt((a^2*b)
^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*
b)^(1/3)/b)/a^2) - ((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 +
2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c -
5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(
2/3)*x + (a^2*b)^(1/3)*a) + 2*((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^
2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a
^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x +
(a^2*b)^(2/3)))/(a^5*b^5*x^8 + 2*a^6*b^4*x^5 + a^7*b^3*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.98, size = 312, normalized size = 1.04

$$\frac{\sqrt{3}\,(20\,b^3c-5\,ab^2d-2\,a^3f-a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}a^3b}+\frac{(20\,b^3c-5\,ab^2d-2\,a^3f-a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{2}{3}}a^3b}+\frac{(20\,b^3c-5\,ab^2d-2\,a^3f-a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4b^2}-\frac{20\,b^4cx^6-5\,ab^3dx^6+7\,a^3bfx^6-a^2b^2x^6e+32\,ab^3cx^3-8\,a^2b^2dx^3+4\,a^4fx^3+2\,a^3bx^3e+9\,a^2b^2c}{18\,\left(bx^4+ax\right)^2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(20*b^3*c - 5*a*b^2*d - 2*a^3*f - a^2*b*e)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3*b) + 1/54*(20*b^3*c
- 5*a*b^2*d - 2*a^3*f - a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(
(-a*b^2)^(2/3)*a^3*b) + 1/27*(20*b^3*c - 5*a*b^2*d - 2*a^3*f - a^2*b*e)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b^2) - 1/18*(20*b^4*c*x^6 - 5*a*b
^3*d*x^6 + 7*a^3*b*f*x^6 - a^2*b^2*x^6*e + 32*a*b^3*c*x^3 - 8*a^2*b^2*d*x^3
 + 4*a^4*f*x^3 + 2*a^3*b*x^3*e + 9*a^2*b^2*c)/((b*x^4 + a*x)^2*a^3*b^2)
```

Mupad [B]
time = 5.16, size = 279, normalized size = 0.93

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(2\,f\,a^3+e\,a^2\,b+5\,d\,a\,b^2-20\,c\,b^3\right)}{27\,a^{11/3}\,b^{7/3}}-\frac{\frac{c}{2\,a}+\frac{x^3\,\left(2\,f\,a^3+e\,a^2\,b-4\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^2\,b^2}+\frac{x^6\,\left(7\,f\,a^3-e\,a^2\,b-5\,d\,a\,b^2+20\,c\,b^3\right)}{18\,a^3\,b}}{a^2\,x^2+2\,a\,b\,x^5+b^2\,x^8}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,f\,a^3+e\,a^2\,b+5\,d\,a\,b^2-20\,c\,b^3\right)}{27\,a^{11/3}\,b^{7/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,f\,a^3+e\,a^2\,b+5\,d\,a\,b^2-20\,c\,b^3\right)}{27\,a^{11/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(2*a^3*f - 20*b^3*c + 5*a*b^2*d + a^2*b*e))/(27*a
^(11/3)*b^(7/3)) - (c/(2*a) + (x^3*(16*b^3*c + 2*a^3*f - 4*a*b^2*d + a^2*b*
e))/(9*a^2*b^2) + (x^6*(20*b^3*c + 7*a^3*f - 5*a*b^2*d - a^2*b*e))/(18*a^3*
b))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
 - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*a^3*f - 20*b^3*c + 5*a*b^2*d + a^2*b*
e))/(27*a^(11/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3)
)*((3^(1/2)*1i)/2 + 1/2)*(2*a^3*f - 20*b^3*c + 5*a*b^2*d + a^2*b*e))/(27*a^
(11/3)*b^(7/3))
```

3.297 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^3}\,dx$

Optimal. Leaf size=317

$$-\frac{c}{4a^3x^4}+\frac{3bc-ad}{a^4x}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^3b\,(a+bx^3)^2}+\frac{(8b^3c-5ab^2d+2a^2be+a^3f)\,x^2}{9a^4b\,(a+bx^3)}-\frac{(35b^3c-14ab^2d+2a$$

```
[Out] -1/4*c/a^3/x^4+(-a*d+3*b*c)/a^4/x+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^
3/b/(b*x^3+a)^2+1/9*(a^3*f+2*a^2*b*e-5*a*b^2*d+8*b^3*c)*x^2/a^4/b/(b*x^3+a)
-1/27*(a^3*f+2*a^2*b*e-14*a*b^2*d+35*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)/
b^(5/3)+1/54*(a^3*f+2*a^2*b*e-14*a*b^2*d+35*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(13/3)/b^(5/3)-1/27*(a^3*f+2*a^2*b*e-14*a*b^2*d+35*b^3*
c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)/b^(5/3)*3^(1/
2)
```

Rubi [A]

time = 0.25, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1843, 1498, 1502, 298, 31, 648, 631, 210, 642}

$$\frac{3bc-ad}{a^4x}-\frac{c}{4a^3x^4}+\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^3b\,(a+bx^3)^2}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(a^3f+2a^2be-14ab^2d+35b^3c)}{9\sqrt{3}\,a^{13/3}b^{5/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)(a^3f+2a^2be-14ab^2d+35b^3c)}{54a^{13/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(a^3f+2a^2be-14ab^2d+35b^3c)}{27a^{13/3}b^{5/3}}+\frac{x^2(a^3f+2a^2be-5ab^2d+8b^3c)}{9a^4b\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] -1/4*c/(a^3*x^4) + (3*b*c - a*d)/(a^4*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*x^2)/(6*a^3*b*(a + b*x^3)^2) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3
*f)*x^2)/(9*a^4*b*(a + b*x^3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*
f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(13/3)*b
^(5/3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3
)*x])/(27*a^(13/3)*b^(5/3)) + ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1498

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1))), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n))*
(n*(-d)^(-(m - Mod[m, n])/n + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p
- ((c*d^2 - b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*
(Mod[m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &
& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-2b^2\left(\frac{2b^3c}{a^2}-\frac{2b^2d}{a}+2be+af\right)x^6}{x^5\left(a+bx^3\right)^2}\,dx}{6ab^3}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)x^2}{9a^4b\left(a+bx^3\right)} - \frac{\int \frac{-18a^2b^5c-}{}}{}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)x^2}{9a^4b\left(a+bx^3\right)} - \frac{\int \left(-\frac{18ab^5}{x^5}\right.}{}$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be\right.}{9a^4b\left(a+bx^3\right)}$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be\right.}{9a^4b\left(a+bx^3\right)}$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be\right.}{9a^4b\left(a+bx^3\right)}$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be\right.}{9a^4b\left(a+bx^3\right)}$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be\right.}{9a^4b\left(a+bx^3\right)}$$

Mathematica [A]

time = 0.15, size = 303, normalized size = 0.96

$$\frac{-\frac{27a^{4/3}c}{x^4}-\frac{108\sqrt[3]{a}\,(-3bc+ad)}{x}-\frac{18a^{4/3}\left(-b^3c+ab^2d-a^2be+a^3f\right)x^2}{b\left(a+bx^3\right)^2}+\frac{12\sqrt[3]{a}\left(8b^3c-5ab^2d+2a^2be+a^3f\right)x^2}{b\left(a+bx^3\right)}-\frac{4\sqrt{3}\left(35b^3c-14ab^2d+2a^2be+a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}}-\frac{4\left(35b^3c-14ab^2d+2a^2be+a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{5/3}}+\frac{2\left(35b^3c-14ab^2d+2a^2be+a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{5/3}}}{108a^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(4/3)*c)/x^4 - (108*a^(1/3)*(-3*b*c + a*d))/x - (18*a^(4/3)*(-(b^3*
c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)^2) + (12*a^(1/3)*(8*b^3
*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)) - (4*Sqrt[3]*(35*b
^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]])/b^(5/3) - (4*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/
3) + b^(1/3)*x])/b^(5/3) + (2*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3))/(108*a^(13/3))
```

Maple [A]
time = 0.38, size = 230, normalized size = 0.73

method	result
default	$\frac{\frac{\left(\frac{1}{9}a^3f+\frac{2}{9}a^2be-\frac{5}{9}a\,b^2d+\frac{8}{9}b^3c\right)x^5-\frac{a\left(a^3f-7a^2be+13a\,b^2d-19b^3c\right)x^2}{18b}}{\left(b\,x^3+a\right)^2}+\frac{\left(a^3f+2a^2be-14a\,b^2d+35b^3c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{9b}}{a^4}$
risch	$\frac{\frac{\left(a^3f+2a^2be-14a\,b^2d+35b^3c\right)x^9}{9a^4}-\frac{\left(2a^3f-14a^2be+98a\,b^2d-245b^3c\right)x^6}{36a^3b}-\frac{(2ad-5bc)x^3}{2a^2}-\frac{c}{4a}}{x^4\left(b\,x^3+a\right)^2}+\left(\_R=\text{RootOf}\left(a^{13}b^5\_Z^3+a^9f^3+6a^8be\,f^2-42a\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(((1/9*a^3*f+2/9*a^2*b*e-5/9*a*b^2*d+8/9*b^3*c)*x^5-1/18*a*(a^3*f-7*a
^2*b*e+13*a*b^2*d-19*b^3*c)/b*x^2)/(b*x^3+a)^2+1/9*(a^3*f+2*a^2*b*e-14*a*b^
2*d+35*b^3*c)/b*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))))-1/4*c/a^3/x^4-(a*d-3*b*c)/a^4/x
```

Maxima [A]
time = 0.51, size = 322, normalized size = 1.02

$$\frac{4\,(35\,b^4c-14\,ab^3d+a^3bf+2\,a^2b^2e)x^9+(245\,ab^3c-98\,a^2b^2d-2\,a^4f+14\,a^3be)x^6-9\,a^3bc+18\,(5\,a^2b^2c-2\,a^3bd)x^3}{36\,(a^4b^3x^{10}+2\,a^5b^2x^7+a^6bx^4)}+\frac{\sqrt{3}\,(35\,b^3c-14\,ab^2d+a^3f+2\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(35\,b^3c-14\,ab^2d+a^3f+2\,a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(35\,b^3c-14\,ab^2d+a^3f+2\,a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(4*(35*b^4*c - 14*a*b^3*d + a^3*b*f + 2*a^2*b^2*e)*x^9 + (245*a*b^3*c
- 98*a^2*b^2*d - 2*a^4*f + 14*a^3*b*e)*x^6 - 9*a^3*b*c + 18*(5*a^2*b^2*c -
2*a^3*b*d)*x^3)/(a^4*b^3*x^10 + 2*a^5*b^2*x^7 + a^6*b*x^4) + 1/27*sqrt(3)*(
35*b^3*c - 14*a*b^2*d + a^3*f + 2*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(a^4*b^2*(a/b)^(1/3)) + 1/54*(35*b^3*c - 14*a*b^2*d + a
^3*f + 2*a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b^2*(a/b)^(1/
3)) - 1/27*(35*b^3*c - 14*a*b^2*d + a^3*f + 2*a^2*b*e)*log(x + (a/b)^(1/3))
/(a^4*b^2*(a/b)^(1/3))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(274) = 548$.
time = 0.42, size = 1254, normalized size = 3.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/108*(12*(35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^9 - 27*a
^4*b^3*c + 3*(245*a^2*b^5*c - 98*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^
6 + 54*(5*a^3*b^4*c - 2*a^4*b^3*d)*x^3 + 6*sqrt(1/3)*((35*a*b^6*c - 14*a^2*
b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^10 + 2*(35*a^2*b^5*c - 14*a^3*b^4*d + 2*
a^4*b^3*e + a^5*b^2*f)*x^7 + (35*a^3*b^4*c - 14*a^4*b^3*d + 2*a^5*b^2*e + a
^6*b*f)*x^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b
*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-
a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((35*b^5*c - 14*a*b^4*d + 2*a^2*b^3*e + a^
3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e + a^4*b*f)*x^7 +
 (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(-a*b^2)^(2/3)*log(
b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*((35*b^5*c - 14*a*b^4*d
+ 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*
e + a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(
-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^5*x^10 + 2*a^6*b^4*x^7 + a^
7*b^3*x^4), 1/108*(12*(35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)
*x^9 - 27*a^4*b^3*c + 3*(245*a^2*b^5*c - 98*a^3*b^4*d + 14*a^4*b^3*e - 2*a^
5*b^2*f)*x^6 + 54*(5*a^3*b^4*c - 2*a^4*b^3*d)*x^3 + 12*sqrt(1/3)*((35*a*b^6
*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^10 + 2*(35*a^2*b^5*c - 14*a^
3*b^4*d + 2*a^4*b^3*e + a^5*b^2*f)*x^7 + (35*a^3*b^4*c - 14*a^4*b^3*d + 2*a
^5*b^2*e + a^6*b*f)*x^4)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x +
(-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*((35*b^5*c - 14*a*b^4*d + 2*
a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e +
a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(-a*b
^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*((35*b^5*c
 - 14*a*b^4*d + 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*
d + 2*a^3*b^2*e + a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e +
```

```
a^5*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^5*x^10 + 2*a^
6*b^4*x^7 + a^7*b^3*x^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.36, size = 357, normalized size = 1.13

$$\frac{\sqrt{3}\,(35\,b^3c-14\,ab^2d+a^3f+2\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}a^4b}-\frac{(35\,b^3c-14\,ab^2d+a^3f+2\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}a^4b}-\frac{\left(35\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5b}+\frac{16\,b^4cx^5-10\,ab^3dx^5+2\,a^3bfx^5+4\,a^2b^2x^5e+19\,ab^3cx^2-13\,a^2b^2dx^2-a^4fx^2+7\,a^3bx^2e}{18\,\left(bx^3+a\right)^2a^4b}+\frac{12\,bcx^3-4\,adx^3-ac}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(35*b^3*c - 14*a*b^2*d + a^3*f + 2*a^2*b*e)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^4*b) - 1/54*(35*b^3*c
 - 14*a*b^2*d + a^3*f + 2*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/((-a*b^2)^(1/3)*a^4*b) - 1/27*(35*b^3*c*(-a/b)^(1/3) - 14*a*b^2*d*(-a/b)^(
1/3) + a^3*f*(-a/b)^(1/3) + 2*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^5*b) + 1/18*(16*b^4*c*x^5 - 10*a*b^3*d*x^5 + 2*a^3*b*f*
x^5 + 4*a^2*b^2*x^5*e + 19*a*b^3*c*x^2 - 13*a^2*b^2*d*x^2 - a^4*f*x^2 + 7*a
^3*b*x^2*e)/((b*x^3 + a)^2*a^4*b) + 1/4*(12*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^4
*x^4)
```

Mupad [**B**]
time = 5.23, size = 293, normalized size = 0.92

$$-\frac{\frac{c}{4\,a}-\frac{x^9\,\left(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3\right)}{9\,a^4}+\frac{x^3\,\left(2\,a\,d-5\,b\,c\right)}{2\,a^2}-\frac{x^6\,\left(-2\,f\,a^3+14\,e\,a^2\,b-98\,d\,a\,b^2+245\,c\,b^3\right)}{36\,a^3\,b}}{a^2\,x^4+2\,a\,b\,x^7+b^2\,x^{10}}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3\right)}{27\,a^{13/3}\,b^{5/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3\right)}{27\,a^{13/3}\,b^{5/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3\right)}{27\,a^{13/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(35
*b^3*c + a^3*f - 14*a*b^2*d + 2*a^2*b*e))/(27*a^(13/3)*b^(5/3)) - (log(b^(1
/3)*x + a^(1/3))*(35*b^3*c + a^3*f - 14*a*b^2*d + 2*a^2*b*e))/(27*a^(13/3)*
b^(5/3)) - (c/(4*a) - (x^9*(35*b^3*c + a^3*f - 14*a*b^2*d + 2*a^2*b*e))/(9*
a^4) + (x^3*(2*a*d - 5*b*c))/(2*a^2) - (x^6*(245*b^3*c - 2*a^3*f - 98*a*b^2
*d + 14*a^2*b*e))/(36*a^3*b))/(a^2*x^4 + b^2*x^10 + 2*a*b*x^7) - (log(3^(1/
2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(35*b^3*c + a
^3*f - 14*a*b^2*d + 2*a^2*b*e))/(27*a^(13/3)*b^(5/3))
```

3.298 $\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3} \, dx$

Optimal. Leaf size=316

$$-\frac{c}{5a^3x^5}+\frac{3bc-ad}{2a^4x^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2}+\frac{\left(17b^3c-11ab^2d+5a^2be+a^3f\right)x}{18a^4b\left(a+bx^3\right)}-\frac{\left(44b^3c-20ab^2d+5a\right.}{9}$$

```
[Out] -1/5*c/a^3/x^5+1/2*(-a*d+3*b*c)/a^4/x^2+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
x/a^3/b/(b*x^3+a)^2+1/18*(a^3*f+5*a^2*b*e-11*a*b^2*d+17*b^3*c)*x/a^4/b/(b*x
^3+a)+1/27*(a^3*f+5*a^2*b*e-20*a*b^2*d+44*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(1
4/3)/b^(4/3)-1/54*(a^3*f+5*a^2*b*e-20*a*b^2*d+44*b^3*c)*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)/b^(4/3)-1/27*(a^3*f+5*a^2*b*e-20*a*b^2*d+44
*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)/b^(4/3)*
3^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1843, 1498, 1502, 206, 31, 648, 631, 210, 642}

$$\frac{3bc-ad}{2a^4x^2}-\frac{c}{5a^3x^5}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{9\sqrt{3}a^{14/3}b^{4/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{54a^{14/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{27a^{14/3}b^{4/3}}+\frac{x\left(a^3f+5a^2be-11ab^2d+17b^3c\right)}{18a^4b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] -1/5*c/(a^3*x^5) + (3*b*c - a*d)/(2*a^4*x^2) + ((b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*x)/(6*a^3*b*(a + b*x^3)^2) + ((17*b^3*c - 11*a*b^2*d + 5*a^2*b*e +
 a^3*f)*x)/(18*a^4*b*(a + b*x^3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a
^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(14/3
)*b^(4/3)) + ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(
1/3)*x])/(27*a^(14/3)*b^(4/3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*
f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(14/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1498

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1))), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n))*
(n*(-d)^(-(m - Mod[m, n])/n + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p
- ((c*d^2 - b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*
(Mod[m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &
& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-b^2\left(\frac{5b^3c}{a^2}-\frac{5b^2d}{a}+5be+af\right)x^6}{x^6\left(a+bx^3\right)^2}\,dx}{6ab^3}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+a^3f\right)x}{18a^4b\left(a+bx^3\right)} - \frac{\int \frac{-18a^2b^5c \cdots}{\cdots}}{\cdots}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+a^3f\right)x}{18a^4b\left(a+bx^3\right)} - \frac{\int \left(-\frac{18ab^5 \cdots}{x^6} \cdots\right)}{\cdots}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2b \cdots\right.}{18a^4b\left(a+bx^3 \cdots\right.}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2b \cdots\right.}{18a^4b\left(a+bx^3 \cdots\right.}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2b \cdots\right.}{18a^4b\left(a+bx^3 \cdots\right.}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2b \cdots\right.}{18a^4b\left(a+bx^3 \cdots\right.}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2b \cdots\right.}{18a^4b\left(a+bx^3 \cdots\right.}$$

Mathematica [A]

time = 0.15, size = 299, normalized size = 0.95

$$\frac{-\frac{54a^{5/3}c}{x^5}-\frac{135a^{2/3}(-3bc+ad)}{x^2}-\frac{45a^{5/3}\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{b\left(a+bx^3\right)^2}+\frac{15a^{2/3}\left(17b^3c-11ab^2d+5a^2be+a^3f\right)x}{b\left(a+bx^3\right)}-\frac{10\sqrt{3}\left(44b^3c-20ab^2d+5a^2be+a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}}+\frac{10\left(44b^3c-20ab^2d+5a^2be+a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{4/3}}-\frac{5\left(44b^3c-20ab^2d+5a^2be+a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{4/3}}}{270a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(5/3)*c)/x^5 - (135*a^(2/3)*(-3*b*c + a*d))/x^2 - (45*a^(5/3)*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*x))/(b*(a + b*x^3)^2) + (15*a^(2/3)*(17*b^
3*c - 11*a*b^2*d + 5*a^2*b*e + a^3*f)*x))/(b*(a + b*x^3)) - (10*Sqrt[3]*(44*
b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(4/3) + (10*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(
1/3) + b^(1/3)*x])/b^(4/3) - (5*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3))/(270*a^(14/3))
```

Maple [A]
time = 0.38, size = 228, normalized size = 0.72

method	result
default	$\frac{\frac{\left(\frac{1}{18}a^{3}f+\frac{5}{18}a^{2}be-\frac{11}{18}a\,b^{2}d+\frac{17}{18}b^{3}c\right)x^{4}-\frac{a\left(a^{3}f-4a^{2}be+7a\,b^{2}d-10b^{3}c\right)x}{9b}}{\left(b\,x^{3}+a\right)^{2}}+\frac{\left(a^{3}f+5a^{2}be-20a\,b^{2}d+44b^{3}c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{9b}}{a^{4}}$
risch	$\frac{\frac{\left(a^{3}f+5a^{2}be-20a\,b^{2}d+44b^{3}c\right)x^{9}}{18a^{4}}-\frac{\left(5a^{3}f-20a^{2}be+80a\,b^{2}d-176b^{3}c\right)x^{6}}{45a^{3}b}-\frac{\left(5ad-11bc\right)x^{3}}{10a^{2}}-\frac{c}{5a}}{x^{5}\left(b\,x^{3}+a\right)^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{14}b^{4}\_Z^{3}-a^{9}f^{3}-15a^{8}be\,f^{2}+6\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(((1/18*a^3*f+5/18*a^2*b*e-11/18*a*b^2*d+17/18*b^3*c)*x^4-1/9*a*(a^3*
f-4*a^2*b*e+7*a*b^2*d-10*b^3*c)/b*x)/(b*x^3+a)^2+1/9*(a^3*f+5*a^2*b*e-20*a*
b^2*d+44*b^3*c)/b*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))))-1/5*c/a^3/x^5-1/2*(a*d-3*b*c)/a^4/x^2
```

Maxima [A]
time = 0.50, size = 323, normalized size = 1.02

$$\frac{5\,(44\,b^4c-20\,ab^3d+a^3bf+5\,a^2b^2e)x^9+2\,(176\,ab^3c-80\,a^2b^2d-5\,a^4f+20\,a^3be)x^6-18\,a^3bc+9\,(11\,a^2b^2c-5\,a^3bd)x^3}{90\,(a^4b^3x^{11}+2\,a^5b^2x^8+a^6bx^5)}+\frac{\sqrt{3}\,(44\,b^3c-20\,ab^2d+a^3f+5\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(44\,b^3c-20\,ab^2d+a^3f+5\,a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(44\,b^3c-20\,ab^2d+a^3f+5\,a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/90*(5*(44*b^4*c - 20*a*b^3*d + a^3*b*f + 5*a^2*b^2*e)*x^9 + 2*(176*a*b^3*
c - 80*a^2*b^2*d - 5*a^4*f + 20*a^3*b*e)*x^6 - 18*a^3*b*c + 9*(11*a^2*b^2*c
 - 5*a^3*b*d)*x^3)/(a^4*b^3*x^11 + 2*a^5*b^2*x^8 + a^6*b*x^5) + 1/27*sqrt(3
)*(44*b^3*c - 20*a*b^2*d + a^3*f + 5*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/(a^4*b^2*(a/b)^(2/3)) - 1/54*(44*b^3*c - 20*a*b^2*d
+ a^3*f + 5*a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b^2*(a/b)^
(2/3)) + 1/27*(44*b^3*c - 20*a*b^2*d + a^3*f + 5*a^2*b*e)*log(x + (a/b)^(1/
3))/(a^4*b^2*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(271) = 542$.
time = 0.44, size = 1247, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/270*(15*(44*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f)*x^9 - 54
*a^5*b^2*c + 6*(176*a^3*b^4*c - 80*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x^
6 + 27*(11*a^4*b^3*c - 5*a^5*b^2*d)*x^3 + 15*sqrt(1/3)*((44*a*b^6*c - 20*a^
2*b^5*d + 5*a^3*b^4*e + a^4*b^3*f)*x^11 + 2*(44*a^2*b^5*c - 20*a^3*b^4*d +
5*a^4*b^3*e + a^5*b^2*f)*x^8 + (44*a^3*b^4*c - 20*a^4*b^3*d + 5*a^5*b^2*e +
 a^6*b*f)*x^5)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x
- a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(
a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*((44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e + a
^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e + a^4*b*f)*x^8
+ (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*((44*b^5*c - 20*a*b^4*d +
 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e
 + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5)*(a
^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^4*x^11 + 2*a^7*b^3*x^8 + a^8
*b^2*x^5), 1/270*(15*(44*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f
)*x^9 - 54*a^5*b^2*c + 6*(176*a^3*b^4*c - 80*a^4*b^3*d + 20*a^5*b^2*e - 5*a
^6*b*f)*x^6 + 27*(11*a^4*b^3*c - 5*a^5*b^2*d)*x^3 + 30*sqrt(1/3)*((44*a*b^6
*c - 20*a^2*b^5*d + 5*a^3*b^4*e + a^4*b^3*f)*x^11 + 2*(44*a^2*b^5*c - 20*a^
3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f)*x^8 + (44*a^3*b^4*c - 20*a^4*b^3*d + 5*a
^5*b^2*e + a^6*b*f)*x^5)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^
(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*((44*b^5*c - 20*a
*b^4*d + 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a
^3*b^2*e + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)
*x^5)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(
(44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20
*a^2*b^3*d + 5*a^3*b^2*e + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*
```

```
a^4*b*e + a^5*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^4*x^
11 + 2*a^7*b^3*x^8 + a^8*b^2*x^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.02, size = 310, normalized size = 0.98

$$-\frac{\sqrt{3}\,(44\,b^3c-20\,ab^2d+a^3f+5\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^4}-\frac{(44\,b^3c-20\,ab^2d+a^3f+5\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^4}-\frac{(44\,b^3c-20\,ab^2d+a^3f+5\,a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5b}+\frac{17\,b^4cx^4-11\,ab^3dx^4+a^3bfx^4+5\,a^2b^2x^4e+20\,ab^3cx-14\,a^2b^2dx-2\,a^4fx+8\,a^3bxe}{18\,(bx^3+a)^2a^4b}+\frac{15\,bcx^3-5\,adx^3-2\,ac}{10\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(44*b^3*c - 20*a*b^2*d + a^3*f + 5*a^2*b*e)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^4) - 1/54*(44*b^3*c
- 20*a*b^2*d + a^3*f + 5*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
((-a*b^2)^(2/3)*a^4) - 1/27*(44*b^3*c - 20*a*b^2*d + a^3*f + 5*a^2*b*e)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b) + 1/18*(17*b^4*c*x^4 - 11*a*b^
3*d*x^4 + a^3*b*f*x^4 + 5*a^2*b^2*x^4*e + 20*a*b^3*c*x - 14*a^2*b^2*d*x - 2
*a^4*f*x + 8*a^3*b*x*e)/((b*x^3 + a)^2*a^4*b) + 1/10*(15*b*c*x^3 - 5*a*d*x^
3 - 2*a*c)/(a^4*x^5)
```

Mupad [B]
time = 5.20, size = 293, normalized size = 0.93

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{27\,a^{14/3}\,b^{4/3}}-\frac{\frac{c}{5\,a}-\frac{x^9\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{18\,a^4}+\frac{x^3\,\left(5\,a\,d-11\,b\,c\right)}{10\,a^2}-\frac{x^6\,\left(-5\,f\,a^3+20\,e\,a^2\,b-80\,d\,a\,b^2+176\,c\,b^3\right)}{45\,a^3\,b}}{a^2\,x^5+2\,a\,b\,x^8+b^2\,x^{11}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{27\,a^{14/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{27\,a^{14/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(44*b^3*c + a^3*f - 20*a*b^2*d + 5*a^2*b*e))/(27*
a^(14/3)*b^(4/3)) - (c/(5*a) - (x^9*(44*b^3*c + a^3*f - 20*a*b^2*d + 5*a^2*
b*e))/(18*a^4) + (x^3*(5*a*d - 11*b*c))/(10*a^2) - (x^6*(176*b^3*c - 5*a^3*
f - 80*a*b^2*d + 20*a^2*b*e))/(45*a^3*b))/(a^2*x^5 + b^2*x^11 + 2*a*b*x^8)
+ (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(
44*b^3*c + a^3*f - 20*a*b^2*d + 5*a^2*b*e))/(27*a^(14/3)*b^(4/3)) - (log(3^
(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(44*b^3*c
+ a^3*f - 20*a*b^2*d + 5*a^2*b*e))/(27*a^(14/3)*b^(4/3))
```

3.299 $\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=343

$$-\frac{c}{7a^3x^7}+\frac{3bc-ad}{4a^4x^4}-\frac{6b^2c-3abd+a^2e}{a^5x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2}-\frac{\left(11b^3c-8ab^2d+5a^2be-2a^3f\right)x^2}{9a^5\left(a+bx^3\right)}$$

```
[Out] -1/7*c/a^3/x^7+1/4*(-a*d+3*b*c)/a^4/x^4+(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x-1/6*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^4/(b*x^3+a)^2-1/9*(-2*a^3*f+5*a^2*b*e-
8*a*b^2*d+11*b^3*c)*x^2/a^5/(b*x^3+a)+1/27*(-2*a^3*f+14*a^2*b*e-35*a*b^2*d+
65*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(16/3)/b^(2/3)-1/54*(-2*a^3*f+14*a^2*b*e-
35*a*b^2*d+65*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(16/3)/b^(
2/3)+1/27*(-2*a^3*f+14*a^2*b*e-35*a*b^2*d+65*b^3*c)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(16/3)/b^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1848, 298, 31, 648, 631, 210, 642}

$$\frac{3bc-ad}{4a^4x^4}-\frac{c}{7a^3x^7}-\frac{a^2e-3abd+6b^2c}{a^5x}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^3f+14a^2be-35ab^2d+65b^3c\right)}{9\sqrt{3}a^{16/3}b^{2/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^3f+14a^2be-35ab^2d+65b^3c\right)}{54a^{16/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^3f+14a^2be-35ab^2d+65b^3c\right)}{27a^{16/3}b^{2/3}}-\frac{x^2\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{9a^5\left(a+bx^3\right)}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]
```

```
[Out] -1/7*c/(a^3*x^7) + (3*b*c - a*d)/(4*a^4*x^4) - (6*b^2*c - 3*a*b*d + a^2*e)/
(a^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^4*(a + b*x^3)^2) -
 ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*x^2)/(9*a^5*(a + b*x^3)) + (
(65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(16/3)*b^(2/3)) + ((65*b^3*c - 35*a*b^2
*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(16/3)*b^(2/3))
- ((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(54*a^(16/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^3}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{4b^3\left(b^3c-a\right.}{}}{x^8\left(a+bx^3\right)^2}}{6ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\left(11b^3c-8ab^2d+5a^2be-2a^3f\right)x^2}{9a^5\left(a+bx^3\right)} + \frac{\int \frac{18b^6}{}}{}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\left(11b^3c-8ab^2d+5a^2be-2a^3f\right)x^2}{9a^5\left(a+bx^3\right)} + \frac{\int \left(\frac{18}{a}\right.}{}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{4a^4x^4} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \left(\right.$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{4a^4x^4} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \left(\right.$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{4a^4x^4} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \left(\right.$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{4a^4x^4} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \left(\right.$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{4a^4x^4} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \left(\right.$$

Mathematica [A]
time = 0.15, size = 328, normalized size = 0.96

$$\frac{-\frac{108a^{7/3}c}{x^7}-\frac{189a^{4/3}(-3bc+ad)}{x^4}-\frac{756\sqrt[3]{a}\left(6b^2c-3abd+a^2e\right)}{x}+\frac{126a^{4/3}\left(-b^3c+ab^2d-a^2be+a^3f\right)x^2}{\left(a+bx^3\right)^2}+\frac{84\sqrt[3]{a}\left(-11b^3c+8ab^2d-5a^2be+2a^3f\right)x^2}{a+bx^3}+\frac{28\sqrt{3}\left(65b^3c-35ab^2d+14a^2be-2a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{28\left(65b^3c-35ab^2d+14a^2be-2a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{14\left(-65b^3c+35ab^2d-14a^2be+2a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}}{756a^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]
```

```
[Out] ((-108*a^(7/3)*c)/x^7 - (189*a^(4/3)*(-3*b*c + a*d))/x^4 - (756*a^(1/3)*(6*
b^2*c - 3*a*b*d + a^2*e))/x + (126*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*x^2)/(a + b*x^3)^2 + (84*a^(1/3)*(-11*b^3*c + 8*a*b^2*d - 5*a^2*b*e
+ 2*a^3*f)*x^2)/(a + b*x^3) + (28*Sqrt[3]*(65*b^3*c - 35*a*b^2*d + 14*a^2*b
*e - 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(6
5*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2
```

```
/3) + (14*(-65*b^3*c + 35*a*b^2*d - 14*a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(756*a^(16/3))
```

Maple [A]
time = 0.44, size = 253, normalized size = 0.74

method	result
default	$\frac{\frac{\left(\frac{2}{9}a^3bf-\frac{5}{9}a^2e\,b^2+\frac{8}{9}ad\,b^3-\frac{11}{9}c\,b^4\right)x^5+\frac{a\left(7a^3f-13a^2be+19a\,b^2d-25b^3c\right)x^2}{18}}{\left(b\,x^3+a\right)^2}+\left(\frac{2}{9}a^3f-\frac{14}{9}a^2be+\frac{35}{9}a\,b^2d-\frac{65}{9}b^3c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\right.}{}\right.}{a^5}$
risch	$\frac{\frac{b\left(2a^3f-14a^2be+35a\,b^2d-65b^3c\right)x^{12}}{9a^5}+\frac{7\left(2a^3f-14a^2be+35a\,b^2d-65b^3c\right)x^9}{36a^4}-\frac{\left(14a^2e-35abd+65b^2c\right)x^6}{14a^3}-\frac{(7ad-13bc)x^3}{28a^2}-\frac{c}{7a}}{x^7\left(b\,x^3+a\right)^2}+\left(\frac{}{\_R=\mathrm{RootOf}(}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(((2/9*a^3*b*f-5/9*a^2*e*b^2+8/9*a*d*b^3-11/9*c*b^4)*x^5+1/18*a*(7*a^
3*f-13*a^2*b*e+19*a*b^2*d-25*b^3*c)*x^2)/(b*x^3+a)^2+(2/9*a^3*f-14/9*a^2*b*
e+35/9*a*b^2*d-65/9*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b
)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/7*c/a^3/x^7-1/4*(a*d-3*b*c)/a^4/x^4-(a
^2*e-3*a*b*d+6*b^2*c)/a^5/x
```

Maxima [A]
time = 0.50, size = 349, normalized size = 1.02

$$-\frac{28\,(65\,b^4c-35\,ab^3d-2\,a^3bf+14\,a^2b^2e)x^{12}+49\,(65\,ab^3c-35\,a^2b^2d-2\,a^4f+14\,a^3be)x^9+18\,(65\,a^2b^2c-35\,a^3bd+14\,a^4e)x^6+36\,a^4c-9\,(13\,a^3bc-7\,a^4d)x^3}{252\,(a^5b^2x^{13}+2\,a^6bx^{10}+a^7x^7)}-\frac{\sqrt{3}\,(65\,b^3c-35\,ab^2d-2\,a^3f+14\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(65\,b^3c-35\,ab^2d-2\,a^3f+14\,a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^5b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(65\,b^3c-35\,ab^2d-2\,a^3f+14\,a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^5b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/252*(28*(65*b^4*c - 35*a*b^3*d - 2*a^3*b*f + 14*a^2*b^2*e)*x^12 + 49*(65
*a*b^3*c - 35*a^2*b^2*d - 2*a^4*f + 14*a^3*b*e)*x^9 + 18*(65*a^2*b^2*c - 35
*a^3*b*d + 14*a^4*e)*x^6 + 36*a^4*c - 9*(13*a^3*b*c - 7*a^4*d)*x^3)/(a^5*b^
2*x^13 + 2*a^6*b*x^10 + a^7*x^7) - 1/27*sqrt(3)*(65*b^3*c - 35*a*b^2*d - 2*
a^3*f + 14*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^
5*b*(a/b)^(1/3)) - 1/54*(65*b^3*c - 35*a*b^2*d - 2*a^3*f + 14*a^2*b*e)*log(
```

```
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*b*(a/b)^(1/3)) + 1/27*(65*b^3*c - 3
5*a*b^2*d - 2*a^3*f + 14*a^2*b*e)*log(x + (a/b)^(1/3))/(a^5*b*(a/b)^(1/3))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(298) = 596$.
time = 0.41, size = 1340, normalized size = 3.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/756*(84*(65*a*b^6*c - 35*a^2*b^5*d + 14*a^3*b^4*e - 2*a^4*b^3*f)*x^12 +
 147*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^9 + 108*a
^5*b^2*c + 54*(65*a^3*b^4*c - 35*a^4*b^3*d + 14*a^5*b^2*e)*x^6 - 27*(13*a^4
*b^3*c - 7*a^5*b^2*d)*x^3 + 42*sqrt(1/3)*((65*a*b^6*c - 35*a^2*b^5*d + 14*a
^3*b^4*e - 2*a^4*b^3*f)*x^13 + 2*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*b^3*
e - 2*a^5*b^2*f)*x^10 + (65*a^3*b^4*c - 35*a^4*b^3*d + 14*a^5*b^2*e - 2*a^6
*b*f)*x^7)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
 + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*
b^2)^(2/3)*x)/(b*x^3 + a)) + 14*((65*b^5*c - 35*a*b^4*d + 14*a^2*b^3*e - 2*
a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*
x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^(
2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((65*b^5*c - 3
5*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d
 + 14*a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b
*e - 2*a^5*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^6*b^4*x^13
+ 2*a^7*b^3*x^10 + a^8*b^2*x^7), -1/756*(84*(65*a*b^6*c - 35*a^2*b^5*d + 14
*a^3*b^4*e - 2*a^4*b^3*f)*x^12 + 147*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*
b^3*e - 2*a^5*b^2*f)*x^9 + 108*a^5*b^2*c + 54*(65*a^3*b^4*c - 35*a^4*b^3*d
+ 14*a^5*b^2*e)*x^6 - 27*(13*a^4*b^3*c - 7*a^5*b^2*d)*x^3 + 84*sqrt(1/3)*((
65*a*b^6*c - 35*a^2*b^5*d + 14*a^3*b^4*e - 2*a^4*b^3*f)*x^13 + 2*(65*a^2*b^
5*c - 35*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^10 + (65*a^3*b^4*c - 35*
a^4*b^3*d + 14*a^5*b^2*e - 2*a^6*b*f)*x^7)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(s
qrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*((65*b^5*
c - 35*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*
b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*
a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (
-a*b^2)^(2/3)) - 28*((65*b^5*c - 35*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x
^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a
^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^(2/3)*log(b*x
 - (-a*b^2)^(1/3)))/(a^6*b^4*x^13 + 2*a^7*b^3*x^10 + a^8*b^2*x^7)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.68, size = 380, normalized size = 1.11

$$-\frac{\sqrt{3}\,(65\,b^3c-35\,ab^2d-2\,a^3f+14\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{1}{3}}\,a^5}+\frac{(65\,b^3c-35\,ab^2d-2\,a^3f+14\,a^2be)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{1}{3}}\,a^5}+\frac{\left(65\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-35\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+14\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^6}-\frac{22\,b^4cx^5-16\,ab^3dx^5-4\,a^3bfx^5+10\,a^2b^2x^5e+25\,ab^3cx^2-19\,a^2b^2dx^2-7\,a^4fx^2+13\,a^3bx^2e}{18\,(bx^3+a)^2a^5}-\frac{168\,b^2cx^6-84\,abdx^6+28\,a^2x^6e-21\,abcx^3+7\,a^2dx^3+4\,a^2c}{28\,a^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(65*b^3*c - 35*a*b^2*d - 2*a^3*f + 14*a^2*b*e)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^5) + 1/54*(65*b^3
*c - 35*a*b^2*d - 2*a^3*f + 14*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(1/3)*a^5) + 1/27*(65*b^3*c*(-a/b)^(1/3) - 35*a*b^2*d*(-a/b
)^(1/3) - 2*a^3*f*(-a/b)^(1/3) + 14*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(
abs(x - (-a/b)^(1/3)))/a^6 - 1/18*(22*b^4*c*x^5 - 16*a*b^3*d*x^5 - 4*a^3*b*
f*x^5 + 10*a^2*b^2*x^5*e + 25*a*b^3*c*x^2 - 19*a^2*b^2*d*x^2 - 7*a^4*f*x^2
+ 13*a^3*b*x^2*e)/((b*x^3 + a)^2*a^5) - 1/28*(168*b^2*c*x^6 - 84*a*b*d*x^6
+ 28*a^2*x^6*e - 21*a*b*c*x^3 + 7*a^2*d*x^3 + 4*a^2*c)/(a^5*x^7)
```

Mupad [B]
time = 5.26, size = 321, normalized size = 0.94

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+65\,c\,b^3\right)}{27\,a^{16/3}\,b^{2/3}}-\frac{\frac{c}{7\,a}+\frac{7\,x^9\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+65\,c\,b^3\right)}{36\,a^4}+\frac{x^3\,\left(7\,a\,d-13\,b\,c\right)}{28\,a^2}+\frac{x^6\,\left(14\,e\,a^2-35\,d\,a\,b+65\,c\,b^2\right)}{14\,a^3}+\frac{b\,x^{12}\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+65\,c\,b^3\right)}{9\,a^5}}{a^2\,x^7+2\,a\,b\,x^{10}+b^2\,x^{13}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+65\,c\,b^3\right)}{27\,a^{16/3}\,b^{2/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+65\,c\,b^3\right)}{27\,a^{16/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(65*b^3*c - 2*a^3*f - 35*a*b^2*d + 14*a^2*b*e))/(
27*a^(16/3)*b^(2/3)) - (c/(7*a) + (7*x^9*(65*b^3*c - 2*a^3*f - 35*a*b^2*d +
 14*a^2*b*e))/(36*a^4) + (x^3*(7*a*d - 13*b*c))/(28*a^2) + (x^6*(65*b^2*c +
 14*a^2*e - 35*a*b*d))/(14*a^3) + (b*x^12*(65*b^3*c - 2*a^3*f - 35*a*b^2*d
+ 14*a^2*b*e))/(9*a^5))/(a^2*x^7 + b^2*x^13 + 2*a*b*x^10) - (log(3^(1/2)*a^
(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(65*b^3*c - 2*a^3*
f - 35*a*b^2*d + 14*a^2*b*e))/(27*a^(16/3)*b^(2/3)) + (log(3^(1/2)*a^(1/3)*
1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(65*b^3*c - 2*a^3*f - 35
*a*b^2*d + 14*a^2*b*e))/(27*a^(16/3)*b^(2/3))
```

3.300 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^3}\,dx$

Optimal. Leaf size=341

$$-\frac{c}{8a^3x^8}+\frac{3bc-ad}{5a^4x^5}-\frac{6b^2c-3abd+a^2e}{2a^5x^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^4\,(a+bx^3)^2}-\frac{(23b^3c-17ab^2d+11a^2be-5a^3f)\,x}{18a^5\,(a+bx^3)}$$

```
[Out] -1/8*c/a^3/x^8+1/5*(-a*d+3*b*c)/a^4/x^5+1/2*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x^
2-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^3+a)^2-1/18*(-5*a^3*f+11*a^
2*b*e-17*a*b^2*d+23*b^3*c)*x/a^5/(b*x^3+a)-1/27*(-5*a^3*f+20*a^2*b*e-44*a*b
^2*d+77*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(17/3)/b^(1/3)+1/54*(-5*a^3*f+20*a^2
*b*e-44*a*b^2*d+77*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(17/3
)/b^(1/3)+1/27*(-5*a^3*f+20*a^2*b*e-44*a*b^2*d+77*b^3*c)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(17/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1848, 206, 31, 648, 631, 210, 642}

$$\frac{3bc-ad}{5a^4x^5}-\frac{c}{8a^3x^8}-\frac{a^2e-3abd+6b^2c}{2a^5x^2}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)(-5a^3f+20a^2be-44ab^2d+77b^3c)}{9\sqrt{3}\,a^{17/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)(-5a^3f+20a^2be-44ab^2d+77b^3c)}{27a^{17/3}\sqrt[3]{b}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)(-5a^3f+20a^2be-44ab^2d+77b^3c)}{54a^{17/3}\sqrt[3]{b}}-\frac{x(-5a^3f+11a^2be-17ab^2d+23b^3c)}{18a^5\,(a+bx^3)}-\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^4\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x]
```

```
[Out] -1/8*c/(a^3*x^8) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/
(2*a^5*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^4*(a + b*x^3)^2)
 - ((23*b^3*c - 17*a*b^2*d + 11*a^2*b*e - 5*a^3*f)*x)/(18*a^5*(a + b*x^3))
+ ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(17/3)*b^(1/3)) - ((77*b^3*c - 44*a*
b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(17/3)*b^(1/3
)) + ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(54*a^(17/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^3}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{5b^3\left(b^3c-ab\cdots\right.}{\cdots}}{x^9\left(a+bx^3\right)^2}}{6ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\left(23b^3c-17ab^2d+11a^2be-5a^3f\right)x}{18a^5\left(a+bx^3\right)} + \frac{\int 18b^{\cdots}}{\cdots}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\left(23b^3c-17ab^2d+11a^2be-5a^3f\right)x}{18a^5\left(a+bx^3\right)} + \frac{\int\left(\frac{18\cdots}{a\cdots}\right.}{\cdots}$$

$$= -\frac{c}{8a^3x^8} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{(2\cdots}{\cdots}$$

$$= -\frac{c}{8a^3x^8} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{(2\cdots}{\cdots}$$

$$= -\frac{c}{8a^3x^8} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{(2\cdots}{\cdots}$$

$$= -\frac{c}{8a^3x^8} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{(2\cdots}{\cdots}$$

$$= -\frac{c}{8a^3x^8} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{(2\cdots}{\cdots}$$

Mathematica [A]
time = 0.15, size = 324, normalized size = 0.95

$$\frac{-\frac{135a^{8/3}c}{x^8}-\frac{216a^{5/3}(-3bc+ad)}{x^5}-\frac{540a^{2/3}\left(6b^2c-3abd+a^2e\right)}{x^2}+\frac{180a^{5/3}\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{\left(a+bx^3\right)^2}+\frac{60a^{2/3}\left(-23b^3c+17ab^2d-11a^2be+5a^3f\right)x}{a+bx^3}+\frac{40\sqrt{3}\left(77b^3c-44ab^2d+20a^2be-5a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{40\left(-77b^3c+44ab^2d-20a^2be+5a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{20\left(77b^3c-44ab^2d+20a^2be-5a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}}{1080a^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x]
```

```
[Out] ((-135*a^(8/3)*c)/x^8 - (216*a^(5/3)*(-3*b*c + a*d))/x^5 - (540*a^(2/3)*(6*
b^2*c - 3*a*b*d + a^2*e))/x^2 + (180*a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e
+ a^3*f)*x)/(a + b*x^3)^2 + (60*a^(2/3)*(-23*b^3*c + 17*a*b^2*d - 11*a^2*b*
e + 5*a^3*f)*x)/(a + b*x^3) + (40*Sqrt[3]*(77*b^3*c - 44*a*b^2*d + 20*a^2*b
*e - 5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-
77*b^3*c + 44*a*b^2*d - 20*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(
```

```
1/3) + (20*(77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(1080*a^(17/3))
```

Maple [A]

time = 0.40, size = 252, normalized size = 0.74

method	result
default	$\frac{\frac{\left(\frac{5}{18}a^3bf-\frac{11}{18}a^2eb^2+\frac{17}{18}adb^3-\frac{23}{18}cb^4\right)x^4+\frac{a\left(4a^3f-7a^2be+10ab^2d-13b^3c\right)x}{9}}{\left(bx^3+a\right)^2}+\frac{\left(5a^3f-20a^2be+44ab^2d-77b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\cdots\right.}{6b\left(\frac{a}{b}\right)\cdots}\cdots\right.}{9}}{a^5}\cdots$
risch	$\frac{\frac{b\left(5a^3f-20a^2be+44ab^2d-77b^3c\right)x^{12}}{18a^5}+\frac{4\left(5a^3f-20a^2be+44ab^2d-77b^3c\right)x^9}{45a^4}-\frac{\left(20a^2e-44abd+77b^2c\right)x^6}{40a^3}-\frac{(4ad-7bc)x^3}{20a^2}-\frac{c}{8a}}{x^8\left(bx^3+a\right)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\cdots\right.}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(((5/18*a^3*b*f-11/18*a^2*e*b^2+17/18*a*d*b^3-23/18*c*b^4)*x^4+1/9*a*
(4*a^3*f-7*a^2*b*e+10*a*b^2*d-13*b^3*c)*x)/(b*x^3+a)^2+1/9*(5*a^3*f-20*a^2*
b*e+44*a*b^2*d-77*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(
2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/8*c/a^3/x^8-1/5*(a*d-3*b*c)/a^4/x^5-1/2*(
a^2*e-3*a*b*d+6*b^2*c)/a^5/x^2
```

Maxima [A]

time = 0.51, size = 349, normalized size = 1.02

$$-\frac{20\,(77\,b^4c-44\,ab^3d-5\,a^3bf+20\,a^2b^2e)x^{12}+32\,(77\,ab^3c-44\,a^2b^2d-5\,a^4f+20\,a^3be)x^9+9\,(77\,a^2b^2c-44\,a^3bd+20\,a^4e)x^6+45\,a^4c-18\,(7\,a^3bc-4\,a^4d)x^3}{360\,(a^5b^2x^{14}+2\,a^6bx^{11}+a^7x^8)}-\frac{\sqrt{3}\,(77\,b^3c-44\,ab^2d-5\,a^3f+20\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(77\,b^3c-44\,ab^2d-5\,a^3f+20\,a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^5b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(77\,b^3c-44\,ab^2d-5\,a^3f+20\,a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^5b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/360*(20*(77*b^4*c - 44*a*b^3*d - 5*a^3*b*f + 20*a^2*b^2*e)*x^12 + 32*(77
*a*b^3*c - 44*a^2*b^2*d - 5*a^4*f + 20*a^3*b*e)*x^9 + 9*(77*a^2*b^2*c - 44*
a^3*b*d + 20*a^4*e)*x^6 + 45*a^4*c - 18*(7*a^3*b*c - 4*a^4*d)*x^3)/(a^5*b^2
*x^14 + 2*a^6*b*x^11 + a^7*x^8) - 1/27*sqrt(3)*(77*b^3*c - 44*a*b^2*d - 5*a
^3*f + 20*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5
*b*(a/b)^(2/3)) + 1/54*(77*b^3*c - 44*a*b^2*d - 5*a^3*f + 20*a^2*b*e)*log(x
```

```
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*b*(a/b)^(2/3)) - 1/27*(77*b^3*c - 44
*a*b^2*d - 5*a^3*f + 20*a^2*b*e)*log(x + (a/b)^(1/3))/(a^5*b*(a/b)^(2/3))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(294) = 588$.
time = 0.44, size = 1317, normalized size = 3.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/1080*(60*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e - 5*a^5*b^2*f)*x^1
2 + 96*(77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x^9 + 135*a
^6*b*c + 27*(77*a^4*b^3*c - 44*a^5*b^2*d + 20*a^6*b*e)*x^6 - 54*(7*a^5*b^2*
c - 4*a^6*b*d)*x^3 + 60*sqrt(1/3)*((77*a*b^6*c - 44*a^2*b^5*d + 20*a^3*b^4*
e - 5*a^4*b^3*f)*x^14 + 2*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e - 5*a
^5*b^2*f)*x^11 + (77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x
^8)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*s
qrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3
)/b))/(b*x^3 + a)) - 20*((77*b^5*c - 44*a*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*
f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b^2*e - 5*a^4*b*f)*x^11 + (
77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*f)*x^8)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*((77*b^5*c - 44*a*b^4*d +
 20*a^2*b^3*e - 5*a^3*b^2*f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b
^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*
f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^7*b^3*x^14 + 2*a^8*b^2
*x^11 + a^9*b*x^8), -1/1080*(60*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e
 - 5*a^5*b^2*f)*x^12 + 96*(77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a
^6*b*f)*x^9 + 135*a^6*b*c + 27*(77*a^4*b^3*c - 44*a^5*b^2*d + 20*a^6*b*e)*x
^6 - 54*(7*a^5*b^2*c - 4*a^6*b*d)*x^3 + 120*sqrt(1/3)*((77*a*b^6*c - 44*a^2
*b^5*d + 20*a^3*b^4*e - 5*a^4*b^3*f)*x^14 + 2*(77*a^2*b^5*c - 44*a^3*b^4*d
+ 20*a^4*b^3*e - 5*a^5*b^2*f)*x^11 + (77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*
b^2*e - 5*a^6*b*f)*x^8)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(
2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*((77*b^5*c - 44*a
*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d +
20*a^3*b^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e
- 5*a^5*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) + 40*((77*b^5*c - 44*a*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*f)*x^14 + 2*(77
*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c -
44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)
^(2/3)))/(a^7*b^3*x^14 + 2*a^8*b^2*x^11 + a^9*b*x^8)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.65, size = 394, normalized size = 1.16

$$\frac{\left(77\,b^{3}c-44\,ab^{2}d-5\,a^{3}f+20\,a^{2}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{6}}-\frac{\sqrt{3}\left(77\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-44\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-5\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+20\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{6}b}-\frac{\left(77\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-44\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-5\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+20\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{6}b}-\frac{23\,b^{4}cx^{4}-17\,ab^{3}dx^{4}-5\,a^{3}bfx^{4}+11\,a^{2}b^{2}x^{4}e+26\,ab^{3}cx-20\,a^{2}b^{2}dx-8\,a^{4}fx+14\,a^{3}bxe}{18\left(bx^{3}+a\right)^{2}a^{5}}-\frac{120\,b^{2}cx^{6}-60\,abdx^{6}+20\,a^{2}x^{6}e-24\,abcx^{3}+8\,a^{2}dx^{3}+5\,a^{2}c}{40\,a^{5}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*(77*b^3*c - 44*a*b^2*d - 5*a^3*f + 20*a^2*b*e)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/a^6 - 1/27*sqrt(3)*(77*(-a*b^2)^(1/3)*b^3*c - 44*(-a*b^2)^
(1/3)*a*b^2*d - 5*(-a*b^2)^(1/3)*a^3*f + 20*(-a*b^2)^(1/3)*a^2*b*e)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^6*b) - 1/54*(77*(-a*b^2)^
(1/3)*b^3*c - 44*(-a*b^2)^(1/3)*a*b^2*d - 5*(-a*b^2)^(1/3)*a^3*f + 20*(-a*b
^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b) - 1/18*
(23*b^4*c*x^4 - 17*a*b^3*d*x^4 - 5*a^3*b*f*x^4 + 11*a^2*b^2*x^4*e + 26*a*b^
3*c*x - 20*a^2*b^2*d*x - 8*a^4*f*x + 14*a^3*b*x*e)/((b*x^3 + a)^2*a^5) - 1/
40*(120*b^2*c*x^6 - 60*a*b*d*x^6 + 20*a^2*x^6*e - 24*a*b*c*x^3 + 8*a^2*d*x^
3 + 5*a^2*c)/(a^5*x^8)
```

Mupad [B]
time = 5.22, size = 321, normalized size = 0.94

$$-\frac{\frac{c}{8\,a}+\frac{4\,x^9\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+77\,c\,b^3\right)}{45\,a^4}+\frac{x^3\,\left(4\,a\,d-7\,b\,c\right)}{20\,a^2}+\frac{x^6\,\left(20\,e\,a^2-44\,d\,a\,b+77\,c\,b^2\right)}{40\,a^3}+\frac{b\,x^{12}\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+77\,c\,b^3\right)}{18\,a^5}}{a^2\,x^8+2\,a\,b\,x^{11}+b^2\,x^{14}}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+77\,c\,b^3\right)}{27\,a^{17/3}\,b^{1/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+77\,c\,b^3\right)}{27\,a^{17/3}\,b^{1/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+77\,c\,b^3\right)}{27\,a^{17/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(77
*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e))/(27*a^(17/3)*b^(1/3)) - (log(b
^(1/3)*x + a^(1/3))*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e))/(27*a^(
17/3)*b^(1/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)
*1i)/2 - 1/2)*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e))/(27*a^(17/3)*
b^(1/3)) - (c/(8*a) + (4*x^9*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e)
)/(45*a^4) + (x^3*(4*a*d - 7*b*c))/(20*a^2) + (x^6*(77*b^2*c + 20*a^2*e - 4
4*a*b*d))/(40*a^3) + (b*x^12*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e)
)/(18*a^5))/(a^2*x^8 + b^2*x^14 + 2*a*b*x^11)
```

3.301 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=381

$$-\frac{c}{10a^3x^{10}}+\frac{3bc-ad}{7a^4x^7}-\frac{6b^2c-3abd+a^2e}{4a^5x^4}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x}+\frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^5\left(a+bx^3\right)^2}+$$

```
[Out] -1/10*c/a^3/x^10+1/7*(-a*d+3*b*c)/a^4/x^7+1/4*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/
x^4+(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x+1/6*b*(-a^3*f+a^2*b*e-a*b^2
*d+b^3*c)*x^2/a^5/(b*x^3+a)^2+1/9*b*(-5*a^3*f+8*a^2*b*e-11*a*b^2*d+14*b^3*c
)*x^2/a^6/(b*x^3+a)-1/27*b^(1/3)*(-14*a^3*f+35*a^2*b*e-65*a*b^2*d+104*b^3*c
)*ln(a^(1/3)+b^(1/3)*x)/a^(19/3)+1/54*b^(1/3)*(-14*a^3*f+35*a^2*b*e-65*a*b^
2*d+104*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(19/3)-1/27*b^(1
/3)*(-14*a^3*f+35*a^2*b*e-65*a*b^2*d+104*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/a^(19/3)*3^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1848, 298, 31, 648, 631, 210, 642}

$$\frac{3bc-ad}{7a^4x^7}-\frac{c}{10a^3x^{10}}-\frac{a^2e-3abd+6b^2c}{4a^5x^4}-\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-14a^3f+35a^2be-65ab^2d+104b^3c\right)}{9\sqrt{3}a^{19/3}}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-14a^3f+35a^2be-65ab^2d+104b^3c\right)}{27a^{19/3}}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-14a^3f+35a^2be-65ab^2d+104b^3c\right)}{54a^{19/3}}+\frac{bx^2\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)}{9a^6\left(a+bx^3\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{a^6x}+\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]
```

```
[Out] -1/10*c/(a^3*x^10) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e
)/(4*a^5*x^4) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^5*(a + b*x^3)^2) + (b*(14*b^3*c
- 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*x^2)/(9*a^6*(a + b*x^3)) - (b^(1/3)*(10
4*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(19/3)) - (b^(1/3)*(104*b^3*c - 65*a*b^2
*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(19/3)) + (b^(1
/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(54*a^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx = \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^5\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{6b^3\left(b^3c-ab\cdots\right.}{\cdots}}{x^{11}(a+bx^3)}}{6ab^3}$$

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^5\left(a+bx^3\right)^2} + \frac{b(14b^3c-11ab^2d+8a^2be-5a^3f)\,x^2}{9a^6\left(a+bx^3\right)} + \frac{\int \frac{18b\cdots}{\cdots}}{\cdots}$$

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^5\left(a+bx^3\right)^2} + \frac{b(14b^3c-11ab^2d+8a^2be-5a^3f)\,x^2}{9a^6\left(a+bx^3\right)} + \frac{\int \left(\frac{1\cdots}{a\cdots}\right.}{\cdots}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \cdots$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \cdots$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \cdots$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \cdots$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \cdots$$

Mathematica [A]

time = 0.27, size = 366, normalized size = 0.96

$$\frac{-\frac{378a^{10/3}c}{x^{10}}-\frac{540a^{7/3}(-3bc+ad)}{x^7}-\frac{945a^{4/3}(6b^2c-3abd+a^2e)}{x^4}-\frac{3780\sqrt[3]{a}\,(-10b^3c+6ab^2d-3a^2be+a^3f)}{x}-\frac{630a^{4/3}b(-b^3c+ab^2d-a^2be+a^3f)x^2}{(a+bx^3)^2}-\frac{420\sqrt[3]{a}\,b(-14b^3c+11ab^2d-8a^2be+5a^3f)x^2}{a+bx^3}-140\sqrt{3}\,\sqrt[3]{b}\,(104b^3c-65ab^2d+35a^2be-14a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+140\sqrt[3]{b}\,(-104b^3c+65ab^2d-35a^2be+14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+70\sqrt[3]{b}\,(104b^3c-65ab^2d+35a^2be-14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{3780a^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]
```

```
[Out] ((-378*a^(10/3)*c)/x^10 - (540*a^(7/3)*(-3*b*c + a*d))/x^7 - (945*a^(4/3)*(
6*b^2*c - 3*a*b*d + a^2*e))/x^4 - (3780*a^(1/3)*(-10*b^3*c + 6*a*b^2*d - 3*
a^2*b*e + a^3*f))/x - (630*a^(4/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)
*x^2)/(a + b*x^3)^2 - (420*a^(1/3)*b*(-14*b^3*c + 11*a*b^2*d - 8*a^2*b*e +
5*a^3*f)*x^2)/(a + b*x^3) - 140*Sqrt[3]*b^(1/3)*(104*b^3*c - 65*a*b^2*d + 3
```

```
5*a^2*b*e - 14*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(
1/3)*(-104*b^3*c + 65*a*b^2*d - 35*a^2*b*e + 14*a^3*f)*Log[a^(1/3) + b^(1/3
)*x] + 70*b^(1/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3780*a^(19/3))
```

Maple [A]
time = 0.38, size = 289, normalized size = 0.76

method	result
default	$-\frac{b\left(\frac{\frac{b\left(5a^3f-8a^2be+11a\,b^2d-14b^3c\right)x^5}{9}+\left(\frac{13}{18}a^4f-\frac{19}{18}a^3be+\frac{25}{18}a^2b^2d-\frac{31}{18}a\,b^3c\right)x^2}{\left(b\,x^3+a\right)^2}+\left(\frac{14}{9}a^3f-\frac{35}{9}a^2be+\frac{65}{9}a\,b^2d-\frac{104}{9}b^3c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.\right.}{a^6}$
risch	$\frac{-\frac{c}{10a}-\frac{(5ad-8bc)x^3}{35a^2}-\frac{\left(35a^2e-65abd+104b^2c\right)x^6}{140a^3}-\frac{\left(14a^3f-35a^2be+65a\,b^2d-104b^3c\right)x^9}{14a^4}-\frac{7b\left(14a^3f-35a^2be+65a\,b^2d-104b^3c\right)x^{12}}{36a^5}-\frac{b^2\left(14a^3f-\right.}{}}{x^{10}\left(b\,x^3+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^6*((1/9*b*(5*a^3*f-8*a^2*b*e+11*a*b^2*d-14*b^3*c)*x^5+(13/18*a^4*f-19/
18*a^3*b*e+25/18*a^2*b^2*d-31/18*a*b^3*c)*x^2)/(b*x^3+a)^2+(14/9*a^3*f-35/9
*a^2*b*e+65/9*a*b^2*d-104/9*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/
6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/10*c/a^3/x^10-1/7*(a*d-3*b*c)/
a^4/x^7-1/4*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^4-(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b
^3*c)/a^6/x
```

Maxima [A]
time = 0.51, size = 383, normalized size = 1.01

$$\frac{140\,(104\,b^5c-65\,ab^4d-14\,a^3b^2f+35\,a^2b^3e)x^{15}+245\,(104\,ab^4c-65\,a^2b^3d-14\,a^4bf+35\,a^3b^2e)x^{12}+90\,(104\,a^2b^3c-65\,a^3b^2d-14\,a^5f+35\,a^4be)x^9-9\,(104\,a^3b^2c-65\,a^4bd+35\,a^5e)x^6-126\,a^5c+36\,(8\,a^4bc-5\,a^5d)x^3}{1260\,(a^6b^2x^{16}+2\,a^7bx^{13}+a^8x^{10})}+\frac{\sqrt{3}\,(104\,b^3c-65\,ab^2d-14\,a^3f+35\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(104\,b^3c-65\,ab^2d-14\,a^3f+35\,a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(104\,b^3c-65\,ab^2d-14\,a^3f+35\,a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1260*(140*(104*b^5*c - 65*a*b^4*d - 14*a^3*b^2*f + 35*a^2*b^3*e)*x^15 + 2
45*(104*a*b^4*c - 65*a^2*b^3*d - 14*a^4*b*f + 35*a^3*b^2*e)*x^12 + 90*(104*
a^2*b^3*c - 65*a^3*b^2*d - 14*a^5*f + 35*a^4*b*e)*x^9 - 9*(104*a^3*b^2*c -
65*a^4*b*d + 35*a^5*e)*x^6 - 126*a^5*c + 36*(8*a^4*b*c - 5*a^5*d)*x^3)/(a^6
```

```
*b^2*x^16 + 2*a^7*b*x^13 + a^8*x^10) + 1/27*sqrt(3)*(104*b^3*c - 65*a*b^2*d
 - 14*a^3*f + 35*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a^6*(a/b)^(1/3)) + 1/54*(104*b^3*c - 65*a*b^2*d - 14*a^3*f + 35*a^2*b*e
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(1/3)) - 1/27*(104*b^3*
c - 65*a*b^2*d - 14*a^3*f + 35*a^2*b*e)*log(x + (a/b)^(1/3))/(a^6*(a/b)^(1/
3))
```

Fricas [A]
time = 0.43, size = 621, normalized size = 1.63

$$\frac{420\,(104\,b^5c-65\,ab^4d+35\,a^2b^3e-14\,a^3b^2f)x^{15}+735\,(104\,ab^4c-65\,a^2b^3d+35\,a^3b^2e-14\,a^4bf)x^{12}+270\,(104\,a^2b^3c-65\,a^3b^2d+35\,a^4be-14\,a^5f)x^9-27\,(104\,a^3b^2c-65\,a^4bd+35\,a^5e)x^6-378\,a^5c+108\,(8\,a^4bc-5\,a^5d)x^3+140\,\sqrt{3}\left((104\,b^5c-65\,ab^4d+35\,a^2b^3e-14\,a^3b^2f)x^{16}+2\,(104\,ab^4c-65\,a^2b^3d+35\,a^3b^2e-14\,a^4bf)x^{13}+(104\,a^2b^3c-65\,a^3b^2d+35\,a^4be-14\,a^5f)x^{10}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+70\left((104\,b^5c-65\,ab^4d+35\,a^2b^3e-14\,a^3b^2f)x^{16}+2\,(104\,ab^4c-65\,a^2b^3d+35\,a^3b^2e-14\,a^4bf)x^{13}+(104\,a^2b^3c-65\,a^3b^2d+35\,a^4be-14\,a^5f)x^{10}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-140\left((104\,b^5c-65\,ab^4d+35\,a^2b^3e-14\,a^3b^2f)x^{16}+2\,(104\,ab^4c-65\,a^2b^3d+35\,a^3b^2e-14\,a^4bf)x^{13}+(104\,a^2b^3c-65\,a^3b^2d+35\,a^4be-14\,a^5f)x^{10}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{3780\,(a^6b^2x^{16}+2\,a^7bx^{13}+a^8x^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3780*(420*(104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^15 + 7
35*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^12 + 270*(104
*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^9 - 27*(104*a^3*b^2*c
- 65*a^4*b*d + 35*a^5*e)*x^6 - 378*a^5*c + 108*(8*a^4*b*c - 5*a^5*d)*x^3 +
140*sqrt(3)*((104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^16 +
2*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^13 + (104*a^2*
b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^10)*(b/a)^(1/3)*arctan(2/3*
sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*((104*b^5*c - 65*a*b^4*d + 35*a^2
*b^3*e - 14*a^3*b^2*f)*x^16 + 2*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e
- 14*a^4*b*f)*x^13 + (104*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)
*x^10)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 140*((104
*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^16 + 2*(104*a*b^4*c -
65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^13 + (104*a^2*b^3*c - 65*a^3*b^
2*d + 35*a^4*b*e - 14*a^5*f)*x^10)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)))/(a
^6*b^2*x^16 + 2*a^7*b*x^13 + a^8*x^10)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 486, normalized size = 1.28

$$-\frac{\left(104\,b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-65\,ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}}+35\,a^2b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7}-\frac{\sqrt{3}\left(104\,(-ab^2)^{\frac{2}{3}}\,b^3c-65\,(-ab^2)^{\frac{2}{3}}\,ab^2d-14\,(-ab^2)^{\frac{2}{3}}\,a^3f+35\,(-ab^2)^{\frac{2}{3}}\,a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^7b}+\frac{\left(104\,(-ab^2)^{\frac{2}{3}}\,b^3c-65\,(-ab^2)^{\frac{2}{3}}\,ab^2d-14\,(-ab^2)^{\frac{2}{3}}\,a^3f+35\,(-ab^2)^{\frac{2}{3}}\,a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^7b}+\frac{28\,b^5cx^5-22\,ab^4dx^5-10\,a^3b^2fx^5+16\,a^2b^3x^5e+31\,ab^4cx^2-25\,a^2b^3dx^2-13\,a^4bfx^2+19\,a^3b^2x^2e}{18\,(bx^3+a)^2a^6}+\frac{1400\,b^3cx^9-840\,ab^2dx^9-140\,a^3fx^9+420\,a^2bx^9e-210\,ab^2cx^6+105\,a^2bdx^6-35\,a^3x^6e+60\,a^2bcx^3-20\,a^3dx^3-14\,a^3c}{140\,a^6x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(104*b^4*c*(-a/b)^(1/3) - 65*a*b^3*d*(-a/b)^(1/3) - 14*a^3*b*f*(-a/b)
^(1/3) + 35*a^2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))
/a^7 - 1/27*sqrt(3)*(104*(-a*b^2)^(2/3)*b^3*c - 65*(-a*b^2)^(2/3)*a*b^2*d -
 14*(-a*b^2)^(2/3)*a^3*f + 35*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^7*b) + 1/54*(104*(-a*b^2)^(2/3)*b^3*c -
 65*(-a*b^2)^(2/3)*a*b^2*d - 14*(-a*b^2)^(2/3)*a^3*f + 35*(-a*b^2)^(2/3)*a^
2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^7*b) + 1/18*(28*b^5*c*x^
5 - 22*a*b^4*d*x^5 - 10*a^3*b^2*f*x^5 + 16*a^2*b^3*x^5*e + 31*a*b^4*c*x^2 -
 25*a^2*b^3*d*x^2 - 13*a^4*b*f*x^2 + 19*a^3*b^2*x^2*e)/((b*x^3 + a)^2*a^6)
+ 1/140*(1400*b^3*c*x^9 - 840*a*b^2*d*x^9 - 140*a^3*f*x^9 + 420*a^2*b*x^9*e
 - 210*a*b^2*c*x^6 + 105*a^2*b*d*x^6 - 35*a^3*x^6*e + 60*a^2*b*c*x^3 - 20*a
^3*d*x^3 - 14*a^3*c)/(a^6*x^10)
```

Mupad [B]
time = 5.28, size = 359, normalized size = 0.94

$$-\frac{\frac{c}{10\,a}-\frac{x^9\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{14\,a^4}+\frac{x^3\,\left(5\,a\,d-8\,b\,c\right)}{35\,a^2}+\frac{x^6\,\left(35\,e\,a^2-65\,d\,a\,b+104\,c\,b^2\right)}{140\,a^3}-\frac{7\,b\,x^{12}\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{36\,a^5}-\frac{b^2\,x^{15}\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{9\,a^6}}{a^2\,x^{10}+2\,a\,b\,x^{13}+b^2\,x^{16}}-\frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{27\,a^{19/3}}+\frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{27\,a^{19/3}}-\frac{b^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{27\,a^{19/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x)
```

```
[Out] (b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(104*b^3*c - 14*a^3*f - 65*a*b^2*d + 35*a^2*b*e))/(27*a^(19/3)) - (b^(
1/3)*log(b^(1/3)*x + a^(1/3))*(104*b^3*c - 14*a^3*f - 65*a*b^2*d + 35*a^2*b
*e))/(27*a^(19/3)) - (c/(10*a) - (x^9*(104*b^3*c - 14*a^3*f - 65*a*b^2*d +
35*a^2*b*e))/(14*a^4) + (x^3*(5*a*d - 8*b*c))/(35*a^2) + (x^6*(104*b^2*c +
35*a^2*e - 65*a*b*d))/(140*a^3) - (7*b*x^12*(104*b^3*c - 14*a^3*f - 65*a*b^
2*d + 35*a^2*b*e))/(36*a^5) - (b^2*x^15*(104*b^3*c - 14*a^3*f - 65*a*b^2*d
+ 35*a^2*b*e))/(9*a^6))/(a^2*x^10 + b^2*x^16 + 2*a*b*x^13) - (b^(1/3)*log(3
^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(104*b^3*
c - 14*a^3*f - 65*a*b^2*d + 35*a^2*b*e))/(27*a^(19/3))
```

3.302 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=380

$$-\frac{c}{11a^3x^{11}}+\frac{3bc-ad}{8a^4x^8}-\frac{6b^2c-3abd+a^2e}{5a^5x^5}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2}+\frac{b(}{}$$

```
[Out] -1/11*c/a^3/x^11+1/8*(-a*d+3*b*c)/a^4/x^8+1/5*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/
x^5+1/2*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x^2+1/6*b*(-a^3*f+a^2*b*e
-a*b^2*d+b^3*c)*x/a^5/(b*x^3+a)^2+1/18*b*(-11*a^3*f+17*a^2*b*e-23*a*b^2*d+2
9*b^3*c)*x/a^6/(b*x^3+a)+1/27*b^(2/3)*(-20*a^3*f+44*a^2*b*e-77*a*b^2*d+119*
b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(20/3)-1/54*b^(2/3)*(-20*a^3*f+44*a^2*b*e-77
*a*b^2*d+119*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(20/3)-1/27
*b^(2/3)*(-20*a^3*f+44*a^2*b*e-77*a*b^2*d+119*b^3*c)*arctan(1/3*(a^(1/3)-2*
b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(20/3)*3^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1848, 206, 31, 648, 631, 210, 642}

$$\frac{3bc-ad}{8a^4x^8}-\frac{c}{11a^3x^{11}}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-20a^3f+44a^2be-77ab^2d+119b^3c\right)}{9\sqrt{3}\,a^{20/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-20a^3f+44a^2be-77ab^2d+119b^3c\right)}{54a^{20/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(-20a^3f+44a^2be-77ab^2d+119b^3c\right)}{27a^{20/3}}+\frac{bx\left(-11a^3f+17a^2be-23ab^2d+29b^3c\right)}{18a^6\left(a+bx^3\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{2a^6x^2}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x]
```

```
[Out] -1/11*c/(a^3*x^11) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d + a^2*e
)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(2*a^6*x^2) + (b
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + (b*(29*b^3*
c - 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(18*a^6*(a + b*x^3)) - (b^(2/3)*
(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3
)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(20/3)) + (b^(2/3)*(119*b^3*c - 77*a*
b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(20/3)) - (b
^(2/3)*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(20/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx = \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{6b^3\left(b^3c-ab^2\cdots\right.}{}}{x^{12}(a+bx^3)^2}}{6ab^3}$$

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2} + \frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6\left(a+bx^3\right)} +$$

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2} + \frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6\left(a+bx^3\right)} +$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} +$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} +$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} +$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} +$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} +$$

Mathematica [A]
time = 0.29, size = 376, normalized size = 0.99

$$-\frac{c}{11a^3x^{11}}+\frac{3bc-ad}{8a^4x^8}-\frac{6b^2c-3abd+a^2e}{5a^5x^5}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2}+\frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6\left(a+bx^3\right)}+\frac{b^{2/3}(-119b^3c+77ab^2d-44a^2be+20a^3f)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{20/3}}+\frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{20/3}}+\frac{b^{2/3}(-119b^3c+77ab^2d-44a^2be+20a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x]
```

```
[Out] -1/11*c/(a^3*x^11) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d + a^2*e
)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(2*a^6*x^2) + (b
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + (b*(29*b^3*
c - 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(18*a^6*(a + b*x^3)) + (b^(2/3)*
(-119*b^3*c + 77*a*b^2*d - 44*a^2*b*e + 20*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)
```

```
/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*a^(20/3)) + (b^(2/3)*(119*b^3*c - 77*a*b^2*d
 + 44*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(20/3)) + (b^(2/3
)*(-119*b^3*c + 77*a*b^2*d - 44*a^2*b*e + 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*a^(20/3))
```

Maple [A]
time = 0.37, size = 288, normalized size = 0.76

method	result
default	$-\frac{b\left(\frac{\left(\frac{11}{18}a^3bf-\frac{17}{18}a^2eb^2+\frac{23}{18}adb^3-\frac{29}{18}cb^4\right)x^4+\frac{a\left(7a^3f-10a^2be+13ab^2d-16b^3c\right)x}{9}}{\left(bx^3+a\right)^2}+\frac{\left(20a^3f-44a^2be+77ab^2d-119b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x \ldots\right.}{\ldots}\ldots\right.}{\ldots}\ldots\right.}{a^6}\ldots$
risch	$\frac{-\frac{c}{11a}-\frac{(11ad-17bc)x^3}{88a^2}-\frac{\left(44a^2e-77abd+119b^2c\right)x^6}{220a^3}-\frac{\left(20a^3f-44a^2be+77ab^2d-119b^3c\right)x^9}{40a^4}-\frac{4b\left(20a^3f-44a^2be+77ab^2d-119b^3c\right)x^{12}}{45a^5}-\frac{b^2\left(20a^3 \ldots\right.}{\ldots}}{x^{11}\left(bx^3+a\right)^2}\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^6*(((11/18*a^3*b*f-17/18*a^2*e*b^2+23/18*a*d*b^3-29/18*c*b^4)*x^4+1/9*
a*(7*a^3*f-10*a^2*b*e+13*a*b^2*d-16*b^3*c)*x)/(b*x^3+a)^2+1/9*(20*a^3*f-44*
a^2*b*e+77*a*b^2*d-119*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/11*c/a^3/x^11-1/8*(a*d-3*b*c)/a^4/x^
8-1/5*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^5-1/2*(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3
*c)/a^6/x^2
```

Maxima [A]
time = 0.52, size = 383, normalized size = 1.01

$$\frac{220\,(119\,b^5c-77\,ab^4d-20\,a^3b^2f+44\,a^2b^3e)x^{15}+352\,(119\,ab^4c-77\,a^2b^3d-20\,a^4bf+44\,a^3b^2e)x^{12}+99\,(119\,a^2b^3c-77\,a^3b^2d-20\,a^5f+44\,a^4be)x^9-18\,(119\,a^3b^2c-77\,a^4bd+44\,a^5e)x^6-360\,a^5c+45\,(17\,a^4bc-11\,a^5d)x^3}{3960\,(a^6b^2x^{17}+2\,a^7bx^{14}+a^8x^{11})}+\frac{\sqrt{3}\,(119\,b^3c-77\,ab^2d-20\,a^3f+44\,a^2be)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(119\,b^3c-77\,ab^2d-20\,a^3f+44\,a^2be)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(119\,b^3c-77\,ab^2d-20\,a^3f+44\,a^2be)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/3960*(220*(119*b^5*c - 77*a*b^4*d - 20*a^3*b^2*f + 44*a^2*b^3*e)*x^15 + 352*(119*a*b^4*c - 77*a^2*b^3*d - 20*a^4*b*f + 44*a^3*b^2*e)*x^12 + 99*(119*a^2*b^3*c - 77*a^3*b^2*d - 20*a^5*f + 44*a^4*b*e)*x^9 - 18*(119*a^3*b^2*c - 77*a^4*b*d + 44*a^5*e)*x^6 - 360*a^5*c + 45*(17*a^4*b*c - 11*a^5*d)*x^3)/(a^6*b^2*x^17 + 2*a^7*b*x^14 + a^8*x^11) + 1/27*sqrt(3)*(119*b^3*c - 77*a*b^2*d - 20*a^3*f + 44*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^6*(a/b)^(2/3)) - 1/54*(119*b^3*c - 77*a*b^2*d - 20*a^3*f + 44*a^2*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(2/3)) + 1/27*(119*b^3*c - 77*a*b^2*d - 20*a^3*f + 44*a^2*b*e)*log(x + (a/b)^(1/3))/(a^6*(a/b)^(2/3))

Fricas [A]
time = 0.38, size = 654, normalized size = 1.72

$$\frac{660\,(119\,b^5c-77\,ab^4d+44\,a^2b^3e-20\,a^3b^2f)x^{15}+1056\,(119\,ab^4c-77\,a^2b^3d+44\,a^3b^2e-20\,a^4bf)x^{12}+297\,(119\,a^2b^3c-77\,a^3b^2d+44\,a^4be-20\,a^5f)x^9-54\,(119\,a^3b^2c-77\,a^4bd+44\,a^5e)x^6-1080\,a^5c+135\,(17\,a^4bc-11\,a^5d)x^3-440\,\sqrt{3}\,((119\,b^5c-77\,ab^4d+44\,a^2b^3e-20\,a^3b^2f)x^{17}+2\,(119\,ab^4c-77\,a^2b^3d+44\,a^3b^2e-20\,a^4bf)x^{14}+(119\,a^2b^3c-77\,a^3b^2d+44\,a^4be-20\,a^5f)x^{11})\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)+220\,((119\,b^5c-77\,ab^4d+44\,a^2b^3e-20\,a^3b^2f)x^{17}+2\,(119\,ab^4c-77\,a^2b^3d+44\,a^3b^2e-20\,a^4bf)x^{14}+(119\,a^2b^3c-77\,a^3b^2d+44\,a^4be-20\,a^5f)x^{11})\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-440\,((119\,b^5c-77\,ab^4d+44\,a^2b^3e-20\,a^3b^2f)x^{17}+2\,(119\,ab^4c-77\,a^2b^3d+44\,a^3b^2e-20\,a^4bf)x^{14}+(119\,a^2b^3c-77\,a^3b^2d+44\,a^4be-20\,a^5f)x^{11})\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)}{11880\,(a^6b^2x^{17}+2\,a^7bx^{14}+a^8x^{11})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/11880*(660*(119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^15 + 1056*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^12 + 297*(119*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^9 - 54*(119*a^3*b^2*c - 77*a^4*b*d + 44*a^5*e)*x^6 - 1080*a^5*c + 135*(17*a^4*b*c - 11*a^5*d)*x^3 - 440*sqrt(3)*((119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^14 + (119*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 220*((119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^14 + (119*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 440*((119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^14 + (119*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^6*b^2*x^17 + 2*a^7*b*x^14 + a^8*x^11)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.66, size = 440, normalized size = 1.16

$$\frac{\sqrt{3}\left(119\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-77\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-20\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+44\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^{7}}-\frac{\left(119b^{4}c-77ab^{3}d-20a^{3}bf+44a^{2}b^{2}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^{7}}+\frac{\left(119\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-77\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-20\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+44\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^{7}}+\frac{29b^{5}cx^{4}-23ab^{4}dx^{4}-11a^{3}b^{2}fx^{4}+17a^{2}b^{3}x^{4}e+32ab^{4}cx-26a^{2}b^{3}dx-14a^{4}bfx+20a^{3}b^{2}xe}{18\left(bx^{3}+a\right)^{2}a^{6}}+\frac{2200b^{3}cx^{9}-1320ab^{2}dx^{9}-220a^{3}fx^{9}+660a^{2}bx^{9}e-528ab^{2}cx^{6}+264a^{2}bdx^{6}-88a^{3}x^{6}e+165a^{2}bcx^{3}-55a^{3}dx^{3}-40a^{3}c}{440a^{6}x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(119*(-a*b^2)^(1/3)*b^3*c - 77*(-a*b^2)^(1/3)*a*b^2*d - 20*(-a
*b^2)^(1/3)*a^3*f + 44*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-
a/b)^(1/3))/(-a/b)^(1/3))/a^7 - 1/27*(119*b^4*c - 77*a*b^3*d - 20*a^3*b*f +
 44*a^2*b^2*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^7 + 1/54*(119*(-a*
b^2)^(1/3)*b^3*c - 77*(-a*b^2)^(1/3)*a*b^2*d - 20*(-a*b^2)^(1/3)*a^3*f + 44
*(-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7 + 1/1
8*(29*b^5*c*x^4 - 23*a*b^4*d*x^4 - 11*a^3*b^2*f*x^4 + 17*a^2*b^3*x^4*e + 32
*a*b^4*c*x - 26*a^2*b^3*d*x - 14*a^4*b*f*x + 20*a^3*b^2*x*e)/((b*x^3 + a)^2
*a^6) + 1/440*(2200*b^3*c*x^9 - 1320*a*b^2*d*x^9 - 220*a^3*f*x^9 + 660*a^2*
b*x^9*e - 528*a*b^2*c*x^6 + 264*a^2*b*d*x^6 - 88*a^3*x^6*e + 165*a^2*b*c*x^
3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^6*x^11)
```

Mupad [B]
time = 5.18, size = 359, normalized size = 0.94

$$\frac{b^{2/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(-20fa^{3}+44ea^{2}b-77dab^{2}+119cb^{3}\right)}{27a^{20/3}}-\frac{\frac{c}{11a}-\frac{x^{9}\left(-20fa^{3}+44ea^{2}b-77dab^{2}+119cb^{3}\right)}{40a^{4}}+\frac{x^{3}\left(11ad-17bc\right)}{88a^{2}}+\frac{x^{6}\left(44ea^{2}-77dab+119cb^{2}\right)}{220a^{3}}-\frac{4bx^{12}\left(-20fa^{3}+44ea^{2}b-77dab^{2}+119cb^{3}\right)}{45a^{5}}-\frac{b^{2}x^{15}\left(-20fa^{3}+44ea^{2}b-77dab^{2}+119cb^{3}\right)}{18a^{6}}}{a^{2}x^{11}+2abx^{14}+b^{2}x^{17}}+\frac{b^{2/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1i\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-20fa^{3}+44ea^{2}b-77dab^{2}+119cb^{3}\right)}{27a^{20/3}}-\frac{b^{2/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}\,1i\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-20fa^{3}+44ea^{2}b-77dab^{2}+119cb^{3}\right)}{27a^{20/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x)
```

```
[Out] (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(119*b^3*c - 20*a^3*f - 77*a*b^2*d + 44*a
^2*b*e))/(27*a^(20/3)) - (c/(11*a) - (x^9*(119*b^3*c - 20*a^3*f - 77*a*b^2*
d + 44*a^2*b*e))/(40*a^4) + (x^3*(11*a*d - 17*b*c))/(88*a^2) + (x^6*(119*b^
2*c + 44*a^2*e - 77*a*b*d))/(220*a^3) - (4*b*x^12*(119*b^3*c - 20*a^3*f - 7
7*a*b^2*d + 44*a^2*b*e))/(45*a^5) - (b^2*x^15*(119*b^3*c - 20*a^3*f - 77*a*
b^2*d + 44*a^2*b*e))/(18*a^6))/(a^2*x^11 + b^2*x^17 + 2*a*b*x^14) + (b^(2/3
)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1
19*b^3*c - 20*a^3*f - 77*a*b^2*d + 44*a^2*b*e))/(27*a^(20/3)) - (b^(2/3)*lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(119*b
^3*c - 20*a^3*f - 77*a*b^2*d + 44*a^2*b*e))/(27*a^(20/3))
```

3.303 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=424

$$-\frac{c}{13a^3x^{13}}+\frac{3bc-ad}{10a^4x^{10}}-\frac{6b^2c-3abd+a^2e}{7a^5x^7}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4}-\frac{b(15b^3c-10ab^2d+6a^2be-3a^3f}{a^7x}$$

```
[Out] -1/13*c/a^3/x^13+1/10*(-a*d+3*b*c)/a^4/x^10+1/7*(-a^2*e+3*a*b*d-6*b^2*c)/a^
5/x^7+1/4*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x^4-b*(-3*a^3*f+6*a^2*b
*e-10*a*b^2*d+15*b^3*c)/a^7/x-1/6*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^
6/(b*x^3+a)^2-1/9*b^2*(-8*a^3*f+11*a^2*b*e-14*a*b^2*d+17*b^3*c)*x^2/a^7/(b*
x^3+a)+1/27*b^(4/3)*(-35*a^3*f+65*a^2*b*e-104*a*b^2*d+152*b^3*c)*ln(a^(1/3)
+b^(1/3)*x)/a^(22/3)-1/54*b^(4/3)*(-35*a^3*f+65*a^2*b*e-104*a*b^2*d+152*b^3
*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(22/3)+1/27*b^(4/3)*(-35*a^
3*f+65*a^2*b*e-104*a*b^2*d+152*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1
/3)*3^(1/2))/a^(22/3)*3^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1848, 298, 31, 648, 631, 210, 642}

$$\frac{3bc-ad}{10a^4x^{10}}-\frac{c}{13a^3x^{13}}-\frac{a^2e-3abd+6b^2c}{7a^5x^7}+\frac{b^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{9\sqrt{3}a^{22/3}}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{54a^{22/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{27a^{22/3}}-\frac{b^2x^2\left(-8a^3f+11a^2be-14ab^2d+17b^3c\right)}{9a^7\left(a+bx^3\right)}-\frac{b\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{a^7x}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{4a^6x^4}-\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^6\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x]
```

```
[Out] -1/13*c/(a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b*d + a^2
*e)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(4*a^6*x^4) -
(b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^7*x) - (b^2*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^6*(a + b*x^3)^2) - (b^2*(17*b^3*c - 14*
a*b^2*d + 11*a^2*b*e - 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/3)*(152*b^
3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a*b^2*d
 + 65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(22/3)) - (b^(4/3
)*(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*a^(22/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
```

```
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx = -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^6\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{6b^3\left(b^3c \cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^6\left(a+bx^3\right)^2} - \frac{b^2\left(17b^3c-14ab^2d+11a^2be-8a^3f\right)x^2}{9a^7\left(a+bx^3\right)} + \cdots$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^6\left(a+bx^3\right)^2} - \frac{b^2\left(17b^3c-14ab^2d+11a^2be-8a^3f\right)x^2}{9a^7\left(a+bx^3\right)} + \cdots$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \cdots$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \cdots$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \cdots$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \cdots$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \cdots$$

Mathematica [A]
time = 0.32, size = 419, normalized size = 0.99

$$-\frac{c}{13a^3x^{13}}+\frac{3bc-ad}{10a^4x^{10}}-\frac{6b^2c-3abd+a^2e}{7a^5x^7}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4}+\frac{b\left(-15b^3c+10ab^2d-6a^2be+3a^3f\right)}{a^7x}+\frac{b^2\left(-b^3c+ab^2d-a^2be+a^3f\right)x^2}{6a^6\left(a+bx^3\right)^2}+\frac{b^2\left(-17b^3c+14ab^2d-11a^2be+8a^3f\right)x^2}{9a^7\left(a+bx^3\right)}+\frac{b^{4/3}\left(152b^3c-104ab^2d+65a^2be-35a^3f\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{22/3}}+\frac{b^{4/3}\left(152b^3c-104ab^2d+65a^2be-35a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{22/3}}+\frac{b^{4/3}\left(-152b^3c+104ab^2d-65a^2be+35a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{22/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x]
```

```
[Out] -1/13*c/(a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b*d + a^2
*e)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(4*a^6*x^4) +
(b*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f))/(a^7*x) + (b^2*(-(b^3*c)
```

```
 + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(6*a^6*(a + b*x^3)^2) + (b^2*(-17*b^3*c
+ 14*a*b^2*d - 11*a^2*b*e + 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/3)*(1
52*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a*b^2*d
+ 65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(22/3)) + (b^(4/3)
*(-152*b^3*c + 104*a*b^2*d - 65*a^2*b*e + 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*a^(22/3))
```

Maple [A]
time = 0.38, size = 325, normalized size = 0.77

method	result
default	$\frac{b^2\left(\frac{\frac{b\left(8a^3f-11a^2be+14a\,b^2d-17b^3c\right)x^5}{9}+\left(\frac{19}{18}a^4f-\frac{25}{18}a^3be+\frac{31}{18}a^2b^2d-\frac{37}{18}a\,b^3c\right)x^2}{\left(b\,x^3+a\right)^2}+\left(\frac{35}{9}a^3f-\frac{65}{9}a^2be+\frac{104}{9}a\,b^2d-\frac{152}{9}b^3c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.\right.}{a^7}$
risch	$\frac{-\frac{c}{13a}-\frac{(13ad-19bc)x^3}{130a^2}-\frac{\left(65a^2e-104abd+152b^2c\right)x^6}{455a^3}-\frac{\left(35a^3f-65a^2be+104a\,b^2d-152b^3c\right)x^9}{140a^4}+\frac{b\left(35a^3f-65a^2be+104a\,b^2d-152b^3c\right)x^{12}}{14a^5}+\frac{7b^2(35}{}}{x^{13}\left(b\,x^3+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/a^7*((1/9*b*(8*a^3*f-11*a^2*b*e+14*a*b^2*d-17*b^3*c)*x^5+(19/18*a^4*f-2
5/18*a^3*b*e+31/18*a^2*b^2*d-37/18*a*b^3*c)*x^2)/(b*x^3+a)^2+(35/9*a^3*f-65
/9*a^2*b*e+104/9*a*b^2*d-152/9*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))
+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/13*c/a^3/x^13-1/10*(a*d-3*b
*c)/a^4/x^10-1/7*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^7-1/4*(a^3*f-3*a^2*b*e+6*a*b
^2*d-10*b^3*c)/a^6/x^4+b*(3*a^3*f-6*a^2*b*e+10*a*b^2*d-15*b^3*c)/a^7/x
```

Maxima [A]
time = 0.51, size = 435, normalized size = 1.03

$$-\frac{1820\,(152\,b^6c-104\,ab^5d-35\,a^3b^3f+65\,a^2b^4e)x^{18}+3185\,(152\,ab^5c-104\,a^2b^4d-35\,a^4b^2f+65\,a^3b^3e)x^{15}+1170\,(152\,a^2b^4c-104\,a^3b^3d-35\,a^5bf+65\,a^4b^2e)x^{12}-117\,(152\,a^3b^3c-104\,a^4b^2d-35\,a^6f+65\,a^5be)x^9+1260\,a^6c+36\,(152\,a^4b^2c-104\,a^5bd+65\,a^6e)x^6-126\,(19\,a^5bc-13\,a^6d)x^3}{16380\,(a^7b^2x^{19}+2\,a^8bx^{16}+a^9x^{13})}-\frac{\sqrt{3}\,(152\,b^4c-104\,ab^3d-35\,a^3bf+65\,a^2b^2e)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(152\,b^4c-104\,ab^3d-35\,a^3bf+65\,a^2b^2e)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(152\,b^4c-104\,ab^3d-35\,a^3bf+65\,a^2b^2e)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16380*(1820*(152*b^6*c - 104*a*b^5*d - 35*a^3*b^3*f + 65*a^2*b^4*e)*x^18
 + 3185*(152*a*b^5*c - 104*a^2*b^4*d - 35*a^4*b^2*f + 65*a^3*b^3*e)*x^15 +
```

```
1170*(152*a^2*b^4*c - 104*a^3*b^3*d - 35*a^5*b*f + 65*a^4*b^2*e)*x^12 - 117
*(152*a^3*b^3*c - 104*a^4*b^2*d - 35*a^6*f + 65*a^5*b*e)*x^9 + 1260*a^6*c +
 36*(152*a^4*b^2*c - 104*a^5*b*d + 65*a^6*e)*x^6 - 126*(19*a^5*b*c - 13*a^6
*d)*x^3)/(a^7*b^2*x^19 + 2*a^8*b*x^16 + a^9*x^13) - 1/27*sqrt(3)*(152*b^4*c
 - 104*a*b^3*d - 35*a^3*b*f + 65*a^2*b^2*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)
^(1/3))/(a/b)^(1/3))/(a^7*(a/b)^(1/3)) - 1/54*(152*b^4*c - 104*a*b^3*d - 35
*a^3*b*f + 65*a^2*b^2*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^7*(a/b)^
(1/3)) + 1/27*(152*b^4*c - 104*a*b^3*d - 35*a^3*b*f + 65*a^2*b^2*e)*log(x +
 (a/b)^(1/3))/(a^7*(a/b)^(1/3))
```

Fricas [**A**]
time = 0.40, size = 686, normalized size = 1.62

$$-\frac{5460\,(152\,b^6c-104\,ab^5d+65\,a^2b^4e-35\,a^3b^3f)x^{18}+9555\,(152\,ab^5c-104\,a^2b^4d+65\,a^3b^3e-35\,a^4b^2f)x^{15}+3510\,(152\,a^2b^4c-104\,a^3b^3d+65\,a^4b^2e-35\,a^5bf)x^{12}-351\,(152\,a^3b^3c-104\,a^4b^2d+65\,a^5be-35\,a^6f)x^9+3780\,a^6c+108\,(152\,a^4b^2c-104\,a^5bd+65\,a^6e)x^6-378\,(19\,a^5bc-13\,a^6d)x^3+1820\,\sqrt{3}\,((152\,b^6c-104\,ab^5d+65\,a^2b^4e-35\,a^3b^3f)x^{19}+2\,(152\,ab^5c-104\,a^2b^4d+65\,a^3b^3e-35\,a^4b^2f)x^{16}+(152\,a^2b^4c-104\,a^3b^3d+65\,a^4b^2e-35\,a^5bf)x^{13})\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)-910\,((152\,b^6c-104\,ab^5d+65\,a^2b^4e-35\,a^3b^3f)x^{19}+2\,(152\,ab^5c-104\,a^2b^4d+65\,a^3b^3e-35\,a^4b^2f)x^{16}+(152\,a^2b^4c-104\,a^3b^3d+65\,a^4b^2e-35\,a^5bf)x^{13})\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+1820\,((152\,b^6c-104\,ab^5d+65\,a^2b^4e-35\,a^3b^3f)x^{19}+2\,(152\,ab^5c-104\,a^2b^4d+65\,a^3b^3e-35\,a^4b^2f)x^{16}+(152\,a^2b^4c-104\,a^3b^3d+65\,a^4b^2e-35\,a^5bf)x^{13})\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{49140\,(a^7b^2x^{19}+2\,a^8bx^{16}+a^9x^{13})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/49140*(5460*(152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^18
 + 9555*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^15 +
3510*(152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^12 - 351
*(152*a^3*b^3*c - 104*a^4*b^2*d + 65*a^5*b*e - 35*a^6*f)*x^9 + 3780*a^6*c +
 108*(152*a^4*b^2*c - 104*a^5*b*d + 65*a^6*e)*x^6 - 378*(19*a^5*b*c - 13*a^
6*d)*x^3 + 1820*sqrt(3)*((152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b
^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*
x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-
b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*((152*b^6
*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104
*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b
^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)
^(2/3) - a*(-b/a)^(1/3)) + 1820*((152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e -
35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4
*b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*
x^13)*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^7*b^2*x^19 + 2*a^8*b*x^16
+ a^9*x^13)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.63, size = 531, normalized size = 1.25

$$\frac{\sqrt{3}\left(152\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-104\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-35\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+65\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^{8}}+\frac{\left(152b^{5}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-104ab^{4}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-35a^{3}b^{2}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+65a^{2}b^{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^{8}}-\frac{\left(152\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-104\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-35\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+65\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^{8}}-\frac{34b^{6}cx^{5}-28ab^{5}dx^{5}-16a^{3}b^{3}fx^{5}+22a^{2}b^{4}x^{5}e+37ab^{5}cx^{2}-31a^{2}b^{4}dx^{2}-19a^{4}b^{2}fx^{2}+25a^{3}b^{3}x^{2}e}{18\left(bx^{3}+a\right)^{2}a^{7}}-\frac{27300b^{4}cx^{12}-18200ab^{3}dx^{12}-5460a^{3}bfx^{12}+10920a^{2}b^{2}x^{12}e-4550ab^{3}cx^{9}+2730a^{2}b^{2}dx^{9}+455a^{4}fx^{9}-1365a^{3}bx^{9}e+1560a^{2}b^{2}cx^{6}-780a^{3}bdx^{6}+260a^{4}x^{6}e-546a^{3}bcx^{3}+182a^{4}dx^{3}+140a^{4}c}{1820a^{7}x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(152*(-a*b^2)^(2/3)*b^3*c - 104*(-a*b^2)^(2/3)*a*b^2*d - 35*(-
a*b^2)^(2/3)*a^3*f + 65*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/a^8 + 1/27*(152*b^5*c*(-a/b)^(1/3) - 104*a*b^4*d
*(-a/b)^(1/3) - 35*a^3*b^2*f*(-a/b)^(1/3) + 65*a^2*b^3*(-a/b)^(1/3)*e)*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^8 - 1/54*(152*(-a*b^2)^(2/3)*b^3*c -
104*(-a*b^2)^(2/3)*a*b^2*d - 35*(-a*b^2)^(2/3)*a^3*f + 65*(-a*b^2)^(2/3)*a^
2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^8 - 1/18*(34*b^6*c*x^5 -
28*a*b^5*d*x^5 - 16*a^3*b^3*f*x^5 + 22*a^2*b^4*x^5*e + 37*a*b^5*c*x^2 - 31*
a^2*b^4*d*x^2 - 19*a^4*b^2*f*x^2 + 25*a^3*b^3*x^2*e)/((b*x^3 + a)^2*a^7) -
1/1820*(27300*b^4*c*x^12 - 18200*a*b^3*d*x^12 - 5460*a^3*b*f*x^12 + 10920*a
^2*b^2*x^12*e - 4550*a*b^3*c*x^9 + 2730*a^2*b^2*d*x^9 + 455*a^4*f*x^9 - 136
5*a^3*b*x^9*e + 1560*a^2*b^2*c*x^6 - 780*a^3*b*d*x^6 + 260*a^4*x^6*e - 546*
a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)/(a^7*x^13)
```

Mupad [B]
time = 5.30, size = 397, normalized size = 0.94

$$\frac{b^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{27\,a^{22/3}}-\frac{\frac{c}{13\,a}-\frac{x^9\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{140\,a^4}+\frac{x^3\,\left(13\,a\,d-19\,b\,c\right)}{130\,a^2}+\frac{x^6\,\left(65\,e\,a^2-104\,d\,a\,b+152\,c\,b^2\right)}{455\,a^3}+\frac{b\,x^{12}\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{14\,a^5}+\frac{7\,b^2\,x^{15}\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{36\,a^6}+\frac{b^3\,x^{18}\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{9\,a^7}}{a^2\,x^{13}+2\,a\,b\,x^{16}+b^2\,x^{19}}-\frac{b^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{27\,a^{22/3}}+\frac{b^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{27\,a^{22/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x)
```

```
[Out] (b^(4/3)*log(b^(1/3)*x + a^(1/3))*(152*b^3*c - 35*a^3*f - 104*a*b^2*d + 65*
a^2*b*e))/(27*a^(22/3)) - (c/(13*a) - (x^9*(152*b^3*c - 35*a^3*f - 104*a*b^
2*d + 65*a^2*b*e))/(140*a^4) + (x^3*(13*a*d - 19*b*c))/(130*a^2) + (x^6*(15
2*b^2*c + 65*a^2*e - 104*a*b*d))/(455*a^3) + (b*x^12*(152*b^3*c - 35*a^3*f
- 104*a*b^2*d + 65*a^2*b*e))/(14*a^5) + (7*b^2*x^15*(152*b^3*c - 35*a^3*f -
 104*a*b^2*d + 65*a^2*b*e))/(36*a^6) + (b^3*x^18*(152*b^3*c - 35*a^3*f - 10
4*a*b^2*d + 65*a^2*b*e))/(9*a^7))/(a^2*x^13 + b^2*x^19 + 2*a*b*x^16) - (b^(
4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)
*(152*b^3*c - 35*a^3*f - 104*a*b^2*d + 65*a^2*b*e))/(27*a^(22/3)) + (b^(4/3
)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1
52*b^3*c - 35*a^3*f - 104*a*b^2*d + 65*a^2*b*e))/(27*a^(22/3))
```

3.304 $\int \frac{(1-x)x^4}{1+x^3}\,dx$

Optimal. Leaf size=54

$$\frac{x^2}{2}-\frac{x^3}{3}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] 1/2*x^2-1/3*x^3+2/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))
*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1901, 1888, 31, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{x^3}{3}+\frac{x^2}{2}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x^4)/(1 + x^3),x]
```

```
[Out] x^2/2 - x^3/3 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log[
1 - x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)x^4}{1+x^3}\,dx &= \int \left(x-x^2+\frac{(-1+x)x}{1+x^3}\right)dx\\
&= \frac{x^2}{2}-\frac{x^3}{3}+\int \frac{(-1+x)x}{1+x^3}\,dx\\
&= \frac{x^2}{2}-\frac{x^3}{3}+\frac{1}{3}\int \frac{-2+x}{1-x+x^2}\,dx+\frac{2}{3}\int \frac{1}{1+x}\,dx\\
&= \frac{x^2}{2}-\frac{x^3}{3}+\frac{2}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\
&= \frac{x^2}{2}-\frac{x^3}{3}+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= \frac{x^2}{2}-\frac{x^3}{3}-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A]

time = 0.01, size = 59, normalized size = 1.09

$$\frac{1}{6}\left(3x^2-2x^3-2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)+2\log(1+x)-\log\left(1-x+x^2\right)+2\log\left(1+x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x^4)/(1 + x^3),x]
```

```
[Out] (3*x^2 - 2*x^3 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Log[
1 - x + x^2] + 2*Log[1 + x^3])/6
```

Maple [A]
time = 0.34, size = 45, normalized size = 0.83

method	result	size
default	$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{2\ln(x+1)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	45
risch	$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{\ln(4x^2-4x+4)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{2\ln(x+1)}{3}$	47
meijerg	$\frac{x^2}{2}-\frac{x^2\left(-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right)}{\left(x^3\right)^{\frac{2}{3}}}+\frac{\ln\left(1-\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2\left(x^3\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2-\left(x^3\right)^{\frac{1}{3}}}\right)}{\left(x^3\right)^{\frac{2}{3}}}\right)}{3}-\frac{x^3}{3}+\frac{\ln(x^3+1)}{3}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x^4/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3+1/2*x^2+2/3*ln(x+1)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)
*3^(1/2))
```

Maxima [A]
time = 0.50, size = 44, normalized size = 0.81

$$-\frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{6}\,\log\left(x^2-x+1\right)+\frac{2}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^
2 - x + 1) + 2/3*log(x + 1)
```

Fricas [A]
time = 0.39, size = 44, normalized size = 0.81

$$-\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^
2 - x + 1) + 2/3*log(x + 1)
```

Sympy [A]
time = 0.07, size = 53, normalized size = 0.98

$$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{2\log(x+1)}{3}+\frac{\log(x^2-x+1)}{6}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x**4/(x**3+1),x)
```

```
[Out] -x**3/3 + x**2/2 + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sq
rt(3)*x/3 - sqrt(3)/3)/3
```

Giac [A]
time = 0.78, size = 45, normalized size = 0.83

$$-\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^
2 - x + 1) + 2/3*log(abs(x + 1))
```

Mupad [B]
time = 0.10, size = 56, normalized size = 1.04

$$\frac{2\ln(x+1)}{3}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)+\frac{x^2}{2}-\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(x - 1))/(x^3 + 1),x)
```

```
[Out] (2*log(x + 1))/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - l
og(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) + x^2/2 - x^3/3
```

3.305 $\int \frac{(1-x)x^3}{1+x^3} \, dx$

Optimal. Leaf size=30

$$x-\frac{x^2}{2}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)$$

```
[Out] x-1/2*x^2-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1901, 1874, 31, 642}

$$-\frac{x^2}{2}+\frac{1}{3}\log\left(x^2-x+1\right)+x-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x^3)/(1 + x^3),x]
```

```
[Out] x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)x^3}{1+x^3}\,dx &= \int \left(1-x-\frac{1-x}{1+x^3}\right)dx \\ &= x-\frac{x^2}{2}-\int \frac{1-x}{1+x^3}\,dx \\ &= x-\frac{x^2}{2}-\frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1}{1+x}\,dx \\ &= x-\frac{x^2}{2}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$x-\frac{x^2}{2}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x^3)/(1 + x^3),x]
[Out] x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Maple [A]
time = 0.33, size = 25, normalized size = 0.83

method	result
default	$x-\frac{x^2}{2}-\frac{2\ln(x+1)}{3}+\frac{\ln(x^2-x+1)}{3}$
norman	$x-\frac{x^2}{2}-\frac{2\ln(x+1)}{3}+\frac{\ln(x^2-x+1)}{3}$
risch	$x-\frac{x^2}{2}-\frac{2\ln(x+1)}{3}+\frac{\ln(x^2-x+1)}{3}$
meijerg	$x-\frac{x\left(\frac{\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{(x^3)^{\frac{1}{3}}}-\frac{\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2(x^3)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{(x^3)^{\frac{1}{3}}}\right)}{3}-\frac{x^2}{2}+\frac{x^2\left(-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{(x^3)^{\frac{2}{3}}}+\frac{\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3\right.}{2(x^3)^{\frac{2}{3}}}\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x^3/(x^3+1),x,method=_RETURNVERBOSE)
[Out] x-1/2*x^2-2/3*ln(x+1)+1/3*ln(x^2-x+1)
```

Maxima [A]
time = 0.50, size = 24, normalized size = 0.80

$$-\frac{1}{2}\,x^2+x+\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{2}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

Fricas [A]
time = 0.37, size = 24, normalized size = 0.80

$$-\frac{1}{2}x^2 + x + \frac{1}{3}\log\left(x^2 - x + 1\right) - \frac{2}{3}\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

Sympy [A]
time = 0.04, size = 24, normalized size = 0.80

$$-\frac{x^2}{2} + x - \frac{2\log\left(x + 1\right)}{3} + \frac{\log\left(x^2 - x + 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x**3/(x**3+1),x)
```

```
[Out] -x**2/2 + x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3
```

Giac [A]
time = 0.63, size = 25, normalized size = 0.83

$$-\frac{1}{2}x^2 + x + \frac{1}{3}\log\left(x^2 - x + 1\right) - \frac{2}{3}\log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(abs(x + 1))
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$x - \frac{2\,\ln\left(x + 1\right)}{3} + \frac{\ln\left(x^2 - x + 1\right)}{3} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(x - 1))/(x^3 + 1),x)
```

```
[Out] x - (2*log(x + 1))/3 + log(x^2 - x + 1)/3 - x^2/2
```

3.306 $\int \frac{(1-x)x^2}{1+x^3} \, dx$

Optimal. Leaf size=44

$$-x-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] -x+2/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1901, 1888, 31, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^2-x+1\right)-x+\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x^2)/(1 + x^3),x]
```

```
[Out] -x - ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log[1 - x + x^2
]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)x^2}{1+x^3}\,dx &= \int \left(-1+\frac{1+x^2}{1+x^3}\right)dx\\
&= -x+\int \frac{1+x^2}{1+x^3}\,dx\\
&= -x+\frac{1}{3}\int \frac{1+x}{1-x+x^2}\,dx+\frac{2}{3}\int \frac{1}{1+x}\,dx\\
&= -x+\frac{2}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\
&= -x+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -x+\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A]

time = 0.01, size = 53, normalized size = 1.20

$$-x+\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)+\frac{1}{3}\log\left(1+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x^2)/(1 + x^3),x]
```

```
[Out] -x + ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x]/3 - Log[1 - x + x^2]/6
 + Log[1 + x^3]/3
```

Maple [A]
time = 0.36, size = 38, normalized size = 0.86

method	result	size
risch	$-x+\frac{2\ln(x+1)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	36
default	$-x+\frac{2\ln(x+1)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	38
meijerg	$\frac{\ln(x^3+1)}{3}-x+\frac{x\left(\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right)}{\left(x^3\right)^{\frac{1}{3}}}-\frac{\ln\left(1-\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2\left(x^3\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^3\right)^{\frac{1}{3}}}{2-\left(x^3\right)^{\frac{1}{3}}}\right)}{\left(x^3\right)^{\frac{1}{3}}}\right)}{3}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x^2/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -x+2/3*ln(x+1)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.49, size = 37, normalized size = 0.84

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-x+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*
log(x + 1)
```

Fricas [A]
time = 0.39, size = 37, normalized size = 0.84

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-x+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*
log(x + 1)
```

Sympy [A]
time = 0.06, size = 44, normalized size = 1.00

$$-x+\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x**2/(x**3+1),x)
```

```
[Out] -x + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sq
rt(3)/3)/3
```

Giac [A]
time = 0.76, size = 38, normalized size = 0.86

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-x+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*
log(abs(x + 1))
```

Mupad [B]
time = 4.96, size = 49, normalized size = 1.11

$$\frac{2\ln\left(x+1\right)}{3}-x-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(x - 1))/(x^3 + 1),x)
```

```
[Out] (2*log(x + 1))/3 - x - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6)
 + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6)
```

3.307 $\int \frac{(1-x)x}{1+x^3} \, dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] -2/3*ln(1+x)-1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1888, 31, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x)/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - (2*Log[1 + x])/3 - Log[1 - x + x^2]/
6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)x}{1+x^3}\,dx &= \frac{1}{3}\int \frac{2-x}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1}{1+x}\,dx\\ &= -\frac{2}{3}\log(1+x)-\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\ &= -\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.22

$$\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{3}\log\left(1+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x)/(1 + x^3),x]
```

```
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6 - Lo
g[1 + x^3]/3
```

Maple [A]
time = 0.34, size = 35, normalized size = 0.85

method	result	size
risch	$-\frac{2\ln(x+1)}{3}-\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\ \arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$-\frac{2\ln(x+1)}{3}-\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	35
meijerg	$-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ (x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}-\frac{\ln(x^3+1)}{3}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x/(x^3+1),x,method=_RETURNVERBOSE)
[Out] -2/3*ln(x+1)-1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.48, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(x^3+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(
x + 1)
```

Fricas [A]
time = 0.38, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(
x + 1)
```

Sympy [A]
time = 0.05, size = 42, normalized size = 1.02

$$-\frac{2\log\left(x+1\right)}{3}-\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(x**3+1),x)
```

```
[Out] -2*log(x + 1)/3 - log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3
)/3)/3
```

Giac [A]
time = 0.73, size = 35, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(
abs(x + 1))
```

Mupad [B]
time = 0.08, size = 63, normalized size = 1.54

$$-\frac{\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6}-\frac{\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6}-\frac{2\ln\left(x+1\right)}{3}-\frac{\sqrt{3}\,\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x - 1))/(x^3 + 1),x)
```

```
[Out] (3^(1/2)*log(x + (3^(1/2)*1i)/2 - 1/2)*1i)/6 - log(x + (3^(1/2)*1i)/2 - 1/2
)/6 - (2*log(x + 1))/3 - (3^(1/2)*log(x - (3^(1/2)*1i)/2 - 1/2)*1i)/6 - log
(x - (3^(1/2)*1i)/2 - 1/2)/6
```

3.308 $\int \frac{1-x}{x(1+x^3)} dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] ln(x)-2/3*ln(1+x)-1/6*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1848, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^2-x+1\right)+\log(x)-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(x*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[x] - (2*Log[1 + x])/3 - Log[1 - x +
 x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x}{x(1+x^3)}\,dx &= \int \left(\frac{1}{x}-\frac{2}{3(1+x)}+\frac{-1-x}{3(1-x+x^2)}\right)dx\\
&= \log(x)-\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-1-x}{1-x+x^2}\,dx\\
&= \log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\
&= \log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 1.26

$$
-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{3}\log\left(1+x^3\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(x*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 + x]/3 + Log[1 - x +
 x^2]/6 - Log[1 + x^3]/3
```

Maple [A]
time = 0.35, size = 37, normalized size = 0.88

method	result	size

risch	$-\frac{2\ln(x+1)}{3}-\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}+\ln(x)$	35
default	$-\frac{2\ln(x+1)}{3}+\ln(x)-\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	37
meijerg	$-\frac{\ln(x^3+1)}{3}+\ln(x)-\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{1}{3}}}-\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/x/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*ln(x+1)+ln(x)-1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.49, size = 36, normalized size = 0.86

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(x + 1) + log(x)
```

Fricas [A]
time = 0.39, size = 36, normalized size = 0.86

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(x + 1) + log(x)
```

Sympy [A]
time = 0.08, size = 46, normalized size = 1.10

$$\log(x)-\frac{2\log(x+1)}{3}-\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x/(x**3+1),x)
```

```
[Out] log(x) - 2*log(x + 1)/3 - log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3
- sqrt(3)/3)/3
```

Giac [A]
time = 0.78, size = 38, normalized size = 0.90

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(|x+1|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(abs(x + 1)) + log(abs(x))
```

Mupad [B]
time = 4.96, size = 48, normalized size = 1.14

$$\ln(x)-\frac{2\ln(x+1)}{3}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/(x*(x^3 + 1)),x)
```

```
[Out] log(x) - (2*log(x + 1))/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 -
 1/6) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6)
```

3.309 $\int \frac{1-x}{x^2(1+x^3)}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] -1/x-ln(x)+2/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1
/2)
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1848, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{x}-\log(x)+\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(x^2*(1 + x^3)),x]
```

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + (2*Log[1 + x])/3 + L
og[1 - x + x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x}{x^2\left(1+x^3\right)}\,dx &= \int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{2}{3(1+x)}+\frac{-2+x}{3\left(1-x+x^2\right)}\right)dx\\
&= -\frac{1}{x}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-2+x}{1-x+x^2}\,dx\\
&= -\frac{1}{x}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\
&= -\frac{1}{x}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -\frac{1}{x}-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 1.22

$$-\frac{1}{x}-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(x)+\frac{1}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)+\frac{1}{3}\log\left(1+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(x^2*(1 + x^3)),x]
[Out] -x^(-1) - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + Log[1 + x]/3 - Log[
1 - x + x^2]/6 + Log[1 + x^3]/3
```

Maple [A]
time = 0.35, size = 44, normalized size = 0.90

method	result	size
risch	$-\frac{1}{x}+\frac{2\ln(x+1)}{3}-\ln(x)+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	42
default	$\frac{2\ln(x+1)}{3}-\frac{1}{x}-\ln(x)+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	44
meijerg	$-\frac{1}{x}-\frac{x^2\left(-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{(x^3)^{\frac{2}{3}}}+\frac{\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2(x^3)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{(x^3)^{\frac{2}{3}}}\right)}{3}+\frac{\ln(x^3+1)}{3}-\ln(x)$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/x^2/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*ln(x+1)-1/x-ln(x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))
```

Maxima [A]
time = 0.47, size = 43, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{1}{x}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(x+1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^2/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/x + 1/6*log(x^2 - x + 1) + 2
/3*log(x + 1) - log(x)
```

Fricas [A]
time = 0.38, size = 48, normalized size = 0.98

$$-\frac{2\sqrt{3}\,x\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-x\log\left(x^2-x+1\right)-4x\log\left(x+1\right)+6x\log\left(x\right)+6}{6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^2/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x - 1)) - x*log(x^2 - x + 1) - 4*x*
log(x + 1) + 6*x*log(x) + 6)/x
```

Sympy [A]
time = 0.11, size = 49, normalized size = 1.00

$$-\log(x)+\frac{2\log(x+1)}{3}+\frac{\log(x^2-x+1)}{6}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x**2/(x**3+1),x)
```

```
[Out] -log(x) + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3
 - sqrt(3)/3)/3 - 1/x
```

Giac [A]
time = 0.62, size = 45, normalized size = 0.92

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{x}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(|x+1|\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^2/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/x + 1/6*log(x^2 - x + 1) + 2
/3*log(abs(x + 1)) - log(abs(x))
```

Mupad [B]
time = 0.08, size = 55, normalized size = 1.12

$$\frac{2\ln(x+1)}{3}-\ln(x)+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/(x^2*(x^3 + 1)),x)
```

```
[Out] (2*log(x + 1))/3 - log(x) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 +
 1/6) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) - 1/x
```

3.310 $\int \frac{1-x}{x^3(1+x^3)} dx$

Optimal. Leaf size=32

$$-\frac{1}{2x^2}+\frac{1}{x}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)$$

```
[Out] -1/2/x^2+1/x-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1848, 642}

$$-\frac{1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{x}-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(x^3*(1 + x^3)),x]
```

```
[Out] -1/2*1/x^2 + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x}{x^3\left(1+x^3\right)}\,dx &= \int \left(\frac{1}{x^3}-\frac{1}{x^2}-\frac{2}{3(1+x)}+\frac{-1+2x}{3\left(1-x+x^2\right)}\right) dx\\
&= -\frac{1}{2x^2}+\frac{1}{x}-\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= -\frac{1}{2x^2}+\frac{1}{x}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 32, normalized size = 1.00

$$-\frac{1}{2x^2}+\frac{1}{x}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(x^3*(1 + x^3)),x]
```

```
[Out] -1/2*1/x^2 + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Maple [A]
time = 0.34, size = 27, normalized size = 0.84

method	result
norman	$\frac{x-\frac{1}{2}}{x^{2}}-\frac{2\ln(x+1)}{3}+\frac{\ln(x^{2}-x+1)}{3}$
risch	$\frac{x-\frac{1}{2}}{x^{2}}-\frac{2\ln(x+1)}{3}+\frac{\ln(x^{2}-x+1)}{3}$
default	$-\frac{1}{2x^{2}}+\frac{1}{x}-\frac{2\ln(x+1)}{3}+\frac{\ln(x^{2}-x+1)}{3}$
meijerg	$-\frac{1}{2x^{2}}-\frac{x\left(\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}-\frac{\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2\left(x^{3}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{3}\right)^{\frac{1}{3}}}{2-\left(x^{3}\right)^{\frac{1}{3}}}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}\right)}{3}+\frac{1}{x}+$ $x^{2}\left(-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{\left(x^{3}\right)^{\frac{2}{3}}}+\frac{\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right.}{2\left(x^{3}\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/x^3/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2+1/x-2/3*ln(x+1)+1/3*ln(x^2-x+1)
```

Maxima [A]
time = 0.49, size = 28, normalized size = 0.88

$$\frac{2\,x-1}{2\,x^2}+\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{2}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^3/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/2*(2*x - 1)/x^2 + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

Fricas [A]
time = 0.36, size = 33, normalized size = 1.03

$$\frac{2\,x^2\log\left(x^2-x+1\right)-4\,x^2\log\left(x+1\right)+6\,x-3}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^3/(x^3+1),x, algorithm="fricas")
[Out] 1/6*(2*x^2*log(x^2 - x + 1) - 4*x^2*log(x + 1) + 6*x - 3)/x^2
```

Sympy [A]
time = 0.04, size = 27, normalized size = 0.84

$$-\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^{2}-x+1\right)}{3}-\frac{1-2x}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x**3/(x**3+1),x)
[Out] -2*log(x + 1)/3 + log(x**2 - x + 1)/3 - (1 - 2*x)/(2*x**2)
```

Giac [A]
time = 0.60, size = 29, normalized size = 0.91

$$\frac{2\,x-1}{2\,x^{2}}+\frac{1}{3}\,\log\left(x^{2}-x+1\right)-\frac{2}{3}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^3/(x^3+1),x, algorithm="giac")
[Out] 1/2*(2*x - 1)/x^2 + 1/3*log(x^2 - x + 1) - 2/3*log(abs(x + 1))
```

Mupad [B]
time = 0.07, size = 25, normalized size = 0.78

$$\frac{\ln\left(x^{2}-x+1\right)}{3}-\frac{2\,\ln\left(x+1\right)}{3}+\frac{x-\frac{1}{2}}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/(x^3*(x^3 + 1)),x)
[Out] log(x^2 - x + 1)/3 - (2*log(x + 1))/3 + (x - 1/2)/x^2
```

3.311 $\int \frac{x(1+2x)}{1+x^3} dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1+x)+\frac{5}{6}\log\left(1-x+x^2\right)$$

```
[Out] 1/3*ln(1+x)+5/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1888, 31, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + 2*x))/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x]/3 + (5*Log[1 - x + x^2])/
6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(1+2x)}{1+x^3}\,dx &= \frac{1}{3}\int \frac{1}{1+x}\,dx+\frac{1}{3}\int \frac{-1+5x}{1-x+x^2}\,dx\\ &= \frac{1}{3}\log(1+x)+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx+\frac{5}{6}\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= \frac{1}{3}\log(1+x)+\frac{5}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1+x)+\frac{5}{6}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.15

$$\frac{1}{6}\left(2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-2\log(1+x)+\log\left(1-x+x^2\right)+4\log\left(1+x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + 2*x))/(1 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + Log[1 - x + x^2] + 4
*Log[1 + x^3])/6
```

Maple [A]
time = 0.39, size = 35, normalized size = 0.85

method	result	size
default	$\frac{\ln(x+1)}{3}+\frac{5\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	35
risch	$\frac{5\ln(4x^2-4x+4)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{\ln(x+1)}{3}$	37
meijerg	$\frac{2\ln(x^3+1)}{3}-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x+1)/(x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(x+1)+5/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.48, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log(
x + 1)
```

Fricas [A]
time = 0.37, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log(
x + 1)
```

Sympy [A]
time = 0.05, size = 42, normalized size = 1.02

$$\frac{\log\left(x+1\right)}{3}+\frac{5\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(x**3+1),x)
```

```
[Out] log(x + 1)/3 + 5*log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)
/3)/3
```

Giac [A]
time = 0.65, size = 35, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log(
abs(x + 1))
```

Mupad [B]
time = 4.96, size = 63, normalized size = 1.54

$$\frac{5\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6}+\frac{5\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6}+\frac{\ln\left(x+1\right)}{3}-\frac{\sqrt{3}\,\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*x + 1))/(x^3 + 1),x)
```

```
[Out] (5*log(x - (3^(1/2)*1i)/2 - 1/2))/6 + (5*log(x + (3^(1/2)*1i)/2 - 1/2))/6 +
 log(x + 1)/3 - (3^(1/2)*log(x - (3^(1/2)*1i)/2 - 1/2)*1i)/6 + (3^(1/2)*log
(x + (3^(1/2)*1i)/2 - 1/2)*1i)/6
```

3.312 $\int \frac{x(1+2x)}{1-x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(1-x)-\frac{1}{2}\log\left(1+x+x^2\right)$$

```
[Out] -ln(1-x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1889, 31, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + 2*x))/(1 - x^3),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x] - Log[1 + x + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q
 + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q -
 C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b
*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
 && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(1+2x)}{1-x^3}\,dx &= \frac{1}{3}\int \frac{-3-3x}{1+x+x^2}\,dx+\int \frac{1}{1-x}\,dx\\ &= -\log(1-x)-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx\\ &= -\log(1-x)-\frac{1}{2}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(1-x)-\frac{1}{2}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 53, normalized size = 1.36

$$-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1-x)+\frac{1}{6}\log\left(1+x+x^2\right)-\frac{2}{3}\log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + 2*x))/(1 - x^3),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x]/3 + Log[1 + x + x^2]/6 -
(2*Log[1 - x^3])/3
```

Maple [**A**]
time = 0.38, size = 33, normalized size = 0.85

method	result	size
risch	$-\ln(x-1)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\ln(x^2+x+1)}{2}$	31
default	$-\frac{\ln(x^2+x+1)}{2}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\ln(x-1)$	33
meijerg	$-\frac{2\ln(-x^3+1)}{3}-\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(x^2+x+1)-1/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-ln(x-1)
```

Maxima [A]
time = 0.48, size = 32, normalized size = 0.82

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(x -
 1)
```

Fricas [A]
time = 0.38, size = 32, normalized size = 0.82

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(x -
 1)
```

Sympy [A]
time = 0.05, size = 41, normalized size = 1.05

$$-\log(x-1)-\frac{\log(x^2+x+1)}{2}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(-x**3+1),x)
```

```
[Out] -log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)
/3
```

Giac [A]
time = 0.60, size = 33, normalized size = 0.85

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(-x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(abs
(x - 1))
```

Mupad [B]
time = 0.09, size = 63, normalized size = 1.62

$$-\frac{\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2}-\frac{\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2}-\ln\left(x-1\right)+\frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6}-\frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(2*x + 1))/(x^3 - 1),x)
```

```
[Out] (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/6 - log(x + (3^(1/2)*1i)/2 + 1/2
)/2 - log(x - 1) - log(x - (3^(1/2)*1i)/2 + 1/2)/2 - (3^(1/2)*log(x + (3^(1
/2)*1i)/2 + 1/2)*1i)/6
```

3.313 $\int x^2(c+dx+ex^2)\left(a+bx^3\right)\,dx$

Optimal. Leaf size=55

$$\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

```
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*b*c*x^6+1/7*b*d*x^7+1/8*b*e*x^8
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.048$, Rules used = {1642}

$$\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e*
x^8)/8
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx&=\int\left(acx^2+adx^3+aex^4+bcx^5+bdx^6+bex^7\right)\,dx\\&=\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8\end{aligned}$$

Mathematica [A]
time = 0.00, size = 55, normalized size = 1.00

$$\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e*
x^8)/8
```

Maple [A]
time = 0.14, size = 44, normalized size = 0.80

method	result	size
gosper	$\frac{1}{3}ac\,x^{3}+\frac{1}{4}ad\,x^{4}+\frac{1}{5}ae\,x^{5}+\frac{1}{6}bc\,x^{6}+\frac{1}{7}bd\,x^{7}+\frac{1}{8}be\,x^{8}$	44
default	$\frac{1}{3}ac\,x^{3}+\frac{1}{4}ad\,x^{4}+\frac{1}{5}ae\,x^{5}+\frac{1}{6}bc\,x^{6}+\frac{1}{7}bd\,x^{7}+\frac{1}{8}be\,x^{8}$	44
norman	$\frac{1}{3}ac\,x^{3}+\frac{1}{4}ad\,x^{4}+\frac{1}{5}ae\,x^{5}+\frac{1}{6}bc\,x^{6}+\frac{1}{7}bd\,x^{7}+\frac{1}{8}be\,x^{8}$	44
risch	$\frac{1}{3}ac\,x^{3}+\frac{1}{4}ad\,x^{4}+\frac{1}{5}ae\,x^{5}+\frac{1}{6}bc\,x^{6}+\frac{1}{7}bd\,x^{7}+\frac{1}{8}be\,x^{8}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*b*c*x^6+1/7*b*d*x^7+1/8*b*e*x^8
```

Maxima [A]
time = 0.26, size = 45, normalized size = 0.82

$$\frac{1}{8}\,bx^8e+\frac{1}{7}\,bdx^7+\frac{1}{6}\,bcx^6+\frac{1}{5}\,ax^5e+\frac{1}{4}\,adx^4+\frac{1}{3}\,acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/8*b*x^8*e + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*x^5*e + 1/4*a*d*x^4 + 1/3*a
*c*x^3
```

Fricas [A]
time = 0.37, size = 43, normalized size = 0.78

$$\frac{1}{8}\,bex^8+\frac{1}{7}\,bdx^7+\frac{1}{6}\,bcx^6+\frac{1}{5}\,aex^5+\frac{1}{4}\,adx^4+\frac{1}{3}\,acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/8*b*e*x^8 + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*e*x^5 + 1/4*a*d*x^4 + 1/3*a
*c*x^3
```

Sympy [A]
time = 0.01, size = 49, normalized size = 0.89

$$\frac{acx^3}{3}+\frac{adx^4}{4}+\frac{aex^5}{5}+\frac{bcx^6}{6}+\frac{bdx^7}{7}+\frac{bex^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*c*x**6/6 + b*d*x**7/7 + b*e*x**8/8
```

Giac [A]
time = 0.61, size = 45, normalized size = 0.82

$$\frac{1}{8}\,bx^8e + \frac{1}{7}\,bdx^7 + \frac{1}{6}\,bcx^6 + \frac{1}{5}\,ax^5e + \frac{1}{4}\,adx^4 + \frac{1}{3}\,acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/8*b*x^8*e + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*x^5*e + 1/4*a*d*x^4 + 1/3*a
*c*x^3
```

Mupad [B]
time = 0.03, size = 43, normalized size = 0.78

$$\frac{b\,e\,x^8}{8}+\frac{b\,d\,x^7}{7}+\frac{b\,c\,x^6}{6}+\frac{a\,e\,x^5}{5}+\frac{a\,d\,x^4}{4}+\frac{a\,c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)*(c + d*x + e*x^2),x)
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (b*c*x^6)/6 + (a*e*x^5)/5 + (b*d*x^7)/7 + (b*e*
x^8)/8
```

3.314 $\int x(c+dx+ex^2)\left(a+bx^3\right)\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

```
[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x^7
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.053$, Rules used = {1642}

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*
x^7)/7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx&=\int\left(acx+adx^2+aex^3+bcx^4+bdx^5+bex^6\right)\,dx\\&=\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7\end{aligned}$$

Mathematica [A]
time = 0.00, size = 55, normalized size = 1.00

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*
x^7)/7
```

Maple [A]
time = 0.14, size = 44, normalized size = 0.80

method	result	size
gosper	$\frac{1}{2}c\,x^{2}a+\frac{1}{3}ad\,x^{3}+\frac{1}{4}ae\,x^{4}+\frac{1}{5}bc\,x^{5}+\frac{1}{6}bd\,x^{6}+\frac{1}{7}be\,x^{7}$	44
default	$\frac{1}{2}c\,x^{2}a+\frac{1}{3}ad\,x^{3}+\frac{1}{4}ae\,x^{4}+\frac{1}{5}bc\,x^{5}+\frac{1}{6}bd\,x^{6}+\frac{1}{7}be\,x^{7}$	44
norman	$\frac{1}{2}c\,x^{2}a+\frac{1}{3}ad\,x^{3}+\frac{1}{4}ae\,x^{4}+\frac{1}{5}bc\,x^{5}+\frac{1}{6}bd\,x^{6}+\frac{1}{7}be\,x^{7}$	44
risch	$\frac{1}{2}c\,x^{2}a+\frac{1}{3}ad\,x^{3}+\frac{1}{4}ae\,x^{4}+\frac{1}{5}bc\,x^{5}+\frac{1}{6}bd\,x^{6}+\frac{1}{7}be\,x^{7}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/2*c*x^2*a+1/3*a*d*x^3+1/4*a*e*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x^7
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.82

$$\frac{1}{7}\,bx^7e+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,ax^4e+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
[Out] 1/7*b*x^7*e + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*x^4*e + 1/3*a*d*x^3 + 1/2*a
*c*x^2
```

Fricas [A]
time = 0.39, size = 43, normalized size = 0.78

$$\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,aex^4+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
[Out] 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*e*x^4 + 1/3*a*d*x^3 + 1/2*a
*c*x^2
```

Sympy [A]
time = 0.01, size = 49, normalized size = 0.89

$$\frac{acx^2}{2}+\frac{adx^3}{3}+\frac{aex^4}{4}+\frac{bcx^5}{5}+\frac{bdx^6}{6}+\frac{bex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*c*x**5/5 + b*d*x**6/6 + b*e*x**7/7
```

Giac [A]
time = 0.56, size = 45, normalized size = 0.82

$$\frac{1}{7}\,bx^7e+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,ax^4e+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/7*b*x^7*e + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*x^4*e + 1/3*a*d*x^3 + 1/2*a
*c*x^2
```

Mupad [B]
time = 0.03, size = 43, normalized size = 0.78

$$\frac{b\,e\,x^7}{7}+\frac{b\,d\,x^6}{6}+\frac{b\,c\,x^5}{5}+\frac{a\,e\,x^4}{4}+\frac{a\,d\,x^3}{3}+\frac{a\,c\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)*(c + d*x + e*x^2),x)
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (b*c*x^5)/5 + (a*e*x^4)/4 + (b*d*x^6)/6 + (b*e*
x^7)/7
```

3.315 $\int (c+dx+ex^2)(a+bx^3)\,dx$

Optimal. Leaf size=50

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*b*c*x^4+1/5*b*d*x^5+1/6*b*e*x^6
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1671}

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3),x]
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/6
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (c+dx+ex^2)\left(a+bx^3\right)\,dx &= \int \left(ac+adx+aex^2+bcx^3+bdx^4+bex^5\right)\,dx\\ &= acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6\end{aligned}$$

Mathematica [A]
time = 0.00, size = 50, normalized size = 1.00

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3),x]
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/6
```

Maple [A]
time = 0.15, size = 41, normalized size = 0.82

method	result	size
gosper	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}ae\,x^{3}+\frac{1}{4}bc\,x^{4}+\frac{1}{5}bd\,x^{5}+\frac{1}{6}be\,x^{6}$	41
default	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}ae\,x^{3}+\frac{1}{4}bc\,x^{4}+\frac{1}{5}bd\,x^{5}+\frac{1}{6}be\,x^{6}$	41
norman	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}ae\,x^{3}+\frac{1}{4}bc\,x^{4}+\frac{1}{5}bd\,x^{5}+\frac{1}{6}be\,x^{6}$	41
risch	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}ae\,x^{3}+\frac{1}{4}bc\,x^{4}+\frac{1}{5}bd\,x^{5}+\frac{1}{6}be\,x^{6}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*b*c*x^4+1/5*b*d*x^5+1/6*b*e*x^6
```

Maxima [A]
time = 0.28, size = 42, normalized size = 0.84

$$\frac{1}{6}\,bx^{6}e+\frac{1}{5}\,bdx^{5}+\frac{1}{4}\,bcx^{4}+\frac{1}{3}\,ax^{3}e+\frac{1}{2}\,adx^{2}+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
[Out] 1/6*b*x^6*e + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*x^3*e + 1/2*a*d*x^2 + a*c*x
```

Fricas [A]
time = 0.38, size = 40, normalized size = 0.80

$$\frac{1}{6}\,bex^{6}+\frac{1}{5}\,bdx^{5}+\frac{1}{4}\,bcx^{4}+\frac{1}{3}\,aex^{3}+\frac{1}{2}\,adx^{2}+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
[Out] 1/6*b*e*x^6 + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Sympy [A]
time = 0.01, size = 46, normalized size = 0.92

$$acx+\frac{adx^{2}}{2}+\frac{aex^{3}}{3}+\frac{bcx^{4}}{4}+\frac{bdx^{5}}{5}+\frac{bex^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*c*x**4/4 + b*d*x**5/5 + b*e*x**6/6
```

Giac [A]
time = 0.54, size = 42, normalized size = 0.84

$$\frac{1}{6}bx^6e + \frac{1}{5}bdx^5 + \frac{1}{4}bcx^4 + \frac{1}{3}ax^3e + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
[Out] 1/6*b*x^6*e + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*x^3*e + 1/2*a*d*x^2 + a*c*x
```

Mupad [B]
time = 0.02, size = 40, normalized size = 0.80

$$\frac{b\,e\,x^6}{6} + \frac{b\,d\,x^5}{5} + \frac{b\,c\,x^4}{4} + \frac{a\,e\,x^3}{3} + \frac{a\,d\,x^2}{2} + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x + e*x^2),x)
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^4)/4 + (a*e*x^3)/3 + (b*d*x^5)/5 + (b*e*x^6)/6
```

3.316 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x} \, dx$

Optimal. Leaf size=46

$$adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5 + ac\log(x)$$

```
[Out] a*d*x+1/2*a*e*x^2+1/3*b*c*x^3+1/4*b*d*x^4+1/5*b*e*x^5+a*c*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{1642\}$

$$ac\log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx+ex^2)(a+bx^3)}{x} \, dx &= \int \left(ad + \frac{ac}{x} + aex + bcx^2 + bdx^3 + bex^4\right) dx \\
&= adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5 + ac\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 46, normalized size = 1.00

$$adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5 + ac\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]
```

Maple [A]
time = 0.02, size = 39, normalized size = 0.85

method	result	size
default	$adx+\frac{ae\,x^{2}}{2}+\frac{bc\,x^{3}}{3}+\frac{bd\,x^{4}}{4}+\frac{be\,x^{5}}{5}+ac\ln\left(x\right)$	39
norman	$adx+\frac{ae\,x^{2}}{2}+\frac{bc\,x^{3}}{3}+\frac{bd\,x^{4}}{4}+\frac{be\,x^{5}}{5}+ac\ln\left(x\right)$	39
risch	$adx+\frac{ae\,x^{2}}{2}+\frac{bc\,x^{3}}{3}+\frac{bd\,x^{4}}{4}+\frac{be\,x^{5}}{5}+ac\ln\left(x\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*b*c*x^3+1/4*b*d*x^4+1/5*b*e*x^5+a*c*ln(x)
```

Maxima [A]
time = 0.29, size = 40, normalized size = 0.87

$$\frac{1}{5}\,bx^{5}e+\frac{1}{4}\,bdx^{4}+\frac{1}{3}\,bcx^{3}+\frac{1}{2}\,ax^{2}e+adx+ac\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5*e + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*x^2*e + a*d*x + a*c*log(x)
```

Fricas [A]
time = 0.38, size = 38, normalized size = 0.83

$$\frac{1}{5}\,bex^{5}+\frac{1}{4}\,bdx^{4}+\frac{1}{3}\,bcx^{3}+\frac{1}{2}\,aex^{2}+adx+ac\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="fricas")
```

```
[Out] 1/5*b*e*x^5 + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*e*x^2 + a*d*x + a*c*log(x)
```

Sympy [A]
time = 0.04, size = 44, normalized size = 0.96

$$ac\log\left(x\right)+adx+\frac{aex^{2}}{2}+\frac{bcx^{3}}{3}+\frac{bdx^{4}}{4}+\frac{bex^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)/x,x)
```

```
[Out] a*c*log(x) + a*d*x + a*e*x**2/2 + b*c*x**3/3 + b*d*x**4/4 + b*e*x**5/5
```

Giac [**A**]
time = 0.76, size = 41, normalized size = 0.89

$$\frac{1}{5}\,bx^5e + \frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,ax^2e + adx + ac\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="giac")
```

```
[Out] 1/5*b*x^5*e + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*x^2*e + a*d*x + a*c*log(abs
(x))
```

Mupad [**B**]
time = 0.03, size = 38, normalized size = 0.83

$$a\,c\,\ln\left(x\right) + a\,d\,x + \frac{b\,c\,x^3}{3} + \frac{a\,e\,x^2}{2} + \frac{b\,d\,x^4}{4} + \frac{b\,e\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2))/x,x)
```

```
[Out] a*c*log(x) + a*d*x + (b*c*x^3)/3 + (a*e*x^2)/2 + (b*d*x^4)/4 + (b*e*x^5)/5
```

3.317 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx$

Optimal. Leaf size=44

$$-\frac{ac}{x}+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+ad\log(x)$$

```
[Out] -a*c/x+a*e*x+1/2*b*c*x^2+1/3*b*d*x^3+1/4*b*e*x^4+a*d*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1642}

$$-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx &= \int \left(ae+\frac{ac}{x^2}+\frac{ad}{x}+bcx+bdx^2+bex^3\right)dx\\ &= -\frac{ac}{x}+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+ad\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 1.00

$$-\frac{ac}{x}+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+ad\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x^2,x]
```

[Out] -((a*c)/x) + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*Log[x]

Maple [A]
time = 0.03, size = 39, normalized size = 0.89

method	result	size
default	$-\frac{ac}{x}+aex+\frac{c\,x^{2} b}{2}+\frac{bd\,x^{3}}{3}+\frac{be\,x^{4}}{4}+ad\ln\left(x\right)$	39
risch	$-\frac{ac}{x}+aex+\frac{c\,x^{2} b}{2}+\frac{bd\,x^{3}}{3}+\frac{be\,x^{4}}{4}+ad\ln\left(x\right)$	39
norman	$\frac{ae\,x^{2}-ac+\frac{1}{2}bc\,x^{3}+\frac{1}{3}bd\,x^{4}+\frac{1}{4}be\,x^{5}}{x}+ad\ln\left(x\right)$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)/x^2,x,method=_RETURNVERBOSE)

[Out] -a*c/x+a*e*x+1/2*c*x^2*b+1/3*b*d*x^3+1/4*b*e*x^4+a*d*ln(x)

Maxima [A]
time = 0.28, size = 40, normalized size = 0.91

$$\frac{1}{4}\,bx^{4}e+\frac{1}{3}\,bdx^{3}+\frac{1}{2}\,bcx^{2}+axe+ad\log\left(x\right)-\frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="maxima")

[Out] 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/2*b*c*x^2 + a*x*e + a*d*log(x) - a*c/x

Fricas [A]
time = 0.37, size = 45, normalized size = 1.02

$$\frac{3\,bex^{5}+4\,bdx^{4}+6\,bcx^{3}+12\,aex^{2}+12\,adx\log\left(x\right)-12\,ac}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="fricas")

[Out] 1/12*(3*b*e*x^5 + 4*b*d*x^4 + 6*b*c*x^3 + 12*a*e*x^2 + 12*a*d*x*log(x) - 12
*a*c)/x

Sympy [A]
time = 0.06, size = 41, normalized size = 0.93

$$-\frac{ac}{x}+ad\log\left(x\right)+aex+\frac{bcx^{2}}{2}+\frac{bdx^{3}}{3}+\frac{bex^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)/x**2,x)
[Out] -a*c/x + a*d*log(x) + a*e*x + b*c*x**2/2 + b*d*x**3/3 + b*e*x**4/4
```

Giac [A]
time = 0.76, size = 41, normalized size = 0.93

$$\frac{1}{4}bx^4e+\frac{1}{3}bdx^3+\frac{1}{2}bcx^2+axe+ad\log\left(|x|\right)-\frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="giac")
[Out] 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/2*b*c*x^2 + a*x*e + a*d*log(abs(x)) - a*c/x
```

Mupad [B]
time = 0.03, size = 38, normalized size = 0.86

$$a\,d\,\ln\left(x\right)+a\,e\,x-\frac{a\,c}{x}+\frac{b\,c\,x^2}{2}+\frac{b\,d\,x^3}{3}+\frac{b\,e\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2))/x^2,x)
[Out] a*d*log(x) + a*e*x - (a*c)/x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4
```

3.318 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3} dx$

Optimal. Leaf size=44

$$-\frac{ac}{2x^2}-\frac{ad}{x}+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3+ae\log(x)$$

```
[Out] -1/2*a*c/x^2-a*d/x+b*c*x+1/2*b*d*x^2+1/3*b*e*x^3+a*e*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1642}

$$-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3))/x^3,x]
```

```
[Out] -1/2*(a*c)/x^2 - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx &= \int \left(bc+\frac{ac}{x^3}+\frac{ad}{x^2}+\frac{ae}{x}+bdx+bex^2\right)dx\\ &= -\frac{ac}{2x^2}-\frac{ad}{x}+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3+ae\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 1.00

$$-\frac{ac}{2x^2}-\frac{ad}{x}+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3+ae\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x^3,x]
```

```
[Out] -1/2*(a*c)/x^2 - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*Log[x]
```

Maple [A]
time = 0.03, size = 39, normalized size = 0.89

method	result	size
default	$-\frac{ac}{2x^2}-\frac{ad}{x}+bcx+\frac{bd\,x^2}{2}+\frac{be\,x^3}{3}+ae\ln\left(x\right)$	39
risch	$\frac{be\,x^3}{3}+\frac{bd\,x^2}{2}+bcx+\frac{-adx-\frac{1}{2}ac}{x^2}+ae\ln\left(x\right)$	39
norman	$\frac{bc\,x^3-\frac{1}{2}ac-adx+\frac{1}{2}bd\,x^4+\frac{1}{3}be\,x^5}{x^2}+ae\ln\left(x\right)$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a*c/x^2-a*d/x+b*c*x+1/2*b*d*x^2+1/3*b*e*x^3+a*e*ln(x)
```

Maxima [A]
time = 0.28, size = 40, normalized size = 0.91

$$\frac{1}{3}\,bx^3e+\frac{1}{2}\,bdx^2+bcx+ae\log\left(x\right)-\frac{2\,adx+ac}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="maxima")
[Out] 1/3*b*x^3*e + 1/2*b*d*x^2 + b*c*x + a*e*log(x) - 1/2*(2*a*d*x + a*c)/x^2
```

Fricas [A]
time = 0.37, size = 45, normalized size = 1.02

$$\frac{2\,bex^5+3\,bdx^4+6\,bcx^3+6\,aex^2\log\left(x\right)-6\,adx-3\,ac}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="fricas")
[Out] 1/6*(2*b*e*x^5 + 3*b*d*x^4 + 6*b*c*x^3 + 6*a*e*x^2*log(x) - 6*a*d*x - 3*a*c
)/x^2
```

Sympy [A]
time = 0.11, size = 44, normalized size = 1.00

$$ae\log\left(x\right)+bcx+\frac{bdx^2}{2}+\frac{bex^3}{3}+\frac{-ac-2adx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)/x**3,x)
[Out] a*e*log(x) + b*c*x + b*d*x**2/2 + b*e*x**3/3 + (-a*c - 2*a*d*x)/(2*x**2)
```

Giac [A]
time = 0.78, size = 41, normalized size = 0.93

$$\frac{1}{3}\,bx^3e+\frac{1}{2}\,bdx^2+bcx+ae\log\left(|x|\right)-\frac{2\,adx+ac}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="giac")
[Out] 1/3*b*x^3*e + 1/2*b*d*x^2 + b*c*x + a*e*log(abs(x)) - 1/2*(2*a*d*x + a*c)/x
^2
```

Mupad [B]
time = 0.03, size = 38, normalized size = 0.86

$$a\,e\,\ln\left(x\right)-\frac{\frac{a\,c}{2}+a\,d\,x}{x^2}+b\,c\,x+\frac{b\,d\,x^2}{2}+\frac{b\,e\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2))/x^3,x)
[Out] a*e*log(x) - ((a*c)/2 + a*d*x)/x^2 + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3
```

3.319 $\int x^2(c+dx+ex^2)\left(a+bx^3\right)^2\,dx$

Optimal. Leaf size=82

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{c\left(a+bx^3\right)^3}{9b}$$

```
[Out] 1/4*a^2*d*x^4+1/5*a^2*e*x^5+2/7*a*b*d*x^7+1/4*a*b*e*x^8+1/10*b^2*d*x^10+1/1
1*b^2*e*x^11+1/9*c*(b*x^3+a)^3/b
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1596, 1864}

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{c\left(a+bx^3\right)^3}{9b}+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (2*a*b*d*x^7)/7 + (a*b*e*x^8)/4 + (b^2*d*x^
10)/10 + (b^2*e*x^11)/11 + (c*(a + b*x^3)^3)/(9*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx=\frac{c(a+bx^3)^3}{9b}+\int\left(a+bx^3\right)^2\left(-cx^2+x^2\left(c+dx+ex^2\right)\right)\,dx$$

$$=\frac{c(a+bx^3)^3}{9b}+\int\left(a^2dx^3+a^2ex^4+2abdx^6+2abex^7+b^2dx^9+b^2ex^{10}\right)$$

$$=\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{c(a+}{9b}$$

Mathematica [A]
time = 0.00, size = 97, normalized size = 1.18

$$\frac{1}{3}a^2cx^3+\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{3}abcx^6+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a*b*c*x^6)/3 + (2*a*b*d*x^
7)/7 + (a*b*e*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11
```

Maple [A]
time = 0.36, size = 80, normalized size = 0.98

method	result	size
gosper	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}x^9b^2c+\frac{1}{4}abe\,x^8+\frac{2}{7}abd\,x^7+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x^3$	80
default	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}x^9b^2c+\frac{1}{4}abe\,x^8+\frac{2}{7}abd\,x^7+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x^3$	80
norman	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}x^9b^2c+\frac{1}{4}abe\,x^8+\frac{2}{7}abd\,x^7+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x^3$	80
risch	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}x^9b^2c+\frac{1}{4}abe\,x^8+\frac{2}{7}abd\,x^7+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x^3$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*x^9*b^2*c+1/4*a*b*e*x^8+2/7*a*b*d*x^7+1
/3*a*b*c*x^6+1/5*a^2*e*x^5+1/4*a^2*d*x^4+1/3*a^2*c*x^3
```

Maxima [A]
time = 0.28, size = 82, normalized size = 1.00

$$\frac{1}{11}\,b^2x^{11}e+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abx^8e+\frac{2}{7}\,abdx^7+\frac{1}{3}\,abcx^6+\frac{1}{5}\,a^2x^5e+\frac{1}{4}\,a^2dx^4+\frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*x^8*e + 2/7*a*b
*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*x^5*e + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

Fricas [A]
time = 0.36, size = 79, normalized size = 0.96

$$\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abex^8+\frac{2}{7}\,abdx^7+\frac{1}{3}\,abcx^6+\frac{1}{5}\,a^2ex^5+\frac{1}{4}\,a^2dx^4+\frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*e*x^8 + 2/7*a*b
*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*e*x^5 + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

Sympy [A]
time = 0.01, size = 92, normalized size = 1.12

$$\frac{a^2cx^3}{3}+\frac{a^2dx^4}{4}+\frac{a^2ex^5}{5}+\frac{abcx^6}{3}+\frac{2abdx^7}{7}+\frac{abex^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + a*b*c*x**6/3 + 2*a*b*d*x**7
/7 + a*b*e*x**8/4 + b**2*c*x**9/9 + b**2*d*x**10/10 + b**2*e*x**11/11
```

Giac [A]
time = 0.63, size = 82, normalized size = 1.00

$$\frac{1}{11}\,b^2x^{11}e+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abx^8e+\frac{2}{7}\,abdx^7+\frac{1}{3}\,abcx^6+\frac{1}{5}\,a^2x^5e+\frac{1}{4}\,a^2dx^4+\frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*x^8*e + 2/7*a*b
*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*x^5*e + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

Mupad [B]
time = 0.04, size = 79, normalized size = 0.96

$$\frac{e\,a^2\,x^5}{5}+\frac{d\,a^2\,x^4}{4}+\frac{c\,a^2\,x^3}{3}+\frac{e\,a\,b\,x^8}{4}+\frac{2\,d\,a\,b\,x^7}{7}+\frac{c\,a\,b\,x^6}{3}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^2*(c + d*x + e*x^2),x)
```

```
[Out] (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (b^2*c*x^9)/9 + (a^2*e*x^5)/5 + (b^2*d*x^10
)/10 + (b^2*e*x^11)/11 + (a*b*c*x^6)/3 + (2*a*b*d*x^7)/7 + (a*b*e*x^8)/4
```

3.320 $\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}+\frac{d\left(a+bx^3\right)^3}{9b}$$

```
[Out] 1/2*a^2*c*x^2+1/4*a^2*e*x^4+2/5*a*b*c*x^5+2/7*a*b*e*x^7+1/8*b^2*c*x^8+1/10*
b^2*e*x^10+1/9*d*(b*x^3+a)^3/b
```

Rubi [A]

time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1596, 1864}

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{d\left(a+bx^3\right)^3}{9b}+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 + (2*a*b*e*x^7)/7 + (b^2*c*
x^8)/8 + (b^2*e*x^10)/10 + (d*(a + b*x^3)^3)/(9*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx=\frac{d(a+bx^3)^3}{9b}+\int\left(a+bx^3\right)^2\left(-dx^2+x\left(c+dx+ex^2\right)\right)\,dx$$
$$=\frac{d(a+bx^3)^3}{9b}+\int\left(a^2cx+a^2ex^3+2abcx^4+2abex^6+b^2cx^7+b^2ex^9\right)\,d$$
$$=\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}+\frac{d(a+ba}{9b}$$

Mathematica [A]
time = 0.00, size = 97, normalized size = 1.18

$$\frac{1}{2}a^2cx^2+\frac{1}{3}a^2dx^3+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{9}b^2dx^9+\frac{1}{10}b^2ex^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 + (a*b*d*x^
6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^8)/8 + (b^2*d*x^9)/9 + (b^2*e*x^10)/10
```

Maple [A]
time = 0.39, size = 80, normalized size = 0.98

method	result	size
gosper	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}x^5abc+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$	80
default	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}x^5abc+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$	80
norman	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}x^5abc+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$	80
risch	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}x^5abc+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^2*e*x^10+1/9*b^2*d*x^9+1/8*b^2*c*x^8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5
*x^5*a*b*c+1/4*a^2*e*x^4+1/3*a^2*d*x^3+1/2*a^2*c*x^2
```

Maxima [A]
time = 0.28, size = 82, normalized size = 1.00

$$\frac{1}{10}\,b^2x^{10}e+\frac{1}{9}\,b^2dx^9+\frac{1}{8}\,b^2cx^8+\frac{2}{7}\,abx^7e+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2x^4e+\frac{1}{3}\,a^2dx^3+\frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*x^10*e + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d
*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*x^4*e + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

Fricas [A]
time = 0.36, size = 79, normalized size = 0.96

$$\frac{1}{10}\,b^2ex^{10}+\frac{1}{9}\,b^2dx^9+\frac{1}{8}\,b^2cx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2ex^4+\frac{1}{3}\,a^2dx^3+\frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*b^2*e*x^10 + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d
*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*e*x^4 + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

Sympy [A]
time = 0.01, size = 94, normalized size = 1.15

$$\frac{a^2cx^2}{2}+\frac{a^2dx^3}{3}+\frac{a^2ex^4}{4}+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{b^2cx^8}{8}+\frac{b^2dx^9}{9}+\frac{b^2ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + 2*a*b*c*x**5/5 + a*b*d*x**6
/3 + 2*a*b*e*x**7/7 + b**2*c*x**8/8 + b**2*d*x**9/9 + b**2*e*x**10/10
```

Giac [A]
time = 0.65, size = 82, normalized size = 1.00

$$\frac{1}{10}\,b^2x^{10}e+\frac{1}{9}\,b^2dx^9+\frac{1}{8}\,b^2cx^8+\frac{2}{7}\,abx^7e+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2x^4e+\frac{1}{3}\,a^2dx^3+\frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^2*x^10*e + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d
*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*x^4*e + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

Mupad [B]
time = 0.04, size = 79, normalized size = 0.96

$$\frac{e\,a^2\,x^4}{4}+\frac{d\,a^2\,x^3}{3}+\frac{c\,a^2\,x^2}{2}+\frac{2\,e\,a\,b\,x^7}{7}+\frac{d\,a\,b\,x^6}{3}+\frac{2\,c\,a\,b\,x^5}{5}+\frac{e\,b^2\,x^{10}}{10}+\frac{d\,b^2\,x^9}{9}+\frac{c\,b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^2*(c + d*x + e*x^2),x)
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (b^2*c*x^8)/8 + (a^2*e*x^4)/4 + (b^2*d*x^9)
/9 + (b^2*e*x^10)/10 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7
```

3.321 $\int (c+dx+ex^2)(a+bx^3)^2 \, dx$

Optimal. Leaf size=77

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{e(a+bx^3)^3}{9b}$$

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/2*a*b*c*x^4+2/5*a*b*d*x^5+1/7*b^2*c*x^7+1/8*b^2*d*x
^8+1/9*e*(b*x^3+a)^3/b
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1596, 1864}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{e(a+bx^3)^3}{9b}+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8 + (e*(a + b*x^3)^3)/(9*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx&=\frac{e(a+bx^3)^3}{9b}+\int(c+dx)\left(a+bx^3\right)^2\,dx\\&=\frac{e(a+bx^3)^3}{9b}+\int\left(a^2c+a^2dx+2abcx^3+2abdx^4+b^2cx^6+b^2dx^7\right)\,dx\\&=a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{e(a+bx^3)^3}{9b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 92, normalized size = 1.19

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{3}abex^6+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{1}{9}b^2ex^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 +
 (a*b*e*x^6)/3 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + (b^2*e*x^9)/9
```

Maple [A]
time = 0.37, size = 77, normalized size = 1.00

method	result	size
gosper	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}abe\,x^6+\frac{2}{5}abd\,x^5+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77
default	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}abe\,x^6+\frac{2}{5}abd\,x^5+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77
norman	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}abe\,x^6+\frac{2}{5}abd\,x^5+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77
risch	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}abe\,x^6+\frac{2}{5}abd\,x^5+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^2*e*x^9+1/8*b^2*d*x^8+1/7*b^2*c*x^7+1/3*a*b*e*x^6+2/5*a*b*d*x^5+1/2*a
*b*c*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c*x
```

Maxima [A]
time = 0.28, size = 79, normalized size = 1.03

$$\frac{1}{9}\,b^2x^9e+\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{1}{3}\,abx^6e+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{3}\,a^2x^3e+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*x^9*e + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*x^6*e + 2/5*a*b*d*x
^5 + 1/2*a*b*c*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

Fricas [A]
time = 0.35, size = 76, normalized size = 0.99

$$\frac{1}{9}\,b^2ex^9+\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{1}{3}\,abex^6+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*b^2*e*x^9 + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*e*x^6 + 2/5*a*b*d*x
^5 + 1/2*a*b*c*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Sympy [A]
time = 0.01, size = 88, normalized size = 1.14

$$a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{abcx^4}{2}+\frac{2abdx^5}{5}+\frac{abex^6}{3}+\frac{b^2cx^7}{7}+\frac{b^2dx^8}{8}+\frac{b^2ex^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a*b*c*x**4/2 + 2*a*b*d*x**5/5 +
a*b*e*x**6/3 + b**2*c*x**7/7 + b**2*d*x**8/8 + b**2*e*x**9/9
```

Giac [A]
time = 0.66, size = 79, normalized size = 1.03

$$\frac{1}{9}\,b^2x^9e+\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{1}{3}\,abx^6e+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{3}\,a^2x^3e+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*x^9*e + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*x^6*e + 2/5*a*b*d*x
^5 + 1/2*a*b*c*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

Mupad [B]
time = 0.04, size = 76, normalized size = 0.99

$$\frac{e\,a^2\,x^3}{3}+\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{e\,a\,b\,x^6}{3}+\frac{2\,d\,a\,b\,x^5}{5}+\frac{c\,a\,b\,x^4}{2}+\frac{e\,b^2\,x^9}{9}+\frac{d\,b^2\,x^8}{8}+\frac{c\,b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x + e*x^2),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*c*x^7)/7 + (a^2*e*x^3)/3 + (b^2*d*x^8)/8 + (b^2*e*x^9)
/9 + a^2*c*x + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (a*b*e*x^6)/3
```

3.322 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x}\,dx$

Optimal. Leaf size=88

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8+a^2c\log(x)$$

```
[Out] a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*c*x^3+1/2*a*b*d*x^4+2/5*a*b*e*x^5+1/6*b^2*c*x
^6+1/7*b^2*d*x^7+1/8*b^2*e*x^8+a^2*c*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1642}

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5
 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x}\,dx &= \int \left(a^2d+\frac{a^2c}{x}+a^2ex+2abcx^2+2abdx^3+2abex^4+b^2cx^5+b^2dx^6+b^2e\right.\\ &= a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex\end{aligned}$$

Mathematica [A]
time = 0.01, size = 88, normalized size = 1.00

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8+a^2c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5
 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]
```

Maple [A]
time = 0.33, size = 75, normalized size = 0.85

method	result	size
default	$a^2dx+\frac{a^2e\,x^2}{2}+\frac{2abc\,x^3}{3}+\frac{abd\,x^4}{2}+\frac{2abe\,x^5}{5}+\frac{b^2c\,x^6}{6}+\frac{b^2d\,x^7}{7}+\frac{b^2e\,x^8}{8}+a^2c\ln\left(x\right)$	75
norman	$a^2dx+\frac{a^2e\,x^2}{2}+\frac{2abc\,x^3}{3}+\frac{abd\,x^4}{2}+\frac{2abe\,x^5}{5}+\frac{b^2c\,x^6}{6}+\frac{b^2d\,x^7}{7}+\frac{b^2e\,x^8}{8}+a^2c\ln\left(x\right)$	75
risch	$a^2dx+\frac{a^2e\,x^2}{2}+\frac{2abc\,x^3}{3}+\frac{abd\,x^4}{2}+\frac{2abe\,x^5}{5}+\frac{b^2c\,x^6}{6}+\frac{b^2d\,x^7}{7}+\frac{b^2e\,x^8}{8}+a^2c\ln\left(x\right)$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*c*x^3+1/2*a*b*d*x^4+2/5*a*b*e*x^5+1/6*b^2*c*x
^6+1/7*b^2*d*x^7+1/8*b^2*e*x^8+a^2*c*ln(x)
```

Maxima [A]
time = 0.27, size = 77, normalized size = 0.88

$$\frac{1}{8}\,b^2x^8e+\frac{1}{7}\,b^2dx^7+\frac{1}{6}\,b^2cx^6+\frac{2}{5}\,abx^5e+\frac{1}{2}\,abdx^4+\frac{2}{3}\,abcx^3+\frac{1}{2}\,a^2x^2e+a^2dx+a^2c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8*e + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*x^5*e + 1/2*a*b*d*x
^4 + 2/3*a*b*c*x^3 + 1/2*a^2*x^2*e + a^2*d*x + a^2*c*log(x)
```

Fricas [A]
time = 0.38, size = 74, normalized size = 0.84

$$\frac{1}{8}\,b^2ex^8+\frac{1}{7}\,b^2dx^7+\frac{1}{6}\,b^2cx^6+\frac{2}{5}\,abex^5+\frac{1}{2}\,abdx^4+\frac{2}{3}\,abcx^3+\frac{1}{2}\,a^2ex^2+a^2dx+a^2c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*b^2*e*x^8 + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*e*x^5 + 1/2*a*b*d*x
^4 + 2/3*a*b*c*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(x)
```

Sympy [A]
time = 0.06, size = 88, normalized size = 1.00

$$a^2c\log\left(x\right)+a^2dx+\frac{a^2ex^2}{2}+\frac{2abcx^3}{3}+\frac{abdx^4}{2}+\frac{2abex^5}{5}+\frac{b^2cx^6}{6}+\frac{b^2dx^7}{7}+\frac{b^2ex^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x,x)
```

```
[Out] a**2*c*log(x) + a**2*d*x + a**2*e*x**2/2 + 2*a*b*c*x**3/3 + a*b*d*x**4/2 +
2*a*b*e*x**5/5 + b**2*c*x**6/6 + b**2*d*x**7/7 + b**2*e*x**8/8
```

Giac [A]
time = 0.68, size = 78, normalized size = 0.89

$$\frac{1}{8}\,b^2x^8e+\frac{1}{7}\,b^2dx^7+\frac{1}{6}\,b^2cx^6+\frac{2}{5}\,abx^5e+\frac{1}{2}\,abdx^4+\frac{2}{3}\,abcx^3+\frac{1}{2}\,a^2x^2e+a^2dx+a^2c\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8*e + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*x^5*e + 1/2*a*b*d*x
^4 + 2/3*a*b*c*x^3 + 1/2*a^2*x^2*e + a^2*d*x + a^2*c*log(abs(x))
```

Mupad [B]
time = 0.04, size = 74, normalized size = 0.84

$$\frac{b^2\,c\,x^6}{6}+\frac{a^2\,e\,x^2}{2}+\frac{b^2\,d\,x^7}{7}+\frac{b^2\,e\,x^8}{8}+a^2\,c\,\ln\left(x\right)+a^2\,d\,x+\frac{2\,a\,b\,c\,x^3}{3}+\frac{a\,b\,d\,x^4}{2}+\frac{2\,a\,b\,e\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2))/x,x)
```

```
[Out] (b^2*c*x^6)/6 + (a^2*e*x^2)/2 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*log(x
) + a^2*d*x + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5
```

3.323 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2} dx$

Optimal. Leaf size=83

$$-\frac{a^2c}{x} + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7 + a^2d\log(x)$$

```
[Out] -a^2*c/x+a^2*e*x+a*b*c*x^2+2/3*a*b*d*x^3+1/2*a*b*e*x^4+1/5*b^2*c*x^5+1/6*b^
2*d*x^6+1/7*b^2*e*x^7+a^2*d*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1642}

$$-\frac{a^2c}{x} + a^2d\log(x) + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (b^2
*c*x^5)/5 + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2} dx = \int \left(a^2e + \frac{a^2c}{x^2} + \frac{a^2d}{x} + 2abcx + 2abdx^2 + 2abex^3 + b^2cx^4 + b^2dx^5 + b^2ex$$
$$= -\frac{a^2c}{x} + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 1.00

$$-\frac{a^2c}{x} + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7 + a^2d\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (b^2
*c*x^5)/5 + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*Log[x]
```

Maple [A]
time = 0.35, size = 74, normalized size = 0.89

method	result	size
default	$-\frac{a^{2} c}{x}+a^{2} e x+a b c\,x^{2}+\frac{2 a b d\,x^{3}}{3}+\frac{a b e\,x^{4}}{2}+\frac{b^{2} c\,x^{5}}{5}+\frac{b^{2} d\,x^{6}}{6}+\frac{b^{2} e\,x^{7}}{7}+a^{2} d \ln \left(x \right)$	74
risch	$-\frac{a^{2} c}{x}+a^{2} e x+a b c\,x^{2}+\frac{2 a b d\,x^{3}}{3}+\frac{a b e\,x^{4}}{2}+\frac{b^{2} c\,x^{5}}{5}+\frac{b^{2} d\,x^{6}}{6}+\frac{b^{2} e\,x^{7}}{7}+a^{2} d \ln \left(x \right)$	74
norman	$\frac{a^{2} e\,x^{2}+a b c\,x^{3}-a^{2} c+\frac{1}{5} b^{2} c\,x^{6}+\frac{1}{6} b^{2} d\,x^{7}+\frac{1}{7} b^{2} e\,x^{8}+\frac{2}{3} a b d\,x^{4}+\frac{1}{2} a b e\,x^{5}}{x}+a^{2} d \ln \left(x \right)$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*c/x+a^2*e*x+a*b*c*x^2+2/3*a*b*d*x^3+1/2*a*b*e*x^4+1/5*b^2*c*x^5+1/6*b^
2*d*x^6+1/7*b^2*e*x^7+a^2*d*ln(x)
```

Maxima [A]
time = 0.26, size = 76, normalized size = 0.92

$$\frac{1}{7}\,b^{2}x^{7}e+\frac{1}{6}\,b^{2}dx^{6}+\frac{1}{5}\,b^{2}cx^{5}+\frac{1}{2}\,abx^{4}e+\frac{2}{3}\,abdx^{3}+abcx^{2}+a^{2}xe+a^{2}d\log\left(x\right)-\frac{a^{2}c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7*e + 1/6*b^2*d*x^6 + 1/5*b^2*c*x^5 + 1/2*a*b*x^4*e + 2/3*a*b*d*x
^3 + a*b*c*x^2 + a^2*x*e + a^2*d*log(x) - a^2*c/x
```

Fricas [A]
time = 0.37, size = 81, normalized size = 0.98

$$\frac{30\,b^{2}ex^{8}+35\,b^{2}dx^{7}+42\,b^{2}cx^{6}+105\,abex^{5}+140\,abdx^{4}+210\,abcx^{3}+210\,a^{2}ex^{2}+210\,a^{2}dx\log\left(x\right)-210\,a^{2}c}{210\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/210*(30*b^2*e*x^8 + 35*b^2*d*x^7 + 42*b^2*c*x^6 + 105*a*b*e*x^5 + 140*a*b
*d*x^4 + 210*a*b*c*x^3 + 210*a^2*e*x^2 + 210*a^2*d*x*log(x) - 210*a^2*c)/x
```

Sympy [A]
time = 0.07, size = 82, normalized size = 0.99

$$-\frac{a^{2}c}{x}+a^{2}d\log\left(x\right)+a^{2}ex+abcx^{2}+\frac{2abdx^{3}}{3}+\frac{abex^{4}}{2}+\frac{b^{2}cx^{5}}{5}+\frac{b^{2}dx^{6}}{6}+\frac{b^{2}ex^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x**2,x)
```

```
[Out] -a**2*c/x + a**2*d*log(x) + a**2*e*x + a*b*c*x**2 + 2*a*b*d*x**3/3 + a*b*e*
x**4/2 + b**2*c*x**5/5 + b**2*d*x**6/6 + b**2*e*x**7/7
```

Giac [A]
time = 0.81, size = 77, normalized size = 0.93

$$\frac{1}{7}\,b^2x^7e+\frac{1}{6}\,b^2dx^6+\frac{1}{5}\,b^2cx^5+\frac{1}{2}\,abx^4e+\frac{2}{3}\,abdx^3+abcx^2+a^2xe+a^2d\log\left(|x|\right)-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7*e + 1/6*b^2*d*x^6 + 1/5*b^2*c*x^5 + 1/2*a*b*x^4*e + 2/3*a*b*d*x
^3 + a*b*c*x^2 + a^2*x*e + a^2*d*log(abs(x)) - a^2*c/x
```

Mupad [B]
time = 0.04, size = 73, normalized size = 0.88

$$\frac{b^2\,c\,x^5}{5}-\frac{a^2\,c}{x}+\frac{b^2\,d\,x^6}{6}+\frac{b^2\,e\,x^7}{7}+a^2\,d\,\ln\left(x\right)+a^2\,e\,x+a\,b\,c\,x^2+\frac{2\,a\,b\,d\,x^3}{3}+\frac{a\,b\,e\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2))/x^2,x)
```

```
[Out] (b^2*c*x^5)/5 - (a^2*c)/x + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*log(x) +
a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2
```

3.324 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3} dx$

Optimal. Leaf size=84

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6+a^2e\log(x)$$

```
[Out] -1/2*a^2*c/x^2-a^2*d/x+2*a*b*c*x+a*b*d*x^2+2/3*a*b*e*x^3+1/4*b^2*c*x^4+1/5*
b^2*d*x^5+1/6*b^2*e*x^6+a^2*e*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1642}

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3,x]
```

```
[Out] -1/2*(a^2*c)/x^2 - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (b
^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx &= \int \left(2abc+\frac{a^2c}{x^3}+\frac{a^2d}{x^2}+\frac{a^2e}{x}+2abdx+2abex^2+b^2cx^3+b^2dx^4+b^2ex^5\right)\\ &= -\frac{a^2c}{2x^2}-\frac{a^2d}{x}+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6+a\end{aligned}$$

Mathematica [A]
time = 0.01, size = 84, normalized size = 1.00

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6+a^2e\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3,x]
```

```
[Out] -1/2*(a^2*c)/x^2 - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (b
^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*Log[x]
```

Maple [A]
time = 0.50, size = 75, normalized size = 0.89

method	result	size
default	$-\frac{a^{2} c}{2 x^{2}}-\frac{a^{2} d}{x}+2 a b c x+a b d\,x^{2}+\frac{2 a b e\,x^{3}}{3}+\frac{b^{2} c\,x^{4}}{4}+\frac{b^{2} d\,x^{5}}{5}+\frac{b^{2} e\,x^{6}}{6}+a^{2} e \ln \left(x \right)$	75
risch	$\frac{b^{2} e\,x^{6}}{6}+\frac{b^{2} d\,x^{5}}{5}+\frac{b^{2} c\,x^{4}}{4}+\frac{2 a b e\,x^{3}}{3}+a b d\,x^{2}+2 a b c x+\frac{-a^{2} d x-\frac{1}{2} a^{2} c}{x^{2}}+a^{2} e \ln \left(x \right)$	75
norman	$\frac{a b d\,x^{4}-\frac{1}{2} a^{2} c-a^{2} d x+\frac{1}{4} b^{2} c\,x^{6}+\frac{1}{5} b^{2} d\,x^{7}+\frac{1}{6} b^{2} e\,x^{8}+2 a b c\,x^{3}+\frac{2}{3} a b e\,x^{5}}{x^{2}}+a^{2} e \ln \left(x \right)$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*c/x^2-a^2*d/x+2*a*b*c*x+a*b*d*x^2+2/3*a*b*e*x^3+1/4*b^2*c*x^4+1/5*
b^2*d*x^5+1/6*b^2*e*x^6+a^2*e*ln(x)
```

Maxima [A]
time = 0.27, size = 77, normalized size = 0.92

$$\frac{1}{6}\,b^{2}x^{6}e+\frac{1}{5}\,b^{2}dx^{5}+\frac{1}{4}\,b^{2}cx^{4}+\frac{2}{3}\,abx^{3}e+abdx^{2}+2\,abcx+a^{2}e\log\left(x\right)-\frac{2\,a^{2}dx+a^{2}c}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*e + 1/5*b^2*d*x^5 + 1/4*b^2*c*x^4 + 2/3*a*b*x^3*e + a*b*d*x^2 +
 2*a*b*c*x + a^2*e*log(x) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

Fricas [A]
time = 0.36, size = 81, normalized size = 0.96

$$\frac{10\,b^{2}ex^{8}+12\,b^{2}dx^{7}+15\,b^{2}cx^{6}+40\,abex^{5}+60\,abdx^{4}+120\,abcx^{3}+60\,a^{2}ex^{2}\log\left(x\right)-60\,a^{2}dx-30\,a^{2}c}{60\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*e*x^8 + 12*b^2*d*x^7 + 15*b^2*c*x^6 + 40*a*b*e*x^5 + 60*a*b*d*
x^4 + 120*a*b*c*x^3 + 60*a^2*e*x^2*log(x) - 60*a^2*d*x - 30*a^2*c)/x^2
```

Sympy [A]
time = 0.13, size = 87, normalized size = 1.04

$$a^{2}e\log\left(x\right)+2abcx+abdx^{2}+\frac{2abex^{3}}{3}+\frac{b^{2}cx^{4}}{4}+\frac{b^{2}dx^{5}}{5}+\frac{b^{2}ex^{6}}{6}+\frac{-a^{2}c-2a^{2}dx}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x**3,x)
```

```
[Out] a**2*e*log(x) + 2*a*b*c*x + a*b*d*x**2 + 2*a*b*e*x**3/3 + b**2*c*x**4/4 + b
**2*d*x**5/5 + b**2*e*x**6/6 + (-a**2*c - 2*a**2*d*x)/(2*x**2)
```

Giac [A]
time = 0.75, size = 78, normalized size = 0.93

$$\frac{1}{6}\,b^2x^6e+\frac{1}{5}\,b^2dx^5+\frac{1}{4}\,b^2cx^4+\frac{2}{3}\,abx^3e+abdx^2+2\,abcx+a^2e\log\left(|x|\right)-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6*e + 1/5*b^2*d*x^5 + 1/4*b^2*c*x^4 + 2/3*a*b*x^3*e + a*b*d*x^2 +
 2*a*b*c*x + a^2*e*log(abs(x)) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

Mupad [B]
time = 0.04, size = 74, normalized size = 0.88

$$\frac{b^2\,c\,x^4}{4}-\frac{\frac{a^2\,c}{2}+a^2\,d\,x}{x^2}+\frac{b^2\,d\,x^5}{5}+\frac{b^2\,e\,x^6}{6}+a^2\,e\,\ln\left(x\right)+a\,b\,d\,x^2+\frac{2\,a\,b\,e\,x^3}{3}+2\,a\,b\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2))/x^3,x)
```

```
[Out] (b^2*c*x^4)/4 - ((a^2*c)/2 + a^2*d*x)/x^2 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 +
 a^2*e*log(x) + a*b*d*x^2 + (2*a*b*e*x^3)/3 + 2*a*b*c*x
```

3.325 $\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx$

Optimal. Leaf size=110

$$\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}+\frac{c(a+bx^3)^4}{12b}$$

```
[Out] 1/4*a^3*d*x^4+1/5*a^3*e*x^5+3/7*a^2*b*d*x^7+3/8*a^2*b*e*x^8+3/10*a*b^2*d*x^
10+3/11*a*b^2*e*x^11+1/13*b^3*d*x^13+1/14*b^3*e*x^14+1/12*c*(b*x^3+a)^4/b
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1596, 1864}

$$\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{c(a+bx^3)^4}{12b}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (3*a^2*b*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (3*
a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*d*x^13)/13 + (b^3*e*x^14)/14
+ (c*(a + b*x^3)^4)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx = \frac{c(a+bx^3)^4}{12b} + \int \left(a+bx^3\right)^3\left(-cx^2+x^2(c+dx+ex^2)\right)\,dx$$
$$= \frac{c(a+bx^3)^4}{12b} + \int \left(a^3dx^3+a^3ex^4+3a^2bdx^6+3a^2bex^7+3ab^2dx^9+3ab\right.$$
$$= \frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}$$

Mathematica [A]
time = 0.00, size = 139, normalized size = 1.26

$$\frac{1}{3}a^3cx^3+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{2}a^2bcx^6+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{12}b^3cx^{12}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*b*c*x^6)/2 + (3*a^2*b*
d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a
*b^2*e*x^11)/11 + (b^3*c*x^12)/12 + (b^3*d*x^13)/13 + (b^3*e*x^14)/14
```

Maple [A]
time = 0.40, size = 116, normalized size = 1.05

method	result
gosper	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}c\,a^2b\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}a\,b^2e$
default	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}c\,a^2b\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}a\,b^2e$
norman	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}c\,a^2b\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}a\,b^2e$
risch	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}c\,a^2b\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}a\,b^2e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*c*a^3*x^3+1/4*a^3*d*x^4+1/5*a^3*e*x^5+1/2*c*a^2*b*x^6+3/7*a^2*b*d*x^7+3
/8*a^2*b*e*x^8+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/11*a*b^2*e*x^11+1/12*b^3
*c*x^12+1/13*b^3*d*x^13+1/14*b^3*e*x^14
```

Maxima [A]
time = 0.27, size = 119, normalized size = 1.08

$$\frac{1}{14}b^3x^{14}e+\frac{1}{13}b^3dx^{13}+\frac{1}{12}b^3cx^{12}+\frac{3}{11}ab^2x^{11}e+\frac{3}{10}ab^2dx^{10}+\frac{1}{3}ab^2cx^9+\frac{3}{8}a^2bx^8e+\frac{3}{7}a^2bdx^7+\frac{1}{2}a^2bcx^6+\frac{1}{5}a^3x^5e+\frac{1}{4}a^3dx^4+\frac{1}{3}a^3cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^3*x^14*e + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*x^11*e + 3
/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*x^8*e + 3/7*a^2*b*d*x^7 + 1/
2*a^2*b*c*x^6 + 1/5*a^3*x^5*e + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

Fricas [A]
time = 0.38, size = 115, normalized size = 1.05

$$\frac{1}{14}\,b^3ex^{14}+\frac{1}{13}\,b^3dx^{13}+\frac{1}{12}\,b^3cx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bex^8+\frac{3}{7}\,a^2bdx^7+\frac{1}{2}\,a^2bcx^6+\frac{1}{5}\,a^3ex^5+\frac{1}{4}\,a^3dx^4+\frac{1}{3}\,a^3cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*b^3*e*x^14 + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*e*x^11 + 3
/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*e*x^8 + 3/7*a^2*b*d*x^7 + 1/
2*a^2*b*c*x^6 + 1/5*a^3*e*x^5 + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

Sympy [A]
time = 0.01, size = 138, normalized size = 1.25

$$\frac{a^3cx^3}{3}+\frac{a^3dx^4}{4}+\frac{a^3ex^5}{5}+\frac{a^2bcx^6}{2}+\frac{3a^2bdx^7}{7}+\frac{3a^2bex^8}{8}+\frac{ab^2cx^9}{3}+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{b^3cx^{12}}{12}+\frac{b^3dx^{13}}{13}+\frac{b^3ex^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + a**2*b*c*x**6/2 + 3*a**2*b*
d*x**7/7 + 3*a**2*b*e*x**8/8 + a*b**2*c*x**9/3 + 3*a*b**2*d*x**10/10 + 3*a*
b**2*e*x**11/11 + b**3*c*x**12/12 + b**3*d*x**13/13 + b**3*e*x**14/14
```

Giac [A]
time = 0.73, size = 119, normalized size = 1.08

$$\frac{1}{14}\,b^3x^{14}e+\frac{1}{13}\,b^3dx^{13}+\frac{1}{12}\,b^3cx^{12}+\frac{3}{11}\,ab^2x^{11}e+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bx^8e+\frac{3}{7}\,a^2bdx^7+\frac{1}{2}\,a^2bcx^6+\frac{1}{5}\,a^3x^5e+\frac{1}{4}\,a^3dx^4+\frac{1}{3}\,a^3cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14*e + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*x^11*e + 3
/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*x^8*e + 3/7*a^2*b*d*x^7 + 1/
2*a^2*b*c*x^6 + 1/5*a^3*x^5*e + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

Mupad [B]
time = 0.08, size = 115, normalized size = 1.05

$$\frac{e\,a^3\,x^5}{5}+\frac{d\,a^3\,x^4}{4}+\frac{c\,a^3\,x^3}{3}+\frac{3\,e\,a^2\,b\,x^8}{8}+\frac{3\,d\,a^2\,b\,x^7}{7}+\frac{c\,a^2\,b\,x^6}{2}+\frac{3\,e\,a\,b^2\,x^{11}}{11}+\frac{3\,d\,a\,b^2\,x^{10}}{10}+\frac{c\,a\,b^2\,x^9}{3}+\frac{e\,b^3\,x^{14}}{14}+\frac{d\,b^3\,x^{13}}{13}+\frac{c\,b^3\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^3*(c + d*x + e*x^2),x)
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (b^3*c*x^12)/12 + (a^3*e*x^5)/5 + (b^3*d*x^
13)/13 + (b^3*e*x^14)/14 + (a^2*b*c*x^6)/2 + (a*b^2*c*x^9)/3 + (3*a^2*b*d*x
^7)/7 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^8)/8 + (3*a*b^2*e*x^11)/11
```

3.326 $\int x(c+dx+ex^2)\left(a+bx^3\right)^3\,dx$

Optimal. Leaf size=110

$$\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{3}{10}ab^2ex^{10}+\frac{1}{11}b^3cx^{11}+\frac{1}{13}b^3ex^{13}+\frac{d(a+bx^3)^4}{12b}$$

```
[Out] 1/2*a^3*c*x^2+1/4*a^3*e*x^4+3/5*a^2*b*c*x^5+3/7*a^2*b*e*x^7+3/8*a*b^2*c*x^8
+3/10*a*b^2*e*x^10+1/11*b^3*c*x^11+1/13*b^3*e*x^13+1/12*d*(b*x^3+a)^4/b
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1596, 1864}

$$\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{3}{10}ab^2ex^{10}+\frac{d(a+bx^3)^4}{12b}+\frac{1}{11}b^3cx^{11}+\frac{1}{13}b^3ex^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (3*a^2*b*e*x^7)/7 + (3*
a*b^2*c*x^8)/8 + (3*a*b^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*e*x^13)/13 +
(d*(a + b*x^3)^4)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx=\frac{d\left(a+bx^3\right)^4}{12b}+\int\left(a+bx^3\right)^3\left(-dx^2+x\left(c+dx+ex^2\right)\right)\,dx$$
$$=\frac{d\left(a+bx^3\right)^4}{12b}+\int\left(a^3cx+a^3ex^3+3a^2bcx^4+3a^2bex^6+3ab^2cx^7+3ab^2e\right.$$
$$=\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{3}{10}ab^2ex^{10}+\frac{1}{11}b^3c$$

Mathematica [A]
time = 0.00, size = 139, normalized size = 1.26

$$\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{1}{3}ab^2dx^9+\frac{3}{10}ab^2ex^{10}+\frac{1}{11}b^3cx^{11}+\frac{1}{12}b^3dx^{12}+\frac{1}{13}b^3ex^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (a^2*b*
d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*c*x^8)/8 + (a*b^2*d*x^9)/3 + (3*a*b
^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*d*x^12)/12 + (b^3*e*x^13)/13
```

Maple [A]
time = 0.38, size = 116, normalized size = 1.05

method	result
gosper	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}a\,b^2d\,x^9+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6+\frac{3}{5}a$
default	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}a\,b^2d\,x^9+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6+\frac{3}{5}a$
norman	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}a\,b^2d\,x^9+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6+\frac{3}{5}a$
risch	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}a\,b^2d\,x^9+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6+\frac{3}{5}a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^3*e*x^13+1/12*b^3*d*x^12+1/11*b^3*c*x^11+3/10*a*b^2*e*x^10+1/3*a*b^2
*d*x^9+3/8*a*b^2*c*x^8+3/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^5+1/4*
a^3*e*x^4+1/3*a^3*d*x^3+1/2*a^3*c*x^2
```

Maxima [A]
time = 0.29, size = 119, normalized size = 1.08

$$\frac{1}{13}b^3x^{13}e+\frac{1}{12}b^3dx^{12}+\frac{1}{11}b^3cx^{11}+\frac{3}{10}ab^2x^{10}e+\frac{1}{3}ab^2dx^9+\frac{3}{8}ab^2cx^8+\frac{3}{7}a^2bx^7e+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{4}a^3x^4e+\frac{1}{3}a^3dx^3+\frac{1}{2}a^3cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13*e + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*x^10*e + 1
/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*
a^2*b*c*x^5 + 1/4*a^3*x^4*e + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

Fricas [A]
time = 0.36, size = 115, normalized size = 1.05

$$\frac{1}{13}\,b^3ex^{13}+\frac{1}{12}\,b^3dx^{12}+\frac{1}{11}\,b^3cx^{11}+\frac{3}{10}\,ab^2ex^{10}+\frac{1}{3}\,ab^2dx^9+\frac{3}{8}\,ab^2cx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3ex^4+\frac{1}{3}\,a^3dx^3+\frac{1}{2}\,a^3cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/13*b^3*e*x^13 + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*e*x^10 + 1
/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*
a^2*b*c*x^5 + 1/4*a^3*e*x^4 + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

Sympy [A]
time = 0.01, size = 138, normalized size = 1.25

$$\frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\frac{3a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{3ab^2cx^8}{8}+\frac{ab^2dx^9}{3}+\frac{3ab^2ex^{10}}{10}+\frac{b^3cx^{11}}{11}+\frac{b^3dx^{12}}{12}+\frac{b^3ex^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + 3*a**2*b*c*x**5/5 + a**2*b*
d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a*b**2*c*x**8/8 + a*b**2*d*x**9/3 + 3*a*b*
*2*e*x**10/10 + b**3*c*x**11/11 + b**3*d*x**12/12 + b**3*e*x**13/13
```

Giac [A]
time = 0.72, size = 119, normalized size = 1.08

$$\frac{1}{13}\,b^3x^{13}e+\frac{1}{12}\,b^3dx^{12}+\frac{1}{11}\,b^3cx^{11}+\frac{3}{10}\,ab^2x^{10}e+\frac{1}{3}\,ab^2dx^9+\frac{3}{8}\,ab^2cx^8+\frac{3}{7}\,a^2bx^7e+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3x^4e+\frac{1}{3}\,a^3dx^3+\frac{1}{2}\,a^3cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13*e + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*x^10*e + 1
/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*
a^2*b*c*x^5 + 1/4*a^3*x^4*e + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

Mupad [B]
time = 0.07, size = 115, normalized size = 1.05

$$\frac{e\,a^3\,x^4}{4}+\frac{d\,a^3\,x^3}{3}+\frac{c\,a^3\,x^2}{2}+\frac{3\,e\,a^2\,b\,x^7}{7}+\frac{d\,a^2\,b\,x^6}{2}+\frac{3\,c\,a^2\,b\,x^5}{5}+\frac{3\,e\,a\,b^2\,x^{10}}{10}+\frac{d\,a\,b^2\,x^9}{3}+\frac{3\,c\,a\,b^2\,x^8}{8}+\frac{e\,b^3\,x^{13}}{13}+\frac{d\,b^3\,x^{12}}{12}+\frac{c\,b^3\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^3*(c + d*x + e*x^2),x)
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (b^3*c*x^11)/11 + (a^3*e*x^4)/4 + (b^3*d*x^
12)/12 + (b^3*e*x^13)/13 + (3*a^2*b*c*x^5)/5 + (3*a*b^2*c*x^8)/8 + (a^2*b*d
*x^6)/2 + (a*b^2*d*x^9)/3 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*e*x^10)/10
```

3.327 $\int (c+dx+ex^2)(a+bx^3)^3\,dx$

Optimal. Leaf size=105

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}+\frac{e(a+bx^3)^4}{12b}$$

```
[Out] a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x^5+3/7*a*b^2*c*x^7+3/8*a
*b^2*d*x^8+1/10*b^3*c*x^10+1/11*b^3*d*x^11+1/12*e*(b*x^3+a)^4/b
```

Rubi [A]
time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1596, 1864}

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{e(a+bx^3)^4}{12b}+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (e*(a +
b*x^3)^4)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (c+dx+ex^2)\left(a+bx^3\right)^3\,dx = \frac{e(a+bx^3)^4}{12b} + \int (c+dx)\left(a+bx^3\right)^3\,dx$$
$$= \frac{e(a+bx^3)^4}{12b} + \int \left(a^3c + a^3dx + 3a^2bcx^3 + 3a^2bdx^4 + 3ab^2cx^6 + 3ab^2dx^7 \right.$$
$$= a^3cx + \frac{1}{2}a^3dx^2 + \frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{10}b^3cx^{10} +$$

Mathematica [A]
time = 0.00, size = 134, normalized size = 1.28

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{1}{2}a^2bex^6+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{3}ab^2ex^9+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}+\frac{1}{12}b^3ex^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^
5)/5 + (a^2*b*e*x^6)/2 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (a*b^2*e*x
^9)/3 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (b^3*e*x^12)/12
```

Maple [A]
time = 0.38, size = 113, normalized size = 1.08

method	result
gosper	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}a\,b^2d\,x^8+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}a^2bd\,x^5+\frac{3}{4}a^2b$
default	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}a\,b^2d\,x^8+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}a^2bd\,x^5+\frac{3}{4}a^2b$
norman	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}a\,b^2d\,x^8+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}a^2bd\,x^5+\frac{3}{4}a^2b$
risch	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}a\,b^2d\,x^8+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}a^2bd\,x^5+\frac{3}{4}a^2b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^3*e*x^12+1/11*b^3*d*x^11+1/10*b^3*c*x^10+1/3*a*b^2*e*x^9+3/8*a*b^2*d
*x^8+3/7*a*b^2*c*x^7+1/2*a^2*b*e*x^6+3/5*a^2*b*d*x^5+3/4*a^2*b*c*x^4+1/3*a^
3*e*x^3+1/2*a^3*d*x^2+a^3*c*x
```

Maxima [A]
time = 0.28, size = 116, normalized size = 1.10

$$\frac{1}{12}b^3x^{12}e+\frac{1}{11}b^3dx^{11}+\frac{1}{10}b^3cx^{10}+\frac{1}{3}ab^2x^9e+\frac{3}{8}ab^2dx^8+\frac{3}{7}ab^2cx^7+\frac{1}{2}a^2bx^6e+\frac{3}{5}a^2bdx^5+\frac{3}{4}a^2bcx^4+\frac{1}{3}a^3x^3e+\frac{1}{2}a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^3*x^12*e + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*x^9*e + 3/8
*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*x^6*e + 3/5*a^2*b*d*x^5 + 3/4*a^
2*b*c*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

Fricas [A]
time = 0.37, size = 112, normalized size = 1.07

$$\frac{1}{12}\,b^3ex^{12}+\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{1}{3}\,ab^2ex^9+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{1}{2}\,a^2bex^6+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{3}\,a^3ex^3+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*b^3*e*x^12 + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*e*x^9 + 3/8
*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*e*x^6 + 3/5*a^2*b*d*x^5 + 3/4*a^
2*b*c*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Sympy [A]
time = 0.01, size = 134, normalized size = 1.28

$$a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{3a^2bcx^4}{4}+\frac{3a^2bdx^5}{5}+\frac{a^2bex^6}{2}+\frac{3ab^2cx^7}{7}+\frac{3ab^2dx^8}{8}+\frac{ab^2ex^9}{3}+\frac{b^3cx^{10}}{10}+\frac{b^3dx^{11}}{11}+\frac{b^3ex^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*x
**5/5 + a**2*b*e*x**6/2 + 3*a*b**2*c*x**7/7 + 3*a*b**2*d*x**8/8 + a*b**2*e*
x**9/3 + b**3*c*x**10/10 + b**3*d*x**11/11 + b**3*e*x**12/12
```

Giac [A]
time = 0.70, size = 116, normalized size = 1.10

$$\frac{1}{12}\,b^3x^{12}e+\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{1}{3}\,ab^2x^9e+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{1}{2}\,a^2bx^6e+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{3}\,a^3x^3e+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*x^12*e + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*x^9*e + 3/8
*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*x^6*e + 3/5*a^2*b*d*x^5 + 3/4*a^
2*b*c*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B]
time = 0.07, size = 112, normalized size = 1.07

$$\frac{e\,a^3\,x^3}{3}+\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{e\,a^2\,b\,x^6}{2}+\frac{3\,d\,a^2\,b\,x^5}{5}+\frac{3\,c\,a^2\,b\,x^4}{4}+\frac{e\,a\,b^2\,x^9}{3}+\frac{3\,d\,a\,b^2\,x^8}{8}+\frac{3\,c\,a\,b^2\,x^7}{7}+\frac{e\,b^3\,x^{12}}{12}+\frac{d\,b^3\,x^{11}}{11}+\frac{c\,b^3\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3*(c + d*x + e*x^2),x)
```

```
[Out] (a^3*d*x^2)/2 + (b^3*c*x^10)/10 + (a^3*e*x^3)/3 + (b^3*d*x^11)/11 + (b^3*e*
x^12)/12 + a^3*c*x + (3*a^2*b*c*x^4)/4 + (3*a*b^2*c*x^7)/7 + (3*a^2*b*d*x^5
)/5 + (3*a*b^2*d*x^8)/8 + (a^2*b*e*x^6)/2 + (a*b^2*e*x^9)/3
```

3.328 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x} dx$

Optimal. Leaf size=127

$$a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^3dx^{10}+\frac{1}{11}b^3ex^{11}+a^3$$

```
[Out] a^3*d*x+1/2*a^3*e*x^2+a^2*b*c*x^3+3/4*a^2*b*d*x^4+3/5*a^2*b*e*x^5+1/2*a*b^2
*c*x^6+3/7*a*b^2*d*x^7+3/8*a*b^2*e*x^8+1/9*b^3*c*x^9+1/10*b^3*d*x^10+1/11*b
^3*e*x^11+a^3*c*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1642}

$$a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^3dx^{10}+\frac{1}{11}b^3ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5)
/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9)/
9 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx &= \int \left(a^3d+\frac{a^3c}{x}+a^3ex+3a^2bcx^2+3a^2bdx^3+3a^2bex^4+3ab^2cx^5+3ab^2\right.\\ &= a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 127, normalized size = 1.00

$$a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^3dx^{10}+\frac{1}{11}b^3ex^{11}+a^3c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5)
/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9)/
9 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*Log[x]
```

Maple [A]
time = 0.35, size = 110, normalized size = 0.87

method	result
default	$a^3dx+\frac{a^3e\,x^2}{2}+a^2bc\,x^3+\frac{3a^2bd\,x^4}{4}+\frac{3a^2be\,x^5}{5}+\frac{a\,b^2c\,x^6}{2}+\frac{3a\,b^2d\,x^7}{7}+\frac{3a\,b^2e\,x^8}{8}+\frac{b^3c\,x^9}{9}+\frac{b^3d\,x^{10}}{10}+\frac{b^3e\,x^{11}}{11}+$
norman	$a^3dx+\frac{a^3e\,x^2}{2}+a^2bc\,x^3+\frac{3a^2bd\,x^4}{4}+\frac{3a^2be\,x^5}{5}+\frac{a\,b^2c\,x^6}{2}+\frac{3a\,b^2d\,x^7}{7}+\frac{3a\,b^2e\,x^8}{8}+\frac{b^3c\,x^9}{9}+\frac{b^3d\,x^{10}}{10}+\frac{b^3e\,x^{11}}{11}+$
risch	$a^3dx+\frac{a^3e\,x^2}{2}+a^2bc\,x^3+\frac{3a^2bd\,x^4}{4}+\frac{3a^2be\,x^5}{5}+\frac{a\,b^2c\,x^6}{2}+\frac{3a\,b^2d\,x^7}{7}+\frac{3a\,b^2e\,x^8}{8}+\frac{b^3c\,x^9}{9}+\frac{b^3d\,x^{10}}{10}+\frac{b^3e\,x^{11}}{11}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*d*x+1/2*a^3*e*x^2+a^2*b*c*x^3+3/4*a^2*b*d*x^4+3/5*a^2*b*e*x^5+1/2*a*b^2
*c*x^6+3/7*a*b^2*d*x^7+3/8*a*b^2*e*x^8+1/9*b^3*c*x^9+1/10*b^3*d*x^10+1/11*b
^3*e*x^11+a^3*c*ln(x)
```

Maxima [A]
time = 0.29, size = 113, normalized size = 0.89

$$\frac{1}{11}\,b^3x^{11}e+\frac{1}{10}\,b^3dx^{10}+\frac{1}{9}\,b^3cx^9+\frac{3}{8}\,ab^2x^8e+\frac{3}{7}\,ab^2dx^7+\frac{1}{2}\,ab^2cx^6+\frac{3}{5}\,a^2bx^5e+\frac{3}{4}\,a^2bdx^4+a^2bcx^3+\frac{1}{2}\,a^3x^2e+a^3dx+a^3c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*x^11*e + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*x^8*e + 3/7*a
*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*x^5*e + 3/4*a^2*b*d*x^4 + a^2*b*c*
x^3 + 1/2*a^3*x^2*e + a^3*d*x + a^3*c*log(x)
```

Fricas [A]
time = 0.37, size = 109, normalized size = 0.86

$$\frac{1}{11}\,b^3ex^{11}+\frac{1}{10}\,b^3dx^{10}+\frac{1}{9}\,b^3cx^9+\frac{3}{8}\,ab^2ex^8+\frac{3}{7}\,ab^2dx^7+\frac{1}{2}\,ab^2cx^6+\frac{3}{5}\,a^2bex^5+\frac{3}{4}\,a^2bdx^4+a^2bcx^3+\frac{1}{2}\,a^3ex^2+a^3dx+a^3c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/11*b^3*e*x^11 + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*e*x^8 + 3/7*a
*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*e*x^5 + 3/4*a^2*b*d*x^4 + a^2*b*c*
x^3 + 1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(x)
```

Sympy [A]
time = 0.07, size = 131, normalized size = 1.03

$$a^3c\log{(x)} + a^3dx + \frac{a^3ex^2}{2} + a^2bcx^3 + \frac{3a^2bdx^4}{4} + \frac{3a^2bex^5}{5} + \frac{ab^2cx^6}{2} + \frac{3ab^2dx^7}{7} + \frac{3ab^2ex^8}{8} + \frac{b^3cx^9}{9} + \frac{b^3dx^{10}}{10} + \frac{b^3ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x,x)
```

```
[Out] a**3*c*log(x) + a**3*d*x + a**3*e*x**2/2 + a**2*b*c*x**3 + 3*a**2*b*d*x**4/
4 + 3*a**2*b*e*x**5/5 + a*b**2*c*x**6/2 + 3*a*b**2*d*x**7/7 + 3*a*b**2*e*x*
*8/8 + b**3*c*x**9/9 + b**3*d*x**10/10 + b**3*e*x**11/11
```

Giac [A]
time = 0.54, size = 114, normalized size = 0.90

$$\frac{1}{11}b^3x^{11}e + \frac{1}{10}b^3dx^{10} + \frac{1}{9}b^3cx^9 + \frac{3}{8}ab^2x^8e + \frac{3}{7}ab^2dx^7 + \frac{1}{2}ab^2cx^6 + \frac{3}{5}a^2bx^5e + \frac{3}{4}a^2bdx^4 + a^2bcx^3 + \frac{1}{2}a^3x^2e + a^3dx + a^3c\log{(|x|)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11*e + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*x^8*e + 3/7*a
*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*x^5*e + 3/4*a^2*b*d*x^4 + a^2*b*c*
x^3 + 1/2*a^3*x^2*e + a^3*d*x + a^3*c*log(abs(x))
```

Mupad [B]
time = 0.08, size = 109, normalized size = 0.86

$$\frac{b^3\,c\,x^9}{9} + \frac{a^3\,e\,x^2}{2} + \frac{b^3\,d\,x^{10}}{10} + \frac{b^3\,e\,x^{11}}{11} + a^3\,c\,\ln{(x)} + a^3\,d\,x + a^2\,b\,c\,x^3 + \frac{a\,b^2\,c\,x^6}{2} + \frac{3\,a^2\,b\,d\,x^4}{4} + \frac{3\,a\,b^2\,d\,x^7}{7} + \frac{3\,a^2\,b\,e\,x^5}{5} + \frac{3\,a\,b^2\,e\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2))/x,x)
```

```
[Out] (b^3*c*x^9)/9 + (a^3*e*x^2)/2 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*l
og(x) + a^3*d*x + a^2*b*c*x^3 + (a*b^2*c*x^6)/2 + (3*a^2*b*d*x^4)/4 + (3*a*
b^2*d*x^7)/7 + (3*a^2*b*e*x^5)/5 + (3*a*b^2*e*x^8)/8
```

3.329 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2} dx$

Optimal. Leaf size=125

$$-\frac{a^3c}{x}+a^3ex+\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab^2ex^7+\frac{1}{8}b^3cx^8+\frac{1}{9}b^3dx^9+\frac{1}{10}b^3ex^{10}+a^3d\log($$

```
[Out] -a^3*c/x+a^3*e*x+3/2*a^2*b*c*x^2+a^2*b*d*x^3+3/4*a^2*b*e*x^4+3/5*a*b^2*c*x^
5+1/2*a*b^2*d*x^6+3/7*a*b^2*e*x^7+1/8*b^3*c*x^8+1/9*b^3*d*x^9+1/10*b^3*e*x^
10+a^3*d*ln(x)
```

Rubi [A]
time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1642}

$$-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab^2ex^7+\frac{1}{8}b^3cx^8+\frac{1}{9}b^3dx^9+\frac{1}{10}b^3ex^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2,x]
```

```
[Out] -((a^3*c)/x) + a^3*e*x + (3*a^2*b*c*x^2)/2 + a^2*b*d*x^3 + (3*a^2*b*e*x^4)/
4 + (3*a*b^2*c*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b^2*e*x^7)/7 + (b^3*c*x^8)/8
 + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2} dx = \int \left(a^3e+\frac{a^3c}{x^2}+\frac{a^3d}{x}+3a^2bcx+3a^2bdx^2+3a^2bex^3+3ab^2cx^4+3ab^2dx^5\right.$$
$$= -\frac{a^3c}{x}+a^3ex+\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab$$

Mathematica [A]
time = 0.01, size = 125, normalized size = 1.00

$$-\frac{a^3c}{x}+a^3ex+\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab^2ex^7+\frac{1}{8}b^3cx^8+\frac{1}{9}b^3dx^9+\frac{1}{10}b^3ex^{10}+a^3d\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2,x]
```

```
[Out] -((a^3*c)/x) + a^3*e*x + (3*a^2*b*c*x^2)/2 + a^2*b*d*x^3 + (3*a^2*b*e*x^4)/
4 + (3*a*b^2*c*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b^2*e*x^7)/7 + (b^3*c*x^8)/8
 + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*Log[x]
```

Maple [A]
time = 0.34, size = 110, normalized size = 0.88

method	result
default	$-\frac{a^3 c}{x}+a^3 e x+\frac{3 a^2 b c x^2}{2}+a^2 b d x^3+\frac{3 a^2 b e x^4}{4}+\frac{3 a b^2 c x^5}{5}+\frac{a b^2 d x^6}{2}+\frac{3 a b^2 e x^7}{7}+\frac{b^3 c x^8}{8}+\frac{b^3 d x^9}{9}+\frac{b^3 e x^{10}}{10}$
risch	$-\frac{a^3 c}{x}+a^3 e x+\frac{3 a^2 b c x^2}{2}+a^2 b d x^3+\frac{3 a^2 b e x^4}{4}+\frac{3 a b^2 c x^5}{5}+\frac{a b^2 d x^6}{2}+\frac{3 a b^2 e x^7}{7}+\frac{b^3 c x^8}{8}+\frac{b^3 d x^9}{9}+\frac{b^3 e x^{10}}{10}$
norman	$\frac{a^3 e x^2+a^2 b d x^4-c a^3+\frac{1}{8} b^3 c x^9+\frac{1}{9} b^3 d x^{10}+\frac{1}{10} b^3 e x^{11}+\frac{3}{5} a b^2 c x^6+\frac{1}{2} a b^2 d x^7+\frac{3}{7} a b^2 e x^8+\frac{3}{2} a^2 b c x^3+\frac{3}{4} a^2 b e x^5}{x}+a^3 d \ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*c/x+a^3*e*x+3/2*a^2*b*c*x^2+a^2*b*d*x^3+3/4*a^2*b*e*x^4+3/5*a*b^2*c*x^
5+1/2*a*b^2*d*x^6+3/7*a*b^2*e*x^7+1/8*b^3*c*x^8+1/9*b^3*d*x^9+1/10*b^3*e*x^
10+a^3*d*ln(x)
```

Maxima [A]
time = 0.28, size = 113, normalized size = 0.90

$$\frac{1}{10} b^3 x^{10} e+\frac{1}{9} b^3 d x^9+\frac{1}{8} b^3 c x^8+\frac{3}{7} a b^2 x^7 e+\frac{1}{2} a b^2 d x^6+\frac{3}{5} a b^2 c x^5+\frac{3}{4} a^2 b x^4 e+a^2 b d x^3+\frac{3}{2} a^2 b c x^2+a^3 x e+a^3 d \log(x)-\frac{a^3 c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10*e + 1/9*b^3*d*x^9 + 1/8*b^3*c*x^8 + 3/7*a*b^2*x^7*e + 1/2*a*b
^2*d*x^6 + 3/5*a*b^2*c*x^5 + 3/4*a^2*b*x^4*e + a^2*b*d*x^3 + 3/2*a^2*b*c*x^
2 + a^3*x*e + a^3*d*log(x) - a^3*c/x
```

Fricas [A]
time = 0.41, size = 117, normalized size = 0.94

$$\frac{252 b^3 e x^{11}+280 b^3 d x^{10}+315 b^3 c x^9+1080 a b^2 e x^8+1260 a b^2 d x^7+1512 a b^2 c x^6+1890 a^2 b e x^5+2520 a^2 b d x^4+3780 a^2 b c x^3+2520 a^3 e x^2+2520 a^3 d x \log(x)-2520 a^3 c}{2520 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/2520*(252*b^3*e*x^11 + 280*b^3*d*x^10 + 315*b^3*c*x^9 + 1080*a*b^2*e*x^8
+ 1260*a*b^2*d*x^7 + 1512*a*b^2*c*x^6 + 1890*a^2*b*e*x^5 + 2520*a^2*b*d*x^4
 + 3780*a^2*b*c*x^3 + 2520*a^3*e*x^2 + 2520*a^3*d*x*log(x) - 2520*a^3*c)/x
```

Sympy [A]
time = 0.08, size = 128, normalized size = 1.02

$$-\frac{a^3c}{x}+a^3d\log{(x)}+a^3ex+\frac{3a^2bcx^2}{2}+a^2bdx^3+\frac{3a^2bex^4}{4}+\frac{3ab^2cx^5}{5}+\frac{ab^2dx^6}{2}+\frac{3ab^2ex^7}{7}+\frac{b^3cx^8}{8}+\frac{b^3dx^9}{9}+\frac{b^3ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x**2,x)
```

```
[Out] -a**3*c/x + a**3*d*log(x) + a**3*e*x + 3*a**2*b*c*x**2/2 + a**2*b*d*x**3 +
3*a**2*b*e*x**4/4 + 3*a*b**2*c*x**5/5 + a*b**2*d*x**6/2 + 3*a*b**2*e*x**7/7
 + b**3*c*x**8/8 + b**3*d*x**9/9 + b**3*e*x**10/10
```

Giac [A]
time = 0.66, size = 114, normalized size = 0.91

$$\frac{1}{10}b^3x^{10}e+\frac{1}{9}b^3dx^9+\frac{1}{8}b^3cx^8+\frac{3}{7}ab^2x^7e+\frac{1}{2}ab^2dx^6+\frac{3}{5}ab^2cx^5+\frac{3}{4}a^2bx^4e+a^2bdx^3+\frac{3}{2}a^2bcx^2+a^3xe+a^3d\log(|x|)-\frac{a^3c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/10*b^3*x^10*e + 1/9*b^3*d*x^9 + 1/8*b^3*c*x^8 + 3/7*a*b^2*x^7*e + 1/2*a*b
^2*d*x^6 + 3/5*a*b^2*c*x^5 + 3/4*a^2*b*x^4*e + a^2*b*d*x^3 + 3/2*a^2*b*c*x^
2 + a^3*x*e + a^3*d*log(abs(x)) - a^3*c/x
```

Mupad [B]
time = 0.08, size = 109, normalized size = 0.87

$$\frac{b^3\,c\,x^8}{8}-\frac{a^3\,c}{x}+\frac{b^3\,d\,x^9}{9}+\frac{b^3\,e\,x^{10}}{10}+a^3\,d\,\ln{(x)}+a^3\,e\,x+\frac{3\,a^2\,b\,c\,x^2}{2}+\frac{3\,a\,b^2\,c\,x^5}{5}+a^2\,b\,d\,x^3+\frac{a\,b^2\,d\,x^6}{2}+\frac{3\,a^2\,b\,e\,x^4}{4}+\frac{3\,a\,b^2\,e\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2))/x^2,x)
```

```
[Out] (b^3*c*x^8)/8 - (a^3*c)/x + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*log(x)
+ a^3*e*x + (3*a^2*b*c*x^2)/2 + (3*a*b^2*c*x^5)/5 + a^2*b*d*x^3 + (a*b^2*d*
x^6)/2 + (3*a^2*b*e*x^4)/4 + (3*a*b^2*e*x^7)/7
```

3.330 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3}\,dx$

Optimal. Leaf size=126

$$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^3cx^7+\frac{1}{8}b^3dx^8+\frac{1}{9}b^3ex^9+a^3e\log(x)$$

```
[Out] -1/2*a^3*c/x^2-a^3*d/x+3*a^2*b*c*x+3/2*a^2*b*d*x^2+a^2*b*e*x^3+3/4*a*b^2*c*
x^4+3/5*a*b^2*d*x^5+1/2*a*b^2*e*x^6+1/7*b^3*c*x^7+1/8*b^3*d*x^8+1/9*b^3*e*x
^9+a^3*e*ln(x)
```

Rubi [A]
time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1642}

$$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^3cx^7+\frac{1}{8}b^3dx^8+\frac{1}{9}b^3ex^9$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^3,x]
```

```
[Out] -1/2*(a^3*c)/x^2 - (a^3*d)/x + 3*a^2*b*c*x + (3*a^2*b*d*x^2)/2 + a^2*b*e*x^
3 + (3*a*b^2*c*x^4)/4 + (3*a*b^2*d*x^5)/5 + (a*b^2*e*x^6)/2 + (b^3*c*x^7)/7
 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 + a^3*e*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3}\,dx = \int \left(3a^2bc+\frac{a^3c}{x^3}+\frac{a^3d}{x^2}+\frac{a^3e}{x}+3a^2bdx+3a^2bex^2+3ab^2cx^3+3ab^2dx^4\right.$$
$$= -\frac{a^3c}{2x^2}-\frac{a^3d}{x}+3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab$$

Mathematica [A]
time = 0.01, size = 126, normalized size = 1.00

$$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^3cx^7+\frac{1}{8}b^3dx^8+\frac{1}{9}b^3ex^9+a^3e\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^3,x]
```

```
[Out] -1/2*(a^3*c)/x^2 - (a^3*d)/x + 3*a^2*b*c*x + (3*a^2*b*d*x^2)/2 + a^2*b*e*x^
3 + (3*a*b^2*c*x^4)/4 + (3*a*b^2*d*x^5)/5 + (a*b^2*e*x^6)/2 + (b^3*c*x^7)/7
 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 + a^3*e*Log[x]
```

Maple [A]
time = 0.39, size = 111, normalized size = 0.88

method	result
default	$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+3a^2bcx+\frac{3a^2bd\,x^2}{2}+a^2be\,x^3+\frac{3a\,b^2c\,x^4}{4}+\frac{3a\,b^2d\,x^5}{5}+\frac{a\,b^2e\,x^6}{2}+\frac{b^3c\,x^7}{7}+\frac{b^3d\,x^8}{8}+\frac{b^3e\,x^9}{9}+a^3$
risch	$\frac{b^3e\,x^9}{9}+\frac{b^3d\,x^8}{8}+\frac{b^3c\,x^7}{7}+\frac{a\,b^2e\,x^6}{2}+\frac{3a\,b^2d\,x^5}{5}+\frac{3a\,b^2c\,x^4}{4}+a^2be\,x^3+\frac{3a^2bd\,x^2}{2}+3a^2bcx+\frac{-a^3dx-\frac{1}{2}c\,a^3}{x^2}+a^3$
norman	$\frac{a^2be\,x^5-\frac{1}{2}c\,a^3-a^3dx+\frac{1}{7}b^3c\,x^9+\frac{1}{8}b^3d\,x^{10}+\frac{1}{9}b^3e\,x^{11}+\frac{3}{4}a\,b^2c\,x^6+\frac{3}{5}a\,b^2d\,x^7+\frac{1}{2}a\,b^2e\,x^8+3a^2bc\,x^3+\frac{3}{2}a^2bd\,x^4}{x^2}+a^3e\ln\left(x\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^3*c/x^2-a^3*d/x+3*a^2*b*c*x+3/2*a^2*b*d*x^2+a^2*b*e*x^3+3/4*a*b^2*c*
x^4+3/5*a*b^2*d*x^5+1/2*a*b^2*e*x^6+1/7*b^3*c*x^7+1/8*b^3*d*x^8+1/9*b^3*e*x
^9+a^3*e*ln(x)
```

Maxima [A]
time = 0.27, size = 114, normalized size = 0.90

$$\frac{1}{9}\,b^3x^9e+\frac{1}{8}\,b^3dx^8+\frac{1}{7}\,b^3cx^7+\frac{1}{2}\,ab^2x^6e+\frac{3}{5}\,ab^2dx^5+\frac{3}{4}\,ab^2cx^4+a^2bx^3e+\frac{3}{2}\,a^2bdx^2+3\,a^2bcx+a^3e\log\left(x\right)-\frac{2\,a^3dx+a^3c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^3*x^9*e + 1/8*b^3*d*x^8 + 1/7*b^3*c*x^7 + 1/2*a*b^2*x^6*e + 3/5*a*b^2
*d*x^5 + 3/4*a*b^2*c*x^4 + a^2*b*x^3*e + 3/2*a^2*b*d*x^2 + 3*a^2*b*c*x + a^
3*e*log(x) - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

Fricas [A]
time = 0.39, size = 117, normalized size = 0.93

$$\frac{280\,b^3ex^{11}+315\,b^3dx^{10}+360\,b^3cx^9+1260\,ab^2ex^8+1512\,ab^2dx^7+1890\,ab^2cx^6+2520\,a^2bex^5+3780\,a^2bdx^4+7560\,a^2bcx^3+2520\,a^3ex^2\log\left(x\right)-2520\,a^3dx-1260\,a^3c}{2520\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/2520*(280*b^3*e*x^11 + 315*b^3*d*x^10 + 360*b^3*c*x^9 + 1260*a*b^2*e*x^8
+ 1512*a*b^2*d*x^7 + 1890*a*b^2*c*x^6 + 2520*a^2*b*e*x^5 + 3780*a^2*b*d*x^4
```

```
 + 7560*a^2*b*c*x^3 + 2520*a^3*e*x^2*log(x) - 2520*a^3*d*x - 1260*a^3*c)/x^
2
```

Sympy [A]
time = 0.14, size = 131, normalized size = 1.04

$$a^3e\log(x) + 3a^2bcx + \frac{3a^2bdx^2}{2} + a^2bex^3 + \frac{3ab^2cx^4}{4} + \frac{3ab^2dx^5}{5} + \frac{ab^2ex^6}{2} + \frac{b^3cx^7}{7} + \frac{b^3dx^8}{8} + \frac{b^3ex^9}{9} + \frac{-a^3c - 2a^3dx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x**3,x)
```

```
[Out] a**3*e*log(x) + 3*a**2*b*c*x + 3*a**2*b*d*x**2/2 + a**2*b*e*x**3 + 3*a*b**2
*c*x**4/4 + 3*a*b**2*d*x**5/5 + a*b**2*e*x**6/2 + b**3*c*x**7/7 + b**3*d*x*
*8/8 + b**3*e*x**9/9 + (-a**3*c - 2*a**3*d*x)/(2*x**2)
```

Giac [A]
time = 0.79, size = 115, normalized size = 0.91

$$\frac{1}{9}b^3x^9e + \frac{1}{8}b^3dx^8 + \frac{1}{7}b^3cx^7 + \frac{1}{2}ab^2x^6e + \frac{3}{5}ab^2dx^5 + \frac{3}{4}ab^2cx^4 + a^2bx^3e + \frac{3}{2}a^2bdx^2 + 3a^2bcx + a^3e\log(|x|) - \frac{2a^3dx + a^3c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*e + 1/8*b^3*d*x^8 + 1/7*b^3*c*x^7 + 1/2*a*b^2*x^6*e + 3/5*a*b^2
*d*x^5 + 3/4*a*b^2*c*x^4 + a^2*b*x^3*e + 3/2*a^2*b*d*x^2 + 3*a^2*b*c*x + a^
3*e*log(abs(x)) - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

Mupad [B]
time = 4.90, size = 110, normalized size = 0.87

$$\frac{b^3cx^7}{7} - \frac{\frac{a^3c}{2} + a^3dx}{x^2} + \frac{b^3dx^8}{8} + \frac{b^3ex^9}{9} + a^3e\ln(x) + 3a^2bcx + \frac{3ab^2cx^4}{4} + \frac{3a^2bdx^2}{2} + \frac{3ab^2dx^5}{5} + a^2bex^3 + \frac{ab^2ex^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2))/x^3,x)
```

```
[Out] (b^3*c*x^7)/7 - ((a^3*c)/2 + a^3*d*x)/x^2 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 +
 a^3*e*log(x) + 3*a^2*b*c*x + (3*a*b^2*c*x^4)/4 + (3*a^2*b*d*x^2)/2 + (3*a*
b^2*d*x^5)/5 + a^2*b*e*x^3 + (a*b^2*e*x^6)/2
```

3.331 $\int x^2(c+dx+ex^2)(a+bx^3)^4\,dx$

Optimal. Leaf size=138

$$\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3dx^{13}+\frac{2}{7}ab^3ex^{14}+\frac{1}{16}b^4dx^{16}+\frac{1}{17}b^4ex^{17}+$$

```
[Out] 1/4*a^4*d*x^4+1/5*a^4*e*x^5+4/7*a^3*b*d*x^7+1/2*a^3*b*e*x^8+3/5*a^2*b^2*d*x
^10+6/11*a^2*b^2*e*x^11+4/13*a*b^3*d*x^13+2/7*a*b^3*e*x^14+1/16*b^4*d*x^16+
1/17*b^4*e*x^17+1/15*c*(b*x^3+a)^5/b
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1596, 1864}

$$\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3dx^{13}+\frac{2}{7}ab^3ex^{14}+\frac{c(a+bx^3)^5}{15b}+\frac{1}{16}b^4dx^{16}+\frac{1}{17}b^4ex^{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (4*a^3*b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (3*a^
2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (4*a*b^3*d*x^13)/13 + (2*a*b^3*e*
x^14)/7 + (b^4*d*x^16)/16 + (b^4*e*x^17)/17 + (c*(a + b*x^3)^5)/(15*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^2(c+dx+ex^2)\,(a+bx^3)^4\,dx = \frac{c(a+bx^3)^5}{15b} + \int (a+bx^3)^4\,(-cx^2+x^2(c+dx+ex^2))\,dx$$
$$= \frac{c(a+bx^3)^5}{15b} + \int (a^4dx^3 + a^4ex^4 + 4a^3bdx^6 + 4a^3bex^7 + 6a^2b^2dx^9 + 6$$
$$= \frac{1}{4}a^4dx^4 + \frac{1}{5}a^4ex^5 + \frac{4}{7}a^3bdx^7 + \frac{1}{2}a^3bex^8 + \frac{3}{5}a^2b^2dx^{10} + \frac{6}{11}a^2b^2ex^{11} +$$

Mathematica [A]
time = 0.00, size = 181, normalized size = 1.31

$$\frac{1}{3}a^4cx^3+\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{2}{3}a^3bcx^6+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{1}{3}ab^3cx^{12}+\frac{4}{13}ab^3dx^{13}+\frac{2}{7}ab^3ex^{14}+\frac{1}{15}b^4cx^{15}+\frac{1}{16}b^4dx^{16}+\frac{1}{17}b^4ex^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^3)/3 + (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (2*a^3*b*c*x^6)/3 + (4*a^3*
b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 +
 (6*a^2*b^2*e*x^11)/11 + (a*b^3*c*x^12)/3 + (4*a*b^3*d*x^13)/13 + (2*a*b^3*
e*x^14)/7 + (b^4*c*x^15)/15 + (b^4*d*x^16)/16 + (b^4*e*x^17)/17
```

Maple [A]
time = 0.40, size = 152, normalized size = 1.10

method	result
gosper	$\frac{1}{3}c\,a^4x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}c\,a^3b\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}a^2b$
default	$\frac{1}{3}c\,a^4x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}c\,a^3b\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}a^2b$
norman	$\frac{1}{3}c\,a^4x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}c\,a^3b\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}a^2b$
risch	$\frac{1}{3}c\,a^4x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}c\,a^3b\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}a^2b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*c*a^4*x^3+1/4*a^4*d*x^4+1/5*a^4*e*x^5+2/3*c*a^3*b*x^6+4/7*a^3*b*d*x^7+1
/2*a^3*b*e*x^8+2/3*a^2*b^2*c*x^9+3/5*a^2*b^2*d*x^10+6/11*a^2*b^2*e*x^11+1/3
*a*b^3*c*x^12+4/13*a*b^3*d*x^13+2/7*a*b^3*e*x^14+1/15*c*b^4*x^15+1/16*b^4*d
*x^16+1/17*b^4*e*x^17
```

Maxima [A]
time = 0.26, size = 156, normalized size = 1.13

$$\frac{1}{17}b^4x^{17}e+\frac{1}{16}b^4dx^{16}+\frac{1}{15}b^4cx^{15}+\frac{2}{7}ab^3x^{14}e+\frac{4}{13}ab^3dx^{13}+\frac{1}{3}ab^3cx^{12}+\frac{6}{11}a^2b^2x^{11}e+\frac{3}{5}a^2b^2dx^{10}+\frac{2}{3}a^2b^2cx^9+\frac{1}{2}a^3bx^8e+\frac{4}{7}a^3bdx^7+\frac{2}{3}a^3bcx^6+\frac{1}{5}a^4x^5e+\frac{1}{4}a^4dx^4+\frac{1}{3}a^4cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/17*b^4*x^17*e + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*x^14*e + 4/
13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*x^11*e + 3/5*a^2*b^2*d*x^
10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*x^8*e + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^
6 + 1/5*a^4*x^5*e + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3
```

Fricas [A]
time = 0.36, size = 151, normalized size = 1.09

$$\frac{1}{17}b^4ex^{17}+\frac{1}{16}b^4dx^{16}+\frac{1}{15}b^4cx^{15}+\frac{2}{7}ab^3ex^{14}+\frac{4}{13}ab^3dx^{13}+\frac{1}{3}ab^3cx^{12}+\frac{6}{11}a^2b^2ex^{11}+\frac{3}{5}a^2b^2dx^{10}+\frac{2}{3}a^2b^2cx^9+\frac{1}{2}a^3bex^8+\frac{4}{7}a^3bdx^7+\frac{2}{3}a^3bcx^6+\frac{1}{5}a^4ex^5+\frac{1}{4}a^4dx^4+\frac{1}{3}a^4cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/17*b^4*e*x^17 + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*e*x^14 + 4/
13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^
10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*e*x^8 + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^
6 + 1/5*a^4*e*x^5 + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3
```

Sympy [A]
time = 0.02, size = 184, normalized size = 1.33

$$\frac{a^4cx^3}{3}+\frac{a^4dx^4}{4}+\frac{a^4ex^5}{5}+\frac{2a^3bcx^6}{3}+\frac{4a^3bdx^7}{7}+\frac{a^3bex^8}{2}+\frac{2a^2b^2cx^9}{3}+\frac{3a^2b^2dx^{10}}{5}+\frac{6a^2b^2ex^{11}}{11}+\frac{ab^3cx^{12}}{3}+\frac{4ab^3dx^{13}}{13}+\frac{2ab^3ex^{14}}{7}+\frac{b^4cx^{15}}{15}+\frac{b^4dx^{16}}{16}+\frac{b^4ex^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x**3/3 + a**4*d*x**4/4 + a**4*e*x**5/5 + 2*a**3*b*c*x**6/3 + 4*a**3*
b*d*x**7/7 + a**3*b*e*x**8/2 + 2*a**2*b**2*c*x**9/3 + 3*a**2*b**2*d*x**10/5
 + 6*a**2*b**2*e*x**11/11 + a*b**3*c*x**12/3 + 4*a*b**3*d*x**13/13 + 2*a*b*
*3*e*x**14/7 + b**4*c*x**15/15 + b**4*d*x**16/16 + b**4*e*x**17/17
```

Giac [A]
time = 0.74, size = 156, normalized size = 1.13

$$\frac{1}{17}b^4x^{17}e+\frac{1}{16}b^4dx^{16}+\frac{1}{15}b^4cx^{15}+\frac{2}{7}ab^3x^{14}e+\frac{4}{13}ab^3dx^{13}+\frac{1}{3}ab^3cx^{12}+\frac{6}{11}a^2b^2x^{11}e+\frac{3}{5}a^2b^2dx^{10}+\frac{2}{3}a^2b^2cx^9+\frac{1}{2}a^3bx^8e+\frac{4}{7}a^3bdx^7+\frac{2}{3}a^3bcx^6+\frac{1}{5}a^4x^5e+\frac{1}{4}a^4dx^4+\frac{1}{3}a^4cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 1/17*b^4*x^17*e + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*x^14*e + 4/
13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*x^11*e + 3/5*a^2*b^2*d*x^
10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*x^8*e + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^
6 + 1/5*a^4*x^5*e + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3
```

Mupad [B]
time = 5.07, size = 151, normalized size = 1.09

$$\frac{e a^4 x^5}{5}+\frac{d a^4 x^4}{4}+\frac{c a^4 x^3}{3}+\frac{e a^3 b x^8}{2}+\frac{4 d a^3 b x^7}{7}+\frac{2 c a^3 b x^6}{3}+\frac{6 e a^2 b^2 x^{11}}{11}+\frac{3 d a^2 b^2 x^{10}}{5}+\frac{2 c a^2 b^2 x^9}{3}+\frac{2 e a b^3 x^{14}}{7}+\frac{4 d a b^3 x^{13}}{13}+\frac{c a b^3 x^{12}}{3}+\frac{e b^4 x^{17}}{17}+\frac{d b^4 x^{16}}{16}+\frac{c b^4 x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^4*(c + d*x + e*x^2),x)
```

```
[Out] (a^4*c*x^3)/3 + (a^4*d*x^4)/4 + (b^4*c*x^15)/15 + (a^4*e*x^5)/5 + (b^4*d*x^
16)/16 + (b^4*e*x^17)/17 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*
a^2*b^2*e*x^11)/11 + (2*a^3*b*c*x^6)/3 + (a*b^3*c*x^12)/3 + (4*a^3*b*d*x^7)
/7 + (4*a*b^3*d*x^13)/13 + (a^3*b*e*x^8)/2 + (2*a*b^3*e*x^14)/7
```

3.332 $\int x(c+dx+ex^2)\left(a+bx^3\right)^4\,dx$

Optimal. Leaf size=138

$$\frac{1}{2}a^4cx^2+\frac{1}{4}a^4ex^4+\frac{4}{5}a^3bcx^5+\frac{4}{7}a^3bex^7+\frac{3}{4}a^2b^2cx^8+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{11}ab^3cx^{11}+\frac{4}{13}ab^3ex^{13}+\frac{1}{14}b^4cx^{14}+\frac{1}{16}b^4ex^{16}+\frac{d(}{}$$

```
[Out] 1/2*a^4*c*x^2+1/4*a^4*e*x^4+4/5*a^3*b*c*x^5+4/7*a^3*b*e*x^7+3/4*a^2*b^2*c*x
^8+3/5*a^2*b^2*e*x^10+4/11*a*b^3*c*x^11+4/13*a*b^3*e*x^13+1/14*b^4*c*x^14+1
/16*b^4*e*x^16+1/15*d*(b*x^3+a)^5/b
```

Rubi [A]
time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1596, 1864}

$$\frac{1}{2}a^4cx^2+\frac{1}{4}a^4ex^4+\frac{4}{5}a^3bcx^5+\frac{4}{7}a^3bex^7+\frac{3}{4}a^2b^2cx^8+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{11}ab^3cx^{11}+\frac{4}{13}ab^3ex^{13}+\frac{d\left(a+bx^3\right)^5}{15b}+\frac{1}{14}b^4cx^{14}+\frac{1}{16}b^4ex^{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^2)/2 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5 + (4*a^3*b*e*x^7)/7 + (3*
a^2*b^2*c*x^8)/4 + (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11 + (4*a*b^3*e*
x^13)/13 + (b^4*c*x^14)/14 + (b^4*e*x^16)/16 + (d*(a + b*x^3)^5)/(15*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx = \frac{d(a+bx^3)^5}{15b} + \int \left(a+bx^3\right)^4\left(-dx^2+x\left(c+dx+ex^2\right)\right)\,dx$$
$$= \frac{d(a+bx^3)^5}{15b} + \int \left(a^4cx + a^4ex^3 + 4a^3bcx^4 + 4a^3bex^6 + 6a^2b^2cx^7 + 6a^2$$
$$= \frac{1}{2}a^4cx^2 + \frac{1}{4}a^4ex^4 + \frac{4}{5}a^3bcx^5 + \frac{4}{7}a^3bex^7 + \frac{3}{4}a^2b^2cx^8 + \frac{3}{5}a^2b^2ex^{10} + \frac{4}{11}$$

Mathematica [A]
time = 0.00, size = 181, normalized size = 1.31

$$\frac{1}{2}a^4cx^2+\frac{1}{3}a^4dx^3+\frac{1}{4}a^4ex^4+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{3}{4}a^2b^2cx^8+\frac{2}{3}a^2b^2dx^9+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{11}ab^3cx^{11}+\frac{1}{3}ab^3dx^{12}+\frac{4}{13}ab^3ex^{13}+\frac{1}{14}b^4cx^{14}+\frac{1}{15}b^4dx^{15}+\frac{1}{16}b^4ex^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^2)/2 + (a^4*d*x^3)/3 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5 + (2*a^3*
b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (3*a^2*b^2*c*x^8)/4 + (2*a^2*b^2*d*x^9)/3
+ (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11 + (a*b^3*d*x^12)/3 + (4*a*b^3*
e*x^13)/13 + (b^4*c*x^14)/14 + (b^4*d*x^15)/15 + (b^4*e*x^16)/16
```

Maple [A]
time = 0.40, size = 152, normalized size = 1.10

method	result
gosper	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}d\,a^4x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a^2b^2e$
default	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}d\,a^4x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a^2b^2e$
norman	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}d\,a^4x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a^2b^2e$
risch	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}d\,a^4x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a^2b^2e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^4*c*x^2+1/3*d*a^4*x^3+1/4*a^4*e*x^4+4/5*a^3*b*c*x^5+2/3*a^3*b*d*x^6+4
/7*a^3*b*e*x^7+3/4*a^2*b^2*c*x^8+2/3*a^2*b^2*d*x^9+3/5*a^2*b^2*e*x^10+4/11*
a*b^3*c*x^11+1/3*a*d*b^3*x^12+4/13*a*b^3*e*x^13+1/14*b^4*c*x^14+1/15*d*b^4*
x^15+1/16*b^4*e*x^16
```

Maxima [A]
time = 0.27, size = 156, normalized size = 1.13

$$\frac{1}{16}\,b^4x^{16}e+\frac{1}{15}\,b^4dx^{15}+\frac{1}{14}\,b^4cx^{14}+\frac{4}{13}\,ab^3x^{13}e+\frac{1}{3}\,ab^3dx^{12}+\frac{4}{11}\,ab^3cx^{11}+\frac{3}{5}\,a^2b^2x^{10}e+\frac{2}{3}\,a^2b^2dx^9+\frac{3}{4}\,a^2b^2cx^8+\frac{4}{7}\,a^3bx^7e+\frac{2}{3}\,a^3bdx^6+\frac{4}{5}\,a^3bcx^5+\frac{1}{4}\,a^4x^4e+\frac{1}{3}\,a^4dx^3+\frac{1}{2}\,a^4cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/16*b^4*x^16*e + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*x^13*e + 1
/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*x^10*e + 2/3*a^2*b^2*d*x^
9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*x^7*e + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5
 + 1/4*a^4*x^4*e + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2
```

Fricas [A]
time = 0.40, size = 151, normalized size = 1.09

$$\frac{1}{16}\,b^{4}ex^{16}+\frac{1}{15}\,b^{4}dx^{15}+\frac{1}{14}\,b^{4}cx^{14}+\frac{4}{13}\,ab^{3}ex^{13}+\frac{1}{3}\,ab^{3}dx^{12}+\frac{4}{11}\,ab^{3}cx^{11}+\frac{3}{5}\,a^{2}b^{2}ex^{10}+\frac{2}{3}\,a^{2}b^{2}dx^{9}+\frac{3}{4}\,a^{2}b^{2}cx^{8}+\frac{4}{7}\,a^{3}bex^{7}+\frac{2}{3}\,a^{3}bdx^{6}+\frac{4}{5}\,a^{3}bcx^{5}+\frac{1}{4}\,a^{4}ex^{4}+\frac{1}{3}\,a^{4}dx^{3}+\frac{1}{2}\,a^{4}cx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/16*b^4*e*x^16 + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*e*x^13 + 1
/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*e*x^10 + 2/3*a^2*b^2*d*x^
9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5
 + 1/4*a^4*e*x^4 + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2
```

Sympy [A]
time = 0.02, size = 185, normalized size = 1.34

$$\frac{a^{4}cx^{2}}{2}+\frac{a^{4}dx^{3}}{3}+\frac{a^{4}ex^{4}}{4}+\frac{4a^{3}bcx^{5}}{5}+\frac{2a^{3}bdx^{6}}{3}+\frac{4a^{3}bex^{7}}{7}+\frac{3a^{2}b^{2}cx^{8}}{4}+\frac{2a^{2}b^{2}dx^{9}}{3}+\frac{3a^{2}b^{2}ex^{10}}{5}+\frac{4ab^{3}cx^{11}}{11}+\frac{ab^{3}dx^{12}}{3}+\frac{4ab^{3}ex^{13}}{13}+\frac{b^{4}cx^{14}}{14}+\frac{b^{4}dx^{15}}{15}+\frac{b^{4}ex^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x**2/2 + a**4*d*x**3/3 + a**4*e*x**4/4 + 4*a**3*b*c*x**5/5 + 2*a**3*
b*d*x**6/3 + 4*a**3*b*e*x**7/7 + 3*a**2*b**2*c*x**8/4 + 2*a**2*b**2*d*x**9/
3 + 3*a**2*b**2*e*x**10/5 + 4*a*b**3*c*x**11/11 + a*b**3*d*x**12/3 + 4*a*b*
*3*e*x**13/13 + b**4*c*x**14/14 + b**4*d*x**15/15 + b**4*e*x**16/16
```

Giac [A]
time = 0.72, size = 156, normalized size = 1.13

$$\frac{1}{16}\,b^{4}x^{16}e+\frac{1}{15}\,b^{4}dx^{15}+\frac{1}{14}\,b^{4}cx^{14}+\frac{4}{13}\,ab^{3}x^{13}e+\frac{1}{3}\,ab^{3}dx^{12}+\frac{4}{11}\,ab^{3}cx^{11}+\frac{3}{5}\,a^{2}b^{2}x^{10}e+\frac{2}{3}\,a^{2}b^{2}dx^{9}+\frac{3}{4}\,a^{2}b^{2}cx^{8}+\frac{4}{7}\,a^{3}bx^{7}e+\frac{2}{3}\,a^{3}bdx^{6}+\frac{4}{5}\,a^{3}bcx^{5}+\frac{1}{4}\,a^{4}x^{4}e+\frac{1}{3}\,a^{4}dx^{3}+\frac{1}{2}\,a^{4}cx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*b^4*x^16*e + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*x^13*e + 1
/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*x^10*e + 2/3*a^2*b^2*d*x^
9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*x^7*e + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5
 + 1/4*a^4*x^4*e + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2
```

Mupad [B]
time = 0.13, size = 151, normalized size = 1.09

$$\frac{e\,a^4\,x^4}{4}+\frac{d\,a^4\,x^3}{3}+\frac{c\,a^4\,x^2}{2}+\frac{4\,e\,a^3\,b\,x^7}{7}+\frac{2\,d\,a^3\,b\,x^6}{3}+\frac{4\,c\,a^3\,b\,x^5}{5}+\frac{3\,e\,a^2\,b^2\,x^{10}}{5}+\frac{2\,d\,a^2\,b^2\,x^9}{3}+\frac{3\,c\,a^2\,b^2\,x^8}{4}+\frac{4\,e\,a\,b^3\,x^{13}}{13}+\frac{d\,a\,b^3\,x^{12}}{3}+\frac{4\,c\,a\,b^3\,x^{11}}{11}+\frac{e\,b^4\,x^{16}}{16}+\frac{d\,b^4\,x^{15}}{15}+\frac{c\,b^4\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^4*(c + d*x + e*x^2),x)
```

```
[Out] (a^4*c*x^2)/2 + (a^4*d*x^3)/3 + (b^4*c*x^14)/14 + (a^4*e*x^4)/4 + (b^4*d*x^
15)/15 + (b^4*e*x^16)/16 + (3*a^2*b^2*c*x^8)/4 + (2*a^2*b^2*d*x^9)/3 + (3*a
^2*b^2*e*x^10)/5 + (4*a^3*b*c*x^5)/5 + (4*a*b^3*c*x^11)/11 + (2*a^3*b*d*x^6
)/3 + (a*b^3*d*x^12)/3 + (4*a^3*b*e*x^7)/7 + (4*a*b^3*e*x^13)/13
```

3.333 $\int (c+dx+ex^2)(a+bx^3)^4\,dx$

Optimal. Leaf size=130

$$a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}+\frac{e(a+b$$

```
[Out] a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*a^3*b*d*x^5+6/7*a^2*b^2*c*x^7+3/4*a^2
*b^2*d*x^8+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/13*b^4*c*x^13+1/14*b^4*d*x^
14+1/15*e*(b*x^3+a)^5/b
```

Rubi [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1596, 1864}

$$a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{e(a+bx^3)^5}{15b}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14 + (e*(a + b*x^3)^5)/(15*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx &= \frac{e\left(a+bx^3\right)^5}{15b}+\int \left(c+dx\right)\left(a+bx^3\right)^4\,dx\\ &= \frac{e\left(a+bx^3\right)^5}{15b}+\int \left(a^4c+a^4dx+4a^3bcx^3+4a^3bdx^4+6a^2b^2cx^6+6a^2b^2d\right.\\ &= a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 173, normalized size = 1.33

$$a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{2}{3}a^3bex^6+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{3}a^2b^2ex^9+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{3}ab^3ex^{12}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}+\frac{1}{15}b^4ex^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 +
 (2*a^3*b*e*x^6)/3 + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a^2*b^2
*e*x^9)/3 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (a*b^3*e*x^12)/3 + (
b^4*c*x^13)/13 + (b^4*d*x^14)/14 + (b^4*e*x^15)/15
```

Maple [A]
time = 0.52, size = 148, normalized size = 1.14

method	result
gosper	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}a^3bd\,x^5+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}a^2b^2d\,x^8+\frac{2}{3}a^2e\,b^2x^9+$
default	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}a^3bd\,x^5+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}a^2b^2d\,x^8+\frac{2}{3}a^2e\,b^2x^9+$
norman	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}a^3bd\,x^5+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}a^2b^2d\,x^8+\frac{2}{3}a^2e\,b^2x^9+$
risch	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}a^3bd\,x^5+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}a^2b^2d\,x^8+\frac{2}{3}a^2e\,b^2x^9+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*c*x+1/2*a^4*d*x^2+1/3*a^4*e*x^3+a^3*b*c*x^4+4/5*a^3*b*d*x^5+2/3*a^3*b*e
*x^6+6/7*a^2*b^2*c*x^7+3/4*a^2*b^2*d*x^8+2/3*a^2*e*b^2*x^9+2/5*a*b^3*c*x^10
+4/11*a*b^3*d*x^11+1/3*a*b^3*e*x^12+1/13*b^4*c*x^13+1/14*b^4*d*x^14+1/15*b^
4*e*x^15
```

Maxima [A]
time = 0.29, size = 152, normalized size = 1.17

$$\frac{1}{15}\,b^4x^{15}e+\frac{1}{14}\,b^4dx^{14}+\frac{1}{13}\,b^4cx^{13}+\frac{1}{3}\,ab^3x^{12}e+\frac{4}{11}\,ab^3dx^{11}+\frac{2}{5}\,ab^3cx^{10}+\frac{2}{3}\,a^2b^2x^9e+\frac{3}{4}\,a^2b^2dx^8+\frac{6}{7}\,a^2b^2cx^7+\frac{2}{3}\,a^3bx^6e+\frac{4}{5}\,a^3bdx^5+a^3bcx^4+\frac{1}{3}\,a^4x^3e+\frac{1}{2}\,a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/15*b^4*x^15*e + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*x^12*e + 4/
11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*x^9*e + 3/4*a^2*b^2*d*x^8
+ 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*x^6*e + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/3
*a^4*x^3*e + 1/2*a^4*d*x^2 + a^4*c*x
```

Fricas [A]
time = 0.38, size = 147, normalized size = 1.13

$$\frac{1}{15}b^4ex^{15}+\frac{1}{14}b^4dx^{14}+\frac{1}{13}b^4cx^{13}+\frac{1}{3}ab^3ex^{12}+\frac{4}{11}ab^3dx^{11}+\frac{2}{5}ab^3cx^{10}+\frac{2}{3}a^2b^2ex^9+\frac{3}{4}a^2b^2dx^8+\frac{6}{7}a^2b^2cx^7+\frac{2}{3}a^3bex^6+\frac{4}{5}a^3bdx^5+a^3bcx^4+\frac{1}{3}a^4ex^3+\frac{1}{2}a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/15*b^4*e*x^15 + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*e*x^12 + 4/
11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*e*x^9 + 3/4*a^2*b^2*d*x^8
+ 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*e*x^6 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/3
*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Sympy [A]
time = 0.02, size = 178, normalized size = 1.37

$$a^4cx+\frac{a^4dx^2}{2}+\frac{a^4ex^3}{3}+a^3bcx^4+\frac{4a^3bdx^5}{5}+\frac{2a^3bex^6}{3}+\frac{6a^2b^2cx^7}{7}+\frac{3a^2b^2dx^8}{4}+\frac{2a^2b^2ex^9}{3}+\frac{2ab^3cx^{10}}{5}+\frac{4ab^3dx^{11}}{11}+\frac{ab^3ex^{12}}{3}+\frac{b^4cx^{13}}{13}+\frac{b^4dx^{14}}{14}+\frac{b^4ex^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**3*b*c*x**4 + 4*a**3*b*d*x**5/
5 + 2*a**3*b*e*x**6/3 + 6*a**2*b**2*c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a**
2*b**2*e*x**9/3 + 2*a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/11 + a*b**3*e*x**12
/3 + b**4*c*x**13/13 + b**4*d*x**14/14 + b**4*e*x**15/15
```

Giac [A]
time = 0.99, size = 152, normalized size = 1.17

$$\frac{1}{15}b^4x^{15}e+\frac{1}{14}b^4dx^{14}+\frac{1}{13}b^4cx^{13}+\frac{1}{3}ab^3x^{12}e+\frac{4}{11}ab^3dx^{11}+\frac{2}{5}ab^3cx^{10}+\frac{2}{3}a^2b^2x^9e+\frac{3}{4}a^2b^2dx^8+\frac{6}{7}a^2b^2cx^7+\frac{2}{3}a^3bx^6e+\frac{4}{5}a^3bdx^5+a^3bcx^4+\frac{1}{3}a^4x^3e+\frac{1}{2}a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 1/15*b^4*x^15*e + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*x^12*e + 4/
11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*x^9*e + 3/4*a^2*b^2*d*x^8
+ 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*x^6*e + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/3
*a^4*x^3*e + 1/2*a^4*d*x^2 + a^4*c*x
```

Mupad [B]
time = 0.15, size = 147, normalized size = 1.13

$$\frac{e\,a^4\,x^3}{3}+\frac{d\,a^4\,x^2}{2}+c\,a^4\,x+\frac{2\,e\,a^3\,b\,x^6}{3}+\frac{4\,d\,a^3\,b\,x^5}{5}+c\,a^3\,b\,x^4+\frac{2\,e\,a^2\,b^2\,x^9}{3}+\frac{3\,d\,a^2\,b^2\,x^8}{4}+\frac{6\,c\,a^2\,b^2\,x^7}{7}+\frac{e\,a\,b^3\,x^{12}}{3}+\frac{4\,d\,a\,b^3\,x^{11}}{11}+\frac{2\,c\,a\,b^3\,x^{10}}{5}+\frac{e\,b^4\,x^{15}}{15}+\frac{d\,b^4\,x^{14}}{14}+\frac{c\,b^4\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^4*(c + d*x + e*x^2),x)
```

```
[Out] (a^4*d*x^2)/2 + (b^4*c*x^13)/13 + (a^4*e*x^3)/3 + (b^4*d*x^14)/14 + (b^4*e*
x^15)/15 + a^4*c*x + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a^2*b^2
*e*x^9)/3 + a^3*b*c*x^4 + (2*a*b^3*c*x^10)/5 + (4*a^3*b*d*x^5)/5 + (4*a*b^3
*d*x^11)/11 + (2*a^3*b*e*x^6)/3 + (a*b^3*e*x^12)/3
```

3.334 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x} dx$

Optimal. Leaf size=166

$$a^4dx+\frac{1}{2}a^4ex^2+\frac{4}{3}a^3bcx^3+a^3bdx^4+\frac{4}{5}a^3bex^5+a^2b^2cx^6+\frac{6}{7}a^2b^2dx^7+\frac{3}{4}a^2b^2ex^8+\frac{4}{9}ab^3cx^9+\frac{2}{5}ab^3dx^{10}+\frac{4}{11}ab^3ex^{11}+$$

```
[Out] a^4*d*x+1/2*a^4*e*x^2+4/3*a^3*b*c*x^3+a^3*b*d*x^4+4/5*a^3*b*e*x^5+a^2*b^2*c
*x^6+6/7*a^2*b^2*d*x^7+3/4*a^2*b^2*e*x^8+4/9*a*b^3*c*x^9+2/5*a*b^3*d*x^10+4
/11*a*b^3*e*x^11+1/12*b^4*c*x^12+1/13*b^4*d*x^13+1/14*b^4*e*x^14+a^4*c*ln(x
)
```

Rubi [A]
time = 0.07, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1642}

$$a^4c\log(x)+a^4dx+\frac{1}{2}a^4ex^2+\frac{4}{3}a^3bcx^3+a^3bdx^4+\frac{4}{5}a^3bex^5+a^2b^2cx^6+\frac{6}{7}a^2b^2dx^7+\frac{3}{4}a^2b^2ex^8+\frac{4}{9}ab^3cx^9+\frac{2}{5}ab^3dx^{10}+\frac{4}{11}ab^3ex^{11}+\frac{1}{12}b^4cx^{12}+\frac{1}{13}b^4dx^{13}+\frac{1}{14}b^4ex^{14}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x,x]
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5)
/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3*c
*x^9)/9 + (2*a*b^3*d*x^10)/5 + (4*a*b^3*e*x^11)/11 + (b^4*c*x^12)/12 + (b^4
*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx &= \int \left(a^4d+\frac{a^4c}{x}+a^4ex+4a^3bcx^2+4a^3bdx^3+4a^3bex^4+6a^2b^2cx^5+6a^2b^2\right.\\ &= a^4dx+\frac{1}{2}a^4ex^2+\frac{4}{3}a^3bcx^3+a^3bdx^4+\frac{4}{5}a^3bex^5+a^2b^2cx^6+\frac{6}{7}a^2b^2dx^7+\frac{3}{4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 166, normalized size = 1.00

$$a^4dx+\frac{1}{2}a^4ex^2+\frac{4}{3}a^3bcx^3+a^3bdx^4+\frac{4}{5}a^3bex^5+a^2b^2cx^6+\frac{6}{7}a^2b^2dx^7+\frac{3}{4}a^2b^2ex^8+\frac{4}{9}ab^3cx^9+\frac{2}{5}ab^3dx^{10}+\frac{4}{11}ab^3ex^{11}+\frac{1}{12}b^4cx^{12}+\frac{1}{13}b^4dx^{13}+\frac{1}{14}b^4ex^{14}+a^4c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x,x]
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5)
/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3*c
*x^9)/9 + (2*a*b^3*d*x^10)/5 + (4*a*b^3*e*x^11)/11 + (b^4*c*x^12)/12 + (b^4
*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c*Log[x]
```

Maple [A]
time = 0.34, size = 145, normalized size = 0.87

method	result
default	$a^4dx+\frac{a^4e\,x^2}{2}+\frac{4a^3bc\,x^3}{3}+a^3bd\,x^4+\frac{4a^3be\,x^5}{5}+a^2b^2c\,x^6+\frac{6a^2b^2d\,x^7}{7}+\frac{3a^2b^2e\,x^8}{4}+\frac{4a\,b^3c\,x^9}{9}+\frac{2a\,b^3d\,x^{10}}{5}+$
norman	$a^4dx+\frac{a^4e\,x^2}{2}+\frac{4a^3bc\,x^3}{3}+a^3bd\,x^4+\frac{4a^3be\,x^5}{5}+a^2b^2c\,x^6+\frac{6a^2b^2d\,x^7}{7}+\frac{3a^2b^2e\,x^8}{4}+\frac{4a\,b^3c\,x^9}{9}+\frac{2a\,b^3d\,x^{10}}{5}+$
risch	$a^4dx+\frac{a^4e\,x^2}{2}+\frac{4a^3bc\,x^3}{3}+a^3bd\,x^4+\frac{4a^3be\,x^5}{5}+a^2b^2c\,x^6+\frac{6a^2b^2d\,x^7}{7}+\frac{3a^2b^2e\,x^8}{4}+\frac{4a\,b^3c\,x^9}{9}+\frac{2a\,b^3d\,x^{10}}{5}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*d*x+1/2*a^4*e*x^2+4/3*a^3*b*c*x^3+a^3*b*d*x^4+4/5*a^3*b*e*x^5+a^2*b^2*c
*x^6+6/7*a^2*b^2*d*x^7+3/4*a^2*b^2*e*x^8+4/9*a*b^3*c*x^9+2/5*a*b^3*d*x^10+4
/11*a*b^3*e*x^11+1/12*b^4*c*x^12+1/13*b^4*d*x^13+1/14*b^4*e*x^14+a^4*c*ln(x
)
```

Maxima [A]
time = 0.27, size = 149, normalized size = 0.90

$$\frac{1}{14}\,b^4x^{14}e+\frac{1}{13}\,b^4dx^{13}+\frac{1}{12}\,b^4cx^{12}+\frac{4}{11}\,ab^3x^{11}e+\frac{2}{5}\,ab^3dx^{10}+\frac{4}{9}\,ab^3cx^9+\frac{3}{4}\,a^2b^2x^8e+\frac{6}{7}\,a^2b^2dx^7+a^2b^2cx^6+\frac{4}{5}\,a^3bx^5e+a^3bdx^4+\frac{4}{3}\,a^3bcx^3+\frac{1}{2}\,a^4x^2e+a^4dx+a^4c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="maxima")
```

```
[Out] 1/14*b^4*x^14*e + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*x^11*e + 2
/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*x^8*e + 6/7*a^2*b^2*d*x^7 +
 a^2*b^2*c*x^6 + 4/5*a^3*b*x^5*e + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*
x^2*e + a^4*d*x + a^4*c*log(x)
```

Fricas [A]
time = 0.39, size = 144, normalized size = 0.87

$$\frac{1}{14}\,b^4ex^{14}+\frac{1}{13}\,b^4dx^{13}+\frac{1}{12}\,b^4cx^{12}+\frac{4}{11}\,ab^3ex^{11}+\frac{2}{5}\,ab^3dx^{10}+\frac{4}{9}\,ab^3cx^9+\frac{3}{4}\,a^2b^2ex^8+\frac{6}{7}\,a^2b^2dx^7+a^2b^2cx^6+\frac{4}{5}\,a^3bex^5+a^3bdx^4+\frac{4}{3}\,a^3bcx^3+\frac{1}{2}\,a^4ex^2+a^4dx+a^4c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="fricas")
```

```
[Out] 1/14*b^4*e*x^14 + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*e*x^11 + 2
/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*e*x^8 + 6/7*a^2*b^2*d*x^7 +
 a^2*b^2*c*x^6 + 4/5*a^3*b*e*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*
e*x^2 + a^4*d*x + a^4*c*log(x)
```

Sympy [A]
time = 0.10, size = 175, normalized size = 1.05

$$a^4c\log(x) + a^4dx + \frac{a^4ex^2}{2} + \frac{4a^3bcx^3}{3} + a^3bdx^4 + \frac{4a^3bex^5}{5} + a^2b^2cx^6 + \frac{6a^2b^2dx^7}{7} + \frac{3a^2b^2ex^8}{4} + \frac{4ab^3cx^9}{9} + \frac{2ab^3dx^{10}}{5} + \frac{4ab^3ex^{11}}{11} + \frac{b^4cx^{12}}{12} + \frac{b^4dx^{13}}{13} + \frac{b^4ex^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x,x)
```

```
[Out] a**4*c*log(x) + a**4*d*x + a**4*e*x**2/2 + 4*a**3*b*c*x**3/3 + a**3*b*d*x**
4 + 4*a**3*b*e*x**5/5 + a**2*b**2*c*x**6 + 6*a**2*b**2*d*x**7/7 + 3*a**2*b*
*2*e*x**8/4 + 4*a*b**3*c*x**9/9 + 2*a*b**3*d*x**10/5 + 4*a*b**3*e*x**11/11
+ b**4*c*x**12/12 + b**4*d*x**13/13 + b**4*e*x**14/14
```

Giac [A]
time = 1.38, size = 150, normalized size = 0.90

$$\frac{1}{14}b^4x^{14}e + \frac{1}{13}b^4dx^{13} + \frac{1}{12}b^4cx^{12} + \frac{4}{11}ab^3x^{11}e + \frac{2}{5}ab^3dx^{10} + \frac{4}{9}ab^3cx^9 + \frac{3}{4}a^2b^2x^8e + \frac{6}{7}a^2b^2dx^7 + a^2b^2cx^6 + \frac{4}{5}a^3bx^5e + a^3bdx^4 + \frac{4}{3}a^3bcx^3 + \frac{1}{2}a^4x^2e + a^4dx + a^4c\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="giac")
```

```
[Out] 1/14*b^4*x^14*e + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*x^11*e + 2
/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*x^8*e + 6/7*a^2*b^2*d*x^7 +
 a^2*b^2*c*x^6 + 4/5*a^3*b*x^5*e + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*
x^2*e + a^4*d*x + a^4*c*log(abs(x))
```

Mupad [B]
time = 0.14, size = 144, normalized size = 0.87

$$\frac{b^4\,c\,x^{12}}{12} + \frac{a^4\,e\,x^2}{2} + \frac{b^4\,d\,x^{13}}{13} + \frac{b^4\,e\,x^{14}}{14} + a^4\,c\,\ln(x) + a^4\,d\,x + a^2\,b^2\,c\,x^6 + \frac{6\,a^2\,b^2\,d\,x^7}{7} + \frac{3\,a^2\,b^2\,e\,x^8}{4} + \frac{4\,a^3\,b\,c\,x^3}{3} + \frac{4\,a\,b^3\,c\,x^9}{9} + a^3\,b\,d\,x^4 + \frac{2\,a\,b^3\,d\,x^{10}}{5} + \frac{4\,a^3\,b\,e\,x^5}{5} + \frac{4\,a\,b^3\,e\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^4*(c + d*x + e*x^2))/x,x)
```

```
[Out] (b^4*c*x^12)/12 + (a^4*e*x^2)/2 + (b^4*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c
*log(x) + a^4*d*x + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)
/4 + (4*a^3*b*c*x^3)/3 + (4*a*b^3*c*x^9)/9 + a^3*b*d*x^4 + (2*a*b^3*d*x^10)
/5 + (4*a^3*b*e*x^5)/5 + (4*a*b^3*e*x^11)/11
```

3.335 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx$

Optimal. Leaf size=162

$$-\frac{a^4c}{x}+a^4ex+2a^3bcx^2+\frac{4}{3}a^3bdx^3+a^3bex^4+\frac{6}{5}a^2b^2cx^5+a^2b^2dx^6+\frac{6}{7}a^2b^2ex^7+\frac{1}{2}ab^3cx^8+\frac{4}{9}ab^3dx^9+\frac{2}{5}ab^3ex^{10}+\frac{1}{11}$$

```
[Out] -a^4*c/x+a^4*e*x+2*a^3*b*c*x^2+4/3*a^3*b*d*x^3+a^3*b*e*x^4+6/5*a^2*b^2*c*x^
5+a^2*b^2*d*x^6+6/7*a^2*b^2*e*x^7+1/2*a*b^3*c*x^8+4/9*a*b^3*d*x^9+2/5*a*b^3
*e*x^10+1/11*b^4*c*x^11+1/12*b^4*d*x^12+1/13*b^4*e*x^13+a^4*d*ln(x)
```

Rubi [A]
time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1642}

$$-\frac{a^4c}{x}+a^4d\log(x)+a^4ex+2a^3bcx^2+\frac{4}{3}a^3bdx^3+a^3bex^4+\frac{6}{5}a^2b^2cx^5+a^2b^2dx^6+\frac{6}{7}a^2b^2ex^7+\frac{1}{2}ab^3cx^8+\frac{4}{9}ab^3dx^9+\frac{2}{5}ab^3ex^{10}+\frac{1}{11}b^4cx^{11}+\frac{1}{12}b^4dx^{12}+\frac{1}{13}b^4ex^{13}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^2,x]
```

```
[Out] -((a^4*c)/x) + a^4*e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 +
(6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)/2
 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^12)/
12 + (b^4*e*x^13)/13 + a^4*d*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx &= \int \left(a^4e+\frac{a^4c}{x^2}+\frac{a^4d}{x}+4a^3bcx+4a^3bdx^2+4a^3bex^3+6a^2b^2cx^4+6a^2b^2 \right.\\ &= -\frac{a^4c}{x}+a^4ex+2a^3bcx^2+\frac{4}{3}a^3bdx^3+a^3bex^4+\frac{6}{5}a^2b^2cx^5+a^2b^2dx^6+\frac{6}{7}a\end{aligned}$$

Mathematica [A]
time = 0.01, size = 162, normalized size = 1.00

$$-\frac{a^4c}{x}+a^4ex+2a^3bcx^2+\frac{4}{3}a^3bdx^3+a^3bex^4+\frac{6}{5}a^2b^2cx^5+a^2b^2dx^6+\frac{6}{7}a^2b^2ex^7+\frac{1}{2}ab^3cx^8+\frac{4}{9}ab^3dx^9+\frac{2}{5}ab^3ex^{10}+\frac{1}{11}b^4cx^{11}+\frac{1}{12}b^4dx^{12}+\frac{1}{13}b^4ex^{13}+a^4d\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^2,x]
```

```
[Out] -((a^4*c)/x) + a^4*e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 +
(6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)/2
 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^12)/
12 + (b^4*e*x^13)/13 + a^4*d*Log[x]
```

Maple [A]
time = 0.35, size = 145, normalized size = 0.90

method	result
default	$-\frac{a^4c}{x}+a^4ex+2a^3bc\,x^2+\frac{4a^3bd\,x^3}{3}+a^3be\,x^4+\frac{6a^2b^2c\,x^5}{5}+a^2b^2d\,x^6+\frac{6a^2b^2e\,x^7}{7}+\frac{a\,b^3c\,x^8}{2}+\frac{4a\,b^3d\,x^9}{9}+$
risch	$-\frac{a^4c}{x}+a^4ex+2a^3bc\,x^2+\frac{4a^3bd\,x^3}{3}+a^3be\,x^4+\frac{6a^2b^2c\,x^5}{5}+a^2b^2d\,x^6+\frac{6a^2b^2e\,x^7}{7}+\frac{a\,b^3c\,x^8}{2}+\frac{4a\,b^3d\,x^9}{9}+$
norman	$\frac{a^4e\,x^2+a^2b^2d\,x^7+a^3be\,x^5-c\,a^4+\frac{1}{11}b^4c\,x^{12}+\frac{1}{12}b^4d\,x^{13}+\frac{1}{13}b^4e\,x^{14}+\frac{1}{2}a\,b^3c\,x^9+\frac{4}{9}a\,b^3d\,x^{10}+\frac{2}{5}a\,b^3e\,x^{11}+\frac{6}{5}a^2b^2c\,x^6+\frac{6}{7}a^2b^2e\,x^8+2a^3bc}{x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^4*c/x+a^4*e*x+2*a^3*b*c*x^2+4/3*a^3*b*d*x^3+a^3*b*e*x^4+6/5*a^2*b^2*c*x^
5+a^2*b^2*d*x^6+6/7*a^2*b^2*e*x^7+1/2*a*b^3*c*x^8+4/9*a*b^3*d*x^9+2/5*a*b^3
*e*x^10+1/11*b^4*c*x^11+1/12*b^4*d*x^12+1/13*b^4*e*x^13+a^4*d*ln(x)
```

Maxima [A]
time = 0.28, size = 149, normalized size = 0.92

$$\frac{1}{13}\,b^4x^{13}e+\frac{1}{12}\,b^4dx^{12}+\frac{1}{11}\,b^4cx^{11}+\frac{2}{5}\,ab^3x^{10}e+\frac{4}{9}\,ab^3dx^9+\frac{1}{2}\,ab^3cx^8+\frac{6}{7}\,a^2b^2x^7e+a^2b^2dx^6+\frac{6}{5}\,a^2b^2cx^5+a^3bx^4e+\frac{4}{3}\,a^3bdx^3+2\,a^3bcx^2+a^4xe+a^4d\log\left(x\right)-\frac{a^4c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^4*x^13*e + 1/12*b^4*d*x^12 + 1/11*b^4*c*x^11 + 2/5*a*b^3*x^10*e + 4/
9*a*b^3*d*x^9 + 1/2*a*b^3*c*x^8 + 6/7*a^2*b^2*x^7*e + a^2*b^2*d*x^6 + 6/5*a
^2*b^2*c*x^5 + a^3*b*x^4*e + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + a^4*x*e + a^
4*d*log(x) - a^4*c/x
```

Fricas [A]
time = 0.40, size = 153, normalized size = 0.94

$$\frac{13860\,b^4ex^{14}+15015\,b^4dx^{13}+16380\,b^4cx^{12}+72072\,ab^3ex^{11}+80080\,ab^3dx^{10}+90090\,ab^3cx^9+154440\,a^2b^2ex^8+180180\,a^2b^2dx^7+216216\,a^2b^2cx^6+180180\,a^3bex^5+240240\,a^3bdx^4+360360\,a^3bcx^3+180180\,a^4ex^2+180180\,a^4dx\log\left(x\right)-180180\,a^4c}{180180\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="fricas")
```

```
[Out] 1/180180*(13860*b^4*e*x^14 + 15015*b^4*d*x^13 + 16380*b^4*c*x^12 + 72072*a*
b^3*e*x^11 + 80080*a*b^3*d*x^10 + 90090*a*b^3*c*x^9 + 154440*a^2*b^2*e*x^8
+ 180180*a^2*b^2*d*x^7 + 216216*a^2*b^2*c*x^6 + 180180*a^3*b*e*x^5 + 240240
*a^3*b*d*x^4 + 360360*a^3*b*c*x^3 + 180180*a^4*e*x^2 + 180180*a^4*d*x*log(x
) - 180180*a^4*c)/x
```

Sympy [A]
time = 0.10, size = 168, normalized size = 1.04

$$-\frac{a^4c}{x}+a^4d\log(x)+a^4ex+2a^3bcx^2+\frac{4a^3bdx^3}{3}+a^3bex^4+\frac{6a^2b^2cx^5}{5}+a^2b^2dx^6+\frac{6a^2b^2ex^7}{7}+\frac{ab^3cx^8}{2}+\frac{4ab^3dx^9}{9}+\frac{2ab^3ex^{10}}{5}+\frac{b^4cx^{11}}{11}+\frac{b^4dx^{12}}{12}+\frac{b^4ex^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x**2,x)
```

```
[Out] -a**4*c/x + a**4*d*log(x) + a**4*e*x + 2*a**3*b*c*x**2 + 4*a**3*b*d*x**3/3
+ a**3*b*e*x**4 + 6*a**2*b**2*c*x**5/5 + a**2*b**2*d*x**6 + 6*a**2*b**2*e*x
**7/7 + a*b**3*c*x**8/2 + 4*a*b**3*d*x**9/9 + 2*a*b**3*e*x**10/5 + b**4*c*x
**11/11 + b**4*d*x**12/12 + b**4*e*x**13/13
```

Giac [A]
time = 1.34, size = 150, normalized size = 0.93

$$\frac{1}{13}b^4x^{13}e+\frac{1}{12}b^4dx^{12}+\frac{1}{11}b^4cx^{11}+\frac{2}{5}ab^3x^{10}e+\frac{4}{9}ab^3dx^9+\frac{1}{2}ab^3cx^8+\frac{6}{7}a^2b^2x^7e+a^2b^2dx^6+\frac{6}{5}a^2b^2cx^5+a^3bx^4e+\frac{4}{3}a^3bdx^3+2\,a^3bcx^2+a^4xe+a^4d\log(|x|)-\frac{a^4c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="giac")
```

```
[Out] 1/13*b^4*x^13*e + 1/12*b^4*d*x^12 + 1/11*b^4*c*x^11 + 2/5*a*b^3*x^10*e + 4/
9*a*b^3*d*x^9 + 1/2*a*b^3*c*x^8 + 6/7*a^2*b^2*x^7*e + a^2*b^2*d*x^6 + 6/5*a
^2*b^2*c*x^5 + a^3*b*x^4*e + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + a^4*x*e + a^
4*d*log(abs(x)) - a^4*c/x
```

Mupad [B]
time = 4.99, size = 144, normalized size = 0.89

$$\frac{b^4\,c\,x^{11}}{11}-\frac{a^4\,c}{x}+\frac{b^4\,d\,x^{12}}{12}+\frac{b^4\,e\,x^{13}}{13}+a^4\,d\,\ln(x)+a^4\,e\,x+\frac{6\,a^2\,b^2\,c\,x^5}{5}+a^2\,b^2\,d\,x^6+\frac{6\,a^2\,b^2\,e\,x^7}{7}+2\,a^3\,b\,c\,x^2+\frac{a\,b^3\,c\,x^8}{2}+\frac{4\,a^3\,b\,d\,x^3}{3}+\frac{4\,a\,b^3\,d\,x^9}{9}+a^3\,b\,e\,x^4+\frac{2\,a\,b^3\,e\,x^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^4*(c + d*x + e*x^2))/x^2,x)
```

```
[Out] (b^4*c*x^11)/11 - (a^4*c)/x + (b^4*d*x^12)/12 + (b^4*e*x^13)/13 + a^4*d*log
(x) + a^4*e*x + (6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 +
 2*a^3*b*c*x^2 + (a*b^3*c*x^8)/2 + (4*a^3*b*d*x^3)/3 + (4*a*b^3*d*x^9)/9 +
a^3*b*e*x^4 + (2*a*b^3*e*x^10)/5
```

3.336 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx$

Optimal. Leaf size=166

$$-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3+\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3ex^9+\frac{1}{10}b^4c$$

```
[Out] -1/2*a^4*c/x^2-a^4*d/x+4*a^3*b*c*x+2*a^3*b*d*x^2+4/3*a^3*b*e*x^3+3/2*a^2*b^
2*c*x^4+6/5*a^2*b^2*d*x^5+a^2*b^2*e*x^6+4/7*a*b^3*c*x^7+1/2*a*b^3*d*x^8+4/9
*a*b^3*e*x^9+1/10*b^4*c*x^10+1/11*b^4*d*x^11+1/12*b^4*e*x^12+a^4*e*ln(x)
```

Rubi [A]

time = 0.08, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1642}

$$-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+a^4e\log(x)+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3+\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3ex^9+\frac{1}{10}b^4cx^{10}+\frac{1}{11}b^4dx^{11}+\frac{1}{12}b^4ex^{12}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3,x]
```

```
[Out] -1/2*(a^4*c)/x^2 - (a^4*d)/x + 4*a^3*b*c*x + 2*a^3*b*d*x^2 + (4*a^3*b*e*x^3
)/3 + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e*x^6 + (4*a*b^3*
c*x^7)/7 + (a*b^3*d*x^8)/2 + (4*a*b^3*e*x^9)/9 + (b^4*c*x^10)/10 + (b^4*d*x
^11)/11 + (b^4*e*x^12)/12 + a^4*e*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = \int \left(4a^3bc+\frac{a^4c}{x^3}+\frac{a^4d}{x^2}+\frac{a^4e}{x}+4a^3bdx+4a^3bex^2+6a^2b^2cx^3+6a^2b^2dx^4\right.$$

$$= -\frac{a^4c}{2x^2}-\frac{a^4d}{x}+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3+\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b$$

Mathematica [A]

time = 0.01, size = 166, normalized size = 1.00

$$-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3+\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3ex^9+\frac{1}{10}b^4cx^{10}+\frac{1}{11}b^4dx^{11}+\frac{1}{12}b^4ex^{12}+a^4e\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3,x]
```

```
[Out] -1/2*(a^4*c)/x^2 - (a^4*d)/x + 4*a^3*b*c*x + 2*a^3*b*d*x^2 + (4*a^3*b*e*x^3
)/3 + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e*x^6 + (4*a*b^3*
c*x^7)/7 + (a*b^3*d*x^8)/2 + (4*a*b^3*e*x^9)/9 + (b^4*c*x^10)/10 + (b^4*d*x
^11)/11 + (b^4*e*x^12)/12 + a^4*e*Log[x]
```

Maple [A]
time = 0.49, size = 147, normalized size = 0.89

method	result
default	$-\frac{a^4 c}{2x^2}-\frac{a^4 d}{x}+4a^3 bcx+2a^3 bd\,x^2+\frac{4a^3 be\,x^3}{3}+\frac{3a^2 b^2 c\,x^4}{2}+\frac{6a^2 b^2 d\,x^5}{5}+a^2 b^2 e\,x^6+\frac{4a\,b^3 c\,x^7}{7}+\frac{a\,b^3 d\,x^8}{2}+$
risch	$\frac{b^4 e\,x^{12}}{12}+\frac{b^4 d\,x^{11}}{11}+\frac{b^4 c\,x^{10}}{10}+\frac{4a\,b^3 e\,x^9}{9}+\frac{a\,b^3 d\,x^8}{2}+\frac{4a\,b^3 c\,x^7}{7}+a^2 b^2 e\,x^6+\frac{6a^2 b^2 d\,x^5}{5}+\frac{3a^2 b^2 c\,x^4}{2}+\frac{4a^3 be\,x^3}{3}+$
norman	$\frac{a^2 b^2 e\,x^8-\frac{1}{2}c\,a^4-a^4 dx+\frac{1}{10}b^4 c\,x^{12}+\frac{1}{11}b^4 d\,x^{13}+\frac{1}{12}b^4 e\,x^{14}+\frac{4}{7}a\,b^3 c\,x^9+\frac{1}{2}a\,b^3 d\,x^{10}+\frac{4}{9}a\,b^3 e\,x^{11}+\frac{3}{2}a^2 b^2 c\,x^6+\frac{6}{5}a^2 b^2 d\,x^7+4a^3 bc\,x^3+2a}{x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^4*c/x^2-a^4*d/x+4*a^3*b*c*x+2*a^3*b*d*x^2+4/3*a^3*b*e*x^3+3/2*a^2*b^
2*c*x^4+6/5*a^2*b^2*d*x^5+a^2*b^2*e*x^6+4/7*a*b^3*c*x^7+1/2*a*b^3*d*x^8+4/9
*a*b^3*e*x^9+1/10*b^4*c*x^10+1/11*b^4*d*x^11+1/12*b^4*e*x^12+a^4*e*ln(x)
```

Maxima [A]
time = 0.26, size = 151, normalized size = 0.91

$$\frac{1}{12}\,b^4x^{12}e+\frac{1}{11}\,b^4dx^{11}+\frac{1}{10}\,b^4cx^{10}+\frac{4}{9}\,ab^3x^9e+\frac{1}{2}\,ab^3dx^8+\frac{4}{7}\,ab^3cx^7+a^2b^2x^6e+\frac{6}{5}\,a^2b^2dx^5+\frac{3}{2}\,a^2b^2cx^4+\frac{4}{3}\,a^3bx^3e+2\,a^3bdx^2+4\,a^3bcx+a^4e\log\left(x\right)-\frac{2\,a^4dx+a^4c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*x^12*e + 1/11*b^4*d*x^11 + 1/10*b^4*c*x^10 + 4/9*a*b^3*x^9*e + 1/2
*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b^2*x^6*e + 6/5*a^2*b^2*d*x^5 + 3/2*a^
2*b^2*c*x^4 + 4/3*a^3*b*x^3*e + 2*a^3*b*d*x^2 + 4*a^3*b*c*x + a^4*e*log(x)
- 1/2*(2*a^4*d*x + a^4*c)/x^2
```

Fricas [A]
time = 0.37, size = 153, normalized size = 0.92

$$\frac{1155\,b^4ex^{14}+1260\,b^4dx^{13}+1386\,b^4cx^{12}+6160\,ab^3ex^{11}+6930\,ab^3dx^{10}+7920\,ab^3cx^9+13860\,a^2b^2ex^8+16632\,a^2b^2dx^7+20790\,a^2b^2cx^6+18480\,a^3bex^5+27720\,a^3bdx^4+55440\,a^3bcx^3+13860\,a^4ex^2\log\left(x\right)-13860\,a^4dx-6930\,a^4c}{13860\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="fricas")
```

```
[Out] 1/13860*(1155*b^4*e*x^14 + 1260*b^4*d*x^13 + 1386*b^4*c*x^12 + 6160*a*b^3*e
*x^11 + 6930*a*b^3*d*x^10 + 7920*a*b^3*c*x^9 + 13860*a^2*b^2*e*x^8 + 16632*
a^2*b^2*d*x^7 + 20790*a^2*b^2*c*x^6 + 18480*a^3*b*e*x^5 + 27720*a^3*b*d*x^4
 + 55440*a^3*b*c*x^3 + 13860*a^4*e*x^2*log(x) - 13860*a^4*d*x - 6930*a^4*c)
/x^2
```

Sympy [A]
time = 0.14, size = 175, normalized size = 1.05

$$a^4e\log(x)+4a^3bcx+2a^3bdx^2+\frac{4a^3bex^3}{3}+\frac{3a^2b^2cx^4}{2}+\frac{6a^2b^2dx^5}{5}+a^2b^2ex^6+\frac{4ab^3cx^7}{7}+\frac{ab^3dx^8}{2}+\frac{4ab^3ex^9}{9}+\frac{b^4cx^{10}}{10}+\frac{b^4dx^{11}}{11}+\frac{b^4ex^{12}}{12}+\frac{-a^4c-2a^4dx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x**3,x)
```

```
[Out] a**4*e*log(x) + 4*a**3*b*c*x + 2*a**3*b*d*x**2 + 4*a**3*b*e*x**3/3 + 3*a**2
*b**2*c*x**4/2 + 6*a**2*b**2*d*x**5/5 + a**2*b**2*e*x**6 + 4*a*b**3*c*x**7/
7 + a*b**3*d*x**8/2 + 4*a*b**3*e*x**9/9 + b**4*c*x**10/10 + b**4*d*x**11/11
 + b**4*e*x**12/12 + (-a**4*c - 2*a**4*d*x)/(2*x**2)
```

Giac [A]
time = 1.37, size = 152, normalized size = 0.92

$$\frac{1}{12}b^4x^{12}e+\frac{1}{11}b^4dx^{11}+\frac{1}{10}b^4cx^{10}+\frac{4}{9}ab^3x^9e+\frac{1}{2}ab^3dx^8+\frac{4}{7}ab^3cx^7+a^2b^2x^6e+\frac{6}{5}a^2b^2dx^5+\frac{3}{2}a^2b^2cx^4+\frac{4}{3}a^3bx^3e+2a^3bdx^2+4a^3bcx+a^4e\log(|x|)-\frac{2a^4dx+a^4c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="giac")
```

```
[Out] 1/12*b^4*x^12*e + 1/11*b^4*d*x^11 + 1/10*b^4*c*x^10 + 4/9*a*b^3*x^9*e + 1/2
*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b^2*x^6*e + 6/5*a^2*b^2*d*x^5 + 3/2*a^
2*b^2*c*x^4 + 4/3*a^3*b*x^3*e + 2*a^3*b*d*x^2 + 4*a^3*b*c*x + a^4*e*log(abs
(x)) - 1/2*(2*a^4*d*x + a^4*c)/x^2
```

Mupad [B]
time = 4.99, size = 146, normalized size = 0.88

$$\frac{b^4cx^{10}}{10}-\frac{\frac{a^4c}{2}+a^4dx}{x^2}+\frac{b^4dx^{11}}{11}+\frac{b^4ex^{12}}{12}+a^4e\ln(x)+\frac{3a^2b^2cx^4}{2}+\frac{6a^2b^2dx^5}{5}+a^2b^2ex^6+4a^3bcx+\frac{4ab^3cx^7}{7}+2a^3bdx^2+\frac{ab^3dx^8}{2}+\frac{4a^3bex^3}{3}+\frac{4ab^3ex^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^4*(c + d*x + e*x^2))/x^3,x)
```

```
[Out] (b^4*c*x^10)/10 - ((a^4*c)/2 + a^4*d*x)/x^2 + (b^4*d*x^11)/11 + (b^4*e*x^12
)/12 + a^4*e*log(x) + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e
*x^6 + 4*a^3*b*c*x + (4*a*b^3*c*x^7)/7 + 2*a^3*b*d*x^2 + (a*b^3*d*x^8)/2 +
(4*a^3*b*e*x^3)/3 + (4*a*b^3*e*x^9)/9
```

3.337 $\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx$

Optimal. Leaf size=205

$$\frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\ (}{}$$

```
[Out] c*x/b+1/2*d*x^2/b+1/3*e*x^3/b-1/3*a^(1/3)*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+
b^(1/3)*x)/b^(5/3)+1/6*a^(1/3)*(c-a^(1/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/b^(4/3)-1/3*a*e*ln(b*x^3+a)/b^2+1/3*a^(1/3)*(b^(1/3)*c+
a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^(1/2
)
```

Rubi [A]
time = 0.17, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}}+\frac{\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)}{\sqrt{3}\,b^{5/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}-\frac{ae\log\left(a+bx^3\right)}{3b^2}+\frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (c*x)/b + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (a^(1/3)*(b^(1/3)*c + a^(1/3)*d)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(
1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/
3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(4/3)) - (a*e*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx &= \int \left(\frac{c}{b}+\frac{dx}{b}+\frac{ex^2}{b}-\frac{ac+adx+aex^2}{b\,(a+bx^3)}\right)\,dx \\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\int \frac{ac+adx+aex^2}{a+bx^3}\,dx}{b} \\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\int \frac{ac+adx}{a+bx^3}\,dx}{b}-\frac{(ae)\int \frac{x^2}{a+bx^3}\,dx}{b} \\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{ae\log\left(a+bx^3\right)}{3b^2}-\frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}\,c+a^{4/3}d\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}\,c+a^{4/3}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}}{3a^{2/3}b^{4/3}} \\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}}-\frac{ae\log\left(a+bx^3\right)}{3b^2}-\frac{(a}{} \\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log(}{} \\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}-\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right.}{3}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 191, normalized size = 0.93

$$
\frac{6bcx+3bdx^2+2bex^3+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{b}\left(-\sqrt[3]{a}\sqrt[3]{b}\,c+a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}\,c-a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2ae\log\left(a+bx^3\right)}{6b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (6*b*c*x + 3*b*d*x^2 + 2*b*e*x^3 + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a
^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(-(a^(1/3
)*b^(1/3)*c) + a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(1/3)*b^(1/
3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*e*Lo
g[a + b*x^3])/(6*b^2)
```

Maple [A]
time = 0.38, size = 227, normalized size = 1.11

method	result

risch	$\frac{e\,x^3}{3b}+\frac{d\,x^2}{2b}+\frac{cx}{b}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-\_R^2e-\_Rd-c\right)\ln\left(x-\_R\right)}{\_R^2}\right)}{3b^2}$
default	$\frac{\frac{1}{3}e\,x^3+\frac{1}{2}d\,x^2+cx}{b}-\frac{\left(\left(c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2\right.}{}\right.\right.\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3*e*x^3+1/2*d*x^2+c*x)-(c*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b
/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+d*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)
^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*e*ln(b*x^3+a)/b)*a/b
```

Maxima [A]
time = 0.51, size = 193, normalized size = 0.94

$$\frac{2\,x^3e+3\,dx^2+6\,cx}{6\,b}-\frac{\sqrt{3}\,\left(abd\left(\frac{a}{b}\right)^{\frac{2}{3}}+abc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\,\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}-\frac{\left(2\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}e+ad\left(\frac{a}{b}\right)^{\frac{1}{3}}-ac\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(a\left(\frac{a}{b}\right)^{\frac{2}{3}}e-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}+ac\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*x^3*e + 3*d*x^2 + 6*c*x)/b - 1/3*sqrt(3)*(a*b*d*(a/b)^(2/3) + a*b*c*
(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) -
1/6*(2*a*(a/b)^(2/3)*e + a*d*(a/b)^(1/3) - a*c)*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b^2*(a/b)^(2/3)) - 1/3*(a*(a/b)^(2/3)*e - a*d*(a/b)^(1/3) + a*
c)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.19, size = 4798, normalized size = 23.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/36*(12*b*e*x^3 + 18*b*d*x^2 - 2*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d
 + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*
b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*
b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)
*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^
3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^2*log(1/36*((-I*sqrt(3) + 1)*
(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 +
a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3
 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6
 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*
b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4*d
+ 2*a*b*c*d^2 - a*b*c^2*e + a^2*d*e^2 + 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sq
rt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/5
4*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^
3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*
a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6
 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^
2) + (b^2*c^3 + a*b*d^3)*x) + 36*b*c*x + (((-I*sqrt(3) + 1)*(a^2*e^2/b^4 -
(a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 +
1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d
*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3
+ a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e
^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^2 + 3*sqrt(1/3)*b^2*s
qrt(-(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*
e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1
/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3)
 + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2
*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/
3) + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*
e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d +
 a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)
^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5
+ 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c
*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2)/b^
4) - 18*a*e)*log(-1/36*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)
/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2
*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/
3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/
18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e
)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*
e^2 - 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d
 + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*
b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*
b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)
*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^
```

```
3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2) + 2*(b^2*c^3 + a*b*d^3)*x + 1/1
2*sqrt(1/3)*(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/
27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/
b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*
sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*
d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b
^6)^(1/3) + 6*a*e/b^2)*b^4*d - 6*b^3*c^2 - 6*a*b^2*d*e)*sqrt(-(((-I*sqrt(3)
 + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*
c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a
^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e
^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/
54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*
b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a
^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6
- 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt
(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d +
a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^
(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2)/b^4)) + (((-I*sqrt(3
) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b
*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 +
a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*
e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*...
```

Sympy [**A**]
time = 0.89, size = 178, normalized size = 0.87

$$\operatorname{RootSum}\left(27t^3b^6 + 27t^2ab^4e + t\left(9a^2b^2e^2 + 9ab^3cd\right) + a^3e^3 + 3a^2bcde - a^2bd^3 + ab^2c^3, \left(t \mapsto t\log\left(x + \frac{9t^2b^4d + 6tab^2de - 3tb^3c^2 + a^2de^2 - abc^2e + 2abcd^2}{abd^3 + b^2c^3}\right)\right)\right) + \frac{cx}{b} + \frac{dx^2}{2b} + \frac{ex^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**6 + 27*_t**2*a*b**4*e + _t*(9*a**2*b**2*e**2 + 9*a*b**3
*c*d) + a**3*e**3 + 3*a**2*b*c*d*e - a**2*b*d**3 + a*b**2*c**3, Lambda(_t,
_t*log(x + (9*_t**2*b**4*d + 6*_t*a*b**2*d*e - 3*_t*b**3*c**2 + a**2*d*e**2
 - a*b*c**2*e + 2*a*b*c*d**2)/(a*b*d**3 + b**2*c**3)))) + c*x/b + d*x**2/(2
*b) + e*x**3/(3*b)
```

Giac [**A**]
time = 1.33, size = 208, normalized size = 1.01

$$-\frac{ae\log\left(|bx^3 + a|\right)}{3\,b^2} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bc + \left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3} + \frac{2\,b^2x^3e + 3\,b^2dx^2 + 6\,b^2cx}{6\,b^3} + \frac{\left(ab^6d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ab^6c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*a*e*log(abs(b*x^3 + a))/b^2 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*c - (-a*b^
2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 - 1/6
*((-a*b^2)^(1/3)*b*c + (-a*b^2)^(2/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^
(2/3))/b^3 + 1/6*(2*b^2*x^3*e + 3*b^2*d*x^2 + 6*b^2*c*x)/b^3 + 1/3*(a*b^6*d
*(-a/b)^(1/3) + a*b^6*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7)
```

Mupad [B]
time = 5.07, size = 319, normalized size = 1.56

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,b^6\,z^3+27\,a\,b^4\,e\,z^2+9\,a\,b^3\,c\,d\,z+9\,a^2\,b^2\,e^2\,z+3\,a^2\,b\,c\,d\,e+a\,b^2\,c^3+a^3\,e^3-a^2\,b\,d^3,z,k\right)\,\left(6\,a^2\,e+\mathrm{root}\left(27\,b^6\,z^3+27\,a\,b^4\,e\,z^2+9\,a\,b^3\,c\,d\,z+9\,a^2\,b^2\,e^2\,z+3\,a^2\,b\,c\,d\,e+a\,b^2\,c^3+a^3\,e^3-a^2\,b\,d^3,z,k\right)\,a\,b^2\,9-3\,a\,b\,c\,x\right)+\frac{a^3\,e^2+b\,c\,d\,a^2}{b^2}+\frac{x\,\left(a^2\,d^2-a^2\,c\,e\right)}{b}\right)\,\mathrm{root}\left(27\,b^6\,z^3+27\,a\,b^4\,e\,z^2+9\,a\,b^3\,c\,d\,z+9\,a^2\,b^2\,e^2\,z+3\,a^2\,b\,c\,d\,e+a\,b^2\,c^3+a^3\,e^3-a^2\,b\,d^3,z,k\right)\right)+\frac{d\,x^2}{2\,b}+\frac{e\,x^3}{3\,b}+\frac{c\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2))/(a + b*x^3),x)
```

```
[Out] symsum(log(root(27*b^6*z^3 + 27*a*b^4*e*z^2 + 9*a*b^3*c*d*z + 9*a^2*b^2*e^2
*z + 3*a^2*b*c*d*e + a*b^2*c^3 + a^3*e^3 - a^2*b*d^3, z, k)*(6*a^2*e + 9*ro
ot(27*b^6*z^3 + 27*a*b^4*e*z^2 + 9*a*b^3*c*d*z + 9*a^2*b^2*e^2*z + 3*a^2*b*
c*d*e + a*b^2*c^3 + a^3*e^3 - a^2*b*d^3, z, k)*a*b^2 - 3*a*b*c*x) + (a^3*e^
2 + a^2*b*c*d)/b^2 + (x*(a^2*d^2 - a^2*c*e))/b)*root(27*b^6*z^3 + 27*a*b^4*
e*z^2 + 9*a*b^3*c*d*z + 9*a^2*b^2*e^2*z + 3*a^2*b*c*d*e + a*b^2*c^3 + a^3*e
^3 - a^2*b*d^3, z, k), k, 1, 3) + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (c*x)/b
```

3.338 $\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx$

Optimal. Leaf size=193

$$\frac{dx}{b}+\frac{ex^2}{2b}+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}+$$

```
[Out] d*x/b+1/2*e*x^2/b-1/3*a^(1/3)*(b^(1/3)*d-a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/b
^(5/3)+1/6*a^(1/3)*(d-a^(1/3)*e/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/b^(4/3)+1/3*c*ln(b*x^3+a)/b+1/3*a^(1/3)*(b^(1/3)*d+a^(1/3)*e)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}}+\frac{\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,e+\sqrt[3]{b}\,d\right)}{\sqrt{3}\,b^{5/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}+\frac{c\log\left(a+bx^3\right)}{3b}+\frac{dx}{b}+\frac{ex^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (d*x)/b + (e*x^2)/(2*b) + (a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(1/3)*(b^(1/3)*d
- a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/3)*(d - (a^(1/3)
*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)) +
(c*Log[a + b*x^3])/(3*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx = \int \left(\frac{d}{b}+\frac{ex}{b}-\frac{ad+aex-bcx^2}{b\,(a+bx^3)}\right)dx$$

$$= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\int \frac{ad+aex-bcx^2}{a+bx^3}\,dx}{b}$$

$$= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\int \frac{ad+aex}{a+bx^3}\,dx}{b}+c\int \frac{x^2}{a+bx^3}\,dx$$

$$= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{c\log(a+bx^3)}{3b}-\frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}\,d+a^{4/3}e\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}\,d+a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}}-\frac{\Big(\sqrt[3]{}}{}$$

$$= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}}+\frac{c\log(a+bx^3)}{3b}-\frac{\left(a^{2/3}\left(\sqrt[3]{b}\,d\right.\right.}{}$$

$$= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\right.}{6b^{4/3}}$$

$$= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}-\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{}\right.}{3b^{4/3}}$$

Mathematica [A]
time = 0.06, size = 184, normalized size = 0.95

$$\frac{6b^{2/3}dx+3b^{2/3}ex^2+2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(-\sqrt[3]{a}\sqrt[3]{b}\,d+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-\left(-\sqrt[3]{a}\sqrt[3]{b}\,d+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2b^{2/3}c\log(a+bx^3)}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (6*b^(2/3)*d*x + 3*b^(2/3)*e*x^2 + 2*Sqrt[3]*a^(1/3)*(b^(1/3)*d + a^(1/3)*e
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-(a^(1/3)*b^(1/3)*d) + a
^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] - (-(a^(1/3)*b^(1/3)*d) + a^(2/3)*e)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c*Log[a + b*x^3])/(
6*b^(5/3))
```

Maple [A]
time = 0.40, size = 220, normalized size = 1.14

method	result

risch	$\frac{e\,x^2}{2b}+\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_R^2bc-\_Rae-ad\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^2}$
default	$\frac{\frac{1}{2}e\,x^2+dx}{b}+\frac{-ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-ae\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\right.\right.}{}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*e*x^2+d*x)+(-a*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^
(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-a*e*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1
/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*c*ln(b*x^3+a))/b
```

Maxima [A]
time = 0.51, size = 185, normalized size = 0.96

$$-\frac{\sqrt{3}\left(a\left(\frac{a}{b}\right)^{\frac{2}{3}}e+ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{x^2e+2\,dx}{2\,b}+\frac{\left(2\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a\left(\frac{a}{b}\right)^{\frac{1}{3}}e+ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+a\left(\frac{a}{b}\right)^{\frac{1}{3}}e-ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(a*(a/b)^(2/3)*e + a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/2*(x^2*e + 2*d*x)/b + 1/6*(2*b*c*(a/b)^
(2/3) - a*(a/b)^(1/3)*e + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*
(a/b)^(2/3)) + 1/3*(b*c*(a/b)^(2/3) + a*(a/b)^(1/3)*e - a*d)*log(x + (a/b)^
(1/3))/(b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.11, size = 4261, normalized size = 22.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

[Out] 1/12*(6*e*x^2 - 2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3*e + b*c*d^2 + b*c^2*e + 2*a*d*e^2 + 1/2*(b^2*d^2 + 2*b^2*c*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + (b*d^3 + a*e^3)*x) + 12*d*x + ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^2*c + 4*b*c^2 + 16*a*d*e)/b^3) + 6*c)*log(-1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 - 1/2*(b^2*d^2 + 2*b^2*c*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + 2*(b*d^3 + a*e^3)*x + 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^3*e - 2*b^2*d^2 + 2*b^2*c*e)*sqrt(-((2

```
*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 -
3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3
 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b
*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*
(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d
^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) +
(1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 +
 a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/
b)*b^2*c + 4*b*c^2 + 16*a*d*e)/b^3)) + ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^
2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3
+ a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*
e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*
b - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2
+ a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5
+ (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqr
t(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b
^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(
b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(...
```

Sympy [A]
time = 0.83, size = 150, normalized size = 0.78

$$\operatorname{RootSum}\left(27t^3b^5 - 27t^2b^4c + t(9ab^2de + 9b^3c^2) - a^2e^3 - 3abcde + abd^3 - b^2c^3, \left(t \mapsto t\log\left(x + \frac{9t^2b^3e - 6tb^2ce - 3tb^2d^2 + 2ade^2 + bc^2e + bcd^2}{ae^3 + bd^3}\right)\right)\right) + \frac{dx}{b} + \frac{ex^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c + _t*(9*a*b**2*d*e + 9*b**3*c**2) -
 a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*c**3, Lambda(_t, _t*log(x + (9*_
t**2*b**3*e - 6*_t*b**2*c*e - 3*_t*b**2*d**2 + 2*a*d*e**2 + b*c**2*e + b*c*
d**2)/(a*e**3 + b*d**3)))) + d*x/b + e*x**2/(2*b)
```

Giac [A]
time = 1.08, size = 195, normalized size = 1.01

$$\frac{c\log\left(|bx^3 + a|\right)}{3\,b} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bd - \left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} + \frac{bx^2e + 2\,bdx}{2\,b^2} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bd + \left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3} + \frac{\left(ab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}e + ab^4d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*c*log(abs(b*x^3 + a))/b - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*d - (-a*b^2)^(2
/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/2*(b*x
```

```
^2*e + 2*b*d*x)/b^2 - 1/6*((-a*b^2)^(1/3)*b*d + (-a*b^2)^(2/3)*e)*log(x^2 +
 x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 1/3*(a*b^4*(-a/b)^(1/3)*e + a*b^4*d)*
(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5)
```

Mupad [B]
time = 5.13, size = 340, normalized size = 1.76

$$\left(\sum_{k=1}^{3}\ln\left(\frac{a\left(b\,c^2+\mathrm{root}(27\,b^5\,z^3-27\,b^4\,c\,z^2+9\,a\,b^2\,d\,e\,z+9\,b^3\,c^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k)^2\,b^3\,9+a\,d\,e-\mathrm{root}(27\,b^5\,z^3-27\,b^4\,c\,z^2+9\,a\,b^2\,d\,e\,z+9\,b^3\,c^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k)\,b^2\,c\,6+a\,e^2\,x+b\,c\,d\,x-\mathrm{root}(27\,b^5\,z^3-27\,b^4\,c\,z^2+9\,a\,b^2\,d\,e\,z+9\,b^3\,c^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k)\,b^2\,d\,x\,3\right)}{b}\right)\mathrm{root}\left(27\,b^5\,z^3-27\,b^4\,c\,z^2+9\,a\,b^2\,d\,e\,z+9\,b^3\,c^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k\right)\right)+\frac{e\,x^2}{2\,b}+\frac{d\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3),x)
```

```
[Out] symsum(log((a*(b*c^2 + 9*root(27*b^5*z^3 - 27*b^4*c*z^2 + 9*a*b^2*d*e*z + 9
*b^3*c^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)^2*b^3 + a*d*e
 - 6*root(27*b^5*z^3 - 27*b^4*c*z^2 + 9*a*b^2*d*e*z + 9*b^3*c^2*z - 3*a*b*c
*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*b^2*c + a*e^2*x + b*c*d*x - 3*roo
t(27*b^5*z^3 - 27*b^4*c*z^2 + 9*a*b^2*d*e*z + 9*b^3*c^2*z - 3*a*b*c*d*e + a
*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*b^2*d*x))/b)*root(27*b^5*z^3 - 27*b^4*c*z
^2 + 9*a*b^2*d*e*z + 9*b^3*c^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^
3, z, k), k, 1, 3) + (e*x^2)/(2*b) + (d*x)/b
```

3.339 $\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx$

Optimal. Leaf size=183

$$\frac{ex}{b}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{4/3}}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{4/3}}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(a^{2/3}-\right.}{6\sqrt[3]{a}\,b^{4/3}}$$

```
[Out] e*x/b-1/3*(b^(2/3)*c+a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(4/3)+1/6*(
b^(2/3)*c+a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(4
/3)+1/3*d*ln(b*x^3+a)/b-1/3*(b^(2/3)*c-a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(b^{2/3}c-a^{2/3}e\right)}{\sqrt{3}\sqrt[3]{a}\,b^{4/3}}+\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{4/3}}+\frac{d\log\left(a+bx^3\right)}{3b}+\frac{ex}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (e*x)/b - ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)) - ((b^(2/3)*c + a^(2/3)*e)*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(1/3)*b^(4/3)) + ((b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(4/3)) + (d*Log[a + b*x^3])
/(3*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx = \int \left(\frac{e}{b} - \frac{ae-bcx-bdx^2}{b\,(a+bx^3)}\right)\,dx$$

$$= \frac{ex}{b} - \frac{\int \frac{ae-bcx-bdx^2}{a+bx^3}\,dx}{b}$$

$$= \frac{ex}{b} - \frac{\int \frac{ae-bcx}{a+bx^3}\,dx}{b} + d\int \frac{x^2}{a+bx^3}\,dx$$

$$= \frac{ex}{b} + \frac{d\log\left(a+bx^3\right)}{3b} - \frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}\,bc+2a\sqrt[3]{b}\,e\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}\,bc-a\sqrt[3]{b}\,e\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}} - \frac{\left(b^{2/3}c \cdots\right.}{\cdots}$$

$$= \frac{ex}{b} - \frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{4/3}} + \frac{d\log\left(a+bx^3\right)}{3b} + \frac{\left(b^{2/3}c-a^{2/3}e\right)\int \frac{\cdots}{a^{2/3}\cdots}}{2b\cdots}$$

$$= \frac{ex}{b} - \frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{4/3}} + \frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+\cdots\right.}{6\sqrt[3]{a}\,b^{4/3}}$$

$$= \frac{ex}{b} - \frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{4/3}} - \frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{4/3}} + \cdots$$

Mathematica [A]
time = 0.04, size = 200, normalized size = 1.09

$$\frac{ex}{b} + \frac{\left(a^{2/3}bc-a^{4/3}\sqrt[3]{b}\,e\right)\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,ab^{5/3}} + \frac{\left(-a^{2/3}bc-a^{4/3}\sqrt[3]{b}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3ab^{5/3}} - \frac{\left(-a^{2/3}bc-a^{4/3}\sqrt[3]{b}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6ab^{5/3}} + \frac{d\log\left(a+bx^3\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (e*x)/b + ((a^(2/3)*b*c - a^(4/3)*b^(1/3)*e)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a*b^(5/3)) + ((-(a^(2/3)*b*c) - a^(4/3)*b^(1
/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a*b^(5/3)) - ((-(a^(2/3)*b*c) - a^(4/3)
*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a*b^(5/3)) +
 (d*Log[a + b*x^3])/(3*b)
```

Maple [A]
time = 0.39, size = 211, normalized size = 1.15

method	result

risch	$\frac{ex}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_R^2bd+\_Rbc-ae\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^2}$
default	$\frac{ex}{b}+\frac{-ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+bc\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] e*x/b+(-a*e*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1)))+b*c*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*d*ln(b*x^3+a))/b
```

Maxima [A]
time = 0.49, size = 177, normalized size = 0.97

$$\frac{xe}{b}+\frac{\sqrt{3}\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a\left(\frac{a}{b}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab}+\frac{\left(2bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] x*e/b + 1/3*sqrt(3)*(b*c*(a/b)^(2/3) - a*(a/b)^(1/3)*e)*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*b*d*(a/b)^(2/3) + b*c*(a/b)
^(1/3) + a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/
3*(b*d*(a/b)^(2/3) - b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(b^2*(a/b)
^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.14, size = 4628, normalized size = 25.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3
/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b
^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d
^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a
*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*log(-1/4*(2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*
d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*
e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e
)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^
2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*a*b^3*c - a*b*c*d^2 + 2*a*b*c^2*e + a^2*d*
e^2 - 1/2*(2*a*b^2*c*d - a^2*b*e^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^
2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)
^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^
3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d
/b) - (b^2*c^3 - a^2*e^3)*x) - 12*e*x - ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d
^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) +
(1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3)
- 2*d/b)*b - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2
- (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(
1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b
)^2*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*
d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(
a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(
2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)
/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*d + 4*d^2 - 16*c*e
)/b^2) + 6*d)*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e
)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d
*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt
(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*a*b^3*c
+ a*b*c*d^2 - 2*a*b*c^2*e - a^2*d*e^2 + 1/2*(2*a*b^2*c*d - a^2*b*e^2)*(2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2
- c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3
 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^
2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c
^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b) - 2*(b^2*c^3 - a^2*e^3)*x + 3/4*sqrt(
1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^
3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4)
```

```
 - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/
b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^
4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*a*b^3*c + 2*a*b^2*c*d + 2*
a^2*b*e^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^
2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*
a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*b^2 + 4*(2*(
1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2
 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^
3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d
^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*
c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*d + 4*d^2 - 16*c*e)/b^2)) - ((2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2
- c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3
 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^
2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c
^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*
d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*
e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e
)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^
2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d
^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 -...
```

Sympy [A]
time = 0.80, size = 160, normalized size = 0.87

$$\operatorname{RootSum}\left(27t^3ab^4-27t^2ab^3d+t\left(-9ab^2ce+9ab^2d^2\right)+a^2e^3+3abcde-abd^3+b^2c^3,\ \left(t\mapsto t\log\left(x+\frac{-9t^2ab^3c-3ta^2be^2+6tab^2cd+a^2de^2+2abc^2e-abcd^2}{a^2e^3-b^2c^3}\right)\right)\right)+\frac{ex}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a*b**4 - 27*_t**2*a*b**3*d + _t*(-9*a*b**2*c*e + 9*a*b**2*
d**2) + a**2*e**3 + 3*a*b*c*d*e - a*b*d**3 + b**2*c**3, Lambda(_t, _t*log(x
 + (-9*_t**2*a*b**3*c - 3*_t*a**2*b*e**2 + 6*_t*a*b**2*c*d + a**2*d*e**2 +
2*a*b*c**2*e - a*b*c*d**2)/(a**2*e**3 - b**2*c**3)))) + e*x/b
```

Giac [A]
time = 0.96, size = 178, normalized size = 0.97

$$\frac{\sqrt{3}\left(ae+\left(-ab^2\right)^{\frac{1}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}}+\frac{\left(ae-\left(-ab^2\right)^{\frac{1}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}}+\frac{xe}{b}+\frac{d\log\left(\left|bx^3+a\right|\right)}{3b}-\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(a*e + (-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(-a*b^2)^(2/3) + 1/6*(a*e - (-a*b^2)^(1/3)*c)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) + x*e/b + 1/3*d*log(abs(b*x^3 +
a))/b - 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a*b^3)
```

Mupad [B]
time = 5.16, size = 266, normalized size = 1.45

$$\left(\sum_{k=1}^{3}\ln\left(x\left(b\,c^2+a\,d\,e\right)-\mathrm{root}\left(27\,a\,b^4\,z^3-27\,a\,b^3\,d\,z^2-9\,a\,b^2\,c\,e\,z+9\,a\,b^2\,d^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\,\left(6\,a\,b\,d-\mathrm{root}\left(27\,a\,b^4\,z^3-27\,a\,b^3\,d\,z^2-9\,a\,b^2\,c\,e\,z+9\,a\,b^2\,d^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\,a\,b^2\,9+3\,a\,b\,e\,x\right)+a\,d^2-a\,c\,e\right)\,\mathrm{root}\left(27\,a\,b^4\,z^3-27\,a\,b^3\,d\,z^2-9\,a\,b^2\,c\,e\,z+9\,a\,b^2\,d^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\right)+\frac{e\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3),x)
```

```
[Out] symsum(log(x*(b*c^2 + a*d*e) - root(27*a*b^4*z^3 - 27*a*b^3*d*z^2 - 9*a*b^2
*c*e*z + 9*a*b^2*d^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*(
6*a*b*d - 9*root(27*a*b^4*z^3 - 27*a*b^3*d*z^2 - 9*a*b^2*c*e*z + 9*a*b^2*d^
2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a*b^2 + 3*a*b*e*x) +
 a*d^2 - a*c*e)*root(27*a*b^4*z^3 - 27*a*b^3*d*z^2 - 9*a*b^2*c*e*z + 9*a*b^
2*d^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k), k, 1, 3) + (e*x
)/b
```

3.340 $\int \frac{c+dx+ex^2}{a+bx^3} \, dx$

Optimal. Leaf size=177

$$-\frac{\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}+\frac{\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{6a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1/6*(c-a^(1
/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)+1/
3*e*ln(b*x^3+a)/b-1/3*(b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x
)/a^(1/3)*3^(1/2))/a^(2/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}-\frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{2/3}}+\frac{e\log\left(a+bx^3\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3),x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) + (e*Log[a + b*x^3])/(3*b
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx = e\int \frac{x^2}{a+bx^3}\,dx + \int \frac{c+dx}{a+bx^3}\,dx$$

$$= \frac{e\log\left(a+bx^3\right)}{3b} + \frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}} + \frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\cdots}}{3a^{2/3}\cdots}$$

$$= \frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{e\log\left(a+bx^3\right)}{3b} - \frac{\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b\cdots}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+\cdots}}{6a^{2/3}b^{2/3}\cdots}$$

$$= \frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}} + \cdots$$

$$= -\frac{\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}} + \frac{\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}\,c-\cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.06, size = 176, normalized size = 0.99

$$\frac{-2\sqrt{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{b}\left(\sqrt[3]{a}\,\sqrt[3]{b}\,c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-\sqrt[3]{b}\left(\sqrt[3]{a}\,\sqrt[3]{b}\,c-a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2ae\log\left(a+bx^3\right)}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(1/3
) + b^(1/3)*x] - b^(1/3)*(a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*e*Log[a + b*x^3])/(6*a*b)
```

Maple [A]
time = 0.38, size = 200, normalized size = 1.13

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_R^2e+\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^2}}{3b}$

default	$c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\right.\right.}{6}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))+d*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1)))+1/3*e*ln(b*x^3+a)/b
```

Maxima [A]
time = 0.52, size = 161, normalized size = 0.91

$$\frac{\sqrt{3}\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab}+\frac{\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}e+d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}e-d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b*d*(a/b)^(2/3) + b*c*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*(a/b)^(2/3)*e + d*(a/b)^(1/3) - c)*
log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*((a/b)^(2/3)*e
 - d*(a/b)^(1/3) + c)*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.17, size = 4671, normalized size = 26.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)
)/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b
^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2
```

```
*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)
*b*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^
2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) +
(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I
*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a
^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/
b)^2*a^2*b^2*d + 2*a*b*c*d^2 - a*b*c^2*e + a^2*d*e^2 - 1/2*(a*b^2*c^2 - 2*a
^2*b*d*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2
))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (
b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*
sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^
2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b
) + (b^2*c^3 + a*b*d^3)*x) - ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b
*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 +
a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/
3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) +
 (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2
*b^3))^(1/3) - 2*e/b)*b + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a
*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a
*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(
3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*
e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d
 + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3
 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b
^2)) + 6*e)*log(-1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*
e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a
^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2
)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 +
 a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1
/3) - 2*e/b)^2*a^2*b^2*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*e^2 + 1/2*(a*b^2
*c^2 - 2*a^2*b*d*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e
^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^
2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)
^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 +
a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/
3) - 2*e/b) + 2*(b^2*c^3 + a*b*d^3)*x + 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e
^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b
*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a^2*b^2*d + 2*a*b^2*c^2 + 2*
```

```
a^2*b*d*e)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2
)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*
b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(
1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*
d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3)
 - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a
*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(
a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3
+ a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(
1/3) - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b^2))) - ((2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e
^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b
*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*b - 3*sqrt(1/3)*b*sqrt(-((2*
(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3
 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3...
```

Sympy [A]
time = 0.79, size = 160, normalized size = 0.90

$$\operatorname{RootSum}\left(27t^3a^2b^3-27t^2a^2b^2e+t\left(9a^2be^2+9ab^2cd\right)-a^2e^3-3abcde+abd^3-b^2c^3,\ \left(t\mapsto t\log\left(x+\frac{9t^2a^2b^2d-6ta^2bde+3tab^2c^2+a^2de^2-abc^2e+2abcd^2}{abd^3+b^2c^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**3 - 27*_t**2*a**2*b**2*e + _t*(9*a**2*b*e**2 + 9*a
*b**2*c*d) - a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*c**3, Lambda(_t, _t*
log(x + (9*_t**2*a**2*b**2*d - 6*_t*a**2*b*d*e + 3*_t*a*b**2*c**2 + a**2*d*
e**2 - a*b*c**2*e + 2*a*b*c*d**2)/(a*b*d**3 + b**2*c**3))))
```

Giac [A]
time = 0.80, size = 163, normalized size = 0.92

$$-\frac{\sqrt{3}\left(bc-\left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}}-\frac{\left(bc+\left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}}+\frac{e\log\left(\left|bx^3+a\right|\right)}{3b}-\frac{\left(bd\left(-\frac{a}{b}\right)^{\frac{1}{3}}+bc\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c + (-a*b^2)^(1/3)*d)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) + 1/3*e*log(abs(b*x^3 + a))/b -
 1/3*(b*d*(-a/b)^(1/3) + b*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b)
```

Mupad [B]
time = 0.26, size = 274, normalized size = 1.55

$$\sum_{k=1}^{3}\ln\left(x\,\left(b\,d^2-b\,c\,e\right)+\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,a^2\,b^2\,e\,z^2+9\,a\,b^2\,c\,d\,z+9\,a^2\,b\,e^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k\right)\,\left(-6\,a\,b\,e+\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,a^2\,b^2\,e\,z^2+9\,a\,b^2\,c\,d\,z+9\,a^2\,b\,e^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k\right)\,a\,b^2\,9+3\,b^2\,c\,x\right)+a\,e^2+b\,c\,d\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,a^2\,b^2\,e\,z^2+9\,a\,b^2\,c\,d\,z+9\,a^2\,b\,e^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^3),x)
```

```
[Out] symsum(log(x*(b*d^2 - b*c*e) + root(27*a^2*b^3*z^3 - 27*a^2*b^2*e*z^2 + 9*a
*b^2*c*d*z + 9*a^2*b*e^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z,
k)*(9*root(27*a^2*b^3*z^3 - 27*a^2*b^2*e*z^2 + 9*a*b^2*c*d*z + 9*a^2*b*e^2*
z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a*b^2 - 6*a*b*e + 3*b^
2*c*x) + a*e^2 + b*c*d)*root(27*a^2*b^3*z^3 - 27*a^2*b^2*e*z^2 + 9*a*b^2*c*
d*z + 9*a^2*b*e^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k), k,
1, 3)
```

3.341 $\int \frac{c+dx+ex^2}{x(a+bx^3)}\,dx$

Optimal. Leaf size=184

$$-\frac{\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}+\frac{c\log(x)}{a}+\frac{\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log}{}$$

```
[Out] c*ln(x)/a+1/3*(b^(1/3)*d-a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1
/6*(d-a^(1/3)*e/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/
b^(1/3)-1/3*c*ln(b*x^3+a)/a-1/3*(b^(1/3)*d+a^(1/3)*e)*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,e+\sqrt[3]{b}\,d\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}-\frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{2/3}}-\frac{c\log\left(a+bx^3\right)}{3a}+\frac{c\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)),x]
```

```
[Out] -(((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + (c*Log[x])/a + ((b^(1/3)*d - a^(1/3)*e)*Log
[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((d - (a^(1/3)*e)/b^(1/3))*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) - (c*Log[a
 + b*x^3])/(3*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax}+\frac{ad+aex-bcx^2}{a\left(a+bx^3\right)}\right)\,dx$$

$$= \frac{c\log(x)}{a}+\frac{\int \frac{ad+aex-bcx^2}{a+bx^3}\,dx}{a}$$

$$= \frac{c\log(x)}{a}+\frac{\int \frac{ad+aex}{a+bx^3}\,dx}{a}-\frac{(bc)\int \frac{x^2}{a+bx^3}\,dx}{a}$$

$$= \frac{c\log(x)}{a}-\frac{c\log\left(a+bx^3\right)}{3a}+\frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}\,d+a^{4/3}e\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}\,d+a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{5/3}\sqrt[3]{b}}+\frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right.\cdots}{\cdots}$$

$$= \frac{c\log(x)}{a}+\frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{c\log\left(a+bx^3\right)}{3a}+\frac{1}{2}\left(\frac{d}{\sqrt[3]{a}}+\frac{e}{\sqrt[3]{b}}\right)\int \cdots$$

$$= \frac{c\log(x)}{a}+\frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b\cdots\right.}{6a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}+\frac{c\log(x)}{a}+\frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\cdots\right.}{3a^{2/3}\sqrt[3]{b}}$$

Mathematica [A]
time = 0.06, size = 176, normalized size = 0.96

$$\frac{-2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6b^{2/3}c\log(x)+2\left(\sqrt[3]{a}\sqrt[3]{b}\,d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+\left(-\sqrt[3]{a}\sqrt[3]{b}\,d+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2b^{2/3}c\log\left(a+bx^3\right)}{6ab^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] + 6*b^(2/3)*c*Log[x] + 2*(a^(1/3)*b^(1/3)*d - a^(2/3)*e)*Log[a
^(1/3) + b^(1/3)*x] + (-(a^(1/3)*b^(1/3)*d) + a^(2/3)*e)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b^(2/3)*c*Log[a + b*x^3])/(6*a*b^(2/3))
```

Maple [A]
time = 0.38, size = 211, normalized size = 1.15

method	result

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^3b^2\_Z^3+3a^2b^2c\_Z^2+\left(3a^2bde+3b^2c^2a\right)\_Z+a^2e^3+3abcde-ab\,d^3+b^2c^3\right)}\_R\ln\left(\left(-4\_R^3a^2b^2-8\_R^2a\,b^2c+(-10abde-\right.\right.\right.}{3}$
default	$\frac{ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+ae\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (a*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1)))+a*e*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1)))-1/3*c*ln(b*x^3+a))/a+c*ln(x)/a
```

Maxima [A]
time = 0.50, size = 179, normalized size = 0.97

$$\frac{c\log\left(x\right)}{a}+\frac{\sqrt{3}\left(a\left(\frac{a}{b}\right)^{\frac{2}{3}}e+ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2}-\frac{\left(2bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a\left(\frac{a}{b}\right)^{\frac{1}{3}}e+ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+a\left(\frac{a}{b}\right)^{\frac{1}{3}}e-ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] c*log(x)/a + 1/3*sqrt(3)*(a*(a/b)^(2/3)*e + a*d*(a/b)^(1/3))*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^2 - 1/6*(2*b*c*(a/b)^(2/3) - a*(a/b
)^(1/3)*e + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)) -
 1/3*(b*c*(a/b)^(2/3) + a*(a/b)^(1/3)*e - a*d)*log(x + (a/b)^(1/3))/(a*b*(a
/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.27, size = 4588, normalized size = 24.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a
^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54
*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3)
+ 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)
/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3
) + 6*c/a)*a*log(1/36*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))
/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^
2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) +
9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b
*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a
^3*b^2))^(1/3) + 6*c/a)^2*a^2*b*e + b*c*d^2 + b*c^2*e + 2*a*d*e^2 - 1/6*(a*
b*d^2 + 2*a*b*c*e)*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-
1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b
^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(
I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^
3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*
b^2))^(1/3) + 6*c/a) + (b*d^3 + a*e^3)*x) - (((-I*sqrt(3) + 1)*(c^2/a^2 - (
b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1
/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a
*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d
*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a + 3*sqrt(1/3)*a*sqrt(-(((-
I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b
*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 +
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*
c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)^2
*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^
3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1
/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(
3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e
^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(
1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b)) - 18*c)*log(-1/36*((-I
*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*
c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c
^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)^2*
a^2*b*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 + 1/6*(a*b*d^2 + 2*a*b*c*e)*((-I*sq
rt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2
 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*
e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/
a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/5
4*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a) + 2*(
b*d^3 + a*e^3)*x + 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*
```

```
d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3
 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b
^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3
*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a^2*b*e + 6*a*b*d^2 - 6*a*b*c*e)*sqrt(
-(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/
18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c
^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-
1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b
^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c
/a)^2*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/
27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2
) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*
sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3
+ a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^
2))^(1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b))) - (((-I*sqrt(3)
+ 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d
*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 +
1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2
*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a - 3*sqrt(
1/3)*a*sqrt(-(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.70, size = 179, normalized size = 0.97

$$-\frac{\sqrt{3}\left(bd-\left(-ab^2\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}}-\frac{\left(bd+\left(-ab^2\right)^{\frac{1}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}}-\frac{c\log\left(|bx^3+a|\right)}{3a}+\frac{c\log\left(|x|\right)}{a}-\frac{\left(a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+a^2bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*d - (-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*d + (-a*b^2)^(1/3)*e)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*c*log(abs(b*x^3 + a))/a +
```

```
c*log(abs(x))/a - 1/3*(a^2*b*(-a/b)^(1/3)*e + a^2*b*d)*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a^3*b)
```

Mupad [B]
time = 5.25, size = 716, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)),x)
```

```
[Out] symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - 36*root(27*a^3*b^2*z^3 + 27*
a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2
*e^3 + b^2*c^3, z, k)^3*a^2*b^3*x - a*b*e^3*x - root(27*a^3*b^2*z^3 + 27*a^
2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e
^3 + b^2*c^3, z, k)*a*b^2*d^2 - 4*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 +
9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3,
z, k)*b^3*c^2*x + 3*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z
+ 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)^2*a^2*b^
2*e - 24*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c
^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)^2*a*b^3*c*x - 2*roo
t(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b
*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a*b^2*c*e - 2*b^2*c*d*e*x - 10*
root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*
a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a*b^2*d*e*x)*root(27*a^3*b^2
*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b
*d^3 + a^2*e^3 + b^2*c^3, z, k), k, 1, 3) + (c*log(x))/a
```

3.342 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=192

$$-\frac{c}{ax}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}\sqrt[3]{b}}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}\sqrt[3]{b}}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\mathrm{lo}}{}$$

```
[Out] -c/a/x+d*ln(x)/a+1/3*(b^(2/3)*c+a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^
(1/3)-1/6*(b^(2/3)*c+a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a
^(4/3)/b^(1/3)-1/3*d*ln(b*x^3+a)/a+1/3*(b^(2/3)*c-a^(2/3)*e)*arctan(1/3*(a^
(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(b^{2/3}c-a^{2/3}e\right)}{\sqrt{3}\,a^{4/3}\sqrt[3]{b}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}}+\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}\sqrt[3]{b}}-\frac{d\log\left(a+bx^3\right)}{3a}-\frac{c}{ax}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(c/(a*x)) + ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(1/3)) + (d*Log[x])/a + ((b^(2/3)*c + a^(2
/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*b^(1/3)) - ((b^(2/3)*c + a^(2/3
)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(1/3)) -
(d*Log[a + b*x^3])/(3*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = \int \left(\frac{c}{ax^2}+\frac{d}{ax}+\frac{ae-bcx-bdx^2}{a(a+bx^3)}\right)dx$$
$$= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{\int \frac{ae-bcx-bdx^2}{a+bx^3}\,dx}{a}$$
$$= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{\int \frac{ae-bcx}{a+bx^3}\,dx}{a}-\frac{(bd)\int \frac{x^2}{a+bx^3}\,dx}{a}$$
$$= -\frac{c}{ax}+\frac{d\log(x)}{a}-\frac{d\log(a+bx^3)}{3a}+\frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}\,bc+2a\sqrt[3]{b}\,e\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}\,bc-a\sqrt[3]{b}\,e\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{5/3}\sqrt[3]{b}}$$
$$= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{(b^{2/3}c+a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}\sqrt[3]{b}}-\frac{d\log(a+bx^3)}{3a}-\frac{(b^{2/3}c-a^{2/3}}{}$$
$$= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{(b^{2/3}c+a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}\sqrt[3]{b}}-\frac{(b^{2/3}c+a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{6a^{4/3}\sqrt[3]{b}}$$
$$= -\frac{c}{ax}+\frac{(b^{2/3}c-a^{2/3}e)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}\sqrt[3]{b}}+\frac{d\log(x)}{a}+\frac{(b^{2/3}c+a^{2/3}e)\log\left(\sqrt[3]{a}+\right.}{3a^{4/3}\sqrt[3]{b}}$$

Mathematica [A]
time = 0.16, size = 184, normalized size = 0.96

$$-\frac{\frac{6ac}{x}+\frac{2\sqrt{3}\,a^{2/3}(-b^{2/3}c+a^{2/3}e)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-6ad\log(x)-\frac{2(a^{2/3}b^{2/3}c+a^{4/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+\frac{(a^{2/3}b^{2/3}c+a^{4/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+2ad\log(a+bx^3)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -1/6*((6*a*c)/x + (2*Sqrt[3]*a^(2/3)*(-(b^(2/3)*c) + a^(2/3)*e)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - 6*a*d*Log[x] - (2*(a^(2/3)*b^(2
/3)*c + a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((a^(2/3)*b^(2/3)*c
+ a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 2*a*
d*Log[a + b*x^3])/a^2
```

Maple [A]
time = 0.38, size = 221, normalized size = 1.15

method	result

risch	$-\frac{c}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{4}b\_Z^{3}+3a^{3}bd\_Z^{2}+\left(-3a^{2}bce+3a^{2}b\,d^{2}\right)\_Z-a^{2}e^{3}-3abcde+ab\,d^{3}-b^{2}c^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{4}b-8\_R^{2}a^{3}bd+\right.\right.\right.}{3}$
default	$\frac{ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-bc\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (a*e*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1)))-b*c*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1)))-1/3*d*ln(b*x^3+a))/a-c/a/x+d*ln(x)/a
```

Maxima [A]
time = 0.50, size = 189, normalized size = 0.98

$$\frac{d\log\left(x\right)}{a}-\frac{\sqrt{3}\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a\left(\frac{a}{b}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}}-\frac{\left(2\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] d*log(x)/a - 1/3*sqrt(3)*(b*c*(a/b)^(2/3) - a*(a/b)^(1/3)*e)*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^2 - 1/6*(2*b*d*(a/b)^(2/3) + b*c*(a
/b)^(1/3) + a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)) -
 1/3*(b*d*(a/b)^(2/3) - b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(a*b*(a
/b)^(2/3)) - c/(a*x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.26, size = 4524, normalized size = 23.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/1
8*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b
) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/
a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*x*log(-1/36
*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 -
 c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54
*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/
18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*
b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^3*b*c - a*b*c*d^2
 + 2*a*b*c^2*e + a^2*d*e^2 + 1/6*(2*a^2*b*c*d - a^3*e^2)*((-I*sqrt(3) + 1)*
(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*
(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3
)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^
3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3
- a^2*e^3)/(a^4*b))^(1/3) + 6*d/a) - (b^2*c^3 - a^2*e^3)*x) - 36*d*x*log(x)
 - (((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^
2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1
/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 +
 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a
^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*x - 3*sqrt(1/3)*
a*x*sqrt(-(((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1
/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4
*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^
3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*
a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^2 - 12*
((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 -
c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*
(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/1
8*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b
) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*d + 36*d^2 - 144*c*e
)/a^2) - 18*d*x)*log(1/36*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1
/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3)
 + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (
d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d
/a)^2*a^3*b*c + a*b*c*d^2 - 2*a*b*c^2*e - a^2*d*e^2 - 1/6*(2*a^2*b*c*d - a^
3*e^2)*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*
(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b)
- 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^
3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)
/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a) - 2*(b^2*c^3 -
a^2*e^3)*x + 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/
(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt
```

```
(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) +
6*d/a)*a^3*b*c - 6*a^2*b*c*d - 6*a^3*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(d^2/a^2
 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3
 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b
))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54
*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^
3)/(a^4*b))^(1/3) + 6*d/a)^2*a^2 - 12*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c
*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) +
9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 +
 a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))
^(1/3) + 6*d/a)*a*d + 36*d^2 - 144*c*e)/a^2)) - (((-I*sqrt(3) + 1)*(d^2/a^2
 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3
 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b
))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54
*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^
3)/(a^4*b))^(1/3) + 6*d/a)*a*x + 3*sqrt(1/3)*a*x*sqrt(-(((-I*sqrt(3) + 1)*(
d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(
b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)
/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3
 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.02, size = 201, normalized size = 1.05

$$-\frac{d\log\left(|bx^3+a|\right)}{3\,a}+\frac{d\log\left(|x|\right)}{a}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ae+\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b}-\frac{c}{ax}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ae-\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b}+\frac{\left(ab^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a + d*log(abs(x))/a + 1/3*sqrt(3)*((-a*b^2)^(1/3
)*a*e + (-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(a^2*b) - c/(a*x) + 1/6*((-a*b^2)^(1/3)*a*e - (-a*b^2)^(2/3)*c)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) + 1/3*(a*b^2*c*(-a/b)^(1/3) - a^
2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b)
```

Mupad [B]
time = 5.06, size = 723, normalized size = 3.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)),x)
```

```
[Out] symsum(log((b^4*c^3*x + a^2*b^2*d*e^2 - 36*root(27*a^4*b*z^3 + 27*a^3*b*d*z
^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*
c^3, z, k)^3*a^4*b^3*x + a^2*b^2*e^3*x + a*b^3*c*d^2 - 3*root(27*a^4*b*z^3
+ 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 -
a^2*e^3 - b^2*c^3, z, k)^2*a^3*b^3*c - root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 -
 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3,
 z, k)*a^3*b^2*e^2 - 4*root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z +
 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a^2*b^3*d
^2*x - 24*root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*
z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)^2*a^3*b^3*d*x + 2*root
(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*
e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a^2*b^3*c*d + 2*a*b^3*c*d*e*x + 10*r
oot(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c
*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a^2*b^3*c*e*x)/a^2)*root(27*a^4*b
*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d
^3 - a^2*e^3 - b^2*c^3, z, k), k, 1, 3) - c/(a*x) + (d*log(x))/a
```

3.343 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=203

$$-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{\sqrt[3]{b}\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{e\log(x)}{a}-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}}$$

```
[Out] -1/2*c/a/x^2-d/a/x+e*ln(x)/a-1/3*b^(1/3)*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b
^(1/3)*x)/a^(5/3)+1/6*b^(2/3)*(c-a^(1/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(5/3)-1/3*e*ln(b*x^3+a)/a+1/3*b^(1/3)*(b^(1/3)*c+a^(1/
3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)}{\sqrt{3}\,a^{5/3}}+\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}}-\frac{e\log\left(a+bx^3\right)}{3a}-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*c/(a*x^2) - d/(a*x) + (b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (e*Log[x])/a - (b^(
1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)) + (b^(2/
3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*a^(5/3)) - (e*Log[a + b*x^3])/(3*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^3\,(a+bx^3)}\,dx = \int \left(\frac{c}{ax^3}+\frac{d}{ax^2}+\frac{e}{ax}-\frac{b(c+dx+ex^2)}{a\,(a+bx^3)}\right)dx$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b\int \frac{c+dx+ex^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b\int \frac{c+dx}{a+bx^3}\,dx}{a}-\frac{(be)\int \frac{x^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{e\log(a+bx^3)}{3a}-\frac{b^{2/3}\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}\,c+\sqrt[3]{\ldots}\right.}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}}{3a^{5/3}}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}}-\frac{e\log(a+bx^3)}{3a}+\ldots$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}}+\frac{\sqrt[3]{b}\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\ldots\right.}{\ldots}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{\sqrt[3]{b}\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{e\log(x)}{a}-\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}}{\sqrt[3]{\ldots}}\right.}{\ldots}$$

Mathematica [A]

time = 0.12, size = 192, normalized size = 0.95

$$\frac{-\frac{3ac}{x^2}-\frac{6ad}{x}+2\sqrt{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6ae\log(x)+2\sqrt[3]{b}\left(-\sqrt[3]{a}\,\sqrt[3]{b}\,c+a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+\sqrt[3]{b}\left(\sqrt[3]{a}\,\sqrt[3]{b}\,c-a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2ae\log(a+bx^3)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a*c)/x^2 - (6*a*d)/x + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a^(1/3)*
d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 6*a*e*Log[x] + 2*b^(1/3)*(
-(a^(1/3)*b^(1/3)*c) + a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(1/
3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] -
2*a*e*Log[a + b*x^3])/(6*a^2)
```

Maple [A]

time = 0.38, size = 232, normalized size = 1.14

method	result

risch	$\frac{-\frac{xd}{a}-\frac{c}{2a}}{x^2}+\frac{e\ln(-x)}{a}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^5\_Z^3+3a^4e\_Z^2+\left(3a^3e^2+3bcd\,a^2\right)\_Z+a^2e^3+3abcde-ab\,d^3+b^2c^3\right)}\_R\ln\left(\left(-4\_R^3a^5-8\_\ldots\right.\right.\right.}{\ldots}$
default	$-\frac{\left(c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\ldots\right.\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/
3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1)))+d*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1)))+1/3*e*ln(b*x^3+a)/b)*b/a-1/2*c/a/x^2-d/a/x+e*ln(x)/a
```

Maxima [A]
time = 0.54, size = 180, normalized size = 0.89

$$\frac{e\log(x)}{a}-\frac{\sqrt{3}\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2}-\frac{\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}e+d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}e-d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2dx+c}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] e*log(x)/a - 1/3*sqrt(3)*(b*d*(a/b)^(2/3) + b*c*(a/b)^(1/3))*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^2 - 1/6*(2*(a/b)^(2/3)*e + d*(a/b)^
(1/3) - c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*((a
/b)^(2/3)*e - d*(a/b)^(1/3) + c)*log(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2
*(2*d*x + c)/(a*x^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.19, size = 4279, normalized size = 21.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 +
 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 +
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^
3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3
 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a*x^2*log(1/36*((-I*s
qrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d +
a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3
- 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d
 + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d
^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^4*d + 2*a*b*c*d^2 - a*b*c^2*e +
a^2*d*e^2 + 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d
 + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 +
a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) +
9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3
 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3)
 + 6*e/a) + (b^2*c^3 + a*b*d^3)*x) - 36*e*x^2*log(x) + 36*d*x - (((-I*sqrt(
3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^
2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*
c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a
*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a*x^2 + 3*sqrt(1/3)*a*x^2*sqrt(-(((-I*sq
rt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a
*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d
+ a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^
3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/a^2
 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*
(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)
^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/
54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a
^5)^(1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3) - 18*e*x^2)*log(-1/36
*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b
*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18
*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e
^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^4*d - 2*a*b*c*d^2 + a*b*c
^2*e - a^2*d*e^2 - 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*sqrt(3) + 1)*(e^2/a^2 -
 (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b
*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(
1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54
*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5
)^(1/3) + 6*e/a) + 2*(b^2*c^3 + a*b*d^3)*x + 1/12*sqrt(1/3)*(((-I*sqrt(3) +
 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e
/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*
e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2
```

```
)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a^4*d - 6*a^2*b*c^2 - 6*a^3*d*e)*sqrt(-(((-I
*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d
+ a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^
3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c
*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/
a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/
54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a
^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 +
 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b
)/a^5)^(1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3)) - (((-I*sqrt(3) +
 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e
/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*
e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2
)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a*x^2 - 3*sqrt(1/3)*a*x^2*sqrt(-(((-I*sqrt(3
) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2
)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*
e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.34, size = 204, normalized size = 1.00

$$-\frac{e\log\left(|bx^3+a|\right)}{3\,a}+\frac{e\log\left(|x|\right)}{a}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc-\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b}+\frac{\left(ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b}-\frac{2\,dx+c}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a + e*log(abs(x))/a - 1/3*sqrt(3)*((-a*b^2)^(1/3
)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(a^2*b) - 1/6*((-a*b^2)^(1/3)*b*c + (-a*b^2)^(2/3)*d)*log(x^2 + x*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a^2*b) + 1/3*(a*b^2*d*(-a/b)^(1/3) + a*b^2*c)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) - 1/2*(2*d*x + c)/(a*x^2)
```

Mupad [B]
time = 0.13, size = 701, normalized size = 3.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)),x)
```

```
[Out] symsum(log(-(b^5*c^3*x - a^2*b^3*d*e^2 + 36*root(27*a^5*z^3 + 27*a^4*e*z^2
+ 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3,
z, k)^3*a^5*b^3*x - a*b^4*c^2*e - a*b^4*d^3*x + root(27*a^5*z^3 + 27*a^4*e*
z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c
^3, z, k)*a^2*b^4*c^2 + 3*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z +
9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)^2*a^4*b^3*d
+ 4*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*
d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a^3*b^3*e^2*x + 24*root(27*a^5*z^3
 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2
*e^3 + b^2*c^3, z, k)^2*a^4*b^3*e*x - 2*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*
a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k
)*a^3*b^3*d*e + 2*a*b^4*c*d*e*x + 10*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2
*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a
^2*b^4*c*d*x)/a^3)*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z + 9*a^3*e
^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k), k, 1, 3) - c/(2*a*
x^2) - d/(a*x) + (e*log(x))/a
```

3.344 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=190

$$-\frac{c+dx+ex^2}{3b\,(a+bx^3)}-\frac{\left(\sqrt[3]{b}\,d+2\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{5/3}}+\frac{\left(\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{5/3}}-\frac{\left(d-\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{}$$

```
[Out] 1/3*(-e*x^2-d*x-c)/b/(b*x^3+a)+1/9*(b^(1/3)*d-2*a^(1/3)*e)*ln(a^(1/3)+b^(1/
3)*x)/a^(2/3)/b^(5/3)-1/18*(d-2*a^(1/3)*e/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)-1/9*(b^(1/3)*d+2*a^(1/3)*e)*arctan(1/3*(a
^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(5/3)*3^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1837, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(2\sqrt[3]{a}\,e+\sqrt[3]{b}\,d\right)}{3\sqrt{3}\,a^{2/3}b^{5/3}}-\frac{\left(d-\frac{2\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{5/3}}-\frac{c+dx+ex^2}{3b\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(c + d*x + e*x^2)/(b*(a + b*x^3)) - ((b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[
(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(5/3)) + (
(b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(5/3)) - (
(d - (2*a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(18*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx = -\frac{c+dx+ex^2}{3b\,(a+bx^3)} + \frac{\int \frac{d+2ex}{a+bx^3}\,dx}{3b}$$

$$= -\frac{c+dx+ex^2}{3b\,(a+bx^3)} + \frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}\,d+2\sqrt[3]{a}\,e\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}\,d+2\sqrt[3]{a}\,e\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{2/3}b^{4/3}} + \frac{\left(d-\frac{2\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)}{9a}$$

$$= -\frac{c+dx+ex^2}{3b\,(a+bx^3)} + \frac{\left(d-\frac{2\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{4/3}} + \frac{\left(\frac{\sqrt[3]{b}\,d}{\sqrt[3]{a}}+2e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}}}{6b^{4/3}}$$

$$= -\frac{c+dx+ex^2}{3b\,(a+bx^3)} + \frac{\left(d-\frac{2\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{4/3}} - \frac{\left(\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(a^{2/3}\right.}{18a^{2/3}b^{5}}$$

$$= -\frac{c+dx+ex^2}{3b\,(a+bx^3)} - \frac{\left(\sqrt[3]{b}\,d+2\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{5/3}} + \frac{\left(d-\frac{2\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}\right.}{9a^{2/3}b^{4/3}}$$

Mathematica [A]
time = 0.09, size = 174, normalized size = 0.92

$$\frac{-\frac{6b^{2/3}(c+x(d+ex))}{a+bx^3} - \frac{2\sqrt{3}\left(\sqrt[3]{b}\,d+2\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2\left(\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}} + \frac{\left(-\sqrt[3]{b}\,d+2\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}}}{18b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3) - (2*Sqrt[3]*(b^(1/3)*d + 2*a^(
1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(b^(1/3)*
d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(b^(1/3)*d) + 2*a^(
1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(5/3
))
```

Maple [A]
time = 0.36, size = 226, normalized size = 1.19

method	result
risch	$\frac{-\frac{e\,x^2}{3b}-\frac{dx}{3b}-\frac{c}{3b}}{b\,x^3+a} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^3+a\right)} \frac{\left(2e\,\_R+d\right)\ln\left(x-\_R\right)}{\_R^2}}{9b^2}$

default	$\frac{-\frac{e x^2}{3b}-\frac{dx}{3b}-\frac{c}{3b}}{b x^3+a}+\frac{d\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+2e\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2\right.}{}\right.}{3b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*e*x^2/b-1/3*d*x/b-1/3*c/b)/(b*x^3+a)+1/3/b*(d*(1/3/b/(a/b)^(2/3)*ln(x
+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+2*e*(-1/3/b/(a/b)
^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.51, size = 167, normalized size = 0.88

$$-\frac{x^2e+dx+c}{3\left(b^2x^3+ab\right)}+\frac{\sqrt{3}\left(2\left(\frac{a}{b}\right)^{\frac{1}{3}}e+d\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2\left(\frac{a}{b}\right)^{\frac{1}{3}}e-d\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2\left(\frac{a}{b}\right)^{\frac{1}{3}}e-d\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(x^2*e + d*x + c)/(b^2*x^3 + a*b) + 1/9*sqrt(3)*(2*(a/b)^(1/3)*e + d)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) + 1/1
8*(2*(a/b)^(1/3)*e - d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(
2/3)) - 1/9*(2*(a/b)^(1/3)*e - d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.12, size = 2077, normalized size = 10.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*e*x^2 + 2*(b^2*x^3 + a*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 +
8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e
*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a
```

```
^2*b^5))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a
^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3)
 - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1
/3)))^2*a^2*b^3*e - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^
2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3)
- 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/
3)))*a*b^2*d^2 + 8*a*d*e^2 + (b*d^3 + 8*a*e^3)*x) + 12*d*x - ((b^2*x^3 + a*
b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a
*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 +
 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3))) + 3*sqrt(1/3)*(b
^2*x^3 + a*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b
^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1
)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3))
)^2*a*b^3 + 32*d*e)/(a*b^3)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3
 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*
d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)
/(a^2*b^5))^(1/3)))^2*a^2*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3
+ 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d
*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/
(a^2*b^5))^(1/3)))*a*b^2*d^2 - 8*a*d*e^2 + 2*(b*d^3 + 8*a*e^3)*x + 3/2*sqrt
(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 -
 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d
^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a^2*b^3*e +
a*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5)
 + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(
a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2
*a*b^3 + 32*d*e)/(a*b^3))) - ((b^2*x^3 + a*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*
((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)
^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8
*a*e^3)/(a^2*b^5))^(1/3))) - 3*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-(((1/2)^(1/3
)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5
))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2
*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a*b^3 + 32*d*e)/(a*b^3)))*lo
g(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 -
 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d
^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a^2*b^3*e
+ 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 -
8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^
3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a*b^2*d^2 - 8
*a*d*e^2 + 2*(b*d^3 + 8*a*e^3)*x - 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) +
 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(
1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3
 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a^2*b^3*e + a*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(
I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^
(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^
```

```
5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a*b^3 + 32*d*e)/(a*b^3))) + 12*
c)/(b^2*x^3 + a*b)
```

Sympy [A]
time = 1.16, size = 110, normalized size = 0.58

$$\operatorname{RootSum}\left(729t^3a^2b^5 + 54tab^2de + 8ae^3 - bd^3, \left(t \mapsto t\log\left(x + \frac{162t^2a^2b^3e + 9tab^2d^2 + 8ade^2}{8ae^3 + bd^3}\right)\right)\right) + \frac{-c - dx - ex^2}{3ab + 3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**2*b**5 + 54*_t*a*b**2*d*e + 8*a*e**3 - b*d**3, Lambda(
_t, _t*log(x + (162*_t**2*a**2*b**3*e + 9*_t*a*b**2*d**2 + 8*a*d*e**2)/(8*a
*e**3 + b*d**3)))) + (-c - d*x - e*x**2)/(3*a*b + 3*b**2*x**3)
```

Giac [A]
time = 1.13, size = 180, normalized size = 0.95

$$-\frac{\sqrt{3}\left(bd - 2\left(-ab^2\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}b} - \frac{\left(bd + 2\left(-ab^2\right)^{\frac{1}{3}}e\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}b} - \frac{\left(2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e + d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab} - \frac{x^2e + dx + c}{3\left(bx^3 + a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(b*d - 2*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/18*(b*d + 2*(-a*b^2)^(1/3)*e)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/9*(2*(-a/b)^(1
/3)*e + d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 1/3*(x^2*e + d*x
 + c)/((b*x^3 + a)*b)
```

Mupad [B]
time = 0.22, size = 180, normalized size = 0.95

$$\left(\sum_{k=1}^{3}\ln\left(\frac{2\,d\,e + 4\,e^2\,x + \mathrm{root}(729\,a^2\,b^5\,z^3 + 54\,a\,b^2\,d\,e\,z + 8\,a\,e^3 - b\,d^3, z, k)^2\,a\,b^3\,81 + \mathrm{root}(729\,a^2\,b^5\,z^3 + 54\,a\,b^2\,d\,e\,z + 8\,a\,e^3 - b\,d^3, z, k)\;b^2\,d\,x\,9}{b\,9}\right)\mathrm{root}\left(729\,a^2\,b^5\,z^3 + 54\,a\,b^2\,d\,e\,z + 8\,a\,e^3 - b\,d^3, z, k\right)\right) - \frac{\frac{c}{3\,b} + \frac{e\,x^2}{3\,b} + \frac{d\,x}{3\,b}}{b\,x^3 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((2*d*e + 4*e^2*x + 81*root(729*a^2*b^5*z^3 + 54*a*b^2*d*e*z + 8*
a*e^3 - b*d^3, z, k)^2*a*b^3 + 9*root(729*a^2*b^5*z^3 + 54*a*b^2*d*e*z + 8*
a*e^3 - b*d^3, z, k)*b^2*d*x)/(9*b))*root(729*a^2*b^5*z^3 + 54*a*b^2*d*e*z
+ 8*a*e^3 - b*d^3, z, k), k, 1, 3) - (c/(3*b) + (e*x^2)/(3*b) + (d*x)/(3*b)
)/(a + b*x^3)
```

3.345 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2} dx$

Optimal. Leaf size=200

$$-\frac{x(ae-bcx-bdx^2)}{3ab(a+bx^3)}-\frac{(b^{2/3}c+a^{2/3}e)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{4/3}}-\frac{(b^{2/3}c-a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}}+\frac{(b^{2/3}c-a}{}$$

```
[Out] -1/3*x*(-b*d*x^2-b*c*x+a*e)/a/b/(b*x^3+a)-1/9*(b^(2/3)*c-a^(2/3)*e)*ln(a^(1
/3)+b^(1/3)*x)/a^(4/3)/b^(4/3)+1/18*(b^(2/3)*c-a^(2/3)*e)*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(4/3)-1/9*(b^(2/3)*c+a^(2/3)*e)*arctan(1
/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(4/3)*3^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1842, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^{2/3}e+b^{2/3}c)}{3\sqrt{3}a^{4/3}b^{4/3}}+\frac{(b^{2/3}c-a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{4/3}}-\frac{(b^{2/3}c-a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}}-\frac{x(ae-bcx-bdx^2)}{3ab(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(x*(a*e - b*c*x - b*d*x^2))/(a*b*(a + b*x^3)) - ((b^(2/3)*c + a^(2/3)*
e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^
(4/3)) - ((b^(2/3)*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(4
/3)) + ((b^(2/3)*c - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(4/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx = -\frac{x(ae-bcx-bdx^2)}{3ab\,(a+bx^3)} - \frac{\int \frac{-ae-bcx}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{x(ae-bcx-bdx^2)}{3ab\,(a+bx^3)} - \frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}\,bc-2a\sqrt[3]{b}\,e\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}\,bc+a\sqrt[3]{b}\,e\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{5/3}b^{4/3}} - \frac{(b^{2/3}c}{}$$

$$= -\frac{x(ae-bcx-bdx^2)}{3ab\,(a+bx^3)} - \frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{4/3}} + \frac{\left(b^{2/3}c-a^{2/3}e\right)\int \frac{-}{a^{2/3}-}}{18a^{4/3}}$$

$$= -\frac{x(ae-bcx-bdx^2)}{3ab\,(a+bx^3)} - \frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{4/3}} + \frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(a\right.}{18a}$$

$$= -\frac{x(ae-bcx-bdx^2)}{3ab\,(a+bx^3)} - \frac{\left(b^{2/3}c+a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{4/3}} - \frac{\left(b^{2/3}c-a^{2/3}e\right)\log}{9a^{4/3}b}$$

Mathematica [A]

time = 0.12, size = 186, normalized size = 0.93

$$\frac{-\frac{6ab^{2/3}(-bcx^2+a(d+ex))}{a+bx^3}-2\sqrt{3}\left(a^{2/3}bc+a^{4/3}\sqrt[3]{b}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(-a^{2/3}bc+a^{4/3}\sqrt[3]{b}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-\left(-a^{2/3}bc+a^{4/3}\sqrt[3]{b}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^2b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a*b^(2/3)*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3) - 2*Sqrt[3]*(a^(2/3)
*b*c + a^(4/3)*b^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(
-(a^(2/3)*b*c) + a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] - (-(a^(2/3)*b
*c) + a^(4/3)*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1
8*a^2*b^(5/3))
```

Maple [A]

time = 0.37, size = 230, normalized size = 1.15

method	result
risch	$\frac{\frac{c\,x^2}{3a}-\frac{ex}{3b}-\frac{d}{3b}}{b\,x^3+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^3+a\right)}\frac{\left(\frac{c\,\_R}{a}+\frac{e}{b}\right)\ln\left(x-\_R\right)}{\_R^2}}{9b}$

default	$\frac{\frac{c\,x^{2}}{3a}-\frac{ex}{3b}-\frac{d}{3b}}{b\,x^{3}+a}+\frac{ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+bc\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\right.}{}\right.}{3ba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/3/a*c*x^2-1/3*e*x/b-1/3*d/b)/(b*x^3+a)+1/3/b/a*(a*e*(1/3/b/(a/b)^(2/3)*l
n(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/
(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+b*c*(-1/3/b/(a
/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.51, size = 189, normalized size = 0.94

$$\frac{bcx^{2}-axe-ad}{3\left(ab^{2}x^{3}+a^{2}b\right)}+\frac{\sqrt{3}\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*c*x^2 - a*x*e - a*d)/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(b*c*(a/b)^(1
/3) + a*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)
^(2/3)) + 1/18*(b*c*(a/b)^(1/3) - a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(a*b^2*(a/b)^(2/3)) - 1/9*(b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(
a*b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.17, size = 2358, normalized size = 11.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b*c*x^2 - 12*a*e*x - 2*(a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
```

- 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
- 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c - 1/2*((1/2)^(1/3)*
(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b
^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^
3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 + 2*a*b*c^2
*e + (b^2*c^3 + a^2*e^3)*x) - 12*a*d + ((a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I
sqrt(3) + 1)((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4
))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)
/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3))) + 3*sqrt(1/3)*(a*b^2*x^
3 + a^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) +
1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4)
)^(1/3)))^2*a^2*b^2 + 16*c*e)/(a^2*b^2))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
- 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4
) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c + 1/2*((1/2)^(1/3)*(
I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^
4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3
)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 - 2*a*b*c^2*
e + 2*(b^2*c^3 + a^2*e^3)*x + 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(
(b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1
/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b
^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b^3*c + 2*a^3*b*e^2)*sqrt(-(((1/2)
^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)
/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 +
a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^2*b^2 + 16
*c*e)/(a^2*b^2))) + ((a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2
*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^
(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c
^3 - a^2*e^3)/(a^4*b^4))^(1/3))) - 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-((
(1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2
*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*
c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^2*b^2
+ 16*c*e)/(a^2*b^2)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^
2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e
*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*
e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^
2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)
^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*
c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 - 2*a*b*c^2*e + 2*(b^2*c^3 + a^
2*e^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)
/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*s

```
qrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(
a^4*b^4))^(1/3)))*a^3*b^3*c + 2*a^3*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) +
 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) -
 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4)
 - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^2*b^2 + 16*c*e)/(a^2*b^2))))/
(a*b^2*x^3 + a^2*b)
```

Sympy [A]
time = 0.91, size = 124, normalized size = 0.62

$$\operatorname{RootSum}\left(729t^3a^4b^4 + 27ta^2b^2ce - a^2e^3 + b^2c^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^3b^3c + 9ta^3be^2 + 2abc^2e}{a^2e^3 + b^2c^3}\right)\right)\right) + \frac{-ad - aex + bcx^2}{3a^2b + 3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**4*b**4 + 27*_t*a**2*b**2*c*e - a**2*e**3 + b**2*c**3,
Lambda(_t, _t*log(x + (81*_t**2*a**3*b**3*c + 9*_t*a**3*b*e**2 + 2*a*b*c**2
*e)/(a**2*e**3 + b**2*c**3)))) + (-a*d - a*e*x + b*c*x**2)/(3*a**2*b + 3*a*
b**2*x**3)
```

Giac [A]
time = 1.09, size = 190, normalized size = 0.95

$$-\frac{\sqrt{3}\left(ae - \left(-ab^2\right)^{\frac{1}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{\left(ae + \left(-ab^2\right)^{\frac{1}{3}}c\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{\left(bc\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} + \frac{bcx^2 - axe - ad}{3\left(bx^3 + a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(a*e - (-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(a*e + (-a*b^2)^(1/3)*c)*log(x^2
 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(b*c*(-a/b)^(1/3
) + a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) + 1/3*(b*c*x^2 - a
*x*e - a*d)/((b*x^3 + a)*a*b)
```

Mupad [B]
time = 5.17, size = 194, normalized size = 0.97

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(729\,a^4\,b^4\,z^3 + 27\,a^2\,b^2\,c\,e\,z + b^2\,c^3 - a^2\,e^3, z, k\right)\,\left(b\,e\,x + \mathrm{root}\left(729\,a^4\,b^4\,z^3 + 27\,a^2\,b^2\,c\,e\,z + b^2\,c^3 - a^2\,e^3, z, k\right)\,a\,b^2\,9\right) + \frac{c\,e}{9\,a} + \frac{b\,c^2\,x}{9\,a^2}\right)\,\mathrm{root}\left(729\,a^4\,b^4\,z^3 + 27\,a^2\,b^2\,c\,e\,z + b^2\,c^3 - a^2\,e^3, z, k\right)\right) - \frac{\frac{d}{3\,b} - \frac{c\,x^2}{3\,a} + \frac{e\,x}{3\,b}}{b\,x^3 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log(root(729*a^4*b^4*z^3 + 27*a^2*b^2*c*e*z + b^2*c^3 - a^2*e^3, z,
k)*(b*e*x + 9*root(729*a^4*b^4*z^3 + 27*a^2*b^2*c*e*z + b^2*c^3 - a^2*e^3,
z, k)*a*b^2) + (c*e)/(9*a) + (b*c^2*x)/(9*a^2))*root(729*a^4*b^4*z^3 + 27*a
^2*b^2*c*e*z + b^2*c^3 - a^2*e^3, z, k), k, 1, 3) - (d/(3*b) - (c*x^2)/(3*a
) + (e*x)/(3*b))/(a + b*x^3)
```

3.346 $\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=199

$$-\frac{ae-bx(c+dx)}{3ab\,(a+bx^3)}-\frac{\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(2\sqrt[3]{b}\,c\right.}{}$$

```
[Out] 1/3*(-a*e+b*x*(d*x+c))/a/b/(b*x^3+a)+1/9*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)
+b^(1/3)*x)/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/9*(2*b^(1/3)*c+a^(1/3)*d)*arctan(
1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1868, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,d+2\sqrt[3]{b}\,c\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}}-\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}}-\frac{ae-bx(c+dx)}{3ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(a*e - b*x*(c + d*x))/(a*b*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3
)) + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3
)) - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(5/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx = -\frac{ae-bx(c+dx)}{3ab\,(a+bx^3)} - \frac{\int \frac{-2c-dx}{a+bx^3}\,dx}{3a}$$

$$= -\frac{ae-bx(c+dx)}{3ab\,(a+bx^3)} - \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}} + \frac{\left(2c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int}{9a^{5/}}$$

$$= -\frac{ae-bx(c+dx)}{3ab\,(a+bx^3)} + \frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\int \frac{-\sqrt[3]{}}{a^{2/3}-}}{18a^{5/3}b^{2/}}$$

$$= -\frac{ae-bx(c+dx)}{3ab\,(a+bx^3)} + \frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/}\right.}{18a^{5/}}$$

$$= -\frac{ae-bx(c+dx)}{3ab\,(a+bx^3)} - \frac{\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\right.}{9a^{5/3}b^{2/3}}$$

Mathematica [A]

time = 0.12, size = 189, normalized size = 0.95

$$\frac{\frac{6a(-ae+bx(c+dx))}{a+bx^3} - 2\sqrt{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + \left(4\sqrt[3]{a}\,b^{2/3}c-2a^{2/3}\sqrt[3]{b}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) + \sqrt[3]{a}\,\sqrt[3]{b}\left(-2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^2,x]
```

```
[Out] ((6*a*(-(a*e) + b*x*(c + d*x)))/(a + b*x^3) - 2*Sqrt[3]*a^(1/3)*b^(1/3)*(2*
b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (4*a^(
1/3)*b^(2/3)*c - 2*a^(2/3)*b^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*b^
(1/3)*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(18*a^2*b)
```

Maple [A]

time = 0.38, size = 245, normalized size = 1.23

method	result
risch	$\frac{\frac{d\,x^2}{3a}+\frac{cx}{3a}-\frac{e}{3b}}{b\,x^3+a} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{\left(\_R d+2c\right)\ln\left(x-\_R\right)}{\_R^2}}{9ba}$

default	$c\left(\frac{x}{3a(b\,x^3+a)}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}\right)+d\left(\frac{x^2}{3a(b\,x^3+a)}+\frac{-\frac{\ln(}{}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/3*x/a/(b*x^3+a)+2/3/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)
^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))+d*(1/3*x^2/a/(b*x^3+a)+1/3/a*(-1/3/b/(a/b
)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/3*
e/b/(b*x^3+a)
```

Maxima [A]
time = 0.58, size = 180, normalized size = 0.90

$$\frac{bdx^2+bcx-ae}{3\,(ab^2x^3+a^2b)}+\frac{\sqrt{3}\,\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\arctan\left(\frac{\sqrt{3}\,\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*d*x^2 + b*c*x - a*e)/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(d*(a/b)^(1/3
) + 2*c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/
3)) + 1/18*(d*(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*
b*(a/b)^(2/3)) - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a/b)^
(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.16, size = 2118, normalized size = 10.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b*d*x^2 + 12*b*c*x - 2*(a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4
*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b
*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b
*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3
)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^
3)/(a^5*b^2))^(1/3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
+ a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d
*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a
^5*b^2))^(1/3)))*a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) - 12*a*e + ((
a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2
) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/
(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))
+ 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b
*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3
)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^
3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*
sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1
/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2)
 + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) +
 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8
*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d
^3)*x + 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5
*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) -
 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3
)))*a^4*b*d + 8*a^2*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(
I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b))) + ((a*b^2*x^3 + a^2*b)*((1/2)^(1
/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b
^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a
^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) - 3*sqrt(1/3)*(a*b^2*x^3 + a
^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8
*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b
*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*
b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d
^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*s
qrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^
2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(
a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3
) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(
1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d^3)*x - 3/4*sqrt(1/3)*(((1/2
)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a
^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3
```

```
)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2)*s
qrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3
- a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*
c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*
c*d)/(a^3*b))))/(a*b^2*x^3 + a^2*b)
```

Sympy [A]
time = 0.67, size = 116, normalized size = 0.58

$$\operatorname{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{-ae + bcx + bdx^2}{3a^2b + 3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(
_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3
 + 8*b*c**3)))) + (-a*e + b*c*x + b*d*x**2)/(3*a**2*b + 3*a*b**2*x**3)
```

Giac [A]
time = 0.88, size = 184, normalized size = 0.92

$$-\frac{\sqrt{3}\left(2\,bc - \left(-ab^2\right)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}\,a} - \frac{\left(2\,bc + \left(-ab^2\right)^{\frac{1}{3}}\,d\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}\,a} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{bdx^2 + bcx - ae}{3\,(bx^3 + a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)^(1
/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(b*d*x^2 + b*c
*x - a*e)/((b*x^3 + a)*a*b)
```

Mupad [B]
time = 0.25, size = 175, normalized size = 0.88

$$\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(2\,c\,d + d^2\,x + \mathrm{root}(729\,a^5\,b^2\,z^3 + 54\,a^2\,b\,c\,d\,z - 8\,b\,c^3 + a\,d^3, z, k)^2\,a^3\,b\,81 + \mathrm{root}(729\,a^5\,b^2\,z^3 + 54\,a^2\,b\,c\,d\,z - 8\,b\,c^3 + a\,d^3, z, k)\,a\,b\,c\,x\,18\right)}{a^2\,9}\right)\,\mathrm{root}\left(729\,a^5\,b^2\,z^3 + 54\,a^2\,b\,c\,d\,z - 8\,b\,c^3 + a\,d^3, z, k\right)\right) + \frac{\frac{d\,x^2}{3\,a} - \frac{e}{3\,b} + \frac{c\,x}{3\,a}}{b\,x^3 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z - 8
*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*b*c
*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) - e/(3*b) + (c*x)/
(3*a))/(a + b*x^3)
```

3.347 $\int \frac{c+dx+ex^2}{x(a+bx^3)^2} \, dx$

Optimal. Leaf size=222

$$\frac{x(ad+aex-bcx^2)}{3a^2(a+bx^3)}-\frac{\left(2\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}}+\frac{c\log(x)}{a^2}+\frac{\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}}$$

```
[Out] 1/3*x*(-b*c*x^2+a*e*x+a*d)/a^2/(b*x^3+a)+c*ln(x)/a^2+1/9*(2*b^(1/3)*d-a^(1/
3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*d-a^(1/3)*e)*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/3*c*ln(b*x^3+a)/a
^2-1/9*(2*b^(1/3)*d+a^(1/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(
1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,e+2\sqrt[3]{b}\,d\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}}-\frac{\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}}+\frac{x(ad+aex-bcx^2)}{3a^2(a+bx^3)}-\frac{c\log(a+bx^3)}{3a^2}+\frac{c\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^2),x]
```

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(3*a^2*(a + b*x^3)) - ((2*b^(1/3)*d + a^(1/3)*e
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(
2/3)) + (c*Log[x])/a^2 + ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x
])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx &= \frac{x(ad+aex-bcx^2)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3bc-2bdx-bex^2}{x(a+bx^3)}\,dx}{3ab} \\
&= \frac{x(ad+aex-bcx^2)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3bc}{ax} - \frac{b(2ad+aex-3bcx^2)}{a(a+bx^3)}\right)dx}{3ab} \\
&= \frac{x(ad+aex-bcx^2)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\int \frac{2ad+aex-3bcx^2}{a+bx^3}\,dx}{3a^2} \\
&= \frac{x(ad+aex-bcx^2)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\int \frac{2ad+aex}{a+bx^3}\,dx}{3a^2} - \frac{(bc)\int \frac{x^2}{a+bx^3}\,dx}{a^2} \\
&= \frac{x(ad+aex-bcx^2)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} - \frac{c\log\left(a+bx^3\right)}{3a^2} + \frac{\int \frac{\sqrt[3]{a}\left(4a\sqrt[3]{b}\,d+a^{4/3}e\right)+\sqrt[3]{b}\left(-2a\sqrt[3]{b}\,d \cdots\right.}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}}{9a^{8/3}\sqrt[3]{b}} \cdots \\
&= \frac{x(ad+aex-bcx^2)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}} - \frac{c\log\left(a+bx^{\cdots}\right.}{3a^2} \cdots \\
&= \frac{x(ad+aex-bcx^2)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}\,d-\sqrt[3]{\cdots}\right.}{\cdots} \cdots \\
&= \frac{x(ad+aex-bcx^2)}{3a^2\left(a+bx^3\right)} - \frac{\left(2\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}\,d-\cdots\right.}{\cdots} \cdots
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 199, normalized size = 0.90

$$\frac{\frac{6a(c+x(d+ex))}{a+bx^3} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + 18c\log(x) + \frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}\,d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}} + \frac{\left(-2\sqrt[3]{a}\sqrt[3]{b}\,d+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}} - 6c\log\left(a+bx^3\right)}{18a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((6*a*(c + x*(d + e*x)))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*d + a^
(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 18*c*Log[x]
 + (2*(2*a^(1/3)*b^(1/3)*d - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) +
 ((-2*a^(1/3)*b^(1/3)*d + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/b^(2/3) - 6*c*Log[a + b*x^3])/(18*a^2)
```

Maple [A]
time = 0.52, size = 240, normalized size = 1.08

method	result
risch	$\frac{\frac{e x^2}{3a}+\frac{x d}{3a}+\frac{c}{3a}}{b x^3+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^6 b^2 \_Z^3+9a^4 b^2 c \_Z^2+\left(6a^3 bde+27a^2 b^2 c^2\right)\_Z+a^2 e^3+18abcde-8ab d^3+27b^2 c^3\right)} \_R \ln\left(\left(-4\_R^3 a^5 b^2 \right.\right.\right.}{}$
default	$\frac{\frac{\frac{1}{3}ae x^2+\frac{1}{3}adx+\frac{1}{3}ac}{b x^3+a}+\frac{2ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}+\frac{ae\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x \right.}{}\right.}{}}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*((1/3*a*e*x^2+1/3*a*d*x+1/3*a*c)/(b*x^3+a)+2/3*a*d*(1/3/b/(a/b)^(2/3)
*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/
b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*a*e*(-1/
3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
))-1/3*c*ln(b*x^3+a))+c*ln(x)/a^2
```

Maxima [A]
time = 0.51, size = 207, normalized size = 0.93

$$\frac{x^2 e+dx+c}{3\left(abx^3+a^2\right)}+\frac{c\log(x)}{a^2}+\frac{\sqrt{3}\left(a\left(\frac{a}{b}\right)^{\frac{2}{3}}e+2ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^3}-\frac{\left(6bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a\left(\frac{a}{b}\right)^{\frac{1}{3}}e+2ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+a\left(\frac{a}{b}\right)^{\frac{1}{3}}e-2ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(x^2*e + d*x + c)/(a*b*x^3 + a^2) + c*log(x)/a^2 + 1/9*sqrt(3)*(a*(a/b)
^(2/3)*e + 2*a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/a^3 - 1/18*(6*b*c*(a/b)^(2/3) - a*(a/b)^(1/3)*e + 2*a*d)*log(x^2 - x
```

```
*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/9*(3*b*c*(a/b)^(2/3) +
a*(a/b)^(1/3)*e - 2*a*d)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.23, size = 5018, normalized size = 22.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/324*(108*a*e*x^2 + 108*a*d*x - 2*(a^2*b*x^3 + a^3)*((-I*sqrt(3) + 1)*(9*c
^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a
*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 +
 a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(
-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e
^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^
6*b^2))^(1/3) + 54*c/a^2)*log(1/324*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2
 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) +
 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d
^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1
/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1
/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 5
4*c/a^2)^2*a^4*b*e + 12*b*c*d^2 + 9*b*c^2*e + 4*a*d*e^2 - 1/9*(2*a^2*b*d^2
+ 3*a^2*b*c*e)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/
(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*
e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a
^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d
*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a
^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2) + (8*b*d^3 +
 a*e^3)*x) + 108*a*c - (162*b*c*x^3 - (a^2*b*x^3 + a^3)*((-I*sqrt(3) + 1)*(
9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 +
2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^
3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1
)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 +
a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/
(a^6*b^2))^(1/3) + 54*c/a^2) + 162*a*c - 3*sqrt(1/3)*(a^2*b*x^3 + a^3)*sqrt
(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a
^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^
2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/
3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b
) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(
4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)^2*a^4*b - 108*((-I*sqrt(
3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9
*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(
```

```
27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sq
rt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8
*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d
*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)*a^2*b*c + 2916*b*c^2 + 2592*a*d*e)/(a
^4*b)))*log(-1/324*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*
b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3
+ a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b
)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2
*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3
 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)^2*a^4*b*
e - 12*b*c*d^2 - 9*b*c^2*e - 4*a*d*e^2 + 1/9*(2*a^2*b*d^2 + 3*a^2*b*c*e)*((
-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 +
1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) -
1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) +
81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1
/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3
 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2) + 2*(8*b*d^3 + a*e^3)*x + 1/1
08*sqrt(1/3)*(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(
-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e
^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^
6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*
e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^
2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)*a^4*b*e + 72*
a^2*b*d^2 - 54*a^2*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 +
2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/
1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3
- 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/16
2*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/14
58*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c
/a^2)^2*a^4*b - 108*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4
*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3
 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*
b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 0.81, size = 217, normalized size = 0.98

$$-\frac{\sqrt{3}\left(2\,bd-\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{\left(2\,bd+\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{c\log\left(\left|bx^{3}+a\right|\right)}{3\,a^{2}}+\frac{c\log\left(\left|x\right|\right)}{a^{2}}+\frac{ax^{2}e+adx+ac}{3\,\left(bx^{3}+a\right)a^{2}}-\frac{\left(a^{3}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+2\,a^{3}bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{5}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*d - (-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*d + (-a*b^2)^(1/3)*e)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/3*c*log(abs(b*
x^3 + a))/a^2 + c*log(abs(x))/a^2 + 1/3*(a*x^2*e + a*d*x + a*c)/((b*x^3 + a
)*a^2) - 1/9*(a^3*b*(-a/b)^(1/3)*e + 2*a^3*b*d)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a^5*b)
```

Mupad [B]
time = 0.38, size = 490, normalized size = 2.21

$$\frac{\frac{c}{3a}+\frac{e x^2}{3a}+\frac{d x}{3a}}{b x^3+a}+\left(\sum_{k=1}^{3}\ln\left(\frac{4 b^2 c d^2-3 b^2 c^2 e}{9 a^3}-\mathrm{root}\left(729 a^6 b^2 z^3+729 a^4 b^2 c z^2+54 a^3 b d e z+243 a^2 b^2 c^2 z+18 a b c d e-8 a b d^3+27 b^2 c^3+a^2 e^3,z,k\right)\left(\mathrm{root}\left(729 a^6 b^2 z^3+729 a^4 b^2 c z^2+54 a^3 b d e z+243 a^2 b^2 c^2 z+18 a b c d e-8 a b d^3+27 b^2 c^3+a^2 e^3,z,k\right)\left(-a b^2 e+24 b^3 c x+\mathrm{root}\left(729 a^6 b^2 z^3+729 a^4 b^2 c z^2+54 a^3 b d e z+243 a^2 b^2 c^2 z+18 a b c d e-8 a b d^3+27 b^2 c^3+a^2 e^3,z,k\right) a^2 b^3 x\, 36\right)+\frac{4 a^2 b^2 d^2+6 c e a^2 b^2}{9 a^3}+\frac{x\left(60 d e a^2 b^2+108 a b^3 c^2\right)}{27 a^3}\right)-\frac{x\left(-8 b^2 d^3+12 c b^2 d e+a b e^3\right)}{27 a^3}\right)\mathrm{root}\left(729 a^6 b^2 z^3+729 a^4 b^2 c z^2+54 a^3 b d e z+243 a^2 b^2 c^2 z+18 a b c d e-8 a b d^3+27 b^2 c^3+a^2 e^3,z,k\right)\right)+\frac{c\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)^2),x)
```

```
[Out] (c/(3*a) + (e*x^2)/(3*a) + (d*x)/(3*a))/(a + b*x^3) + symsum(log((4*b^2*c*d
^2 - 3*b^2*c^2*e)/(9*a^3) - root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a
^3*b*d*e*z + 243*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^
2*e^3, z, k)*(root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a^3*b*d*e*z + 2
43*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^2*e^3, z, k)*(
24*b^3*c*x - a*b^2*e + 36*root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a^3
*b*d*e*z + 243*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^2*
e^3, z, k)*a^2*b^3*x) + (4*a^2*b^2*d^2 + 6*a^2*b^2*c*e)/(9*a^3) + (x*(108*a
*b^3*c^2 + 60*a^2*b^2*d*e))/(27*a^3)) - (x*(a*b*e^3 - 8*b^2*d^3 + 12*b^2*c*
d*e))/(27*a^3))*root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a^3*b*d*e*z +
 243*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^2*e^3, z, k)
, k, 1, 3) + (c*log(x))/a^2
```

3.348 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx$

Optimal. Leaf size=231

$$-\frac{c}{a^2x}+\frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)}+\frac{2(2b^{2/3}c-a^{2/3}e)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}\sqrt[3]{b}}+\frac{d\log(x)}{a^2}+\frac{2(2b^{2/3}c+a^{2/3}e)\log\left(\sqrt[3]{a}\right.}{9a^{7/3}\sqrt[3]{b}}$$

```
[Out] -c/a^2/x+1/3*x*(-b*d*x^2-b*c*x+a*e)/a^2/(b*x^3+a)+d*ln(x)/a^2+2/9*(2*b^(2/3
)*c+a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(1/3)-1/9*(2*b^(2/3)*c+a^(2/
3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(1/3)-1/3*d*ln(b*
x^3+a)/a^2+2/9*(2*b^(2/3)*c-a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/a^(7/3)/b^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(2b^{2/3}c-a^{2/3}e)}{3\sqrt{3}\,a^{7/3}\sqrt[3]{b}}-\frac{(a^{2/3}e+2b^{2/3}c)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{2(a^{2/3}e+2b^{2/3}c)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)}-\frac{d\log(a+bx^3)}{3a^2}-\frac{c}{a^2x}+\frac{d\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -(c/(a^2*x)) + (x*(a*e - b*c*x - b*d*x^2))/(3*a^2*(a + b*x^3)) + (2*(2*b^(2
/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sq
rt[3]*a^(7/3)*b^(1/3)) + (d*Log[x])/a^2 + (2*(2*b^(2/3)*c + a^(2/3)*e)*Log[
a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(1/3)) - ((2*b^(2/3)*c + a^(2/3)*e)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(7/3)*b^(1/3)) - (d*Log[a
+ b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^2}\,dx &= \frac{x(ae-bcx-bdx^2)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3bc-3bdx-2bex^2+\frac{b^2cx^3}{a}}{x^2(a+bx^3)}\,dx}{3ab} \\
&= \frac{x(ae-bcx-bdx^2)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3bc}{ax^2}-\frac{3bd}{ax}-\frac{b(2ae-4bcx-3bdx^2)}{a(a+bx^3)}\right)\,dx}{3ab} \\
&= -\frac{c}{a^2x} + \frac{x(ae-bcx-bdx^2)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{\int \frac{2ae-4bcx-3bdx^2}{a+bx^3}\,dx}{3a^2} \\
&= -\frac{c}{a^2x} + \frac{x(ae-bcx-bdx^2)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{\int \frac{2ae-4bcx}{a+bx^3}\,dx}{3a^2} - \frac{(bd)\int \frac{x^2}{a+bx^3}\,dx}{a^2} \\
&= -\frac{c}{a^2x} + \frac{x(ae-bcx-bdx^2)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} - \frac{d\log\left(a+bx^3\right)}{3a^2} + \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{a}\,bc+4a\sqrt[3]{b}\,e\right)}{a^{2/3}-\sqrt[3]{a}}}{9a^8} \\
&= -\frac{c}{a^2x} + \frac{x(ae-bcx-bdx^2)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{2\left(2b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}\sqrt[3]{b}} - \frac{d\,l}{} \\
&= -\frac{c}{a^2x} + \frac{x(ae-bcx-bdx^2)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{2\left(2b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}\sqrt[3]{b}} - \frac{(2}{} \\
&= -\frac{c}{a^2x} + \frac{x(ae-bcx-bdx^2)}{3a^2\left(a+bx^3\right)} + \frac{2\left(2b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}\sqrt[3]{b}} + \frac{d\log(x)}{a^2} +
\end{aligned}$$

Mathematica [A]

time = 0.18, size = 213, normalized size = 0.92

$$-\frac{\frac{9ac}{x}-\frac{3a\left(-bcx^2+a(d+ex)\right)}{a+bx^3}+\frac{2\sqrt{3}\,a^{2/3}\left(-2b^{2/3}c+a^{2/3}e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-9ad\log(x)-\frac{2\left(2a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+\frac{\left(2a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+3ad\log\left(a+bx^3\right)}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -1/9*((9*a*c)/x - (3*a*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3) + (2*Sqrt[3]
*a^(2/3)*(-2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]])/b^(1/3) - 9*a*d*Log[x] - (2*(2*a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[a^(
1/3) + b^(1/3)*x])/b^(1/3) + ((2*a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 3*a*d*Log[a + b*x^3])/a^3
```

Maple [A]
time = 0.37, size = 249, normalized size = 1.08

method	result
risch	$\frac{-\frac{4bc\,x^3}{3a^2}+\frac{e\,x^2}{3a}+\frac{xd}{3a}-\frac{c}{a}}{x(b\,x^3+a)}+\frac{d\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^7b\_Z^3+9a^5bd\_Z^2+\left(-24a^3bce+27a^3b\,d^2\right)\_Z-8a^2e^3-72abcde+27ab\,d^3-64b^2c^3\right)}\right.}{}$ —
default	$\frac{\frac{-\frac{1}{3}c\,x^2b+\frac{1}{3}aex+\frac{1}{3}ad}{b\,x^3+a}+\frac{2ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}-\frac{4bc\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x\right.}{}\right.}{}}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*((-1/3*c*x^2*b+1/3*a*e*x+1/3*a*d)/(b*x^3+a)+2/3*a*e*(1/3/b/(a/b)^(2/3
)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3
/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-4/3*b*c*(-1
/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
)))-1/3*d*ln(b*x^3+a))-c/a^2/x+d*ln(x)/a^2
```

Maxima [A]
time = 0.52, size = 226, normalized size = 0.98

$$-\frac{4\,bcx^3-ax^2e-adx+3\,ac}{3\,(a^2bx^4+a^3x)}+\frac{d\log(x)}{a^2}-\frac{2\,\sqrt{3}\,\left(2\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a\left(\frac{a}{b}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3}-\frac{\left(3\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(4*b*c*x^3 - a*x^2*e - a*d*x + 3*a*c)/(a^2*b*x^4 + a^3*x) + d*log(x)/a
^2 - 2/9*sqrt(3)*(2*b*c*(a/b)^(2/3) - a*(a/b)^(1/3)*e)*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^3 - 1/9*(3*b*d*(a/b)^(2/3) + 2*b*c*(a/b)^
```

(1/3) + a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1
/9*(3*b*d*(a/b)^(2/3) - 4*b*c*(a/b)^(1/3) - 2*a*e)*log(x + (a/b)^(1/3))/(a^
2*b*(a/b)^(2/3))

Fricas [C] Result contains complex when optimal does not.
time = 1.31, size = 4976, normalized size = 21.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/324*(432*b*c*x^3 - 108*a*e*x^2 - 108*a*d*x + 2*(a^2*b*x^4 + a^3*x)*((-I*
sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^
2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b
)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)
*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*
e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b
))^(1/3) + 54*d/a^2)*log(-1/324*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c
*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3
+ 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3
)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e
)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b)
 - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^5*b*c - 9*a*b
*c*d^2 + 16*a*b*c^2*e + 3*a^2*d*e^2 + 1/18*(6*a^3*b*c*d - a^4*e^2)*((-I*sqr
t(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 -
 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(
a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-
1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3
 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^
(1/3) + 54*d/a^2) - 2*(8*b^2*c^3 - a^2*e^3)*x) + 324*a*c + (162*b*d*x^4 + 1
62*a*d*x - (a^2*b*x^4 + a^3*x)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*
e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 +
 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)
/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)
*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b)
- 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2) + 3*sqrt(1/3)*(a^2
*b*x^4 + a^3*x)*sqrt(-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/
(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e
^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b)
)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 +
 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*
(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) +
 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e
)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b)
```

```
 - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d
^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(
3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3)
+ 54*d/a^2)*a^2*d + 2916*d^2 - 10368*c*e)/a^4))*log(1/324*((-I*sqrt(3) + 1)
*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d
/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) -
4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/
a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d
^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 5
4*d/a^2)^2*a^5*b*c + 9*a*b*c*d^2 - 16*a*b*c^2*e - 3*a^2*d*e^2 - 1/18*(6*a^3
*b*c*d - a^4*e^2)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/2
7*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 -
9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/
3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/14
58*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^
2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2) - 4*(8*b^2*c^3 - a^2*e^3)*x + 1
/108*sqrt(1/3)*(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*
d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*
(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3)
 + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458
*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*
c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*a^5*b*c - 54*a^3*b*c*d - 18*a^4*e
^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a
^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^
3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81
*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*
b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 -
 a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) + 1)*(9*d^2/a
^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/
1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*
b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/1
62*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*
d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*
a^2*d + 2916*d^2 - 10368*c*e)/a^4)) + (162*b*d*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 0.68, size = 237, normalized size = 1.03

$$-\frac{d\log\left(|bx^3+a|\right)}{3\,a^2}+\frac{d\log\left(|x|\right)}{a^2}+\frac{2\,\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ae+2\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b}-\frac{4\,bcx^3-ax^2e-adx+3\,ac}{3\left(bx^4+ax\right)a^2}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ae-2\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3b}+\frac{2\left(2\,a^2b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^2 + d*log(abs(x))/a^2 + 2/9*sqrt(3)*((-a*b^2)^
(1/3)*a*e + 2*(-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a
/b)^(1/3))/(a^3*b) - 1/3*(4*b*c*x^3 - a*x^2*e - a*d*x + 3*a*c)/((b*x^4 + a*
x)*a^2) + 1/9*((-a*b^2)^(1/3)*a*e - 2*(-a*b^2)^(2/3)*c)*log(x^2 + x*(-a/b)^
(1/3) + (-a/b)^(2/3))/(a^3*b) + 2/9*(2*a^2*b^2*c*(-a/b)^(1/3) - a^3*b*e)*(-
a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b)
```

Mupad [B]
time = 5.47, size = 488, normalized size = 2.11

$$\left(\sum_{k=1}^{3}\ln\left(-\mathrm{root}\left(729\,a^7\,b\,z^3+729\,a^5\,b\,d\,z^2-216\,a^3\,b\,c\,e\,z+243\,a^3\,b\,d^2\,z-72\,a\,b\,c\,d\,e+27\,a\,b\,d^3-8\,a^2\,e^3-64\,b^2\,c^3,z,k\right)\left(\mathrm{root}\left(729\,a^7\,b\,z^3+729\,a^5\,b\,d\,z^2-216\,a^3\,b\,c\,e\,z+243\,a^3\,b\,d^2\,z-72\,a\,b\,c\,d\,e+27\,a\,b\,d^3-8\,a^2\,e^3-64\,b^2\,c^3,z,k\right)\left(4\,b^3\,c+24\,b^3\,d\,x+\mathrm{root}\left(729\,a^7\,b\,z^3+729\,a^5\,b\,d\,z^2-216\,a^3\,b\,c\,e\,z+243\,a^3\,b\,d^2\,z-72\,a\,b\,c\,d\,e+27\,a\,b\,d^3-8\,a^2\,e^3-64\,b^2\,c^3,z,k\right)\,a^2\,b^3\,x\,36\right)+\frac{4\left(a^3\,b^2\,e^2-6\,a^2\,b^3\,c\,d\right)}{9\,a^4}+\frac{4\,x\left(27\,a^3\,b^3\,d^2-60\,a^3\,b^3\,c\,e\right)}{27\,a^5}\right)+\frac{4\left(3\,c\,b^3\,d^2+a\,b^2\,d\,e^2\right)}{9\,a^4}+\frac{4\,x\left(2\,a^2\,b^2\,e^3+12\,d\,a\,b^3\,c\,e+16\,b^4\,c^3\right)}{27\,a^5}\right)\mathrm{root}\left(729\,a^7\,b\,z^3+729\,a^5\,b\,d\,z^2-216\,a^3\,b\,c\,e\,z+243\,a^3\,b\,d^2\,z-72\,a\,b\,c\,d\,e+27\,a\,b\,d^3-8\,a^2\,e^3-64\,b^2\,c^3,z,k\right)\right)-\frac{\frac{c}{a}-\frac{e\,x^2}{3\,a}-\frac{d\,x}{3\,a}+\frac{4\,b\,c\,x^3}{3\,a^2}}{b\,x^4+a\,x}+\frac{d\,\ln\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] symsum(log((4*(3*b^3*c*d^2 + a*b^2*d*e^2))/(9*a^4) - root(729*a^7*b*z^3 + 7
29*a^5*b*d*z^2 - 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*
d^3 - 8*a^2*e^3 - 64*b^2*c^3, z, k)*(root(729*a^7*b*z^3 + 729*a^5*b*d*z^2 -
 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*d^3 - 8*a^2*e^3
- 64*b^2*c^3, z, k)*(4*b^3*c + 24*b^3*d*x + 36*root(729*a^7*b*z^3 + 729*a^5
*b*d*z^2 - 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*d^3 -
8*a^2*e^3 - 64*b^2*c^3, z, k)*a^2*b^3*x) + (4*(a^3*b^2*e^2 - 6*a^2*b^3*c*d)
)/(9*a^4) + (4*x*(27*a^3*b^3*d^2 - 60*a^3*b^3*c*e))/(27*a^5)) + (4*x*(16*b^
4*c^3 + 2*a^2*b^2*e^3 + 12*a*b^3*c*d*e))/(27*a^5))*root(729*a^7*b*z^3 + 729
*a^5*b*d*z^2 - 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*d^
3 - 8*a^2*e^3 - 64*b^2*c^3, z, k), k, 1, 3) - (c/a - (e*x^2)/(3*a) - (d*x)/
(3*a) + (4*b*c*x^3)/(3*a^2))/(a*x + b*x^4) + (d*log(x))/a^2
```

3.349 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^2}\,dx$

Optimal. Leaf size=242

$$-\frac{c}{2a^2x^2}-\frac{d}{a^2x}-\frac{x(bc+bdx+bex^2)}{3a^2\,(a+bx^3)}+\frac{\sqrt[3]{b}\,\left(5\sqrt[3]{b}\,c+4\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}+\frac{e\log(x)}{a^2}-\frac{\sqrt[3]{b}\,\left(5\sqrt[3]{b}\,c-4\right.}{}$$

```
[Out] -1/2*c/a^2/x^2-d/a^2/x-1/3*x*(b*e*x^2+b*d*x+b*c)/a^2/(b*x^3+a)+e*ln(x)/a^2-
1/9*b^(1/3)*(5*b^(1/3)*c-4*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)+1/18*b^
(1/3)*(5*b^(1/3)*c-4*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a
^(8/3)-1/3*e*ln(b*x^3+a)/a^2+1/9*b^(1/3)*(5*b^(1/3)*c+4*a^(1/3)*d)*arctan(1
/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)*3^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(4\sqrt[3]{a}\,d+5\sqrt[3]{b}\,c\right)}{3\sqrt{3}\,a^{8/3}}+\frac{\sqrt[3]{b}\,\left(5\sqrt[3]{b}\,c-4\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{\sqrt[3]{b}\,\left(5\sqrt[3]{b}\,c-4\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}}-\frac{x(bc+bdx+bex^2)}{3a^2\,(a+bx^3)}-\frac{e\log\left(a+bx^3\right)}{3a^2}-\frac{c}{2a^2x^2}-\frac{d}{a^2x}+\frac{e\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/2*c/(a^2*x^2) - d/(a^2*x) - (x*(b*c + b*d*x + b*e*x^2))/(3*a^2*(a + b*x^
3)) + (b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) + (e*Log[x])/a^2 - (b^(1/3)*(5*b^(1/
3)*c - 4*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)) + (b^(1/3)*(5*b^(
1/3)*c - 4*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a
^(8/3)) - (e*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx &= -\frac{x(bc+bdx+bex^2)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3bc-3bdx-3bex^2+\frac{2b^2cx^3}{a}+\frac{b^2dx^4}{a}}{x^3(a+bx^3)}\,dx}{3ab} \\
&= -\frac{x(bc+bdx+bex^2)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3bc}{ax^3}-\frac{3bd}{ax^2}-\frac{3be}{ax}+\frac{b^2(5c+4dx+3ex^2)}{a(a+bx^3)}\right)dx}{3ab} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc+bdx+bex^2)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{b\int \frac{5c+4dx+3ex^2}{a+bx^3}\,dx}{3a^2} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc+bdx+bex^2)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{b\int \frac{5c+4dx}{a+bx^3}\,dx}{3a^2} - \frac{(be)\int \frac{x^2}{a+bx^3}\,dx}{a^2} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc+bdx+bex^2)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{e\log\left(a+bx^3\right)}{3a^2} - \frac{b^{2/3}\int \frac{\sqrt[3]{a}\left(10\sqrt[3]{b}\right.}{}}{} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc+bdx+bex^2)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,c-4\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\right.}{9a^{8/3}} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc+bdx+bex^2)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,c-4\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\right.}{9a^{8/3}} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc+bdx+bex^2)}{3a^2\left(a+bx^3\right)} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,c+4\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}} +
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 221, normalized size = 0.91

$$
\frac{-\frac{9ac}{x^2}-\frac{18ad}{x}+\frac{6a(ae-bx(c+dx))}{a+bx^3}+2\sqrt{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(5\sqrt[3]{b}\,c+4\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+18ae\log(x)+2\sqrt[3]{b}\left(-5\sqrt[3]{a}\,\sqrt[3]{b}\,c+4a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+\sqrt[3]{b}\left(5\sqrt[3]{a}\,\sqrt[3]{b}\,c-4a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-6ae\log\left(a+bx^3\right)}{18a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a*c)/x^2 - (18*a*d)/x + (6*a*(a*e - b*x*(c + d*x)))/(a + b*x^3) + 2*Sq
rt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)
/a^(1/3))/Sqrt[3]] + 18*a*e*Log[x] + 2*b^(1/3)*(-5*a^(1/3)*b^(1/3)*c + 4*a^
(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(5*a^(1/3)*b^(1/3)*c - 4*a^(2/3
)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 6*a*e*Log[a + b*x^3])
/(18*a^3)
```

Maple [A]

time = 0.35, size = 262, normalized size = 1.08

method	result
risch	$\frac{-\frac{4bd\,x^4}{3a^2}-\frac{5bc\,x^3}{6a^2}+\frac{e\,x^2}{3a}-\frac{xd}{a}-\frac{c}{2a}}{x^2(b\,x^3+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^8\_Z^3+9a^6e\_Z^2+\left(27a^4e^2+60a^3bcd\right)\_Z+27a^2e^3+180abcde-64ab\,d^3+125b^2c^3\right)}\_R\right.}{}$
default	$-\frac{b\left(\frac{\frac{d\,x^2}{3}+\frac{cx}{3}-\frac{ae}{3b}}{b\,x^3+a}+\frac{5c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}+\frac{4d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2\right.}{}\right.}{}\right.}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^2*((1/3*d*x^2+1/3*c*x-1/3/b*a*e)/(b*x^3+a)+5/3*c*(1/3/b/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(
a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+4/3*d*(-1/3/b/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/
3*e*ln(b*x^3+a)/b)-1/2*c/a^2/x^2-d/a^2/x+e*ln(x)/a^2
```

Maxima [A]

time = 0.52, size = 224, normalized size = 0.93

$$-\frac{8\,bdx^4+5\,bcx^3-2\,ax^2e+6\,adx+3\,ac}{6\,(a^2bx^5+a^3x^2)}+\frac{e\log(x)}{a^2}-\frac{\sqrt{3}\left(4\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3}-\frac{\left(6\left(\frac{a}{b}\right)^{\frac{2}{3}}e+4\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}e-4\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(8*b*d*x^4 + 5*b*c*x^3 - 2*a*x^2*e + 6*a*d*x + 3*a*c)/(a^2*b*x^5 + a^3
*x^2) + e*log(x)/a^2 - 1/9*sqrt(3)*(4*b*d*(a/b)^(2/3) + 5*b*c*(a/b)^(1/3))*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^3 - 1/18*(6*(a/b)^(2/
3)*e + 4*d*(a/b)^(1/3) - 5*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(
a/b)^(2/3)) - 1/9*(3*(a/b)^(2/3)*e - 4*d*(a/b)^(1/3) + 5*c)*log(x + (a/b)^(
1/3))/(a^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.27, size = 4774, normalized size = 19.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/324*(432*b*d*x^4 + 270*b*c*x^3 - 108*a*e*x^2 + 324*a*d*x + 2*(a^2*b*x^5
+ a^3*x^2)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*
e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*
b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^
(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^
7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3
- 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*log(1/81*((-I*sqrt(3) +
 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d
 + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c
^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1
)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 6
4*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a
*b)/a^8)^(1/3) + 54*e/a^2)^2*a^6*d + 160*a*b*c*d^2 - 75*a*b*c^2*e + 36*a^2*
d*e^2 + 1/18*(25*a^3*b*c^2 - 24*a^4*d*e)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20
*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 +
1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*
(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1
/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1
458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*
e/a^2) + (125*b^2*c^3 + 64*a*b*d^3)*x) + 162*a*c + (162*b*e*x^5 + 162*a*e*x
^2 - (a^2*b*x^5 + a^3*x^2)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e
^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*
c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c
*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d
 + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c
^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2) - 3*sqr
t(1/3)*(a^2*b*x^5 + a^3*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*
```

```
d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/145
8*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d
^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*
(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(
125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2
)^2*a^5 - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/
27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^
3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^
8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e
/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e
^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^3*e + 25920*b*c*d
 + 2916*a*e^2)/a^5))*log(-1/81*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*
a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125
*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 4
5*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*
c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^
2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^
6*d - 160*a*b*c*d^2 + 75*a*b*c^2*e - 36*a^2*d*e^2 - 1/18*(25*a^3*b*c^2 - 24
*a^4*d*e)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e
^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b
/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(
1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7
 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 -
 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2) + 2*(125*b^2*c^3 + 64*a*
b*d^3)*x + 1/54*sqrt(1/3)*(2*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a
*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*
b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45
*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c
*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2
*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^6*d
 - 225*a^3*b*c^2 - 108*a^4*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b
*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/
1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(1
6*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/1
62*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/145
8*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/
a^2)^2*a^5 - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(
-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a
*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)
/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.71, size = 248, normalized size = 1.02

$$-\frac{e\log\left(|bx^3+a|\right)}{3a^2}+\frac{e\log\left(|x|\right)}{a^2}-\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}bc-4\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^3b}-\frac{\left(5\left(-ab^2\right)^{\frac{1}{3}}bc+4\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^3b}+\frac{\left(4a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5a^2b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^5b}-\frac{8bdx^4+5bcx^3-2ax^2e+6adx+3ac}{6\left(bx^3+a\right)a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^2 + e*log(abs(x))/a^2 - 1/9*sqrt(3)*(5*(-a*b^2
)^(1/3)*b*c - 4*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/(a^3*b) - 1/18*(5*(-a*b^2)^(1/3)*b*c + 4*(-a*b^2)^(2/3)*d)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b) + 1/9*(4*a^2*b^2*d*(-a/b)^(1/
3) + 5*a^2*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b) - 1/6*(8*
b*d*x^4 + 5*b*c*x^3 - 2*a*x^2*e + 6*a*d*x + 3*a*c)/((b*x^3 + a)*a^2*x^2)
```

Mupad [B]
time = 5.39, size = 733, normalized size = 3.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] symsum(log(-(b^3*(108*root(729*a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z +
243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k
)^2*a^6*d - 36*a^2*d*e^2 + 972*root(729*a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b
*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*
c^3, z, k)^3*a^8*x + 125*b^2*c^3*x - 72*root(729*a^8*z^3 + 729*a^6*e*z^2 +
540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 27*a^2*e^3 +
 125*b^2*c^3, z, k)*a^4*d*e - 75*a*b*c^2*e - 64*a*b*d^3*x + 75*root(729*a^8
*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64
*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k)*a^3*b*c^2 + 108*root(729*a^8*z^3
 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b
*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k)*a^4*e^2*x + 648*root(729*a^8*z^3 + 7
29*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3
 + 27*a^2*e^3 + 125*b^2*c^3, z, k)^2*a^6*e*x + 600*root(729*a^8*z^3 + 729*a
^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 2
7*a^2*e^3 + 125*b^2*c^3, z, k)*a^3*b*c*d*x + 120*a*b*c*d*e*x))/(27*a^6))*ro
ot(729*a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*
c*d*e - 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k), k, 1, 3) - (c/(2*a) -
 (e*x^2)/(3*a) + (d*x)/a + (5*b*c*x^3)/(6*a^2) + (4*b*d*x^4)/(3*a^2))/(a*x^
2 + b*x^5) + (e*log(x))/a^2
```

3.350 $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2}\,dx$

Optimal. Leaf size=262

$$-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)}+\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,d+4\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}-\frac{2bc\log(x)}{a^3}-\sqrt[3]{}$$

```
[Out] -1/3*c/a^2/x^3-1/2*d/a^2/x^2-e/a^2/x-1/3*x*(b*d+b*x*e-b^2*c*x^2/a)/a^2/(b*x
^3+a)-2*b*c*ln(x)/a^3-1/9*b^(1/3)*(5*b^(1/3)*d-4*a^(1/3)*e)*ln(a^(1/3)+b^(1
/3)*x)/a^(8/3)+1/18*b^(1/3)*(5*b^(1/3)*d-4*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/a^(8/3)+2/3*b*c*ln(b*x^3+a)/a^3+1/9*b^(1/3)*(5*b^(1/3)
*d+4*a^(1/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)*3
^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(4\sqrt[3]{a}\,e+5\sqrt[3]{b}\,d\right)}{3\sqrt{3}\,a^{8/3}}+\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}}+\frac{2bc\log\left(a+bx^3\right)}{3a^3}-\frac{2bc\log(x)}{a^3}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{3a^2\left(a+bx^3\right)}-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d + b*e*x - (b^2*c*x^2
)/a))/(3*a^2*(a + b*x^3)) + (b^(1/3)*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) - (2*b*c*Log[x
])/a^3 - (b^(1/3)*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*
a^(8/3)) + (b^(1/3)*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(18*a^(8/3)) + (2*b*c*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
```

```
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^2}\,dx = -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3bc-3bdx-3bex^2+\frac{3b^2cx^3}{a}+\frac{2b^2dx^4}{a}+\frac{b^2ex^5}{a}}{x^4\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3bc}{ax^4}-\frac{3bd}{ax^3}-\frac{3be}{ax^2}+\frac{6b^2c}{a^2x}+\frac{b^2\left(5ad+4aex-6bcx^2\right)}{a^2\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)} - \frac{2bc\log(x)}{a^3} - \frac{b\int \frac{5ad+4aex-6bcx^2}{a+bx^3}\,dx}{3a^3}$$

$$= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)} - \frac{2bc\log(x)}{a^3} - \frac{b\int \frac{5ad+4aex}{a+bx^3}\,dx}{3a^3} + (2b$$

$$= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)} - \frac{2bc\log(x)}{a^3} + \frac{2bc\log\left(a+bx^3\right)}{3a^3} - b$$

$$= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)} - \frac{2bc\log(x)}{a^3} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right.}{9a}$$

$$= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)} - \frac{2bc\log(x)}{a^3} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right.}{9a}$$

$$= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,d+4\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}}{\sqrt{}}\right.}{3\sqrt{3}\,a^{8/3}}$$

Mathematica [A]

time = 0.12, size = 225, normalized size = 0.86

$$\frac{-\frac{6ac}{x^3}-\frac{9ad}{x^2}-\frac{18ae}{x}-\frac{6ab(c+x(d+ex))}{a+bx^3}+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(5\sqrt[3]{b}\,d+4\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-36bc\log(x)+2\sqrt[3]{b}\left(-5\sqrt[3]{a}\sqrt[3]{b}\,d+4a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+\sqrt[3]{b}\left(5\sqrt[3]{a}\sqrt[3]{b}\,d-4a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+12bc\log\left(a+bx^3\right)}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x - (6*a*b*(c + x*(d + e*x)))/(a + b
*x^3) + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 36*b*c*Log[x] + 2*b^(1/3)*(-5*a^(1/3)*b^(1
/3)*d + 4*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(5*a^(1/3)*b^(1/3)*
d - 4*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 12*b*c*Lo
g[a + b*x^3])/(18*a^3)
```

Maple [A]
time = 0.36, size = 271, normalized size = 1.03

method	result
default	$-\frac{b\left(\frac{\frac{1}{3}aex^2+\frac{1}{3}adx+\frac{1}{3}ac}{bx^3+a}+\frac{5ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}+\frac{4ae\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.}{}\right)}{a^3}$
risch	$\frac{-\frac{4bex^5}{3a^2}-\frac{5bdx^4}{6a^2}-\frac{2bcx^3}{3a^2}-\frac{ex^2}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^3(bx^3+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^9\_Z^3-18a^6bc\_Z^2+\left(60a^4bde+108a^3b^2c^2\right)\_Z-64a^2be^3-360ab^2cde+125ab^2d^3\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*b*((1/3*a*e*x^2+1/3*a*d*x+1/3*a*c)/(b*x^3+a)+5/3*a*d*(1/3/b/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1
/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+4/3*a*e*(
-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1)))-2/3*c*ln(b*x^3+a))-e/a^2/x-1/3*c/a^2/x^3-1/2*d/a^2/x^2-2*b*c*ln(x)/a^
3
```

Maxima [A]
time = 0.53, size = 241, normalized size = 0.92

$$-\frac{8\,bx^5e+5\,bdx^4+4\,bcx^3+6\,ax^2e+3\,adx+2\,ac}{6\,(a^2bx^6+a^3x^3)}-\frac{2\,bc\log{(x)}}{a^3}-\frac{\sqrt{3}\,\left(4\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}e+5\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4}+\frac{\left(12\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}e+5\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(6\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}e-5\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(8*b*x^5*e + 5*b*d*x^4 + 4*b*c*x^3 + 6*a*x^2*e + 3*a*d*x + 2*a*c)/(a^2
*b*x^6 + a^3*x^3) - 2*b*c*log(x)/a^3 - 1/9*sqrt(3)*(4*a*(a/b)^(2/3)*e + 5*a
*d*(a/b)^(1/3))*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^4 +
 1/18*(12*b*c*(a/b)^(2/3) - 4*a*(a/b)^(1/3)*e + 5*a*d)*log(x^2 - x*(a/b)^(1
/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/3)) + 1/9*(6*b*c*(a/b)^(2/3) + 4*a*(a/b)^(
1/3)*e - 5*a*d)*log(x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.47, size = 5373, normalized size = 20.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(48*a*b*e*x^5 + 30*a*b*d*x^4 + 24*a*b*c*x^3 + 36*a^2*e*x^2 + 18*a^2*d
*x + 12*a^2*c + 2*(a^3*b*x^6 + a^4*x^3)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*
b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 +
64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^
2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*
b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)
/a^9)^(1/3) - 12*b*c/a^3)*log((8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^
6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*
b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 -
5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*
b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c
/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3
) - 12*b*c/a^3)^2*a^6*e + 150*b^2*c*d^2 + 144*b^2*c^2*e + 160*a*b*d*e^2 + 1
/2*(25*a^3*b*d^2 + 48*a^3*b*c*e)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2
/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^
3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3
 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(4
32*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*
b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(
1/3) - 12*b*c/a^3) + (125*b^2*d^3 + 64*a*b*e^3)*x) - (36*b^2*c*x^6 + 36*a*b
*c*x^3 + (a^3*b*x^6 + a^4*x^3)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a
```

```
^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)
*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 -
 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432
*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*
c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/
3) - 12*b*c/a^3) + 3*sqrt(1/3)*(a^3*b*x^6 + a^4*x^3)*sqrt(-((8*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3
/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 +
(216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 -
72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^
3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)^2*a^6 + 24*(8*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^
9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (21
6*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^
(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*
(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 -
 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^3*b*c + 144*b^2*c^2 + 320*a*b*
d*e)/a^6))*log(-(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2
 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*
b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72
*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 +
(125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^
3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3
)^2*a^6*e - 150*b^2*c*d^2 - 144*b^2*c^2*e - 160*a*b*d*e^2 - 1/2*(25*a^3*b*d
^2 + 48*a^3*b*c*e)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*
c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*
(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 -
 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9
 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216
*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/
a^3) + 2*(125*b^2*d^3 + 64*a*b*e^3)*x + 3/2*sqrt(1/3)*(2*(8*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^
9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (21
6*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^
(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*
(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 -
 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^6*e - 25*a^3*b*d^2 + 24*a^3*b*
c*e)*sqrt(-((8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5
*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*
c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d
*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125
*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^
3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)^2*
a^6 + 24*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.69, size = 269, normalized size = 1.03

$$\frac{2\,bc\log\left(|bx^3+a|\right)}{3\,a^3}-\frac{2\,bc\log\left(|x|\right)}{a^3}-\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}bd-4\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b}-\frac{\left(5\left(-ab^2\right)^{\frac{1}{3}}bd+4\left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b}+\frac{\left(4\,a^4b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+5\,a^4b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^7b}-\frac{8\,abx^5e+5\,abdx^4+4\,abcx^3+6\,a^2x^2e+3\,a^2dx+2\,a^2c}{6\,\left(bx^3+a\right)a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*b*c*log(abs(b*x^3 + a))/a^3 - 2*b*c*log(abs(x))/a^3 - 1/9*sqrt(3)*(5*(-
a*b^2)^(1/3)*b*d - 4*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^3*b) - 1/18*(5*(-a*b^2)^(1/3)*b*d + 4*(-a*b^2)^(2/3)*e
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b) + 1/9*(4*a^4*b^2*(-a/b)^
(1/3)*e + 5*a^4*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b) - 1/
6*(8*a*b*x^5*e + 5*a*b*d*x^4 + 4*a*b*c*x^3 + 6*a^2*x^2*e + 3*a^2*d*x + 2*a^
2*c)/((b*x^3 + a)*a^3*x^3)
```

Mupad [**B**]
time = 5.48, size = 537, normalized size = 2.05

$$\left(\sum_{k=1}^{3}\ln\left(-\frac{50\,b^5\,c\,d^2-48\,b^5\,c^2\,e}{9\,a^6}-\mathrm{root}\left(729\,a^9\,z^3-1458\,a^6\,b\,c\,z^2+540\,a^4\,b\,d\,e\,z+972\,a^3\,b^2\,c^2\,z-360\,a\,b^2\,c\,d\,e-64\,a^2\,b\,e^3+125\,a\,b^2\,d^3-216\,b^3\,c^3,z,k\right)\left(\frac{25\,a^3\,b^4\,d^2+48\,c\,e\,a^3\,b^4}{9\,a^6}+\mathrm{root}\left(729\,a^9\,z^3-1458\,a^6\,b\,c\,z^2+540\,a^4\,b\,d\,e\,z+972\,a^3\,b^2\,c^2\,z-360\,a\,b^2\,c\,d\,e-64\,a^2\,b\,e^3+125\,a\,b^2\,d^3-216\,b^3\,c^3,z,k\right)\left(4\,b^3\,e+\mathrm{root}\left(729\,a^9\,z^3-1458\,a^6\,b\,c\,z^2+540\,a^4\,b\,d\,e\,z+972\,a^3\,b^2\,c^2\,z-360\,a\,b^2\,c\,d\,e-64\,a^2\,b\,e^3+125\,a\,b^2\,d^3-216\,b^3\,c^3,z,k\right)\,a^2\,b^3\,x\,36-\frac{48\,b^4\,c\,x}{a}\right)+\frac{x\,\left(600\,d\,e\,a^3\,b^4+432\,a^2\,b^5\,c^2\right)}{27\,a^6}\right)+\frac{x\,\left(-125\,b^5\,d^3+240\,c\,b^5\,d\,e+64\,a\,b^4\,e^3\right)}{27\,a^6}\right)\,\mathrm{root}\left(729\,a^9\,z^3-1458\,a^6\,b\,c\,z^2+540\,a^4\,b\,d\,e\,z+972\,a^3\,b^2\,c^2\,z-360\,a\,b^2\,c\,d\,e-64\,a^2\,b\,e^3+125\,a\,b^2\,d^3-216\,b^3\,c^3,z,k\right)\right)-\frac{\frac{c}{3\,a}+\frac{e\,x^2}{a}+\frac{d\,x}{2\,a}+\frac{2\,b\,c\,x^3}{3\,a^2}+\frac{5\,b\,d\,x^4}{6\,a^2}+\frac{4\,b\,e\,x^5}{3\,a^2}}{b\,x^6+a\,x^3}-\frac{2\,b\,c\,\ln\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] symsum(log((x*(64*a*b^4*e^3 - 125*b^5*d^3 + 240*b^5*c*d*e))/(27*a^6) - root
(729*a^9*z^3 - 1458*a^6*b*c*z^2 + 540*a^4*b*d*e*z + 972*a^3*b^2*c^2*z - 360
*a*b^2*c*d*e - 64*a^2*b*e^3 + 125*a*b^2*d^3 - 216*b^3*c^3, z, k)*((25*a^3*b
^4*d^2 + 48*a^3*b^4*c*e)/(9*a^6) + root(729*a^9*z^3 - 1458*a^6*b*c*z^2 + 54
0*a^4*b*d*e*z + 972*a^3*b^2*c^2*z - 360*a*b^2*c*d*e - 64*a^2*b*e^3 + 125*a*
b^2*d^3 - 216*b^3*c^3, z, k)*(4*b^3*e + 36*root(729*a^9*z^3 - 1458*a^6*b*c*
z^2 + 540*a^4*b*d*e*z + 972*a^3*b^2*c^2*z - 360*a*b^2*c*d*e - 64*a^2*b*e^3
+ 125*a*b^2*d^3 - 216*b^3*c^3, z, k)*a^2*b^3*x - (48*b^4*c*x)/a) + (x*(432*
a^2*b^5*c^2 + 600*a^3*b^4*d*e))/(27*a^6)) - (50*b^5*c*d^2 - 48*b^5*c^2*e)/(
9*a^6))*root(729*a^9*z^3 - 1458*a^6*b*c*z^2 + 540*a^4*b*d*e*z + 972*a^3*b^2
*c^2*z - 360*a*b^2*c*d*e - 64*a^2*b*e^3 + 125*a*b^2*d^3 - 216*b^3*c^3, z, k
), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (d*x)/(2*a) + (2*b*c*x^3)/(3*a^2) + (5
*b*d*x^4)/(6*a^2) + (4*b*e*x^5)/(3*a^2))/(a*x^3 + b*x^6) - (2*b*c*log(x))/a
^3
```

3.351 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=215

$$-\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}-\frac{\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{5/3}}+\frac{\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{5/3}}$$

```
[Out] 1/6*(-e*x^2-d*x-c)/b/(b*x^3+a)^2+1/18*x*(2*e*x+d)/a/b/(b*x^3+a)+1/27*(b^(1/
3)*d-a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(5/3)-1/54*(d-a^(1/3)*e/b^(
1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(4/3)-1/27*(b^(1/
3)*d+a^(1/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b
^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1837, 1869, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,e+\sqrt[3]{b}\,d\right)}{9\sqrt{3}\,a^{5/3}b^{5/3}}-\frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{5/3}}-\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(c + d*x + e*x^2)/(b*(a + b*x^3)^2) + (x*(d + 2*e*x))/(18*a*b*(a + b*x
^3)) - ((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(9*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3)
+ b^(1/3)*x])/(27*a^(5/3)*b^(5/3)) - ((d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx = -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{\int \frac{d+2ex}{(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} - \frac{\int \frac{-2d-2ex}{a+bx^3}\,dx}{18ab}$$

$$= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} - \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{54a^{5/3}b^{4/3}}$$

$$= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} + \frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}} + \frac{\left(\sqrt[3]{b}\,d+\sqrt[3]{\ldots}\right.}{\ldots}$$

$$= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} + \frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}} - \frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\ldots\right.}{\ldots}$$

$$= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} - \frac{\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{5/3}} + \frac{\left(d-\ldots\right.}{\ldots}$$

Mathematica [A]
time = 0.12, size = 198, normalized size = 0.92

$$\frac{\frac{3b^{2/3}x(d+2ex)}{a(a+bx^3)} - \frac{9b^{2/3}(c+x(d+ex))}{(a+bx^3)^2} - \frac{2\sqrt{3}\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{2\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}} + \frac{\left(-\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}}{54b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] ((3*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)) - (9*b^(2/3)*(c + x*(d + e*x)))/
(a + b*x^3)^2 - (2*Sqrt[3]*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*(b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(
1/3)*x])/a^(5/3) + ((-(b^(1/3)*d) + a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/a^(5/3))/(54*b^(5/3))
```

Maple [A]
time = 0.36, size = 246, normalized size = 1.14

method	result

risch	$\frac{\frac{e x^5}{9a}+\frac{d x^4}{18a}-\frac{e x^2}{18b}-\frac{dx}{9b}-\frac{c}{6b}}{(b x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(e\_R+d)\ln(x-\_R)}{\_R^2}}{27a b^2}$
default	$\frac{\frac{e x^5}{9a}+\frac{d x^4}{18a}-\frac{e x^2}{18b}-\frac{dx}{9b}-\frac{c}{6b}}{(b x^3+a)^2}+\frac{d\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+e\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{9ba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/9/a*e*x^5+1/18*d/a*x^4-1/18*e*x^2/b-1/9*d*x/b-1/6*c/b)/(b*x^3+a)^2+1/9/b
/a*(d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1)))+e*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.53, size = 208, normalized size = 0.97

$$\frac{2\,bx^5e+bdx^4-ax^2e-2\,adx-3\,ac}{18\,(ab^3x^6+2\,a^2b^2x^3+a^3b)}+\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}e+d\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}e-d\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}e-d\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*b*x^5*e + b*d*x^4 - a*x^2*e - 2*a*d*x - 3*a*c)/(a*b^3*x^6 + 2*a^2*b
^2*x^3 + a^3*b) + 1/27*sqrt(3)*((a/b)^(1/3)*e + d)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) + 1/54*((a/b)^(1/3)*e - d)*
log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) - 1/27*((a/b)^(1
/3)*e - d)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.24, size = 2163, normalized size = 10.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/108*(12*b*e*x^5 + 6*b*d*x^4 - 6*a*e*x^2 - 12*a*d*x - 2*(a*b^3*x^6 + 2*a^2
*b^2*x^3 + a^3*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) +
 (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3
*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*log(1/
4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)
/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a
*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^4*b^3*e - 1/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*
b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/
(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*a^2*b^2*d^2 + 2*a*d*e^2 + (b
*d^3 + a*e^3)*x) - 18*a*c + ((a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^
(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b
^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3))) + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b
^2*x^3 + a^3*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b
^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)
/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2
*a^3*b^3 + 16*d*e)/(a^3*b^3)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^
3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e
*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^
5*b^5))^(1/3)))^2*a^4*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*
e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*s
qrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5)
)^(1/3)))*a^2*b^2*d^2 - 2*a*d*e^2 + 2*(b*d^3 + a*e^3)*x + 3/4*sqrt(1/3)*(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^
5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3
)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*a^4*b^3*e + 2*a^2*b^2*d^2)
*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 -
a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d
^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^3*b^3 + 16*d
*e)/(a^3*b^3))) + ((a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*((1/2)^(1/3)*(I*sqrt
(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*
(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d
^3 - a*e^3)/(a^5*b^5))^(1/3))) - 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a
^3*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d
^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*
((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^3*b^3 +
 16*d*e)/(a^3*b^3)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)
/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(
3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1
/3)))^2*a^4*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*
b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1
)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*
```

```
a^2*b^2*d^2 - 2*a*d*e^2 + 2*(b*d^3 + a*e^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)
*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1
/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5
) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*a^4*b^3*e + 2*a^2*b^2*d^2)*sqrt(-(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^
5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3
)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^3*b^3 + 16*d*e)/(a^3*b
^3))))/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)
```

Sympy [A]
time = 4.97, size = 148, normalized size = 0.69

$$\operatorname{RootSum}\left(19683t^3a^5b^5 + 81ta^2b^2de + ae^3 - bd^3, \left(t \mapsto t\log\left(x + \frac{729t^2a^4b^3e + 27ta^2b^2d^2 + 2ade^2}{ae^3 + bd^3}\right)\right)\right) + \frac{-3ac - 2adx - aex^2 + bdx^4 + 2bex^5}{18a^3b + 36a^2b^2x^3 + 18ab^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**5*b**5 + 81*_t*a**2*b**2*d*e + a*e**3 - b*d**3, Lamb
da(_t, _t*log(x + (729*_t**2*a**4*b**3*e + 27*_t*a**2*b**2*d**2 + 2*a*d*e**
2)/(a*e**3 + b*d**3)))) + (-3*a*c - 2*a*d*x - a*e*x**2 + b*d*x**4 + 2*b*e*x
**5)/(18*a**3*b + 36*a**2*b**2*x**3 + 18*a*b**3*x**6)
```

Giac [A]
time = 0.68, size = 208, normalized size = 0.97

$$-\frac{\sqrt{3}\left(bd - (-ab^2)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x + (-\frac{a}{b})^{\frac{1}{3}}\right)}{3(-\frac{a}{b})^{\frac{1}{3}}}\right)}{27(-ab^2)^{\frac{2}{3}}ab} - \frac{\left(bd + (-ab^2)^{\frac{1}{3}}e\right)\log\left(x^2 + x(-\frac{a}{b})^{\frac{1}{3}} + (-\frac{a}{b})^{\frac{2}{3}}\right)}{54(-ab^2)^{\frac{2}{3}}ab} - \frac{\left((-\frac{a}{b})^{\frac{1}{3}}e + d\right)(-\frac{a}{b})^{\frac{1}{3}}\log\left(\left|x - (-\frac{a}{b})^{\frac{1}{3}}\right|\right)}{27a^2b} + \frac{2bx^5e + bdx^4 - ax^2e - 2adx - 3ac}{18(bx^3 + a)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(b*d - (-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/54*(b*d + (-a*b^2)^(1/3)*e)*log(
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) - 1/27*((-a/b)^(1
/3)*e + d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) + 1/18*(2*b*x^5*
e + b*d*x^4 - a*x^2*e - 2*a*d*x - 3*a*c)/((b*x^3 + a)^2*a*b)
```

Mupad [B]
time = 0.23, size = 216, normalized size = 1.00

$$\left(\sum_{k=1}^{3}\ln\left(\frac{d\,e + e^2\,x + \mathrm{root}(19683\,a^5\,b^5\,z^3 + 81\,a^2\,b^2\,d\,e\,z + a\,e^3 - b\,d^3, z, k)^2\,a^3\,b^3\,729 + \mathrm{root}(19683\,a^5\,b^5\,z^3 + 81\,a^2\,b^2\,d\,e\,z + a\,e^3 - b\,d^3, z, k)\,a\,b^2\,d\,x\,27}{a^2\,b\,81}\right)\mathrm{root}(19683\,a^5\,b^5\,z^3 + 81\,a^2\,b^2\,d\,e\,z + a\,e^3 - b\,d^3, z, k)\right) - \frac{\frac{c}{6\,b} - \frac{d\,x^4}{18\,a} - \frac{e\,x^5}{9\,a} + \frac{e\,x^2}{18\,b} + \frac{d\,x}{9\,b}}{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x)
```

```
[Out] symsum(log((d*e + e^2*x + 729*root(19683*a^5*b^5*z^3 + 81*a^2*b^2*d*e*z + a
*e^3 - b*d^3, z, k)^2*a^3*b^3 + 27*root(19683*a^5*b^5*z^3 + 81*a^2*b^2*d*e*
```

```
z + a*e^3 - b*d^3, z, k)*a*b^2*d*x)/(81*a^2*b))*root(19683*a^5*b^5*z^3 + 81
*a^2*b^2*d*e*z + a*e^3 - b*d^3, z, k), k, 1, 3) - (c/(6*b) - (d*x^4)/(18*a)
 - (e*x^5)/(9*a) + (e*x^2)/(18*b) + (d*x)/(9*b))/(a^2 + b^2*x^6 + 2*a*b*x^3
)
```

3.352 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{x(ae-bcx-bdx^2)}{6ab\,(a+bx^3)^2}-\frac{3ad-x(ae+4bcx)}{18a^2b\,(a+bx^3)}-\frac{(2b^{2/3}c+a^{2/3}e)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{4/3}}-\frac{(2b^{2/3}c-a^{2/3}e)\log\left(\cdots\right.}{27a^{7/3}b^{4/3}}$$

```
[Out] -1/6*x*(-b*d*x^2-b*c*x+a*e)/a/b/(b*x^3+a)^2+1/18*(-3*a*d+x*(4*b*c*x+a*e))/a
^2/b/(b*x^3+a)-1/27*(2*b^(2/3)*c-a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b
^(4/3)+1/54*(2*b^(2/3)*c-a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/a^(7/3)/b^(4/3)-1/27*(2*b^(2/3)*c+a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1842, 1868, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(a^{2/3}e+2b^{2/3}c)}{9\sqrt{3}\,a^{7/3}b^{4/3}}+\frac{(2b^{2/3}c-a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{7/3}b^{4/3}}-\frac{(2b^{2/3}c-a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{4/3}}-\frac{3ad-x(ae+4bcx)}{18a^2b\,(a+bx^3)}-\frac{x(ae-bcx-bdx^2)}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(x*(a*e - b*c*x - b*d*x^2))/(a*b*(a + b*x^3)^2) - (3*a*d - x*(a*e + 4*
b*c*x))/(18*a^2*b*(a + b*x^3)) - ((2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(4/3)) - ((2*b^(2/
3)*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(4/3)) + ((2*b^(2
/3)*c - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7
/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx = -\frac{x(ae-bcx-bdx^2)}{6ab(a+bx^3)^2} - \frac{\int \frac{-ae-4bcx-3bdx^2}{(a+bx^3)^2}\,dx}{6ab}$$

$$= -\frac{x(ae-bcx-bdx^2)}{6ab(a+bx^3)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b(a+bx^3)} + \frac{\int \frac{2ae+4bcx}{a+bx^3}\,dx}{18a^2b}$$

$$= -\frac{x(ae-bcx-bdx^2)}{6ab(a+bx^3)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b(a+bx^3)} + \frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{a}\,bc+4a\sqrt[3]{b}\,e\right)+\sqrt[3]{b}\left(4\sqrt[3]{a}\right.}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}}{54a^{8/3}b^{4/3}}$$

$$= -\frac{x(ae-bcx-bdx^2)}{6ab(a+bx^3)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b(a+bx^3)} - \frac{(2b^{2/3}c-a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{4/3}}$$

$$= -\frac{x(ae-bcx-bdx^2)}{6ab(a+bx^3)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b(a+bx^3)} - \frac{(2b^{2/3}c-a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{4/3}}$$

$$= -\frac{x(ae-bcx-bdx^2)}{6ab(a+bx^3)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b(a+bx^3)} - \frac{(2b^{2/3}c+a^{2/3}e)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right.}{9\sqrt{3}\,a^{7/3}b^{4/3}}$$

Mathematica [A]
time = 0.19, size = 214, normalized size = 0.90

$$\frac{\frac{3ab^{2/3}(4b^2cx^5-a^2(3d+2ex)+abx^2(7c+ex^2))}{(a+bx^3)^2} - 2\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,(2b^{2/3}c+a^{2/3}e)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\left(-2a^{2/3}bc+a^{4/3}\sqrt[3]{b}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) + \left(2a^{2/3}bc-a^{4/3}\sqrt[3]{b}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^3b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] ((3*a*b^(2/3)*(4*b^2*c*x^5 - a^2*(3*d + 2*e*x) + a*b*x^2*(7*c + e*x^2)))/(a
 + b*x^3)^2 - 2*Sqrt[3]*a^(2/3)*b^(1/3)*(2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-2*a^(2/3)*b*c + a^(4/3)*b^(1/3)*e)
*Log[a^(1/3) + b^(1/3)*x] + (2*a^(2/3)*b*c - a^(4/3)*b^(1/3)*e)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^3*b^(5/3))
```

Maple [A]
time = 0.41, size = 250, normalized size = 1.05

method	result

risch	$\frac{\frac{2bc\,x^5}{9a^2}+\frac{e\,x^4}{18a}+\frac{7c\,x^2}{18a}-\frac{ex}{9b}-\frac{d}{6b}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\left(\frac{2c\_R}{a}+\frac{e}{b}\right)\ln(x-\_R)}{\_R^2}}{27ba}$
default	$\frac{\frac{2bc\,x^5}{9a^2}+\frac{e\,x^4}{18a}+\frac{7c\,x^2}{18a}-\frac{ex}{9b}-\frac{d}{6b}}{(b\,x^3+a)^2}+\frac{ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+2bc\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right.\right.}{3b\left(\frac{a}{b}\right.}\right.}{9a^2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2/9*b*c/a^2*x^5+1/18/a*e*x^4+7/18/a*c*x^2-1/9*e*x/b-1/6*d/b)/(b*x^3+a)^2+1
/9/a^2/b*(a*e*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1)))+2*b*c*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)
^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.51, size = 228, normalized size = 0.95

$$\frac{4\,b^2cx^5+abx^4e+7\,abcx^2-2\,a^2xe-3\,a^2d}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}+\frac{\sqrt{3}\left(2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b^2*c*x^5 + a*b*x^4*e + 7*a*b*c*x^2 - 2*a^2*x*e - 3*a^2*d)/(a^2*b^3
*x^6 + 2*a^3*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(2*b*c*(a/b)^(1/3) + a*e)*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) + 1/5
4*(2*b*c*(a/b)^(1/3) - a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2
*(a/b)^(2/3)) - 1/27*(2*b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(a^2*b^
2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.25, size = 2519, normalized size = 10.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(24*b^2*c*x^5 + 6*a*b*e*x^4 + 42*a*b*c*x^2 - 12*a^2*e*x - 18*a^2*d -
2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^
2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/
2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8
*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)
*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)
+ 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^
4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^5*b^3*c - 1/2*((1/2)^(1/3
)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/
(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 +
 a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*a^4*b*e^2 +
8*a*b*c^2*e + (8*b^2*c^3 + a^2*e^3)*x) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^
4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2
*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b
^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3
))) + 3*sqrt(1/3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)
*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(
a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 +
a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2 + 3
2*c*e)/(a^4*b^2)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*
e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e
*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a
^2*e^3)/(a^7*b^4))^(1/3)))^2*a^5*b^3*c + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(
(8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) +
4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4)
 - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*a^4*b*e^2 - 8*a*b*c^2*e + 2*(8*
b^2*c^3 + a^2*e^3)*x + 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*
c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)
^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b
^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*a^5*b^3*c + a^4*b*e^2)*sqrt(-(((1/2)^(
1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^
3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^
3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2
 + 32*c*e)/(a^4*b^2))) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3
)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/
(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 +
 a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3))) - 3*sqrt(1/3
)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)
*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)
+ 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^
4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2 + 32*c*e)/(a^4*b^2)
```

```
))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) -
 (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)
/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4
))^(1/3)))^2*a^5*b^3*c + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2
*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*
e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 -
a^2*e^3)/(a^7*b^4))^(1/3)))*a^4*b*e^2 - 8*a*b*c^2*e + 2*(8*b^2*c^3 + a^2*e^
3)*x - 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(
a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sq
rt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3
)/(a^7*b^4))^(1/3)))*a^5*b^3*c + a^4*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1
/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^
7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2 + 32*c*e)/(a^4*
b^2))))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
```

Sympy [A]
time = 1.87, size = 170, normalized size = 0.71

$$\mathrm{RootSum}\left(19683t^3a^7b^4 + 162ta^3b^2ce - a^2e^3 + 8b^2c^3, \left(t \mapsto t\log\left(x + \frac{1458t^2a^5b^3c + 27ta^4be^2 + 8abc^2e}{a^2e^3 + 8b^2c^3}\right)\right)\right) + \frac{-3a^2d - 2a^2ex + 7abcx^2 + abex^4 + 4b^2cx^5}{18a^4b + 36a^3b^2x^3 + 18a^2b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**7*b**4 + 162*_t*a**3*b**2*c*e - a**2*e**3 + 8*b**2*c
**3, Lambda(_t, _t*log(x + (1458*_t**2*a**5*b**3*c + 27*_t*a**4*b*e**2 + 8*
a*b*c**2*e)/(a**2*e**3 + 8*b**2*c**3)))) + (-3*a**2*d - 2*a**2*e*x + 7*a*b*
c*x**2 + a*b*e*x**4 + 4*b**2*c*x**5)/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a*
*2*b**3*x**6)
```

Giac [A]
time = 0.67, size = 215, normalized size = 0.90

$$-\frac{\sqrt{3}\left(ae - 2\left(-ab^2\right)^{\frac{1}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{\left(ae + 2\left(-ab^2\right)^{\frac{1}{3}}c\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{\left(2bc\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^3b} + \frac{4b^2cx^5 + abx^4e + 7abcx^2 - 2a^2xe - 3a^2d}{18\left(bx^3 + a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(a*e - 2*(-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(a*e + 2*(-a*b^2)^(1/3)*c)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(2*b*c
*(-a/b)^(1/3) + a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18
*(4*b^2*c*x^5 + a*b*x^4*e + 7*a*b*c*x^2 - 2*a^2*x*e - 3*a^2*d)/((b*x^3 + a)
^2*a^2*b)
```

Mupad [B]
time = 0.23, size = 232, normalized size = 0.97

$$\frac{\frac{7\,c\,x^2}{18\,a}-\frac{d}{6\,b}+\frac{e\,x^4}{18\,a}-\frac{e\,x}{9\,b}+\frac{2\,b\,c\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\left(\sum_{k=1}^{3}\ln\left(\frac{2\,a\,c\,e+\mathrm{root}(19683\,a^7\,b^4\,z^3+162\,a^3\,b^2\,c\,e\,z+8\,b^2\,c^3-a^2\,e^3,z,k)^2\,a^5\,b^2\,729+4\,b\,c^2\,x+\mathrm{root}(19683\,a^7\,b^4\,z^3+162\,a^3\,b^2\,c\,e\,z+8\,b^2\,c^3-a^2\,e^3,z,k)\,a^3\,b\,e\,x\,27}{a^4\,81}\right)\,\mathrm{root}\left(19683\,a^7\,b^4\,z^3+162\,a^3\,b^2\,c\,e\,z+8\,b^2\,c^3-a^2\,e^3,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3)^3,x)
```

```
[Out] ((7*c*x^2)/(18*a) - d/(6*b) + (e*x^4)/(18*a) - (e*x)/(9*b) + (2*b*c*x^5)/(9
*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((2*a*c*e + 729*root(19683*a
^7*b^4*z^3 + 162*a^3*b^2*c*e*z + 8*b^2*c^3 - a^2*e^3, z, k)^2*a^5*b^2 + 4*b
*c^2*x + 27*root(19683*a^7*b^4*z^3 + 162*a^3*b^2*c*e*z + 8*b^2*c^3 - a^2*e^
3, z, k)*a^3*b*e*x)/(81*a^4))*root(19683*a^7*b^4*z^3 + 162*a^3*b^2*c*e*z +
8*b^2*c^3 - a^2*e^3, z, k), k, 1, 3)
```

3.353 $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=225

$$\frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}-\frac{\left(5\sqrt[3]{b}\,c+2\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\right.}{27a^{8/3}b^{2/3}}$$

```
[Out] 1/18*x*(4*d*x+5*c)/a^2/(b*x^3+a)+1/6*(-a*e+b*x*(d*x+c))/a/b/(b*x^3+a)^2+1/2
7*(5*b^(1/3)*c-2*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(2/3)-1/54*(5*b
^(1/3)*c-2*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(
2/3)-1/27*(5*b^(1/3)*c+2*a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3
)*3^(1/2))/a^(8/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1868, 1869, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(2\sqrt[3]{a}\,d+5\sqrt[3]{b}\,c\right)}{9\sqrt{3}\,a^{8/3}b^{2/3}}-\frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{2/3}}+\frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^3,x]
```

```
[Out] (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3)) - (a*e - b*x*(c + d*x))/(6*a*b*(a +
b*x^3)^2) - ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)
*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)
*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx = -\frac{ae-bx(c+dx)}{6ab\,(a+bx^3)^2} - \frac{\int \frac{-5c-4dx}{(a+bx^3)^2}\,dx}{6a}$$

$$= \frac{x(5c+4dx)}{18a^2\,(a+bx^3)} - \frac{ae-bx(c+dx)}{6ab\,(a+bx^3)^2} + \frac{\int \frac{10c+4dx}{a+bx^3}\,dx}{18a^2}$$

$$= \frac{x(5c+4dx)}{18a^2\,(a+bx^3)} - \frac{ae-bx(c+dx)}{6ab\,(a+bx^3)^2} + \frac{\int \frac{\sqrt[3]{a}\left(20\sqrt[3]{b}\,c+4\sqrt[3]{a}\,d\right)+\sqrt[3]{b}\left(-10\sqrt[3]{b}\,c+4\sqrt[3]{a}\,d\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{54a^{8/3}\sqrt[3]{b}}$$

$$= \frac{x(5c+4dx)}{18a^2\,(a+bx^3)} - \frac{ae-bx(c+dx)}{6ab\,(a+bx^3)^2} + \frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}\,c\right.}{}$$

$$= \frac{x(5c+4dx)}{18a^2\,(a+bx^3)} - \frac{ae-bx(c+dx)}{6ab\,(a+bx^3)^2} + \frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}\,c\right.}{}$$

$$= \frac{x(5c+4dx)}{18a^2\,(a+bx^3)} - \frac{ae-bx(c+dx)}{6ab\,(a+bx^3)^2} - \frac{\left(5\sqrt[3]{b}\,c+2\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}\right.}{}$$

Mathematica [A]
time = 0.19, size = 213, normalized size = 0.95

$$\frac{\frac{3a\left(-3a^2e+b^2x^4(5c+4dx)+abx(8c+7dx)\right)}{(a+bx^3)^2} - 2\sqrt{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(5\sqrt[3]{b}\,c+2\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\sqrt[3]{b}\left(5\sqrt[3]{a}\,\sqrt[3]{b}\,c-2a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) + \sqrt[3]{a}\,\sqrt[3]{b}\left(-5\sqrt[3]{b}\,c+2\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^3,x]
```

```
[Out] ((3*a*(-3*a^2*e + b^2*x^4*(5*c + 4*d*x) + a*b*x*(8*c + 7*d*x)))/(a + b*x^3)
^2 - 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(5*a^(1/3)*b^(1/3)*c - 2*a^(2/3)*d)
*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*b^(1/3)*(-5*b^(1/3)*c + 2*a^(1/3)*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^3*b)
```

Maple [A]
time = 0.38, size = 311, normalized size = 1.38

method	result

risch	$\frac{\frac{2bd\,x^5}{9a^2}+\frac{5bc\,x^4}{18a^2}+\frac{7d\,x^2}{18a}+\frac{4cx}{9a}-\frac{e}{6b}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(2\_Rd+5c)\ln(x-\_R)}{\_R^2}}{27a^2b}$
default	$c\left(\frac{x}{6a(b\,x^3+a)^2}+\frac{\frac{5x}{18a(b\,x^3+a)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{6a}}{a}\right)+d\left(\frac{}{6}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/6*x/a/(b*x^3+a)^2+5/6/a*(1/3*x/a/(b*x^3+a)+2/3/a*(1/3/b/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(
a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))))+d*(1/6*x^2/a/
(b*x^3+a)^2+2/3/a*(1/3*x^2/a/(b*x^3+a)+1/3/a*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)
^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(
a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))))+e*(1/6/a*x^3/(b*x^3+a
)^2-1/6/a/b/(b*x^3+a))
```

Maxima [A]

time = 0.49, size = 220, normalized size = 0.98

$$\frac{4\,b^2dx^5+5\,b^2cx^4+7\,abdx^2+8\,abcx-3\,a^2e}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}+\frac{\sqrt{3}\,\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\arctan\left(\frac{\sqrt{3}\,\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b^2*d*x^5 + 5*b^2*c*x^4 + 7*a*b*d*x^2 + 8*a*b*c*x - 3*a^2*e)/(a^2*b
^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(2*d*(a/b)^(1/3) + 5*c)*arct
```

an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) + 1/54*(2*d*(a/b)^(1/3) - 5*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/27*(2*d*(a/b)^(1/3) - 5*c)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))

Fricas [C] Result contains complex when optimal does not.
time = 1.12, size = 2251, normalized size = 10.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/108*(24*b^2*d*x^5 + 30*b^2*c*x^4 + 42*a*b*d*x^2 + 48*a*b*c*x - 18*a^2*e - 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d - 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 + 40*a*c*d^2 + (125*b*c^3 + 8*a*d^3)*x) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3))) + 3*sqrt(1/3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^3 + 8*a*d^3)*x + 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^6*b*d + 25*a^3*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8

```
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*
c*d)/(a^5*b))) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2
))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3
)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3))) - 3*sqrt(1/3)*(a^2*b
^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b
*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/
2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (12
5*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2
*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*(
(125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^
2*a^6*b*d + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b
^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(
3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^
8*b^2))^(1/3)))*a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^3 + 8*a*d^3)*x - 3/2*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (1
25*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/
(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^
(1/3)))*a^6*b*d + 25*a^3*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b
*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/
2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (12
5*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b))))/(a^2*b^
3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
```

Sympy [A]
time = 1.14, size = 163, normalized size = 0.72

$$\operatorname{RootSum}\left(19683t^3a^8b^2 + 810ta^3bcd + 8ad^3 - 125bc^3, \left(t \mapsto t\log\left(x + \frac{1458t^2a^6bd + 675ta^3bc^2 + 40acd^2}{8ad^3 + 125bc^3}\right)\right)\right) + \frac{-3a^2e + 8abcx + 7abdx^2 + 5b^2cx^4 + 4b^2dx^5}{18a^4b + 36a^3b^2x^3 + 18a^2b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 125*b*c**3,
Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**3*b*c**2 + 40*a*c*d*
*2)/(8*a*d**3 + 125*b*c**3)))) + (-3*a**2*e + 8*a*b*c*x + 7*a*b*d*x**2 + 5*
b**2*c*x**4 + 4*b**2*d*x**5)/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a**2*b**3*
x**6)
```

Giac [A]
time = 0.82, size = 210, normalized size = 0.93

$$-\frac{\sqrt{3}\left(5\,bc - 2\,(-ab^2)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{\left(5\,bc + 2\,(-ab^2)^{\frac{1}{3}}\,d\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{\left(2\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3} + \frac{4\,b^2dx^5 + 5\,b^2cx^4 + 7\,abdx^2 + 8\,abcx - 3\,a^2e}{18\,(bx^3 + a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(5*b*c - 2*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(5*b*c + 2*(-a*b^2)^(1/3)
*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(2
*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/18*(
4*b^2*d*x^5 + 5*b^2*c*x^4 + 7*a*b*d*x^2 + 8*a*b*c*x - 3*a^2*e)/((b*x^3 + a)
^2*a^2*b)
```

Mupad [B]
time = 0.26, size = 212, normalized size = 0.94

$$\frac{\frac{7\,d\,x^2}{18\,a}-\frac{e}{6\,b}+\frac{4\,c\,x}{9\,a}+\frac{5\,b\,c\,x^4}{18\,a^2}+\frac{2\,b\,d\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\left(\sum_{k=1}^{3}\ln\left(\frac{b\,\left(10\,c\,d+4\,d^2\,x+\mathrm{root}(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)^2\,a^5\,b\,729+\mathrm{root}(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)\,a^2\,b\,c\,x\,135\right)}{a^4\,81}\right)\,\mathrm{root}\left(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^3)^3,x)
```

```
[Out] ((7*d*x^2)/(18*a) - e/(6*b) + (4*c*x)/(9*a) + (5*b*c*x^4)/(18*a^2) + (2*b*d
*x^5)/(9*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((b*(10*c*d + 4*d^2*
x + 729*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3, z,
k)^2*a^5*b + 135*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a
*d^3, z, k)*a^2*b*c*x))/(81*a^4))*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z
- 125*b*c^3 + 8*a*d^3, z, k), k, 1, 3)
```

3.354 $\int \frac{c+dx+ex^2}{x(a+bx^3)^3} \, dx$

Optimal. Leaf size=257

$$\frac{x(ad+aex-bcx^2)}{6a^2\left(a+bx^3\right)^2}+\frac{x(5ad+4aex-9bcx^2)}{18a^3\left(a+bx^3\right)}-\frac{\left(5\sqrt[3]{b}\,d+2\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{2/3}}+\frac{c\log(x)}{a^3}+\frac{\left(5\sqrt[3]{b}\,d\right.}{}$$

```
[Out] 1/6*x*(-b*c*x^2+a*e*x+a*d)/a^2/(b*x^3+a)^2+1/18*x*(-9*b*c*x^2+4*a*e*x+5*a*d
)/a^3/(b*x^3+a)+c*ln(x)/a^3+1/27*(5*b^(1/3)*d-2*a^(1/3)*e)*ln(a^(1/3)+b^(1/
3)*x)/a^(8/3)/b^(2/3)-1/54*(5*b^(1/3)*d-2*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(2/3)-1/3*c*ln(b*x^3+a)/a^3-1/27*(5*b^(1/3)*d
+2*a^(1/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(
2/3)*3^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(2\sqrt[3]{a}\,e+5\sqrt[3]{b}\,d\right)}{9\sqrt{3}\,a^{8/3}b^{2/3}}-\frac{\left(5\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{2/3}}+\frac{x(5ad+4aex-9bcx^2)}{18a^3\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^3}+\frac{c\log(x)}{a^3}+\frac{x(ad+aex-bcx^2)}{6a^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*d + 4*a*e*x - 9
*b*c*x^2))/(18*a^3*(a + b*x^3)) - ((5*b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + (c*Log
[x])/a^3 + ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/
3)*b^(2/3)) - ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
```

```
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx = \frac{x(ad+aex-bcx^2)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6bc-5bdx-4bex^2+\frac{3b^2cx^3}{a}}{x\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3\left(a+bx^3\right)} + \frac{\int \frac{18b^2c+10b^2dx+4b^2ex^2}{x(a+bx^3)}\,dx}{18a^2b^2}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^2c}{ax} + \frac{2b^2(5ad+2aex-9bcx^2)}{a(a+bx^3)}\right)dx}{18a^2b^2}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\int \frac{5ad+2aex-9bcx^2}{a+bx^3}\,dx}{9a^3}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\int \frac{5ad+2aex}{a+bx^3}\,dx}{9a^3} - \frac{(bc)\int \frac{\cdots}{a+\cdots}}{a^3}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} - \frac{c\log\left(a+bx^3\right)}{3a^3} + \frac{\int \frac{\sqrt[3]{a}\,(\cdots}{\cdots}}{\cdots}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\left(5\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(\cdots\right.}{27a^{8/3}b^{2/3}}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\left(5\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(\cdots\right.}{27a^{8/3}b^{2/3}}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3\left(a+bx^3\right)} - \frac{\left(5\sqrt[3]{b}\,d+2\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{\cdots}}{\sqrt{3}\,\sqrt[3]{a}\cdots}\right.}{9\sqrt{3}\,a^{8/3}b^{2/3}}$$

Mathematica [A]

time = 0.12, size = 229, normalized size = 0.89

$$\frac{\frac{9a^2(c+x(d+ex))}{(a+bx^3)^2}+\frac{3a(6c+x(5d+4ex))}{a+bx^3}-\frac{2\sqrt{3}\,\sqrt[3]{a}\left(5\sqrt[3]{b}\,d+2\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+54c\log(x)+\frac{2\left(5\sqrt[3]{a}\,\sqrt[3]{b}\,d-2a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{\left(-5\sqrt[3]{a}\,\sqrt[3]{b}\,d+2a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}-18c\log\left(a+bx^3\right)}{54a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((9*a^2*(c + x*(d + e*x)))/(a + b*x^3)^2 + (3*a*(6*c + x*(5*d + 4*e*x)))/(a
 + b*x^3) - (2*Sqrt[3]*a^(1/3)*(5*b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 54*c*Log[x] + (2*(5*a^(1/3)*b^(1/3)*
d - 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/3)*d
 + 2*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 1
8*c*Log[a + b*x^3])/(54*a^3)
```

Maple [A]

time = 0.37, size = 270, normalized size = 1.05

method	result
risch	$\frac{\frac{2be\,x^5}{9a^2}+\frac{5bd\,x^4}{18a^2}+\frac{bc\,x^3}{3a^2}+\frac{7e\,x^2}{18a}+\frac{4xd}{9a}+\frac{c}{2a}}{(b\,x^3+a)^2}+\frac{c\ln(-x)}{a^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^9b^2\_Z^3+27a^6b^2c\_Z^2+\left(30a^4bde+243a^3b^2c^2\right)\_Z+8a^2e^3+270abc\right.}\right.}{}$
default	$\frac{\frac{\frac{2}{9}abe\,x^5+\frac{5}{18}abd\,x^4+\frac{1}{3}abc\,x^3+\frac{7}{18}a^2e\,x^2+\frac{4}{9}a^2dx+\frac{1}{2}a^2c}{\left(b\,x^3+a\right)^2}+\frac{5ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{9}}{a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*((2/9*a*b*e*x^5+5/18*a*b*d*x^4+1/3*a*b*c*x^3+7/18*a^2*e*x^2+4/9*a^2*d
*x+1/2*a^2*c)/(b*x^3+a)^2+5/9*a*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/
b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+2/9*a*e*(-1/3/b/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/
b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-1/3*c*ln(b*x^3+a))+c
*ln(x)/a^3
```

Maxima [A]
time = 0.48, size = 251, normalized size = 0.98

$$\frac{4\,bx^5e+5\,bdx^4+6\,bcx^3+7\,ax^2e+8\,adx+9\,ac}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)}+\frac{c\log(x)}{a^3}+\frac{\sqrt{3}\left(2\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}e+5\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4}-\frac{\left(18\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}e+5\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(9\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}e-5\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b*x^5*e + 5*b*d*x^4 + 6*b*c*x^3 + 7*a*x^2*e + 8*a*d*x + 9*a*c)/(a^2
*b^2*x^6 + 2*a^3*b*x^3 + a^4) + c*log(x)/a^3 + 1/27*sqrt(3)*(2*a*(a/b)^(2/3
)*e + 5*a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/a^4 - 1/54*(18*b*c*(a/b)^(2/3) - 2*a*(a/b)^(1/3)*e + 5*a*d)*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) - 1/27*(9*b*c*(a/b)^(2/3) + 2
*a*(a/b)^(1/3)*e - 5*a*d)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.29, size = 5229, normalized size = 20.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2916*(648*a*b*e*x^5 + 810*a*b*d*x^4 + 972*a*b*c*x^3 + 1134*a^2*e*x^2 + 12
96*a^2*d*x + 1458*a^2*c - 2*(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)*((-I*sqrt(3)
+ 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(
81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) -
1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1
/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(
a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8
*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*log(1/1
458*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c
^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*
e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a
*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^
2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366
*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 4
86*c/a^3)^2*a^6*b*e + 225*b*c*d^2 + 162*b*c^2*e + 40*a*d*e^2 - 1/54*(25*a^3
*b*d^2 + 36*a^3*b*c*e)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e
)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/3936
6*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(2
5*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^
9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/
(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(
a^9*b^2))^(1/3) + 486*c/a^3) + (125*b*d^3 + 8*a*e^3)*x) - (1458*b^2*c*x^6 +
```

2916*a*b*c*x^3 + 1458*a^2*c - (a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3) - 3*sqrt(1/3)*(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)*sqrt(-(((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*b - 972*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*a^3*b*c + 236196*b*c^2 + 116640*a*d*e)/(a^6*b)))*log(-1/1458*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*b*e - 225*b*c*d^2 - 162*b*c^2*e - 40*a*d*e^2 + 1/54*(25*a^3*b*d^2 + 36*a^3*b*c*e)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3) + 2*(125*b*d^3 + 8*a*e^3)*x + 1/486*sqrt(1/3)*(((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*a^6*b*e + 675*a^3*b*d^2 - 486*a^3*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.91, size = 253, normalized size = 0.98

$$-\frac{\sqrt{3}\left(5\,bd-2\,(-ab^2)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+(-\frac{a}{b})^{\frac{1}{3}}\right)}{3\,(-\frac{a}{b})^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}a^2}-\frac{\left(5\,bd+2\,(-ab^2)^{\frac{1}{3}}e\right)\log\left(x^2+x(-\frac{a}{b})^{\frac{1}{3}}+(-\frac{a}{b})^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}a^2}-\frac{c\log\left(|bx^3+a|\right)}{3\,a^3}+\frac{c\log\left(|x|\right)}{a^3}+\frac{4\,abx^5e+5\,abdx^4+6\,abcx^3+7\,a^2x^2e+8\,a^2dx+9\,a^2c}{18\,(bx^3+a)^2a^3}-\frac{\left(2\,a^4b(-\frac{a}{b})^{\frac{1}{3}}e+5\,a^4bd\right)(-\frac{a}{b})^{\frac{1}{3}}\log\left(\left|x-(-\frac{a}{b})^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(5*b*d - 2*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(5*b*d + 2*(-a*b^2)^(1/3)
*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/3*c*l
og(abs(b*x^3 + a))/a^3 + c*log(abs(x))/a^3 + 1/18*(4*a*b*x^5*e + 5*a*b*d*x^
4 + 6*a*b*c*x^3 + 7*a^2*x^2*e + 8*a^2*d*x + 9*a^2*c)/((b*x^3 + a)^2*a^3) -
1/27*(2*a^4*b*(-a/b)^(1/3)*e + 5*a^4*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(
1/3)))/(a^7*b)
```

Mupad [**B**]
time = 5.44, size = 540, normalized size = 2.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)^3),x)
```

```
[Out] (c/(2*a) + (7*e*x^2)/(18*a) + (4*d*x)/(9*a) + (b*c*x^3)/(3*a^2) + (5*b*d*x^
4)/(18*a^2) + (2*b*e*x^5)/(9*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log
((25*b^2*c*d^2 - 18*b^2*c^2*e)/(81*a^6) - root(19683*a^9*b^2*z^3 + 19683*a^
6*b^2*c*z^2 + 810*a^4*b*d*e*z + 6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*
b*d^3 + 8*a^2*e^3 + 729*b^2*c^3, z, k)*((25*a^3*b^2*d^2 + 36*a^3*b^2*c*e)/(
81*a^6) + root(19683*a^9*b^2*z^3 + 19683*a^6*b^2*c*z^2 + 810*a^4*b*d*e*z +
6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*b*d^3 + 8*a^2*e^3 + 729*b^2*c^3,
 z, k)*(36*root(19683*a^9*b^2*z^3 + 19683*a^6*b^2*c*z^2 + 810*a^4*b*d*e*z +
 6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*b*d^3 + 8*a^2*e^3 + 729*b^2*c^3
, z, k)*a^2*b^3*x - (2*b^2*e)/3 + (24*b^3*c*x)/a) + (x*(2916*a^2*b^3*c^2 +
900*a^3*b^2*d*e))/(729*a^6)) - (x*(8*a*b*e^3 - 125*b^2*d^3 + 180*b^2*c*d*e)
)/(729*a^6))*root(19683*a^9*b^2*z^3 + 19683*a^6*b^2*c*z^2 + 810*a^4*b*d*e*z
 + 6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*b*d^3 + 8*a^2*e^3 + 729*b^2*c
^3, z, k), k, 1, 3) + (c*log(x))/a^3
```

3.355 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3} \, dx$

Optimal. Leaf size=267

$$-\frac{c}{a^3x}+\frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2}+\frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)}+\frac{(14b^{2/3}c-5a^{2/3}e)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt[3]{b}}+\frac{d\log(x)}{a^3}+$$

```
[Out] -c/a^3/x+1/6*x*(-b*d*x^2-b*c*x+a*e)/a^2/(b*x^3+a)^2+1/18*x*(-9*b*d*x^2-10*b
*c*x+5*a*e)/a^3/(b*x^3+a)+d*ln(x)/a^3+1/27*(14*b^(2/3)*c+5*a^(2/3)*e)*ln(a^
(1/3)+b^(1/3)*x)/a^(10/3)/b^(1/3)-1/54*(14*b^(2/3)*c+5*a^(2/3)*e)*ln(a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(1/3)-1/3*d*ln(b*x^3+a)/a^3+1/2
7*(14*b^(2/3)*c-5*a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/
2))/a^(10/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(14b^{2/3}c-5a^{2/3}e)}{9\sqrt{3}a^{10/3}\sqrt[3]{b}}-\frac{(5a^{2/3}e+14b^{2/3}c)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}\sqrt[3]{b}}+\frac{(5a^{2/3}e+14b^{2/3}c)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt[3]{b}}+\frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)}-\frac{d\log(a+bx^3)}{3a^3}-\frac{c}{a^3x}+\frac{d\log(x)}{a^3}+\frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -(c/(a^3*x)) + (x*(a*e - b*c*x - b*d*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*
e - 10*b*c*x - 9*b*d*x^2))/(18*a^3*(a + b*x^3)) + ((14*b^(2/3)*c - 5*a^(2/3
)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)
*b^(1/3)) + (d*Log[x])/a^3 + ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(1/3) + b^
(1/3)*x])/(27*a^(10/3)*b^(1/3)) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(1/3)) - (d*Log[a + b*x
^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
```

```
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^3}\,dx = \frac{x(ae-bcx-bdx^2)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6bc-6bdx-5bex^2+\frac{4b^2cx^3}{a}+\frac{3b^2dx^4}{a}}{x^2\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{x(ae-bcx-bdx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3\left(a+bx^3\right)} + \frac{\int \frac{18b^3c+18b^3dx+10b^3ex^2-\frac{10b^4cx^3}{a}}{x^2\left(a+bx^3\right)}\,dx}{18a^2b^3}$$

$$= \frac{x(ae-bcx-bdx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^3c}{ax^2} + \frac{18b^3d}{ax} + \frac{2b^3\left(5ae-14bcx-9bdx^2\right)}{a\left(a+bx^3\right)}\right.}{18a^2b^3}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3\left(a+bx^3\right)} + \frac{d\log(x)}{a^3} + \frac{\int \frac{5ae-14bcx-9bdx^2}{a+bx^3}}{9a^3}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3\left(a+bx^3\right)} + \frac{d\log(x)}{a^3} + \frac{\int \frac{5ae-14bcx}{a+bx^3}\,dx}{9a^3} -$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3\left(a+bx^3\right)} + \frac{d\log(x)}{a^3} - \frac{d\log\left(a+bx^3\right)}{3a^3} +$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3\left(a+bx^3\right)} + \frac{d\log(x)}{a^3} + \frac{\left(14b^{2/3}c+5a^{2/3}\right.}{27a}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3\left(a+bx^3\right)} + \frac{d\log(x)}{a^3} + \frac{\left(14b^{2/3}c+5a^{2/3}\right.}{27a}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2\left(a+bx^3\right)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3\left(a+bx^3\right)} + \frac{\left(14b^{2/3}c-5a^{2/3}e\right)\tan^{-1}\left(\right.}{9\sqrt{3}\,a^{10/3}\sqrt[3]{b}}$$

Mathematica [A]

time = 0.14, size = 248, normalized size = 0.93

$$\frac{-\frac{54ac}{x}+\frac{3a(6ad+5aex-10bcx^2)}{a+bx^3}+\frac{9a^2(-bcx^2+a(d+ex))}{(a+bx^3)^2}-\frac{2\sqrt{3}\,a^{2/3}(-14b^{2/3}c+5a^{2/3}e)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+54ad\log(x)+\frac{2(14a^{2/3}b^{2/3}c+5a^{4/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{(14a^{2/3}b^{2/3}c+5a^{4/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}}-18ad\log(a+bx^3)}{54a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a*c)/x + (3*a*(6*a*d + 5*a*e*x - 10*b*c*x^2))/(a + b*x^3) + (9*a^2*(-
(b*c*x^2) + a*(d + e*x)))/(a + b*x^3)^2 - (2*Sqrt[3]*a^(2/3)*(-14*b^(2/3)*c
 + 5*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + 54*a
*d*Log[x] + (2*(14*a^(2/3)*b^(2/3)*c + 5*a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x
])/b^(1/3) - ((14*a^(2/3)*b^(2/3)*c + 5*a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/b^(1/3) - 18*a*d*Log[a + b*x^3])/(54*a^4)
```

Maple [A]

time = 0.36, size = 279, normalized size = 1.04

method	result
risch	$\frac{-\frac{14c\,b^2x^6}{9a^3}+\frac{5be\,x^5}{18a^2}+\frac{bd\,x^4}{3a^2}-\frac{49bc\,x^3}{18a^2}+\frac{4e\,x^2}{9a}+\frac{xd}{2a}-\frac{c}{a}}{x(b\,x^3+a)^2}+\frac{d\ln(x)}{a^3}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^{10}b\_Z^3+27a^7bd\_Z^2+\left(-210a^4bce+243a^4b\,d^2\right)\_Z-1\right.}\right.}{}$
default	$\frac{\frac{-\frac{5}{9}b^2c\,x^5+\frac{5}{18}abe\,x^4+\frac{1}{3}abd\,x^3-\frac{13}{18}abc\,x^2+\frac{4}{9}a^2ex+\frac{1}{2}a^2d}{(b\,x^3+a)^2}+\frac{5ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{9}}{a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*((-5/9*b^2*c*x^5+5/18*a*b*e*x^4+1/3*a*b*d*x^3-13/18*a*b*c*x^2+4/9*a^2
*e*x+1/2*a^2*d)/(b*x^3+a)^2+5/9*a*e*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/
6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-14/9*b*c*(-1/3/b/(a/b)^(1/3)*ln(x
+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/
2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-1/3*d*ln(b*x^3+a)
)-c/a^3/x+d*ln(x)/a^3
```

Maxima [A]
time = 0.52, size = 271, normalized size = 1.01

$$-\frac{28\,b^2cx^6-5\,abx^5e-6\,abdx^4+49\,abcx^3-8\,a^2x^2e-9\,a^2dx+18\,a^2c}{18\,(a^3b^2x^7+2\,a^4bx^4+a^5x)}+\frac{d\log(x)}{a^3}-\frac{\sqrt{3}\left(14\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4}-\frac{\left(18\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(9\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(28*b^2*c*x^6 - 5*a*b*x^5*e - 6*a*b*d*x^4 + 49*a*b*c*x^3 - 8*a^2*x^2*
e - 9*a^2*d*x + 18*a^2*c)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x) + d*log(x)/a^
3 - 1/27*sqrt(3)*(14*b*c*(a/b)^(2/3) - 5*a*(a/b)^(1/3)*e)*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^4 - 1/54*(18*b*d*(a/b)^(2/3) + 14*b*c*
(a/b)^(1/3) + 5*a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2
/3)) - 1/27*(9*b*d*(a/b)^(2/3) - 14*b*c*(a/b)^(1/3) - 5*a*e)*log(x + (a/b)^
(1/3))/(a^3*b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.40, size = 5112, normalized size = 19.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2916*(4536*b^2*c*x^6 - 810*a*b*e*x^5 - 972*a*b*d*x^4 + 7938*a*b*c*x^3 -
1296*a^2*e*x^2 - 1458*a^2*d*x + 2916*a^2*c + 2*(a^3*b^2*x^7 + 2*a^4*b*x^4 +
 a^5*x)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a
^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 -
 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3
)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 7
0*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)
*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486
*d/a^3)*log(-7/1458*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/
(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 1
25*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 -
 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458
*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3
 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))
^(1/3) + 486*d/a^3)^2*a^7*b*c - 1134*a*b*c*d^2 + 1960*a*b*c^2*e + 225*a^2*d
*e^2 + 1/54*(252*a^4*b*c*d - 25*a^5*e^2)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (8
1*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39
366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/
39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-
1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125
*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 1
```

25*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3) - (2744*b^2*c^3 - 125*a^2*e^3)*x)
+ (1458*b^2*d*x^7 + 2916*a*b*d*x^4 + 1458*a^2*d*x - (a^3*b^2*x^7 + 2*a^4*b*
x^4 + a^5*x)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*
d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*
e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^
2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^
2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c
*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3)
+ 486*d/a^3) + 3*sqrt(1/3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*sqrt(-(((-I*
sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(
81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 -
 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(
1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9
+ 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b
) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)^2*a^6
 - 972*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^
9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 -
27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)
/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70
*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*
a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*
d/a^3)*a^3*d + 236196*d^2 - 816480*c*e)/a^6))*log(7/1458*((-I*sqrt(3) + 1)*
(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c
*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*
b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I
sqrt(3) + 1)(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(27
44*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(
2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)^2*a^7*b*c + 1134*a
*b*c*d^2 - 1960*a*b*c^2*e - 225*a^2*d*e^2 - 1/54*(252*a^4*b*c*d - 25*a^5*e^
2)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 +
1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(
27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^
10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e
)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)
/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^
3) - 2*(2744*b^2*c^3 - 125*a^2*e^3)*x + 1/486*sqrt(1/3)*(7*((-I*sqrt(3) + 1
)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70
*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*
a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*
(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(
2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366
*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)*a^7*b*c - 3402*a
^4*b*c*d - 675*a^5*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70
*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*
b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(274

```
4*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.28, size = 273, normalized size = 1.02

$$-\frac{d \log\left(|bx^3+a|\right)}{3\,a^3}+\frac{d\log\left(|x|\right)}{a^3}+\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}ae+14\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b}+\frac{\left(5\left(-ab^2\right)^{\frac{1}{3}}ae-14\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b}-\frac{28\,b^2cx^6-5\,abx^5e-6\,abdx^4+49\,abcx^3-8\,a^2x^2e-9\,a^2dx+18\,a^2c}{18\left(bx^3+a\right)^2a^3x}+\frac{\left(14\,a^3b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^4be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^3 + d*log(abs(x))/a^3 + 1/27*sqrt(3)*(5*(-a*b^
2)^(1/3)*a*e + 14*(-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))/(a^4*b) + 1/54*(5*(-a*b^2)^(1/3)*a*e - 14*(-a*b^2)^(2/3)*c)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/18*(28*b^2*c*x^6 - 5*a
*b*x^5*e - 6*a*b*d*x^4 + 49*a*b*c*x^3 - 8*a^2*x^2*e - 9*a^2*d*x + 18*a^2*c)
/((b*x^3 + a)^2*a^3*x) + 1/27*(14*a^3*b^2*c*(-a/b)^(1/3) - 5*a^4*b*e)*(-a/b
)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

Mupad [B]
time = 5.46, size = 793, normalized size = 2.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x)
```

```
[Out] ((4*e*x^2)/(9*a) - c/a + (d*x)/(2*a) - (14*b^2*c*x^6)/(9*a^3) - (49*b*c*x^3
)/(18*a^2) + (b*d*x^4)/(3*a^2) + (5*b*e*x^5)/(18*a^2))/(a^2*x + b^2*x^7 + 2
*a*b*x^4) + symsum(log((b^2*(225*a^2*d*e^2 - 225*root(19683*a^10*b*z^3 + 19
683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 72
9*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)*a^5*e^2 + 2744*b^2*c^3*x + 12
5*a^2*e^3*x + 1134*a*b*c*d^2 - 3402*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z
^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 1
25*a^2*e^3 - 2744*b^2*c^3, z, k)^2*a^7*b*c - 26244*root(19683*a^10*b*z^3 +
19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e +
729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)^3*a^10*b*x - 2916*root(1968
```

```
3*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 18
90*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)*a^4*b*d^2*x
- 17496*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561
*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z
, k)^2*a^7*b*d*x + 2268*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^
4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 -
 2744*b^2*c^3, z, k)*a^4*b*c*d + 6300*root(19683*a^10*b*z^3 + 19683*a^7*b*d
*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 -
 125*a^2*e^3 - 2744*b^2*c^3, z, k)*a^4*b*c*e*x + 1260*a*b*c*d*e*x))/(729*a^
8))*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4
*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)
, k, 1, 3) + (d*log(x))/a^3
```

3.356 $\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=276

$$-\frac{c}{2a^3x^2}-\frac{d}{a^3x}-\frac{x(bc+bdx+bex^2)}{6a^2\left(a+bx^3\right)^2}-\frac{x(11bc+10bdx+9bex^2)}{18a^3\left(a+bx^3\right)}+\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}\,c+7\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{11/3}}$$

```
[Out] -1/2*c/a^3/x^2-d/a^3/x-1/6*x*(b*e*x^2+b*d*x+b*c)/a^2/(b*x^3+a)^2-1/18*x*(9*
b*e*x^2+10*b*d*x+11*b*c)/a^3/(b*x^3+a)+e*ln(x)/a^3-2/27*b^(1/3)*(10*b^(1/3)
*c-7*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)+1/27*b^(1/3)*(10*b^(1/3)*c-7
*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)-1/3*e*ln(b*x
^3+a)/a^3+2/27*b^(1/3)*(10*b^(1/3)*c+7*a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)*3^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{2\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}\,d+10\sqrt[3]{b}\,c\right)}{9\sqrt{3}\,a^{11/3}}+\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{11/3}}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{11/3}}-\frac{x(11bc+10bdx+9bex^2)}{18a^3\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^3}-\frac{c}{2a^3x^2}-\frac{d}{a^3x}+\frac{e\log(x)}{a^3}-\frac{x(bc+bdx+bex^2)}{6a^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -1/2*c/(a^3*x^2) - d/(a^3*x) - (x*(b*c + b*d*x + b*e*x^2))/(6*a^2*(a + b*x^
3)^2) - (x*(11*b*c + 10*b*d*x + 9*b*e*x^2))/(18*a^3*(a + b*x^3)) + (2*b^(1/
3)*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(9*Sqrt[3]*a^(11/3)) + (e*Log[x])/a^3 - (2*b^(1/3)*(10*b^(1/3)*c -
7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)) + (b^(1/3)*(10*b^(1/3)
*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(11
/3)) - (e*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
```

```
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{x(bc+bdx+bex^2)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6bc-6bdx-6bex^2+\frac{5b^2cx^3}{a}+\frac{4b^2dx^4}{a}+\frac{3b^2ex^5}{a}}{x^3\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= -\frac{x(bc+bdx+bex^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3\left(a+bx^3\right)} + \frac{\int \frac{18b^3c+18b^3dx+18b^3ex^2-\frac{22b^4cx^3}{a}-\frac{10b^4dx^4}{a}}{x^3\left(a+bx^3\right)}}{18a^2b^3}$$

$$= -\frac{x(bc+bdx+bex^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^3c}{ax^3}+\frac{18b^3d}{ax^2}+\frac{18b^3e}{ax}-\frac{2b^4(20c+1}{a(a-}\right.}{18a^2b^3}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc+bdx+bex^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - \frac{b\int \frac{20c-}{}}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc+bdx+bex^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - \frac{b\int \frac{20c-}{a+}}{9}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc+bdx+bex^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - \frac{e\log(a}{3}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc+bdx+bex^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - \frac{2\sqrt[3]{b}\,\left(\right.}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc+bdx+bex^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - \frac{2\sqrt[3]{b}\,\left(\right.}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc+bdx+bex^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3\left(a+bx^3\right)} + \frac{2\sqrt[3]{b}\,\left(10\sqrt[3]{b}\,c+7\right.}{}$$

Mathematica [A]

time = 0.14, size = 253, normalized size = 0.92

$$\frac{-\frac{27ac}{x^2}-\frac{54ad}{x}+\frac{9a^2(ae-bx(c+dx))}{(a+bx^3)^2}+\frac{3a(6ae-bx(11c+10dx))}{a+bx^3}+4\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(10\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+54ae\log(x)+4\sqrt[3]{b}\left(-10\sqrt[3]{a}\sqrt[3]{b}c+7a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt[3]{b}\left(10\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-18ae\log\left(a+bx^3\right)}{54a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a*c)/x^2 - (54*a*d)/x + (9*a^2*(a*e - b*x*(c + d*x)))/(a + b*x^3)^2 +
 (3*a*(6*a*e - b*x*(11*c + 10*d*x)))/(a + b*x^3) + 4*Sqrt[3]*a^(1/3)*b^(1/3
)*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
+ 54*a*e*Log[x] + 4*b^(1/3)*(-10*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(1/
3) + b^(1/3)*x] + 2*b^(1/3)*(10*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 18*a*e*Log[a + b*x^3])/(54*a^4)
```

Maple [A]

time = 0.37, size = 287, normalized size = 1.04

method	result
risch	$\frac{-\frac{14 d b^2 x^7}{9a^3}-\frac{10 c b^2 x^6}{9a^3}+\frac{b e x^5}{3a^2}-\frac{49 b d x^4}{18a^2}-\frac{16 b c x^3}{9a^2}+\frac{e x^2}{2a}-\frac{x d}{a}-\frac{c}{2a}}{x^2\left(b x^3+a\right)^2}+\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^3+27a^8 e\_Z^2+\left(243a^5e^2+840a^4bcd\right)\_Z+729a^2e \cdots\right.} \cdots\right.$
default	$-\frac{b\left(\frac{\frac{5bd x^5}{9}+\frac{11bc x^4}{18}-\frac{ae x^3}{3}+\frac{13ad x^2}{18}+\frac{7acx}{9}-\frac{a^2e}{2b}}{\left(b x^3+a\right)^2}+\frac{20c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} \cdots\right.}{9} \cdots\right.}{a^3 \cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*b*((5/9*b*d*x^5+11/18*b*c*x^4-1/3*a*e*x^3+13/18*a*d*x^2+7/9*a*c*x-1/
2*a^2/b*e)/(b*x^3+a)^2+20/9*c*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a
```

```
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+14/9*d*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/
b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*e*ln(b*x^3+a)/b)-1/2*
c/a^3/x^2-d/a^3/x+e*ln(x)/a^3
```

Maxima [A]
time = 0.51, size = 270, normalized size = 0.98

$$-\frac{28\,b^2dx^7+20\,b^2cx^6-6\,abx^5e+49\,abdx^4+32\,abcx^3-9\,a^2x^2e+18\,a^2dx+9\,a^2c}{18\,(a^3b^2x^8+2\,a^4bx^5+a^5x^2)}+\frac{e\log(x)}{a^3}-\frac{2\,\sqrt{3}\left(7\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+10\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4}-\frac{\left(9\left(\frac{a}{b}\right)^{\frac{2}{3}}e+7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-10\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(9\left(\frac{a}{b}\right)^{\frac{2}{3}}e-14\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(28*b^2*d*x^7 + 20*b^2*c*x^6 - 6*a*b*x^5*e + 49*a*b*d*x^4 + 32*a*b*c*
x^3 - 9*a^2*x^2*e + 18*a^2*d*x + 9*a^2*c)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*
x^2) + e*log(x)/a^3 - 2/27*sqrt(3)*(7*b*d*(a/b)^(2/3) + 10*b*c*(a/b)^(1/3))
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^4 - 1/27*(9*(a/b)^(2
/3)*e + 7*d*(a/b)^(1/3) - 10*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3
*(a/b)^(2/3)) - 1/27*(9*(a/b)^(2/3)*e - 14*d*(a/b)^(1/3) + 20*c)*log(x + (a
/b)^(1/3))/(a^3*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.27, size = 4911, normalized size = 17.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2916*(4536*b^2*d*x^7 + 3240*b^2*c*x^6 - 972*a*b*e*x^5 + 7938*a*b*d*x^4 +
 5184*a*b*c*x^3 - 1458*a^2*e*x^2 + 2916*a^2*d*x + 1458*a^2*c + 2*(a^3*b^2*x
^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81
*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/1968
3*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 5
6*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^
9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)
*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b
)/a^11)^(1/3) + 486*e/a^3)*log(7/2916*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*
b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^1
0 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a
^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1
/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 +
343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*
c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)^2*a^8*d + 3920*a*b*c*d^2 - 1800*a*b*c^
```

```
2*e + 567*a^2*d*e^2 + 1/27*(100*a^4*b*c^2 - 63*a^5*d*e)*((-I*sqrt(3) + 1)*(
81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d
 + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(80
00*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(
I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19
683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 -
 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3) + 4*(1000*b^2*c^3 +
343*a*b*d^3)*x) + (1458*b^2*e*x^8 + 2916*a*b*e*x^5 + 1458*a^2*e*x^2 - (a^3*
b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d
 + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4
/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^
3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e
^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a
*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e
)*a*b)/a^11)^(1/3) + 486*e/a^3) - 3*sqrt(1/3)*(a^3*b^2*x^8 + 2*a^4*b*x^5 +
a^5*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)
/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^
3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 -
135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(
280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/
39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3
) + 486*e/a^3)^2*a^7 - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*
a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683
*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56
*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9
 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*
b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)
/a^11)^(1/3) + 486*e/a^3)*a^4*e + 3265920*b*c*d + 236196*a*e^2)/a^7))*log(-
7/2916*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e
^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a
*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e
)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d
 + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(80
00*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e
/a^3)^2*a^8*d - 3920*a*b*c*d^2 + 1800*a*b*c^2*e - 567*a^2*d*e^2 - 1/27*(100
*a^4*b*c^2 - 63*a^5*d*e)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*
e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(
1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(
49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 +
 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/
a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a
^11)^(1/3) + 486*e/a^3) + 8*(1000*b^2*c^3 + 343*a*b*d^3)*x + 1/972*sqrt(1/3
)*(7*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3
/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d
^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*
```

```
a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d +
 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000
*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a
^3)*a^8*d - 10800*a^4*b*c^2 - 3402*a^5*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(81*e^
2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81
*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^
2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.31, size = 282, normalized size = 1.02

$$-\frac{e\log\left(|bx^3+a|\right)}{3\,a^3}+\frac{e\log\left(|x|\right)}{a^3}-\frac{2\,\sqrt{3}\left(10\left(-ab^2\right)^{\frac{1}{3}}bc-7\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b}-\frac{\left(10\left(-ab^2\right)^{\frac{1}{3}}bc+7\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4b}-\frac{28\,b^2dx^7+20\,b^2cx^6-6\,abx^5e+49\,abdx^4+32\,abcx^3-9\,a^2x^2e+18\,a^2dx+9\,a^2c}{18\left(bx^4+ax\right)^2a^3}+\frac{2\left(7\,a^3b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+10\,a^3b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^3 + e*log(abs(x))/a^3 - 2/27*sqrt(3)*(10*(-a*b
^2)^(1/3)*b*c - 7*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))/(a^4*b) - 1/27*(10*(-a*b^2)^(1/3)*b*c + 7*(-a*b^2)^(2/3)*d)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/18*(28*b^2*d*x^7 + 20*
b^2*c*x^6 - 6*a*b*x^5*e + 49*a*b*d*x^4 + 32*a*b*c*x^3 - 9*a^2*x^2*e + 18*a^
2*d*x + 9*a^2*c)/((b*x^4 + a*x)^2*a^3) + 2/27*(7*a^3*b^2*d*(-a/b)^(1/3) + 1
0*a^3*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

Mupad [B]
time = 5.36, size = 778, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x)
```

```
[Out] symsum(log(-(2*b^3*(1701*root(19683*a^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*
b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 80
00*b^2*c^3, z, k)^2*a^8*d - 567*a^2*d*e^2 + 13122*root(19683*a^11*z^3 + 196
83*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a
```

```
*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)^3*a^11*x + 4000*b^2*c^3*x - 1134
*root(19683*a^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z
 + 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^5*d*
e - 1800*a*b*c^2*e - 1372*a*b*d^3*x + 1800*root(19683*a^11*z^3 + 19683*a^8*
e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a*b*d^3
+ 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^4*b*c^2 + 1458*root(19683*a^11*z^3 +
19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 274
4*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^5*e^2*x + 8748*root(19683*a
^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c
*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)^2*a^8*e*x + 12600*r
oot(19683*a^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z +
 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^4*b*c*
d*x + 2520*a*b*c*d*e*x))/(729*a^9))*root(19683*a^11*z^3 + 19683*a^8*e*z^2 +
 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a
^2*e^3 + 8000*b^2*c^3, z, k), k, 1, 3) - (c/(2*a) - (e*x^2)/(2*a) + (d*x)/a
 + (10*b^2*c*x^6)/(9*a^3) + (14*b^2*d*x^7)/(9*a^3) + (16*b*c*x^3)/(9*a^2) +
 (49*b*d*x^4)/(18*a^2) - (b*e*x^5)/(3*a^2))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5)
 + (e*log(x))/a^3
```

3.357 $\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=298

$$-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)}+\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}\,d+7\sqrt[3]{a}\,e\right)\tan^{-1}\left(\right.}{9\sqrt{3}\,a^{11/3}}$$

```
[Out] -1/3*c/a^3/x^3-1/2*d/a^3/x^2-e/a^3/x-1/6*x*(b*d+b*x*e-b^2*c*x^2/a)/a^2/(b*x
^3+a)^2-1/18*x*(11*b*d+10*b*x*e-15*b^2*c*x^2/a)/a^3/(b*x^3+a)-3*b*c*ln(x)/a
^4-2/27*b^(1/3)*(10*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)+1
/27*b^(1/3)*(10*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/a^(11/3)+b*c*ln(b*x^3+a)/a^4+2/27*b^(1/3)*(10*b^(1/3)*d+7*a^(1/3)*e)*
arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)*3^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{2\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}\,e+10\sqrt[3]{b}\,d\right)}{9\sqrt{3}\,a^{11/3}}+\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{11/3}}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{11/3}}+\frac{bc\log\left(a+bx^3\right)}{a^4}-\frac{3bc\log(x)}{a^4}-\frac{x\left(-\frac{15b^2cx^2}{a}+11bd+10bex\right)}{18a^3\left(a+bx^3\right)}-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{6a^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -1/3*c/(a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d + b*e*x - (b^2*c*x^2
)/a))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d + 10*b*e*x - (15*b^2*c*x^2)/a))/(1
8*a^3*(a + b*x^3)) + (2*b^(1/3)*(10*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)) - (3*b*c*Log[x])/
a^4 - (2*b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27
*a^(11/3)) + (b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(27*a^(11/3)) + (b*c*Log[a + b*x^3])/a^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
```

```
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx = -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{\int \frac{-6bc-6bdx-6bex^2+\frac{6b^2cx^3}{a}+\frac{5b^2dx^4}{a}+\frac{4b^2ex^5}{a}-\frac{3b^3cx^6}{a^2}}{x^4\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)}+\frac{\int \frac{18b^3c+18b^3dx+18b^3ex^2-\frac{36b^4cx^3}{a}-\cdots}{x^4\left(a+bx^3\right)}}{18a^2b^3}\cdots$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)}+\frac{\int\left(\frac{18b^3c}{ax^4}+\frac{18b^3d}{ax^3}+\frac{18b^3e}{ax^2}-\frac{54b^4c}{a^2x}\cdots\right.}{18a^2\cdots}$$

$$= -\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)}-\frac{3bc\log\cdots}{a\cdots}$$

$$= -\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)}-\frac{3bc\log\cdots}{a\cdots}$$

$$= -\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)}-\frac{3bc\log\cdots}{a\cdots}$$

$$= -\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)}-\frac{3bc\log\cdots}{a\cdots}$$

$$= -\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)}-\frac{3bc\log\cdots}{a\cdots}$$

$$= -\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)}+\frac{2\sqrt[3]{b}\cdots}{\cdots}$$

Mathematica [A]

time = 0.24, size = 255, normalized size = 0.86

$$-\frac{\frac{18ac}{x^3}+\frac{27ad}{x^2}+\frac{54ae}{x}+\frac{9a^2b(c+x(d+ex))}{(a+bx^3)^2}+\frac{3ab(12c+x(11d+10ex))}{a+bx^3}-4\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(10\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+162bc\log(x)+4\sqrt[3]{b}\left(10\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt[3]{b}\left(10\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-54bc\log\left(a+bx^3\right)}{54a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -1/54*((18*a*c)/x^3 + (27*a*d)/x^2 + (54*a*e)/x + (9*a^2*b*(c + x*(d + e*x)
))/(a + b*x^3)^2 + (3*a*b*(12*c + x*(11*d + 10*e*x)))/(a + b*x^3) - 4*Sqrt[
3]*a^(1/3)*b^(1/3)*(10*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 162*b*c*Log[x] + 4*b^(1/3)*(10*a^(1/3)*b^(1/3)*d - 7*a^(
2/3)*e)*Log[a^(1/3) + b^(1/3)*x] - 2*b^(1/3)*(10*a^(1/3)*b^(1/3)*d - 7*a^(2
/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 54*b*c*Log[a + b*x^
3])/a^4
```

Maple [A]

time = 0.38, size = 301, normalized size = 1.01

method	result
default	$-\frac{b\left(\frac{\frac{5}{9}abe\,x^5+\frac{11}{18}abd\,x^4+\frac{2}{3}abc\,x^3+\frac{13}{18}a^2e\,x^2+\frac{7}{9}a^2dx+\frac{5}{6}a^2c}{\left(b\,x^3+a\right)^2}+\frac{20ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)\cdots}\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\cdots\right.}{9}\cdots\right)}{a^4}\cdots$
risch	$\frac{-\frac{14e\,b^2x^8}{9a^3}-\frac{10d\,b^2x^7}{9a^3}-\frac{c\,b^2x^6}{a^3}-\frac{49be\,x^5}{18a^2}-\frac{16bd\,x^4}{9a^2}-\frac{3bc\,x^3}{2a^2}-\frac{e\,x^2}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^3\left(b\,x^3+a\right)^2}-\frac{3bc\ln(x)}{a^4}+\left(\_R=\mathrm{RootOf}\left(a^{12}\_Z^3-81a^8bc\_Z^2+(840a^5\cdots\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^4*b*((5/9*a*b*e*x^5+11/18*a*b*d*x^4+2/3*a*b*c*x^3+13/18*a^2*e*x^2+7/9*
a^2*d*x+5/6*a^2*c)/(b*x^3+a)^2+20/9*a*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+14/9*a*e*(-1/3/b/(a/b)^(1/3)*
ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3
^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-c*ln(b*x^3+a)
)-1/2*d/a^3/x^2-1/3*c/a^3/x^3-e/a^3/x-3*b*c*ln(x)/a^4
```

Maxima [A]
time = 0.51, size = 289, normalized size = 0.97

$$-\frac{28\,b^2x^8e+20\,b^2dx^7+18\,b^2cx^6+49\,abx^5e+32\,abdx^4+27\,abcx^3+18\,a^2x^2e+9\,a^2dx+6\,a^2c}{18\,(a^3b^2x^9+2\,a^4bx^6+a^5x^3)}-\frac{3\,bc\log(x)}{a^4}-\frac{2\,\sqrt{3}\,\left(7\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}e+10\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5}+\frac{\left(27\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-7\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}e+10\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(27\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}e-20\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(28*b^2*x^8*e + 20*b^2*d*x^7 + 18*b^2*c*x^6 + 49*a*b*x^5*e + 32*a*b*d
*x^4 + 27*a*b*c*x^3 + 18*a^2*x^2*e + 9*a^2*d*x + 6*a^2*c)/(a^3*b^2*x^9 + 2*
a^4*b*x^6 + a^5*x^3) - 3*b*c*log(x)/a^4 - 2/27*sqrt(3)*(7*a*(a/b)^(2/3)*e +
 10*a*d*(a/b)^(1/3))*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/
a^5 + 1/27*(27*b*c*(a/b)^(2/3) - 7*a*(a/b)^(1/3)*e + 10*a*d)*log(x^2 - x*(a
/b)^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) + 1/27*(27*b*c*(a/b)^(2/3) + 14*
a*(a/b)^(1/3)*e - 20*a*d)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.57, size = 5550, normalized size = 18.62

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(168*a*b^2*e*x^8 + 120*a*b^2*d*x^7 + 108*a*b^2*c*x^6 + 294*a^2*b*e*x
^5 + 192*a^2*b*d*x^4 + 162*a^2*b*c*x^3 + 108*a^3*e*x^2 + 54*a^3*d*x + 36*a^
3*c + 2*(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) +
1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12
+ 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^
12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12
)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 +
 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c
^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/
a^4)*log(7/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^
2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^
11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*
e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
```

+ 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*
c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3
 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8*e + 5400*b^2*c*d^2 + 5
103*b^2*c^2*e + 3920*a*b*d*e^2 + (100*a^4*b*d^2 + 189*a^4*b*c*e)*(2*(1/2)^(
2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(
39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 +
280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567
*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^1
2 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/
a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^
12)^(1/3) - 54*b*c/a^4) + 4*(1000*b^2*d^3 + 343*a*b*e^3)*x) - (162*b^3*c*x^
9 + 324*a*b^2*c*x^6 + 162*a^2*b*c*x^3 + (a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^
3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*
b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(7
29*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(
200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(3936
6*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*
a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d
*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4) + 3*sqrt(1/3)*(a^4*b^2*x^9 + 2*a^5*b*x
^6 + a^6*x^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (72
9*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e
^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 274
4*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*
sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(
729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*
(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8 + 108*(2*(1/2)
^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)
/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2
+ 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 5
67*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a
^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*
c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/
a^12)^(1/3) - 54*b*c/a^4)*a^4*b*c + 2916*b^2*c^2 + 4480*a*b*d*e)/a^8))*log(
-7/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*
a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*
(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40
*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39
366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 28
0*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c
*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8*e - 5400*b^2*c*d^2 - 5103*b^2*
c^2*e - 3920*a*b*d*e^2 - (100*a^4*b*d^2 + 189*a^4*b*c*e)*(2*(1/2)^(2/3)*(-I
sqrt(3) + 1)(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^
3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*
d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*
a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1

```
000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (
19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3
) - 54*b*c/a^4) + 8*(1000*b^2*d^3 + 343*a*b*e^3)*x + 3/4*sqrt(1/3)*(7*(2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a
^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c
^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3
- 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^
3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)
*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 4...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.41, size = 305, normalized size = 1.02

$$\frac{bc\log(|bx^3+a|)}{a^4}-\frac{3\,bc\log(|x|)}{a^4}-\frac{2\sqrt{3}\left(10\,(-ab^2)^{\frac{1}{3}}bd-7\,(-ab^2)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+(-\frac{a}{b})^{\frac{1}{3}}\right)}{3\,(-\frac{a}{b})^{\frac{1}{3}}}\right)}{27\,a^4b}-\frac{\left(10\,(-ab^2)^{\frac{1}{3}}bd+7\,(-ab^2)^{\frac{2}{3}}e\right)\log\left(x^2+x(-\frac{a}{b})^{\frac{1}{3}}+(-\frac{a}{b})^{\frac{2}{3}}\right)}{27\,a^4b}+\frac{2\left(7\,a^5b^2(-\frac{a}{b})^{\frac{1}{3}}e+10\,a^5b^2d\right)(-\frac{a}{b})^{\frac{1}{3}}\log\left(\left|x-(-\frac{a}{b})^{\frac{1}{3}}\right|\right)}{27\,a^9b}-\frac{28\,ab^2x^8e+20\,ab^2dx^7+18\,ab^2cx^6+49\,a^2bx^5e+32\,a^2bdx^4+27\,a^2bcx^3+18\,a^3x^2e+9\,a^3dx+6\,a^3c}{18\,(bx^3+a)^2a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] b*c*log(abs(b*x^3 + a))/a^4 - 3*b*c*log(abs(x))/a^4 - 2/27*sqrt(3)*(10*(-a*
b^2)^(1/3)*b*d - 7*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(a^4*b) - 1/27*(10*(-a*b^2)^(1/3)*b*d + 7*(-a*b^2)^(2/3)*e)
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) + 2/27*(7*a^5*b^2*(-a/b)^
(1/3)*e + 10*a^5*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b) - 1
/18*(28*a*b^2*x^8*e + 20*a*b^2*d*x^7 + 18*a*b^2*c*x^6 + 49*a^2*b*x^5*e + 32
*a^2*b*d*x^4 + 27*a^2*b*c*x^3 + 18*a^3*x^2*e + 9*a^3*d*x + 6*a^3*c)/((b*x^3
 + a)^2*a^4*x^3)
```

Mupad [B]
time = 0.46, size = 870, normalized size = 2.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x)
```

```
[Out] symsum(log(-(2*b^3*(1701*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^
5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000
```

```
*a*b^2*d^3 - 19683*b^3*c^3, z, k)^2*a^8*e + 5400*b^2*c*d^2 - 5103*b^2*c^2*e
 + 13122*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 5904
9*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 196
83*b^3*c^3, z, k)^3*a^11*x + 4000*b^2*d^3*x - 1372*a*b*e^3*x + 1800*root(19
683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z
 - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k
)*a^4*b*d^2 - 26244*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d
*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^
2*d^3 - 19683*b^3*c^3, z, k)^2*a^7*b*c*x + 13122*root(19683*a^12*z^3 - 5904
9*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e
 - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k)*a^3*b^2*c^2*x + 3
402*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4
*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^
3*c^3, z, k)*a^4*b*c*e - 7560*b^2*c*d*e*x + 12600*root(19683*a^12*z^3 - 590
49*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*
e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k)*a^4*b*d*e*x))/(7
29*a^9))*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 5904
9*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 196
83*b^3*c^3, z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (d*x)/(2*a) + (b^2*c*x
^6)/a^3 + (10*b^2*d*x^7)/(9*a^3) + (14*b^2*e*x^8)/(9*a^3) + (3*b*c*x^3)/(2*
a^2) + (16*b*d*x^4)/(9*a^2) + (49*b*e*x^5)/(18*a^2))/(a^2*x^3 + b^2*x^9 + 2
*a*b*x^6) - (3*b*c*log(x))/a^4
```

3.358 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx$

Optimal. Leaf size=248

$$-\frac{c+dx+ex^2}{9b\,(a+bx^3)^3}+\frac{x(d+2ex)}{54ab\,(a+bx^3)^2}+\frac{x(5d+8ex)}{162a^2b\,(a+bx^3)}-\frac{\left(5\sqrt[3]{b}\,d+4\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{81\sqrt{3}\,a^{8/3}b^{5/3}}+\frac{\left(5\sqrt[3]{b}\,d-4\sqrt[3]{\cdots}\right.}{\cdots}$$

```
[Out] 1/9*(-e*x^2-d*x-c)/b/(b*x^3+a)^3+1/54*x*(2*e*x+d)/a/b/(b*x^3+a)^2+1/162*x*(
8*e*x+5*d)/a^2/b/(b*x^3+a)+1/243*(5*b^(1/3)*d-4*a^(1/3)*e)*ln(a^(1/3)+b^(1/
3)*x)/a^(8/3)/b^(5/3)-1/486*(5*b^(1/3)*d-4*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(5/3)-1/243*(5*b^(1/3)*d+4*a^(1/3)*e)*arctan
(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1837, 1869, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(4\sqrt[3]{a}\,e+5\sqrt[3]{b}\,d\right)}{81\sqrt{3}\,a^{8/3}b^{5/3}}-\frac{\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{486a^{8/3}b^{5/3}}+\frac{\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{8/3}b^{5/3}}+\frac{x(5d+8ex)}{162a^2b\,(a+bx^3)}-\frac{c+dx+ex^2}{9b\,(a+bx^3)^3}+\frac{x(d+2ex)}{54ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] -1/9*(c + d*x + e*x^2)/(b*(a + b*x^3)^3) + (x*(d + 2*e*x))/(54*a*b*(a + b*x
^3)^2) + (x*(5*d + 8*e*x))/(162*a^2*b*(a + b*x^3)) - ((5*b^(1/3)*d + 4*a^(1
/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(8/
3)*b^(5/3)) + ((5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a
^(8/3)*b^(5/3)) - ((5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(486*a^(8/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx = -\frac{c+dx+ex^2}{9b\,(a+bx^3)^3} + \frac{\int \frac{d+2ex}{(a+bx^3)^3}\,dx}{9b}$$

$$= -\frac{c+dx+ex^2}{9b\,(a+bx^3)^3} + \frac{x(d+2ex)}{54ab\,(a+bx^3)^2} - \frac{\int \frac{-5d-8ex}{(a+bx^3)^2}\,dx}{54ab}$$

$$= -\frac{c+dx+ex^2}{9b\,(a+bx^3)^3} + \frac{x(d+2ex)}{54ab\,(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b\,(a+bx^3)} + \frac{\int \frac{10d+8ex}{a+bx^3}\,dx}{162a^2b}$$

$$= -\frac{c+dx+ex^2}{9b\,(a+bx^3)^3} + \frac{x(d+2ex)}{54ab\,(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b\,(a+bx^3)} + \frac{\int \frac{\sqrt[3]{a}\left(20\sqrt[3]{b}\,d+8\sqrt[3]{a}\,e\right)+\sqrt[3]{\ }}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}}}{486a^{8/3}}$$

$$= -\frac{c+dx+ex^2}{9b\,(a+bx^3)^3} + \frac{x(d+2ex)}{54ab\,(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b\,(a+bx^3)} + \frac{\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{\ }\right.}{243a^{8/3}b^{5/3}}$$

$$= -\frac{c+dx+ex^2}{9b\,(a+bx^3)^3} + \frac{x(d+2ex)}{54ab\,(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b\,(a+bx^3)} + \frac{\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{\ }\right.}{243a^{8/3}b^{5/3}}$$

$$= -\frac{c+dx+ex^2}{9b\,(a+bx^3)^3} + \frac{x(d+2ex)}{54ab\,(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b\,(a+bx^3)} - \frac{\left(5\sqrt[3]{b}\,d+4\sqrt[3]{a}\,e\right)\tan^{-1}\left(\right.}{81\sqrt{3}\,a^{8/3}b^{5/}}$$

Mathematica [A]
time = 0.15, size = 230, normalized size = 0.93

$$\frac{\frac{9b^{2/3}x(d+2ex)}{a(a+bx^3)^2}+\frac{3b^{2/3}x(5d+8ex)}{a^2(a+bx^3)}-\frac{54b^{2/3}(c+x(d+ex))}{(a+bx^3)^3}-\frac{2\sqrt{3}\left(5\sqrt[3]{b}\,d+4\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}}+\frac{2\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{8/3}}+\frac{\left(-5\sqrt[3]{b}\,d+4\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{8/3}}}{486b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] ((9*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)^2) + (3*b^(2/3)*x*(5*d + 8*e*x))/
(a^2*(a + b*x^3)) - (54*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3)^3 - (2*Sqrt[
3]*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])
/a^(8/3) + (2*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^(8/3)
 + ((-5*b^(1/3)*d + 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/a^(8/3))/(486*b^(5/3))
```

Maple [A]
time = 0.36, size = 268, normalized size = 1.08

method	result
risch	$\frac{\frac{4be\,x^8}{81a^2}+\frac{5bd\,x^7}{162a^2}+\frac{11e\,x^5}{81a}+\frac{13d\,x^4}{162a}-\frac{2e\,x^2}{81b}-\frac{5dx}{81b}-\frac{c}{9b}}{(b\,x^3+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(4e\_R+5d)\ln(x-\_R)}{\_R^2}}{243a^2b^2}$
default	$\frac{\frac{4be\,x^8}{81a^2}+\frac{5bd\,x^7}{162a^2}+\frac{11e\,x^5}{81a}+\frac{13d\,x^4}{162a}-\frac{2e\,x^2}{81b}-\frac{5dx}{81b}-\frac{c}{9b}}{(b\,x^3+a)^3}+\frac{5d\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (4/81*b*e/a^2*x^8+5/162/a^2*b*d*x^7+11/81/a*e*x^5+13/162*d/a*x^4-2/81*e*x^2
/b-5/81*d*x/b-1/9*c/b)/(b*x^3+a)^3+1/81/a^2/b*(5*d*(1/3/b/(a/b)^(2/3)*ln(x+
(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+4*e*(-1/3/b/(a/b)^
(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.49, size = 254, normalized size = 1.02

$$\frac{8\,b^2x^8e+5\,b^2dx^7+22\,abx^5e+13\,abdx^4-4\,a^2x^2e-10\,a^2dx-18\,a^2c}{162\,(a^2b^4x^9+3\,a^3b^3x^6+3\,a^4b^2x^3+a^5b)}+\frac{\sqrt{3}\left(4\left(\frac{a}{b}\right)^{\frac{1}{3}}e+5\,d\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(4\left(\frac{a}{b}\right)^{\frac{1}{3}}e-5\,d\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(4\left(\frac{a}{b}\right)^{\frac{1}{3}}e-5\,d\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/162*(8*b^2*x^8*e + 5*b^2*d*x^7 + 22*a*b*x^5*e + 13*a*b*d*x^4 - 4*a^2*x^2*
e - 10*a^2*d*x - 18*a^2*c)/(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a
^5*b) + 1/243*sqrt(3)*(4*(a/b)^(1/3)*e + 5*d)*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) + 1/486*(4*(a/b)^(1/3)*e - 5*d
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) - 1/243*(4*(
a/b)^(1/3)*e - 5*d)*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.43, size = 2364, normalized size = 9.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(48*b^2*e*x^8 + 30*b^2*d*x^7 + 132*a*b*e*x^5 + 78*a*b*d*x^4 - 24*a^2*
e*x^2 - 60*a^2*d*x - 108*a^2*c - 2*(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2
*x^3 + a^5*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5
) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3
) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/
(a^8*b^5))^(1/3)))*log(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)
/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(
-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 6
4*a*e^3)/(a^8*b^5))^(1/3)))^2*a^6*b^3*e - 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)
*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3
) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a
^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))*a^3*b^2*d^2 + 160*a*d*e
^2 + (125*b*d^3 + 64*a*e^3)*x) + ((a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*
x^3 + a^5*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5)
 + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3)
 + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(
a^8*b^5))^(1/3))) + 3*sqrt(1/3)*(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^
3 + a^5*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8
*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sq
rt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e
^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 320*d*e)/(a^5*b^3)))*log(-((1/2)^(1/3)*(
I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(
a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3
+ 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^6*b^3
*e + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) +
(125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) +
1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8
*b^5))^(1/3)))*a^3*b^2*d^2 - 160*a*d*e^2 + 2*(125*b*d^3 + 64*a*e^3)*x + 3/2
*sqrt(1/3)*(2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5
) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3
) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/
(a^8*b^5))^(1/3)))*a^6*b^3*e + 25*a^3*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(
3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5
))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*
e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 320*
d*e)/(a^5*b^3))) + ((a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a^5*b)*(
(1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3
- 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3
*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3
))) - 3*sqrt(1/3)*(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a^5*b)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b
```

```
*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^
5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))
^(1/3)))^2*a^5*b^3 + 320*d*e)/(a^5*b^3)))*log(-((1/2)^(1/3)*(I*sqrt(3) + 1)
*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3
) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a
^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^6*b^3*e + 25/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 6
4*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((
125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))
*a^3*b^2*d^2 - 160*a*d*e^2 + 2*(125*b*d^3 + 64*a*e^3)*x - 3/2*sqrt(1/3)*(2*
((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3
 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^
3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/
3)))*a^6*b^3*e + 25*a^3*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*
b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*
(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5)
 + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 320*d*e)/(a^5*b^3)
)))/(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a^5*b)
```

Sympy [A]
time = 153.88, size = 201, normalized size = 0.81

$$\operatorname{RootSum}\left(14348907t^3a^8b^5+14580ta^3b^2de+64ae^3-125bd^3,\left(t\mapsto t\log\left(x+\frac{236196t^2a^6b^3e+6075ta^3b^2d^2+160ade^2}{64ae^3+125bd^3}\right)\right)\right)+\frac{-18a^2c-10a^2dx-4a^2ex^2+13abdx^4+22abex^5+5b^2dx^7+8b^2ex^8}{162a^5b+486a^4b^2x^3+486a^3b^3x^6+162a^2b^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**8*b**5 + 14580*_t*a**3*b**2*d*e + 64*a*e**3 - 125
*b*d**3, Lambda(_t, _t*log(x + (236196*_t**2*a**6*b**3*e + 6075*_t*a**3*b**
2*d**2 + 160*a*d*e**2)/(64*a*e**3 + 125*b*d**3)))) + (-18*a**2*c - 10*a**2*
d*x - 4*a**2*e*x**2 + 13*a*b*d*x**4 + 22*a*b*e*x**5 + 5*b**2*d*x**7 + 8*b**
2*e*x**8)/(162*a**5*b + 486*a**4*b**2*x**3 + 486*a**3*b**3*x**6 + 162*a**2*
b**4*x**9)
```

Giac [A]
time = 0.94, size = 242, normalized size = 0.98

$$-\frac{\sqrt{3}\left(5\,bd-4\left(-ab^2\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,\left(-ab^2\right)^{\frac{2}{3}}a^2b}-\frac{\left(5\,bd+4\left(-ab^2\right)^{\frac{1}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,\left(-ab^2\right)^{\frac{2}{3}}a^2b}-\frac{\left(4\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+5\,d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^3b}+\frac{8\,b^2x^8e+5\,b^2dx^7+22\,abx^5e+13\,abdx^4-4\,a^2x^2e-10\,a^2dx-18\,a^2c}{162\,\left(bx^3+a\right)^3a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/243*sqrt(3)*(5*b*d - 4*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b) - 1/486*(5*b*d + 4*(-a*b^2)^(
1/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) - 1
```

```
/243*(4*(-a/b)^(1/3)*e + 5*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*
b) + 1/162*(8*b^2*x^8*e + 5*b^2*d*x^7 + 22*a*b*x^5*e + 13*a*b*d*x^4 - 4*a^2
*x^2*e - 10*a^2*d*x - 18*a^2*c)/((b*x^3 + a)^3*a^2*b)
```

Mupad [B]
time = 0.27, size = 253, normalized size = 1.02

$$\left(\sum_{k=1}^{3}\ln\left(\frac{20\,d\,e+16\,e^2\,x+\mathrm{root}(14348907\,a^8\,b^5\,z^3+14580\,a^3\,b^2\,d\,e\,z-125\,b\,d^3+64\,a\,e^3,z,k)^2\,a^5\,b^3\,59049+\mathrm{root}(14348907\,a^8\,b^5\,z^3+14580\,a^3\,b^2\,d\,e\,z-125\,b\,d^3+64\,a\,e^3,z,k)\,a^2\,b^2\,d\,x\,1215}{a^4\,b\,6561}\right)\mathrm{root}\left(14348907\,a^8\,b^5\,z^3+14580\,a^3\,b^2\,d\,e\,z-125\,b\,d^3+64\,a\,e^3,z,k\right)\right)+\frac{\frac{13\,d\,x^4}{162\,a}-\frac{c}{9\,b}+\frac{11\,e\,x^5}{81\,a}-\frac{2\,e\,x^2}{81\,b}-\frac{5\,d\,x}{81\,b}+\frac{5\,b\,d\,x^7}{162\,a^2}+\frac{4\,b\,e\,x^8}{81\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x)
```

```
[Out] symsum(log((20*d*e + 16*e^2*x + 59049*root(14348907*a^8*b^5*z^3 + 14580*a^3
*b^2*d*e*z - 125*b*d^3 + 64*a*e^3, z, k)^2*a^5*b^3 + 1215*root(14348907*a^8
*b^5*z^3 + 14580*a^3*b^2*d*e*z - 125*b*d^3 + 64*a*e^3, z, k)*a^2*b^2*d*x)/(
6561*a^4*b))*root(14348907*a^8*b^5*z^3 + 14580*a^3*b^2*d*e*z - 125*b*d^3 +
64*a*e^3, z, k), k, 1, 3) + ((13*d*x^4)/(162*a) - c/(9*b) + (11*e*x^5)/(81*
a) - (2*e*x^2)/(81*b) - (5*d*x)/(81*b) + (5*b*d*x^7)/(162*a^2) + (4*b*e*x^8
)/(81*a^2))/(a^3 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6)
```

3.359 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4} \, dx$

Optimal. Leaf size=270

$$-\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}+\frac{x(5ae+28bcx)}{162a^3b\,(a+bx^3)}-\frac{6ad-x(ae+7bcx)}{54a^2b\,(a+bx^3)^2}-\frac{(14b^{2/3}c+5a^{2/3}e)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{81\sqrt{3}\,a^{10/3}b^{4/3}}-$$

```
[Out] -1/9*x*(-b*d*x^2-b*c*x+a*e)/a/b/(b*x^3+a)^3+1/162*x*(28*b*c*x+5*a*e)/a^3/b/
(b*x^3+a)+1/54*(-6*a*d+x*(7*b*c*x+a*e))/a^2/b/(b*x^3+a)^2-1/243*(14*b^(2/3)
*c-5*a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(4/3)+1/486*(14*b^(2/3)*c-
5*a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(4/3)-1/2
43*(14*b^(2/3)*c+5*a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1
/2))/a^(10/3)/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1842, 1868, 1869, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(5a^{2/3}e+14b^{2/3}c)}{81\sqrt{3}\,a^{10/3}b^{4/3}}+\frac{(14b^{2/3}c-5a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{486a^{10/3}b^{4/3}}-\frac{(14b^{2/3}c-5a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{10/3}b^{4/3}}+\frac{x(5ae+28bcx)}{162a^3b\,(a+bx^3)}-\frac{6ad-x(ae+7bcx)}{54a^2b\,(a+bx^3)^2}-\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] -1/9*(x*(a*e - b*c*x - b*d*x^2))/(a*b*(a + b*x^3)^3) + (x*(5*a*e + 28*b*c*x
))/(162*a^3*b*(a + b*x^3)) - (6*a*d - x*(a*e + 7*b*c*x))/(54*a^2*b*(a + b*x
^3)^2) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(81*Sqrt[3]*a^(10/3)*b^(4/3)) - ((14*b^(2/3)*c - 5*a^(2/3)*e
)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(10/3)*b^(4/3)) + ((14*b^(2/3)*c - 5*a^(
2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(486*a^(10/3)*b^(4/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx &= -\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}-\frac{\int \frac{-ae-7bcx-6bdx^2}{(a+bx^3)^3}\,dx}{9ab}\\
&= -\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}-\frac{6ad-x(ae+7bcx)}{54a^2b\,(a+bx^3)^2}+\frac{\int \frac{5ae+28bcx}{(a+bx^3)^2}\,dx}{54a^2b}\\
&= -\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}+\frac{x(5ae+28bcx)}{162a^3b\,(a+bx^3)}-\frac{6ad-x(ae+7bcx)}{54a^2b\,(a+bx^3)^2}-\frac{\int \frac{-10ae-28bcx}{a+bx^3}\,d}{162a^3b}\\
&= -\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}+\frac{x(5ae+28bcx)}{162a^3b\,(a+bx^3)}-\frac{6ad-x(ae+7bcx)}{54a^2b\,(a+bx^3)^2}-\frac{\int \sqrt[3]{a}\left(-28\sqrt[3]{a}\right.}{}\\
&= -\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}+\frac{x(5ae+28bcx)}{162a^3b\,(a+bx^3)}-\frac{6ad-x(ae+7bcx)}{54a^2b\,(a+bx^3)^2}-\frac{(14b^{2/3}c-5a^2}{24}\\
&= -\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}+\frac{x(5ae+28bcx)}{162a^3b\,(a+bx^3)}-\frac{6ad-x(ae+7bcx)}{54a^2b\,(a+bx^3)^2}-\frac{(14b^{2/3}c-5a^2}{24}\\
&= -\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}+\frac{x(5ae+28bcx)}{162a^3b\,(a+bx^3)}-\frac{6ad-x(ae+7bcx)}{54a^2b\,(a+bx^3)^2}-\frac{(14b^{2/3}c+5a^2}{81}
\end{aligned}
$$

Mathematica [A]
time = 0.24, size = 241, normalized size = 0.89

$$
\frac{\frac{3ab^{2/3}\left(28b^3cx^8-2a^3(9d+5ex)+ab^2x^5\left(77c+5ex^2\right)+a^2bx^2\left(67c+13ex^2\right)\right)}{(a+bx^3)^3}-2\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,\left(14b^{2/3}c+5a^{2/3}e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(-14a^{2/3}bc+5a^{4/3}\sqrt[3]{b}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+a^{2/3}\sqrt[3]{b}\,\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{486a^4b^{5/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] ((3*a*b^(2/3)*(28*b^3*c*x^8 - 2*a^3*(9*d + 5*e*x) + a*b^2*x^5*(77*c + 5*e*x
^2) + a^2*b*x^2*(67*c + 13*e*x^2)))/(a + b*x^3)^3 - 2*Sqrt[3]*a^(2/3)*b^(1/
```

```
3)*(14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
 + 2*(-14*a^(2/3)*b*c + 5*a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] + a^(
2/3)*b^(1/3)*(14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(486*a^4*b^(5/3))
```

Maple [A]
time = 0.34, size = 273, normalized size = 1.01

method	result
risch	$\frac{\frac{14 c\, b^{2} x^{8}}{81 a^{3}}+\frac{5 b e\, x^{7}}{162 a^{2}}+\frac{77 b c\, x^{5}}{162 a^{2}}+\frac{13 e\, x^{4}}{162 a}+\frac{67 c\, x^{2}}{162 a}-\frac{5 e x}{81 b}-\frac{d}{9 b}}{\left(b\, x^{3}+a \right)^{3}}+\frac{\sum_{\_R =\mathit{RootOf}\left(b\, \_Z^{3}+a \right)}\frac{\left(\frac{14 c \_R}{a}+\frac{5 e}{b}\right) \ln \left(x -\_R \right)}{\_R^{2}}}{243 a^{2} b}$
default	$\frac{\frac{14 c\, b^{2} x^{8}}{81 a^{3}}+\frac{5 b e\, x^{7}}{162 a^{2}}+\frac{77 b c\, x^{5}}{162 a^{2}}+\frac{13 e\, x^{4}}{162 a}+\frac{67 c\, x^{2}}{162 a}-\frac{5 e x}{81 b}-\frac{d}{9 b}}{\left(b\, x^{3}+a \right)^{3}}+\frac{5 a e \left(\frac{\ln \left(x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x +\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\right.}{3}\right.}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (14/81*c/a^3*b^2*x^8+5/162*b*e/a^2*x^7+77/162*b*c/a^2*x^5+13/162/a*e*x^4+67
/162/a*c*x^2-5/81*e*x/b-1/9*d/b)/(b*x^3+a)^3+1/81/a^3/b*(5*a*e*(1/3/b/(a/b)
^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+14*b*c
*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))))
```

Maxima [A]
time = 0.50, size = 266, normalized size = 0.99

$$\frac{28 \, b^{3} c x^{8} + 5 \, a b^{2} x^{7} e + 77 \, a b^{2} c x^{5} + 13 \, a^{2} b x^{4} e + 67 \, a^{2} b c x^{2} - 10 \, a^{3} x e - 18 \, a^{3} d}{162 \, {\left(a^{3} b^{4} x^{9} + 3 \, a^{4} b^{3} x^{6} + 3 \, a^{5} b^{2} x^{3} + a^{6} b\right)}} + \frac{\sqrt{3} {\left(14 \, b c \left(\frac{a}{b}\right)^{\frac{1}{3}} + 5 \, a e\right)} \arctan\left(\frac{\sqrt{3} {\left(2 \, x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}}{3 \, \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243 \, a^{3} b^{2} \left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{{\left(14 \, b c \left(\frac{a}{b}\right)^{\frac{1}{3}} - 5 \, a e\right)} \log\left(x^{2} - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486 \, a^{3} b^{2} \left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{{\left(14 \, b c \left(\frac{a}{b}\right)^{\frac{1}{3}} - 5 \, a e\right)} \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243 \, a^{3} b^{2} \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/162*(28*b^3*c*x^8 + 5*a*b^2*x^7*e + 77*a*b^2*c*x^5 + 13*a^2*b*x^4*e + 67*
a^2*b*c*x^2 - 10*a^3*x*e - 18*a^3*d)/(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b
^2*x^3 + a^6*b) + 1/243*sqrt(3)*(14*b*c*(a/b)^(1/3) + 5*a*e)*arctan(1/3*sqr
```

```
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3)) + 1/486*(14*b*c
*(a/b)^(1/3) - 5*a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)
^(2/3)) - 1/243*(14*b*c*(a/b)^(1/3) - 5*a*e)*log(x + (a/b)^(1/3))/(a^3*b^2*
(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.45, size = 2646, normalized size = 9.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(168*b^3*c*x^8 + 30*a*b^2*e*x^7 + 462*a*b^2*c*x^5 + 78*a^2*b*e*x^4 +
402*a^2*b*c*x^2 - 60*a^3*e*x - 108*a^3*d - 2*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 +
 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a
^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(
1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^1
0*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))*log(7/2*((1/2)^(1
/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^
3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(
a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*
e^3)/(a^10*b^4))^(1/3)))^2*a^7*b^3*c - 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((
2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10
*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3
 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)
))*a^5*b*e^2 + 1960*a*b*c^2*e + (2744*b^2*c^3 + 125*a^2*e^3)*x) + ((a^3*b^4
*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*
((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^
10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c
^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/
3))) + 3*sqrt(1/3)*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*sq
rt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4)
- (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*
sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*
c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^6*b^2 + 1120*c*e)/(a^6*b^2)))*lo
g(-7/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4
) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-
I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^
2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^7*b^3*c + 25/2*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 12
5*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^
2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(
a^10*b^4))^(1/3)))*a^5*b*e^2 - 1960*a*b*c^2*e + 2*(2744*b^2*c^3 + 125*a^2*e
^3)*x + 3/2*sqrt(1/3)*(7*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*
```

```
a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*
(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^
10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))*a^7*b^3*c + 25*a
^5*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)
/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2
/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4)
- (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^6*b^2 + 1120*c*e)/(a
^6*b^2))) + ((a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(
1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c
^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/
(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2
*e^3)/(a^10*b^4))^(1/3))) - 3*sqrt(1/3)*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^
5*b^2*x^3 + a^6*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125
*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140
*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a
^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^6*b^2 + 112
0*c*e)/(a^6*b^2)))*log(-7/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 1
25*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 1
40*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/
(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^7*b^3*c +
 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4)
 - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I
*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2
*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))*a^5*b*e^2 - 1960*a*b*c^2*e + 2*(274
4*b^2*c^3 + 125*a^2*e^3)*x - 3/2*sqrt(1/3)*(7*((1/2)^(1/3)*(I*sqrt(3) + 1)*
((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^
10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c
^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/
3)))*a^7*b^3*c + 25*a^5*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b
^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))
^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125
*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a
^6*b^2 + 1120*c*e)/(a^6*b^2))))/(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^
3 + a^6*b)
```

Sympy [A]
time = 6.27, size = 214, normalized size = 0.79

$$\operatorname{RootSum}\left(14348907t^3a^{10}b^4 + 51030ta^4b^2ce - 125a^2e^3 + 2744b^2c^3, \left(t \mapsto t\log\left(x + \frac{826686t^2a^7b^3c + 6075ta^5be^2 + 1960abc^2e}{125a^2e^3 + 2744b^2c^3}\right)\right)\right) + \frac{-18a^3d - 10a^3ex + 67a^2bcx^2 + 13a^2bex^4 + 77ab^2cx^5 + 5ab^2ex^7 + 28b^3cx^8}{162a^6b + 486a^5b^2x^3 + 486a^4b^3x^6 + 162a^3b^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**10*b**4 + 51030*_t*a**4*b**2*c*e - 125*a**2*e**3
+ 2744*b**2*c**3, Lambda(_t, _t*log(x + (826686*_t**2*a**7*b**3*c + 6075*_t
```

```
*a**5*b*e**2 + 1960*a*b*c**2*e)/(125*a**2*e**3 + 2744*b**2*c**3)))) + (-18*
a**3*d - 10*a**3*e*x + 67*a**2*b*c*x**2 + 13*a**2*b*e*x**4 + 77*a*b**2*c*x*
*5 + 5*a*b**2*e*x**7 + 28*b**3*c*x**8)/(162*a**6*b + 486*a**5*b**2*x**3 + 4
86*a**4*b**3*x**6 + 162*a**3*b**4*x**9)
```

Giac [A]
time = 1.56, size = 244, normalized size = 0.90

$$-\frac{\sqrt{3}\left(5\,ae-14\,(-ab^2)^{\frac{1}{3}}\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+(-\frac{a}{b})^{\frac{1}{3}}\right)}{3\,(-\frac{a}{b})^{\frac{1}{3}}}\right)}{243\,(-ab^2)^{\frac{2}{3}}\,a^3}-\frac{\left(5\,ae+14\,(-ab^2)^{\frac{1}{3}}\,c\right)\log\left(x^2+x(-\frac{a}{b})^{\frac{1}{3}}+(-\frac{a}{b})^{\frac{2}{3}}\right)}{486\,(-ab^2)^{\frac{2}{3}}\,a^3}-\frac{\left(14\,bc(-\frac{a}{b})^{\frac{1}{3}}+5\,ae\right)(-\frac{a}{b})^{\frac{1}{3}}\log\left(\left|x-(-\frac{a}{b})^{\frac{1}{3}}\right|\right)}{243\,a^4b}+\frac{28\,b^3cx^8+5\,ab^2x^7e+77\,ab^2cx^5+13\,a^2bx^4e+67\,a^2bcx^2-10\,a^3xe-18\,a^3d}{162\,(bx^3+a)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/243*sqrt(3)*(5*a*e - 14*(-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/486*(5*a*e + 14*(-a*b^2)^(
1/3)*c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 1/2
43*(14*b*c*(-a/b)^(1/3) + 5*a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a
^4*b) + 1/162*(28*b^3*c*x^8 + 5*a*b^2*x^7*e + 77*a*b^2*c*x^5 + 13*a^2*b*x^4
*e + 67*a^2*b*c*x^2 - 10*a^3*x*e - 18*a^3*d)/((b*x^3 + a)^3*a^3*b)
```

Mupad [B]
time = 0.24, size = 265, normalized size = 0.98

$$\frac{\frac{67\,c\,x^2}{162\,a}-\frac{d}{9\,b}+\frac{13\,e\,x^4}{162\,a}-\frac{5\,e\,x}{81\,b}+\frac{14\,b^2\,c\,x^8}{81\,a^3}+\frac{77\,b\,c\,x^5}{162\,a^2}+\frac{5\,b\,e\,x^7}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}+\left(\sum_{k=1}^{3}\ln\left(\frac{70\,a\,c\,e+\mathrm{root}(14348907\,a^{10}\,b^4\,z^3+51030\,a^4\,b^2\,c\,e\,z-125\,a^2\,e^3+2744\,b^2\,c^3,z,k)^2\,a^7\,b^2\,59049+196\,b\,c^2\,x+\mathrm{root}(14348907\,a^{10}\,b^4\,z^3+51030\,a^4\,b^2\,c\,e\,z-125\,a^2\,e^3+2744\,b^2\,c^3,z,k)\,a^4\,b\,e\,x\,1215}{a^6\,6561}\right)\,\mathrm{root}\left(14348907\,a^{10}\,b^4\,z^3+51030\,a^4\,b^2\,c\,e\,z-125\,a^2\,e^3+2744\,b^2\,c^3,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3)^4,x)
```

```
[Out] ((67*c*x^2)/(162*a) - d/(9*b) + (13*e*x^4)/(162*a) - (5*e*x)/(81*b) + (14*b
^2*c*x^8)/(81*a^3) + (77*b*c*x^5)/(162*a^2) + (5*b*e*x^7)/(162*a^2))/(a^3 +
 b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6) + symsum(log((70*a*c*e + 59049*root(1
4348907*a^10*b^4*z^3 + 51030*a^4*b^2*c*e*z - 125*a^2*e^3 + 2744*b^2*c^3, z,
 k)^2*a^7*b^2 + 196*b*c^2*x + 1215*root(14348907*a^10*b^4*z^3 + 51030*a^4*b
^2*c*e*z - 125*a^2*e^3 + 2744*b^2*c^3, z, k)*a^4*b*e*x)/(6561*a^6))*root(14
348907*a^10*b^4*z^3 + 51030*a^4*b^2*c*e*z - 125*a^2*e^3 + 2744*b^2*c^3, z,
k), k, 1, 3)
```

3.360 $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^4}\,dx$

Optimal. Leaf size=250

$$\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}-\frac{2\left(20\sqrt[3]{b}\,c+7\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{81\sqrt{3}\,a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}\,c-\right.}{}$$

```
[Out] 1/54*x*(7*d*x+8*c)/a^2/(b*x^3+a)^2+2/81*x*(7*d*x+10*c)/a^3/(b*x^3+a)+1/9*(-
a*e+b*x*(d*x+c))/a/b/(b*x^3+a)^3+2/243*(20*b^(1/3)*c-7*a^(1/3)*d)*ln(a^(1/3
)+b^(1/3)*x)/a^(11/3)/b^(2/3)-1/243*(20*b^(1/3)*c-7*a^(1/3)*d)*ln(a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(2/3)-2/243*(20*b^(1/3)*c+7*a^(1/3
)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3)*3^(
1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1868, 1869, 1874, 31, 648, 631, 210, 642}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}\,d+20\sqrt[3]{b}\,c\right)}{81\sqrt{3}\,a^{11/3}b^{2/3}}-\frac{\left(20\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{11/3}b^{2/3}}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}+\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^4,x]
```

```
[Out] (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2) + (2*x*(10*c + 7*d*x))/(81*a^3*(a
 + b*x^3)) - (a*e - b*x*(c + d*x))/(9*a*b*(a + b*x^3)^3) - (2*(20*b^(1/3)*c
 + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[
3]*a^(11/3)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3
)*x])/(243*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx = -\frac{ae-bx(c+dx)}{9ab\,(a+bx^3)^3} - \frac{\int \frac{-8c-7dx}{(a+bx^3)^3}\,dx}{9a}$$

$$= \frac{x(8c+7dx)}{54a^2\,(a+bx^3)^2} - \frac{ae-bx(c+dx)}{9ab\,(a+bx^3)^3} + \frac{\int \frac{40c+28dx}{(a+bx^3)^2}\,dx}{54a^2}$$

$$= \frac{x(8c+7dx)}{54a^2\,(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3\,(a+bx^3)} - \frac{ae-bx(c+dx)}{9ab\,(a+bx^3)^3} - \frac{\int \frac{-80c-28dx}{a+bx^3}\,dx}{162a^3}$$

$$= \frac{x(8c+7dx)}{54a^2\,(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3\,(a+bx^3)} - \frac{ae-bx(c+dx)}{9ab\,(a+bx^3)^3} - \frac{\int \frac{\sqrt[3]{a}\left(-160\sqrt[3]{b}\,c-28\sqrt[3]{a}\,d\right)+\sqrt[3]{b}}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b}}{486a^{11/3}\sqrt[3]{b}}$$

$$= \frac{x(8c+7dx)}{54a^2\,(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3\,(a+bx^3)} - \frac{ae-bx(c+dx)}{9ab\,(a+bx^3)^3} + \frac{2\left(20\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}\right.}{243a^{11/3}b^{2/3}}$$

$$= \frac{x(8c+7dx)}{54a^2\,(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3\,(a+bx^3)} - \frac{ae-bx(c+dx)}{9ab\,(a+bx^3)^3} + \frac{2\left(20\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}\right.}{243a^{11/3}b^{2/3}}$$

$$= \frac{x(8c+7dx)}{54a^2\,(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3\,(a+bx^3)} - \frac{ae-bx(c+dx)}{9ab\,(a+bx^3)^3} - \frac{2\left(20\sqrt[3]{b}\,c+7\sqrt[3]{a}\,d\right)\tan^{-1}\left(\right.}{81\sqrt{3}\,a^{11/3}b^{2/3}}$$

Mathematica [A]
time = 0.15, size = 239, normalized size = 0.96

$$\frac{\frac{9a^2x(8c+7dx)}{(a+bx^3)^2}+\frac{12ax(10c+7dx)}{a+bx^3}-\frac{54a^3(ae-bx(c+dx))}{b(a+bx^3)^3}-\frac{4\sqrt{3}\sqrt[3]{a}\left(20\sqrt[3]{b}\,c+7\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}\,c-7a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{2\left(-20\sqrt[3]{a}\sqrt[3]{b}\,c+7a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}}{486a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^4,x]
```

```
[Out] ((9*a^2*x*(8*c + 7*d*x))/(a + b*x^3)^2 + (12*a*x*(10*c + 7*d*x))/(a + b*x^3
) - (54*a^3*(a*e - b*x*(c + d*x)))/(b*(a + b*x^3)^3) - (4*Sqrt[3]*a^(1/3)*(
20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(2/3) + (4*(20*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b
^(2/3) + (2*(-20*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/b^(2/3))/(486*a^4)
```

Maple [A]
time = 0.36, size = 378, normalized size = 1.51

method	result
risch	$\frac{\frac{14d\,b^2x^8}{81a^3}+\frac{20c\,b^2x^7}{81a^3}+\frac{77bd\,x^5}{162a^2}+\frac{52bc\,x^4}{81a^2}+\frac{67d\,x^2}{162a}+\frac{41cx}{81a}-\frac{e}{9b}}{(b\,x^3+a)^3}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(7\_Rd+20c)\ln(x-\_R)}{\_R^2}\right)}{243a^3b}$
default	$c\left(\frac{x}{9a(b\,x^3+a)^3}+\frac{\frac{4x}{27a(b\,x^3+a)^2}+\frac{8\left(\frac{5x}{18a(b\,x^3+a)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right.}{3}\right.}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{6a}\right.}{9a}}{a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
[Out] c*(1/9/a*x/(b*x^3+a)^3+8/9/a*(1/6*x/a/(b*x^3+a)^2+5/6/a*(1/3*x/a/(b*x^3+a)+
```

```
2/3/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))))))+d*(1/9/a*x^2/(b*x^3+a)^3+7/9/a*(1/6*x^2/a/(b*x^3+a)^2+2/3/a*
(1/3*x^2/a/(b*x^3+a)+1/3/a*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b
)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))))+e*(1/9/a*x^3/(b*x^3+a)^3+2/3/a*(1/6/a*
x^3/(b*x^3+a)^2-1/6/a/b/(b*x^3+a)))
```

Maxima [A]
time = 0.51, size = 255, normalized size = 1.02

$$\frac{28\,b^3dx^8+40\,b^3cx^7+77\,ab^2dx^5+104\,ab^2cx^4+67\,a^2bdx^2+82\,a^2bcx-18\,a^3e}{162\,(a^3b^4x^9+3\,a^4b^3x^6+3\,a^5b^2x^3+a^6b)}+\frac{2\,\sqrt{3}\,\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/162*(28*b^3*d*x^8 + 40*b^3*c*x^7 + 77*a*b^2*d*x^5 + 104*a*b^2*c*x^4 + 67*
a^2*b*d*x^2 + 82*a^2*b*c*x - 18*a^3*e)/(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5
*b^2*x^3 + a^6*b) + 2/243*sqrt(3)*(7*d*(a/b)^(1/3) + 20*c)*arctan(1/3*sqrt(
3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) + 1/243*(7*d*(a/b)^
(1/3) - 20*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) -
2/243*(7*d*(a/b)^(1/3) - 20*c)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.15, size = 2344, normalized size = 9.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(168*b^3*d*x^8 + 240*b^3*c*x^7 + 462*a*b^2*d*x^5 + 624*a*b^2*c*x^4 +
402*a^2*b*d*x^2 + 492*a^2*b*c*x - 108*a^3*e - 2*(a^3*b^4*x^9 + 3*a^4*b^3*x^
6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^
3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*
d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*log(7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*
b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)
- 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d - 400*(4^(1/3)
*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a
*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*
c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*
a^4*b*c^2 + 7840*a*c*d^2 + 4*(8000*b*c^3 + 343*a*d^3)*x) + ((a^3*b^4*x^9 +
3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^
```

```
3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 14
0*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2)
+ (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3))) + 3*sqrt(1/3)*(a^3*b^4*x^9 +
 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8
000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1
/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^
11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/
(a^7*b)))*log(-7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sq
rt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*
a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d + 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*
b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)
- 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2
 + 8*(8000*b*c^3 + 343*a*d^3)*x + 3/4*sqrt(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)
*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2)
)^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)
/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^8*b*d + 1600*a
^4*b*c^2)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt
(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*
d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b))) + ((a^3*b^4*x^9 + 3*
a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3
+ 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*
4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +
(8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3))) - 3*sqrt(1/3)*(a^3*b^4*x^9 + 3
*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((800
0*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3
) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a
^7*b)))*log(-7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt
(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*
d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d + 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*
c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) -
140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2
) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2 +
 8*(8000*b*c^3 + 343*a*d^3)*x - 3/4*sqrt(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)*(
(8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^
(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(
a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^8*b*d + 1600*a^4
*b*c^2)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2
) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3
) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^
3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b))))/(a^3*b^4*x^9 + 3*a^4*
```

```
b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)
```

Sympy [A]
time = 1.95, size = 202, normalized size = 0.81

$$\operatorname{RootSum}\left(14348907t^3a^{11}b^2+408240ta^4bcd+2744ad^3-64000bc^3,\left(t\mapsto t\log\left(x+\frac{413343t^2a^8bd+194400ta^4bc^2+7840acd^2}{1372ad^3+32000bc^3}\right)\right)\right)+\frac{-18a^3e+82a^2bcx+67a^2bdx^2+104ab^2cx^4+77ab^2dx^5+40b^3cx^7+28b^3dx^8}{162a^6b+486a^5b^2x^3+486a^4b^3x^6+162a^3b^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a*d**3 - 64
000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*d + 194400*_t*a**4*b
*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b*c**3)))) + (-18*a**3*e + 82*a
**2*b*c*x + 67*a**2*b*d*x**2 + 104*a*b**2*c*x**4 + 77*a*b**2*d*x**5 + 40*b*
*3*c*x**7 + 28*b**3*d*x**8)/(162*a**6*b + 486*a**5*b**2*x**3 + 486*a**4*b**
3*x**6 + 162*a**3*b**4*x**9)
```

Giac [A]
time = 1.27, size = 234, normalized size = 0.94

$$-\frac{2\,\sqrt{3}\left(20\,bc-7\,(-ab^2)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,(-ab^2)^{\frac{2}{3}}\,a^3}-\frac{\left(20\,bc+7\,(-ab^2)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,(-ab^2)^{\frac{2}{3}}\,a^3}-\frac{2\left(7\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4}+\frac{28\,b^3dx^8+40\,b^3cx^7+77\,ab^2dx^5+104\,ab^2cx^4+67\,a^2bdx^2+82\,a^2bcx-18\,a^3e}{162\,(bx^3+a)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -2/243*sqrt(3)*(20*b*c - 7*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/243*(20*b*c + 7*(-a*b^2)^(
1/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 2/2
43*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 +
1/162*(28*b^3*d*x^8 + 40*b^3*c*x^7 + 77*a*b^2*d*x^5 + 104*a*b^2*c*x^4 + 67*
a^2*b*d*x^2 + 82*a^2*b*c*x - 18*a^3*e)/((b*x^3 + a)^3*a^3*b)
```

Mupad [B]
time = 0.28, size = 247, normalized size = 0.99

$$\frac{\frac{67\,d\,x^2}{162\,a}-\frac{e}{9\,b}+\frac{41\,c\,x}{81\,a}+\frac{20\,b^2\,c\,x^7}{81\,a^3}+\frac{14\,b^2\,d\,x^8}{81\,a^3}+\frac{52\,b\,c\,x^4}{81\,a^2}+\frac{77\,b\,d\,x^5}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}+\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(560\,c\,d+196\,d^2\,x+\mathrm{root}(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k)^2\,a^7\,b\,59049+\mathrm{root}(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k)\,a^3\,b\,c\,x\,9720\right)}{a^6\,6561}\right)\mathrm{root}\left(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^3)^4,x)
```

```
[Out] ((67*d*x^2)/(162*a) - e/(9*b) + (41*c*x)/(81*a) + (20*b^2*c*x^7)/(81*a^3) +
 (14*b^2*d*x^8)/(81*a^3) + (52*b*c*x^4)/(81*a^2) + (77*b*d*x^5)/(162*a^2))/
(a^3 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6) + symsum(log((b*(560*c*d + 196*
d^2*x + 59049*root(14348907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 64000*b*c^3
 + 2744*a*d^3, z, k)^2*a^7*b + 9720*root(14348907*a^11*b^2*z^3 + 408240*a^4
*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)*a^3*b*c*x))/(6561*a^6))*root(143
48907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k),
k, 1, 3)
```

3.361 $\int \frac{c+dx+ex^2}{x(a+bx^3)^4}\,dx$

Optimal. Leaf size=291

$$\frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3}+\frac{x(8ad+7aex-15bcx^2)}{54a^3(a+bx^3)^2}+\frac{x(40ad+28aex-99bcx^2)}{162a^4(a+bx^3)}-\frac{2\left(20\sqrt[3]{b}\,d+7\sqrt[3]{a}\,e\right)\tan^{-1}\left(\right.}{81\sqrt{3}\,a^{11/3}b^{2/3}}$$

```
[Out] 1/9*x*(-b*c*x^2+a*e*x+a*d)/a^2/(b*x^3+a)^3+1/54*x*(-15*b*c*x^2+7*a*e*x+8*a*
d)/a^3/(b*x^3+a)^2+1/162*x*(-99*b*c*x^2+28*a*e*x+40*a*d)/a^4/(b*x^3+a)+c*ln
(x)/a^4+2/243*(20*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(
2/3)-1/243*(20*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/a^(11/3)/b^(2/3)-1/3*c*ln(b*x^3+a)/a^4-2/243*(20*b^(1/3)*d+7*a^(1/3)*e
)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3)*3^(1/2
)
```

Rubi [A]
time = 0.35, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}\,e+20\sqrt[3]{b}\,d\right)}{81\sqrt{3}\,a^{11/3}b^{2/3}}-\frac{\left(20\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{11/3}b^{2/3}}+\frac{x(40ad+28aex-99bcx^2)}{162a^4(a+bx^3)}-\frac{c\log(a+bx^3)}{3a^4}+\frac{c\log(x)}{a^4}+\frac{x(8ad+7aex-15bcx^2)}{54a^3(a+bx^3)^2}+\frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^4),x]
```

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*d + 7*a*e*x - 1
5*b*c*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*d + 28*a*e*x - 99*b*c*x^2))/(
162*a^4*(a + b*x^3)) - (2*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (c*Log[x])/a
^4 + (2*(20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(11/3
)*b^(2/3)) - ((20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
```

```
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx = \frac{x(ad+aex-bcx^2)}{9a^2\left(a+bx^3\right)^3} - \frac{\int \frac{-9bc-8bdx-7bex^2+\frac{6b^2cx^3}{a}}{x\left(a+bx^3\right)^3}\,dx}{9ab}$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2\left(a+bx^3\right)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3\left(a+bx^3\right)^2} + \frac{\int \frac{54b^2c+40b^2dx+28b^2ex^2-\frac{45b^3cx^3}{a}}{x\left(a+bx^3\right)^2}\,dx}{54a^2b^2}$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2\left(a+bx^3\right)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3\left(a+bx^3\right)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4\left(a+bx^3\right)} - \cdots$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2\left(a+bx^3\right)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3\left(a+bx^3\right)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4\left(a+bx^3\right)} - \cdots$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2\left(a+bx^3\right)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3\left(a+bx^3\right)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4\left(a+bx^3\right)} + \cdots$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2\left(a+bx^3\right)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3\left(a+bx^3\right)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4\left(a+bx^3\right)} + \cdots$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2\left(a+bx^3\right)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3\left(a+bx^3\right)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4\left(a+bx^3\right)} + \cdots$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2\left(a+bx^3\right)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3\left(a+bx^3\right)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4\left(a+bx^3\right)} + \cdots$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2\left(a+bx^3\right)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3\left(a+bx^3\right)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4\left(a+bx^3\right)} + \cdots$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2\left(a+bx^3\right)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3\left(a+bx^3\right)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4\left(a+bx^3\right)} - \cdots$$

Mathematica [A]

time = 0.16, size = 259, normalized size = 0.89

$$\frac{\frac{54a^3(c+x(d+ex))}{(a+bx^3)^3}+\frac{9a^2(9c+x(8d+7ex))}{(a+bx^3)^2}+\frac{6a(27c+2x(10d+7ex))}{a+bx^3}-\frac{4\sqrt{3}\,\sqrt[3]{a}\left(20\sqrt[3]{b}\,d+7\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+486c\log(x)+\frac{4\left(20\sqrt[3]{a}\,\sqrt[3]{b}\,d-7a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{2\left(-20\sqrt[3]{a}\,\sqrt[3]{b}\,d+7a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}-162c\log\left(a+bx^3\right)}{486a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^4),x]
```

```
[Out] ((54*a^3*(c + x*(d + e*x)))/(a + b*x^3)^3 + (9*a^2*(9*c + x*(8*d + 7*e*x)))
/(a + b*x^3)^2 + (6*a*(27*c + 2*x*(10*d + 7*e*x)))/(a + b*x^3) - (4*Sqrt[3]
*a^(1/3)*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(2/3) + 486*c*Log[x] + (4*(20*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Lo
g[a^(1/3) + b^(1/3)*x])/b^(2/3) + (2*(-20*a^(1/3)*b^(1/3)*d + 7*a^(2/3)*e)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 162*c*Log[a + b*x
^3])/(486*a^4)
```

Maple [A]

time = 0.38, size = 306, normalized size = 1.05

method	result
risch	$\frac{\frac{14e\,b^2x^8}{81a^3}+\frac{20d\,b^2x^7}{81a^3}+\frac{c\,b^2x^6}{3a^3}+\frac{77be\,x^5}{162a^2}+\frac{52bd\,x^4}{81a^2}+\frac{5bc\,x^3}{6a^2}+\frac{67e\,x^2}{162a}+\frac{41xd}{81a}+\frac{11c}{18a}}{\left(b\,x^3+a\right)^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{12}b^2\_Z^3+243a^8b^2c\_Z^2+\left(1680a^5bde+196\right.\right.}\right.}{}$
default	$\frac{\frac{14}{81}a\,b^2e\,x^8+\frac{20}{81}a\,b^2d\,x^7+\frac{1}{3}a\,b^2c\,x^6+\frac{77}{162}a^2be\,x^5+\frac{52}{81}a^2bd\,x^4+\frac{5}{6}a^2bc\,x^3+\frac{67}{162}a^3e\,x^2+\frac{41}{81}a^3dx+\frac{11}{18}c\,a^3}{\left(b\,x^3+a\right)^3}+\frac{40ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\right.\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*((14/81*a*b^2*e*x^8+20/81*a*b^2*d*x^7+1/3*a*b^2*c*x^6+77/162*a^2*b*e*
x^5+52/81*a^2*b*d*x^4+5/6*a^2*b*c*x^3+67/162*a^3*e*x^2+41/81*a^3*d*x+11/18*
c*a^3)/(b*x^3+a)^3+40/81*a*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/
b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+14/81*a*e*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)
^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(
```

```
a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-1/3*c*ln(b*x^3+a))+c*ln
(x)/a^4
```

Maxima [A]
time = 0.51, size = 299, normalized size = 1.03

$$\frac{28\,b^2x^8e+40\,b^2dx^7+54\,b^2cx^6+77\,abx^5e+104\,abdx^4+135\,abcx^3+67\,a^2x^2e+82\,a^2dx+99\,a^2c}{162\,(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6)}+\frac{c\log(x)}{a^4}+\frac{2\,\sqrt{3}\left(7\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}e+20\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5}-\frac{\left(81\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-7\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}e+20\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(81\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}e-40\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/162*(28*b^2*x^8*e + 40*b^2*d*x^7 + 54*b^2*c*x^6 + 77*a*b*x^5*e + 104*a*b*
d*x^4 + 135*a*b*c*x^3 + 67*a^2*x^2*e + 82*a^2*d*x + 99*a^2*c)/(a^3*b^3*x^9
+ 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6) + c*log(x)/a^4 + 2/243*sqrt(3)*(7*a*(a
/b)^(2/3)*e + 20*a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a
/b)^(1/3))/a^5 - 1/243*(81*b*c*(a/b)^(2/3) - 7*a*(a/b)^(1/3)*e + 20*a*d)*lo
g(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) - 1/243*(81*b*c*(a
/b)^(2/3) + 14*a*(a/b)^(1/3)*e - 40*a*d)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^
(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.36, size = 5370, normalized size = 18.45

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/236196*(40824*a*b^2*e*x^8 + 58320*a*b^2*d*x^7 + 78732*a*b^2*c*x^6 + 11226
6*a^2*b*e*x^5 + 151632*a^2*b*d*x^4 + 196830*a^2*b*c*x^3 + 97686*a^3*e*x^2 +
 119556*a^3*d*x + 144342*a^3*c - 2*(a^4*b^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x
^3 + a^7)*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b
))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348
907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744
*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt
(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4
/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3
+ 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c
/a^4)*log(7/236196*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*
e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b)
+ 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c
^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 5904
9*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^
12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441
*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3)
```

+ 39366*c/a^4)^2*a^8*b*e + 64800*b*c*d^2 + 45927*b*c^2*e + 7840*a*d*e^2 - 1
/243*(400*a^4*b*d^2 + 567*a^4*b*c*e)*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (656
1*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*
a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/2869
7814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*
b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2
+ 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) -
1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/
(a^12*b^2))^(1/3) + 39366*c/a^4) + 4*(8000*b*d^3 + 343*a*e^3)*x) - (118098*
b^3*c*x^9 + 354294*a*b^2*c*x^6 + 354294*a^2*b*c*x^3 + 118098*a^3*c - (a^4*b
^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x^3 + a^7)*((-I*sqrt(3) + 1)*(6561*c^2/a^8
 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2
 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) -
 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)
/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561
*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*
b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e
)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4) - 3*sqrt(1/3)*(a^4*b^3*x^9 + 3*a^5*
b^2*x^6 + 3*a^6*b*x^3 + a^7)*sqrt(-(((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561
*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a
*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697
814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b
^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 +
 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1
/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(
a^12*b^2))^(1/3) + 39366*c/a^4)^2*a^8*b - 78732*((-I*sqrt(3) + 1)*(6561*c^2
/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b
*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^
2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*
a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(
6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a
^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c
*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4)*a^4*b*c + 1549681956*b*c^2 + 52
9079040*a*d*e)/(a^8*b)))*log(-7/236196*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6
561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 56
0*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28
697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^1
2*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^
2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2)
- 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b
)/(a^12*b^2))^(1/3) + 39366*c/a^4)^2*a^8*b*e - 64800*b*c*d^2 - 45927*b*c^2*
e - 7840*a*d*e^2 + 1/243*(400*a^4*b*d^2 + 567*a^4*b*c*e)*((-I*sqrt(3) + 1)*
(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/11809
8*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)
/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 170

```
1*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1
/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*
a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3
 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4) + 8*(8000*b*d^3 + 343*
a*e^3)*x + 1/78732*sqrt(1/3)*(7*((-I*sqrt(3) + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.02, size = 290, normalized size = 1.00

$$-\frac{2\sqrt{3}\left(20\,bd-7\,(-ab^2)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,(-ab^2)^{\frac{2}{3}}a^3}-\frac{\left(20\,bd+7\,(-ab^2)^{\frac{1}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,(-ab^2)^{\frac{2}{3}}a^3}-\frac{c\log\left(|bx^3+a|\right)}{3\,a^4}+\frac{c\log\left(|x|\right)}{a^4}+\frac{28\,ab^2x^8e+40\,ab^2dx^7+54\,ab^2cx^6+77\,a^2bx^5e+104\,a^2bdx^4+135\,a^2bcx^3+67\,a^3x^2e+82\,a^3dx+99\,a^3c}{162\,(bx^3+a)^3a^4}-\frac{2\left(7\,a^5b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+20\,a^5bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -2/243*sqrt(3)*(20*b*d - 7*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/243*(20*b*d + 7*(-a*b^2)^(
1/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 1/3
*c*log(abs(b*x^3 + a))/a^4 + c*log(abs(x))/a^4 + 1/162*(28*a*b^2*x^8*e + 40
*a*b^2*d*x^7 + 54*a*b^2*c*x^6 + 77*a^2*b*x^5*e + 104*a^2*b*d*x^4 + 135*a^2*
b*c*x^3 + 67*a^3*x^2*e + 82*a^3*d*x + 99*a^3*c)/((b*x^3 + a)^3*a^4) - 2/243
*(7*a^5*b*(-a/b)^(1/3)*e + 20*a^5*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3
)))/(a^9*b)
```

Mupad [**B**]
time = 5.40, size = 871, normalized size = 2.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)^4),x)
```

```
[Out] ((11*c)/(18*a) + (67*e*x^2)/(162*a) + (41*d*x)/(81*a) + (b^2*c*x^6)/(3*a^3)
 + (20*b^2*d*x^7)/(81*a^3) + (14*b^2*e*x^8)/(81*a^3) + (5*b*c*x^3)/(6*a^2)
+ (52*b*d*x^4)/(81*a^2) + (77*b*e*x^5)/(162*a^2))/(a^3 + b^3*x^9 + 3*a^2*b*
x^3 + 3*a*b^2*x^6) + symsum(log(-(2*b*(45927*b*c^2*e - 64800*b*c*d^2 + 1372
*a*e^3*x - 32000*b*d^3*x + 9565938*root(14348907*a^12*b^2*z^3 + 14348907*a^
```

```
8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e
 - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)^3*a^11*b^2*x + 6480
0*root(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z
+ 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 +
 531441*b^2*c^3, z, k)*a^4*b*d^2 - 137781*root(14348907*a^12*b^2*z^3 + 1434
8907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*
b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)^2*a^8*b*e +
45360*b*c*d*e*x + 1062882*root(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z
^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*
a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)*a^3*b^2*c^2*x + 6377292*root
(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782
969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 53144
1*b^2*c^3, z, k)^2*a^7*b^2*c*x + 91854*root(14348907*a^12*b^2*z^3 + 1434890
7*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c
*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)*a^4*b*c*e + 226
800*root(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*
z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3
 + 531441*b^2*c^3, z, k)*a^4*b*d*e*x))/(531441*a^9))*root(14348907*a^12*b^2
*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z
+ 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k),
k, 1, 3) + (c*log(x))/a^4
```

3.362 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^4}\,dx$

Optimal. Leaf size=301

$$-\frac{c}{a^4x}+\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}+\frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2}+\frac{x(40ae-118bcx-99bdx^2)}{162a^4(a+bx^3)}+\frac{20(7b^{2/3}c-2a^{2/3}e)\,\mathrm{t}}{81\sqrt{3}\,a}$$

```
[Out] -c/a^4/x+1/9*x*(-b*d*x^2-b*c*x+a*e)/a^2/(b*x^3+a)^3+1/54*x*(-15*b*d*x^2-16*
b*c*x+8*a*e)/a^3/(b*x^3+a)^2+1/162*x*(-99*b*d*x^2-118*b*c*x+40*a*e)/a^4/(b*
x^3+a)+d*ln(x)/a^4+20/243*(7*b^(2/3)*c+2*a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a
^(13/3)/b^(1/3)-10/243*(7*b^(2/3)*c+2*a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/a^(13/3)/b^(1/3)-1/3*d*ln(b*x^3+a)/a^4+20/243*(7*b^(2/3)*c-
2*a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)/b^(
1/3)*3^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{20\,\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(7b^{2/3}c-2a^{2/3}e)}{81\sqrt{3}\,a^{13/3}\sqrt[3]{b}}-\frac{10(2a^{2/3}e+7b^{2/3}c)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20(2a^{2/3}e+7b^{2/3}c)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{x(40ae-118bcx-99bdx^2)}{162a^4(a+bx^3)}-\frac{d\log(a+bx^3)}{3a^4}-\frac{c}{a^4x}+\frac{d\log(x)}{a^4}+\frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2}+\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x]
```

```
[Out] -(c/(a^4*x)) + (x*(a*e - b*c*x - b*d*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*
e - 16*b*c*x - 15*b*d*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*e - 118*b*c*x
 - 99*b*d*x^2))/(162*a^4*(a + b*x^3)) + (20*(7*b^(2/3)*c - 2*a^(2/3)*e)*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(13/3)*b^(1/3
)) + (d*Log[x])/a^4 + (20*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)
*x])/(243*a^(13/3)*b^(1/3)) - (10*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(13/3)*b^(1/3)) - (d*Log[a + b*x^
3])/(3*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
```

```
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^4}\,dx = \frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3} - \frac{\int \frac{-9bc-9bdx-8bex^2+\frac{7b^2cx^3}{a}+\frac{6b^2dx^4}{a}}{x^2(a+bx^3)^3}\,dx}{9ab}$$

$$= \frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2} + \frac{\int \frac{54b^3c+54b^3dx+40b^3ex^2-\frac{64b^4cx^3}{a}-\frac{45b^4dx^4}{a}}{x^2(a+bx^3)^2}}{54a^2b^3}$$

$$= \frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ae-118bcx-99bdx^2)}{162a^4(a+bx^3)} - \frac{\int}{}$$

$$= \frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ae-118bcx-99bdx^2)}{162a^4(a+bx^3)} - \frac{\int (}{}$$

$$= -\frac{c}{a^4x} + \frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ae-118bcx-99bd}{162a^4(a+bx^3)}$$

$$= -\frac{c}{a^4x} + \frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ae-118bcx-99bd}{162a^4(a+bx^3)}$$

$$= -\frac{c}{a^4x} + \frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ae-118bcx-99bd}{162a^4(a+bx^3)}$$

$$= -\frac{c}{a^4x} + \frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ae-118bcx-99bd}{162a^4(a+bx^3)}$$

$$= -\frac{c}{a^4x} + \frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ae-118bcx-99bd}{162a^4(a+bx^3)}$$

$$= -\frac{c}{a^4x} + \frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ae-118bcx-99bd}{162a^4(a+bx^3)}$$

Mathematica [A]

time = 0.19, size = 279, normalized size = 0.93

$$\frac{-\frac{486ac}{x}+\frac{9a^2(9ad+8aex-16bcx^2)}{(a+bx^3)^2}+\frac{6a(27ad+20aex-59bcx^2)}{a+bx^3}+\frac{54a^3(-bcx^2+a(d+ex))}{(a+bx^3)^3}-\frac{40\sqrt{3}\,a^{2/3}(-7b^{2/3}c+2a^{2/3}e)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+486ad\log(x)+\frac{40(7a^{2/3}b^{2/3}c+2a^{4/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{20(7a^{2/3}b^{2/3}c+2a^{4/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}}-162ad\log\left(a+bx^3\right)}{486a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x]
```

```
[Out] ((-486*a*c)/x + (9*a^2*(9*a*d + 8*a*e*x - 16*b*c*x^2))/(a + b*x^3)^2 + (6*a
*(27*a*d + 20*a*e*x - 59*b*c*x^2))/(a + b*x^3) + (54*a^3*(-(b*c*x^2) + a*(d
 + e*x)))/(a + b*x^3)^3 - (40*Sqrt[3]*a^(2/3)*(-7*b^(2/3)*c + 2*a^(2/3)*e)*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + 486*a*d*Log[x] + (40
*(7*a^(2/3)*b^(2/3)*c + 2*a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (2
0*(7*a^(2/3)*b^(2/3)*c + 2*a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/b^(1/3) - 162*a*d*Log[a + b*x^3])/(486*a^5)
```

Maple [A]

time = 0.38, size = 315, normalized size = 1.05

method	result
risch	$\frac{-\frac{140b^3c\,x^9}{81a^4}+\frac{20e\,b^2x^8}{81a^3}+\frac{d\,b^2x^7}{3a^3}-\frac{385c\,b^2x^6}{81a^3}+\frac{52be\,x^5}{81a^2}+\frac{5bd\,x^4}{6a^2}-\frac{335bc\,x^3}{81a^2}+\frac{41e\,x^2}{81a}+\frac{11xd}{18a}-\frac{c}{a}}{x(b\,x^3+a)^3}+\frac{d\ln(x)}{a^4}+\frac{\left(\_R=\text{RootOf}\left(a^{13}b\_Z^3+243a^9\right.\right.}{}$
default	$\frac{\frac{-\frac{59}{81}b^3c\,x^8+\frac{20}{81}a\,b^2e\,x^7+\frac{1}{3}a\,b^2d\,x^6-\frac{142}{81}a\,b^2c\,x^5+\frac{52}{81}a^2be\,x^4+\frac{5}{6}a^2bd\,x^3-\frac{92}{81}a^2bc\,x^2+\frac{41}{81}a^3ex+\frac{11}{18}a^3d}{\left(b\,x^3+a\right)^3}+\frac{40ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*((-59/81*b^3*c*x^8+20/81*a*b^2*e*x^7+1/3*a*b^2*d*x^6-142/81*a*b^2*c*x
^5+52/81*a^2*b*e*x^4+5/6*a^2*b*d*x^3-92/81*a^2*b*c*x^2+41/81*a^3*e*x+11/18*
a^3*d)/(b*x^3+a)^3+40/81*a*e*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/
b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-140/81*b*c*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/
```

```
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-1/3*d*ln(b*x^3+a))-c/a
^4/x+d*ln(x)/a^4
```

Maxima [A]
time = 0.51, size = 319, normalized size = 1.06

$$-\frac{280\,b^3cx^9-40\,ab^2x^8e-54\,ab^2dx^7+770\,ab^2cx^6-104\,a^2bx^5e-135\,a^2bdx^4+670\,a^2bcx^3-82\,a^3x^2e-99\,a^3dx+162\,a^3c}{162\,(a^4b^3x^{10}+3\,a^5b^2x^7+3\,a^6bx^4+a^7x)}+\frac{d\log(x)}{a^4}-\frac{20\,\sqrt{3}\left(7\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5}-\frac{\left(81\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+70\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(81\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-140\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-40\,ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] -1/162*(280*b^3*c*x^9 - 40*a*b^2*x^8*e - 54*a*b^2*d*x^7 + 770*a*b^2*c*x^6 -
 104*a^2*b*x^5*e - 135*a^2*b*d*x^4 + 670*a^2*b*c*x^3 - 82*a^3*x^2*e - 99*a^
3*d*x + 162*a^3*c)/(a^4*b^3*x^10 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x) + d
*log(x)/a^4 - 20/243*sqrt(3)*(7*b*c*(a/b)^(2/3) - 2*a*(a/b)^(1/3)*e)*arctan
(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5 - 1/243*(81*b*d*(a/b)^(2/
3) + 70*b*c*(a/b)^(1/3) + 20*a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
^4*b*(a/b)^(2/3)) - 1/243*(81*b*d*(a/b)^(2/3) - 140*b*c*(a/b)^(1/3) - 40*a*
e)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.40, size = 5250, normalized size = 17.44

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] -1/236196*(408240*b^3*c*x^9 - 58320*a*b^2*e*x^8 - 78732*a*b^2*d*x^7 + 11226
60*a*b^2*c*x^6 - 151632*a^2*b*e*x^5 - 196830*a^2*b*d*x^4 + 976860*a^2*b*c*x
^3 - 119556*a^3*e*x^2 - 144342*a^3*d*x + 236196*a^3*c + 2*(a^4*b^3*x^10 + 3
*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561
*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^
12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d
*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3)
 + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d
/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*
c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1
/3) + 39366*d/a^4)*log(-7/236196*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^
2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12
+ 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)
*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) +
59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^
12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d
*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3)
 + 39366*d/a^4)^2*a^9*b*c - 45927*a*b*c*d^2 + 78400*a*b*c^2*e + 6480*a^2*d*
```

e^2 + 1/243*(567*a^5*b*c*d - 40*a^6*e^2)*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4) - 400*(343*b^2*c^3 - 8*a^2*e^3)*x) + (118098*b^3*d*x^10 + 354294*a*b^2*d*x^7 + 354294*a^2*b*d*x^4 + 118098*a^3*d*x - (a^4*b^3*x^10 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4) + 3*sqrt(1/3)*(a^4*b^3*x^10 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)*sqrt(-(((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)^2*a^8 - 78732*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)*a^4*d + 1549681956*d^2 - 5290790400*c*e)/a^8))*log(7/236196*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)^2*a^9*b*c + 45927*a*b*c*d^2 - 78400*a*b*c^2*e - 6480*a^2*d*e^2 - 1/243*(567*a^5*b*c*d - 40*a^6*e^2)*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 +

64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4) - 800*(343*b^2*c^3 - 8*a^2*e^3)*x + 1/78732*sqrt(1/3)*(7*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/11...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.05, size = 310, normalized size = 1.03

$$-\frac{d\log\left(|bx^3+a|\right)}{3\,a^4}+\frac{d\log\left(|x|\right)}{a^4}+\frac{20\,\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}ae+7\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5b}+\frac{10\left(2\left(-ab^2\right)^{\frac{1}{3}}ae-7\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b}-\frac{280\,b^3cx^9-40\,ab^2x^8e-54\,ab^2dx^7+770\,ab^2cx^6-104\,a^2bx^5e-135\,a^2bdx^4+670\,a^2bcx^3-82\,a^3x^2e-99\,a^3dx+162\,a^3c}{162\left(bx^3+a\right)^3a^4x}+\frac{20\left(7\,a^4b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^5be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^4 + d*log(abs(x))/a^4 + 20/243*sqrt(3)*(2*(-a*b^2)^(1/3)*a*e + 7*(-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) + 10/243*(2*(-a*b^2)^(1/3)*a*e - 7*(-a*b^2)^(2/3)*c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/162*(280*b^3*c*x^9 - 40*a*b^2*x^8*e - 54*a*b^2*d*x^7 + 770*a*b^2*c*x^6 - 104*a^2*b*x^5*e - 135*a^2*b*d*x^4 + 670*a^2*b*c*x^3 - 82*a^3*x^2*e - 99*a^3*d*x + 162*a^3*c)/((b*x^3 + a)^3*a^4*x) + 20/243*(7*a^4*b^2*c*(-a/b)^(1/3) - 2*a^5*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b)
```

Mupad [**B**]
time = 5.43, size = 840, normalized size = 2.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x)
```

```
[Out] ((41*e*x^2)/(81*a) - c/a + (11*d*x)/(18*a) - (385*b^2*c*x^6)/(81*a^3) - (140*b^3*c*x^9)/(81*a^4) + (b^2*d*x^7)/(3*a^3) + (20*b^2*e*x^8)/(81*a^3) - (335*b*c*x^3)/(81*a^2) + (5*b*d*x^4)/(6*a^2) + (52*b*e*x^5)/(81*a^2))/(a^3*x + b^3*x^10 + 3*a^2*b*x^4 + 3*a*b^2*x^7) + symsum(log((4*b^2*(32400*a^2*d*e^2 - 32400*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*
```

```
e^3 - 2744000*b^2*c^3, z, k)*a^6*e^2 + 686000*b^2*c^3*x + 16000*a^2*e^3*x +
 229635*a*b*c*d^2 - 688905*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2
- 4082400*a^5*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*
b*d^3 - 64000*a^2*e^3 - 2744000*b^2*c^3, z, k)^2*a^9*b*c - 4782969*root(143
48907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 4782969*a^5
*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3 - 2744000*b^2
*c^3, z, k)^3*a^13*b*x - 531441*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d
*z^2 - 4082400*a^5*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 5314
41*a*b*d^3 - 64000*a^2*e^3 - 2744000*b^2*c^3, z, k)*a^5*b*d^2*x - 3188646*r
oot(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 4782
969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3 - 2744
000*b^2*c^3, z, k)^2*a^9*b*d*x + 459270*root(14348907*a^13*b*z^3 + 14348907
*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*
e + 531441*a*b*d^3 - 64000*a^2*e^3 - 2744000*b^2*c^3, z, k)*a^5*b*c*d + 113
4000*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z
+ 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3
- 2744000*b^2*c^3, z, k)*a^5*b*c*e*x + 226800*a*b*c*d*e*x))/(531441*a^11))*
root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 478
2969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3 - 274
4000*b^2*c^3, z, k), k, 1, 3) + (d*log(x))/a^4
```

3.363 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^4} dx$

Optimal. Leaf size=310

$$-\frac{c}{2a^4x^2}-\frac{d}{a^4x}-\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}-\frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2}-\frac{x(139bc+118bdx+99bex^2)}{162a^4(a+bx^3)}+\frac{20\sqrt[3]{b}\,(11\sqrt[3]{}}{}$$

```
[Out] -1/2*c/a^4/x^2-d/a^4/x-1/9*x*(b*e*x^2+b*d*x+b*c)/a^2/(b*x^3+a)^3-1/54*x*(15
*b*e*x^2+16*b*d*x+17*b*c)/a^3/(b*x^3+a)^2-1/162*x*(99*b*e*x^2+118*b*d*x+139
*b*c)/a^4/(b*x^3+a)+e*ln(x)/a^4-20/243*b^(1/3)*(11*b^(1/3)*c-7*a^(1/3)*d)*l
n(a^(1/3)+b^(1/3)*x)/a^(14/3)+10/243*b^(1/3)*(11*b^(1/3)*c-7*a^(1/3)*d)*ln(
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)-1/3*e*ln(b*x^3+a)/a^4+20/24
3*b^(1/3)*(11*b^(1/3)*c+7*a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/
3)*3^(1/2))/a^(14/3)*3^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{20\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}d+11\sqrt[3]{b}c\right)}{81\sqrt{3}a^{14/3}}+\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}}-\frac{x(139bc+118bdx+99bex^2)}{162a^4(a+bx^3)}-\frac{e\log(a+bx^3)}{3a^4}-\frac{c}{2a^4x^2}-\frac{d}{a^4x}+\frac{e\log(x)}{a^4}-\frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2}-\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x]
```

```
[Out] -1/2*c/(a^4*x^2) - d/(a^4*x) - (x*(b*c + b*d*x + b*e*x^2))/(9*a^2*(a + b*x^
3)^3) - (x*(17*b*c + 16*b*d*x + 15*b*e*x^2))/(54*a^3*(a + b*x^3)^2) - (x*(1
39*b*c + 118*b*d*x + 99*b*e*x^2))/(162*a^4*(a + b*x^3)) + (20*b^(1/3)*(11*b
^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(81*Sqrt[3]*a^(14/3)) + (e*Log[x])/a^4 - (20*b^(1/3)*(11*b^(1/3)*c - 7*a^(1
/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*c
 - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(14/
3)) - (e*Log[a + b*x^3])/(3*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
```

```
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^4}\,dx = -\frac{x(bc+bdx+bex^2)}{9a^2\left(a+bx^3\right)^3} - \frac{\int \frac{-9bc-9bdx-9bex^2+\frac{8b^2cx^3}{a}+\frac{7b^2dx^4}{a}+\frac{6b^2ex^5}{a}}{x^3(a+bx^3)^3}\,dx}{9ab}$$

$$= -\frac{x(bc+bdx+bex^2)}{9a^2\left(a+bx^3\right)^3} - \frac{x(17bc+16bdx+15bex^2)}{54a^3\left(a+bx^3\right)^2} + \frac{\int \frac{54b^3c+54b^3dx+54b^3ex^2-\frac{85b^4cx^3}{a}-\frac{64b^4}{}}{x^3(a+bx^3)^2}}{54a^2b^3}$$

$$= -\frac{x(bc+bdx+bex^2)}{9a^2\left(a+bx^3\right)^3} - \frac{x(17bc+16bdx+15bex^2)}{54a^3\left(a+bx^3\right)^2} - \frac{x(139bc+118bdx+99bex^2)}{162a^4\left(a+bx^3\right)} -$$

$$= -\frac{x(bc+bdx+bex^2)}{9a^2\left(a+bx^3\right)^3} - \frac{x(17bc+16bdx+15bex^2)}{54a^3\left(a+bx^3\right)^2} - \frac{x(139bc+118bdx+99bex^2)}{162a^4\left(a+bx^3\right)} -$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x(bc+bdx+bex^2)}{9a^2\left(a+bx^3\right)^3} - \frac{x(17bc+16bdx+15bex^2)}{54a^3\left(a+bx^3\right)^2} - \frac{x(139bc+118b}{162a^4(a}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x(bc+bdx+bex^2)}{9a^2\left(a+bx^3\right)^3} - \frac{x(17bc+16bdx+15bex^2)}{54a^3\left(a+bx^3\right)^2} - \frac{x(139bc+118b}{162a^4(a}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x(bc+bdx+bex^2)}{9a^2\left(a+bx^3\right)^3} - \frac{x(17bc+16bdx+15bex^2)}{54a^3\left(a+bx^3\right)^2} - \frac{x(139bc+118b}{162a^4(a}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x(bc+bdx+bex^2)}{9a^2\left(a+bx^3\right)^3} - \frac{x(17bc+16bdx+15bex^2)}{54a^3\left(a+bx^3\right)^2} - \frac{x(139bc+118b}{162a^4(a}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x(bc+bdx+bex^2)}{9a^2\left(a+bx^3\right)^3} - \frac{x(17bc+16bdx+15bex^2)}{54a^3\left(a+bx^3\right)^2} - \frac{x(139bc+118b}{162a^4(a}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x(bc+bdx+bex^2)}{9a^2\left(a+bx^3\right)^3} - \frac{x(17bc+16bdx+15bex^2)}{54a^3\left(a+bx^3\right)^2} - \frac{x(139bc+118b}{162a^4(a}$$

Mathematica [A]

time = 0.18, size = 284, normalized size = 0.92

$$\frac{-\frac{243ac}{x^2}-\frac{486ad}{x}+\frac{54a^3(ae-bx(c+dx))}{(a+bx^3)^3}+\frac{9a^2(9ae-bx(17c+16dx))}{(a+bx^3)^2}+\frac{3a(54ae-bx(139c+118dx))}{a+bx^3}+40\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(11\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+486ae\log(x)+40\sqrt[3]{b}\left(-11\sqrt[3]{a}\sqrt[3]{b}c+7a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+20\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-162ae\log\left(a+bx^3\right)}{486a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x]
```

```
[Out] ((-243*a*c)/x^2 - (486*a*d)/x + (54*a^3*(a*e - b*x*(c + d*x)))/(a + b*x^3)^
3 + (9*a^2*(9*a*e - b*x*(17*c + 16*d*x)))/(a + b*x^3)^2 + (3*a*(54*a*e - b*
x*(139*c + 118*d*x)))/(a + b*x^3) + 40*Sqrt[3]*a^(1/3)*b^(1/3)*(11*b^(1/3)*
c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 486*a*e*Log[
x] + 40*b^(1/3)*(-11*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)
*x] + 20*b^(1/3)*(11*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2] - 162*a*e*Log[a + b*x^3])/(486*a^5)
```

Maple [A]

time = 0.40, size = 321, normalized size = 1.04

method	result
risch	$\frac{-\frac{140b^3dx^{10}}{81a^4}-\frac{110b^3cx^9}{81a^4}+\frac{eb^2x^8}{3a^3}-\frac{385db^2x^7}{81a^3}-\frac{286cb^2x^6}{81a^3}+\frac{5bex^5}{6a^2}-\frac{335bdx^4}{81a^2}-\frac{451bcx^3}{162a^2}+\frac{11ex^2}{18a}-\frac{xd}{a}-\frac{c}{2a}}{x^2(bx^3+a)^3}+\frac{\left(\_R=\text{RootOf}\left(a^{14}\_Z^3+243a^1\right.\right.}{}$
default	$-\frac{b\left(\frac{\frac{59b^2dx^8}{81}+\frac{139b^2cx^7}{162}-\frac{abex^6}{3}+\frac{142abdx^5}{81}+\frac{329abcx^4}{162}-\frac{5a^2ex^3}{6}+\frac{92a^2dx^2}{81}+\frac{104a^2cx}{81}-\frac{11a^3e}{18b}}{(bx^3+a)^3}+\frac{220c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right.\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^4*b*((59/81*b^2*d*x^8+139/162*b^2*c*x^7-1/3*a*b*e*x^6+142/81*a*b*d*x^5
+329/162*a*b*c*x^4-5/6*a^2*e*x^3+92/81*a^2*d*x^2+104/81*a^2*c*x-11/18*a^3*e
/b)/(b*x^3+a)^3+220/81*c*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(
2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+140/81*d*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3
))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^
(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*e*ln(b*x^3+a)/b)-1/2*c/a
^4/x^2-d/a^4/x+e*ln(x)/a^4
```

Maxima [A]
time = 0.62, size = 318, normalized size = 1.03

$$-\frac{280\,b^3dx^{10}+220\,b^3cx^9-54\,ab^2x^8e+770\,ab^2dx^7+572\,ab^2cx^6-135\,a^2bx^5e+670\,a^2bdx^4+451\,a^2bcx^3-99\,a^3x^2e+162\,a^3dx+81\,a^3c}{162\,(a^4b^3x^{11}+3\,a^5b^2x^8+3\,a^6bx^5+a^7x^2)}+\frac{e\log(x)}{a^4}-\frac{20\sqrt{3}\left(7\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+11\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5}-\frac{\left(81\left(\frac{a}{b}\right)^{\frac{2}{3}}e+70\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-110\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(81\left(\frac{a}{b}\right)^{\frac{2}{3}}e-140\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+220\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] -1/162*(280*b^3*d*x^10 + 220*b^3*c*x^9 - 54*a*b^2*x^8*e + 770*a*b^2*d*x^7 +
 572*a*b^2*c*x^6 - 135*a^2*b*x^5*e + 670*a^2*b*d*x^4 + 451*a^2*b*c*x^3 - 99
*a^3*x^2*e + 162*a^3*d*x + 81*a^3*c)/(a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*
b*x^5 + a^7*x^2) + e*log(x)/a^4 - 20/243*sqrt(3)*(7*b*d*(a/b)^(2/3) + 11*b*
c*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5 - 1/
243*(81*(a/b)^(2/3)*e + 70*d*(a/b)^(1/3) - 110*c)*log(x^2 - x*(a/b)^(1/3) +
 (a/b)^(2/3))/(a^4*(a/b)^(2/3)) - 1/243*(81*(a/b)^(2/3)*e - 140*d*(a/b)^(1/
3) + 220*c)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.28, size = 5049, normalized size = 16.29

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] -1/236196*(408240*b^3*d*x^10 + 320760*b^3*c*x^9 - 78732*a*b^2*e*x^8 + 11226
60*a*b^2*d*x^7 + 833976*a*b^2*c*x^6 - 196830*a^2*b*e*x^5 + 976860*a^2*b*d*x
^4 + 657558*a^2*b*c*x^3 - 144342*a^3*e*x^2 + 236196*a^3*d*x + 118098*a^3*c
+ 2*(a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)*((-I*sqrt(3) + 1
)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/11809
8*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3
)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 -
2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/11
8098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*
d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3
 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)*log(7/236196*((-I*sqrt(3) +
1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/1180
```

98*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^
3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 -
 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/1
18098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a
*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^
3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)^2*a^10*d + 431200*a*b*c*d^2
 - 196020*a*b*c^2*e + 45927*a^2*d*e^2 + 1/243*(1210*a^5*b*c^2 - 567*a^6*d*e
)*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*
e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331
*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3
 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/
27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1
331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e
^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4) + 400*(133
1*b^2*c^3 + 343*a*b*d^3)*x) + (118098*b^3*e*x^11 + 354294*a*b^2*e*x^8 + 354
294*a^2*b*e*x^5 + 118098*a^3*e*x^2 - (a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*
b*x^5 + a^7*x^2)*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^
2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000
/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 +
531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sq
rt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4
000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3
 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/
a^4) - 3*sqrt(1/3)*(a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)*s
qrt(-(((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1
/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(
1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*
e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*
(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/1434890
7*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a
^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)^2*a^9
 - 78732*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(
-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907
*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^
2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1
)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348
907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441
*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)*a^5*
e + 29099347200*b*c*d + 1549681956*a*e^2)/a^9))*log(-7/236196*((-I*sqrt(3)
 + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/11
8098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*
d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3
 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1
/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343
*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*

```
d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)^2*a^10*d - 431200*a*b*c*d
^2 + 196020*a*b*c^2*e - 45927*a^2*d*e^2 - 1/243*(1210*a^5*b*c^2 - 567*a^6*d
*e)*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/2
7*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(13
31*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^
3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-
1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*
(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2
*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4) + 800*(1
331*b^2*c^3 + 343*a*b*d^3)*x + 1/78732*sqrt(1/3...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.01, size = 320, normalized size = 1.03

$$-\frac{e\log\left(|bx^3+a|\right)}{3\,a^4}+\frac{e\log\left(|x|\right)}{a^4}-\frac{20\,\sqrt{3}\left(11\,(-ab^2)^{\frac{1}{3}}\,bc-7\,(-ab^2)^{\frac{2}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5b}-\frac{10\left(11\,(-ab^2)^{\frac{1}{3}}\,bc+7\,(-ab^2)^{\frac{2}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b}+\frac{20\left(7\,a^4b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+11\,a^4b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^9b}-\frac{280\,b^3dx^{10}+220\,b^3cx^9-54\,ab^2x^8e+770\,ab^2dx^7+572\,ab^2cx^6-135\,a^2bx^5e+670\,a^2bdx^4+451\,a^2bcx^3-99\,a^3x^2e+162\,a^3dx+81\,a^3c}{162\,(bx^3+a)^3a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^4 + e*log(abs(x))/a^4 - 20/243*sqrt(3)*(11*(-a
*b^2)^(1/3)*b*c - 7*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(a^5*b) - 10/243*(11*(-a*b^2)^(1/3)*b*c + 7*(-a*b^2)^(2/3)
*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) + 20/243*(7*a^4*b^2*d*
(-a/b)^(1/3) + 11*a^4*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b
) - 1/162*(280*b^3*d*x^10 + 220*b^3*c*x^9 - 54*a*b^2*x^8*e + 770*a*b^2*d*x^
7 + 572*a*b^2*c*x^6 - 135*a^2*b*x^5*e + 670*a^2*b*d*x^4 + 451*a^2*b*c*x^3 -
 99*a^3*x^2*e + 162*a^3*d*x + 81*a^3*c)/((b*x^3 + a)^3*a^4*x^2)
```

Mupad [B]
time = 5.38, size = 825, normalized size = 2.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x)
```

```
[Out] symsum(log(-(4*b^3*(688905*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 + 2
2453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d
^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k)^2*a^10*d - 229635*a^2*d*e^2 +
 4782969*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*
z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^
3 + 10648000*b^2*c^3, z, k)^3*a^14*x + 2662000*b^2*c^3*x - 459270*root(1434
8907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e^
2*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*c
^3, z, k)*a^6*d*e - 980100*a*b*c^2*e - 686000*a*b*d^3*x + 980100*root(14348
907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e^2
*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*c^
3, z, k)*a^5*b*c^2 + 531441*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 +
22453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*
d^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k)*a^6*e^2*x + 3188646*root(143
48907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e
^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*
c^3, z, k)^2*a^10*e*x + 6237000*root(14348907*a^14*z^3 + 14348907*a^10*e*z^
2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*
a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k)*a^5*b*c*d*x + 1247400*a*
b*c*d*e*x))/(531441*a^12))*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 + 2
2453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d
^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k), k, 1, 3) - (c/(2*a) - (11*e*
x^2)/(18*a) + (d*x)/a + (286*b^2*c*x^6)/(81*a^3) + (110*b^3*c*x^9)/(81*a^4)
 + (385*b^2*d*x^7)/(81*a^3) + (140*b^3*d*x^10)/(81*a^4) - (b^2*e*x^8)/(3*a^
3) + (451*b*c*x^3)/(162*a^2) + (335*b*d*x^4)/(81*a^2) - (5*b*e*x^5)/(6*a^2)
)/(a^3*x^2 + b^3*x^11 + 3*a^2*b*x^5 + 3*a*b^2*x^8) + (e*log(x))/a^4
```

3.364 $\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx$

Optimal. Leaf size=340

$$-\frac{c}{3a^4x^3}-\frac{d}{2a^4x^2}-\frac{e}{a^4x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3}-\frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2}-\frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4\left(a+bx^3\right)}+$$

```
[Out] -1/3*c/a^4/x^3-1/2*d/a^4/x^2-e/a^4/x-1/9*x*(b*d+b*x*e-b^2*c*x^2/a)/a^2/(b*x
^3+a)^3-1/54*x*(17*b*d+16*b*x*e-24*b^2*c*x^2/a)/a^3/(b*x^3+a)^2-1/162*x*(13
9*b*d+118*b*x*e-234*b^2*c*x^2/a)/a^4/(b*x^3+a)-4*b*c*ln(x)/a^5-20/243*b^(1/
3)*(11*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)+10/243*b^(1/3)
*(11*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14
/3)+4/3*b*c*ln(b*x^3+a)/a^5+20/243*b^(1/3)*(11*b^(1/3)*d+7*a^(1/3)*e)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)*3^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{20\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}\,e+11\sqrt[3]{b}\,d\right)}{81\sqrt{3}\,a^{14/3}}+\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{243a^{14/3}}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{14/3}}+\frac{4bc\log\left(a+bx^3\right)}{3a^5}-\frac{4bc\log(x)}{a^5}-\frac{x\left(-\frac{234b^2cx^2}{a}+139bd+118bex\right)}{162a^4\left(a+bx^3\right)}-\frac{c}{3a^4x^3}-\frac{d}{2a^4x^2}-\frac{e}{a^4x}-\frac{x\left(-\frac{24b^2cx^2}{a}+17bd+16bex\right)}{54a^3\left(a+bx^3\right)^2}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x]
```

```
[Out] -1/3*c/(a^4*x^3) - d/(2*a^4*x^2) - e/(a^4*x) - (x*(b*d + b*e*x - (b^2*c*x^2
)/a))/(9*a^2*(a + b*x^3)^3) - (x*(17*b*d + 16*b*e*x - (24*b^2*c*x^2)/a))/(5
4*a^3*(a + b*x^3)^2) - (x*(139*b*d + 118*b*e*x - (234*b^2*c*x^2)/a))/(162*a
^4*(a + b*x^3)) + (20*b^(1/3)*(11*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(14/3)) - (4*b*c*Log[x])/a
^5 - (20*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(24
3*a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(14/3)) + (4*b*c*Log[a + b*x^3])/(3*a^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
```

```
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx = -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{\int \frac{-9bc-9bdx-9bex^2+\frac{9b^2cx^3}{a}+\frac{8b^2dx^4}{a}+\frac{7b^2ex^5}{a}-\frac{6b^3cx^6}{a^2}}{x^4\left(a+bx^3\right)^3}\,dx}{9ab}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} + \frac{\int \frac{54b^3c+54b^3dx+54b^3ex^2-\frac{108b^4cx^3}{a}-85}{x^4\left(a+bx^3\right)^2}}{54a^2b^3}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4\left(a+bx^3\right)}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4\left(a+bx^3\right)}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - x\left(13\right.$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - x\left(13\right.$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - x\left(13\right.$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - x\left(13\right.$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - x\left(13\right.$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - x\left(13\right.$$

Mathematica [A]
time = 0.30, size = 284, normalized size = 0.84

$$-\frac{\frac{162ac}{x^3}+\frac{243ad}{x^2}+\frac{486ae}{x}+\frac{54a^3b(c+x(d+ex))}{(a+bx^3)^3}+\frac{9a^2b(18c+x(17d+16ex))}{(a+bx^3)^2}+\frac{3ab(162c+x(139d+118ex))}{a+bx^3}-40\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(11\sqrt[3]{b}\,d+7\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+1944bc\log(x)+40\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{b}\,d-7a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-20\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{b}\,d-7a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-648bc\log\left(a+bx^3\right)}{486a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x]
```

[Out] -1/486*((162*a*c)/x^3 + (243*a*d)/x^2 + (486*a*e)/x + (54*a^3*b*(c + x*(d + e*x)))/(a + b*x^3)^3 + (9*a^2*b*(18*c + x*(17*d + 16*e*x)))/(a + b*x^3)^2 + (3*a*b*(162*c + x*(139*d + 118*e*x)))/(a + b*x^3) - 40*Sqrt[3]*a^(1/3)*b^(1/3)*(11*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 1944*b*c*Log[x] + 40*b^(1/3)*(11*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] - 20*b^(1/3)*(11*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 648*b*c*Log[a + b*x^3]])/a^5

Maple [A]
time = 0.39, size = 336, normalized size = 0.99

method	result
default	$-\frac{b\left(\frac{\frac{59}{81}a\,b^{2}e\,x^{8}+\frac{139}{162}a\,b^{2}d\,x^{7}+a\,b^{2}c\,x^{6}+\frac{142}{81}a^{2}be\,x^{5}+\frac{329}{162}a^{2}bd\,x^{4}+\frac{7}{3}a^{2}bc\,x^{3}+\frac{92}{81}a^{3}e\,x^{2}+\frac{104}{81}a^{3}dx+\frac{13}{9}c\,a^{3}}{\left(b\,x^{3}+a\right)^{3}}+\frac{220ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\right.}{}\right.}{}\right.}{}$
risch	$\frac{-\frac{140b^{3}e\,x^{11}}{81a^{4}}-\frac{110b^{3}d\,x^{10}}{81a^{4}}-\frac{4b^{3}c\,x^{9}}{3a^{4}}-\frac{385e\,b^{2}x^{8}}{81a^{3}}-\frac{286d\,b^{2}x^{7}}{81a^{3}}-\frac{10c\,b^{2}x^{6}}{3a^{3}}-\frac{335be\,x^{5}}{81a^{2}}-\frac{451bd\,x^{4}}{162a^{2}}-\frac{22bc\,x^{3}}{9a^{2}}-\frac{e\,x^{2}}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^{3}\left(b\,x^{3}+a\right)^{3}}+\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x,method=_RETURNVERBOSE)

[Out] -1/a^5*b*((59/81*a*b^2*e*x^8+139/162*a*b^2*d*x^7+a*b^2*c*x^6+142/81*a^2*b*e*x^5+329/162*a^2*b*d*x^4+7/3*a^2*b*c*x^3+92/81*a^3*e*x^2+104/81*a^3*d*x+13/9*c*a^3)/(b*x^3+a)^3+220/81*a*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+140/81*a*e*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-4/3*c*ln(b*x^3+a))-1/3*c/a^4/x^3-1/2*d/a^4/x^2-e/a^4/x-4*b*c*ln(x)/a^5

Maxima [A]
time = 0.58, size = 337, normalized size = 0.99

$$-\frac{280\,b^3x^{11}e+220\,b^3dx^{10}+216\,b^3cx^9+770\,ab^2x^8e+572\,ab^2dx^7+540\,ab^2cx^6+670\,a^2bx^5e+451\,a^2bdx^4+396\,a^2bcx^3+162\,a^3x^2e+81\,a^3dx+54\,a^3c}{162\,(a^4b^3x^{12}+3\,a^5b^2x^9+3\,a^6bx^6+a^7x^3)}-\frac{4\,bc\log(x)}{a^5}-\frac{20\,\sqrt{3}\left(7\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}e+11\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^6}+\frac{2\left(162\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-35\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}e+55\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{4\left(81\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+35\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}e-55\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] -1/162*(280*b^3*x^11*e + 220*b^3*d*x^10 + 216*b^3*c*x^9 + 770*a*b^2*x^8*e +
 572*a*b^2*d*x^7 + 540*a*b^2*c*x^6 + 670*a^2*b*x^5*e + 451*a^2*b*d*x^4 + 39
6*a^2*b*c*x^3 + 162*a^3*x^2*e + 81*a^3*d*x + 54*a^3*c)/(a^4*b^3*x^12 + 3*a^
5*b^2*x^9 + 3*a^6*b*x^6 + a^7*x^3) - 4*b*c*log(x)/a^5 - 20/243*sqrt(3)*(7*a
*(a/b)^(2/3)*e + 11*a*d*(a/b)^(1/3))*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3
))/(a/b)^(1/3))/a^6 + 2/243*(162*b*c*(a/b)^(2/3) - 35*a*(a/b)^(1/3)*e + 55*
a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(2/3)) + 4/243*(81*b
*c*(a/b)^(2/3) + 35*a*(a/b)^(1/3)*e - 55*a*d)*log(x + (a/b)^(1/3))/(a^5*(a/
b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.90, size = 5670, normalized size = 16.68

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] -1/486*(840*a*b^3*e*x^11 + 660*a*b^3*d*x^10 + 648*a*b^3*c*x^9 + 2310*a^2*b^
2*e*x^8 + 1716*a^2*b^2*d*x^7 + 1620*a^2*b^2*c*x^6 + 2010*a^3*b*e*x^5 + 1353
*a^3*b*d*x^4 + 1188*a^3*b*c*x^3 + 486*a^4*e*x^2 + 243*a^4*d*x + 162*a^4*c +
 2*(a^5*b^3*x^12 + 3*a^6*b^2*x^9 + 3*a^7*b*x^6 + a^8*x^3)*(4^(2/3)*(-I*sqrt
(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*
b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 19
25*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1
701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^
15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e
)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)
*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*log(7*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^
2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 12
5*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a
^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)
/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*
d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531
441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/
3) - 324*b*c/a^5)^2*a^10*e + 784080*b^2*c*d^2 + 734832*b^2*c^2*e + 431200*a
*b*d*e^2 + 4*(605*a^5*b*d^2 + 1134*a^5*b*c*e)*(4^(2/3)*(-I*sqrt(3) + 1)*(65
```

61*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15
 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*
b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a
b^2)/a^15)^(1/3) + 4^(1/3)(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(13
31*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 +
 (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15
)^(1/3) - 324*b*c/a^5) + 400*(1331*b^2*d^3 + 343*a*b*e^3)*x) - (972*b^4*c*x
^12 + 2916*a*b^3*c*x^9 + 2916*a^2*b^2*c*x^6 + 972*a^3*b*c*x^3 + (a^5*b^3*x^
12 + 3*a^6*b^2*x^9 + 3*a^7*b*x^6 + a^8*x^3)*(4^(2/3)*(-I*sqrt(3) + 1)*(6561
*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 +
 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*
c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b
^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331
*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (
531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^
(1/3) - 324*b*c/a^5) + 3*sqrt(1/3)*(a^5*b^3*x^12 + 3*a^6*b^2*x^9 + 3*a^7*b*
x^6 + a^8*x^3)*sqrt(-((4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*
b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343
*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c
^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1
/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/
a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875
*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)^2
a^10 + 648(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 +
1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/
a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875
*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqr
t(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243
*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3
 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*a^5*b*c + 1
04976*b^2*c^2 + 123200*a*b*d*e)/a^10))*log(-7*(4^(2/3)*(-I*sqrt(3) + 1)*(65
61*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15
 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*
b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a
b^2)/a^15)^(1/3) + 4^(1/3)(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(13
31*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 +
 (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15
)^(1/3) - 324*b*c/a^5)^2*a^10*e - 784080*b^2*c*d^2 - 734832*b^2*c^2*e - 431
200*a*b*d*e^2 - 4*(605*a^5*b*d^2 + 1134*a^5*b*c*e)*(4^(2/3)*(-I*sqrt(3) + 1
)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3
/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*
d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d
*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 12
5*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a
^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)

```
/a^15)^(1/3) - 324*b*c/a^5) + 800*(1331*b^2*d^3 + 343*a*b*e^3)*x + 3*sqrt(1
/3)*(7*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*
a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(133...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.17, size = 333, normalized size = 0.98

$$\frac{4\,bc\log\left(|bx^3+a|\right)}{3\,a^5}-\frac{4\,bc\log\left(|x|\right)}{a^5}-\frac{20\,\sqrt{3}\left(11\,(-ab^2)^{\frac{1}{3}}\,bd-7\,(-ab^2)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5b}-\frac{10\left(11\,(-ab^2)^{\frac{1}{3}}\,bd+7\,(-ab^2)^{\frac{2}{3}}\,e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b}-\frac{280\,b^3x^{11}e+220\,b^3dx^{10}+216\,b^3cx^9+770\,ab^2x^8e+572\,ab^2dx^7+540\,ab^2cx^6+670\,a^2bx^5e+451\,a^2bdx^4+396\,a^2bcx^3+162\,a^3x^2e+81\,a^3dx+54\,a^3c}{162\,(bx^4+ax)^3a^4}+\frac{20\left(7\,a^6b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+11\,a^6b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^{11}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 4/3*b*c*log(abs(b*x^3 + a))/a^5 - 4*b*c*log(abs(x))/a^5 - 20/243*sqrt(3)*(1
1*(-a*b^2)^(1/3)*b*d - 7*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/(a^5*b) - 10/243*(11*(-a*b^2)^(1/3)*b*d + 7*(-a*b^2)^
(2/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/162*(280*b^3*
x^11*e + 220*b^3*d*x^10 + 216*b^3*c*x^9 + 770*a*b^2*x^8*e + 572*a*b^2*d*x^7
 + 540*a*b^2*c*x^6 + 670*a^2*b*x^5*e + 451*a^2*b*d*x^4 + 396*a^2*b*c*x^3 +
162*a^3*x^2*e + 81*a^3*d*x + 54*a^3*c)/((b*x^4 + a*x)^3*a^4) + 20/243*(7*a^
6*b^2*(-a/b)^(1/3)*e + 11*a^6*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3))
)/(a^11*b)
```

Mupad [B]
time = 0.52, size = 918, normalized size = 2.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x)
```

```
[Out] symsum(log(-(4*b^3*(688905*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 +
 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 274
4000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)^2*a^10*e + 39
20400*b^2*c*d^2 - 3674160*b^2*c^2*e + 4782969*root(14348907*a^15*z^3 - 5739
5628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 2993760
0*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3,
```

z, k)^3*a^14*x + 2662000*b^2*d^3*x - 686000*a*b*e^3*x + 980100*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)*a^5*b*d^2 - 12754584*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)^2*a^9*b*c*x + 8503056*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)*a^4*b^2*c^2*x + 1837080*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)*a^5*b*c*e - 4989600*b^2*c*d*e*x + 6237000*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)*a^5*b*d*e*x))/(531441*a^12))*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (d*x)/(2*a) + (10*b^2*c*x^6)/(3*a^3) + (4*b^3*c*x^9)/(3*a^4) + (286*b^2*d*x^7)/(81*a^3) + (110*b^3*d*x^10)/(81*a^4) + (385*b^2*e*x^8)/(81*a^3) + (140*b^3*e*x^11)/(81*a^4) + (22*b*c*x^3)/(9*a^2) + (451*b*d*x^4)/(162*a^2) + (335*b*e*x^5)/(81*a^2))/(a^3*x^3 + b^3*x^12 + 3*a^2*b*x^6 + 3*a*b^2*x^9) - (4*b*c*log(x))/a^5

3.365 $\int \frac{2ax-x^2}{a^3+x^3} \, dx$

Optimal. Leaf size=29

$$-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}-\log(a+x)$$

```
[Out] -ln(a+x)-2/3*arctan(1/3*(a-2*x)/a*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1607, 1882, 31, 631, 210}

$$-\frac{2\text{ArcTan}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}-\log(a+x)$$

Antiderivative was successfully verified.

```
[In] Int[(2*a*x - x^2)/(a^3 + x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1882

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2ax-x^2}{a^3+x^3}\,dx &= \int \frac{(2a-x)x}{a^3+x^3}\,dx\\
&= a\int \frac{1}{a^2-ax+x^2}\,dx - \int \frac{1}{a+x}\,dx\\
&= -\log(a+x) + 2\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{a}\right)\\
&= -\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}} - \log(a+x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 1.97

$$\frac{1}{3}\left(2\sqrt{3}\,\tan^{-1}\left(\frac{-a+2x}{\sqrt{3}\,a}\right) - 2\log(a+x) + \log\left(a^2-ax+x^2\right) - \log\left(a^3+x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a*x - x^2)/(a^3 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x +
x^2] - Log[a^3 + x^3])/3
```

Maple [A]
time = 0.35, size = 29, normalized size = 1.00

method	result	size
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3} - \ln(a+x)$	29
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3} - \ln(a+x)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x-x^2)/(a^3+x^3),x,method=_RETURNVERBOSE)
[Out] 2/3*3^(1/2)*arctan(1/3*(-a+2*x)*3^(1/2)/a)-ln(a+x)
```

Maxima [A]
time = 0.48, size = 26, normalized size = 0.90

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

Fricas [A]
time = 0.39, size = 26, normalized size = 0.90

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.06, size = 54, normalized size = 1.86

$$-\log(a+x)-\frac{\sqrt{3}\,i\log\left(-\frac{a}{2}-\frac{\sqrt{3}\,ia}{2}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(-\frac{a}{2}+\frac{\sqrt{3}\,ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-x**2)/(a**3+x**3),x)
[Out] -log(a + x) - sqrt(3)*I*log(-a/2 - sqrt(3)*I*a/2 + x)/3 + sqrt(3)*I*log(-a/
2 + sqrt(3)*I*a/2 + x)/3
```

Giac [A]
time = 1.51, size = 27, normalized size = 0.93

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(|a+x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(abs(a + x))
```

Mupad [B]
time = 4.97, size = 26, normalized size = 0.90

$$-\ln(a+x)-\frac{2\sqrt{3}\,\mathrm{atan}\left(-\frac{\sqrt{3}\,a}{a-2\,x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x - x^2)/(a^3 + x^3),x)
```

```
[Out] - log(a + x) - (2*3^(1/2)*atan(-(3^(1/2)*a)/(a - 2*x)))/3
```

3.366 $\int \frac{(2a-x)x}{a^3+x^3}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}-\log(a+x)$$

```
[Out] -ln(a+x)-2/3*arctan(1/3*(a-2*x)/a*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {1882, 31, 631, 210}

$$-\frac{2\text{ArcTan}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}-\log(a+x)$$

Antiderivative was successfully verified.

```
[In] Int[((2*a - x)*x)/(a^3 + x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1882

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
```

```
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{(2a-x)x}{a^3+x^3}\,dx &= a\int \frac{1}{a^2-ax+x^2}\,dx-\int \frac{1}{a+x}\,dx\\ &= -\log(a+x)+2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}-\log(a+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 57, normalized size = 1.97

$$\frac{1}{3}\left(2\sqrt{3}\,\tan^{-1}\left(\frac{-a+2x}{\sqrt{3}\,a}\right)-2\log(a+x)+\log\left(a^2-ax+x^2\right)-\log\left(a^3+x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2*a - x)*x)/(a^3 + x^3),x]
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x +
x^2] - Log[a^3 + x^3])/3
```

Maple [A]
time = 0.34, size = 29, normalized size = 1.00

method	result	size
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3}-\ln(a+x)$	29
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3}-\ln(a+x)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a-x)*x/(a^3+x^3),x,method=_RETURNVERBOSE)
[Out] 2/3*3^(1/2)*arctan(1/3*(-a+2*x)*3^(1/2)/a)-ln(a+x)
```

Maxima [A]
time = 0.52, size = 26, normalized size = 0.90

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-x)*x/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

Fricas [A]
time = 0.40, size = 26, normalized size = 0.90

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-x)*x/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.06, size = 54, normalized size = 1.86

$$-\log(a+x)-\frac{\sqrt{3}\,i\log\left(-\frac{a}{2}-\frac{\sqrt{3}\,ia}{2}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(-\frac{a}{2}+\frac{\sqrt{3}\,ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-x)*x/(a**3+x**3),x)
```

```
[Out] -log(a + x) - sqrt(3)*I*log(-a/2 - sqrt(3)*I*a/2 + x)/3 + sqrt(3)*I*log(-a/
2 + sqrt(3)*I*a/2 + x)/3
```

Giac [A]
time = 1.19, size = 27, normalized size = 0.93

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(|a+x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-x)*x/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(abs(a + x))
```

Mupad [B]
time = 0.03, size = 26, normalized size = 0.90

$$-\ln(a+x)-\frac{2\sqrt{3}\,\mathrm{atan}\left(-\frac{\sqrt{3}\,a}{a-2\,x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*a - x))/(a^3 + x^3),x)
[Out] - log(a + x) - (2*3^(1/2)*atan(-(3^(1/2)*a)/(a - 2*x)))/3
```

3.367 $\int \frac{2ax+x^2}{a^3-x^3}\,dx$

Optimal. Leaf size=31

$$-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}-\log(a-x)$$

```
[Out] -ln(a-x)-2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1607, 1882, 31, 631, 210}

$$-\frac{2\text{ArcTan}\left(\frac{a+2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}-\log(a-x)$$

Antiderivative was successfully verified.

```
[In] Int[(2*a*x + x^2)/(a^3 - x^3),x]
```

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1882

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{2ax+x^2}{a^3-x^3}\,dx &= \int \frac{x(2a+x)}{a^3-x^3}\,dx\\ &= -\left(a\int \frac{1}{a^2+ax+x^2}\,dx\right)-\int \frac{1}{-a+x}\,dx\\ &= -\log(a-x)+2\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{a}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}-\log(a-x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 58, normalized size = 1.87

$$\frac{1}{3}\left(-2\sqrt{3}\,\tan^{-1}\left(\frac{a+2x}{\sqrt{3}\,a}\right)-2\log(-a+x)+\log\left(a^2+ax+x^2\right)-\log\left(-a^3+x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a*x + x^2)/(a^3 - x^3),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a^2 + a*x +
 x^2] - Log[-a^3 + x^3])/3
```

Maple [A]
time = 0.38, size = 29, normalized size = 0.94

method	result	size
default	$-\ln\left(a-x\right)-\frac{2\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3}$	29
risch	$-\ln\left(-a+x\right)-\frac{2\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x+x^2)/(a^3-x^3),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(a-x)-2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.48, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Fricas [A]
time = 0.39, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.06, size = 54, normalized size = 1.74

$$-\log(-a+x)+\frac{\sqrt{3}\,i\log\left(\frac{a}{2}-\frac{\sqrt{3}\,ia}{2}+x\right)}{3}-\frac{\sqrt{3}\,i\log\left(\frac{a}{2}+\frac{\sqrt{3}\,ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+x**2)/(a**3-x**3),x)
```

```
[Out] -log(-a + x) + sqrt(3)*I*log(a/2 - sqrt(3)*I*a/2 + x)/3 - sqrt(3)*I*log(a/2
 + sqrt(3)*I*a/2 + x)/3
```

Giac [A]
time = 1.23, size = 29, normalized size = 0.94

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(|-a+x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(abs(-a + x))
```

Mupad [B]
time = 4.95, size = 27, normalized size = 0.87

$$\frac{2\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}\ a}{a+2\,x}\right)}{3}-\ln\left(x-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x + x^2)/(a^3 - x^3),x)
```

```
[Out] (2*3^(1/2)*atan((3^(1/2)*a)/(a + 2*x)))/3 - log(x - a)
```

3.368 $\int \frac{x(2a+x)}{a^3-x^3}\,dx$

Optimal. Leaf size=31

$$-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}-\log(a-x)$$

```
[Out] -ln(a-x)-2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1882, 31, 631, 210}

$$-\frac{2\text{ArcTan}\left(\frac{a+2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}-\log(a-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*a + x))/(a^3 - x^3),x]
```

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1882

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
```

```
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{x(2a+x)}{a^3-x^3}\,dx &= -\left(a\int \frac{1}{a^2+ax+x^2}\,dx\right)-\int \frac{1}{-a+x}\,dx\\ &= -\log(a-x)+2\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{a}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}-\log(a-x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 58, normalized size = 1.87

$$\frac{1}{3}\left(-2\sqrt{3}\,\tan^{-1}\left(\frac{a+2x}{\sqrt{3}\,a}\right)-2\log(-a+x)+\log\left(a^2+ax+x^2\right)-\log\left(-a^3+x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*a + x))/(a^3 - x^3),x]
[Out] (-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a^2 + a*x +
 x^2] - Log[-a^3 + x^3])/3
```

Maple [A]
time = 0.38, size = 29, normalized size = 0.94

method	result	size
default	$-\ln\left(a-x\right)-\frac{2\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3}$	29
risch	$-\ln\left(-a+x\right)-\frac{2\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*a+x)/(a^3-x^3),x,method=_RETURNVERBOSE)
[Out] -ln(a-x)-2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.49, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Fricas [A]
time = 0.39, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.06, size = 54, normalized size = 1.74

$$-\log(-a+x)+\frac{\sqrt{3}\,i\log\left(\frac{a}{2}-\frac{\sqrt{3}\,ia}{2}+x\right)}{3}-\frac{\sqrt{3}\,i\log\left(\frac{a}{2}+\frac{\sqrt{3}\,ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*a+x)/(a**3-x**3),x)
```

```
[Out] -log(-a + x) + sqrt(3)*I*log(a/2 - sqrt(3)*I*a/2 + x)/3 - sqrt(3)*I*log(a/2
 + sqrt(3)*I*a/2 + x)/3
```

Giac [A]
time = 1.21, size = 29, normalized size = 0.94

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(|-a+x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(abs(-a + x))
```

Mupad [B]
time = 0.03, size = 27, normalized size = 0.87

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,a}{a+2\,x}\right)}{3}-\ln\left(x-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*a + x))/(a^3 - x^3),x)
[Out] (2*3^(1/2)*atan((3^(1/2)*a)/(a + 2*x)))/3 - log(x - a)
```

3.369 $\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}\,C+Cx\right)}{a+bx^3}\,dx$

Optimal. Leaf size=50

$$\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

```
[Out] C*ln((a/b)^(1/3)+x)/b+2/3*C*arctan(1/3*(1-2*x/(a/b)^(1/3))*3^(1/2))/b*3^(1/
2)
```

Rubi [A]
time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1881, 31, 631, 210}

$$\frac{2C\,\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^(1
/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1881

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}\,C+Cx\right)}{a+bx^3}\,dx=\frac{C\int\frac{1}{\sqrt[3]{\frac{a}{b}}+x}\,dx}{b}-\frac{\left(\sqrt[3]{\frac{a}{b}}\,C\right)\int\frac{1}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}\,x+x^2}\,dx}{b}$$

$$=\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{(2C)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$

$$=\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 146 vs. 2(50) = 100.
time = 0.03, size = 146, normalized size = 2.92

$$\frac{C\left(2\sqrt{3}\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a+bx^3\right)\right)}{3\sqrt[3]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] - (a/b)^(1/3)*b^(1/3)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a + b*x^3]))/(3
*a^(1/3)*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(43) = 86$.
time = 0.32, size = 116, normalized size = 2.32

method	result	si
default	$C\left(-2\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+\frac{\ln(b\,x^3+a)}{3b}\right)$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-2*(a/b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1)))+1/3*ln(b*x^3+a)/b)
```

Maxima [A]
time = 0.49, size = 51, normalized size = 1.02

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}+\frac{C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/b + C*lo
g(x + (a/b)^(1/3))/b
```

Fricas [A]
time = 0.38, size = 52, normalized size = 1.04

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-3\,C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) - 3
*C*log(x + (a/b)^(1/3)))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.13, size = 100, normalized size = 2.00

$$\frac{C\left(\log\left(\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(a/b)**(2/3)) + x) + sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) - sqrt
(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(43) = 86$.
time = 1.34, size = 96, normalized size = 1.92

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}-\frac{\left(Cb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-2\left(ab^{2}\right)^{\frac{1}{3}}C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/3
*(C*b*(-a/b)^(2/3) - 2*(a*b^2)^(1/3)*C*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a*b)
```

Mupad [B]
time = 5.22, size = 154, normalized size = 3.08

$$\sum_{k=1}^{3}\ln\left(\frac{C^2\,a+\mathrm{root}(27\,a\,b^3\,z^3-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k)^2\,a\,b^2\,9-C\,\mathrm{root}(27\,a\,b^3\,z^3-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k)\,a\,b\,6+4\,C^2\,b\,x\,{\left(\frac{a}{b}\right)}^{2/3}}{b^3}\right)\,\mathrm{root}(27\,a\,b^3\,z^3-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(C*x - 2*C*(a/b)^(1/3)))/(a + b*x^3),x)
```

```
[Out] symsum(log((C^2*a + 9*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z - 9*
C^3*a, z, k)^2*a*b^2 - 6*C*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z
 - 9*C^3*a, z, k)*a*b + 4*C^2*b*x*(a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 - 27*
C*a*b^2*z^2 + 9*C^2*a*b*z - 9*C^3*a, z, k), k, 1, 3)
```

3.370 $\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

```
[Out] -C*ln((-a/b)^(1/3)+x)/b-2/3*C*arctan(1/3*(1-2*x/(-a/b)^(1/3))*3^(1/2))/b*3^
(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1881, 31, 631, 210}

$$-\frac{2C\,\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(-(a
/b))^(1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1881

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}\,C+Cx\right)}{a-bx^3}\,dx=-\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}+x}\,dx}{b}+\frac{\left(\sqrt[3]{-\frac{a}{b}}\,C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}-\sqrt[3]{-\frac{a}{b}}\,x+x^2}\,dx}{b}$$

$$=-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}+\frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$=-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 149 vs. 2(53) = 106.
time = 0.05, size = 149, normalized size = 2.81

$$-\frac{C\left(-2\sqrt{3}\,\sqrt[3]{-\frac{a}{b}}\,\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{-\frac{a}{b}}\,\sqrt[3]{b}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)+\sqrt[3]{-\frac{a}{b}}\,\sqrt[3]{b}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a-bx^3\right)\right)}{3\sqrt[3]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] -1/3*(C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3)*x] + (-(a/b))^
(1/3)*b^(1/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[
a - b*x^3]))/(a^(1/3)*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(46) = 92$.
time = 0.32, size = 119, normalized size = 2.25

method	result
default	$C\left(-2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-\frac{\ln\left(-b\,x^{3}+a\right)}{3b}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-2*(-a/b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*
ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*(1+2
/(a/b)^(1/3)*x)*3^(1/2)))-1/3*ln(-b*x^3+a)/b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(46) = 92$.
time = 0.50, size = 166, normalized size = 3.13

$$-\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}+C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^{2}+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\sqrt{3}\left(Ca-\left(3\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*(C*(a/b)^(1/3) + C*(-a/b)^(1/3))*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3)
)/(b*(a/b)^(1/3)) - 1/3*(C*(a/b)^(1/3) - 2*C*(-a/b)^(1/3))*log(x - (a/b)^(1
/3))/(b*(a/b)^(1/3)) - 2/9*sqrt(3)*(C*a - (3*C*(a/b)^(2/3)*(-a/b)^(1/3) + C
*a/b)*b)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

Fricas [A]
time = 0.38, size = 53, normalized size = 1.00

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+3\,C\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt(3)*a)/a) +
3*C*log(x + (-a/b)^(1/3)))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.14, size = 110, normalized size = 2.08

$$-\frac{C\left(\log\left(-\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(-a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(-a/b)**(2/3)) + x) - sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3)) - s
qrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

Giac [C] Result contains complex when optimal does not.
time = 1.22, size = 165, normalized size = 3.11

$$-\frac{\left(Cb\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\left(-ab^2\right)^{\frac{1}{3}}C\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}+\frac{\sqrt{3}\left(ab^2+i\sqrt{3}\sqrt{a^2b^4}\right)C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}-\frac{\left(3\,ab^2+i\sqrt{3}\sqrt{a^2b^4}\right)C\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(C*b*(a/b)^(2/3) - 2*(-a*b^2)^(1/3)*C*(a/b)^(1/3))*(a/b)^(1/3)*log(abs
(x - (a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*(a*b^2 + I*sqrt(3)*sqrt(a^2*b^4))*C*
arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) - 1/6*(3*a*b^2
+ I*sqrt(3)*sqrt(a^2*b^4))*C*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3)
```

Mupad [B]
time = 5.25, size = 156, normalized size = 2.94

$$\sum_{k=1}^{3}\ln\left(-\frac{C^2\,a+\mathrm{root}(27\,a\,b^3\,z^3+27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k)^2\,a\,b^2\,9+C\,\mathrm{root}(27\,a\,b^3\,z^3+27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k)\,a\,b\,6-4\,C^2\,b\,x\,{\left(-\frac{a}{b}\right)}^{2/3}}{b^3}\right)\,\mathrm{root}\left(27\,a\,b^3\,z^3+27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(C*x - 2*C*(-a/b)^(1/3)))/(a - b*x^3),x)
```

```
[Out] symsum(log(-(C^2*a + 9*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*z + 9
*C^3*a, z, k)^2*a*b^2 + 6*C*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*
z + 9*C^3*a, z, k)*a*b - 4*C^2*b*x*(-a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 + 2
7*C*a*b^2*z^2 + 9*C^2*a*b*z + 9*C^3*a, z, k), k, 1, 3)
```

3.371 $\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}\,C+Cx\right)}{a+bx^3}\,dx$

Optimal. Leaf size=54

$$\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

```
[Out] C*ln((-a/b)^(1/3)-x)/b+2/3*C*arctan(1/3*(1+2*x/(-a/b)^(1/3))*3^(1/2))/b*3^(
1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1883, 31, 631, 210}

$$\frac{2C\,\text{ArcTan}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(-(a/
b))^(1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1883

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Dist[-C/b
, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]
] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &&
PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}\,C+Cx\right)}{a+bx^3}\,dx=-\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}\,dx}{b}+\frac{\left(\sqrt[3]{-\frac{a}{b}}\,C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}+\sqrt[3]{-\frac{a}{b}}\,x+x^2}\,dx}{b}$$

$$=\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$=\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(54) = 108.
time = 0.03, size = 148, normalized size = 2.74

$$\frac{C\left(-2\sqrt{3}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a+bx^3\right)\right)}{3\sqrt[3]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] + (-(a/b))^(1/3)
*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a + b
*x^3]))/(3*a^(1/3)*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(47) = 94$.
time = 0.34, size = 117, normalized size = 2.17

method	result	si
default	$C\left(2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+\frac{\ln\left(b\,x^{3}+a\right)}{3b}\right)$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(2*(-a/b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1)))+1/3*ln(b*x^3+a)/b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(47) = 94$.
time = 0.48, size = 167, normalized size = 3.09

$$\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}+C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\sqrt{3}\left(Ca-\left(3\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(C*(a/b)^(1/3) + C*(-a/b)^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(b*(a/b)^(1/3)) + 1/3*(C*(a/b)^(1/3) - 2*C*(-a/b)^(1/3))*log(x + (a/b)^(1/
3))/(b*(a/b)^(1/3)) - 2/9*sqrt(3)*(C*a - (3*C*(a/b)^(2/3)*(-a/b)^(1/3) + C*
a/b)*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

Fricas [A]
time = 0.39, size = 56, normalized size = 1.04

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-3\,C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) -
3*C*log(x - (-a/b)^(1/3)))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.13, size = 109, normalized size = 2.02

$$\frac{C\left(\log\left(\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(-a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(-a/b)**(2/3)) + x) + sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3)) - sq
rt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(47) = 94.
time = 1.65, size = 97, normalized size = 1.80

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(-ab^2\right)^{\frac{1}{3}}C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/3
*(C*b*(-a/b)^(2/3) + 2*(-a*b^2)^(1/3)*C*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(
x - (-a/b)^(1/3)))/(a*b)
```

Mupad [B]
time = 5.22, size = 155, normalized size = 2.87

$$\sum_{k=1}^{3}\ln\left(\frac{C^2\,a+\mathrm{root}(27\,a\,b^3\,z^3-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k)^2\,a\,b^2\,9-C\,\mathrm{root}(27\,a\,b^3\,z^3-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k)\,a\,b\,6+4\,C^2\,b\,x\,{\left(-\frac{a}{b}\right)}^{2/3}}{b^3}\right)\,\mathrm{root}\left(27\,a\,b^3\,z^3-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(C*x + 2*C*(-a/b)^(1/3)))/(a + b*x^3),x)
```

```
[Out] symsum(log((C^2*a + 9*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z - 9*
C^3*a, z, k)^2*a*b^2 - 6*C*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z
 - 9*C^3*a, z, k)*a*b + 4*C^2*b*x*(-a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 - 27
*C*a*b^2*z^2 + 9*C^2*a*b*z - 9*C^3*a, z, k), k, 1, 3)
```

3.372 $\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}\,C+Cx\right)}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$-\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

```
[Out] -C*ln((a/b)^(1/3)-x)/b-2/3*C*arctan(1/3*(1+2*x/(a/b)^(1/3))*3^(1/2))/b*3^(1
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1883, 31, 631, 210}

$$-\frac{2C\,\text{ArcTan}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^(
1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1883

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Dist[-C/b
, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]
] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &&
PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx &= \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}\,dx}{b}-\frac{\left(\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}+\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}\\ &= -\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}+\frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}\\ &= -\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 147 vs. 2(53) = 106.
time = 0.04, size = 147, normalized size = 2.77

$$-\frac{C\left(2\sqrt{3}\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)-\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a-bx^3\right)\right)}{3\sqrt[3]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] -1/3*(C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3)*x] - (a/b)^(1/3)*b^(1
/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a - b*x^3]
))/(a^(1/3)*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(46) = 92$.
time = 0.33, size = 118, normalized size = 2.23

method	result	si
default	$C\left(2\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-\frac{\ln\left(-b\,x^{3}+a\right)}{3b}\right)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(2*(a/b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln
(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*(1+2/(
a/b)^(1/3)*x)*3^(1/2)))-1/3*ln(-b*x^3+a)/b)
```

Maxima [A]
time = 0.48, size = 52, normalized size = 0.98

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - C*lo
g(x - (a/b)^(1/3))/b
```

Fricas [A]
time = 0.39, size = 53, normalized size = 1.00

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+3\,C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) + sqrt(3)*a)/a) + 3
*C*log(x - (a/b)^(1/3)))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.14, size = 102, normalized size = 1.92

$$-\frac{C\left(\log\left(-\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(a/b)**(2/3)) + x) - sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) - sqr
t(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

Giac [A]
time = 1.34, size = 90, normalized size = 1.70

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}-\frac{\left(Cb\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^{2}\right)^{\frac{1}{3}}C\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - 1/3*
(C*b*(a/b)^(2/3) + 2*(a*b^2)^(1/3)*C*(a/b)^(1/3))*(a/b)^(1/3)*log(abs(x - (
a/b)^(1/3)))/(a*b)
```

Mupad [B]
time = 5.23, size = 155, normalized size = 2.92

$$\sum_{k=1}^{3}\ln\left(-\frac{C^2\,a+\mathrm{root}(27\,a\,b^3\,z^3+27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k)^2\,a\,b^2\,9+C\,\mathrm{root}(27\,a\,b^3\,z^3+27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k)\,a\,b\,6-4\,C^2\,b\,x\,\left(\frac{a}{b}\right)^{2/3}}{b^3}\right)\,\mathrm{root}\left(27\,a\,b^3\,z^3+27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(C*x + 2*C*(a/b)^(1/3)))/(a - b*x^3),x)
```

```
[Out] symsum(log(-(C^2*a + 9*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*z + 9
*C^3*a, z, k)^2*a*b^2 + 6*C*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*
z + 9*C^3*a, z, k)*a*b - 4*C^2*b*x*(a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 + 27
*C*a*b^2*z^2 + 9*C^2*a*b*z + 9*C^3*a, z, k), k, 1, 3)
```

3.373 $\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=97

$$\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{8}(bc+af)x^8+\frac{1}{9}(bd+ag)x^9+\frac{1}{10}(be+ah)x^{10}+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}$$

```
[Out] 1/5*a*c*x^5+1/6*a*d*x^6+1/7*a*e*x^7+1/8*(a*f+b*c)*x^8+1/9*(a*g+b*d)*x^9+1/1
0*(a*h+b*e)*x^10+1/11*b*f*x^11+1/12*b*g*x^12+1/13*b*h*x^13
```

Rubi [A]
time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1834}

$$\frac{1}{8}x^8(af+bc)+\frac{1}{9}x^9(ag+bd)+\frac{1}{10}x^{10}(ah+be)+\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*g
)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x^1
3)/13
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \int \left(acx^4+adx^5+aex^6+(bc+af)x^7+(bd+ag)x^8\right.\\ &= \frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{8}(bc+af)x^8+\frac{1}{9}(bd+a\end{aligned}$$

Mathematica [A]
time = 0.02, size = 97, normalized size = 1.00

$$\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{8}(bc+af)x^8+\frac{1}{9}(bd+ag)x^9+\frac{1}{10}(be+ah)x^{10}+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*g
)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x^1
3)/13
```

Maple [A]
time = 2.00, size = 80, normalized size = 0.82

method	result
default	$\frac{ac\,x^{5}}{5}+\frac{ad\,x^{6}}{6}+\frac{ae\,x^{7}}{7}+\frac{(af+bc)x^{8}}{8}+\frac{(ag+bd)x^{9}}{9}+\frac{(ah+be)x^{10}}{10}+\frac{bf\,x^{11}}{11}+\frac{bg\,x^{12}}{12}+\frac{bh\,x^{13}}{13}$
norman	$\frac{bh\,x^{13}}{13}+\frac{bg\,x^{12}}{12}+\frac{bf\,x^{11}}{11}+\left(\frac{ah}{10}+\frac{be}{10}\right)x^{10}+\left(\frac{ag}{9}+\frac{bd}{9}\right)x^{9}+\left(\frac{af}{8}+\frac{bc}{8}\right)x^{8}+\frac{ae\,x^{7}}{7}+\frac{ad\,x^{6}}{6}+\frac{ac\,x^{5}}{5}$
gosper	$\frac{1}{13}bh\,x^{13}+\frac{1}{12}bg\,x^{12}+\frac{1}{11}bf\,x^{11}+\frac{1}{10}x^{10}ah+\frac{1}{10}be\,x^{10}+\frac{1}{9}x^{9}ag+\frac{1}{9}bd\,x^{9}+\frac{1}{8}x^{8}af+\frac{1}{8}bc\,x^{8}+\frac{1}{7}ae\,x$
risch	$\frac{1}{13}bh\,x^{13}+\frac{1}{12}bg\,x^{12}+\frac{1}{11}bf\,x^{11}+\frac{1}{10}x^{10}ah+\frac{1}{10}be\,x^{10}+\frac{1}{9}x^{9}ag+\frac{1}{9}bd\,x^{9}+\frac{1}{8}x^{8}af+\frac{1}{8}bc\,x^{8}+\frac{1}{7}ae\,x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*c*x^5+1/6*a*d*x^6+1/7*a*e*x^7+1/8*(a*f+b*c)*x^8+1/9*(a*g+b*d)*x^9+1/1
0*(a*h+b*e)*x^10+1/11*b*f*x^11+1/12*b*g*x^12+1/13*b*h*x^13
```

Maxima [A]
time = 0.26, size = 81, normalized size = 0.84

$$\frac{1}{13}\,bhx^{13}+\frac{1}{12}\,bgx^{12}+\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,(ah+be)x^{10}+\frac{1}{9}\,(bd+ag)x^{9}+\frac{1}{8}\,(bc+af)x^{8}+\frac{1}{7}\,ax^{7}e+\frac{1}{6}\,adx^{6}+\frac{1}{5}\,acx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*(a*h + b*e)*x^10 + 1/9
*(b*d + a*g)*x^9 + 1/8*(b*c + a*f)*x^8 + 1/7*a*x^7*e + 1/6*a*d*x^6 + 1/5*a*
c*x^5
```

Fricas [A]
time = 0.40, size = 79, normalized size = 0.81

$$\frac{1}{13}\,bhx^{13}+\frac{1}{12}\,bgx^{12}+\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,(be+ah)x^{10}+\frac{1}{9}\,(bd+ag)x^{9}+\frac{1}{7}\,aex^{7}+\frac{1}{8}\,(bc+af)x^{8}+\frac{1}{6}\,adx^{6}+\frac{1}{5}\,acx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*(b*e + a*h)*x^10 + 1/9
*(b*d + a*g)*x^9 + 1/7*a*e*x^7 + 1/8*(b*c + a*f)*x^8 + 1/6*a*d*x^6 + 1/5*a*
c*x^5
```

Sympy [A]
time = 0.01, size = 90, normalized size = 0.93

$$\frac{acx^5}{5}+\frac{adx^6}{6}+\frac{aex^7}{7}+\frac{bfx^{11}}{11}+\frac{bgx^{12}}{12}+\frac{bhx^{13}}{13}+x^{10}\left(\frac{ah}{10}+\frac{be}{10}\right)+x^9\left(\frac{ag}{9}+\frac{bd}{9}\right)+x^8\left(\frac{af}{8}+\frac{bc}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**5/5 + a*d*x**6/6 + a*e*x**7/7 + b*f*x**11/11 + b*g*x**12/12 + b*h*x*
*13/13 + x**10*(a*h/10 + b*e/10) + x**9*(a*g/9 + b*d/9) + x**8*(a*f/8 + b*c
/8)
```

Giac [A]
time = 1.18, size = 87, normalized size = 0.90

$$\frac{1}{13}\,bhx^{13}+\frac{1}{12}\,bgx^{12}+\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,ahx^{10}+\frac{1}{10}\,bx^{10}e+\frac{1}{9}\,bdx^9+\frac{1}{9}\,agx^9+\frac{1}{8}\,bcx^8+\frac{1}{8}\,afx^8+\frac{1}{7}\,ax^7e+\frac{1}{6}\,adx^6+\frac{1}{5}\,acx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*a*h*x^10 + 1/10*b*x^10
*e + 1/9*b*d*x^9 + 1/9*a*g*x^9 + 1/8*b*c*x^8 + 1/8*a*f*x^8 + 1/7*a*x^7*e +
1/6*a*d*x^6 + 1/5*a*c*x^5
```

Mupad [B]
time = 0.05, size = 82, normalized size = 0.85

$$\frac{b\,h\,x^{13}}{13}+\frac{b\,g\,x^{12}}{12}+\frac{b\,f\,x^{11}}{11}+\left(\frac{b\,e}{10}+\frac{a\,h}{10}\right)\,x^{10}+\left(\frac{b\,d}{9}+\frac{a\,g}{9}\right)\,x^9+\left(\frac{b\,c}{8}+\frac{a\,f}{8}\right)\,x^8+\frac{a\,e\,x^7}{7}+\frac{a\,d\,x^6}{6}+\frac{a\,c\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^8*((b*c)/8 + (a*f)/8) + x^9*((b*d)/9 + (a*g)/9) + x^10*((b*e)/10 + (a*h)/
10) + (b*h*x^13)/13 + (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + (b*f*x^11)/
11 + (b*g*x^12)/12
```

3.374 $\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=97

$$\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}(bc+af)x^7+\frac{1}{8}(bd+ag)x^8+\frac{1}{9}(be+ah)x^9+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

```
[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*(a*f+b*c)*x^7+1/8*(a*g+b*d)*x^8+1/9
*(a*h+b*e)*x^9+1/10*b*f*x^10+1/11*b*g*x^11+1/12*b*h*x^12
```

Rubi [A]
time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1834}

$$\frac{1}{7}x^7(af+bc)+\frac{1}{8}x^8(ag+bd)+\frac{1}{9}x^9(ah+be)+\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*g
)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^12)
/12
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \int \left(acx^3+adx^4+aex^5+(bc+af)x^6+(bd+ag)\right.\\ &= \frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}(bc+af)x^7+\frac{1}{8}(bd+\end{aligned}$$

Mathematica [A]
time = 0.02, size = 97, normalized size = 1.00

$$\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}(bc+af)x^7+\frac{1}{8}(bd+ag)x^8+\frac{1}{9}(be+ah)x^9+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*g
)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^12)
/12
```

Maple [A]
time = 1.98, size = 80, normalized size = 0.82

method	result
default	$\frac{ac\,x^4}{4}+\frac{ad\,x^5}{5}+\frac{ae\,x^6}{6}+\frac{(af+bc)x^7}{7}+\frac{(ag+bd)x^8}{8}+\frac{(ah+be)x^9}{9}+\frac{bf\,x^{10}}{10}+\frac{bg\,x^{11}}{11}+\frac{bh\,x^{12}}{12}$
norman	$\frac{bh\,x^{12}}{12}+\frac{bg\,x^{11}}{11}+\frac{bf\,x^{10}}{10}+\left(\frac{ah}{9}+\frac{be}{9}\right)x^9+\left(\frac{ag}{8}+\frac{bd}{8}\right)x^8+\left(\frac{af}{7}+\frac{bc}{7}\right)x^7+\frac{ae\,x^6}{6}+\frac{ad\,x^5}{5}+\frac{ac\,x^4}{4}$
gosper	$\frac{1}{12}bh\,x^{12}+\frac{1}{11}bg\,x^{11}+\frac{1}{10}bf\,x^{10}+\frac{1}{9}x^9ah+\frac{1}{9}x^9be+\frac{1}{8}x^8ag+\frac{1}{8}bd\,x^8+\frac{1}{7}af\,x^7+\frac{1}{7}x^7bc+\frac{1}{6}ae\,x^6+\frac{1}{5}$
risch	$\frac{1}{12}bh\,x^{12}+\frac{1}{11}bg\,x^{11}+\frac{1}{10}bf\,x^{10}+\frac{1}{9}x^9ah+\frac{1}{9}x^9be+\frac{1}{8}x^8ag+\frac{1}{8}bd\,x^8+\frac{1}{7}af\,x^7+\frac{1}{7}x^7bc+\frac{1}{6}ae\,x^6+\frac{1}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*(a*f+b*c)*x^7+1/8*(a*g+b*d)*x^8+1/9
*(a*h+b*e)*x^9+1/10*b*f*x^10+1/11*b*g*x^11+1/12*b*h*x^12
```

Maxima [A]
time = 0.28, size = 81, normalized size = 0.84

$$\frac{1}{12}\,bhx^{12}+\frac{1}{11}\,bgx^{11}+\frac{1}{10}\,bfx^{10}+\frac{1}{9}\,(ah+be)x^9+\frac{1}{8}\,(bd+ag)x^8+\frac{1}{7}\,(bc+af)x^7+\frac{1}{6}\,ax^6e+\frac{1}{5}\,adx^5+\frac{1}{4}\,acx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*(a*h + b*e)*x^9 + 1/8*(
b*d + a*g)*x^8 + 1/7*(b*c + a*f)*x^7 + 1/6*a*x^6*e + 1/5*a*d*x^5 + 1/4*a*c*
x^4
```

Fricas [A]
time = 0.39, size = 79, normalized size = 0.81

$$\frac{1}{12}\,bhx^{12}+\frac{1}{11}\,bgx^{11}+\frac{1}{10}\,bfx^{10}+\frac{1}{9}\,(be+ah)x^9+\frac{1}{8}\,(bd+ag)x^8+\frac{1}{6}\,aex^6+\frac{1}{7}\,(bc+af)x^7+\frac{1}{5}\,adx^5+\frac{1}{4}\,acx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*(b*e + a*h)*x^9 + 1/8*(
b*d + a*g)*x^8 + 1/6*a*e*x^6 + 1/7*(b*c + a*f)*x^7 + 1/5*a*d*x^5 + 1/4*a*c*
x^4
```

Sympy [A]
time = 0.01, size = 90, normalized size = 0.93

$$\frac{acx^4}{4} + \frac{adx^5}{5} + \frac{aex^6}{6} + \frac{bfx^{10}}{10} + \frac{bgx^{11}}{11} + \frac{bhx^{12}}{12} + x^9\left(\frac{ah}{9} + \frac{be}{9}\right) + x^8\left(\frac{ag}{8} + \frac{bd}{8}\right) + x^7\left(\frac{af}{7} + \frac{bc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + b*f*x**10/10 + b*g*x**11/11 + b*h*x*
*12/12 + x**9*(a*h/9 + b*e/9) + x**8*(a*g/8 + b*d/8) + x**7*(a*f/7 + b*c/7)
```

Giac [A]
time = 1.12, size = 87, normalized size = 0.90

$$\frac{1}{12}bhx^{12} + \frac{1}{11}bgx^{11} + \frac{1}{10}bfx^{10} + \frac{1}{9}ahx^9 + \frac{1}{9}bx^9e + \frac{1}{8}bdx^8 + \frac{1}{8}agx^8 + \frac{1}{7}bcx^7 + \frac{1}{7}afx^7 + \frac{1}{6}ax^6e + \frac{1}{5}adx^5 + \frac{1}{4}acx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*a*h*x^9 + 1/9*b*x^9*e +
 1/8*b*d*x^8 + 1/8*a*g*x^8 + 1/7*b*c*x^7 + 1/7*a*f*x^7 + 1/6*a*x^6*e + 1/5*
a*d*x^5 + 1/4*a*c*x^4
```

Mupad [B]
time = 0.04, size = 82, normalized size = 0.85

$$\frac{b\,h\,x^{12}}{12} + \frac{b\,g\,x^{11}}{11} + \frac{b\,f\,x^{10}}{10} + \left(\frac{b\,e}{9} + \frac{a\,h}{9}\right)\,x^9 + \left(\frac{b\,d}{8} + \frac{a\,g}{8}\right)\,x^8 + \left(\frac{b\,c}{7} + \frac{a\,f}{7}\right)\,x^7 + \frac{a\,e\,x^6}{6} + \frac{a\,d\,x^5}{5} + \frac{a\,c\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^7*((b*c)/7 + (a*f)/7) + x^8*((b*d)/8 + (a*g)/8) + x^9*((b*e)/9 + (a*h)/9)
 + (b*h*x^12)/12 + (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (b*f*x^10)/10
+ (b*g*x^11)/11
```

3.375 $\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=97

$$\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}(bc+af)x^6+\frac{1}{7}(bd+ag)x^7+\frac{1}{8}(be+ah)x^8+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

```
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*(a*f+b*c)*x^6+1/7*(a*g+b*d)*x^7+1/8
*(a*h+b*e)*x^8+1/9*b*f*x^9+1/10*b*g*x^10+1/11*b*h*x^11
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1834}

$$\frac{1}{6}x^6(af+bc)+\frac{1}{7}x^7(ag+bd)+\frac{1}{8}x^8(ah+be)+\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*g
)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)/1
1
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^2\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx=\int \left(acx^2+adx^3+aex^4+(bc+af)x^5+(bd+ag)x^6\right.$$
$$=\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}(bc+af)x^6+\frac{1}{7}(bd+a$$

Mathematica [A]
time = 0.02, size = 97, normalized size = 1.00

$$\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}(bc+af)x^6+\frac{1}{7}(bd+ag)x^7+\frac{1}{8}(be+ah)x^8+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*g
)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)/1
1
```

Maple [A]
time = 1.98, size = 80, normalized size = 0.82

method	result
default	$\frac{ac\,x^{3}}{3}+\frac{ad\,x^{4}}{4}+\frac{ae\,x^{5}}{5}+\frac{(af+bc)x^{6}}{6}+\frac{(ag+bd)x^{7}}{7}+\frac{(ah+be)x^{8}}{8}+\frac{bf\,x^{9}}{9}+\frac{bg\,x^{10}}{10}+\frac{bh\,x^{11}}{11}$
norman	$\frac{bh\,x^{11}}{11}+\frac{bg\,x^{10}}{10}+\frac{bf\,x^{9}}{9}+\left(\frac{ah}{8}+\frac{be}{8}\right)x^{8}+\left(\frac{ag}{7}+\frac{bd}{7}\right)x^{7}+\left(\frac{af}{6}+\frac{bc}{6}\right)x^{6}+\frac{ae\,x^{5}}{5}+\frac{ad\,x^{4}}{4}+\frac{ac\,x^{3}}{3}$
gosper	$\frac{1}{11}bh\,x^{11}+\frac{1}{10}bg\,x^{10}+\frac{1}{9}bf\,x^{9}+\frac{1}{8}x^{8}ah+\frac{1}{8}be\,x^{8}+\frac{1}{7}x^{7}ag+\frac{1}{7}bd\,x^{7}+\frac{1}{6}x^{6}af+\frac{1}{6}bc\,x^{6}+\frac{1}{5}ae\,x^{5}+$
risch	$\frac{1}{11}bh\,x^{11}+\frac{1}{10}bg\,x^{10}+\frac{1}{9}bf\,x^{9}+\frac{1}{8}x^{8}ah+\frac{1}{8}be\,x^{8}+\frac{1}{7}x^{7}ag+\frac{1}{7}bd\,x^{7}+\frac{1}{6}x^{6}af+\frac{1}{6}bc\,x^{6}+\frac{1}{5}ae\,x^{5}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*(a*f+b*c)*x^6+1/7*(a*g+b*d)*x^7+1/8
*(a*h+b*e)*x^8+1/9*b*f*x^9+1/10*b*g*x^10+1/11*b*h*x^11
```

Maxima [A]
time = 0.27, size = 81, normalized size = 0.84

$$\frac{1}{11}\,bhx^{11}+\frac{1}{10}\,bgx^{10}+\frac{1}{9}\,bfx^{9}+\frac{1}{8}\,(ah+be)x^{8}+\frac{1}{7}\,(bd+ag)x^{7}+\frac{1}{6}\,(bc+af)x^{6}+\frac{1}{5}\,ax^{5}e+\frac{1}{4}\,adx^{4}+\frac{1}{3}\,acx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*(a*h + b*e)*x^8 + 1/7*(b*
d + a*g)*x^7 + 1/6*(b*c + a*f)*x^6 + 1/5*a*x^5*e + 1/4*a*d*x^4 + 1/3*a*c*x^
3
```

Fricas [A]
time = 0.36, size = 79, normalized size = 0.81

$$\frac{1}{11}\,bhx^{11}+\frac{1}{10}\,bgx^{10}+\frac{1}{9}\,bfx^{9}+\frac{1}{8}\,(be+ah)x^{8}+\frac{1}{7}\,(bd+ag)x^{7}+\frac{1}{5}\,aex^{5}+\frac{1}{6}\,(bc+af)x^{6}+\frac{1}{4}\,adx^{4}+\frac{1}{3}\,acx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*(b*e + a*h)*x^8 + 1/7*(b*
d + a*g)*x^7 + 1/5*a*e*x^5 + 1/6*(b*c + a*f)*x^6 + 1/4*a*d*x^4 + 1/3*a*c*x^
3
```

Sympy [A]
time = 0.01, size = 90, normalized size = 0.93

$$\frac{acx^3}{3} + \frac{adx^4}{4} + \frac{aex^5}{5} + \frac{bfx^9}{9} + \frac{bgx^{10}}{10} + \frac{bhx^{11}}{11} + x^8\left(\frac{ah}{8} + \frac{be}{8}\right) + x^7\left(\frac{ag}{7} + \frac{bd}{7}\right) + x^6\left(\frac{af}{6} + \frac{bc}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*f*x**9/9 + b*g*x**10/10 + b*h*x**1
1/11 + x**8*(a*h/8 + b*e/8) + x**7*(a*g/7 + b*d/7) + x**6*(a*f/6 + b*c/6)
```

Giac [A]
time = 1.11, size = 87, normalized size = 0.90

$$\frac{1}{11}\,bhx^{11} + \frac{1}{10}\,bgx^{10} + \frac{1}{9}\,bfx^9 + \frac{1}{8}\,ahx^8 + \frac{1}{8}\,bx^8e + \frac{1}{7}\,bdx^7 + \frac{1}{7}\,agx^7 + \frac{1}{6}\,bcx^6 + \frac{1}{6}\,afx^6 + \frac{1}{5}\,ax^5e + \frac{1}{4}\,adx^4 + \frac{1}{3}\,acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*a*h*x^8 + 1/8*b*x^8*e + 1
/7*b*d*x^7 + 1/7*a*g*x^7 + 1/6*b*c*x^6 + 1/6*a*f*x^6 + 1/5*a*x^5*e + 1/4*a*
d*x^4 + 1/3*a*c*x^3
```

Mupad [B]
time = 0.04, size = 82, normalized size = 0.85

$$\frac{b\,h\,x^{11}}{11} + \frac{b\,g\,x^{10}}{10} + \frac{b\,f\,x^9}{9} + \left(\frac{b\,e}{8} + \frac{a\,h}{8}\right)\,x^8 + \left(\frac{b\,d}{7} + \frac{a\,g}{7}\right)\,x^7 + \left(\frac{b\,c}{6} + \frac{a\,f}{6}\right)\,x^6 + \frac{a\,e\,x^5}{5} + \frac{a\,d\,x^4}{4} + \frac{a\,c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^6*((b*c)/6 + (a*f)/6) + x^7*((b*d)/7 + (a*g)/7) + x^8*((b*e)/8 + (a*h)/8)
 + (b*h*x^11)/11 + (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*f*x^9)/9 +
(b*g*x^10)/10
```

3.376 $\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=97

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}(bc+af)x^5+\frac{1}{6}(bd+ag)x^6+\frac{1}{7}(be+ah)x^7+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

```
[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*(a*f+b*c)*x^5+1/6*(a*g+b*d)*x^6+1/7
*(a*h+b*e)*x^7+1/8*b*f*x^8+1/9*b*g*x^9+1/10*b*h*x^10
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1834}

$$\frac{1}{5}x^5(af+bc)+\frac{1}{6}x^6(ag+bd)+\frac{1}{7}x^7(ah+be)+\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*g
)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/10
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx=\int \left(acx+adx^2+aex^3+(bc+af)x^4+(bd+ag)x^5\right.$$
$$=\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}(bc+af)x^5+\frac{1}{6}(bd+$$

Mathematica [A]
time = 0.01, size = 97, normalized size = 1.00

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}(bc+af)x^5+\frac{1}{6}(bd+ag)x^6+\frac{1}{7}(be+ah)x^7+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*g
)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/10
```

Maple [A]
time = 2.03, size = 80, normalized size = 0.82

method	result
default	$\frac{c\,x^{2} a}{2}+\frac{a d\,x^{3}}{3}+\frac{a e\,x^{4}}{4}+\frac{\left(a f+b c\right) x^{5}}{5}+\frac{\left(a g+b d\right) x^{6}}{6}+\frac{\left(a h+b e\right) x^{7}}{7}+\frac{b f\,x^{8}}{8}+\frac{b g\,x^{9}}{9}+\frac{b h\,x^{10}}{10}$
norman	$\frac{b h\,x^{10}}{10}+\frac{b g\,x^{9}}{9}+\frac{b f\,x^{8}}{8}+\left(\frac{a h}{7}+\frac{b e}{7}\right) x^{7}+\left(\frac{a g}{6}+\frac{b d}{6}\right) x^{6}+\left(\frac{a f}{5}+\frac{b c}{5}\right) x^{5}+\frac{a e\,x^{4}}{4}+\frac{a d\,x^{3}}{3}+\frac{c\,x^{2} a}{2}$
gosper	$\frac{1}{10} b h\,x^{10}+\frac{1}{9} b g\,x^{9}+\frac{1}{8} b f\,x^{8}+\frac{1}{7} x^{7} a h+\frac{1}{7} b e\,x^{7}+\frac{1}{6} x^{6} a g+\frac{1}{6} b d\,x^{6}+\frac{1}{5} x^{5} a f+\frac{1}{5} b c\,x^{5}+\frac{1}{4} a e\,x^{4}+\frac{1}{3} a d$
risch	$\frac{1}{10} b h\,x^{10}+\frac{1}{9} b g\,x^{9}+\frac{1}{8} b f\,x^{8}+\frac{1}{7} x^{7} a h+\frac{1}{7} b e\,x^{7}+\frac{1}{6} x^{6} a g+\frac{1}{6} b d\,x^{6}+\frac{1}{5} x^{5} a f+\frac{1}{5} b c\,x^{5}+\frac{1}{4} a e\,x^{4}+\frac{1}{3} a d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c*x^2*a+1/3*a*d*x^3+1/4*a*e*x^4+1/5*(a*f+b*c)*x^5+1/6*(a*g+b*d)*x^6+1/7
*(a*h+b*e)*x^7+1/8*b*f*x^8+1/9*b*g*x^9+1/10*b*h*x^10
```

Maxima [A]
time = 0.36, size = 81, normalized size = 0.84

$$\frac{1}{10}\,bhx^{10}+\frac{1}{9}\,bgx^{9}+\frac{1}{8}\,bfx^{8}+\frac{1}{7}\,(ah+be)x^{7}+\frac{1}{6}\,(bd+ag)x^{6}+\frac{1}{5}\,(bc+af)x^{5}+\frac{1}{4}\,ax^{4}e+\frac{1}{3}\,adx^{3}+\frac{1}{2}\,acx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*(a*h + b*e)*x^7 + 1/6*(b*d
+ a*g)*x^6 + 1/5*(b*c + a*f)*x^5 + 1/4*a*x^4*e + 1/3*a*d*x^3 + 1/2*a*c*x^2
```

Fricas [A]
time = 0.35, size = 79, normalized size = 0.81

$$\frac{1}{10}\,bhx^{10}+\frac{1}{9}\,bgx^{9}+\frac{1}{8}\,bfx^{8}+\frac{1}{7}\,(be+ah)x^{7}+\frac{1}{6}\,(bd+ag)x^{6}+\frac{1}{4}\,aex^{4}+\frac{1}{5}\,(bc+af)x^{5}+\frac{1}{3}\,adx^{3}+\frac{1}{2}\,acx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*(b*e + a*h)*x^7 + 1/6*(b*d
+ a*g)*x^6 + 1/4*a*e*x^4 + 1/5*(b*c + a*f)*x^5 + 1/3*a*d*x^3 + 1/2*a*c*x^2
```

Sympy [A]
time = 0.01, size = 90, normalized size = 0.93

$$\frac{acx^2}{2}+\frac{adx^3}{3}+\frac{aex^4}{4}+\frac{bfx^8}{8}+\frac{bgx^9}{9}+\frac{bhx^{10}}{10}+x^7\left(\frac{ah}{7}+\frac{be}{7}\right)+x^6\left(\frac{ag}{6}+\frac{bd}{6}\right)+x^5\left(\frac{af}{5}+\frac{bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*f*x**8/8 + b*g*x**9/9 + b*h*x**10/
10 + x**7*(a*h/7 + b*e/7) + x**6*(a*g/6 + b*d/6) + x**5*(a*f/5 + b*c/5)
```

Giac [A]
time = 1.18, size = 87, normalized size = 0.90

$$\frac{1}{10}bhx^{10}+\frac{1}{9}bgx^9+\frac{1}{8}bfx^8+\frac{1}{7}ahx^7+\frac{1}{7}bx^7e+\frac{1}{6}bdx^6+\frac{1}{6}agx^6+\frac{1}{5}bcx^5+\frac{1}{5}afx^5+\frac{1}{4}ax^4e+\frac{1}{3}adx^3+\frac{1}{2}acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*a*h*x^7 + 1/7*b*x^7*e + 1/6
*b*d*x^6 + 1/6*a*g*x^6 + 1/5*b*c*x^5 + 1/5*a*f*x^5 + 1/4*a*x^4*e + 1/3*a*d*
x^3 + 1/2*a*c*x^2
```

Mupad [B]
time = 0.04, size = 82, normalized size = 0.85

$$\frac{b\,h\,x^{10}}{10}+\frac{b\,g\,x^9}{9}+\frac{b\,f\,x^8}{8}+\left(\frac{b\,e}{7}+\frac{a\,h}{7}\right)\,x^7+\left(\frac{b\,d}{6}+\frac{a\,g}{6}\right)\,x^6+\left(\frac{b\,c}{5}+\frac{a\,f}{5}\right)\,x^5+\frac{a\,e\,x^4}{4}+\frac{a\,d\,x^3}{3}+\frac{a\,c\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^5*((b*c)/5 + (a*f)/5) + x^6*((b*d)/6 + (a*g)/6) + x^7*((b*e)/7 + (a*h)/7)
 + (b*h*x^10)/10 + (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*f*x^8)/8 +
(b*g*x^9)/9
```

3.377 $\int (a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

Optimal. Leaf size=92

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}(bc+af)x^4+\frac{1}{5}(bd+ag)x^5+\frac{1}{6}(be+ah)x^6+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*(a*f+b*c)*x^4+1/5*(a*g+b*d)*x^5+1/6*(a*h+
b*e)*x^6+1/7*b*f*x^7+1/8*b*g*x^8+1/9*b*h*x^9
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {1864}

$$\frac{1}{4}x^4(af+bc)+\frac{1}{5}x^5(ag+bd)+\frac{1}{6}x^6(ah+be)+acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5)
/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \int (ac+adx+aex^2+(bc+af)x^3+(bd+ag)x^4+(b$$
$$= acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}(bc+af)x^4+\frac{1}{5}(bd+ag)x^5$$

Mathematica [A]
time = 0.01, size = 92, normalized size = 1.00

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}(bc+af)x^4+\frac{1}{5}(bd+ag)x^5+\frac{1}{6}(be+ah)x^6+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5)
/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9
```

Maple [A]
time = 0.12, size = 77, normalized size = 0.84

method	result
default	$acx+\frac{a d \,x^{2}}{2}+\frac{a e \,x^{3}}{3}+\frac{\left(a f+b c \right) x^{4}}{4}+\frac{\left(a g+b d \right) x^{5}}{5}+\frac{\left(a h+b e \right) x^{6}}{6}+\frac{b f \,x^{7}}{7}+\frac{b g \,x^{8}}{8}+\frac{b h \,x^{9}}{9}$
norman	$\frac{b h \,x^{9}}{9}+\frac{b g \,x^{8}}{8}+\frac{b f \,x^{7}}{7}+\left(\frac{a h}{6}+\frac{b e}{6}\right) x^{6}+\left(\frac{a g}{5}+\frac{b d}{5}\right) x^{5}+\left(\frac{a f}{4}+\frac{b c}{4}\right) x^{4}+\frac{a e \,x^{3}}{3}+\frac{a d \,x^{2}}{2}+acx$
gosper	$\frac{1}{9}b h \,x^{9}+\frac{1}{8}b g \,x^{8}+\frac{1}{7}b f \,x^{7}+\frac{1}{6}x^{6} a h+\frac{1}{6}b e \,x^{6}+\frac{1}{5}x^{5} a g+\frac{1}{5}b d \,x^{5}+\frac{1}{4}a f \,x^{4}+\frac{1}{4}b c \,x^{4}+\frac{1}{3}a e \,x^{3}+\frac{1}{2}ad$
risch	$\frac{1}{9}b h \,x^{9}+\frac{1}{8}b g \,x^{8}+\frac{1}{7}b f \,x^{7}+\frac{1}{6}x^{6} a h+\frac{1}{6}b e \,x^{6}+\frac{1}{5}x^{5} a g+\frac{1}{5}b d \,x^{5}+\frac{1}{4}a f \,x^{4}+\frac{1}{4}b c \,x^{4}+\frac{1}{3}a e \,x^{3}+\frac{1}{2}ad$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*(a*f+b*c)*x^4+1/5*(a*g+b*d)*x^5+1/6*(a*h+
b*e)*x^6+1/7*b*f*x^7+1/8*b*g*x^8+1/9*b*h*x^9
```

Maxima [A]
time = 0.33, size = 78, normalized size = 0.85

$$\frac{1}{9}\,bhx^9+\frac{1}{8}\,bgx^8+\frac{1}{7}\,bfx^7+\frac{1}{6}\,(ah+be)x^6+\frac{1}{5}\,(bd+ag)x^5+\frac{1}{4}\,(bc+af)x^4+\frac{1}{3}\,ax^3e+\frac{1}{2}\,adx^2+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] 1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*(a*h + b*e)*x^6 + 1/5*(b*d +
a*g)*x^5 + 1/4*(b*c + a*f)*x^4 + 1/3*a*x^3*e + 1/2*a*d*x^2 + a*c*x
```

Fricas [A]
time = 0.37, size = 76, normalized size = 0.83

$$\frac{1}{9}\,bhx^9+\frac{1}{8}\,bgx^8+\frac{1}{7}\,bfx^7+\frac{1}{6}\,(be+ah)x^6+\frac{1}{5}\,(bd+ag)x^5+\frac{1}{3}\,aex^3+\frac{1}{4}\,(bc+af)x^4+\frac{1}{2}\,adx^2+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] 1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*(b*e + a*h)*x^6 + 1/5*(b*d +
a*g)*x^5 + 1/3*a*e*x^3 + 1/4*(b*c + a*f)*x^4 + 1/2*a*d*x^2 + a*c*x
```

Sympy [A]
time = 0.01, size = 87, normalized size = 0.95

$$acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{bfx^7}{7}+\frac{bgx^8}{8}+\frac{bhx^9}{9}+x^6\left(\frac{ah}{6}+\frac{be}{6}\right)+x^5\left(\frac{ag}{5}+\frac{bd}{5}\right)+x^4\left(\frac{af}{4}+\frac{bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*f*x**7/7 + b*g*x**8/8 + b*h*x**9/9 + x*
*6*(a*h/6 + b*e/6) + x**5*(a*g/5 + b*d/5) + x**4*(a*f/4 + b*c/4)
```

Giac [A]
time = 1.93, size = 84, normalized size = 0.91

$$\frac{1}{9}\,bhx^9+\frac{1}{8}\,bgx^8+\frac{1}{7}\,bfx^7+\frac{1}{6}\,ahx^6+\frac{1}{6}\,bx^6e+\frac{1}{5}\,bdx^5+\frac{1}{5}\,agx^5+\frac{1}{4}\,bcx^4+\frac{1}{4}\,afx^4+\frac{1}{3}\,ax^3e+\frac{1}{2}\,adx^2+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*a*h*x^6 + 1/6*b*x^6*e + 1/5*b
*d*x^5 + 1/5*a*g*x^5 + 1/4*b*c*x^4 + 1/4*a*f*x^4 + 1/3*a*x^3*e + 1/2*a*d*x^
2 + a*c*x
```

Mupad [B]
time = 0.04, size = 79, normalized size = 0.86

$$\frac{b\,h\,x^9}{9}+\frac{b\,g\,x^8}{8}+\frac{b\,f\,x^7}{7}+\left(\frac{b\,e}{6}+\frac{a\,h}{6}\right)\,x^6+\left(\frac{b\,d}{5}+\frac{a\,g}{5}\right)\,x^5+\left(\frac{b\,c}{4}+\frac{a\,f}{4}\right)\,x^4+\frac{a\,e\,x^3}{3}+\frac{a\,d\,x^2}{2}+a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^4*((b*c)/4 + (a*f)/4) + x^5*((b*d)/5 + (a*g)/5) + x^6*((b*e)/6 + (a*h)/6)
 + (b*h*x^9)/9 + a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*f*x^7)/7 + (b*g*x^8
)/8
```

3.378 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

Optimal. Leaf size=88

$$adx+\frac{1}{2}aex^2+\frac{1}{3}(bc+af)x^3+\frac{1}{4}(bd+ag)x^4+\frac{1}{5}(be+ah)x^5+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8+ac\log(x)$$

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*c)*x^3+1/4*(a*g+b*d)*x^4+1/5*(a*h+b*e)*x^5+1/6
*b*f*x^6+1/7*b*g*x^7+1/8*b*h*x^8+a*c*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1834}

$$\frac{1}{3}x^3(af+bc)+\frac{1}{4}x^4(ag+bd)+\frac{1}{5}x^5(ah+be)+ac\log(x)+adx+\frac{1}{2}aex^2+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e + a
*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx &= \int \left(ad+\frac{ac}{x}+aex+(bc+af)x^2+(bd+ag)x^3+(be\right.\\ &= adx+\frac{1}{2}aex^2+\frac{1}{3}(bc+af)x^3+\frac{1}{4}(bd+ag)x^4+\frac{1}{5}(be\end{aligned}$$

Mathematica [A]
time = 0.02, size = 88, normalized size = 1.00

$$adx+\frac{1}{2}aex^2+\frac{1}{3}(bc+af)x^3+\frac{1}{4}(bd+ag)x^4+\frac{1}{5}(be+ah)x^5+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8+ac\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e + a
*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]
```

Maple [A]
time = 0.03, size = 81, normalized size = 0.92

method	result	size
norman	$\left(\frac{af}{3}+\frac{bc}{3}\right)x^{3}+\left(\frac{ag}{4}+\frac{bd}{4}\right)x^{4}+\left(\frac{ah}{5}+\frac{be}{5}\right)x^{5}+adx+\frac{ae\,x^{2}}{2}+\frac{bf\,x^{6}}{6}+\frac{bg\,x^{7}}{7}+\frac{bh\,x^{8}}{8}+ac\ln\left(x\right)$	78
default	$\frac{bh\,x^{8}}{8}+\frac{bg\,x^{7}}{7}+\frac{bf\,x^{6}}{6}+\frac{ah\,x^{5}}{5}+\frac{be\,x^{5}}{5}+\frac{ag\,x^{4}}{4}+\frac{bd\,x^{4}}{4}+\frac{af\,x^{3}}{3}+\frac{bc\,x^{3}}{3}+\frac{ae\,x^{2}}{2}+adx+ac\ln\left(x\right)$	81
risch	$\frac{bh\,x^{8}}{8}+\frac{bg\,x^{7}}{7}+\frac{bf\,x^{6}}{6}+\frac{ah\,x^{5}}{5}+\frac{be\,x^{5}}{5}+\frac{ag\,x^{4}}{4}+\frac{bd\,x^{4}}{4}+\frac{af\,x^{3}}{3}+\frac{bc\,x^{3}}{3}+\frac{ae\,x^{2}}{2}+adx+ac\ln\left(x\right)$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b*h*x^8+1/7*b*g*x^7+1/6*b*f*x^6+1/5*a*h*x^5+1/5*b*e*x^5+1/4*a*g*x^4+1/4
*b*d*x^4+1/3*a*f*x^3+1/3*b*c*x^3+1/2*a*e*x^2+a*d*x+a*c*ln(x)
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.86

$$\frac{1}{8}\,bhx^{8}+\frac{1}{7}\,bgx^{7}+\frac{1}{6}\,bfx^{6}+\frac{1}{5}\,(ah+be)x^{5}+\frac{1}{4}\,(bd+ag)x^{4}+\frac{1}{3}\,(bc+af)x^{3}+\frac{1}{2}\,ax^{2}e+adx+ac\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxima"
)
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*(a*h + b*e)*x^5 + 1/4*(b*d +
a*g)*x^4 + 1/3*(b*c + a*f)*x^3 + 1/2*a*x^2*e + a*d*x + a*c*log(x)
```

Fricas [A]
time = 0.37, size = 74, normalized size = 0.84

$$\frac{1}{8}\,bhx^{8}+\frac{1}{7}\,bgx^{7}+\frac{1}{6}\,bfx^{6}+\frac{1}{5}\,(be+ah)x^{5}+\frac{1}{4}\,(bd+ag)x^{4}+\frac{1}{2}\,aex^{2}+\frac{1}{3}\,(bc+af)x^{3}+adx+ac\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas"
)
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*(b*e + a*h)*x^5 + 1/4*(b*d +
a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*c + a*f)*x^3 + a*d*x + a*c*log(x)
```

Sympy [A]
time = 0.07, size = 85, normalized size = 0.97

$$ac\log(x) + adx + \frac{aex^2}{2} + \frac{bfx^6}{6} + \frac{bgx^7}{7} + \frac{bhx^8}{8} + x^5\left(\frac{ah}{5} + \frac{be}{5}\right) + x^4\left(\frac{ag}{4} + \frac{bd}{4}\right) + x^3\left(\frac{af}{3} + \frac{bc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a*c*log(x) + a*d*x + a*e*x**2/2 + b*f*x**6/6 + b*g*x**7/7 + b*h*x**8/8 + x*
*5*(a*h/5 + b*e/5) + x**4*(a*g/4 + b*d/4) + x**3*(a*f/3 + b*c/3)
```

Giac [A]
time = 1.51, size = 83, normalized size = 0.94

$$\frac{1}{8}bhx^8 + \frac{1}{7}bgx^7 + \frac{1}{6}bfx^6 + \frac{1}{5}ahx^5 + \frac{1}{5}bx^5e + \frac{1}{4}bdx^4 + \frac{1}{4}agx^4 + \frac{1}{3}bcx^3 + \frac{1}{3}afx^3 + \frac{1}{2}ax^2e + adx + ac\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*a*h*x^5 + 1/5*b*x^5*e + 1/4*b
*d*x^4 + 1/4*a*g*x^4 + 1/3*b*c*x^3 + 1/3*a*f*x^3 + 1/2*a*x^2*e + a*d*x + a*
c*log(abs(x))
```

Mupad [B]
time = 0.05, size = 77, normalized size = 0.88

$$x^3\left(\frac{b\,c}{3} + \frac{a\,f}{3}\right) + x^4\left(\frac{b\,d}{4} + \frac{a\,g}{4}\right) + x^5\left(\frac{b\,e}{5} + \frac{a\,h}{5}\right) + \frac{b\,h\,x^8}{8} + a\,c\,\ln(x) + a\,d\,x + \frac{a\,e\,x^2}{2} + \frac{b\,f\,x^6}{6} + \frac{b\,g\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x)
```

```
[Out] x^3*((b*c)/3 + (a*f)/3) + x^4*((b*d)/4 + (a*g)/4) + x^5*((b*e)/5 + (a*h)/5)
 + (b*h*x^8)/8 + a*c*log(x) + a*d*x + (a*e*x^2)/2 + (b*f*x^6)/6 + (b*g*x^7)
/7
```

3.379 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} \, dx$

Optimal. Leaf size=86

$$-\frac{ac}{x}+aex+\frac{1}{2}(bc+af)x^2+\frac{1}{3}(bd+ag)x^3+\frac{1}{4}(be+ah)x^4+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7+ad\log(x)$$

```
[Out] -a*c/x+a*e*x+1/2*(a*f+b*c)*x^2+1/3*(a*g+b*d)*x^3+1/4*(a*h+b*e)*x^4+1/5*b*f*
x^5+1/6*b*g*x^6+1/7*b*h*x^7+a*d*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1834}

$$\frac{1}{2}x^2(af+bc)+\frac{1}{3}x^3(ag+bd)+\frac{1}{4}x^4(ah+be)-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 + ((b*e + a*
h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a*d*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} \, dx = \int \left(ae+\frac{ac}{x^2}+\frac{ad}{x}+(bc+af)x+(bd+ag)x^2+(be+\right.$$

$$= -\frac{ac}{x}+aex+\frac{1}{2}(bc+af)x^2+\frac{1}{3}(bd+ag)x^3+\frac{1}{4}(be+a$$

Mathematica [A]
time = 0.03, size = 86, normalized size = 1.00

$$-\frac{ac}{x}+aex+\frac{1}{2}(bc+af)x^2+\frac{1}{3}(bd+ag)x^3+\frac{1}{4}(be+ah)x^4+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7+ad\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 + ((b*e + a*
h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a*d*Log[x]
```

Maple [A]
time = 0.03, size = 81, normalized size = 0.94

method	result	size
default	$\frac{bh\,x^{7}}{7}+\frac{bg\,x^{6}}{6}+\frac{bf\,x^{5}}{5}+\frac{ah\,x^{4}}{4}+\frac{be\,x^{4}}{4}+\frac{ag\,x^{3}}{3}+\frac{bd\,x^{3}}{3}+\frac{x^{2}af}{2}+\frac{c\,x^{2}b}{2}+aex+ad\ln\left(x\right)-\frac{ac}{x}$	81
risch	$\frac{bh\,x^{7}}{7}+\frac{bg\,x^{6}}{6}+\frac{bf\,x^{5}}{5}+\frac{ah\,x^{4}}{4}+\frac{be\,x^{4}}{4}+\frac{ag\,x^{3}}{3}+\frac{bd\,x^{3}}{3}+\frac{x^{2}af}{2}+\frac{c\,x^{2}b}{2}+aex+ad\ln\left(x\right)-\frac{ac}{x}$	81
norman	$\frac{\left(\frac{af}{2}+\frac{bc}{2}\right)x^{3}+\left(\frac{ag}{3}+\frac{bd}{3}\right)x^{4}+\left(\frac{ah}{4}+\frac{be}{4}\right)x^{5}+ae\,x^{2}-ac+\frac{bf\,x^{6}}{5}+\frac{bg\,x^{7}}{6}+\frac{bh\,x^{8}}{7}}{x}+ad\ln\left(x\right)$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b*h*x^7+1/6*b*g*x^6+1/5*b*f*x^5+1/4*a*h*x^4+1/4*b*e*x^4+1/3*a*g*x^3+1/3
*b*d*x^3+1/2*x^2*a*f+1/2*c*x^2*b+a*e*x+a*d*ln(x)-a*c/x
```

Maxima [A]
time = 0.32, size = 76, normalized size = 0.88

$$\frac{1}{7}\,bhx^{7}+\frac{1}{6}\,bgx^{6}+\frac{1}{5}\,bfx^{5}+\frac{1}{4}\,(ah+be)x^{4}+\frac{1}{3}\,(bd+ag)x^{3}+\frac{1}{2}\,(bc+af)x^{2}+axe+ad\log\left(x\right)-\frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="maxim
a")
```

```
[Out] 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*(a*h + b*e)*x^4 + 1/3*(b*d +
a*g)*x^3 + 1/2*(b*c + a*f)*x^2 + a*x*e + a*d*log(x) - a*c/x
```

Fricas [A]
time = 0.39, size = 81, normalized size = 0.94

$$\frac{60\,bhx^{8}+70\,bgx^{7}+84\,bfx^{6}+105\,(be+ah)x^{5}+140\,(bd+ag)x^{4}+420\,aex^{2}+210\,(bc+af)x^{3}+420\,adx\log\left(x\right)-420\,ac}{420\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="frica
s")
```

```
[Out] 1/420*(60*b*h*x^8 + 70*b*g*x^7 + 84*b*f*x^6 + 105*(b*e + a*h)*x^5 + 140*(b*
d + a*g)*x^4 + 420*a*e*x^2 + 210*(b*c + a*f)*x^3 + 420*a*d*x*log(x) - 420*a
*c)/x
```

Sympy [A]
time = 0.08, size = 82, normalized size = 0.95

$$-\frac{ac}{x}+ad\log{(x)}+aex+\frac{bfx^5}{5}+\frac{bgx^6}{6}+\frac{bhx^7}{7}+x^4\left(\frac{ah}{4}+\frac{be}{4}\right)+x^3\left(\frac{ag}{3}+\frac{bd}{3}\right)+x^2\left(\frac{af}{2}+\frac{bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a*c/x + a*d*log(x) + a*e*x + b*f*x**5/5 + b*g*x**6/6 + b*h*x**7/7 + x**4*(
a*h/4 + b*e/4) + x**3*(a*g/3 + b*d/3) + x**2*(a*f/2 + b*c/2)
```

Giac [A]
time = 0.78, size = 83, normalized size = 0.97

$$\frac{1}{7}bhx^7+\frac{1}{6}bgx^6+\frac{1}{5}bfx^5+\frac{1}{4}ahx^4+\frac{1}{4}bx^4e+\frac{1}{3}bdx^3+\frac{1}{3}agx^3+\frac{1}{2}bcx^2+\frac{1}{2}afx^2+axe+ad\log{(|x|)}-\frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="giac"
)
```

```
[Out] 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*a*h*x^4 + 1/4*b*x^4*e + 1/3*b
*d*x^3 + 1/3*a*g*x^3 + 1/2*b*c*x^2 + 1/2*a*f*x^2 + a*x*e + a*d*log(abs(x))
- a*c/x
```

Mupad [B]
time = 0.05, size = 77, normalized size = 0.90

$$x^2\left(\frac{b\,c}{2}+\frac{a\,f}{2}\right)+x^3\left(\frac{b\,d}{3}+\frac{a\,g}{3}\right)+x^4\left(\frac{b\,e}{4}+\frac{a\,h}{4}\right)+\frac{b\,h\,x^7}{7}+a\,d\,\ln{(x)}+a\,e\,x-\frac{a\,c}{x}+\frac{b\,f\,x^5}{5}+\frac{b\,g\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x)
```

```
[Out] x^2*((b*c)/2 + (a*f)/2) + x^3*((b*d)/3 + (a*g)/3) + x^4*((b*e)/4 + (a*h)/4)
 + (b*h*x^7)/7 + a*d*log(x) + a*e*x - (a*c)/x + (b*f*x^5)/5 + (b*g*x^6)/6
```

3.380 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

Optimal. Leaf size=86

$$-\frac{ac}{2x^2}-\frac{ad}{x}+(bc+af)x+\frac{1}{2}(bd+ag)x^2+\frac{1}{3}(be+ah)x^3+\frac{1}{4}bfx^4+\frac{1}{5}bgx^5+\frac{1}{6}bhx^6+ae\log(x)$$

```
[Out] -1/2*a*c/x^2-a*d/x+(a*f+b*c)*x+1/2*(a*g+b*d)*x^2+1/3*(a*h+b*e)*x^3+1/4*b*f*
x^4+1/5*b*g*x^5+1/6*b*h*x^6+a*e*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1834}

$$x(af+bc)+\frac{1}{2}x^2(ag+bd)+\frac{1}{3}x^3(ah+be)-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+\frac{1}{4}bfx^4+\frac{1}{5}bgx^5+\frac{1}{6}bhx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -1/2*(a*c)/x^2 - (a*d)/x + (b*c + a*f)*x + ((b*d + a*g)*x^2)/2 + ((b*e + a*
h)*x^3)/3 + (b*f*x^4)/4 + (b*g*x^5)/5 + (b*h*x^6)/6 + a*e*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx=\int \left(bc\left(1+\frac{af}{bc}\right)+\frac{ac}{x^3}+\frac{ad}{x^2}+\frac{ae}{x}+(bd+ag)x+(be$$

$$=-\frac{ac}{2x^2}-\frac{ad}{x}+(bc+af)x+\frac{1}{2}(bd+ag)x^2+\frac{1}{3}(be+ah$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 0.91

$$bcx+\frac{a(-3c-6dx+6fx^3+3gx^4+2hx^5)}{6x^2}+\frac{1}{60}bx^2(30d+x(20e+15fx+12gx^2+10hx^3))+ae\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] b*c*x + (a*(-3*c - 6*d*x + 6*f*x^3 + 3*g*x^4 + 2*h*x^5))/(6*x^2) + (b*x^2*(
30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3)))/60 + a*e*Log[x]
```

Maple [A]
time = 0.03, size = 78, normalized size = 0.91

method	result	size
default	$\frac{bh\,x^6}{6}+\frac{bg\,x^5}{5}+\frac{bf\,x^4}{4}+\frac{ah\,x^3}{3}+\frac{be\,x^3}{3}+\frac{ag\,x^2}{2}+\frac{bd\,x^2}{2}+afx+bcx-\frac{ac}{2x^2}+ae\ln\left(x\right)-\frac{ad}{x}$	78
risch	$\frac{bh\,x^6}{6}+\frac{bg\,x^5}{5}+\frac{bf\,x^4}{4}+\frac{ah\,x^3}{3}+\frac{be\,x^3}{3}+\frac{ag\,x^2}{2}+\frac{bd\,x^2}{2}+afx+bcx+\frac{-adx-\frac{1}{2}ac}{x^2}+ae\ln\left(x\right)$	78
norman	$\frac{\left(\frac{ag}{2}+\frac{bd}{2}\right)x^4+\left(\frac{ah}{3}+\frac{be}{3}\right)x^5+(af+bc)x^3-\frac{ac}{2}-adx+\frac{bf\,x^6}{4}+\frac{bg\,x^7}{5}+\frac{bh\,x^8}{6}}{x^2}+ae\ln\left(x\right)$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b*h*x^6+1/5*b*g*x^5+1/4*b*f*x^4+1/3*a*h*x^3+1/3*b*e*x^3+1/2*a*g*x^2+1/2
*b*d*x^2+a*f*x+b*c*x-1/2*a*c/x^2+a*e*ln(x)-a*d/x
```

Maxima [A]
time = 0.27, size = 76, normalized size = 0.88

$$\frac{1}{6}\,bhx^6+\frac{1}{5}\,bgx^5+\frac{1}{4}\,bfx^4+\frac{1}{3}\,(ah+be)x^3+\frac{1}{2}\,(bd+ag)x^2+ae\log\left(x\right)+(bc+af)x-\frac{2\,adx+ac}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="maxim
a")
```

```
[Out] 1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*(a*h + b*e)*x^3 + 1/2*(b*d +
a*g)*x^2 + a*e*log(x) + (b*c + a*f)*x - 1/2*(2*a*d*x + a*c)/x^2
```

Fricas [A]
time = 0.37, size = 81, normalized size = 0.94

$$\frac{10\,bhx^8+12\,bgx^7+15\,bfx^6+20\,(be+ah)x^5+30\,(bd+ag)x^4+60\,aex^2\log\left(x\right)+60\,(bc+af)x^3-60\,adx-30\,ac}{60\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="frica
s")
```

```
[Out] 1/60*(10*b*h*x^8 + 12*b*g*x^7 + 15*b*f*x^6 + 20*(b*e + a*h)*x^5 + 30*(b*d +
 a*g)*x^4 + 60*a*e*x^2*log(x) + 60*(b*c + a*f)*x^3 - 60*a*d*x - 30*a*c)/x^2
```

Sympy [A]
time = 0.13, size = 83, normalized size = 0.97

$$ae\log(x) + \frac{bfx^4}{4} + \frac{bgx^5}{5} + \frac{bhx^6}{6} + x^3\left(\frac{ah}{3} + \frac{be}{3}\right) + x^2\left(\frac{ag}{2} + \frac{bd}{2}\right) + x(af + bc) + \frac{-ac - 2adx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a*e*log(x) + b*f*x**4/4 + b*g*x**5/5 + b*h*x**6/6 + x**3*(a*h/3 + b*e/3) +
x**2*(a*g/2 + b*d/2) + x*(a*f + b*c) + (-a*c - 2*a*d*x)/(2*x**2)
```

Giac [A]
time = 0.69, size = 80, normalized size = 0.93

$$\frac{1}{6}bhx^6 + \frac{1}{5}bgx^5 + \frac{1}{4}bfx^4 + \frac{1}{3}ahx^3 + \frac{1}{3}bx^3e + \frac{1}{2}bdx^2 + \frac{1}{2}agx^2 + bcx + afx + ae\log(|x|) - \frac{2\,adx + ac}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="giac"
)
```

```
[Out] 1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*a*h*x^3 + 1/3*b*x^3*e + 1/2*b
*d*x^2 + 1/2*a*g*x^2 + b*c*x + a*f*x + a*e*log(abs(x)) - 1/2*(2*a*d*x + a*c
)/x^2
```

Mupad [B]
time = 0.04, size = 76, normalized size = 0.88

$$x\,(b\,c + a\,f) - \frac{\frac{a\,c}{2} + a\,d\,x}{x^2} + x^2\left(\frac{b\,d}{2} + \frac{a\,g}{2}\right) + x^3\left(\frac{b\,e}{3} + \frac{a\,h}{3}\right) + \frac{b\,h\,x^6}{6} + a\,e\,\ln(x) + \frac{b\,f\,x^4}{4} + \frac{b\,g\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x)
```

```
[Out] x*(b*c + a*f) - ((a*c)/2 + a*d*x)/x^2 + x^2*((b*d)/2 + (a*g)/2) + x^3*((b*e
)/3 + (a*h)/3) + (b*h*x^6)/6 + a*e*log(x) + (b*f*x^4)/4 + (b*g*x^5)/5
```

3.381 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx$

Optimal. Leaf size=86

$$-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+(bd+ag)x+\frac{1}{2}(be+ah)x^2+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4+\frac{1}{5}bhx^5+(bc+af)\log(x)$$

```
[Out] -1/3*a*c/x^3-1/2*a*d/x^2-a*e/x+(a*g+b*d)*x+1/2*(a*h+b*e)*x^2+1/3*b*f*x^3+1/4*b*g*x^4+1/5*b*h*x^5+(a*f+b*c)*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = $\{1834\}$

$$\log(x)(af+bc)+x(ag+bd)+\frac{1}{2}x^2(ah+be)-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4+\frac{1}{5}bhx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/3*(a*c)/x^3 - (a*d)/(2*x^2) - (a*e)/x + (b*d + a*g)*x + ((b*e + a*h)*x^2)/2 + (b*f*x^3)/3 + (b*g*x^4)/4 + (b*h*x^5)/5 + (b*c + a*f)*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx &= \int \left(bd\left(1+\frac{ag}{bd}\right)+\frac{ac}{x^4}+\frac{ad}{x^3}+\frac{ae}{x^2}+\frac{bc+af}{x}+(be+ah\\ &= -\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+(bd+ag)x+\frac{1}{2}(be+ah)x^2+\frac{1}{3}bf\end{aligned}$$

Mathematica [A]
time = 0.04, size = 76, normalized size = 0.88

$$-\frac{a(2c+3x(d+2ex-x^3(2g+hx)))}{6x^3}+\frac{1}{60}bx(60d+x(30e+x(20f+15gx+12hx^2)))+(bc+af)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/6*(a*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/x^3 + (b*x*(60*d + x*(30
*e + x*(20*f + 15*g*x + 12*h*x^2))))/60 + (b*c + a*f)*Log[x]
```

Maple [A]
time = 0.04, size = 76, normalized size = 0.88

method	result	size
default	$\frac{bh\,x^{5}}{5}+\frac{bg\,x^{4}}{4}+\frac{f\,x^{3}b}{3}+\frac{ah\,x^{2}}{2}+\frac{be\,x^{2}}{2}+agx+xbd-\frac{ad}{2x^{2}}-\frac{ac}{3x^{3}}+\left(af+bc\right)\ln\left(x\right)-\frac{ae}{x}$	76
risch	$\frac{bh\,x^{5}}{5}+\frac{bg\,x^{4}}{4}+\frac{f\,x^{3}b}{3}+\frac{ah\,x^{2}}{2}+\frac{be\,x^{2}}{2}+agx+xbd+\frac{-ae\,x^{2}-\frac{1}{2}adx-\frac{1}{3}ac}{x^{3}}+\ln\left(x\right)af+\ln\left(x\right)bc$	76
norman	$\frac{\left(\frac{ah}{2}+\frac{be}{2}\right)x^{5}+\left(ag+bd\right)x^{4}-\frac{ac}{3}-\frac{adx}{2}-ae\,x^{2}+\frac{bf\,x^{6}}{3}+\frac{bg\,x^{7}}{4}+\frac{bh\,x^{8}}{5}}{x^{3}}+\left(af+bc\right)\ln\left(x\right)$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b*h*x^5+1/4*b*g*x^4+1/3*f*x^3*b+1/2*a*h*x^2+1/2*b*e*x^2+a*g*x+x*b*d-1/2
*a*d/x^2-1/3*a*c/x^3+(a*f+b*c)*ln(x)-a*e/x
```

Maxima [A]
time = 0.29, size = 77, normalized size = 0.90

$$\frac{1}{5}\,bhx^{5}+\frac{1}{4}\,bgx^{4}+\frac{1}{3}\,bfx^{3}+\frac{1}{2}\,(ah+be)x^{2}+(bd+ag)x+(bc+af)\log\left(x\right)-\frac{6\,ax^{2}e+3\,adx+2\,ac}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="maxim
a")
```

```
[Out] 1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*(a*h + b*e)*x^2 + (b*d + a*g)
*x + (b*c + a*f)*log(x) - 1/6*(6*a*x^2*e + 3*a*d*x + 2*a*c)/x^3
```

Fricas [A]
time = 0.37, size = 81, normalized size = 0.94

$$\frac{12\,bhx^{8}+15\,bgx^{7}+20\,bfx^{6}+30\,(be+ah)x^{5}+60\,(bd+ag)x^{4}+60\,(bc+af)x^{3}\log\left(x\right)-60\,aex^{2}-30\,adx-20\,ac}{60\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="frica
s")
```

```
[Out] 1/60*(12*b*h*x^8 + 15*b*g*x^7 + 20*b*f*x^6 + 30*(b*e + a*h)*x^5 + 60*(b*d +
 a*g)*x^4 + 60*(b*c + a*f)*x^3*log(x) - 60*a*e*x^2 - 30*a*d*x - 20*a*c)/x^3
```

Sympy [A]
time = 0.33, size = 83, normalized size = 0.97

$$\frac{bfx^3}{3} + \frac{bgx^4}{4} + \frac{bhx^5}{5} + x^2\left(\frac{ah}{2} + \frac{be}{2}\right) + x(ag + bd) + (af + bc)\log\left(x\right) + \frac{-2ac - 3adx - 6aex^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] b*f*x**3/3 + b*g*x**4/4 + b*h*x**5/5 + x**2*(a*h/2 + b*e/2) + x*(a*g + b*d)
 + (a*f + b*c)*log(x) + (-2*a*c - 3*a*d*x - 6*a*e*x**2)/(6*x**3)
```

Giac [A]
time = 0.64, size = 79, normalized size = 0.92

$$\frac{1}{5}\,bhx^5 + \frac{1}{4}\,bgx^4 + \frac{1}{3}\,bfx^3 + \frac{1}{2}\,ahx^2 + \frac{1}{2}\,bx^2e + bdx + agx + (bc + af)\log\left(|x|\right) - \frac{6\,ax^2e + 3\,adx + 2\,ac}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="giac"
)
```

```
[Out] 1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*a*h*x^2 + 1/2*b*x^2*e + b*d*x
 + a*g*x + (b*c + a*f)*log(abs(x)) - 1/6*(6*a*x^2*e + 3*a*d*x + 2*a*c)/x^3
```

Mupad [B]
time = 0.04, size = 75, normalized size = 0.87

$$x\,\left(b\,d+a\,g\right)-\frac{a\,e\,x^2+\frac{a\,d\,x}{2}+\frac{a\,c}{3}}{x^3}+x^2\,\left(\frac{b\,e}{2}+\frac{a\,h}{2}\right)+\ln\left(x\right)\,\left(b\,c+a\,f\right)+\frac{b\,h\,x^5}{5}+\frac{b\,f\,x^3}{3}+\frac{b\,g\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x)
```

```
[Out] x*(b*d + a*g) - ((a*c)/3 + (a*d*x)/2 + a*e*x^2)/x^3 + x^2*((b*e)/2 + (a*h)/
2) + log(x)*(b*c + a*f) + (b*h*x^5)/5 + (b*f*x^3)/3 + (b*g*x^4)/4
```

3.382 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx$

Optimal. Leaf size=86

$$-\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}-\frac{bc+af}{x}+(be+ah)x+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bhx^4+(bd+ag)\log(x)$$

```
[Out] -1/4*a*c/x^4-1/3*a*d/x^3-1/2*a*e/x^2+(-a*f-b*c)/x+(a*h+b*e)*x+1/2*b*f*x^2+1
/3*b*g*x^3+1/4*b*h*x^4+(a*g+b*d)*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1834}

$$-\frac{af+bc}{x}+\log(x)(ag+bd)+x(ah+be)-\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bhx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -1/4*(a*c)/x^4 - (a*d)/(3*x^3) - (a*e)/(2*x^2) - (b*c + a*f)/x + (b*e + a*h
)*x + (b*f*x^2)/2 + (b*g*x^3)/3 + (b*h*x^4)/4 + (b*d + a*g)*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx=\int \left(be\left(1+\frac{ah}{be}\right)+\frac{ac}{x^5}+\frac{ad}{x^4}+\frac{ae}{x^3}+\frac{bc+af}{x^2}+\frac{bd+\right.$$
$$=-\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}-\frac{bc+af}{x}+(be+ah)x+\frac{1}{2}bfx^2$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 0.90

$$b\left(-\frac{c}{x}+ex+\frac{1}{12}x^2(6f+4gx+3hx^2)\right)-\frac{a(3c+4dx+6x^2(e+2fx-2hx^3))}{12x^4}+(bd+ag)\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]

[Out] b*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) - (a*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)))/(12*x^4) + (b*d + a*g)*Log[x]

Maple [A]
time = 0.04, size = 74, normalized size = 0.86

method	result	size
default	$\frac{bh\,x^{4}}{4}+\frac{bg\,x^{3}}{3}+\frac{bf\,x^{2}}{2}+ahx+bex-\frac{ac}{4x^{4}}-\frac{ae}{2x^{2}}-\frac{ad}{3x^{3}}+\left(ag+bd\right)\ln\left(x\right)-\frac{af+bc}{x}$	74
risch	$\frac{bh\,x^{4}}{4}+\frac{bg\,x^{3}}{3}+\frac{bf\,x^{2}}{2}+ahx+bex+\frac{\left(-af-bc\right)x^{3}-\frac{ae\,x^{2}}{2}-\frac{adx}{3}-\frac{ac}{4}}{x^{4}}+\ln\left(x\right)ag+\ln\left(x\right)bd$	75
norman	$\frac{\left(-af-bc\right)x^{3}+\left(ah+be\right)x^{5}-\frac{ac}{4}-\frac{adx}{3}-\frac{ae\,x^{2}}{2}+\frac{bf\,x^{6}}{2}+\frac{bg\,x^{7}}{3}+\frac{bh\,x^{8}}{4}}{x^{4}}+\left(ag+bd\right)\ln\left(x\right)$	78

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x,method=_RETURNVERBOSE)

[Out] 1/4*b*h*x^4+1/3*b*g*x^3+1/2*b*f*x^2+a*h*x+b*e*x-1/4*a*c/x^4-1/2*a*e/x^2-1/3*a*d/x^3+(a*g+b*d)*ln(x)-(a*f+b*c)/x

Maxima [A]
time = 0.28, size = 77, normalized size = 0.90

$$\frac{1}{4}\,bhx^{4}+\frac{1}{3}\,bgx^{3}+\frac{1}{2}\,bfx^{2}+(ah+be)x+(bd+ag)\log\left(x\right)-\frac{12\,(bc+af)x^{3}+6\,ax^{2}e+4\,adx+3\,ac}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="maxima")

[Out] 1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + (a*h + b*e)*x + (b*d + a*g)*log(x) - 1/12*(12*(b*c + a*f)*x^3 + 6*a*x^2*e + 4*a*d*x + 3*a*c)/x^4

Fricas [A]
time = 0.38, size = 81, normalized size = 0.94

$$\frac{3\,bhx^{8}+4\,bgx^{7}+6\,bfx^{6}+12\,(be+ah)x^{5}+12\,(bd+ag)x^{4}\log\left(x\right)-6\,aex^{2}-12\,(bc+af)x^{3}-4\,adx-3\,ac}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fricas")

[Out] 1/12*(3*b*h*x^8 + 4*b*g*x^7 + 6*b*f*x^6 + 12*(b*e + a*h)*x^5 + 12*(b*d + a*g)*x^4*log(x) - 6*a*e*x^2 - 12*(b*c + a*f)*x^3 - 4*a*d*x - 3*a*c)/x^4

Sympy [A]
time = 1.34, size = 83, normalized size = 0.97

$$\frac{bfx^2}{2} + \frac{bgx^3}{3} + \frac{bhx^4}{4} + x(ah + be) + (ag + bd)\log(x) + \frac{-3ac - 4adx - 6aex^2 + x^3(-12af - 12bc)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] b*f*x**2/2 + b*g*x**3/3 + b*h*x**4/4 + x*(a*h + b*e) + (a*g + b*d)*log(x) +
 (-3*a*c - 4*a*d*x - 6*a*e*x**2 + x**3*(-12*a*f - 12*b*c))/(12*x**4)
```

Giac [A]
time = 0.66, size = 77, normalized size = 0.90

$$\frac{1}{4}bhx^4 + \frac{1}{3}bgx^3 + \frac{1}{2}bfx^2 + ahx + bxe + (bd + ag)\log(|x|) - \frac{12(bc + af)x^3 + 6ax^2e + 4adx + 3ac}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="giac"
)
```

```
[Out] 1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + a*h*x + b*x*e + (b*d + a*g)*log(a
bs(x)) - 1/12*(12*(b*c + a*f)*x^3 + 6*a*x^2*e + 4*a*d*x + 3*a*c)/x^4
```

Mupad [B]
time = 4.98, size = 74, normalized size = 0.86

$$x(be + ah) - \frac{(bc + af)x^3 + \frac{aex^2}{2} + \frac{adx}{3} + \frac{ac}{4}}{x^4} + \ln(x)(bd + ag) + \frac{bhx^4}{4} + \frac{bfx^2}{2} + \frac{bgx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x)
```

```
[Out] x*(b*e + a*h) - ((a*c)/4 + x^3*(b*c + a*f) + (a*d*x)/3 + (a*e*x^2)/2)/x^4 +
 log(x)*(b*d + a*g) + (b*h*x^4)/4 + (b*f*x^2)/2 + (b*g*x^3)/3
```

3.383 $\int x^4(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx$

Optimal. Leaf size=163

$$\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{8}a(2bc+af)x^8+\frac{1}{9}a(2bd+ag)x^9+\frac{1}{10}a(2be+ah)x^{10}+\frac{1}{11}b(bc+2af)x^{11}+\frac{1}{12}b(bd+2a$$

```
[Out] 1/5*a^2*c*x^5+1/6*a^2*d*x^6+1/7*a^2*e*x^7+1/8*a*(a*f+2*b*c)*x^8+1/9*a*(a*g+
2*b*d)*x^9+1/10*a*(a*h+2*b*e)*x^10+1/11*b*(2*a*f+b*c)*x^11+1/12*b*(2*a*g+b*
d)*x^12+1/13*b*(2*a*h+b*e)*x^13+1/14*b^2*f*x^14+1/15*b^2*g*x^15+1/16*b^2*h*
x^16
```

Rubi [A]
time = 0.14, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}=0.026$, Rules used = {1834}

$$\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{11}bx^{11}(2af+bc)+\frac{1}{8}ax^8(af+2bc)+\frac{1}{12}bx^{12}(2ag+bd)+\frac{1}{9}ax^9(ag+2bd)+\frac{1}{13}bx^{13}(2ah+be)+\frac{1}{10}ax^{10}(ah+2be)+\frac{1}{14}b^2fx^{14}+\frac{1}{15}b^2gx^{15}+\frac{1}{16}b^2hx^{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 + (
a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^11)
/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^14)/
14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^4\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx=\int\left(a^2cx^4+a^2dx^5+a^2ex^6+a(2bc+af)x^7+a(2ba\right.$$
$$=\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{8}a(2bc+af)x^8+\frac{1}{9}a$$

Mathematica [A]
time = 0.02, size = 163, normalized size = 1.00

$$\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{8}a(2bc+af)x^8+\frac{1}{9}a(2bd+ag)x^9+\frac{1}{10}a(2be+ah)x^{10}+\frac{1}{11}b(bc+2af)x^{11}+\frac{1}{12}b(bd+2ag)x^{12}+\frac{1}{13}b(be+2ah)x^{13}+\frac{1}{14}b^2fx^{14}+\frac{1}{15}b^2gx^{15}+\frac{1}{16}b^2hx^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 + (
a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^11)
/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^14)/
14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

Maple [A]
time = 2.03, size = 152, normalized size = 0.93

method	result
default	$\frac{b^{2} h \,x^{16}}{16}+\frac{b^{2} g \,x^{15}}{15}+\frac{b^{2} f \,x^{14}}{14}+\frac{\left(2 a b h +b^{2} e \right) x^{13}}{13}+\frac{\left(2 a b g +b^{2} d \right) x^{12}}{12}+\frac{\left(2 a b f +b^{2} c \right) x^{11}}{11}+\frac{\left(a^{2} h +2 a b e \right) x^{10}}{10}+\frac{\left(a^{2} g +2 a b d \right) x^{9}}{9}$
norman	$\frac{a^{2} c \,x^{5}}{5}+\frac{a^{2} d \,x^{6}}{6}+\frac{a^{2} e \,x^{7}}{7}+\left(\frac{1}{8} a^{2} f +\frac{1}{4} a b c \right) x^{8}+\left(\frac{1}{9} a^{2} g +\frac{2}{9} a b d \right) x^{9}+\left(\frac{1}{10} a^{2} h +\frac{1}{5} a b e \right) x^{10}+\left(\frac{2}{11} a b f +\right.$
gosper	$\frac{1}{5} a^{2} c \,x^{5}+\frac{1}{6} a^{2} d \,x^{6}+\frac{1}{7} a^{2} e \,x^{7}+\frac{1}{8} x^{8} a^{2} f +\frac{1}{4} x^{8} a b c +\frac{1}{9} x^{9} a^{2} g +\frac{2}{9} x^{9} a b d +\frac{1}{10} x^{10} a^{2} h +\frac{1}{5} a b e \,x^{10}+\frac{2}{11}$
risch	$\frac{1}{5} a^{2} c \,x^{5}+\frac{1}{6} a^{2} d \,x^{6}+\frac{1}{7} a^{2} e \,x^{7}+\frac{1}{8} x^{8} a^{2} f +\frac{1}{4} x^{8} a b c +\frac{1}{9} x^{9} a^{2} g +\frac{2}{9} x^{9} a b d +\frac{1}{10} x^{10} a^{2} h +\frac{1}{5} a b e \,x^{10}+\frac{2}{11}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/16*b^2*h*x^16+1/15*b^2*g*x^15+1/14*b^2*f*x^14+1/13*(2*a*b*h+b^2*e)*x^13+1
/12*(2*a*b*g+b^2*d)*x^12+1/11*(2*a*b*f+b^2*c)*x^11+1/10*(a^2*h+2*a*b*e)*x^1
0+1/9*(a^2*g+2*a*b*d)*x^9+1/8*(a^2*f+2*a*b*c)*x^8+1/7*a^2*e*x^7+1/6*a^2*d*x
^6+1/5*a^2*c*x^5
```

Maxima [A]
time = 0.28, size = 154, normalized size = 0.94

$$\frac{1}{16} \, b^{2} h x^{16} + \frac{1}{15} \, b^{2} g x^{15} + \frac{1}{14} \, b^{2} f x^{14} + \frac{1}{13} \, {\left(2 \, a b h + b^{2} e\right)} x^{13} + \frac{1}{12} \, {\left(b^{2} d + 2 \, a b g\right)} x^{12} + \frac{1}{11} \, {\left(b^{2} c + 2 \, a b f\right)} x^{11} + \frac{1}{10} \, {\left(a^{2} h + 2 \, a b e\right)} x^{10} + \frac{1}{9} \, {\left(2 \, a b d + a^{2} g\right)} x^{9} + \frac{1}{7} \, a^{2} x^{7} e + \frac{1}{6} \, a^{2} d x^{6} + \frac{1}{8} \, {\left(2 \, a b c + a^{2} f\right)} x^{8} + \frac{1}{5} \, a^{2} c x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 1/13*(2*a*b*h + b^2*e
)*x^13 + 1/12*(b^2*d + 2*a*b*g)*x^12 + 1/11*(b^2*c + 2*a*b*f)*x^11 + 1/10*(
a^2*h + 2*a*b*e)*x^10 + 1/9*(2*a*b*d + a^2*g)*x^9 + 1/7*a^2*x^7*e + 1/6*a^2
*d*x^6 + 1/8*(2*a*b*c + a^2*f)*x^8 + 1/5*a^2*c*x^5
```

Fricas [A]
time = 0.36, size = 151, normalized size = 0.93

$$\frac{1}{16} \, b^{2} h x^{16} + \frac{1}{15} \, b^{2} g x^{15} + \frac{1}{14} \, b^{2} f x^{14} + \frac{1}{13} \, {\left(b^{2} e + 2 \, a b h\right)} x^{13} + \frac{1}{12} \, {\left(b^{2} d + 2 \, a b g\right)} x^{12} + \frac{1}{11} \, {\left(b^{2} c + 2 \, a b f\right)} x^{11} + \frac{1}{10} \, {\left(2 \, a b e + a^{2} h\right)} x^{10} + \frac{1}{7} \, a^{2} e x^{7} + \frac{1}{9} \, {\left(2 \, a b d + a^{2} g\right)} x^{9} + \frac{1}{6} \, a^{2} d x^{6} + \frac{1}{8} \, {\left(2 \, a b c + a^{2} f\right)} x^{8} + \frac{1}{5} \, a^{2} c x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 1/13*(b^2*e + 2*a*b*h
)*x^13 + 1/12*(b^2*d + 2*a*b*g)*x^12 + 1/11*(b^2*c + 2*a*b*f)*x^11 + 1/10*(
2*a*b*e + a^2*h)*x^10 + 1/7*a^2*e*x^7 + 1/9*(2*a*b*d + a^2*g)*x^9 + 1/6*a^2
*d*x^6 + 1/8*(2*a*b*c + a^2*f)*x^8 + 1/5*a^2*c*x^5
```

Sympy [A]
time = 0.02, size = 167, normalized size = 1.02

$$\frac{a^2cx^5}{5} + \frac{a^2dx^6}{6} + \frac{a^2ex^7}{7} + \frac{b^2fx^{14}}{14} + \frac{b^2gx^{15}}{15} + \frac{b^2hx^{16}}{16} + x^{13}\cdot\left(\frac{2abh}{13} + \frac{b^2e}{13}\right) + x^{12}\left(\frac{abg}{6} + \frac{b^2d}{12}\right) + x^{11}\cdot\left(\frac{2abf}{11} + \frac{b^2c}{11}\right) + x^{10}\left(\frac{a^2h}{10} + \frac{abe}{5}\right) + x^9\left(\frac{a^2g}{9} + \frac{2abd}{9}\right) + x^8\left(\frac{a^2f}{8} + \frac{abc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**5/5 + a**2*d*x**6/6 + a**2*e*x**7/7 + b**2*f*x**14/14 + b**2*g*x*
*15/15 + b**2*h*x**16/16 + x**13*(2*a*b*h/13 + b**2*e/13) + x**12*(a*b*g/6
+ b**2*d/12) + x**11*(2*a*b*f/11 + b**2*c/11) + x**10*(a**2*h/10 + a*b*e/5)
 + x**9*(a**2*g/9 + 2*a*b*d/9) + x**8*(a**2*f/8 + a*b*c/4)
```

Giac [A]
time = 0.59, size = 160, normalized size = 0.98

$$\frac{1}{16}b^2hx^{16} + \frac{1}{15}b^2gx^{15} + \frac{1}{14}b^2fx^{14} + \frac{2}{13}abhx^{13} + \frac{1}{13}b^2x^{13}e + \frac{1}{12}b^2dx^{12} + \frac{1}{6}abgx^{12} + \frac{1}{11}b^2cx^{11} + \frac{2}{11}abfx^{11} + \frac{1}{10}a^2hx^{10} + \frac{1}{5}abx^{10}e + \frac{2}{9}abdx^9 + \frac{1}{9}a^2gx^9 + \frac{1}{4}abcx^8 + \frac{1}{8}a^2fx^8 + \frac{1}{7}a^2x^7e + \frac{1}{6}a^2dx^6 + \frac{1}{5}a^2cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 2/13*a*b*h*x^13 + 1/1
3*b^2*x^13*e + 1/12*b^2*d*x^12 + 1/6*a*b*g*x^12 + 1/11*b^2*c*x^11 + 2/11*a*
b*f*x^11 + 1/10*a^2*h*x^10 + 1/5*a*b*x^10*e + 2/9*a*b*d*x^9 + 1/9*a^2*g*x^9
 + 1/4*a*b*c*x^8 + 1/8*a^2*f*x^8 + 1/7*a^2*x^7*e + 1/6*a^2*d*x^6 + 1/5*a^2*
c*x^5
```

Mupad [B]
time = 0.10, size = 151, normalized size = 0.93

$$x^8\left(\frac{f\,a^2}{8} + \frac{b\,c\,a}{4}\right) + x^{11}\left(\frac{c\,b^2}{11} + \frac{2\,a\,f\,b}{11}\right) + x^9\left(\frac{g\,a^2}{9} + \frac{2\,b\,d\,a}{9}\right) + x^{12}\left(\frac{d\,b^2}{12} + \frac{a\,g\,b}{6}\right) + x^{10}\left(\frac{h\,a^2}{10} + \frac{b\,e\,a}{5}\right) + x^{13}\left(\frac{e\,b^2}{13} + \frac{2\,a\,h\,b}{13}\right) + \frac{a^2\,c\,x^5}{5} + \frac{a^2\,d\,x^6}{6} + \frac{a^2\,e\,x^7}{7} + \frac{b^2\,f\,x^{14}}{14} + \frac{b^2\,g\,x^{15}}{15} + \frac{b^2\,h\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^8*((a^2*f)/8 + (a*b*c)/4) + x^11*((b^2*c)/11 + (2*a*b*f)/11) + x^9*((a^2*
g)/9 + (2*a*b*d)/9) + x^12*((b^2*d)/12 + (a*b*g)/6) + x^10*((a^2*h)/10 + (a
*b*e)/5) + x^13*((b^2*e)/13 + (2*a*b*h)/13) + (a^2*c*x^5)/5 + (a^2*d*x^6)/6
 + (a^2*e*x^7)/7 + (b^2*f*x^14)/14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

3.384 $\int x^3(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=163

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a(2bc+af)x^7+\frac{1}{8}a(2bd+ag)x^8+\frac{1}{9}a(2be+ah)x^9+\frac{1}{10}b(bc+2af)x^{10}+\frac{1}{11}b(bd+2ag$$

```
[Out] 1/4*a^2*c*x^4+1/5*a^2*d*x^5+1/6*a^2*e*x^6+1/7*a*(a*f+2*b*c)*x^7+1/8*a*(a*g+
2*b*d)*x^8+1/9*a*(a*h+2*b*e)*x^9+1/10*b*(2*a*f+b*c)*x^10+1/11*b*(2*a*g+b*d)
*x^11+1/12*b*(2*a*h+b*e)*x^12+1/13*b^2*f*x^13+1/14*b^2*g*x^14+1/15*b^2*h*x^
15
```

Rubi [A]
time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1834}

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{10}bx^{10}(2af+bc)+\frac{1}{7}ax^7(af+2bc)+\frac{1}{11}bx^{11}(2ag+bd)+\frac{1}{8}ax^8(ag+2bd)+\frac{1}{12}bx^{12}(2ah+be)+\frac{1}{9}ax^9(ah+2be)+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 + (
a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)/1
0 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)/13
 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \int\left(a^2cx^3+a^2dx^4+a^2ex^5+a(2bc+af)x^6+a(2\right.\\ &= \frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a(2bc+af)x^7+\end{aligned}$$

Mathematica [A]
time = 0.02, size = 163, normalized size = 1.00

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a(2bc+af)x^7+\frac{1}{8}a(2bd+ag)x^8+\frac{1}{9}a(2be+ah)x^9+\frac{1}{10}b(bc+2af)x^{10}+\frac{1}{11}b(bd+2ag)x^{11}+\frac{1}{12}b(be+2ah)x^{12}+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 + (
a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)/1
0 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)/13
 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

Maple [A]
time = 2.07, size = 152, normalized size = 0.93

method	result
default	$\frac{b^2 h\,x^{15}}{15}+\frac{b^2 g\,x^{14}}{14}+\frac{b^2 f\,x^{13}}{13}+\frac{(2abh+b^2 e)x^{12}}{12}+\frac{(2abg+b^2 d)x^{11}}{11}+\frac{(2abf+b^2 c)x^{10}}{10}+\frac{(a^2 h+2abe)x^9}{9}+\frac{(a^2 g+2abd)x^8}{8}+$
norman	$\frac{a^2 c\,x^4}{4}+\frac{x^5 a^2 d}{5}+\frac{a^2 e\,x^6}{6}+\left(\frac{1}{7}a^2 f+\frac{2}{7}abc\right)x^7+\left(\frac{1}{8}a^2 g+\frac{1}{4}abd\right)x^8+\left(\frac{1}{9}a^2 h+\frac{2}{9}abe\right)x^9+\left(\frac{1}{5}abf+\frac{1}{10}b\right.$
gosper	$\frac{1}{4}a^2 c\,x^4+\frac{1}{5}x^5 a^2 d+\frac{1}{6}a^2 e\,x^6+\frac{1}{7}a^2 f\,x^7+\frac{2}{7}x^7 abc+\frac{1}{8}x^8 a^2 g+\frac{1}{4}x^8 abd+\frac{1}{9}x^9 a^2 h+\frac{2}{9}x^9 abe+\frac{1}{5}x^{10}ab$
risch	$\frac{1}{4}a^2 c\,x^4+\frac{1}{5}x^5 a^2 d+\frac{1}{6}a^2 e\,x^6+\frac{1}{7}a^2 f\,x^7+\frac{2}{7}x^7 abc+\frac{1}{8}x^8 a^2 g+\frac{1}{4}x^8 abd+\frac{1}{9}x^9 a^2 h+\frac{2}{9}x^9 abe+\frac{1}{5}x^{10}ab$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/15*b^2*h*x^15+1/14*b^2*g*x^14+1/13*b^2*f*x^13+1/12*(2*a*b*h+b^2*e)*x^12+1
/11*(2*a*b*g+b^2*d)*x^11+1/10*(2*a*b*f+b^2*c)*x^10+1/9*(a^2*h+2*a*b*e)*x^9+
1/8*(a^2*g+2*a*b*d)*x^8+1/7*(a^2*f+2*a*b*c)*x^7+1/6*a^2*e*x^6+1/5*x^5*a^2*d
+1/4*a^2*c*x^4
```

Maxima [A]
time = 0.27, size = 154, normalized size = 0.94

$$\frac{1}{15}\,b^2hx^{15}+\frac{1}{14}\,b^2gx^{14}+\frac{1}{13}\,b^2fx^{13}+\frac{1}{12}\,(2\,abh+b^2e)x^{12}+\frac{1}{11}\,(b^2d+2\,abg)x^{11}+\frac{1}{10}\,(b^2c+2\,abf)x^{10}+\frac{1}{9}\,(a^2h+2\,abe)x^9+\frac{1}{8}\,(2\,abd+a^2g)x^8+\frac{1}{6}\,a^2x^6e+\frac{1}{5}\,a^2dx^5+\frac{1}{7}\,(2\,abc+a^2f)x^7+\frac{1}{4}\,a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/12*(2*a*b*h + b^2*e
)*x^12 + 1/11*(b^2*d + 2*a*b*g)*x^11 + 1/10*(b^2*c + 2*a*b*f)*x^10 + 1/9*(a
^2*h + 2*a*b*e)*x^9 + 1/8*(2*a*b*d + a^2*g)*x^8 + 1/6*a^2*x^6*e + 1/5*a^2*d
*x^5 + 1/7*(2*a*b*c + a^2*f)*x^7 + 1/4*a^2*c*x^4
```

Fricas [A]
time = 0.36, size = 151, normalized size = 0.93

$$\frac{1}{15}\,b^2hx^{15}+\frac{1}{14}\,b^2gx^{14}+\frac{1}{13}\,b^2fx^{13}+\frac{1}{12}\,(b^2e+2\,abh)x^{12}+\frac{1}{11}\,(b^2d+2\,abg)x^{11}+\frac{1}{10}\,(b^2c+2\,abf)x^{10}+\frac{1}{9}\,(2\,abe+a^2h)x^9+\frac{1}{6}\,a^2ex^6+\frac{1}{8}\,(2\,abd+a^2g)x^8+\frac{1}{5}\,a^2dx^5+\frac{1}{7}\,(2\,abc+a^2f)x^7+\frac{1}{4}\,a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/12*(b^2*e + 2*a*b*h
)*x^12 + 1/11*(b^2*d + 2*a*b*g)*x^11 + 1/10*(b^2*c + 2*a*b*f)*x^10 + 1/9*(2
*a*b*e + a^2*h)*x^9 + 1/6*a^2*e*x^6 + 1/8*(2*a*b*d + a^2*g)*x^8 + 1/5*a^2*d
*x^5 + 1/7*(2*a*b*c + a^2*f)*x^7 + 1/4*a^2*c*x^4
```

Sympy [A]
time = 0.02, size = 167, normalized size = 1.02

$$\frac{a^2cx^4}{4} + \frac{a^2dx^5}{5} + \frac{a^2ex^6}{6} + \frac{b^2fx^{13}}{13} + \frac{b^2gx^{14}}{14} + \frac{b^2hx^{15}}{15} + x^{12}\left(\frac{abh}{6} + \frac{b^2e}{12}\right) + x^{11}\cdot\left(\frac{2abg}{11} + \frac{b^2d}{11}\right) + x^{10}\left(\frac{abf}{5} + \frac{b^2c}{10}\right) + x^9\left(\frac{a^2h}{9} + \frac{2abe}{9}\right) + x^8\left(\frac{a^2g}{8} + \frac{abd}{4}\right) + x^7\left(\frac{a^2f}{7} + \frac{2abc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + b**2*f*x**13/13 + b**2*g*x*
*14/14 + b**2*h*x**15/15 + x**12*(a*b*h/6 + b**2*e/12) + x**11*(2*a*b*g/11
+ b**2*d/11) + x**10*(a*b*f/5 + b**2*c/10) + x**9*(a**2*h/9 + 2*a*b*e/9) +
x**8*(a**2*g/8 + a*b*d/4) + x**7*(a**2*f/7 + 2*a*b*c/7)
```

Giac [A]
time = 0.56, size = 160, normalized size = 0.98

$$\frac{1}{15}\,b^2hx^{15} + \frac{1}{14}\,b^2gx^{14} + \frac{1}{13}\,b^2fx^{13} + \frac{1}{6}\,abhx^{12} + \frac{1}{12}\,b^2x^{12}e + \frac{1}{11}\,b^2dx^{11} + \frac{2}{11}\,abgx^{11} + \frac{1}{10}\,b^2cx^{10} + \frac{1}{5}\,abfx^{10} + \frac{1}{9}\,a^2hx^9 + \frac{2}{9}\,abx^9e + \frac{1}{4}\,abdx^8 + \frac{1}{8}\,a^2gx^8 + \frac{2}{7}\,abcx^7 + \frac{1}{7}\,a^2fx^7 + \frac{1}{6}\,a^2x^6e + \frac{1}{5}\,a^2dx^5 + \frac{1}{4}\,a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/6*a*b*h*x^12 + 1/12
*b^2*x^12*e + 1/11*b^2*d*x^11 + 2/11*a*b*g*x^11 + 1/10*b^2*c*x^10 + 1/5*a*b
*f*x^10 + 1/9*a^2*h*x^9 + 2/9*a*b*x^9*e + 1/4*a*b*d*x^8 + 1/8*a^2*g*x^8 + 2
/7*a*b*c*x^7 + 1/7*a^2*f*x^7 + 1/6*a^2*x^6*e + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^
4
```

Mupad [B]
time = 0.09, size = 151, normalized size = 0.93

$$x^7\left(\frac{f\,a^2}{7} + \frac{2\,b\,c\,a}{7}\right) + x^{10}\left(\frac{c\,b^2}{10} + \frac{a\,f\,b}{5}\right) + x^8\left(\frac{g\,a^2}{8} + \frac{b\,d\,a}{4}\right) + x^{11}\left(\frac{d\,b^2}{11} + \frac{2\,a\,g\,b}{11}\right) + x^9\left(\frac{h\,a^2}{9} + \frac{2\,b\,e\,a}{9}\right) + x^{12}\left(\frac{e\,b^2}{12} + \frac{a\,h\,b}{6}\right) + \frac{a^2\,c\,x^4}{4} + \frac{a^2\,d\,x^5}{5} + \frac{a^2\,e\,x^6}{6} + \frac{b^2\,f\,x^{13}}{13} + \frac{b^2\,g\,x^{14}}{14} + \frac{b^2\,h\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^7*((a^2*f)/7 + (2*a*b*c)/7) + x^10*((b^2*c)/10 + (a*b*f)/5) + x^8*((a^2*g
)/8 + (a*b*d)/4) + x^11*((b^2*d)/11 + (2*a*b*g)/11) + x^9*((a^2*h)/9 + (2*a
*b*e)/9) + x^12*((b^2*e)/12 + (a*b*h)/6) + (a^2*c*x^4)/4 + (a^2*d*x^5)/5 +
(a^2*e*x^6)/6 + (b^2*f*x^13)/13 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

3.385 $\int x^2(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=158

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{1}{7}a(2bd+ag)x^7+\frac{1}{8}a(2be+ah)x^8+\frac{2}{9}abfx^9+\frac{1}{10}b(bd+2ag)x^{10}+\frac{1}{11}b(be+2ah)x^{11}+$$

```
[Out] 1/4*a^2*d*x^4+1/5*a^2*e*x^5+1/6*a^2*f*x^6+1/7*a*(a*g+2*b*d)*x^7+1/8*a*(a*h+
2*b*e)*x^8+2/9*a*b*f*x^9+1/10*b*(2*a*g+b*d)*x^10+1/11*b*(2*a*h+b*e)*x^11+1/
12*b^2*f*x^12+1/13*b^2*g*x^13+1/14*b^2*h*x^14+1/9*c*(b*x^3+a)^3/b
```

Rubi [A]
time = 0.09, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1596, 1864}

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{c(a+bx^3)^3}{9b}+\frac{1}{10}bx^{10}(2ag+bd)+\frac{1}{7}ax^7(ag+2bd)+\frac{1}{11}bx^{11}(2ah+be)+\frac{1}{8}ax^8(ah+2be)+\frac{2}{9}abfx^9+\frac{1}{12}b^2fx^{12}+\frac{1}{13}b^2gx^{13}+\frac{1}{14}b^2hx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a^2*f*x^6)/6 + (a*(2*b*d + a*g)*x^7)/7 + (
a*(2*b*e + a*h)*x^8)/8 + (2*a*b*f*x^9)/9 + (b*(b*d + 2*a*g)*x^10)/10 + (b*(
b*e + 2*a*h)*x^11)/11 + (b^2*f*x^12)/12 + (b^2*g*x^13)/13 + (b^2*h*x^14)/14
 + (c*(a + b*x^3)^3)/(9*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^2 (a+bx^3)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \frac{c(a+bx^3)^3}{9b} + \int (a+bx^3)^2 \left(-cx^2+x^2(c+dx+\right.$$

$$= \frac{c(a+bx^3)^3}{9b} + \int \left(a^2dx^3+a^2ex^4+a^2fx^5+a(2bd\right.$$

$$= \frac{1}{4}a^2dx^4 + \frac{1}{5}a^2ex^5 + \frac{1}{6}a^2fx^6 + \frac{1}{7}a(2bd+ag)x^7 +$$

Mathematica [A]
time = 0.05, size = 150, normalized size = 0.95

$$a^2\left(\frac{cx^3}{3}+\frac{dx^4}{4}+\frac{ex^5}{5}+\frac{fx^6}{6}+\frac{gx^7}{7}+\frac{hx^8}{8}\right)+ab\left(\frac{cx^6}{3}+\frac{2dx^7}{7}+\frac{ex^8}{4}+\frac{2fx^9}{9}+\frac{gx^{10}}{5}+\frac{2hx^{11}}{11}\right)+\frac{b^2x^9(20020c+3x(6006d+5460ex+55x^2(91f+84gx+78hx^2)))}{180180}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^2*((c*x^3)/3 + (d*x^4)/4 + (e*x^5)/5 + (f*x^6)/6 + (g*x^7)/7 + (h*x^8)/8)
 + a*b*((c*x^6)/3 + (2*d*x^7)/7 + (e*x^8)/4 + (2*f*x^9)/9 + (g*x^10)/5 + (2
*h*x^11)/11) + (b^2*x^9*(20020*c + 3*x*(6006*d + 5460*e*x + 55*x^2*(91*f +
84*g*x + 78*h*x^2))))/180180
```

Maple [A]
time = 2.00, size = 152, normalized size = 0.96

method	result
default	$\frac{b^2hx^{14}}{14}+\frac{b^2gx^{13}}{13}+\frac{b^2fx^{12}}{12}+\frac{(2abh+b^2e)x^{11}}{11}+\frac{(2abg+b^2d)x^{10}}{10}+\frac{(2abf+b^2c)x^9}{9}+\frac{(a^2h+2abe)x^8}{8}+\frac{(a^2g+2abd)x^7}{7}$
norman	$\frac{a^2cx^3}{3}+\frac{a^2dx^4}{4}+\frac{a^2ex^5}{5}+\left(\frac{1}{6}a^2f+\frac{1}{3}abc\right)x^6+\left(\frac{1}{7}a^2g+\frac{2}{7}abd\right)x^7+\left(\frac{1}{8}a^2h+\frac{1}{4}abe\right)x^8+\left(\frac{2}{9}abf+\frac{1}{9}\right.$
gosper	$\frac{1}{3}a^2cx^3+\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{1}{3}abcx^6+\frac{1}{7}x^7a^2g+\frac{2}{7}abdx^7+\frac{1}{8}x^8a^2h+\frac{1}{4}abex^8+\frac{2}{9}ab$
risch	$\frac{1}{3}a^2cx^3+\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{1}{3}abcx^6+\frac{1}{7}x^7a^2g+\frac{2}{7}abdx^7+\frac{1}{8}x^8a^2h+\frac{1}{4}abex^8+\frac{2}{9}ab$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/14*b^2*h*x^14+1/13*b^2*g*x^13+1/12*b^2*f*x^12+1/11*(2*a*b*h+b^2*e)*x^11+1
/10*(2*a*b*g+b^2*d)*x^10+1/9*(2*a*b*f+b^2*c)*x^9+1/8*(a^2*h+2*a*b*e)*x^8+1/
7*(a^2*g+2*a*b*d)*x^7+1/6*(a^2*f+2*a*b*c)*x^6+1/5*a^2*e*x^5+1/4*a^2*d*x^4+1
/3*a^2*c*x^3
```

Maxima [A]
time = 0.27, size = 154, normalized size = 0.97

$$\frac{1}{14}b^2hx^{14}+\frac{1}{13}b^2gx^{13}+\frac{1}{12}b^2fx^{12}+\frac{1}{11}(2abh+b^2e)x^{11}+\frac{1}{10}(b^2d+2abg)x^{10}+\frac{1}{9}(b^2c+2abf)x^9+\frac{1}{8}(a^2h+2abe)x^8+\frac{1}{7}(2abd+a^2g)x^7+\frac{1}{5}a^2x^5e+\frac{1}{4}a^2dx^4+\frac{1}{6}(2abc+a^2f)x^6+\frac{1}{3}a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*(2*a*b*h + b^2*e
)*x^11 + 1/10*(b^2*d + 2*a*b*g)*x^10 + 1/9*(b^2*c + 2*a*b*f)*x^9 + 1/8*(a^2
*h + 2*a*b*e)*x^8 + 1/7*(2*a*b*d + a^2*g)*x^7 + 1/5*a^2*x^5*e + 1/4*a^2*d*x
^4 + 1/6*(2*a*b*c + a^2*f)*x^6 + 1/3*a^2*c*x^3
```

Fricas [A]
time = 0.39, size = 151, normalized size = 0.96

$$\frac{1}{14}\,b^{2}hx^{14}+\frac{1}{13}\,b^{2}gx^{13}+\frac{1}{12}\,b^{2}fx^{12}+\frac{1}{11}\,(b^{2}e+2\,abh)x^{11}+\frac{1}{10}\,(b^{2}d+2\,abg)x^{10}+\frac{1}{9}\,(b^{2}c+2\,abf)x^{9}+\frac{1}{8}\,(2\,abe+a^{2}h)x^{8}+\frac{1}{5}\,a^{2}ex^{5}+\frac{1}{7}\,(2\,abd+a^{2}g)x^{7}+\frac{1}{4}\,a^{2}dx^{4}+\frac{1}{6}\,(2\,abc+a^{2}f)x^{6}+\frac{1}{3}\,a^{2}cx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*(b^2*e + 2*a*b*h
)*x^11 + 1/10*(b^2*d + 2*a*b*g)*x^10 + 1/9*(b^2*c + 2*a*b*f)*x^9 + 1/8*(2*a
*b*e + a^2*h)*x^8 + 1/5*a^2*e*x^5 + 1/7*(2*a*b*d + a^2*g)*x^7 + 1/4*a^2*d*x
^4 + 1/6*(2*a*b*c + a^2*f)*x^6 + 1/3*a^2*c*x^3
```

Sympy [A]
time = 0.02, size = 167, normalized size = 1.06

$$\frac{a^{2}cx^{3}}{3}+\frac{a^{2}dx^{4}}{4}+\frac{a^{2}ex^{5}}{5}+\frac{b^{2}fx^{12}}{12}+\frac{b^{2}gx^{13}}{13}+\frac{b^{2}hx^{14}}{14}+x^{11}\cdot\left(\frac{2abh}{11}+\frac{b^{2}e}{11}\right)+x^{10}\left(\frac{abg}{5}+\frac{b^{2}d}{10}\right)+x^{9}\cdot\left(\frac{2abf}{9}+\frac{b^{2}c}{9}\right)+x^{8}\left(\frac{a^{2}h}{8}+\frac{abe}{4}\right)+x^{7}\left(\frac{a^{2}g}{7}+\frac{2abd}{7}\right)+x^{6}\left(\frac{a^{2}f}{6}+\frac{abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + b**2*f*x**12/12 + b**2*g*x*
*13/13 + b**2*h*x**14/14 + x**11*(2*a*b*h/11 + b**2*e/11) + x**10*(a*b*g/5
+ b**2*d/10) + x**9*(2*a*b*f/9 + b**2*c/9) + x**8*(a**2*h/8 + a*b*e/4) + x*
*7*(a**2*g/7 + 2*a*b*d/7) + x**6*(a**2*f/6 + a*b*c/3)
```

Giac [A]
time = 0.57, size = 160, normalized size = 1.01

$$\frac{1}{14}\,b^{2}hx^{14}+\frac{1}{13}\,b^{2}gx^{13}+\frac{1}{12}\,b^{2}fx^{12}+\frac{2}{11}\,abhx^{11}+\frac{1}{11}\,b^{2}x^{11}e+\frac{1}{10}\,b^{2}dx^{10}+\frac{1}{5}\,abgx^{10}+\frac{1}{9}\,b^{2}cx^{9}+\frac{2}{9}\,abfx^{9}+\frac{1}{8}\,a^{2}hx^{8}+\frac{1}{4}\,abx^{8}e+\frac{2}{7}\,abdx^{7}+\frac{1}{7}\,a^{2}gx^{7}+\frac{1}{3}\,abcx^{6}+\frac{1}{6}\,a^{2}fx^{6}+\frac{1}{5}\,a^{2}x^{5}e+\frac{1}{4}\,a^{2}dx^{4}+\frac{1}{3}\,a^{2}cx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 2/11*a*b*h*x^11 + 1/1
1*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/5*a*b*g*x^10 + 1/9*b^2*c*x^9 + 2/9*a*b*f
*x^9 + 1/8*a^2*h*x^8 + 1/4*a*b*x^8*e + 2/7*a*b*d*x^7 + 1/7*a^2*g*x^7 + 1/3*
a*b*c*x^6 + 1/6*a^2*f*x^6 + 1/5*a^2*x^5*e + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

Mupad [B]
time = 0.09, size = 151, normalized size = 0.96

$$x^6\left(\frac{f\,a^2}{6}+\frac{b\,c\,a}{3}\right)+x^9\left(\frac{c\,b^2}{9}+\frac{2\,a\,f\,b}{9}\right)+x^7\left(\frac{g\,a^2}{7}+\frac{2\,b\,d\,a}{7}\right)+x^{10}\left(\frac{d\,b^2}{10}+\frac{a\,g\,b}{5}\right)+x^8\left(\frac{h\,a^2}{8}+\frac{b\,e\,a}{4}\right)+x^{11}\left(\frac{e\,b^2}{11}+\frac{2\,a\,h\,b}{11}\right)+\frac{a^2\,c\,x^3}{3}+\frac{a^2\,d\,x^4}{4}+\frac{a^2\,e\,x^5}{5}+\frac{b^2\,f\,x^{12}}{12}+\frac{b^2\,g\,x^{13}}{13}+\frac{b^2\,h\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^6*((a^2*f)/6 + (a*b*c)/3) + x^9*((b^2*c)/9 + (2*a*b*f)/9) + x^7*((a^2*g)/
7 + (2*a*b*d)/7) + x^10*((b^2*d)/10 + (a*b*g)/5) + x^8*((a^2*h)/8 + (a*b*e)
/4) + x^11*((b^2*e)/11 + (2*a*b*h)/11) + (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (a
^2*e*x^5)/5 + (b^2*f*x^12)/12 + (b^2*g*x^13)/13 + (b^2*h*x^14)/14
```

3.386 $\int x(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx$

Optimal. Leaf size=158

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{5}a(2bc+af)x^5+\frac{1}{6}a^2gx^6+\frac{1}{7}a(2be+ah)x^7+\frac{1}{8}b(bc+2af)x^8+\frac{2}{9}abgx^9+\frac{1}{10}b(be+2ah)x^{10}+\frac{1}{11}b^2$$

```
[Out] 1/2*a^2*c*x^2+1/4*a^2*e*x^4+1/5*a*(a*f+2*b*c)*x^5+1/6*a^2*g*x^6+1/7*a*(a*h+
2*b*e)*x^7+1/8*b*(2*a*f+b*c)*x^8+2/9*a*b*g*x^9+1/10*b*(2*a*h+b*e)*x^10+1/11
*b^2*f*x^11+1/12*b^2*g*x^12+1/13*b^2*h*x^13+1/9*d*(b*x^3+a)^3/b
```

Rubi [A]
time = 0.09, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1596, 1864}

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{6}a^2gx^6+\frac{1}{8}bx^8(2af+bc)+\frac{1}{5}ax^5(af+2bc)+\frac{d(a+bx^3)^3}{9b}+\frac{1}{10}bx^{10}(2ah+be)+\frac{1}{7}ax^7(ah+2be)+\frac{2}{9}abgx^9+\frac{1}{11}b^2fx^{11}+\frac{1}{12}b^2gx^{12}+\frac{1}{13}b^2hx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (a^2*g*x^6)/6 + (
a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8 + (2*a*b*g*x^9)/9 + (b*(b*
e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (b^2*g*x^12)/12 + (b^2*h*x^13)/13 +
 (d*(a + b*x^3)^3)/(9*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{d\left(a+bx^3\right)^3}{9b}+\int\left(a+bx^3\right)^2\left(-dx^2+x\left(c+dx+e\right.\right.$$
$$= \frac{d\left(a+bx^3\right)^3}{9b}+\int\left(a^2cx+a^2ex^3+a(2bc+af)x^4+\right.$$
$$= \frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{5}a(2bc+af)x^5+\frac{1}{6}a^2gx^6+\frac{1}{7}$$

Mathematica [A]
time = 0.02, size = 163, normalized size = 1.03

$$\frac{1}{2}a^2cx^2+\frac{1}{3}a^2dx^3+\frac{1}{4}a^2ex^4+\frac{1}{5}a(2bc+af)x^5+\frac{1}{6}a(2bd+ag)x^6+\frac{1}{7}a(2be+ah)x^7+\frac{1}{8}b(bc+2af)x^8+\frac{1}{9}b(bd+2ag)x^9+\frac{1}{10}b(be+2ah)x^{10}+\frac{1}{11}b^2fx^{11}+\frac{1}{12}b^2gx^{12}+\frac{1}{13}b^2hx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (
a*(2*b*d + a*g)*x^6)/6 + (a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8
+ (b*(b*d + 2*a*g)*x^9)/9 + (b*(b*e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (
b^2*g*x^12)/12 + (b^2*h*x^13)/13
```

Maple [A]
time = 2.08, size = 152, normalized size = 0.96

method	result
default	$\frac{b^2h\,x^{13}}{13}+\frac{b^2g\,x^{12}}{12}+\frac{b^2f\,x^{11}}{11}+\frac{(2abh+b^2e)x^{10}}{10}+\frac{(2abg+b^2d)x^9}{9}+\frac{(2abf+b^2c)x^8}{8}+\frac{(a^2h+2abe)x^7}{7}+\frac{(a^2g+2abd)x^6}{6}+$
norman	$\frac{b^2h\,x^{13}}{13}+\frac{b^2g\,x^{12}}{12}+\frac{b^2f\,x^{11}}{11}+\left(\frac{1}{5}abh+\frac{1}{10}b^2e\right)x^{10}+\left(\frac{2}{9}abg+\frac{1}{9}b^2d\right)x^9+\left(\frac{1}{4}abf+\frac{1}{8}b^2c\right)x^8+\left(\frac{1}{7}a^2h+\right.$
gosper	$\frac{1}{13}b^2h\,x^{13}+\frac{1}{12}b^2g\,x^{12}+\frac{1}{11}b^2f\,x^{11}+\frac{1}{5}x^{10}abh+\frac{1}{10}b^2e\,x^{10}+\frac{2}{9}abg\,x^9+\frac{1}{9}b^2d\,x^9+\frac{1}{4}abf\,x^8+\frac{1}{8}b^2c\,x$
risch	$\frac{1}{13}b^2h\,x^{13}+\frac{1}{12}b^2g\,x^{12}+\frac{1}{11}b^2f\,x^{11}+\frac{1}{5}x^{10}abh+\frac{1}{10}b^2e\,x^{10}+\frac{2}{9}abg\,x^9+\frac{1}{9}b^2d\,x^9+\frac{1}{4}abf\,x^8+\frac{1}{8}b^2c\,x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^2*h*x^13+1/12*b^2*g*x^12+1/11*b^2*f*x^11+1/10*(2*a*b*h+b^2*e)*x^10+1
/9*(2*a*b*g+b^2*d)*x^9+1/8*(2*a*b*f+b^2*c)*x^8+1/7*(a^2*h+2*a*b*e)*x^7+1/6*
(a^2*g+2*a*b*d)*x^6+1/5*(a^2*f+2*a*b*c)*x^5+1/4*a^2*e*x^4+1/3*a^2*d*x^3+1/2
*a^2*c*x^2
```

Maxima [A]
time = 0.28, size = 154, normalized size = 0.97

$$\frac{1}{13}\,b^2hx^{13}+\frac{1}{12}\,b^2gx^{12}+\frac{1}{11}\,b^2fx^{11}+\frac{1}{10}\,(2\,abh+b^2e)x^{10}+\frac{1}{9}\,(b^2d+2\,abg)x^9+\frac{1}{8}\,(b^2c+2\,abf)x^8+\frac{1}{7}\,(a^2h+2\,abe)x^7+\frac{1}{6}\,(2\,abd+a^2g)x^6+\frac{1}{4}\,a^2x^4e+\frac{1}{3}\,a^2dx^3+\frac{1}{5}\,(2\,abc+a^2f)x^5+\frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/10*(2*a*b*h + b^2*e
)*x^10 + 1/9*(b^2*d + 2*a*b*g)*x^9 + 1/8*(b^2*c + 2*a*b*f)*x^8 + 1/7*(a^2*h
 + 2*a*b*e)*x^7 + 1/6*(2*a*b*d + a^2*g)*x^6 + 1/4*a^2*x^4*e + 1/3*a^2*d*x^3
 + 1/5*(2*a*b*c + a^2*f)*x^5 + 1/2*a^2*c*x^2
```

<u>Fricas</u> [A]
time = 0.36, size = 151, normalized size = 0.96

$$\frac{1}{13}\,b^2hx^{13}+\frac{1}{12}\,b^2gx^{12}+\frac{1}{11}\,b^2fx^{11}+\frac{1}{10}\,(b^2e+2\,abh)x^{10}+\frac{1}{9}\,(b^2d+2\,abg)x^9+\frac{1}{8}\,(b^2c+2\,abf)x^8+\frac{1}{7}\,(2\,abe+a^2h)x^7+\frac{1}{4}\,a^2ex^4+\frac{1}{6}\,(2\,abd+a^2g)x^6+\frac{1}{3}\,a^2dx^3+\frac{1}{5}\,(2\,abc+a^2f)x^5+\frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/10*(b^2*e + 2*a*b*h
)*x^10 + 1/9*(b^2*d + 2*a*b*g)*x^9 + 1/8*(b^2*c + 2*a*b*f)*x^8 + 1/7*(2*a*b
*e + a^2*h)*x^7 + 1/4*a^2*e*x^4 + 1/6*(2*a*b*d + a^2*g)*x^6 + 1/3*a^2*d*x^3
 + 1/5*(2*a*b*c + a^2*f)*x^5 + 1/2*a^2*c*x^2
```

<u>Sympy</u> [A]
time = 0.02, size = 167, normalized size = 1.06

$$\frac{a^2cx^2}{2}+\frac{a^2dx^3}{3}+\frac{a^2ex^4}{4}+\frac{b^2fx^{11}}{11}+\frac{b^2gx^{12}}{12}+\frac{b^2hx^{13}}{13}+x^{10}\left(\frac{abh}{5}+\frac{b^2e}{10}\right)+x^9\cdot\left(\frac{2abg}{9}+\frac{b^2d}{9}\right)+x^8\left(\frac{abf}{4}+\frac{b^2c}{8}\right)+x^7\left(\frac{a^2h}{7}+\frac{2abe}{7}\right)+x^6\left(\frac{a^2g}{6}+\frac{abd}{3}\right)+x^5\left(\frac{a^2f}{5}+\frac{2abc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + b**2*f*x**11/11 + b**2*g*x*
*12/12 + b**2*h*x**13/13 + x**10*(a*b*h/5 + b**2*e/10) + x**9*(2*a*b*g/9 +
b**2*d/9) + x**8*(a*b*f/4 + b**2*c/8) + x**7*(a**2*h/7 + 2*a*b*e/7) + x**6*
(a**2*g/6 + a*b*d/3) + x**5*(a**2*f/5 + 2*a*b*c/5)
```

<u>Giac</u> [A]
time = 0.56, size = 160, normalized size = 1.01

$$\frac{1}{13}\,b^2hx^{13}+\frac{1}{12}\,b^2gx^{12}+\frac{1}{11}\,b^2fx^{11}+\frac{1}{5}\,abhx^{10}+\frac{1}{10}\,b^2x^{10}e+\frac{1}{9}\,b^2dx^9+\frac{2}{9}\,abgx^9+\frac{1}{8}\,b^2cx^8+\frac{1}{4}\,abfx^8+\frac{1}{7}\,a^2hx^7+\frac{2}{7}\,abx^7e+\frac{1}{3}\,abdx^6+\frac{1}{6}\,a^2gx^6+\frac{2}{5}\,abcx^5+\frac{1}{5}\,a^2fx^5+\frac{1}{4}\,a^2x^4e+\frac{1}{3}\,a^2dx^3+\frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/5*a*b*h*x^10 + 1/10
*b^2*x^10*e + 1/9*b^2*d*x^9 + 2/9*a*b*g*x^9 + 1/8*b^2*c*x^8 + 1/4*a*b*f*x^8
 + 1/7*a^2*h*x^7 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 1/6*a^2*g*x^6 + 2/5*a*b*
c*x^5 + 1/5*a^2*f*x^5 + 1/4*a^2*x^4*e + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

Mupad [B]
time = 0.09, size = 151, normalized size = 0.96

$$x^5\left(\frac{f\,a^2}{5}+\frac{2\,b\,c\,a}{5}\right)+x^8\left(\frac{c\,b^2}{8}+\frac{a\,f\,b}{4}\right)+x^6\left(\frac{g\,a^2}{6}+\frac{b\,d\,a}{3}\right)+x^9\left(\frac{d\,b^2}{9}+\frac{2\,a\,g\,b}{9}\right)+x^7\left(\frac{h\,a^2}{7}+\frac{2\,b\,e\,a}{7}\right)+x^{10}\left(\frac{e\,b^2}{10}+\frac{a\,h\,b}{5}\right)+\frac{a^2\,c\,x^2}{2}+\frac{a^2\,d\,x^3}{3}+\frac{a^2\,e\,x^4}{4}+\frac{b^2\,f\,x^{11}}{11}+\frac{b^2\,g\,x^{12}}{12}+\frac{b^2\,h\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^5*((a^2*f)/5 + (2*a*b*c)/5) + x^8*((b^2*c)/8 + (a*b*f)/4) + x^6*((a^2*g)/
6 + (a*b*d)/3) + x^9*((b^2*d)/9 + (2*a*b*g)/9) + x^7*((a^2*h)/7 + (2*a*b*e)
/7) + x^10*((b^2*e)/10 + (a*b*h)/5) + (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*
e*x^4)/4 + (b^2*f*x^11)/11 + (b^2*g*x^12)/12 + (b^2*h*x^13)/13
```

3.387 $\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)\, dx$

Optimal. Leaf size=153

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{4}a(2bc+af)x^4+\frac{1}{5}a(2bd+ag)x^5+\frac{1}{6}a^2hx^6+\frac{1}{7}b(bc+2af)x^7+\frac{1}{8}b(bd+2ag)x^8+\frac{2}{9}abhx^9+\frac{1}{10}b^2fx^{10}$$

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/4*a*(a*f+2*b*c)*x^4+1/5*a*(a*g+2*b*d)*x^5+1/6*a^2*h
*x^6+1/7*b*(2*a*f+b*c)*x^7+1/8*b*(2*a*g+b*d)*x^8+2/9*a*b*h*x^9+1/10*b^2*f*x
^10+1/11*b^2*g*x^11+1/12*b^2*h*x^12+1/9*e*(b*x^3+a)^3/b
```

Rubi [A]
time = 0.09, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1596, 1864}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{6}a^2hx^6+\frac{1}{7}bx^7(2af+bc)+\frac{1}{4}ax^4(af+2bc)+\frac{1}{8}bx^8(2ag+bd)+\frac{1}{5}ax^5(ag+2bd)+\frac{e(a+bx^3)^3}{9b}+\frac{2}{9}abhx^9+\frac{1}{10}b^2fx^{10}+\frac{1}{11}b^2gx^{11}+\frac{1}{12}b^2hx^{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*(2*b*c + a*f)*x^4)/4 + (a*(2*b*d + a*g)*x^5)/5
 + (a^2*h*x^6)/6 + (b*(b*c + 2*a*f)*x^7)/7 + (b*(b*d + 2*a*g)*x^8)/8 + (2*a
*b*h*x^9)/9 + (b^2*f*x^10)/10 + (b^2*g*x^11)/11 + (b^2*h*x^12)/12 + (e*(a +
 b*x^3)^3)/(9*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \frac{e(a+bx^3)^3}{9b} + \int \left(a+bx^3\right)^2 \left(c+dx+fx^3+gx^4+\right.$$

$$= \frac{e(a+bx^3)^3}{9b} + \int \left(a^2c+a^2dx+a(2bc+af)x^3+a(2\right.$$

$$= a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{4}a(2bc+af)x^4 + \frac{1}{5}a(2bd+ag)x^5$$

Mathematica [A]
time = 0.05, size = 125, normalized size = 0.82

$$\frac{b^2x^7(3960c+7x(495d+440ex+6x^2(66f+60gx+55hx^2)))+462a^2x(60c+x(30d+x(20e+15fx+12gx^2+10hx^3)))+22abx^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (b^2*x^7*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55*h*x^2))
) + 462*a^2*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))) +
 22*a*b*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x)))))
)/27720
```

Maple [A]
time = 1.89, size = 149, normalized size = 0.97

method	result
default	$\frac{b^2h\,x^{12}}{12}+\frac{b^2g\,x^{11}}{11}+\frac{b^2f\,x^{10}}{10}+\frac{(2abh+b^2e)x^9}{9}+\frac{(2abg+b^2d)x^8}{8}+\frac{(2abf+b^2c)x^7}{7}+\frac{(a^2h+2abe)x^6}{6}+\frac{(a^2g+2abd)x^5}{5}+$
norman	$\frac{b^2h\,x^{12}}{12}+\frac{b^2g\,x^{11}}{11}+\frac{b^2f\,x^{10}}{10}+\left(\frac{2}{9}abh+\frac{1}{9}b^2e\right)x^9+\left(\frac{1}{4}abg+\frac{1}{8}b^2d\right)x^8+\left(\frac{2}{7}abf+\frac{1}{7}b^2c\right)x^7+\left(\frac{1}{6}a^2h+\right.$
gosper	$\frac{1}{12}b^2h\,x^{12}+\frac{1}{11}b^2g\,x^{11}+\frac{1}{10}b^2f\,x^{10}+\frac{2}{9}abh\,x^9+\frac{1}{9}b^2e\,x^9+\frac{1}{4}x^8abg+\frac{1}{8}b^2d\,x^8+\frac{2}{7}x^7abf+\frac{1}{7}b^2c\,x^7+$
risch	$\frac{1}{12}b^2h\,x^{12}+\frac{1}{11}b^2g\,x^{11}+\frac{1}{10}b^2f\,x^{10}+\frac{2}{9}abh\,x^9+\frac{1}{9}b^2e\,x^9+\frac{1}{4}x^8abg+\frac{1}{8}b^2d\,x^8+\frac{2}{7}x^7abf+\frac{1}{7}b^2c\,x^7+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^2*h*x^12+1/11*b^2*g*x^11+1/10*b^2*f*x^10+1/9*(2*a*b*h+b^2*e)*x^9+1/8
*(2*a*b*g+b^2*d)*x^8+1/7*(2*a*b*f+b^2*c)*x^7+1/6*(a^2*h+2*a*b*e)*x^6+1/5*(a
^2*g+2*a*b*d)*x^5+1/4*(a^2*f+2*a*b*c)*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c
*x
```

Maxima [A]
time = 0.27, size = 151, normalized size = 0.99

$$\frac{1}{12}b^2hx^{12}+\frac{1}{11}b^2gx^{11}+\frac{1}{10}b^2fx^{10}+\frac{1}{9}(2abh+b^2e)x^9+\frac{1}{8}(b^2d+2abg)x^8+\frac{1}{7}(b^2c+2abf)x^7+\frac{1}{6}(a^2h+2abe)x^6+\frac{1}{5}(2abd+a^2g)x^5+\frac{1}{3}a^2x^3e+\frac{1}{2}a^2dx^2+\frac{1}{4}(2abc+a^2f)x^4+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 1/9*(2*a*b*h + b^2*e)
*x^9 + 1/8*(b^2*d + 2*a*b*g)*x^8 + 1/7*(b^2*c + 2*a*b*f)*x^7 + 1/6*(a^2*h +
 2*a*b*e)*x^6 + 1/5*(2*a*b*d + a^2*g)*x^5 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 +
 1/4*(2*a*b*c + a^2*f)*x^4 + a^2*c*x
```

Fricas [A]
time = 0.41, size = 148, normalized size = 0.97

$$\frac{1}{12}\,b^2hx^{12}+\frac{1}{11}\,b^2gx^{11}+\frac{1}{10}\,b^2fx^{10}+\frac{1}{9}\,(b^2e+2\,abh)x^9+\frac{1}{8}\,(b^2d+2\,abg)x^8+\frac{1}{7}\,(b^2c+2\,abf)x^7+\frac{1}{6}\,(2\,abe+a^2h)x^6+\frac{1}{3}\,a^2ex^3+\frac{1}{5}\,(2\,abd+a^2g)x^5+\frac{1}{2}\,a^2dx^2+\frac{1}{4}\,(2\,abc+a^2f)x^4+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 1/9*(b^2*e + 2*a*b*h)
*x^9 + 1/8*(b^2*d + 2*a*b*g)*x^8 + 1/7*(b^2*c + 2*a*b*f)*x^7 + 1/6*(2*a*b*e
 + a^2*h)*x^6 + 1/3*a^2*e*x^3 + 1/5*(2*a*b*d + a^2*g)*x^5 + 1/2*a^2*d*x^2 +
 1/4*(2*a*b*c + a^2*f)*x^4 + a^2*c*x
```

Sympy [A]
time = 0.02, size = 163, normalized size = 1.07

$$a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{b^2fx^{10}}{10}+\frac{b^2gx^{11}}{11}+\frac{b^2hx^{12}}{12}+x^9\cdot\left(\frac{2abh}{9}+\frac{b^2e}{9}\right)+x^8\left(\frac{abg}{4}+\frac{b^2d}{8}\right)+x^7\cdot\left(\frac{2abf}{7}+\frac{b^2c}{7}\right)+x^6\left(\frac{a^2h}{6}+\frac{abe}{3}\right)+x^5\left(\frac{a^2g}{5}+\frac{2abd}{5}\right)+x^4\left(\frac{a^2f}{4}+\frac{abc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + b**2*f*x**10/10 + b**2*g*x**11/1
1 + b**2*h*x**12/12 + x**9*(2*a*b*h/9 + b**2*e/9) + x**8*(a*b*g/4 + b**2*d/
8) + x**7*(2*a*b*f/7 + b**2*c/7) + x**6*(a**2*h/6 + a*b*e/3) + x**5*(a**2*g
/5 + 2*a*b*d/5) + x**4*(a**2*f/4 + a*b*c/2)
```

Giac [A]
time = 0.53, size = 157, normalized size = 1.03

$$\frac{1}{12}\,b^2hx^{12}+\frac{1}{11}\,b^2gx^{11}+\frac{1}{10}\,b^2fx^{10}+\frac{2}{9}\,abhx^9+\frac{1}{9}\,b^2x^9e+\frac{1}{8}\,b^2dx^8+\frac{1}{4}\,abgx^8+\frac{1}{7}\,b^2cx^7+\frac{2}{7}\,abfx^7+\frac{1}{6}\,a^2hx^6+\frac{1}{3}\,abx^6e+\frac{2}{5}\,abdx^5+\frac{1}{5}\,a^2gx^5+\frac{1}{2}\,abcx^4+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2x^3e+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 2/9*a*b*h*x^9 + 1/9*b
^2*x^9*e + 1/8*b^2*d*x^8 + 1/4*a*b*g*x^8 + 1/7*b^2*c*x^7 + 2/7*a*b*f*x^7 +
```

```
1/6*a^2*h*x^6 + 1/3*a*b*x^6*e + 2/5*a*b*d*x^5 + 1/5*a^2*g*x^5 + 1/2*a*b*c*x
^4 + 1/4*a^2*f*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

Mupad [B]
time = 0.09, size = 148, normalized size = 0.97

$$x^4\left(\frac{f\,a^2}{4}+\frac{b\,c\,a}{2}\right)+x^7\left(\frac{c\,b^2}{7}+\frac{2\,a\,f\,b}{7}\right)+x^5\left(\frac{g\,a^2}{5}+\frac{2\,b\,d\,a}{5}\right)+x^8\left(\frac{d\,b^2}{8}+\frac{a\,g\,b}{4}\right)+x^6\left(\frac{h\,a^2}{6}+\frac{b\,e\,a}{3}\right)+x^9\left(\frac{e\,b^2}{9}+\frac{2\,a\,h\,b}{9}\right)+\frac{a^2\,d\,x^2}{2}+\frac{a^2\,e\,x^3}{3}+\frac{b^2\,f\,x^{10}}{10}+\frac{b^2\,g\,x^{11}}{11}+\frac{b^2\,h\,x^{12}}{12}+a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^4*((a^2*f)/4 + (a*b*c)/2) + x^7*((b^2*c)/7 + (2*a*b*f)/7) + x^5*((a^2*g)/
5 + (2*a*b*d)/5) + x^8*((b^2*d)/8 + (a*b*g)/4) + x^6*((a^2*h)/6 + (a*b*e)/3
) + x^9*((b^2*e)/9 + (2*a*b*h)/9) + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (b^2*f*
x^10)/10 + (b^2*g*x^11)/11 + (b^2*h*x^12)/12 + a^2*c*x
```

3.388 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

Optimal. Leaf size=149

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{4}a(2bd+ag)x^4+\frac{1}{5}a(2be+ah)x^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b(bd+2ag)x^7+\frac{1}{8}b(be+2ah)x^8+\frac{1}{10}b^2gx^{10}+$$

```
[Out] a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*c*x^3+1/4*a*(a*g+2*b*d)*x^4+1/5*a*(a*h+2*b*e)
*x^5+1/6*b^2*c*x^6+1/7*b*(2*a*g+b*d)*x^7+1/8*b*(2*a*h+b*e)*x^8+1/10*b^2*g*x
^10+1/11*b^2*h*x^11+1/9*f*(b*x^3+a)^3/b+a^2*c*ln(x)
```

Rubi [A]
time = 0.07, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1597, 1834}

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{7}bx^7(2ag+bd)+\frac{1}{4}ax^4(ag+2bd)+\frac{1}{8}bx^8(2ah+be)+\frac{1}{5}ax^5(ah+2be)+\frac{f\left(a+bx^3\right)^3}{9b}+\frac{1}{6}b^2cx^6+\frac{1}{10}b^2gx^{10}+\frac{1}{11}b^2hx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*(2*b*d + a*g)*x^4)/4 + (a*(2
*b*e + a*h)*x^5)/5 + (b^2*c*x^6)/6 + (b*(b*d + 2*a*g)*x^7)/7 + (b*(b*e + 2*
a*h)*x^8)/8 + (b^2*g*x^10)/10 + (b^2*h*x^11)/11 + (f*(a + b*x^3)^3)/(9*b) +
 a^2*c*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = \frac{f(a+bx^3)^3}{9b} + \int \frac{(a+bx^3)^2(c+dx+ex^2+gx^4+h$$
$$= \frac{f(a+bx^3)^3}{9b} + \int \left(a^2d+\frac{a^2c}{x}+a^2ex+2abcx^2+a(2$$
$$= a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{4}a(2bd+ag)x^4+\frac{1}{5}a(2be$$

Mathematica [A]
time = 0.03, size = 154, normalized size = 1.03

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bc+af)x^3+\frac{1}{4}a(2bd+ag)x^4+\frac{1}{5}a(2be+ah)x^5+\frac{1}{6}b(bc+2af)x^6+\frac{1}{7}b(bd+2ag)x^7+\frac{1}{8}b(be+2ah)x^8+\frac{1}{9}b^2fx^9+\frac{1}{10}b^2gx^{10}+\frac{1}{11}b^2hx^{11}+a^2c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*c + a*f)*x^3)/3 + (a*(2*b*d + a*g)*x^4)/4
 + (a*(2*b*e + a*h)*x^5)/5 + (b*(b*c + 2*a*f)*x^6)/6 + (b*(b*d + 2*a*g)*x^7
)/7 + (b*(b*e + 2*a*h)*x^8)/8 + (b^2*f*x^9)/9 + (b^2*g*x^10)/10 + (b^2*h*x^
11)/11 + a^2*c*Log[x]
```

Maple [A]
time = 0.35, size = 153, normalized size = 1.03

method	result
norman	$\left(\frac{1}{3}a^2f+\frac{2}{3}abc\right)x^3+\left(\frac{1}{4}a^2g+\frac{1}{2}abd\right)x^4+\left(\frac{1}{5}a^2h+\frac{2}{5}abe\right)x^5+\left(\frac{1}{3}abf+\frac{1}{6}b^2c\right)x^6+\left(\frac{2}{7}abg+\frac{1}{7}b^2d\right)$
default	$\frac{b^2hx^{11}}{11}+\frac{b^2gx^{10}}{10}+\frac{fx^9b^2}{9}+\frac{abhx^8}{4}+\frac{b^2ex^8}{8}+\frac{2abgx^7}{7}+\frac{b^2dx^7}{7}+\frac{abfx^6}{3}+\frac{b^2cx^6}{6}+\frac{a^2hx^5}{5}+\frac{2abex^5}{5}+\frac{a^2gx^4}{4}$
risch	$\frac{b^2hx^{11}}{11}+\frac{b^2gx^{10}}{10}+\frac{fx^9b^2}{9}+\frac{abhx^8}{4}+\frac{b^2ex^8}{8}+\frac{2abgx^7}{7}+\frac{b^2dx^7}{7}+\frac{abfx^6}{3}+\frac{b^2cx^6}{6}+\frac{a^2hx^5}{5}+\frac{2abex^5}{5}+\frac{a^2gx^4}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^2*h*x^11+1/10*b^2*g*x^10+1/9*f*x^9*b^2+1/4*a*b*h*x^8+1/8*b^2*e*x^8+2
/7*a*b*g*x^7+1/7*b^2*d*x^7+1/3*a*b*f*x^6+1/6*b^2*c*x^6+1/5*a^2*h*x^5+2/5*a*
b*e*x^5+1/4*a^2*g*x^4+1/2*a*b*d*x^4+1/3*a^2*f*x^3+2/3*a*b*c*x^3+1/2*a^2*e*x
^2+a^2*d*x+a^2*c*ln(x)
```

Maxima [A]
time = 0.28, size = 149, normalized size = 1.00

$$\frac{1}{11}b^2hx^{11}+\frac{1}{10}b^2gx^{10}+\frac{1}{9}b^2fx^9+\frac{1}{8}(2abh+b^2e)x^8+\frac{1}{7}(b^2d+2abg)x^7+\frac{1}{6}(b^2c+2abf)x^6+\frac{1}{5}(a^2h+2abe)x^5+\frac{1}{4}(2abd+a^2g)x^4+\frac{1}{2}a^2x^2e+a^2dx+\frac{1}{3}(2abc+a^2f)x^3+a^2c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxim
a")
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(2*a*b*h + b^2*e)*x
^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*x^6 + 1/5*(a^2*h + 2
*a*b*e)*x^5 + 1/4*(2*a*b*d + a^2*g)*x^4 + 1/2*a^2*x^2*e + a^2*d*x + 1/3*(2*
a*b*c + a^2*f)*x^3 + a^2*c*log(x)
```

Fricas [A]
time = 0.39, size = 146, normalized size = 0.98

$$\frac{1}{11}\,b^{2}hx^{11}+\frac{1}{10}\,b^{2}gx^{10}+\frac{1}{9}\,b^{2}fx^{9}+\frac{1}{8}\,(b^{2}e+2\,abh)x^{8}+\frac{1}{7}\,(b^{2}d+2\,abg)x^{7}+\frac{1}{6}\,(b^{2}c+2\,abf)x^{6}+\frac{1}{5}\,(2\,abe+a^{2}h)x^{5}+\frac{1}{2}\,a^{2}ex^{2}+\frac{1}{4}\,(2\,abd+a^{2}g)x^{4}+a^{2}dx+\frac{1}{3}\,(2\,abc+a^{2}f)x^{3}+a^{2}c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="frica
s")
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(b^2*e + 2*a*b*h)*x
^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*x^6 + 1/5*(2*a*b*e +
 a^2*h)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*d + a^2*g)*x^4 + a^2*d*x + 1/3*(2*
a*b*c + a^2*f)*x^3 + a^2*c*log(x)
```

Sympy [A]
time = 0.12, size = 162, normalized size = 1.09

$$a^{2}c\log\left(x\right)+a^{2}dx+\frac{a^{2}ex^{2}}{2}+\frac{b^{2}fx^{9}}{9}+\frac{b^{2}gx^{10}}{10}+\frac{b^{2}hx^{11}}{11}+x^{8}\left(\frac{abh}{4}+\frac{b^{2}e}{8}\right)+x^{7}\cdot\left(\frac{2abg}{7}+\frac{b^{2}d}{7}\right)+x^{6}\left(\frac{abf}{3}+\frac{b^{2}c}{6}\right)+x^{5}\left(\frac{a^{2}h}{5}+\frac{2abe}{5}\right)+x^{4}\left(\frac{a^{2}g}{4}+\frac{abd}{2}\right)+x^{3}\left(\frac{a^{2}f}{3}+\frac{2abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a**2*c*log(x) + a**2*d*x + a**2*e*x**2/2 + b**2*f*x**9/9 + b**2*g*x**10/10
+ b**2*h*x**11/11 + x**8*(a*b*h/4 + b**2*e/8) + x**7*(2*a*b*g/7 + b**2*d/7)
 + x**6*(a*b*f/3 + b**2*c/6) + x**5*(a**2*h/5 + 2*a*b*e/5) + x**4*(a**2*g/4
 + a*b*d/2) + x**3*(a**2*f/3 + 2*a*b*c/3)
```

Giac [A]
time = 0.52, size = 156, normalized size = 1.05

$$\frac{1}{11}\,b^{2}hx^{11}+\frac{1}{10}\,b^{2}gx^{10}+\frac{1}{9}\,b^{2}fx^{9}+\frac{1}{4}\,abhx^{8}+\frac{1}{8}\,b^{2}x^{8}e+\frac{1}{7}\,b^{2}dx^{7}+\frac{2}{7}\,abgx^{7}+\frac{1}{6}\,b^{2}cx^{6}+\frac{1}{3}\,abfx^{6}+\frac{1}{5}\,a^{2}hx^{5}+\frac{2}{5}\,abx^{5}e+\frac{1}{2}\,abdx^{4}+\frac{1}{4}\,a^{2}gx^{4}+\frac{2}{3}\,abcx^{3}+\frac{1}{3}\,a^{2}fx^{3}+\frac{1}{2}\,a^{2}x^{2}e+a^{2}dx+a^{2}c\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac"
)
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/4*a*b*h*x^8 + 1/8*b^2
*x^8*e + 1/7*b^2*d*x^7 + 2/7*a*b*g*x^7 + 1/6*b^2*c*x^6 + 1/3*a*b*f*x^6 + 1/
5*a^2*h*x^5 + 2/5*a*b*x^5*e + 1/2*a*b*d*x^4 + 1/4*a^2*g*x^4 + 2/3*a*b*c*x^3
 + 1/3*a^2*f*x^3 + 1/2*a^2*x^2*e + a^2*d*x + a^2*c*log(abs(x))
```

Mupad [B]
time = 0.10, size = 146, normalized size = 0.98

$$x^3\left(\frac{f\,a^2}{3}+\frac{2\,b\,c\,a}{3}\right)+x^6\left(\frac{c\,b^2}{6}+\frac{a\,f\,b}{3}\right)+x^4\left(\frac{g\,a^2}{4}+\frac{b\,d\,a}{2}\right)+x^7\left(\frac{d\,b^2}{7}+\frac{2\,a\,g\,b}{7}\right)+x^5\left(\frac{h\,a^2}{5}+\frac{2\,b\,e\,a}{5}\right)+x^8\left(\frac{e\,b^2}{8}+\frac{a\,h\,b}{4}\right)+\frac{a^2\,e\,x^2}{2}+\frac{b^2\,f\,x^9}{9}+\frac{b^2\,g\,x^{10}}{10}+\frac{b^2\,h\,x^{11}}{11}+a^2\,c\,\ln\left(x\right)+a^2\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x)
```

```
[Out] x^3*((a^2*f)/3 + (2*a*b*c)/3) + x^6*((b^2*c)/6 + (a*b*f)/3) + x^4*((a^2*g)/
4 + (a*b*d)/2) + x^7*((b^2*d)/7 + (2*a*b*g)/7) + x^5*((a^2*h)/5 + (2*a*b*e)
/5) + x^8*((b^2*e)/8 + (a*b*h)/4) + (a^2*e*x^2)/2 + (b^2*f*x^9)/9 + (b^2*g*
x^10)/10 + (b^2*h*x^11)/11 + a^2*c*log(x) + a^2*d*x
```

3.389 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$

Optimal. Leaf size=147

$$-\frac{a^2c}{x}+a^2ex+\frac{1}{2}a(2bc+af)x^2+\frac{2}{3}abdx^3+\frac{1}{4}a(2be+ah)x^4+\frac{1}{5}b(bc+2af)x^5+\frac{1}{6}b^2dx^6+\frac{1}{7}b(be+2ah)x^7+\frac{1}{8}b^2fx^8+\frac{1}{10}$$

```
[Out] -a^2*c/x+a^2*e*x+1/2*a*(a*f+2*b*c)*x^2+2/3*a*b*d*x^3+1/4*a*(a*h+2*b*e)*x^4+
1/5*b*(2*a*f+b*c)*x^5+1/6*b^2*d*x^6+1/7*b*(2*a*h+b*e)*x^7+1/8*b^2*f*x^8+1/1
0*b^2*h*x^10+1/9*g*(b*x^3+a)^3/b+a^2*d*ln(x)
```

Rubi [A]

time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1597, 1834}

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{1}{5}bx^5(2af+bc)+\frac{1}{2}ax^2(af+2bc)+\frac{2}{3}abdx^3+\frac{1}{7}bx^7(2ah+be)+\frac{1}{4}ax^4(ah+2be)+\frac{g(a+bx^3)^3}{9b}+\frac{1}{6}b^2dx^6+\frac{1}{8}b^2fx^8+\frac{1}{10}b^2hx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (2*a*b*d*x^3)/3 + (a*(2*
b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b^2*d*x^6)/6 + (b*(b*e + 2*a
*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*h*x^10)/10 + (g*(a + b*x^3)^3)/(9*b) + a^
2*d*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = \frac{g(a+bx^3)^3}{9b} + \int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+h$$
$$= \frac{g(a+bx^3)^3}{9b} + \int \left(a^2e + \frac{a^2c}{x^2} + \frac{a^2d}{x} + a(2bc+af)x\right.$$
$$= -\frac{a^2c}{x} + a^2ex + \frac{1}{2}a(2bc+af)x^2 + \frac{2}{3}abdx^3 + \frac{1}{4}a(2be$$

Mathematica [A]
time = 0.04, size = 152, normalized size = 1.03

$$-\frac{a^2c}{x}+a^2ex+\frac{1}{2}a(2bc+af)x^2+\frac{1}{3}a(2bd+ag)x^3+\frac{1}{4}a(2be+ah)x^4+\frac{1}{5}b(bc+2af)x^5+\frac{1}{6}b(bd+2ag)x^6+\frac{1}{7}b(be+2ah)x^7+\frac{1}{8}b^2fx^8+\frac{1}{9}b^2gx^9+\frac{1}{10}b^2hx^{10}+a^2d\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (a*(2*b*d + a*g)*x^3)/3
+ (a*(2*b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b*(b*d + 2*a*g)*x^6)
/6 + (b*(b*e + 2*a*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*g*x^9)/9 + (b^2*h*x^10)
/10 + a^2*d*Log[x]
```

Maple [A]
time = 0.33, size = 152, normalized size = 1.03

method	result
norman	$\frac{(\frac{1}{2}a^2f+abc)x^3+(\frac{1}{3}a^2g+\frac{2}{3}abd)x^4+(\frac{1}{4}a^2h+\frac{1}{2}abe)x^5+(\frac{2}{5}abf+\frac{1}{5}b^2c)x^6+(\frac{1}{3}abg+\frac{1}{6}b^2d)x^7+(\frac{2}{7}abh+\frac{1}{7}b^2e)x^8+a^2e\,x^2-a^2c+\frac{b^2g\,x^{10}}{9}}{x}$
default	$\frac{b^2h\,x^{10}}{10}+\frac{b^2g\,x^9}{9}+\frac{b^2f\,x^8}{8}+\frac{2abh\,x^7}{7}+\frac{b^2e\,x^7}{7}+\frac{abg\,x^6}{3}+\frac{b^2d\,x^6}{6}+\frac{2abf\,x^5}{5}+\frac{b^2c\,x^5}{5}+\frac{a^2h\,x^4}{4}+\frac{abe\,x^4}{2}+\frac{a^2g\,x^3}{3}$
risch	$\frac{b^2h\,x^{10}}{10}+\frac{b^2g\,x^9}{9}+\frac{b^2f\,x^8}{8}+\frac{2abh\,x^7}{7}+\frac{b^2e\,x^7}{7}+\frac{abg\,x^6}{3}+\frac{b^2d\,x^6}{6}+\frac{2abf\,x^5}{5}+\frac{b^2c\,x^5}{5}+\frac{a^2h\,x^4}{4}+\frac{abe\,x^4}{2}+\frac{a^2g\,x^3}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/10*b^2*h*x^10+1/9*b^2*g*x^9+1/8*b^2*f*x^8+2/7*a*b*h*x^7+1/7*b^2*e*x^7+1/3
*a*b*g*x^6+1/6*b^2*d*x^6+2/5*a*b*f*x^5+1/5*b^2*c*x^5+1/4*a^2*h*x^4+1/2*a*b*
e*x^4+1/3*a^2*g*x^3+2/3*a*b*d*x^3+1/2*x^2*a^2*f+a*b*c*x^2+a^2*e*x+a^2*d*ln(
x)-a^2*c/x
```

Maxima [A]
time = 0.29, size = 149, normalized size = 1.01

$$\frac{1}{10}b^2hx^{10}+\frac{1}{9}b^2gx^9+\frac{1}{8}b^2fx^8+\frac{1}{7}(2abh+b^2e)x^7+\frac{1}{6}(b^2d+2abg)x^6+\frac{1}{5}(b^2c+2abf)x^5+\frac{1}{4}(a^2h+2abe)x^4+\frac{1}{3}(2abd+a^2g)x^3+a^2xe+a^2d\log(x)+\frac{1}{2}(2abc+a^2f)x^2-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="max
ima")
```

```
[Out] 1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 1/7*(2*a*b*h + b^2*e)*x^7
 + 1/6*(b^2*d + 2*a*b*g)*x^6 + 1/5*(b^2*c + 2*a*b*f)*x^5 + 1/4*(a^2*h + 2*a
*b*e)*x^4 + 1/3*(2*a*b*d + a^2*g)*x^3 + a^2*x*e + a^2*d*log(x) + 1/2*(2*a*b
*c + a^2*f)*x^2 - a^2*c/x
```

Fricas [A]
time = 0.39, size = 153, normalized size = 1.04

$$\frac{252\,b^2hx^{11} + 280\,b^2gx^{10} + 315\,b^2fx^9 + 360\,(b^2e + 2\,abh)x^8 + 420\,(b^2d + 2\,abg)x^7 + 504\,(b^2c + 2\,abf)x^6 + 630\,(2\,abe + a^2h)x^5 + 2520\,a^2ex^2 + 840\,(2\,abd + a^2g)x^4 + 2520\,a^2dx\log(x) + 1260\,(2\,abc + a^2f)x^3 - 2520\,a^2c}{2520\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fri
cas")
```

```
[Out] 1/2520*(252*b^2*h*x^11 + 280*b^2*g*x^10 + 315*b^2*f*x^9 + 360*(b^2*e + 2*a*
b*h)*x^8 + 420*(b^2*d + 2*a*b*g)*x^7 + 504*(b^2*c + 2*a*b*f)*x^6 + 630*(2*a
*b*e + a^2*h)*x^5 + 2520*a^2*e*x^2 + 840*(2*a*b*d + a^2*g)*x^4 + 2520*a^2*d
*x*log(x) + 1260*(2*a*b*c + a^2*f)*x^3 - 2520*a^2*c)/x
```

Sympy [A]
time = 0.13, size = 156, normalized size = 1.06

$$-\frac{a^2c}{x} + a^2d\log(x) + a^2ex + \frac{b^2fx^8}{8} + \frac{b^2gx^9}{9} + \frac{b^2hx^{10}}{10} + x^7\cdot\left(\frac{2abh}{7} + \frac{b^2e}{7}\right) + x^6\left(\frac{abg}{3} + \frac{b^2d}{6}\right) + x^5\cdot\left(\frac{2abf}{5} + \frac{b^2c}{5}\right) + x^4\left(\frac{a^2h}{4} + \frac{abe}{2}\right) + x^3\left(\frac{a^2g}{3} + \frac{2abd}{3}\right) + x^2\left(\frac{a^2f}{2} + abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a**2*c/x + a**2*d*log(x) + a**2*e*x + b**2*f*x**8/8 + b**2*g*x**9/9 + b**2
*h*x**10/10 + x**7*(2*a*b*h/7 + b**2*e/7) + x**6*(a*b*g/3 + b**2*d/6) + x**
5*(2*a*b*f/5 + b**2*c/5) + x**4*(a**2*h/4 + a*b*e/2) + x**3*(a**2*g/3 + 2*a
*b*d/3) + x**2*(a**2*f/2 + a*b*c)
```

Giac [A]
time = 0.58, size = 155, normalized size = 1.05

$$\frac{1}{10}\,b^2hx^{10} + \frac{1}{9}\,b^2gx^9 + \frac{1}{8}\,b^2fx^8 + \frac{2}{7}\,abhx^7 + \frac{1}{7}\,b^2x^7e + \frac{1}{6}\,b^2dx^6 + \frac{1}{3}\,abgx^6 + \frac{1}{5}\,b^2cx^5 + \frac{2}{5}\,abfx^5 + \frac{1}{4}\,a^2hx^4 + \frac{1}{2}\,abx^4e + \frac{2}{3}\,abdx^3 + \frac{1}{3}\,a^2gx^3 + abcx^2 + \frac{1}{2}\,a^2fx^2 + a^2xe + a^2d\log(|x|) - \frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="gia
c")
```

```
[Out] 1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 2/7*a*b*h*x^7 + 1/7*b^2*x
^7*e + 1/6*b^2*d*x^6 + 1/3*a*b*g*x^6 + 1/5*b^2*c*x^5 + 2/5*a*b*f*x^5 + 1/4*
a^2*h*x^4 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + 1/3*a^2*g*x^3 + a*b*c*x^2 + 1/2
*a^2*f*x^2 + a^2*x*e + a^2*d*log(abs(x)) - a^2*c/x
```

Mupad [B]
time = 0.10, size = 145, normalized size = 0.99

$$x^2\left(\frac{f\,a^2}{2}+b\,c\,a\right)+x^5\left(\frac{c\,b^2}{5}+\frac{2\,a\,f\,b}{5}\right)+x^3\left(\frac{g\,a^2}{3}+\frac{2\,b\,d\,a}{3}\right)+x^6\left(\frac{d\,b^2}{6}+\frac{a\,g\,b}{3}\right)+x^4\left(\frac{h\,a^2}{4}+\frac{b\,e\,a}{2}\right)+x^7\left(\frac{e\,b^2}{7}+\frac{2\,a\,h\,b}{7}\right)-\frac{a^2\,c}{x}+\frac{b^2\,f\,x^8}{8}+\frac{b^2\,g\,x^9}{9}+\frac{b^2\,h\,x^{10}}{10}+a^2\,d\,\ln\left(x\right)+a^2\,e\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x)
```

```
[Out] x^2*((a^2*f)/2 + a*b*c) + x^5*((b^2*c)/5 + (2*a*b*f)/5) + x^3*((a^2*g)/3 +
(2*a*b*d)/3) + x^6*((b^2*d)/6 + (a*b*g)/3) + x^4*((a^2*h)/4 + (a*b*e)/2) +
x^7*((b^2*e)/7 + (2*a*b*h)/7) - (a^2*c)/x + (b^2*f*x^8)/8 + (b^2*g*x^9)/9 +
 (b^2*h*x^10)/10 + a^2*d*log(x) + a^2*e*x
```

3.390 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

Optimal. Leaf size=147

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a(2bc+af)x+\frac{1}{2}a(2bd+ag)x^2+\frac{2}{3}abex^3+\frac{1}{4}b(bc+2af)x^4+\frac{1}{5}b(bd+2ag)x^5+\frac{1}{6}b^2ex^6+\frac{1}{7}b^2fx^7+\frac{1}{8}b^2g$$

```
[Out] -1/2*a^2*c/x^2-a^2*d/x+a*(a*f+2*b*c)*x+1/2*a*(a*g+2*b*d)*x^2+2/3*a*b*e*x^3+
1/4*b*(2*a*f+b*c)*x^4+1/5*b*(2*a*g+b*d)*x^5+1/6*b^2*e*x^6+1/7*b^2*f*x^7+1/8
*b^2*g*x^8+1/9*h*(b*x^3+a)^3/b+a^2*e*ln(x)
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1597, 1834}

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+\frac{1}{4}bx^4(2af+bc)+ax(af+2bc)+\frac{1}{5}bx^5(2ag+bd)+\frac{1}{2}ax^2(ag+2bd)+\frac{2}{3}abex^3+\frac{h(a+bx^3)^3}{9b}+\frac{1}{6}b^2ex^6+\frac{1}{7}b^2fx^7+\frac{1}{8}b^2gx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -1/2*(a^2*c)/x^2 - (a^2*d)/x + a*(2*b*c + a*f)*x + (a*(2*b*d + a*g)*x^2)/2
+ (2*a*b*e*x^3)/3 + (b*(b*c + 2*a*f)*x^4)/4 + (b*(b*d + 2*a*g)*x^5)/5 + (b^
2*e*x^6)/6 + (b^2*f*x^7)/7 + (b^2*g*x^8)/8 + (h*(a + b*x^3)^3)/(9*b) + a^2*
e*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx = \frac{h(a+bx^3)^3}{9b} + \int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+g$$
$$= \frac{h(a+bx^3)^3}{9b} + \int \Big(a(2bc+af) + \frac{a^2c}{x^3} + \frac{a^2d}{x^2} + \frac{a^2e}{x} +$$
$$= -\frac{a^2c}{2x^2} - \frac{a^2d}{x} + a(2bc+af)x + \frac{1}{2}a(2bd+ag)x^2 + \frac{2}{3}a$$

Mathematica [A]
time = 0.05, size = 127, normalized size = 0.86

$$\frac{a^2(-3c-6dx+x^3(6f+3gx+2hx^2))}{6x^2} + \frac{1}{30}abx(60c+x(30d+x(20e+15fx+12gx^2+10hx^3))) + \frac{b^2x^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))}{2520} + a^2e\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] (a^2*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (a*b*x*(60*c +
 x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))))/30 + (b^2*x^4*(630*c
+ x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x))))))/2520 + a^2*e*Log[
x]
```

Maple [A]
time = 0.33, size = 150, normalized size = 1.02

method	result
norman	$\frac{\left(\frac{1}{2}a^2g+abd\right)x^4+\left(\frac{1}{3}a^2h+\frac{2}{3}abe\right)x^5+\left(\frac{1}{2}abf+\frac{1}{4}b^2c\right)x^6+\left(\frac{2}{5}abg+\frac{1}{5}b^2d\right)x^7+\left(\frac{1}{3}abh+\frac{1}{6}b^2e\right)x^8+\left(a^2f+2abc\right)x^3-\frac{a^2c}{2}-a^2dx+\frac{b^2gx^{10}}{8}+b^2}{x^2}$
default	$\frac{b^2hx^9}{9}+\frac{b^2gx^8}{8}+\frac{b^2fx^7}{7}+\frac{abhx^6}{3}+\frac{b^2ex^6}{6}+\frac{2abgx^5}{5}+\frac{b^2dx^5}{5}+\frac{abfx^4}{2}+\frac{b^2cx^4}{4}+\frac{a^2hx^3}{3}+\frac{2abex^3}{3}+\frac{a^2gx^2}{2}$
risch	$\frac{b^2hx^9}{9}+\frac{b^2gx^8}{8}+\frac{b^2fx^7}{7}+\frac{abhx^6}{3}+\frac{b^2ex^6}{6}+\frac{2abgx^5}{5}+\frac{b^2dx^5}{5}+\frac{abfx^4}{2}+\frac{b^2cx^4}{4}+\frac{a^2hx^3}{3}+\frac{2abex^3}{3}+\frac{a^2gx^2}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/9*b^2*h*x^9+1/8*b^2*g*x^8+1/7*b^2*f*x^7+1/3*a*b*h*x^6+1/6*b^2*e*x^6+2/5*a
*b*g*x^5+1/5*b^2*d*x^5+1/2*a*b*f*x^4+1/4*b^2*c*x^4+1/3*a^2*h*x^3+2/3*a*b*e*
x^3+1/2*a^2*g*x^2+a*b*d*x^2+a^2*f*x+2*a*b*c*x-1/2*a^2*c/x^2+a^2*e*ln(x)-a^2
*d/x
```

Maxima [A]
time = 0.27, size = 149, normalized size = 1.01

$$\frac{1}{9}b^2hx^9 + \frac{1}{8}b^2gx^8 + \frac{1}{7}b^2fx^7 + \frac{1}{6}(2abh+b^2e)x^6 + \frac{1}{5}(b^2d+2abg)x^5 + \frac{1}{4}(b^2c+2abf)x^4 + \frac{1}{3}(a^2h+2abe)x^3 + a^2e\log(x) + \frac{1}{2}(2abd+a^2g)x^2 + (2abc+a^2f)x - \frac{2a^2dx+a^2c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="max
ima")
```

```
[Out] 1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/6*(2*a*b*h + b^2*e)*x^6 +
 1/5*(b^2*d + 2*a*b*g)*x^5 + 1/4*(b^2*c + 2*a*b*f)*x^4 + 1/3*(a^2*h + 2*a*b
*e)*x^3 + a^2*e*log(x) + 1/2*(2*a*b*d + a^2*g)*x^2 + (2*a*b*c + a^2*f)*x -
1/2*(2*a^2*d*x + a^2*c)/x^2
```

Fricas [A]
time = 0.38, size = 153, normalized size = 1.04

$$\frac{280 b^2 h x^{11} + 315 b^2 g x^{10} + 360 b^2 f x^9 + 420 (b^2 e + 2 abh) x^8 + 504 (b^2 d + 2 abg) x^7 + 630 (b^2 c + 2 abf) x^6 + 840 (2 abe + a^2 h) x^5 + 2520 a^2 e x^2 \log(x) + 1260 (2 abd + a^2 g) x^4 - 2520 a^2 d x + 2520 (2 abc + a^2 f) x^3 - 1260 a^2 c}{2520 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fri
cas")
```

```
[Out] 1/2520*(280*b^2*h*x^11 + 315*b^2*g*x^10 + 360*b^2*f*x^9 + 420*(b^2*e + 2*a*
b*h)*x^8 + 504*(b^2*d + 2*a*b*g)*x^7 + 630*(b^2*c + 2*a*b*f)*x^6 + 840*(2*a
*b*e + a^2*h)*x^5 + 2520*a^2*e*x^2*log(x) + 1260*(2*a*b*d + a^2*g)*x^4 - 25
20*a^2*d*x + 2520*(2*a*b*c + a^2*f)*x^3 - 1260*a^2*c)/x^2
```

Sympy [A]
time = 0.17, size = 158, normalized size = 1.07

$$a^2 e \log(x) + \frac{b^2 f x^7}{7} + \frac{b^2 g x^8}{8} + \frac{b^2 h x^9}{9} + x^6 \left(\frac{abh}{3} + \frac{b^2 e}{6}\right) + x^5 \cdot \left(\frac{2abg}{5} + \frac{b^2 d}{5}\right) + x^4 \left(\frac{abf}{2} + \frac{b^2 c}{4}\right) + x^3 \left(\frac{a^2 h}{3} + \frac{2abe}{3}\right) + x^2 \left(\frac{a^2 g}{2} + abd\right) + x (a^2 f + 2abc) + \frac{-a^2 c - 2a^2 dx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**2*e*log(x) + b**2*f*x**7/7 + b**2*g*x**8/8 + b**2*h*x**9/9 + x**6*(a*b*h
/3 + b**2*e/6) + x**5*(2*a*b*g/5 + b**2*d/5) + x**4*(a*b*f/2 + b**2*c/4) +
x**3*(a**2*h/3 + 2*a*b*e/3) + x**2*(a**2*g/2 + a*b*d) + x*(a**2*f + 2*a*b*c
) + (-a**2*c - 2*a**2*d*x)/(2*x**2)
```

Giac [A]
time = 0.57, size = 153, normalized size = 1.04

$$\frac{1}{9} b^2 h x^9 + \frac{1}{8} b^2 g x^8 + \frac{1}{7} b^2 f x^7 + \frac{1}{3} abh x^6 + \frac{1}{6} b^2 x^6 e + \frac{1}{5} b^2 d x^5 + \frac{2}{5} abg x^5 + \frac{1}{4} b^2 c x^4 + \frac{1}{2} abf x^4 + \frac{1}{3} a^2 h x^3 + \frac{2}{3} abx^3 e + abd x^2 + \frac{1}{2} a^2 g x^2 + 2 abcx + a^2 f x + a^2 e \log(|x|) - \frac{2 a^2 d x + a^2 c}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="gia
c")
```

```
[Out] 1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/3*a*b*h*x^6 + 1/6*b^2*x^6
*e + 1/5*b^2*d*x^5 + 2/5*a*b*g*x^5 + 1/4*b^2*c*x^4 + 1/2*a*b*f*x^4 + 1/3*a^
2*h*x^3 + 2/3*a*b*x^3*e + a*b*d*x^2 + 1/2*a^2*g*x^2 + 2*a*b*c*x + a^2*f*x +
 a^2*e*log(abs(x)) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

Mupad [B]
time = 5.01, size = 145, normalized size = 0.99

$$x\,\left(f\,a^2+2\,b\,c\,a\right)-\frac{\frac{a^2\,c}{2}+a^2\,d\,x}{x^2}+x^4\,\left(\frac{c\,b^2}{4}+\frac{a\,f\,b}{2}\right)+x^2\,\left(\frac{g\,a^2}{2}+b\,d\,a\right)+x^5\,\left(\frac{d\,b^2}{5}+\frac{2\,a\,g\,b}{5}\right)+x^3\,\left(\frac{h\,a^2}{3}+\frac{2\,b\,e\,a}{3}\right)+x^6\,\left(\frac{e\,b^2}{6}+\frac{a\,h\,b}{3}\right)+\frac{b^2\,f\,x^7}{7}+\frac{b^2\,g\,x^8}{8}+\frac{b^2\,h\,x^9}{9}+a^2\,e\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x)
```

```
[Out] x*(a^2*f + 2*a*b*c) - ((a^2*c)/2 + a^2*d*x)/x^2 + x^4*((b^2*c)/4 + (a*b*f)/
2) + x^2*((a^2*g)/2 + a*b*d) + x^5*((b^2*d)/5 + (2*a*b*g)/5) + x^3*((a^2*h)
/3 + (2*a*b*e)/3) + x^6*((b^2*e)/6 + (a*b*h)/3) + (b^2*f*x^7)/7 + (b^2*g*x^
8)/8 + (b^2*h*x^9)/9 + a^2*e*log(x)
```

3.391 $\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} \, dx$

Optimal. Leaf size=152

$$-\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+a(2bd+ag)x+\frac{1}{2}a(2be+ah)x^2+\frac{1}{3}b(bc+2af)x^3+\frac{1}{4}b(bd+2ag)x^4+\frac{1}{5}b(be+2ah)x^5+\frac{1}{6}b^2fx^6+\frac{1}{7}$$

```
[Out] -1/3*a^2*c/x^3-1/2*a^2*d/x^2-a^2*e/x+a*(a*g+2*b*d)*x+1/2*a*(a*h+2*b*e)*x^2+
1/3*b*(2*a*f+b*c)*x^3+1/4*b*(2*a*g+b*d)*x^4+1/5*b*(2*a*h+b*e)*x^5+1/6*b^2*f
*x^6+1/7*b^2*g*x^7+1/8*b^2*h*x^8+a*(a*f+2*b*c)*ln(x)
```

Rubi [A]
time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1834}

$$-\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+\frac{1}{3}bx^3(2af+bc)+a\log(x)(af+2bc)+\frac{1}{4}bx^4(2ag+bd)+ax(ag+2bd)+\frac{1}{5}bx^5(2ah+be)+\frac{1}{2}ax^2(ah+2be)+\frac{1}{6}b^2fx^6+\frac{1}{7}b^2gx^7+\frac{1}{8}b^2hx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/3*(a^2*c)/x^3 - (a^2*d)/(2*x^2) - (a^2*e)/x + a*(2*b*d + a*g)*x + (a*(2*
b*e + a*h)*x^2)/2 + (b*(b*c + 2*a*f)*x^3)/3 + (b*(b*d + 2*a*g)*x^4)/4 + (b*
(b*e + 2*a*h)*x^5)/5 + (b^2*f*x^6)/6 + (b^2*g*x^7)/7 + (b^2*h*x^8)/8 + a*(2
*b*c + a*f)*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} \, dx &= \int \left(a(2bd+ag)+\frac{a^2c}{x^4}+\frac{a^2d}{x^3}+\frac{a^2e}{x^2}+\frac{a(2bc+af)}{x}+\right. \\ &= -\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+a(2bd+ag)x+\frac{1}{2}a(2be+ah)x^2\end{aligned}$$

Mathematica [A]
time = 0.05, size = 123, normalized size = 0.81

$$-\frac{a^2(2c+3x(d+2ex-x^3(2g+hx)))}{6x^3}+\frac{1}{30}abx(60d+x(30e+x(20f+15gx+12hx^2)))+\frac{1}{840}b^2x^3(280c+x(210d+x(168e+140fx+120gx^2+105hx^3)))+a(2bc+af)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/6*(a^2*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/x^3 + (a*b*x*(60*d + x
*(30*e + x*(20*f + 15*g*x + 12*h*x^2))))/30 + (b^2*x^3*(280*c + x*(210*d +
x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^3))))/840 + a*(2*b*c + a*f)*Log[x]
```

Maple [A]
time = 0.34, size = 147, normalized size = 0.97

method	result
default	$\frac{b^2 h x^8}{8}+\frac{b^2 g x^7}{7}+\frac{b^2 f x^6}{6}+\frac{2abh x^5}{5}+\frac{b^2 e x^5}{5}+\frac{abg x^4}{2}+\frac{b^2 d x^4}{4}+\frac{2abf x^3}{3}+\frac{b^2 c x^3}{3}+\frac{a^2 h x^2}{2}+x^2 abe+a^2 gx$
norman	$\frac{(\frac{1}{2}a^2h+abe)x^5+(\frac{2}{3}abf+\frac{1}{3}b^2c)x^6+(\frac{1}{2}abg+\frac{1}{4}b^2d)x^7+(\frac{2}{5}abh+\frac{1}{5}b^2e)x^8+(a^2g+2abd)x^4-\frac{a^2c}{3}-\frac{a^2dx}{2}-a^2e x^2+\frac{b^2 g x^{10}}{7}+\frac{b^2 h x^{11}}{8}+f x}{x^3}$
risch	$\frac{b^2 h x^8}{8}+\frac{b^2 g x^7}{7}+\frac{b^2 f x^6}{6}+\frac{2abh x^5}{5}+\frac{b^2 e x^5}{5}+\frac{abg x^4}{2}+\frac{b^2 d x^4}{4}+\frac{2abf x^3}{3}+\frac{b^2 c x^3}{3}+\frac{a^2 h x^2}{2}+x^2 abe+a^2 gx$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/8*b^2*h*x^8+1/7*b^2*g*x^7+1/6*b^2*f*x^6+2/5*a*b*h*x^5+1/5*b^2*e*x^5+1/2*a
*b*g*x^4+1/4*b^2*d*x^4+2/3*a*b*f*x^3+1/3*b^2*c*x^3+1/2*a^2*h*x^2+x^2*a*b*e+
a^2*g*x+2*a*b*d*x-1/2*a^2*d/x^2-1/3*a^2*c/x^3+a*(a*f+2*b*c)*ln(x)-a^2*e/x
```

Maxima [A]
time = 0.28, size = 150, normalized size = 0.99

$$\frac{1}{8}\,b^2hx^8+\frac{1}{7}\,b^2gx^7+\frac{1}{6}\,b^2fx^6+\frac{1}{5}\,(2\,abh+b^2e)x^5+\frac{1}{4}\,(b^2d+2\,abg)x^4+\frac{1}{3}\,(b^2c+2\,abf)x^3+\frac{1}{2}\,(a^2h+2\,abe)x^2+(2\,abd+a^2g)x+(2\,abc+a^2f)\log(x)-\frac{6\,a^2x^2e+3\,a^2dx+2\,a^2c}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="max
ima")
```

```
[Out] 1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 1/5*(2*a*b*h + b^2*e)*x^5 +
 1/4*(b^2*d + 2*a*b*g)*x^4 + 1/3*(b^2*c + 2*a*b*f)*x^3 + 1/2*(a^2*h + 2*a*b
*e)*x^2 + (2*a*b*d + a^2*g)*x + (2*a*b*c + a^2*f)*log(x) - 1/6*(6*a^2*x^2*e
 + 3*a^2*d*x + 2*a^2*c)/x^3
```

Fricas [A]
time = 0.41, size = 153, normalized size = 1.01

$$\frac{105\,b^2hx^{11}+120\,b^2gx^{10}+140\,b^2fx^9+168\,(b^2e+2\,abh)x^8+210\,(b^2d+2\,abg)x^7+280\,(b^2c+2\,abf)x^6+420\,(2\,abe+a^2h)x^5-840\,a^2ex^2+840\,(2\,abd+a^2g)x^4+840\,(2\,abc+a^2f)x^3\log(x)-420\,a^2dx-280\,a^2c}{840\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fri
cas")
```

```
[Out] 1/840*(105*b^2*h*x^11 + 120*b^2*g*x^10 + 140*b^2*f*x^9 + 168*(b^2*e + 2*a*b
*h)*x^8 + 210*(b^2*d + 2*a*b*g)*x^7 + 280*(b^2*c + 2*a*b*f)*x^6 + 420*(2*a*
b*e + a^2*h)*x^5 - 840*a^2*e*x^2 + 840*(2*a*b*d + a^2*g)*x^4 + 840*(2*a*b*c
 + a^2*f)*x^3*log(x) - 420*a^2*d*x - 280*a^2*c)/x^3
```

Sympy [A]
time = 0.40, size = 158, normalized size = 1.04

$$a(af + 2bc)\log{(x)} + \frac{b^2fx^6}{6} + \frac{b^2gx^7}{7} + \frac{b^2hx^8}{8} + x^5\cdot\left(\frac{2abh}{5} + \frac{b^2e}{5}\right) + x^4\left(\frac{abg}{2} + \frac{b^2d}{4}\right) + x^3\cdot\left(\frac{2abf}{3} + \frac{b^2c}{3}\right) + x^2\left(\frac{a^2h}{2} + abe\right) + x(a^2g + 2abd) + \frac{-2a^2c - 3a^2dx - 6a^2ex^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a*(a*f + 2*b*c)*log(x) + b**2*f*x**6/6 + b**2*g*x**7/7 + b**2*h*x**8/8 + x*
*5*(2*a*b*h/5 + b**2*e/5) + x**4*(a*b*g/2 + b**2*d/4) + x**3*(2*a*b*f/3 + b
**2*c/3) + x**2*(a**2*h/2 + a*b*e) + x*(a**2*g + 2*a*b*d) + (-2*a**2*c - 3*
a**2*d*x - 6*a**2*e*x**2)/(6*x**3)
```

Giac [A]
time = 0.51, size = 153, normalized size = 1.01

$$\frac{1}{8}\,b^2hx^8 + \frac{1}{7}\,b^2gx^7 + \frac{1}{6}\,b^2fx^6 + \frac{2}{5}\,abhx^5 + \frac{1}{5}\,b^2x^5e + \frac{1}{4}\,b^2dx^4 + \frac{1}{2}\,abgx^4 + \frac{1}{3}\,b^2cx^3 + \frac{2}{3}\,abfx^3 + \frac{1}{2}\,a^2hx^2 + abx^2e + 2\,abdx + a^2gx + \left(2\,abc + a^2f\right)\log\left(|x|\right) - \frac{6\,a^2x^2e + 3\,a^2dx + 2\,a^2c}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="gia
c")
```

```
[Out] 1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 2/5*a*b*h*x^5 + 1/5*b^2*x^5
*e + 1/4*b^2*d*x^4 + 1/2*a*b*g*x^4 + 1/3*b^2*c*x^3 + 2/3*a*b*f*x^3 + 1/2*a^
2*h*x^2 + a*b*x^2*e + 2*a*b*d*x + a^2*g*x + (2*a*b*c + a^2*f)*log(abs(x)) -
 1/6*(6*a^2*x^2*e + 3*a^2*d*x + 2*a^2*c)/x^3
```

Mupad [B]
time = 0.08, size = 145, normalized size = 0.95

$$x\,\left(g\,a^2 + 2\,b\,d\,a\right) - \frac{e\,a^2\,x^2 + \frac{d\,a^2\,x}{2} + \frac{c\,a^2}{3}}{x^3} + x^3\,\left(\frac{c\,b^2}{3} + \frac{2\,a\,f\,b}{3}\right) + x^4\,\left(\frac{d\,b^2}{4} + \frac{a\,g\,b}{2}\right) + x^2\,\left(\frac{h\,a^2}{2} + b\,e\,a\right) + x^5\,\left(\frac{e\,b^2}{5} + \frac{2\,a\,h\,b}{5}\right) + \ln\left(x\right)\,\left(f\,a^2 + 2\,b\,c\,a\right) + \frac{b^2\,f\,x^6}{6} + \frac{b^2\,g\,x^7}{7} + \frac{b^2\,h\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x)
```

```
[Out] x*(a^2*g + 2*a*b*d) - ((a^2*c)/3 + a^2*e*x^2 + (a^2*d*x)/2)/x^3 + x^3*((b^2
*c)/3 + (2*a*b*f)/3) + x^4*((b^2*d)/4 + (a*b*g)/2) + x^2*((a^2*h)/2 + a*b*e
) + x^5*((b^2*e)/5 + (2*a*b*h)/5) + log(x)*(a^2*f + 2*a*b*c) + (b^2*f*x^6)/
6 + (b^2*g*x^7)/7 + (b^2*h*x^8)/8
```

3.392 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$

Optimal. Leaf size=152

$$-\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}-\frac{a(2bc+af)}{x}+a(2be+ah)x+\frac{1}{2}b(bc+2af)x^2+\frac{1}{3}b(bd+2ag)x^3+\frac{1}{4}b(be+2ah)x^4+\frac{1}{5}b^2fx^5+\frac{1}{6}$$

```
[Out] -1/4*a^2*c/x^4-1/3*a^2*d/x^3-1/2*a^2*e/x^2-a*(a*f+2*b*c)/x+a*(a*h+2*b*e)*x+
1/2*b*(2*a*f+b*c)*x^2+1/3*b*(2*a*g+b*d)*x^3+1/4*b*(2*a*h+b*e)*x^4+1/5*b^2*f
*x^5+1/6*b^2*g*x^6+1/7*b^2*h*x^7+a*(a*g+2*b*d)*ln(x)
```

Rubi [A]
time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1834}

$$-\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}+\frac{1}{2}bx^2(2af+bc)-\frac{a(af+2bc)}{x}+\frac{1}{3}bx^3(2ag+bd)+a\log(x)(ag+2bd)+\frac{1}{4}bx^4(2ah+be)+ax(ah+2be)+\frac{1}{5}b^2fx^5+\frac{1}{6}b^2gx^6+\frac{1}{7}b^2hx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -1/4*(a^2*c)/x^4 - (a^2*d)/(3*x^3) - (a^2*e)/(2*x^2) - (a*(2*b*c + a*f))/x
+ a*(2*b*e + a*h)*x + (b*(b*c + 2*a*f)*x^2)/2 + (b*(b*d + 2*a*g)*x^3)/3 + (
b*(b*e + 2*a*h)*x^4)/4 + (b^2*f*x^5)/5 + (b^2*g*x^6)/6 + (b^2*h*x^7)/7 + a*
(2*b*d + a*g)*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx = \int \left(a(2be+ah)+\frac{a^2c}{x^5}+\frac{a^2d}{x^4}+\frac{a^2e}{x^3}+\frac{a(2bc+af)}{x^2}\right.$$

$$= -\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}-\frac{a(2bc+af)}{x}+a(2be+ah)x+$$

Mathematica [A]
time = 0.05, size = 125, normalized size = 0.82

$$-\frac{2abc}{x}-\frac{a^2(3c+4dx+6x^2(e+2fx-2hx^3))}{12x^4}+\frac{1}{6}abx(12e+x(6f+x(4g+3hx)))+\frac{1}{420}b^2x^2(210c+x(140d+x(105e+84fx+70gx^2+60hx^3)))+a(2bd+ag)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] (-2*a*b*c)/x - (a^2*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)))/(12*x^4) +
 (a*b*x*(12*e + x*(6*f + x*(4*g + 3*h*x))))/6 + (b^2*x^2*(210*c + x*(140*d
+ x*(105*e + 84*f*x + 70*g*x^2 + 60*h*x^3))))/420 + a*(2*b*d + a*g)*Log[x]
```

Maple [A]
time = 0.32, size = 144, normalized size = 0.95

method	result
default	$\frac{b^2 h x^7}{7}+\frac{b^2 g x^6}{6}+\frac{f x^5 b^2}{5}+\frac{abh x^4}{2}+\frac{b^2 e x^4}{4}+\frac{2abg x^3}{3}+\frac{b^2 d x^3}{3}+abf x^2+\frac{b^2 c x^2}{2}+a^2 hx+2abex-\frac{a^2 c}{4x^4}-$
norman	$\frac{(abf+\frac{1}{2}b^2 c)x^6+(\frac{2}{3}abg+\frac{1}{3}b^2 d)x^7+(\frac{1}{2}abh+\frac{1}{4}b^2 e)x^8+(-a^2 f-2abc)x^3+(a^2 h+2abe)x^5-\frac{a^2 c}{4}-\frac{a^2 dx}{3}-\frac{a^2 e x^2}{2}+\frac{b^2 g x^{10}}{6}+\frac{b^2 h x^{11}}{7}+\frac{f x^9 b}{5}}{x^4}$
risch	$\frac{b^2 h x^7}{7}+\frac{b^2 g x^6}{6}+\frac{f x^5 b^2}{5}+\frac{abh x^4}{2}+\frac{b^2 e x^4}{4}+\frac{2abg x^3}{3}+\frac{b^2 d x^3}{3}+abf x^2+\frac{b^2 c x^2}{2}+a^2 hx+2abex+(-a^2 f-2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/7*b^2*h*x^7+1/6*b^2*g*x^6+1/5*f*x^5*b^2+1/2*a*b*h*x^4+1/4*b^2*e*x^4+2/3*a
*b*g*x^3+1/3*b^2*d*x^3+a*b*f*x^2+1/2*b^2*c*x^2+a^2*h*x+2*a*b*e*x-1/4*a^2*c/
x^4-1/2*a^2*e/x^2-1/3*a^2*d/x^3+a*(a*g+2*b*d)*ln(x)-a*(a*f+2*b*c)/x
```

Maxima [A]
time = 0.27, size = 150, normalized size = 0.99

$$\frac{1}{7}\,b^2hx^7+\frac{1}{6}\,b^2gx^6+\frac{1}{5}\,b^2fx^5+\frac{1}{4}\,(2\,abh+b^2e)x^4+\frac{1}{3}\,(b^2d+2\,abg)x^3+\frac{1}{2}\,(b^2c+2\,abf)x^2+(a^2h+2\,abe)x+(2\,abd+a^2g)\log\left(x\right)-\frac{6\,a^2x^2e+4\,a^2dx+12\,(2\,abc+a^2f)x^3+3\,a^2c}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="max
ima")
```

```
[Out] 1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/4*(2*a*b*h + b^2*e)*x^4 +
 1/3*(b^2*d + 2*a*b*g)*x^3 + 1/2*(b^2*c + 2*a*b*f)*x^2 + (a^2*h + 2*a*b*e)*
x + (2*a*b*d + a^2*g)*log(x) - 1/12*(6*a^2*x^2*e + 4*a^2*d*x + 12*(2*a*b*c
+ a^2*f)*x^3 + 3*a^2*c)/x^4
```

Fricas [A]
time = 0.39, size = 153, normalized size = 1.01

$$\frac{60\,b^2hx^{11}+70\,b^2gx^{10}+84\,b^2fx^9+105\,(b^2e+2\,abh)x^8+140\,(b^2d+2\,abg)x^7+210\,(b^2c+2\,abf)x^6+420\,(2\,abe+a^2h)x^5+420\,(2\,abd+a^2g)x^4\log\left(x\right)-210\,a^2ex^2-140\,a^2dx-420\,(2\,abc+a^2f)x^3-105\,a^2c}{420\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fri
cas")
```

```
[Out] 1/420*(60*b^2*h*x^11 + 70*b^2*g*x^10 + 84*b^2*f*x^9 + 105*(b^2*e + 2*a*b*h)
*x^8 + 140*(b^2*d + 2*a*b*g)*x^7 + 210*(b^2*c + 2*a*b*f)*x^6 + 420*(2*a*b*e
 + a^2*h)*x^5 + 420*(2*a*b*d + a^2*g)*x^4*log(x) - 210*a^2*e*x^2 - 140*a^2*
d*x - 420*(2*a*b*c + a^2*f)*x^3 - 105*a^2*c)/x^4
```

Sympy [A]
time = 1.51, size = 156, normalized size = 1.03

$$a(ag+2bd)\log(x)+\frac{b^2fx^5}{5}+\frac{b^2gx^6}{6}+\frac{b^2hx^7}{7}+x^4\left(\frac{abh}{2}+\frac{b^2e}{4}\right)+x^3\cdot\left(\frac{2abg}{3}+\frac{b^2d}{3}\right)+x^2\left(abf+\frac{b^2c}{2}\right)+x\left(a^2h+2abe\right)+\frac{-3a^2c-4a^2dx-6a^2ex^2+x^3(-12a^2f-24abc)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a*(a*g + 2*b*d)*log(x) + b**2*f*x**5/5 + b**2*g*x**6/6 + b**2*h*x**7/7 + x*
*4*(a*b*h/2 + b**2*e/4) + x**3*(2*a*b*g/3 + b**2*d/3) + x**2*(a*b*f + b**2*
c/2) + x*(a**2*h + 2*a*b*e) + (-3*a**2*c - 4*a**2*d*x - 6*a**2*e*x**2 + x**
3*(-12*a**2*f - 24*a*b*c))/(12*x**4)
```

Giac [A]
time = 0.53, size = 152, normalized size = 1.00

$$\frac{1}{7}b^2hx^7+\frac{1}{6}b^2gx^6+\frac{1}{5}b^2fx^5+\frac{1}{2}abhx^4+\frac{1}{4}b^2x^4e+\frac{1}{3}b^2dx^3+\frac{2}{3}abgx^3+\frac{1}{2}b^2cx^2+abfx^2+a^2hx+2abxe+(2abd+a^2g)\log(|x|)-\frac{6a^2x^2e+4a^2dx+12(2abc+a^2f)x^3+3a^2c}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="gia
c")
```

```
[Out] 1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/2*a*b*h*x^4 + 1/4*b^2*x^4
*e + 1/3*b^2*d*x^3 + 2/3*a*b*g*x^3 + 1/2*b^2*c*x^2 + a*b*f*x^2 + a^2*h*x +
2*a*b*x*e + (2*a*b*d + a^2*g)*log(abs(x)) - 1/12*(6*a^2*x^2*e + 4*a^2*d*x +
 12*(2*a*b*c + a^2*f)*x^3 + 3*a^2*c)/x^4
```

Mupad [B]
time = 0.07, size = 145, normalized size = 0.95

$$x\left(h\,a^2+2\,b\,e\,a\right)-\frac{\frac{a^2\,c}{4}+x^3\,\left(f\,a^2+2\,b\,c\,a\right)+\frac{a^2\,e\,x^2}{2}+\frac{a^2\,d\,x}{3}}{x^4}+x^2\,\left(\frac{c\,b^2}{2}+a\,f\,b\right)+x^3\,\left(\frac{d\,b^2}{3}+\frac{2\,a\,g\,b}{3}\right)+x^4\,\left(\frac{e\,b^2}{4}+\frac{a\,h\,b}{2}\right)+\ln\left(x\right)\,\left(g\,a^2+2\,b\,d\,a\right)+\frac{b^2\,f\,x^5}{5}+\frac{b^2\,g\,x^6}{6}+\frac{b^2\,h\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x)
```

```
[Out] x*(a^2*h + 2*a*b*e) - ((a^2*c)/4 + x^3*(a^2*f + 2*a*b*c) + (a^2*e*x^2)/2 +
(a^2*d*x)/3)/x^4 + x^2*((b^2*c)/2 + a*b*f) + x^3*((b^2*d)/3 + (2*a*b*g)/3)
+ x^4*((b^2*e)/4 + (a*b*h)/2) + log(x)*(a^2*g + 2*a*b*d) + (b^2*f*x^5)/5 +
(b^2*g*x^6)/6 + (b^2*h*x^7)/7
```

3.393 $\int x^4(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx$

Optimal. Leaf size=223

$$\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2(3bc+af)x^8+\frac{1}{9}a^2(3bd+ag)x^9+\frac{1}{10}a^2(3be+ah)x^{10}+\frac{3}{11}ab(bc+af)x^{11}+\frac{1}{4}ab(bd+$$

```
[Out] 1/5*a^3*c*x^5+1/6*a^3*d*x^6+1/7*a^3*e*x^7+1/8*a^2*(a*f+3*b*c)*x^8+1/9*a^2*(
a*g+3*b*d)*x^9+1/10*a^2*(a*h+3*b*e)*x^10+3/11*a*b*(a*f+b*c)*x^11+1/4*a*b*(a
*g+b*d)*x^12+3/13*a*b*(a*h+b*e)*x^13+1/14*b^2*(3*a*f+b*c)*x^14+1/15*b^2*(3*
a*g+b*d)*x^15+1/16*b^2*(3*a*h+b*e)*x^16+1/17*b^3*f*x^17+1/18*b^3*g*x^18+1/1
9*b^3*h*x^19
```

Rubi [A]
time = 0.20, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1834}

$$\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2x^8(af+3bc)+\frac{1}{9}a^2x^9(ag+3bd)+\frac{1}{10}a^2x^{10}(ah+3be)+\frac{1}{14}b^2x^{14}(3af+bc)+\frac{1}{15}b^2x^{15}(3ag+bd)+\frac{1}{16}b^2x^{16}(3ah+be)+\frac{3}{11}abx^{11}(af+bc)+\frac{1}{4}abx^{12}(ag+bd)+\frac{3}{13}abx^{13}(ah+be)+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8 +
 (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/10 + (3*a*b*(b*c + a*
f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a*b*(b*e + a*h)*x^13)/13 + (b^2
*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a*h)*
x^16)/16 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^4\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx=\int\left(a^3cx^4+a^3dx^5+a^3ex^6+a^2(3bc+af)x^7+a^2(3$$
$$=\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2(3bc+af)x^8+\frac{1}{9}$$

Mathematica [A]
time = 0.03, size = 223, normalized size = 1.00

$$\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2(3bc+af)x^8+\frac{1}{9}a^2(3bd+ag)x^9+\frac{1}{10}a^2(3be+ah)x^{10}+\frac{3}{11}ab(bc+af)x^{11}+\frac{1}{4}ab(bd+ag)x^{12}+\frac{3}{13}ab(be+ah)x^{13}+\frac{1}{14}b^2(bc+3af)x^{14}+\frac{1}{15}b^2(bd+3ag)x^{15}+\frac{1}{16}b^2(be+3ah)x^{16}+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8 +
 (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/10 + (3*a*b*(b*c + a*
f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a*b*(b*e + a*h)*x^13)/13 + (b^2
*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a*h)*
x^16)/16 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

Maple [A]
time = 2.01, size = 224, normalized size = 1.00

method	result
norman	$\frac{a^{3} c \,x^{5}}{5}+\frac{a^{3} d \,x^{6}}{6}+\frac{a^{3} e \,x^{7}}{7}+\left(\frac{1}{8} a^{3} f +\frac{3}{8} c \,a^{2} b \right) x^{8}+\left(\frac{1}{9} a^{3} g +\frac{1}{3} d \,a^{2} b \right) x^{9}+\left(\frac{1}{10} a^{3} h +\frac{3}{10} a^{2} b e \right) x^{10}+\left(\frac{3}{11} a\right.$
default	$\frac{b^{3} h \,x^{19}}{19}+\frac{b^{3} g \,x^{18}}{18}+\frac{b^{3} f \,x^{17}}{17}+\frac{\left(3 a \,b^{2} h +e \,b^{3}\right) x^{16}}{16}+\frac{\left(3 a \,b^{2} g +b^{3} d \right) x^{15}}{15}+\frac{\left(3 a \,b^{2} f +b^{3} c \right) x^{14}}{14}+\frac{\left(3 a^{2} b h +3 a \,b^{2} e \right) x^{13}}{13}+\frac{(3a^{2}b}{}$
gosper	$\frac{1}{5} a^{3} c \,x^{5}+\frac{1}{6} a^{3} d \,x^{6}+\frac{1}{7} a^{3} e \,x^{7}+\frac{1}{8} x^{8} a^{3} f +\frac{3}{8} x^{8} c \,a^{2} b +\frac{1}{9} x^{9} a^{3} g +\frac{1}{3} a^{2} b d \,x^{9}+\frac{1}{10} x^{10} a^{3} h +\frac{3}{10} a^{2} b e \,x^{10}$
risch	$\frac{1}{5} a^{3} c \,x^{5}+\frac{1}{6} a^{3} d \,x^{6}+\frac{1}{7} a^{3} e \,x^{7}+\frac{1}{8} x^{8} a^{3} f +\frac{3}{8} x^{8} c \,a^{2} b +\frac{1}{9} x^{9} a^{3} g +\frac{1}{3} a^{2} b d \,x^{9}+\frac{1}{10} x^{10} a^{3} h +\frac{3}{10} a^{2} b e \,x^{10}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/19*b^3*h*x^19+1/18*b^3*g*x^18+1/17*b^3*f*x^17+1/16*(3*a*b^2*h+b^3*e)*x^16
+1/15*(3*a*b^2*g+b^3*d)*x^15+1/14*(3*a*b^2*f+b^3*c)*x^14+1/13*(3*a^2*b*h+3*
a*b^2*e)*x^13+1/12*(3*a^2*b*g+3*a*b^2*d)*x^12+1/11*(3*a^2*b*f+3*a*b^2*c)*x^
11+1/10*(a^3*h+3*a^2*b*e)*x^10+1/9*(a^3*g+3*a^2*b*d)*x^9+1/8*(a^3*f+3*a^2*b
*c)*x^8+1/7*a^3*e*x^7+1/6*a^3*d*x^6+1/5*a^3*c*x^5
```

Maxima [A]
time = 0.27, size = 221, normalized size = 0.99

$$\frac{1}{19} b^{3} h x^{19} + \frac{1}{18} b^{3} g x^{18} + \frac{1}{17} b^{3} f x^{17} + \frac{1}{16} \left(3 a b^{2} h + b^{3} e\right) x^{16} + \frac{1}{15} \left(b^{3} d + 3 a b^{2} g\right) x^{15} + \frac{1}{14} \left(b^{3} c + 3 a b^{2} f\right) x^{14} + \frac{3}{13} \left(a^{2} b h + a b^{2} e\right) x^{13} + \frac{1}{4} \left(a b^{2} d + a^{2} b g\right) x^{12} + \frac{3}{11} \left(a b^{2} c + a^{2} b f\right) x^{11} + \frac{1}{10} \left(a^{3} h + 3 a^{2} b e\right) x^{10} + \frac{1}{7} a^{3} x^{7} e + \frac{1}{6} a^{3} d x^{6} + \frac{1}{9} \left(3 a^{2} b d + a^{3} g\right) x^{9} + \frac{1}{5} a^{3} c x^{5} + \frac{1}{8} \left(3 a^{2} b c + a^{3} f\right) x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 1/16*(3*a*b^2*h + b^3
*e)*x^16 + 1/15*(b^3*d + 3*a*b^2*g)*x^15 + 1/14*(b^3*c + 3*a*b^2*f)*x^14 +
3/13*(a^2*b*h + a*b^2*e)*x^13 + 1/4*(a*b^2*d + a^2*b*g)*x^12 + 3/11*(a*b^2*
c + a^2*b*f)*x^11 + 1/10*(a^3*h + 3*a^2*b*e)*x^10 + 1/7*a^3*x^7*e + 1/6*a^3
*d*x^6 + 1/9*(3*a^2*b*d + a^3*g)*x^9 + 1/5*a^3*c*x^5 + 1/8*(3*a^2*b*c + a^3
*f)*x^8
```

Fricas [A]
time = 0.36, size = 217, normalized size = 0.97

$$\frac{1}{19}\,b^{3}hx^{19}+\frac{1}{18}\,b^{3}gx^{18}+\frac{1}{17}\,b^{3}fx^{17}+\frac{1}{16}\,(b^{3}e+3\,ab^{2}h)x^{16}+\frac{1}{15}\,(b^{3}d+3\,ab^{2}g)x^{15}+\frac{1}{14}\,(b^{3}c+3\,ab^{2}f)x^{14}+\frac{3}{13}\,(ab^{2}e+a^{2}bh)x^{13}+\frac{1}{4}\,(ab^{2}d+a^{2}bg)x^{12}+\frac{3}{11}\,(ab^{2}c+a^{2}bf)x^{11}+\frac{1}{7}\,a^{3}ex^{7}+\frac{1}{10}\,(3\,a^{2}be+a^{3}h)x^{10}+\frac{1}{6}\,a^{3}dx^{6}+\frac{1}{9}\,(3\,a^{2}bd+a^{3}g)x^{9}+\frac{1}{5}\,a^{3}cx^{5}+\frac{1}{8}\,(3\,a^{2}bc+a^{3}f)x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 1/16*(b^3*e + 3*a*b^2
*h)*x^16 + 1/15*(b^3*d + 3*a*b^2*g)*x^15 + 1/14*(b^3*c + 3*a*b^2*f)*x^14 +
3/13*(a*b^2*e + a^2*b*h)*x^13 + 1/4*(a*b^2*d + a^2*b*g)*x^12 + 3/11*(a*b^2*
c + a^2*b*f)*x^11 + 1/7*a^3*e*x^7 + 1/10*(3*a^2*b*e + a^3*h)*x^10 + 1/6*a^3
*d*x^6 + 1/9*(3*a^2*b*d + a^3*g)*x^9 + 1/5*a^3*c*x^5 + 1/8*(3*a^2*b*c + a^3
*f)*x^8
```

Sympy [A]
time = 0.03, size = 246, normalized size = 1.10

$$\frac{a^{3}cx^{5}}{5}+\frac{a^{3}dx^{6}}{6}+\frac{a^{3}ex^{7}}{7}+\frac{b^{3}fx^{17}}{17}+\frac{b^{3}gx^{18}}{18}+\frac{b^{3}hx^{19}}{19}+x^{16}\cdot\left(\frac{3ab^{2}h}{16}+\frac{b^{3}e}{16}\right)+x^{15}\left(\frac{ab^{2}g}{5}+\frac{b^{3}d}{15}\right)+x^{14}\cdot\left(\frac{3ab^{2}f}{14}+\frac{b^{3}c}{14}\right)+x^{13}\cdot\left(\frac{3a^{2}bh}{13}+\frac{3ab^{2}e}{13}\right)+x^{12}\left(\frac{a^{2}bg}{4}+\frac{ab^{2}d}{4}\right)+x^{11}\cdot\left(\frac{3a^{2}bf}{11}+\frac{3ab^{2}c}{11}\right)+x^{10}\left(\frac{a^{3}h}{10}+\frac{3a^{2}be}{10}\right)+x^{9}\left(\frac{a^{3}g}{9}+\frac{a^{2}bd}{3}\right)+x^{8}\left(\frac{a^{3}f}{8}+\frac{3a^{2}bc}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**5/5 + a**3*d*x**6/6 + a**3*e*x**7/7 + b**3*f*x**17/17 + b**3*g*x*
*18/18 + b**3*h*x**19/19 + x**16*(3*a*b**2*h/16 + b**3*e/16) + x**15*(a*b**
2*g/5 + b**3*d/15) + x**14*(3*a*b**2*f/14 + b**3*c/14) + x**13*(3*a**2*b*h/
13 + 3*a*b**2*e/13) + x**12*(a**2*b*g/4 + a*b**2*d/4) + x**11*(3*a**2*b*f/1
1 + 3*a*b**2*c/11) + x**10*(a**3*h/10 + 3*a**2*b*e/10) + x**9*(a**3*g/9 + a
**2*b*d/3) + x**8*(a**3*f/8 + 3*a**2*b*c/8)
```

Giac [A]
time = 0.50, size = 233, normalized size = 1.04

$$\frac{1}{19}\,b^{3}hx^{19}+\frac{1}{18}\,b^{3}gx^{18}+\frac{1}{17}\,b^{3}fx^{17}+\frac{3}{16}\,ab^{2}hx^{16}+\frac{1}{16}\,b^{3}x^{16}e+\frac{1}{15}\,b^{3}dx^{15}+\frac{1}{5}\,ab^{2}gx^{15}+\frac{1}{14}\,b^{3}cx^{14}+\frac{3}{14}\,ab^{2}fx^{14}+\frac{3}{13}\,a^{2}bhx^{13}+\frac{3}{13}\,ab^{2}x^{13}e+\frac{1}{4}\,ab^{2}dx^{12}+\frac{1}{4}\,a^{2}bgx^{12}+\frac{3}{11}\,ab^{2}cx^{11}+\frac{3}{11}\,a^{2}bfx^{11}+\frac{1}{10}\,a^{3}hx^{10}+\frac{3}{10}\,a^{2}bx^{10}e+\frac{1}{3}\,a^{2}bdx^{9}+\frac{1}{9}\,a^{3}gx^{9}+\frac{3}{8}\,a^{2}bcx^{8}+\frac{1}{8}\,a^{3}fx^{8}+\frac{1}{7}\,a^{3}x^{7}e+\frac{1}{6}\,a^{3}dx^{6}+\frac{1}{5}\,a^{3}cx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 3/16*a*b^2*h*x^16 + 1
/16*b^3*x^16*e + 1/15*b^3*d*x^15 + 1/5*a*b^2*g*x^15 + 1/14*b^3*c*x^14 + 3/1
4*a*b^2*f*x^14 + 3/13*a^2*b*h*x^13 + 3/13*a*b^2*x^13*e + 1/4*a*b^2*d*x^12 +
 1/4*a^2*b*g*x^12 + 3/11*a*b^2*c*x^11 + 3/11*a^2*b*f*x^11 + 1/10*a^3*h*x^10
 + 3/10*a^2*b*x^10*e + 1/3*a^2*b*d*x^9 + 1/9*a^3*g*x^9 + 3/8*a^2*b*c*x^8 +
1/8*a^3*f*x^8 + 1/7*a^3*x^7*e + 1/6*a^3*d*x^6 + 1/5*a^3*c*x^5
```

Mupad [B]

time = 0.17, size = 205, normalized size = 0.92

$$x^8\left(\frac{f\,a^3}{8}+\frac{3\,b\,c\,a^2}{8}\right)+x^{14}\left(\frac{c\,b^3}{14}+\frac{3\,a\,f\,b^2}{14}\right)+x^9\left(\frac{g\,a^3}{9}+\frac{b\,d\,a^2}{3}\right)+x^{15}\left(\frac{d\,b^3}{15}+\frac{a\,g\,b^2}{5}\right)+x^{10}\left(\frac{h\,a^3}{10}+\frac{3\,b\,e\,a^2}{10}\right)+x^{16}\left(\frac{e\,b^3}{16}+\frac{3\,a\,h\,b^2}{16}\right)+\frac{a^3\,c\,x^5}{5}+\frac{a^3\,d\,x^6}{6}+\frac{a^3\,e\,x^7}{7}+\frac{b^3\,f\,x^{17}}{17}+\frac{b^3\,g\,x^{18}}{18}+\frac{b^3\,h\,x^{19}}{19}+\frac{3\,a\,b\,x^{11}\,(b\,c+a\,f)}{11}+\frac{a\,b\,x^{12}\,(b\,d+a\,g)}{4}+\frac{3\,a\,b\,x^{13}\,(b\,e+a\,h)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^8*((a^3*f)/8 + (3*a^2*b*c)/8) + x^14*((b^3*c)/14 + (3*a*b^2*f)/14) + x^9*
((a^3*g)/9 + (a^2*b*d)/3) + x^15*((b^3*d)/15 + (a*b^2*g)/5) + x^10*((a^3*h)
/10 + (3*a^2*b*e)/10) + x^16*((b^3*e)/16 + (3*a*b^2*h)/16) + (a^3*c*x^5)/5
+ (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*
h*x^19)/19 + (3*a*b*x^11*(b*c + a*f))/11 + (a*b*x^12*(b*d + a*g))/4 + (3*a*
b*x^13*(b*e + a*h))/13
```

3.394 $\int x^3(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=223

$$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2(3bc+af)x^7+\frac{1}{8}a^2(3bd+ag)x^8+\frac{1}{9}a^2(3be+ah)x^9+\frac{3}{10}ab(bc+af)x^{10}+\frac{3}{11}ab(bd+a$$

```
[Out] 1/4*a^3*c*x^4+1/5*a^3*d*x^5+1/6*a^3*e*x^6+1/7*a^2*(a*f+3*b*c)*x^7+1/8*a^2*(
a*g+3*b*d)*x^8+1/9*a^2*(a*h+3*b*e)*x^9+3/10*a*b*(a*f+b*c)*x^10+3/11*a*b*(a*
g+b*d)*x^11+1/4*a*b*(a*h+b*e)*x^12+1/13*b^2*(3*a*f+b*c)*x^13+1/14*b^2*(3*a*
g+b*d)*x^14+1/15*b^2*(3*a*h+b*e)*x^15+1/16*b^3*f*x^16+1/17*b^3*g*x^17+1/18*
b^3*h*x^18
```

Rubi [A]
time = 0.15, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{1834\}$

$$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2x^7(af+3bc)+\frac{1}{8}a^2x^8(ag+3bd)+\frac{1}{9}a^2x^9(ah+3be)+\frac{1}{13}b^2x^{13}(3af+bc)+\frac{1}{14}b^2x^{14}(3ag+bd)+\frac{1}{15}b^2x^{15}(3ah+be)+\frac{3}{10}abx^{10}(af+bc)+\frac{3}{11}abx^{11}(ag+bd)+\frac{1}{4}abx^{12}(ah+be)+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f)*x^7)/7 +
 (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9 + (3*a*b*(b*c + a*f)
*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*b*(b*e + a*h)*x^12)/4 + (b^2*(
b*c + 3*a*f)*x^13)/13 + (b^2*(b*d + 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h)*x^
15)/15 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^3\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx=\int \left(a^3cx^3+a^3dx^4+a^3ex^5+a^2(3bc+af)x^6+a^2(3\right.$$

$$=\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2(3bc+af)x^7+\frac{1}{8}$$

Mathematica [A]
time = 0.03, size = 223, normalized size = 1.00

$$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2(3bc+af)x^7+\frac{1}{8}a^2(3bd+ag)x^8+\frac{1}{9}a^2(3be+ah)x^9+\frac{3}{10}ab(bc+af)x^{10}+\frac{3}{11}ab(bd+ag)x^{11}+\frac{1}{4}ab(be+ah)x^{12}+\frac{1}{13}b^2(bc+3af)x^{13}+\frac{1}{14}b^2(bd+3ag)x^{14}+\frac{1}{15}b^2(be+3ah)x^{15}+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f)*x^7)/7 +
 (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9 + (3*a*b*(b*c + a*f)
*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*b*(b*e + a*h)*x^12)/4 + (b^2*(
b*c + 3*a*f)*x^13)/13 + (b^2*(b*d + 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h)*x^
15)/15 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

Maple [A]
time = 2.03, size = 224, normalized size = 1.00

method	result
norman	$\frac{a^{3} c \,x^{4}}{4}+\frac{a^{3} d \,x^{5}}{5}+\frac{a^{3} e \,x^{6}}{6}+\left(\frac{1}{7} a^{3} f+\frac{3}{7} c \,a^{2} b\right) x^{7}+\left(\frac{1}{8} a^{3} g+\frac{3}{8} d \,a^{2} b\right) x^{8}+\left(\frac{1}{9} a^{3} h+\frac{1}{3} a^{2} b e\right) x^{9}+$
default	$\frac{b^{3} h \,x^{18}}{18}+\frac{b^{3} g \,x^{17}}{17}+\frac{b^{3} f \,x^{16}}{16}+\frac{\left(3 a \,b^{2} h+e \,b^{3}\right) x^{15}}{15}+\frac{\left(3 a \,b^{2} g+b^{3} d\right) x^{14}}{14}+\frac{\left(3 a \,b^{2} f+b^{3} c\right) x^{13}}{13}+\frac{\left(3 a^{2} b h+3 a \,b^{2} e\right) x^{12}}{12}+$
gosper	$\frac{1}{4} a^{3} c \,x^{4}+\frac{1}{5} a^{3} d \,x^{5}+\frac{1}{6} a^{3} e \,x^{6}+\frac{1}{7} a^{3} f \,x^{7}+\frac{3}{7} x^{7} c \,a^{2} b+\frac{1}{8} x^{8} a^{3} g+\frac{3}{8} x^{8} d \,a^{2} b+\frac{1}{9} x^{9} a^{3} h+\frac{1}{3} x^{9} a^{2} b e+$
risch	$\frac{1}{4} a^{3} c \,x^{4}+\frac{1}{5} a^{3} d \,x^{5}+\frac{1}{6} a^{3} e \,x^{6}+\frac{1}{7} a^{3} f \,x^{7}+\frac{3}{7} x^{7} c \,a^{2} b+\frac{1}{8} x^{8} a^{3} g+\frac{3}{8} x^{8} d \,a^{2} b+\frac{1}{9} x^{9} a^{3} h+\frac{1}{3} x^{9} a^{2} b e+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/18*b^3*h*x^18+1/17*b^3*g*x^17+1/16*b^3*f*x^16+1/15*(3*a*b^2*h+b^3*e)*x^15
+1/14*(3*a*b^2*g+b^3*d)*x^14+1/13*(3*a*b^2*f+b^3*c)*x^13+1/12*(3*a^2*b*h+3*
a*b^2*e)*x^12+1/11*(3*a^2*b*g+3*a*b^2*d)*x^11+1/10*(3*a^2*b*f+3*a*b^2*c)*x^
10+1/9*(a^3*h+3*a^2*b*e)*x^9+1/8*(a^3*g+3*a^2*b*d)*x^8+1/7*(a^3*f+3*a^2*b*c
)*x^7+1/6*a^3*e*x^6+1/5*a^3*d*x^5+1/4*a^3*c*x^4
```

Maxima [A]
time = 0.34, size = 221, normalized size = 0.99

$$\frac{1}{18}\,b^{3}hx^{18}+\frac{1}{17}\,b^{3}gx^{17}+\frac{1}{16}\,b^{3}fx^{16}+\frac{1}{15}\,\left(3\,ab^{2}h+b^{3}e\right)x^{15}+\frac{1}{14}\,\left(b^{3}d+3\,ab^{2}g\right)x^{14}+\frac{1}{13}\,\left(b^{3}c+3\,ab^{2}f\right)x^{13}+\frac{1}{4}\,\left(a^{2}bh+ab^{2}e\right)x^{12}+\frac{3}{11}\,\left(ab^{2}d+a^{2}bg\right)x^{11}+\frac{3}{10}\,\left(ab^{2}c+a^{2}bf\right)x^{10}+\frac{1}{9}\,\left(a^{3}h+3\,a^{2}be\right)x^{9}+\frac{1}{6}\,a^{3}x^{6}e+\frac{1}{5}\,a^{3}dx^{5}+\frac{1}{8}\,\left(3\,a^{2}bd+a^{3}g\right)x^{8}+\frac{1}{4}\,a^{3}cx^{4}+\frac{1}{7}\,\left(3\,a^{2}bc+a^{3}f\right)x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*(3*a*b^2*h + b^3
*e)*x^15 + 1/14*(b^3*d + 3*a*b^2*g)*x^14 + 1/13*(b^3*c + 3*a*b^2*f)*x^13 +
1/4*(a^2*b*h + a*b^2*e)*x^12 + 3/11*(a*b^2*d + a^2*b*g)*x^11 + 3/10*(a*b^2*
c + a^2*b*f)*x^10 + 1/9*(a^3*h + 3*a^2*b*e)*x^9 + 1/6*a^3*x^6*e + 1/5*a^3*d
*x^5 + 1/8*(3*a^2*b*d + a^3*g)*x^8 + 1/4*a^3*c*x^4 + 1/7*(3*a^2*b*c + a^3*f
)*x^7
```

Fricas [A]
time = 0.37, size = 217, normalized size = 0.97

$$\frac{1}{18}b^3hx^{18}+\frac{1}{17}b^3gx^{17}+\frac{1}{16}b^3fx^{16}+\frac{1}{15}\left(b^3e+3ab^2h\right)x^{15}+\frac{1}{14}\left(b^3d+3ab^2g\right)x^{14}+\frac{1}{13}\left(b^3c+3ab^2f\right)x^{13}+\frac{1}{4}\left(ab^2e+a^2bh\right)x^{12}+\frac{3}{11}\left(ab^2d+a^2bg\right)x^{11}+\frac{3}{10}\left(ab^2c+a^2bf\right)x^{10}+\frac{1}{6}a^3ex^6+\frac{1}{9}\left(3a^2be+a^3h\right)x^9+\frac{1}{5}a^3dx^5+\frac{1}{8}\left(3a^2bd+a^3g\right)x^8+\frac{1}{4}a^3cx^4+\frac{1}{7}\left(3a^2bc+a^3f\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*(b^3*e + 3*a*b^2
*h)*x^15 + 1/14*(b^3*d + 3*a*b^2*g)*x^14 + 1/13*(b^3*c + 3*a*b^2*f)*x^13 +
1/4*(a*b^2*e + a^2*b*h)*x^12 + 3/11*(a*b^2*d + a^2*b*g)*x^11 + 3/10*(a*b^2*
c + a^2*b*f)*x^10 + 1/6*a^3*e*x^6 + 1/9*(3*a^2*b*e + a^3*h)*x^9 + 1/5*a^3*d
*x^5 + 1/8*(3*a^2*b*d + a^3*g)*x^8 + 1/4*a^3*c*x^4 + 1/7*(3*a^2*b*c + a^3*f
)*x^7
```

Sympy [A]
time = 0.02, size = 246, normalized size = 1.10

$$\frac{a^3cx^4}{4}+\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\frac{b^3fx^{16}}{16}+\frac{b^3gx^{17}}{17}+\frac{b^3hx^{18}}{18}+x^{15}\left(\frac{ab^2h}{5}+\frac{b^3e}{15}\right)+x^{14}\cdot\left(\frac{3ab^2g}{14}+\frac{b^3d}{14}\right)+x^{13}\cdot\left(\frac{3ab^2f}{13}+\frac{b^3c}{13}\right)+x^{12}\left(\frac{a^2bh}{4}+\frac{ab^2e}{4}\right)+x^{11}\cdot\left(\frac{3a^2bg}{11}+\frac{3ab^2d}{11}\right)+x^{10}\cdot\left(\frac{3a^2bf}{10}+\frac{3ab^2c}{10}\right)+x^9\left(\frac{a^3h}{9}+\frac{a^2be}{3}\right)+x^8\left(\frac{a^3g}{8}+\frac{3a^2bd}{8}\right)+x^7\left(\frac{a^3f}{7}+\frac{3a^2bc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + b**3*f*x**16/16 + b**3*g*x*
*17/17 + b**3*h*x**18/18 + x**15*(a*b**2*h/5 + b**3*e/15) + x**14*(3*a*b**2
*g/14 + b**3*d/14) + x**13*(3*a*b**2*f/13 + b**3*c/13) + x**12*(a**2*b*h/4
+ a*b**2*e/4) + x**11*(3*a**2*b*g/11 + 3*a*b**2*d/11) + x**10*(3*a**2*b*f/1
0 + 3*a*b**2*c/10) + x**9*(a**3*h/9 + a**2*b*e/3) + x**8*(a**3*g/8 + 3*a**2
*b*d/8) + x**7*(a**3*f/7 + 3*a**2*b*c/7)
```

Giac [A]
time = 0.59, size = 233, normalized size = 1.04

$$\frac{1}{18}b^3hx^{18}+\frac{1}{17}b^3gx^{17}+\frac{1}{16}b^3fx^{16}+\frac{1}{5}ab^2hx^{15}+\frac{1}{15}b^3x^{15}e+\frac{1}{14}b^3dx^{14}+\frac{3}{14}ab^2gx^{14}+\frac{1}{13}b^3cx^{13}+\frac{3}{13}ab^2fx^{13}+\frac{1}{4}a^2bhx^{12}+\frac{1}{4}ab^2x^{12}e+\frac{3}{11}ab^2dx^{11}+\frac{3}{11}a^2bgx^{11}+\frac{3}{10}ab^2cx^{10}+\frac{3}{10}a^2bfx^{10}+\frac{1}{9}a^3hx^9+\frac{1}{3}a^2bx^9e+\frac{3}{8}a^2bdx^8+\frac{1}{8}a^3gx^8+\frac{3}{7}a^2bcx^7+\frac{1}{7}a^3fx^7+\frac{1}{6}a^3x^6e+\frac{1}{5}a^3dx^5+\frac{1}{4}a^3cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/5*a*b^2*h*x^15 + 1/
15*b^3*x^15*e + 1/14*b^3*d*x^14 + 3/14*a*b^2*g*x^14 + 1/13*b^3*c*x^13 + 3/1
3*a*b^2*f*x^13 + 1/4*a^2*b*h*x^12 + 1/4*a*b^2*x^12*e + 3/11*a*b^2*d*x^11 +
3/11*a^2*b*g*x^11 + 3/10*a*b^2*c*x^10 + 3/10*a^2*b*f*x^10 + 1/9*a^3*h*x^9 +
 1/3*a^2*b*x^9*e + 3/8*a^2*b*d*x^8 + 1/8*a^3*g*x^8 + 3/7*a^2*b*c*x^7 + 1/7*
a^3*f*x^7 + 1/6*a^3*x^6*e + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

Mupad [B]
time = 5.16, size = 205, normalized size = 0.92

$$x^7\left(\frac{f\,a^3}{7}+\frac{3\,b\,c\,a^2}{7}\right)+x^{13}\left(\frac{c\,b^3}{13}+\frac{3\,a\,f\,b^2}{13}\right)+x^8\left(\frac{g\,a^3}{8}+\frac{3\,b\,d\,a^2}{8}\right)+x^{14}\left(\frac{d\,b^3}{14}+\frac{3\,a\,g\,b^2}{14}\right)+x^9\left(\frac{h\,a^3}{9}+\frac{b\,e\,a^2}{3}\right)+x^{15}\left(\frac{e\,b^3}{15}+\frac{a\,h\,b^2}{5}\right)+\frac{a^3\,c\,x^4}{4}+\frac{a^3\,d\,x^5}{5}+\frac{a^3\,e\,x^6}{6}+\frac{b^3\,f\,x^{16}}{16}+\frac{b^3\,g\,x^{17}}{17}+\frac{b^3\,h\,x^{18}}{18}+\frac{3\,a\,b\,x^{10}\,\left(b\,c+a\,f\right)}{10}+\frac{3\,a\,b\,x^{11}\,\left(b\,d+a\,g\right)}{11}+\frac{a\,b\,x^{12}\,\left(b\,e+a\,h\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^7*((a^3*f)/7 + (3*a^2*b*c)/7) + x^13*((b^3*c)/13 + (3*a*b^2*f)/13) + x^8*
((a^3*g)/8 + (3*a^2*b*d)/8) + x^14*((b^3*d)/14 + (3*a*b^2*g)/14) + x^9*((a^
3*h)/9 + (a^2*b*e)/3) + x^15*((b^3*e)/15 + (a*b^2*h)/5) + (a^3*c*x^4)/4 + (
a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x
^18)/18 + (3*a*b*x^10*(b*c + a*f))/10 + (3*a*b*x^11*(b*d + a*g))/11 + (a*b*
x^12*(b*e + a*h))/4
```

3.395 $\int x^2(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

Optimal. Leaf size=212

$$\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{7}a^2(3bd+ag)x^7+\frac{1}{8}a^2(3be+ah)x^8+\frac{1}{3}a^2bfx^9+\frac{3}{10}ab(bd+ag)x^{10}+\frac{3}{11}ab(be+ah)x^{11}$$

```
[Out] 1/4*a^3*d*x^4+1/5*a^3*e*x^5+1/6*a^3*f*x^6+1/7*a^2*(a*g+3*b*d)*x^7+1/8*a^2*(
a*h+3*b*e)*x^8+1/3*a^2*b*f*x^9+3/10*a*b*(a*g+b*d)*x^10+3/11*a*b*(a*h+b*e)*x
^11+1/4*a*b^2*f*x^12+1/13*b^2*(3*a*g+b*d)*x^13+1/14*b^2*(3*a*h+b*e)*x^14+1/
15*b^3*f*x^15+1/16*b^3*g*x^16+1/17*b^3*h*x^17+1/12*c*(b*x^3+a)^4/b
```

Rubi [A]

time = 0.12, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1596, 1864}

$$\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{7}a^2x^7(ag+3bd)+\frac{1}{8}a^2x^8(ah+3be)+\frac{1}{3}a^2bfx^9+\frac{1}{13}b^2x^{13}(3ag+bd)+\frac{1}{14}b^2x^{14}(3ah+be)+\frac{1}{4}ab^2fx^{12}+\frac{c(a+bx^3)^4}{12b}+\frac{3}{10}abx^{10}(ag+bd)+\frac{3}{11}abx^{11}(ah+be)+\frac{1}{15}b^3fx^{15}+\frac{1}{16}b^3gx^{16}+\frac{1}{17}b^3hx^{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^3*f*x^6)/6 + (a^2*(3*b*d + a*g)*x^7)/7 +
 (a^2*(3*b*e + a*h)*x^8)/8 + (a^2*b*f*x^9)/3 + (3*a*b*(b*d + a*g)*x^10)/10
+ (3*a*b*(b*e + a*h)*x^11)/11 + (a*b^2*f*x^12)/4 + (b^2*(b*d + 3*a*g)*x^13)
/13 + (b^2*(b*e + 3*a*h)*x^14)/14 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^
3*h*x^17)/17 + (c*(a + b*x^3)^4)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^2\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{c\left(a+bx^3\right)^4}{12b}+\int\left(a+bx^3\right)^3\left(-cx^2+x^2(c+dx+\right.$$
$$= \frac{c\left(a+bx^3\right)^4}{12b}+\int\left(a^3dx^3+a^3ex^4+a^3fx^5+a^2(3b\right.$$
$$= \frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{7}a^2(3bd+ag)x^7+$$

Mathematica [A]
time = 0.03, size = 223, normalized size = 1.05

$$\frac{1}{3}a^3cx^3+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^2(3bc+af)x^6+\frac{1}{7}a^2(3bd+ag)x^7+\frac{1}{8}a^2(3be+ah)x^8+\frac{1}{3}ab(bc+af)x^9+\frac{3}{10}ab(bd+ag)x^{10}+\frac{3}{11}ab(be+ah)x^{11}+\frac{1}{12}b^2(bc+3af)x^{12}+\frac{1}{13}b^2(bd+3ag)x^{13}+\frac{1}{14}b^2(be+3ah)x^{14}+\frac{1}{15}b^3fx^{15}+\frac{1}{16}b^3gx^{16}+\frac{1}{17}b^3hx^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*(3*b*c + a*f)*x^6)/6 +
 (a^2*(3*b*d + a*g)*x^7)/7 + (a^2*(3*b*e + a*h)*x^8)/8 + (a*b*(b*c + a*f)*x
^9)/3 + (3*a*b*(b*d + a*g)*x^10)/10 + (3*a*b*(b*e + a*h)*x^11)/11 + (b^2*(b
*c + 3*a*f)*x^12)/12 + (b^2*(b*d + 3*a*g)*x^13)/13 + (b^2*(b*e + 3*a*h)*x^1
4)/14 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^3*h*x^17)/17
```

Maple [A]
time = 2.24, size = 224, normalized size = 1.06

method	result
norman	$\frac{c\,a^3x^3}{3}+\frac{a^3d\,x^4}{4}+\frac{a^3e\,x^5}{5}+\left(\frac{1}{6}a^3f+\frac{1}{2}c\,a^2b\right)x^6+\left(\frac{1}{7}a^3g+\frac{3}{7}d\,a^2b\right)x^7+\left(\frac{1}{8}a^3h+\frac{3}{8}a^2be\right)x^8+\left(\frac{1}{3}a^2bf\right.$
default	$\frac{b^3h\,x^{17}}{17}+\frac{b^3g\,x^{16}}{16}+\frac{b^3f\,x^{15}}{15}+\frac{\left(3a\,b^2h+e\,b^3\right)x^{14}}{14}+\frac{\left(3a\,b^2g+b^3d\right)x^{13}}{13}+\frac{\left(3a\,b^2f+b^3c\right)x^{12}}{12}+\frac{\left(3a^2bh+3a\,b^2e\right)x^{11}}{11}+$
gosper	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{6}a^3f\,x^6+\frac{1}{2}c\,a^2b\,x^6+\frac{1}{7}x^7a^3g+\frac{3}{7}a^2bd\,x^7+\frac{1}{8}x^8a^3h+\frac{3}{8}a^2be\,x^8+$
risch	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{6}a^3f\,x^6+\frac{1}{2}c\,a^2b\,x^6+\frac{1}{7}x^7a^3g+\frac{3}{7}a^2bd\,x^7+\frac{1}{8}x^8a^3h+\frac{3}{8}a^2be\,x^8+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/17*b^3*h*x^17+1/16*b^3*g*x^16+1/15*b^3*f*x^15+1/14*(3*a*b^2*h+b^3*e)*x^14
+1/13*(3*a*b^2*g+b^3*d)*x^13+1/12*(3*a*b^2*f+b^3*c)*x^12+1/11*(3*a^2*b*h+3*
a*b^2*e)*x^11+1/10*(3*a^2*b*g+3*a*b^2*d)*x^10+1/9*(3*a^2*b*f+3*a*b^2*c)*x^9
+1/8*(a^3*h+3*a^2*b*e)*x^8+1/7*(a^3*g+3*a^2*b*d)*x^7+1/6*(a^3*f+3*a^2*b*c)*
x^6+1/5*a^3*e*x^5+1/4*a^3*d*x^4+1/3*c*a^3*x^3
```

Maxima [A]
time = 0.30, size = 221, normalized size = 1.04

$$\frac{1}{17}b^3hx^{17}+\frac{1}{16}b^3gx^{16}+\frac{1}{15}b^3fx^{15}+\frac{1}{14}\left(3ab^2h+b^3e\right)x^{14}+\frac{1}{13}\left(b^3d+3ab^2g\right)x^{13}+\frac{1}{12}\left(b^3c+3ab^2f\right)x^{12}+\frac{3}{11}\left(a^2bh+ab^2e\right)x^{11}+\frac{3}{10}\left(ab^2d+a^2bg\right)x^{10}+\frac{1}{3}\left(ab^2c+a^2bf\right)x^9+\frac{1}{8}\left(a^3h+3a^2be\right)x^8+\frac{1}{5}a^3x^5e+\frac{1}{4}a^3dx^4+\frac{1}{7}\left(3a^2bd+a^3g\right)x^7+\frac{1}{3}a^3cx^3+\frac{1}{6}\left(3a^2bc+a^3f\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 1/14*(3*a*b^2*h + b^3
*e)*x^14 + 1/13*(b^3*d + 3*a*b^2*g)*x^13 + 1/12*(b^3*c + 3*a*b^2*f)*x^12 +
3/11*(a^2*b*h + a*b^2*e)*x^11 + 3/10*(a*b^2*d + a^2*b*g)*x^10 + 1/3*(a*b^2*
c + a^2*b*f)*x^9 + 1/8*(a^3*h + 3*a^2*b*e)*x^8 + 1/5*a^3*x^5*e + 1/4*a^3*d*
x^4 + 1/7*(3*a^2*b*d + a^3*g)*x^7 + 1/3*a^3*c*x^3 + 1/6*(3*a^2*b*c + a^3*f)
*x^6
```

Fricas [A]
time = 0.40, size = 217, normalized size = 1.02

$$\frac{1}{17}b^3hx^{17}+\frac{1}{16}b^3gx^{16}+\frac{1}{15}b^3fx^{15}+\frac{1}{14}\left(b^3e+3ab^2h\right)x^{14}+\frac{1}{13}\left(b^3d+3ab^2g\right)x^{13}+\frac{1}{12}\left(b^3c+3ab^2f\right)x^{12}+\frac{3}{11}\left(ab^2e+a^2bh\right)x^{11}+\frac{3}{10}\left(ab^2d+a^2bg\right)x^{10}+\frac{1}{3}\left(ab^2c+a^2bf\right)x^9+\frac{1}{5}a^3ex^5+\frac{1}{8}\left(3a^2be+a^3h\right)x^8+\frac{1}{4}a^3dx^4+\frac{1}{7}\left(3a^2bd+a^3g\right)x^7+\frac{1}{3}a^3cx^3+\frac{1}{6}\left(3a^2bc+a^3f\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 1/14*(b^3*e + 3*a*b^2
*h)*x^14 + 1/13*(b^3*d + 3*a*b^2*g)*x^13 + 1/12*(b^3*c + 3*a*b^2*f)*x^12 +
3/11*(a*b^2*e + a^2*b*h)*x^11 + 3/10*(a*b^2*d + a^2*b*g)*x^10 + 1/3*(a*b^2*
c + a^2*b*f)*x^9 + 1/5*a^3*e*x^5 + 1/8*(3*a^2*b*e + a^3*h)*x^8 + 1/4*a^3*d*
x^4 + 1/7*(3*a^2*b*d + a^3*g)*x^7 + 1/3*a^3*c*x^3 + 1/6*(3*a^2*b*c + a^3*f)
*x^6
```

Sympy [A]
time = 0.02, size = 246, normalized size = 1.16

$$\frac{a^3cx^3}{3}+\frac{a^3dx^4}{4}+\frac{a^3ex^5}{5}+\frac{b^3fx^{15}}{15}+\frac{b^3gx^{16}}{16}+\frac{b^3hx^{17}}{17}+x^{14}\cdot\left(\frac{3ab^2h}{14}+\frac{b^3e}{14}\right)+x^{13}\cdot\left(\frac{3ab^2g}{13}+\frac{b^3d}{13}\right)+x^{12}\left(\frac{ab^2f}{4}+\frac{b^3c}{12}\right)+x^{11}\cdot\left(\frac{3a^2bh}{11}+\frac{3ab^2e}{11}\right)+x^{10}\cdot\left(\frac{3a^2bg}{10}+\frac{3ab^2d}{10}\right)+x^9\left(\frac{a^2bf}{3}+\frac{ab^2c}{3}\right)+x^8\left(\frac{a^3h}{8}+\frac{3a^2be}{8}\right)+x^7\left(\frac{a^3g}{7}+\frac{3a^2bd}{7}\right)+x^6\left(\frac{a^3f}{6}+\frac{a^2bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + b**3*f*x**15/15 + b**3*g*x*
*16/16 + b**3*h*x**17/17 + x**14*(3*a*b**2*h/14 + b**3*e/14) + x**13*(3*a*b
**2*g/13 + b**3*d/13) + x**12*(a*b**2*f/4 + b**3*c/12) + x**11*(3*a**2*b*h/
11 + 3*a*b**2*e/11) + x**10*(3*a**2*b*g/10 + 3*a*b**2*d/10) + x**9*(a**2*b*
f/3 + a*b**2*c/3) + x**8*(a**3*h/8 + 3*a**2*b*e/8) + x**7*(a**3*g/7 + 3*a**
2*b*d/7) + x**6*(a**3*f/6 + a**2*b*c/2)
```

Giac [A]
time = 0.50, size = 233, normalized size = 1.10

$$\frac{1}{17}b^3hx^{17}+\frac{1}{16}b^3gx^{16}+\frac{1}{15}b^3fx^{15}+\frac{3}{14}ab^2hx^{14}+\frac{1}{14}b^3x^{14}e+\frac{1}{13}b^3dx^{13}+\frac{3}{13}ab^2gx^{13}+\frac{1}{12}b^3cx^{12}+\frac{1}{4}ab^2fx^{12}+\frac{3}{11}a^2bhx^{11}+\frac{3}{11}ab^2x^{11}e+\frac{3}{10}ab^2dx^{10}+\frac{3}{10}a^2bgx^{10}+\frac{1}{3}ab^2cx^9+\frac{1}{3}a^2bfx^9+\frac{1}{8}a^3hx^8+\frac{3}{8}a^2bx^8e+\frac{3}{7}a^2bdx^7+\frac{1}{7}a^3gx^7+\frac{1}{2}a^2bcx^6+\frac{1}{6}a^3fx^6+\frac{1}{5}a^3x^5e+\frac{1}{4}a^3dx^4+\frac{1}{3}a^3cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 3/14*a*b^2*h*x^14 + 1
/14*b^3*x^14*e + 1/13*b^3*d*x^13 + 3/13*a*b^2*g*x^13 + 1/12*b^3*c*x^12 + 1/
4*a*b^2*f*x^12 + 3/11*a^2*b*h*x^11 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10
+ 3/10*a^2*b*g*x^10 + 1/3*a*b^2*c*x^9 + 1/3*a^2*b*f*x^9 + 1/8*a^3*h*x^8 + 3
/8*a^2*b*x^8*e + 3/7*a^2*b*d*x^7 + 1/7*a^3*g*x^7 + 1/2*a^2*b*c*x^6 + 1/6*a^
3*f*x^6 + 1/5*a^3*x^5*e + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

Mupad [B]
time = 0.16, size = 205, normalized size = 0.97

$$x^6\left(\frac{f\,a^3}{6}+\frac{b\,c\,a^2}{2}\right)+x^{12}\left(\frac{c\,b^3}{12}+\frac{a\,f\,b^2}{4}\right)+x^7\left(\frac{g\,a^3}{7}+\frac{3\,b\,d\,a^2}{7}\right)+x^{13}\left(\frac{d\,b^3}{13}+\frac{3\,a\,g\,b^2}{13}\right)+x^8\left(\frac{h\,a^3}{8}+\frac{3\,b\,e\,a^2}{8}\right)+x^{14}\left(\frac{e\,b^3}{14}+\frac{3\,a\,h\,b^2}{14}\right)+\frac{a^3\,c\,x^3}{3}+\frac{a^3\,d\,x^4}{4}+\frac{a^3\,e\,x^5}{5}+\frac{b^3\,f\,x^{15}}{15}+\frac{b^3\,g\,x^{16}}{16}+\frac{b^3\,h\,x^{17}}{17}+\frac{a\,b\,x^9\,\left(b\,c+a\,f\right)}{3}+\frac{3\,a\,b\,x^{10}\,\left(b\,d+a\,g\right)}{10}+\frac{3\,a\,b\,x^{11}\,\left(b\,e+a\,h\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^6*((a^3*f)/6 + (a^2*b*c)/2) + x^12*((b^3*c)/12 + (a*b^2*f)/4) + x^7*((a^3
*g)/7 + (3*a^2*b*d)/7) + x^13*((b^3*d)/13 + (3*a*b^2*g)/13) + x^8*((a^3*h)/
8 + (3*a^2*b*e)/8) + x^14*((b^3*e)/14 + (3*a*b^2*h)/14) + (a^3*c*x^3)/3 + (
a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^3*h*x
^17)/17 + (a*b*x^9*(b*c + a*f))/3 + (3*a*b*x^10*(b*d + a*g))/10 + (3*a*b*x^
11*(b*e + a*h))/11
```

3.396 $\int x(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

Optimal. Leaf size=212

$$\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{5}a^2(3bc+af)x^5+\frac{1}{6}a^3gx^6+\frac{1}{7}a^2(3be+ah)x^7+\frac{3}{8}ab(bc+af)x^8+\frac{1}{3}a^2bgx^9+\frac{3}{10}ab(be+ah)x^{10}+\frac{1}{1}$$

```
[Out] 1/2*a^3*c*x^2+1/4*a^3*e*x^4+1/5*a^2*(a*f+3*b*c)*x^5+1/6*a^3*g*x^6+1/7*a^2*(
a*h+3*b*e)*x^7+3/8*a*b*(a*f+b*c)*x^8+1/3*a^2*b*g*x^9+3/10*a*b*(a*h+b*e)*x^1
0+1/11*b^2*(3*a*f+b*c)*x^11+1/4*a*b^2*g*x^12+1/13*b^2*(3*a*h+b*e)*x^13+1/14
*b^3*f*x^14+1/15*b^3*g*x^15+1/16*b^3*h*x^16+1/12*d*(b*x^3+a)^4/b
```

Rubi [A]
time = 0.13, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1596, 1864}

$$\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{6}a^3gx^6+\frac{1}{5}a^2x^5(af+3bc)+\frac{1}{7}a^2x^7(ah+3be)+\frac{1}{3}a^2bgx^9+\frac{1}{11}b^2x^{11}(3af+bc)+\frac{1}{13}b^2x^{13}(3ah+be)+\frac{1}{4}ab^2gx^{12}+\frac{3}{8}abx^8(af+bc)+\frac{d(a+bx^3)^4}{12b}+\frac{3}{10}abx^{10}(ah+be)+\frac{1}{14}b^3fx^{14}+\frac{1}{15}b^3gx^{15}+\frac{1}{16}b^3hx^{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5 + (a^3*g*x^6)/6 +
 (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*f)*x^8)/8 + (a^2*b*g*x^9)/3 +
(3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(b*c + 3*a*f)*x^11)/11 + (a*b^2*g*x^12)/
4 + (b^2*(b*e + 3*a*h)*x^13)/13 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*
h*x^16)/16 + (d*(a + b*x^3)^4)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx=\frac{d\left(a+bx^3\right)^4}{12b}+\int\left(a+bx^3\right)^3\left(-dx^2+x(c+dx+e\right.$$
$$=\frac{d\left(a+bx^3\right)^4}{12b}+\int\left(a^3cx+a^3ex^3+a^2(3bc+af)x^4\right.$$
$$=\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{5}a^2(3bc+af)x^5+\frac{1}{6}a^3gx^6+$$

Mathematica [A]
time = 0.02, size = 223, normalized size = 1.05

$$\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{1}{5}a^2(3bc+af)x^5+\frac{1}{6}a^2(3bd+ag)x^6+\frac{1}{7}a^2(3be+ah)x^7+\frac{3}{8}ab(bc+af)x^8+\frac{1}{3}ab(bd+ag)x^9+\frac{3}{10}ab(be+ah)x^{10}+\frac{1}{11}b^2(bc+3af)x^{11}+\frac{1}{12}b^2(bd+3ag)x^{12}+\frac{1}{13}b^2(be+3ah)x^{13}+\frac{1}{14}b^3fx^{14}+\frac{1}{15}b^3gx^{15}+\frac{1}{16}b^3hx^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5 +
 (a^2*(3*b*d + a*g)*x^6)/6 + (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*f)
*x^8)/8 + (a*b*(b*d + a*g)*x^9)/3 + (3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(b*c
 + 3*a*f)*x^11)/11 + (b^2*(b*d + 3*a*g)*x^12)/12 + (b^2*(b*e + 3*a*h)*x^13)
/13 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*h*x^16)/16
```

Maple [A]
time = 2.09, size = 224, normalized size = 1.06

method	result
norman	$\frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\left(\frac{1}{5}a^3f+\frac{3}{5}ca^2b\right)x^5+\left(\frac{1}{6}a^3g+\frac{1}{2}da^2b\right)x^6+\left(\frac{1}{7}a^3h+\frac{3}{7}a^2be\right)x^7+\left(\frac{3}{8}a^2bf\right.$
default	$\frac{b^3hx^{16}}{16}+\frac{b^3gx^{15}}{15}+\frac{b^3fx^{14}}{14}+\frac{(3ab^2h+eb^3)x^{13}}{13}+\frac{(3ab^2g+b^3d)x^{12}}{12}+\frac{(3ab^2f+b^3c)x^{11}}{11}+\frac{(3a^2bh+3ab^2e)x^{10}}{10}+\frac{(3a^2b}{}$
gosper	$\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{1}{5}x^5a^3f+\frac{3}{5}a^2bcx^5+\frac{1}{6}a^3gx^6+\frac{1}{2}a^2bdx^6+\frac{1}{7}x^7a^3h+\frac{3}{7}a^2bex^7+$
risch	$\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{1}{5}x^5a^3f+\frac{3}{5}a^2bcx^5+\frac{1}{6}a^3gx^6+\frac{1}{2}a^2bdx^6+\frac{1}{7}x^7a^3h+\frac{3}{7}a^2bex^7+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*b^3*h*x^16+1/15*b^3*g*x^15+1/14*b^3*f*x^14+1/13*(3*a*b^2*h+b^3*e)*x^13
+1/12*(3*a*b^2*g+b^3*d)*x^12+1/11*(3*a*b^2*f+b^3*c)*x^11+1/10*(3*a^2*b*h+3*
a*b^2*e)*x^10+1/9*(3*a^2*b*g+3*a*b^2*d)*x^9+1/8*(3*a^2*b*f+3*a*b^2*c)*x^8+1
/7*(a^3*h+3*a^2*b*e)*x^7+1/6*(a^3*g+3*a^2*b*d)*x^6+1/5*(a^3*f+3*a^2*b*c)*x^
5+1/4*a^3*e*x^4+1/3*a^3*d*x^3+1/2*a^3*c*x^2
```

Maxima [A]
time = 0.28, size = 221, normalized size = 1.04

$$\frac{1}{16}\,b^{3}hx^{16}+\frac{1}{15}\,b^{3}gx^{15}+\frac{1}{14}\,b^{3}fx^{14}+\frac{1}{13}\,(3\,ab^{2}h+b^{3}e)x^{13}+\frac{1}{12}\,(b^{3}d+3\,ab^{2}g)x^{12}+\frac{1}{11}\,(b^{3}c+3\,ab^{2}f)x^{11}+\frac{3}{10}\,(a^{2}bh+ab^{2}e)x^{10}+\frac{1}{3}\,(ab^{2}d+a^{2}bg)x^{9}+\frac{3}{8}\,(ab^{2}c+a^{2}bf)x^{8}+\frac{1}{7}\,(a^{3}h+3\,a^{2}be)x^{7}+\frac{1}{4}\,a^{3}x^{4}e+\frac{1}{3}\,a^{3}dx^{3}+\frac{1}{6}\,(3\,a^{2}bd+a^{3}g)x^{6}+\frac{1}{2}\,a^{3}cx^{2}+\frac{1}{5}\,(3\,a^{2}bc+a^{3}f)x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 1/13*(3*a*b^2*h + b^3
*e)*x^13 + 1/12*(b^3*d + 3*a*b^2*g)*x^12 + 1/11*(b^3*c + 3*a*b^2*f)*x^11 +
3/10*(a^2*b*h + a*b^2*e)*x^10 + 1/3*(a*b^2*d + a^2*b*g)*x^9 + 3/8*(a*b^2*c
+ a^2*b*f)*x^8 + 1/7*(a^3*h + 3*a^2*b*e)*x^7 + 1/4*a^3*x^4*e + 1/3*a^3*d*x^
3 + 1/6*(3*a^2*b*d + a^3*g)*x^6 + 1/2*a^3*c*x^2 + 1/5*(3*a^2*b*c + a^3*f)*x
^5
```

Fricas [A]
time = 0.37, size = 217, normalized size = 1.02

$$\frac{1}{16}\,b^{3}hx^{16}+\frac{1}{15}\,b^{3}gx^{15}+\frac{1}{14}\,b^{3}fx^{14}+\frac{1}{13}\,(b^{3}e+3\,ab^{2}h)x^{13}+\frac{1}{12}\,(b^{3}d+3\,ab^{2}g)x^{12}+\frac{1}{11}\,(b^{3}c+3\,ab^{2}f)x^{11}+\frac{3}{10}\,(ab^{2}e+a^{2}bh)x^{10}+\frac{1}{3}\,(ab^{2}d+a^{2}bg)x^{9}+\frac{3}{8}\,(ab^{2}c+a^{2}bf)x^{8}+\frac{1}{4}\,a^{3}ex^{4}+\frac{1}{7}\,(3\,a^{2}be+a^{3}h)x^{7}+\frac{1}{3}\,a^{3}dx^{3}+\frac{1}{6}\,(3\,a^{2}bd+a^{3}g)x^{6}+\frac{1}{2}\,a^{3}cx^{2}+\frac{1}{5}\,(3\,a^{2}bc+a^{3}f)x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 1/13*(b^3*e + 3*a*b^2
*h)*x^13 + 1/12*(b^3*d + 3*a*b^2*g)*x^12 + 1/11*(b^3*c + 3*a*b^2*f)*x^11 +
3/10*(a*b^2*e + a^2*b*h)*x^10 + 1/3*(a*b^2*d + a^2*b*g)*x^9 + 3/8*(a*b^2*c
+ a^2*b*f)*x^8 + 1/4*a^3*e*x^4 + 1/7*(3*a^2*b*e + a^3*h)*x^7 + 1/3*a^3*d*x^
3 + 1/6*(3*a^2*b*d + a^3*g)*x^6 + 1/2*a^3*c*x^2 + 1/5*(3*a^2*b*c + a^3*f)*x
^5
```

Sympy [A]
time = 0.02, size = 246, normalized size = 1.16

$$\frac{a^{3}cx^{2}}{2}+\frac{a^{3}dx^{3}}{3}+\frac{a^{3}ex^{4}}{4}+\frac{b^{3}fx^{14}}{14}+\frac{b^{3}gx^{15}}{15}+\frac{b^{3}hx^{16}}{16}+x^{13}\cdot\left(\frac{3ab^{2}h}{13}+\frac{b^{3}e}{13}\right)+x^{12}\left(\frac{ab^{2}g}{4}+\frac{b^{3}d}{12}\right)+x^{11}\cdot\left(\frac{3ab^{2}f}{11}+\frac{b^{3}c}{11}\right)+x^{10}\cdot\left(\frac{3a^{2}bh}{10}+\frac{3ab^{2}e}{10}\right)+x^{9}\left(\frac{a^{2}bg}{3}+\frac{ab^{2}d}{3}\right)+x^{8}\cdot\left(\frac{3a^{2}bf}{8}+\frac{3ab^{2}c}{8}\right)+x^{7}\left(\frac{a^{3}h}{7}+\frac{3a^{2}be}{7}\right)+x^{6}\left(\frac{a^{3}g}{6}+\frac{a^{2}bd}{2}\right)+x^{5}\left(\frac{a^{3}f}{5}+\frac{3a^{2}bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + b**3*f*x**14/14 + b**3*g*x*
*15/15 + b**3*h*x**16/16 + x**13*(3*a*b**2*h/13 + b**3*e/13) + x**12*(a*b**
2*g/4 + b**3*d/12) + x**11*(3*a*b**2*f/11 + b**3*c/11) + x**10*(3*a**2*b*h/
10 + 3*a*b**2*e/10) + x**9*(a**2*b*g/3 + a*b**2*d/3) + x**8*(3*a**2*b*f/8 +
 3*a*b**2*c/8) + x**7*(a**3*h/7 + 3*a**2*b*e/7) + x**6*(a**3*g/6 + a**2*b*d
/2) + x**5*(a**3*f/5 + 3*a**2*b*c/5)
```

Giac [A]
time = 0.53, size = 233, normalized size = 1.10

$$\frac{1}{16}\,b^3hx^{16}+\frac{1}{15}\,b^3gx^{15}+\frac{1}{14}\,b^3fx^{14}+\frac{3}{13}\,ab^2hx^{13}+\frac{1}{13}\,b^3x^{13}e+\frac{1}{12}\,b^3dx^{12}+\frac{1}{4}\,ab^2gx^{12}+\frac{1}{11}\,b^3cx^{11}+\frac{3}{11}\,ab^2fx^{11}+\frac{3}{10}\,a^2bhx^{10}+\frac{3}{10}\,ab^2x^{10}e+\frac{1}{3}\,ab^2dx^9+\frac{1}{3}\,a^2bgx^9+\frac{3}{8}\,ab^2cx^8+\frac{3}{8}\,a^2bfx^8+\frac{1}{7}\,a^3hx^7+\frac{3}{7}\,a^2bx^7e+\frac{1}{2}\,a^2bdx^6+\frac{1}{6}\,a^3gx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{5}\,a^3fx^5+\frac{1}{4}\,a^3x^4e+\frac{1}{3}\,a^3dx^3+\frac{1}{2}\,a^3cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 3/13*a*b^2*h*x^13 + 1
/13*b^3*x^13*e + 1/12*b^3*d*x^12 + 1/4*a*b^2*g*x^12 + 1/11*b^3*c*x^11 + 3/1
1*a*b^2*f*x^11 + 3/10*a^2*b*h*x^10 + 3/10*a*b^2*x^10*e + 1/3*a*b^2*d*x^9 +
1/3*a^2*b*g*x^9 + 3/8*a*b^2*c*x^8 + 3/8*a^2*b*f*x^8 + 1/7*a^3*h*x^7 + 3/7*a
^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 1/6*a^3*g*x^6 + 3/5*a^2*b*c*x^5 + 1/5*a^3*f*
x^5 + 1/4*a^3*x^4*e + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

Mupad [B]
time = 0.16, size = 205, normalized size = 0.97

$$x^5\,\left(\frac{f\,a^3}{5}+\frac{3\,b\,c\,a^2}{5}\right)+x^{11}\,\left(\frac{c\,b^3}{11}+\frac{3\,a\,f\,b^2}{11}\right)+x^6\,\left(\frac{g\,a^3}{6}+\frac{b\,d\,a^2}{2}\right)+x^{12}\,\left(\frac{d\,b^3}{12}+\frac{a\,g\,b^2}{4}\right)+x^7\,\left(\frac{h\,a^3}{7}+\frac{3\,b\,e\,a^2}{7}\right)+x^{13}\,\left(\frac{e\,b^3}{13}+\frac{3\,a\,h\,b^2}{13}\right)+\frac{a^3\,c\,x^2}{2}+\frac{a^3\,d\,x^3}{3}+\frac{a^3\,e\,x^4}{4}+\frac{b^3\,f\,x^{14}}{14}+\frac{b^3\,g\,x^{15}}{15}+\frac{b^3\,h\,x^{16}}{16}+\frac{3\,a\,b\,x^8\,\left(b\,c+a\,f\right)}{8}+\frac{a\,b\,x^9\,\left(b\,d+a\,g\right)}{3}+\frac{3\,a\,b\,x^{10}\,\left(b\,e+a\,h\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^5*((a^3*f)/5 + (3*a^2*b*c)/5) + x^11*((b^3*c)/11 + (3*a*b^2*f)/11) + x^6*
((a^3*g)/6 + (a^2*b*d)/2) + x^12*((b^3*d)/12 + (a*b^2*g)/4) + x^7*((a^3*h)/
7 + (3*a^2*b*e)/7) + x^13*((b^3*e)/13 + (3*a*b^2*h)/13) + (a^3*c*x^2)/2 + (
a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*h*x
^16)/16 + (3*a*b*x^8*(b*c + a*f))/8 + (a*b*x^9*(b*d + a*g))/3 + (3*a*b*x^10
*(b*e + a*h))/10
```

3.397 $\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

Optimal. Leaf size=207

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{4}a^2(3bc+af)x^4+\frac{1}{5}a^2(3bd+ag)x^5+\frac{1}{6}a^3hx^6+\frac{3}{7}ab(bc+af)x^7+\frac{3}{8}ab(bd+ag)x^8+\frac{1}{3}a^2bhx^9+\frac{1}{10}b^2($$

```
[Out] a^3*c*x+1/2*a^3*d*x^2+1/4*a^2*(a*f+3*b*c)*x^4+1/5*a^2*(a*g+3*b*d)*x^5+1/6*a
^3*h*x^6+3/7*a*b*(a*f+b*c)*x^7+3/8*a*b*(a*g+b*d)*x^8+1/3*a^2*b*h*x^9+1/10*b
^2*(3*a*f+b*c)*x^10+1/11*b^2*(3*a*g+b*d)*x^11+1/4*a*b^2*h*x^12+1/13*b^3*f*x
^13+1/14*b^3*g*x^14+1/15*b^3*h*x^15+1/12*e*(b*x^3+a)^4/b
```

Rubi [A]
time = 0.12, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1596, 1864}

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{6}a^3hx^6+\frac{1}{4}a^2x^4(af+3bc)+\frac{1}{5}a^2x^5(ag+3bd)+\frac{1}{3}a^2bhx^9+\frac{1}{10}b^2x^{10}(3af+bc)+\frac{1}{11}b^2x^{11}(3ag+bd)+\frac{1}{4}ab^2hx^{12}+\frac{3}{7}abx^7(af+bc)+\frac{3}{8}abx^8(ag+bd)+\frac{e(a+bx^3)^4}{12b}+\frac{1}{13}b^3fx^{13}+\frac{1}{14}b^3gx^{14}+\frac{1}{15}b^3hx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^2*(3*b*c + a*f)*x^4)/4 + (a^2*(3*b*d + a*g)*x^
5)/5 + (a^3*h*x^6)/6 + (3*a*b*(b*c + a*f)*x^7)/7 + (3*a*b*(b*d + a*g)*x^8)/
8 + (a^2*b*h*x^9)/3 + (b^2*(b*c + 3*a*f)*x^10)/10 + (b^2*(b*d + 3*a*g)*x^11
)/11 + (a*b^2*h*x^12)/4 + (b^3*f*x^13)/13 + (b^3*g*x^14)/14 + (b^3*h*x^15)/
15 + (e*(a + b*x^3)^4)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{e(a+bx^3)^4}{12b} + \int \left(a+bx^3\right)^3 \left(c+dx+fx^3+gx^4+\right.$$

$$= \frac{e(a+bx^3)^4}{12b} + \int \left(a^3c+a^3dx+a^2(3bc+af)x^3+a^2\right.$$

$$= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{4}a^2(3bc+af)x^4+\frac{1}{5}a^2(3bd+ag)$$

Mathematica [A]
time = 0.07, size = 170, normalized size = 0.82

$$\frac{x(13ab^2x^6(3960c+7x(495d+440ex+6x^2(66f+60gx+55hx^2)))+2002a^3(60c+x(30d+x(20e+15fx+12gx^2+10hx^3)))+2b^3x^9(6006c+x(5460d+11x(455e+420fx+390gx^2+364hx^3)))+143a^2bx^3(630c+x(504d+5x(84e+x(72f+7x(9g+8hx))))))}{120120}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (x*(13*a*b^2*x^6*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55
*h*x^2))) + 2002*a^3*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x
^3))) + 2*b^3*x^9*(6006*c + x*(5460*d + 11*x*(455*e + 420*f*x + 390*g*x^2 +
 364*h*x^3))) + 143*a^2*b*x^3*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x
*(9*g + 8*h*x)))))))/120120
```

Maple [A]
time = 1.93, size = 221, normalized size = 1.07

method	result
norman	$a^3cx+\frac{a^3d\,x^2}{2}+\frac{a^3e\,x^3}{3}+\left(\frac{1}{4}a^3f+\frac{3}{4}c\,a^2b\right)x^4+\left(\frac{1}{5}a^3g+\frac{3}{5}d\,a^2b\right)x^5+\left(\frac{1}{6}a^3h+\frac{1}{2}a^2be\right)x^6+\left(\frac{3}{7}a^2bf\right.$
default	$\frac{b^3h\,x^{15}}{15}+\frac{b^3g\,x^{14}}{14}+\frac{b^3f\,x^{13}}{13}+\frac{(3a\,b^2h+e\,b^3)x^{12}}{12}+\frac{(3a\,b^2g+b^3d)x^{11}}{11}+\frac{(3a\,b^2f+b^3c)x^{10}}{10}+\frac{(3a^2bh+3a\,b^2e)x^9}{9}+\frac{(3a^2bg}{}$
gosper	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}a^3f\,x^4+\frac{3}{4}a^2bc\,x^4+\frac{1}{5}x^5a^3g+\frac{3}{5}a^2bd\,x^5+\frac{1}{6}a^3h\,x^6+\frac{1}{2}a^2be\,x^6+\frac{3}{7}x^7$
risch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}a^3f\,x^4+\frac{3}{4}a^2bc\,x^4+\frac{1}{5}x^5a^3g+\frac{3}{5}a^2bd\,x^5+\frac{1}{6}a^3h\,x^6+\frac{1}{2}a^2be\,x^6+\frac{3}{7}x^7$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*b^3*h*x^15+1/14*b^3*g*x^14+1/13*b^3*f*x^13+1/12*(3*a*b^2*h+b^3*e)*x^12
+1/11*(3*a*b^2*g+b^3*d)*x^11+1/10*(3*a*b^2*f+b^3*c)*x^10+1/9*(3*a^2*b*h+3*a
*b^2*e)*x^9+1/8*(3*a^2*b*g+3*a*b^2*d)*x^8+1/7*(3*a^2*b*f+3*a*b^2*c)*x^7+1/6
*(a^3*h+3*a^2*b*e)*x^6+1/5*(a^3*g+3*a^2*b*d)*x^5+1/4*(a^3*f+3*a^2*b*c)*x^4+
1/3*a^3*e*x^3+1/2*a^3*d*x^2+a^3*c*x
```

Maxima [A]
time = 0.27, size = 218, normalized size = 1.05

$$\frac{1}{15}\,b^3hx^{15}+\frac{1}{14}\,b^3gx^{14}+\frac{1}{13}\,b^3fx^{13}+\frac{1}{12}\,(3\,ab^2h+b^3e)x^{12}+\frac{1}{11}\,(b^3d+3\,ab^2g)x^{11}+\frac{1}{10}\,(b^3c+3\,ab^2f)x^{10}+\frac{1}{3}\,(a^2bh+ab^2e)x^9+\frac{3}{8}\,(ab^2d+a^2bg)x^8+\frac{3}{7}\,(ab^2c+a^2bf)x^7+\frac{1}{6}\,(a^3h+3\,a^2be)x^6+\frac{1}{3}\,a^3x^3e+\frac{1}{2}\,a^3dx^2+\frac{1}{5}\,(3\,a^2bd+a^3g)x^5+a^3cx+\frac{1}{4}\,(3\,a^2bc+a^3f)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/12*(3*a*b^2*h + b^3
*e)*x^12 + 1/11*(b^3*d + 3*a*b^2*g)*x^11 + 1/10*(b^3*c + 3*a*b^2*f)*x^10 +
1/3*(a^2*b*h + a*b^2*e)*x^9 + 3/8*(a*b^2*d + a^2*b*g)*x^8 + 3/7*(a*b^2*c +
a^2*b*f)*x^7 + 1/6*(a^3*h + 3*a^2*b*e)*x^6 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2
+ 1/5*(3*a^2*b*d + a^3*g)*x^5 + a^3*c*x + 1/4*(3*a^2*b*c + a^3*f)*x^4
```

Fricas [A]
time = 0.36, size = 214, normalized size = 1.03

$$\frac{1}{15}\,b^3hx^{15}+\frac{1}{14}\,b^3gx^{14}+\frac{1}{13}\,b^3fx^{13}+\frac{1}{12}\,(b^3e+3\,ab^2h)x^{12}+\frac{1}{11}\,(b^3d+3\,ab^2g)x^{11}+\frac{1}{10}\,(b^3c+3\,ab^2f)x^{10}+\frac{1}{3}\,(ab^2e+a^2bh)x^9+\frac{3}{8}\,(ab^2d+a^2bg)x^8+\frac{3}{7}\,(ab^2c+a^2bf)x^7+\frac{1}{3}\,a^3ex^3+\frac{1}{6}\,(3\,a^2be+a^3h)x^6+\frac{1}{2}\,a^3dx^2+\frac{1}{5}\,(3\,a^2bd+a^3g)x^5+a^3cx+\frac{1}{4}\,(3\,a^2bc+a^3f)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/12*(b^3*e + 3*a*b^2
*h)*x^12 + 1/11*(b^3*d + 3*a*b^2*g)*x^11 + 1/10*(b^3*c + 3*a*b^2*f)*x^10 +
1/3*(a*b^2*e + a^2*b*h)*x^9 + 3/8*(a*b^2*d + a^2*b*g)*x^8 + 3/7*(a*b^2*c +
a^2*b*f)*x^7 + 1/3*a^3*e*x^3 + 1/6*(3*a^2*b*e + a^3*h)*x^6 + 1/2*a^3*d*x^2
+ 1/5*(3*a^2*b*d + a^3*g)*x^5 + a^3*c*x + 1/4*(3*a^2*b*c + a^3*f)*x^4
```

Sympy [A]
time = 0.02, size = 243, normalized size = 1.17

$$a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{b^3fx^{13}}{13}+\frac{b^3gx^{14}}{14}+\frac{b^3hx^{15}}{15}+x^{12}\left(\frac{ab^2h}{4}+\frac{b^3e}{12}\right)+x^{11}\cdot\left(\frac{3ab^2g}{11}+\frac{b^3d}{11}\right)+x^{10}\cdot\left(\frac{3ab^2f}{10}+\frac{b^3c}{10}\right)+x^9\left(\frac{a^2bh}{3}+\frac{ab^2e}{3}\right)+x^8\cdot\left(\frac{3a^2bg}{8}+\frac{3ab^2d}{8}\right)+x^7\cdot\left(\frac{3a^2bf}{7}+\frac{3ab^2c}{7}\right)+x^6\left(\frac{a^3h}{6}+\frac{a^2be}{2}\right)+x^5\left(\frac{a^3g}{5}+\frac{3a^2bd}{5}\right)+x^4\left(\frac{a^3f}{4}+\frac{3a^2bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + b**3*f*x**13/13 + b**3*g*x**14/1
4 + b**3*h*x**15/15 + x**12*(a*b**2*h/4 + b**3*e/12) + x**11*(3*a*b**2*g/11
 + b**3*d/11) + x**10*(3*a*b**2*f/10 + b**3*c/10) + x**9*(a**2*b*h/3 + a*b*
*2*e/3) + x**8*(3*a**2*b*g/8 + 3*a*b**2*d/8) + x**7*(3*a**2*b*f/7 + 3*a*b**
2*c/7) + x**6*(a**3*h/6 + a**2*b*e/2) + x**5*(a**3*g/5 + 3*a**2*b*d/5) + x*
*4*(a**3*f/4 + 3*a**2*b*c/4)
```

Giac [A]
time = 0.51, size = 230, normalized size = 1.11

$$\frac{1}{15}b^3hx^{15}+\frac{1}{14}b^3gx^{14}+\frac{1}{13}b^3fx^{13}+\frac{1}{4}ab^2hx^{12}+\frac{1}{12}b^3x^{12}e+\frac{1}{11}b^3dx^{11}+\frac{3}{11}ab^2gx^{11}+\frac{1}{10}b^3cx^{10}+\frac{3}{10}ab^2fx^{10}+\frac{1}{3}a^2bhx^9+\frac{1}{3}ab^2x^9e+\frac{3}{8}ab^2dx^8+\frac{3}{8}a^2bgx^8+\frac{3}{7}ab^2cx^7+\frac{3}{7}a^2bfx^7+\frac{1}{6}a^3hx^6+\frac{1}{2}a^2bx^6e+\frac{3}{5}a^2bdx^5+\frac{1}{5}a^3gx^5+\frac{3}{4}a^2bcx^4+\frac{1}{4}a^3fx^4+\frac{1}{3}a^3x^3e+\frac{1}{2}a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/4*a*b^2*h*x^12 + 1/
12*b^3*x^12*e + 1/11*b^3*d*x^11 + 3/11*a*b^2*g*x^11 + 1/10*b^3*c*x^10 + 3/1
0*a*b^2*f*x^10 + 1/3*a^2*b*h*x^9 + 1/3*a*b^2*x^9*e + 3/8*a*b^2*d*x^8 + 3/8*
a^2*b*g*x^8 + 3/7*a*b^2*c*x^7 + 3/7*a^2*b*f*x^7 + 1/6*a^3*h*x^6 + 1/2*a^2*b
*x^6*e + 3/5*a^2*b*d*x^5 + 1/5*a^3*g*x^5 + 3/4*a^2*b*c*x^4 + 1/4*a^3*f*x^4
+ 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B]
time = 0.16, size = 202, normalized size = 0.98

$$x^4\left(\frac{f\,a^3}{4}+\frac{3\,b\,c\,a^2}{4}\right)+x^{10}\left(\frac{c\,b^3}{10}+\frac{3\,a\,f\,b^2}{10}\right)+x^5\left(\frac{g\,a^3}{5}+\frac{3\,b\,d\,a^2}{5}\right)+x^{11}\left(\frac{d\,b^3}{11}+\frac{3\,a\,g\,b^2}{11}\right)+x^6\left(\frac{h\,a^3}{6}+\frac{b\,e\,a^2}{2}\right)+x^{12}\left(\frac{e\,b^3}{12}+\frac{a\,h\,b^2}{4}\right)+\frac{a^3\,d\,x^2}{2}+\frac{a^3\,e\,x^3}{3}+\frac{b^3\,f\,x^{13}}{13}+\frac{b^3\,g\,x^{14}}{14}+\frac{b^3\,h\,x^{15}}{15}+a^3\,c\,x+\frac{3\,a\,b\,x^7\,\left(b\,c+a\,f\right)}{7}+\frac{3\,a\,b\,x^8\,\left(b\,d+a\,g\right)}{8}+\frac{a\,b\,x^9\,\left(b\,e+a\,h\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^4*((a^3*f)/4 + (3*a^2*b*c)/4) + x^10*((b^3*c)/10 + (3*a*b^2*f)/10) + x^5*
((a^3*g)/5 + (3*a^2*b*d)/5) + x^11*((b^3*d)/11 + (3*a*b^2*g)/11) + x^6*((a^
3*h)/6 + (a^2*b*e)/2) + x^12*((b^3*e)/12 + (a*b^2*h)/4) + (a^3*d*x^2)/2 + (
a^3*e*x^3)/3 + (b^3*f*x^13)/13 + (b^3*g*x^14)/14 + (b^3*h*x^15)/15 + a^3*c*
x + (3*a*b*x^7*(b*c + a*f))/7 + (3*a*b*x^8*(b*d + a*g))/8 + (a*b*x^9*(b*e +
 a*h))/3
```

3.398 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

Optimal. Leaf size=200

$$a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{1}{4}a^2(3bd+ag)x^4+\frac{1}{5}a^2(3be+ah)x^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab(bd+ag)x^7+\frac{3}{8}ab(be+ah)x^8+\frac{1}{9}b^3cx^9$$

```
[Out] a^3*d*x+1/2*a^3*e*x^2+a^2*b*c*x^3+1/4*a^2*(a*g+3*b*d)*x^4+1/5*a^2*(a*h+3*b*
e)*x^5+1/2*a*b^2*c*x^6+3/7*a*b*(a*g+b*d)*x^7+3/8*a*b*(a*h+b*e)*x^8+1/9*b^3*
c*x^9+1/10*b^2*(3*a*g+b*d)*x^10+1/11*b^2*(3*a*h+b*e)*x^11+1/13*b^3*g*x^13+1
/14*b^3*h*x^14+1/12*f*(b*x^3+a)^4/b+a^3*c*ln(x)
```

Rubi [A]
time = 0.09, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1597, 1834}

$$a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{1}{4}a^2x^4(ag+3bd)+\frac{1}{5}a^2x^5(ah+3be)+\frac{1}{2}ab^2cx^6+\frac{1}{10}b^2x^{10}(3ag+bd)+\frac{1}{11}b^2x^{11}(3ah+be)+\frac{3}{7}abx^7(ag+bd)+\frac{3}{8}abx^8(ah+be)+\frac{f(a+bx^3)^4}{12b}+\frac{1}{9}b^3cx^9+\frac{1}{13}b^3gx^{13}+\frac{1}{14}b^3hx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (a^2*(3*b*d + a*g)*x^4)/4 + (a^2*(3
*b*e + a*h)*x^5)/5 + (a*b^2*c*x^6)/2 + (3*a*b*(b*d + a*g)*x^7)/7 + (3*a*b*(
b*e + a*h)*x^8)/8 + (b^3*c*x^9)/9 + (b^2*(b*d + 3*a*g)*x^10)/10 + (b^2*(b*e
 + 3*a*h)*x^11)/11 + (b^3*g*x^13)/13 + (b^3*h*x^14)/14 + (f*(a + b*x^3)^4)/
(12*b) + a^3*c*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = \frac{f(a+bx^3)^4}{12b} + \int \frac{(a+bx^3)^3\,(c+dx+ex^2+gx^4+h$$

$$= \frac{f(a+bx^3)^4}{12b} + \int \left(a^3d + \frac{a^3c}{x} + a^3ex + 3a^2bcx^2 + a^2$$

$$= a^3dx + \frac{1}{2}a^3ex^2 + a^2bcx^3 + \frac{1}{4}a^2(3bd+ag)x^4 + \frac{1}{5}a^2(3$$

Mathematica [A]
time = 0.05, size = 214, normalized size = 1.07

$$a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{3}a^2(3bc+af)x^3+\frac{1}{4}a^2(3bd+ag)x^4+\frac{1}{5}a^2(3be+ah)x^5+\frac{1}{2}ab(bc+af)x^6+\frac{3}{7}ab(bd+ag)x^7+\frac{3}{8}ab(be+ah)x^8+\frac{1}{9}b^2(bc+3af)x^9+\frac{1}{10}b^2(bd+3ag)x^{10}+\frac{1}{11}b^2(be+3ah)x^{11}+\frac{1}{12}b^3fx^{12}+\frac{1}{13}b^3gx^{13}+\frac{1}{14}b^3hx^{14}+a^3c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*c + a*f)*x^3)/3 + (a^2*(3*b*d + a*g)*x^
4)/4 + (a^2*(3*b*e + a*h)*x^5)/5 + (a*b*(b*c + a*f)*x^6)/2 + (3*a*b*(b*d +
a*g)*x^7)/7 + (3*a*b*(b*e + a*h)*x^8)/8 + (b^2*(b*c + 3*a*f)*x^9)/9 + (b^2*
(b*d + 3*a*g)*x^10)/10 + (b^2*(b*e + 3*a*h)*x^11)/11 + (b^3*f*x^12)/12 + (b
^3*g*x^13)/13 + (b^3*h*x^14)/14 + a^3*c*Log[x]
```

Maple [A]
time = 0.34, size = 224, normalized size = 1.12

method	result
norman	$\left(\frac{1}{3}a^3f+c\,a^2b\right)x^3+\left(\frac{1}{4}a^3g+\frac{3}{4}d\,a^2b\right)x^4+\left(\frac{1}{5}a^3h+\frac{3}{5}a^2be\right)x^5+\left(\frac{1}{3}a\,b^2f+\frac{1}{9}b^3c\right)x^9+\left(\frac{3}{10}a\,b^2g+\right.$
default	$\frac{b^3h\,x^{14}}{14}+\frac{b^3g\,x^{13}}{13}+\frac{f\,x^{12}b^3}{12}+\frac{3a\,b^2h\,x^{11}}{11}+\frac{b^3e\,x^{11}}{11}+\frac{3a\,b^2g\,x^{10}}{10}+\frac{b^3d\,x^{10}}{10}+\frac{a\,b^2f\,x^9}{3}+\frac{b^3c\,x^9}{9}+\frac{3a^2bh\,x^8}{8}+\frac{3a\,b^2}{8}$
risch	$\frac{b^3h\,x^{14}}{14}+\frac{b^3g\,x^{13}}{13}+\frac{f\,x^{12}b^3}{12}+\frac{3a\,b^2h\,x^{11}}{11}+\frac{b^3e\,x^{11}}{11}+\frac{3a\,b^2g\,x^{10}}{10}+\frac{b^3d\,x^{10}}{10}+\frac{a\,b^2f\,x^9}{3}+\frac{b^3c\,x^9}{9}+\frac{3a^2bh\,x^8}{8}+\frac{3a\,b^2}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*b^3*h*x^14+1/13*b^3*g*x^13+1/12*f*x^12*b^3+3/11*a*b^2*h*x^11+1/11*b^3*
e*x^11+3/10*a*b^2*g*x^10+1/10*b^3*d*x^10+1/3*a*b^2*f*x^9+1/9*b^3*c*x^9+3/8*
a^2*b*h*x^8+3/8*a*b^2*e*x^8+3/7*a^2*b*g*x^7+3/7*a*b^2*d*x^7+1/2*a^2*b*f*x^6
+1/2*a*b^2*c*x^6+1/5*a^3*h*x^5+3/5*a^2*b*e*x^5+1/4*a^3*g*x^4+3/4*a^2*b*d*x^
4+1/3*a^3*f*x^3+a^2*b*c*x^3+1/2*a^3*e*x^2+a^3*d*x+a^3*c*ln(x)
```

Maxima [A]
time = 0.28, size = 216, normalized size = 1.08

$$\frac{1}{14}\,b^3hx^{14}+\frac{1}{13}\,b^3gx^{13}+\frac{1}{12}\,b^3fx^{12}+\frac{1}{11}\,(3\,ab^2h+b^3e)x^{11}+\frac{1}{10}\,(b^3d+3\,ab^2g)x^{10}+\frac{1}{9}\,(b^3c+3\,ab^2f)x^9+\frac{3}{8}\,(a^2bh+ab^2e)x^8+\frac{3}{7}\,(ab^2d+a^2bg)x^7+\frac{1}{2}\,(ab^2c+a^2bf)x^6+\frac{1}{5}\,(a^3h+3\,a^2be)x^5+\frac{1}{2}\,a^3x^2e+a^3dx+\frac{1}{4}\,(3\,a^2bd+a^3g)x^4+a^3c\log(x)+\frac{1}{3}\,(3\,a^2bc+a^3f)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxim
a")
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*(3*a*b^2*h + b^3
*e)*x^11 + 1/10*(b^3*d + 3*a*b^2*g)*x^10 + 1/9*(b^3*c + 3*a*b^2*f)*x^9 + 3/
8*(a^2*b*h + a*b^2*e)*x^8 + 3/7*(a*b^2*d + a^2*b*g)*x^7 + 1/2*(a*b^2*c + a^
2*b*f)*x^6 + 1/5*(a^3*h + 3*a^2*b*e)*x^5 + 1/2*a^3*x^2*e + a^3*d*x + 1/4*(3
*a^2*b*d + a^3*g)*x^4 + a^3*c*log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3
```

<u>Fricas</u> [A]
time = 0.38, size = 212, normalized size = 1.06

$$\frac{1}{14}\,b^3hx^{14}+\frac{1}{13}\,b^3gx^{13}+\frac{1}{12}\,b^3fx^{12}+\frac{1}{11}\,(b^3e+3\,ab^2h)x^{11}+\frac{1}{10}\,(b^3d+3\,ab^2g)x^{10}+\frac{1}{9}\,(b^3c+3\,ab^2f)x^9+\frac{3}{8}\,(ab^2e+a^2bh)x^8+\frac{3}{7}\,(ab^2d+a^2bg)x^7+\frac{1}{2}\,(ab^2c+a^2bf)x^6+\frac{1}{2}\,a^3ex^2+\frac{1}{5}\,(3\,a^2be+a^3h)x^5+a^3dx+\frac{1}{4}\,(3\,a^2bd+a^3g)x^4+a^3c\log(x)+\frac{1}{3}\,(3\,a^2bc+a^3f)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="frica
s")
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*(b^3*e + 3*a*b^2
*h)*x^11 + 1/10*(b^3*d + 3*a*b^2*g)*x^10 + 1/9*(b^3*c + 3*a*b^2*f)*x^9 + 3/
8*(a*b^2*e + a^2*b*h)*x^8 + 3/7*(a*b^2*d + a^2*b*g)*x^7 + 1/2*(a*b^2*c + a^
2*b*f)*x^6 + 1/2*a^3*e*x^2 + 1/5*(3*a^2*b*e + a^3*h)*x^5 + a^3*d*x + 1/4*(3
*a^2*b*d + a^3*g)*x^4 + a^3*c*log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3
```

<u>Sympy</u> [A]
time = 0.18, size = 240, normalized size = 1.20

$$a^3c\log(x)+a^3dx+\frac{a^3ex^2}{2}+\frac{b^3fx^{12}}{12}+\frac{b^3gx^{13}}{13}+\frac{b^3hx^{14}}{14}+x^{11}\cdot\left(\frac{3ab^2h}{11}+\frac{b^3e}{11}\right)+x^{10}\cdot\left(\frac{3ab^2g}{10}+\frac{b^3d}{10}\right)+x^9\left(\frac{ab^2f}{3}+\frac{b^3c}{9}\right)+x^8\cdot\left(\frac{3a^2bh}{8}+\frac{3ab^2e}{8}\right)+x^7\cdot\left(\frac{3a^2bg}{7}+\frac{3ab^2d}{7}\right)+x^6\left(\frac{a^2bf}{2}+\frac{ab^2c}{2}\right)+x^5\left(\frac{a^3h}{5}+\frac{3a^2be}{5}\right)+x^4\left(\frac{a^3g}{4}+\frac{3a^2bd}{4}\right)+x^3\left(\frac{a^3f}{3}+a^2bc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a**3*c*log(x) + a**3*d*x + a**3*e*x**2/2 + b**3*f*x**12/12 + b**3*g*x**13/1
3 + b**3*h*x**14/14 + x**11*(3*a*b**2*h/11 + b**3*e/11) + x**10*(3*a*b**2*g
/10 + b**3*d/10) + x**9*(a*b**2*f/3 + b**3*c/9) + x**8*(3*a**2*b*h/8 + 3*a*
b**2*e/8) + x**7*(3*a**2*b*g/7 + 3*a*b**2*d/7) + x**6*(a**2*b*f/2 + a*b**2*
c/2) + x**5*(a**3*h/5 + 3*a**2*b*e/5) + x**4*(a**3*g/4 + 3*a**2*b*d/4) + x*
*3*(a**3*f/3 + a**2*b*c)
```

<u>Giac</u> [A]
time = 0.49, size = 228, normalized size = 1.14

$$\frac{1}{14}\,b^3hx^{14}+\frac{1}{13}\,b^3gx^{13}+\frac{1}{12}\,b^3fx^{12}+\frac{3}{11}\,ab^2hx^{11}+\frac{1}{11}\,b^3x^{11}e+\frac{1}{10}\,b^3dx^{10}+\frac{3}{10}\,ab^2gx^{10}+\frac{1}{9}\,b^3cx^9+\frac{1}{3}\,ab^2fx^9+\frac{3}{8}\,a^2bhx^8+\frac{3}{8}\,ab^2x^8e+\frac{3}{7}\,ab^2dx^7+\frac{3}{7}\,a^2bgx^7+\frac{1}{2}\,ab^2cx^6+\frac{1}{2}\,a^2bfx^6+\frac{1}{5}\,a^3hx^5+\frac{3}{5}\,a^2bx^5e+\frac{3}{4}\,a^2bdx^4+\frac{1}{4}\,a^3gx^4+a^2bcx^3+\frac{1}{3}\,a^3fx^3+\frac{1}{2}\,a^3x^2e+a^3dx+a^3c\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac"
)
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 3/11*a*b^2*h*x^11 + 1
/11*b^3*x^11*e + 1/10*b^3*d*x^10 + 3/10*a*b^2*g*x^10 + 1/9*b^3*c*x^9 + 1/3*
a*b^2*f*x^9 + 3/8*a^2*b*h*x^8 + 3/8*a*b^2*x^8*e + 3/7*a*b^2*d*x^7 + 3/7*a^2
*b*g*x^7 + 1/2*a*b^2*c*x^6 + 1/2*a^2*b*f*x^6 + 1/5*a^3*h*x^5 + 3/5*a^2*b*x^
5*e + 3/4*a^2*b*d*x^4 + 1/4*a^3*g*x^4 + a^2*b*c*x^3 + 1/3*a^3*f*x^3 + 1/2*a
^3*x^2*e + a^3*d*x + a^3*c*log(abs(x))
```

Mupad [B]

time = 5.11, size = 199, normalized size = 1.00

$$x^3\left(\frac{f\,a^3}{3}+b\,c\,a^2\right)+x^9\left(\frac{c\,b^3}{9}+\frac{a\,f\,b^2}{3}\right)+x^4\left(\frac{g\,a^3}{4}+\frac{3\,b\,d\,a^2}{4}\right)+x^{10}\left(\frac{d\,b^3}{10}+\frac{3\,a\,g\,b^2}{10}\right)+x^5\left(\frac{h\,a^3}{5}+\frac{3\,b\,e\,a^2}{5}\right)+x^{11}\left(\frac{e\,b^3}{11}+\frac{3\,a\,h\,b^2}{11}\right)+\frac{a^3\,e\,x^2}{2}+\frac{b^3\,f\,x^{12}}{12}+\frac{b^3\,g\,x^{13}}{13}+\frac{b^3\,h\,x^{14}}{14}+a^3\,c\,\ln\left(x\right)+a^3\,d\,x+\frac{a\,b\,x^6\,\left(b\,c+a\,f\right)}{2}+\frac{3\,a\,b\,x^7\,\left(b\,d+a\,g\right)}{7}+\frac{3\,a\,b\,x^8\,\left(b\,e+a\,h\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x)
```

```
[Out] x^3*((a^3*f)/3 + a^2*b*c) + x^9*((b^3*c)/9 + (a*b^2*f)/3) + x^4*((a^3*g)/4
+ (3*a^2*b*d)/4) + x^10*((b^3*d)/10 + (3*a*b^2*g)/10) + x^5*((a^3*h)/5 + (3
*a^2*b*e)/5) + x^11*((b^3*e)/11 + (3*a*b^2*h)/11) + (a^3*e*x^2)/2 + (b^3*f*
x^12)/12 + (b^3*g*x^13)/13 + (b^3*h*x^14)/14 + a^3*c*log(x) + a^3*d*x + (a*
b*x^6*(b*c + a*f))/2 + (3*a*b*x^7*(b*d + a*g))/7 + (3*a*b*x^8*(b*e + a*h))/
8
```

3.399 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$

Optimal. Leaf size=198

$$-\frac{a^3c}{x}+a^3ex+\frac{1}{2}a^2(3bc+af)x^2+a^2bdx^3+\frac{1}{4}a^2(3be+ah)x^4+\frac{3}{5}ab(bc+af)x^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab(be+ah)x^7+\frac{1}{8}b^2(bc+$$

```
[Out] -a^3*c/x+a^3*e*x+1/2*a^2*(a*f+3*b*c)*x^2+a^2*b*d*x^3+1/4*a^2*(a*h+3*b*e)*x^
4+3/5*a*b*(a*f+b*c)*x^5+1/2*a*b^2*d*x^6+3/7*a*b*(a*h+b*e)*x^7+1/8*b^2*(3*a*
f+b*c)*x^8+1/9*b^3*d*x^9+1/10*b^2*(3*a*h+b*e)*x^10+1/11*b^3*f*x^11+1/13*b^3
*h*x^13+1/12*g*(b*x^3+a)^4/b+a^3*d*ln(x)
```

Rubi [A]
time = 0.12, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1597, 1834}

$$-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{1}{2}a^2x^2(af+3bc)+a^2bdx^3+\frac{1}{4}a^2x^4(ah+3be)+\frac{1}{8}b^2x^8(3af+bc)+\frac{1}{2}ab^2dx^6+\frac{1}{10}b^2x^{10}(3ah+be)+\frac{3}{5}abx^5(af+bc)+\frac{3}{7}abx^7(ah+be)+\frac{g(a+bx^3)^4}{12b}+\frac{1}{9}b^3dx^9+\frac{1}{11}b^3fx^{11}+\frac{1}{13}b^3hx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^3*c)/x) + a^3*e*x + (a^2*(3*b*c + a*f)*x^2)/2 + a^2*b*d*x^3 + (a^2*(3*
b*e + a*h)*x^4)/4 + (3*a*b*(b*c + a*f)*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b*(b
*e + a*h)*x^7)/7 + (b^2*(b*c + 3*a*f)*x^8)/8 + (b^3*d*x^9)/9 + (b^2*(b*e +
3*a*h)*x^10)/10 + (b^3*f*x^11)/11 + (b^3*h*x^13)/13 + (g*(a + b*x^3)^4)/(12
*b) + a^3*d*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = \frac{g(a+bx^3)^4}{12b} + \int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+h$$

$$= \frac{g(a+bx^3)^4}{12b} + \int \left(a^3e + \frac{a^3c}{x^2} + \frac{a^3d}{x} + a^2(3bc+af)x$$

$$= -\frac{a^3c}{x} + a^3ex + \frac{1}{2}a^2(3bc+af)x^2 + a^2bdx^3 + \frac{1}{4}a^2(3be$$

Mathematica [A]
time = 0.09, size = 172, normalized size = 0.87

$$a^3\left(-\frac{c}{x}+ex+\frac{1}{12}x^2(6f+4gx+3hx^2)\right)+\frac{b^3x^8(6435c+5720dx+6x^2(858e+780fx+715gx^2+660hx^3))}{51480}+\frac{1}{140}a^2bx^2(210c+x(140d+x(105e+84fx+70gx^2+60hx^3)))+\frac{1}{840}ab^2x^5(504c+x(420d+x(360e+315fx+280gx^2+252hx^3)))+a^3d\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] a^3*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) + (b^3*x^8*(6435*c +
5720*d*x + 6*x^2*(858*e + 780*f*x + 715*g*x^2 + 660*h*x^3)))/51480 + (a^2*b
*x^2*(210*c + x*(140*d + x*(105*e + 84*f*x + 70*g*x^2 + 60*h*x^3))))/140 +
(a*b^2*x^5*(504*c + x*(420*d + x*(360*e + 315*f*x + 280*g*x^2 + 252*h*x^3))
))/840 + a^3*d*Log[x]
```

Maple [A]
time = 0.33, size = 224, normalized size = 1.13

method	result
norman	$\frac{(\frac{1}{2}a^3f+\frac{3}{2}ca^2b)x^3+(\frac{1}{3}a^3g+da^2b)x^4+(\frac{1}{4}a^3h+\frac{3}{4}a^2be)x^5+(\frac{3}{8}ab^2f+\frac{1}{8}b^3c)x^9+(\frac{1}{3}ab^2g+\frac{1}{9}b^3d)x^{10}+(\frac{3}{10}ab^2h+\frac{1}{10}eb^3)x^{11}+(\frac{3}{5}a^2bf-}{x}$
default	$\frac{b^3hx^{13}}{13}+\frac{b^3gx^{12}}{12}+\frac{b^3fx^{11}}{11}+\frac{3ab^2hx^{10}}{10}+\frac{b^3ex^{10}}{10}+\frac{ab^2gx^9}{3}+\frac{b^3dx^9}{9}+\frac{3ab^2fx^8}{8}+\frac{b^3cx^8}{8}+\frac{3a^2bhx^7}{7}+\frac{3ab^2ex}{7}$
risch	$\frac{b^3hx^{13}}{13}+\frac{b^3gx^{12}}{12}+\frac{b^3fx^{11}}{11}+\frac{3ab^2hx^{10}}{10}+\frac{b^3ex^{10}}{10}+\frac{ab^2gx^9}{3}+\frac{b^3dx^9}{9}+\frac{3ab^2fx^8}{8}+\frac{b^3cx^8}{8}+\frac{3a^2bhx^7}{7}+\frac{3ab^2ex}{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/13*b^3*h*x^13+1/12*b^3*g*x^12+1/11*b^3*f*x^11+3/10*a*b^2*h*x^10+1/10*b^3*
e*x^10+1/3*a*b^2*g*x^9+1/9*b^3*d*x^9+3/8*a*b^2*f*x^8+1/8*b^3*c*x^8+3/7*a^2*
b*h*x^7+3/7*a*b^2*e*x^7+1/2*a^2*b*g*x^6+1/2*a*b^2*d*x^6+3/5*a^2*b*f*x^5+3/5
*a*b^2*c*x^5+1/4*a^3*h*x^4+3/4*a^2*b*e*x^4+1/3*a^3*g*x^3+a^2*b*d*x^3+1/2*a^
3*f*x^2+3/2*a^2*b*c*x^2+a^3*e*x+a^3*d*ln(x)-a^3*c/x
```

Maxima [A]
time = 0.31, size = 216, normalized size = 1.09

$$\frac{1}{13}\,b^{3}hx^{13}+\frac{1}{12}\,b^{3}gx^{12}+\frac{1}{11}\,b^{3}fx^{11}+\frac{1}{10}\,(3\,ab^{2}h+b^{3}e)x^{10}+\frac{1}{9}\,(b^{3}d+3\,ab^{2}g)x^{9}+\frac{1}{8}\,(b^{3}c+3\,ab^{2}f)x^{8}+\frac{3}{7}\,(a^{2}bh+ab^{2}e)x^{7}+\frac{1}{2}\,(ab^{2}d+a^{2}bg)x^{6}+\frac{3}{5}\,(ab^{2}c+a^{2}bf)x^{5}+\frac{1}{4}\,(a^{3}h+3\,a^{2}be)x^{4}+a^{3}xe+a^{3}d\log(x)+\frac{1}{3}\,(3\,a^{2}bd+a^{3}g)x^{3}-\frac{a^{3}c}{x}+\frac{1}{2}\,(3\,a^{2}bc+a^{3}f)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="max
ima")
```

```
[Out] 1/13*b^3*h*x^13 + 1/12*b^3*g*x^12 + 1/11*b^3*f*x^11 + 1/10*(3*a*b^2*h + b^3
*e)*x^10 + 1/9*(b^3*d + 3*a*b^2*g)*x^9 + 1/8*(b^3*c + 3*a*b^2*f)*x^8 + 3/7*
(a^2*b*h + a*b^2*e)*x^7 + 1/2*(a*b^2*d + a^2*b*g)*x^6 + 3/5*(a*b^2*c + a^2*
b*f)*x^5 + 1/4*(a^3*h + 3*a^2*b*e)*x^4 + a^3*x*e + a^3*d*log(x) + 1/3*(3*a^
2*b*d + a^3*g)*x^3 - a^3*c/x + 1/2*(3*a^2*b*c + a^3*f)*x^2
```

Fricas [A]
time = 0.36, size = 219, normalized size = 1.11

$$\frac{27720\,b^{3}hx^{14}+30030\,b^{3}gx^{13}+32760\,b^{3}fx^{12}+36036\,(b^{3}e+3\,ab^{2}h)x^{11}+40040\,(b^{3}d+3\,ab^{2}g)x^{10}+45045\,(b^{3}c+3\,ab^{2}f)x^{9}+154440\,(ab^{2}e+a^{2}bh)x^{8}+180180\,(ab^{2}d+a^{2}bg)x^{7}+216216\,(ab^{2}c+a^{2}bf)x^{6}+360360\,a^{3}ex^{2}+90090\,(3\,a^{2}be+a^{3}h)x^{5}+360360\,a^{3}dx\log(x)+120120\,(3\,a^{2}bd+a^{3}g)x^{4}-360360\,a^{3}c+180180\,(3\,a^{2}bc+a^{3}f)x^{3}}{360360\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fri
cas")
```

```
[Out] 1/360360*(27720*b^3*h*x^14 + 30030*b^3*g*x^13 + 32760*b^3*f*x^12 + 36036*(b
^3*e + 3*a*b^2*h)*x^11 + 40040*(b^3*d + 3*a*b^2*g)*x^10 + 45045*(b^3*c + 3*
a*b^2*f)*x^9 + 154440*(a*b^2*e + a^2*b*h)*x^8 + 180180*(a*b^2*d + a^2*b*g)*
x^7 + 216216*(a*b^2*c + a^2*b*f)*x^6 + 360360*a^3*e*x^2 + 90090*(3*a^2*b*e
+ a^3*h)*x^5 + 360360*a^3*d*x*log(x) + 120120*(3*a^2*b*d + a^3*g)*x^4 - 360
360*a^3*c + 180180*(3*a^2*b*c + a^3*f)*x^3)/x
```

Sympy [A]
time = 0.19, size = 236, normalized size = 1.19

$$-\frac{a^{3}c}{x}+a^{3}d\log(x)+a^{3}ex+\frac{b^{3}fx^{11}}{11}+\frac{b^{3}gx^{12}}{12}+\frac{b^{3}hx^{13}}{13}+x^{10}\cdot\left(\frac{3ab^{2}h}{10}+\frac{b^{3}e}{10}\right)+x^{9}\left(\frac{ab^{2}g}{3}+\frac{b^{3}d}{9}\right)+x^{8}\cdot\left(\frac{3ab^{2}f}{8}+\frac{b^{3}c}{8}\right)+x^{7}\cdot\left(\frac{3a^{2}bh}{7}+\frac{3ab^{2}e}{7}\right)+x^{6}\left(\frac{a^{2}bg}{2}+\frac{ab^{2}d}{2}\right)+x^{5}\cdot\left(\frac{3a^{2}bf}{5}+\frac{3ab^{2}c}{5}\right)+x^{4}\left(\frac{a^{3}h}{4}+\frac{3a^{2}be}{4}\right)+x^{3}\left(\frac{a^{3}g}{3}+a^{2}bd\right)+x^{2}\left(\frac{a^{3}f}{2}+\frac{3a^{2}bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a**3*c/x + a**3*d*log(x) + a**3*e*x + b**3*f*x**11/11 + b**3*g*x**12/12 +
b**3*h*x**13/13 + x**10*(3*a*b**2*h/10 + b**3*e/10) + x**9*(a*b**2*g/3 + b*
*3*d/9) + x**8*(3*a*b**2*f/8 + b**3*c/8) + x**7*(3*a**2*b*h/7 + 3*a*b**2*e/
7) + x**6*(a**2*b*g/2 + a*b**2*d/2) + x**5*(3*a**2*b*f/5 + 3*a*b**2*c/5) +
x**4*(a**3*h/4 + 3*a**2*b*e/4) + x**3*(a**3*g/3 + a**2*b*d) + x**2*(a**3*f/
2 + 3*a**2*b*c/2)
```

Giac [A]
time = 0.53, size = 228, normalized size = 1.15

$$\frac{1}{13}\,b^3hx^{13}+\frac{1}{12}\,b^3gx^{12}+\frac{1}{11}\,b^3fx^{11}+\frac{3}{10}\,ab^2hx^{10}+\frac{1}{10}\,b^3x^{10}e+\frac{1}{9}\,b^3dx^9+\frac{1}{3}\,ab^2gx^9+\frac{1}{8}\,b^3cx^8+\frac{3}{8}\,ab^2fx^8+\frac{3}{7}\,a^2bhx^7+\frac{3}{7}\,ab^2x^7e+\frac{1}{2}\,ab^2dx^6+\frac{1}{2}\,a^2bgx^6+\frac{3}{5}\,ab^2cx^5+\frac{3}{5}\,a^2bfx^5+\frac{1}{4}\,a^3hx^4+\frac{3}{4}\,a^2bx^4e+a^2bdx^3+\frac{1}{3}\,a^3gx^3+\frac{3}{2}\,a^2bcx^2+\frac{1}{2}\,a^3fx^2+a^3xe+a^3d\log\left(|x|\right)-\frac{a^3c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="gia
c")
```

```
[Out] 1/13*b^3*h*x^13 + 1/12*b^3*g*x^12 + 1/11*b^3*f*x^11 + 3/10*a*b^2*h*x^10 + 1
/10*b^3*x^10*e + 1/9*b^3*d*x^9 + 1/3*a*b^2*g*x^9 + 1/8*b^3*c*x^8 + 3/8*a*b^
2*f*x^8 + 3/7*a^2*b*h*x^7 + 3/7*a*b^2*x^7*e + 1/2*a*b^2*d*x^6 + 1/2*a^2*b*g
*x^6 + 3/5*a*b^2*c*x^5 + 3/5*a^2*b*f*x^5 + 1/4*a^3*h*x^4 + 3/4*a^2*b*x^4*e
+ a^2*b*d*x^3 + 1/3*a^3*g*x^3 + 3/2*a^2*b*c*x^2 + 1/2*a^3*f*x^2 + a^3*x*e +
 a^3*d*log(abs(x)) - a^3*c/x
```

Mupad [B]
time = 5.05, size = 199, normalized size = 1.01

$$x^2\,\left(\frac{f\,a^3}{2}+\frac{3\,b\,c\,a^2}{2}\right)+x^8\,\left(\frac{c\,b^3}{8}+\frac{3\,a\,f\,b^2}{8}\right)+x^3\,\left(\frac{g\,a^3}{3}+b\,d\,a^2\right)+x^9\,\left(\frac{d\,b^3}{9}+\frac{a\,g\,b^2}{3}\right)+x^4\,\left(\frac{h\,a^3}{4}+\frac{3\,b\,e\,a^2}{4}\right)+x^{10}\,\left(\frac{e\,b^3}{10}+\frac{3\,a\,h\,b^2}{10}\right)-\frac{a^3\,c}{x}+\frac{b^3\,f\,x^{11}}{11}+\frac{b^3\,g\,x^{12}}{12}+\frac{b^3\,h\,x^{13}}{13}+a^3\,d\,\ln\left(x\right)+a^3\,e\,x+\frac{3\,a\,b\,x^5\,\left(b\,c+a\,f\right)}{5}+\frac{a\,b\,x^6\,\left(b\,d+a\,g\right)}{2}+\frac{3\,a\,b\,x^7\,\left(b\,e+a\,h\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x)
```

```
[Out] x^2*((a^3*f)/2 + (3*a^2*b*c)/2) + x^8*((b^3*c)/8 + (3*a*b^2*f)/8) + x^3*((a
^3*g)/3 + a^2*b*d) + x^9*((b^3*d)/9 + (a*b^2*g)/3) + x^4*((a^3*h)/4 + (3*a^
2*b*e)/4) + x^10*((b^3*e)/10 + (3*a*b^2*h)/10) - (a^3*c)/x + (b^3*f*x^11)/1
1 + (b^3*g*x^12)/12 + (b^3*h*x^13)/13 + a^3*d*log(x) + a^3*e*x + (3*a*b*x^5
*(b*c + a*f))/5 + (a*b*x^6*(b*d + a*g))/2 + (3*a*b*x^7*(b*e + a*h))/7
```

3.400 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

Optimal. Leaf size=198

$$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^2(3bc+af)x+\frac{1}{2}a^2(3bd+ag)x^2+a^2bex^3+\frac{3}{4}ab(bc+af)x^4+\frac{3}{5}ab(bd+ag)x^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^2(bc+3af$$

```
[Out] -1/2*a^3*c/x^2-a^3*d/x+a^2*(a*f+3*b*c)*x+1/2*a^2*(a*g+3*b*d)*x^2+a^2*b*e*x^
3+3/4*a*b*(a*f+b*c)*x^4+3/5*a*b*(a*g+b*d)*x^5+1/2*a*b^2*e*x^6+1/7*b^2*(3*a*
f+b*c)*x^7+1/8*b^2*(3*a*g+b*d)*x^8+1/9*b^3*e*x^9+1/10*b^3*f*x^10+1/11*b^3*g
*x^11+1/12*h*(b*x^3+a)^4/b+a^3*e*ln(x)
```

Rubi [A]
time = 0.13, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1597, 1834}

$$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+a^2x(af+3bc)+\frac{1}{2}a^2x^2(ag+3bd)+a^2bex^3+\frac{1}{7}b^2x^7(3af+bc)+\frac{1}{8}b^2x^8(3ag+bd)+\frac{1}{2}ab^2ex^6+\frac{3}{4}abx^4(af+bc)+\frac{3}{5}abx^5(ag+bd)+\frac{h(a+bx^3)^4}{12b}+\frac{1}{9}b^3ex^9+\frac{1}{10}b^3fx^{10}+\frac{1}{11}b^3gx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -1/2*(a^3*c)/x^2 - (a^3*d)/x + a^2*(3*b*c + a*f)*x + (a^2*(3*b*d + a*g)*x^2
)/2 + a^2*b*e*x^3 + (3*a*b*(b*c + a*f)*x^4)/4 + (3*a*b*(b*d + a*g)*x^5)/5 +
 (a*b^2*e*x^6)/2 + (b^2*(b*c + 3*a*f)*x^7)/7 + (b^2*(b*d + 3*a*g)*x^8)/8 +
(b^3*e*x^9)/9 + (b^3*f*x^10)/10 + (b^3*g*x^11)/11 + (h*(a + b*x^3)^4)/(12*b
) + a^3*e*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx = \frac{h(a+bx^3)^4}{12b} + \int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+g$$
$$= \frac{h(a+bx^3)^4}{12b} + \int \left(a^2(3bc+af) + \frac{a^3c}{x^3} + \frac{a^3d}{x^2} + \frac{a^3e}{x}\right.$$
$$= -\frac{a^3c}{2x^2} - \frac{a^3d}{x} + a^2(3bc+af)x + \frac{1}{2}a^2(3bd+ag)x^2 + a$$

Mathematica [A]
time = 0.08, size = 174, normalized size = 0.88

$$\frac{a^3(-3c-6dx+x^3(6f+3gx+2hx^2))}{6x^2} + \frac{b^3x^7(3960c+7x(495d+440ex+6x^2(66f+60gx+55hx^2)))}{27720} + \frac{1}{20}a^2bx(60c+x(30d+x(20e+15fx+12gx^2+10hx^3))) + \frac{1}{840}ab^2x^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx))))) + a^3e\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] (a^3*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (b^3*x^7*(3960
*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55*h*x^2))))/27720 + (a^
2*b*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))))/20 + (a*
b^2*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x))))))/84
0 + a^3*e*Log[x]
```

Maple [A]
time = 0.35, size = 222, normalized size = 1.12

method	result
norman	$\frac{(\frac{1}{2}a^3g+\frac{3}{2}d\,a^2b)x^4+(\frac{1}{3}a^3h+a^2be)x^5+(\frac{3}{7}a\,b^2f+\frac{1}{7}b^3c)x^9+(\frac{3}{8}a\,b^2g+\frac{1}{8}b^3d)x^{10}+(\frac{1}{3}a\,b^2h+\frac{1}{9}e\,b^3)x^{11}+(\frac{3}{4}a^2bf+\frac{3}{4}ac\,b^2)x^6+(\frac{3}{5}a^2bg+}{x^2}$
default	$\frac{b^3h\,x^{12}}{12}+\frac{b^3g\,x^{11}}{11}+\frac{b^3f\,x^{10}}{10}+\frac{a\,b^2h\,x^9}{3}+\frac{b^3e\,x^9}{9}+\frac{3a\,b^2g\,x^8}{8}+\frac{b^3d\,x^8}{8}+\frac{3a\,b^2f\,x^7}{7}+\frac{b^3c\,x^7}{7}+\frac{a^2bh\,x^6}{2}+\frac{a\,b^2e\,x^6}{2}+$
risch	$\frac{b^3h\,x^{12}}{12}+\frac{b^3g\,x^{11}}{11}+\frac{b^3f\,x^{10}}{10}+\frac{a\,b^2h\,x^9}{3}+\frac{b^3e\,x^9}{9}+\frac{3a\,b^2g\,x^8}{8}+\frac{b^3d\,x^8}{8}+\frac{3a\,b^2f\,x^7}{7}+\frac{b^3c\,x^7}{7}+\frac{a^2bh\,x^6}{2}+\frac{a\,b^2e\,x^6}{2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/12*b^3*h*x^12+1/11*b^3*g*x^11+1/10*b^3*f*x^10+1/3*a*b^2*h*x^9+1/9*b^3*e*x
^9+3/8*a*b^2*g*x^8+1/8*b^3*d*x^8+3/7*a*b^2*f*x^7+1/7*b^3*c*x^7+1/2*a^2*b*h*
x^6+1/2*a*b^2*e*x^6+3/5*a^2*b*g*x^5+3/5*a*b^2*d*x^5+3/4*a^2*b*f*x^4+3/4*a*b
^2*c*x^4+1/3*a^3*h*x^3+a^2*b*e*x^3+1/2*a^3*g*x^2+3/2*a^2*b*d*x^2+a^3*f*x+3*
a^2*b*c*x-1/2*a^3*c/x^2+a^3*e*ln(x)-a^3*d/x
```

Maxima [A]
time = 0.27, size = 216, normalized size = 1.09

$$\frac{1}{12}\,b^{3}hx^{12}+\frac{1}{11}\,b^{3}gx^{11}+\frac{1}{10}\,b^{3}fx^{10}+\frac{1}{9}\,(3\,ab^{2}h+b^{3}e)x^{9}+\frac{1}{8}\,(b^{3}d+3\,ab^{2}g)x^{8}+\frac{1}{7}\,(b^{3}c+3\,ab^{2}f)x^{7}+\frac{1}{2}\,(a^{2}bh+ab^{2}e)x^{6}+\frac{3}{5}\,(ab^{2}d+a^{2}bg)x^{5}+\frac{3}{4}\,(ab^{2}c+a^{2}bf)x^{4}+a^{3}e\log\left(x\right)+\frac{1}{3}\,(a^{3}h+3\,a^{2}be)x^{3}+\frac{1}{2}\,(3\,a^{2}bd+a^{3}g)x^{2}+(3\,a^{2}bc+a^{3}f)x-\frac{2\,a^{3}dx+a^{3}c}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="max
ima")
```

```
[Out] 1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/9*(3*a*b^2*h + b^3*
e)*x^9 + 1/8*(b^3*d + 3*a*b^2*g)*x^8 + 1/7*(b^3*c + 3*a*b^2*f)*x^7 + 1/2*(a
^2*b*h + a*b^2*e)*x^6 + 3/5*(a*b^2*d + a^2*b*g)*x^5 + 3/4*(a*b^2*c + a^2*b*
f)*x^4 + a^3*e*log(x) + 1/3*(a^3*h + 3*a^2*b*e)*x^3 + 1/2*(3*a^2*b*d + a^3*
g)*x^2 + (3*a^2*b*c + a^3*f)*x - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

Fricas [A]
time = 0.39, size = 219, normalized size = 1.11

$$\frac{2310\,b^{3}hx^{14}+2520\,b^{3}gx^{13}+2772\,b^{3}fx^{12}+3080\,(b^{3}e+3\,ab^{2}h)x^{11}+3465\,(b^{3}d+3\,ab^{2}g)x^{10}+3960\,(b^{3}c+3\,ab^{2}f)x^{9}+13860\,(ab^{2}e+a^{2}bh)x^{8}+16632\,(ab^{2}d+a^{2}bg)x^{7}+20790\,(ab^{2}c+a^{2}bf)x^{6}+27720\,a^{3}ex^{2}\log\left(x\right)+9240\,(3\,a^{2}be+a^{3}h)x^{5}-27720\,a^{3}dx+13860\,(3\,a^{2}bd+a^{3}g)x^{4}-13860\,a^{3}c+27720\,(3\,a^{2}bc+a^{3}f)x^{3}}{27720\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fri
cas")
```

```
[Out] 1/27720*(2310*b^3*h*x^14 + 2520*b^3*g*x^13 + 2772*b^3*f*x^12 + 3080*(b^3*e
+ 3*a*b^2*h)*x^11 + 3465*(b^3*d + 3*a*b^2*g)*x^10 + 3960*(b^3*c + 3*a*b^2*f
)*x^9 + 13860*(a*b^2*e + a^2*b*h)*x^8 + 16632*(a*b^2*d + a^2*b*g)*x^7 + 207
90*(a*b^2*c + a^2*b*f)*x^6 + 27720*a^3*e*x^2*log(x) + 9240*(3*a^2*b*e + a^3
*h)*x^5 - 27720*a^3*d*x + 13860*(3*a^2*b*d + a^3*g)*x^4 - 13860*a^3*c + 277
20*(3*a^2*b*c + a^3*f)*x^3)/x^2
```

Sympy [A]
time = 0.23, size = 238, normalized size = 1.20

$$a^{3}e\log\left(x\right)+\frac{b^{3}fx^{10}}{10}+\frac{b^{3}gx^{11}}{11}+\frac{b^{3}hx^{12}}{12}+x^{9}\left(\frac{ab^{2}h}{3}+\frac{b^{3}e}{9}\right)+x^{8}\cdot\left(\frac{3ab^{2}g}{8}+\frac{b^{3}d}{8}\right)+x^{7}\cdot\left(\frac{3ab^{2}f}{7}+\frac{b^{3}c}{7}\right)+x^{6}\left(\frac{a^{2}bh}{2}+\frac{ab^{2}e}{2}\right)+x^{5}\cdot\left(\frac{3a^{2}bg}{5}+\frac{3ab^{2}d}{5}\right)+x^{4}\cdot\left(\frac{3a^{2}bf}{4}+\frac{3ab^{2}c}{4}\right)+x^{3}\left(\frac{a^{3}h}{3}+a^{2}be\right)+x^{2}\left(\frac{a^{3}g}{2}+\frac{3a^{2}bd}{2}\right)+x\left(a^{3}f+3a^{2}bc\right)+\frac{-a^{3}c-2a^{3}dx}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**3*e*log(x) + b**3*f*x**10/10 + b**3*g*x**11/11 + b**3*h*x**12/12 + x**9*
(a*b**2*h/3 + b**3*e/9) + x**8*(3*a*b**2*g/8 + b**3*d/8) + x**7*(3*a*b**2*f
/7 + b**3*c/7) + x**6*(a**2*b*h/2 + a*b**2*e/2) + x**5*(3*a**2*b*g/5 + 3*a*
b**2*d/5) + x**4*(3*a**2*b*f/4 + 3*a*b**2*c/4) + x**3*(a**3*h/3 + a**2*b*e)
 + x**2*(a**3*g/2 + 3*a**2*b*d/2) + x*(a**3*f + 3*a**2*b*c) + (-a**3*c - 2*
a**3*d*x)/(2*x**2)
```

Giac [A]
time = 0.51, size = 226, normalized size = 1.14

$$\frac{1}{12}b^3hx^{12}+\frac{1}{11}b^3gx^{11}+\frac{1}{10}b^3fx^{10}+\frac{1}{3}ab^2hx^9+\frac{1}{9}b^3x^9e+\frac{1}{8}b^3dx^8+\frac{3}{8}ab^2gx^8+\frac{1}{7}b^3cx^7+\frac{3}{7}ab^2fx^7+\frac{1}{2}a^2bhx^6+\frac{1}{2}ab^2x^6e+\frac{3}{5}ab^2dx^5+\frac{3}{5}a^2bgx^5+\frac{3}{4}ab^2cx^4+\frac{3}{4}a^2bfx^4+\frac{1}{3}a^3hx^3+a^2bx^3e+\frac{3}{2}a^2bdx^2+\frac{1}{2}a^3gx^2+3a^2bcx+a^3fx+a^3e\log(|x|)-\frac{2a^3dx+a^3c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="gia
c")
```

```
[Out] 1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/3*a*b^2*h*x^9 + 1/9
*b^3*x^9*e + 1/8*b^3*d*x^8 + 3/8*a*b^2*g*x^8 + 1/7*b^3*c*x^7 + 3/7*a*b^2*f*
x^7 + 1/2*a^2*b*h*x^6 + 1/2*a*b^2*x^6*e + 3/5*a*b^2*d*x^5 + 3/5*a^2*b*g*x^5
 + 3/4*a*b^2*c*x^4 + 3/4*a^2*b*f*x^4 + 1/3*a^3*h*x^3 + a^2*b*x^3*e + 3/2*a^
2*b*d*x^2 + 1/2*a^3*g*x^2 + 3*a^2*b*c*x + a^3*f*x + a^3*e*log(abs(x)) - 1/2
*(2*a^3*d*x + a^3*c)/x^2
```

Mupad [B]
time = 0.14, size = 199, normalized size = 1.01

$$x^7\left(\frac{c\,b^3}{7}+\frac{3\,a\,f\,b^2}{7}\right)+x^2\left(\frac{g\,a^3}{2}+\frac{3\,b\,d\,a^2}{2}\right)+x^8\left(\frac{d\,b^3}{8}+\frac{3\,a\,g\,b^2}{8}\right)+x^3\left(\frac{h\,a^3}{3}+b\,e\,a^2\right)+x^9\left(\frac{e\,b^3}{9}+\frac{a\,h\,b^2}{3}\right)-\frac{\frac{a^3\,c}{2}+a^3\,d\,x}{x^2}+x\left(f\,a^3+3\,b\,c\,a^2\right)+\frac{b^3\,f\,x^{10}}{10}+\frac{b^3\,g\,x^{11}}{11}+\frac{b^3\,h\,x^{12}}{12}+a^3\,e\,\ln\left(x\right)+\frac{3\,a\,b\,x^4\,\left(b\,c+a\,f\right)}{4}+\frac{3\,a\,b\,x^5\,\left(b\,d+a\,g\right)}{5}+\frac{a\,b\,x^6\,\left(b\,e+a\,h\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x)
```

```
[Out] x^7*((b^3*c)/7 + (3*a*b^2*f)/7) + x^2*((a^3*g)/2 + (3*a^2*b*d)/2) + x^8*((b
^3*d)/8 + (3*a*b^2*g)/8) + x^3*((a^3*h)/3 + a^2*b*e) + x^9*((b^3*e)/9 + (a*
b^2*h)/3) - ((a^3*c)/2 + a^3*d*x)/x^2 + x*(a^3*f + 3*a^2*b*c) + (b^3*f*x^10
)/10 + (b^3*g*x^11)/11 + (b^3*h*x^12)/12 + a^3*e*log(x) + (3*a*b*x^4*(b*c +
 a*f))/4 + (3*a*b*x^5*(b*d + a*g))/5 + (a*b*x^6*(b*e + a*h))/2
```

3.401 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx$

Optimal. Leaf size=209

$$-\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2(3bd+ag)x+\frac{1}{2}a^2(3be+ah)x^2+ab(bc+af)x^3+\frac{3}{4}ab(bd+ag)x^4+\frac{3}{5}ab(be+ah)x^5+\frac{1}{6}b^2(bc+3a$$

```
[Out] -1/3*a^3*c/x^3-1/2*a^3*d/x^2-a^3*e/x+a^2*(a*g+3*b*d)*x+1/2*a^2*(a*h+3*b*e)*
x^2+a*b*(a*f+b*c)*x^3+3/4*a*b*(a*g+b*d)*x^4+3/5*a*b*(a*h+b*e)*x^5+1/6*b^2*(
3*a*f+b*c)*x^6+1/7*b^2*(3*a*g+b*d)*x^7+1/8*b^2*(3*a*h+b*e)*x^8+1/9*b^3*f*x^
9+1/10*b^3*g*x^10+1/11*b^3*h*x^11+a^2*(a*f+3*b*c)*ln(x)
```

Rubi [A]
time = 0.12, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1834}

$$-\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2\log(x)(af+3bc)+a^2x(ag+3bd)+\frac{1}{2}a^2x^2(ah+3be)+\frac{1}{6}b^2x^6(3af+bc)+\frac{1}{7}b^2x^7(3ag+bd)+\frac{1}{8}b^2x^8(3ah+be)+abx^3(af+bc)+\frac{3}{4}abx^4(ag+bd)+\frac{3}{5}abx^5(ah+be)+\frac{1}{9}b^3fx^9+\frac{1}{10}b^3gx^{10}+\frac{1}{11}b^3hx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/3*(a^3*c)/x^3 - (a^3*d)/(2*x^2) - (a^3*e)/x + a^2*(3*b*d + a*g)*x + (a^2
*(3*b*e + a*h)*x^2)/2 + a*b*(b*c + a*f)*x^3 + (3*a*b*(b*d + a*g)*x^4)/4 + (
3*a*b*(b*e + a*h)*x^5)/5 + (b^2*(b*c + 3*a*f)*x^6)/6 + (b^2*(b*d + 3*a*g)*x
^7)/7 + (b^2*(b*e + 3*a*h)*x^8)/8 + (b^3*f*x^9)/9 + (b^3*g*x^10)/10 + (b^3*
h*x^11)/11 + a^2*(3*b*c + a*f)*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx = \int \left(a^2(3bd+ag)+\frac{a^3c}{x^4}+\frac{a^3d}{x^3}+\frac{a^3e}{x^2}+\frac{a^2(3bc+af)}{x}\right.$$

$$= -\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2(3bd+ag)x+\frac{1}{2}a^2(3be+ah)x$$

Mathematica [A]
time = 0.08, size = 172, normalized size = 0.82

$$-\frac{a^3(2c+3x(d+2ex-x^3(2g+hx)))}{6x^3}+\frac{1}{20}a^2bx(60d+x(30e+x(20f+15gx+12hx^2)))+\frac{1}{280}ab^2x^3(280c+x(210d+x(168e+140fx+120gx^2+105hx^3)))+\frac{b^3x^6(4620c+x(3960d+7x(495e+4x(110f+99gx+90hx^2))))}{27720}+a^2(3bc+af)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/6*(a^3*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/x^3 + (a^2*b*x*(60*d +
 x*(30*e + x*(20*f + 15*g*x + 12*h*x^2))))/20 + (a*b^2*x^3*(280*c + x*(210*
d + x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^3))))/280 + (b^3*x^6*(4620*c +
 x*(3960*d + 7*x*(495*e + 4*x*(110*f + 99*g*x + 90*h*x^2))))))/27720 + a^2*(
3*b*c + a*f)*Log[x]
```

Maple [A]
time = 0.36, size = 218, normalized size = 1.04

method	result
norman	$\frac{\left(\frac{1}{2}a^3h+\frac{3}{2}a^2be\right)x^5+\left(\frac{1}{2}a\,b^2f+\frac{1}{6}b^3c\right)x^9+\left(\frac{3}{7}a\,b^2g+\frac{1}{7}b^3d\right)x^{10}+\left(\frac{3}{8}a\,b^2h+\frac{1}{8}e\,b^3\right)x^{11}+\left(\frac{3}{4}a^2bg+\frac{3}{4}a\,b^2d\right)x^7+\left(\frac{3}{5}a^2bh+\frac{3}{5}a\,b^2e\right)x^8+(a^2bf-}{x^3}$
default	$\frac{b^3h\,x^{11}}{11}+\frac{b^3g\,x^{10}}{10}+\frac{b^3f\,x^9}{9}+\frac{3a\,b^2h\,x^8}{8}+\frac{b^3e\,x^8}{8}+\frac{3a\,b^2g\,x^7}{7}+\frac{b^3d\,x^7}{7}+\frac{a\,b^2f\,x^6}{2}+\frac{b^3c\,x^6}{6}+\frac{3a^2bh\,x^5}{5}+\frac{3a\,b^2e\,x^5}{5}$
risch	$\frac{b^3h\,x^{11}}{11}+\frac{b^3g\,x^{10}}{10}+\frac{b^3f\,x^9}{9}+\frac{3a\,b^2h\,x^8}{8}+\frac{b^3e\,x^8}{8}+\frac{3a\,b^2g\,x^7}{7}+\frac{b^3d\,x^7}{7}+\frac{a\,b^2f\,x^6}{2}+\frac{b^3c\,x^6}{6}+\frac{3a^2bh\,x^5}{5}+\frac{3a\,b^2e\,x^5}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/11*b^3*h*x^11+1/10*b^3*g*x^10+1/9*b^3*f*x^9+3/8*a*b^2*h*x^8+1/8*b^3*e*x^8
+3/7*a*b^2*g*x^7+1/7*b^3*d*x^7+1/2*a*b^2*f*x^6+1/6*b^3*c*x^6+3/5*a^2*b*h*x^
5+3/5*a*b^2*e*x^5+3/4*a^2*b*g*x^4+3/4*a*b^2*d*x^4+a^2*b*f*x^3+a*b^2*c*x^3+1
/2*a^3*h*x^2+3/2*a^2*b*e*x^2+a^3*g*x+3*a^2*b*d*x-1/2*a^3*d/x^2-1/3*a^3*c/x^
3+a^2*(a*f+3*b*c)*ln(x)-a^3*e/x
```

Maxima [A]
time = 0.28, size = 216, normalized size = 1.03

$$\frac{1}{11}\,b^3hx^{11}+\frac{1}{10}\,b^3gx^{10}+\frac{1}{9}\,b^3fx^9+\frac{1}{8}\,(3\,ab^2h+b^3e)x^8+\frac{1}{7}\,(b^3d+3\,ab^2g)x^7+\frac{1}{6}\,(b^3c+3\,ab^2f)x^6+\frac{3}{5}\,(a^2bh+ab^2e)x^5+\frac{3}{4}\,(ab^2d+a^2bg)x^4+(ab^2c+a^2bf)x^3+\frac{1}{2}\,(a^3h+3\,a^2be)x^2+(3\,a^2bd+a^3g)x+(3\,a^2bc+a^3f)\log\left(x\right)-\frac{6\,a^3x^2e+3\,a^3dx+2\,a^3c}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="max
ima")
```

```
[Out] 1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 1/8*(3*a*b^2*h + b^3*e)
*x^8 + 1/7*(b^3*d + 3*a*b^2*g)*x^7 + 1/6*(b^3*c + 3*a*b^2*f)*x^6 + 3/5*(a^2
*b*h + a*b^2*e)*x^5 + 3/4*(a*b^2*d + a^2*b*g)*x^4 + (a*b^2*c + a^2*b*f)*x^3
 + 1/2*(a^3*h + 3*a^2*b*e)*x^2 + (3*a^2*b*d + a^3*g)*x + (3*a^2*b*c + a^3*f
)*log(x) - 1/6*(6*a^3*x^2*e + 3*a^3*d*x + 2*a^3*c)/x^3
```

Fricas [A]
time = 0.41, size = 219, normalized size = 1.05

$$\frac{2520\,b^3hx^{14}+2772\,b^3gx^{13}+3080\,b^3fx^{12}+3465\,(b^3e+3\,ab^2h)x^{11}+3960\,(b^3d+3\,ab^2g)x^{10}+4620\,(b^3c+3\,ab^2f)x^9+16632\,(ab^2e+a^2bh)x^8+20790\,(ab^2d+a^2bg)x^7+27720\,(ab^2c+a^2bf)x^6-27720\,a^3ex^2+13860\,(3\,a^2be+a^3h)x^5-13860\,a^3dx+27720\,(3\,a^2bd+a^3g)x^4+27720\,(3\,a^2bc+a^3f)x^3\log\left(x\right)-9240\,a^3c}{27720\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fri
cas")
```

```
[Out] 1/27720*(2520*b^3*h*x^14 + 2772*b^3*g*x^13 + 3080*b^3*f*x^12 + 3465*(b^3*e
+ 3*a*b^2*h)*x^11 + 3960*(b^3*d + 3*a*b^2*g)*x^10 + 4620*(b^3*c + 3*a*b^2*f
)*x^9 + 16632*(a*b^2*e + a^2*b*h)*x^8 + 20790*(a*b^2*d + a^2*b*g)*x^7 + 277
20*(a*b^2*c + a^2*b*f)*x^6 - 27720*a^3*e*x^2 + 13860*(3*a^2*b*e + a^3*h)*x^
5 - 13860*a^3*d*x + 27720*(3*a^2*b*d + a^3*g)*x^4 + 27720*(3*a^2*b*c + a^3*
f)*x^3*log(x) - 9240*a^3*c)/x^3
```

Sympy [A]
time = 0.46, size = 236, normalized size = 1.13

$$a^{2}(af+3bc)\log{(x)}+\frac{b^{3}fx^{9}}{9}+\frac{b^{3}gx^{10}}{10}+\frac{b^{3}hx^{11}}{11}+x^{8}\cdot\left(\frac{3ab^{2}h}{8}+\frac{b^{3}e}{8}\right)+x^{7}\cdot\left(\frac{3ab^{2}g}{7}+\frac{b^{3}d}{7}\right)+x^{6}\left(\frac{ab^{2}f}{2}+\frac{b^{3}c}{6}\right)+x^{5}\cdot\left(\frac{3a^{2}bh}{5}+\frac{3ab^{2}e}{5}\right)+x^{4}\cdot\left(\frac{3a^{2}bg}{4}+\frac{3ab^{2}d}{4}\right)+x^{3}(a^{2}bf+ab^{2}c)+x^{2}\left(\frac{a^{3}h}{2}+\frac{3a^{2}be}{2}\right)+x(a^{3}g+3a^{2}bd)+\frac{-2a^{3}c-3a^{3}dx-6a^{3}ex^{2}}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a**2*(a*f + 3*b*c)*log(x) + b**3*f*x**9/9 + b**3*g*x**10/10 + b**3*h*x**11/
11 + x**8*(3*a*b**2*h/8 + b**3*e/8) + x**7*(3*a*b**2*g/7 + b**3*d/7) + x**6
*(a*b**2*f/2 + b**3*c/6) + x**5*(3*a**2*b*h/5 + 3*a*b**2*e/5) + x**4*(3*a**
2*b*g/4 + 3*a*b**2*d/4) + x**3*(a**2*b*f + a*b**2*c) + x**2*(a**3*h/2 + 3*a
**2*b*e/2) + x*(a**3*g + 3*a**2*b*d) + (-2*a**3*c - 3*a**3*d*x - 6*a**3*e*x
**2)/(6*x**3)
```

Giac [A]
time = 0.50, size = 225, normalized size = 1.08

$$\frac{1}{11}b^{3}hx^{11}+\frac{1}{10}b^{3}gx^{10}+\frac{1}{9}b^{3}fx^{9}+\frac{3}{8}ab^{2}hx^{8}+\frac{1}{8}b^{3}x^{8}e+\frac{1}{7}b^{3}dx^{7}+\frac{3}{7}ab^{2}gx^{7}+\frac{1}{6}b^{3}cx^{6}+\frac{1}{2}ab^{2}fx^{6}+\frac{3}{5}a^{2}bhx^{5}+\frac{3}{5}ab^{2}x^{5}e+\frac{3}{4}ab^{2}dx^{4}+\frac{3}{4}a^{2}bgx^{4}+ab^{2}cx^{3}+a^{2}bfx^{3}+\frac{1}{2}a^{3}hx^{2}+\frac{3}{2}a^{2}bx^{2}e+3\,a^{2}bdx+a^{3}gx+(3\,a^{2}bc+a^{3}f)\log(|x|)-\frac{6\,a^{3}x^{2}e+3\,a^{3}dx+2\,a^{3}c}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="gia
c")
```

```
[Out] 1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 3/8*a*b^2*h*x^8 + 1/8*b
^3*x^8*e + 1/7*b^3*d*x^7 + 3/7*a*b^2*g*x^7 + 1/6*b^3*c*x^6 + 1/2*a*b^2*f*x^
6 + 3/5*a^2*b*h*x^5 + 3/5*a*b^2*x^5*e + 3/4*a*b^2*d*x^4 + 3/4*a^2*b*g*x^4 +
 a*b^2*c*x^3 + a^2*b*f*x^3 + 1/2*a^3*h*x^2 + 3/2*a^2*b*x^2*e + 3*a^2*b*d*x
+ a^3*g*x + (3*a^2*b*c + a^3*f)*log(abs(x)) - 1/6*(6*a^3*x^2*e + 3*a^3*d*x
+ 2*a^3*c)/x^3
```

Mupad [B]
time = 0.12, size = 199, normalized size = 0.95

$$x^{6}\left(\frac{c\,b^{3}}{6}+\frac{a\,f\,b^{2}}{2}\right)+x^{7}\left(\frac{d\,b^{3}}{7}+\frac{3\,a\,g\,b^{2}}{7}\right)+x^{2}\left(\frac{h\,a^{3}}{2}+\frac{3\,b\,e\,a^{2}}{2}\right)+x^{8}\left(\frac{e\,b^{3}}{8}+\frac{3\,a\,h\,b^{2}}{8}\right)+\ln(x)\,\left(f\,a^{3}+3\,b\,c\,a^{2}\right)-\frac{e\,a^{3}\,x^{2}+\frac{d\,a^{3}\,x}{2}+\frac{c\,a^{3}}{3}}{x^{3}}+x\,\left(g\,a^{3}+3\,b\,d\,a^{2}\right)+\frac{b^{3}\,f\,x^{9}}{9}+\frac{b^{3}\,g\,x^{10}}{10}+\frac{b^{3}\,h\,x^{11}}{11}+a\,b\,x^{3}\,\left(b\,c+a\,f\right)+\frac{3\,a\,b\,x^{4}\,\left(b\,d+a\,g\right)}{4}+\frac{3\,a\,b\,x^{5}\,\left(b\,e+a\,h\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x)
```

```
[Out] x^6*((b^3*c)/6 + (a*b^2*f)/2) + x^7*((b^3*d)/7 + (3*a*b^2*g)/7) + x^2*((a^3
*h)/2 + (3*a^2*b*e)/2) + x^8*((b^3*e)/8 + (3*a*b^2*h)/8) + log(x)*(a^3*f +
3*a^2*b*c) - ((a^3*c)/3 + a^3*e*x^2 + (a^3*d*x)/2)/x^3 + x*(a^3*g + 3*a^2*b
*d) + (b^3*f*x^9)/9 + (b^3*g*x^10)/10 + (b^3*h*x^11)/11 + a*b*x^3*(b*c + a*
f) + (3*a*b*x^4*(b*d + a*g))/4 + (3*a*b*x^5*(b*e + a*h))/5
```

3.402 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$

Optimal. Leaf size=209

$$-\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^2(3bc+af)}{x}+a^2(3be+ah)x+\frac{3}{2}ab(bc+af)x^2+ab(bd+ag)x^3+\frac{3}{4}ab(be+ah)x^4+\frac{1}{5}b^2(bc+3af$$

```
[Out] -1/4*a^3*c/x^4-1/3*a^3*d/x^3-1/2*a^3*e/x^2-a^2*(a*f+3*b*c)/x+a^2*(a*h+3*b*e
)*x+3/2*a*b*(a*f+b*c)*x^2+a*b*(a*g+b*d)*x^3+3/4*a*b*(a*h+b*e)*x^4+1/5*b^2*(
3*a*f+b*c)*x^5+1/6*b^2*(3*a*g+b*d)*x^6+1/7*b^2*(3*a*h+b*e)*x^7+1/8*b^3*f*x^
8+1/9*b^3*g*x^9+1/10*b^3*h*x^10+a^2*(a*g+3*b*d)*ln(x)
```

Rubi [A]
time = 0.11, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1834}

$$-\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^2(af+3bc)}{x}+a^2\log(x)(ag+3bd)+a^2x(ah+3be)+\frac{1}{5}b^2x^5(3af+bc)+\frac{1}{6}b^2x^6(3ag+bd)+\frac{1}{7}b^2x^7(3ah+be)+\frac{3}{2}abx^2(af+bc)+abx^3(ag+bd)+\frac{3}{4}abx^4(ah+be)+\frac{1}{8}b^3fx^8+\frac{1}{9}b^3gx^9+\frac{1}{10}b^3hx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -1/4*(a^3*c)/x^4 - (a^3*d)/(3*x^3) - (a^3*e)/(2*x^2) - (a^2*(3*b*c + a*f))/
x + a^2*(3*b*e + a*h)*x + (3*a*b*(b*c + a*f)*x^2)/2 + a*b*(b*d + a*g)*x^3 +
 (3*a*b*(b*e + a*h)*x^4)/4 + (b^2*(b*c + 3*a*f)*x^5)/5 + (b^2*(b*d + 3*a*g)
*x^6)/6 + (b^2*(b*e + 3*a*h)*x^7)/7 + (b^3*f*x^8)/8 + (b^3*g*x^9)/9 + (b^3*
h*x^10)/10 + a^2*(3*b*d + a*g)*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx &= \int \left(a^2(3be+ah)+\frac{a^3c}{x^5}+\frac{a^3d}{x^4}+\frac{a^3e}{x^3}+\frac{a^2(3bc+af)}{x^2}\right.\\ &= -\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^2(3bc+af)}{x}+a^2(3be+ah)x+\end{aligned}$$

Mathematica [A]
time = 0.08, size = 170, normalized size = 0.81

$$\frac{-210a^3(3c+4dx+6x^2(e+2fx-2hx^3))+630a^2bx^3(-12c+x^2(12e+6fx+4gx^2+3hx^3))+18ab^2x^6(210c+x(140d+105ex+84fx^2+70gx^3+60hx^4))+b^3x^9(504c+x(420d+360ex+315fx^2+280gx^3+252hx^4))}{2520x^4}+a^2(3bd+ag)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] (-210*a^3*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)) + 630*a^2*b*x^3*(-12*
c + x^2*(12*e + 6*f*x + 4*g*x^2 + 3*h*x^3)) + 18*a*b^2*x^6*(210*c + x*(140*
d + 105*e*x + 84*f*x^2 + 70*g*x^3 + 60*h*x^4)) + b^3*x^9*(504*c + x*(420*d
+ 360*e*x + 315*f*x^2 + 280*g*x^3 + 252*h*x^4)))/(2520*x^4) + a^2*(3*b*d +
a*g)*Log[x]
```

Maple [A]

time = 0.36, size = 215, normalized size = 1.03

method	result
default	$\frac{b^3 h\,x^{10}}{10}+\frac{b^3 g\,x^9}{9}+\frac{b^3 f\,x^8}{8}+\frac{3a\,b^2 h\,x^7}{7}+\frac{b^3 e\,x^7}{7}+\frac{a\,b^2 g\,x^6}{2}+\frac{b^3 d\,x^6}{6}+\frac{3a\,b^2 f\,x^5}{5}+\frac{b^3 c\,x^5}{5}+\frac{3a^2 bh\,x^4}{4}+\frac{3a\,b^2 e\,x^4}{4}+$
norman	$\frac{(\frac{3}{5}a\,b^2 f+\frac{1}{5}b^3 c)x^9+(\frac{1}{2}a\,b^2 g+\frac{1}{6}b^3 d)x^{10}+(\frac{3}{7}a\,b^2 h+\frac{1}{7}e\,b^3)x^{11}+(\frac{3}{2}a^2 bf+\frac{3}{2}ac\,b^2)x^6+(\frac{3}{4}a^2 bh+\frac{3}{4}a\,b^2 e)x^8+(-a^3 f-3c\,a^2 b)x^3+(a^2 bg+}{x^4}$
risch	$\frac{b^3 h\,x^{10}}{10}+\frac{b^3 g\,x^9}{9}+\frac{b^3 f\,x^8}{8}+\frac{3a\,b^2 h\,x^7}{7}+\frac{b^3 e\,x^7}{7}+\frac{a\,b^2 g\,x^6}{2}+\frac{b^3 d\,x^6}{6}+\frac{3a\,b^2 f\,x^5}{5}+\frac{b^3 c\,x^5}{5}+\frac{3a^2 bh\,x^4}{4}+\frac{3a\,b^2 e\,x^4}{4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/10*b^3*h*x^10+1/9*b^3*g*x^9+1/8*b^3*f*x^8+3/7*a*b^2*h*x^7+1/7*b^3*e*x^7+1
/2*a*b^2*g*x^6+1/6*b^3*d*x^6+3/5*a*b^2*f*x^5+1/5*b^3*c*x^5+3/4*a^2*b*h*x^4+
3/4*a*b^2*e*x^4+a^2*b*g*x^3+a*b^2*d*x^3+3/2*a^2*b*f*x^2+3/2*a*b^2*c*x^2+a^3
*h*x+3*a^2*b*e*x-1/4*a^3*c/x^4-1/2*a^3*e/x^2-1/3*a^3*d/x^3+a^2*(a*g+3*b*d)*
ln(x)-a^2*(a*f+3*b*c)/x
```

Maxima [A]

time = 0.29, size = 216, normalized size = 1.03

$$\frac{1}{10}\,b^3hx^{10}+\frac{1}{9}\,b^3gx^9+\frac{1}{8}\,b^3fx^8+\frac{1}{7}\,(3\,ab^2h+b^3e)x^7+\frac{1}{6}\,(b^3d+3\,ab^2g)x^6+\frac{1}{5}\,(b^3c+3\,ab^2f)x^5+\frac{3}{4}\,(a^2bh+ab^2e)x^4+(ab^2d+a^2bg)x^3+\frac{3}{2}\,(ab^2c+a^2bf)x^2+(a^3h+3\,a^2be)x+(3\,a^2bd+a^3g)\log(x)-\frac{6\,a^3x^2e+4\,a^3dx+3\,a^3c+12\,(3\,a^2bc+a^3f)x^3}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="max
ima")
```

```
[Out] 1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 1/7*(3*a*b^2*h + b^3*e)*x
^7 + 1/6*(b^3*d + 3*a*b^2*g)*x^6 + 1/5*(b^3*c + 3*a*b^2*f)*x^5 + 3/4*(a^2*b
*h + a*b^2*e)*x^4 + (a*b^2*d + a^2*b*g)*x^3 + 3/2*(a*b^2*c + a^2*b*f)*x^2 +
 (a^3*h + 3*a^2*b*e)*x + (3*a^2*b*d + a^3*g)*log(x) - 1/12*(6*a^3*x^2*e + 4
*a^3*d*x + 3*a^3*c + 12*(3*a^2*b*c + a^3*f)*x^3)/x^4
```

Fricas [A]
time = 0.39, size = 219, normalized size = 1.05

$$\frac{252\,b^{3}hx^{14}+280\,b^{3}gx^{13}+315\,b^{3}fx^{12}+360\,(b^{3}e+3\,ab^{2}h)x^{11}+420\,(b^{3}d+3\,ab^{2}g)x^{10}+504\,(b^{3}c+3\,ab^{2}f)x^{9}+1890\,(ab^{2}e+a^{2}bh)x^{8}+2520\,(ab^{2}d+a^{2}bg)x^{7}+3780\,(ab^{2}c+a^{2}bf)x^{6}-1260\,a^{3}ex^{2}+2520\,(3\,a^{2}be+a^{3}h)x^{5}+2520\,(3\,a^{2}bd+a^{3}g)x^{4}\log\left(x\right)-840\,a^{3}dx-630\,a^{3}c-2520\,(3\,a^{2}bc+a^{3}f)x^{3}}{2520\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fri
cas")
```

```
[Out] 1/2520*(252*b^3*h*x^14 + 280*b^3*g*x^13 + 315*b^3*f*x^12 + 360*(b^3*e + 3*a
*b^2*h)*x^11 + 420*(b^3*d + 3*a*b^2*g)*x^10 + 504*(b^3*c + 3*a*b^2*f)*x^9 +
 1890*(a*b^2*e + a^2*b*h)*x^8 + 2520*(a*b^2*d + a^2*b*g)*x^7 + 3780*(a*b^2*
c + a^2*b*f)*x^6 - 1260*a^3*e*x^2 + 2520*(3*a^2*b*e + a^3*h)*x^5 + 2520*(3*
a^2*b*d + a^3*g)*x^4*log(x) - 840*a^3*d*x - 630*a^3*c - 2520*(3*a^2*b*c + a
^3*f)*x^3)/x^4
```

Sympy [A]
time = 1.57, size = 235, normalized size = 1.12

$$a^{2}(ag+3bd)\log\left(x\right)+\frac{b^{3}fx^{8}}{8}+\frac{b^{3}gx^{9}}{9}+\frac{b^{3}hx^{10}}{10}+x^{7}\cdot\left(\frac{3ab^{2}h}{7}+\frac{b^{3}e}{7}\right)+x^{6}\left(\frac{ab^{2}g}{2}+\frac{b^{3}d}{6}\right)+x^{5}\cdot\left(\frac{3ab^{2}f}{5}+\frac{b^{3}c}{5}\right)+x^{4}\cdot\left(\frac{3a^{2}bh}{4}+\frac{3ab^{2}e}{4}\right)+x^{3}(a^{2}bg+ab^{2}d)+x^{2}\cdot\left(\frac{3a^{2}bf}{2}+\frac{3ab^{2}c}{2}\right)+x(a^{3}h+3a^{2}be)+\frac{-3a^{3}c-4a^{3}dx-6a^{3}ex^{2}+x^{3}(-12a^{3}f-36a^{2}bc)}{12x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a**2*(a*g + 3*b*d)*log(x) + b**3*f*x**8/8 + b**3*g*x**9/9 + b**3*h*x**10/10
 + x**7*(3*a*b**2*h/7 + b**3*e/7) + x**6*(a*b**2*g/2 + b**3*d/6) + x**5*(3*
a*b**2*f/5 + b**3*c/5) + x**4*(3*a**2*b*h/4 + 3*a*b**2*e/4) + x**3*(a**2*b*
g + a*b**2*d) + x**2*(3*a**2*b*f/2 + 3*a*b**2*c/2) + x*(a**3*h + 3*a**2*b*e
) + (-3*a**3*c - 4*a**3*d*x - 6*a**3*e*x**2 + x**3*(-12*a**3*f - 36*a**2*b*
c))/(12*x**4)
```

Giac [A]
time = 0.60, size = 224, normalized size = 1.07

$$\frac{1}{10}\,b^{3}hx^{10}+\frac{1}{9}\,b^{3}gx^{9}+\frac{1}{8}\,b^{3}fx^{8}+\frac{3}{7}\,ab^{2}hx^{7}+\frac{1}{7}\,b^{3}x^{7}e+\frac{1}{6}\,b^{3}dx^{6}+\frac{1}{2}\,ab^{2}gx^{6}+\frac{1}{5}\,b^{3}cx^{5}+\frac{3}{5}\,ab^{2}fx^{5}+\frac{3}{4}\,a^{2}bhx^{4}+\frac{3}{4}\,ab^{2}x^{4}e+ab^{2}dx^{3}+a^{2}bgx^{3}+\frac{3}{2}\,ab^{2}cx^{2}+\frac{3}{2}\,a^{2}bfx^{2}+a^{3}hx+3\,a^{2}bxe+(3\,a^{2}bd+a^{3}g)\log\left(|x|\right)-\frac{6\,a^{3}x^{2}e+4\,a^{3}dx+3\,a^{3}c+12\,(3\,a^{2}bc+a^{3}f)x^{3}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="gia
c")
```

```
[Out] 1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 3/7*a*b^2*h*x^7 + 1/7*b^3
*x^7*e + 1/6*b^3*d*x^6 + 1/2*a*b^2*g*x^6 + 1/5*b^3*c*x^5 + 3/5*a*b^2*f*x^5
+ 3/4*a^2*b*h*x^4 + 3/4*a*b^2*x^4*e + a*b^2*d*x^3 + a^2*b*g*x^3 + 3/2*a*b^2
*c*x^2 + 3/2*a^2*b*f*x^2 + a^3*h*x + 3*a^2*b*x*e + (3*a^2*b*d + a^3*g)*log(
abs(x)) - 1/12*(6*a^3*x^2*e + 4*a^3*d*x + 3*a^3*c + 12*(3*a^2*b*c + a^3*f)*
x^3)/x^4
```

Mupad [B]
time = 5.03, size = 199, normalized size = 0.95

$$x^5\left(\frac{c\,b^3}{5}+\frac{3\,a\,f\,b^2}{5}\right)+x^6\left(\frac{d\,b^3}{6}+\frac{a\,g\,b^2}{2}\right)+x^7\left(\frac{e\,b^3}{7}+\frac{3\,a\,h\,b^2}{7}\right)+\ln(x)\,\left(g\,a^3+3\,b\,d\,a^2\right)-\frac{x^3\,\left(f\,a^3+3\,b\,c\,a^2\right)+\frac{a^3\,c}{4}+\frac{a^3\,e\,x^2}{2}+\frac{a^3\,d\,x}{3}}{x^4}+x\,\left(h\,a^3+3\,b\,e\,a^2\right)+\frac{b^3\,f\,x^8}{8}+\frac{b^3\,g\,x^9}{9}+\frac{b^3\,h\,x^{10}}{10}+\frac{3\,a\,b\,x^2\,(b\,c+a\,f)}{2}+a\,b\,x^3\,(b\,d+a\,g)+\frac{3\,a\,b\,x^4\,(b\,e+a\,h)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x)
```

```
[Out] x^5*((b^3*c)/5 + (3*a*b^2*f)/5) + x^6*((b^3*d)/6 + (a*b^2*g)/2) + x^7*((b^3
*e)/7 + (3*a*b^2*h)/7) + log(x)*(a^3*g + 3*a^2*b*d) - (x^3*(a^3*f + 3*a^2*b
*c) + (a^3*c)/4 + (a^3*e*x^2)/2 + (a^3*d*x)/3)/x^4 + x*(a^3*h + 3*a^2*b*e)
+ (b^3*f*x^8)/8 + (b^3*g*x^9)/9 + (b^3*h*x^10)/10 + (3*a*b*x^2*(b*c + a*f))
/2 + a*b*x^3*(b*d + a*g) + (3*a*b*x^4*(b*e + a*h))/4
```

3.403 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal. Leaf size=331

$$-\frac{a(be-ah)x}{b^3}+\frac{(bc-af)x^2}{2b^2}+\frac{(bd-ag)x^3}{3b^2}+\frac{(be-ah)x^4}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+\frac{hx^7}{7b}+\frac{a^{2/3}\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{\cdots}\right.}{\sqrt{3}\,b^{10/\cdots}}$$

```
[Out] -a*(-a*h+b*e)*x/b^3+1/2*(-a*f+b*c)*x^2/b^2+1/3*(-a*g+b*d)*x^3/b^2+1/4*(-a*h
+b*e)*x^4/b^2+1/5*f*x^5/b+1/6*g*x^6/b+1/7*h*x^7/b+1/3*a^(2/3)*(b^(2/3)*(-a*
f+b*c)+a^(2/3)*(-a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/b^(10/3)-1/6*a^(2/3)*(b^(2
/3)*(-a*f+b*c)+a^(2/3)*(-a*h+b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/b^(10/3)-1/3*a*(-a*g+b*d)*ln(b*x^3+a)/b^3+1/3*a^(2/3)*(b^(5/3)*c-a^(2/3)*
b*e-a*b^(2/3)*f+a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2)
)/b^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.68, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1850, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}\,b^{10/3}}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6b^{10/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3b^{10/3}}-\frac{a(bd-ag)\log\left(a+bx^3\right)}{3b^3}-\frac{ax(be-ah)}{b^3}+\frac{x^2(bc-af)}{2b^2}+\frac{x^3(bd-ag)}{3b^2}+\frac{x^4(be-ah)}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+\frac{hx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] -((a*(b*e - a*h)*x)/b^3) + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*g)*x^3)/(3
*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*x^6)/(6*b) + (h*x^7)
/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(10/3)) + (a^(2/3)
*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*b
^(10/3)) - (a^(2/3)*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)) - (a*(b*d - a*g)*Log[a +
b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
```

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \frac{hx^7}{7b} + \frac{\int \frac{x^4(7bc+7bdx+7(be-ah)x^2+7bfx^3+7bgx^4)}{a+bx^3}\,dx}{7b}$$

$$= \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \frac{x^4(42b^2c+42b(bd-ag)x+42b(be-ah)x^2+42b^2fx^3)}{a+bx^3}\,dx}{42b^2}$$

$$= \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \frac{x^4(210b^2(bc-af)+210b^2(bd-ag)x+210b^2(be-ah)x^2)}{a+bx^3}}{210b^3}$$

$$= \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \Big(-210a(be-ah)+210b(bc-af)x+2}{}$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} +$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} +$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} +$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} +$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} +$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} +$$

Mathematica [A]
time = 0.29, size = 334, normalized size = 1.01

$$\frac{a(-be+ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{a^{2/3}(b^{5/3}c - a^{2/3}be - ab^{2/3}f + a^{5/3}h)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{10/3}} + \frac{a^{2/3}(b^{5/3}c + a^{2/3}be - ab^{2/3}f - a^{5/3}h)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} + \frac{a^{2/3}(-b^{5/3}c - a^{2/3}be + ab^{2/3}f + a^{5/3}h)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}} + \frac{a(-bd+ag)\log(a+bx^3)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (a*(-(b*e) + a*h)*x)/b^3 + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*g)*x^3)/(3
*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*x^6)/(6*b) + (h*x^7)
/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(10/3)) + (a^(2/3)*(b^(5
/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/(3
*b^(10/3)) + (a^(2/3)*(-(b^(5/3)*c) - a^(2/3)*b*e + a*b^(2/3)*f + a^(5/3)*h
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)) + (a*(-(b*d)
 + a*g)*Log[a + b*x^3])/(3*b^3)
```

Maple [A]
time = 0.38, size = 332, normalized size = 1.00

method	result
risch	$\frac{h\,x^7}{7b}+\frac{g\,x^6}{6b}+\frac{f\,x^5}{5b}-\frac{ah\,x^4}{4b^2}+\frac{e\,x^4}{4b}-\frac{ag\,x^3}{3b^2}+\frac{d\,x^3}{3b}-\frac{af\,x^2}{2b^2}+\frac{c\,x^2}{2b}+\frac{a^2hx}{b^3}-\frac{aex}{b^2}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(b(a \cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{1}{7}b^2h\,x^7+\frac{1}{6}b^2g\,x^6+\frac{1}{5}f\,x^5b^2-\frac{1}{4}abh\,x^4+\frac{1}{4}b^2e\,x^4-\frac{1}{3}abg\,x^3+\frac{1}{3}b^2d\,x^3-\frac{1}{2}abf\,x^2+\frac{1}{2}b^2c\,x^2+a^2hx-abex}{b^3}-\frac{\left((a^2h-abe)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\cdots\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/7*b^2*h*x^7+1/6*b^2*g*x^6+1/5*f*x^5*b^2-1/4*a*b*h*x^4+1/4*b^2*e*x^
4-1/3*a*b*g*x^3+1/3*b^2*d*x^3-1/2*a*b*f*x^2+1/2*b^2*c*x^2+a^2*h*x-a*b*e*x)-
((a^2*h-a*b*e)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1)))+(-a*b*f+b^2*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1
/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(-a*b*g+b^2*d)*ln(b*x^3+a)/b
)*a/b^3
```

Maxima [A]
time = 0.56, size = 383, normalized size = 1.16

$$-\frac{\sqrt{3}\left(ab^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2bf\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^3h\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}+\frac{60\,b^2hx^7+70\,b^2gx^6+84\,b^2fx^5-105\,(abh-b^2e)x^4+140\,(b^2d-abg)x^3+210\,(b^2c-abf)x^2+420\,(a^2h-abe)x}{420\,b^3}-\frac{\left(2\,ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^2bg\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2bf\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^3h+a^2be\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2bg\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2bf\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^3h-a^2be\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] -1/3*sqrt(3)*(a*b^2*c*(a/b)^(2/3) - a^2*b*f*(a/b)^(2/3) + a^3*h*(a/b)^(1/3)
 - a^2*b*(a/b)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a*b^3) + 1/420*(60*b^2*h*x^7 + 70*b^2*g*x^6 + 84*b^2*f*x^5 - 105*(a*b*h -
 b^2*e)*x^4 + 140*(b^2*d - a*b*g)*x^3 + 210*(b^2*c - a*b*f)*x^2 + 420*(a^2*
h - a*b*e)*x)/b^3 - 1/6*(2*a*b^2*d*(a/b)^(2/3) - 2*a^2*b*g*(a/b)^(2/3) + a*
b^2*c*(a/b)^(1/3) - a^2*b*f*(a/b)^(1/3) - a^3*h + a^2*b*e)*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) - 1/3*(a*b^2*d*(a/b)^(2/3) - a^2*b
*g*(a/b)^(2/3) - a*b^2*c*(a/b)^(1/3) + a^2*b*f*(a/b)^(1/3) + a^3*h - a^2*b*
e)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.99, size = 15635, normalized size = 47.24

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] 1/1260*(180*b^2*h*x^7 + 210*b^2*g*x^6 + 252*b^2*f*x^5 + 315*(b^2*e - a*b*h)
*x^4 - 70*((-I*sqrt(3) + 1)*((a*b*d - a^2*g)^2/b^6 - ((g^2 - f*h)*a^4 + (e*
f - 2*d*g + c*h)*a^3*b + (d^2 - c*e)*a^2*b^2)/b^6)/(-1/27*(a*b*d - a^2*g)^3
/b^9 + 1/54*(b^5*c^3 - a^2*b^3*e^3 - 3*a*b^4*c^2*f + 3*a^2*b^3*c*f^2 - a^3*
b^2*f^3 + 3*a^3*b^2*e^2*h - 3*a^4*b*e*h^2 + a^5*h^3)*a^2/b^10 + 1/18*((g^2
- f*h)*a^4 + (e*f - 2*d*g + c*h)*a^3*b + (d^2 - c*e)*a^2*b^2)*(a*b*d - a^2*
g)/b^9 + 1/54*(a^2*b^5*c^3 - a^7*h^3 + (g^3 - 3*f*g*h + 3*e*h^2)*a^6*b - (f
^3 - 3*e*f*g + 3*e^2*h - 3*c*g*h + 3*(g^2 - f*h)*d)*a^5*b^2 + (e^3 - 3*d*e*
f + 3*d^2*g + 3*(f^2 - e*g - d*h)*c)*a^4*b^3 - (d^3 - 3*c*d*e + 3*c^2*f)*a^
3*b^4)/b^10)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*(a*b*d - a^2*g)^3/b^9 + 1/54*
(b^5*c^3 - a^2*b^3*e^3 - 3*a*b^4*c^2*f + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 + 3*
a^3*b^2*e^2*h - 3*a^4*b*e*h^2 + a^5*h^3)*a^2/b^10 + 1/18*((g^2 - f*h)*a^4 +
 (e*f - 2*d*g + c*h)*a^3*b + (d^2 - c*e)*a^2*b^2)*(a*b*d - a^2*g)/b^9 + 1/5
4*(a^2*b^5*c^3 - a^7*h^3  ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

[Out] Timed out

Giac [A]
time = 0.53, size = 380, normalized size = 1.15

$$-\frac{(abd-a^2g)\log(|bx^3+a|)}{3b^3}-\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}a^2h-(-ab^2)^{\frac{1}{3}}abe-(-ab^2)^{\frac{2}{3}}bc+(-ab^2)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2x+(-\frac{a}{b})^{\frac{1}{3}}\right)}{3(-\frac{a}{b})^{\frac{1}{3}}}\right)}{3b^4}-\frac{\left((-ab^2)^{\frac{1}{3}}a^2h-(-ab^2)^{\frac{1}{3}}abe+(-ab^2)^{\frac{2}{3}}bc-(-ab^2)^{\frac{2}{3}}af\right)\log\left(x^2+x(-\frac{a}{b})^{\frac{1}{3}}+(-\frac{a}{b})^{\frac{2}{3}}\right)}{6b^4}+\frac{60b^6hx^7+70b^6gx^6+84b^6fx^5-105ab^5hx^4+105b^6x^4e+140b^6dx^3-140ab^5gx^3+210b^6cx^2-210ab^5fx^2+420a^2b^4hx-420ab^5xe}{420b^7}+\frac{\left(ab^{14}c(-\frac{a}{b})^{\frac{1}{3}}-a^2b^{13}f(-\frac{a}{b})^{\frac{1}{3}}+a^3b^{12}h-a^2b^{13}e\right)(-\frac{a}{b})^{\frac{1}{3}}\log\left(\left|x-(-\frac{a}{b})^{\frac{1}{3}}\right|\right)}{3ab^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] -1/3*(a*b*d - a^2*g)*log(abs(b*x^3 + a))/b^3 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*
a^2*h - (-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2/3)*b*c + (-a*b^2)^(2/3)*a*f)*arc
tan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3
)*a^2*h - (-a*b^2)^(1/3)*a*b*e + (-a*b^2)^(2/3)*b*c - (-a*b^2)^(2/3)*a*f)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/420*(60*b^6*h*x^7 + 70*b^6*
g*x^6 + 84*b^6*f*x^5 - 105*a*b^5*h*x^4 + 105*b^6*x^4*e + 140*b^6*d*x^3 - 14
0*a*b^5*g*x^3 + 210*b^6*c*x^2 - 210*a*b^5*f*x^2 + 420*a^2*b^4*h*x - 420*a*b
^5*x*e)/b^7 + 1/3*(a*b^14*c*(-a/b)^(1/3) - a^2*b^13*f*(-a/b)^(1/3) + a^3*b^
12*h - a^2*b^13*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^15)
```

Mupad [B]
time = 5.09, size = 1271, normalized size = 3.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

```
[Out] x^2*(c/(2*b) - (a*f)/(2*b^2)) + x^3*(d/(3*b) - (a*g)/(3*b^2)) + x^4*(e/(4*b
) - (a*h)/(4*b^2)) + symsum(log(root(27*b^10*z^3 + 27*a*b^8*d*z^2 - 27*a^2*
b^7*g*z^2 - 9*a^4*b^4*f*h*z - 18*a^3*b^5*d*g*z + 9*a^3*b^5*e*f*z + 9*a^3*b^
5*c*h*z - 9*a^2*b^6*c*e*z + 9*a^4*b^4*g^2*z + 9*a^2*b^6*d^2*z + 3*a^6*b*f*g
*h - 3*a^5*b^2*e*f*g - 3*a^5*b^2*d*f*h - 3*a^5*b^2*c*g*h + 3*a^4*b^3*d*e*f
+ 3*a^4*b^3*c*e*g + 3*a^4*b^3*c*d*h - 3*a^3*b^4*c*d*e - 3*a^6*b*e*h^2 + 3*a
^5*b^2*e^2*h + 3*a^5*b^2*d*g^2 - 3*a^4*b^3*d^2*g - 3*a^4*b^3*c*f^2 + 3*a^3*
b^4*c^2*f + a^5*b^2*f^3 + a^3*b^4*d^3 + a^7*h^3 - a^4*b^3*e^3 - a^2*b^5*c^3
 - a^6*b*g^3, z, k)*((6*a^2*b^4*d - 6*a^3*b^3*g)/b^4 + (x*(3*a^2*b^4*e - 3*
a^3*b^3*h))/b^4 + 9*root(27*b^10*z^3 + 27*a*b^8*d*z^2 - 27*a^2*b^7*g*z^2 -
9*a^4*b^4*f*h*z - 18*a^3*b^5*d*g*z + 9*a^3*b^5*e*f*z + 9*a^3*b^5*c*h*z - 9*
a^2*b^6*c*e*z + 9*a^4*b^4*g^2*z + 9*a^2*b^6*d^2*z + 3*a^6*b*f*g*h - 3*a^5*b
^2*e*f*g - 3*a^5*b^2*d*f*h - 3*a^5*b^2*c*g*h + 3*a^4*b^3*d*e*f + 3*a^4*b^3*
c*e*g + 3*a^4*b^3*c*d*h - 3*a^3*b^4*c*d*e - 3*a^6*b*e*h^2 + 3*a^5*b^2*e^2*h
 + 3*a^5*b^2*d*g^2 - 3*a^4*b^3*d^2*g - 3*a^4*b^3*c*f^2 + 3*a^3*b^4*c^2*f +
a^5*b^2*f^3 + a^3*b^4*d^3 + a^7*h^3 - a^4*b^3*e^3 - a^2*b^5*c^3 - a^6*b*g^3
, z, k)*a*b^2) + (a^5*g^2 + a^3*b^2*d^2 - a^5*f*h + a^4*b*c*h - 2*a^4*b*d*g
```

```
 + a^4*b*e*f - a^3*b^2*c*e)/b^4 + (x*(a^4*b*f^2 + a^2*b^3*c^2 + a^5*g*h - a
^4*b*d*h - a^4*b*e*g - 2*a^3*b^2*c*f + a^3*b^2*d*e))/b^4)*root(27*b^10*z^3
+ 27*a*b^8*d*z^2 - 27*a^2*b^7*g*z^2 - 9*a^4*b^4*f*h*z - 18*a^3*b^5*d*g*z +
9*a^3*b^5*e*f*z + 9*a^3*b^5*c*h*z - 9*a^2*b^6*c*e*z + 9*a^4*b^4*g^2*z + 9*a
^2*b^6*d^2*z + 3*a^6*b*f*g*h - 3*a^5*b^2*e*f*g - 3*a^5*b^2*d*f*h - 3*a^5*b^
2*c*g*h + 3*a^4*b^3*d*e*f + 3*a^4*b^3*c*e*g + 3*a^4*b^3*c*d*h - 3*a^3*b^4*c
*d*e - 3*a^6*b*e*h^2 + 3*a^5*b^2*e^2*h + 3*a^5*b^2*d*g^2 - 3*a^4*b^3*d^2*g
- 3*a^4*b^3*c*f^2 + 3*a^3*b^4*c^2*f + a^5*b^2*f^3 + a^3*b^4*d^3 + a^7*h^3 -
 a^4*b^3*e^3 - a^2*b^5*c^3 - a^6*b*g^3, z, k), k, 1, 3) + (f*x^5)/(5*b) + (
g*x^6)/(6*b) + (h*x^7)/(7*b) - (a*x*(e/b - (a*h)/b^2))/b
```

3.404 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal. Leaf size=313

$$\frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}+\frac{\sqrt[3]{a}\left(b^{4/3}c+\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f-a^{4/3}g\right)\tan^{-1}\left(\right.}{\sqrt{3}\,b^{8/3}}$$

```
[Out] (-a*f+b*c)*x/b^2+1/2*(-a*g+b*d)*x^2/b^2+1/3*(-a*h+b*e)*x^3/b^2+1/4*f*x^4/b+
1/5*g*x^5/b+1/6*h*x^6/b-1/3*a^(1/3)*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g+b*d))
*ln(a^(1/3)+b^(1/3)*x)/b^(8/3)+1/6*a^(1/3)*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*
g+b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(8/3)-1/3*a*(-a*h+b*e)*
ln(b*x^3+a)/b^3+1/3*a^(1/3)*(b^(4/3)*c+a^(1/3)*b*d-a*b^(1/3)*f-a^(4/3)*g)*a
rctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1850, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+b^{4/3}c\right)}{\sqrt{3}\,b^{8/3}}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(\sqrt[3]{b}\,(bc-af)-\sqrt[3]{a}\,(bd-ag)\right)}{6b^{8/3}}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(bc-af)-\sqrt[3]{a}\,(bd-ag)\right)}{3b^{8/3}}-\frac{a(be-ah)\log\left(a+bx^3\right)}{3b^3}+\frac{x(bc-af)}{b^2}+\frac{x^2(bd-ag)}{2b^2}+\frac{x^3(be-ah)}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] ((b*c - a*f)*x)/b^2 + ((b*d - a*g)*x^2)/(2*b^2) + ((b*e - a*h)*x^3)/(3*b^2)
 + (f*x^4)/(4*b) + (g*x^5)/(5*b) + (h*x^6)/(6*b) + (a^(1/3)*(b^(4/3)*c + a^
(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*b^(8/3)) - (a^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)*(b
*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*c -
 a*f) - a^(1/3)*(b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(8/3)) - (a*(b*e - a*h)*Log[a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \frac{hx^6}{6b}+\frac{\int \frac{x^3(6bc+6bdx+6(be-ah)x^2+6bfx^3+6bgx^4)}{a+bx^3}\,dx}{6b}$$

$$= \frac{gx^5}{5b}+\frac{hx^6}{6b}+\frac{\int \frac{x^3(30b^2c+30b(bd-ag)x+30b(be-ah)x^2+30b^2fx^3)}{a+bx^3}\,dx}{30b^2}$$

$$= \frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}+\frac{\int \frac{x^3(120b^2(bc-af)+120b^2(bd-ag)x+120b^2(be-ah)x^2}{a+bx^3}}{120b^3}$$

$$= \frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}+\frac{\int \Big(120b(bc-af)+120b(bd-ag)x+12}{}$$

$$= \frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx}{6b}$$

$$= \frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx}{6b}$$

$$= \frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx}{6b}$$

$$= \frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx}{6b}$$

$$= \frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx}{6b}$$

$$= \frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx}{6b}$$

Mathematica [A]

time = 0.17, size = 299, normalized size = 0.96

$$\frac{60b(bc-af)x+30b(bd-ag)x^2+20b(be-ah)x^3+15b^2fx^4+12b^2gx^5+10b^2hx^6-20\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(-b^{4/3}c-\sqrt[3]{a}bd+a\sqrt[3]{b}f+a^{4/3}g\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-20\sqrt[3]{a}\sqrt[3]{b}\left(b^{4/3}c-\sqrt[3]{a}bd-a\sqrt[3]{b}f+a^{4/3}g\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+10\sqrt[3]{a}\sqrt[3]{b}\left(b^{4/3}c-\sqrt[3]{a}bd-a\sqrt[3]{b}f+a^{4/3}g\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+20a(-be+ah)\log\left(a+bx^3\right)}{60b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (60*b*(b*c - a*f)*x + 30*b*(b*d - a*g)*x^2 + 20*b*(b*e - a*h)*x^3 + 15*b^2*
f*x^4 + 12*b^2*g*x^5 + 10*b^2*h*x^6 - 20*Sqrt[3]*a^(1/3)*b^(1/3)*(-(b^(4/3)
*c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]] - 20*a^(1/3)*b^(1/3)*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f +
 a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x] + 10*a^(1/3)*b^(1/3)*(b^(4/3)*c - a^(1
/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2] + 20*a*(-(b*e) + a*h)*Log[a + b*x^3])/(60*b^3)
```

Maple [A]
time = 0.38, size = 291, normalized size = 0.93

method	result
risch	$\frac{h\,x^{6}}{6 b}+\frac{g\,x^{5}}{5 b}+\frac{f\,x^{4}}{4 b}-\frac{a h\,x^{3}}{3 b^{2}}+\frac{e\,x^{3}}{3 b}-\frac{a g\,x^{2}}{2 b^{2}}+\frac{d\,x^{2}}{2 b}-\frac{a f x}{b^{2}}+\frac{c x}{b}+\frac{a\left(\sum_{\_R =\mathrm{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\left(\left(a h -b e \right)\_R^{2}+\left(a g -b d \right)\_R \cdots\right.}{\_R^{2} \cdots}\right.}{3 b^{3}}$
default	$-\frac{-\frac{1}{6} b h\,x^{6}-\frac{1}{5} b g\,x^{5}-\frac{1}{4} b f\,x^{4}+\frac{1}{3} a h\,x^{3}-\frac{1}{3} b e\,x^{3}+\frac{1}{2} a g\,x^{2}-\frac{1}{2} b d\,x^{2}+a f x -b c x}{b^{2}}+\frac{\left(\left(a f -b c \right)\left(\frac{\ln\left(x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x +\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*(-1/6*b*h*x^6-1/5*b*g*x^5-1/4*b*f*x^4+1/3*a*h*x^3-1/3*b*e*x^3+1/2*a*
g*x^2-1/2*b*d*x^2+a*f*x-b*c*x)+((a*f-b*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/
3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(a*g-b*d)*(-1/3/b/(a/b)^(1/
3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/
3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(a*h-b
*e)*ln(b*x^3+a)/b)*a/b^2
```

Maxima [A]
time = 0.52, size = 335, normalized size = 1.07

$$\frac{10\,bhx^{6}+12\,bgx^{5}+15\,bfx^{4}-20\,(ah-be)x^{3}+30\,(bd-ag)x^{2}+60\,(bc-af)x}{60\,b^{2}}-\frac{\sqrt{3}\left(ab^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^{2}bg\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}bf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^{3}}+\frac{\left(2\,a^{2}h\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}e-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}g\left(\frac{a}{b}\right)^{\frac{1}{3}}+abc-a^{2}f\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(a^{2}h\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab\left(\frac{a}{b}\right)^{\frac{2}{3}}e+abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}g\left(\frac{a}{b}\right)^{\frac{1}{3}}-abc+a^{2}f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] 1/60*(10*b*h*x^6 + 12*b*g*x^5 + 15*b*f*x^4 - 20*(a*h - b*e)*x^3 + 30*(b*d -
 a*g)*x^2 + 60*(b*c - a*f)*x)/b^2 - 1/3*sqrt(3)*(a*b^2*d*(a/b)^(2/3) - a^2*
b*g*(a/b)^(2/3) + a*b^2*c*(a/b)^(1/3) - a^2*b*f*(a/b)^(1/3))*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) + 1/6*(2*a^2*h*(a/b)^(2/3) -
2*a*b*(a/b)^(2/3)*e - a*b*d*(a/b)^(1/3) + a^2*g*(a/b)^(1/3) + a*b*c - a^2*f
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(a^2*h*(a/
b)^(2/3) - a*b*(a/b)^(2/3)*e + a*b*d*(a/b)^(1/3) - a^2*g*(a/b)^(1/3) - a*b*
c + a^2*f)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Fricas [**C**] Result contains complex when optimal does not.
time = 1.97, size = 15451, normalized size = 49.36

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] 1/180*(30*b^2*h*x^6 + 36*b^2*g*x^5 + 45*b^2*f*x^4 - 10*((-I*sqrt(3) + 1)*((
a*b*e - a^2*h)^2/b^6 - (a*b^3*c*d + a^4*h^2 + (f*g - 2*e*h)*a^3*b + (e^2 -
d*f - c*g)*a^2*b^2)/b^6)/(-1/27*(a*b*e - a^2*h)^3/b^9 - 1/54*(b^4*c^3 + a*b
^3*d^3 - 3*a*b^3*c^2*f + 3*a^2*b^2*c*f^2 - a^3*b*f^3 - 3*a^2*b^2*d^2*g + 3*
a^3*b*d*g^2 - a^4*g^3)*a/b^8 + 1/18*(a*b^3*c*d + a^4*h^2 + (f*g - 2*e*h)*a^
3*b + (e^2 - d*f - c*g)*a^2*b^2)*(a*b*e - a^2*h)/b^9 - 1/54*(a*b^5*c^3 - a^
6*h^3 + (g^3 - 3*f*g*h + 3*e*h^2)*a^5*b - (f^3 - 3*e*f*g + 3*e^2*h - 3*c*g*
h + 3*(g^2 - f*h)*d)*a^4*b^2 + (e^3 - 3*d*e*f + 3*d^2*g + 3*(f^2 - e*g - d*
h)*c)*a^3*b^3 - (d^3 - 3*c*d*e + 3*c^2*f)*a^2*b^4)/b^9)^(1/3) + 9*(I*sqrt(3
) + 1)*(-1/27*(a*b*e - a^2*h)^3/b^9 - 1/54*(b^4*c^3 + a*b^3*d^3 - 3*a*b^3*c
^2*f + 3*a^2*b^2*c*f^2 - a^3*b*f^3 - 3*a^2*b^2*d^2*g + 3*a^3*b*d*g^2 - a^4*
g^3)*a/b^8 + 1/18*(a*b^3*c*d + a^4*h^2 + (f*g - 2*e*h)*a^3*b + (e^2 - d*f -
 c*g)*a^2*b^2)*(a*b*e - a^2*h)/b^9 - 1/54*(a*b^5*c^3 - a^6*h^3 + (g^3 - 3*f
*g*h + 3*e*h^2)*a^5*b - ( ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 353, normalized size = 1.13

$$\frac{(a^2h-abe)\log(|bx^3+a|)}{3\,b^3}-\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}b^2c-(-ab^2)^{\frac{1}{3}}abf-(-ab^2)^{\frac{2}{3}}bd+(-ab^2)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4}-\frac{\left((-ab^2)^{\frac{1}{3}}b^2c-(-ab^2)^{\frac{1}{3}}abf+(-ab^2)^{\frac{2}{3}}bd-(-ab^2)^{\frac{2}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4}+\frac{10\,b^5hx^6+12\,b^5gx^5+15\,b^5fx^4-20\,ab^4hx^3+20\,b^5x^3e+30\,b^5dx^2-30\,ab^4gx^2+60\,b^5cx-60\,ab^4fx}{60\,b^6}+\frac{\left(ab^{12}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^{11}g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^{12}c-a^2b^{11}f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] 1/3*(a^2*h - a*b*e)*log(abs(b*x^3 + a))/b^3 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b
^2*c - (-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d + (-a*b^2)^(2/3)*a*g)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)
*b^2*c - (-a*b^2)^(1/3)*a*b*f + (-a*b^2)^(2/3)*b*d - (-a*b^2)^(2/3)*a*g)*lo
g(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/60*(10*b^5*h*x^6 + 12*b^5*g*
x^5 + 15*b^5*f*x^4 - 20*a*b^4*h*x^3 + 20*b^5*x^3*e + 30*b^5*d*x^2 - 30*a*b^
4*g*x^2 + 60*b^5*c*x - 60*a*b^4*f*x)/b^6 + 1/3*(a*b^12*d*(-a/b)^(1/3) - a^2
*b^11*g*(-a/b)^(1/3) + a*b^12*c - a^2*b^11*f)*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/(a*b^13)
```

Mupad [B]
time = 4.99, size = 1236, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

```
[Out] x^2*(d/(2*b) - (a*g)/(2*b^2)) + x^3*(e/(3*b) - (a*h)/(3*b^2)) + symsum(log(
root(27*b^9*z^3 + 27*a*b^7*e*z^2 - 27*a^2*b^6*h*z^2 + 9*a*b^6*c*d*z - 18*a^
3*b^4*e*h*z + 9*a^3*b^4*f*g*z - 9*a^2*b^5*d*f*z - 9*a^2*b^5*c*g*z + 9*a^4*b
^3*h^2*z + 9*a^2*b^5*e^2*z - 3*a^5*b*f*g*h + 3*a^4*b^2*e*f*g + 3*a^4*b^2*d*
f*h + 3*a^4*b^2*c*g*h - 3*a^3*b^3*d*e*f - 3*a^3*b^3*c*e*g - 3*a^3*b^3*c*d*h
 + 3*a^2*b^4*c*d*e + 3*a^5*b*e*h^2 - 3*a^4*b^2*e^2*h - 3*a^4*b^2*d*g^2 + 3*
a^3*b^3*d^2*g + 3*a^3*b^3*c*f^2 - 3*a^2*b^4*c^2*f + a^3*b^3*e^3 + a^5*b*g^3
 + a*b^5*c^3 - a^4*b^2*f^3 - a^2*b^4*d^3 - a^6*h^3, z, k)*((6*a^2*b^4*e - 6
*a^3*b^3*h)/b^4 + (x*(3*a^2*b^3*f - 3*a*b^4*c))/b^3 + 9*root(27*b^9*z^3 + 2
7*a*b^7*e*z^2 - 27*a^2*b^6*h*z^2 + 9*a*b^6*c*d*z - 18*a^3*b^4*e*h*z + 9*a^3
*b^4*f*g*z - 9*a^2*b^5*d*f*z - 9*a^2*b^5*c*g*z + 9*a^4*b^3*h^2*z + 9*a^2*b^
5*e^2*z - 3*a^5*b*f*g*h + 3*a^4*b^2*e*f*g + 3*a^4*b^2*d*f*h + 3*a^4*b^2*c*g
*h - 3*a^3*b^3*d*e*f - 3*a^3*b^3*c*e*g - 3*a^3*b^3*c*d*h + 3*a^2*b^4*c*d*e
+ 3*a^5*b*e*h^2 - 3*a^4*b^2*e^2*h - 3*a^4*b^2*d*g^2 + 3*a^3*b^3*d^2*g + 3*a
^3*b^3*c*f^2 - 3*a^2*b^4*c^2*f + a^3*b^3*e^3 + a^5*b*g^3 + a*b^5*c^3 - a^4*
b^2*f^3 - a^2*b^4*d^3 - a^6*h^3, z, k)*a*b^2) + (a^5*h^2 + a^3*b^2*e^2 - 2*
a^4*b*e*h + a^4*b*f*g + a^2*b^3*c*d - a^3*b^2*c*g - a^3*b^2*d*f)/b^4 + (x*(
a^4*g^2 + a^2*b^2*d^2 - a^4*f*h + a^3*b*c*h - 2*a^3*b*d*g + a^3*b*e*f - a^2
```

```
*b^2*c*e))/b^3)*root(27*b^9*z^3 + 27*a*b^7*e*z^2 - 27*a^2*b^6*h*z^2 + 9*a*b
^6*c*d*z - 18*a^3*b^4*e*h*z + 9*a^3*b^4*f*g*z - 9*a^2*b^5*d*f*z - 9*a^2*b^5
*c*g*z + 9*a^4*b^3*h^2*z + 9*a^2*b^5*e^2*z - 3*a^5*b*f*g*h + 3*a^4*b^2*e*f*
g + 3*a^4*b^2*d*f*h + 3*a^4*b^2*c*g*h - 3*a^3*b^3*d*e*f - 3*a^3*b^3*c*e*g -
 3*a^3*b^3*c*d*h + 3*a^2*b^4*c*d*e + 3*a^5*b*e*h^2 - 3*a^4*b^2*e^2*h - 3*a^
4*b^2*d*g^2 + 3*a^3*b^3*d^2*g + 3*a^3*b^3*c*f^2 - 3*a^2*b^4*c^2*f + a^3*b^3
*e^3 + a^5*b*g^3 + a*b^5*c^3 - a^4*b^2*f^3 - a^2*b^4*d^3 - a^6*h^3, z, k),
k, 1, 3) + x*(c/b - (a*f)/b^2) + (f*x^4)/(4*b) + (g*x^5)/(5*b) + (h*x^6)/(6
*b)
```

3.405 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx$

Optimal. Leaf size=294

$$\frac{(bd-ag)x}{b^2}+\frac{(be-ah)x^2}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}+\frac{\sqrt[3]{a}\left(b^{4/3}d+\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g-a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}}-$$

```
[Out] (-a*g+b*d)*x/b^2+1/2*(-a*h+b*e)*x^2/b^2+1/3*f*x^3/b+1/4*g*x^4/b+1/5*h*x^5/b
-1/3*a^(1/3)*(b^(1/3)*(-a*g+b*d)-a^(1/3)*(-a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/
b^(8/3)+1/6*a^(1/3)*(b^(1/3)*(-a*g+b*d)-a^(1/3)*(-a*h+b*e))*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(8/3)+1/3*(-a*f+b*c)*ln(b*x^3+a)/b^2+1/3*a^(1/
3)*(b^(4/3)*d+a^(1/3)*b*e-a*b^(1/3)*g-a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)/a^(1/3)*3^(1/2))/b^(8/3)*3^(1/2)
```

Rubi [A]

time = 0.65, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1850, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g+b^{4/3}d\right)}{\sqrt{3}\,b^{8/3}}+\frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(\sqrt[3]{b}\,(bd-ag)-\sqrt[3]{a}\,(be-ah)\right)}{6b^{8/3}}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(bd-ag)-\sqrt[3]{a}\,(be-ah)\right)}{3b^{8/3}}+\frac{(bc-af)\log\left(a+bx^3\right)}{3b^2}+\frac{x(bd-ag)}{b^2}+\frac{x^2(be-ah)}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] ((b*d - a*g)*x)/b^2 + ((b*e - a*h)*x^2)/(2*b^2) + (f*x^3)/(3*b) + (g*x^4)/(
4*b) + (h*x^5)/(5*b) + (a^(1/3)*(b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3
)) - (a^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(
1/3)*x])/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3)) + ((b*c - a*f)
*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \frac{hx^5}{5b}+\frac{\int \frac{x^2\left(5bc+5bdx+5(be-ah)x^2+5bfx^3+5bgx^4\right)}{a+bx^3}\,dx}{5b}$$

$$= \frac{gx^4}{4b}+\frac{hx^5}{5b}+\frac{\int \frac{x^2\left(20b^2c+20b(bd-ag)x+20b(be-ah)x^2+20b^2fx^3\right)}{a+bx^3}\,dx}{20b^2}$$

$$= \frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}+\frac{\int \frac{x^2\left(60b^2(bc-af)+60b^2(bd-ag)x+60b^2(be-ah)x^2\right)}{a+bx^3}\,dx}{60b^3}$$

$$= \frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}+\frac{\int \Big(60b(bd-ag)+60b(be-ah)x-\frac{60(ab(}{}}{60b^3}$$

$$= \frac{(bd-ag)x}{b^2}+\frac{(be-ah)x^2}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}-\frac{\int \frac{ab(bd-ag)+a}{}}{}$$

$$= \frac{(bd-ag)x}{b^2}+\frac{(be-ah)x^2}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}-\frac{\int \frac{ab(bd-ag)+a}{a+bx}}{b^3}$$

$$= \frac{(bd-ag)x}{b^2}+\frac{(be-ah)x^2}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}+\frac{(bc-af)\log}{3b}$$

$$= \frac{(bd-ag)x}{b^2}+\frac{(be-ah)x^2}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}\,(bc\right.}{}$$

$$= \frac{(bd-ag)x}{b^2}+\frac{(be-ah)x^2}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}\,(bc\right.}{}$$

$$= \frac{(bd-ag)x}{b^2}+\frac{(be-ah)x^2}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}+\frac{\sqrt[3]{a}\left(b^{4/3}d+\right.}{}$$

Mathematica [A]
time = 0.18, size = 290, normalized size = 0.99

$$\frac{60b^{2/3}(bd-ag)x+30b^{2/3}(be-ah)x^2+20b^{5/3}fx^3+15b^{5/3}gx^4+12b^{5/3}hx^5-20\sqrt{3}\,\sqrt[3]{a}\left(-b^{4/3}d-\sqrt[3]{a}\,be+a\sqrt[3]{b}\,g+a^{4/3}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+20\sqrt[3]{a}\left(-b^{4/3}d+\sqrt[3]{a}\,be+a\sqrt[3]{b}\,g-a^{4/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+10\sqrt[3]{a}\left(b^{4/3}d-\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g+a^{4/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+20b^{2/3}(bc-af)\log\left(a+bx^3\right)}{60b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (60*b^(2/3)*(b*d - a*g)*x + 30*b^(2/3)*(b*e - a*h)*x^2 + 20*b^(5/3)*f*x^3 +
 15*b^(5/3)*g*x^4 + 12*b^(5/3)*h*x^5 - 20*Sqrt[3]*a^(1/3)*(-(b^(4/3)*d) - a
^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] + 20*a^(1/3)*(-(b^(4/3)*d) + a^(1/3)*b*e + a*b^(1/3)*g - a^(4/3)*h)*
Log[a^(1/3) + b^(1/3)*x] + 10*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*
g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 20*b^(2/3)*
(b*c - a*f)*Log[a + b*x^3])/(60*b^(8/3))
```

Maple [A]
time = 0.37, size = 285, normalized size = 0.97

method	result
risch	$\frac{h\,x^{5}}{5b}+\frac{g\,x^{4}}{4b}+\frac{f\,x^{3}}{3b}-\frac{ah\,x^{2}}{2b^{2}}+\frac{e\,x^{2}}{2b}-\frac{agx}{b^{2}}+\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(b(-af+bc)\_R^{2}+a(ah-be)\_R+a^{2}g-abd\right)\ln\left(x\right.}{\_R^{2}}}{3b^{3}}$
default	$-\frac{-\frac{1}{5}bh\,x^{5}-\frac{1}{4}bg\,x^{4}-\frac{1}{3}f\,x^{3}b+\frac{1}{2}ah\,x^{2}-\frac{1}{2}be\,x^{2}+agx-xbd}{b^{2}}+\frac{\left(a^{2}g-abd\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\right.}{3b}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*(-1/5*b*h*x^5-1/4*b*g*x^4-1/3*f*x^3*b+1/2*a*h*x^2-1/2*b*e*x^2+a*g*x-
x*b*d)+((a^2*g-a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1)))+(a^2*h-a*b*e)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1
/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(-a*b*f+b^2*c)*ln(b*x^
3+a)/b)/b^2
```

Maxima [A]
time = 0.49, size = 317, normalized size = 1.08

$$\frac{\sqrt{3}\left(a^{2}h\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab\left(\frac{a}{b}\right)^{\frac{2}{3}}e-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^{2}}+\frac{12\,bhx^{5}+15\,bgx^{4}+20\,bfx^{3}-30\,(ah-be)x^{2}+60\,(bd-ag)x}{60\,b^{2}}+\frac{\left(2\,b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e+abd-a^{2}g\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}+ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e-abd+a^{2}g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] 1/3*sqrt(3)*(a^2*h*(a/b)^(2/3) - a*b*(a/b)^(2/3)*e - a*b*d*(a/b)^(1/3) + a^
2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2
) + 1/60*(12*b*h*x^5 + 15*b*g*x^4 + 20*b*f*x^3 - 30*(a*h - b*e)*x^2 + 60*(b
*d - a*g)*x)/b^2 + 1/6*(2*b^2*c*(a/b)^(2/3) - 2*a*b*f*(a/b)^(2/3) + a^2*h*(
a/b)^(1/3) - a*b*(a/b)^(1/3)*e + a*b*d - a^2*g)*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(b^2*c*(a/b)^(2/3) - a*b*f*(a/b)^(2/3)
- a^2*h*(a/b)^(1/3) + a*b*(a/b)^(1/3)*e - a*b*d + a^2*g)*log(x + (a/b)^(1/3
))/(b^3*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.74, size = 14746, normalized size = 50.16

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] 1/60*(12*b*h*x^5 + 15*b*g*x^4 + 20*b*f*x^3 - 10*(2*(1/2)^(2/3)*(-I*sqrt(3)
+ 1)*((b*c - a*f)^2/b^4 - (b^3*c^2 + a^3*g*h + (f^2 - e*g - d*h)*a^2*b + (d
*e - 2*c*f)*a*b^2)/b^5)/(2*(b*c - a*f)^3/b^6 - 3*(b^3*c^2 + a^3*g*h + (f^2
- e*g - d*h)*a^2*b + (d*e - 2*c*f)*a*b^2)*(b*c - a*f)/b^7 - (b^4*d^3 + a*b^
3*e^3 - 3*a*b^3*d^2*g + 3*a^2*b^2*d*g^2 - a^3*b*g^3 - 3*a^2*b^2*e^2*h + 3*a
^3*b*e*h^2 - a^4*h^3)*a/b^8 + (b^5*c^3 - a^5*h^3 + (g^3 - 3*f*g*h + 3*e*h^2
)*a^4*b - (f^3 - 3*e*f*g + 3*e^2*h - 3*c*g*h + 3*(g^2 - f*h)*d)*a^3*b^2 + (
e^3 - 3*d*e*f + 3*d^2*g + 3*(f^2 - e*g - d*h)*c)*a^2*b^3 - (d^3 - 3*c*d*e +
 3*c^2*f)*a*b^4)/b^8)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b*c - a*f)^3/
b^6 - 3*(b^3*c^2 + a^3*g*h + (f^2 - e*g - d*h)*a^2*b + (d*e - 2*c*f)*a*b^2)
*(b*c - a*f)/b^7 - (b^4*d^3 + a*b^3*e^3 - 3*a*b^3*d^2*g + 3*a^2*b^2*d*g^2 -
 a^3*b*g^3 - 3*a^2*b^2*e^2*h + 3*a^3*b*e*h^2 - a^4*h^3)*a/b^8 + (b^5*c^3 -
a^5*h^3 + (g^3 - 3*f*g*h + 3*e*h^2)*a^4*b - (f^3 - 3*e*f*g + 3*e^2*h - 3*c*
g*h + 3*(g^2 - f*h)*d)*a^ ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 333, normalized size = 1.13

$$\frac{(bc-af)\log\left(|bx^3+a|\right)}{3\,b^2}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d-\left(-ab^2\right)^{\frac{1}{3}}abg+\left(-ab^2\right)^{\frac{2}{3}}ah-\left(-ab^2\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d-\left(-ab^2\right)^{\frac{1}{3}}abg-\left(-ab^2\right)^{\frac{2}{3}}ah+\left(-ab^2\right)^{\frac{2}{3}}be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4}+\frac{12\,b^4hx^5+15\,b^4gx^4+20\,b^4fx^3-30\,ab^3hx^2+30\,b^4x^2e+60\,b^4dx-60\,ab^3gx}{60\,b^5}-\frac{\left(a^2b^9h\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^{10}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e-ab^{10}d+a^2b^9g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] 1/3*(b*c - a*f)*log(abs(b*x^3 + a))/b^2 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*d
 - (-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*a*h - (-a*b^2)^(2/3)*b*e)*arctan(1
/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*b^2
*d - (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*a*h + (-a*b^2)^(2/3)*b*e)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/60*(12*b^4*h*x^5 + 15*b^4*g*x^4
+ 20*b^4*f*x^3 - 30*a*b^3*h*x^2 + 30*b^4*x^2*e + 60*b^4*d*x - 60*a*b^3*g*x)
/b^5 - 1/3*(a^2*b^9*h*(-a/b)^(1/3) - a*b^10*(-a/b)^(1/3)*e - a*b^10*d + a^2
*b^9*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^11)
```

Mupad [B]
time = 5.02, size = 1170, normalized size = 3.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

```
[Out] x^2*(e/(2*b) - (a*h)/(2*b^2)) + symsum(log(root(27*b^8*z^3 + 27*a*b^6*f*z^2
 - 27*b^7*c*z^2 - 18*a*b^5*c*f*z + 9*a*b^5*d*e*z + 9*a^3*b^3*g*h*z - 9*a^2*
b^4*e*g*z - 9*a^2*b^4*d*h*z + 9*a^2*b^4*f^2*z + 9*b^6*c^2*z + 3*a^4*b*f*g*h
 - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*
a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4
*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*
f^2 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3
, z, k)*((6*a^2*b^3*f - 6*a*b^4*c)/b^3 + (x*(3*a^2*b^3*g - 3*a*b^4*d))/b^3
+ 9*root(27*b^8*z^3 + 27*a*b^6*f*z^2 - 27*b^7*c*z^2 - 18*a*b^5*c*f*z + 9*a*
b^5*d*e*z + 9*a^3*b^3*g*h*z - 9*a^2*b^4*e*g*z - 9*a^2*b^4*d*h*z + 9*a^2*b^4
*f^2*z + 9*b^6*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*
a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2
*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*
g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3
 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k)*a*b^2) + (a*b^3*c^2 + a^3*b*f^2
 + a^4*g*h - a^3*b*d*h - a^3*b*e*g - 2*a^2*b^2*c*f + a^2*b^2*d*e)/b^3 + (x*
(a^4*h^2 + a^2*b^2*e^2 + a*b^3*c*d - 2*a^3*b*e*h + a^3*b*f*g - a^2*b^2*c*g
 - a^2*b^2*d*f))/b^3)*root(27*b^8*z^3 + 27*a*b^6*f*z^2 - 27*b^7*c*z^2 - 18*a
*b^5*c*f*z + 9*a*b^5*d*e*z + 9*a^3*b^3*g*h*z - 9*a^2*b^4*e*g*z - 9*a^2*b^4*
```

```
d*h*z + 9*a^2*b^4*f^2*z + 9*b^6*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a
^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*
b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2
*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3 + a*
b^4*d^3 + a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k), k, 1, 3) + x*
(d/b - (a*g)/b^2) + (f*x^3)/(3*b) + (g*x^4)/(4*b) + (h*x^5)/(5*b)
```

3.406 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx$

Optimal. Leaf size=275

$$\frac{(be-ah)x}{b^2}+\frac{fx^2}{2b}+\frac{gx^3}{3b}+\frac{hx^4}{4b}-\frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{7/3}}-\frac{(b^{2/3}(bc-af)+a^{2}}{}$$

```
[Out] (-a*h+b*e)*x/b^2+1/2*f*x^2/b+1/3*g*x^3/b+1/4*h*x^4/b-1/3*(b^(2/3)*(-a*f+b*c
)+a^(2/3)*(-a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(7/3)+1/6*(b^(2/3)*(-
a*f+b*c)+a^(2/3)*(-a*h+b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1
/3)/b^(7/3)+1/3*(-a*g+b*d)*ln(b*x^3+a)/b^2-1/3*(b^(5/3)*c-a^(2/3)*b*e-a*b^(
2/3)*f+a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)
/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1850, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{7/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6\sqrt[3]{a}\,b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3\sqrt[3]{a}\,b^{7/3}}+\frac{(bd-ag)\log\left(a+bx^3\right)}{3b^2}+\frac{x(be-ah)}{b^2}+\frac{fx^2}{2b}+\frac{gx^3}{3b}+\frac{hx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] ((b*e - a*h)*x)/b^2 + (f*x^2)/(2*b) + (g*x^3)/(3*b) + (h*x^4)/(4*b) - ((b^(
5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(7/3)) - ((b^(2/3)*(b*c - a*f) +
 a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(7/3)) + ((b^(
2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(6*a^(1/3)*b^(7/3)) + ((b*d - a*g)*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \frac{hx^4}{4b} + \frac{\int \frac{x(4bc+4bdx+4(be-ah)x^2+4bfx^3+4bgx^4)}{a+bx^3}\,dx}{4b}$$

$$= \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \frac{x(12b^2c+12b(bd-ag)x+12b(be-ah)x^2+12b^2fx^3)}{a+bx^3}\,dx}{12b^2}$$

$$= \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \frac{x(24b^2(bc-af)+24b^2(bd-ag)x+24b^2(be-ah)x^2)}{a+bx^3}\,dx}{24b^3}$$

$$= \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \Big(24b(be-ah) - \frac{24(ab(be-ah)-b^2(bc-af)x-}{a+bx^3}}{24b^3}$$

$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\int \frac{ab(be-ah)-b^2(bc-af)x-b^2(bd-}{a+bx^3}}{b^3}$$

$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\int \frac{ab(be-ah)-b^2(bc-af)x}{a+bx^3}\,dx}{b^3} +$$

$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{(bd-ag)\log(a+bx^3)}{3b^2} -$$

$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{(b^{2/3}(bc-af)+a^{2/3}(be-}{3\sqrt[3]{a}\,b}$$

$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{(b^{2/3}(bc-af)+a^{2/3}(be-}{3\sqrt[3]{a}\,b}$$

$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{(b^{5/3}c-a^{2/3}be-ab^{2/3}f+}{\sqrt{3}\,\sqrt[3]{}}$$

Mathematica [A]

time = 0.26, size = 272, normalized size = 0.99

$$\frac{12\sqrt[3]{b}(be-ah)x+6b^{4/3}fx^2+4b^{4/3}gx^3+3b^{4/3}hx^4-\frac{4\sqrt{3}\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{4\left(-b^{5/3}c-a^{2/3}be+ab^{2/3}f+a^{5/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}}+\frac{2\left(b^{5/3}c+a^{2/3}be-ab^{2/3}f-a^{5/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{a}}+4\sqrt[3]{b}(bd-ag)\log\left(a+bx^3\right)}{12b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (12*b^(1/3)*(b*e - a*h)*x + 6*b^(4/3)*f*x^2 + 4*b^(4/3)*g*x^3 + 3*b^(4/3)*h
*x^4 - (4*Sqrt[3]*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (4*(-(b^(5/3)*c) - a^(2/3
)*b*e + a*b^(2/3)*f + a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (2*(b^
(5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/a^(1/3) + 4*b^(1/3)*(b*d - a*g)*Log[a + b*x^3])/(12*b
^(7/3))
```

Maple [A]
time = 0.37, size = 271, normalized size = 0.99

method	result
risch	$\frac{h\,x^{4}}{4b}+\frac{g\,x^{3}}{3b}+\frac{f\,x^{2}}{2b}-\frac{ahx}{b^{2}}+\frac{ex}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(b(-ag+bd)\_R^{2}+b(-af+bc)\_R+a^{2}h-abe\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3b^{3}}$
default	$-\frac{-\frac{1}{4}bh\,x^{4}-\frac{1}{3}bg\,x^{3}-\frac{1}{2}bf\,x^{2}+ahx-bex}{b^{2}}+\frac{\left(a^{2}h-abe\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*(-1/4*b*h*x^4-1/3*b*g*x^3-1/2*b*f*x^2+a*h*x-b*e*x)+((a^2*h-a*b*e)*(1
/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
)))+(-a*b*f+b^2*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(-a*b*g+b^2*d)*ln(b*x^3+a)/b)/b^2
```

Maxima [A]
time = 0.51, size = 304, normalized size = 1.11

$$\frac{\sqrt{3}\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^{2}}+\frac{3\,bhx^{4}+4\,bgx^{3}+6\,bfx^{2}-12\,(ah-be)x}{12\,b^{2}}+\frac{\left(2\,b^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}h+abe\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}-abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}h-abe\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima"
)
```

```
[Out] 1/3*sqrt(3)*(b^2*c*(a/b)^(2/3) - a*b*f*(a/b)^(2/3) + a^2*h*(a/b)^(1/3) - a*
b*(a/b)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2
) + 1/12*(3*b*h*x^4 + 4*b*g*x^3 + 6*b*f*x^2 - 12*(a*h - b*e)*x)/b^2 + 1/6*(
2*b^2*d*(a/b)^(2/3) - 2*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) - a*b*f*(a/b)
^(1/3) - a^2*h + a*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(
2/3)) + 1/3*(b^2*d*(a/b)^(2/3) - a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3) + a*
b*f*(a/b)^(1/3) + a^2*h - a*b*e)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Fricas [**C**] Result contains complex when optimal does not.
time = 1.72, size = 14875, normalized size = 54.09

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas"
)
```

```
[Out] 1/12*(3*b*h*x^4 + 4*b*g*x^3 + 6*b*f*x^2 - 2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)
*((b*d - a*g)^2/b^4 - ((g^2 - f*h)*a^2 + (e*f - 2*d*g + c*h)*a*b + (d^2 - c
*e)*b^2)/b^4)/(2*(b*d - a*g)^3/b^6 - 3*((g^2 - f*h)*a^2 + (e*f - 2*d*g + c*
h)*a*b + (d^2 - c*e)*b^2)*(b*d - a*g)/b^6 + (b^5*c^3 - a^2*b^3*e^3 - 3*a*b^
4*c^2*f + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 + 3*a^3*b^2*e^2*h - 3*a^4*b*e*h^2 +
 a^5*h^3)/(a*b^7) - (b^5*c^3 - a^5*h^3 + (g^3 - 3*f*g*h + 3*e*h^2)*a^4*b -
(f^3 - 3*e*f*g + 3*e^2*h - 3*c*g*h + 3*(g^2 - f*h)*d)*a^3*b^2 + (e^3 - 3*d*
e*f + 3*d^2*g + 3*(f^2 - e*g - d*h)*c)*a^2*b^3 - (d^3 - 3*c*d*e + 3*c^2*f)*
a*b^4)/(a*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b*d - a*g)^3/b^6 -
3*((g^2 - f*h)*a^2 + (e*f - 2*d*g + c*h)*a*b + (d^2 - c*e)*b^2)*(b*d - a*g)
/b^6 + (b^5*c^3 - a^2*b^3*e^3 - 3*a*b^4*c^2*f + 3*a^2*b^3*c*f^2 - a^3*b^2*f
^3 + 3*a^3*b^2*e^2*h - 3*a^4*b*e*h^2 + a^5*h^3)/(a*b^7) - (b^5*c^3 - a^5*h^
3 + (g^3 - 3*f*g*h + 3*e*h^2)*a^4*b - (f^3 - 3*e*f*g + 3*e^2*h - 3*c*g*h +
3*(g^2 - f*h)*d)*a^3*b^2  ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.50, size = 295, normalized size = 1.07

$$-\frac{\sqrt{3}\left(a^2h-abe-(-ab^2)^{\frac{1}{3}}bc+(-ab^2)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2x+(-\frac{a}{b})^{\frac{1}{3}}\right)}{3(-\frac{a}{b})^{\frac{1}{3}}}\right)}{3(-ab^2)^{\frac{2}{3}}b}-\frac{\left(a^2h-abe+(-ab^2)^{\frac{1}{3}}bc-(-ab^2)^{\frac{1}{3}}af\right)\log\left(x^2+x(-\frac{a}{b})^{\frac{1}{3}}+(-\frac{a}{b})^{\frac{2}{3}}\right)}{6(-ab^2)^{\frac{2}{3}}b}+\frac{(bd-ag)\log\left(|bx^3+a|\right)}{3b^2}+\frac{3b^3hx^4+4b^3gx^3+6b^3fx^2-12ab^2hx+12b^3xe}{12b^4}-\frac{\left(b^9c(-\frac{a}{b})^{\frac{1}{3}}-ab^8f(-\frac{a}{b})^{\frac{1}{3}}+a^2b^7h-ab^8e\right)(-\frac{a}{b})^{\frac{1}{3}}\log\left(\left|x-(-\frac{a}{b})^{\frac{1}{3}}\right|\right)}{3ab^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(a^2*h - a*b*e - (-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*f)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/6*
(a^2*h - a*b*e + (-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) + 1/3*(b*d - a*g)*log(abs(b*x^3
+ a))/b^2 + 1/12*(3*b^3*h*x^4 + 4*b^3*g*x^3 + 6*b^3*f*x^2 - 12*a*b^2*h*x +
12*b^3*x*e)/b^4 - 1/3*(b^9*c*(-a/b)^(1/3) - a*b^8*f*(-a/b)^(1/3) + a^2*b^7*
h - a*b^8*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^9)
```

Mupad [B]
time = 4.99, size = 1161, normalized size = 4.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

```
[Out] symsum(log(root(27*a*b^7*z^3 - 27*a*b^6*d*z^2 + 27*a^2*b^5*g*z^2 - 9*a*b^5*
c*e*z - 9*a^3*b^3*f*h*z - 18*a^2*b^4*d*g*z + 9*a^2*b^4*e*f*z + 9*a^2*b^4*c*
h*z + 9*a*b^5*d^2*z + 9*a^3*b^3*g^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a
^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*
b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2
*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^3 + a^
4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k)*((6*a^2*b^2*g
- 6*a*b^3*d)/b^2 + (x*(3*a^2*b^2*h - 3*a*b^3*e))/b^2 + 9*root(27*a*b^7*z^3
- 27*a*b^6*d*z^2 + 27*a^2*b^5*g*z^2 - 9*a*b^5*c*e*z - 9*a^3*b^3*f*h*z - 18*
a^2*b^4*d*g*z + 9*a^2*b^4*e*f*z + 9*a^2*b^4*c*h*z + 9*a*b^5*d^2*z + 9*a^3*b
^3*g^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*
h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h +
 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*
b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f
^3 - a*b^4*d^3 - a^5*h^3, z, k)*a*b^2) + (a^3*g^2 + a*b^2*d^2 - a^3*f*h - a
*b^2*c*e + a^2*b*c*h - 2*a^2*b*d*g + a^2*b*e*f)/b^2 + (x*(b^3*c^2 + a^2*b*f
^2 + a^3*g*h - 2*a*b^2*c*f + a*b^2*d*e - a^2*b*d*h - a^2*b*e*g))/b^2)*root(
27*a*b^7*z^3 - 27*a*b^6*d*z^2 + 27*a^2*b^5*g*z^2 - 9*a*b^5*c*e*z - 9*a^3*b^
3*f*h*z - 18*a^2*b^4*d*g*z + 9*a^2*b^4*e*f*z + 9*a^2*b^4*c*h*z + 9*a*b^5*d^
2*z + 9*a^3*b^3*g^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3
*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^
2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d
*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^
3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k), k, 1, 3) + x*(e/b - (a*h)/b^2
) + (f*x^2)/(2*b) + (g*x^3)/(3*b) + (h*x^4)/(4*b)
```

3.407 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3} \, dx$

Optimal. Leaf size=259

$$\frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}-\frac{\left(b^{4/3}c+\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f-a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{5/3}}+\frac{\left(\sqrt[3]{b}\,(bc-af)-\sqrt[3]{a}\,(bd-ag)\right)}{3a^{2/3}b^{5/3}}$$

```
[Out] f*x/b+1/2*g*x^2/b+1/3*h*x^3/b+1/3*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g+b*d))*l
n(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(5/3)-1/6*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g+
b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(5/3)+1/3*(-a*h+b
*e)*ln(b*x^3+a)/b^2-1/3*(b^(4/3)*c+a^(1/3)*b*d-a*b^(1/3)*f-a^(4/3)*g)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 257, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+b^{4/3}c\right)}{\sqrt{3}\,a^{2/3}b^{5/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}\,(bd-ag)}{\sqrt[3]{b}}-af+bc\right)}{6a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(bc-af)-\sqrt[3]{a}\,(bd-ag)\right)}{3a^{2/3}b^{5/3}}+\frac{(be-ah)\log\left(a+bx^3\right)}{3b^2}+\frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x]
```

```
[Out] (f*x)/b + (g*x^2)/(2*b) + (h*x^3)/(3*b) - ((b^(4/3)*c + a^(1/3)*b*d - a*b^(
1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqr
t[3]*a^(2/3)*b^(5/3)) + ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^
(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b*c - a*f - (a^(1/3)*(b*d - a*g
))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4
/3)) + ((b*e - a*h)*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx = \int \left(\frac{f}{b}+\frac{gx}{b}+\frac{hx^2}{b}+\frac{bc-af+(bd-ag)x+(be-ah)x^2}{b\,(a+bx^3)}\right)dx$$

$$= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{\int \frac{bc-af+(bd-ag)x+(be-ah)x^2}{a+bx^3}\,dx}{b}$$

$$= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{\int \frac{bc-af+(bd-ag)x}{a+bx^3}\,dx}{b}+\frac{(be-ah)\int \frac{x^2}{a+bx^3}\,dx}{b}$$

$$= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{(be-ah)\log(a+bx^3)}{3b^2}+\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}\,(bc-af)+\right.}{}}{}$$

$$= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{\left(bc-af-\frac{\sqrt[3]{a}\,(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}}$$

$$= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{\left(bc-af-\frac{\sqrt[3]{a}\,(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}}$$

$$= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}-\frac{\left(b^{4/3}c+\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f-a^{4/3}g\right)\tan^{-1}\left(\right.}{\sqrt{3}\,a^{2/3}b^{5/3}}$$

Mathematica [A]
time = 0.20, size = 254, normalized size = 0.98

$$\frac{6b^{2/3}fx+3b^{2/3}gx^2+2b^{2/3}hx^3+\frac{2\sqrt{3}\left(-b^{4/3}c-\sqrt[3]{a}\,bd+a\sqrt[3]{b}\,f+a^{4/3}g\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{2\left(b^{4/3}c-\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+a^{4/3}g\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}}-\frac{\left(b^{4/3}c-\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+a^{4/3}g\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{2(be-ah)\log(a+bx^3)}{\sqrt[3]{b}}}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x]
```

```
[Out] (6*b^(2/3)*f*x + 3*b^(2/3)*g*x^2 + 2*b^(2/3)*h*x^3 + (2*Sqrt[3]*(-(b^(4/3)*
c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(2/3) + (2*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)
*g)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) - ((b^(4/3)*c - a^(1/3)*b*d - a*b^(1/
3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) +
 (2*(b*e - a*h)*Log[a + b*x^3])/b^(1/3))/(6*b^(5/3))
```

Maple [A]
time = 0.36, size = 246, normalized size = 0.95

method	result

risch	$\frac{h\,x^{3}}{3b}+\frac{g\,x^{2}}{2b}+\frac{fx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(bc-af+\left(-ag+bd\right)\_R+\left(-ah+be\right)\_R^{2}\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3b^{2}}$
default	$\frac{\frac{1}{3}h\,x^{3}+\frac{1}{2}g\,x^{2}+fx}{b}+\frac{\left(-af+bc\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\left(-ag+bd\right)\left(-\frac{\ln\left(x \cdots\right.}{3 \cdots}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3*h*x^3+1/2*g*x^2+f*x)+((-a*f+b*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/
3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(-a*g+b*d)*(-1/3/b/(a/b)^(1
/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1
/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(-a*h
+b*e)*ln(b*x^3+a)/b)/b
```

Maxima [A]
time = 0.49, size = 268, normalized size = 1.03

$$\frac{2hx^{3}+3gx^{2}+6fx}{6b}+\frac{\sqrt{3}\left(b^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}-abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^{2}}-\frac{\left(2ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-2b\left(\frac{a}{b}\right)^{\frac{2}{3}}e-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc-af\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-b\left(\frac{a}{b}\right)^{\frac{2}{3}}e+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc+af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*h*x^3 + 3*g*x^2 + 6*f*x)/b + 1/3*sqrt(3)*(b^2*d*(a/b)^(2/3) - a*b*g*
(a/b)^(2/3) + b^2*c*(a/b)^(1/3) - a*b*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) - 1/6*(2*a*h*(a/b)^(2/3) - 2*b*(a/b)^
(2/3)*e - b*d*(a/b)^(1/3) + a*g*(a/b)^(1/3) + b*c - a*f)*log(x^2 - x*(a/b)^
(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) - 1/3*(a*h*(a/b)^(2/3) - b*(a/b)^(2/
3)*e + b*d*(a/b)^(1/3) - a*g*(a/b)^(1/3) - b*c + a*f)*log(x + (a/b)^(1/3))/
(b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.82, size = 15235, normalized size = 58.82

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(4*b*h*x^3 + 6*b*g*x^2 - 2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b*e - a*h
)^2/b^4 - (b^3*c*d + a^3*h^2 + (f*g - 2*e*h)*a^2*b + (e^2 - d*f - c*g)*a*b^
2)/(a*b^4))/(2*(b*e - a*h)^3/b^6 - 3*(b^3*c*d + a^3*h^2 + (f*g - 2*e*h)*a^2
*b + (e^2 - d*f - c*g)*a*b^2)*(b*e - a*h)/(a*b^6) - (b^4*c^3 + a*b^3*d^3 -
3*a*b^3*c^2*f + 3*a^2*b^2*c*f^2 - a^3*b*f^3 - 3*a^2*b^2*d^2*g + 3*a^3*b*d*g
^2 - a^4*g^3)/(a^2*b^5) + (b^5*c^3 - a^5*h^3 + (g^3 - 3*f*g*h + 3*e*h^2)*a^
4*b - (f^3 - 3*e*f*g + 3*e^2*h - 3*c*g*h + 3*(g^2 - f*h)*d)*a^3*b^2 + (e^3
- 3*d*e*f + 3*d^2*g + 3*(f^2 - e*g - d*h)*c)*a^2*b^3 - (d^3 - 3*c*d*e + 3*c
^2*f)*a*b^4)/(a^2*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b*e - a*h)^
3/b^6 - 3*(b^3*c*d + a^3*h^2 + (f*g - 2*e*h)*a^2*b + (e^2 - d*f - c*g)*a*b^
2)*(b*e - a*h)/(a*b^6) - (b^4*c^3 + a*b^3*d^3 - 3*a*b^3*c^2*f + 3*a^2*b^2*c
*f^2 - a^3*b*f^3 - 3*a^2*b^2*d^2*g + 3*a^3*b*d*g^2 - a^4*g^3)/(a^2*b^5) + (
b^5*c^3 - a^5*h^3 + (g^3 - 3*f*g*h + 3*e*h^2)*a^4*b - (f^3 - 3*e*f*g + 3*e^
2*h - 3*c*g*h + 3*(g^2 -  ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.56, size = 272, normalized size = 1.05

$$-\frac{\sqrt{3}\left(b^{2}c-abf-\left(-ab^{2}\right)^{\frac{1}{3}}bd+\left(-ab^{2}\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{2}{3}}b}-\frac{\left(b^{2}c-abf+\left(-ab^{2}\right)^{\frac{1}{3}}bd-\left(-ab^{2}\right)^{\frac{1}{3}}ag\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{2}{3}}b}-\frac{(ah-be)\log\left(|bx^{3}+a|\right)}{3b^{2}}+\frac{2b^{2}hx^{3}+3b^{2}gx^{2}+6b^{2}fx}{6b^{3}}-\frac{\left(b^{7}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^{6}g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b^{7}c-ab^{6}f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^2*c - a*b*f - (-a*b^2)^(1/3)*b*d + (-a*b^2)^(1/3)*a*g)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/6*
(b^2*c - a*b*f + (-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/3*(a*h - b*e)*log(abs(b*x^3
+ a))/b^2 + 1/6*(2*b^2*h*x^3 + 3*b^2*g*x^2 + 6*b^2*f*x)/b^3 - 1/3*(b^7*d*(-
a/b)^(1/3) - a*b^6*g*(-a/b)^(1/3) + b^7*c - a*b^6*f)*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a*b^7)
```

Mupad [**B**]
time = 5.03, size = 1150, normalized size = 4.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x)
```

```
[Out] symsum(log((a^3*h^2 + a*b^2*e^2 + b^3*c*d - a*b^2*c*g - a*b^2*d*f - 2*a^2*b
*e*h + a^2*b*f*g)/b^2 + root(27*a^2*b^6*z^3 + 27*a^3*b^4*h*z^2 - 27*a^2*b^5
*e*z^2 + 9*a*b^5*c*d*z - 18*a^3*b^3*e*h*z + 9*a^3*b^3*f*g*z - 9*a^2*b^4*d*f
*z - 9*a^2*b^4*c*g*z + 9*a^4*b^2*h^2*z + 9*a^2*b^4*e^2*z + 3*a^4*b*f*g*h -
3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2
*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^
2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2
 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z
, k)*((6*a^2*b^2*h - 6*a*b^3*e)/b^2 + (x*(3*b^3*c - 3*a*b^2*f))/b + 9*root(
27*a^2*b^6*z^3 + 27*a^3*b^4*h*z^2 - 27*a^2*b^5*e*z^2 + 9*a*b^5*c*d*z - 18*a
^3*b^3*e*h*z + 9*a^3*b^3*f*g*z - 9*a^2*b^4*d*f*z - 9*a^2*b^4*c*g*z + 9*a^4*
b^2*h^2*z + 9*a^2*b^4*e^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f
*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g
+ 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3
*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3 + a*b^4*d^3 +
a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k)*a*b^2) + (x*(b^2*d^2 + a
^2*g^2 - b^2*c*e - a^2*f*h + a*b*c*h - 2*a*b*d*g + a*b*e*f))/b)*root(27*a^2
*b^6*z^3 + 27*a^3*b^4*h*z^2 - 27*a^2*b^5*e*z^2 + 9*a*b^5*c*d*z - 18*a^3*b^3
*e*h*z + 9*a^3*b^3*f*g*z - 9*a^2*b^4*d*f*z - 9*a^2*b^4*c*g*z + 9*a^4*b^2*h^
2*z + 9*a^2*b^4*e^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3
*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^
2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d
*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^
3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k), k, 1, 3) + (g*x^2)/(2*b) + (h
*x^3)/(3*b) + (f*x)/b
```

3.408 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx$

Optimal. Leaf size=258

$$\frac{gx}{b}+\frac{hx^2}{2b}-\frac{\left(b^{4/3}d+\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g-a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{5/3}}+\frac{c\log(x)}{a}+\frac{\left(\sqrt[3]{b}\,(bd-ag)-\sqrt[3]{a}\,(be-\right.}{3a^{2/3}b}$$

```
[Out] g*x/b+1/2*h*x^2/b+c*ln(x)/a+1/3*(b^(1/3)*(-a*g+b*d)-a^(1/3)*(-a*h+b*e))*ln(
a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(5/3)-1/6*(b^(1/3)*(-a*g+b*d)-a^(1/3)*(-a*h+b*
e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(5/3)-1/3*(-a*f+b*c
)*ln(b*x^3+a)/a/b-1/3*(b^(4/3)*d+a^(1/3)*b*e-a*b^(1/3)*g-a^(4/3)*h)*arctan(
1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 256, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g+b^{4/3}d\right)}{\sqrt{3}\,a^{2/3}b^{5/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}\,(be-ah)}{\sqrt[3]{b}}-ag+bd\right)}{6a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(bd-ag)-\sqrt[3]{a}\,(be-ah)\right)}{3a^{2/3}b^{5/3}}-\frac{(bc-af)\log\left(a+bx^3\right)}{3ab}+\frac{c\log(x)}{a}+\frac{gx}{b}+\frac{hx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) - ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^(4/3)
*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(
5/3)) + (c*Log[x])/a + ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(
1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b*d - a*g - (a^(1/3)*(b*e - a*h)
)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/
3)) - ((b*c - a*f)*Log[a + b*x^3])/(3*a*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\,(a+bx^3)}\,dx = \int \left(\frac{g}{b}+\frac{c}{ax}+\frac{hx}{b}+\frac{a(bd-ag)+a(be-ah)x-b(bc-af)x^2}{ab\,(a+bx^3)}\right)$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}+\frac{\int \frac{a(bd-ag)+a(be-ah)x-b(bc-af)x^2}{a+bx^3}\,dx}{ab}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}+\frac{\int \frac{a(bd-ag)+a(be-ah)x}{a+bx^3}\,dx}{ab}-\frac{(bc-af)\int \frac{x^2}{a+bx^3}}{a}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}-\frac{(bc-af)\log\left(a+bx^3\right)}{3ab}+\frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}\,(bd\right.}{}}{}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}+\frac{\left(bd-ag-\frac{\sqrt[3]{a}\,(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{3a^{2/3}b^{4/3}}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}+\frac{\left(bd-ag-\frac{\sqrt[3]{a}\,(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{3a^{2/3}b^{4/3}}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}-\frac{\left(b^{4/3}d+\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g-a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\,\sqrt[3]{a}}\right.}{\sqrt{3}\,a^{2/3}b^{5/3}}$$

Mathematica [A]

time = 0.14, size = 258, normalized size = 1.00

$$\frac{6ab^{2/3}gx+3ab^{2/3}hx^2+2\sqrt{3}\sqrt[3]{a}\left(-b^{4/3}d-\sqrt[3]{a}\,be+a\sqrt[3]{b}\,g+a^{4/3}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6b^{5/3}c\log(x)+2\sqrt[3]{a}\left(b^{4/3}d-\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g+a^{4/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-\sqrt[3]{a}\left(b^{4/3}d-\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g+a^{4/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2b^{2/3}(bc-af)\log\left(a+bx^3\right)}{6ab^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x]
```

```
[Out] (6*a*b^(2/3)*g*x + 3*a*b^(2/3)*h*x^2 + 2*Sqrt[3]*a^(1/3)*(-(b^(4/3)*d) - a^
(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] + 6*b^(5/3)*c*Log[x] + 2*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)
*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e
 - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
- 2*b^(2/3)*(b*c - a*f)*Log[a + b*x^3])/(6*a*b^(5/3))
```

Maple [A]

time = 0.38, size = 259, normalized size = 1.00

method	result

default	$\frac{\frac{1}{2}h\,x^2+gx}{b}+\frac{(-a^2g+abd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(-a^2h+abe)\left(-\frac{\ln\left(x+\right.}{3b\left(\right.}\right.}{ba}$
risch	$\frac{h\,x^2}{2b}+\frac{gx}{b}+\frac{\_R=\mathrm{RootOf}\left(a^3b^2\_Z^3+\left(-3a^3b^2f+3a^2c\,b^3\right)\_Z^2+\left(3a^4bgh-3a^3b^2dh-3a^3b^2eg+3a^3b^2f^2-6a^2b^3cf+3a^2b^3de+3a\,b^4c^2\right)\_Z-a^5\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*h*x^2+g*x)+((-a^2*g+a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/
6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(-a^2*h+a*b*e)*(-1/3/b/(a/b)^(1/3
)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3
*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(a*b*f-
b^2*c)*ln(b*x^3+a)/b)/b/a+c*ln(x)/a
```

Maxima [A]
time = 0.52, size = 293, normalized size = 1.14

$$\frac{c\log(x)}{a}+\frac{hx^2+2\,gx}{2\,b}-\frac{\sqrt{3}\left(a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab\left(\frac{a}{b}\right)^{\frac{2}{3}}e-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b}-\frac{\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e+abd-a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e-abd+a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="maxima"
)
```

```
[Out] c*log(x)/a + 1/2*(h*x^2 + 2*g*x)/b - 1/3*sqrt(3)*(a^2*h*(a/b)^(2/3) - a*b*(
a/b)^(2/3)*e - a*b*d*(a/b)^(1/3) + a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b) - 1/6*(2*b^2*c*(a/b)^(2/3) - 2*a*b*f
*(a/b)^(2/3) + a^2*h*(a/b)^(1/3) - a*b*(a/b)^(1/3)*e + a*b*d - a^2*g)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) - 1/3*(b^2*c*(a/b)^(2
/3) - a*b*f*(a/b)^(2/3) - a^2*h*(a/b)^(1/3) + a*b*(a/b)^(1/3)*e - a*b*d + a
^2*g)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 60.85, size = 15327, normalized size = 59.41

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="fricas"
)
```

```
[Out] 1/36*(18*a*h*x^2 - 2*((-I*sqrt(3) + 1)*((b*c - a*f)^2/(a^2*b^2) - (b^3*c^2
+ a^3*g*h + (f^2 - e*g - d*h)*a^2*b + (d*e - 2*c*f)*a*b^2)/(a^2*b^3))/(-1/2
7*(b*c - a*f)^3/(a^3*b^3) + 1/18*(b^3*c^2 + a^3*g*h + (f^2 - e*g - d*h)*a^2
*b + (d*e - 2*c*f)*a*b^2)*(b*c - a*f)/(a^3*b^4) - 1/54*(b^4*d^3 + a*b^3*e^3
 - 3*a*b^3*d^2*g + 3*a^2*b^2*d*g^2 - a^3*b*g^3 - 3*a^2*b^2*e^2*h + 3*a^3*b*
e*h^2 - a^4*h^3)/(a^2*b^5) - 1/54*(b^5*c^3 - a^5*h^3 + (g^3 - 3*f*g*h + 3*e
*h^2)*a^4*b - (f^3 - 3*e*f*g + 3*e^2*h - 3*c*g*h + 3*(g^2 - f*h)*d)*a^3*b^2
 + (e^3 - 3*d*e*f + 3*d^2*g + 3*(f^2 - e*g - d*h)*c)*a^2*b^3 - (d^3 - 3*c*d
*e + 3*c^2*f)*a*b^4)/(a^3*b^5))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*(b*c - a*f
)^3/(a^3*b^3) + 1/18*(b^3*c^2 + a^3*g*h + (f^2 - e*g - d*h)*a^2*b + (d*e -
2*c*f)*a*b^2)*(b*c - a*f)/(a^3*b^4) - 1/54*(b^4*d^3 + a*b^3*e^3 - 3*a*b^3*d
^2*g + 3*a^2*b^2*d*g^2 - a^3*b*g^3 - 3*a^2*b^2*e^2*h + 3*a^3*b*e*h^2 - a^4*
h^3)/(a^2*b^5) - 1/54*(b^5*c^3 - a^5*h^3 + (g^3 - 3*f*g*h + 3*e*h^2)*a^4*b
- (f^3 - 3*e*f*g + 3*e^2* ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.65, size = 281, normalized size = 1.09

$$\frac{c\log\left(|x|\right)}{a}-\frac{\sqrt{3}\left(b^2d-abg+\left(-ab^2\right)^{\frac{1}{3}}ah-\left(-ab^2\right)^{\frac{1}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b}-\frac{\left(b^2d-abg-\left(-ab^2\right)^{\frac{1}{3}}ah+\left(-ab^2\right)^{\frac{1}{3}}be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}b}-\frac{(bc-af)\log\left(|bx^3+a|\right)}{3\,ab}+\frac{bhx^2+2\,bgx}{2\,b^2}+\frac{\left(a^3b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}e-a^2b^3d+a^3b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] c*log(abs(x))/a - 1/3*sqrt(3)*(b^2*d - a*b*g + (-a*b^2)^(1/3)*a*h - (-a*b^2
)^(1/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2
)^(2/3)*b) - 1/6*(b^2*d - a*b*g - (-a*b^2)^(1/3)*a*h + (-a*b^2)^(1/3)*b*e)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/3*(b*c - a*
f)*log(abs(b*x^3 + a))/(a*b) + 1/2*(b*h*x^2 + 2*b*g*x)/b^2 + 1/3*(a^3*b^2*h
*(-a/b)^(1/3) - a^2*b^3*(-a/b)^(1/3)*e - a^2*b^3*d + a^3*b^2*g)*(-a/b)^(1/3
)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3)
```

Mupad [B]
time = 5.10, size = 1731, normalized size = 6.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x)
[Out] symsum(log(b^2*c*d^2 - root(27*a^3*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5*
c*z^2 + 9*a^4*b^2*g*h*z - 9*a^3*b^3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c*
f*z + 9*a^2*b^4*d*e*z + 9*a*b^5*c^2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h + 3
*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*
b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2
*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2
+ a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z,
 k)*(a^3*g^2 - root(27*a^3*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5*c*z^2 +
9*a^4*b^2*g*h*z - 9*a^3*b^3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c*f*z + 9*
a^2*b^4*d*e*z + 9*a*b^5*c^2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c
*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*
f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a
^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^
3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k)*((x*
(33*a^2*b^4*f - 24*a*b^5*c))/b^2 + 3*a^2*b^2*e - 3*a^3*b*h - 36*root(27*a^3
*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5*c*z^2 + 9*a^4*b^2*g*h*z - 9*a^3*b^
3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c*f*z + 9*a^2*b^4*d*e*z + 9*a*b^5*c^
2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3
*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^
2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d
*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^
3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k)*a^2*b^3*x) + (x*(4*b^5*c^2 + 1
0*a^2*b^3*f^2 - 14*a*b^4*c*f + 10*a*b^4*d*e - 10*a^2*b^3*d*h - 10*a^2*b^3*e
*g + 10*a^3*b^2*g*h))/b^2 + a*b^2*d^2 - a^3*f*h + 2*a*b^2*c*e - 2*a^2*b*c*h
 - 2*a^2*b*d*g + a^2*b*e*f) - b^2*c^2*e + a^2*c*g^2 + (x*(b^4*d^3 + a^4*h^3
 - a*b^3*e^3 - a^3*b*g^3 + b^4*c^2*f + a^2*b^2*f^3 + 3*a^2*b^2*d*g^2 + 3*a^
2*b^2*e^2*h - 2*b^4*c*d*e - 2*a*b^3*c*f^2 - 3*a*b^3*d^2*g - 3*a^3*b*e*h^2 -
 2*a^2*b^2*c*g*h - 3*a^2*b^2*d*f*h - 3*a^2*b^2*e*f*g + 2*a*b^3*c*d*h + 2*a*
b^3*c*e*g + 3*a*b^3*d*e*f + 3*a^3*b*f*g*h))/b^2 + a*b*c^2*h - a^2*c*f*h - 2
*a*b*c*d*g + a*b*c*e*f)*root(27*a^3*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5
*c*z^2 + 9*a^4*b^2*g*h*z - 9*a^3*b^3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c
*f*z + 9*a^2*b^4*d*e*z + 9*a*b^5*c^2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h +
3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2
*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^
2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2
 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z
, k), k, 1, 3) + (h*x^2)/(2*b) + (c*log(x))/a + (g*x)/b
```

3.409 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=253

$$-\frac{c}{ax}+\frac{hx}{b}+\frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{4/3}}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-a\right.}{3a^{4/3}b^4}$$

```
[Out] -c/a/x+h*x/b+d*ln(x)/a+1/3*(b^(2/3)*(-a*f+b*c)+a^(2/3)*(-a*h+b*e))*ln(a^(1/
3)+b^(1/3)*x)/a^(4/3)/b^(4/3)-1/6*(b^(2/3)*(-a*f+b*c)+a^(2/3)*(-a*h+b*e))*l
n(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(4/3)-1/3*(-a*g+b*d)*ln(
b*x^3+a)/a/b+1/3*(b^(5/3)*c-a^(2/3)*b*e-a*b^(2/3)*f+a^(5/3)*h)*arctan(1/3*(
a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(4/3)*3^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}\,a^{4/3}b^{4/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6a^{4/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3a^{4/3}b^{4/3}}-\frac{(bd-ag)\log\left(a+bx^3\right)}{3ab}-\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{hx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(c/(a*x)) + (h*x)/b + ((b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(4/3
)) + (d*Log[x])/a + ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(4/3)*b^(4/3)) - ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e
- a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(4/3))
 - ((b*d - a*g)*Log[a + b*x^3])/(3*a*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx = \int \left(\frac{h}{b}+\frac{c}{ax^2}+\frac{d}{ax}+\frac{a(be-ah)-b(bc-af)x-b(bd-ag)x^2}{ab(a+bx^3)}\right)$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\int \frac{a(be-ah)-b(bc-af)x-b(bd-ag)x^2}{a+bx^3}\,dx}{ab}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\int \frac{a(be-ah)-b(bc-af)x}{a+bx^3}\,dx}{ab}-\frac{(bd-ag)\int \frac{x^2}{a+b}}{a}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}-\frac{(bd-ag)\log(a+bx^3)}{3ab}+\frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}\,b(\right.}{}}{}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}\right.}{3a^{4/3}b^{4/3}}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}\right.}{3a^{4/3}b^{4/3}}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right.}{\sqrt{3}\,a^{4/3}b^{4/3}}$$

Mathematica [A]
time = 0.19, size = 257, normalized size = 1.02

$$\frac{1}{6}\left(-\frac{6c}{ax}+\frac{6hx}{b}+\frac{2\sqrt{3}\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}b^{4/3}}+\frac{6d\log(x)}{a}+\frac{2\left(b^{5/3}c+a^{2/3}be-ab^{2/3}f-a^{5/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}b^{4/3}}+\frac{\left(-b^{5/3}c-a^{2/3}be+ab^{2/3}f+a^{5/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{4/3}b^{4/3}}+\frac{2(-bd+ag)\log(a+bx^3)}{ab}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x]
```

```
[Out] ((-6*c)/(a*x) + (6*h*x)/b + (2*Sqrt[3]*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)
*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(4/3)*b^(4/
3)) + (6*d*Log[x])/a + (2*(b^(5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*
h)*Log[a^(1/3) + b^(1/3)*x])/(a^(4/3)*b^(4/3)) + ((-(b^(5/3)*c) - a^(2/3)*b
*e + a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(a^(4/3)*b^(4/3)) + (2*(-(b*d) + a*g)*Log[a + b*x^3])/(a*b))/6
```

Maple [A]
time = 0.38, size = 260, normalized size = 1.03

method	result

default	$\frac{hx}{b}+\frac{(-a^2h+abe)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(abf-b^2c)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.}{ba}$
risch	$\frac{hx}{b}-\frac{c}{ax}+\frac{\_R=\mathrm{RootOf}\left(a^4b\_Z^3+\left(-3a^4bg+3d\,a^3b^2\right)\_Z^2+\left(-3a^4bfh+3a^4b\,g^2+3a^3b^2ch-6a^3b^2dg+3a^3b^2ef-3a^2b^3ce+3a^2b^3d^2\right)\_Z+a^5h^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] h*x/b+((-a^2*h+a*b*e)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1)))+(a*b*f-b^2*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1
/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(a*b*g-b^2*d)*ln(b*x^3
+a)/b)/b/a-c/a/x+d*ln(x)/a
```

Maxima [A]
time = 0.49, size = 293, normalized size = 1.16

$$\frac{hx}{b}+\frac{d\log(x)}{a}-\frac{\sqrt{3}\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b}-\frac{c}{ax}-\frac{\left(2b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-2abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h+abe\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h-abe\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] h*x/b + d*log(x)/a - 1/3*sqrt(3)*(b^2*c*(a/b)^(2/3) - a*b*f*(a/b)^(2/3) + a
^2*h*(a/b)^(1/3) - a*b*(a/b)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(a^2*b) - c/(a*x) - 1/6*(2*b^2*d*(a/b)^(2/3) - 2*a*b*g*(a/b)
^(2/3) + b^2*c*(a/b)^(1/3) - a*b*f*(a/b)^(1/3) - a^2*h + a*b*e)*log(x^2 - x
*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) - 1/3*(b^2*d*(a/b)^(2/3) -
a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3) + a*b*f*(a/b)^(1/3) + a^2*h - a*b*e)*
log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 65.20, size = 15238, normalized size = 60.23

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*((b*d - a*g)^2/(a^2*b^2) - ((g^2 - f*h)*a^2 + (e
*f - 2*d*g + c*h)*a*b + (d^2 - c*e)*b^2)/(a^2*b^2))/(-1/27*(b*d - a*g)^3/(a
^3*b^3) + 1/18*((g^2 - f*h)*a^2 + (e*f - 2*d*g + c*h)*a*b + (d^2 - c*e)*b^2
)*(b*d - a*g)/(a^3*b^3) + 1/54*(b^5*c^3 - a^2*b^3*e^3 - 3*a*b^4*c^2*f + 3*a
^2*b^3*c*f^2 - a^3*b^2*f^3 + 3*a^3*b^2*e^2*h - 3*a^4*b*e*h^2 + a^5*h^3)/(a^
4*b^4) + 1/54*(b^5*c^3 - a^5*h^3 + (g^3 - 3*f*g*h + 3*e*h^2)*a^4*b - (f^3 -
 3*e*f*g + 3*e^2*h - 3*c*g*h + 3*(g^2 - f*h)*d)*a^3*b^2 + (e^3 - 3*d*e*f +
3*d^2*g + 3*(f^2 - e*g - d*h)*c)*a^2*b^3 - (d^3 - 3*c*d*e + 3*c^2*f)*a*b^4)
/(a^4*b^4))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*(b*d - a*g)^3/(a^3*b^3) + 1/18
*((g^2 - f*h)*a^2 + (e*f - 2*d*g + c*h)*a*b + (d^2 - c*e)*b^2)*(b*d - a*g)/
(a^3*b^3) + 1/54*(b^5*c^3 - a^2*b^3*e^3 - 3*a*b^4*c^2*f + 3*a^2*b^3*c*f^2 -
 a^3*b^2*f^3 + 3*a^3*b^2*e^2*h - 3*a^4*b*e*h^2 + a^5*h^3)/(a^4*b^4) + 1/54*
(b^5*c^3 - a^5*h^3 + (g^3 - 3*f*g*h + 3*e*h^2)*a^4*b - (f^3 - 3*e*f*g + 3*e
^2*h - 3*c*g*h + 3*(g^2 - ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.54, size = 277, normalized size = 1.09

$$\frac{hx}{b}+\frac{d\log\left(|x|\right)}{a}+\frac{\sqrt{3}\left(a^2h-abe-\left(-ab^2\right)^{\frac{1}{3}}bc+\left(-ab^2\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}a}+\frac{\left(a^2h-abe+\left(-ab^2\right)^{\frac{1}{3}}bc-\left(-ab^2\right)^{\frac{1}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}a}-\frac{(bd-ag)\log\left(|bx^3+a|\right)}{3ab}-\frac{c}{ax}+\frac{\left(ab^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3b^2h-a^2b^3e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] h*x/b + d*log(abs(x))/a + 1/3*sqrt(3)*(a^2*h - a*b*e - (-a*b^2)^(1/3)*b*c +
 (-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/
((-a*b^2)^(2/3)*a) + 1/6*(a^2*h - a*b*e + (-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/
3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/3*(
b*d - a*g)*log(abs(b*x^3 + a))/(a*b) - c/(a*x) + 1/3*(a*b^4*c*(-a/b)^(1/3)
- a^2*b^3*f*(-a/b)^(1/3) + a^3*b^2*h - a^2*b^3*e)*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/(a^3*b^3)
```

Mupad [B]
time = 5.09, size = 1802, normalized size = 7.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x)
[Out] symsum(log((b^3*c*d^2 + a^3*d*h^2 + a*b^2*d*e^2 - a*b^2*d^2*f - a*b^2*c*d*g
 - 2*a^2*b*d*e*h + a^2*b*d*f*g)/a - root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2
+ 27*a^3*b^4*d*z^2 - 9*a^4*b^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z +
 9*a^3*b^3*c*h*z - 9*a^2*b^4*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*
a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2
*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^
2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3
*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^
3 + a^5*h^3, z, k)*(root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2 + 27*a^3*b^4*d*z
^2 - 9*a^4*b^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z + 9*a^3*b^3*c*h*z
 - 9*a^2*b^4*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*a^4*b*f*g*h - 3*
a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b
^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*
f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 -
 a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z,
k)*((3*a^2*b^3*c - 3*a^3*b^2*f)/a + (x*(24*a^3*b^4*d - 33*a^4*b^3*g))/(a^2*
b) + 36*root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2 + 27*a^3*b^4*d*z^2 - 9*a^4*b
^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z + 9*a^3*b^3*c*h*z - 9*a^2*b^4
*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e
- 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3
*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^
2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3
 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*a^2*b^3*x
) + (a^4*h^2 + a^2*b^2*e^2 - 2*a*b^3*c*d - 2*a^3*b*e*h + a^3*b*f*g - a^2*b^
2*c*g + 2*a^2*b^2*d*f)/a + (x*(4*a^2*b^4*d^2 + 10*a^4*b^2*g^2 - 10*a^2*b^4*
c*e + 10*a^3*b^3*c*h - 14*a^3*b^3*d*g + 10*a^3*b^3*e*f - 10*a^4*b^2*f*h))/(
a^2*b)) + (x*(b^5*c^3 - a^5*h^3 + a^4*b*g^3 + a^2*b^3*e^3 - a^3*b^2*f^3 + 3
*a^2*b^3*c*f^2 + a^2*b^3*d^2*g - 2*a^3*b^2*d*g^2 - 3*a^3*b^2*e^2*h - 3*a*b^
4*c^2*f + 3*a^4*b*e*h^2 - 2*a^2*b^3*c*d*h - 3*a^2*b^3*c*e*g - 2*a^2*b^3*d*e
*f + 3*a^3*b^2*c*g*h + 2*a^3*b^2*d*f*h + 3*a^3*b^2*e*f*g + 2*a*b^4*c*d*e -
3*a^4*b*f*g*h))/(a^2*b))*root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2 + 27*a^3*b^
4*d*z^2 - 9*a^4*b^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z + 9*a^3*b^3*
c*h*z - 9*a^2*b^4*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*a^4*b*f*g*h
 - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*
a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4
*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*
f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3
, z, k), k, 1, 3) + (h*x)/b - c/(a*x) + (d*log(x))/a
```

3.410 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=260

$$-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{\left(b^{4/3}c+\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f-a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}b^{2/3}}+\frac{e\log(x)}{a}-\frac{\left(\sqrt[3]{b}\,(bc-af)-\sqrt[3]{a}\,(ba\right.}{3a^5}$$

```
[Out] -1/2*c/a/x^2-d/a/x+e*ln(x)/a-1/3*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g+b*d))*ln
(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(2/3)+1/6*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g+b
*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/3*(-a*h+b*
e)*ln(b*x^3+a)/a/b+1/3*(b^(4/3)*c+a^(1/3)*b*d-a*b^(1/3)*f-a^(4/3)*g)*arctan
(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 258, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+b^{4/3}c\right)}{\sqrt{3}\,a^{5/3}b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}\,(bd-ag)}{\sqrt[3]{b}}-af+bc\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(bc-af)-\sqrt[3]{a}\,(bd-ag)\right)}{3a^{5/3}b^{2/3}}-\frac{(be-ah)\log\left(a+bx^3\right)}{3ab}-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*c/(a*x^2) - d/(a*x) + ((b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f - a^(4/3
)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*b^
(2/3)) + (e*Log[x])/a - ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^
(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b*c - a*f - (a^(1/3)*(b*d - a*g
))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(1
/3)) - ((b*e - a*h)*Log[a + b*x^3])/(3*a*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)}\,dx &= \int \left(\frac{c}{ax^3}+\frac{d}{ax^2}+\frac{e}{ax}+\frac{-bc+af-(bd-ag)x-(be-ah)x^2}{a\left(a+bx^3\right)}\right) \\
&= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}+\frac{\int \frac{-bc+af-(bd-ag)x-(be-ah)x^2}{a+bx^3}\,dx}{a} \\
&= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}+\frac{\int \frac{-bc+af+(-bd+ag)x}{a+bx^3}\,dx}{a}+\frac{(-be+ah)\int}{a} \\
&= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{(be-ah)\log\left(a+bx^3\right)}{3ab}+\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}\right.}{}}{} \\
&= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{\left(bc-af-\frac{\sqrt[3]{a}\,(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{3a^{5/3}\sqrt[3]{b}} \\
&= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{\left(bc-af-\frac{\sqrt[3]{a}\,(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{3a^{5/3}\sqrt[3]{b}} \\
&= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{\left(b^{4/3}c+\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f-a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2}{\sqrt{3}}\right.}{\sqrt{3}\,a^{5/3}b^{2/3}}
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 257, normalized size = 0.99

$$\frac{-\frac{3a^{2/3}c}{x^2}-\frac{6a^{2/3}d}{x}+\frac{2\sqrt{3}\left(b^{4/3}c+\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f-a^{4/3}g\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+6a^{2/3}e\log(x)-\frac{2\left(b^{4/3}c-\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+a^{4/3}g\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{\left(b^{4/3}c-\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+a^{4/3}g\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{2a^{2/3}(-be+ah)\log\left(a+bx^3\right)}{b}}{6a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(2/3)*c)/x^2 - (6*a^(2/3)*d)/x + (2*Sqrt[3]*(b^(4/3)*c + a^(1/3)*b*d
 - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(2/3) + 6*a^(2/3)*e*Log[x] - (2*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^
(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((b^(4/3)*c - a^(1/3)*b*d - a*
b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2
/3) + (2*a^(2/3)*(-(b*e) + a*h)*Log[a + b*x^3])/b)/(6*a^(5/3))
```

Maple [A]
time = 0.42, size = 251, normalized size = 0.97

method	result

default	$\frac{(af-bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(ag-bd)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x \cdots\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}} \cdots\right.}{a}$
risch	$\frac{-\frac{xd}{a}-\frac{c}{2a}}{x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5b^3\_Z^3+\left(-3a^5b^2h+3a^4b^3e\right)\_Z^2+\left(3a^5bh^2-6a^4b^2eh+3a^4b^2fg-3a^3b^3cg-3a^3b^3df+3a^3b^3e^2+3a^2b^4cd\right)\_Z-a^5 \cdots\right)} \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] ((a*f-b*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1)))+(a*g-b*d)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b
)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(a*h-b*e)*ln(b*x^3+a)/b)/a-1/2*c/a/x^
2-d/a/x+e*ln(x)/a
```

Maxima [A]
time = 0.49, size = 274, normalized size = 1.05

$$\frac{e\log(x)}{a}-\frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b}+\frac{\left(2ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-2b\left(\frac{a}{b}\right)^{\frac{2}{3}}e-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc-af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-b\left(\frac{a}{b}\right)^{\frac{2}{3}}e+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc+af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2dx+c}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] e*log(x)/a - 1/3*sqrt(3)*(b^2*d*(a/b)^(2/3) - a*b*g*(a/b)^(2/3) + b^2*c*(a/
b)^(1/3) - a*b*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/(a^2*b) + 1/6*(2*a*h*(a/b)^(2/3) - 2*b*(a/b)^(2/3)*e - b*d*(a/b)^(1/
3) + a*g*(a/b)^(1/3) + b*c - a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
*b*(a/b)^(2/3)) + 1/3*(a*h*(a/b)^(2/3) - b*(a/b)^(2/3)*e + b*d*(a/b)^(1/3)
- a*g*(a/b)^(1/3) - b*c + a*f)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3)) - 1/2
*(2*d*x + c)/(a*x^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 35.63, size = 15424, normalized size = 59.32

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*((b*e - a*h)^2/(a^2*b^2) - (b^3*c*d + a^3*h^2 +
(f*g - 2*e*h)*a^2*b + (e^2 - d*f - c*g)*a*b^2)/(a^3*b^2))/(-1/27*(b*e - a*h
)^3/(a^3*b^3) + 1/18*(b^3*c*d + a^3*h^2 + (f*g - 2*e*h)*a^2*b + (e^2 - d*f
- c*g)*a*b^2)*(b*e - a*h)/(a^4*b^3) - 1/54*(b^4*c^3 + a*b^3*d^3 - 3*a*b^3*c
^2*f + 3*a^2*b^2*c*f^2 - a^3*b*f^3 - 3*a^2*b^2*d^2*g + 3*a^3*b*d*g^2 - a^4*
g^3)/(a^5*b^2) - 1/54*(b^5*c^3 - a^5*h^3 + (g^3 - 3*f*g*h + 3*e*h^2)*a^4*b
- (f^3 - 3*e*f*g + 3*e^2*h - 3*c*g*h + 3*(g^2 - f*h)*d)*a^3*b^2 + (e^3 - 3*
d*e*f + 3*d^2*g + 3*(f^2 - e*g - d*h)*c)*a^2*b^3 - (d^3 - 3*c*d*e + 3*c^2*f
)*a*b^4)/(a^5*b^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*(b*e - a*h)^3/(a^3*b^3
) + 1/18*(b^3*c*d + a^3*h^2 + (f*g - 2*e*h)*a^2*b + (e^2 - d*f - c*g)*a*b^2
)*(b*e - a*h)/(a^4*b^3) - 1/54*(b^4*c^3 + a*b^3*d^3 - 3*a*b^3*c^2*f + 3*a^2
*b^2*c*f^2 - a^3*b*f^3 - 3*a^2*b^2*d^2*g + 3*a^3*b*d*g^2 - a^4*g^3)/(a^5*b^
2) - 1/54*(b^5*c^3 - a^5*h^3 + (g^3 - 3*f*g*h + 3*e*h^2)*a^4*b - (f^3 - 3*e
*f*g + 3*e^2*h - 3*c*g*h  ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 269, normalized size = 1.03

$$\frac{e\log\left(|x|\right)}{a}+\frac{\sqrt{3}\left(b^2c-abf-\left(-ab^2\right)^{\frac{1}{3}}bd+\left(-ab^2\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}a}+\frac{\left(b^2c-abf+\left(-ab^2\right)^{\frac{1}{3}}bd-\left(-ab^2\right)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}a}+\frac{(ah-be)\log\left(|bx^3+a|\right)}{3ab}+\frac{\left(ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^2c-a^2bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b}-\frac{2dx+c}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] e*log(abs(x))/a + 1/3*sqrt(3)*(b^2*c - a*b*f - (-a*b^2)^(1/3)*b*d + (-a*b^2
)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2
)^(2/3)*a) + 1/6*(b^2*c - a*b*f + (-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) + 1/3*(a*h - b*
e)*log(abs(b*x^3 + a))/(a*b) + 1/3*(a*b^2*d*(-a/b)^(1/3) - a^2*b*g*(-a/b)^(
1/3) + a*b^2*c - a^2*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) -
 1/2*(2*d*x + c)/(a*x^2)
```

Mupad [B]
time = 5.20, size = 2500, normalized size = 9.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x)
[Out] symsum(log(-(b^5*c^3*x - a^5*h^3*x - a^2*b^3*d*e^2 + 36*root(27*a^5*b^3*z^3
 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z
 - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*
a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2
*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*
d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3
*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*
b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^3*a^5*b^3*x - a^3*b^2*e*f^2 + a^3*b^2*
e^2*g - a^3*b^2*f^3*x - a*b^4*c^2*e - a*b^4*d^3*x + a^4*b*g^3*x + root(27*a
^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4
*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*
h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g +
 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*
a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2
*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*
h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^2*b^4*c^2 + 3*root(27*a^5*
b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^
2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2
*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*
a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2
*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*
g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3
 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^2*a^4*b^3*d + root(27*a^5*b^3*z
^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g
*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z +
9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b
^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*
c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 +
 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^
2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^4*b^2*f^2 - 3*root(27*a^5*b^3*z^3
- 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z
- 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a
^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*
d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d
*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*
a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b
^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^2*a^5*b^2*g + 2*a^2*b^3*c*e*f + a^3*b^2
```

*d*e*h + 10*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^5*b*h^2*x - 3*a*b^4*c^2*f*x + 2*a^4*b*e*h^2*x + 4*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^3*b^3*e^2*x + 24*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^2*a^4*b^3*e*x - 33*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^2*a^5*b^2*h*x + 3*a^2*b^3*c*f^2*x + 3*a^2*b^3*d^2*g*x - 3*a^3*b^2*d*g^2*x - a^3*b^2*e^2*h*x + ro...

3.411 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx$

Optimal. Leaf size=276

$$-\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}+\frac{\left(b^{4/3}d+\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g-a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}b^{2/3}}-\frac{(bc-af)\log(x)}{a^2}-\frac{\left(\sqrt[3]{b}\,(bd-\right.}{}$$

```
[Out] -1/3*c/a/x^3-1/2*d/a/x^2-e/a/x-(-a*f+b*c)*ln(x)/a^2-1/3*(b^(1/3)*(-a*g+b*d)
-a^(1/3)*(-a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(2/3)+1/6*(b^(1/3)*(-a
*g+b*d)-a^(1/3)*(-a*h+b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/
3)/b^(2/3)+1/3*(-a*f+b*c)*ln(b*x^3+a)/a^2+1/3*(b^(4/3)*d+a^(1/3)*b*e-a*b^(1
/3)*g-a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/
b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 274, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g+b^{4/3}d\right)}{\sqrt{3}\,a^{5/3}b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}\,(be-ah)}{\sqrt[3]{b}}-ag+bd\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(bd-ag)-\sqrt[3]{a}\,(be-ah)\right)}{3a^{5/3}b^{2/3}}+\frac{(bc-af)\log\left(a+bx^3\right)}{3a^2}-\frac{\log(x)(bc-af)}{a^2}-\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*c/(a*x^3) - d/(2*a*x^2) - e/(a*x) + ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1
/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*a^(5/3)*b^(2/3)) - ((b*c - a*f)*Log[x])/a^2 - ((b^(1/3)*(b*d - a*g) - a
^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b*d -
 a*g - (a^(1/3)*(b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(6*a^(5/3)*b^(1/3)) + ((b*c - a*f)*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^4}+\frac{d}{ax^3}+\frac{e}{ax^2}+\frac{-bc+af}{a^2x}+\frac{-a(bd-ag)-a(be-ah)x\,\cdots}{a^2\left(a+bx^3\right)}\right.$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}+\frac{\int \frac{-a(bd-ag)-a(be-ah)x+b(b\cdots}{a+bx^3}}{a^2}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}+\frac{\int \frac{-a(bd-ag)-a(be-ah)x}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}+\frac{(bc-af)\log\left(a+bx^3\right)}{3a^2}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}-\frac{\left(bd-ag-\frac{\sqrt[3]{a}\,(be-ah)}{\sqrt[3]{b}}\cdots\right.}{3a^{5/3}\cdots}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}-\frac{\left(bd-ag-\frac{\sqrt[3]{a}\,(be-ah)}{\sqrt[3]{b}}\cdots\right.}{3a^{5/3}\cdots}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}+\frac{\left(b^{4/3}d+\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g-a^{4/3}h\right)\tan^{-1}\left(\cdots\right.}{\sqrt{3}\,a^{5/3}b^{2/3}}$$

Mathematica [A]
time = 0.31, size = 264, normalized size = 0.96

$$-\frac{\frac{2ac}{x^3}+\frac{3ad}{x^2}+\frac{6ae}{x}+\frac{2\sqrt{3}\,\sqrt[3]{a}\left(-b^{4/3}d-\sqrt[3]{a}\,be+a\sqrt[3]{b}\,g+a^{4/3}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+6(bc-af)\log(x)+\frac{2\sqrt[3]{a}\left(b^{4/3}d-\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g+a^{4/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}-\frac{\sqrt[3]{a}\left(b^{4/3}d-\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g+a^{4/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}-2(bc-af)\log\left(a+bx^3\right)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/6*((2*a*c)/x^3 + (3*a*d)/x^2 + (6*a*e)/x + (2*Sqrt[3]*a^(1/3)*(-(b^(4/3)
*d) - a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(2/3) + 6*(b*c - a*f)*Log[x] + (2*a^(1/3)*(b^(4/3)*d - a^(
1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^
(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 2*(b*c - a*f)*Log[a + b*x^3])/a^2
```

Maple [A]
time = 0.42, size = 276, normalized size = 1.00

method	result

default	$\frac{(a^2g-abd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(a^2h-abe)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\right.}{}\right.}{a^2}$
risch	$\frac{-\frac{e\,x^2}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^6b^2\_Z^3+\left(3a^5b^2f-3a^4b^3c\right)\_Z^2+\left(3a^5bgh-3a^4b^2dh-3a^4b^2eg+3a^4b^2f^2-6a^3b^3cf+3a^3b^3de+3a^2b^4c^2\right)\_\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] ((a^2*g-a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1)))+(a^2*h-a*b*e)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/
6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(-a*b*f+b^2*c)*ln(b*x^3+a)/b)
/a^2-e/a/x-1/3*c/a/x^3-1/2*d/a/x^2+(a*f-b*c)/a^2*ln(x)
```

Maxima [A]
time = 0.50, size = 306, normalized size = 1.11

$$-\frac{(bc-af)\log(x)}{a^2}+\frac{\sqrt{3}\left(a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab\left(\frac{a}{b}\right)^{\frac{2}{3}}e-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3}+\frac{\left(2b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e+abd-a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e-abd+a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{6x^2e+3dx+2c}{6ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] -(b*c - a*f)*log(x)/a^2 + 1/3*sqrt(3)*(a^2*h*(a/b)^(2/3) - a*b*(a/b)^(2/3)*
e - a*b*d*(a/b)^(1/3) + a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/a^3 + 1/6*(2*b^2*c*(a/b)^(2/3) - 2*a*b*f*(a/b)^(2/3) +
a^2*h*(a/b)^(1/3) - a*b*(a/b)^(1/3)*e + a*b*d - a^2*g)*log(x^2 - x*(a/b)^(1
/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) + 1/3*(b^2*c*(a/b)^(2/3) - a*b*f*(a/
b)^(2/3) - a^2*h*(a/b)^(1/3) + a*b*(a/b)^(1/3)*e - a*b*d + a^2*g)*log(x + (
a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) - 1/6*(6*x^2*e + 3*d*x + 2*c)/(a*x^3)
```

Fricas [C] Result contains complex when optimal does not.
time = 64.23, size = 15204, normalized size = 55.09

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b*c - a*f)^2/a^4 - (b^3*c^2 + a^
3*g*h + (f^2 - e*g - d*h)*a^2*b + (d*e - 2*c*f)*a*b^2)/(a^4*b))/(2*(b*c - a
*f)^3/a^6 - 3*(b^3*c^2 + a^3*g*h + (f^2 - e*g - d*h)*a^2*b + (d*e - 2*c*f)*
a*b^2)*(b*c - a*f)/(a^6*b) - (b^4*d^3 + a*b^3*e^3 - 3*a*b^3*d^2*g + 3*a^2*b
^2*d*g^2 - a^3*b*g^3 - 3*a^2*b^2*e^2*h + 3*a^3*b*e*h^2 - a^4*h^3)/(a^5*b^2)
 + (b^5*c^3 - a^5*h^3 + (g^3 - 3*f*g*h + 3*e*h^2)*a^4*b - (f^3 - 3*e*f*g +
3*e^2*h - 3*c*g*h + 3*(g^2 - f*h)*d)*a^3*b^2 + (e^3 - 3*d*e*f + 3*d^2*g + 3
*(f^2 - e*g - d*h)*c)*a^2*b^3 - (d^3 - 3*c*d*e + 3*c^2*f)*a*b^4)/(a^6*b^2))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b*c - a*f)^3/a^6 - 3*(b^3*c^2 + a^
3*g*h + (f^2 - e*g - d*h)*a^2*b + (d*e - 2*c*f)*a*b^2)*(b*c - a*f)/(a^6*b)
- (b^4*d^3 + a*b^3*e^3 - 3*a*b^3*d^2*g + 3*a^2*b^2*d*g^2 - a^3*b*g^3 - 3*a^
2*b^2*e^2*h + 3*a^3*b*e*h^2 - a^4*h^3)/(a^5*b^2) + (b^5*c^3 - a^5*h^3 + (g^
3 - 3*f*g*h + 3*e*h^2)*a^4*b - (f^3 - 3*e*f*g + 3*e^2*h - 3*c*g*h + 3*(g^2
- f*h)*d)*a^3*b^2 + (e^3  ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.54, size = 291, normalized size = 1.05

$$\frac{\sqrt{3}\left(b^2d-abg+\left(-ab^2\right)^{\frac{1}{3}}ah-\left(-ab^2\right)^{\frac{1}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}a}+\frac{\left(b^2d-abg-\left(-ab^2\right)^{\frac{1}{3}}ah+\left(-ab^2\right)^{\frac{1}{3}}be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}a}+\frac{(bc-af)\log\left(|bx^3+a|\right)}{3a^2}-\frac{(bc-af)\log\left(|x|\right)}{a^2}-\frac{\left(a^4bh\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e-a^3b^2d+a^4bg\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^5b}-\frac{6ax^2e+3adx+2ac}{6a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] 1/3*sqrt(3)*(b^2*d - a*b*g + (-a*b^2)^(1/3)*a*h - (-a*b^2)^(1/3)*b*e)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) + 1/6*(
b^2*d - a*b*g - (-a*b^2)^(1/3)*a*h + (-a*b^2)^(1/3)*b*e)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) + 1/3*(b*c - a*f)*log(abs(b*x^3 +
 a))/a^2 - (b*c - a*f)*log(abs(x))/a^2 - 1/3*(a^4*b*h*(-a/b)^(1/3) - a^3*b^
2*(-a/b)^(1/3)*e - a^3*b^2*d + a^4*b*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/
3)))/(a^5*b) - 1/6*(6*a*x^2*e + 3*a*d*x + 2*a*c)/(a^2*x^3)
```

Mupad [B]
time = 5.87, size = 1842, normalized size = 6.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x)
[Out] symsum(log(- (b^5*c*d^2 - b^5*c^2*e + a^2*b^3*c*g^2 - a^2*b^3*e*f^2 - a^3*b
^2*f*g^2 + a^3*b^2*f^2*h - a*b^4*d^2*f + a*b^4*c^2*h - 2*a^2*b^3*c*f*h + 2*
a^2*b^3*d*f*g - 2*a*b^4*c*d*g + 2*a*b^4*c*e*f)/a^3 - root(27*a^6*b^2*z^3 +
27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g*h*z - 9*a^4*b^2*e*g*z - 9*a
^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b^3*d*e*z + 9*a^4*b^2*f^2*z + 9*a^2
*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*
f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h
 - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^
2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2
*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*((a^2*b^4*d^2 + a^4*b^2*g^2 + 2*a^2*b^4*c
*e - 2*a^3*b^3*c*h - 2*a^3*b^3*d*g - 2*a^3*b^3*e*f + 2*a^4*b^2*f*h)/a^3 + r
oot(27*a^6*b^2*z^3 + 27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g*h*z -
9*a^4*b^2*e*g*z - 9*a^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b^3*d*e*z + 9*
a^4*b^2*f^2*z + 9*a^2*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2
*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*
e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3
*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^
3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*((3*a^4*b^3*e - 3*a^
5*b^2*h)/a^3 - (x*(24*a^3*b^4*c - 24*a^4*b^3*f))/a^3 + 36*root(27*a^6*b^2*z
^3 + 27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g*h*z - 9*a^4*b^2*e*g*z
- 9*a^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b^3*d*e*z + 9*a^4*b^2*f^2*z +
9*a^2*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b
^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*
c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 -
 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^
3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*a^2*b^3*x) + (x*(4*a*b^5*c^2 + 4*a^3
*b^3*f^2 - 8*a^2*b^4*c*f + 10*a^2*b^4*d*e - 10*a^3*b^3*d*h - 10*a^3*b^3*e*g
 + 10*a^4*b^2*g*h))/a^3) - (x*(b^5*d^3 - a*b^4*e^3 + a^4*b*h^3 - a^3*b^2*g^
3 + 3*a^2*b^3*d*g^2 + 3*a^2*b^3*e^2*h - 3*a^3*b^2*e*h^2 - 2*b^5*c*d*e - 3*a
*b^4*d^2*g - 2*a^2*b^3*c*g*h - 2*a^2*b^3*d*f*h - 2*a^2*b^3*e*f*g + 2*a^3*b^
2*f*g*h + 2*a*b^4*c*d*h + 2*a*b^4*c*e*g + 2*a*b^4*d*e*f))/a^3)*root(27*a^6*
b^2*z^3 + 27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g*h*z - 9*a^4*b^2*e
*g*z - 9*a^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b^3*d*e*z + 9*a^4*b^2*f^2
*z + 9*a^2*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*
a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2
*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*
g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3
```

```
 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/
a + (d*x)/(2*a))/x^3 - (log(x)*(b*c - a*f))/a^2
```

3.412 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=337

$$\frac{(be-2ah)x}{b^3}+\frac{fx^2}{2b^2}+\frac{gx^3}{3b^2}+\frac{hx^4}{4b^2}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3b^3\,(a+bx^3)}-\frac{(2b^{5/3}c-4a^{2/3}be-5ab^{2/3}f}{3\sqrt{3}}$$

```
[Out] (-2*a*h+b*e)*x/b^3+1/2*f*x^2/b^2+1/3*g*x^3/b^2+1/4*h*x^4/b^2+1/3*x*(a*(-a*h
+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/b^3/(b*x^3+a)-1/9*(b^(2/3)*(-5*a*f+2
*b*c)+a^(2/3)*(-7*a*h+4*b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(10/3)+1/18*(
b^(2/3)*(-5*a*f+2*b*c)+a^(2/3)*(-7*a*h+4*b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/a^(1/3)/b^(10/3)+1/3*(-2*a*g+b*d)*ln(b*x^3+a)/b^3-1/9*(2*b^(5
/3)*c-4*a^(2/3)*b*e-5*a*b^(2/3)*f+7*a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3
)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1842, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-4a^{2/3}be+7a^{5/3}h-5ab^{2/3}f+2b^{5/3}c\right)}{3\sqrt{3}\sqrt[3]{a}\,b^{10/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{18\sqrt[3]{a}\,b^{10/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{9\sqrt[3]{a}\,b^{10/3}}+\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{3b^3\,(a+bx^3)}+\frac{(bd-2ag)\log\left(a+bx^3\right)}{3b^3}+\frac{x(be-2ah)}{b^3}+\frac{fx^2}{2b^2}+\frac{gx^3}{3b^2}+\frac{hx^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*h)*x)/b^3 + (f*x^2)/(2*b^2) + (g*x^3)/(3*b^2) + (h*x^4)/(4*b^2)
 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*b^3*(a + b*
x^3)) - ((2*b^(5/3)*c - 4*a^(2/3)*b*e - 5*a*b^(2/3)*f + 7*a^(5/3)*h)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(10/3)) -
 ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(1/3) + b^(1/3)
*x])/(9*a^(1/3)*b^(10/3)) + ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*
a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(10/3))
 + ((b*d - 2*a*g)*Log[a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3b^3(a+bx^3)} - \frac{\int \frac{a^2(be-ah)-2ab \cdots}{\cdots}}{\cdots}$$

$$= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3b^3(a+bx^3)} - \frac{\int \left(-3a(be- \cdots\right.}{\cdots}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be-ah)-b(bc-a \cdots}{3b^3(a+ \cdots}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be-ah)-b(bc-a \cdots}{3b^3(a+ \cdots}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be-ah)-b(bc-a \cdots}{3b^3(a+ \cdots}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be-ah)-b(bc-a \cdots}{3b^3(a+ \cdots}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be-ah)-b(bc-a \cdots}{3b^3(a+ \cdots}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be-ah)-b(bc-a \cdots}{3b^3(a+ \cdots}$$

Mathematica [A]
time = 0.26, size = 334, normalized size = 0.99

$$\frac{36b^{2/3}(be-2ah)x+18b^{5/3}fx^2+12b^{5/3}gx^3+9b^{5/3}hx^4-\frac{12b^{2/3}(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{a+bx^3}-\frac{4\sqrt{3}\left(2b^2c-4a^{2/3}b^{4/3}e-5abf+7a^{5/3}\sqrt[3]{b}\,h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{4\left(-2b^2c-4a^{2/3}b^{4/3}e+5abf+7a^{5/3}\sqrt[3]{b}\,h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}}+\frac{2\left(2b^2c+4a^{2/3}b^{4/3}e-5abf-7a^{5/3}\sqrt[3]{b}\,h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{a}}+12b^{2/3}(bd-2ag)\log\left(a+bx^3\right)}{36b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(2/3)*(b*e - 2*a*h)*x + 18*b^(5/3)*f*x^2 + 12*b^(5/3)*g*x^3 + 9*b^(5/
3)*h*x^4 - (12*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x))))
/(a + b*x^3) - (4*Sqrt[3]*(2*b^2*c - 4*a^(2/3)*b^(4/3)*e - 5*a*b*f + 7*a^(5
/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (4*(-
2*b^2*c - 4*a^(2/3)*b^(4/3)*e + 5*a*b*f + 7*a^(5/3)*b^(1/3)*h)*Log[a^(1/3)
+ b^(1/3)*x])/a^(1/3) + (2*(2*b^2*c + 4*a^(2/3)*b^(4/3)*e - 5*a*b*f - 7*a^(
5/3)*b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3) + 1
2*b^(2/3)*(b*d - 2*a*g)*Log[a + b*x^3])/(36*b^(11/3))
```

Maple [A]

time = 0.38, size = 329, normalized size = 0.98

method	result
risch	$\frac{h\,x^{4}}{4b^{2}}+\frac{g\,x^{3}}{3b^{2}}+\frac{f\,x^{2}}{2b^{2}}-\frac{2ahx}{b^{3}}+\frac{ex}{b^{2}}+\frac{\left(\frac{1}{3}abf-\frac{1}{3}b^{2}c\right)x^{2}+\left(-\frac{1}{3}a^{2}h+\frac{1}{3}abe\right)x-\frac{a(ag-bd)}{3}}{b^{3}\left(b\,x^{3}+a\right)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(3b(-2ag+bd)\_\right.}{}}{}$
default	$-\frac{-\frac{1}{4}bh\,x^{4}-\frac{1}{3}bg\,x^{3}-\frac{1}{2}bf\,x^{2}+2ahx-bex}{b^{3}}+\frac{\frac{\left(\frac{1}{3}abf-\frac{1}{3}b^{2}c\right)x^{2}+\left(-\frac{1}{3}a^{2}h+\frac{1}{3}abe\right)x-\frac{a(ag-bd)}{3}}{b\,x^{3}+a}+\left(7a^{2}h-4abe\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)\right.}{6b(}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/b^3*(-1/4*b*h*x^4-1/3*b*g*x^3-1/2*b*f*x^2+2*a*h*x-b*e*x)+1/b^3*(((1/3*a*
b*f-1/3*b^2*c)*x^2+(-1/3*a^2*h+1/3*a*b*e)*x-1/3*a*(a*g-b*d))/(b*x^3+a)+1/3*
(7*a^2*h-4*a*b*e)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1)))+1/3*(-5*a*b*f+2*b^2*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/
b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b
/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/9*(-6*a*b*g+3*b^2*d
)*ln(b*x^3+a)/b)
```

Maxima [A]

time = 0.51, size = 371, normalized size = 1.10

$$\frac{abd-a^{2}g-(b^{2}c-abf)x^{2}-(a^{2}h-abe)x}{3\,(b^{4}x^{3}+ab^{3})}+\frac{\sqrt{3}\left(2\,b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+7\,a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{3}}+\frac{3\,bhx^{4}+4\,bgx^{3}+6\,bfx^{2}-12\,(2\,ah-be)x}{12\,b^{3}}+\frac{\left(6\,b^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}-12\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-7\,a^{2}h+4\,abe\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(3\,b^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}-6\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+7\,a^{2}h-4\,abe\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] 1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 - (a^2*h - a*b*e)*x)/(b^4*x^3 + a*
b^3) + 1/9*sqrt(3)*(2*b^2*c*(a/b)^(2/3) - 5*a*b*f*(a/b)^(2/3) + 7*a^2*h*(a/
b)^(1/3) - 4*a*b*(a/b)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(a*b^3) + 1/12*(3*b*h*x^4 + 4*b*g*x^3 + 6*b*f*x^2 - 12*(2*a*h - b*
e)*x)/b^3 + 1/18*(6*b^2*d*(a/b)^(2/3) - 12*a*b*g*(a/b)^(2/3) + 2*b^2*c*(a/b
)^(1/3) - 5*a*b*f*(a/b)^(1/3) - 7*a^2*h + 4*a*b*e)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/9*(3*b^2*d*(a/b)^(2/3) - 6*a*b*g*(a/b)
^(2/3) - 2*b^2*c*(a/b)^(1/3) + 5*a*b*f*(a/b)^(1/3) + 7*a^2*h - 4*a*b*e)*log
(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

Fricas [**C**] Result contains complex when optimal does not.
time = 2.10, size = 16147, normalized size = 47.91

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] 1/36*(9*b^2*h*x^7 + 12*b^2*g*x^6 + 18*b^2*f*x^5 + 12*a*b*g*x^3 + 9*(4*b^2*e
 - 7*a*b*h)*x^4 + 12*a*b*d - 12*a^2*g - 6*(2*b^2*c - 5*a*b*f)*x^2 - 2*(b^4*
x^3 + a*b^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*(b*d - 2*a*g)^2/b^6 - ((36*
g^2 - 35*f*h)*a^2 + 2*(10*e*f - 18*d*g + 7*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/
b^6)/(54*(b*d - 2*a*g)^3/b^9 - 9*((36*g^2 - 35*f*h)*a^2 + 2*(10*e*f - 18*d*
g + 7*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*(b*d - 2*a*g)/b^9 + (8*b^5*c^3 - 64*a
^2*b^3*e^3 - 60*a*b^4*c^2*f + 150*a^2*b^3*c*f^2 - 125*a^3*b^2*f^3 + 336*a^3
*b^2*e^2*h - 588*a^4*b*e*h^2 + 343*a^5*h^3)/(a*b^10) - (8*b^5*c^3 - 343*a^5
*h^3 + 6*(36*g^3 - 105*f*g*h + 98*e*h^2)*a^4*b - (125*f^3 - 360*e*f*g + 336
*e^2*h - 252*c*g*h + 9*(36*g^2 - 35*f*h)*d)*a^3*b^2 + 2*(32*e^3 - 90*d*e*f
+ 81*d^2*g + 3*(25*f^2 - 24*e*g - 21*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e
+ 20*c^2*f)*a*b^4)/(a*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*(b*d -
 2*a*g)^3/b^9 - 9*((36*g^2 - 35*f*h)*a^2 + 2*(10*e*f - 18*d*g + 7*c*h)*a*b
+ (9*d^2 - 8*c*e)*b^2)*(b ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.56, size = 357, normalized size = 1.06

$$-\frac{\sqrt{3}\left(7\,a^{2}h-4\,abe-2\left(-ab^{2}\right)^{\frac{1}{3}}bc+5\left(-ab^{2}\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^{2}\right)^{\frac{2}{3}}b^{2}}-\frac{\left(7\,a^{2}h-4\,abe+2\left(-ab^{2}\right)^{\frac{1}{3}}bc-5\left(-ab^{2}\right)^{\frac{1}{3}}af\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^{2}\right)^{\frac{2}{3}}b^{2}}+\frac{(bd-2\,ag)\log\left(|bx^{3}+a|\right)}{3\,b^{3}}+\frac{abd-a^{2}g-(b^{2}c-abf)x^{2}-(a^{2}h-abe)x}{3\,(bx^{3}+a)b^{3}}-\frac{\left(2\,b^{6}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^{5}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+7\,a^{2}b^{4}h-4\,ab^{5}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^{7}}+\frac{3\,b^{6}hx^{4}+4\,b^{6}gx^{3}+6\,b^{6}fx^{2}-24\,ab^{5}hx+12\,b^{6}xe}{12\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/9*sqrt(3)*(7*a^2*h - 4*a*b*e - 2*(-a*b^2)^(1/3)*b*c + 5*(-a*b^2)^(1/3)*a
*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b
^2) - 1/18*(7*a^2*h - 4*a*b*e + 2*(-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*f
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) + 1/3*(b*d
- 2*a*g)*log(abs(b*x^3 + a))/b^3 + 1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2
 - (a^2*h - a*b*e)*x)/((b*x^3 + a)*b^3) - 1/9*(2*b^6*c*(-a/b)^(1/3) - 5*a*b
^5*f*(-a/b)^(1/3) + 7*a^2*b^4*h - 4*a*b^5*e)*(-a/b)^(1/3)*log(abs(x - (-a/b
)^(1/3)))/(a*b^7) + 1/12*(3*b^6*h*x^4 + 4*b^6*g*x^3 + 6*b^6*f*x^2 - 24*a*b^
5*h*x + 12*b^6*x*e)/b^8
```

Mupad [B]
time = 5.11, size = 1241, normalized size = 3.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log(root(729*a*b^10*z^3 - 729*a*b^8*d*z^2 + 1458*a^2*b^7*g*z^2 - 216
*a*b^6*c*e*z - 945*a^3*b^4*f*h*z - 972*a^2*b^5*d*g*z + 540*a^2*b^5*e*f*z +
378*a^2*b^5*c*h*z + 243*a*b^6*d^2*z + 972*a^3*b^4*g^2*z - 630*a^4*b*f*g*h +
 72*a*b^4*c*d*e + 360*a^3*b^2*e*f*g + 315*a^3*b^2*d*f*h + 252*a^3*b^2*c*g*h
 - 180*a^2*b^3*d*e*f - 144*a^2*b^3*c*e*g - 126*a^2*b^3*c*d*h + 588*a^4*b*e*
h^2 - 60*a*b^4*c^2*f - 336*a^3*b^2*e^2*h - 324*a^3*b^2*d*g^2 + 162*a^2*b^3*
d^2*g + 150*a^2*b^3*c*f^2 - 125*a^3*b^2*f^3 + 64*a^2*b^3*e^3 + 216*a^4*b*g^
3 - 27*a*b^4*d^3 - 343*a^5*h^3 + 8*b^5*c^3, z, k)*((108*a^2*b^3*g - 54*a*b^
4*d)/(9*b^4) + (x*(63*a^2*b^3*h - 36*a*b^4*e))/(9*b^4) + 9*root(729*a*b^10*
z^3 - 729*a*b^8*d*z^2 + 1458*a^2*b^7*g*z^2 - 216*a*b^6*c*e*z - 945*a^3*b^4*
f*h*z - 972*a^2*b^5*d*g*z + 540*a^2*b^5*e*f*z + 378*a^2*b^5*c*h*z + 243*a*b
^6*d^2*z + 972*a^3*b^4*g^2*z - 630*a^4*b*f*g*h + 72*a*b^4*c*d*e + 360*a^3*b
^2*e*f*g + 315*a^3*b^2*d*f*h + 252*a^3*b^2*c*g*h - 180*a^2*b^3*d*e*f - 144*
a^2*b^3*c*e*g - 126*a^2*b^3*c*d*h + 588*a^4*b*e*h^2 - 60*a*b^4*c^2*f - 336*
a^3*b^2*e^2*h - 324*a^3*b^2*d*g^2 + 162*a^2*b^3*d^2*g + 150*a^2*b^3*c*f^2 -
 125*a^3*b^2*f^3 + 64*a^2*b^3*e^3 + 216*a^4*b*g^3 - 27*a*b^4*d^3 - 343*a^5*
h^3 + 8*b^5*c^3, z, k)*a*b^2) + (36*a^3*g^2 + 9*a*b^2*d^2 - 35*a^3*f*h - 8*
```

```
a*b^2*c*e + 14*a^2*b*c*h - 36*a^2*b*d*g + 20*a^2*b*e*f)/(9*b^4) + (x*(4*b^3
*c^2 + 25*a^2*b*f^2 + 42*a^3*g*h - 20*a*b^2*c*f + 12*a*b^2*d*e - 21*a^2*b*d
*h - 24*a^2*b*e*g))/(9*b^4))*root(729*a*b^10*z^3 - 729*a*b^8*d*z^2 + 1458*a
^2*b^7*g*z^2 - 216*a*b^6*c*e*z - 945*a^3*b^4*f*h*z - 972*a^2*b^5*d*g*z + 54
0*a^2*b^5*e*f*z + 378*a^2*b^5*c*h*z + 243*a*b^6*d^2*z + 972*a^3*b^4*g^2*z -
 630*a^4*b*f*g*h + 72*a*b^4*c*d*e + 360*a^3*b^2*e*f*g + 315*a^3*b^2*d*f*h +
 252*a^3*b^2*c*g*h - 180*a^2*b^3*d*e*f - 144*a^2*b^3*c*e*g - 126*a^2*b^3*c*
d*h + 588*a^4*b*e*h^2 - 60*a*b^4*c^2*f - 336*a^3*b^2*e^2*h - 324*a^3*b^2*d*
g^2 + 162*a^2*b^3*d^2*g + 150*a^2*b^3*c*f^2 - 125*a^3*b^2*f^3 + 64*a^2*b^3*
e^3 + 216*a^4*b*g^3 - 27*a*b^4*d^3 - 343*a^5*h^3 + 8*b^5*c^3, z, k), k, 1,
3) + x*(e/b^2 - (2*a*h)/b^3) - (x*((a^2*h)/3 - (a*b*e)/3) + (a^2*g)/3 + x^2
*((b^2*c)/3 - (a*b*f)/3) - (a*b*d)/3)/(a*b^3 + b^4*x^3) + (f*x^2)/(2*b^2) +
 (g*x^3)/(3*b^2) + (h*x^4)/(4*b^2)
```

3.413 $\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=311

$$\frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2\left(a+bx^3\right)}-\frac{\left(b^{4/3}c+2\sqrt[3]{a}\,bd-4a\sqrt[3]{b}\,f-5a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}}{\sqrt{}}\right.}{3\sqrt{3}\,a^{2/3}b^{8/3}}$$

```
[Out] f*x/b^2+1/2*g*x^2/b^2+1/3*h*x^3/b^2-1/3*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*
x^2)/b^2/(b*x^3+a)+1/9*(b^(1/3)*(-4*a*f+b*c)-a^(1/3)*(-5*a*g+2*b*d))*ln(a^(
1/3)+b^(1/3)*x)/a^(2/3)/b^(8/3)-1/18*(b^(1/3)*(-4*a*f+b*c)-a^(1/3)*(-5*a*g+
2*b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(8/3)+1/3*(-2*a
*h+b*e)*ln(b*x^3+a)/b^3-1/9*(b^(4/3)*c+2*a^(1/3)*b*d-4*a*b^(1/3)*f-5*a^(4/3
)*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(8/3)*3^(1
/2)
```

Rubi [A]
time = 0.41, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1842, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(-5a^{4/3}g+2\sqrt[3]{a}\,bd-4a\sqrt[3]{b}\,f+b^{4/3}c\right)}{3\sqrt{3}\,a^{2/3}b^{8/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(\sqrt[3]{b}\,(bc-4af)-\sqrt[3]{a}\,(2bd-5ag)\right)}{18a^{2/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(bc-4af)-\sqrt[3]{a}\,(2bd-5ag)\right)}{9a^{2/3}b^{8/3}}+\frac{(be-2ah)\log\left(a+bx^3\right)}{3b^3}-\frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{3b^2\left(a+bx^3\right)}+\frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (f*x)/b^2 + (g*x^2)/(2*b^2) + (h*x^3)/(3*b^2) - (x*(b*c - a*f + (b*d - a*g)
*x + (b*e - a*h)*x^2))/(3*b^2*(a + b*x^3)) - ((b^(4/3)*c + 2*a^(1/3)*b*d -
4*a*b^(1/3)*f - 5*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(3*Sqrt[3]*a^(2/3)*b^(8/3)) + ((b^(1/3)*(b*c - 4*a*f) - a^(1/3)*(2*b*d
 - 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b^(1/3)*(b*c -
 4*a*f) - a^(1/3)*(2*b*d - 5*a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*a^(2/3)*b^(8/3)) + ((b*e - 2*a*h)*Log[a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} - \frac{\int \frac{-ab(bc-af)-2ab(bd-}{}}{}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} - \frac{\int \Big(-3abf-3abg}{}$$

$$= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} +$$

$$= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} +$$

$$= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} +$$

$$= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} +$$

$$= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} +$$

$$= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} -$$

Mathematica [A]
time = 0.14, size = 294, normalized size = 0.95

$$\frac{18bfx+9bgx^2+6bhx^3-\frac{6(a^2h+b^2x(c+dx)-ab(e+x(f+gx)))}{a+bx^3}+\frac{2\sqrt{3}\sqrt[3]{b}\left(-b^{4/3}c-2\sqrt[3]{a}bd+4a\sqrt[3]{b}f+5a^{4/3}g\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{2\sqrt[3]{b}\left(b^{4/3}c-2\sqrt[3]{a}bd-4a\sqrt[3]{b}f+5a^{4/3}g\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}-\frac{\sqrt[3]{b}\left(b^{4/3}c-2\sqrt[3]{a}bd-4a\sqrt[3]{b}f+5a^{4/3}g\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}+6(be-2ah)\log(a+bx^3)}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b*f*x + 9*b*g*x^2 + 6*b*h*x^3 - (6*(a^2*h + b^2*x*(c + d*x) - a*b*(e +
x*(f + g*x))))/(a + b*x^3) + (2*Sqrt[3]*b^(1/3)*(-(b^(4/3)*c) - 2*a^(1/3)*b
*d + 4*a*b^(1/3)*f + 5*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]])/a^(2/3) + (2*b^(1/3)*(b^(4/3)*c - 2*a^(1/3)*b*d - 4*a*b^(1/3)*f + 5*a^(
4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) - (b^(1/3)*(b^(4/3)*c - 2*a^(1/3)
*b*d - 4*a*b^(1/3)*f + 5*a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/a^(2/3) + 6*(b*e - 2*a*h)*Log[a + b*x^3])/(18*b^3)
```

Maple [A]
time = 0.42, size = 301, normalized size = 0.97

method	result
risch	$\frac{h\,x^3}{3b^2}+\frac{g\,x^2}{2b^2}+\frac{fx}{b^2}+\frac{\left(\frac{ag}{3}-\frac{bd}{3}\right)x^2+\left(\frac{af}{3}-\frac{bc}{3}\right)x-\frac{a(ah-be)}{3b}}{b^2(b\,x^3+a)}+\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^3+a\right)}\frac{\left(3(-2ah+be)\_R^2+(-5ag+2bd)\_R-4af+\ldots\right.}{\_R^2}}{9b^3}$
default	$\frac{\frac{1}{3}h\,x^3+\frac{1}{2}g\,x^2+fx}{b^2}-\frac{\left(-\frac{ag}{3}+\frac{bd}{3}\right)x^2+\left(-\frac{af}{3}+\frac{bc}{3}\right)x+\frac{a(ah-be)}{3b}}{b\,x^3+a}+\frac{(4af-bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\ldots\right)}{3b\left(\ldots\right.}\ldots\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/b^2*(1/3*h*x^3+1/2*g*x^2+f*x)-1/b^2*(((-1/3*a*g+1/3*b*d)*x^2+(-1/3*a*f+1/
3*b*c)*x+1/3*a*(a*h-b*e)/b)/(b*x^3+a)+1/3*(4*a*f-b*c)*(1/3/b/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(
a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(5*a*g-2*b*
d)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/
3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1)))+1/9*(6*a*h-3*b*e)*ln(b*x^3+a)/b)
```

Maxima [A]
time = 0.50, size = 330, normalized size = 1.06

$$-\frac{a^2h+(b^2d-abg)x^2-abe+(b^2c-abf)x}{3\,(b^4x^3+ab^3)}+\frac{2\,hx^3+3\,gx^2+6\,fx}{6\,b^2}+\frac{\sqrt{3}\left(2\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3}-\frac{\left(12\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}e-2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc-4\,af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(6\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}e+2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc+4\,af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] -1/3*(a^2*h + (b^2*d - a*b*g)*x^2 - a*b*e + (b^2*c - a*b*f)*x)/(b^4*x^3 + a
*b^3) + 1/6*(2*h*x^3 + 3*g*x^2 + 6*f*x)/b^2 + 1/9*sqrt(3)*(2*b^2*d*(a/b)^(2
/3) - 5*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) - 4*a*b*f*(a/b)^(1/3))*arctan
(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) - 1/18*(12*a*h*(a/b)^
(2/3) - 6*b*(a/b)^(2/3)*e - 2*b*d*(a/b)^(1/3) + 5*a*g*(a/b)^(1/3) + b*c - 4
*a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 1/9*(6*a*h
*(a/b)^(2/3) - 3*b*(a/b)^(2/3)*e + 2*b*d*(a/b)^(1/3) - 5*a*g*(a/b)^(1/3) -
b*c + 4*a*f)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 2.22, size = 16285, normalized size = 52.36

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] 1/36*(12*b^2*h*x^6 + 18*b^2*g*x^5 + 36*b^2*f*x^4 + 12*a*b*h*x^3 + 12*a*b*e
- 12*a^2*h - 6*(2*b^2*d - 5*a*b*g)*x^2 - 2*(b^4*x^3 + a*b^3)*(2*(1/2)^(2/3)
*(-I*sqrt(3) + 1)*(9*(b*e - 2*a*h)^2/b^6 - (2*b^3*c*d + 36*a^3*h^2 + 4*(5*f
*g - 9*e*h)*a^2*b + (9*e^2 - 8*d*f - 5*c*g)*a*b^2)/(a*b^6))/(54*(b*e - 2*a*
h)^3/b^9 - 9*(2*b^3*c*d + 36*a^3*h^2 + 4*(5*f*g - 9*e*h)*a^2*b + (9*e^2 - 8
*d*f - 5*c*g)*a*b^2)*(b*e - 2*a*h)/(a*b^9) - (b^4*c^3 + 8*a*b^3*d^3 - 12*a*
b^3*c^2*f + 48*a^2*b^2*c*f^2 - 64*a^3*b*f^3 - 60*a^2*b^2*d^2*g + 150*a^3*b*
d*g^2 - 125*a^4*g^3)/(a^2*b^8) + (b^5*c^3 - 216*a^5*h^3 + (125*g^3 - 360*f*
g*h + 324*e*h^2)*a^4*b - 2*(32*f^3 - 90*e*f*g + 81*e^2*h - 45*c*g*h + 3*(25
*g^2 - 24*f*h)*d)*a^3*b^2 + 3*(9*e^3 - 24*d*e*f + 20*d^2*g + (16*f^2 - 15*e
*g - 12*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e + 6*c^2*f)*a*b^4)/(a^2*b^9))^(
1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*(b*e - 2*a*h)^3/b^9 - 9*(2*b^3*c*d +
 36*a^3*h^2 + 4*(5*f*g - 9*e*h)*a^2*b + (9*e^2 - 8*d*f - 5*c*g)*a*b^2)*(b*e
 - 2*a*h)/(a*b^9) - (b^4* ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 330, normalized size = 1.06

$$-\frac{\sqrt{3}\left(b^{2}c-4\,abf-2\,(-ab^{2})^{\frac{1}{3}}\,bd+5\,(-ab^{2})^{\frac{1}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^{2})^{\frac{2}{3}}\,b^{2}}-\frac{\left(b^{2}c-4\,abf+2\,(-ab^{2})^{\frac{1}{3}}\,bd-5\,(-ab^{2})^{\frac{1}{3}}\,ag\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^{2})^{\frac{2}{3}}\,b^{2}}-\frac{(2\,ah-be)\log\left(|bx^{3}+a|\right)}{3\,b^{3}}-\frac{a^{2}h+(b^{2}d-abg)x^{2}-abe+(b^{2}c-abf)x}{3\,(bx^{3}+a)b^{3}}-\frac{\left(2\,b^{4}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^{3}g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b^{4}c-4\,ab^{3}f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^{5}}+\frac{2\,b^{4}hx^{3}+3\,b^{4}gx^{2}+6\,b^{4}fx}{6\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/9*sqrt(3)*(b^2*c - 4*a*b*f - 2*(-a*b^2)^(1/3)*b*d + 5*(-a*b^2)^(1/3)*a*g
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2
) - 1/18*(b^2*c - 4*a*b*f + 2*(-a*b^2)^(1/3)*b*d - 5*(-a*b^2)^(1/3)*a*g)*lo
g(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) - 1/3*(2*a*h -
b*e)*log(abs(b*x^3 + a))/b^3 - 1/3*(a^2*h + (b^2*d - a*b*g)*x^2 - a*b*e + (
b^2*c - a*b*f)*x)/((b*x^3 + a)*b^3) - 1/9*(2*b^4*d*(-a/b)^(1/3) - 5*a*b^3*g
*(-a/b)^(1/3) + b^4*c - 4*a*b^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^5) + 1/6*(2*b^4*h*x^3 + 3*b^4*g*x^2 + 6*b^4*f*x)/b^6
```

Mupad [B]
time = 0.15, size = 1229, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((36*a^3*h^2 + 9*a*b^2*e^2 + 2*b^3*c*d - 5*a*b^2*c*g - 8*a*b^2*d*
f - 36*a^2*b*e*h + 20*a^2*b*f*g)/(9*b^4) + root(729*a^2*b^9*z^3 + 1458*a^3*
b^6*h*z^2 - 729*a^2*b^7*e*z^2 + 54*a*b^6*c*d*z - 972*a^3*b^4*e*h*z + 540*a^
3*b^4*f*g*z - 216*a^2*b^5*d*f*z - 135*a^2*b^5*c*g*z + 972*a^4*b^3*h^2*z + 2
43*a^2*b^5*e^2*z + 360*a^4*b*f*g*h - 18*a*b^4*c*d*e - 180*a^3*b^2*e*f*g - 1
44*a^3*b^2*d*f*h - 90*a^3*b^2*c*g*h + 72*a^2*b^3*d*e*f + 45*a^2*b^3*c*e*g +
 36*a^2*b^3*c*d*h - 324*a^4*b*e*h^2 + 12*a*b^4*c^2*f + 162*a^3*b^2*e^2*h +
150*a^3*b^2*d*g^2 - 60*a^2*b^3*d^2*g - 48*a^2*b^3*c*f^2 + 64*a^3*b^2*f^3 -
27*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8*a*b^4*d^3 + 216*a^5*h^3 - b^5*c^3, z, k)
*((108*a^2*b^3*h - 54*a*b^4*e)/(9*b^4) + (x*(9*b^4*c - 36*a*b^3*f))/(9*b^3)
 + 9*root(729*a^2*b^9*z^3 + 1458*a^3*b^6*h*z^2 - 729*a^2*b^7*e*z^2 + 54*a*b
^6*c*d*z - 972*a^3*b^4*e*h*z + 540*a^3*b^4*f*g*z - 216*a^2*b^5*d*f*z - 135*
a^2*b^5*c*g*z + 972*a^4*b^3*h^2*z + 243*a^2*b^5*e^2*z + 360*a^4*b*f*g*h - 1
8*a*b^4*c*d*e - 180*a^3*b^2*e*f*g - 144*a^3*b^2*d*f*h - 90*a^3*b^2*c*g*h +
72*a^2*b^3*d*e*f + 45*a^2*b^3*c*e*g + 36*a^2*b^3*c*d*h - 324*a^4*b*e*h^2 +
12*a*b^4*c^2*f + 162*a^3*b^2*e^2*h + 150*a^3*b^2*d*g^2 - 60*a^2*b^3*d^2*g -
 48*a^2*b^3*c*f^2 + 64*a^3*b^2*f^3 - 27*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8*a*b
^4*d^3 + 216*a^5*h^3 - b^5*c^3, z, k)*a*b^2) + (x*(4*b^2*d^2 + 25*a^2*g^2 -
 3*b^2*c*e - 24*a^2*f*h + 6*a*b*c*h - 20*a*b*d*g + 12*a*b*e*f))/(9*b^3))*ro
```

```
ot(729*a^2*b^9*z^3 + 1458*a^3*b^6*h*z^2 - 729*a^2*b^7*e*z^2 + 54*a*b^6*c*d*
z - 972*a^3*b^4*e*h*z + 540*a^3*b^4*f*g*z - 216*a^2*b^5*d*f*z - 135*a^2*b^5
*c*g*z + 972*a^4*b^3*h^2*z + 243*a^2*b^5*e^2*z + 360*a^4*b*f*g*h - 18*a*b^4
*c*d*e - 180*a^3*b^2*e*f*g - 144*a^3*b^2*d*f*h - 90*a^3*b^2*c*g*h + 72*a^2*
b^3*d*e*f + 45*a^2*b^3*c*e*g + 36*a^2*b^3*c*d*h - 324*a^4*b*e*h^2 + 12*a*b^
4*c^2*f + 162*a^3*b^2*e^2*h + 150*a^3*b^2*d*g^2 - 60*a^2*b^3*d^2*g - 48*a^2
*b^3*c*f^2 + 64*a^3*b^2*f^3 - 27*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8*a*b^4*d^3
+ 216*a^5*h^3 - b^5*c^3, z, k), k, 1, 3) - (x*((b*c)/3 - (a*f)/3) + (a^2*h
- a*b*e)/(3*b) + x^2*((b*d)/3 - (a*g)/3))/(a*b^2 + b^3*x^3) + (g*x^2)/(2*b^
2) + (h*x^3)/(3*b^2) + (f*x)/b^2
```

3.414 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=290

$$\frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}-\frac{\left(b^{4/3}d+2\sqrt[3]{a}\,be-4a\sqrt[3]{b}\,g-5a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{8/3}}$$

```
[Out] 4/3*g*x/b^2+5/6*h*x^2/b^2+1/3*(-h*x^5-g*x^4-f*x^3-e*x^2-d*x-c)/b/(b*x^3+a)+
1/9*(b^(1/3)*(-4*a*g+b*d)-a^(1/3)*(-5*a*h+2*b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(
2/3)/b^(8/3)-1/18*(b^(1/3)*(-4*a*g+b*d)-a^(1/3)*(-5*a*h+2*b*e))*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(8/3)+1/3*f*ln(b*x^3+a)/b^2-1/9*(b
^(4/3)*d+2*a^(1/3)*b*e-4*a*b^(1/3)*g-5*a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 288, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1837, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(-5a^{4/3}h+2\sqrt[3]{a}\,be-4a\sqrt[3]{b}\,g+b^{4/3}d\right)}{3\sqrt{3}\,a^{2/3}b^{8/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}\,(2be-5ah)}{\sqrt[3]{b}}-4ag+bd\right)}{18a^{2/3}b^{7/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(bd-4ag)-\sqrt[3]{a}\,(2be-5ah)\right)}{9a^{2/3}b^{8/3}}+\frac{f\log\left(a+bx^3\right)}{3b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}+\frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (4*g*x)/(3*b^2) + (5*h*x^2)/(6*b^2) - (c + d*x + e*x^2 + f*x^3 + g*x^4 + h*
x^5)/(3*b*(a + b*x^3)) - ((b^(4/3)*d + 2*a^(1/3)*b*e - 4*a*b^(1/3)*g - 5*a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2
/3)*b^(8/3)) + ((b^(1/3)*(b*d - 4*a*g) - a^(1/3)*(2*b*e - 5*a*h))*Log[a^(1/
3) + b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b*d - 4*a*g - (a^(1/3)*(2*b*e - 5*
a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(2/3)*
b^(7/3)) + (f*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = -\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\,(a+bx^3)} + \frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{a+bx^3}\,d\cdots}{3b}$$

$$= -\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\,(a+bx^3)} + \frac{\int \left(\frac{4g}{b}+\frac{5hx}{b}+\frac{bd-4ag+(2\cdots}{b(\cdots}\right.}{3b}$$

$$= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\,(a+bx^3)} + \frac{\int \frac{bd-4ag+(2\cdots}{}}{}$$

$$= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\,(a+bx^3)} + \frac{\int \frac{bd-4ag+(\cdots}{a+\cdots}}{3\cdots}$$

$$= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\,(a+bx^3)} + \frac{f\log(a+\cdots}{3b^2}$$

$$= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\,(a+bx^3)} + \frac{\left(bd-4ag\cdots\right.}{}$$

$$= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\,(a+bx^3)} + \frac{\left(bd-4ag\cdots\right.}{}$$

$$= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\,(a+bx^3)} - \frac{\left(b^{4/3}d+2\cdots\right.}{}$$

Mathematica [A]
time = 0.13, size = 280, normalized size = 0.97

$$\frac{18b^{2/3}gx+9b^{2/3}hx^2-\frac{6b^{2/3}(b(c+x(d+ex))-a(f+x(g+hx)))}{a+bx^3}+\frac{2\sqrt{3}\left(-b^{4/3}d-2\sqrt[3]{a}\,be+4a\sqrt[3]{b}\,g+5a^{4/3}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{2\left(b^{4/3}d-2\sqrt[3]{a}\,be-4a\sqrt[3]{b}\,g+5a^{4/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}}-\frac{\left(b^{4/3}d-2\sqrt[3]{a}\,be-4a\sqrt[3]{b}\,g+5a^{4/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}}+6b^{2/3}f\log\left(a+bx^3\right)}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(2/3)*g*x + 9*b^(2/3)*h*x^2 - (6*b^(2/3)*(b*(c + x*(d + e*x)) - a*(f
+ x*(g + h*x))))/(a + b*x^3) + (2*Sqrt[3]*(-(b^(4/3)*d) - 2*a^(1/3)*b*e + 4
*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^
```

(2/3) + (2*(b^(4/3)*d - 2*a^(1/3)*b*e - 4*a*b^(1/3)*g + 5*a^(4/3)*h)*Log[a^
(1/3) + b^(1/3)*x])/a^(2/3) - ((b^(4/3)*d - 2*a^(1/3)*b*e - 4*a*b^(1/3)*g +
 5*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) + 6*b
^(2/3)*f*Log[a + b*x^3])/(18*b^(8/3))

Maple [A]
time = 0.41, size = 278, normalized size = 0.96

method	result
risch	$\frac{h\,x^{2}}{2b^{2}}+\frac{gx}{b^{2}}+\frac{\left(\frac{ah}{3}-\frac{be}{3}\right)x^{2}+\left(\frac{ag}{3}-\frac{bd}{3}\right)x+\frac{af}{3}-\frac{bc}{3}}{b^{2}(b\,x^{3}+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(3bf\_R^{2}+(-5ah+2be)\_R-4ag+bd\right)\ln\left(x-\_R\right)}{\_R^{2}}}{9b^{3}}$
default	$\frac{\frac{1}{2}h\,x^{2}+gx}{b^{2}}-\frac{\frac{\left(-\frac{ah}{3}+\frac{be}{3}\right)x^{2}+\left(-\frac{ag}{3}+\frac{bd}{3}\right)x-\frac{af}{3}+\frac{bc}{3}}{b\,x^{3}+a}+\frac{(4ag-bd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{3}}{b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE
)

[Out] 1/b^2*(1/2*h*x^2+g*x)-1/b^2*(((-1/3*a*h+1/3*b*e)*x^2+(-1/3*a*g+1/3*b*d)*x-1
/3*a*f+1/3*b*c)/(b*x^3+a)+1/3*(4*a*g-b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/
3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(5*a*h-2*b*e)*(-1/3/b/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-1/
3*f*ln(b*x^3+a))

Maxima [A]
time = 0.53, size = 289, normalized size = 1.00

$$\frac{(ah-be)x^{2}-bc+af-(bd-ag)x}{3\,(b^{3}x^{3}+ab^{2})}-\frac{\sqrt{3}\left(5\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}e-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+4\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{2}}+\frac{hx^{2}+2\,gx}{2\,b^{2}}+\frac{\left(6\,bf\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}e-bd+4\,ag\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(3\,bf\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}e+bd-4\,ag\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="max
ima")

[Out] 1/3*((a*h - b*e)*x^2 - b*c + a*f - (b*d - a*g)*x)/(b^3*x^3 + a*b^2) - 1/9*s
qrt(3)*(5*a*h*(a/b)^(2/3) - 2*b*(a/b)^(2/3)*e - b*d*(a/b)^(1/3) + 4*a*g*(a/

```
b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) + 1/2
*(h*x^2 + 2*g*x)/b^2 + 1/18*(6*b*f*(a/b)^(2/3) - 5*a*h*(a/b)^(1/3) + 2*b*(a
/b)^(1/3)*e - b*d + 4*a*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b
)^(2/3)) + 1/9*(3*b*f*(a/b)^(2/3) + 5*a*h*(a/b)^(1/3) - 2*b*(a/b)^(1/3)*e +
 b*d - 4*a*g)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.76, size = 12153, normalized size = 41.91

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] 1/36*(18*b*h*x^5 + 36*b*g*x^4 - 6*(2*b*e - 5*a*h)*x^2 - 2*(b^3*x^3 + a*b^2)
*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*
f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h
+ (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^
3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*
h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*
h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)
*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1
/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*
g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^
2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h
^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a
^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4
*e^3 - 9*d*e*f + 6*d^2*g) ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 307, normalized size = 1.06

$$\frac{f\log\left(|bx^3+a|\right)}{3\,b^2}-\frac{\sqrt{3}\left(b^2d-4\,abg+5\left(-ab^2\right)^{\frac{1}{3}}ah-2\left(-ab^2\right)^{\frac{1}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}b^2}-\frac{\left(b^2d-4\,abg-5\left(-ab^2\right)^{\frac{1}{3}}ah+2\left(-ab^2\right)^{\frac{1}{3}}be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}b^2}+\frac{(ah-be)x^2-bc+af-(bd-ag)x}{3\,(bx^3+a)b^2}+\frac{b^2hx^2+2\,b^2gx}{2\,b^4}+\frac{\left(5\,ab^3h\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,b^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}e-b^4d+4\,ab^3g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] 1/3*f*log(abs(b*x^3 + a))/b^2 - 1/9*sqrt(3)*(b^2*d - 4*a*b*g + 5*(-a*b^2)^(
1/3)*a*h - 2*(-a*b^2)^(1/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/18*(b^2*d - 4*a*b*g - 5*(-a*b^2)^(1/3)
*a*h + 2*(-a*b^2)^(1/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*
b^2)^(2/3)*b^2) + 1/3*((a*h - b*e)*x^2 - b*c + a*f - (b*d - a*g)*x)/((b*x^3
 + a)*b^2) + 1/2*(b^2*h*x^2 + 2*b^2*g*x)/b^4 + 1/9*(5*a*b^3*h*(-a/b)^(1/3)
- 2*b^4*(-a/b)^(1/3)*e - b^4*d + 4*a*b^3*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a*b^5)
```

Mupad [B]
time = 0.14, size = 816, normalized size = 2.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((9*a*b*f^2 + 2*b^2*d*e + 20*a^2*g*h - 5*a*b*d*h - 8*a*b*e*g)/(9*
b^3) + root(729*a^2*b^8*z^3 - 729*a^2*b^6*f*z^2 + 54*a*b^5*d*e*z + 540*a^3*
b^3*g*h*z - 216*a^2*b^4*e*g*z - 135*a^2*b^4*d*h*z + 243*a^2*b^4*f^2*z - 180
*a^3*b*f*g*h - 18*a*b^3*d*e*f + 72*a^2*b^2*e*f*g + 45*a^2*b^2*d*f*h + 150*a
^3*b*e*h^2 + 12*a*b^3*d^2*g - 60*a^2*b^2*e^2*h - 48*a^2*b^2*d*g^2 - 27*a^2*
b^2*f^3 + 64*a^3*b*g^3 + 8*a*b^3*e^3 - 125*a^4*h^3 - b^4*d^3, z, k)*((x*(9*
b^4*d - 36*a*b^3*g))/(9*b^3) - 6*a*f + 9*root(729*a^2*b^8*z^3 - 729*a^2*b^6
*f*z^2 + 54*a*b^5*d*e*z + 540*a^3*b^3*g*h*z - 216*a^2*b^4*e*g*z - 135*a^2*b
^4*d*h*z + 243*a^2*b^4*f^2*z - 180*a^3*b*f*g*h - 18*a*b^3*d*e*f + 72*a^2*b^
2*e*f*g + 45*a^2*b^2*d*f*h + 150*a^3*b*e*h^2 + 12*a*b^3*d^2*g - 60*a^2*b^2*
e^2*h - 48*a^2*b^2*d*g^2 - 27*a^2*b^2*f^3 + 64*a^3*b*g^3 + 8*a*b^3*e^3 - 12
5*a^4*h^3 - b^4*d^3, z, k)*a*b^2) + (x*(4*b^2*e^2 + 25*a^2*h^2 - 3*b^2*d*f
- 20*a*b*e*h + 12*a*b*f*g))/(9*b^3))*root(729*a^2*b^8*z^3 - 729*a^2*b^6*f*z
^2 + 54*a*b^5*d*e*z + 540*a^3*b^3*g*h*z - 216*a^2*b^4*e*g*z - 135*a^2*b^4*d
*h*z + 243*a^2*b^4*f^2*z - 180*a^3*b*f*g*h - 18*a*b^3*d*e*f + 72*a^2*b^2*e*
f*g + 45*a^2*b^2*d*f*h + 150*a^3*b*e*h^2 + 12*a*b^3*d^2*g - 60*a^2*b^2*e^2*
h - 48*a^2*b^2*d*g^2 - 27*a^2*b^2*f^3 + 64*a^3*b*g^3 + 8*a*b^3*e^3 - 125*a^
4*h^3 - b^4*d^3, z, k), k, 1, 3) - ((b*c)/3 - (a*f)/3 + x*((b*d)/3 - (a*g)/
3) + x^2*((b*e)/3 - (a*h)/3))/(a*b^2 + b^3*x^3) + (h*x^2)/(2*b^2) + (g*x)/b
^2
```

3.415 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2} dx$

Optimal. Leaf size=289

$$\frac{hx}{b^2}-\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)}-\frac{(b^{5/3}c+a^{2/3}be+2ab^{2/3}f-4a^{5/3}h)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{7/3}}$$

```
[Out] h*x/b^2-1/3*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/a/b^2/(b*x^3+a
)-1/9*(b^(2/3)*(2*a*f+b*c)-a^(2/3)*(-4*a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(4
/3)/b^(7/3)+1/18*(b^(2/3)*(2*a*f+b*c)-a^(2/3)*(-4*a*h+b*e))*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(7/3)+1/3*g*ln(b*x^3+a)/b^2-1/9*(b^(5/
3)*c+a^(2/3)*b*e+2*a*b^(2/3)*f-4*a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x
)/a^(1/3)*3^(1/2))/a^(4/3)/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1842, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^{2/3}be-4a^{5/3}h+2ab^{2/3}f+b^{5/3}c)}{3\sqrt{3}a^{4/3}b^{7/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(b^{2/3}(2af+bc)-a^{2/3}(be-4ah))}{18a^{4/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(b^{2/3}(2af+bc)-a^{2/3}(be-4ah))}{9a^{4/3}b^{7/3}}-\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{3ab^2(a+bx^3)}+\frac{g\log(a+bx^3)}{3b^2}+\frac{hx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (h*x)/b^2 - (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*a*
b^2*(a + b*x^3)) - ((b^(5/3)*c + a^(2/3)*b*e + 2*a*b^(2/3)*f - 4*a^(5/3)*h)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(7
/3)) - ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h))*Log[a^(1/3) + b^(1/
3)*x])/(9*a^(4/3)*b^(7/3)) + ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h
))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(7/3)) + (
g*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
```

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} - \frac{\int \frac{-a(be-ah)-b \cdots}{\cdots}}{\cdots}$$

$$= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} - \frac{\int \left(-3ah - \cdots\right.}{\cdots}$$

$$= \frac{hx}{b^2} - \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} + \frac{\int \frac{a(be-4a \cdots}{\cdots}}{\cdots}$$

$$= \frac{hx}{b^2} - \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} + \frac{\int \frac{a(be-4a \cdots}{\cdots}}{\cdots}$$

$$= \frac{hx}{b^2} - \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} + \frac{g\log(a \cdots}{3b^{2}\cdots}$$

$$= \frac{hx}{b^2} - \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} - \frac{\left(b^{2/3}(bc \cdots\right.}{\cdots}$$

$$= \frac{hx}{b^2} - \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} - \frac{\left(b^{2/3}(bc \cdots\right.}{\cdots}$$

$$= \frac{hx}{b^2} - \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} - \frac{\left(b^{5/3}c + \cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.13, size = 285, normalized size = 0.99

$$\frac{18b^{2/3}hx + \frac{6b^{2/3}(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{a(a+bx^3)} - \frac{2\sqrt{3}\left(b^2c+a^{2/3}b^{4/3}e+2abf-4a^{5/3}\sqrt[3]{b}\,h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} - \frac{2\left(b^2c-a^{2/3}b^{4/3}e+2abf+4a^{5/3}\sqrt[3]{b}\,h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{4/3}} + \frac{\left(b^2c-a^{2/3}b^{4/3}e+2abf+4a^{5/3}\sqrt[3]{b}\,h\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{4/3}} + 6b^{2/3}g\log(a+bx^3)}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(2/3)*h*x + (6*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f
*x))))/(a*(a + b*x^3)) - (2*Sqrt[3]*(b^2*c + a^(2/3)*b^(4/3)*e + 2*a*b*f -
4*a^(5/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4/3) -
 (2*(b^2*c - a^(2/3)*b^(4/3)*e + 2*a*b*f + 4*a^(5/3)*b^(1/3)*h)*Log[a^(1/3)
 + b^(1/3)*x])/a^(4/3) + ((b^2*c - a^(2/3)*b^(4/3)*e + 2*a*b*f + 4*a^(5/3)*
b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) + 6*b^(2
/3)*g*Log[a + b*x^3])/(18*b^(8/3))
```

Maple [A]
time = 0.40, size = 285, normalized size = 0.99

method	result
risch	$\frac{hx}{b^{2}}+\frac{-\frac{b(af-bc)x^{2}}{3a}+\left(\frac{ah}{3}-\frac{be}{3}\right)x+\frac{ag}{3}-\frac{bd}{3}}{b^{2}(bx^{3}+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(3bg\_R^{2}+\frac{b(2af+bc)\_R}{a}-4ah+be\right)\ln\left(x-\_R\right)}{\_R^{2}}}{9b^{3}}$
default	$\frac{hx}{b^{2}}-\frac{\frac{b(af-bc)x^{2}}{3a}+\left(-\frac{ah}{3}+\frac{be}{3}\right)x-\frac{ag}{3}+\frac{bd}{3}}{bx^{3}+a}+\frac{\left(4a^{2}h-abe\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] h*x/b^2-1/b^2*((1/3*b*(a*f-b*c)/a*x^2+(-1/3*a*h+1/3*b*e)*x-1/3*a*g+1/3*b*d)
/(b*x^3+a)+1/3/a*((4*a^2*h-a*b*e)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/
b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(-2*a*b*f-b^2*c)*(-1/3/b/(a/b)^(1/3
)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3
*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-a*g*ln(b*x^
3+a))
```

Maxima [A]
time = 0.53, size = 316, normalized size = 1.09

$$-\frac{abd-a^{2}g-(b^{2}c-abf)x^{2}-(a^{2}h-abe)x}{3\left(ab^{3}x^{3}+a^{2}b^{2}\right)}+\frac{hx}{b^{2}}+\frac{\sqrt{3}\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}+ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{2}b^{2}}+\frac{\left(6\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+4\,a^{2}h-abe\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(3\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^{2}h+abe\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxim
a")
```

```
[Out] -1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 - (a^2*h - a*b*e)*x)/(a*b^3*x^3 +
 a^2*b^2) + h*x/b^2 + 1/9*sqrt(3)*(b^2*c*(a/b)^(2/3) + 2*a*b*f*(a/b)^(2/3)
- 4*a^2*h*(a/b)^(1/3) + a*b*(a/b)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(a^2*b^2) + 1/18*(6*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/
3) + 2*a*b*f*(a/b)^(1/3) + 4*a^2*h - a*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)
^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/9*(3*a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3)
- 2*a*b*f*(a/b)^(1/3) - 4*a^2*h + a*b*e)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^
(2/3))
```

Fricas [**C**] Result contains complex when optimal does not.
time = 1.91, size = 12617, normalized size = 43.66

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="frica
s")
```

```
[Out] 1/36*(36*a*b*h*x^4 - 12*a*b*d + 12*a^2*g + 12*(b^2*c - a*b*f)*x^2 - 2*(a*b^
3*x^3 + a^2*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9
*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*
e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2
*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^
2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 6
4*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*
e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*
f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*
b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*
e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(
9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*
h)*a^3*b^2 - (e^3 - 3*(4* ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 318, normalized size = 1.10

$$\frac{hx}{b^2}+\frac{g\log\left(|bx^3+a|\right)}{3\,b^2}+\frac{\sqrt{3}\left(4\,a^2h-abe+\left(-ab^2\right)^{\frac{1}{3}}bc+2\left(-ab^2\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab}+\frac{\left(4\,a^2h-abe-\left(-ab^2\right)^{\frac{1}{3}}bc-2\left(-ab^2\right)^{\frac{1}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}ab}-\frac{abd-a^2g-(b^2c-abf)x^2-(a^2h-abe)x}{3\,(bx^3+a)ab^2}-\frac{\left(ab^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2b^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^3b^3h+a^2b^4e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac"
)
```

```
[Out] h*x/b^2 + 1/3*g*log(abs(b*x^3 + a))/b^2 + 1/9*sqrt(3)*(4*a^2*h - a*b*e + (-
a*b^2)^(1/3)*b*c + 2*(-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) + 1/18*(4*a^2*h - a*b*e - (-a*b^2)
^(1/3)*b*c - 2*(-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/((-a*b^2)^(2/3)*a*b) - 1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 - (a^2*h -
 a*b*e)*x)/((b*x^3 + a)*a*b^2) - 1/9*(a*b^5*c*(-a/b)^(1/3) + 2*a^2*b^4*f*(-
a/b)^(1/3) - 4*a^3*b^3*h + a^2*b^4*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)
))/(a^3*b^5)
```

Mupad [B]
time = 5.39, size = 827, normalized size = 2.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((9*a^2*g^2 + b^2*c*e - 8*a^2*f*h - 4*a*b*c*h + 2*a*b*e*f)/(9*a*b
^2) - root(729*a^4*b^7*z^3 - 729*a^4*b^5*g*z^2 - 216*a^4*b^3*f*h*z - 108*a^
3*b^4*c*h*z + 54*a^3*b^4*e*f*z + 27*a^2*b^5*c*e*z + 243*a^4*b^3*g^2*z + 72*
a^4*b*f*g*h + 36*a^3*b^2*c*g*h - 18*a^3*b^2*e*f*g - 9*a^2*b^3*c*e*g - 48*a^
4*b*e*h^2 + 6*a*b^4*c^2*f + 12*a^3*b^2*e^2*h + 12*a^2*b^3*c*f^2 + 8*a^3*b^2
*f^3 - 27*a^4*b*g^3 + 64*a^5*h^3 + b^5*c^3 - a^2*b^3*e^3, z, k)*(6*a*g - b*
e*x + 4*a*h*x - 9*root(729*a^4*b^7*z^3 - 729*a^4*b^5*g*z^2 - 216*a^4*b^3*f*
h*z - 108*a^3*b^4*c*h*z + 54*a^3*b^4*e*f*z + 27*a^2*b^5*c*e*z + 243*a^4*b^3
*g^2*z + 72*a^4*b*f*g*h + 36*a^3*b^2*c*g*h - 18*a^3*b^2*e*f*g - 9*a^2*b^3*c
*e*g - 48*a^4*b*e*h^2 + 6*a*b^4*c^2*f + 12*a^3*b^2*e^2*h + 12*a^2*b^3*c*f^2
 + 8*a^3*b^2*f^3 - 27*a^4*b*g^3 + 64*a^5*h^3 + b^5*c^3 - a^2*b^3*e^3, z, k)
*a*b^2) + (x*(b^3*c^2 + 4*a^2*b*f^2 + 12*a^3*g*h + 4*a*b^2*c*f - 3*a^2*b*e*
g))/(9*a^2*b^2))*root(729*a^4*b^7*z^3 - 729*a^4*b^5*g*z^2 - 216*a^4*b^3*f*h
*z - 108*a^3*b^4*c*h*z + 54*a^3*b^4*e*f*z + 27*a^2*b^5*c*e*z + 243*a^4*b^3*
g^2*z + 72*a^4*b*f*g*h + 36*a^3*b^2*c*g*h - 18*a^3*b^2*e*f*g - 9*a^2*b^3*c*
e*g - 48*a^4*b*e*h^2 + 6*a*b^4*c^2*f + 12*a^3*b^2*e^2*h + 12*a^2*b^3*c*f^2
+ 8*a^3*b^2*f^3 - 27*a^4*b*g^3 + 64*a^5*h^3 + b^5*c^3 - a^2*b^3*e^3, z, k),
 k, 1, 3) - ((b*d)/3 - (a*g)/3 + x*((b*e)/3 - (a*h)/3) - (b*x^2*(b*c - a*f)
)/(3*a))/(a*b^2 + b^3*x^3) + (h*x)/b^2
```

3.416 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=276

$$\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab\left(a+bx^3\right)}-\frac{\left(2b^{4/3}c+\sqrt[3]{a}\,bd+a\sqrt[3]{b}\,f+2a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{5/3}}+\frac{\left(\sqrt[3]{b}\right.}{}$$

```
[Out] 1/3*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/a/b/(b*x^3+a)+1/9*(b^(1/3)*(a*f
+2*b*c)-a^(1/3)*(2*a*g+b*d))*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(5/3)-1/18*(b^
(1/3)*(a*f+2*b*c)-a^(1/3)*(2*a*g+b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/a^(5/3)/b^(5/3)+1/3*h*ln(b*x^3+a)/b^2-1/9*(2*b^(4/3)*c+a^(1/3)*b*d+a*
b^(1/3)*f+2*a^(4/3)*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^
(5/3)/b^(5/3)*3^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1872, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(2a^{4/3}g+\sqrt[3]{a}\,bd+a\sqrt[3]{b}\,f+2b^{4/3}c\right)}{3\sqrt{3}\,a^{5/3}b^{5/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(\sqrt[3]{b}\,(af+2bc)-\sqrt[3]{a}\,(2ag+bd)\right)}{18a^{5/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(af+2bc)-\sqrt[3]{a}\,(2ag+bd)\right)}{9a^{5/3}b^{5/3}}+\frac{h\log\left(a+bx^3\right)}{3b^2}+\frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]
```

```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*a*b*(a + b*x^3)) - ((2
*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)*(2*b*
c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(5
/3)) - ((b^(1/3)*(2*b*c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(5/3)) + (h*Log[a + b*x^3])/(3*b
^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx = \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab\,(a+bx^3)} - \frac{\int \frac{-b(2bc+af)-b(bd+2ag)x-3}{a+bx^3}}{3ab^2}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab\,(a+bx^3)} - \frac{\int \frac{-b(2bc+af)-b(bd+2ag)x}{a+bx^3}\,d}{3ab^2}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab\,(a+bx^3)} + \frac{h\log(a+bx^3)}{3b^2} - \frac{\int \frac{\sqrt[3]{a}}{}}{}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab\,(a+bx^3)} + \frac{\left(\sqrt[3]{b}\,(2bc+af)-\sqrt[3]{a}\right.}{}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab\,(a+bx^3)} + \frac{\left(\sqrt[3]{b}\,(2bc+af)-\sqrt[3]{a}\right.}{}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab\,(a+bx^3)} - \frac{\left(2b^{4/3}c+\sqrt[3]{a}\,bd+a\sqrt[3]{\right.}{}$$

Mathematica [**A**]
time = 0.11, size = 268, normalized size = 0.97

$$\frac{\frac{6(a^2h+b^2x(c+dx)-ab(e+x(f+gx)))}{a(a+bx^3)} - \frac{2\sqrt{3}\,\sqrt[3]{b}\,\left(2b^{4/3}c+\sqrt[3]{a}\,bd+a\sqrt[3]{b}\,f+2a^{4/3}g\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{2\sqrt[3]{b}\,\left(2b^{4/3}c-\sqrt[3]{a}\,bd+a\sqrt[3]{b}\,f-2a^{4/3}g\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}} + \frac{\sqrt[3]{b}\,\left(-2b^{4/3}c+\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+2a^{4/3}g\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}} + 6h\log(a+bx^3)}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]
```

```
[Out] ((6*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a*(a + b*x^3)) - (2
*Sqrt[3]*b^(1/3)*(2*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*b^(1/3)*(2*b^(4/3)*
c - a^(1/3)*b*d + a*b^(1/3)*f - 2*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(5
/3) + (b^(1/3)*(-2*b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f + 2*a^(4/3)*g)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + 6*h*Log[a + b*x^3])/
(18*b^2)
```

Maple [**A**]
time = 0.38, size = 283, normalized size = 1.03

method	result
risch	$\frac{-\frac{(ag-bd)x^2}{3ab}-\frac{(af-bc)x}{3ab}+\frac{ah-be}{3b^2}}{b\,x^3+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(3h\_R^2+\frac{(2ag+bd)\_R}{a}+\frac{af+2bc}{a}\right)\ln\left(x-\_R\right)}{\_R^2}}{9b^2}$
default	$\frac{-\frac{(ag-bd)x^2}{3ab}-\frac{(af-bc)x}{3ab}+\frac{ah-be}{3b^2}}{b\,x^3+a}+\frac{(af+2bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(2ag+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*(a*g-b*d)/a/b*x^2-1/3*(a*f-b*c)/a/b*x+1/3*(a*h-b*e)/b^2)/(b*x^3+a)+1/
3/b/a*((a*f+2*b*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1)))+(2*a*g+b*d)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+
1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/
3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+a*h*ln(b*x^3+a)/b)
```

Maxima [A]

time = 0.49, size = 291, normalized size = 1.05

$$\frac{a^2h+(b^2d-abg)x^2-abe+(b^2c-abf)x}{3\,(ab^3x^3+a^2b^2)}+\frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2}+\frac{\left(6\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,bc-af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(3\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,bc+af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/3*(a^2*h + (b^2*d - a*b*g)*x^2 - a*b*e + (b^2*c - a*b*f)*x)/(a*b^3*x^3 +
a^2*b^2) + 1/9*sqrt(3)*(b^2*d*(a/b)^(2/3) + 2*a*b*g*(a/b)^(2/3) + 2*b^2*c*(
a/b)^(1/3) + a*b*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(a^2*b^2) + 1/18*(6*a*h*(a/b)^(2/3) + b*d*(a/b)^(1/3) + 2*a*g*(a/b
)^(1/3) - 2*b*c - a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^
(2/3)) + 1/9*(3*a*h*(a/b)^(2/3) - b*d*(a/b)^(1/3) - 2*a*g*(a/b)^(1/3) + 2*b
*c + a*f)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.

time = 1.79, size = 12636, normalized size = 45.78

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas"
)
```

```
[Out] -1/36*(12*a*b*e - 12*a^2*h - 12*(b^2*d - a*b*g)*x^2 + 2*(a*b^3*x^3 + a^2*b^
2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g +
9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*
a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b
^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 1
2*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 -
 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g
 - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1
/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*
h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*
c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*
a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b +
(f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c
)*a^2*b^3 - (d^3 - 12*c^2 ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 302, normalized size = 1.09

$$\frac{h\log\left(|bx^3+a|\right)}{3\,b^2}-\frac{\sqrt{3}\left(2\,b^2c+abf-\left(-ab^2\right)^{\frac{1}{3}}bd-2\left(-ab^2\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab}-\frac{\left(2\,b^2c+abf+\left(-ab^2\right)^{\frac{1}{3}}bd+2\left(-ab^2\right)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}ab}+\frac{(bd-ag)x^2+(bc-af)x+\frac{a^2h-abe}{b}}{3\,(bx^3+a)ab}-\frac{\left(ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2b^2g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab^3c+a^2b^2f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*h*log(abs(b*x^3 + a))/b^2 - 1/9*sqrt(3)*(2*b^2*c + a*b*f - (-a*b^2)^(1/
3)*b*d - 2*(-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/18*(2*b^2*c + a*b*f + (-a*b^2)^(1/3)*b*d
 + 2*(-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)
^(2/3)*a*b) + 1/3*((b*d - a*g)*x^2 + (b*c - a*f)*x + (a^2*h - a*b*e)/b)/((b
*x^3 + a)*a*b) - 1/9*(a*b^3*d*(-a/b)^(1/3) + 2*a^2*b^2*g*(-a/b)^(1/3) + 2*a
*b^3*c + a^2*b^2*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3)
```

Mupad [B]
time = 5.54, size = 835, normalized size = 3.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((root(729*a^5*b^6*z^3 - 729*a^5*b^4*h*z^2 + 54*a^4*b^3*f*g*z + 1
08*a^3*b^4*c*g*z + 27*a^3*b^4*d*f*z + 54*a^2*b^5*c*d*z + 243*a^5*b^2*h^2*z
- 18*a^4*b*f*g*h - 36*a^3*b^2*c*g*h - 9*a^3*b^2*d*f*h - 18*a^2*b^3*c*d*h -
12*a*b^4*c^2*f + 12*a^3*b^2*d*g^2 + 6*a^2*b^3*d^2*g - 6*a^2*b^3*c*f^2 + 8*a
^4*b*g^3 + a*b^4*d^3 - 27*a^5*h^3 - 8*b^5*c^3 - a^3*b^2*f^3, z, k)*(9*root(
729*a^5*b^6*z^3 - 729*a^5*b^4*h*z^2 + 54*a^4*b^3*f*g*z + 108*a^3*b^4*c*g*z
+ 27*a^3*b^4*d*f*z + 54*a^2*b^5*c*d*z + 243*a^5*b^2*h^2*z - 18*a^4*b*f*g*h
- 36*a^3*b^2*c*g*h - 9*a^3*b^2*d*f*h - 18*a^2*b^3*c*d*h - 12*a*b^4*c^2*f +
12*a^3*b^2*d*g^2 + 6*a^2*b^3*d^2*g - 6*a^2*b^3*c*f^2 + 8*a^4*b*g^3 + a*b^4*
d^3 - 27*a^5*h^3 - 8*b^5*c^3 - a^3*b^2*f^3, z, k)*a^2*b^2 - 6*a^2*h + 2*b^2
*c*x + a*b*f*x))/a + (9*a^3*h^2 + 2*b^3*c*d + 4*a*b^2*c*g + a*b^2*d*f + 2*a
^2*b*f*g)/(9*a^2*b^2) + (x*(b^2*d^2 + 4*a^2*g^2 - 3*a^2*f*h - 6*a*b*c*h + 4
*a*b*d*g))/(9*a^2*b))*root(729*a^5*b^6*z^3 - 729*a^5*b^4*h*z^2 + 54*a^4*b^3
*f*g*z + 108*a^3*b^4*c*g*z + 27*a^3*b^4*d*f*z + 54*a^2*b^5*c*d*z + 243*a^5*
b^2*h^2*z - 18*a^4*b*f*g*h - 36*a^3*b^2*c*g*h - 9*a^3*b^2*d*f*h - 18*a^2*b^
3*c*d*h - 12*a*b^4*c^2*f + 12*a^3*b^2*d*g^2 + 6*a^2*b^3*d^2*g - 6*a^2*b^3*c
*f^2 + 8*a^4*b*g^3 + a*b^4*d^3 - 27*a^5*h^3 - 8*b^5*c^3 - a^3*b^2*f^3, z, k
), k, 1, 3) + ((x*(b*c - a*f))/(3*a*b) - (b*e - a*h)/(3*b^2) + (x^2*(b*d -
a*g))/(3*a*b))/(a + b*x^3)
```

3.417 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2}\,dx$

Optimal. Leaf size=289

$$\frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\,(a+bx^3)}-\frac{\left(2b^{4/3}d+\sqrt[3]{a}\,be+a\sqrt[3]{b}\,g+2a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{5/3}}+$$

```
[Out] 1/3*x*(a*(-a*g+b*d)+a*(-a*h+b*e)*x-b*(-a*f+b*c)*x^2)/a^2/b/(b*x^3+a)+c*ln(x
)/a^2+1/9*(b^(1/3)*(a*g+2*b*d)-a^(1/3)*(2*a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a
^(5/3)/b^(5/3)-1/18*(b^(1/3)*(a*g+2*b*d)-a^(1/3)*(2*a*h+b*e))*ln(a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(5/3)-1/3*c*ln(b*x^3+a)/a^2-1/9*(2*b
^(4/3)*d+a^(1/3)*b*e+a*b^(1/3)*g+2*a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)
*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 287, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(2a^{4/3}h+\sqrt[3]{a}\,be+a\sqrt[3]{b}\,g+2b^{4/3}d\right)}{3\sqrt{3}\,a^{5/3}b^{5/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}\,(2ah+be)}{\sqrt[3]{b}}+ag+2bd\right)}{18a^{5/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(ag+2bd)-\sqrt[3]{a}\,(2ah+be)\right)}{9a^{5/3}b^{5/3}}+\frac{x(-bx^2(bc-af)+a(bd-ag)+ax(be-ah))}{3a^2b\,(a+bx^3)}-\frac{c\log\left(a+bx^3\right)}{3a^2}+\frac{c\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x]
```

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(3*a^2*b*(a + b*x
^3)) - ((2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h)*ArcTan[(a^(
1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + (c*Lo
g[x])/a^2 + ((b^(1/3)*(2*b*d + a*g) - a^(1/3)*(b*e + 2*a*h))*Log[a^(1/3) +
b^(1/3)*x])/(9*a^(5/3)*b^(5/3)) - ((2*b*d + a*g - (a^(1/3)*(b*e + 2*a*h))/b
^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3)
) - (c*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
```

```
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^2}\,dx = \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\left(a+bx^3\right)} - \frac{\int \frac{-3b^2c-b(2bd+ag)x-}{x(a+bx^3}}{3ab^2}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^2c}{ax}+\frac{b(-a(2}{}\right.}{}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} - \frac{\int \frac{-a(2b}{}}{}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} - \frac{\int \frac{-a(2b}{}}{}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} - \frac{c\log(a}{3}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\left(2bd-\right.}{}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\left(2bd-\right.}{}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\left(a+bx^3\right)} - \frac{\left(2b^{4/3}d+\sqrt[3]{a}\,be\right.}{}$$

Mathematica [A]

time = 0.13, size = 269, normalized size = 0.93

$$\frac{-\frac{6a(-b(c+x(d+ex))+a(f+x(g+hx)))}{b(a+bx^3)} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2b^{4/3}d+\sqrt[3]{a}\,be+a\sqrt[3]{b}\,g+2a^{4/3}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} + 18c\log(x) + \frac{2\sqrt[3]{a}\left(2b^{4/3}d-\sqrt[3]{a}\,be+a\sqrt[3]{b}\,g-2a^{4/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{5/3}} + \frac{\sqrt[3]{a}\left(-2b^{4/3}d+\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g+2a^{4/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{5/3}} - 6c\log\left(a+bx^3\right)}{18a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*(-(b*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(b*(a + b*x^3)) - (2
*Sqrt[3]*a^(1/3)*(2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) + 18*c*Log[x] + (2*a^(1/
3)*(2*b^(4/3)*d - a^(1/3)*b*e + a*b^(1/3)*g - 2*a^(4/3)*h)*Log[a^(1/3) + b^
(1/3)*x])/b^(5/3) + (a^(1/3)*(-2*b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g + 2*
a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - 6*c*Lo
g[a + b*x^3])/(18*a^2)
```

Maple [A]

time = 0.43, size = 293, normalized size = 1.01

method	result
default	$\frac{\frac{-\frac{a(ah-be)x^2}{3b}-\frac{a(ag-bd)x}{3b}-\frac{a(af-bc)}{3b}}{b\,x^3+a}+\frac{\left(a^2g+2abd\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\left(2a\right.}{}}{a^2}$
risch	$\frac{-\frac{(ah-be)x^2}{3ab}-\frac{(ag-bd)x}{3ab}-\frac{af-bc}{3ab}}{b\,x^3+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^6b^5\_Z^3+9a^4b^5c\_Z^2+\left(6a^5b^2gh+12a^4b^3dh+3a^4b^3eg+6a^3b^4de+27a^2b^5c^2\right)\_Z+8a^5h^3+\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*((-1/3*a*(a*h-b*e)/b*x^2-1/3*a*(a*g-b*d)/b*x-1/3*a*(a*f-b*c)/b)/(b*x^
3+a)+1/3/b*((a^2*g+2*a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b
)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(2*a^2*h+a*b*e)*(-1/3/b/(a/b)^(1/3)*ln(x+
(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2
)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-b*c*ln(b*x^3+a))+
c*ln(x)/a^2
```

Maxima [A]

time = 0.50, size = 307, normalized size = 1.06

$$-\frac{(ah-be)x^2-bc+af-(bd-ag)x}{3\left(ab^2x^3+a^2b\right)}+\frac{c\log\left(x\right)}{a^2}+\frac{\sqrt{3}\left(2\,a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab\left(\frac{a}{b}\right)^{\frac{2}{3}}e+2\,abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b}-\frac{\left(6\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e+2\,abd+a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e-2\,abd-a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxim
a")
```

```
[Out] -1/3*((a*h - b*e)*x^2 - b*c + a*f - (b*d - a*g)*x)/(a*b^2*x^3 + a^2*b) + c*
log(x)/a^2 + 1/9*sqrt(3)*(2*a^2*h*(a/b)^(2/3) + a*b*(a/b)^(2/3)*e + 2*a*b*d
*(a/b)^(1/3) + a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a
/b)^(1/3))/(a^3*b) - 1/18*(6*b^2*c*(a/b)^(2/3) - 2*a^2*h*(a/b)^(1/3) - a*b*
(a/b)^(1/3)*e + 2*a*b*d + a^2*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^
2*b^2*(a/b)^(2/3)) - 1/9*(3*b^2*c*(a/b)^(2/3) + 2*a^2*h*(a/b)^(1/3) + a*b*(
a/b)^(1/3)*e - 2*a*b*d - a^2*g)*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

Fricas [**C**] Result contains complex when optimal does not.
time = 20.80, size = 12541, normalized size = 43.39

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="frica
s")
```

```
[Out] 1/324*(108*a*b*c - 108*a^2*f + 108*(a*b*e - a^2*h)*x^2 - 2*(a^2*b^2*x^3 + a
^3*b)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h +
 (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^
2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 +
a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h
+ 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 -
(g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^
2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1
/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e +
2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^
3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3
*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 1
2*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*
(e*g + 4*d*h)*c)*a^2*b^3  ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 319, normalized size = 1.10

$$-\frac{c\log\left(|bx^3+a|\right)}{3\,a^2}+\frac{c\log\left(|x|\right)}{a^2}-\frac{\sqrt{3}\left(2\,b^2d+abg-2\left(-ab^2\right)^{\frac{1}{3}}ah-\left(-ab^2\right)^{\frac{1}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab}-\frac{\left(2\,b^2d+abg+2\left(-ab^2\right)^{\frac{1}{3}}ah+\left(-ab^2\right)^{\frac{1}{3}}be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}ab}+\frac{abc-a^2f-(a^2h-abe)x^2+(abd-a^2g)x}{3\,(bx^3+a)a^2b}-\frac{\left(2\,a^4b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+2\,a^3b^3d+a^4b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac"
)
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a^2 + c*log(abs(x))/a^2 - 1/9*sqrt(3)*(2*b^2*d +
 a*b*g - 2*(-a*b^2)^(1/3)*a*h - (-a*b^2)^(1/3)*b*e)*arctan(1/3*sqrt(3)*(2*x
 + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/18*(2*b^2*d + a*b*g
 + 2*(-a*b^2)^(1/3)*a*h + (-a*b^2)^(1/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) + 1/3*(a*b*c - a^2*f - (a^2*h - a*b*e)*x^2
 + (a*b*d - a^2*g)*x)/((b*x^3 + a)*a^2*b) - 1/9*(2*a^4*b^2*h*(-a/b)^(1/3) +
 a^3*b^3*(-a/b)^(1/3)*e + 2*a^3*b^3*d + a^4*b^2*g)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a^5*b^3)
```

Mupad [B]
time = 5.60, size = 1660, normalized size = 5.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x)
```

```
[Out] ((b*c - a*f)/(3*a*b) + (x*(b*d - a*g))/(3*a*b) + (x^2*(b*e - a*h))/(3*a*b))
/(a + b*x^3) + symsum(log((c*(4*b^2*d^2 + a^2*g^2 - 3*b^2*c*e - 6*a*b*c*h +
 4*a*b*d*g))/(9*a^3) - (root(729*a^6*b^5*z^3 + 729*a^4*b^5*c*z^2 + 54*a^5*b
^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3*b^4*d*e*z + 243*a^
2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*
b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2
*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z,
 k)*(a^3*g^2 + 4*a*b^2*d^2 + 36*b^3*c^2*x + 324*root(729*a^6*b^5*z^3 + 729*
a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 5
4*a^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 3
6*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a
^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b
^5*c^3 + a^2*b^3*e^3, z, k)^2*a^4*b^3*x - 18*root(729*a^6*b^5*z^3 + 729*a^4
*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a
^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a
^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*
b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*
c^3 + a^2*b^3*e^3, z, k)*a^4*b*h + 6*a*b^2*c*e + 12*a^2*b*c*h + 4*a^2*b*d*g
 + 20*a^3*g*h*x - 9*root(729*a^6*b^5*z^3 + 729*a^4*b^5*c*z^2 + 54*a^5*b^2*g
*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3*b^4*d*e*z + 243*a^2*b^
5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*b^3*
c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2*d*g
^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z, k)*
a^3*b^2*e + 216*root(729*a^6*b^5*z^3 + 729*a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z
```

```
 + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3*b^4*d*e*z + 243*a^2*b^5*c^
2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*
g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 -
 a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z, k)*a^2*
b^3*c*x + 20*a*b^2*d*e*x + 40*a^2*b*d*h*x + 10*a^2*b*e*g*x))/(9*a^2) - (x*(
8*a^4*h^3 - 8*b^4*d^3 + a*b^3*e^3 - a^3*b*g^3 - 6*a^2*b^2*d*g^2 + 6*a^2*b^2
*e^2*h + 12*b^4*c*d*e - 12*a*b^3*d^2*g + 12*a^3*b*e*h^2 + 12*a^2*b^2*c*g*h
+ 24*a*b^3*c*d*h + 6*a*b^3*c*e*g))/(27*a^3*b^2))*root(729*a^6*b^5*z^3 + 729
*a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z +
54*a^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h +
36*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*
a^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*
b^5*c^3 + a^2*b^3*e^3, z, k), k, 1, 3) + (c*log(x))/a^2
```

3.418 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2} dx$

Optimal. Leaf size=301

$$-\frac{c}{a^2x}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\,(a+bx^3)}+\frac{(4b^{5/3}c-2a^{2/3}be-ab^{2/3}f-a^{5/3}h)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right.}{3\sqrt{3}\,a^{7/3}b^{4/3}}$$

```
[Out] -c/a^2/x+1/3*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/a^2/b/(b*x^3+
a)+d*ln(x)/a^2+1/9*(b^(2/3)*(-a*f+4*b*c)+a^(2/3)*(a*h+2*b*e))*ln(a^(1/3)+b^
(1/3)*x)/a^(7/3)/b^(4/3)-1/18*(b^(2/3)*(-a*f+4*b*c)+a^(2/3)*(a*h+2*b*e))*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(4/3)-1/3*d*ln(b*x^3+a)/a
^2+1/9*(4*b^(5/3)*c-2*a^(2/3)*b*e-a*b^(2/3)*f-a^(5/3)*h)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-2a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+4b^{5/3}c)}{3\sqrt{3}\,a^{7/3}b^{4/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^{2/3}(ah+2be)+b^{2/3}(4bc-af))}{18a^{7/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^{2/3}(ah+2be)+b^{2/3}(4bc-af))}{9a^{7/3}b^{4/3}}+\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{3a^2b\,(a+bx^3)}-\frac{d\log(a+bx^3)}{3a^2}-\frac{c}{a^2x}+\frac{d\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -(c/(a^2*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3
*a^2*b*(a + b*x^3)) + ((4*b^(5/3)*c - 2*a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)
*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b
^(4/3)) + (d*Log[x])/a^2 + ((b^(2/3)*(4*b*c - a*f) + a^(2/3)*(2*b*e + a*h))
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(4/3)) - ((b^(2/3)*(4*b*c - a*f) +
a^(2/3)*(2*b*e + a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*
a^(7/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
```

```
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^2}\,dx = \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\left(a+bx^3\right)} - \frac{\int \frac{-3b^2c-3b^2dx-b(2be+\cdots}{x^2(a+\cdots}}{3a\cdots}$$

$$= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^2c}{ax^2}-\frac{3b^2d}{ax}+\cdots\right.}{\cdots}$$

$$= -\frac{c}{a^2x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} - \cdots$$

$$= -\frac{c}{a^2x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} - \cdots$$

$$= -\frac{c}{a^2x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} - \cdots$$

$$= -\frac{c}{a^2x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \cdots$$

$$= -\frac{c}{a^2x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \cdots$$

$$= -\frac{c}{a^2x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\left(a+bx^3\right)} + \frac{\left(4b^{5/3}c-2\cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.20, size = 285, normalized size = 0.95

$$-\frac{\frac{18ac}{x}+\frac{6a\left(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx))\right)}{b\left(a+bx^3\right)}+\frac{2\sqrt{3}\,a^{2/3}\left(-4b^{5/3}c+2a^{2/3}be+ab^{2/3}f+a^{5/3}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}}-18ad\log(x)-\frac{2a^{2/3}\left(4b^{5/3}c+2a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{4/3}}+\frac{a^{2/3}\left(4b^{5/3}c+2a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{4/3}}+6ad\log\left(a+bx^3\right)}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -1/18*((18*a*c)/x + (6*a*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x))
))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(2/3)*(-4*b^(5/3)*c + 2*a^(2/3)*b*e + a*b
^(2/3)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(4/3)
- 18*a*d*Log[x] - (2*a^(2/3)*(4*b^(5/3)*c + 2*a^(2/3)*b*e - a*b^(2/3)*f + a
^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(2/3)*(4*b^(5/3)*c + 2*a^(
2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/b^(4/3) + 6*a*d*Log[a + b*x^3])/a^3
```

Maple [A]
time = 0.43, size = 298, normalized size = 0.99

method	result
default	$\frac{\left(\frac{af}{3}-\frac{bc}{3}\right)x^2-\frac{a(ah-be)x}{3b}-\frac{a(ag-bd)}{3b}}{b\,x^3+a}+\frac{\left(a^2h+2abe\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\left(a\right.}{a^2}$
risch	$\frac{\frac{(af-4bc)x^3}{3a^2}-\frac{(ah-be)x^2}{3ab}-\frac{(ag-bd)x}{3ab}-\frac{c}{a}}{x(b\,x^3+a)}+\frac{d\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^7b^4\_Z^3+9a^5b^4d\_Z^2+\left(3a^5b^2fh-12a^4b^3ch+6a^4b^3ef-24a^3b^4ce+\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/a^2*(((1/3*a*f-1/3*b*c)*x^2-1/3*a*(a*h-b*e)/b*x-1/3*a*(a*g-b*d)/b)/(b*x^3
+a)+1/3/b*((a^2*h+2*a*b*e)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)
^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(a*b*f-4*b^2*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(
a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)
/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-b*d*ln(b*x^3+a)))-c
/a^2/x+d*ln(x)/a^2
```

Maxima [A]
time = 0.50, size = 332, normalized size = 1.10

$$-\frac{(4\,b^2c-abf)x^3+3\,abc+(a^2h-abe)x^2-(abd-a^2g)x}{3\,(a^2b^2x^4+a^3bx)}+\frac{d\log(x)}{a^2}-\frac{\sqrt{3}\left(4\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b}-\frac{\left(6\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h+2\,abe\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h-2\,abe\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] -1/3*((4*b^2*c - a*b*f)*x^3 + 3*a*b*c + (a^2*h - a*b*e)*x^2 - (a*b*d - a^2*
g)*x)/(a^2*b^2*x^4 + a^3*b*x) + d*log(x)/a^2 - 1/9*sqrt(3)*(4*b^2*c*(a/b)^(
2/3) - a*b*f*(a/b)^(2/3) - a^2*h*(a/b)^(1/3) - 2*a*b*(a/b)^(1/3)*e)*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b) - 1/18*(6*b^2*d*(a/b)^
(2/3) + 4*b^2*c*(a/b)^(1/3) - a*b*f*(a/b)^(1/3) + a^2*h + 2*a*b*e)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) - 1/9*(3*b^2*d*(a/b)^(
2/3) - 4*b^2*c*(a/b)^(1/3) + a*b*f*(a/b)^(1/3) - a^2*h - 2*a*b*e)*log(x + (
a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 21.85, size = 12556, normalized size = 41.71

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] -1/324*(108*(4*b^2*c - a*b*f)*x^3 + 324*a*b*c - 108*(a*b*e - a^2*h)*x^2 + 2
*(a^2*b^2*x^4 + a^3*b*x)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^2*f*h + 2*(e*f -
 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^6 + 1/162*(a^2*f
*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^
5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 1
2*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 +
 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 -
9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^
4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(a^2*f*h +
2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3
 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3
*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^
4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e
*f + 6*(f^2 + 3*d*h)*c)*a ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 328, normalized size = 1.09

$$-\frac{d\log\left(|bx^3+a|\right)}{3\,a^2}+\frac{d\log\left(|x|\right)}{a^2}-\frac{\sqrt{3}\left(a^2h+2\,abe+4\,(-ab^2)^{\frac{1}{3}}\,bc-(-ab^2)^{\frac{1}{3}}\,af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+(-\frac{a}{b})^{\frac{1}{3}}\right)}{3\,(-\frac{a}{b})^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a^2}-\frac{\left(a^2h+2\,abe-4\,(-ab^2)^{\frac{1}{3}}\,bc+(-ab^2)^{\frac{1}{3}}\,af\right)\log\left(x^2+x(-\frac{a}{b})^{\frac{1}{3}}+(-\frac{a}{b})^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a^2}-\frac{4\,b^2cx^3-abfx^3+a^2hx^2-abx^2e-abdx+a^2gx+3\,abc}{3\,(bx^4+ax)a^2b}+\frac{\left(4\,a^2b^4c(-\frac{a}{b})^{\frac{1}{3}}-a^3b^3f(-\frac{a}{b})^{\frac{1}{3}}-a^4b^2h-2\,a^3b^3e\right)(-\frac{a}{b})^{\frac{1}{3}}\log\left(\left|x-(-\frac{a}{b})^{\frac{1}{3}}\right|\right)}{9\,a^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^2 + d*log(abs(x))/a^2 - 1/9*sqrt(3)*(a^2*h + 2
*a*b*e + 4*(-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x
 + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/18*(a^2*h + 2*a*b*e
 - 4*(-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/3*(4*b^2*c*x^3 - a*b*f*x^3 + a^2*h*x^2
 - a*b*x^2*e - a*b*d*x + a^2*g*x + 3*a*b*c)/((b*x^4 + a*x)*a^2*b) + 1/9*(4*
a^2*b^4*c*(-a/b)^(1/3) - a^3*b^3*f*(-a/b)^(1/3) - a^4*b^2*h - 2*a^3*b^3*e)*
(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b^3)
```

Mupad [B]
time = 5.77, size = 1684, normalized size = 5.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] symsum(log((d*(a^3*h^2 + 4*a*b^2*e^2 + 12*b^3*c*d - 3*a*b^2*d*f + 4*a^2*b*e
*h))/(9*a^4) - (root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f*h*z
 - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b^4*d
^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3*d*e
*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*f^2 -
 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z, k)*(
a^3*h^2 + 4*a*b^2*e^2 + 36*b^3*d^2*x - 24*b^3*c*d + 324*root(729*a^7*b^4*z^
3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e
*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d
*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f
 - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*f^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5
*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z, k)^2*a^4*b^3*x + 6*a*b^2*d*f + 4*a^2*b*
e*h - 80*b^3*c*e*x + 36*root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b
^2*f*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a
^3*b^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2
*b^3*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3
*c*f^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3,
 z, k)*a^2*b^3*c - 9*root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*
f*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*
```

```
b^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^
3*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*
f^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z,
 k)*a^3*b^2*f + 216*root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f
*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b
^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3
*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*f
^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z,
k)*a^2*b^3*d*x - 40*a*b^2*c*h*x + 20*a*b^2*e*f*x + 10*a^2*b*f*h*x))/(9*a^2)
 + (x*(64*b^5*c^3 + a^5*h^3 + 8*a^2*b^3*e^3 - a^3*b^2*f^3 + 12*a^2*b^3*c*f^
2 + 12*a^3*b^2*e^2*h - 48*a*b^4*c^2*f + 6*a^4*b*e*h^2 + 24*a^2*b^3*c*d*h -
12*a^2*b^3*d*e*f - 6*a^3*b^2*d*f*h + 48*a*b^4*c*d*e))/(27*a^5*b))*root(729*
a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f*h*z - 108*a^4*b^3*c*h*z + 54
*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b^4*d^2*z - 72*a*b^4*c*d*e + 9
*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3*d*e*f - 6*a^4*b*e*h^2 + 48*a
*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*f^2 - 8*a^2*b^3*e^3 + 27*a*b^4
*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z, k), k, 1, 3) - (c/a + (x^3*(4
*b*c - a*f))/(3*a^2) - (x*(b*d - a*g))/(3*a*b) - (x^2*(b*e - a*h))/(3*a*b))
/(a*x + b*x^4) + (d*log(x))/a^2
```

3.419 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2}\,dx$

Optimal. Leaf size=306

$$-\frac{c}{2a^2x^2}-\frac{d}{a^2x}-\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2\,(a+bx^3)}+\frac{\left(5b^{4/3}c+4\sqrt[3]{a}\,bd-2a\sqrt[3]{b}\,f-a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}}{\sqrt{}}\right.}{3\sqrt{3}\,a^{8/3}b^{2/3}}$$

```
[Out] -1/2*c/a^2/x^2-d/a^2/x-1/3*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/a^2/(b*x
^3+a)+e*ln(x)/a^2-1/9*(b^(1/3)*(-2*a*f+5*b*c)-a^(1/3)*(-a*g+4*b*d))*ln(a^(1
/3)+b^(1/3)*x)/a^(8/3)/b^(2/3)+1/18*(b^(1/3)*(-2*a*f+5*b*c)-a^(1/3)*(-a*g+4
*b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(2/3)-1/3*e*ln(b
*x^3+a)/a^2+1/9*(5*b^(4/3)*c+4*a^(1/3)*b*d-2*a*b^(1/3)*f-a^(4/3)*g)*arctan(
1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.38, antiderivative size = 304, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+4\sqrt[3]{a}\,bd-2a\sqrt[3]{b}\,f+5b^{4/3}c\right)}{3\sqrt{3}\,a^{8/3}b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}\,(4bd-ag)}{\sqrt[3]{b}}-2af+5bc\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(5bc-2af)-\sqrt[3]{a}\,(4bd-ag)\right)}{9a^{8/3}b^{2/3}}-\frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{3a^2\,(a+bx^3)}-\frac{e\log\left(a+bx^3\right)}{3a^2}-\frac{c}{2a^2x^2}-\frac{d}{a^2x}+\frac{e\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/2*c/(a^2*x^2) - d/(a^2*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*
x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*c + 4*a^(1/3)*b*d - 2*a*b^(1/3)*f -
 a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a
^(8/3)*b^(2/3)) + (e*Log[x])/a^2 - ((b^(1/3)*(5*b*c - 2*a*f) - a^(1/3)*(4*b
*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b^(2/3)) + ((5*b*c - 2*a*f
- (a^(1/3)*(4*b*d - a*g))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*a^(8/3)*b^(1/3)) - (e*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
```

```
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3b^2c-3b^2dx-3b^2ex^2+2\cdots}{x^3(a\cdots}}{3\cdots}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^2c}{ax^3}-\frac{3b^2d}{ax^2}-\frac{3b^2\cdots}{a\cdots}\right.}{\cdots}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2\left(a+bx^3\right)} + \frac{e\log\cdots}{a^2\cdots}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2\left(a+bx^3\right)} + \frac{e\log\cdots}{a^2\cdots}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2\left(a+bx^3\right)} + \frac{e\log\cdots}{a^2\cdots}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2\left(a+bx^3\right)} + \frac{e\log\cdots}{a^2\cdots}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2\left(a+bx^3\right)} + \frac{e\log\cdots}{a^2\cdots}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2\left(a+bx^3\right)} + \frac{\left(5b^{4/\cdots}\right.}{\cdots}$$

Mathematica [A]

time = 0.32, size = 292, normalized size = 0.95

$$-\frac{\frac{9ac}{x^2}+\frac{18ad}{x}+\frac{6a\left(a^2h+b^2x(c+dx)-ab(e+x(f+gx))\right)}{b(a+bx^3)}+\frac{2\sqrt{3}\,\sqrt[3]{a}\left(-5b^{4/3}c-4\sqrt[3]{a}\,bd+2a\sqrt[3]{b}\,f+a^{4/3}g\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}-18ae\log(x)+\frac{2\sqrt[3]{a}\left(5b^{4/3}c-4\sqrt[3]{a}\,bd-2a\sqrt[3]{b}\,f+a^{4/3}g\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}-\frac{\sqrt[3]{a}\left(5b^{4/3}c-4\sqrt[3]{a}\,bd-2a\sqrt[3]{b}\,f+a^{4/3}g\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}+6ae\log\left(a+bx^3\right)}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/18*((9*a*c)/x^2 + (18*a*d)/x + (6*a*(a^2*h + b^2*x*(c + d*x) - a*b*(e + 
x*(f + g*x))))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(1/3)*(-5*b^(4/3)*c - 4*a^(1/
3)*b*d + 2*a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]])/b^(2/3) - 18*a*e*Log[x] + (2*a^(1/3)*(5*b^(4/3)*c - 4*a^(1/3)*b*d - 2
*a*b^(1/3)*f + a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1/3)*(5*b
^(4/3)*c - 4*a^(1/3)*b*d - 2*a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 6*a*e*Log[a + b*x^3])/a^3
```

Maple [A]

time = 0.42, size = 293, normalized size = 0.96

method	result
default	$\frac{\frac{\left(\frac{ag}{3}-\frac{bd}{3}\right)x^{2}+\left(\frac{af}{3}-\frac{bc}{3}\right)x-\frac{a(ah-be)}{3b}}{b\,x^{3}+a}+\frac{(2af-5bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}+\frac{(ag-4\cdots}{\cdots}}{a^{2}}$
risch	$\frac{\frac{(ag-4bd)x^{4}}{3a^{2}}+\frac{(2af-5bc)x^{3}}{6a^{2}}-\frac{(ah-be)x^{2}}{3ab}-\frac{xd}{a}-\frac{c}{2a}}{x^{2}(b\,x^{3}+a)}+\frac{e\ln(-x)}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{8}b^{2}\_Z^{3}+9a^{6}b^{2}e\_Z^{2}+\left(6a^{5}bfg-15a^{4}b^{2}cg-24a^{4}b^{2}df+27\cdots\right.\right.}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/a^2*(((1/3*a*g-1/3*b*d)*x^2+(1/3*a*f-1/3*b*c)*x-1/3*a*(a*h-b*e)/b)/(b*x^3
+a)+1/3*(2*a*f-5*b*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(a*g-4*b*d)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^
(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a
/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-1/3*e*ln(b*x^3+a))-1/2*c
/a^2/x^2-d/a^2/x+e*ln(x)/a^2
```

Maxima [A]

time = 0.50, size = 320, normalized size = 1.05

$$-\frac{2\,(4\,b^{2}d-abg)x^{4}+6\,abdx+(5\,b^{2}c-2\,abf)x^{3}+3\,abc+2\,(a^{2}h-abe)x^{2}}{6\,(a^{2}b^{2}x^{5}+a^{3}bx^{2})}+\frac{e\log(x)}{a^{2}}-\frac{\sqrt{3}\left(4\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-ag\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,af\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}}-\frac{\left(6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}e+4\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,bc+2\,af\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}e-4\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,bc-2\,af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] -1/6*(2*(4*b^2*d - a*b*g)*x^4 + 6*a*b*d*x + (5*b^2*c - 2*a*b*f)*x^3 + 3*a*b
*c + 2*(a^2*h - a*b*e)*x^2)/(a^2*b^2*x^5 + a^3*b*x^2) + e*log(x)/a^2 - 1/9*
sqrt(3)*(4*b*d*(a/b)^(2/3) - a*g*(a/b)^(2/3) + 5*b*c*(a/b)^(1/3) - 2*a*f*(a
/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^3 - 1/18*(
6*b*(a/b)^(2/3)*e + 4*b*d*(a/b)^(1/3) - a*g*(a/b)^(1/3) - 5*b*c + 2*a*f)*lo
g(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/9*(3*b*(a/b)^(
2/3)*e - 4*b*d*(a/b)^(1/3) + a*g*(a/b)^(1/3) + 5*b*c - 2*a*f)*log(x + (a/b)
^(1/3))/(a^2*b*(a/b)^(2/3))
```

Fricas [**C**] Result contains complex when optimal does not.
time = 15.87, size = 12231, normalized size = 39.97

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] -1/324*(108*(4*b^2*d - a*b*g)*x^4 + 324*a*b*d*x + 54*(5*b^2*c - 2*a*b*f)*x^
3 + 162*a*b*c - 108*(a*b*e - a^2*h)*x^2 + 2*(a^2*b^2*x^5 + a^3*b*x^2)*((-I*
sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)
*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*
d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^
3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^
2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g
 + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a
^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*s
qrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f
 - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c
^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 -
 a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g +
6*d*g^2)*a^3*b + 3*(9*e^3 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 336, normalized size = 1.10

$$-\frac{e\log\left(|bx^3+a|\right)}{3\,a^2}+\frac{e\log\left(|x|\right)}{a^2}+\frac{\sqrt{3}\left(5\,b^2c-2\,abf-4\left(-ab^2\right)^{\frac{1}{3}}bd+\left(-ab^2\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a^2}+\frac{\left(5\,b^2c-2\,abf+4\left(-ab^2\right)^{\frac{1}{3}}bd-\left(-ab^2\right)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a^2}+\frac{\left(4\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b^2c-2\,a^3bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b}-\frac{2\left(4\,b^2d-abg\right)x^4+6\,abdx+\left(5\,b^2c-2\,abf\right)x^3+3\,abc+2\left(a^2h-abe\right)x^2}{6\left(bx^3+a\right)a^2bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^2 + e*log(abs(x))/a^2 + 1/9*sqrt(3)*(5*b^2*c -
 2*a*b*f - 4*(-a*b^2)^(1/3)*b*d + (-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) + 1/18*(5*b^2*c - 2*a
*b*f + 4*(-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) + 1/9*(4*a^2*b^2*d*(-a/b)^(1/3) - a^3*
b*g*(-a/b)^(1/3) + 5*a^2*b^2*c - 2*a^3*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^5*b) - 1/6*(2*(4*b^2*d - a*b*g)*x^4 + 6*a*b*d*x + (5*b^2*c - 2*
a*b*f)*x^3 + 3*a*b*c + 2*(a^2*h - a*b*e)*x^2)/((b*x^3 + a)*a^2*b*x^2)
```

Mupad [B]
time = 5.71, size = 1632, normalized size = 5.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] symsum(log((b^2*e*(25*b^2*c^2 + 4*a^2*f^2 - 3*a^2*e*g - 20*a*b*c*f + 12*a*b
*d*e))/(9*a^5) - (root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z
 - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*
e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*
e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^
2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z,
 k)*b^2*(25*b^2*c^2 + 4*a^2*f^2 - 9*root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^
2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*
d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e
*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g
 + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4
*c^3 + a^4*g^3, z, k)*a^4*g + 6*a^2*e*g + 36*root(729*a^8*b^2*z^3 + 729*a^6
*b^2*e*z^2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a
^3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^
2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*
b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3
+ 125*b^4*c^3 + a^4*g^3, z, k)*a^3*b*d + 36*a*b*e^2*x + 200*b^2*c*d*x + 20*
a^2*f*g*x + 324*root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z -
 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*e^
```

```
2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*e*
g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^2
+ 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z, k
)^2*a^5*b*x - 20*a*b*c*f - 24*a*b*d*e - 50*a*b*c*g*x - 80*a*b*d*f*x + 216*r
oot(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*
z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*
f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^
2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3
- 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z, k)*a^3*b*e*x))/(9*
a^3) - (b*x*(125*b^4*c^3 + a^4*g^3 - 64*a*b^3*d^3 - 8*a^3*b*f^3 + 60*a^2*b^
2*c*f^2 + 48*a^2*b^2*d^2*g - 150*a*b^3*c^2*f - 12*a^3*b*d*g^2 - 30*a^2*b^2*
c*e*g - 48*a^2*b^2*d*e*f + 120*a*b^3*c*d*e + 12*a^3*b*e*f*g))/(27*a^6))*roo
t(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z
- 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*
g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2
- 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 -
8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z, k), k, 1, 3) - (c/(2
*a) + (x^3*(5*b*c - 2*a*f))/(6*a^2) + (x^4*(4*b*d - a*g))/(3*a^2) + (d*x)/a
 - (x^2*(b*e - a*h))/(3*a*b))/(a*x^2 + b*x^5) + (e*log(x))/a^2
```

3.420 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx$

Optimal. Leaf size=338

$$-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)}+\frac{\left(5b^{4/3}d+4\sqrt[3]{a}\,be-2a\sqrt[3]{b}\,g-a^{4/3}h\right)\tan}{3\sqrt{3}\,a^{8/3}b^{2/3}}$$

```
[Out] -1/3*c/a^2/x^3-1/2*d/a^2/x^2-e/a^2/x-1/3*x*(b*d-a*g+(-a*h+b*e)*x-b*(b*c/a-f
)*x^2)/a^2/(b*x^3+a)-(-a*f+2*b*c)*ln(x)/a^3-1/9*(b^(1/3)*(-2*a*g+5*b*d)-a^(
1/3)*(-a*h+4*b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(2/3)+1/18*(b^(1/3)*(-2*
a*g+5*b*d)-a^(1/3)*(-a*h+4*b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
a^(8/3)/b^(2/3)+1/3*(-a*f+2*b*c)*ln(b*x^3+a)/a^3+1/9*(5*b^(4/3)*d+4*a^(1/3)
*b*e-2*a*b^(1/3)*g-a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1
/2))/a^(8/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 336, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+4\sqrt[3]{a}\,be-2a\sqrt[3]{b}\,g+5b^{4/3}d\right)}{3\sqrt{3}\,a^{8/3}b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}\,(4be-ah)}{\sqrt[3]{b}}-2ag+5bd\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(5bd-2ag)-\sqrt[3]{a}\,(4be-ah)\right)}{9a^{8/3}b^{2/3}}+\frac{(2bc-af)\log\left(a+bx^3\right)}{3a^3}-\frac{\log(x)(2bc-af)}{a^3}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{3a^2\left(a+bx^3\right)}-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d - a*g + (b*e - a*h)*
x - b*((b*c)/a - f)*x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*d + 4*a^(1/3)*b
*e - 2*a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(3*Sqrt[3]*a^(8/3)*b^(2/3)) - ((2*b*c - a*f)*Log[x])/a^3 - ((b^(1/3
)*(5*b*d - 2*a*g) - a^(1/3)*(4*b*e - a*h))*Log[a^(1/3) + b^(1/3)*x]])/(9*a^(
8/3)*b^(2/3)) + ((5*b*d - 2*a*g - (a^(1/3)*(4*b*e - a*h))/b^(1/3))*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]])/(18*a^(8/3)*b^(1/3)) + ((2*b*c - a*
f)*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
```

```
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2}\,dx = -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3b^2c-3b^2dx-3b^2ex^2+3b\ldots}{\ldots}}{\ldots}$$

$$= -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^2c}{ax^4}-\frac{3b^2d}{ax^3}-\frac{3b^2\ldots}{ax^2\ldots}\right.}{\ldots}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)}\ldots$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)}\ldots$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)}\ldots$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)}\ldots$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)}\ldots$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)}\ldots$$

Mathematica [A]
time = 0.30, size = 303, normalized size = 0.90

$$\frac{-\frac{6ac}{x^3}-\frac{9ad}{x^2}-\frac{18ae}{x}+\frac{a(-6b(c+x(d+ex))+6a(f+x(g+hx)))}{a+bx^3}-\frac{2\sqrt{3}\sqrt[3]{a}\left(-5b^{4/3}d-4\sqrt[3]{a}be+2a\sqrt[3]{b}g+a^{4/3}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+18(-2bc+af)\log(x)-\frac{2\sqrt[3]{a}\left(5b^{4/3}d-4\sqrt[3]{a}be-2a\sqrt[3]{b}g+a^{4/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{\sqrt[3]{a}\left(5b^{4/3}d-4\sqrt[3]{a}be-2a\sqrt[3]{b}g+a^{4/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+6(2bc-af)\log\left(a+bx^3\right)}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x + (a*(-6*b*(c + x*(d + e*x)) + 6*a
*(f + x*(g + h*x))))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(-5*b^(4/3)*d - 4*a^(
1/3)*b*e + 2*a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(2/3) + 18*(-2*b*c + a*f)*Log[x] - (2*a^(1/3)*(5*b^(4/3)*d - 4*a^
(1/3)*b*e - 2*a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) +
(a^(1/3)*(5*b^(4/3)*d - 4*a^(1/3)*b*e - 2*a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 6*(2*b*c - a*f)*Log[a + b
*x^3])/(18*a^3)
```

Maple [A]
time = 0.58, size = 332, normalized size = 0.98

method	result
default	$\frac{\frac{\left(\frac{1}{3}a^{2}h-\frac{1}{3}abe\right)x^{2}+\left(\frac{1}{3}a^{2}g-\frac{1}{3}abd\right)x+\frac{a(af-bc)}{3}}{b\,x^{3}+a}+\frac{\left(2a^{2}g-5abd\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1 \right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{3}}{a^{3}}$
risch	$\frac{\frac{(ah-4be)x^{5}}{3a^{2}}+\frac{(2ag-5bd)x^{4}}{6a^{2}}+\frac{(af-2bc)x^{3}}{3a^{2}}-\frac{e\,x^{2}}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^{3}(b\,x^{3}+a)}+\left(\_R=\operatorname{RootOf}\left(a^{9}b^{2}\_Z^{3}+\left(9a^{7}b^{2}f-18a^{6}b^{3}c\right)\_Z^{2}+\left(6a^{6}bgh-15a^{5}b^{2}dh-24a^{5}\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/a^3*((((1/3*a^2*h-1/3*a*b*e)*x^2+(1/3*a^2*g-1/3*a*b*d)*x+1/3*a*(a*f-b*c))/
(b*x^3+a)+1/3*(2*a^2*g-5*a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/
(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(a^2*h-4*a*b*e)*(-1/3/b/(a/b)^(1/
3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/
3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/9*(-3*a*
b*f+6*b^2*c)*ln(b*x^3+a)/b)-e/a^2/x-1/3*c/a^2/x^3-1/2*d/a^2/x^2+(a*f-2*b*c)
/a^3*ln(x)
```

Maxima [A]
time = 0.49, size = 369, normalized size = 1.09

$$\frac{2\,(ah-4\,be)x^{5}-(5\,bd-2\,ag)x^{4}-2\,(2\,bc-af)x^{3}-6\,ax^{2}e-3\,adx-2\,ac}{6\,(a^{2}bx^{6}+a^{3}x^{3})}-\frac{(2\,bc-af)\log(x)}{a^{3}}+\frac{\sqrt{3}\left(a^{2}h\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}e-5\,abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^{2}g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{4}}+\frac{\left(12\,b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-6\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e+5\,abd-2\,a^{2}g\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{3}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(6\,b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-3\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}+4\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e-5\,abd+2\,a^{2}g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{3}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] 1/6*(2*(a*h - 4*b*e)*x^5 - (5*b*d - 2*a*g)*x^4 - 2*(2*b*c - a*f)*x^3 - 6*a*
x^2*e - 3*a*d*x - 2*a*c)/(a^2*b*x^6 + a^3*x^3) - (2*b*c - a*f)*log(x)/a^3 +
 1/9*sqrt(3)*(a^2*h*(a/b)^(2/3) - 4*a*b*(a/b)^(2/3)*e - 5*a*b*d*(a/b)^(1/3)
 + 2*a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/a^4 + 1/18*(12*b^2*c*(a/b)^(2/3) - 6*a*b*f*(a/b)^(2/3) + a^2*h*(a/b)^(1/3)
 - 4*a*b*(a/b)^(1/3)*e + 5*a*b*d - 2*a^2*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)
^(2/3))/(a^3*b*(a/b)^(2/3)) + 1/9*(6*b^2*c*(a/b)^(2/3) - 3*a*b*f*(a/b)^(2/3
) - a^2*h*(a/b)^(1/3) + 4*a*b*(a/b)^(1/3)*e - 5*a*b*d + 2*a^2*g)*log(x + (a
/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 67.33, size = 16568, normalized size = 49.02

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] -1/36*(12*(4*a*b*e - a^2*h)*x^5 + 36*a^2*e*x^2 + 6*(5*a*b*d - 2*a^2*g)*x^4
+ 18*a^2*d*x + 12*(2*a*b*c - a^2*f)*x^3 + 12*a^2*c + 2*(a^3*b*x^6 + a^4*x^3
)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*(2*b*c - a*f)^2/a^6 - (36*b^3*c^2 + 2*
a^3*g*h + (9*f^2 - 8*e*g - 5*d*h)*a^2*b + 4*(5*d*e - 9*c*f)*a*b^2)/(a^6*b))
/(54*(2*b*c - a*f)^3/a^9 - 9*(36*b^3*c^2 + 2*a^3*g*h + (9*f^2 - 8*e*g - 5*d
*h)*a^2*b + 4*(5*d*e - 9*c*f)*a*b^2)*(2*b*c - a*f)/(a^9*b) - (125*b^4*d^3 +
 64*a*b^3*e^3 - 150*a*b^3*d^2*g + 60*a^2*b^2*d*g^2 - 8*a^3*b*g^3 - 48*a^2*b
^2*e^2*h + 12*a^3*b*e*h^2 - a^4*h^3)/(a^8*b^2) + (216*b^5*c^3 - a^5*h^3 + 2
*(4*g^3 - 9*f*g*h + 6*e*h^2)*a^4*b - 3*(9*f^3 - 24*e*f*g + 16*e^2*h - 12*c*
g*h + 5*(4*g^2 - 3*f*h)*d)*a^3*b^2 + 2*(32*e^3 - 90*d*e*f + 75*d^2*g + 9*(9
*f^2 - 8*e*g - 5*d*h)*c)*a^2*b^3 - (125*d^3 - 360*c*d*e + 324*c^2*f)*a*b^4)
/(a^9*b^2))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*(2*b*c - a*f)^3/a^9 - 9
*(36*b^3*c^2 + 2*a^3*g*h + (9*f^2 - 8*e*g - 5*d*h)*a^2*b + 4*(5*d*e - 9*c*f
)*a*b^2)*(2*b*c - a*f)/(a ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
```

[Out] Timed out

Giac [A]
time = 0.49, size = 363, normalized size = 1.07

$$\frac{\sqrt{3}\left(5\,b^2d-2\,abg+\left(-ab^2\right)^{\frac{1}{3}}ah-4\left(-ab^2\right)^{\frac{1}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a^2}+\frac{\left(5\,b^2d-2\,abg-\left(-ab^2\right)^{\frac{1}{3}}ah+4\left(-ab^2\right)^{\frac{1}{3}}be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a^2}+\frac{(2\,bc-af)\log\left(|bx^3+a|\right)}{3\,a^3}-\frac{(2\,bc-af)\log\left(|x|\right)}{a^3}-\frac{\left(a^5bh\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^4b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e-5\,a^4b^2d+2\,a^5bg\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^7b}+\frac{2\left(a^2h-4\,abe\right)x^5-\left(5\,abd-2\,a^2g\right)x^4-6\,a^2x^2e-3\,a^2dx-2\left(2\,abc-a^2f\right)x^3-2\,a^2c}{6\left(bx^3+a\right)a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] 1/9*sqrt(3)*(5*b^2*d - 2*a*b*g + (-a*b^2)^(1/3)*a*h - 4*(-a*b^2)^(1/3)*b*e)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2)
 + 1/18*(5*b^2*d - 2*a*b*g - (-a*b^2)^(1/3)*a*h + 4*(-a*b^2)^(1/3)*b*e)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) + 1/3*(2*b*c - a
*f)*log(abs(b*x^3 + a))/a^3 - (2*b*c - a*f)*log(abs(x))/a^3 - 1/9*(a^5*b*h*
(-a/b)^(1/3) - 4*a^4*b^2*(-a/b)^(1/3)*e - 5*a^4*b^2*d + 2*a^5*b*g)*(-a/b)^(
1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b) + 1/6*(2*(a^2*h - 4*a*b*e)*x^5 - (5
*a*b*d - 2*a^2*g)*x^4 - 6*a^2*x^2*e - 3*a^2*d*x - 2*(2*a*b*c - a^2*f)*x^3 -
 2*a^2*c)/((b*x^3 + a)*a^3*x^3)
```

Mupad [B]
time = 5.96, size = 1924, normalized size = 5.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] symsum(log(- (50*b^5*c*d^2 - 48*b^5*c^2*e + 8*a^2*b^3*c*g^2 - 12*a^2*b^3*e*
f^2 - 4*a^3*b^2*f*g^2 + 3*a^3*b^2*f^2*h - 25*a*b^4*d^2*f + 12*a*b^4*c^2*h -
 12*a^2*b^3*c*f*h + 20*a^2*b^3*d*f*g - 40*a*b^4*c*d*g + 48*a*b^4*c*e*f)/(9*
a^6) - root(729*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1458*a^6*b^3*c*z^2 + 54*a
^6*b*g*h*z - 216*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z - 972*a^4*b^3*c*f*z + 54
0*a^4*b^3*d*e*z + 243*a^5*b^2*f^2*z + 972*a^3*b^4*c^2*z + 18*a^4*b*f*g*h -
360*a*b^4*c*d*e - 72*a^3*b^2*e*f*g - 45*a^3*b^2*d*f*h - 36*a^3*b^2*c*g*h +
180*a^2*b^3*d*e*f + 144*a^2*b^3*c*e*g + 90*a^2*b^3*c*d*h - 12*a^4*b*e*h^2 +
 324*a*b^4*c^2*f + 48*a^3*b^2*e^2*h - 150*a^2*b^3*d^2*g + 60*a^3*b^2*d*g^2
- 162*a^2*b^3*c*f^2 + 27*a^3*b^2*f^3 - 64*a^2*b^3*e^3 - 8*a^4*b*g^3 + 125*a
*b^4*d^3 - 216*b^5*c^3 + a^5*h^3, z, k)*((25*a^3*b^4*d^2 + 4*a^5*b^2*g^2 +
48*a^3*b^4*c*e - 12*a^4*b^3*c*h - 20*a^4*b^3*d*g - 24*a^4*b^3*e*f + 6*a^5*b
^2*f*h)/(9*a^6) + root(729*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1458*a^6*b^3*c
*z^2 + 54*a^6*b*g*h*z - 216*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z - 972*a^4*b^3
*c*f*z + 540*a^4*b^3*d*e*z + 243*a^5*b^2*f^2*z + 972*a^3*b^4*c^2*z + 18*a^4
*b*f*g*h - 360*a*b^4*c*d*e - 72*a^3*b^2*e*f*g - 45*a^3*b^2*d*f*h - 36*a^3*b
^2*c*g*h + 180*a^2*b^3*d*e*f + 144*a^2*b^3*c*e*g + 90*a^2*b^3*c*d*h - 12*a^
```

4*b*e*h^2 + 324*a*b^4*c^2*f + 48*a^3*b^2*e^2*h - 150*a^2*b^3*d^2*g + 60*a^3
*b^2*d*g^2 - 162*a^2*b^3*c*f^2 + 27*a^3*b^2*f^3 - 64*a^2*b^3*e^3 - 8*a^4*b*
g^3 + 125*a*b^4*d^3 - 216*b^5*c^3 + a^5*h^3, z, k)*((36*a^6*b^3*e - 9*a^7*b
^2*h)/(9*a^6) - (x*(1296*a^5*b^4*c - 648*a^6*b^3*f))/(27*a^6) + 36*root(729
*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1458*a^6*b^3*c*z^2 + 54*a^6*b*g*h*z - 21
6*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z - 972*a^4*b^3*c*f*z + 540*a^4*b^3*d*e*z
 + 243*a^5*b^2*f^2*z + 972*a^3*b^4*c^2*z + 18*a^4*b*f*g*h - 360*a*b^4*c*d*e
 - 72*a^3*b^2*e*f*g - 45*a^3*b^2*d*f*h - 36*a^3*b^2*c*g*h + 180*a^2*b^3*d*e
*f + 144*a^2*b^3*c*e*g + 90*a^2*b^3*c*d*h - 12*a^4*b*e*h^2 + 324*a*b^4*c^2*
f + 48*a^3*b^2*e^2*h - 150*a^2*b^3*d^2*g + 60*a^3*b^2*d*g^2 - 162*a^2*b^3*c
*f^2 + 27*a^3*b^2*f^3 - 64*a^2*b^3*e^3 - 8*a^4*b*g^3 + 125*a*b^4*d^3 - 216*
b^5*c^3 + a^5*h^3, z, k)*a^2*b^3*x) + (x*(432*a^2*b^5*c^2 + 108*a^4*b^3*f^2
 - 432*a^3*b^4*c*f + 600*a^3*b^4*d*e - 150*a^4*b^3*d*h - 240*a^4*b^3*e*g +
60*a^5*b^2*g*h))/(27*a^6)) - (x*(125*b^5*d^3 - 64*a*b^4*e^3 + a^4*b*h^3 - 8
*a^3*b^2*g^3 + 60*a^2*b^3*d*g^2 + 48*a^2*b^3*e^2*h - 12*a^3*b^2*e*h^2 - 240
*b^5*c*d*e - 150*a*b^4*d^2*g - 24*a^2*b^3*c*g*h - 30*a^2*b^3*d*f*h - 48*a^2
*b^3*e*f*g + 12*a^3*b^2*f*g*h + 60*a*b^4*c*d*h + 96*a*b^4*c*e*g + 120*a*b^4
*d*e*f))/(27*a^6))*root(729*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1458*a^6*b^3*
c*z^2 + 54*a^6*b*g*h*z - 216*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z - 972*a^4*b^
3*c*f*z + 540*a^4*b^3*d*e*z + 243*a^5*b^2*f^2*z + 972*a^3*b^4*c^2*z + 18*a^
4*b*f*g*h - 360*a*b^4*c*d*e - 72*a^3*b^2*e*f*g - 45*a^3*b^2*d*f*h - 36*a^3*
b^2*c*g*h + 180*a^2*b^3*d*e*f + 144*a^2*b^3*c*e*g + 90*a^2*b^3*c*d*h - 12*a
^4*b*e*h^2 + 324*a*b^4*c^2*f + 48*a^3*b^2*e^2*h - 150*a^2*b^3*d^2*g + 60*a^
3*b^2*d*g^2 - 162*a^2*b^3*c*f^2 + 27*a^3*b^2*f^3 - 64*a^2*b^3*e^3 - 8*a^4*b
*g^3 + 125*a*b^4*d^3 - 216*b^5*c^3 + a^5*h^3, z, k), k, 1, 3) - (c/(3*a) +
(e*x^2)/a + (x^3*(2*b*c - a*f))/(3*a^2) + (x^4*(5*b*d - 2*a*g))/(6*a^2) + (
x^5*(4*b*e - a*h))/(3*a^2) + (d*x)/(2*a))/(a*x^3 + b*x^6) - (log(x)*(2*b*c
- a*f))/a^3

3.421 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=345

$$\frac{hx}{b^3}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3\left(a+bx^3\right)^2}-\frac{x(a(7be-13ah)-2b(bc-4af)x-3b(bd-3ag)x^2)}{18ab^3\left(a+bx^3\right)}-$$

```
[Out] h*x/b^3+1/6*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/b^3/(b*x^3+a)^
2-1/18*x*(a*(-13*a*h+7*b*e)-2*b*(-4*a*f+b*c)*x-3*b*(-3*a*g+b*d)*x^2)/a/b^3/
(b*x^3+a)-1/27*(b^(2/3)*(5*a*f+b*c)-2*a^(2/3)*(-7*a*h+b*e))*ln(a^(1/3)+b^(1
/3)*x)/a^(4/3)/b^(10/3)+1/54*(b^(2/3)*(5*a*f+b*c)-2*a^(2/3)*(-7*a*h+b*e))*l
n(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(10/3)+1/3*g*ln(b*x^3+a)
/b^3-1/27*(b^(5/3)*c+2*a^(2/3)*b*e+5*a*b^(2/3)*f-14*a^(5/3)*h)*arctan(1/3*(
a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {1842, 1872, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{2/3}be-14a^{5/3}h+5ab^{2/3}f+b^{5/3}c\right)}{9\sqrt{3}a^{4/3}b^{10/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{54a^{4/3}b^{10/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{27a^{4/3}b^{10/3}}-\frac{x(-2bx(bc-4af)-3bx^2(bd-3ag)+a(7be-13ah))}{18ab^3\left(a+bx^3\right)}+\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{6b^3\left(a+bx^3\right)^2}+\frac{g\log\left(a+bx^3\right)}{3b^3}+\frac{hx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] (h*x)/b^3 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*b^
3*(a + b*x^3)^2) - (x*(a*(7*b*e - 13*a*h) - 2*b*(b*c - 4*a*f)*x - 3*b*(b*d
- 3*a*g)*x^2))/(18*a*b^3*(a + b*x^3)) - ((b^(5/3)*c + 2*a^(2/3)*b*e + 5*a*b
^(2/3)*f - 14*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(9*Sqrt[3]*a^(4/3)*b^(10/3)) - ((b^(2/3)*(b*c + 5*a*f) - 2*a^(2/3)*(b*e -
7*a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(10/3)) + ((b^(2/3)*(b*c +
5*a*f) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(54*a^(4/3)*b^(10/3)) + (g*Log[a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \int \frac{a^2(be-ah)-2ab(b \ldots}{\ldots}$$

$$= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{x(a(7be-13ah \ldots}{\ldots}$$

$$= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{x(a(7be-13ah \ldots}{\ldots}$$

$$= \frac{hx}{b^3} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{x(a(7be \ldots}{\ldots}$$

$$= \frac{hx}{b^3} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{x(a(7be \ldots}{\ldots}$$

$$= \frac{hx}{b^3} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{x(a(7be \ldots}{\ldots}$$

$$= \frac{hx}{b^3} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{x(a(7be \ldots}{\ldots}$$

$$= \frac{hx}{b^3} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{x(a(7be \ldots}{\ldots}$$

$$= \frac{hx}{b^3} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{x(a(7be \ldots}{\ldots}$$

Mathematica [A]
time = 0.20, size = 342, normalized size = 0.99

$$\frac{54b^{2/3}hx - \frac{9b^{2/3}(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{(a+bx^3)^2} + \frac{3b^{2/3}(2b^2cx^2+a^2(12g+13hx)-ab(6d+x(7e+8fx)))}{a(a+bx^3)} - \frac{2\sqrt{3}\left(b^2c+2a^{2/3}b^{4/3}e+5abf-14a^{5/3}\sqrt[3]{b}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} - \frac{2\left(b^2c-2a^{2/3}b^{4/3}e+5abf+14a^{5/3}\sqrt[3]{b}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}} + \frac{\left(b^2c-2a^{2/3}b^{4/3}e+5abf+14a^{5/3}\sqrt[3]{b}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{4/3}} + 18b^{2/3}g\log\left(a+bx^3\right)}{54b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(2/3)*h*x - (9*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f
*x))))/(a + b*x^3)^2 + (3*b^(2/3)*(2*b^2*c*x^2 + a^2*(12*g + 13*h*x) - a*b*
(6*d + x*(7*e + 8*f*x))))/(a*(a + b*x^3)) - (2*Sqrt[3]*(b^2*c + 2*a^(2/3)*b
^(4/3)*e + 5*a*b*f - 14*a^(5/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
```

```
))/Sqrt[3]])/a^(4/3) - (2*(b^2*c - 2*a^(2/3)*b^(4/3)*e + 5*a*b*f + 14*a^(5/
3)*b^(1/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + ((b^2*c - 2*a^(2/3)*b^(4/
3)*e + 5*a*b*f + 14*a^(5/3)*b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/a^(4/3) + 18*b^(2/3)*g*Log[a + b*x^3])/(54*b^(11/3))
```

Maple [A]
time = 0.40, size = 339, normalized size = 0.98

method	result
risch	$\frac{hx}{b^3}+\frac{-\frac{b^2(4af-bc)x^5}{9a}+\left(\frac{13}{18}abh-\frac{7}{18}b^2e\right)x^4+\left(\frac{2}{3}abg-\frac{1}{3}b^2d\right)x^3-\frac{b(5af+bc)x^2}{18}+\frac{a(5ah-2be)x}{9}+\frac{a^2g}{2}-\frac{abd}{6}}{b^3\left(bx^3+a\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\left(9\right.}{}$
default	$\frac{hx}{b^3}-\frac{\frac{b^2(4af-bc)x^5}{9a}+\left(-\frac{13}{18}abh+\frac{7}{18}b^2e\right)x^4+\left(-\frac{2}{3}abg+\frac{1}{3}b^2d\right)x^3+\frac{b(5af+bc)x^2}{18}-\frac{a(5ah-2be)x}{9}-\frac{a^2g}{2}+\frac{abd}{6}}{\left(bx^3+a\right)^2}+\frac{\left(14a^2h-2abe\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] h*x/b^3-1/b^3*((1/9*b^2*(4*a*f-b*c)/a*x^5+(-13/18*a*b*h+7/18*b^2*e)*x^4+(-2
/3*a*b*g+1/3*b^2*d)*x^3+1/18*b*(5*a*f+b*c)*x^2-1/9*a*(5*a*h-2*b*e)*x-1/2*a^
2*g+1/6*a*b*d)/(b*x^3+a)^2+1/9/a*((14*a^2*h-2*a*b*e)*(1/3/b/(a/b)^(2/3)*ln(
x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(-5*a*b*f-b^2*c)
*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))-3*a*g*ln(b*x^3+a)))
```

Maxima [A]
time = 0.54, size = 395, normalized size = 1.14

$$\frac{2\left(b^3c-4ab^2f\right)x^5+\left(13a^2bh-7ab^2e\right)x^4-3a^2bd+9a^3g-6\left(ab^2d-2a^2bg\right)x^3-\left(ab^2c+5a^2bf\right)x^2+2\left(5a^3h-2a^2be\right)x}{18\left(ab^5x^6+2a^2b^4x^3+a^3b^3\right)}+\frac{hx}{b^3}+\frac{\sqrt{3}\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}+5abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-14a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+2ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^2b^3}+\frac{\left(18abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+5abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+14a^2h-2abe\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(9abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-5abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-14a^2h+2abe\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] 1/18*(2*(b^3*c - 4*a*b^2*f)*x^5 + (13*a^2*b*h - 7*a*b^2*e)*x^4 - 3*a^2*b*d
+ 9*a^3*g - 6*(a*b^2*d - 2*a^2*b*g)*x^3 - (a*b^2*c + 5*a^2*b*f)*x^2 + 2*(5*
a^3*h - 2*a^2*b*e)*x)/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3) + h*x/b^3 + 1/2
7*sqrt(3)*(b^2*c*(a/b)^(2/3) + 5*a*b*f*(a/b)^(2/3) - 14*a^2*h*(a/b)^(1/3) +
 2*a*b*(a/b)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(
a^2*b^3) + 1/54*(18*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) + 5*a*b*f*(a/b)^(
1/3) + 14*a^2*h - 2*a*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^4*(a
/b)^(2/3)) + 1/27*(9*a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3) - 5*a*b*f*(a/b)^
(1/3) - 14*a^2*h + 2*a*b*e)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3))
```

Fricas [**C**] Result contains complex when optimal does not.
time = 2.64, size = 12967, normalized size = 37.59

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] 1/108*(108*a*b^2*h*x^7 + 12*(b^3*c - 4*a*b^2*f)*x^5 - 42*(a*b^2*e - 7*a^2*b
*h)*x^4 - 18*a^2*b*d + 54*a^3*g - 36*(a*b^2*d - 2*a^2*b*g)*x^3 - 6*(a*b^2*c
 + 5*a^2*b*f)*x^2 - 2*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*(2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*
f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h
)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*
a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176
*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*
a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g
+ 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)
/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*
c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3
 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 16
8*a^3*b^2*e^2*h + 1176*a^ ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 385, normalized size = 1.12

$$\frac{hx}{b^3} + \frac{g\log\left(|bx^3+a|\right)}{3\,b^3} + \frac{\sqrt{3}\left(14\,a^2h - 2\,abe + (-ab^2)^{\frac{1}{3}}\,bc + 5\,(-ab^2)^{\frac{1}{3}}\,af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,ab^2} + \frac{\left(14\,a^2h - 2\,abe - (-ab^2)^{\frac{1}{3}}\,bc - 5\,(-ab^2)^{\frac{1}{3}}\,af\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,ab^2} + \frac{2\,(b^3c - 4\,ab^2f)x^5 + (13\,a^2bh - 7\,ab^2e)x^4 - 3\,a^2bd + 9\,a^3g - 6\,(ab^2d - 2\,a^2bg)x^3 - (ab^2c + 5\,a^2bf)x^2 + 2\,(5\,a^3h - 2\,a^2be)x}{18\,(bx^3+a)^2ab^3} - \frac{\left(ab^6c\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,a^2b^5f\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 14\,a^3b^4h + 2\,a^2b^5e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] h*x/b^3 + 1/3*g*log(abs(b*x^3 + a))/b^3 + 1/27*sqrt(3)*(14*a^2*h - 2*a*b*e
+ (-a*b^2)^(1/3)*b*c + 5*(-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) + 1/54*(14*a^2*h - 2*a*b*e -
 (-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) + 1/18*(2*(b^3*c - 4*a*b^2*f)*x^5 + (13*a^
2*b*h - 7*a*b^2*e)*x^4 - 3*a^2*b*d + 9*a^3*g - 6*(a*b^2*d - 2*a^2*b*g)*x^3
- (a*b^2*c + 5*a^2*b*f)*x^2 + 2*(5*a^3*h - 2*a^2*b*e)*x)/((b*x^3 + a)^2*a*b
^3) - 1/27*(a*b^6*c*(-a/b)^(1/3) + 5*a^2*b^5*f*(-a/b)^(1/3) - 14*a^3*b^4*h
+ 2*a^2*b^5*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^7)
```

Mupad [B]
time = 0.58, size = 916, normalized size = 2.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

```
[Out] symsum(log(root(19683*a^4*b^10*z^3 - 19683*a^4*b^7*g*z^2 - 5670*a^4*b^4*f*h
*z - 1134*a^3*b^5*c*h*z + 810*a^3*b^5*e*f*z + 162*a^2*b^6*c*e*z + 6561*a^4*
b^4*g^2*z + 1890*a^4*b*f*g*h + 378*a^3*b^2*c*g*h - 270*a^3*b^2*e*f*g - 54*a
^2*b^3*c*e*g - 1176*a^4*b*e*h^2 + 15*a*b^4*c^2*f + 168*a^3*b^2*e^2*h + 75*a
^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 8*a^2*b^3*e^3 - 729*a^4*b*g^3 + 2744*a^5*h
^3 + b^5*c^3, z, k)*(9*root(19683*a^4*b^10*z^3 - 19683*a^4*b^7*g*z^2 - 5670
*a^4*b^4*f*h*z - 1134*a^3*b^5*c*h*z + 810*a^3*b^5*e*f*z + 162*a^2*b^6*c*e*z
 + 6561*a^4*b^4*g^2*z + 1890*a^4*b*f*g*h + 378*a^3*b^2*c*g*h - 270*a^3*b^2*
e*f*g - 54*a^2*b^3*c*e*g - 1176*a^4*b*e*h^2 + 15*a*b^4*c^2*f + 168*a^3*b^2*
e^2*h + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 8*a^2*b^3*e^3 - 729*a^4*b*g^3
+ 2744*a^5*h^3 + b^5*c^3, z, k)*a*b^2 - (6*a*g)/b + (x*(54*a^2*b^4*e - 378*
a^3*b^3*h))/(81*a^2*b^4)) + (81*a^2*g^2 + 2*b^2*c*e - 70*a^2*f*h - 14*a*b*c
*h + 10*a*b*e*f)/(81*a*b^4) + (x*(b^3*c^2 + 25*a^2*b*f^2 + 126*a^3*g*h + 10
*a*b^2*c*f - 18*a^2*b*e*g))/(81*a^2*b^4))*root(19683*a^4*b^10*z^3 - 19683*a
^4*b^7*g*z^2 - 5670*a^4*b^4*f*h*z - 1134*a^3*b^5*c*h*z + 810*a^3*b^5*e*f*z
+ 162*a^2*b^6*c*e*z + 6561*a^4*b^4*g^2*z + 1890*a^4*b*f*g*h + 378*a^3*b^2*c
*g*h - 270*a^3*b^2*e*f*g - 54*a^2*b^3*c*e*g - 1176*a^4*b*e*h^2 + 15*a*b^4*c
^2*f + 168*a^3*b^2*e^2*h + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 8*a^2*b^3*e
^3 - 729*a^4*b*g^3 + 2744*a^5*h^3 + b^5*c^3, z, k), k, 1, 3) - (x^2*((b^2*c
)/18 + (5*a*b*f)/18) - (a^2*g)/2 - x*((5*a^2*h)/9 - (2*a*b*e)/9) + x^3*((b^
2*d)/3 - (2*a*b*g)/3) + (b*x^4*(7*b*e - 13*a*h))/18 + (a*b*d)/6 - (b*x^5*(b
^2*c - 4*a*b*f))/(9*a))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (h*x)/b^3
```

3.422 $\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=325

$$-\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2\left(a+bx^3\right)^2}+\frac{x(bc-7af+2(bd-4ag)x+3(be-3ah)x^2)}{18ab^2\left(a+bx^3\right)}-\frac{\left(b^{4/3}c+\sqrt[3]{a}\,bd+\right.}{}$$

```
[Out] -1/6*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/b^2/(b*x^3+a)^2+1/18*x*(b*c-7*
a*f+2*(-4*a*g+b*d)*x+3*(-3*a*h+b*e)*x^2)/a/b^2/(b*x^3+a)+1/27*(b^(1/3)*(2*a
*f+b*c)-a^(1/3)*(5*a*g+b*d))*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(8/3)-1/54*(b^
(1/3)*(2*a*f+b*c)-a^(1/3)*(5*a*g+b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/a^(5/3)/b^(8/3)+1/3*h*ln(b*x^3+a)/b^3-1/27*(b^(4/3)*c+a^(1/3)*b*d+2*a
*b^(1/3)*f+5*a^(4/3)*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(5/3)/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1842, 1872, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^{4/3}g+\sqrt[3]{a}\,bd+2a\sqrt[3]{b}\,f+b^{4/3}c\right)}{9\sqrt{3}\,a^{5/3}b^{8/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{27a^{5/3}b^{8/3}}+\frac{h\log\left(a+bx^3\right)}{3b^3}+\frac{x(2x(bd-4ag)+3x^2(be-3ah)-7af+bc)}{18ab^2\left(a+bx^3\right)}-\frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{6b^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(b^2*(a + b*x^3)^2)
+ (x*(b*c - 7*a*f + 2*(b*d - 4*a*g)*x + 3*(b*e - 3*a*h)*x^2))/(18*a*b^2*(a
+ b*x^3)) - ((b^(4/3)*c + a^(1/3)*b*d + 2*a*b^(1/3)*f + 5*a^(4/3)*g)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(8/3)) +
((b^(1/3)*(b*c + 2*a*f) - a^(1/3)*(b*d + 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/
(27*a^(5/3)*b^(8/3)) - ((b^(1/3)*(b*c + 2*a*f) - a^(1/3)*(b*d + 5*a*g))*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3)) + (h*Log[
a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} - \frac{\int \frac{-ab(bc-af)-2ab(bd-}{}}{}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} + \frac{x(bc-7af+2(bd}{18a}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} + \frac{x(bc-7af+2(bd}{18a}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} + \frac{x(bc-7af+2(bd}{18a}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} + \frac{x(bc-7af+2(bd}{18a}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} + \frac{x(bc-7af+2(bd}{18a}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} + \frac{x(bc-7af+2(bd}{18a}$$

Mathematica [A]
time = 0.17, size = 315, normalized size = 0.97

$$\frac{-\frac{9(a^2h+b^2x(c+dx)-ab(e+x(f+gx)))}{(a+bx^3)^2}+\frac{36a^2h+3b^2x(c+2dx)-3ab(6e+x(7f+8gx))}{a(a+bx^3)}-\frac{2\sqrt{3}\sqrt[3]{b}\left(b^{4/3}c+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+5a^{4/3}g\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{2\sqrt[3]{b}\left(b^{4/3}c-\sqrt[3]{a}bd+2a\sqrt[3]{b}f-5a^{4/3}g\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}+\frac{\sqrt[3]{b}\left(-b^{4/3}c+\sqrt[3]{a}bd-2a\sqrt[3]{b}f+5a^{4/3}g\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}+18h\log(a+bx^3)}{54b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a + b*x^3)^2 + (36
*a^2*h + 3*b^2*x*(c + 2*d*x) - 3*a*b*(6*e + x*(7*f + 8*g*x)))/(a*(a + b*x^3
)) - (2*Sqrt[3]*b^(1/3)*(b^(4/3)*c + a^(1/3)*b*d + 2*a*b^(1/3)*f + 5*a^(4/3
)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*b^(1/3)*(b^(
4/3)*c - a^(1/3)*b*d + 2*a*b^(1/3)*f - 5*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x
])/a^(5/3) + (b^(1/3)*(-(b^(4/3)*c) + a^(1/3)*b*d - 2*a*b^(1/3)*f + 5*a^(4/
3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + 18*h*Log[a
+ b*x^3])/(54*b^3)
```

Maple [A]
time = 0.40, size = 337, normalized size = 1.04

method	result
risch	$\frac{-\frac{(4ag-bd)x^5}{9ab}-\frac{(7af-bc)x^4}{18ab}+\frac{(2ah-be)x^3}{3b^2}-\frac{(5ag+bd)x^2}{18b^2}-\frac{(2af+bc)x}{9b^2}+\frac{a(3ah-be)}{6b^3}}{(bx^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\left(9h\_R^2+\frac{(5ag+bd)\_R}{a}+\right.}{\_R^2}}{27b^3}$
default	$\frac{-\frac{(4ag-bd)x^5}{9ab}-\frac{(7af-bc)x^4}{18ab}+\frac{(2ah-be)x^3}{3b^2}-\frac{(5ag+bd)x^2}{18b^2}-\frac{(2af+bc)x}{9b^2}+\frac{a(3ah-be)}{6b^3}}{(bx^3+a)^2}+\frac{(2af+bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] (-1/9*(4*a*g-b*d)/a/b*x^5-1/18*(7*a*f-b*c)/a/b*x^4+1/3*(2*a*h-b*e)/b^2*x^3-
1/18*(5*a*g+b*d)/b^2*x^2-1/9*(2*a*f+b*c)/b^2*x+1/6*a*(3*a*h-b*e)/b^3)/(b*x^
3+a)^2+1/9/a/b^2*((2*a*f+b*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(5*a*g+b*d)*(-1/3/b/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/
b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+3*a*h*ln(b*x^3+a)/b)
```

Maxima [A]
time = 0.53, size = 369, normalized size = 1.14

$$\frac{2\,(b^3d-4\,ab^2g)x^5+(b^3c-7\,ab^2f)x^4+9\,a^3h+6\,(2\,a^2bh-ab^2e)x^3-3\,a^2be-(ab^2d+5\,a^2bg)x^2-2\,(ab^2c+2\,a^2bf)x}{18\,(ab^5x^6+2\,a^2b^4x^3+a^3b^3)}+\frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3}+\frac{\left(18\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc-2\,af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(9\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc+2\,af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] 1/18*(2*(b^3*d - 4*a*b^2*g)*x^5 + (b^3*c - 7*a*b^2*f)*x^4 + 9*a^3*h + 6*(2*
a^2*b*h - a*b^2*e)*x^3 - 3*a^2*b*e - (a*b^2*d + 5*a^2*b*g)*x^2 - 2*(a*b^2*c
 + 2*a^2*b*f)*x)/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3) + 1/27*sqrt(3)*(b^2*
d*(a/b)^(2/3) + 5*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) + 2*a*b*f*(a/b)^(1/
3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3) + 1/54*(1
8*a*h*(a/b)^(2/3) + b*d*(a/b)^(1/3) + 5*a*g*(a/b)^(1/3) - b*c - 2*a*f)*log(
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/27*(9*a*h*(a/b)^
(2/3) - b*d*(a/b)^(1/3) - 5*a*g*(a/b)^(1/3) + b*c + 2*a*f)*log(x + (a/b)^(1
/3))/(a*b^3*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 2.48, size = 12939, normalized size = 39.81

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] 1/108*(12*(b^3*d - 4*a*b^2*g)*x^5 + 6*(b^3*c - 7*a*b^2*f)*x^4 - 18*a^2*b*e
+ 54*a^3*h - 36*(a*b^2*e - 2*a^2*b*h)*x^3 - 6*(a*b^2*d + 5*a^2*b*g)*x^2 - 2
*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*
h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^
3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f +
5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b
^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(
a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 +
135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c
)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3
) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5
*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^
2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a
^5*b^8) + (b^5*c^3 + 729* ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 363, normalized size = 1.12

$$\frac{h\log\left(|bx^3+a|\right)}{3\,b^3}-\frac{\sqrt{3}\left(b^2c+2\,abf-\left(-ab^2\right)^{\frac{1}{3}}bd-5\left(-ab^2\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}ab^2}-\frac{\left(b^2c+2\,abf+\left(-ab^2\right)^{\frac{1}{3}}bd+5\left(-ab^2\right)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}ab^2}+\frac{2\left(b^2d-4\,abg\right)x^5+\left(b^2c-7\,abf\right)x^4+6\left(2\,a^2h-abe\right)x^3-\left(abd+5\,a^2g\right)x^2-2\left(abc+2\,a^2f\right)x+\frac{3\left(3\,a^3h-a^2be\right)}{b}}{18\left(bx^3+a\right)^2ab^2}-\frac{\left(ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b^3g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^4c+2\,a^2b^3f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] 1/3*h*log(abs(b*x^3 + a))/b^3 - 1/27*sqrt(3)*(b^2*c + 2*a*b*f - (-a*b^2)^(1
/3)*b*d - 5*(-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a
/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1/54*(b^2*c + 2*a*b*f + (-a*b^2)^(1/3)*
b*d + 5*(-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b
^2)^(2/3)*a*b^2) + 1/18*(2*(b^2*d - 4*a*b*g)*x^5 + (b^2*c - 7*a*b*f)*x^4 +
6*(2*a^2*h - a*b*e)*x^3 - (a*b*d + 5*a^2*g)*x^2 - 2*(a*b*c + 2*a^2*f)*x + 3
*(3*a^3*h - a^2*b*e)/b)/((b*x^3 + a)^2*a*b^2) - 1/27*(a*b^4*d*(-a/b)^(1/3)
+ 5*a^2*b^3*g*(-a/b)^(1/3) + a*b^4*c + 2*a^2*b^3*f)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^3*b^5)
```

Mupad [B]
time = 5.66, size = 908, normalized size = 2.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

```
[Out] ((3*a^2*h - a*b*e)/(6*b^3) - (x*(b*c + 2*a*f))/(9*b^2) - (x^2*(b*d + 5*a*g)
)/(18*b^2) - (x^3*(b*e - 2*a*h))/(3*b^2) + (x^4*(b*c - 7*a*f))/(18*a*b) + (
x^5*(b*d - 4*a*g))/(9*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log(root(1
9683*a^5*b^9*z^3 - 19683*a^5*b^6*h*z^2 + 810*a^4*b^4*f*g*z + 405*a^3*b^5*c*
g*z + 162*a^3*b^5*d*f*z + 81*a^2*b^6*c*d*z + 6561*a^5*b^3*h^2*z - 270*a^4*b
*f*g*h - 135*a^3*b^2*c*g*h - 54*a^3*b^2*d*f*h - 27*a^2*b^3*c*d*h - 6*a*b^4*
c^2*f + 75*a^3*b^2*d*g^2 + 15*a^2*b^3*d^2*g - 12*a^2*b^3*c*f^2 - 8*a^3*b^2*
f^3 + 125*a^4*b*g^3 + a*b^4*d^3 - 729*a^5*h^3 - b^5*c^3, z, k)*(9*root(1968
3*a^5*b^9*z^3 - 19683*a^5*b^6*h*z^2 + 810*a^4*b^4*f*g*z + 405*a^3*b^5*c*g*z
 + 162*a^3*b^5*d*f*z + 81*a^2*b^6*c*d*z + 6561*a^5*b^3*h^2*z - 270*a^4*b*f*
g*h - 135*a^3*b^2*c*g*h - 54*a^3*b^2*d*f*h - 27*a^2*b^3*c*d*h - 6*a*b^4*c^2
*f + 75*a^3*b^2*d*g^2 + 15*a^2*b^3*d^2*g - 12*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3
 + 125*a^4*b*g^3 + a*b^4*d^3 - 729*a^5*h^3 - b^5*c^3, z, k)*a*b^2 - (6*a*h)
/b + (x*(54*a^2*b^3*f + 27*a*b^4*c))/(81*a^2*b^3)) + (81*a^3*h^2 + b^3*c*d
+ 5*a*b^2*c*g + 2*a*b^2*d*f + 10*a^2*b*f*g)/(81*a^2*b^4) + (x*(b^2*d^2 + 25
*a^2*g^2 - 18*a^2*f*h - 9*a*b*c*h + 10*a*b*d*g))/(81*a^2*b^3))*root(19683*a
^5*b^9*z^3 - 19683*a^5*b^6*h*z^2 + 810*a^4*b^4*f*g*z + 405*a^3*b^5*c*g*z +
```

```
162*a^3*b^5*d*f*z + 81*a^2*b^6*c*d*z + 6561*a^5*b^3*h^2*z - 270*a^4*b*f*g*h
 - 135*a^3*b^2*c*g*h - 54*a^3*b^2*d*f*h - 27*a^2*b^3*c*d*h - 6*a*b^4*c^2*f
+ 75*a^3*b^2*d*g^2 + 15*a^2*b^3*d^2*g - 12*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 +
125*a^4*b*g^3 + a*b^4*d^3 - 729*a^5*h^3 - b^5*c^3, z, k), k, 1, 3)
```

3.423 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=297

$$\frac{x(bd-4ag+(2be-5ah)x+3bfx^2)}{18ab^2(a+bx^3)}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b(a+bx^3)^2}-\frac{\left(b^{4/3}d+\sqrt[3]{a}\,be+2a\sqrt[3]{b}\,g+5a^{4/\cdots}\right.}{9\sqrt{3}\,a^{5/3}b^{\cdots}}$$

```
[Out] 1/18*x*(b*d-4*a*g+(-5*a*h+2*b*e)*x+3*b*f*x^2)/a/b^2/(b*x^3+a)+1/6*(-h*x^5-g
*x^4-f*x^3-e*x^2-d*x-c)/b/(b*x^3+a)^2+1/27*(b^(1/3)*(2*a*g+b*d)-a^(1/3)*(5*
a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(8/3)-1/54*(b^(1/3)*(2*a*g+b*d)-a
^(1/3)*(5*a*h+b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(8/
3)-1/27*(b^(4/3)*d+a^(1/3)*b*e+2*a*b^(1/3)*g+5*a^(4/3)*h)*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1837, 1872, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(5a^{4/3}h+\sqrt[3]{a}\,be+2a\sqrt[3]{b}\,g+b^{4/3}d\right)}{9\sqrt{3}\,a^{5/3}b^{8/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(\sqrt[3]{b}\,(2ag+bd)-\sqrt[3]{a}\,(5ah+be)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(2ag+bd)-\sqrt[3]{a}\,(5ah+be)\right)}{27a^{5/3}b^{8/3}}+\frac{x(x(2be-5ah)-4ag+bd+3bfx^2)}{18ab^2(a+bx^3)}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] (x*(b*d - 4*a*g + (2*b*e - 5*a*h)*x + 3*b*f*x^2))/(18*a*b^2*(a + b*x^3)) -
(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(6*b*(a + b*x^3)^2) - ((b^(4/3)*d
 + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(8/3)) + ((b^(1/3)*(b*d + 2*a*g)
 - a^(1/3)*(b*e + 5*a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(8/3)) -
((b^(1/3)*(b*d + 2*a*g) - a^(1/3)*(b*e + 5*a*h))*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = -\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\,(a+bx^3)^2} + \frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{(a+bx^3)^2}\,d}{6b}$$

$$= \frac{x(bd-4ag+(2be-5ah)x+3bfx^2)}{18ab^2\,(a+bx^3)} - \frac{c+dx+ex^2+fx^3+}{6b\,(a+bx^3)}$$

$$= \frac{x(bd-4ag+(2be-5ah)x+3bfx^2)}{18ab^2\,(a+bx^3)} - \frac{c+dx+ex^2+fx^3+}{6b\,(a+bx^3)}$$

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$$= \frac{x(bd-4ag+(2be-5ah)x+3bfx^2)}{18ab^2\,(a+bx^3)} - \frac{c+dx+ex^2+fx^3+}{6b\,(a+bx^3)}$$

Mathematica [A]
time = 0.16, size = 287, normalized size = 0.97

$$\frac{-\frac{9b^{2/3}(b(c+x(d+ex))-a(f+x(g+hx)))}{(a+bx^3)^2}+\frac{3b^{2/3}(bx(d+2ex)-a(6f+x(7g+8hx)))}{a(a+bx^3)}-\frac{2\sqrt{3}\left(b^{4/3}d+\sqrt[3]{a}\,be+2a\sqrt[3]{b}\,g+5a^{4/3}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{2\left(b^{4/3}d-\sqrt[3]{a}\,be+2a\sqrt[3]{b}\,g-5a^{4/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}}+\frac{\left(-b^{4/3}d+\sqrt[3]{a}\,be-2a\sqrt[3]{b}\,g+5a^{4/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}}{54b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(2/3)*(b*(c + x*(d + e*x)) - a*(f + x*(g + h*x))))/(a + b*x^3)^2 + (
3*b^(2/3)*(b*x*(d + 2*e*x) - a*(6*f + x*(7*g + 8*h*x))))/(a*(a + b*x^3)) -
(2*Sqrt[3]*(b^(4/3)*d + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*(b^(4/3)*d - a^(1/3)*b*e
+ 2*a*b^(1/3)*g - 5*a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) + ((-(b^(4
/3)*d) + a^(1/3)*b*e - 2*a*b^(1/3)*g + 5*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(54*b^(8/3))
```

Maple [A]
time = 0.40, size = 311, normalized size = 1.05

method	result

risch	$\frac{-\frac{(4ah-be)x^5}{9ab}-\frac{(7ag-bd)x^4}{18ab}-\frac{f x^3}{3b}-\frac{(5ah+be)x^2}{18b^2}-\frac{(2ag+bd)x}{9b^2}-\frac{af+bc}{6b^2}}{(b x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left((5ah+be)\_R+2ag+bd\right)\ln\left(x-\_R\right)}{\_R^2}}{27a b^3}$
default	$\frac{-\frac{(4ah-be)x^5}{9ab}-\frac{(7ag-bd)x^4}{18ab}-\frac{f x^3}{3b}-\frac{(5ah+be)x^2}{18b^2}-\frac{(2ag+bd)x}{9b^2}-\frac{af+bc}{6b^2}}{(b x^3+a)^2}+\frac{(2ag+bd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\ \mathrm{arct}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] (-1/9*(4*a*h-b*e)/a/b*x^5-1/18*(7*a*g-b*d)/a/b*x^4-1/3*f*x^3/b-1/18*(5*a*h+
b*e)/b^2*x^2-1/9*(2*a*g+b*d)/b^2*x-1/6*(a*f+b*c)/b^2)/(b*x^3+a)^2+1/9/a/b^2
*((2*a*g+b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1)))+(5*a*h+b*e)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b
/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]

time = 0.50, size = 314, normalized size = 1.06

$$-\frac{6\,abfx^3+2\,(4\,abh-b^2e)x^5-(b^2d-7\,abg)x^4+3\,abc+3\,a^2f+(5\,a^2h+abe)x^2+2\,(abd+2\,a^2g)x}{18\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)}+\frac{\sqrt{3}\left(5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}+b\left(\frac{a}{b}\right)^{\frac{1}{3}}e+bd+2\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}+b\left(\frac{a}{b}\right)^{\frac{1}{3}}e-bd-2\,ag\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}+b\left(\frac{a}{b}\right)^{\frac{1}{3}}e-bd-2\,ag\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] -1/18*(6*a*b*f*x^3 + 2*(4*a*b*h - b^2*e)*x^5 - (b^2*d - 7*a*b*g)*x^4 + 3*a*
b*c + 3*a^2*f + (5*a^2*h + a*b*e)*x^2 + 2*(a*b*d + 2*a^2*g)*x)/(a*b^4*x^6 +
 2*a^2*b^3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(5*a*h*(a/b)^(1/3) + b*(a/b)^(1/3)
*e + b*d + 2*a*g)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^
3*(a/b)^(2/3)) + 1/54*(5*a*h*(a/b)^(1/3) + b*(a/b)^(1/3)*e - b*d - 2*a*g)*l
og(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) - 1/27*(5*a*h*(a/
b)^(1/3) + b*(a/b)^(1/3)*e - b*d - 2*a*g)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)
^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.93, size = 6926, normalized size = 23.32

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] -1/108*(36*a*b*f*x^3 - 12*(b^2*e - 4*a*b*h)*x^5 - 6*(b^2*d - 7*a*b*g)*x^4 +
 18*a*b*c + 18*a^2*f + 6*(a*b*e + 5*a^2*h)*x^2 + 2*(a*b^4*x^6 + 2*a^2*b^3*x
^3 + a^3*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*
d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2
+ 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*
b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)
 - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) +
 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a
^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^
4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*
b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)))*log(2*a*b^3*d*e^2 + 4*a^2*b
^2*e^2*g + 1/4*(a^4*b^6*e + 5*a^5*b^5*h)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4
*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*
b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^
3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2
*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g
+ 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2
*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 1
25*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b +
 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)))^
2 + 50*(a^3*b*d + 2*a^4*g)*h^2 - 1/2*(a^2*b^5*d^2 + 4*a^3*b^4*d*g + 4*a^4*b
^3*g^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g
+ 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*
a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*
(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(
1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a
^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g
^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3
- 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 -
(e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3))) + 20*(a^2*b^2*d*e + 2*a^3*b*e*g)*
h + (b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 +
 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)*x) + 12*(a*b*d + 2*a^2*g)
*x - ((a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((
b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a
^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4
*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*
```

```
d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e
*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*
d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2
+ 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*
b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)
)) + 3*sqrt(1/3)*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*sqrt(-(((1/2)^(1/3)*
(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 +
8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) +
(b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a
^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e +
 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*
b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h
 + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^
3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3
)/(a^5*b^8))^(1/3)))^2*a^3*b^5 + 16*b^2*d*e + 32*a*b*e*g + 80*(a*b*d + 2*a^
2*g)*h)/(a^3*b^5)))*log(-2*a*b^3*d*e^2 - 4*a^2*b^2*e^2*g - 1/4*(a^4*b^6*e +
 5*a^5*b^5*h)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*
d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2
+ 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*
b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)
 - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) +
 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a
^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^
4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*
b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)))^2 - 50*(a^3*b*d + 2*a^4*g)*
h^2 + 1/2*(a^2*b^5*d^2 + 4*a^3*b^4*d*g + 4*a^4*b^3*g^2)*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*
b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d
^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2
 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^
2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 320, normalized size = 1.08

$$-\frac{\sqrt{3}\left(b^{2}d+2\,abg-5\,\left(-ab^{2}\right)^{\frac{1}{3}}\,ah-\left(-ab^{2}\right)^{\frac{1}{3}}\,be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^{2}\right)^{\frac{2}{3}}\,ab^{2}}-\frac{\left(b^{2}d+2\,abg+5\,\left(-ab^{2}\right)^{\frac{1}{3}}\,ah+\left(-ab^{2}\right)^{\frac{1}{3}}\,be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^{2}\right)^{\frac{2}{3}}\,ab^{2}}-\frac{\left(5\,ah\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+bd+2\,ag\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{2}b^{2}}-\frac{8\,abhx^{5}-2\,b^{2}x^{5}e-b^{2}dx^{4}+7\,abgx^{4}+6\,abfx^{3}+5\,a^{2}hx^{2}+abx^{2}e+2\,abdx+4\,a^{2}gx+3\,abc+3\,a^{2}f}{18\,\left(bx^{3}+a\right)^{2}ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] -1/27*sqrt(3)*(b^2*d + 2*a*b*g - 5*(-a*b^2)^(1/3)*a*h - (-a*b^2)^(1/3)*b*e)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^
2) - 1/54*(b^2*d + 2*a*b*g + 5*(-a*b^2)^(1/3)*a*h + (-a*b^2)^(1/3)*b*e)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) - 1/27*(5*a*h*
(-a/b)^(1/3) + b*(-a/b)^(1/3)*e + b*d + 2*a*g)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a^2*b^2) - 1/18*(8*a*b*h*x^5 - 2*b^2*x^5*e - b^2*d*x^4 + 7*a*b
*g*x^4 + 6*a*b*f*x^3 + 5*a^2*h*x^2 + a*b*x^2*e + 2*a*b*d*x + 4*a^2*g*x + 3*
a*b*c + 3*a^2*f)/((b*x^3 + a)^2*a*b^2)
```

Mupad [B]

time = 5.69, size = 627, normalized size = 2.11

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(19683\,a^5\,b^8\,z^3+810\,a^4\,b^3\,g\,h\,z+405\,a^3\,b^4\,d\,h\,z+162\,a^3\,b^4\,e\,g\,z+81\,a^2\,b^5\,d\,e\,z+75\,a^3\,b\,e\,h^2-6\,a\,b^3\,d^2\,g+15\,a^2\,b^2\,e^2\,h-12\,a^2\,b^2\,d\,g^2-8\,a^3\,b\,g^3+a\,b^3\,e^3+125\,a^4\,h^3-b^4\,d^3,z,k\right)\,\left(\mathrm{root}\left(19683\,a^5\,b^8\,z^3+810\,a^4\,b^3\,g\,h\,z+405\,a^3\,b^4\,d\,h\,z+162\,a^3\,b^4\,e\,g\,z+81\,a^2\,b^5\,d\,e\,z+75\,a^3\,b\,e\,h^2-6\,a\,b^3\,d^2\,g+15\,a^2\,b^2\,e^2\,h-12\,a^2\,b^2\,d\,g^2-8\,a^3\,b\,g^3+a\,b^3\,e^3+125\,a^4\,h^3-b^4\,d^3,z,k\right)\,a\,b^2\,9+\frac{x\,\left(54\,g\,a^2\,b^3+27\,d\,a\,b^4\right)}{81\,a^2\,b^3}\right)+\frac{b^2\,d\,e+10\,a^2\,g\,h+5\,a\,b\,d\,h+2\,a\,b\,e\,g}{81\,a^2\,b^3}+\frac{x\,\left(25\,a^2\,h^2+10\,a\,b\,e\,h+b^2\,e^2\right)}{81\,a^2\,b^3}\right)\,\mathrm{root}\left(19683\,a^5\,b^8\,z^3+810\,a^4\,b^3\,g\,h\,z+405\,a^3\,b^4\,d\,h\,z+162\,a^3\,b^4\,e\,g\,z+81\,a^2\,b^5\,d\,e\,z+75\,a^3\,b\,e\,h^2-6\,a\,b^3\,d^2\,g+15\,a^2\,b^2\,e^2\,h-12\,a^2\,b^2\,d\,g^2-8\,a^3\,b\,g^3+a\,b^3\,e^3+125\,a^4\,h^3-b^4\,d^3,z,k\right)\right)-\frac{\frac{b\,c+a\,f}{6\,b^2}+\frac{x\,\left(b\,d+2\,a\,g\right)}{9\,b^2}+\frac{f\,x^3}{3\,b}+\frac{x^2\,\left(b\,e+5\,a\,h\right)}{18\,b^2}-\frac{x^4\,\left(b\,d-7\,a\,g\right)}{18\,a\,b}-\frac{x^5\,\left(b\,e-4\,a\,h\right)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

```
[Out] symsum(log(root(19683*a^5*b^8*z^3 + 810*a^4*b^3*g*h*z + 405*a^3*b^4*d*h*z +
 162*a^3*b^4*e*g*z + 81*a^2*b^5*d*e*z + 75*a^3*b*e*h^2 - 6*a*b^3*d^2*g + 15
*a^2*b^2*e^2*h - 12*a^2*b^2*d*g^2 - 8*a^3*b*g^3 + a*b^3*e^3 + 125*a^4*h^3 -
 b^4*d^3, z, k)*(9*root(19683*a^5*b^8*z^3 + 810*a^4*b^3*g*h*z + 405*a^3*b^4
*d*h*z + 162*a^3*b^4*e*g*z + 81*a^2*b^5*d*e*z + 75*a^3*b*e*h^2 - 6*a*b^3*d^
2*g + 15*a^2*b^2*e^2*h - 12*a^2*b^2*d*g^2 - 8*a^3*b*g^3 + a*b^3*e^3 + 125*a
^4*h^3 - b^4*d^3, z, k)*a*b^2 + (x*(54*a^2*b^3*g + 27*a*b^4*d))/(81*a^2*b^3
)) + (b^2*d*e + 10*a^2*g*h + 5*a*b*d*h + 2*a*b*e*g)/(81*a^2*b^3) + (x*(b^2*
e^2 + 25*a^2*h^2 + 10*a*b*e*h))/(81*a^2*b^3))*root(19683*a^5*b^8*z^3 + 810*
a^4*b^3*g*h*z + 405*a^3*b^4*d*h*z + 162*a^3*b^4*e*g*z + 81*a^2*b^5*d*e*z +
75*a^3*b*e*h^2 - 6*a*b^3*d^2*g + 15*a^2*b^2*e^2*h - 12*a^2*b^2*d*g^2 - 8*a^
3*b*g^3 + a*b^3*e^3 + 125*a^4*h^3 - b^4*d^3, z, k), k, 1, 3) - ((b*c + a*f)
/(6*b^2) + (x*(b*d + 2*a*g))/(9*b^2) + (f*x^3)/(3*b) + (x^2*(b*e + 5*a*h))/
(18*b^2) - (x^4*(b*d - 7*a*g))/(18*a*b) - (x^5*(b*e - 4*a*h))/(9*a*b))/(a^2
 + b^2*x^6 + 2*a*b*x^3)
```

3.424 $\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=323

$$-\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2\left(a+bx^3\right)^2}+\frac{x(a(be-7ah)+2b(2bc+af)x+3b(bd+ag)x^2)}{18a^2b^2\left(a+bx^3\right)}-\frac{\left(2b^{5/3}c+\right.}{}$$

```
[Out] -1/6*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/a/b^2/(b*x^3+a)^2+1/1
8*x*(a*(-7*a*h+b*e)+2*b*(a*f+2*b*c)*x+3*b*(a*g+b*d)*x^2)/a^2/b^2/(b*x^3+a)-
1/27*(b^(2/3)*(a*f+2*b*c)-a^(2/3)*(2*a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(7/3
)/b^(7/3)+1/54*(b^(2/3)*(a*f+2*b*c)-a^(2/3)*(2*a*h+b*e))*ln(a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(7/3)-1/27*(2*b^(5/3)*c+a^(2/3)*b*e+a*b^(
2/3)*f+2*a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/
3)/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1842, 1872, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{2/3}be+2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)}{9\sqrt{3}a^{7/3}b^{7/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{54a^{7/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{27a^{7/3}b^{7/3}}+\frac{x(2bx(af+2bc)+3bx^2(ag+bd)+a(be-7ah))}{18a^2b^2\left(a+bx^3\right)}-\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{6ab^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(a*b^2*(a +
b*x^3)^2) + (x*(a*(b*e - 7*a*h) + 2*b*(2*b*c + a*f)*x + 3*b*(b*d + a*g)*x^2
))/(18*a^2*b^2*(a + b*x^3)) - ((2*b^(5/3)*c + a^(2/3)*b*e + a*b^(2/3)*f + 2
*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a
^(7/3)*b^(7/3)) - ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b*e + 2*a*h))*Log[a^(1
/3) + b^(1/3)*x])/(27*a^(7/3)*b^(7/3)) + ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*
(b*e + 2*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*
b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
```

```
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} - \frac{\int \frac{-a(be-ah)-2b(}{}}{}$$

$$= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} + \frac{x(a(be-7ah)}{}$$

$$= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} + \frac{x(a(be-7ah)}{}$$

$$= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} + \frac{x(a(be-7ah)}{}$$

$$= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} + \frac{x(a(be-7ah)}{}$$

$$= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} + \frac{x(a(be-7ah)}{}$$

Mathematica [A]
time = 0.20, size = 297, normalized size = 0.92

$$\frac{-\frac{3\sqrt[3]{a}\sqrt[3]{b}\left(-4b^2cx^2-abx(e+2fx)+a^2(6g+7hx)\right)}{a+bx^3}+\frac{9a^{4/3}\sqrt[3]{b}\left(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx))\right)}{(a+bx^3)^2}-2\sqrt{3}\left(2b^{5/3}c+a^{2/3}be+ab^{2/3}f+2a^{5/3}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(-2b^{5/3}c+a^{2/3}be-ab^{2/3}f+2a^{5/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\left(2b^{5/3}c-a^{2/3}be+ab^{2/3}f-2a^{5/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-3*a^(1/3)*b^(1/3)*(-4*b^2*c*x^2 - a*b*x*(e + 2*f*x) + a^2*(6*g + 7*h*x))
)/(a + b*x^3) + (9*a^(4/3)*b^(1/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*
(e + f*x))))/(a + b*x^3)^2 - 2*Sqrt[3]*(2*b^(5/3)*c + a^(2/3)*b*e + a*b^(2/
3)*f + 2*a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-2*b^(
5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f + 2*a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x]
+ (2*b^(5/3)*c - a^(2/3)*b*e + a*b^(2/3)*f - 2*a^(5/3)*h)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(7/3))
```

Maple [A]

time = 0.41, size = 316, normalized size = 0.98

method	result
risch	$\frac{\frac{(af+2bc)x^5}{9a^2}-\frac{(7ah-be)x^4}{18ab}-\frac{g\,x^3}{3b}-\frac{(af-7bc)x^2}{18ab}-\frac{(2ah+be)x}{9b^2}-\frac{ag+bd}{6b^2}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\left(\frac{(af+2bc)\_R}{a}+\frac{2ah+be}{b}\right)\ln(x-\_R)}{\_R^2}}{27a\,b^2}$
default	$\frac{\frac{(af+2bc)x^5}{9a^2}-\frac{(7ah-be)x^4}{18ab}-\frac{g\,x^3}{3b}-\frac{(af-7bc)x^2}{18ab}-\frac{(2ah+be)x}{9b^2}-\frac{ag+bd}{6b^2}}{(b\,x^3+a)^2}+\frac{(2a^2h+abe)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\ \ldots}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/9*(a*f+2*b*c)/a^2*x^5-1/18*(7*a*h-b*e)/a/b*x^4-1/3*g*x^3/b-1/18*(a*f-7*b
*c)/a/b*x^2-1/9*(2*a*h+b*e)/b^2*x-1/6*(a*g+b*d)/b^2)/(b*x^3+a)^2+1/9/a^2/b^
2*((2*a^2*h+a*b*e)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1)))+(a*b*f+2*b^2*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)
^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]

time = 0.48, size = 349, normalized size = 1.08

$$-\frac{6\,a^2bgx^3-2\,(2\,b^3c+ab^2f)x^5+(7\,a^2bh-ab^2e)x^4+3\,a^2bd+3\,a^3g-(7\,ab^2c-a^2bf)x^2+2\,(2\,a^3h+a^2be)x}{18\,(a^2b^4x^6+2\,a^3b^3x^3+a^4b^2)}+\frac{\sqrt{3}\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2h+abe\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2h-abe\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2h-abe\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxim
a")
```

```
[Out] -1/18*(6*a^2*b*g*x^3 - 2*(2*b^3*c + a*b^2*f)*x^5 + (7*a^2*b*h - a*b^2*e)*x^
4 + 3*a^2*b*d + 3*a^3*g - (7*a*b^2*c - a^2*b*f)*x^2 + 2*(2*a^3*h + a^2*b*e)
*x)/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2) + 1/27*sqrt(3)*(2*b^2*c*(a/b)^(
1/3) + a*b*f*(a/b)^(1/3) + 2*a^2*h + a*b*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)
^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3)) + 1/54*(2*b^2*c*(a/b)^(1/3) + a*
b*f*(a/b)^(1/3) - 2*a^2*h - a*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(
```

```
a^2*b^3*(a/b)^(2/3)) - 1/27*(2*b^2*c*(a/b)^(1/3) + a*b*f*(a/b)^(1/3) - 2*a^
2*h - a*b*e)*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 2.12, size = 7190, normalized size = 22.26

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="frica
s")
```

```
[Out] -1/108*(36*a^2*b*g*x^3 - 12*(2*b^3*c + a*b^2*f)*x^5 - 6*(a*b^2*e - 7*a^2*b*
h)*x^4 + 18*a^2*b*d + 18*a^3*g - 6*(7*a*b^2*c - a^2*b*f)*x^2 + 2*(a^2*b^4*x
^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^
2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*
h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 1
2*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b
^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)
*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f
+ 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*
h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 +
 (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))*log(
8*a*b^4*c^2*e + 8*a^2*b^3*c*e*f + 2*a^3*b^2*e*f^2 + 1/4*(2*a^5*b^6*c + a^6*
b^5*f)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^
2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*
a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h
^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) -
2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/
(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3
*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5
*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^
2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))^2 - 1/2*(a^4*b^4*e^2 + 4*a^
5*b^3*e*h + 4*a^6*b^2*h^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b
^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h +
 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a
^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)
/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*
b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6
*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3
)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f
^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3))) + 4*(4*
a^2*b^3*c^2 + 4*a^3*b^2*c*f + a^4*b*f^2)*h + (8*b^5*c^3 + a^2*b^3*e^3 + 12*
a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*
h^2 + 8*a^5*h^3)*x) + 12*(a^2*b*e + 2*a^3*h)*x - ((a^2*b^4*x^6 + 2*a^3*b^3*
```

```
x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*
a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*
h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 -
 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^
(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(
3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f
^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7)
- (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h
)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3))) + 3*sqrt(1/3)*(a^2*
b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*
b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*
a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b
^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*
c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (
e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*
a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*
h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 -
 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^
(1/3)))^2*a^4*b^4 + 32*b^2*c*e + 16*a*b*e*f + 32*(2*a*b*c + a^2*f)*h)/(a^4*
b^4)))*log(-8*a*b^4*c^2*e - 8*a^2*b^3*c*e*f - 2*a^3*b^2*e*f^2 - 1/4*(2*a^5*
b^6*c + a^6*b^5*f)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 +
 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*
b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h
^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^
7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*s
qrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3
*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b
^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e
^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))^2 + 1/2*(a^4*b^
4*e^2 + 4*a^5*b^3*e*h + 4*a^6*b^2*h^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5
*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3
*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*
c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f
^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Giac [**A**]

time = 0.65, size = 340, normalized size = 1.05

$$-\frac{\sqrt{3}\left(2a^2h+abe-2\left(-ab^2\right)^{\frac{1}{3}}bc-\left(-ab^2\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2b}-\frac{\left(2a^2h+abe+2\left(-ab^2\right)^{\frac{1}{3}}bc+\left(-ab^2\right)^{\frac{1}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2b}-\frac{\left(2b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2a^2h+abe\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^3b^2}+\frac{4b^3cx^5+2ab^2fx^5-7a^2bhx^4+ab^2x^4e-6a^2bgx^3+7ab^2cx^2-a^2bfx^2-4a^3hx-2a^2bxe-3a^2bd-3a^3g}{18\left(bx^3+a\right)^2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac"
)
```

```
[Out] -1/27*sqrt(3)*(2*a^2*h + a*b*e - 2*(-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*f)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*
b) - 1/54*(2*a^2*h + a*b*e + 2*(-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*f)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) - 1/27*(2*b^2*
c*(-a/b)^(1/3) + a*b*f*(-a/b)^(1/3) + 2*a^2*h + a*b*e)*(-a/b)^(1/3)*log(abs
(x - (-a/b)^(1/3)))/(a^3*b^2) + 1/18*(4*b^3*c*x^5 + 2*a*b^2*f*x^5 - 7*a^2*b
*h*x^4 + a*b^2*x^4*e - 6*a^2*b*g*x^3 + 7*a*b^2*c*x^2 - a^2*b*f*x^2 - 4*a^3*
h*x - 2*a^2*b*x*e - 3*a^2*b*d - 3*a^3*g)/((b*x^3 + a)^2*a^2*b^2)
```

Mupad [B]
time = 5.36, size = 640, normalized size = 1.98

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(19683\,a^7\,b^7\,z^3+162\,a^5\,b^3\,f\,h\,z+324\,a^4\,b^4\,c\,h\,z+81\,a^4\,b^4\,e\,f\,z+162\,a^3\,b^5\,c\,e\,z-12\,a^4\,b\,e\,h^2+12\,a\,b^4\,c^2\,f-6\,a^3\,b^2\,e^2\,h+6\,a^2\,b^3\,c\,f^2+a^3\,b^2\,f^3-8\,a^5\,h^3+8\,b^5\,c^3-a^2\,b^3\,e^3,z,k\right)\left(\mathrm{root}\left(19683\,a^7\,b^7\,z^3+162\,a^5\,b^3\,f\,h\,z+324\,a^4\,b^4\,c\,h\,z+81\,a^4\,b^4\,e\,f\,z+162\,a^3\,b^5\,c\,e\,z-12\,a^4\,b\,e\,h^2+12\,a\,b^4\,c^2\,f-6\,a^3\,b^2\,e^2\,h+6\,a^2\,b^3\,c\,f^2+a^3\,b^2\,f^3-8\,a^5\,h^3+8\,b^5\,c^3-a^2\,b^3\,e^3,z,k\right)\,a\,b^2\,9+\frac{x\,\left(27\,a^3\,b^2\,e+54\,a^4\,b\,h\right)}{81\,a^4\,b}\right)+\frac{2\,b^2\,c\,e+2\,a^2\,f\,h+4\,a\,b\,c\,h+a\,b\,e\,f}{81\,a^3\,b^2}+\frac{x\,\left(4\,b^2\,c^2+a^2\,f^2+4\,a\,b\,c\,f\right)}{81\,a^4\,b}\right)\,\mathrm{root}\left(19683\,a^7\,b^7\,z^3+162\,a^5\,b^3\,f\,h\,z+324\,a^4\,b^4\,c\,h\,z+81\,a^4\,b^4\,e\,f\,z+162\,a^3\,b^5\,c\,e\,z-12\,a^4\,b\,e\,h^2+12\,a\,b^4\,c^2\,f-6\,a^3\,b^2\,e^2\,h+6\,a^2\,b^3\,c\,f^2+a^3\,b^2\,f^3-8\,a^5\,h^3+8\,b^5\,c^3-a^2\,b^3\,e^3,z,k\right)\right)-\frac{\frac{b\,d+a\,g}{6\,b^2}+\frac{x\,\left(b\,e+2\,a\,h\right)}{9\,b^2}+\frac{g\,x^3}{3\,b}-\frac{x^5\,\left(2\,b\,c+a\,f\right)}{9\,a^2}-\frac{x^2\,\left(7\,b\,c-a\,f\right)}{18\,a\,b}-\frac{x^4\,\left(b\,e-7\,a\,h\right)}{18\,a\,b}}{a^2+b^2\,x^6+2\,a\,b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

```
[Out] symsum(log(root(19683*a^7*b^7*z^3 + 162*a^5*b^3*f*h*z + 324*a^4*b^4*c*h*z +
 81*a^4*b^4*e*f*z + 162*a^3*b^5*c*e*z - 12*a^4*b*e*h^2 + 12*a*b^4*c^2*f - 6
*a^3*b^2*e^2*h + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 - 8*a^5*h^3 + 8*b^5*c^3 - a^
2*b^3*e^3, z, k)*(9*root(19683*a^7*b^7*z^3 + 162*a^5*b^3*f*h*z + 324*a^4*b^
4*c*h*z + 81*a^4*b^4*e*f*z + 162*a^3*b^5*c*e*z - 12*a^4*b*e*h^2 + 12*a*b^4*
c^2*f - 6*a^3*b^2*e^2*h + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 - 8*a^5*h^3 + 8*b^5
*c^3 - a^2*b^3*e^3, z, k)*a*b^2 + (x*(27*a^3*b^2*e + 54*a^4*b*h))/(81*a^4*b
)) + (2*b^2*c*e + 2*a^2*f*h + 4*a*b*c*h + a*b*e*f)/(81*a^3*b^2) + (x*(4*b^2
*c^2 + a^2*f^2 + 4*a*b*c*f))/(81*a^4*b))*root(19683*a^7*b^7*z^3 + 162*a^5*b
^3*f*h*z + 324*a^4*b^4*c*h*z + 81*a^4*b^4*e*f*z + 162*a^3*b^5*c*e*z - 12*a^
4*b*e*h^2 + 12*a*b^4*c^2*f - 6*a^3*b^2*e^2*h + 6*a^2*b^3*c*f^2 + a^3*b^2*f^
3 - 8*a^5*h^3 + 8*b^5*c^3 - a^2*b^3*e^3, z, k), k, 1, 3) - ((b*d + a*g)/(6*
b^2) + (x*(b*e + 2*a*h))/(9*b^2) + (g*x^3)/(3*b) - (x^5*(2*b*c + a*f))/(9*a
^2) - (x^2*(7*b*c - a*f))/(18*a*b) - (x^4*(b*e - 7*a*h))/(18*a*b))/(a^2 + b
^2*x^6 + 2*a*b*x^3)
```

3.425 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3} dx$

Optimal. Leaf size=313

$$\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\,(a+bx^3)^2}-\frac{3a(be+ah)-bx(5bc+af+2(2bd+ag)x)}{18a^2b^2\,(a+bx^3)}-\frac{\left(5b^{4/3}c+2\sqrt[3]{a}\,bd+\right.}{}$$

```
[Out] 1/6*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/a/b/(b*x^3+a)^2+1/18*(-3*a*(a*h
+b*e)+b*x*(5*b*c+a*f+2*(a*g+2*b*d)*x))/a^2/b^2/(b*x^3+a)+1/27*(b^(1/3)*(a*f
+5*b*c)-a^(1/3)*(a*g+2*b*d))*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(5/3)-1/54*(b^
(1/3)*(a*f+5*b*c)-a^(1/3)*(a*g+2*b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/a^(8/3)/b^(5/3)-1/27*(5*b^(4/3)*c+2*a^(1/3)*b*d+a*b^(1/3)*f+a^(4/3)*g
)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1872, 1868, 1874, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(a^{4/3}g+2\sqrt[3]{a}\,bd+a\sqrt[3]{b}\,f+5b^{4/3}c\right)}{9\sqrt{3}\,a^{8/3}b^{5/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(\sqrt[3]{b}\,(af+5bc)-\sqrt[3]{a}\,(ag+2bd)\right)}{54a^{8/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(af+5bc)-\sqrt[3]{a}\,(ag+2bd)\right)}{27a^{8/3}b^{5/3}}-\frac{3a(ah+be)-bx(2x(ag+2bd)+af+5bc)}{18a^2b^2\,(a+bx^3)}+\frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]
```

```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*a*b*(a + b*x^3)^2) - (
3*a*(b*e + a*h) - b*x*(5*b*c + a*f + 2*(2*b*d + a*g)*x))/(18*a^2*b^2*(a + b
*x^3)) - ((5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(5/3)) + ((b
^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^(1/3) + b^(1/3)*x])/(27
*a^(8/3)*b^(5/3)) - ((b^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx = \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\,(a+bx^3)^2} - \frac{\int \frac{-b(5bc+af)-2b(2bd+ag)x-}{(a+bx^3)^2}}{6ab^2}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\,(a+bx^3)^2} - \frac{3a(be+ah)-bx(5bc+}{18a^2b^2\,(a}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\,(a+bx^3)^2} - \frac{3a(be+ah)-bx(5bc+}{18a^2b^2\,(a}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\,(a+bx^3)^2} - \frac{3a(be+ah)-bx(5bc+}{18a^2b^2\,(a}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\,(a+bx^3)^2} - \frac{3a(be+ah)-bx(5bc+}{18a^2b^2\,(a}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\,(a+bx^3)^2} - \frac{3a(be+ah)-bx(5bc+}{18a^2b^2\,(a}$$

Mathematica [A]
time = 0.15, size = 295, normalized size = 0.94

$$\frac{\frac{3a^{2/3}(-6a^2h+b^2x(5c+4dx)+abx(f+2gx))}{a+bx^3}+\frac{9a^{5/3}(a^2h+b^2x(c+dx)-ab(e+x(f+gx)))}{(a+bx^3)^2}-2\sqrt{3}\,\sqrt[3]{b}\left(5b^{4/3}c+2\sqrt[3]{a}\,bd+a\sqrt[3]{b}\,f+a^{4/3}g\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{b}\left(5b^{4/3}c-2\sqrt[3]{a}\,bd+a\sqrt[3]{b}\,f-a^{4/3}g\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+\sqrt[3]{b}\left(-5b^{4/3}c+2\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+a^{4/3}g\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]
```

```
[Out] ((3*a^(2/3)*(-6*a^2*h + b^2*x*(5*c + 4*d*x) + a*b*x*(f + 2*g*x)))/(a + b*x^
3) + (9*a^(5/3)*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a + b*x
^3)^2 - 2*Sqrt[3]*b^(1/3)*(5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^(1/3)*f + a^(4
/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(5*b^(4/3)*c
 - 2*a^(1/3)*b*d + a*b^(1/3)*f - a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x] + b^(1
/3)*(-5*b^(4/3)*c + 2*a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^2)
```

Maple [A]
time = 0.44, size = 309, normalized size = 0.99

method	result

risch	$\frac{\frac{(ag+2bd)x^5}{9a^2}+\frac{(af+5bc)x^4}{18a^2}-\frac{h\,x^3}{3b}-\frac{(ag-7bd)x^2}{18ab}-\frac{(af-4bc)x}{9ab}-\frac{ah+be}{6b^2}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{((ag+2bd)\_R+af+5bc)\ln(x-\_R)}{\_R^2}}{27a^2b^2}$
default	$\frac{\frac{(ag+2bd)x^5}{9a^2}+\frac{(af+5bc)x^4}{18a^2}-\frac{h\,x^3}{3b}-\frac{(ag-7bd)x^2}{18ab}-\frac{(af-4bc)x}{9ab}-\frac{ah+be}{6b^2}}{(b\,x^3+a)^2}+\frac{(af+5bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\ \mathrm{arctan}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/9*(a*g+2*b*d)/a^2*x^5+1/18*(a*f+5*b*c)/a^2*x^4-1/3*h*x^3/b-1/18*(a*g-7*b
*d)/a/b*x^2-1/9*(a*f-4*b*c)/a/b*x-1/6*(a*h+b*e)/b^2)/(b*x^3+a)^2+1/9/a^2/b*
((a*f+5*b*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1)))+(a*g+2*b*d)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/
(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.49, size = 328, normalized size = 1.05

$$-\frac{6\,a^2bhx^3-2\,(2\,b^3d+ab^2g)x^5-(5\,b^3c+ab^2f)x^4+3\,a^3h+3\,a^2be-(7\,ab^2d-a^2bg)x^2-2\,(4\,ab^2c-a^2bf)x}{18\,(a^2b^4x^6+2\,a^3b^3x^3+a^4b^2)}+\frac{\sqrt{3}\left(2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,bc+af\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,bc-af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,bc-af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima"
)
```

```
[Out] -1/18*(6*a^2*b*h*x^3 - 2*(2*b^3*d + a*b^2*g)*x^5 - (5*b^3*c + a*b^2*f)*x^4
+ 3*a^3*h + 3*a^2*b*e - (7*a*b^2*d - a^2*b*g)*x^2 - 2*(4*a*b^2*c - a^2*b*f)
*x)/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2) + 1/27*sqrt(3)*(2*b*d*(a/b)^(1/
3) + a*g*(a/b)^(1/3) + 5*b*c + a*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) + 1/54*(2*b*d*(a/b)^(1/3) + a*g*(a/b)^(1
/3) - 5*b*c - a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2
/3)) - 1/27*(2*b*d*(a/b)^(1/3) + a*g*(a/b)^(1/3) - 5*b*c - a*f)*log(x + (a/
b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.92, size = 6984, normalized size = 22.31

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas"
)

[Out] -1/108*(36*a^2*b*h*x^3 - 12*(2*b^3*d + a*b^2*g)*x^5 - 6*(5*b^3*c + a*b^2*f)
*x^4 + 18*a^2*b*e + 18*a^3*h - 6*(7*a*b^2*d - a^2*b*g)*x^2 + 2*(a^2*b^4*x^6
 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*
a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*
g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*
d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a
^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)
*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 1
5*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(
a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*
d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))*log(40*a*b^3*
c*d^2 + 8*a^2*b^2*d^2*f + 1/4*(2*a^6*b^4*d + a^7*b^3*g)*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 +
 a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b
^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 -
(8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^
2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*
b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g
 + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*
g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8
*b^5))^(1/3)))^2 + 2*(5*a^3*b*c + a^4*f)*g^2 - 1/2*(25*a^3*b^4*c^2 + 10*a^4
*b^3*c*f + a^5*b^2*f^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^
3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g +
6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^
2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b
^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I
*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^
2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*
b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*
g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3))) + 8*(5*a^2*b^2*c*
d + a^3*b*d*f)*g + (125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2
*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)*x) - 12*(4
*a*b^2*c - a^2*b*f)*x - ((a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^
2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5)
 + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a
^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2
*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c
^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b
^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f
```

```
^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*
b^3)/(a^8*b^5))^(1/3))) + 3*sqrt(1/3)*(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^
2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b
^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2
+ a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(
5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2
*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/
(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 +
a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^
4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (
8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 160*b^2*c*d + 32*a*
b*d*f + 16*(5*a*b*c + a^2*f)*g)/(a^5*b^3)))*log(-40*a*b^3*c*d^2 - 8*a^2*b^2
*d^2*f - 1/4*(2*a^6*b^4*d + a^7*b^3*g)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b
^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a
^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3
+ (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f
)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f +
5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^
3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 +
 a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5
*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2
- 2*(5*a^3*b*c + a^4*f)*g^2 + 1/2*(25*a^3*b^4*c^2 + 10*a^4*b^3*c*f + a^5*b^
2*f^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*
c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a
^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c
*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1
/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*...
```

<u>**Sympy**</u> **[F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[A]**
time = 0.48, size = 330, normalized size = 1.05

$$-\frac{\sqrt{3}\left(5\,b^{2}c+abf-2\,\left(-ab^{2}\right)^{\frac{1}{3}}bd-\left(-ab^{2}\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}b}-\frac{\left(5\,b^{2}c+abf+2\,\left(-ab^{2}\right)^{\frac{1}{3}}bd+\left(-ab^{2}\right)^{\frac{1}{3}}ag\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}b}-\frac{\left(2\,bd\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,bc+af\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{3}b}+\frac{4\,b^{3}dx^{5}+2\,ab^{2}gx^{5}+5\,b^{3}cx^{4}+ab^{2}fx^{4}-6\,a^{2}bhx^{3}+7\,ab^{2}dx^{2}-a^{2}bgx^{2}+8\,ab^{2}cx-2\,a^{2}bfx-3\,a^{3}h-3\,a^{2}be}{18\,\left(bx^{3}+a\right)^{2}a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(5*b^2*c + a*b*f - 2*(-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*
b) - 1/54*(5*b^2*c + a*b*f + 2*(-a*b^2)^(1/3)*b*d + (-a*b^2)^(1/3)*a*g)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) - 1/27*(2*b*d*
(-a/b)^(1/3) + a*g*(-a/b)^(1/3) + 5*b*c + a*f)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a^3*b) + 1/18*(4*b^3*d*x^5 + 2*a*b^2*g*x^5 + 5*b^3*c*x^4 + a*b
^2*f*x^4 - 6*a^2*b*h*x^3 + 7*a*b^2*d*x^2 - a^2*b*g*x^2 + 8*a*b^2*c*x - 2*a^
2*b*f*x - 3*a^3*h - 3*a^2*b*e)/((b*x^3 + a)^2*a^2*b^2)
```

Mupad [B]

time = 0.43, size = 630, normalized size = 2.01

$$\frac{\frac{x^4\,\left(5\,b\,c+a\,f\right)}{18\,a^2}-\frac{h\,x^3}{3\,b}-\frac{b\,e+a\,h}{6\,b^2}+\frac{x^5\,\left(2\,b\,d+a\,g\right)}{9\,a^2}+\frac{x\,\left(4\,b\,c-a\,f\right)}{9\,a\,b}+\frac{x^2\,\left(7\,b\,d-a\,g\right)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(19683\,a^8\,b^5\,z^3+81\,a^5\,b^2\,f\,g\,z+405\,a^4\,b^3\,c\,g\,z+162\,a^4\,b^3\,d\,f\,z+810\,a^3\,b^4\,c\,d\,z+6\,a^3\,b\,d\,g^2-75\,a\,b^3\,c^2\,f+12\,a^2\,b^2\,d^2\,g-15\,a^2\,b^2\,c\,f^2+8\,a\,b^3\,d^3+a^4\,g^3-125\,b^4\,c^3-a^3\,b\,f^3,z,k\right)\,\left(\mathrm{root}\left(19683\,a^8\,b^5\,z^3+81\,a^5\,b^2\,f\,g\,z+405\,a^4\,b^3\,c\,g\,z+162\,a^4\,b^3\,d\,f\,z+810\,a^3\,b^4\,c\,d\,z+6\,a^3\,b\,d\,g^2-75\,a\,b^3\,c^2\,f+12\,a^2\,b^2\,d^2\,g-15\,a^2\,b^2\,c\,f^2+8\,a\,b^3\,d^3+a^4\,g^3-125\,b^4\,c^3-a^3\,b\,f^3,z,k\right)\,a\,b^2\,9+\frac{x\,\left(27\,f\,a^3\,b^2+135\,c\,a^2\,b^3\right)}{81\,a^4\,b}\right)+\frac{10\,b^2\,c\,d+a^2\,f\,g+5\,a\,b\,c\,g+2\,a\,b\,d\,f}{81\,a^4\,b}+\frac{x\,\left(a^2\,g^2+4\,a\,b\,d\,g+4\,b^2\,d^2\right)}{81\,a^4\,b}\right)\,\mathrm{root}\left(19683\,a^8\,b^5\,z^3+81\,a^5\,b^2\,f\,g\,z+405\,a^4\,b^3\,c\,g\,z+162\,a^4\,b^3\,d\,f\,z+810\,a^3\,b^4\,c\,d\,z+6\,a^3\,b\,d\,g^2-75\,a\,b^3\,c^2\,f+12\,a^2\,b^2\,d^2\,g-15\,a^2\,b^2\,c\,f^2+8\,a\,b^3\,d^3+a^4\,g^3-125\,b^4\,c^3-a^3\,b\,f^3,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x)
```

```
[Out] ((x^4*(5*b*c + a*f))/(18*a^2) - (h*x^3)/(3*b) - (b*e + a*h)/(6*b^2) + (x^5*
(2*b*d + a*g))/(9*a^2) + (x*(4*b*c - a*f))/(9*a*b) + (x^2*(7*b*d - a*g))/(1
8*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log(root(19683*a^8*b^5*z^3 + 8
1*a^5*b^2*f*g*z + 405*a^4*b^3*c*g*z + 162*a^4*b^3*d*f*z + 810*a^3*b^4*c*d*z
 + 6*a^3*b*d*g^2 - 75*a*b^3*c^2*f + 12*a^2*b^2*d^2*g - 15*a^2*b^2*c*f^2 + 8
*a*b^3*d^3 + a^4*g^3 - 125*b^4*c^3 - a^3*b*f^3, z, k)*(9*root(19683*a^8*b^5
*z^3 + 81*a^5*b^2*f*g*z + 405*a^4*b^3*c*g*z + 162*a^4*b^3*d*f*z + 810*a^3*b
^4*c*d*z + 6*a^3*b*d*g^2 - 75*a*b^3*c^2*f + 12*a^2*b^2*d^2*g - 15*a^2*b^2*c
*f^2 + 8*a*b^3*d^3 + a^4*g^3 - 125*b^4*c^3 - a^3*b*f^3, z, k)*a*b^2 + (x*(1
35*a^2*b^3*c + 27*a^3*b^2*f))/(81*a^4*b)) + (10*b^2*c*d + a^2*f*g + 5*a*b*c
*g + 2*a*b*d*f)/(81*a^4*b) + (x*(4*b^2*d^2 + a^2*g^2 + 4*a*b*d*g))/(81*a^4*
b))*root(19683*a^8*b^5*z^3 + 81*a^5*b^2*f*g*z + 405*a^4*b^3*c*g*z + 162*a^4
*b^3*d*f*z + 810*a^3*b^4*c*d*z + 6*a^3*b*d*g^2 - 75*a*b^3*c^2*f + 12*a^2*b^
2*d^2*g - 15*a^2*b^2*c*f^2 + 8*a*b^3*d^3 + a^4*g^3 - 125*b^4*c^3 - a^3*b*f^
3, z, k), k, 1, 3)
```

3.426 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^3}\,dx$

Optimal. Leaf size=347

$$\frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\,(a+bx^3)^2}+\frac{x(a(5bd+ag)+2a(2be+ah)x-3b(3bc-af)x^2)}{18a^3b\,(a+bx^3)}-\frac{\left(5b^{4/3}d+\right.}{}$$

```
[Out] 1/6*x*(a*(-a*g+b*d)+a*(-a*h+b*e)*x-b*(-a*f+b*c)*x^2)/a^2/b/(b*x^3+a)^2+1/18
*x*(a*(a*g+5*b*d)+2*a*(a*h+2*b*e)*x-3*b*(-a*f+3*b*c)*x^2)/a^3/b/(b*x^3+a)+c
*ln(x)/a^3+1/27*(b^(1/3)*(a*g+5*b*d)-a^(1/3)*(a*h+2*b*e))*ln(a^(1/3)+b^(1/3
)*x)/a^(8/3)/b^(5/3)-1/54*(b^(1/3)*(a*g+5*b*d)-a^(1/3)*(a*h+2*b*e))*ln(a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(5/3)-1/3*c*ln(b*x^3+a)/a^3-1/
27*(5*b^(4/3)*d+2*a^(1/3)*b*e+a*b^(1/3)*g+a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*
b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(5/3)*3^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 345, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(a^{4/3}h+2\sqrt[3]{a}\,be+a\sqrt[3]{b}\,g+5b^{4/3}d\right)}{9\sqrt{3}\,a^{8/3}b^{5/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}\,(ah+2be)}{\sqrt[3]{b}}+ag+5bd\right)}{54a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(ag+5bd)-\sqrt[3]{a}\,(ah+2be)\right)}{27a^{8/3}b^{5/3}}+\frac{x(-3bx^2(3bc-af)+a(ag+5bd)+2ax(ah+2be))}{18a^3b\,(a+bx^3)}-\frac{c\log\left(a+bx^3\right)}{3a^3}+\frac{c\log(x)}{a^3}+\frac{x(-bx^2(bc-af)+a(bd-ag)+ax(be-ah))}{6a^2b\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x]
```

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(6*a^2*b*(a + b*x
^3)^2) + (x*(a*(5*b*d + a*g) + 2*a*(2*b*e + a*h)*x - 3*b*(3*b*c - a*f)*x^2)
)/(18*a^3*b*(a + b*x^3)) - ((5*b^(4/3)*d + 2*a^(1/3)*b*e + a*b^(1/3)*g + a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8
/3)*b^(5/3)) + (c*Log[x])/a^3 + ((b^(1/3)*(5*b*d + a*g) - a^(1/3)*(2*b*e +
a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(5/3)) - ((5*b*d + a*g - (a^(
1/3)*(2*b*e + a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(54*a^(8/3)*b^(4/3)) - (c*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
```

```
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx = \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-b(5bd+ag)x-2\cdots}{x(\cdots}}{\cdots}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5bd+ag)+2\cdots}{1\cdots}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5bd+ag)+2\cdots}{1\cdots}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5bd+ag)+2\cdots}{1\cdots}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5bd+ag)+2\cdots}{1\cdots}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5bd+ag)+2\cdots}{1\cdots}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5bd+ag)+2\cdots}{1\cdots}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5bd+ag)+2\cdots}{1\cdots}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5bd+ag)+2\cdots}{1\cdots}$$

Mathematica [A]

time = 0.18, size = 311, normalized size = 0.90

$$\frac{\frac{3a(6bc+bx(5d+4ex)+ax(g+2hx))}{b(a+bx^3)}-\frac{9a^2(-b(c+x(d+ex))+a(f+x(g+hx)))}{b(a+bx^3)^2}-\frac{2\sqrt{3}\sqrt[3]{a}\left(5b^{4/3}d+2\sqrt[3]{a}be+a\sqrt[3]{b}g+a^{4/3}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}}+54c\log(x)+\frac{2\sqrt[3]{a}\left(5b^{4/3}d-2\sqrt[3]{a}be+a\sqrt[3]{b}g-a^{4/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{5/3}}+\frac{\sqrt[3]{a}\left(-5b^{4/3}d+2\sqrt[3]{a}be-a\sqrt[3]{b}g+a^{4/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{5/3}}-18c\log\left(a+bx^3\right)}{54a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((3*a*(6*b*c + b*x*(5*d + 4*e*x) + a*x*(g + 2*h*x)))/(b*(a + b*x^3)) - (9*a
^2*(-(b*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(b*(a + b*x^3)^2) - (2*S
qrt[3]*a^(1/3)*(5*b^(4/3)*d + 2*a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcT
an[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) + 54*c*Log[x] + (2*a^(1/3)
*(5*b^(4/3)*d - 2*a^(1/3)*b*e + a*b^(1/3)*g - a^(4/3)*h)*Log[a^(1/3) + b^(1
/3)*x])/b^(5/3) + (a^(1/3)*(-5*b^(4/3)*d + 2*a^(1/3)*b*e - a*b^(1/3)*g + a^
(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - 18*c*Log
[a + b*x^3])/(54*a^3)
```

Maple [A]
time = 0.42, size = 339, normalized size = 0.98

method	result
default	$\frac{\frac{\left(\frac{1}{9}a^2h+\frac{2}{9}abe\right)x^5+\left(\frac{1}{18}a^2g+\frac{5}{18}abd\right)x^4+\frac{abc\,x^3}{3}-\frac{a^2(ah-7be)x^2}{18b}-\frac{a^2(ag-4bd)x}{9b}-\frac{a^2(af-3bc)}{6b}}{\left(b\,x^3+a\right)^2}+\frac{\left(a^2g+5abd\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}}{}$
risch	$\frac{\frac{(ah+2be)x^5}{9a^2}+\frac{(ag+5bd)x^4}{18a^2}+\frac{bc\,x^3}{3a^2}-\frac{(ah-7be)x^2}{18ab}-\frac{(ag-4bd)x}{9ab}-\frac{af-3bc}{6ab}}{(b\,x^3+a)^2}+\frac{c\ln(-x)}{a^3}+\frac{\left(\_R=\text{RootOf}\left(a^9b^5\_Z^3+27a^6b^5c\_Z^2+\left(3a^6b^2gh\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*((((1/9*a^2*h+2/9*a*b*e)*x^5+(1/18*a^2*g+5/18*a*b*d)*x^4+1/3*a*b*c*x^3
-1/18*a^2*(a*h-7*b*e)/b*x^2-1/9*a^2*(a*g-4*b*d)/b*x-1/6*a^2*(a*f-3*b*c)/b)/
(b*x^3+a)^2+1/9/b*((a^2*g+5*a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6
/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(a^2*h+2*a*b*e)*(-1/3/b/(a/b)^(1/3
)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3
```

```
*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-3*b*c*ln(b*
x^3+a)))+c*ln(x)/a^3
```

Maxima [A]
time = 0.50, size = 373, normalized size = 1.07

$$\frac{6\,b^2cx^3+2\,(abh+2\,b^2e)x^5+(5\,b^2d+abg)x^4+9\,abc-3\,a^2f-(a^2h-7\,abe)x^2+2\,(4\,abd-a^2g)x}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}+\frac{c\log(x)}{a^3}+\frac{\sqrt{3}\left(a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}e+5\,abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b}-\frac{\left(18\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e+5\,abd+a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(9\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e-5\,abd-a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxim
a")
```

```
[Out] 1/18*(6*b^2*c*x^3 + 2*(a*b*h + 2*b^2*e)*x^5 + (5*b^2*d + a*b*g)*x^4 + 9*a*b
*c - 3*a^2*f - (a^2*h - 7*a*b*e)*x^2 + 2*(4*a*b*d - a^2*g)*x)/(a^2*b^3*x^6
+ 2*a^3*b^2*x^3 + a^4*b) + c*log(x)/a^3 + 1/27*sqrt(3)*(a^2*h*(a/b)^(2/3) +
 2*a*b*(a/b)^(2/3)*e + 5*a*b*d*(a/b)^(1/3) + a^2*g*(a/b)^(1/3))*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b) - 1/54*(18*b^2*c*(a/b)^(2/
3) - a^2*h*(a/b)^(1/3) - 2*a*b*(a/b)^(1/3)*e + 5*a*b*d + a^2*g)*log(x^2 - x
*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) - 1/27*(9*b^2*c*(a/b)^(2/
3) + a^2*h*(a/b)^(1/3) + 2*a*b*(a/b)^(1/3)*e - 5*a*b*d - a^2*g)*log(x + (a/
b)^(1/3))/(a^3*b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 21.81, size = 12815, normalized size = 36.93

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="frica
s")
```

```
[Out] 1/2916*(972*a*b^2*c*x^3 + 324*(2*a*b^2*e + a^2*b*h)*x^5 + 162*(5*a*b^2*d +
a^2*b*g)*x^4 + 1458*a^2*b*c - 486*a^3*f + 162*(7*a^2*b*e - a^3*h)*x^2 - 2*(
a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b^
3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/27*c
^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*
b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a
^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8
*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2
 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a
^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3) + 1
)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5
*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^
2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4
*h^3)/(a^8*b^5) - 1/39366 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 376, normalized size = 1.08

$$-\frac{c\log\left(|bx^3+a|\right)}{3\,a^3}+\frac{c\log\left(|x|\right)}{a^3}-\frac{\sqrt{3}\left(5\,b^2d+abg-\left(-ab^2\right)^{\frac{1}{3}}ah-2\left(-ab^2\right)^{\frac{1}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2b}-\frac{\left(5\,b^2d+abg+\left(-ab^2\right)^{\frac{1}{3}}ah+2\left(-ab^2\right)^{\frac{1}{3}}be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2b}+\frac{6\,ab^2cx^3+2\,(a^2bh+2\,ab^2e)x^5+(5\,ab^2d+a^2bg)x^4+9\,a^2bc-3\,a^3f-(a^3h-7\,a^2be)x^2+2\,(4\,a^2bd-a^3g)x}{18\,(bx^3+a)^2a^3b}-\frac{\left(a^5b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^4b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+5\,a^4b^3d+a^5b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac"
)
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a^3 + c*log(abs(x))/a^3 - 1/27*sqrt(3)*(5*b^2*d
+ a*b*g - (-a*b^2)^(1/3)*a*h - 2*(-a*b^2)^(1/3)*b*e)*arctan(1/3*sqrt(3)*(2*
x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b) - 1/54*(5*b^2*d + a*
b*g + (-a*b^2)^(1/3)*a*h + 2*(-a*b^2)^(1/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) + 1/18*(6*a*b^2*c*x^3 + 2*(a^2*b*h +
2*a*b^2*e)*x^5 + (5*a*b^2*d + a^2*b*g)*x^4 + 9*a^2*b*c - 3*a^3*f - (a^3*h -
 7*a^2*b*e)*x^2 + 2*(4*a^2*b*d - a^3*g)*x)/((b*x^3 + a)^2*a^3*b) - 1/27*(a^
5*b^2*h*(-a/b)^(1/3) + 2*a^4*b^3*(-a/b)^(1/3)*e + 5*a^4*b^3*d + a^5*b^2*g)*
(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b^3)
```

Mupad [**B**]
time = 5.70, size = 1716, normalized size = 4.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x)
```

```
[Out] ((3*b*c - a*f)/(6*a*b) + (x^4*(5*b*d + a*g))/(18*a^2) + (x^5*(2*b*e + a*h))
/(9*a^2) + (x*(4*b*d - a*g))/(9*a*b) + (x^2*(7*b*e - a*h))/(18*a*b) + (b*c*
x^3)/(3*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((c*(25*b^2*d^2 + a^2
*g^2 - 18*b^2*c*e - 9*a*b*c*h + 10*a*b*d*g))/(81*a^6) - (root(19683*a^9*b^5
*z^3 + 19683*a^6*b^5*c*z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5
*b^3*e*g*z + 810*a^4*b^4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*
a^3*b^2*c*g*h + 135*a^2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a
^3*b^2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*
```

g^3 - 125*a*b^4*d^3 + 729*b^5*c^3 + a^5*h^3, z, k)*(a^3*g^2 + 25*a*b^2*d^2 + 324*b^3*c^2*x + 2916*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c*z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 729*b^5*c^3 + a^5*h^3, z, k)^2*a^6*b^3*x - 27*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c*z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 729*b^5*c^3 + a^5*h^3, z, k)*a^5*b*h + 36*a*b^2*c*e + 18*a^2*b*c*h + 10*a^2*b*d*g + 10*a^3*g*h*x - 54*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c*z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 729*b^5*c^3 + a^5*h^3, z, k)*a^4*b^2*e + 1944*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c*z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 729*b^5*c^3 + a^5*h^3, z, k)*a^3*b^3*c*x + 100*a*b^2*d*e*x + 50*a^2*b*d*h*x + 20*a^2*b*e*g*x))/(81*a^4) - (x*(a^4*h^3 - 125*b^4*d^3 + 8*a*b^3*e^3 - a^3*b*g^3 - 15*a^2*b^2*d*g^2 + 12*a^2*b^2*e^2*h + 180*b^4*c*d*e - 75*a*b^3*d^2*g + 6*a^3*b*e*h^2 + 18*a^2*b^2*c*g*h + 90*a*b^3*c*d*h + 36*a*b^3*c*e*g))/(729*a^6*b^2))*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c*z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 729*b^5*c^3 + a^5*h^3, z, k), k, 1, 3) + (c*log(x))/a^3

3.427 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^3}\,dx$

Optimal. Leaf size=362

$$-\frac{c}{a^3x}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\,(a+bx^3)^2}+\frac{x(a(5be+ah)-2b(5bc-2af)x-3b(3bd-ag)x^2)}{18a^3b\,(a+bx^3)}+$$

```
[Out] -c/a^3/x+1/6*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/a^2/b/(b*x^3+
a)^2+1/18*x*(a*(a*h+5*b*e)-2*b*(-2*a*f+5*b*c)*x-3*b*(-a*g+3*b*d)*x^2)/a^3/b
/(b*x^3+a)+d*ln(x)/a^3+1/27*(2*b^(2/3)*(-a*f+7*b*c)+a^(2/3)*(a*h+5*b*e))*ln
(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(4/3)-1/54*(2*b^(2/3)*(-a*f+7*b*c)+a^(2/3)*(
a*h+5*b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(4/3)-1/3*
d*ln(b*x^3+a)/a^3+1/27*(14*b^(5/3)*c-5*a^(2/3)*b*e-2*a*b^(2/3)*f-a^(5/3)*h)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.89, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(-5a^{2/3}be+a^{5/3}(-h)-2ab^{2/3}f+14b^{5/3}c\right)}{9\sqrt{3}\,a^{10/3}b^{4/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{54a^{10/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{27a^{10/3}b^{4/3}}+\frac{x(-2bx(5bc-2af)-3bx^2(3bd-ag)+a(ah+5be))}{18a^3b\,(a+bx^3)}-\frac{d\log\left(a+bx^3\right)}{3a^3}-\frac{c}{a^3x}+\frac{d\log(x)}{a^3}+\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{6a^2b\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -(c/(a^3*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6
*a^2*b*(a + b*x^3)^2) + (x*(a*(5*b*e + a*h) - 2*b*(5*b*c - 2*a*f)*x - 3*b*(
3*b*d - a*g)*x^2))/(18*a^3*b*(a + b*x^3)) + ((14*b^(5/3)*c - 5*a^(2/3)*b*e
- 2*a*b^(2/3)*f - a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(9*Sqrt[3]*a^(10/3)*b^(4/3)) + (d*Log[x])/a^3 + ((2*b^(2/3)*(7*b*c - a
*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(4/3)
) - ((2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a
^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx = \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-6b^2dx-b(5be+}{}}{}$$

$$= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5be+ah)-2b}{1}$$

$$= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5be+ah)-2b}{1}$$

$$= -\frac{c}{a^3x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5be+}{}$$

$$= -\frac{c}{a^3x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5be+}{}$$

$$= -\frac{c}{a^3x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5be+}{}$$

$$= -\frac{c}{a^3x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5be+}{}$$

$$= -\frac{c}{a^3x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5be+}{}$$

$$= -\frac{c}{a^3x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5be+}{}$$

Mathematica [A]
time = 0.35, size = 336, normalized size = 0.93

$$-\frac{\frac{54ac}{x}+\frac{9a^2(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{b(a+bx^3)^2}-\frac{3a(a^2hx-10b^2cx^2+ab(6d+x(5e+4fx)))}{b(a+bx^3)}+\frac{2\sqrt{3}\,a^{2/3}(-14b^{5/3}c+5a^{2/3}be+2ab^{2/3}f+a^{5/3}h)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}}-54ad\log(x)-\frac{2a^{2/3}(14b^{5/3}c+5a^{2/3}be-2ab^{2/3}f+a^{5/3}h)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{4/3}}+\frac{a^{2/3}(14b^{5/3}c+5a^{2/3}be-2ab^{2/3}f+a^{5/3}h)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{4/3}}+18ad\log\left(a+bx^3\right)}{54a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -1/54*((54*a*c)/x + (9*a^2*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x
))))/(b*(a + b*x^3)^2) - (3*a*(a^2*h*x - 10*b^2*c*x^2 + a*b*(6*d + x*(5*e +
 4*f*x))))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(2/3)*(-14*b^(5/3)*c + 5*a^(2/3)*
```

```
b*e + 2*a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/b^(4/3) - 54*a*d*Log[x] - (2*a^(2/3)*(14*b^(5/3)*c + 5*a^(2/3)*b*e - 2*a
*b^(2/3)*f + a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(2/3)*(14*b^
(5/3)*c + 5*a^(2/3)*b*e - 2*a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/b^(4/3) + 18*a*d*Log[a + b*x^3])/a^4
```

Maple [A]
time = 0.42, size = 345, normalized size = 0.95

method	result
default	$\frac{\frac{\left(\frac{2}{9}abf-\frac{5}{9}b^2c\right)x^5+\left(\frac{1}{18}a^2h+\frac{5}{18}abe\right)x^4+\frac{abd\,x^3}{3}+\frac{a(7af-13bc)x^2}{18}-\frac{a^2(ah-4be)x}{9b}-\frac{a^2(ag-3bd)}{6b}}{\left(b\,x^3+a\right)^2}+\frac{\left(a^2h+5abe\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x\right.}{6b\left(\frac{a}{b}\right)}\right.}{}}{}$
risch	$\frac{\frac{2b(af-7bc)x^6}{9a^3}+\frac{(ah+5be)x^5}{18a^2}+\frac{bd\,x^4}{3a^2}+\frac{7(af-7bc)x^3}{18a^2}-\frac{(ah-4be)x^2}{9ab}-\frac{(ag-3bd)x}{6ab}-\frac{c}{a}}{x\left(b\,x^3+a\right)^2}+\frac{d\ln(x)}{a^3}+\frac{\left(\left(\_R=\text{RootOf}\left(a^{10}b^4\_Z^3+27a^7b^4d\_Z^2-\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/a^3*(((2/9*a*b*f-5/9*b^2*c)*x^5+(1/18*a^2*h+5/18*a*b*e)*x^4+1/3*a*b*d*x^3
+1/18*a*(7*a*f-13*b*c)*x^2-1/9*a^2*(a*h-4*b*e)/b*x-1/6*a^2*(a*g-3*b*d)/b)/(
b*x^3+a)^2+1/9/b*((a^2*h+5*a*b*e)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/
b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(2*a*b*f-14*b^2*c)*(-1/3/b/(a/b)^(1
/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1
/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-3*b*d*ln(
b*x^3+a)))-c/a^3/x+d*ln(x)/a^3
```

Maxima [A]
time = 0.51, size = 404, normalized size = 1.12

$$\frac{6\,ab^2dx^4-4\,(7\,b^3c-ab^2f)x^6+(a^2bh+5\,ab^2e)x^5-18\,a^2bc-7\,(7\,ab^2c-a^2bf)x^3-2\,(a^3h-4\,a^2be)x^2+3\,(3\,a^2bd-a^3g)x}{18\,(a^3b^3x^7+2\,a^4b^2x^4+a^5bx)}+\frac{d\log(x)}{a^3}-\frac{\sqrt{3}\left(14\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b}-\frac{\left(18\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h+5\,abe\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(9\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-14\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h-5\,abe\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] 1/18*(6*a*b^2*d*x^4 - 4*(7*b^3*c - a*b^2*f)*x^6 + (a^2*b*h + 5*a*b^2*e)*x^5
 - 18*a^2*b*c - 7*(7*a*b^2*c - a^2*b*f)*x^3 - 2*(a^3*h - 4*a^2*b*e)*x^2 + 3
*(3*a^2*b*d - a^3*g)*x)/(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b*x) + d*log(x)/
a^3 - 1/27*sqrt(3)*(14*b^2*c*(a/b)^(2/3) - 2*a*b*f*(a/b)^(2/3) - a^2*h*(a/b
)^(1/3) - 5*a*b*(a/b)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/(a^4*b) - 1/54*(18*b^2*d*(a/b)^(2/3) + 14*b^2*c*(a/b)^(1/3) - 2*a*b
*f*(a/b)^(1/3) + a^2*h + 5*a*b*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
^3*b^2*(a/b)^(2/3)) - 1/27*(9*b^2*d*(a/b)^(2/3) - 14*b^2*c*(a/b)^(1/3) + 2*
a*b*f*(a/b)^(1/3) - a^2*h - 5*a*b*e)*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(2
/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 22.79, size = 12951, normalized size = 35.78

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] 1/2916*(972*a*b^2*d*x^4 - 648*(7*b^3*c - a*b^2*f)*x^6 + 162*(5*a*b^2*e + a^
2*b*h)*x^5 - 2916*a^2*b*c - 1134*(7*a*b^2*c - a^2*b*f)*x^3 + 324*(4*a^2*b*e
 - a^3*h)*x^2 - 2*(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b*x)*((-I*sqrt(3) + 1)
*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/
(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81
*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 -
 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h -
15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h
^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f
 + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^
4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f
*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(
2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a
^3*b^2*f^3 - 75*a^3*b^2*e ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 390, normalized size = 1.08

$$-\frac{d\log\left(|bx^3+a|\right)}{3\,a^3}+\frac{d\log\left(|x|\right)}{a^3}-\frac{\sqrt{3}\left(a^2h+5\,abe+14\left(-ab^2\right)^{\frac{1}{3}}bc-2\left(-ab^2\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^3}-\frac{\left(a^2h+5\,abe-14\left(-ab^2\right)^{\frac{1}{3}}bc+2\left(-ab^2\right)^{\frac{1}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^3}+\frac{6\,ab^2dx^4-4\left(7\,b^3c-ab^2f\right)x^6+\left(a^2bh+5\,ab^2e\right)x^5-18\,a^2bc-7\left(7\,ab^2c-a^2bf\right)x^3-2\left(a^3h-4\,a^2be\right)x^2+3\left(3\,a^2bd-a^3g\right)x}{18\left(bx^3+a\right)^2a^3bx}+\frac{\left(14\,a^3b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^4b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^5b^2h-5\,a^4b^3e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="gia
c")

[Out] -1/3*d*log(abs(b*x^3 + a))/a^3 + d*log(abs(x))/a^3 - 1/27*sqrt(3)*(a^2*h +
5*a*b*e + 14*(-a*b^2)^(1/3)*b*c - 2*(-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/54*(a^2*h + 5*a
*b*e - 14*(-a*b^2)^(1/3)*b*c + 2*(-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) + 1/18*(6*a*b^2*d*x^4 - 4*(7*b^3*c
- a*b^2*f)*x^6 + (a^2*b*h + 5*a*b^2*e)*x^5 - 18*a^2*b*c - 7*(7*a*b^2*c - a^
2*b*f)*x^3 - 2*(a^3*h - 4*a^2*b*e)*x^2 + 3*(3*a^2*b*d - a^3*g)*x)/((b*x^3 +
 a)^2*a^3*b*x) + 1/27*(14*a^3*b^4*c*(-a/b)^(1/3) - 2*a^4*b^3*f*(-a/b)^(1/3)
 - a^5*b^2*h - 5*a^4*b^3*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b^
3)
```

Mupad [B]
time = 5.75, size = 1747, normalized size = 4.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x)

[Out] symsum(log((d*(a^3*h^2 + 25*a*b^2*e^2 + 126*b^3*c*d - 18*a*b^2*d*f + 10*a^2
*b*e*h))/(81*a^7) - (root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^
6*b^2*f*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z +
 6561*a^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d
*h + 270*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2
*h - 168*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 -
a^5*h^3 - 2744*b^5*c^3, z, k)*(a^3*h^2 + 25*a*b^2*e^2 + 324*b^3*d^2*x - 252
*b^3*c*d + 2916*root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^6*b^2
*f*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z + 6561
*a^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d*h +
270*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2*h -
168*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h
^3 - 2744*b^5*c^3, z, k)^2*a^6*b^3*x + 36*a*b^2*d*f + 10*a^2*b*e*h - 700*b^
3*c*e*x + 378*root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^6*b^2*f
*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z + 6561*a
^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d*h + 27
0*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2*h - 16
```

```
8*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h^3
 - 2744*b^5*c^3, z, k)*a^3*b^3*c - 54*root(19683*a^10*b^4*z^3 + 19683*a^7*b
^4*d*z^2 + 162*a^6*b^2*f*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 567
0*a^4*b^4*c*e*z + 6561*a^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h
- 378*a^2*b^3*c*d*h + 270*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f
 - 75*a^3*b^2*e^2*h - 168*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 +
 729*a*b^4*d^3 - a^5*h^3 - 2744*b^5*c^3, z, k)*a^4*b^2*f + 1944*root(19683*
a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^6*b^2*f*h*z - 1134*a^5*b^3*c*h*z
 + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z + 6561*a^4*b^4*d^2*z - 1890*a*b^4
*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d*h + 270*a^2*b^3*d*e*f - 15*a^4*
b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2*h - 168*a^2*b^3*c*f^2 + 8*a^3*b
^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h^3 - 2744*b^5*c^3, z, k)*a^
3*b^3*d*x - 140*a*b^2*c*h*x + 100*a*b^2*e*f*x + 20*a^2*b*f*h*x))/(81*a^4) +
 (x*(2744*b^5*c^3 + a^5*h^3 + 125*a^2*b^3*e^3 - 8*a^3*b^2*f^3 + 168*a^2*b^3
*c*f^2 + 75*a^3*b^2*e^2*h - 1176*a*b^4*c^2*f + 15*a^4*b*e*h^2 + 252*a^2*b^3
*c*d*h - 180*a^2*b^3*d*e*f - 36*a^3*b^2*d*f*h + 1260*a*b^4*c*d*e))/(729*a^8
*b))*root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^6*b^2*f*h*z - 11
34*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z + 6561*a^4*b^4*d^
2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d*h + 270*a^2*b^3
*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2*h - 168*a^2*b^3
*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h^3 - 2744*b
^5*c^3, z, k), k, 1, 3) + ((x^5*(5*b*e + a*h))/(18*a^2) - (7*x^3*(7*b*c - a
*f))/(18*a^2) - c/a - (2*b*x^6*(7*b*c - a*f))/(9*a^3) + (x*(3*b*d - a*g))/(
6*a*b) + (x^2*(4*b*e - a*h))/(9*a*b) + (b*d*x^4)/(3*a^2))/(a^2*x + b^2*x^7
+ 2*a*b*x^4) + (d*log(x))/a^3
```

3.428 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^3}\,dx$

Optimal. Leaf size=360

$$-\frac{c}{2a^3x^2}-\frac{d}{a^3x}-\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2\left(a+bx^3\right)^2}-\frac{x(11bc-5af+2(5bd-2ag)x+3(3be-ah)x^2)}{18a^3\left(a+bx^3\right)}+$$

```
[Out] -1/2*c/a^3/x^2-d/a^3/x-1/6*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/a^2/(b*x
^3+a)^2-1/18*x*(11*b*c-5*a*f+2*(-2*a*g+5*b*d)*x+3*(-a*h+3*b*e)*x^2)/a^3/(b*
x^3+a)+e*ln(x)/a^3-1/27*(5*b^(1/3)*(-a*f+4*b*c)-2*a^(1/3)*(-a*g+7*b*d))*ln(
a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(2/3)+1/54*(5*b^(1/3)*(-a*f+4*b*c)-2*a^(1/3)*
(-a*g+7*b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(2/3)-1/
3*e*ln(b*x^3+a)/a^3+1/27*(20*b^(4/3)*c+14*a^(1/3)*b*d-5*a*b^(1/3)*f-2*a^(4/
3)*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3)*3^
(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 357, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{4/3}g+14\sqrt[3]{a}bd-5a\sqrt[3]{b}f+20b^{4/3}c\right)}{9\sqrt{3}a^{11/3}b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{2\sqrt[3]{a}(7bd-ag)}{\sqrt[3]{b}}-5af+20bc\right)}{54a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)}{27a^{11/3}b^{2/3}}-\frac{x(2x(5bd-2ag)+3x^2(3be-ah)-5af+11bc)}{18a^3\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^3}-\frac{c}{2a^3x^2}-\frac{d}{a^3x}+\frac{e\log(x)}{a^3}-\frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{6a^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -1/2*c/(a^3*x^2) - d/(a^3*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*
x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*c - 5*a*f + 2*(5*b*d - 2*a*g)*x + 3*
(3*b*e - a*h)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b^(4/3)*c + 14*a^(1/3)*b*d
- 5*a*b^(1/3)*f - 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(11/3)*b^(2/3)) + (e*Log[x])/a^3 - ((5*b^(1/3)*(4*b*c -
 a*f) - 2*a^(1/3)*(7*b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(
2/3)) + ((20*b*c - 5*a*f - (2*a^(1/3)*(7*b*d - a*g))/b^(1/3))*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(1/3)) - (e*Log[a + b*x^3
])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-6b^2dx-6b^2ex^2+5b^2}{}}{}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc-5af+2(5bd}{18a^3}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc-5af+2(5bd}{18a^3}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2\left(a+bx^3\right)^2} - \frac{x(11bc}{}$$

Mathematica [A]
time = 0.37, size = 337, normalized size = 0.94

$$-\frac{\frac{27ac}{x^2}+\frac{54ad}{x}-\frac{3a(6ae-bx(11c+10dx)+ax(5f+4gx))}{a+bx^3}+\frac{9a^2(a^2h+b^2x(c+dx)-ab(e+x(f+gx)))}{b(a+bx^3)^2}+\frac{2\sqrt{3}\sqrt[3]{a}\left(-20b^{4/3}c-14\sqrt[3]{a}bd+5a\sqrt[3]{b}f+2a^{4/3}g\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}-54ae\log(x)+\frac{2\sqrt[3]{a}\left(20b^{4/3}c-14\sqrt[3]{a}bd-5a\sqrt[3]{b}f+2a^{4/3}g\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}-\frac{\sqrt[3]{a}\left(20b^{4/3}c-14\sqrt[3]{a}bd-5a\sqrt[3]{b}f+2a^{4/3}g\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+18ae\log\left(a+bx^3\right)}{54a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -1/54*((27*a*c)/x^2 + (54*a*d)/x - (3*a*(6*a*e - b*x*(11*c + 10*d*x) + a*x*
(5*f + 4*g*x)))/(a + b*x^3) + (9*a^2*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*
(f + g*x))))/(b*(a + b*x^3)^2) + (2*Sqrt[3]*a^(1/3)*(-20*b^(4/3)*c - 14*a^(
```

```
1/3)*b*d + 5*a*b^(1/3)*f + 2*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(2/3) - 54*a*e*Log[x] + (2*a^(1/3)*(20*b^(4/3)*c - 14*a^(1/3)*b
*d - 5*a*b^(1/3)*f + 2*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1
/3)*(20*b^(4/3)*c - 14*a^(1/3)*b*d - 5*a*b^(1/3)*f + 2*a^(4/3)*g)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 18*a*e*Log[a + b*x^3])/a^4
```

Maple [A]

time = 0.40, size = 340, normalized size = 0.94

method	result
default	$\frac{\frac{\left(\frac{2}{9}abg-\frac{5}{9}b^2d\right)x^5+\left(\frac{5}{18}abf-\frac{11}{18}b^2c\right)x^4+\frac{abe\,x^3}{3}+\frac{a(7ag-13bd)x^2}{18}+\frac{a(4af-7bc)x}{9}-\frac{a^2(ah-3be)}{6b}}{\left(b\,x^3+a\right)^2}+\frac{(5af-20bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}}{}$
risch	$\frac{\frac{2b(ag-7bd)x^7}{9a^3}+\frac{5b(af-4bc)x^6}{18a^3}+\frac{be\,x^5}{3a^2}+\frac{7(ag-7bd)x^4}{18a^2}+\frac{4(af-4bc)x^3}{9a^2}-\frac{(ah-3be)x^2}{6ab}-\frac{xd}{a}-\frac{c}{2a}}{x^2(b\,x^3+a)^2}+\frac{e\ln(-x)}{a^3}+\frac{\left(\_R=\mathrm{RootOf}\left(a^{11}b^2\_Z^3+27\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/a^3*(((2/9*a*b*g-5/9*b^2*d)*x^5+(5/18*a*b*f-11/18*b^2*c)*x^4+1/3*a*b*e*x^
3+1/18*a*(7*a*g-13*b*d)*x^2+1/9*a*(4*a*f-7*b*c)*x-1/6*a^2*(a*h-3*b*e)/b)/(b
*x^3+a)^2+1/9*(5*a*f-20*b*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/
b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/9*(2*a*g-14*b*d)*(-1/3/b/(a/b)^(1/3)*l
n(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^
(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-1/3*e*ln(b*x^3
+a))-1/2*c/a^3/x^2-d/a^3/x+e*ln(x)/a^3
```

Maxima [A]

time = 0.50, size = 394, normalized size = 1.09

$$\frac{6\,ab^2x^5e-4\,(7\,b^3d-ab^2g)x^7-5\,(4\,b^3c-ab^2f)x^6-18\,a^2bdx-7\,(7\,ab^2d-a^2bg)x^4-9\,a^2bc-8\,(4\,ab^2c-a^2bf)x^3-3\,(a^3h-3\,a^2be)x^2}{18\,(a^3b^3x^8+2\,a^4b^2x^5+a^5bx^2)}+\frac{e\log(x)}{a^3}-\frac{\sqrt{3}\left(14\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ag\left(\frac{a}{b}\right)^{\frac{2}{3}}+20\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,af\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4}-\frac{\left(18\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}e+14\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,bc+5\,af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(9\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}e-14\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,bc-5\,af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] 1/18*(6*a*b^2*x^5*e - 4*(7*b^3*d - a*b^2*g)*x^7 - 5*(4*b^3*c - a*b^2*f)*x^6
 - 18*a^2*b*d*x - 7*(7*a*b^2*d - a^2*b*g)*x^4 - 9*a^2*b*c - 8*(4*a*b^2*c -
a^2*b*f)*x^3 - 3*(a^3*h - 3*a^2*b*e)*x^2)/(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^
5*b*x^2) + e*log(x)/a^3 - 1/27*sqrt(3)*(14*b*d*(a/b)^(2/3) - 2*a*g*(a/b)^(2
/3) + 20*b*c*(a/b)^(1/3) - 5*a*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/a^4 - 1/54*(18*b*(a/b)^(2/3)*e + 14*b*d*(a/b)^(1/3)
- 2*a*g*(a/b)^(1/3) - 20*b*c + 5*a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(a^3*b*(a/b)^(2/3)) - 1/27*(9*b*(a/b)^(2/3)*e - 14*b*d*(a/b)^(1/3) + 2*a*
g*(a/b)^(1/3) + 20*b*c - 5*a*f)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 17.06, size = 12435, normalized size = 34.54

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] 1/2916*(972*a*b^2*e*x^5 - 648*(7*b^3*d - a*b^2*g)*x^7 - 810*(4*b^3*c - a*b^
2*f)*x^6 - 2916*a^2*b*d*x - 1134*(7*a*b^2*d - a^2*b*g)*x^4 - 1458*a^2*b*c -
 1296*(4*a*b^2*c - a^2*b*f)*x^3 + 486*(3*a^2*b*e - a^3*h)*x^2 - 2*(a^3*b^3*
x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c
*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 +
 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10
*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*
b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^
3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g +
168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g
)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3)
 + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g +
(81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a
*b^3*d^3 - 6000*a*b^3*c^2 ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 399, normalized size = 1.11

$$-\frac{e\log\left(|bx^3+a|\right)}{3\,a^3}+\frac{e\log\left(|x|\right)}{a^3}+\frac{\sqrt{3}\left(20\,b^2c-5\,abf-14\,(-ab^2)^{\frac{1}{3}}\,bd+2\,(-ab^2)^{\frac{1}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^3}+\frac{\left(20\,b^2c-5\,abf+14\,(-ab^2)^{\frac{1}{3}}\,bd-2\,(-ab^2)^{\frac{1}{3}}\,ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^3}-\frac{28\,b^3dx^7-4\,ab^2gx^7+20\,b^3cx^6-5\,ab^2fx^6-6\,ab^2x^5e+49\,ab^2dx^4-7\,a^2bgx^4+32\,ab^2cx^3-8\,a^2bfx^3+3\,a^3hx^2-9\,a^2bx^2e+18\,a^2bdx+9\,a^2bc}{18\,(bx^4+ax)^2a^3b}+\frac{\left(14\,a^3b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^4bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,a^3b^2c-5\,a^4bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^3 + e*log(abs(x))/a^3 + 1/27*sqrt(3)*(20*b^2*c
 - 5*a*b*f - 14*(-a*b^2)^(1/3)*b*d + 2*(-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) + 1/54*(20*b^2*c
 - 5*a*b*f + 14*(-a*b^2)^(1/3)*b*d - 2*(-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/
b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 1/18*(28*b^3*d*x^7 - 4*a*b^
2*g*x^7 + 20*b^3*c*x^6 - 5*a*b^2*f*x^6 - 6*a*b^2*x^5*e + 49*a*b^2*d*x^4 - 7
*a^2*b*g*x^4 + 32*a*b^2*c*x^3 - 8*a^2*b*f*x^3 + 3*a^3*h*x^2 - 9*a^2*b*x^2*e
 + 18*a^2*b*d*x + 9*a^2*b*c)/((b*x^4 + a*x)^2*a^3*b) + 1/27*(14*a^3*b^2*d*(
-a/b)^(1/3) - 2*a^4*b*g*(-a/b)^(1/3) + 20*a^3*b^2*c - 5*a^4*b*f)*(-a/b)^(1/
3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

Mupad [B]
time = 5.66, size = 1697, normalized size = 4.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x)
```

```
[Out] symsum(log((b^2*e*(400*b^2*c^2 + 25*a^2*f^2 - 18*a^2*e*g - 200*a*b*c*f + 12
6*a*b*d*e))/(81*a^8) - (root(19683*a^11*b^2*z^3 + 19683*a^8*b^2*e*z^2 + 810
*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c*g*z + 22680*a^4*b^3*c*d*
z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*b^3*c*d*e - 1890*a^2*b^2*
d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 6000*a*b^3*c^2*f + 1176*a^2*
b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 - 125*a^3*b*f^3 - 2744*a*b
^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z, k)*b^2*(400*b^2*c^2 + 25*a^2*f^2 - 54
*root(19683*a^11*b^2*z^3 + 19683*a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5
*b^2*d*f*z - 3240*a^5*b^2*c*g*z + 22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z
+ 270*a^3*b*e*f*g + 7560*a*b^3*c*d*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*
e*g - 168*a^3*b*d*g^2 - 6000*a*b^3*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^
2*c*f^2 + 729*a^2*b^2*e^3 - 125*a^3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 80
00*b^4*c^3, z, k)*a^5*g + 36*a^2*e*g + 378*root(19683*a^11*b^2*z^3 + 19683*
a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c*g*z +
 22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*b^3*c*
d*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 6000*a*b^
3*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 - 125*a
^3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z, k)*a^4*b*d + 324*a
*b*e^2*x + 2800*b^2*c*d*x + 100*a^2*f*g*x + 2916*root(19683*a^11*b^2*z^3 +
19683*a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c
*g*z + 22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*
b^3*c*d*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 600
```

```
0*a*b^3*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 -
 125*a^3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z, k)^2*a^7*b*x
 - 200*a*b*c*f - 252*a*b*d*e - 400*a*b*c*g*x - 700*a*b*d*f*x + 1944*root(19
683*a^11*b^2*z^3 + 19683*a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2*d*f
*z - 3240*a^5*b^2*c*g*z + 22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z + 270*a^
3*b*e*f*g + 7560*a*b^3*c*d*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*e*g - 16
8*a^3*b*d*g^2 - 6000*a*b^3*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2
+ 729*a^2*b^2*e^3 - 125*a^3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c
^3, z, k)*a^4*b*e*x))/(81*a^5) - (b*x*(8000*b^4*c^3 + 8*a^4*g^3 - 2744*a*b^
3*d^3 - 125*a^3*b*f^3 + 1500*a^2*b^2*c*f^2 + 1176*a^2*b^2*d^2*g - 6000*a*b^
3*c^2*f - 168*a^3*b*d*g^2 - 720*a^2*b^2*c*e*g - 1260*a^2*b^2*d*e*f + 5040*a
*b^3*c*d*e + 180*a^3*b*e*f*g))/(729*a^9))*root(19683*a^11*b^2*z^3 + 19683*a
^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c*g*z +
22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*b^3*c*d
*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 6000*a*b^3
*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 - 125*a^
3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z, k), k, 1, 3) - (c/(
2*a) + (4*x^3*(4*b*c - a*f))/(9*a^2) + (7*x^4*(7*b*d - a*g))/(18*a^2) + (d*
x)/a + (5*b*x^6*(4*b*c - a*f))/(18*a^3) + (2*b*x^7*(7*b*d - a*g))/(9*a^3) -
 (x^2*(3*b*e - a*h))/(6*a*b) - (b*e*x^5)/(3*a^2))/(a^2*x^2 + b^2*x^8 + 2*a*
b*x^5) + (e*log(x))/a^3
```

3.429 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^3}\,dx$

Optimal. Leaf size=395

$$-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd-5ag+2(5be-2ah)x-3b\left(\frac{5bc}{a}\right.\right.}{18a^3\left(a+bx^3\right)}$$

```
[Out] -1/3*c/a^3/x^3-1/2*d/a^3/x^2-e/a^3/x-1/6*x*(b*d-a*g+(-a*h+b*e)*x-b*(b*c/a-f
)*x^2)/a^2/(b*x^3+a)^2-1/18*x*(11*b*d-5*a*g+2*(-2*a*h+5*b*e)*x-3*b*(5*b*c/a
-3*f)*x^2)/a^3/(b*x^3+a)-(-a*f+3*b*c)*ln(x)/a^4-1/27*(5*b^(1/3)*(-a*g+4*b*d
)-2*a^(1/3)*(-a*h+7*b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(2/3)+1/54*(5*b^
(1/3)*(-a*g+4*b*d)-2*a^(1/3)*(-a*h+7*b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/a^(11/3)/b^(2/3)+1/3*(-a*f+3*b*c)*ln(b*x^3+a)/a^4+1/27*(20*b^(4/3
)*d+14*a^(1/3)*b*e-5*a*b^(1/3)*g-2*a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)
*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 392, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$,
Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{4/3}h+14\sqrt[3]{a}be-5a\sqrt[3]{b}g+20b^{4/3}d\right)}{9\sqrt{3}a^{11/3}b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{2\sqrt[3]{a}(7be-ah)}{\sqrt[3]{b}}-5ag+20bd\right)}{54a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)}{27a^{11/3}b^{2/3}}+\frac{(3bc-af)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3bc-af)}{a^4}-\frac{x\left(-3bx^2\left(\frac{5bc}{a}-3f\right)+2x(5be-2ah)-5ag+11bd\right)}{18a^3\left(a+bx^3\right)}-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{6a^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -1/3*c/(a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d - a*g + (b*e - a*h)*
x - b*((b*c)/a - f)*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d - 5*a*g + 2*(5
*b*e - 2*a*h)*x - 3*b*((5*b*c)/a - 3*f)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b
^(4/3)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(2/3)) - ((3*b*c - a
*f)*Log[x])/a^4 - ((5*b^(1/3)*(4*b*d - a*g) - 2*a^(1/3)*(7*b*e - a*h))*Log[
a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(2/3)) + ((20*b*d - 5*a*g - (2*a^(1/3)
*(7*b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(5
4*a^(11/3)*b^(1/3)) + ((3*b*c - a*f)*Log[a + b*x^3])/(3*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^3}\,dx = -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-6b^2dx-6b^2ex^2+6b}{}}{}$$

$$= -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd-5ag+2(5be\right.}{18a}$$

$$= -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd-5ag+2(5be\right.}{18a}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2}$$

<u>Mathematica</u> [A]
time = 0.37, size = 352, normalized size = 0.89

$$\frac{-\frac{18ac}{x^3}-\frac{27ad}{x^2}-\frac{54ae}{x}+\frac{3a(-12bc+6af-bx(11d+10ex)+ax(5g+4hx))}{a+bx^3}+\frac{a^2(-9b(c+x(d+ex))+9a(f+x(g+hx)))}{(a+bx^3)^2}+\frac{2\sqrt{3}\sqrt[3]{a}\left(20b^{4/3}d+14\sqrt[3]{a}be-5a\sqrt[3]{b}g-2a^{4/3}h\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+54(-3bc+af)\log(x)-\frac{2\sqrt[3]{a}\left(20b^{4/3}d-14\sqrt[3]{a}be-5a\sqrt[3]{b}g+2a^{4/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{\sqrt[3]{a}\left(20b^{4/3}d-14\sqrt[3]{a}be-5a\sqrt[3]{b}g+2a^{4/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+18(3bc-af)\log\left(a+bx^3\right)}{54a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-18*a*c)/x^3 - (27*a*d)/x^2 - (54*a*e)/x + (3*a*(-12*b*c + 6*a*f - b*x*(1
1*d + 10*e*x) + a*x*(5*g + 4*h*x)))/(a + b*x^3) + (a^2*(-9*b*(c + x*(d + e*
x)) + 9*a*(f + x*(g + h*x))))/(a + b*x^3)^2 + (2*Sqrt[3]*a^(1/3)*(20*b^(4/3
```

```
)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 54*(-3*b*c + a*f)*Log[x] - (2*a^(1/3)*(20*b
^(4/3)*d - 14*a^(1/3)*b*e - 5*a*b^(1/3)*g + 2*a^(4/3)*h)*Log[a^(1/3) + b^(1
/3)*x])/b^(2/3) + (a^(1/3)*(20*b^(4/3)*d - 14*a^(1/3)*b*e - 5*a*b^(1/3)*g +
 2*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 18*
(3*b*c - a*f)*Log[a + b*x^3])/(54*a^4)
```

Maple [A]

time = 0.41, size = 394, normalized size = 1.00

method	result
default	$\frac{\frac{\left(\frac{2}{9}a^2bh-\frac{5}{9}ab^2e\right)x^5+\left(\frac{5}{18}a^2bg-\frac{11}{18}ab^2d\right)x^4+\left(\frac{1}{3}a^2bf-\frac{2}{3}acb^2\right)x^3+\frac{a^2(7ah-13be)x^2}{18}+\frac{a^2(4ag-7bd)x}{9}+\frac{a^3f}{2}-\frac{5ca^2b}{6}}{\left(bx^3+a\right)^2}+\frac{\left(5a^2g-20abd\right)\left(\frac{\ln\left(x+\left(\right.\right.}{3b\left(\frac{a}{b}\right.}\right.}{}$
risch	$\frac{\frac{2b(ah-7be)x^8}{9a^3}+\frac{5b(ag-4bd)x^7}{18a^3}+\frac{b(af-3bc)x^6}{3a^3}+\frac{7(ah-7be)x^5}{18a^2}+\frac{4(ag-4bd)x^4}{9a^2}+\frac{(af-3bc)x^3}{2a^2}-\frac{ex^2}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^3\left(bx^3+a\right)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{12}b^2\_Z^3+\left(\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/a^4*(((2/9*a^2*b*h-5/9*a*b^2*e)*x^5+(5/18*a^2*b*g-11/18*a*b^2*d)*x^4+(1/3
*a^2*b*f-2/3*a*c*b^2)*x^3+1/18*a^2*(7*a*h-13*b*e)*x^2+1/9*a^2*(4*a*g-7*b*d)
*x+1/2*a^3*f-5/6*c*a^2*b)/(b*x^3+a)^2+1/9*(5*a^2*g-20*a*b*d)*(1/3/b/(a/b)^(
2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+
1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/9*(2*a
^2*h-14*a*b*e)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1)))+1/27*(-9*a*b*f+27*b^2*c)*ln(b*x^3+a)/b)-1/2*d/a^3/x^2
-1/3*c/a^3/x^3-e/a^3/x+(a*f-3*b*c)/a^4*ln(x)
```

Maxima [A]

time = 0.50, size = 448, normalized size = 1.13

$$\frac{4\left(abh-7b^2e\right)x^8-5\left(4b^2d-abg\right)x^7-6\left(3b^2c-abf\right)x^6+7\left(a^2h-7abe\right)x^5-8\left(4abd-a^2g\right)x^4-18a^2x^2e-9a^2dx-9\left(3abc-a^2f\right)x^3-6a^2c}{18\left(a^3b^2x^9+2a^4bx^6+a^5x^3\right)}-\frac{(3bc-af)\log(x)}{a^4}+\frac{\sqrt{3}\left(2a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}-14ab\left(\frac{a}{b}\right)^{\frac{2}{3}}e-20abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+5a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^5}+\frac{\left(54b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-18abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+2a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-14ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e+20abd-5a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(27b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-9abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+14ab\left(\frac{a}{b}\right)^{\frac{1}{3}}e-20abd+5a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] 1/18*(4*(a*b*h - 7*b^2*e)*x^8 - 5*(4*b^2*d - a*b*g)*x^7 - 6*(3*b^2*c - a*b*
f)*x^6 + 7*(a^2*h - 7*a*b*e)*x^5 - 8*(4*a*b*d - a^2*g)*x^4 - 18*a^2*x^2*e -
 9*a^2*d*x - 9*(3*a*b*c - a^2*f)*x^3 - 6*a^2*c)/(a^3*b^2*x^9 + 2*a^4*b*x^6
+ a^5*x^3) - (3*b*c - a*f)*log(x)/a^4 + 1/27*sqrt(3)*(2*a^2*h*(a/b)^(2/3) -
 14*a*b*(a/b)^(2/3)*e - 20*a*b*d*(a/b)^(1/3) + 5*a^2*g*(a/b)^(1/3))*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5 + 1/54*(54*b^2*c*(a/b)^(2/
3) - 18*a*b*f*(a/b)^(2/3) + 2*a^2*h*(a/b)^(1/3) - 14*a*b*(a/b)^(1/3)*e + 20
*a*b*d - 5*a^2*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)
) + 1/27*(27*b^2*c*(a/b)^(2/3) - 9*a*b*f*(a/b)^(2/3) - 2*a^2*h*(a/b)^(1/3)
+ 14*a*b*(a/b)^(1/3)*e - 20*a*b*d + 5*a^2*g)*log(x + (a/b)^(1/3))/(a^4*b*(a
/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 72.05, size = 16697, normalized size = 42.27

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] -1/108*(24*(7*a*b^2*e - a^2*b*h)*x^8 + 30*(4*a*b^2*d - a^2*b*g)*x^7 + 36*(3
*a*b^2*c - a^2*b*f)*x^6 + 108*a^3*e*x^2 + 42*(7*a^2*b*e - a^3*h)*x^5 + 54*a
^3*d*x + 48*(4*a^2*b*d - a^3*g)*x^4 + 36*a^3*c + 54*(3*a^2*b*c - a^3*f)*x^3
 + 2*(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*
(81*(3*b*c - a*f)^2/a^8 - (729*b^3*c^2 + 10*a^3*g*h + (81*f^2 - 70*e*g - 40
*d*h)*a^2*b + 2*(140*d*e - 243*c*f)*a*b^2)/(a^8*b))/(1458*(3*b*c - a*f)^3/a
^12 - 27*(729*b^3*c^2 + 10*a^3*g*h + (81*f^2 - 70*e*g - 40*d*h)*a^2*b + 2*(
140*d*e - 243*c*f)*a*b^2)*(3*b*c - a*f)/(a^12*b) - (8000*b^4*d^3 + 2744*a*b
^3*e^3 - 6000*a*b^3*d^2*g + 1500*a^2*b^2*d*g^2 - 125*a^3*b*g^3 - 1176*a^2*b
^2*e^2*h + 168*a^3*b*e*h^2 - 8*a^4*h^3)/(a^11*b^2) + (19683*b^5*c^3 - 8*a^5
*h^3 + (125*g^3 - 270*f*g*h + 168*e*h^2)*a^4*b - 3*(243*f^3 - 630*e*f*g + 3
92*e^2*h - 270*c*g*h + 20*(25*g^2 - 18*f*h)*d)*a^3*b^2 + (2744*e^3 - 7560*d
*e*f + 6000*d^2*g + 81*(81*f^2 - 70*e*g - 40*d*h)*c)*a^2*b^3 - (8000*d^3 -
22680*c*d*e + 19683*c^2*f ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 431, normalized size = 1.09

$$\frac{\sqrt{3}\left(20\,b^{2}d-5\,abg+2\,(-ab^{2})^{\frac{1}{3}}\,ah-14\,(-ab^{2})^{\frac{1}{3}}\,be\right)\arctan\left(\frac{\sqrt{3}\left(2x+(-\frac{a}{b})^{\frac{1}{3}}\right)}{3\,(-\frac{a}{b})^{\frac{1}{3}}}\right)}{27\,(-ab^{2})^{\frac{2}{3}}\,a^{3}}+\frac{\left(20\,b^{2}d-5\,abg-2\,(-ab^{2})^{\frac{1}{3}}\,ah+14\,(-ab^{2})^{\frac{1}{3}}\,be\right)\log\left(x^{2}+x(-\frac{a}{b})^{\frac{1}{3}}+(-\frac{a}{b})^{\frac{2}{3}}\right)}{54\,(-ab^{2})^{\frac{2}{3}}\,a^{3}}+\frac{(3\,bc-af)\log\left(|bx^{3}+a|\right)}{3\,a^{4}}-\frac{(3\,bc-af)\log\left(|x|\right)}{a^{4}}-\frac{\left(2\,a^{6}bh(-\frac{a}{b})^{\frac{1}{3}}-14\,a^{5}b^{2}(-\frac{a}{b})^{\frac{1}{3}}\,e-20\,a^{5}b^{2}d+5\,a^{6}bg\right)(-\frac{a}{b})^{\frac{1}{3}}\log\left(\left|x-(-\frac{a}{b})^{\frac{1}{3}}\right|\right)}{27\,a^{9}b}+\frac{4\,(a^{2}bh-7\,ab^{2}e)x^{8}-5\,(4\,ab^{2}d-a^{2}bg)x^{7}-6\,(3\,ab^{2}c-a^{2}bf)x^{6}+7\,(a^{3}h-7\,a^{2}be)x^{5}-18\,a^{3}x^{2}e-9\,a^{3}dx-8\,(4\,a^{2}bd-a^{3}g)x^{4}-6\,a^{3}c-9\,(3\,a^{2}bc-a^{3}f)x^{3}}{18\,(bx^{3}+a)^{2}a^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] 1/27*sqrt(3)*(20*b^2*d - 5*a*b*g + 2*(-a*b^2)^(1/3)*a*h - 14*(-a*b^2)^(1/3)
*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)
*a^3) + 1/54*(20*b^2*d - 5*a*b*g - 2*(-a*b^2)^(1/3)*a*h + 14*(-a*b^2)^(1/3)
*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) + 1/3*(
3*b*c - a*f)*log(abs(b*x^3 + a))/a^4 - (3*b*c - a*f)*log(abs(x))/a^4 - 1/27
*(2*a^6*b*h*(-a/b)^(1/3) - 14*a^5*b^2*(-a/b)^(1/3)*e - 20*a^5*b^2*d + 5*a^6
*b*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b) + 1/18*(4*(a^2*b*h -
7*a*b^2*e)*x^8 - 5*(4*a*b^2*d - a^2*b*g)*x^7 - 6*(3*a*b^2*c - a^2*b*f)*x^6
+ 7*(a^3*h - 7*a^2*b*e)*x^5 - 18*a^3*x^2*e - 9*a^3*d*x - 8*(4*a^2*b*d - a^3
*g)*x^4 - 6*a^3*c - 9*(3*a^2*b*c - a^3*f)*x^3)/((b*x^3 + a)^2*a^4*x^3)
```

Mupad [B]
time = 6.32, size = 1994, normalized size = 5.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x)
```

```
[Out] symsum(log(- (1200*b^5*c*d^2 - 1134*b^5*c^2*e + 75*a^2*b^3*c*g^2 - 126*a^2*
b^3*e*f^2 - 25*a^3*b^2*f*g^2 + 18*a^3*b^2*f^2*h - 400*a*b^4*d^2*f + 162*a*b
^4*c^2*h - 108*a^2*b^3*c*f*h + 200*a^2*b^3*d*f*g - 600*a*b^4*c*d*g + 756*a*
b^4*c*e*f)/(81*a^9) - root(19683*a^12*b^2*z^3 + 19683*a^9*b^2*f*z^2 - 59049
*a^8*b^3*c*z^2 + 810*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z - 3240*a^6*b^2*d*h*z
- 39366*a^5*b^3*c*f*z + 22680*a^5*b^3*d*e*z + 6561*a^6*b^2*f^2*z + 59049*a^
4*b^4*c^2*z + 270*a^4*b*f*g*h - 22680*a*b^4*c*d*e - 1890*a^3*b^2*e*f*g - 10
80*a^3*b^2*d*f*h - 810*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*f + 5670*a^2*b^3*c*
e*g + 3240*a^2*b^3*c*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4*c^2*f + 1176*a^3*b
^2*e^2*h - 6000*a^2*b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 6561*a^2*b^3*c*f^2 + 7
29*a^3*b^2*f^3 - 2744*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8000*a*b^4*d^3 + 8*a^5*
h^3 - 19683*b^5*c^3, z, k)*((400*a^4*b^4*d^2 + 25*a^6*b^2*g^2 + 756*a^4*b^4
*c*e - 108*a^5*b^3*c*h - 200*a^5*b^3*d*g - 252*a^5*b^3*e*f + 36*a^6*b^2*f*h
)/(81*a^9) + root(19683*a^12*b^2*z^3 + 19683*a^9*b^2*f*z^2 - 59049*a^8*b^3*
c*z^2 + 810*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z - 3240*a^6*b^2*d*h*z - 39366*a
^5*b^3*c*f*z + 22680*a^5*b^3*d*e*z + 6561*a^6*b^2*f^2*z + 59049*a^4*b^4*c^2
```

*z + 270*a^4*b*f*g*h - 22680*a*b^4*c*d*e - 1890*a^3*b^2*e*f*g - 1080*a^3*b^2*d*f*h - 810*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*f + 5670*a^2*b^3*c*e*g + 3240*a^2*b^3*c*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4*c^2*f + 1176*a^3*b^2*e^2*h - 6000*a^2*b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 6561*a^2*b^3*c*f^2 + 729*a^3*b^2*f^3 - 2744*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8000*a*b^4*d^3 + 8*a^5*h^3 - 19683*b^5*c^3, z, k)*((378*a^8*b^3*e - 54*a^9*b^2*h)/(81*a^9) - (x*(52488*a^7*b^4*c - 17496*a^8*b^3*f))/(729*a^9) + 36*root(19683*a^12*b^2*z^3 + 19683*a^9*b^2*f*z^2 - 59049*a^8*b^3*c*z^2 + 810*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z - 3240*a^6*b^2*d*h*z - 39366*a^5*b^3*c*f*z + 22680*a^5*b^3*d*e*z + 6561*a^6*b^2*f^2*z + 59049*a^4*b^4*c^2*z + 270*a^4*b*f*g*h - 22680*a*b^4*c*d*e - 1890*a^3*b^2*e*f*g - 1080*a^3*b^2*d*f*h - 810*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*f + 5670*a^2*b^3*c*e*g + 3240*a^2*b^3*c*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4*c^2*f + 1176*a^3*b^2*e^2*h - 6000*a^2*b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 6561*a^2*b^3*c*f^2 + 729*a^3*b^2*f^3 - 2744*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8000*a*b^4*d^3 + 8*a^5*h^3 - 19683*b^5*c^3, z, k)*a^2*b^3*x) + (x*(26244*a^3*b^5*c^2 + 2916*a^5*b^3*f^2 - 17496*a^4*b^4*c*f + 25200*a^4*b^4*d*e - 3600*a^5*b^3*d*h - 6300*a^5*b^3*e*g + 900*a^6*b^2*g*h))/(729*a^9)) - (x*(8000*b^5*d^3 - 2744*a*b^4*e^3 + 8*a^4*b*h^3 - 125*a^3*b^2*g^3 + 1500*a^2*b^3*d*g^2 + 1176*a^2*b^3*e^2*h - 168*a^3*b^2*e*h^2 - 15120*b^5*c*d*e - 6000*a*b^4*d^2*g - 540*a^2*b^3*c*g*h - 720*a^2*b^3*d*f*h - 1260*a^2*b^3*e*f*g + 180*a^3*b^2*f*g*h + 2160*a*b^4*c*d*h + 3780*a*b^4*c*e*g + 5040*a*b^4*d*e*f))/(729*a^9))*root(19683*a^12*b^2*z^3 + 19683*a^9*b^2*f*z^2 - 59049*a^8*b^3*c*z^2 + 810*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z - 3240*a^6*b^2*d*h*z - 39366*a^5*b^3*c*f*z + 22680*a^5*b^3*d*e*z + 6561*a^6*b^2*f^2*z + 59049*a^4*b^4*c^2*z + 270*a^4*b*f*g*h - 22680*a*b^4*c*d*e - 1890*a^3*b^2*e*f*g - 1080*a^3*b^2*d*f*h - 810*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*f + 5670*a^2*b^3*c*e*g + 3240*a^2*b^3*c*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4*c^2*f + 1176*a^3*b^2*e^2*h - 6000*a^2*b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 6561*a^2*b^3*c*f^2 + 729*a^3*b^2*f^3 - 2744*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8000*a*b^4*d^3 + 8*a^5*h^3 - 19683*b^5*c^3, z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (x^3*(3*b*c - a*f))/(2*a^2) + (4*x^4*(4*b*d - a*g))/(9*a^2) + (7*x^5*(7*b*e - a*h))/(18*a^2) + (d*x)/(2*a) + (b*x^6*(3*b*c - a*f))/(3*a^3) + (5*b*x^7*(4*b*d - a*g))/(18*a^3) + (2*b*x^8*(7*b*e - a*h))/(9*a^3))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6) - (log(x)*(3*b*c - a*f))/a^4

3.430 $\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=583

$$-\frac{4ae\sqrt{a+bx^3}}{9b^2}+\frac{2cx\sqrt{a+bx^3}}{5b}+\frac{2dx^2\sqrt{a+bx^3}}{7b}+\frac{2ex^3\sqrt{a+bx^3}}{9b}-\frac{8ad\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+ 4\sqrt[4]{3}\cdots$$

```
[Out] -4/9*a*e*(b*x^3+a)^(1/2)/b^2+2/5*c*x*(b*x^3+a)^(1/2)/b+2/7*d*x^2*(b*x^3+a)^
(1/2)/b+2/9*e*x^3*(b*x^3+a)^(1/2)/b-8/7*a*d*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))+4/7*3^(1/4)*a^(4/3)*d*(a^(1/3)+b^(1/3)*x)*Elliptic
E((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-4/105*a*(a^(1/3)+
b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*c-10*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2
)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 583, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1902, 1608, 1900, 267, 1892, 224, 1891}

$$-\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(7\sqrt[3]{b}\,c-10\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{35\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{8ad\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{4ae\sqrt{a+bx^3}}{9b^2}+\frac{2cx\sqrt{a+bx^3}}{5b}+\frac{2dx^2\sqrt{a+bx^3}}{7b}+\frac{2ex^3\sqrt{a+bx^3}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (-4*a*e*Sqrt[a + b*x^3])/(9*b^2) + (2*c*x*Sqrt[a + b*x^3])/(5*b) + (2*d*x^2
*Sqrt[a + b*x^3])/(7*b) + (2*e*x^3*Sqrt[a + b*x^3])/(9*b) - (8*a*d*Sqrt[a +
 b*x^3])/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[
2 - Sqrt[3]]*a^(4/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
```

```
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*Sqrt[2 + Sqrt[3]]*a*(7*b^
(1/3)*c - 10*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) +
 b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{2\int \frac{-3aex^2+\frac{9}{2}bcx^3+\frac{9}{2}bdx^4}{\sqrt{a+bx^3}}\,dx}{9b}$$

$$= \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{2\int \frac{x^2\left(-3ae+\frac{9bcx}{2}+\frac{9}{2}bdx^2\right)}{\sqrt{a+bx^3}}\,dx}{9b}$$

$$= \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{4\int \frac{-9abdx-\frac{21}{2}abex^2+\frac{63}{4}b^2cx^3}{\sqrt{a+bx^3}}\,dx}{63b^2}$$

$$= \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{4\int \frac{x\left(-9abd-\frac{21}{2}abex+\frac{63}{4}b^2cx^2\right)}{\sqrt{a+bx^3}}\,dx}{63b^2}$$

$$= \frac{2cx\sqrt{a+bx^3}}{5b} + \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{8\int \frac{-\frac{63}{4}ab^2c-\frac{45}{2}ab^2dx-\frac{105}{4}ab^2ex^2}{\sqrt{a+bx^3}}\,}{315b^3}$$

$$= \frac{2cx\sqrt{a+bx^3}}{5b} + \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{8\int \frac{-\frac{63}{4}ab^2c-\frac{45}{2}ab^2dx}{\sqrt{a+bx^3}}\,dx}{315b^3} - \frac{(2a}{}$$

$$= -\frac{4ae\sqrt{a+bx^3}}{9b^2} + \frac{2cx\sqrt{a+bx^3}}{5b} + \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} - \frac{(4ad)\int}{}$$

$$= -\frac{4ae\sqrt{a+bx^3}}{9b^2} + \frac{2cx\sqrt{a+bx^3}}{5b} + \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} - \frac{}{7b^{5/3}\left(\left(\right.\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.10, size = 132, normalized size = 0.23

$$\frac{-2(a+bx^3)(70ae-bx(63c+5x(9d+7ex)))-126abcx\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-90abdx^2\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{315b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (-2*(a + b*x^3)*(70*a*e - b*x*(63*c + 5*x*(9*d + 7*e*x))) - 126*a*b*c*x*Sqr
t[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 90*a*b*d*
```

```
x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(31
5*b^2*Sqrt[a + b*x^3])
```

Maple [A]
time = 0.38, size = 793, normalized size = 1.36 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(2/9*x^3*(b*x^3+a)^(1/2)/b-4/9*a*(b*x^3+a)^(1/2)/b^2)+d*(2/7*x^2*(b*x^3+a
)^(1/2)/b+8/21*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b
^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+c*(2/5*x*
(b*x^3+a)^(1/2)/b+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d*x + c)*x^3/sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 81, normalized size = 0.14

$$-\frac{2\left(126\,a\sqrt{b}\,c\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-180\,a\sqrt{b}\,d\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\left(35\,bex^3+45\,bdx^2+63\,bcx-70\,ae\right)\sqrt{bx^3+a}\right)}{315\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(126*a*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 180*a*sqrt(b)*d
*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (35*b*e*x^
3 + 45*b*d*x^2 + 63*b*c*x - 70*a*e)*sqrt(b*x^3 + a))/b^2
```

Sympy [A]
time = 1.91, size = 129, normalized size = 0.22

$$e\left(\begin{cases}-\frac{4a\sqrt{a+bx^3}}{9b^2}+\frac{2x^3\sqrt{a+bx^3}}{9b} & \text{for } b\neq 0\\ \frac{x^6}{6\sqrt{a}} & \text{otherwise}\end{cases}\right)+\frac{cx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)}+\frac{dx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] e*Piecewise((-4*a*sqrt(a + b*x**3)/(9*b**2) + 2*x**3*sqrt(a + b*x**3)/(9*b)
, Ne(b, 0)), (x**6/(6*sqrt(a)), True)) + c*x**4*gamma(4/3)*hyper((1/2, 4/3)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + d*x**5*gamma(5
/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/
3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)*x^3/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(e\,x^2+d\,x+c\right)}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int((x^3*(c + d*x + e*x^2))/(a + b*x^3)^(1/2), x)
```

3.431 $\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=560

$$\frac{2c\sqrt{a+bx^3}}{3b}+\frac{2dx\sqrt{a+bx^3}}{5b}+\frac{2ex^2\sqrt{a+bx^3}}{7b}-\frac{8ae\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}e\big(}{}$$

```
[Out] 2/3*c*(b*x^3+a)^(1/2)/b+2/5*d*x*(b*x^3+a)^(1/2)/b+2/7*e*x^2*(b*x^3+a)^(1/2)
/b-8/7*a*e*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+4/7*3^(1
/4)*a^(4/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)-4/105*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/
3)*d-10*a^(1/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(
5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1902, 1608, 1900, 267, 1892, 224, 1891}

$$-\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(7\sqrt[3]{b}\,d-10\left(1-\sqrt{3}\right)\sqrt[3]{a}\,e\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{35\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{8ae\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2c\sqrt{a+bx^3}}{3b}+\frac{2dx\sqrt{a+bx^3}}{5b}+\frac{2ex^2\sqrt{a+bx^3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*c*Sqrt[a + b*x^3])/(3*b) + (2*d*x*Sqrt[a + b*x^3])/(5*b) + (2*e*x^2*Sqrt
[a + b*x^3])/(7*b) - (8*a*e*Sqrt[a + b*x^3])/(7*b^(5/3)*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
```

```
x^3]) - (4*Sqrt[2 + Sqrt[3]]*a*(7*b^(1/3)*d - 10*(1 - Sqrt[3])*a^(1/3)*e)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1
/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \frac{2ex^2\sqrt{a+bx^3}}{7b} + \frac{2\int \frac{-2aex+\frac{7}{2}bcx^2+\frac{7}{2}bdx^3}{\sqrt{a+bx^3}}\,dx}{7b}$$

$$= \frac{2ex^2\sqrt{a+bx^3}}{7b} + \frac{2\int \frac{x\left(-2ae+\frac{7bcx}{2}+\frac{7}{2}bdx^2\right)}{\sqrt{a+bx^3}}\,dx}{7b}$$

$$= \frac{2dx\sqrt{a+bx^3}}{5b} + \frac{2ex^2\sqrt{a+bx^3}}{7b} + \frac{4\int \frac{-\frac{7}{2}abd-5abex+\frac{35}{4}b^2cx^2}{\sqrt{a+bx^3}}\,dx}{35b^2}$$

$$= \frac{2dx\sqrt{a+bx^3}}{5b} + \frac{2ex^2\sqrt{a+bx^3}}{7b} + \frac{4\int \frac{-\frac{7}{2}abd-5abex}{\sqrt{a+bx^3}}\,dx}{35b^2} + c\int \frac{x^2}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2c\sqrt{a+bx^3}}{3b} + \frac{2dx\sqrt{a+bx^3}}{5b} + \frac{2ex^2\sqrt{a+bx^3}}{7b} - \frac{(4ae)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,d}{7b^{4/3}}$$

$$= \frac{2c\sqrt{a+bx^3}}{3b} + \frac{2dx\sqrt{a+bx^3}}{5b} + \frac{2ex^2\sqrt{a+bx^3}}{7b} - \frac{8ae\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.07, size = 121, normalized size = 0.22

$$\frac{2(a+bx^3)(35c+3x(7d+5ex))-42adx\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-30aex^2\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{105b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(a + b*x^3)*(35*c + 3*x*(7*d + 5*e*x)) - 42*a*d*x*Sqrt[1 + (b*x^3)/a]*Hy
pergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 30*a*e*x^2*Sqrt[1 + (b*x^3)/
a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(105*b*Sqrt[a + b*x^3])
```

Maple [A]
time = 0.40, size = 773, normalized size = 1.38 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(2/7*x^2*(b*x^3+a)^(1/2)/b+8/21*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))))+d*(2/5*x*(b*x^3+a)^(1/2)/b+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+2/3*c*(b*x^3+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

[Out] 2/3*sqrt(b*x^3 + a)*c/b + integrate((x^4*e + d*x^3)/sqrt(b*x^3 + a), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 74, normalized size = 0.13

$$-\frac{2\left(42\,a\sqrt{b}\,d\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-60\,a\sqrt{b}\,e\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-\left(15\,bex^2+21\,bdx+35\,bc\right)\sqrt{bx^3+a}\right)}{105\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(42*a*sqrt(b)*d*weierstrassPInverse(0, -4*a/b, x) - 60*a*sqrt(b)*e*w
eierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (15*b*e*x^2
+ 21*b*d*x + 35*b*c)*sqrt(b*x^3 + a))/b^2
```

Sympy [A]
time = 1.80, size = 107, normalized size = 0.19

$$c\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for}\ b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right)+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)}+\frac{ex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] c*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True))
 + d*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3
*sqrt(a)*gamma(7/3)) + e*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)*x^2/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\left(e\,x^2+d\,x+c\right)}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^(1/2), x)
```

3.432 $\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=537

$$\frac{2d\sqrt{a+bx^3}}{3b}+\frac{2ex\sqrt{a+bx^3}}{5b}+\frac{2c\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/}}{((1}}}{b^{2/3}\sqrt{\frac{}{((}}}$$

```
[Out] 2/3*d*(b*x^3+a)^(1/2)/b+2/5*e*x*(b*x^3+a)^(1/2)/b+2*c*(b*x^3+a)^(1/2)/b^(2/
3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)*c*(a^(1/3)+b^(1/3)*x)*El
lipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-2/15*3^(3/4
)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(2*a^(2/3)*e+5*b^(2/3)*c*(1-3^(
1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1902, 1900, 267, 1892, 224, 1891}

$$-\frac{2\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(2a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2c\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2d\sqrt{a+bx^3}}{3b}+\frac{2ex\sqrt{a+bx^3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*d*Sqrt[a + b*x^3])/(3*b) + (2*e*x*Sqrt[a + b*x^3])/(5*b) + (2*c*Sqrt[a +
 b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 -
Sqrt[3]]*a^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*a^(1/3)*(5*(1
 - Sqrt[3])*b^(2/3)*c + 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
```

```
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx &= \frac{2ex\sqrt{a+bx^3}}{5b}+\frac{2\int \frac{-ae+\frac{5bcx}{2}+\frac{5}{2}bdx^2}{\sqrt{a+bx^3}}\,dx}{5b}\\ &= \frac{2ex\sqrt{a+bx^3}}{5b}+\frac{2\int \frac{-ae+\frac{5bcx}{2}}{\sqrt{a+bx^3}}\,dx}{5b}+d\int \frac{x^2}{\sqrt{a+bx^3}}\,dx\\ &= \frac{2d\sqrt{a+bx^3}}{3b}+\frac{2ex\sqrt{a+bx^3}}{5b}+\frac{c\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}}-\frac{\left(\sqrt[3]{a}\left(5\left(1-\sqrt{3}\right)\right.\right.\ldots}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\ldots}\\ &= \frac{2d\sqrt{a+bx^3}}{3b}+\frac{2ex\sqrt{a+bx^3}}{5b}+\frac{2c\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\ldots\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 114, normalized size = 0.21

$$\frac{4(5d+3ex)\left(a+bx^3\right)-12aex\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+15bcx^2\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{30b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (4*(5*d + 3*e*x)*(a + b*x^3) - 12*a*e*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 15*b*c*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(30*b*Sqrt[a + b*x^3])
```

Maple [A]
time = 0.40, size = 753, normalized size = 1.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(2/5*x*(b*x^3+a)^(1/2)/b+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))+2/3*d*(b*x^3+a)^(1/2)/b-2/3*I*c*3^(1/2)/b*(-a*b
^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d*x + c)*x/sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 66, normalized size = 0.12

$$-\frac{2\left(6\,a\sqrt{b}\,e\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+15\,b^{\frac{3}{2}}c\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\sqrt{bx^3+a}\,(3\,bex+5\,bd)\right)}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(6*a*sqrt(b)*e*weierstrassPInverse(0, -4*a/b, x) + 15*b^(3/2)*c*weier
strassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - sqrt(b*x^3 + a)*
(3*b*e*x + 5*b*d))/b^2
```

Sympy [A]
time = 1.77, size = 107, normalized size = 0.20

$$d\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for } b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right)+\frac{cx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] d*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True))
 + c*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3
*sqrt(a)*gamma(5/3)) + e*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)*x/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(e\,x^2+d\,x+c\right)}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int((x*(c + d*x + e*x^2))/(a + b*x^3)^(1/2), x)
```

3.433 $\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=509

$$\frac{2e\sqrt{a+bx^3}}{3b}+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x\,-}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\,+\right.}}}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}\,+\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}}}$$

```
[Out] 2/3*e*(b*x^3+a)^(1/2)/b+2*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))-3^(1/4)*a^(1/3)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^
(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*(b^(1/3)*c-a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)
/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1900, 267, 1892, 224, 1891}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2e\sqrt{a+bx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b
*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
```

```
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx = e\int \frac{x^2}{\sqrt{a+bx^3}}\,dx + \int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2e\sqrt{a+bx^3}}{3b} + \frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} + \left(c - \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}$$

$$= \frac{2e\sqrt{a+bx^3}}{3b} + \frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 107, normalized size = 0.21

$$\frac{4e(a+bx^3)+6bcx\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+3bdx^2\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{6b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x^3],x]
```

```
[Out] (4*e*(a + b*x^3) + 6*b*c*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2,
4/3, -((b*x^3)/a)] + 3*b*d*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2
/3, 5/3, -((b*x^3)/a)])/(6*b*Sqrt[a + b*x^3])
```

Maple [A]
time = 0.39, size = 735, normalized size = 1.44 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*e*(b*x^3+a)^(1/2)/b-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
```

```
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
-2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2*e + d*x + c)/sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 55, normalized size = 0.11

$$\frac{2\left(3\sqrt{b}\,c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-3\sqrt{b}\,d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\sqrt{bx^3+a}\,e\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] 2/3*(3*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 3*sqrt(b)*d*weierstras
sZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + sqrt(b*x^3 + a)*e)/b
```

Sympy [A]
time = 1.29, size = 105, normalized size = 0.21

$$e\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for}\ b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right)+\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True))
 + c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d\,x+c}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^3)^(1/2),x)
```

```
[Out] int((c + d*x + e*x^2)/(a + b*x^3)^(1/2), x)
```

3.434 $\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=518

$$\frac{2e\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\cdots}{\left(\left(1+\sqrt{\cdots}\right.\right.}}}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\cdots}{\left(\left(1+\cdots\right.\right.}}}$$

```
[Out] -2/3*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+2*e*(b*x^3+a)^(1/2)/b^(2/3)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)*e*(a^(1/3)+b^(1/3)*x)*Elli
pticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/3*(a^(1/3)+
b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*d-a^(1/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2
*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1846, 272, 65, 214, 1892, 224, 1891}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{b}\,d-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,e\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2e\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*e*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*c*
ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*
a^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]
])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - (1 - Sqrt
[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
```

```
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
```

```
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx &= c\int \frac{1}{x\sqrt{a+bx^3}}\,dx+\int \frac{d+ex}{\sqrt{a+bx^3}}\,dx \\
&= \frac{1}{3}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)+\frac{e\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}}+\left(d-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{b}}\cdots\right. \\
&= \frac{2e\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}\cdots}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\cdots\right.}{\left(\left(1+\sqrt{3}\cdots\right.\right.}}\cdots} \\
&= \frac{2e\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,e\left(\sqrt[3]{\cdots}\right.}{\cdots}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.69, size = 493, normalized size = 0.95

$$
-\frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{2d\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}-\frac{2\sqrt{2}\sqrt[3]{a}\,e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}}\left(\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)+F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x + e*x^2)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (2*d*((-1)^(1/3)*a^(1
/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqr
t[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*E
llipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(
1/3))]], (-1)^(1/3)])/(b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]) - (2*Sqrt[2]*a^(1/3)*e*((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(
(-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]
/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6)
 - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(b^(2/3
)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a
+ b*x^3])
```

Maple [A]
time = 0.38, size = 740, normalized size = 1.43 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*I*d*3^(1/2)/b*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*c*arctanh((b*x^3+a)^(
1/2)/a^(1/2))/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d*x + c)/(sqrt(b*x^3 + a)*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 190, normalized size = 0.37

$$\left[\frac{\sqrt{a}\, bc \log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\,\sqrt{a}+8\,a^2}{x^6}\right)+12\,a\sqrt{b}\,d\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-12\,a\sqrt{b}\,e\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{6\,ab}, \frac{\sqrt{-a}\, bc \arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-a}}{bx^3+2\,a}\right)+6\,a\sqrt{b}\,d\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-6\,a\sqrt{b}\,e\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{3\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*b*c*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 +
a)*sqrt(a) + 8*a^2)/x^6) + 12*a*sqrt(b)*d*weierstrassPInverse(0, -4*a/b, x)
 - 12*a*sqrt(b)*e*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b,
 x)))/(a*b), 1/3*(sqrt(-a)*b*c*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2
*a)) + 6*a*sqrt(b)*d*weierstrassPInverse(0, -4*a/b, x) - 6*a*sqrt(b)*e*weie
rstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/(a*b)]
```

Sympy [A]
time = 2.25, size = 105, normalized size = 0.20

$$-\frac{2c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}+\frac{dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{ex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(1/2),x)
```

```
[Out] -2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)) + d*x*gamma(1/3)*hyper((
1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + e*x**
2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)
*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)/(sqrt(b*x^3 + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d\,x+c}{x\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)^(1/2)),x)
[Out] int((c + d*x + e*x^2)/(x*(a + b*x^3)^(1/2)), x)
```

3.435 $\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=547

$$-\frac{c\sqrt{a+bx^3}}{ax}+\frac{\sqrt[3]{b}\,c\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2}$$

```
[Out] -2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-c*(b*x^3+a)^(1/2)/a/x+b^(1/
3)*c*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-1/2*3^(1/4)*b^(1/3)*
c*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3
)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)-1/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(-2*a^(2/3)*e+b^(2/3)*c*
(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(1/3)/(b*x^
3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{c\sqrt{a+bx^3}}{ax}+\frac{\sqrt[3]{b}\,c\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] -((c*Sqrt[a + b*x^3])/(a*x)) + (b^(1/3)*c*Sqrt[a + b*x^3])/(a*((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]
) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (Sqrt[2 +
```

```
Sqrt[3]]*((1 - Sqrt[3])*b^(2/3)*c - 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*b^(1/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
```

```
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = -\frac{c\sqrt{a+bx^3}}{ax} - \frac{\int \frac{-2ad-2aex-bcx^2}{x\sqrt{a+bx^3}}\,dx}{2a}$$

$$= -\frac{c\sqrt{a+bx^3}}{ax} - \frac{\int \frac{-2ae-bcx}{\sqrt{a+bx^3}}\,dx}{2a} + d\int \frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$= -\frac{c\sqrt{a+bx^3}}{ax} + \frac{\left(b^{2/3}c\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{2a} + \frac{1}{3}d\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right.$$

$$= -\frac{c\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}\,c\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a}$$

$$= -\frac{c\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}\,c\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{\sqrt[4]{3}\sqrt{2}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.64, size = 513, normalized size = 0.94

$$-\frac{c\sqrt{a+bx^3}}{ax}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{2e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}-\frac{\sqrt{2}\sqrt[3]{b}\,c\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}}\left(\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)+F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] -((c*Sqrt[a + b*x^3])/(a*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sq
rt[a]) - (2*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)
/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(b^(1/3)*Sqrt[(a^(1/3)
+ (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]) - (Sqr
```

```
t[2]*b^(1/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) -
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/
a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1
/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))] + Elli
pticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/
(-1 + (-1)^(1/3))]))/(a^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [A]
time = 0.39, size = 759, normalized size = 1.39 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/
3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+c*(-(b*x^3+a)^(1/2)/a/x-1/3*I/
a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d*x + c)/(sqrt(b*x^3 + a)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 235, normalized size = 0.43

$$\left[\frac{\sqrt{a}\, b d x \log\left(\frac{b^{2} x^{6} + 8\, a b x^{3} - 4\, {\left(b x^{3} + 2\, a\right)} \sqrt{b x^{3} + a} \sqrt{a} + 8\, a^{2}}{x^{6}}\right) + 12\, a \sqrt{b} e x {\rm weierstrassPInverse}\left(0, -\frac{4\, a}{b}, x\right) - 6\, b^{\frac{3}{2}} c x {\rm weierstrassZeta}\left(0, -\frac{4\, a}{b}, {\rm weierstrassPInverse}\left(0, -\frac{4\, a}{b}, x\right)\right) - 6\, \sqrt{b x^{3} + a} b c}{6\, a b x}, \frac{\sqrt{-a}\, b d x \arctan\left(\frac{{\left(b x^{3} + 2\, a\right)} \sqrt{b x^{3} + a} \sqrt{-a}}{2\, {\left(a b x^{3} + a^{2}\right)}}\right) + 6\, a \sqrt{b} e x {\rm weierstrassPInverse}\left(0, -\frac{4\, a}{b}, x\right) - 3\, b^{\frac{3}{2}} c x {\rm weierstrassZeta}\left(0, -\frac{4\, a}{b}, {\rm weierstrassPInverse}\left(0, -\frac{4\, a}{b}, x\right)\right) - 3\, \sqrt{b x^{3} + a} b c}{3\, a b x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*b*d*x*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 +
 a)*sqrt(a) + 8*a^2)/x^6) + 12*a*sqrt(b)*e*x*weierstrassPInverse(0, -4*a/b,
 x) - 6*b^(3/2)*c*x*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/
b, x)) - 6*sqrt(b*x^3 + a)*b*c)/(a*b*x), 1/3*(sqrt(-a)*b*d*x*arctan(1/2*(b*
x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) + 6*a*sqrt(b)*e*x*weie
rstrassPInverse(0, -4*a/b, x) - 3*b^(3/2)*c*x*weierstrassZeta(0, -4*a/b, we
ierstrassPInverse(0, -4*a/b, x)) - 3*sqrt(b*x^3 + a)*b*c)/(a*b*x)]
```

Sympy [A]
time = 1.62, size = 107, normalized size = 0.20

$$\frac{c\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,x\Gamma\left(\frac{2}{3}\right)}-\frac{2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}+\frac{ex\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(1/2),x)
```

```
[Out] c*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x*gamma(2/3)) - 2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)) + e*x*
gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*g
amma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)/(sqrt(b*x^3 + a)*x^2), x)
```

Mupad [B]
time = 5.96, size = 121, normalized size = 0.22

$$\frac{d\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,\sqrt{a}}-\frac{2\,c\,\sqrt{\frac{a}{b\,x^3}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{6};\ \frac{11}{6};\ -\frac{a}{b\,x^3}\right)}{5\,x\,\sqrt{b\,x^3+a}}+\frac{e\,x\,\sqrt{\frac{b\,x^3}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^(1/2)),x)
```

```
[Out] (d*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)
)/(3*a^(1/2)) - (2*c*(a/(b*x^3) + 1)^(1/2)*hypergeom([1/2, 5/6], 11/6, -a/(
b*x^3)))/(5*x*(a + b*x^3)^(1/2)) + (e*x*((b*x^3)/a + 1)^(1/2)*hypergeom([1/
3, 1/2], 4/3, -(b*x^3)/a))/(a + b*x^3)^(1/2)
```

3.436 $\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=569

$$-\frac{c\sqrt{a+bx^3}}{2ax^2}-\frac{d\sqrt{a+bx^3}}{ax}+\frac{\sqrt[3]{b}\,d\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}}{}$$

```
[Out] -2/3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/2*c*(b*x^3+a)^(1/2)/a/x^2
-d*(b*x^3+a)^(1/2)/a/x+b^(1/3)*d*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))-1/2*3^(1/4)*b^(1/3)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1
/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2
)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/6*3^(3/4)*b^(1/3)*(a^(1/3)+b^(1/
3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*(b^(1/3)*c+2*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$-\frac{\sqrt{2+\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(2\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{c\sqrt{a+bx^3}}{2ax^2}-\frac{d\sqrt{a+bx^3}}{ax}+\frac{\sqrt[3]{b}\,d\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/2*(c*Sqrt[a + b*x^3])/(a*x^2) - (d*Sqrt[a + b*x^3])/(a*x) + (b^(1/3)*d*S
qrt[a + b*x^3])/(a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*e*ArcTanh[Sqrt
[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*d*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a^(2/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
```

```
2]*Sqrt[a + b*x^3]) - (Sqrt[2 + Sqrt[3]]*b^(1/3)*(b^(1/3)*c + 2*(1 - Sqrt[3
])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
```

```
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx = -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{\int \frac{-4ad-4aex+bcx^2}{x^2\sqrt{a+bx^3}}\,dx}{4a}$$

$$= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\int \frac{8a^2e-2abcx+4abdx^2}{x\sqrt{a+bx^3}}\,dx}{8a^2}$$

$$= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\int \frac{-2abc+4abdx}{\sqrt{a+bx^3}}\,dx}{8a^2} + e\int \frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\left(b^{2/3}d\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{2a} - \frac{\left(b^{2/3}\left(\sqrt[3]{b}\,c+2\left(1\right.\right.\right.}{}$$

$$= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}\,d\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\,d\left(\right.}{}$$

$$= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}\,d\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right.}{3\sqrt{a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.34, size = 525, normalized size = 0.92

$$-\frac{(c+2dx)\sqrt{a+bx^3}}{2ax^2} - \frac{2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} + \frac{b^{2/3}c\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{2a\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}} - \frac{\sqrt{2}\sqrt[3]{b}\,d\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}}\left(\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)+F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/2*((c + 2*d*x)*Sqrt[a + b*x^3])/(a*x^2) - (2*e*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]])/(3*Sqrt[a]) + (b^(2/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^
(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (
-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(
```

```
1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(2*
a*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a
+ b*x^3]) - (Sqrt[2]*b^(1/3)*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^
(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(
1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*EllipticE[Arc
Sin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1
)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/
4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(a^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [**A**]
time = 0.39, size = 778, normalized size = 1.37 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/2*(b*x^3+a)^(1/2)/a/x^2+1/6*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)))-2/3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+d*
(-(b*x^3+a)^(1/2)/a/x-1/3*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/
b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d*x + c)/(sqrt(b*x^3 + a)*x^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 246, normalized size = 0.43

$$\left[\frac{\sqrt{a}\,ex^{2}\log\left(\frac{b^{2}x^{6}+8\,abx^{3}-4\,(bx^{3}+2\,a)\sqrt{bx^{3}+a}\,\sqrt{a}+8\,a^{2}}{x^{6}}\right)-3\,\sqrt{b}\,cx^{2}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-6\,\sqrt{b}\,dx^{2}\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-3\,\sqrt{bx^{3}+a}\,(2\,dx+c)}{6\,ax^{2}},\ \frac{2\,\sqrt{-a}\,ex^{2}\arctan\left(\frac{(bx^{3}+2\,a)\sqrt{bx^{3}+a}\,\sqrt{-a}}{2\,(abx^{3}+a^{2})}\right)-3\,\sqrt{b}\,cx^{2}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-6\,\sqrt{b}\,dx^{2}\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-3\,\sqrt{bx^{3}+a}\,(2\,dx+c)}{6\,ax^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*e*x^2*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 +
 a)*sqrt(a) + 8*a^2)/x^6) - 3*sqrt(b)*c*x^2*weierstrassPInverse(0, -4*a/b,
x) - 6*sqrt(b)*d*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a
/b, x)) - 3*sqrt(b*x^3 + a)*(2*d*x + c))/(a*x^2), 1/6*(2*sqrt(-a)*e*x^2*arc
tan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) - 3*sqrt(b)
*c*x^2*weierstrassPInverse(0, -4*a/b, x) - 6*sqrt(b)*d*x^2*weierstrassZeta(
0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - 3*sqrt(b*x^3 + a)*(2*d*x +
c))/(a*x^2)]
```

Sympy [A]
time = 1.81, size = 112, normalized size = 0.20

$$\frac{c\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,x^{2}\Gamma\left(\frac{1}{3}\right)}+\frac{d\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,x\Gamma\left(\frac{2}{3}\right)}-\frac{2e\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**(1/2),x)
```

```
[Out] c*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x**2*gamma(1/3)) + d*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*x*gamma(2/3)) - 2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2
)))/(3*sqrt(a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)/(sqrt(b*x^3 + a)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d\,x+c}{x^3\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^(1/2)),x)
[Out] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^(1/2)), x)
```

3.437 $\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=594

4

$$\frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}}+\frac{4c\sqrt{a+bx^3}}{3b^2}+\frac{2dx\sqrt{a+bx^3}}{5b^2}+\frac{2ex^2\sqrt{a+bx^3}}{7b^2}-\frac{80ae\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+-$$

```
[Out] 2/3*x*(-b*c*x^2+a*e*x+a*d)/b^2/(b*x^3+a)^(1/2)+4/3*c*(b*x^3+a)^(1/2)/b^2+2/
5*d*x*(b*x^3+a)^(1/2)/b^2+2/7*e*x^2*(b*x^3+a)^(1/2)/b^2-80/21*a*e*(b*x^3+a)
^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+40/21*a^(4/3)*e*(a^(1/3)+b^(
1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(8/3)/(b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)-16/315*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(14*b^(1/3)*d-25*a^(1/3)*
e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(8/3)/(b*x^3+a)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$,
Rules used = {1842, 1902, 1900, 267, 1892, 224, 1891}

$$-\frac{16\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(14\sqrt[3]{b}\,d-25\left(1-\sqrt{3}\right)\sqrt[3]{a}\,e\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{105\sqrt[4]{3}\,b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{40\sqrt{2-\sqrt{3}}\,a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\,3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{80ae\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}}+\frac{4c\sqrt{a+bx^3}}{3b^2}+\frac{2dx\sqrt{a+bx^3}}{5b^2}+\frac{2ex^2\sqrt{a+bx^3}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*c*Sqrt[a + b*x^3
])/(3*b^2) + (2*d*x*Sqrt[a + b*x^3])/(5*b^2) + (2*e*x^2*Sqrt[a + b*x^3])/(7
*b^2) - (80*a*e*Sqrt[a + b*x^3])/(21*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) + (40*Sqrt[2 - Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3)
```

```
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (1
6*Sqrt[2 + Sqrt[3]]*a*(14*b^(1/3)*d - 25*(1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(105*3^(1/4)*b^(
8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}} - \frac{2\int \frac{a^2bd+2a^2bex-3ab^2cx^2-\frac{3}{2}ab^2dx^3-\frac{3}{2}ab^2ex^4}{\sqrt{a+bx^3}}\,dx}{3ab^3}$$

$$= \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2} - \frac{4\int \frac{\frac{7}{2}a^2b^2d+10a^2b^2ex-\frac{21}{2}ab^3cx^2-\frac{21}{4}ab^3dx^3}{\sqrt{a+bx^3}}\,dx}{21ab^4}$$

$$= \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}} + \frac{2dx\sqrt{a+bx^3}}{5b^2} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2} - \frac{8\int \frac{14a^2b^3d+25a^2b^3ex-\frac{10\cdots}{4}}{\sqrt{a+bx^3}}}{105ab^5}$$

$$= \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}} + \frac{2dx\sqrt{a+bx^3}}{5b^2} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2} - \frac{8\int \frac{14a^2b^3d+25a^2b^3ex}{\sqrt{a+bx^3}}\,dx}{105ab^5}$$

$$= \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}} + \frac{4c\sqrt{a+bx^3}}{3b^2} + \frac{2dx\sqrt{a+bx^3}}{5b^2} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2} - \frac{(40a\cdots}{}$$

$$= \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}} + \frac{4c\sqrt{a+bx^3}}{3b^2} + \frac{2dx\sqrt{a+bx^3}}{5b^2} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2} - \frac{}{21b^{8/\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 134, normalized size = 0.23

$$\frac{2\left(70ac+56adx-150aex^2+35bcx^3+21bdx^4+15bex^5-56adx\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+150aex^2\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{105b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(70*a*c + 56*a*d*x - 150*a*e*x^2 + 35*b*c*x^3 + 21*b*d*x^4 + 15*b*e*x^5
- 56*a*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a
)] + 150*a*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*
x^3)/a)]))/(105*b^2*Sqrt[a + b*x^3])
```

Maple [A]
time = 0.40, size = 836, normalized size = 1.41 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(2/3/b^2*a*x^2/((x^3+a/b)*b)^(1/2)+2/7*x^2*(b*x^3+a)^(1/2)/b^2+80/63*I*a/
b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+d*(2/3/b^2*a*x/((x^3+a/b)*b)
^(1/2)+2/5*x*(b*x^3+a)^(1/2)/b^2+32/45*I*a/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+c*(2/3/b^2*a/((x^3+a/b)*b)^(1/2)+2/3*(b*
x^3+a)^(1/2)/b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*c*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2)) + integrate((x^7*e +
d*x^6)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 138, normalized size = 0.23

$$-\frac{2\left(112\,(abdx^3+a^2d)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-200\,(abex^3+a^2e)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-(15\,b^2ex^5+21\,b^2dx^4+35\,b^2cx^3+50\,abex^2+56\,abdx+70\,abc)\sqrt{bx^3+a}\right)}{105\,(b^4x^3+ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/105*(112*(a*b*d*x^3 + a^2*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) -
 200*(a*b*e*x^3 + a^2*e)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInv
```

```
erse(0, -4*a/b, x)) - (15*b^2*e*x^5 + 21*b^2*d*x^4 + 35*b^2*c*x^3 + 50*a*b*
e*x^2 + 56*a*b*d*x + 70*a*b*c)*sqrt(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

Sympy [A]
time = 10.64, size = 129, normalized size = 0.22

$$c\left(\begin{cases}\frac{4a}{3b^2\sqrt{a+bx^3}}+\frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0\\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{dx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}+\frac{ex^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] c*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)),
 Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + d*x**7*gamma(7/3)*hyper((3/2, 7/3)
, (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(10/3)) + e*x**8*gamm
a(8/3)*hyper((3/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gam
ma(11/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)*x^5/(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(e\,x^2+d\,x+c\right)}{{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
```

```
[Out] int((x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.438 $\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=574

$$\frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}}+\frac{4d\sqrt{a+bx^3}}{3b^2}+\frac{2ex\sqrt{a+bx^3}}{5b^2}+\frac{8c\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,c\Big(}{}$$

```
[Out] 2/3*x*(-b*d*x^2-b*c*x+a*e)/b^2/(b*x^3+a)^(1/2)+4/3*d*(b*x^3+a)^(1/2)/b^2+2/
5*e*x*(b*x^3+a)^(1/2)/b^2+8/3*c*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))-4/3*a^(1/3)*c*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-8/45*a^(1/3)*(a^(1/3)+b^(1/3)
*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(4*a^(2/3)*e+5*b^(2/3)*c*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)*3^(3/4)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1842, 1902, 1900, 267, 1892, 224, 1891}

$$-\frac{8\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(4a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{8c\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}}+\frac{4d\sqrt{a+bx^3}}{3b^2}+\frac{2ex\sqrt{a+bx^3}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*d*Sqrt[a + b*x^3
])/(3*b^2) + (2*e*x*Sqrt[a + b*x^3])/(5*b^2) + (8*c*Sqrt[a + b*x^3])/(3*b^(
5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*c*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4
)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
```

```
1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2 + Sqrt[3]]*a^(1/3)*(5*(1 - Sqrt[3])
*b^(2/3)*c + 4*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(15*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}} - \frac{2\int \frac{a^2e-2abcx-3abdx^2-\frac{3}{2}abex^3}{\sqrt{a+bx^3}}\,dx}{3ab^2}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}} + \frac{2ex\sqrt{a+bx^3}}{5b^2} - \frac{4\int \frac{4a^2be-5ab^2cx-\frac{15}{2}ab^2dx^2}{\sqrt{a+bx^3}}\,dx}{15ab^3}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}} + \frac{2ex\sqrt{a+bx^3}}{5b^2} - \frac{4\int \frac{4a^2be-5ab^2cx}{\sqrt{a+bx^3}}\,dx}{15ab^3} + \frac{(2d)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{b}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}} + \frac{4d\sqrt{a+bx^3}}{3b^2} + \frac{2ex\sqrt{a+bx^3}}{5b^2} + \frac{(4c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}{\sqrt{a+bx^3}}}{3b^{4/3}}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}} + \frac{4d\sqrt{a+bx^3}}{3b^2} + \frac{2ex\sqrt{a+bx^3}}{5b^2} + \frac{8c\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 127, normalized size = 0.22

$$\frac{2\left(10ad+8aex+15bcx^2+5bdx^3+3bex^4-8aex\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-15bcx^2\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{15b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(10*a*d + 8*a*e*x + 15*b*c*x^2 + 5*b*d*x^3 + 3*b*e*x^4 - 8*a*e*x*Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 15*b*c*x^2*Sq
rt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(15*b^2*
Sqrt[a + b*x^3])
```

Maple [A]
time = 0.39, size = 817, normalized size = 1.42 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(2/3/b^2*a*x/((x^3+a/b)*b)^(1/2)+2/5*x*(b*x^3+a)^(1/2)/b^2+32/45*I*a/b^3*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(2/3/b^2*a
/((x^3+a/b)*b)^(1/2)+2/3*(b*x^3+a)^(1/2)/b^2)+c*(-2/3/b*x^2/((x^3+a/b)*b)^(
1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d*x + c)*x^4/(b*x^3 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 130, normalized size = 0.23

$$-\frac{2\left(16\,(abex^3+a^2e)\sqrt{b}\ \mathrm{weierstrassPInverse}(0,-\frac{4a}{b},x)+20\,(b^2cx^3+abc)\sqrt{b}\ \mathrm{weierstrassZeta}(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}(0,-\frac{4a}{b},x))-(3\,b^2ex^4+5\,b^2dx^3-5\,b^2cx^2+8\,abex+10\,abd)\sqrt{bx^3+a}\right)}{15\,(b^4x^3+ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(16*(a*b*e*x^3 + a^2*e)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + 2
0*(b^2*c*x^3 + a*b*c)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInvers
e(0, -4*a/b, x)) - (3*b^2*e*x^4 + 5*b^2*d*x^3 - 5*b^2*c*x^2 + 8*a*b*e*x + 1
0*a*b*d)*sqrt(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

Sympy [A]
time = 7.94, size = 129, normalized size = 0.22

$$d\left(\begin{cases}\frac{4a}{3b^2\sqrt{a+bx^3}}+\frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0\\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}+\frac{ex^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] d*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)),
 Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + c*x**5*gamma(5/3)*hyper((3/2, 5/3)
, (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3)) + e*x**7*gamma(
7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma
(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)*x^4/(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(e\,x^2+d\,x+c\right)}{{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
[Out] int((x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.439 $\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=542

$$-\frac{2x(c+dx+ex^2)}{3b\sqrt{a+bx^3}}+\frac{4e\sqrt{a+bx^3}}{3b^2}+\frac{8d\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\cdots}}{3^{3/4}b^{5\cdots}}$$

```
[Out] -2/3*x*(e*x^2+d*x+c)/b/(b*x^3+a)^(1/2)+4/3*e*(b*x^3+a)^(1/2)/b^2+8/3*d*(b*x
^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-4/3*a^(1/3)*d*(a^(1/3)+
b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(
b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)+4/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c-2*a^(1/3)*d*(1-3
^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(
a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1842, 1900, 267, 1892, 224, 1891}

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{b}\,c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{8d\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{4e\sqrt{a+bx^3}}{3b^2}-\frac{2x(c+dx+ex^2)}{3b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*x*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (4*e*Sqrt[a + b*x^3])/(3*b
^2) + (8*d*Sqrt[a + b*x^3])/(3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*Sqrt[2 +
 Sqrt[3]]*(b^(1/3)*c - 2*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqr
```

```
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = -\frac{2x(c+dx+ex^2)}{3b\sqrt{a+bx^3}} - \frac{2\int \frac{-abc-2abdx-3abex^2}{\sqrt{a+bx^3}}\,dx}{3ab^2}$$

$$= -\frac{2x(c+dx+ex^2)}{3b\sqrt{a+bx^3}} - \frac{2\int \frac{-abc-2abdx}{\sqrt{a+bx^3}}\,dx}{3ab^2} + \frac{(2e)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{b}$$

$$= -\frac{2x(c+dx+ex^2)}{3b\sqrt{a+bx^3}} + \frac{4e\sqrt{a+bx^3}}{3b^2} + \frac{(4d)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{3b^{4/3}} + \frac{\left(2\left(\sqrt[3]{b}\,c-\right.\right.}{}$$

$$4\sqrt{2-}$$

$$= -\frac{2x(c+dx+ex^2)}{3b\sqrt{a+bx^3}} + \frac{4e\sqrt{a+bx^3}}{3b^2} + \frac{8d\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 118, normalized size = 0.22

$$\frac{2\left(2ae-bcx+3bdx^2+bex^3+bcx\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-3bdx^2\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(2*a*e - b*c*x + 3*b*d*x^2 + b*e*x^3 + b*c*x*Sqrt[1 + (b*x^3)/a]*Hyperge
ometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 3*b*d*x^2*Sqrt[1 + (b*x^3)/a]*Hyp
ergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(3*b^2*Sqrt[a + b*x^3])
```

Maple [A]
time = 0.40, size = 800, normalized size = 1.48 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(2/3/b^2*a/((x^3+a/b)*b)^(1/2)+2/3*(b*x^3+a)^(1/2)/b^2)+d*(-2/3/b*x^2/((x
^3+a/b)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(
-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(
-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+c*(-2
/3/b*x/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d*x + c)*x^3/(b*x^3 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 107, normalized size = 0.20

$$\frac{2\left(2\,(bcx^3+ac)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-4\,(bdx^3+ad)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(bex^3-bdx^2-bcx+2\,ae)\sqrt{bx^3+a}\right)}{3\,(b^3x^3+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*(b*c*x^3 + a*c)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - 4*(b*d*x
^3 + a*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b,
 x)) + (b*e*x^3 - b*d*x^2 - b*c*x + 2*a*e)*sqrt(b*x^3 + a))/(b^3*x^3 + a*b^
2)
```

Sympy [A]
time = 5.72, size = 129, normalized size = 0.24

$$e\left(\begin{cases}\frac{4a}{3b^2\sqrt{a+bx^3}}+\frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for}\ b\neq 0\\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{dx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] e*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)),
 Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + c*x**4*gamma(4/3)*hyper((4/3, 3/2)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + d*x**5*gamma(
5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(
8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)*x^3/(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(e\,x^2+d\,x+c\right)}{{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
```

```
[Out] int((x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.440 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=522

$$-\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}}+\frac{8e\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}}}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\,\left(\right.}{\left(\left(1+\sqrt{3}\right.\right.}}}$$

```
[Out] -2/3*(e*x^2+d*x+c)/b/(b*x^3+a)^(1/2)+8/3*e*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))-4/3*a^(1/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/9*(a^(1/3)+b^(1/
3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*(b^(1/3)*d-2*a^(1/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1837, 1892, 224, 1891}

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{b}\,d-2\left(1-\sqrt{3}\right)\sqrt[3]{a}\,e\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{8e\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (8*e*Sqrt[a + b*x^3])/(3*b^(
5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*e*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4
)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - 2*(1 - Sqrt
[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
```

```
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}} + \frac{2\int \frac{d+2ex}{\sqrt{a+bx^3}}\,dx}{3b}$$

$$= -\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}} + \frac{(4e)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{3b^{4/3}} + \frac{\left(2\left(d-\frac{2\left(1-\sqrt{3}\right)\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\right)}{3b}$$

$$= -\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}} + \frac{8e\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,e\left(\sqrt[3]{a}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 107, normalized size = 0.20

$$\frac{2dx\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-2\left(c+x(d-3ex)+3ex^2\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{3b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] -
 2*(c + x*(d - 3*e*x) + 3*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3,
3/2, 5/3, -((b*x^3)/a)]))/(3*b*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 778 vs. 2(388) = 776.
time = 0.44, size = 779, normalized size = 1.49 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-2/3/b*x^2/((x^3+a/b)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
```

```
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))))+d*(-2/3/b*x/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*c/b/(b*x^3+a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*c/(sqrt(b*x^3 + a)*b) + integrate((x^4*e + d*x^3)*sqrt(b*x^3 + a)/(b^2
*x^6 + 2*a*b*x^3 + a^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 99, normalized size = 0.19

$$\frac{2\left(2\,(bdx^3+ad)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-4\,(bex^3+ae)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-(bex^2+bdx+bc)\sqrt{bx^3+a}\right)}{3\,(b^3x^3+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*(b*d*x^3 + a*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - 4*(b*e*x
^3 + a*e)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b,
 x)) - (b*e*x^2 + b*d*x + b*c)*sqrt(b*x^3 + a))/(b^3*x^3 + a*b^2)
```

Sympy [A]
time = 5.01, size = 109, normalized size = 0.21

$$c\left(\begin{cases}-\frac{2}{3b\sqrt{a+bx^3}} & \text{for}\ b\neq 0\\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{ex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] c*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True
)) + d*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/
(3*a**(3/2)*gamma(7/3)) + e*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**
3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)*x^2/(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(e\,x^2+d\,x+c\right)}{{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
```

```
[Out] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.441 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=561

$$-\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab}-\frac{2c\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt{2-\sqrt{3}}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\ldots}}{3^{3/4}a^{2}\ldots}$$

```
[Out] -2/3*x*(-b*d*x^2-b*c*x+a*e)/a/b/(b*x^3+a)^(1/2)-2/3*d*(b*x^3+a)^(1/2)/a/b-2
/3*c*(b*x^3+a)^(1/2)/a/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/3*c*(a^(1/
3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3
)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(2*a^(2/3)*e+b^(
2/3)*(c-c*3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(4/3
)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1842, 1900, 267, 1892, 224, 1891}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(2a^{2/3}e+b^{2/3}\left(c-\sqrt{3}\,c\right)\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a^{2/3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2c\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*x*(a*e - b*c*x - b*d*x^2))/(3*a*b*Sqrt[a + b*x^3]) - (2*d*Sqrt[a + b*x^
3])/(3*a*b) - (2*c*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*
```

```
Sqrt[2 + Sqrt[3]]*(b^(2/3)*(c - Sqrt[3]*c) + 2*a^(2/3)*e)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(2/3)*b^(4/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-ae+\frac{bcx}{2}+\frac{3}{2}bdx^2}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-ae+\frac{bcx}{2}}{\sqrt{a+bx^3}}\,dx}{3ab} - \frac{d\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{a}$$

$$= -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab} - \frac{c\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{3a\sqrt[3]{b}} + \frac{\left(\frac{\left(1-\sqrt{3}\right)b^{2}\ldots}{a^{2/3}}\right.\ldots}{\ldots}$$

$$= -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab} - \frac{2c\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\sqrt{2\ldots}}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 108, normalized size = 0.19

$$\frac{-4a(d+ex)+4aex\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+3bcx^2\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)}{6ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-4*a*(d + e*x) + 4*a*e*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4
/3, -((b*x^3)/a)] + 3*b*c*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/
2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqrt[a + b*x^3])
```

Maple [A]
time = 0.38, size = 782, normalized size = 1.39 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-2/3/b*x/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*d/b/(b*x^3+a)^(1/2)+c*(2/3/a*x^2/((x^3+a/
b)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^
2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^
2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d*x + c)*x/(b*x^3 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 112, normalized size = 0.20

$$\frac{2\left(2\left(abex^3+a^2e\right)\sqrt{b}\,\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\left(b^2cx^3+abc\right)\sqrt{b}\,\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(b^2cx^2-abex-abd\right)\sqrt{bx^3+a}\right)}{3\left(ab^3x^3+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*(a*b*e*x^3 + a^2*e)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (b^2
*c*x^3 + a*b*c)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -
4*a/b, x)) + (b^2*c*x^2 - a*b*e*x - a*b*d)*sqrt(b*x^3 + a))/(a*b^3*x^3 + a^
2*b^2)
```

Sympy [A]
time = 4.51, size = 109, normalized size = 0.19

$$d\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for}\ b\neq 0\\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] d*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True
)) + c*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/
(3*a**(3/2)*gamma(5/3)) + e*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**
3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)*x/(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(e\,x^2+d\,x+c\right)}{{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
```

```
[Out] int((x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.442 $\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}} \, dx$

Optimal. Leaf size=532

$$-\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\ldots}}}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\ldots}{\left(\left(1+\ldots\right.\right.}}}$$

```
[Out] -2/3*(a*e-b*x*(d*x+c))/a/b/(b*x^3+a)^(1/2)-2/3*d*(b*x^3+a)^(1/2)/a/b^(2/3)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/3*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/9*(a^(1/
3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c+a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+
1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1868, 1892, 224, 1891}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*d*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
(2*(a*e - b*x*(c + d*x)))/(3*a*b*Sqrt[a + b*x^3]) + (Sqrt[2 - Sqrt[3]]*d*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*
a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c + (1 -
Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
```

```
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx = -\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{c}{2}+\frac{dx}{2}}{\sqrt{a+bx^3}}\,dx}{3a}$$

$$= -\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}} - \frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{3a\sqrt[3]{b}} + \frac{\left(c+\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+b}}}{3a}$$

$$= -\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}} + \frac{\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 109, normalized size = 0.20

$$\frac{-4ae+4bcx+2bcx\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+3bdx^2\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)}{6ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^(3/2),x]
```

```
[Out] (-4*a*e + 4*b*c*x + 2*b*c*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2,
 4/3, -((b*x^3)/a)] + 3*b*d*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3,
3/2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqrt[a + b*x^3])
```

Maple [**A**]
time = 0.36, size = 785, normalized size = 1.48 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*e/b/(b*x^3+a)^(1/2)+d*(2/3/a*x^2/((x^3+a/b)*b)^(1/2)+2/9*I/a*3^(1/2)/b
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1
```

```
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+c*(2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*
3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((x^2*e + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 98, normalized size = 0.18

$$\frac{2\left((bcx^3+ac)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+(bdx^3+ad)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(bdx^2+bcx-ae)\sqrt{bx^3+a}\right)}{3\,(ab^2x^3+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] 2/3*((b*c*x^3 + a*c)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (b*d*x^3 +
 a*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
 + (b*d*x^2 + b*c*x - a*e)*sqrt(b*x^3 + a))/(a*b^2*x^3 + a^2*b)
```

Sympy [A]
time = 4.06, size = 107, normalized size = 0.20

$$e\left(\begin{cases}-\frac{2}{3b\sqrt{a+bx^3}} & \text{for}\ b\neq 0\\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] e*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True
)) + c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d\,x+c}{{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^3)^(3/2),x)
```

```
[Out] int((c + d*x + e*x^2)/(a + b*x^3)^(3/2), x)
```

3.443 $\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=579

$$\frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}}+\frac{2c\sqrt{a+bx^3}}{3a^2}-\frac{2e\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}+\frac{\sqrt{2-\sqrt{}}\cdots}{\cdots}$$

```
[Out] -2/3*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)+2/3*x*(-b*c*x^2+a*e*x+a*d)/
a^2/(b*x^3+a)^(1/2)+2/3*c*(b*x^3+a)^(1/2)/a^2-2/3*e*(b*x^3+a)^(1/2)/a/b^(2/
3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/3*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(
1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/9*(a^
(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*d+a^(1/3)*e*(1-3^(1/2)))*(1/2*6^(1/
2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1843, 1846, 272, 65, 214, 1900, 267, 1892, 224, 1891}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\,e+\sqrt[3]{b}\,d\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\,e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}+\frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}}+\frac{2c\sqrt{a+bx^3}}{3a^2}-\frac{2e\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*c*Sqrt[a + b*x^3
])/(3*a^2) - (2*e*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)) - (2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)) + (Sqrt[2 -
Sqrt[3]]*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3
]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
```

```
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1
/3)*d + (1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
```

```
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx &= \frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{3bc}{2}-\frac{bdx}{2}+\frac{1}{2}bex^2-\frac{3b^2cx^3}{2a}}{x\sqrt{a+bx^3}}\,dx}{3ab} \\
&= \frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{bd}{2}+\frac{bex}{2}-\frac{3b^2cx^2}{2a}}{\sqrt{a+bx^3}}\,dx}{3ab} + \frac{c\int \frac{1}{x\sqrt{a+bx^3}}\,dx}{a} \\
&= \frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{bd}{2}+\frac{bex}{2}}{\sqrt{a+bx^3}}\,dx}{3ab} + \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{3a} + \frac{(bc)\cdots}{\cdots} \\
&= \frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}} + \frac{2c\sqrt{a+bx^3}}{3a^2} + \frac{(2c)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3ab} - \frac{e\cdots}{\cdots} \\
&= \frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}} + \frac{2c\sqrt{a+bx^3}}{3a^2} - \frac{2e\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2c\tanh\cdots}{\cdots}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.14, size = 518, normalized size = 0.89

$$
\frac{2\left(\frac{c+x(d+ex)}{\sqrt{a+bx^3}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{d\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}+\frac{\sqrt{2}\sqrt[3]{a}e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}}\left(\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)+F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}\right)}{3a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*((c + x*(d + e*x))/Sqrt[a + b*x^3] - (c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]
)/Sqrt[a] - (d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/
((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b
^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(b^(1/3)*Sqrt[(a^(1/3)
 + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]) + (Sq
rt[2]*a^(1/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) -
 (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)
/a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(
```

```
1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))] + Ell
ipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)
/(-1 + (-1)^(1/3))]))/(b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])))/(3*a)
```

Maple [A]
time = 0.37, size = 810, normalized size = 1.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(2/3/a*x^2/((x^3+a/b)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))))+d*(2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+c*(2/3/a/((x^3+a/b)*b)^(1/2)-2/3*ar
ctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d*x + c)/((b*x^3 + a)^(3/2)*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 345, normalized size = 0.60

$$\left[\frac{(b^2cx^3+abc)\sqrt{a}\log\left(\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+4\,(abdx^3+a^2d)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+4\,(abex^3+a^2e)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+4\,(abex^2+abdx+abc)\sqrt{bx^3+a}}{6\,(a^2b^2x^3+a^3b)}, \frac{(b^2cx^3+abc)\sqrt{-a}\arctan\left(\frac{(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{-a}}{2\,(abx^3+a^2)}\right)+2\,(abdx^3+a^2d)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+2\,(abex^3+a^2e)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+2\,(abex^2+abdx+abc)\sqrt{bx^3+a}}{3\,(a^2b^2x^3+a^3b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*((b^2*c*x^3 + a*b*c)*sqrt(a)*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a
)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 4*(a*b*d*x^3 + a^2*d)*sqrt(b)*wei
erstrassPInverse(0, -4*a/b, x) + 4*(a*b*e*x^3 + a^2*e)*sqrt(b)*weierstrassZ
eta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + 4*(a*b*e*x^2 + a*b*d*x
+ a*b*c)*sqrt(b*x^3 + a))/(a^2*b^2*x^3 + a^3*b), 1/3*((b^2*c*x^3 + a*b*c)*s
qrt(-a)*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2))
+ 2*(a*b*d*x^3 + a^2*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + 2*(a*b*
e*x^3 + a^2*e)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4
*a/b, x)) + 2*(a*b*e*x^2 + a*b*d*x + a*b*c)*sqrt(b*x^3 + a))/(a^2*b^2*x^3 +
 a^3*b)]
```

Sympy [A]
time = 6.16, size = 265, normalized size = 0.46

$$c\left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}\right)+\frac{dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{ex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(3/2),x)
```

```
[Out] c*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*
x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) +
1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2)
 + 3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/
2) + 3*a**(7/2)*b*x**3)) + d*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3)) + e*x**2*gamma(2/3)*hyper((2/3,
3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)/((b*x^3 + a)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d\,x+c}{x\,{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)),x)
[Out] int((c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)), x)
```

3.444 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=607

$$\frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}}+\frac{2d\sqrt{a+bx^3}}{3a^2}-\frac{c\sqrt{a+bx^3}}{a^2x}+\frac{5\sqrt[3]{b}\,c\sqrt{a+bx^3}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right.}{3a^{3/2}}$$

```
[Out] -2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)+2/3*x*(-b*d*x^2-b*c*x+a*e)/
a^2/(b*x^3+a)^(1/2)+2/3*d*(b*x^3+a)^(1/2)/a^2-c*(b*x^3+a)^(1/2)/a^2/x+5/3*b
^(1/3)*c*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-5/6*b^(1/3)*c*
(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/
a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)-1/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(-2*a^(2/3)*e+5*b^
(2/3)*c*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b^(1/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1843, 1849, 1846, 272, 65, 214, 1900, 267, 1892, 224, 1891}

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a^{5/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}+\frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}}-\frac{c\sqrt{a+bx^3}}{a^2x}+\frac{5\sqrt[3]{b}\,c\sqrt{a+bx^3}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2d\sqrt{a+bx^3}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*d*Sqrt[a + b*x^3
])/(3*a^2) - (c*Sqrt[a + b*x^3])/(a^2*x) + (5*b^(1/3)*c*Sqrt[a + b*x^3])/(3
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/Sq
rt[a]])/(3*a^(3/2)) - (5*Sqrt[2 - Sqrt[3]]*b^(1/3)*c*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*Sqrt[(a^(1
```

```
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) - (Sqrt[2 + Sqrt[3]]*(5*(1 - Sqrt[3])*b^(2/3)*c - 2*a^(2/3)*e)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*
a^(5/3)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
```

```
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
```

```
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{3bc}{2}-\frac{3bdx}{2}-\frac{1}{2}bex^2-\frac{b^2cx^3}{2a}-\frac{3b^2dx^4}{2a}}{x^2\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{\int \frac{3abd+abex+\frac{5}{2}b^2cx^2+3b^2dx^3}{x\sqrt{a+bx^3}}\,dx}{3a^2b}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{\int \frac{abe+\frac{5}{2}b^2cx+3b^2dx^2}{\sqrt{a+bx^3}}\,dx}{3a^2b} + \frac{d\int \frac{1}{x\sqrt{a+bx^3}}\,dx}{a}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{\int \frac{abe+\frac{5}{2}b^2cx}{\sqrt{a+bx^3}}\,dx}{3a^2b} + \frac{d\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x\right.}{3a}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}} + \frac{2d\sqrt{a+bx^3}}{3a^2} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{\left(5b^{2/3}c\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}}{6a^2}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}} + \frac{2d\sqrt{a+bx^3}}{3a^2} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{5\sqrt[3]{b}\,c\sqrt{a+bx^3}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.61, size = 542, normalized size = 0.89

$$\frac{-3ac-5bcx^3+2ax(d+ex)}{3a^2x\sqrt{a+bx^3}} - \frac{4\sqrt{a}\,d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) + \frac{4ae\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}} + \frac{10\sqrt{2}\sqrt[3]{a}\sqrt[3]{b}c\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}}\left(\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)+F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)\right)}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}}{6a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-3*a*c - 5*b*c*x^3 + 2*a*x*(d + e*x))/(3*a^2*x*Sqrt[a + b*x^3]) - (4*Sqrt[
a]*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + (4*a*e*((-1)^(1/3)*a^(1/3) - b^(1/3
)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3
```

```
)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[Arc
Sin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1
)^(1/3)])/(b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*
a^(1/3))]*Sqrt[a + b*x^3]) + (10*Sqrt[2]*a^(1/3)*b^(1/3)*c*((-1)^(1/3)*a^(1
/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-1
 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^
(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) -
(I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(Sqrt[(a^(
1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]))/
(6*a^2)
```

Maple [A]
time = 0.40, size = 825, normalized size = 1.36 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(2/3/a/((x^3+a/b)*b)^(1/2)-2/3*arctanh((b*x^
3+a)^(1/2)/a^(1/2))/a^(3/2))+c*(-2/3*b*x^2/a^2/((x^3+a/b)*b)^(1/2)-(b*x^3+a
)^(1/2)/a^2/x-5/9*I/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*
b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(
-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*
b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d*x + c)/((b*x^3 + a)^(3/2)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 377, normalized size = 0.62

$$\left[\frac{\left(b^{2} d x^{4} + a b d x\right) \sqrt{a} \log\left(\frac{b^{2} x^{6} + 8 \, a b x^{3} - 4 \, \left(b x^{3} + 2 \, a\right) \sqrt{b x^{3} + a} \sqrt{a} + 8 \, a^{2}}{x^{6}}\right) + 4 \, \left(a b e x^{4} + a^{2} e x\right) \sqrt{b} \, \mathrm{weierstrassPInverse}\left(0, -\frac{4 \, a}{b}, x\right) - 10 \, \left(b^{2} c x^{4} + a b c x\right) \sqrt{b} \, \mathrm{weierstrassZeta}\left(0, -\frac{4 \, a}{b}, \mathrm{weierstrassPInverse}\left(0, -\frac{4 \, a}{b}, x\right)\right) - 2 \, \left(5 \, b^{2} c x^{3} - 2 \, a b e x^{2} - 2 \, a b d x + 3 \, a b c\right) \sqrt{b x^{3} + a}}{6 \, \left(a^{2} b^{2} x^{4} + a^{3} b x\right)}, \frac{\left(b^{2} d x^{4} + a b d x\right) \sqrt{-a} \arctan\left(\frac{\left(b x^{3} + 2 \, a\right) \sqrt{b x^{3} + a} \sqrt{-a}}{2 \, \left(a b x^{3} + a^{2}\right)}\right) + 2 \, \left(a b e x^{4} + a^{2} e x\right) \sqrt{b} \, \mathrm{weierstrassPInverse}\left(0, -\frac{4 \, a}{b}, x\right) - 5 \, \left(b^{2} c x^{4} + a b c x\right) \sqrt{b} \, \mathrm{weierstrassZeta}\left(0, -\frac{4 \, a}{b}, \mathrm{weierstrassPInverse}\left(0, -\frac{4 \, a}{b}, x\right)\right) - \left(5 \, b^{2} c x^{3} - 2 \, a b e x^{2} - 2 \, a b d x + 3 \, a b c\right) \sqrt{b x^{3} + a}}{3 \, \left(a^{2} b^{2} x^{4} + a^{3} b x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*((b^2*d*x^4 + a*b*d*x)*sqrt(a)*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2
*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 4*(a*b*e*x^4 + a^2*e*x)*sqrt(b)
*weierstrassPInverse(0, -4*a/b, x) - 10*(b^2*c*x^4 + a*b*c*x)*sqrt(b)*weier
strassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - 2*(5*b^2*c*x^3 -
 2*a*b*e*x^2 - 2*a*b*d*x + 3*a*b*c)*sqrt(b*x^3 + a))/(a^2*b^2*x^4 + a^3*b*x
), 1/3*((b^2*d*x^4 + a*b*d*x)*sqrt(-a)*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3
+ a)*sqrt(-a)/(a*b*x^3 + a^2)) + 2*(a*b*e*x^4 + a^2*e*x)*sqrt(b)*weierstras
sPInverse(0, -4*a/b, x) - 5*(b^2*c*x^4 + a*b*c*x)*sqrt(b)*weierstrassZeta(0
, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (5*b^2*c*x^3 - 2*a*b*e*x^2 -
 2*a*b*d*x + 3*a*b*c)*sqrt(b*x^3 + a))/(a^2*b^2*x^4 + a^3*b*x)]
```

Sympy [A]
time = 6.30, size = 267, normalized size = 0.44

$$d \left(\frac{2 a^{3} \sqrt{1 + \frac{b x^{3}}{a}}}{3 a^{\frac{9}{2}} + 3 a^{\frac{7}{2}} b x^{3}} + \frac{a^{3} \log{\left(\frac{b x^{3}}{a} \right)}}{3 a^{\frac{9}{2}} + 3 a^{\frac{7}{2}} b x^{3}} - \frac{2 a^{3} \log{\left(\sqrt{1 + \frac{b x^{3}}{a}} + 1 \right)}}{3 a^{\frac{9}{2}} + 3 a^{\frac{7}{2}} b x^{3}} + \frac{a^{2} b x^{3} \log{\left(\frac{b x^{3}}{a} \right)}}{3 a^{\frac{9}{2}} + 3 a^{\frac{7}{2}} b x^{3}} - \frac{2 a^{2} b x^{3} \log{\left(\sqrt{1 + \frac{b x^{3}}{a}} + 1 \right)}}{3 a^{\frac{9}{2}} + 3 a^{\frac{7}{2}} b x^{3}}\right) + \frac{c \Gamma\left(- \frac{1}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{1}{3}, \frac{3}{2} \\ \frac{2}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}}{3 a^{\frac{3}{2}} x \Gamma\left(\frac{2}{3}\right)} + \frac{e x \Gamma\left(\frac{1}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{3}{2} \\ \frac{4}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}}{3 a^{\frac{3}{2}} \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(3/2),x)
```

```
[Out] d*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*
x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) +
1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2)
 + 3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/
2) + 3*a**(7/2)*b*x**3)) + c*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*x*gamma(2/3)) + e*x*gamma(1/3)*hyper((1/3, 3
/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
```

[Out] integrate((x^2*e + d*x + c)/((b*x^3 + a)^(3/2)*x^2), x)

Mupad [B]
time = 5.80, size = 136, normalized size = 0.22

$$\frac{2\,d}{3\,a\,\sqrt{b\,x^3+a}}+\frac{d\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{3/2}}-\frac{2\,c\,{\left(\frac{a}{b\,x^3}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{11}{6};\ \frac{17}{6};\ -\frac{a}{b\,x^3}\right)}{11\,x\,{\left(b\,x^3+a\right)}^{3/2}}+\frac{e\,x\,{\left(\frac{b\,x^3}{a}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{3}{2};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{{\left(b\,x^3+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)),x)

[Out] (2*d)/(3*a*(a + b*x^3)^(1/2)) + (d*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(3/2)) - (2*c*(a/(b*x^3) + 1)^(3/2)*hypergeom([3/2, 11/6], 17/6, -a/(b*x^3)))/(11*x*(a + b*x^3)^(3/2)) + (e*x*((b*x^3)/a + 1)^(3/2)*hypergeom([1/3, 3/2], 4/3, -(b*x^3)/a))/(a + b*x^3)^(3/2)

3.445 $\int x^3\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)\,dx$

Optimal. Leaf size=733

$$-\frac{4a^2e\sqrt{a+bx^3}}{45b^2}+\frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2}+\frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2}+\frac{2aex^3\sqrt{a+bx^3}}{45b}+\frac{6afx^4\sqrt{a}}{187}$$

```
[Out] -4/45*a^2*e*(b*x^3+a)^(1/2)/b^2+6/935*a*(-8*a*f+17*b*c)*x*(b*x^3+a)^(1/2)/b
^2+6/1729*a*(-10*a*g+19*b*d)*x^2*(b*x^3+a)^(1/2)/b^2+2/45*a*e*x^3*(b*x^3+a)
^(1/2)/b+6/187*a*f*x^4*(b*x^3+a)^(1/2)/b+6/247*a*g*x^5*(b*x^3+a)^(1/2)/b+2/
692835*x^3*(36465*g*x^5+40755*f*x^4+46189*e*x^3+53295*d*x^2+62985*c*x)*(b*x
^3+a)^(1/2)-24/1729*a^2*(-10*a*g+19*b*d)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))+12/1729*3^(1/4)*a^(7/3)*(-10*a*g+19*b*d)*(a^(1/3)+b^(
1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1
/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-4
/1616615*3^(3/4)*a^2*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1729*b^(1/3)*(-8*a*
f+17*b*c)-1870*a^(1/3)*(-10*a*g+19*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/
2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 1.30, antiderivative size = 733, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1850, 1902, 1608, 1900, 267, 1892, 224, 1891}

$$\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(19bd-10ag)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{24a^2\sqrt{a+bx^3}\,(19bd-10ag)}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4a^2e\sqrt{a+bx^3}}{45b^2}-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(17bc-8af)-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}(19bd-10ag)\right)}{1616615b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{6ax\sqrt{a+bx^3}\,(17bc-8af)}{935b^2}+\frac{6ax^2\sqrt{a+bx^3}\,(19bd-10ag)}{1729b^2}+\frac{2x^3\sqrt{a+bx^3}\,(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835}+\frac{2aex^3\sqrt{a+bx^3}}{45b}+\frac{6afx^4\sqrt{a+bx^3}}{187b}+\frac{6agx^5\sqrt{a+bx^3}}{247b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (-4*a^2*e*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*c - 8*a*f)*x*Sqrt[a + b*x^
3])/(935*b^2) + (6*a*(19*b*d - 10*a*g)*x^2*Sqrt[a + b*x^3])/(1729*b^2) + (2
*a*e*x^3*Sqrt[a + b*x^3])/(45*b) + (6*a*f*x^4*Sqrt[a + b*x^3])/(187*b) + (6
*a*g*x^5*Sqrt[a + b*x^3])/(247*b) - (24*a^2*(19*b*d - 10*a*g)*Sqrt[a + b*x^
3])/(1729*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^3*Sqrt[a + b*
```

```
x^3]*(62985*c*x + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))/6
92835 + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*d - 10*a*g)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(8/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1729*b^(1/3)*(17*b*c - 8*
a*f) - 1870*(1 - Sqrt[3])*a^(1/3)*(19*b*d - 10*a*g))*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1616615*b^(8/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
```

```
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{2x^3\sqrt{a+bx^3}\,(62985cx+53295dx^2+46189ex^3+40755\ldots}{692835}$$

$$= \frac{6agx^5\sqrt{a+bx^3}}{247b} + \frac{2x^3\sqrt{a+bx^3}\,(62985cx+53295dx^2+\ldots}{692\ldots}$$

$$= \frac{6afx^4\sqrt{a+bx^3}}{187b} + \frac{6agx^5\sqrt{a+bx^3}}{247b} + \frac{2x^3\sqrt{a+bx^3}\,(62\ldots}{\ldots}$$

$$= \frac{2aex^3\sqrt{a+bx^3}}{45b} + \frac{6afx^4\sqrt{a+bx^3}}{187b} + \frac{6agx^5\sqrt{a+bx^3}}{247b}\ldots$$

$$= \frac{2aex^3\sqrt{a+bx^3}}{45b} + \frac{6afx^4\sqrt{a+bx^3}}{187b} + \frac{6agx^5\sqrt{a+bx^3}}{247b}\ldots$$

$$= \frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2aex^3\sqrt{a+bx^3}}{45b} + \frac{6af\ldots}{\ldots}$$

$$= \frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2aex^3\sqrt{a+bx^3}}{45b} + \frac{6af\ldots}{\ldots}$$

$$= \frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2} + \frac{6a(19bd-10ag)x^2\sqrt{a+b\ldots}}{1729b^2}$$

$$= \frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2} + \frac{6a(19bd-10ag)x^2\sqrt{a+b\ldots}}{1729b^2}$$

$$= -\frac{4a^2e\sqrt{a+bx^3}}{45b^2} + \frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2} + \frac{6a(19\ldots}{\ldots}$$

$$= -\frac{4a^2e\sqrt{a+bx^3}}{45b^2} + \frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2} + \frac{6a(19\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 7.92, size = 172, normalized size = 0.23

$$\frac{2\sqrt{a+bx^3}\left(-\left((a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,(a(92378e+90x(988f+935gx))-3bx(62985c+11x(4845d+13x(323e+285fx+255gx^2))))\right)+11115a(-17bc+8af)x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+8415a(-19bd+10ag)x^2\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2078505b^2\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(a*(92378*e + 90*x*(9
88*f + 935*g*x)) - 3*b*x*(62985*c + 11*x*(4845*d + 13*x*(323*e + 285*f*x +
255*g*x^2))))) + 11115*a*(-17*b*c + 8*a*f)*x*Hypergeometric2F1[-1/2, 1/3, 4
/3, -((b*x^3)/a)] + 8415*a*(-19*b*d + 10*a*g)*x^2*Hypergeometric2F1[-1/2, 2
/3, 5/3, -((b*x^3)/a)]))/(2078505*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1673 vs. 2(573) = 1146.
time = 0.37, size = 1674, normalized size = 2.28

method	result	size
elliptic	Expression too large to display	956
risch	Expression too large to display	1138
default	Expression too large to display	1674

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/19*x^8*(b*x^3+a)^(1/2)+6/247*a*x^5*(b*x^3+a)^(1/2)/b-60/1729*a^2*x^2*(
b*x^3+a)^(1/2)/b^2-80/1729*I*a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))))+f*(2/17*x^7*(b*x^3+a)^(1/2)+6/187*a*x^4*(b*x^3+a)^(1/2)/b-48/935*a^2*x
*(b*x^3+a)^(1/2)/b^2-32/935*I*a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
```

```
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)))+e*(2/15*x^6*(b*x^3+a)^(1/2)+2/45*a/b*x^3*(b*x^3+a
)^(1/2)-4/45*a^2/b^2*(b*x^3+a)^(1/2))+d*(2/13*x^5*(b*x^3+a)^(1/2)+6/91*a*x^
2*(b*x^3+a)^(1/2)/b+8/91*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
))+c*(2/11*x^4*(b*x^3+a)^(1/2)+6/55*a*x*(b*x^3+a)^(1/2)/b+4/55*I/b^2*a^2*3^
(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)*x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 202, normalized size = 0.28

$$-\frac{2\left(93366\,(17\,a^2bc-8\,a^3f)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-100980\,(19\,a^2bd-10\,a^3g)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(765765\,b^3gx^8+855855\,b^3fx^7+969969\,b^3ex^6+323323\,ab^2ex^3+58905\,(19\,b^3d+3\,ab^2g)x^5+77805\,(17\,b^3c+3\,ab^2f)x^4-646646\,a^2be+25245\,(19\,ab^2d-10\,a^2bg)x^2+46683\,(17\,ab^2c-8\,a^2bf)x)\sqrt{bx^3+a}\right)}{14549535\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] -2/14549535*(93366*(17*a^2*b*c - 8*a^3*f)*sqrt(b)*weierstrassPInverse(0, -4
*a/b, x) - 100980*(19*a^2*b*d - 10*a^3*g)*sqrt(b)*weierstrassZeta(0, -4*a/b
, weierstrassPInverse(0, -4*a/b, x)) - (765765*b^3*g*x^8 + 855855*b^3*f*x^7
```

```
 + 969969*b^3*e*x^6 + 323323*a*b^2*e*x^3 + 58905*(19*b^3*d + 3*a*b^2*g)*x^5
 + 77805*(17*b^3*c + 3*a*b^2*f)*x^4 - 646646*a^2*b*e + 25245*(19*a*b^2*d -
10*a^2*b*g)*x^2 + 46683*(17*a*b^2*c - 8*a^2*b*f)*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A]
time = 2.52, size = 238, normalized size = 0.32

$$\frac{\sqrt{a}\, c x^{4} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}\, d x^{5} \Gamma\left(\frac{5}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}\, f x^{7} \Gamma\left(\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a}\, g x^{8} \Gamma\left(\frac{8}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{11}{3}\right)} + e \left(\begin{cases} - \frac{4 a^{2} \sqrt{a + b x^{3}}}{45 b^{2}} + \frac{2 a x^{3} \sqrt{a + b x^{3}}}{45 b} + \frac{2 x^{6} \sqrt{a + b x^{3}}}{15} & \text{for}\: b \neq 0 \\ \frac{\sqrt{a} x^{6}}{6} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3)) + sqrt(a)*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*f*x**7*gamma(7/3)*hyper((
-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*g*
x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*ga
mma(11/3)) + e*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqr
t(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6
/6, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

```
[Out] int(x^3*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.446 $\int x^2\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)\,dx$

Optimal. Leaf size=681

$$\frac{2a(5bc-2af)\sqrt{a+bx^3}}{45b^2}+\frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2}+\frac{6aex^2\sqrt{a+bx^3}}{91b}+\frac{2afx^3\sqrt{a+bx^3}}{45b}+\frac{6agx^4\sqrt{a+b}}{187b}$$

```
[Out] 2/45*a*(-2*a*f+5*b*c)*(b*x^3+a)^(1/2)/b^2+6/935*a*(-8*a*g+17*b*d)*x*(b*x^3+
a)^(1/2)/b^2+6/91*a*e*x^2*(b*x^3+a)^(1/2)/b+2/45*a*f*x^3*(b*x^3+a)^(1/2)/b+
6/187*a*g*x^4*(b*x^3+a)^(1/2)/b+2/109395*x^2*(6435*g*x^5+7293*f*x^4+8415*e*
x^3+9945*d*x^2+12155*c*x)*(b*x^3+a)^(1/2)-24/91*a^2*e*(b*x^3+a)^(1/2)/b^(5/
3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+12/91*3^(1/4)*a^(7/3)*e*(a^(1/3)+b^(1/3)
*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/
(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-4/850
85*3^(3/4)*a^2*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1547*b*d-728*a*g-1870*a^(
1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/(b*x^3
+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1850, 1902, 1608, 1900, 267, 1892, 224, 1891}

$$\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{24a^2e\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-728ag+1547bd\right)}{85085b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2a\sqrt{a+bx^3}(5bc-2af)}{45b^2}+\frac{6ax\sqrt{a+bx^3}(17bd-8ag)}{935b^2}+\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}+\frac{6aex^2\sqrt{a+bx^3}}{91b}+\frac{2afx^3\sqrt{a+bx^3}}{45b}+\frac{6agx^4\sqrt{a+bx^3}}{187b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a*(5*b*c - 2*a*f)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*d - 8*a*g)*x*Sq
rt[a + b*x^3])/(935*b^2) + (6*a*e*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*f*x^3*
Sqrt[a + b*x^3])/(45*b) + (6*a*g*x^4*Sqrt[a + b*x^3])/(187*b) - (24*a^2*e*S
qrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^2*S
qrt[a + b*x^3]*(12155*c*x + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 + 6435*g*x
^5))/109395 + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1/3) + b^(1/3)*x)
```

```
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
) - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1547*b*d - 1870*(1 - Sqrt[3])*a^(1/3)
*b^(2/3)*e - 728*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(85085*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
```

```
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{2x^2\sqrt{a+bx^3}\,(12155cx+9945dx^2+8415ex^3+7293fx^4+}{109395}$$

$$= \frac{6agx^4\sqrt{a+bx^3}}{187b} + \frac{2x^2\sqrt{a+bx^3}\,(12155cx+9945dx^2+8}{109395}$$

$$= \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4\sqrt{a+bx^3}}{187b} + \frac{2x^2\sqrt{a+bx^3}\,(121}{}$$

$$= \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4\sqrt{a+bx^3}}{187b} +$$

$$= \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4\sqrt{a+bx^3}}{187b} +$$

$$= \frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2} + \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{}}{45}$$

$$= \frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2} + \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{}}{45}$$

$$= \frac{2a(5bc-2af)\sqrt{a+bx^3}}{45b^2} + \frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2} +$$

$$= \frac{2a(5bc-2af)\sqrt{a+bx^3}}{45b^2} + \frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.20, size = 158, normalized size = 0.23

$$\frac{2\sqrt{a+bx^3}\left(-\left((a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,(26a(187f+180gx)-b(12155c+9945dx+33x^2(255e+13x(17f+15gx))))\right)+585a(-17bd+8ag)x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)-8415abex^2\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{109395b^2\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(26*a*(187*f + 180*g*
x) - b*(12155*c + 9945*d*x + 33*x^2*(255*e + 13*x*(17*f + 15*g*x))))) + 585
*a*(-17*b*d + 8*a*g)*x*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)] - 84
15*a*b*e*x^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)]))/(109395*b^2*
Sqrt[1 + (b*x^3)/a])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1196 vs. 2(525) = 1050.
time = 0.38, size = 1197, normalized size = 1.76

method	result	size
elliptic	Expression too large to display	920
risch	Expression too large to display	1115
default	Expression too large to display	1197

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/17*x^7*(b*x^3+a)^(1/2)+6/187*a*x^4*(b*x^3+a)^(1/2)/b-48/935*a^2*x*(b*x
^3+a)^(1/2)/b^2-32/935*I*a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)))+f*(2/15*x^6*(b*x^3+a)^(1/2)+2/45*a/b*x^3*(b*x^3+a)^(1/
2)-4/45*a^2/b^2*(b*x^3+a)^(1/2))+e*(2/13*x^5*(b*x^3+a)^(1/2)+6/91*a*x^2*(b*
x^3+a)^(1/2)/b+8/91*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/
b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+d*
(2/11*x^4*(b*x^3+a)^(1/2)+6/55*a*x*(b*x^3+a)^(1/2)/b+4/55*I/b^2*a^2*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
```

```
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/9*c*(b*x^3+a)^(3/
2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)*c/b + integrate((g*x^6 + f*x^5 + x^4*e + d*x^3)*sqrt(
b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 177, normalized size = 0.26

$$\frac{2\left(100980\,a^{2}b^{\frac{3}{2}}e\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-4914\,(17\,a^{2}bd-8\,a^{3}g)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+(45045\,b^{3}gx^{7}+51051\,b^{3}fx^{6}+58905\,b^{3}ex^{5}+25245\,ab^{2}ex^{2}+4095\,(17\,b^{3}d+3\,ab^{2}g)x^{4}+85085\,ab^{2}c-34034\,a^{2}bf+17017\,(5\,b^{3}c+ab^{2}f)x^{3}+2457\,(17\,ab^{2}d-8\,a^{2}bg)x)\sqrt{bx^{3}+a}\right)}{765765\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/765765*(100980*a^2*b^(3/2)*e*weierstrassZeta(0, -4*a/b, weierstrassPInver
se(0, -4*a/b, x)) - 4914*(17*a^2*b*d - 8*a^3*g)*sqrt(b)*weierstrassPInverse
(0, -4*a/b, x) + (45045*b^3*g*x^7 + 51051*b^3*f*x^6 + 58905*b^3*e*x^5 + 252
45*a*b^2*e*x^2 + 4095*(17*b^3*d + 3*a*b^2*g)*x^4 + 85085*a*b^2*c - 34034*a^
2*b*f + 17017*(5*b^3*c + a*b^2*f)*x^3 + 2457*(17*a*b^2*d - 8*a^2*b*g)*x)*sq
rt(b*x^3 + a))/b^3
```

Sympy [A]
time = 2.37, size = 223, normalized size = 0.33

$$\frac{\sqrt{a}\,dx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}\,ex^{5}\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{\sqrt{a}\,gx^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+c\left(\begin{cases}\frac{\sqrt{a}\,x^{3}}{3} & \text{for}\;b=0\\ \frac{2(a+bx^{3})^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)+f\left(\begin{cases}-\frac{4a^{2}\sqrt{a+bx^{3}}}{45b^{2}}+\frac{2ax^{3}\sqrt{a+bx^{3}}}{45b}+\frac{2x^{6}\sqrt{a+bx^{3}}}{15} & \text{for}\;b\neq0\\ \frac{\sqrt{a}\,x^{6}}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3)) + sqrt(a)*e*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*g*x**7*gamma(7/3)*hyper((
-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + c*Piecewis
e((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + f*Piec
```

```
ewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b
) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac"
)
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
[Out] int(x^2*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.447 $\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=667

$$\frac{2a(5bd-2ag)\sqrt{a+bx^3}}{45b^2}+\frac{6aex\sqrt{a+bx^3}}{55b}+\frac{6afx^2\sqrt{a+bx^3}}{91b}+\frac{2agx^3\sqrt{a+bx^3}}{45b}+\frac{6a(13bc-4af)\sqrt{a+b}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}$$

```
[Out] 2/45*a*(-2*a*g+5*b*d)*(b*x^3+a)^(1/2)/b^2+6/55*a*e*x*(b*x^3+a)^(1/2)/b+6/91
*a*f*x^2*(b*x^3+a)^(1/2)/b+2/45*a*g*x^3*(b*x^3+a)^(1/2)/b+2/45045*x*(3003*g
*x^5+3465*f*x^4+4095*e*x^3+5005*d*x^2+6435*c*x)*(b*x^3+a)^(1/2)+6/91*a*(-4*
a*f+13*b*c)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3/91*3^
(1/4)*a^(4/3)*(-4*a*f+13*b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1
/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2
)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-2/5005*3^(3/4)*a^(4/3)*(a^(1/3)+b^
(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(182*a^(2/3)*b^(1/3)*e+55*(-4*a*f+13*b*c)*(1-3^(1/2)
))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.69, antiderivative size = 667, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1840, 1850, 1902, 1900, 267, 1892, 224, 1891}

$$-\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(182a^{2/3}\sqrt[3]{b}e+55\left(1-\sqrt{3}\right)(13bc-4af)\right)}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13bc-4af)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{6a\sqrt{a+bx^3}(13bc-4af)}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a\sqrt{a+bx^3}(5bd-2ag)}{45b^2}+\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}+\frac{6aex\sqrt{a+bx^3}}{55b}+\frac{6afx^2\sqrt{a+bx^3}}{91b}+\frac{2agx^3\sqrt{a+bx^3}}{45b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a*(5*b*d - 2*a*g)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*e*x*Sqrt[a + b*x^3])/
(55*b) + (6*a*f*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*g*x^3*Sqrt[a + b*x^3])/(
45*b) + (6*a*(13*b*c - 4*a*f)*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)) + (2*x*Sqrt[a + b*x^3]*(6435*c*x + 5005*d*x^2 + 4095*e
*x^3 + 3465*f*x^4 + 3003*g*x^5))/45045 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/
3)*(13*b*c - 4*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
```

```
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a
^(4/3)*(182*a^(2/3)*b^(1/3)*e + 55*(1 - Sqrt[3])*(13*b*c - 4*a*f))*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(5/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{2x\sqrt{a+bx^3}\,(6435cx+5005dx^2+4095ex^3+3465fx^4+3}{45045}$$

$$= \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}\,(6435cx+5005dx^2+409}{45045}$$

$$= \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}\,(6435}{}$$

$$= \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} +$$

$$= \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} +$$

$$= \frac{2a(5bd-2ag)\sqrt{a+bx^3}}{45b^2} + \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+}}{91b}$$

$$= \frac{2a(5bd-2ag)\sqrt{a+bx^3}}{45b^2} + \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+}}{91b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.85, size = 143, normalized size = 0.21

$$\frac{\sqrt{a+bx^3}\left(-4(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,(286ag-b(715d+585ex+495fx^2+429gx^3))-2340abex\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+495b(13bc-4af)x^2\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{12870b^2\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (Sqrt[a + b*x^3]*(-4*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(286*a*g - b*(715*d +
585*e*x + 495*f*x^2 + 429*g*x^3)) - 2340*a*b*e*x*Hypergeometric2F1[-1/2, 1/
3, 4/3, -((b*x^3)/a)] + 495*b*(13*b*c - 4*a*f)*x^2*Hypergeometric2F1[-1/2,
2/3, 5/3, -((b*x^3)/a)]))/(12870*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1310 vs. $2(513) = 1026$.
time = 0.38, size = 1311, normalized size = 1.97 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/15*x^6*(b*x^3+a)^(1/2)+2/45*a/b*x^3*(b*x^3+a)^(1/2)-4/45*a^2/b^2*(b*x^
3+a)^(1/2))+f*(2/13*x^5*(b*x^3+a)^(1/2)+6/91*a*x^2*(b*x^3+a)^(1/2)/b+8/91*I
/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+e*(2/11*x^4*(b*x^3+a)^(
1/2)+6/55*a*x*(b*x^3+a)^(1/2)/b+4/55*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+2/9*d*(b*x^3+a)^(3/2)/b+c*(2/7*x^2*(b*x^
3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2
)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)*x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 147, normalized size = 0.22

$$-\frac{2\left(4914\,a^{2}\sqrt{b}\,e\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+1485\,(13\,abc-4\,a^{2}f)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-(3003\,b^{2}gx^{6}+3465\,b^{2}fx^{5}+4095\,b^{2}ex^{4}+2457\,abex+1001\,(5\,b^{2}d+abg)x^{3}+5005\,abd-2002\,a^{2}g+495\,(13\,b^{2}c+3\,abf)x^{2})\sqrt{bx^{3}+a}\right)}{45045\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/45045*(4914*a^2*sqrt(b)*e*weierstrassPInverse(0, -4*a/b, x) + 1485*(13*a
*b*c - 4*a^2*f)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -
4*a/b, x)) - (3003*b^2*g*x^6 + 3465*b^2*f*x^5 + 4095*b^2*e*x^4 + 2457*a*b*e
*x + 1001*(5*b^2*d + a*b*g)*x^3 + 5005*a*b*d - 2002*a^2*g + 495*(13*b^2*c +
 3*a*b*f)*x^2)*sqrt(b*x^3 + a))/b^2
```

Sympy [A]
time = 2.29, size = 223, normalized size = 0.33

$$\frac{\sqrt{a}\,cx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt{a}\,ex^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}\,fx^{5}\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+d\left(\begin{cases}\frac{\sqrt{a}\,x^{3}}{3} & \text{for}\ b=0\\ \frac{2(a+bx^{3})^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)+g\left(\begin{cases}-\frac{4a^{2}\sqrt{a+bx^{3}}}{45b^{2}}+\frac{2ax^{3}\sqrt{a+bx^{3}}}{45b}+\frac{2x^{6}\sqrt{a+bx^{3}}}{15} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^{6}}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(5/3)) + sqrt(a)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*f*x**5*gamma(5/3)*hyper((
-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + d*Piecewise(
(sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + g*Piecew
ise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b)
+ 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
[Out] int(x*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.448 $\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=639

$$\frac{2ae\sqrt{a+bx^3}}{9b}+\frac{6afx\sqrt{a+bx^3}}{55b}+\frac{6agx^2\sqrt{a+bx^3}}{91b}+\frac{6a(13bd-4ag)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2\sqrt{a+bx^3}\,(9009c}{}$$

```
[Out] 2/9*a*e*(b*x^3+a)^(1/2)/b+6/55*a*f*x*(b*x^3+a)^(1/2)/b+6/91*a*g*x^2*(b*x^3+
a)^(1/2)/b+2/45045*(3465*g*x^5+4095*f*x^4+5005*e*x^3+6435*d*x^2+9009*c*x)*(
b*x^3+a)^(1/2)+6/91*a*(-4*a*g+13*b*d)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))-3/91*3^(1/4)*a^(4/3)*(-4*a*g+13*b*d)*(a^(1/3)+b^(1/3)*x)
*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/5005*3
^(3/4)*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(91*b^(1/3)*(-2*a*f+11*b*c)-55*a
^(1/3)*(-4*a*g+13*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3
)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 639, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1867, 1902, 1900, 267, 1892, 224, 1891}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(91\sqrt[3]{b}\,(11bc-2af)-55\left(1-\sqrt{3}\right)\sqrt[3]{a}\,(13bd-4ag)\right)}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(13bd-4ag)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{6a\sqrt{a+bx^3}\,(13bd-4ag)}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}+\frac{2ae\sqrt{a+bx^3}}{9b}+\frac{6afx\sqrt{a+bx^3}}{55b}+\frac{6agx^2\sqrt{a+bx^3}}{91b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a*e*Sqrt[a + b*x^3])/(9*b) + (6*a*f*x*Sqrt[a + b*x^3])/(55*b) + (6*a*g*x
^2*Sqrt[a + b*x^3])/(91*b) + (6*a*(13*b*d - 4*a*g)*Sqrt[a + b*x^3])/(91*b^(
5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqrt[a + b*x^3]*(9009*c*x +
6435*d*x^2 + 5005*e*x^3 + 4095*f*x^4 + 3465*g*x^5))/45045 - (3*3^(1/4)*Sqrt
[2 - Sqrt[3]]*a^(4/3)*(13*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
```

```
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*
Sqrt[2 + Sqrt[3]]*a*(91*b^(1/3)*(11*b*c - 2*a*f) - 55*(1 - Sqrt[3])*a^(1/3)
*(13*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(5005*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{2\sqrt{a+bx^3}\,(9009cx+6435dx^2+5005ex^3+4095fx^4+3465}{45045}$$

$$= \frac{6agx^2\sqrt{a+bx^3}}{91b} + \frac{2\sqrt{a+bx^3}\,(9009cx+6435dx^2+5005ex}{45045}$$

$$= \frac{6afx\sqrt{a+bx^3}}{55b} + \frac{6agx^2\sqrt{a+bx^3}}{91b} + \frac{2\sqrt{a+bx^3}\,(9009cx+}{}$$

$$= \frac{6afx\sqrt{a+bx^3}}{55b} + \frac{6agx^2\sqrt{a+bx^3}}{91b} + \frac{2\sqrt{a+bx^3}\,(9009cx+}{}$$

$$= \frac{2ae\sqrt{a+bx^3}}{9b} + \frac{6afx\sqrt{a+bx^3}}{55b} + \frac{6agx^2\sqrt{a+bx^3}}{91b} + \frac{2\sqrt{a}}{}$$

$$= \frac{2ae\sqrt{a+bx^3}}{9b} + \frac{6afx\sqrt{a+bx^3}}{55b} + \frac{6agx^2\sqrt{a+bx^3}}{91b} + \frac{6}{91b^{5/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.53, size = 135, normalized size = 0.21

$$\frac{\sqrt{a+bx^3}\left(4(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,(143e+9x(13f+11gx))+234(11bc-2af)x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+99(13bd-4ag)x^2\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2574b\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (Sqrt[a + b*x^3]*(4*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(143*e + 9*x*(13*f + 11
*g*x)) + 234*(11*b*c - 2*a*f)*x*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)
/a)] + 99*(13*b*d - 4*a*g)*x^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2574*b*Sqrt[1 + (b*x^3)/a])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1556 vs. 2(491) = 982.
time = 0.38, size = 1557, normalized size = 2.44

method	result
elliptic	$\frac{2 g \,x^{5} \sqrt{b \,x^{3}+a}}{13}+\frac{2 f \,x^{4} \sqrt{b \,x^{3}+a}}{11}+\frac{2 e \,x^{3} \sqrt{b \,x^{3}+a}}{9}+\frac{2 \left(\frac{3 a g}{13}+b d \right) x^{2} \sqrt{b \,x^{3}+a}}{7 b}+\frac{2 \left(\frac{3 a f}{11}+b c \right) x \sqrt{b \,x^{3}+}}{5 b}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/13*x^5*(b*x^3+a)^(1/2)+6/91*a*x^2*(b*x^3+a)^(1/2)/b+8/91*I/b^2*a^2*3^(
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+f*(2/11*x^4*(b*x^3+a)^(1/2)+6/55*a*
x*(b*x^3+a)^(1/2)/b+4/55*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)))+2/9*e*(b*x^3+a)^(3/2)/b+d*(2/7*x^2*(b*x^3+a)^(1/2)-2
/7*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
```

```
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+c*(2/5*x*(b*x^3+a)^(1/2
)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 141, normalized size = 0.22

$$\frac{2\left(2457\,(11\,abc-2\,a^2f)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-1485\,(13\,abd-4\,a^2g)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(3465\,b^2gx^5+4095\,b^2fx^4+5005\,b^2ex^3+5005\,abe+495\,(13\,b^2d+3\,abg)x^2+819\,(11\,b^2c+3\,abf)x)\sqrt{bx^3+a}\right)}{45045\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] 2/45045*(2457*(11*a*b*c - 2*a^2*f)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x
) - 1485*(13*a*b*d - 4*a^2*g)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstras
sPInverse(0, -4*a/b, x)) + (3465*b^2*g*x^5 + 4095*b^2*f*x^4 + 5005*b^2*e*x^
3 + 5005*a*b*e + 495*(13*b^2*d + 3*a*b*g)*x^2 + 819*(11*b^2*c + 3*a*b*f)*x)
*sqrt(b*x^3 + a))/b^2
```

Sympy [A]
time = 2.31, size = 194, normalized size = 0.30

$$\frac{\sqrt{a}\,cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{a}\,dx^2\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt{a}\,fx^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}\,gx^5\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+e\left(\begin{cases}\frac{\sqrt{a}\,x^3}{3} & \text{for}\: b=0\\\frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) + sqrt(a)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*f*x**4*gamma(4/3)*hyper((-1/
2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*g*x**5*
gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/
3)) + e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b),
 True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

```
[Out] int((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.449 $\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$

Optimal. Leaf size=620

$$\frac{2af\sqrt{a+bx^3}}{9b}+\frac{6agx\sqrt{a+bx^3}}{55b}+\frac{6ae\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2\sqrt{a+bx^3}\,(1155cx+693dx^2+495ex^3}{3465x}$$

[Out] -2/3*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/9*a*f*(b*x^3+a)^(1/2)/b+6/55*a*g*x*(b*x^3+a)^(1/2)/b+2/3465*(315*g*x^5+385*f*x^4+495*e*x^3+693*d*x^2+1155*c*x)*(b*x^3+a)^(1/2)/x+6/7*a*e*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3/7*3^(1/4)*a^(4/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/385*3^(3/4)*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(77*b*d-14*a*g-55*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.36, antiderivative size = 620, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {1840, 1846, 272, 65, 214, 1902, 1900, 267, 1892, 224, 1891}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(-55\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-14ag+77bd\right)}{385b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{6ae\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2\sqrt{a+bx^3}\,(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x}-\frac{2}{3}\sqrt{a}\,c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2af\sqrt{a+bx^3}}{9b}+\frac{6agx\sqrt{a+bx^3}}{55b}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]

[Out] (2*a*f*Sqrt[a + b*x^3])/(9*b) + (6*a*g*x*Sqrt[a + b*x^3])/(55*b) + (6*a*e*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqrt[a + b*x^3]*(1155*c*x + 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g*x^5))/(3465*x) - (2*Sqrt[a]*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7

```
- 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a
*(77*b*d - 55*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 14*a*g)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(385*b^(4/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
```

```
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
```

```
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \frac{2\sqrt{a+bx^3}\,(1155cx+693dx^2+495ex^3+385fx^4+315gx^5}{3465x}$$

$$= \frac{2\sqrt{a+bx^3}\,(1155cx+693dx^2+495ex^3+385fx^4+315gx^5}{3465x}$$

$$= \frac{6agx\sqrt{a+bx^3}}{55b} + \frac{2\sqrt{a+bx^3}\,(1155cx+693dx^2+495ex^3}{3465x}$$

$$= \frac{6agx\sqrt{a+bx^3}}{55b} + \frac{2\sqrt{a+bx^3}\,(1155cx+693dx^2+495ex^3}{3465x}$$

$$= \frac{2af\sqrt{a+bx^3}}{9b} + \frac{6agx\sqrt{a+bx^3}}{55b} + \frac{2\sqrt{a+bx^3}\,(1155cx+6}{}$$

$$= \frac{2af\sqrt{a+bx^3}}{9b} + \frac{6agx\sqrt{a+bx^3}}{55b} + \frac{6ae\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.27, size = 714, normalized size = 1.15

$$\frac{2\sqrt{a+bx^3}\,(1155bc+7a(55f+27gx)+bx(693d+5x(99e+7x(11f+9gx))))}{3465b} - \frac{2\sqrt{a}\left(385b^{4/3}c\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+693\sqrt{a}bd\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}F\left(\sin^{-1}\left(\cdots\right)\right)\cdots\right)}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(1155*b*c + 7*a*(55*f + 27*g*x) + b*x*(693*d + 5*x*(99*e
 + 7*x*(11*f + 9*g*x)))))/(3465*b) - (2*Sqrt[a]*(385*b^(4/3)*c*Sqrt[(a^(1/3
) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTa
nh[Sqrt[a + b*x^3]/Sqrt[a]] + 693*Sqrt[a]*b*d*((-1)^(1/3)*a^(1/3) - b^(1/3)
*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)
*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[Ar
```

```
cSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-
1)^(1/3)] - 126*a^(3/2)*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) +
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/
3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 495*Sqrt
[2]*a^(5/6)*b^(2/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^
(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^
(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[S
qrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/
3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)],
 (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(1155*b^(4/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1117 vs. $2(468) = 936$.
time = 0.37, size = 1118, normalized size = 1.80

method	result
elliptic	$\frac{2 g x^4 \sqrt{b x^3+a}}{11}+\frac{2 f x^3 \sqrt{b x^3+a}}{9}+\frac{2 e x^2 \sqrt{b x^3+a}}{7}+\frac{2\left(\frac{3 a g}{11}+b d\right) x \sqrt{b x^3+a}}{5 b}+\frac{2\left(\frac{a f}{3}+b c\right) \sqrt{b x^3+a}}{3 b}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/11*x^4*(b*x^3+a)^(1/2)+6/55*a*x*(b*x^3+a)^(1/2)/b+4/55*I/b^2*a^2*3^(1/
2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
```

```
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/9*f*(b*x^3+a)^(
3/2)/b+e*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))))+d*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+c*(-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))
*a^(1/2)+2/3*(b*x^3+a)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 340, normalized size = 0.55

$$\left[\frac{1155\,\sqrt{a}\,b^{2}c\log\left(-\frac{b^{2}x^{6}+8\,abx^{3}-4\,(bx^{3}+2\,a)\sqrt{bx^{3}+a}\,\sqrt{a}+8\,a^{2}}{x^{6}}\right)-5940\,ab^{\frac{3}{2}}e\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+756\,(11\,abd-2\,a^{2}g)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+4\,(315\,b^{2}gx^{4}+385\,b^{2}fx^{3}+495\,b^{2}ex^{2}+1155\,b^{2}c+385\,abf+63\,(11\,b^{2}d+3\,abg)x)\sqrt{bx^{3}+a}}{6930\,b^{2}},\ \frac{1155\,\sqrt{-a}\,b^{2}c\arctan\left(\frac{2\sqrt{bx^{3}+a}\,\sqrt{-a}}{bx^{3}+2\,a}\right)-2970\,ab^{\frac{3}{2}}e\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+378\,(11\,abd-2\,a^{2}g)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+2\,(315\,b^{2}gx^{4}+385\,b^{2}fx^{3}+495\,b^{2}ex^{2}+1155\,b^{2}c+385\,abf+63\,(11\,b^{2}d+3\,abg)x)\sqrt{bx^{3}+a}}{3465\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] [1/6930*(1155*sqrt(a)*b^2*c*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqr
t(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 5940*a*b^(3/2)*e*weierstrassZeta(0, -4
*a/b, weierstrassPInverse(0, -4*a/b, x)) + 756*(11*a*b*d - 2*a^2*g)*sqrt(b)
*weierstrassPInverse(0, -4*a/b, x) + 4*(315*b^2*g*x^4 + 385*b^2*f*x^3 + 495
```

```
*b^2*e*x^2 + 1155*b^2*c + 385*a*b*f + 63*(11*b^2*d + 3*a*b*g)*x)*sqrt(b*x^3
 + a))/b^2, 1/3465*(1155*sqrt(-a)*b^2*c*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(
b*x^3 + 2*a)) - 2970*a*b^(3/2)*e*weierstrassZeta(0, -4*a/b, weierstrassPInv
erse(0, -4*a/b, x)) + 378*(11*a*b*d - 2*a^2*g)*sqrt(b)*weierstrassPInverse(
0, -4*a/b, x) + 2*(315*b^2*g*x^4 + 385*b^2*f*x^3 + 495*b^2*e*x^2 + 1155*b^2
*c + 385*a*b*f + 63*(11*b^2*d + 3*a*b*g)*x)*sqrt(b*x^3 + a))/b^2]
```

Sympy [A]
time = 5.00, size = 235, normalized size = 0.38

$$- \frac{2 \sqrt{a} c \operatorname{asinh}{\left(\frac{\sqrt{a}}{\sqrt{b} x^{\frac{3}{2}}} \right)}}{3} + \frac{\sqrt{a} d x \Gamma\left(\frac{1}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}}{3 \Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a} e x^{2} \Gamma\left(\frac{2}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}}{3 \Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a} g x^{4} \Gamma\left(\frac{4}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}}{3 \Gamma\left(\frac{7}{3}\right)} + \frac{2 a c}{3 \sqrt{b} x^{\frac{3}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} + \frac{2 \sqrt{b} c x^{\frac{3}{2}}}{3 \sqrt{\frac{a}{b x^{3}} + 1}} + f \left(\begin{cases} \frac{\sqrt{a} x^{3}}{3} & \text{for}\: b = 0 \\ \frac{2 \left(a + b x^{3}\right)^{\frac{3}{2}}}{9 b} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*d*x*gamma(1/3)*h
yper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a
)*e*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(5/3)) + sqrt(a)*g*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a*c/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**
3) + 1)) + 2*sqrt(b)*c*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + f*Piecewise((sqr
t(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x)
```

```
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x, x)
```

3.450 $\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$

Optimal. Leaf size=638

$$\frac{2ag\sqrt{a+bx^3}}{9b}-\frac{3c\sqrt{a+bx^3}}{x}+\frac{3(7bc+2af)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2\sqrt{a+bx^3}\,(315cx+105dx^2+63ex^3+4}{315x^2}$$

```
[Out] -2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/9*a*g*(b*x^3+a)^(1/2)/b-3
*c*(b*x^3+a)^(1/2)/x+2/315*(35*g*x^5+45*f*x^4+63*e*x^3+105*d*x^2+315*c*x)*(
b*x^3+a)^(1/2)/x^2+3/7*(2*a*f+7*b*c)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))-3/14*3^(1/4)*a^(1/3)*(2*a*f+7*b*c)*(a^(1/3)+b^(1/3)*x)*El
lipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/35*3^(3/4
)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(14*a^(2/3)*b^(1/3)*e-5*(2*a*f+
7*b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2
)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.43, antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {1840, 1849, 1846, 272, 65, 214, 1900, 267, 1892, 224, 1891}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(14a^{2/3}\sqrt[3]{b}\,e-5\left(1-\sqrt{3}\right)(2af+7bc)\right)}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(2af+7bc)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{3\sqrt{a+bx^3}\,(2af+7bc)}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2\sqrt{a+bx^3}\,(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2}-\frac{3c\sqrt{a+bx^3}}{x}-\frac{2}{3}\sqrt{a}\,d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2ag\sqrt{a+bx^3}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

```
[Out] (2*a*g*Sqrt[a + b*x^3])/(9*b) - (3*c*Sqrt[a + b*x^3])/x + (3*(7*b*c + 2*a*f
)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqr
t[a + b*x^3]*(315*c*x + 105*d*x^2 + 63*e*x^3 + 45*f*x^4 + 35*g*x^5))/(315*x
^2) - (2*Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(1/3)*(7*b*c + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
```

```
(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2
 + Sqrt[3]]*a^(1/3)*(14*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 2*a*f)
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*b
^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
```

```
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
```

```
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \frac{2\sqrt{a+bx^3}\,(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2}+\frac{1}{2}($$

$$= -\frac{3c\sqrt{a+bx^3}}{x}+\frac{2\sqrt{a+bx^3}\,(315cx+105dx^2+63ex^3+45f}{315x^2}$$

$$= -\frac{3c\sqrt{a+bx^3}}{x}+\frac{2\sqrt{a+bx^3}\,(315cx+105dx^2+63ex^3+45f}{315x^2}$$

$$= -\frac{3c\sqrt{a+bx^3}}{x}+\frac{2\sqrt{a+bx^3}\,(315cx+105dx^2+63ex^3+45f}{315x^2}$$

$$= \frac{2ag\sqrt{a+bx^3}}{9b}-\frac{3c\sqrt{a+bx^3}}{x}+\frac{2\sqrt{a+bx^3}\,(315cx+105dx}{31}$$

$$= \frac{2ag\sqrt{a+bx^3}}{9b}-\frac{3c\sqrt{a+bx^3}}{x}+\frac{3(7bc+2af)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.18, size = 810, normalized size = 1.27

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

```
[Out] (Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(a + b*x
^3)*(-315*b*c + 70*a*g*x + 2*b*x*(105*d + x*(63*e + 5*x*(9*f + 7*g*x)))) -
210*Sqrt[a]*b*d*x*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a
^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 378*a*b^(2/3)*e
*x*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(
1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-
1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/
```

```
((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + 945*Sqrt[2]*a^(1/3)*b^(4/3)*c*x
*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^
(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3
*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b
^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[Arc
Sin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1
)^(1/3))]) + 270*Sqrt[2]*a^(4/3)*b^(1/3)*f*x*((-1)^(1/3)*a^(1/3) - b^(1/3)*
x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(
1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2
/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-
1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3
)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(315*b*x*Sqrt[(a^(1
/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1247 vs. $2(486) = 972$.
time = 0.44, size = 1248, normalized size = 1.96

method	result
elliptic	$-\frac{c\sqrt{b\,x^3+a}}{x}+\frac{2g\,x^3\sqrt{b\,x^3+a}}{9}+\frac{2f\,x^2\sqrt{b\,x^3+a}}{7}+\frac{2ex\sqrt{b\,x^3+a}}{5}+\frac{2\left(\frac{ag}{3}+bd\right)\sqrt{b\,x^3+a}}{3b}-$ ... $2iae\sqrt{}$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*g*(b*x^3+a)^(3/2)/b+f*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-a*b^
2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
```

```
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))))+e*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-2/3*arctanh((b*x^3+
a)^(1/2)/a^(1/2))*a^(1/2)+2/3*(b*x^3+a)^(1/2))+c*(-(b*x^3+a)^(1/2)/x-I*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 307, normalized size = 0.48

$$\left[\frac{105\,\sqrt{a}\,bdx\log\left(-\frac{b^{2}x^{6}+8\,abx^{3}-4\,(bx^{3}+2\,a)\sqrt{bx^{3}+a}\,\sqrt{a}+8\,a^{2}}{x^{6}}\right)+756\,a\sqrt{b}\,ex\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-270\,(7\,bc+2\,af)\sqrt{b}\,x\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+2\,(70\,bgx^{4}+90\,bfx^{3}+126\,bex^{2}-315\,bc+70\,(3\,bd+ag)x)\sqrt{bx^{3}+a}}{630\,bx},\ \frac{105\,\sqrt{-a}\,bdx\arctan\left(\frac{2\,\sqrt{bx^{3}+a}\,\sqrt{-a}}{bx^{3}+2\,a}\right)+378\,a\sqrt{b}\,ex\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-135\,(7\,bc+2\,af)\sqrt{b}\,x\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(70\,bgx^{4}+90\,bfx^{3}+126\,bex^{2}-315\,bc+70\,(3\,bd+ag)x)\sqrt{bx^{3}+a}}{315\,bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] [1/630*(105*sqrt(a)*b*d*x*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(
b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 756*a*sqrt(b)*e*x*weierstrassPInverse(0,
 -4*a/b, x) - 270*(7*b*c + 2*a*f)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weie
rstrassPInverse(0, -4*a/b, x)) + 2*(70*b*g*x^4 + 90*b*f*x^3 + 126*b*e*x^2 -
 315*b*c + 70*(3*b*d + a*g)*x)*sqrt(b*x^3 + a))/(b*x), 1/315*(105*sqrt(-a)*
b*d*x*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 378*a*sqrt(b)*e*x*
weierstrassPInverse(0, -4*a/b, x) - 135*(7*b*c + 2*a*f)*sqrt(b)*x*weierstra
ssZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (70*b*g*x^4 + 90*b*f
*x^3 + 126*b*e*x^2 - 315*b*c + 70*(3*b*d + a*g)*x)*sqrt(b*x^3 + a))/(b*x)]
```

Sympy [**A**]
time = 3.18, size = 236, normalized size = 0.37

$$\frac{\sqrt{a}\,c\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}-\frac{2\sqrt{a}\,d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3}+\frac{\sqrt{a}\,ex\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{a}\,fx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{2ad}{3\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx^{3}}+1}}+\frac{2\sqrt{b}\,dx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^{3}}+1}}+g\left(\begin{cases}\frac{\sqrt{a}\,x^{3}}{3} & \text{for}\ b=0\\ \frac{2(a+bx^{3})^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**2,x)
```

```
[Out] sqrt(a)*c*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x*gamma(2/3)) - 2*sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(
a)*e*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(4/3)) + sqrt(a)*f*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a*d/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3)
 + 1)) + 2*sqrt(b)*d*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + g*Piecewise((sqrt(
a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x)
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2, x)
```

3.451 $\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$

Optimal. Leaf size=640

$$\frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{3d\sqrt{a+bx^3}}{x}+\frac{3(7bd+2ag)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2\sqrt{a+bx^3}\,(105cx-105dx^2-35ex^3-21}{105x^3}$$

```
[Out] -2/3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+3/2*c*(b*x^3+a)^(1/2)/x^2-3
*d*(b*x^3+a)^(1/2)/x-2/105*(-15*g*x^5-21*f*x^4-35*e*x^3-105*d*x^2+105*c*x)*
(b*x^3+a)^(1/2)/x^3+3/7*(2*a*g+7*b*d)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))-3/14*3^(1/4)*a^(1/3)*(2*a*g+7*b*d)*(a^(1/3)+b^(1/3)*x)*E
llipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*
3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/70*3^(3/
4)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*(4*a*f+5*b*c)-10*a^(1/3)*(2
*a*g+7*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)
^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)
```

Rubi [A]
time = 0.52, antiderivative size = 640, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}\,(4af+5bc)-10\left(1-\sqrt{3}\right)\sqrt[3]{a}\,(2ag+7bd)\right)}{70b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(2ag+7bd)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{3\sqrt{a+bx^3}\,(2ag+7bd)}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2\sqrt{a+bx^3}\,(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3}+\frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{3d\sqrt{a+bx^3}}{x}-\frac{2}{3}\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

```
[Out] (3*c*Sqrt[a + b*x^3])/(2*x^2) - (3*d*Sqrt[a + b*x^3])/x + (3*(7*b*d + 2*a*g
)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*Sqr
t[a + b*x^3]*(105*c*x - 105*d*x^2 - 35*e*x^3 - 21*f*x^4 - 15*g*x^5))/(105*x
^3) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(1/3)*(7*b*d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
```

```
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2
 + Sqrt[3]]*(7*b^(1/3)*(5*b*c + 4*a*f) - 10*(1 - Sqrt[3])*a^(1/3)*(7*b*d +
2*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(70*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = -\frac{2\sqrt{a+bx^3}\,(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3} +$$

$$= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{2\sqrt{a+bx^3}\,(105cx-105dx^2-35ex^3-21fx}{105x^3}$$

$$= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{3d\sqrt{a+bx^3}}{x} - \frac{2\sqrt{a+bx^3}\,(105cx-105dx^2}{105}$$

$$= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{3d\sqrt{a+bx^3}}{x} - \frac{2\sqrt{a+bx^3}\,(105cx-105dx^2}{105}$$

$$= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{3d\sqrt{a+bx^3}}{x} - \frac{2\sqrt{a+bx^3}\,(105cx-105dx^2}{105}$$

$$= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{3d\sqrt{a+bx^3}}{x} + \frac{3(7bd+2ag)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}$$

$$= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{3d\sqrt{a+bx^3}}{x} + \frac{3(7bd+2ag)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.87, size = 962, normalized size = 1.50

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

```
[Out] (b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*
(a + b*x^3)*(-105*c + 2*x*(-105*d + 70*e*x + 42*f*x^2 + 30*g*x^3)) - 140*Sq
```

```
rt[a]*b^(2/3)*e*x^2*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 315*b^(4/3)*c
*x^2*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 +
(-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 252*a*b^(1/3)*f*x^2*((-1)^(1/
3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^
(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(
1/3))*a^(1/3))]], (-1)^(1/3)] + 630*Sqrt[2]*a^(1/3)*b*d*x^2*((-1)^(1/3)*a^(
1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(
-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3
)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1
/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) + 180
*Sqrt[2]*a^(4/3)*g*x^2*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a
^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b
^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[
Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1
/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)]
, (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(210*b^(2/3)*x^2*Sqrt[(a^(1/3) + (-1)^(2/
3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1528 vs. $2(490) = 980$.
time = 0.41, size = 1529, normalized size = 2.39

method	result
elliptic	$-\frac{c\sqrt{b\,x^3+a}}{2x^2}-\frac{d\sqrt{b\,x^3+a}}{x}+\frac{2g\,x^2\sqrt{b\,x^3+a}}{7}+\frac{2fx\sqrt{b\,x^3+a}}{5}+\frac{2e\sqrt{b\,x^3+a}}{3}-\frac{2i\left(\frac{3af}{5}+\frac{3bc}{4}\right)\sqrt{3}}{}$
default	Expression too large to display

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))))+f*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))+c*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-a*b
^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+e*(-2/3*arctanh((b*x^3+a)
^(1/2)/a^(1/2))*a^(1/2)+2/3*(b*x^3+a)^(1/2))+d*(-(b*x^3+a)^(1/2)/x-I*3^(1/2
)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 320, normalized size = 0.50

$$\left[\frac{35\,\sqrt{a}\,bex^2\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\,\sqrt{a}+8\,a^2}{x^6}\right)+63\,(5\,bc+4\,af)\sqrt{b}\,x^2\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-90\,(7\,bd+2\,ag)\sqrt{b}\,x^2\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(60\,bgx^4+84\,bfx^3+140\,bex^2-210\,bdx-105\,bc)\sqrt{bx^3+a}}{210\,bx^2},\ \frac{70\,\sqrt{-a}\,bex^2\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-a}}{bx^3+2\,a}\right)+63\,(5\,bc+4\,af)\sqrt{b}\,x^2\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-90\,(7\,bd+2\,ag)\sqrt{b}\,x^2\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(60\,bgx^4+84\,bfx^3+140\,bex^2-210\,bdx-105\,bc)\sqrt{bx^3+a}}{210\,bx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] [1/210*(35*sqrt(a)*b*e*x^2*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt
(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 63*(5*b*c + 4*a*f)*sqrt(b)*x^2*weierstr
assPInverse(0, -4*a/b, x) - 90*(7*b*d + 2*a*g)*sqrt(b)*x^2*weierstrassZeta(
0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (60*b*g*x^4 + 84*b*f*x^3 +
140*b*e*x^2 - 210*b*d*x - 105*b*c)*sqrt(b*x^3 + a))/(b*x^2), 1/210*(70*sqrt
(-a)*b*e*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 63*(5*b*c +
 4*a*f)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) - 90*(7*b*d + 2*a*g)*
sqrt(b)*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) +
 (60*b*g*x^4 + 84*b*f*x^3 + 140*b*e*x^2 - 210*b*d*x - 105*b*c)*sqrt(b*x^3 +
 a))/(b*x^2)]
```

Sympy [A]
time = 3.24, size = 255, normalized size = 0.40

$$\frac{\sqrt{a}\,c\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{\sqrt{a}\,d\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}-\frac{2\sqrt{a}\,e\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3}+\frac{\sqrt{a}\,fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{a}\,gx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{2ae}{3\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2\sqrt{b}\,ex^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] sqrt(a)*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3)) + sqrt(a)*d*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*
x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(a)*e*asinh(sqrt(a)/(sqrt(
b)*x**(3/2)))/3 + sqrt(a)*f*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*g*x**2*gamma(2/3)*hyper((-1/2,
2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a*e/(3*sqrt(b)*x
```

```
**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*sqrt(b)*e*x**(3/2)/(3*sqrt(a/(b*x**3) + 1
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x)
```

```
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3, x)
```

3.452 $\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$

Optimal. Leaf size=637

$$\frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}+\frac{3\sqrt[3]{b}\,e\sqrt{a+bx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}-\frac{2\sqrt{a+bx^3}\,(5cx+15dx^2-15ex^3-5$$

[Out] -1/3*(2*a*f+b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+1/3*c*(b*x^3+a)^(1/2)/x^3+3/2*d*(b*x^3+a)^(1/2)/x^2-3*e*(b*x^3+a)^(1/2)/x-2/15*(-3*g*x^5-5*f*x^4-15*e*x^3+15*d*x^2+5*c*x)*(b*x^3+a)^(1/2)/x^4+3*b^(1/3)*e*(b*x^3+a)^(1/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3/2*3^(1/4)*a^(1/3)*b^(1/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/10*3^(3/4)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(5*b*d+4*a*g-10*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.57, antiderivative size = 637, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(-10\left(1-\sqrt{3}\right)\sqrt[3]{a}\,b^{2/3}e+4ag+5bd\right)}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\,e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{a+bx^3}\,(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4}-\frac{(2af+bc)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}+\frac{3\sqrt[3]{b}\,e\sqrt{a+bx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]

[Out] (c*Sqrt[a + b*x^3])/(3*x^3) + (3*d*Sqrt[a + b*x^3])/(2*x^2) - (3*e*Sqrt[a + b*x^3])/x + (3*b^(1/3)*e*Sqrt[a + b*x^3])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x) - (2*Sqrt[a + b*x^3]*(5*c*x + 15*d*x^2 - 15*e*x^3 - 5*f*x^4 - 3*g*x^5))/(15*x^4) - ((b*c + 2*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*

```
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqr
t[2 + Sqrt[3]]*(5*b*d - 10*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10*b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = -\frac{2\sqrt{a+bx^3}\,(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4}+\frac{1}{2}(3a)$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}-\frac{2\sqrt{a+bx^3}\,(5cx+15dx^2-15ex^3-5fx^4-3g}{15x^4}$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{2\sqrt{a+bx^3}\,(5cx+15dx^2-15}{15x^4}$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}-\frac{2\sqrt{a+bx^3}\,(5}{}$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}-\frac{2\sqrt{a+bx^3}\,(5}{}$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}-\frac{2\sqrt{a+bx^3}\,(5}{}$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}+\frac{3\sqrt[3]{b}\,e\sqrt{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{}}$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}+\frac{3\sqrt[3]{b}\,e\sqrt{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.08, size = 769, normalized size = 1.21

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

```
[Out] Sqrt[a + b*x^3]*((2*f)/3 - (10*c + 3*x*(5*d + 10*e*x - 4*g*x^3))/(30*x^3))
- (b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (2*Sqrt[a]*f*ArcTanh
[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*b^(2/3)*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x
)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a
^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin
[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(
1/3)])/(2*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]
*Sqrt[a + b*x^3]) - (6*a*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) +
 b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/
3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) +
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(5*b^(1/3)
*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a +
 b*x^3]) - (3*Sqrt[2]*a^(1/3)*b^(1/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*
Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*Elli
pticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/
(-1 + (-1)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/
3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(Sqrt[(a^(1/3) + (-1)^(2/3)*b
^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1113 vs. $2(485) = 970$.
time = 0.41, size = 1114, normalized size = 1.75

method	result
elliptic	$-\frac{c\sqrt{b\,x^{3}+a}}{3x^{3}}-\frac{d\sqrt{b\,x^{3}+a}}{2x^{2}}-\frac{e\sqrt{b\,x^{3}+a}}{x}+\frac{2gx\sqrt{b\,x^{3}+a}}{5}+\frac{2f\sqrt{b\,x^{3}+a}}{3}-\frac{2i\left(\frac{3ag}{5}+\frac{3bd}{4}\right)\sqrt{3}\,(-}{}$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)))+d*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-a*b^2)^(
1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+c*(-1/3*b*arctanh((b*x^3+a)^(1
/2)/a^(1/2))/a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3)+f*(-2/3*arctanh((b*x^3+a)^(1/
2)/a^(1/2))*a^(1/2)+2/3*(b*x^3+a)^(1/2))+e*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-
a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^4, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 348, normalized size = 0.55

$$\left[-\frac{180\,ab^{\frac{3}{2}}ex^3\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-5\left(b^2c+2\,abf\right)\sqrt{a}\,x^3\log\left(-\frac{b^2x^6+8\,abx^3-4\left(bx^3+2\,a\right)\sqrt{bx^3+a}\,\sqrt{a}+8\,a^2}{x^6}\right)-18\left(5\,abd+4\,a^2g\right)\sqrt{b}\,x^3\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-2\left(12\,abgx^4+20\,abfx^3-30\,abex^2-15\,abdx-10\,abc\right)\sqrt{bx^3+a}}{60\,abx^3},-\frac{90\,ab^{\frac{3}{2}}ex^3\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-5\left(b^2c+2\,abf\right)\sqrt{-a}\,x^3\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-a}}{bx^3+2\,a}\right)-9\left(5\,abd+4\,a^2g\right)\sqrt{b}\,x^3\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\left(12\,abgx^4+20\,abfx^3-30\,abex^2-15\,abdx-10\,abc\right)\sqrt{bx^3+a}}{30\,abx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] [-1/60*(180*a*b^(3/2)*e*x^3*weierstrassZeta(0, -4*a/b, weierstrassPInverse(
0, -4*a/b, x)) - 5*(b^2*c + 2*a*b*f)*sqrt(a)*x^3*log(-(b^2*x^6 + 8*a*b*x^3
- 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 18*(5*a*b*d + 4*a
^2*g)*sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) - 2*(12*a*b*g*x^4 + 20*
a*b*f*x^3 - 30*a*b*e*x^2 - 15*a*b*d*x - 10*a*b*c)*sqrt(b*x^3 + a))/(a*b*x^3
), -1/30*(90*a*b^(3/2)*e*x^3*weierstrassZeta(0, -4*a/b, weierstrassPInverse
(0, -4*a/b, x)) - 5*(b^2*c + 2*a*b*f)*sqrt(-a)*x^3*arctan(2*sqrt(b*x^3 + a)
*sqrt(-a)/(b*x^3 + 2*a)) - 9*(5*a*b*d + 4*a^2*g)*sqrt(b)*x^3*weierstrassPIn
verse(0, -4*a/b, x) - (12*a*b*g*x^4 + 20*a*b*f*x^3 - 30*a*b*e*x^2 - 15*a*b*
d*x - 10*a*b*c)*sqrt(b*x^3 + a))/(a*b*x^3)]
```

Sympy [A]

time = 3.78, size = 265, normalized size = 0.42

$$\frac{\sqrt{a}\, d\Gamma\left(-\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{2}\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}\, e\Gamma\left(-\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^{3}e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}\, f \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\, x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}\, gx\Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2af}{3\sqrt{b}\, x^{\frac{3}{2}} \sqrt{\frac{a}{bx^{3}} + 1}} - \frac{\sqrt{b}\, c\sqrt{\frac{a}{bx^{3}} + 1}}{3x^{\frac{3}{2}}} + \frac{2\sqrt{b}\, fx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^{3}} + 1}} - \frac{bc \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\, x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] sqrt(a)*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3)) + sqrt(a)*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*
x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(a)*f*asinh(sqrt(a)/(sqrt(
b)*x**(3/2)))/3 + sqrt(a)*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(4/3)) + 2*a*f/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**
3) + 1)) - sqrt(b)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*sqrt(b)*f*x**(3/
2)/(3*sqrt(a/(b*x**3) + 1)) - b*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt
(a))
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^4, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x)
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4, x)
```

3.453 $\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx$

Optimal. Leaf size=694

$$\frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3(bc+8af)\sqrt{a+bx^3}}{8ax}+\frac{3\sqrt[3]{b}\,(bc+8af)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2\sqrt{a+}}{}$$

```
[Out] -1/3*(2*a*g+b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+3/20*c*(b*x^3+a)^
(1/2)/x^4+1/3*d*(b*x^3+a)^(1/2)/x^3+3/2*e*(b*x^3+a)^(1/2)/x^2-3/8*(8*a*f+b*
c)*(b*x^3+a)^(1/2)/a/x-2/15*(-5*g*x^5-15*f*x^4+15*e*x^3+5*d*x^2+3*c*x)*(b*x
^3+a)^(1/2)/x^5+3/8*b^(1/3)*(8*a*f+b*c)*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))-3/16*3^(1/4)*b^(1/3)*(8*a*f+b*c)*(a^(1/3)+b^(1/3)*x)*Ellipti
cE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/8*3^(3/4)*b^(1
/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(4*a^(2/3)*b^(1/3)*e-(8*a*f+b*c)*(1-3
^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.72, antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(4a^{2/3}\sqrt[3]{b}\,e-\left(1-\sqrt{3}\right)(8af+bc)\right)}{8a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(8af+bc)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}-\frac{3\sqrt{a+bx^3}\,(8af+bc)}{8ax}+\frac{3\sqrt[3]{b}\,\sqrt{a+bx^3}\,(8af+bc)}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{3c\sqrt{a+bx^3}}{20x^4}-\frac{(2ag+bd)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] (3*c*Sqrt[a + b*x^3])/(20*x^4) + (d*Sqrt[a + b*x^3])/(3*x^3) + (3*e*Sqrt[a
+ b*x^3])/(2*x^2) - (3*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(1/3)*
(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
(2*Sqrt[a + b*x^3]*(3*c*x + 5*d*x^2 + 15*e*x^3 - 15*f*x^4 - 5*g*x^5))/(15*x
^5) - ((b*d + 2*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(
1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^
```

```
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3
/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(4*a^(2/3)*b^(1/3)*e - (1 - Sqrt[3])*(b*c + 8
*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(8*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
```

```
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx = -\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}+\frac{1}{2}(3a)$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5\right.}{15x^5}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15e\right.}{15x^5}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{2\sqrt{a+bx^3}\left(3\right.}{}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3(bc+8af)\sqrt{}}{8ax}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3(bc+8af)\sqrt{}}{8ax}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3(bc+8af)\sqrt{}}{8ax}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3(bc+8af)\sqrt{}}{8ax}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3(bc+8af)\sqrt{}}{8ax}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.45, size = 855, normalized size = 1.23

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-6*a*c - 9*b*c*x^3 - 4*a*x*(2*d + x*(3*e + 6*f*x - 4*g*x^
2))))/(24*a*x^4) - (8*Sqrt[a]*b*d*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] +
 16*a^(3/2)*g*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/
3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 36*a*b^(2/3)*e*((-1
)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a
^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3
))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 9*Sqrt[2]*a^(1/3)*b^(4/3)*c*((-1)^(1/3
)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3
])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a
^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-
1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])
- 72*Sqrt[2]*a^(4/3)*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^
(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I
*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE
[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 +
 (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/
3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(24*a*Sqrt[(a^(1/3) + (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1285 vs. $2(532) = 1064$.
time = 0.52, size = 1286, normalized size = 1.85

method	result
elliptic	$ie\sqrt{3}\,(-a\,b^2)$ $-\frac{c\sqrt{b\,x^3+a}}{4x^4}-\frac{d\sqrt{b\,x^3+a}}{3x^3}-\frac{e\sqrt{b\,x^3+a}}{2x^2}-\frac{(8af+3bc)\sqrt{b\,x^3+a}}{8ax}+\frac{2g\sqrt{b\,x^3+a}}{3}-$
risch	Expression too large to display

default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/4*(b*x^3+a)^(1/2)/x^4-3/8*b*(b*x^3+a)^(1/2)/a/x-1/8*I*b/a*3^(1/2)*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))))+e*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-1/3*b*arctanh((b*
x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3)+g*(-2/3*arctanh((b*x
^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/3*(b*x^3+a)^(1/2))+f*(-(b*x^3+a)^(1/2)/x-I*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 330, normalized size = 0.48

$$\left[\frac{36\,a\sqrt{b}\,ex^4\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+2\,(bd+2\,ag)\sqrt{a}\,x^4\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\,\sqrt{a}+8\,a^2}{x^6}\right)-9\,(bc+8\,af)\sqrt{b}\,x^4\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(16\,agx^4-12\,aex^2-3\,(3\,bc+8\,af)x^3-8\,adx-6\,ac)\sqrt{bx^3+a}}{24\,ax^4},\frac{36\,a\sqrt{b}\,ex^4\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+4\,(bd+2\,ag)\sqrt{-a}\,x^4\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{-a}}{bx^3+2\,a}\right)-9\,(bc+8\,af)\sqrt{b}\,x^4\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(16\,agx^4-12\,aex^2-3\,(3\,bc+8\,af)x^3-8\,adx-6\,ac)\sqrt{bx^3+a}}{24\,ax^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="frica
s")
```

```
[Out] [1/24*(36*a*sqrt(b)*e*x^4*weierstrassPInverse(0, -4*a/b, x) + 2*(b*d + 2*a*
g)*sqrt(a)*x^4*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*
sqrt(a) + 8*a^2)/x^6) - 9*(b*c + 8*a*f)*sqrt(b)*x^4*weierstrassZeta(0, -4*a
/b, weierstrassPInverse(0, -4*a/b, x)) + (16*a*g*x^4 - 12*a*e*x^2 - 3*(3*b*
c + 8*a*f)*x^3 - 8*a*d*x - 6*a*c)*sqrt(b*x^3 + a))/(a*x^4), 1/24*(36*a*sqrt
(b)*e*x^4*weierstrassPInverse(0, -4*a/b, x) + 4*(b*d + 2*a*g)*sqrt(-a)*x^4*
arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 9*(b*c + 8*a*f)*sqrt(b)*
x^4*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (16*a*g
*x^4 - 12*a*e*x^2 - 3*(3*b*c + 8*a*f)*x^3 - 8*a*d*x - 6*a*c)*sqrt(b*x^3 + a
))/(a*x^4)]
```

Sympy [A]
time = 3.84, size = 274, normalized size = 0.39

$$\frac{\sqrt{a}\,c\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt{a}\,e\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{\sqrt{a}\,f\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}-\frac{2\sqrt{a}\,g\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3}+\frac{2ag}{3\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}\,d\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}+\frac{2\sqrt{b}\,gx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}-\frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**5,x)
```

```
[Out] sqrt(a)*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**4*gamma(-1/3)) + sqrt(a)*e*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*f*gamma(-1/3)*hyper
((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(
a)*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + 2*a*g/(3*sqrt(b)*x**(3/2)*sqrt(a
/(b*x**3) + 1)) - sqrt(b)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*sqrt(b)*g
*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - b*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/
(3*sqrt(a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x)
```

```
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5, x)
```

3.454 $\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$

Optimal. Leaf size=652

$$-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{20ax^2}-\frac{3bd\sqrt{a+bx^3}}{8ax}+\frac{3\sqrt[3]{b}\,(bd+8ag)\sqrt{a-}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}$$

[Out] -1/3*b*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/60*(12*c/x^5+15*d/x^4+20*e/x^3+30*f/x^2+60*g/x)*(b*x^3+a)^(1/2)-3/20*b*c*(b*x^3+a)^(1/2)/a/x^2-3/8*b*d*(b*x^3+a)^(1/2)/a/x+3/8*b^(1/3)*(8*a*g+b*d)*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3/16*3^(1/4)*b^(1/3)*(8*a*g+b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/40*3^(3/4)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(2*b^(1/3)*(-10*a*f+b*c)+5*a^(1/3)*(8*a*g+b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.56, antiderivative size = 652, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)\left(2\sqrt[3]{b}\,(bc-10af)+5\left(1-\sqrt{3}\right)\sqrt[3]{a}\,(8ag+bd)\right)}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(8ag+bd)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)-\frac{3bc\sqrt{a+bx^3}}{20ax^2}+\frac{3\sqrt[3]{b}\sqrt{a+bx^3}\,(8ag+bd)}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{3bd\sqrt{a+bx^3}}{8ax}-\frac{be\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]

[Out] -1/60*(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*Sqrt[a + b*x^3]) - (3*b*c*Sqrt[a + b*x^3])/(20*a*x^2) - (3*b*d*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(1/3)*(b*d + 8*a*g)*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*d + 8*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1

```
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) -
(3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(2*b^(1/3)*(b*c - 10*a*f) + 5*(1 - Sqrt[
3])*a^(1/3)*(b*d + 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
], -7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
```

```
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^6}\,dx = -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3} - \frac{1}{2}(3b)\int$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a}}{20a}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a}}{20a}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a}}{20a}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a}}{20a}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a}}{20a}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a}}{20a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.60, size = 934, normalized size = 1.43

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

```
[Out] -1/120*(Sqrt[a + b*x^3]*(24*a*c + 9*b*x^3*(2*c + 5*d*x) + 10*a*x*(3*d + 4*e
*x + 6*x^2*(f + 2*g*x))))/(a*x^5) - (b^(1/3)*(40*Sqrt[a]*b^(2/3)*e*Sqrt[(a^
```

```
(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*A
rcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 18*b^(4/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3
)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3
)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[A
rcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (
-1)^(1/3)] + 180*a*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3)
 + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^
(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3
) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 45*Sq
rt[2]*a^(1/3)*b*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3
) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3
)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[
(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]
) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1
)^(1/3)/(-1 + (-1)^(1/3))]) - 360*Sqrt[2]*a^(4/3)*g*((-1)^(1/3)*a^(1/3) - b
^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/
3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 +
(-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/
4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I
*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(120*a*Sqrt
[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^
3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1570 vs. 2(500) = 1000.
time = 0.48, size = 1571, normalized size = 2.41

method	result
default	$2i\left(bf\right.$
elliptic	$-\frac{c\sqrt{b\,x^3+a}}{5x^5}-\frac{d\sqrt{b\,x^3+a}}{4x^4}-\frac{e\sqrt{b\,x^3+a}}{3x^3}-\frac{(10af+3bc)\sqrt{b\,x^3+a}}{20a\,x^2}-\frac{(8ag+3bd)\sqrt{b\,x^3+a}}{8ax}-$
risch	Expression too large to display

default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b*(b*x^3+a)^(1/2)/a/x^2+1/20*I*b/a*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-1/4*(b*x^3+a)^(
1/2)/x^4-3/8*b*(b*x^3+a)^(1/2)/a/x-1/8*I*b/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))))+f*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+e*(-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2
))/a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3)+g*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-a*b^2
)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 346, normalized size = 0.53

$$\left[\frac{10\,\sqrt{a}\,bex^{5}\log\left(-\frac{b^{2}x^{6}+8\,abx^{3}-4\,(bx^{3}+2\,a)\sqrt{bx^{3}+a}\,\sqrt{a}+8\,a^{2}}{x^{6}}\right)-18\,(bc-10\,af)\sqrt{b}\,x^{5}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-45\,(bd+8\,ag)\sqrt{b}\,x^{5}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-\left(15\,(3\,bd+8\,ag)x^{4}+40\,aex^{2}+6\,(3\,bc+10\,af)x^{3}+30\,adx+24\,ac\right)\sqrt{bx^{3}+a}}{120\,ax^{5}},\ \frac{20\,\sqrt{-a}\,bex^{5}\arctan\left(\frac{2\sqrt{bx^{3}+a}\,\sqrt{-a}}{bx^{3}+2a}\right)-18\,(bc-10\,af)\sqrt{b}\,x^{5}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-45\,(bd+8\,ag)\sqrt{b}\,x^{5}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-\left(15\,(3\,bd+8\,ag)x^{4}+40\,aex^{2}+6\,(3\,bc+10\,af)x^{3}+30\,adx+24\,ac\right)\sqrt{bx^{3}+a}}{120\,ax^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="frica
s")
```

```
[Out] [1/120*(10*sqrt(a)*b*e*x^5*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt
(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 18*(b*c - 10*a*f)*sqrt(b)*x^5*weierstra
ssPInverse(0, -4*a/b, x) - 45*(b*d + 8*a*g)*sqrt(b)*x^5*weierstrassZeta(0,
-4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (15*(3*b*d + 8*a*g)*x^4 + 40*a
*e*x^2 + 6*(3*b*c + 10*a*f)*x^3 + 30*a*d*x + 24*a*c)*sqrt(b*x^3 + a))/(a*x^
5), 1/120*(20*sqrt(-a)*b*e*x^5*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2
*a)) - 18*(b*c - 10*a*f)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) - 45
*(b*d + 8*a*g)*sqrt(b)*x^5*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0
, -4*a/b, x)) - (15*(3*b*d + 8*a*g)*x^4 + 40*a*e*x^2 + 6*(3*b*c + 10*a*f)*x
^3 + 30*a*d*x + 24*a*c)*sqrt(b*x^3 + a))/(a*x^5)]
```

Sympy [A]
time = 3.51, size = 240, normalized size = 0.37

$$\frac{\sqrt{a}\,c\Gamma\left(-\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{5}\Gamma\left(-\frac{2}{3}\right)}+\frac{\sqrt{a}\,d\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt{a}\,f\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{2}\Gamma\left(\frac{1}{3}\right)}+\frac{\sqrt{a}\,g\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}-\frac{\sqrt{b}\,e\sqrt{\frac{a}{bx^{3}}+1}}{3x^{\frac{3}{2}}}-\frac{be\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**6,x)
```

```
[Out] sqrt(a)*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**5*gamma(-2/3)) + sqrt(a)*d*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*f*gamma(-2/3)*hyp
er((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sq
rt(a)*g*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(
```

```
3*x*gamma(2/3)) - sqrt(b)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b*e*asinh(s
qrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x)
```

```
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6, x)
```

3.455 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}\,dx$

Optimal. Leaf size=659

$$-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+bx^3}}{20ax^2}-\frac{3be\sqrt{a+bx^3}}{8ax}+\frac{3}{8a\left(\left(1\right.\right.}$$

```
[Out] 1/12*b*(-4*a*f+b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/60*(10*c/x^6
+12*d/x^5+15*e/x^4+20*f/x^3+30*g/x^2)*(b*x^3+a)^(1/2)-1/12*b*c*(b*x^3+a)^(1
/2)/a/x^3-3/20*b*d*(b*x^3+a)^(1/2)/a/x^2-3/8*b*e*(b*x^3+a)^(1/2)/a/x+3/8*b^
(4/3)*e*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3/16*3^(1/4)*b^(4
/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^
(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)-1/40*3^(3/4)*b^(2/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(2*
b*d-20*a*g+5*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A]

time = 0.64, antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-20ag+2bd\right)}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{b(bc-4af)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}+\frac{3b^{4/3}e\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+bx^3}}{20ax^2}-\frac{3be\sqrt{a+bx^3}}{8ax}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] -1/60*(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3 + (30*g)/x^2)*Sqr
t[a + b*x^3]) - (b*c*Sqrt[a + b*x^3])/(12*a*x^3) - (3*b*d*Sqrt[a + b*x^3])/
(20*a*x^2) - (3*b*e*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(4/3)*e*Sqrt[a + b*x^3]
)/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b*(b*c - 4*a*f)*ArcTanh[Sqrt
[a + b*x^3]/Sqrt[a]])/(12*a^(3/2)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*e
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
```

```
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3)
 + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^
(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d + 5*(1
- Sqrt[3])*a^(1/3)*b^(2/3)*e - 20*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
```

```
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3} - \frac{1}{2}(3b)\int$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3} - \frac{bc\sqrt{a+}}{12a}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3} - \frac{bc\sqrt{a+}}{12a}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3} - \frac{bc\sqrt{a+}}{12a}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3} - \frac{bc\sqrt{a+}}{12a}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3} - \frac{bc\sqrt{a+}}{12a}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3} - \frac{bc\sqrt{a+}}{12a}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3} - \frac{bc\sqrt{a+}}{12a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.78, size = 800, normalized size = 1.21

$$-\frac{\sqrt{a+bx^3}\,(bx^3(10c+9x(2d+5ex))+a(20c+2x(12d+5x(3e+4fx+6gx^2))))}{120ax^6}+\frac{b\left(\frac{20bc\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{80}{3}\sqrt{a}\,f\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{12b^{2/3}d\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\sqrt{a+bx^3}}-\text{[illegible]}\right)}{80a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] -1/120*(Sqrt[a + b*x^3]*(b*x^3*(10*c + 9*x*(2*d + 5*e*x)) + a*(20*c + 2*x*(
12*d + 5*x*(3*e + 4*f*x + 6*g*x^2)))))/(a*x^6) + (b*((20*b*c*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (80*Sqrt[a]*f*ArcTanh[Sqrt[a + b*x^3]/Sqr
t[a]])/3 + (12*b^(2/3)*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b
^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)
*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(Sqrt[(a^(1/
3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]) - (
120*a*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
 (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*
x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(b^(1/3)*Sqrt[(a^(1/3) + (-1)
^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]) - (30*Sqrt[2
]*a^(1/3)*b^(1/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/
3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3
)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-
1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))] +
 EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(
1/3)/(-1 + (-1)^(1/3))]))/(Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])))/(80*a)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1179 vs. $2(503) = 1006$.
time = 0.42, size = 1180, normalized size = 1.79

method	result
elliptic	$-\frac{c\sqrt{b\,x^3+a}}{6x^6}-\frac{d\sqrt{b\,x^3+a}}{5x^5}-\frac{e\sqrt{b\,x^3+a}}{4x^4}-\frac{(4af+bc)\sqrt{b\,x^3+a}}{12a\,x^3}-\frac{(10ag+3bd)\sqrt{b\,x^3+a}}{20a\,x^2}-\frac{3be\sqrt{b}}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b*(b*x^3+a)^(1/2)/a/x^2+1/20*I*b/a*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+e*(-1/4*(b*x^3+a)^(
1/2)/x^4-3/8*b*(b*x^3+a)^(1/2)/a/x-1/8*I*b/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))))+c*(1/12*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/6*(b*x^
3+a)^(1/2)/x^6-1/12*b*(b*x^3+a)^(1/2)/a/x^3)+g*(-1/2*(b*x^3+a)^(1/2)/x^2-1/
2*I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+f*(-1/3*
b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="maxim
a")
```

```
[Out] -1/24*(b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(
3/2) + 2*((b*x^3 + a)^(3/2)*b^2 + sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a -
```

```
 2*(b*x^3 + a)*a^2 + a^3))*c + integrate(sqrt(b*x^3 + a)*(g*x^3 + f*x^2 + x
*e + d)/x^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 404, normalized size = 0.61

$$\left[-\frac{90\,ab^{\frac{3}{2}}ex^{6}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+5\left(b^{2}c-4\,abf\right)\sqrt{a}\,x^{6}\log\left(\frac{b^{2}x^{6}+8\,abx^{3}-4\left(bx^{3}+2a\right)\sqrt{bx^{3}+a}\,\sqrt{a}+8a^{2}}{x^{6}}\right)+36\left(abd-10\,a^{2}g\right)\sqrt{b}\,x^{6}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+2\left(45\,abex^{5}+30\,a^{2}ex^{2}+6\left(3\,abd+10\,a^{2}g\right)x^{4}+24\,a^{2}dx+10\left(abc+4\,a^{2}f\right)x^{3}+20\,a^{2}c\right)\sqrt{bx^{3}+a}}{240\,a^{2}x^{6}},\ -\frac{45\,ab^{\frac{3}{2}}ex^{6}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+5\left(b^{2}c-4\,abf\right)\sqrt{-a}\,x^{6}\arctan\left(\frac{\left(bx^{3}+2a\right)\sqrt{bx^{3}+a}\,\sqrt{-a}}{2\left(abx^{3}+a^{2}\right)}\right)+18\left(abd-10\,a^{2}g\right)\sqrt{b}\,x^{6}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\left(45\,abex^{5}+30\,a^{2}ex^{2}+6\left(3\,abd+10\,a^{2}g\right)x^{4}+24\,a^{2}dx+10\left(abc+4\,a^{2}f\right)x^{3}+20\,a^{2}c\right)\sqrt{bx^{3}+a}}{120\,a^{2}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="frica
s")
```

```
[Out] [-1/240*(90*a*b^(3/2)*e*x^6*weierstrassZeta(0, -4*a/b, weierstrassPInverse(
0, -4*a/b, x)) + 5*(b^2*c - 4*a*b*f)*sqrt(a)*x^6*log((b^2*x^6 + 8*a*b*x^3 -
 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 36*(a*b*d - 10*a^2
*g)*sqrt(b)*x^6*weierstrassPInverse(0, -4*a/b, x) + 2*(45*a*b*e*x^5 + 30*a^
2*e*x^2 + 6*(3*a*b*d + 10*a^2*g)*x^4 + 24*a^2*d*x + 10*(a*b*c + 4*a^2*f)*x^
3 + 20*a^2*c)*sqrt(b*x^3 + a))/(a^2*x^6), -1/120*(45*a*b^(3/2)*e*x^6*weiers
trassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + 5*(b^2*c - 4*a*b*
f)*sqrt(-a)*x^6*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3
+ a^2)) + 18*(a*b*d - 10*a^2*g)*sqrt(b)*x^6*weierstrassPInverse(0, -4*a/b,
x) + (45*a*b*e*x^5 + 30*a^2*e*x^2 + 6*(3*a*b*d + 10*a^2*g)*x^4 + 24*a^2*d*x
 + 10*(a*b*c + 4*a^2*f)*x^3 + 20*a^2*c)*sqrt(b*x^3 + a))/(a^2*x^6)]
```

Sympy [A]
time = 5.16, size = 304, normalized size = 0.46

$$\frac{\sqrt{a}\,d\Gamma\left(-\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{5}\Gamma\left(-\frac{2}{3}\right)}+\frac{\sqrt{a}\,e\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt{a}\,g\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{2}\Gamma\left(\frac{1}{3}\right)}-\frac{ac}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{\sqrt{b}\,c}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{\sqrt{b}\,f\sqrt{\frac{a}{bx^{3}}+1}}{3x^{\frac{3}{2}}}-\frac{b^{\frac{3}{2}}c}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}+\frac{b^{2}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] sqrt(a)*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**5*gamma(-2/3)) + sqrt(a)*e*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*g*gamma(-2/3)*hyp
er((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) - a*
c/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)*c/(4*x**(9/2)*sqrt(a
/(b*x**3) + 1)) - sqrt(b)*f*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*c/
(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b*f*asinh(sqrt(a)/(sqrt(b)*x**(3/2))
)/(3*sqrt(a)) + b**2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x)
```

```
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7, x)
```

3.456 $\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$

Optimal. Leaf size=711

$$-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\frac{bd\sqrt{a+bx^3}}{12ax^3}-\frac{3be\sqrt{a+bx^3}}{20ax^2}+\frac{3b(5$$

```
[Out] 1/12*b*(-4*a*g+b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/420*(60*c/x^
7+70*d/x^6+84*e/x^5+105*f/x^4+140*g/x^3)*(b*x^3+a)^(1/2)-3/56*b*c*(b*x^3+a)
^(1/2)/a/x^4-1/12*b*d*(b*x^3+a)^(1/2)/a/x^3-3/20*b*e*(b*x^3+a)^(1/2)/a/x^2+
3/112*b*(-14*a*f+5*b*c)*(b*x^3+a)^(1/2)/a^2/x-3/112*b^(4/3)*(-14*a*f+5*b*c)
*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+3/224*3^(1/4)*b^(4/3)*
(-14*a*f+5*b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)-1/560*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*Ellipt
icF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/
2)+2*I)*(28*a^(2/3)*b^(1/3)*e-5*(-14*a*f+5*b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(5bc-14af)\right)}{560a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5bc-14af)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{b(bd-4ag)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}-\frac{3b^{4/3}\sqrt{a+bx^3}(5bc-14af)}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{3b\sqrt{a+bx^3}(5bc-14af)}{112a^2x}-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\frac{bd\sqrt{a+bx^3}}{12ax^3}-\frac{3be\sqrt{a+bx^3}}{20ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] -1/420*(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x^3)*
Sqrt[a + b*x^3]) - (3*b*c*Sqrt[a + b*x^3])/(56*a*x^4) - (b*d*Sqrt[a + b*x^3
])/(12*a*x^3) - (3*b*e*Sqrt[a + b*x^3])/(20*a*x^2) + (3*b*(5*b*c - 14*a*f)*
Sqrt[a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5*b*c - 14*a*f)*Sqrt[a + b*x^3])
/(112*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b*(b*d - 4*a*g)*ArcTanh[S
```

```
qrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3
)*(5*b*c - 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(
4/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(5*b*c - 14*a*f))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(560*a^(5/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^8}\,dx = -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} - \frac{1}{2}(3$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} - \frac{3bc}{}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} - \frac{3bc}{}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} - \frac{3bc}{}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} - \frac{3bc}{}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} - \frac{3bc}{}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} - \frac{3bc}{}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} - \frac{3bc}{}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} - \frac{3bc}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.52, size = 892, normalized size = 1.25

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^3]*(225*b^2*c*x^6 - 2*a*b*x^3*(45*c + 7*x*(10*d + 9*x*(2*e +
5*f*x))) - 4*a^2*(60*c + 7*x*(10*d + x*(12*e + 5*x*(3*f + 4*g*x))))))/(1680
*a^2*x^7) + (b*(140*Sqrt[a]*b*d*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 +
 (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 5
60*a^(3/2)*g*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 252*a*b^(2/3)*e*((-1
)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a
^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3
))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 225*Sqrt[2]*a^(1/3)*b^(4/3)*c*((-1)^(1
/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))
/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt
[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)
/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[
(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]
) + 630*Sqrt[2]*a^(4/3)*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-
1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt
[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*Ellipt
icE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-
1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3
)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(1680*a^2*Sqrt[(a^(1/3) + (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1375 vs. $2(549) = 1098$.
time = 0.42, size = 1376, normalized size = 1.94

method	result

elliptic	$-\frac{c\sqrt{b\,x^3+a}}{7x^7}-\frac{d\sqrt{b\,x^3+a}}{6x^6}-\frac{e\sqrt{b\,x^3+a}}{5x^5}-\frac{(14af+3bc)\sqrt{b\,x^3+a}}{56a\,x^4}-\frac{(4ag+bd)\sqrt{b\,x^3+a}}{12a\,x^3}-\frac{3be\sqrt{b\,x}}{20a}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b*(b*x^3+a)^(1/2)/a/x^2+1/20*I*b/a*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+f*(-1/4*(b*x^3+a)^(
1/2)/x^4-3/8*b*(b*x^3+a)^(1/2)/a/x-1/8*I*b/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))))+d*(1/12*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/6*(b*x^
3+a)^(1/2)/x^6-1/12*b*(b*x^3+a)^(1/2)/a/x^3)+c*(-1/7*(b*x^3+a)^(1/2)/x^7-3/
56*b/a*(b*x^3+a)^(1/2)/x^4+15/112*b^2/a^2*(b*x^3+a)^(1/2)/x+5/112*I*b^2/a^2
```

```
*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+g*(-1/3*b*arctanh((b*x^3+a)^(1/
2)/a^(1/2))/a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^8, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 439, normalized size = 0.62

$$\left[-\frac{252\,ab^{\frac{3}{2}}ex^{7}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+35\left(b^{2}d-4abg\right)\sqrt{a}\,x^{7}\log\left(\frac{b^{2}x^{6}+8abx^{3}-4\left(bx^{3}+2a\right)\sqrt{bx^{3}+a}\sqrt{a}+8a^{2}}{x^{6}}\right)-45\left(5b^{2}c-14abf\right)\sqrt{b}\,x^{7}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(252abex^{5}-45\left(5b^{2}c-14abf\right)x^{6}+336a^{2}ex^{2}+140\left(abd+4a^{2}g\right)x^{4}+280a^{2}dx+30\left(3abc+14a^{2}f\right)x^{3}+240a^{2}c\right)\sqrt{bx^{3}+a}}{1680\,a^{2}x^{7}},\ -\frac{252\,ab^{\frac{3}{2}}ex^{7}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+70\left(b^{2}d-4abg\right)\sqrt{-a}\,x^{7}\arctan\left(\frac{\left(bx^{3}+2a\right)\sqrt{bx^{3}+a}\sqrt{-a}}{2\left(abx^{3}+a^{2}\right)}\right)-45\left(5b^{2}c-14abf\right)\sqrt{b}\,x^{7}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(252abex^{5}-45\left(5b^{2}c-14abf\right)x^{6}+336a^{2}ex^{2}+140\left(abd+4a^{2}g\right)x^{4}+280a^{2}dx+30\left(3abc+14a^{2}f\right)x^{3}+240a^{2}c\right)\sqrt{bx^{3}+a}}{1680\,a^{2}x^{7}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="frica
s")
```

```
[Out] [-1/1680*(252*a*b^(3/2)*e*x^7*weierstrassPInverse(0, -4*a/b, x) + 35*(b^2*d
 - 4*a*b*g)*sqrt(a)*x^7*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x
^3 + a)*sqrt(a) + 8*a^2)/x^6) - 45*(5*b^2*c - 14*a*b*f)*sqrt(b)*x^7*weierst
rassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (252*a*b*e*x^5 - 4
5*(5*b^2*c - 14*a*b*f)*x^6 + 336*a^2*e*x^2 + 140*(a*b*d + 4*a^2*g)*x^4 + 28
0*a^2*d*x + 30*(3*a*b*c + 14*a^2*f)*x^3 + 240*a^2*c)*sqrt(b*x^3 + a))/(a^2*
x^7), -1/1680*(252*a*b^(3/2)*e*x^7*weierstrassPInverse(0, -4*a/b, x) + 70*(
b^2*d - 4*a*b*g)*sqrt(-a)*x^7*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt
(-a)/(a*b*x^3 + a^2)) - 45*(5*b^2*c - 14*a*b*f)*sqrt(b)*x^7*weierstrassZeta
(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (252*a*b*e*x^5 - 45*(5*b^2
*c - 14*a*b*f)*x^6 + 336*a^2*e*x^2 + 140*(a*b*d + 4*a^2*g)*x^4 + 280*a^2*d*
x + 30*(3*a*b*c + 14*a^2*f)*x^3 + 240*a^2*c)*sqrt(b*x^3 + a))/(a^2*x^7)]
```

Sympy [A]
time = 5.35, size = 308, normalized size = 0.43

$$\frac{\sqrt{a}\,c\,\Gamma\left(-\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)}+\frac{\sqrt{a}\,e\,\Gamma\left(-\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{5}\Gamma\left(-\frac{2}{3}\right)}+\frac{\sqrt{a}\,f\,\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)}-\frac{ad}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{\sqrt{b}\,d}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{\sqrt{b}\,g\sqrt{\frac{a}{bx^{3}}+1}}{3x^{\frac{3}{2}}}-\frac{b^{\frac{3}{2}}d}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{bg\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}+\frac{b^{2}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**8,x)
```

```
[Out] sqrt(a)*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**7*gamma(-4/3)) + sqrt(a)*e*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*f*gamma(-4/3)*hyp
er((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) -
a*d/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)*d/(4*x**(9/2)*sqrt
(a/(b*x**3) + 1)) - sqrt(b)*g*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*
d/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2
)))/(3*sqrt(a)) + b**2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x)
```

```
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8, x)
```

3.457 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx$

Optimal. Leaf size=743

$$-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{80ax^5}-\frac{3bd\sqrt{a+bx^3}}{56ax^4}-\frac{be\sqrt{a+bx^3}}{12ax^3}+$$

```
[Out] 1/12*b^2*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/840*(105*c/x^8+120*d/
x^7+140*e/x^6+168*f/x^5+210*g/x^4)*(b*x^3+a)^(1/2)-3/80*b*c*(b*x^3+a)^(1/2)
/a/x^5-3/56*b*d*(b*x^3+a)^(1/2)/a/x^4-1/12*b*e*(b*x^3+a)^(1/2)/a/x^3+3/320*
b*(-16*a*f+7*b*c)*(b*x^3+a)^(1/2)/a^2/x^2+3/112*b*(-14*a*g+5*b*d)*(b*x^3+a)
^(1/2)/a^2/x-3/112*b^(4/3)*(-14*a*g+5*b*d)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))+3/224*3^(1/4)*b^(4/3)*(-14*a*g+5*b*d)*(a^(1/3)+b^(1/3)*
x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(
a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/2240
*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*(-16*a*f+7*b*
c)+20*a^(1/3)*(-14*a*g+5*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
/a^2/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.88, antiderivative size = 743, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5bd-14ag)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{b^2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}-\frac{3b^{4/3}\sqrt{a+bx^3}(5bd-14ag)}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{3b\sqrt{a+bx^3}(7bc-16af)}{320a^2x^2}+\frac{3b\sqrt{a+bx^3}(5bd-14ag)}{112a^2x}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(7bc-16af)+20\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd-14ag)\right)}{2240a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)-\frac{3bc\sqrt{a+bx^3}}{80ax^5}-\frac{3bd\sqrt{a+bx^3}}{56ax^4}-\frac{be\sqrt{a+bx^3}}{12ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] -1/840*(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210*g)/x^
4)*Sqrt[a + b*x^3]) - (3*b*c*Sqrt[a + b*x^3])/(80*a*x^5) - (3*b*d*Sqrt[a +
b*x^3])/(56*a*x^4) - (b*e*Sqrt[a + b*x^3])/(12*a*x^3) + (3*b*(7*b*c - 16*a*
f)*Sqrt[a + b*x^3])/(320*a^2*x^2) + (3*b*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/
```

```
(112*a^2*x) - (3*b^(4/3)*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/(112*a^2*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) + (b^2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(1
2*a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*b*d - 14*a*g)*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(7*b^(1/3)*(7*b*c - 16
*a*f) + 20*(1 - Sqrt[3])*a^(1/3)*(5*b*d - 14*a*g))*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2240*a^2*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} -$$

Mathematica [C] Result contains complex when optimal does not.

time = 11.77, size = 979, normalized size = 1.32

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] (Sqrt[a + b*x^3]*(9*b^2*x^6*(49*c + 100*d*x) - 4*a*b*x^3*(63*c + 2*x*(45*d
+ 7*x*(10*e + 9*x*(2*f + 5*g*x)))) - 8*a^2*(105*c + 2*x*(60*d + 7*x*(10*e +
 3*x*(4*f + 5*g*x))))))/(6720*a^2*x^8) + (b^(4/3)*(560*Sqrt[a]*b^(2/3)*e*Sq
rt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*
x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 441*b^(4/3)*c*((-1)^(1/3)*a^(1/3) -
 b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-
1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Elli
pticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]], (-1)^(1/3)] + 1008*a*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[
(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3)
 - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt
[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]
 - 900*Sqrt[2]*a^(1/3)*b*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3
)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1
+ (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[Arc
Sin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1
)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1
/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) + 2520*Sqrt[2]*a^(4/3)*g*((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1
+ (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]
*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1
/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^
(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(
6720*a^2*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*
Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1678 vs. 2(579) = 1158.
time = 0.42, size = 1679, normalized size = 2.26

method	result	size
elliptic	Expression too large to display	931
risch	Expression too large to display	1579
default	Expression too large to display	1679

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] f*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b*(b*x^3+a)^(1/2)/a/x^2+1/20*I*b/a*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+g*(-1/4*(b*x^3+a)^(
1/2)/x^4-3/8*b*(b*x^3+a)^(1/2)/a/x-1/8*I*b/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))))+e*(1/12*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/6*(b*x^
3+a)^(1/2)/x^6-1/12*b*(b*x^3+a)^(1/2)/a/x^3)+c*(-1/8*(b*x^3+a)^(1/2)/x^8-3/
80*b*(b*x^3+a)^(1/2)/a/x^5+21/320*b^2*(b*x^3+a)^(1/2)/a^2/x^2-7/320*I*b^2/a
^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^
3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-1/7*(
b*x^3+a)^(1/2)/x^7-3/56*b/a*(b*x^3+a)^(1/2)/x^4+15/112*b^2/a^2*(b*x^3+a)^(1
/2)/x+5/112*I*b^2/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^
2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^
2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="maxim
a")

[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^9, x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 482, normalized size = 0.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="frica
s")

[Out] [1/6720*(140*sqrt(a)*b^2*e*x^8*log((b^2*x^6 + 8*a*b*x^3 + 4*(b*x^3 + 2*a)*s
qrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 63*(7*b^2*c - 16*a*b*f)*sqrt(b)*x^8*
weierstrassPInverse(0, -4*a/b, x) + 180*(5*b^2*d - 14*a*b*g)*sqrt(b)*x^8*we
ierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (560*a*b*e*x^
5 - 180*(5*b^2*d - 14*a*b*g)*x^7 - 63*(7*b^2*c - 16*a*b*f)*x^6 + 1120*a^2*e
x^2 + 120(3*a*b*d + 14*a^2*g)*x^4 + 960*a^2*d*x + 84*(3*a*b*c + 16*a^2*f)
*x^3 + 840*a^2*c)*sqrt(b*x^3 + a))/(a^2*x^8), -1/6720*(280*sqrt(-a)*b^2*e*x
^8*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) - 63*
(7*b^2*c - 16*a*b*f)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b, x) - 180*(5
*b^2*d - 14*a*b*g)*sqrt(b)*x^8*weierstrassZeta(0, -4*a/b, weierstrassPInver
se(0, -4*a/b, x)) + (560*a*b*e*x^5 - 180*(5*b^2*d - 14*a*b*g)*x^7 - 63*(7*b
^2*c - 16*a*b*f)*x^6 + 1120*a^2*e*x^2 + 120*(3*a*b*d + 14*a^2*g)*x^4 + 960*
a^2*d*x + 84*(3*a*b*c + 16*a^2*f)*x^3 + 840*a^2*c)*sqrt(b*x^3 + a))/(a^2*x^
8)]

Sympy [A]
time = 5.05, size = 304, normalized size = 0.41

$$\frac{\sqrt{a}\,c\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)}+\frac{\sqrt{a}\,d\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{\sqrt{a}\,f\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{\sqrt{a}\,g\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}-\frac{ae}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}\,e}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{b^{\frac{3}{2}}e}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b^2e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**9,x)

[Out] sqrt(a)*c*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**8*gamma(-5/3)) + sqrt(a)*d*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*f*gamma(-5/3)*hyp
er((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) +
sqrt(a)*g*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**4*gamma(-1/3)) - a*e/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - s

```
qrt(b)*e/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*e/(12*a*x**(3/2)*sqrt
(a/(b*x**3) + 1)) + b**2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*sqrt(b*x^3 + a)/x^9, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x)
```

```
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9, x)
```

3.458 $\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=791

$$-\frac{4a^3e\sqrt{a+bx^3}}{105b^2}+\frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2}+\frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2}+\frac{2a^2ex^3\sqrt{a+bx^3}}{105b}+\frac{54a^2f$$

```
[Out] 2/3900225*x^3*(b*x^3+a)^(3/2)*(156009*g*x^5+169575*f*x^4+185725*e*x^3+20527
5*d*x^2+229425*c*x)-4/105*a^3*e*(b*x^3+a)^(1/2)/b^2+54/21505*a^2*(-8*a*f+23
*b*c)*x*(b*x^3+a)^(1/2)/b^2+54/8645*a^2*(-2*a*g+5*b*d)*x^2*(b*x^3+a)^(1/2)/
b^2+2/105*a^2*e*x^3*(b*x^3+a)^(1/2)/b+54/4301*a^2*f*x^4*(b*x^3+a)^(1/2)/b+5
4/6175*a^2*g*x^5*(b*x^3+a)^(1/2)/b+2/185910725*a*x^3*(3522519*g*x^5+4279275
*f*x^4+5311735*e*x^3+6774075*d*x^2+8947575*c*x)*(b*x^3+a)^(1/2)-216/8645*a^
3*(-2*a*g+5*b*d)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+10
8/8645*3^(1/4)*a^(10/3)*(-2*a*g+5*b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/
3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1
/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^
(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-36/37182145*3^(3/4)*a^3*(
a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1729*b^(1/3)*(-8*a*f+23*b*c)-8602*a^(1/3)
*(-2*a*g+5*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^
3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A]
time = 1.44, antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1850, 1902, 1608, 1900, 267, 1892, 224, 1891}

$$\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5bd-2ag)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{216a^3\sqrt{a+bx^3}(5bd-2ag)}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4a^3e\sqrt{a+bx^3}}{105b^2}+\frac{54a^2x\sqrt{a+bx^3}(23bc-8af)}{21505b^2}+\frac{54a^2x^2\sqrt{a+bx^3}(5bd-2ag)}{8645b^2}+\frac{2a^2ex^3\sqrt{a+bx^3}}{105b}+\frac{54a^2fx^4\sqrt{a+bx^3}}{4301b}+\frac{54a^2gx^5\sqrt{a+bx^3}}{6175b}-\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(23bc-8af)-8602\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd-2ag)\right)}{37182145b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x^3(a+bx^3)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}+\frac{2ax^3\sqrt{a+bx^3}\left(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5\right)}{185910725}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (-4*a^3*e*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b*c - 8*a*f)*x*Sqrt[a +
b*x^3])/(21505*b^2) + (54*a^2*(5*b*d - 2*a*g)*x^2*Sqrt[a + b*x^3])/(8645*b^
2) + (2*a^2*e*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*f*x^4*Sqrt[a + b*x^3])
```

```
/(4301*b) + (54*a^2*g*x^5*Sqrt[a + b*x^3])/(6175*b) - (216*a^3*(5*b*d - 2*a
*g)*Sqrt[a + b*x^3])/(8645*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (
2*x^3*(a + b*x^3)^(3/2)*(229425*c*x + 205275*d*x^2 + 185725*e*x^3 + 169575*
f*x^4 + 156009*g*x^5))/3900225 + (2*a*x^3*Sqrt[a + b*x^3]*(8947575*c*x + 67
74075*d*x^2 + 5311735*e*x^3 + 4279275*f*x^4 + 3522519*g*x^5))/185910725 + (
108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*(5*b*d - 2*a*g)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3]) - (36*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(1729*b^(1/3)*(23*b*c - 8*a*f) - 8
602*(1 - Sqrt[3])*a^(1/3)*(5*b*d - 2*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(37182145*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int x^3\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\frac{2x^3(a+bx^3)^{3/2}\left(229425cx+205275dx^2+185725ex^3+\right.}{3900225}$$

$$=\frac{2x^3(a+bx^3)^{3/2}\left(229425cx+205275dx^2+185725ex^3+\right.}{3900225}$$

$$=\frac{54a^2gx^5\sqrt{a+bx^3}}{6175b}+\frac{2x^3(a+bx^3)^{3/2}\left(229425cx+2052\right.}{}$$

$$=\frac{54a^2fx^4\sqrt{a+bx^3}}{4301b}+\frac{54a^2gx^5\sqrt{a+bx^3}}{6175b}+\frac{2x^3(a+bx^3)}{}$$

$$=\frac{2a^2ex^3\sqrt{a+bx^3}}{105b}+\frac{54a^2fx^4\sqrt{a+bx^3}}{4301b}+\frac{54a^2gx^5\sqrt{a+}}{6175b}$$

$$=\frac{2a^2ex^3\sqrt{a+bx^3}}{105b}+\frac{54a^2fx^4\sqrt{a+bx^3}}{4301b}+\frac{54a^2gx^5\sqrt{a+}}{6175b}$$

$$=\frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2}+\frac{2a^2ex^3\sqrt{a+bx^3}}{105b}+\frac{54a}{}$$

$$=\frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2}+\frac{2a^2ex^3\sqrt{a+bx^3}}{105b}+\frac{54a}{}$$

$$=\frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2}+\frac{54a^2(5bd-2ag)x^2\sqrt{a+}}{8645b^2}$$

$$=\frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2}+\frac{54a^2(5bd-2ag)x^2\sqrt{a+}}{8645b^2}$$

$$=-\frac{4a^3e\sqrt{a+bx^3}}{105b^2}+\frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2}+\frac{54a}{}$$

$$=-\frac{4a^3e\sqrt{a+bx^3}}{105b^2}+\frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2}+\frac{54a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

4 in optimal.
time = 9.87, size = 179, normalized size = 0.23

$$\frac{2\sqrt{a+bx^3}\left(-(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}\,(10a(7429e+21x(380f+391gx))-bx(229425c+17x(12075d+19x(575e+525fx+483gx^2))))+9975a^2(-23bc+8af)x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+41055a^2(-5bd+2ag)x^2\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{3900225b^2\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(10*a*(7429*e + 21*
x*(380*f + 391*g*x)) - b*x*(229425*c + 17*x*(12075*d + 19*x*(575*e + 525*f*
x + 483*g*x^2))))) + 9975*a^2*(-23*b*c + 8*a*f)*x*Hypergeometric2F1[-3/2, 1
/3, 4/3, -((b*x^3)/a)] + 41055*a^2*(-5*b*d + 2*a*g)*x^2*Hypergeometric2F1[-
3/2, 2/3, 5/3, -((b*x^3)/a)]))/(3900225*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1763 vs. 2(627) = 1254.
time = 0.40, size = 1764, normalized size = 2.23

method	result	size
elliptic	Expression too large to display	1161
risch	Expression too large to display	1198
default	Expression too large to display	1764

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/25*b*x^11*(b*x^3+a)^(1/2)+56/475*a*x^8*(b*x^3+a)^(1/2)+54/6175*a^2*x^5
*(b*x^3+a)^(1/2)/b-108/8645*a^3*x^2*(b*x^3+a)^(1/2)/b^2-144/8645*I*a^4/b^3*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+f*(2/23*b*x^10*(b*x^3+a)^(1/2)+5
2/391*a*x^7*(b*x^3+a)^(1/2)+54/4301*a^2*x^4*(b*x^3+a)^(1/2)/b-432/21505*a^3
*x*(b*x^3+a)^(1/2)/b^2-288/21505*I*a^4/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
```

```
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)))+e*(2/21*b*x^9*(b*x^3+a)^(1/2)+16/105*a*x^6*(
b*x^3+a)^(1/2)+2/105/b*a^2*x^3*(b*x^3+a)^(1/2)-4/105*a^3/b^2*(b*x^3+a)^(1/2
))+d*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729*a^2*x
^2*(b*x^3+a)^(1/2)/b+72/1729*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))))+c*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935*a^
2*x*(b*x^3+a)^(1/2)/b+36/935*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)*x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 262, normalized size = 0.33

$$-\frac{2\left(1400490\,(23\,a^{3}bc-8\,a^{4}f)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-6967620\,(5\,a^{3}bd-2\,a^{4}g)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-(22309287\,b^{4}gx^{11}+24249225\,b^{4}fx^{10}+26558675\,b^{4}ex^{9}+42493880\,ab^{3}ex^{6}+1174173\,(25\,b^{4}d+28\,ab^{3}g)x^{8}+5311735\,a^{2}b^{2}ex^{3}+1426425\,(23\,b^{4}c+26\,ab^{3}f)x^{7}+90321\,(550\,ab^{3}d+27\,a^{2}b^{2}g)x^{5}-10623470\,a^{3}be+129675\,(460\,ab^{3}c+27\,a^{2}b^{2}f)x^{4}+1741905\,(5\,a^{2}b^{2}d-2\,a^{3}bg)x^{2}+700245\,(23\,a^{2}b^{2}c-8\,a^{3}bf)x)\sqrt{bx^{3}+a}\right)}{557732175\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] -2/557732175*(1400490*(23*a^3*b*c - 8*a^4*f)*sqrt(b)*weierstrassPInverse(0,
 -4*a/b, x) - 6967620*(5*a^3*b*d - 2*a^4*g)*sqrt(b)*weierstrassZeta(0, -4*a
/b, weierstrassPInverse(0, -4*a/b, x)) - (22309287*b^4*g*x^11 + 24249225*b^
4*f*x^10 + 26558675*b^4*e*x^9 + 42493880*a*b^3*e*x^6 + 1174173*(25*b^4*d +
28*a*b^3*g)*x^8 + 5311735*a^2*b^2*e*x^3 + 1426425*(23*b^4*c + 26*a*b^3*f)*x
^7 + 90321*(550*a*b^3*d + 27*a^2*b^2*g)*x^5 - 10623470*a^3*b*e + 129675*(46
0*a*b^3*c + 27*a^2*b^2*f)*x^4 + 1741905*(5*a^2*b^2*d - 2*a^3*b*g)*x^2 + 700
245*(23*a^2*b^2*c - 8*a^3*b*f)*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A]
time = 4.66, size = 512, normalized size = 0.65

$$\frac{a^{\frac{3}{2}} c x^{4} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{3}{2}} d x^{5} \Gamma\left(\frac{5}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{8}{3}\right)} + \frac{a^{\frac{3}{2}} f x^{7} \Gamma\left(\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{10}{3}\right)} + \frac{a^{\frac{3}{2}} g x^{8} \Gamma\left(\frac{8}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{11}{3}\right)} + \frac{\sqrt{a} b c x^{7} \Gamma\left(\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a} b d x^{8} \Gamma\left(\frac{8}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{11}{3}\right)} + \frac{\sqrt{a} b f x^{10} \Gamma\left(\frac{10}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{10}{3} \\ \frac{13}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{13}{3}\right)} + \frac{\sqrt{a} b g x^{11} \Gamma\left(\frac{11}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{11}{3} \\ \frac{14}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{14}{3}\right)} + a e \left(\begin{cases} - \frac{4 a^{2} \sqrt{a + b x^{3}}}{45 b^{2}} + \frac{2 a x^{3} \sqrt{a + b x^{3}}}{45 b} + \frac{2 x^{6} \sqrt{a + b x^{3}}}{15} & \text{for}\: b \neq 0 \\ \frac{\sqrt{a} x^{6}}{6} & \text{otherwise} \end{cases}\right) + b e \left(\begin{cases} \frac{16 a^{3} \sqrt{a + b x^{3}}}{315 b^{3}} - \frac{8 a^{2} x^{3} \sqrt{a + b x^{3}}}{315 b^{2}} + \frac{2 a x^{6} \sqrt{a + b x^{3}}}{105 b} + \frac{2 x^{9} \sqrt{a + b x^{3}}}{21} & \text{for}\: b \neq 0 \\ \frac{\sqrt{a} x^{9}}{9} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + a**(3/2)*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + a**(3/2)*f*x**7*gamma(7/3)*hype
r((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(3/2
)*g*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(11/3)) + sqrt(a)*b*c*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*d*x**8*gamma(8/3)*hyper
((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + sqrt(a)*
b*f*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(13/3)) + sqrt(a)*b*g*x**11*gamma(11/3)*hyper((-1/2, 11/3), (14/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(14/3)) + a*e*Piecewise((-4*a**2*sqrt
(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a +
 b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*e*Piecewise((16*a**3*sq
rt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*a*x
**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt
(a)*x**9/9, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)*x^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(b\,x^3+a\right)^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
[Out] int(x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.459 $\int x^2 (a+bx^3)^{3/2} (c+dx+ex^2+fx^3+gx^4)\, dx$

Optimal. Leaf size=742

$$\frac{2a^2(7bc-2af)\sqrt{a+bx^3}}{105b^2}+\frac{54a^2(23bd-8ag)x\sqrt{a+bx^3}}{21505b^2}+\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2gx}{4}$$

```
[Out] 2/780045*x^2*(b*x^3+a)^(3/2)*(33915*g*x^5+37145*f*x^4+41055*e*x^3+45885*d*x
^2+52003*c*x)+2/105*a^2*(-2*a*f+7*b*c)*(b*x^3+a)^(1/2)/b^2+54/21505*a^2*(-8
*a*g+23*b*d)*x*(b*x^3+a)^(1/2)/b^2+54/1729*a^2*e*x^2*(b*x^3+a)^(1/2)/b+2/10
5*a^2*f*x^3*(b*x^3+a)^(1/2)/b+54/4301*a^2*g*x^4*(b*x^3+a)^(1/2)/b+2/1115464
35*a*x^2*(2567565*g*x^5+3187041*f*x^4+4064445*e*x^3+5368545*d*x^2+7436429*c
*x)*(b*x^3+a)^(1/2)-216/1729*a^3*e*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))+108/1729*3^(1/4)*a^(10/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((
b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+36/37182145*3^(3/4)*
a^3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(13832*a*g-39767*b*d+43010*a^(1/3)*b^
(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/(b*x^3+a)^(1/
2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 1.04, antiderivative size = 742, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1850, 1902, 1608, 1900, 267, 1892, 224, 1891}

$$\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{10/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{216a^3e\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2\sqrt{a+bx^3}(7bc-2af)}{105b^2}+\frac{54a^2x\sqrt{a+bx^3}(23bd-8ag)}{21505b^2}+\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2gx^4\sqrt{a+bx^3}}{4301b}+\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(43010\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-1729(23bd-8ag)\right)}{37182145b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x^2(a+bx^3)^{3/2}(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5)}{780045}+\frac{2ax^2\sqrt{a+bx^3}(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5)}{111546435}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a^2*(7*b*c - 2*a*f)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b*d - 8*a*g
)*x*Sqrt[a + b*x^3])/(21505*b^2) + (54*a^2*e*x^2*Sqrt[a + b*x^3])/(1729*b)
+ (2*a^2*f*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*g*x^4*Sqrt[a + b*x^3])/(4
301*b) - (216*a^3*e*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) + (2*x^2*(a + b*x^3)^(3/2)*(52003*c*x + 45885*d*x^2 + 41055*e*
```

```
x^3 + 37145*f*x^4 + 33915*g*x^5))/780045 + (2*a*x^2*Sqrt[a + b*x^3]*(743642
9*c*x + 5368545*d*x^2 + 4064445*e*x^3 + 3187041*f*x^4 + 2567565*g*x^5))/111
546435 + (108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*e*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
 + (36*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(43010*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e
 - 1729*(23*b*d - 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(37182145*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
```

```
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int x^2\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+371\cdots\right.}{780045}$$

$$=\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+371\cdots\right.}{780045}$$

$$=\frac{54a^2gx^4\sqrt{a+bx^3}}{4301b}+\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885\cdots\right.}{\cdots}$$

$$=\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2gx^4\sqrt{a+bx^3}}{4301b}+\frac{2x^2\left(a+bx^3\right)^{3\cdots}}{\cdots}$$

$$=\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2gx^4\sqrt{a+\cdots}}{4301b}$$

$$=\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2gx^4\sqrt{a+\cdots}}{4301b}$$

$$=\frac{54a^2(23bd-8ag)x\sqrt{a+bx^3}}{21505b^2}+\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}+\frac{2a\cdots}{\cdots}$$

$$=\frac{54a^2(23bd-8ag)x\sqrt{a+bx^3}}{21505b^2}+\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}+\frac{2a\cdots}{\cdots}$$

$$=\frac{2a^2(7bc-2af)\sqrt{a+bx^3}}{105b^2}+\frac{54a^2(23bd-8ag)x\sqrt{a+bx\cdots}}{21505b^2}$$

$$=\frac{2a^2(7bc-2af)\sqrt{a+bx^3}}{105b^2}+\frac{54a^2(23bd-8ag)x\sqrt{a+bx\cdots}}{21505b^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.12, size = 162, normalized size = 0.22

$$\frac{2\left((a+bx^3)^3\left(52003bc-38a(391f+420gx)+5bx(9177d+17x(483e+19x(23f+21gx)))\right)+1995a^3(-23bd+8ag)x\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)-41055a^3bex^2\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{780045b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*((a + b*x^3)^3*(52003*b*c - 38*a*(391*f + 420*g*x) + 5*b*x*(9177*d + 17*
x*(483*e + 19*x*(23*f + 21*g*x)))) + 1995*a^3*(-23*b*d + 8*a*g)*x*Sqrt[1 +
(b*x^3)/a]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] - 41055*a^3*b*e*
x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(
780045*b^2*Sqrt[a + b*x^3])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1268 vs. 2(579) = 1158.
time = 0.38, size = 1269, normalized size = 1.71

method	result	size
elliptic	Expression too large to display	1103
risch	Expression too large to display	1175
default	Expression too large to display	1269

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/23*b*x^10*(b*x^3+a)^(1/2)+52/391*a*x^7*(b*x^3+a)^(1/2)+54/4301*a^2*x^4
*(b*x^3+a)^(1/2)/b-432/21505*a^3*x*(b*x^3+a)^(1/2)/b^2-288/21505*I*a^4/b^3*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+f*(2/21*b*x^
9*(b*x^3+a)^(1/2)+16/105*a*x^6*(b*x^3+a)^(1/2)+2/105/b*a^2*x^3*(b*x^3+a)^(1
/2)-4/105*a^3/b^2*(b*x^3+a)^(1/2))+e*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x
^5*(b*x^3+a)^(1/2)+54/1729*a^2*x^2*(b*x^3+a)^(1/2)/b+72/1729*I/b^2*a^3*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+d*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187
*a*x^4*(b*x^3+a)^(1/2)+54/935*a^2*x*(b*x^3+a)^(1/2)/b+36/935*I/b^2*a^3*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
```

```
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/15*c*(b*x^3+a)
^(5/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)*c/b + integrate((b*g*x^9 + b*f*x^8 + b*x^7*e + a*f*x
^5 + (b*d + a*g)*x^6 + a*x^4*e + a*d*x^3)*sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 237, normalized size = 0.32

$$\frac{2\left(6967620\,a^{3}b^{\frac{3}{2}}e\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-280098\left(23a^{3}bd-8a^{4}g\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\left(4849845b^{4}gx^{10}+5311735b^{4}fx^{9}+5870865b^{4}ex^{8}+9935310ab^{3}ex^{5}+285285\left(23b^{4}d+26ab^{3}g\right)x^{7}+1741905a^{2}b^{2}ex^{2}+1062347\left(7b^{4}c+8ab^{3}f\right)x^{6}+7436429a^{2}b^{2}c-2124694a^{3}bf+25935\left(460ab^{3}d+27a^{2}b^{2}g\right)x^{4}+1062347\left(14ab^{3}c+a^{2}b^{2}f\right)x^{3}+140049\left(23a^{2}b^{2}d-8a^{3}bg\right)x\right)\sqrt{bx^{3}+a}\right)}{111546435\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 2/111546435*(6967620*a^3*b^(3/2)*e*weierstrassZeta(0, -4*a/b, weierstrassPI
nverse(0, -4*a/b, x)) - 280098*(23*a^3*b*d - 8*a^4*g)*sqrt(b)*weierstrassPI
nverse(0, -4*a/b, x) + (4849845*b^4*g*x^10 + 5311735*b^4*f*x^9 + 5870865*b^
4*e*x^8 + 9935310*a*b^3*e*x^5 + 285285*(23*b^4*d + 26*a*b^3*g)*x^7 + 174190
5*a^2*b^2*e*x^2 + 1062347*(7*b^4*c + 8*a*b^3*f)*x^6 + 7436429*a^2*b^2*c - 2
124694*a^3*b*f + 25935*(460*a*b^3*d + 27*a^2*b^2*g)*x^4 + 1062347*(14*a*b^3
*c + a^2*b^2*f)*x^3 + 140049*(23*a^2*b^2*d - 8*a^3*b*g)*x)*sqrt(b*x^3 + a))
/b^3
```

Sympy [A]
time = 4.29, size = 525, normalized size = 0.71

$$\frac{a^{\frac{3}{2}}dx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{3}{2}}ex^{5}\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{a^{\frac{3}{2}}gx^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt{a}\,bdx^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt{a}\,bex^{8}\Gamma\left(\frac{8}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}+\frac{\sqrt{a}\,bgx^{10}\Gamma\left(\frac{10}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\\frac{13}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}+ac\left(\begin{cases}\frac{\sqrt{a}\,x^{3}}{3} & \text{for}\: b=0\\\frac{2\left(a+bx^{3}\right)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)+af\left(\begin{cases}-\frac{4a^{2}\sqrt{a+bx^{3}}}{45b^{2}}+\frac{2ax^{3}\sqrt{a+bx^{3}}}{45b}+\frac{2x^{6}\sqrt{a+bx^{3}}}{15} & \text{for}\: b\neq 0\\\frac{\sqrt{a}\,x^{6}}{6} & \text{otherwise}\end{cases}\right)+bc\left(\begin{cases}-\frac{4a^{2}\sqrt{a+bx^{3}}}{45b^{2}}+\frac{2ax^{3}\sqrt{a+bx^{3}}}{45b}+\frac{2x^{6}\sqrt{a+bx^{3}}}{15} & \text{for}\: b\neq 0\\\frac{\sqrt{a}\,x^{6}}{6} & \text{otherwise}\end{cases}\right)+bf\left(\begin{cases}\frac{16a^{3}\sqrt{a+bx^{3}}}{315b^{3}}-\frac{8a^{2}x^{3}\sqrt{a+bx^{3}}}{315b^{2}}+\frac{2ax^{6}\sqrt{a+bx^{3}}}{105b}+\frac{2x^{9}\sqrt{a+bx^{3}}}{21} & \text{for}\: b\neq 0\\\frac{\sqrt{a}\,x^{9}}{9} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + a**(3/2)*e*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + a**(3/2)*g*x**7*gamma(7/3)*hype
r((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)
*b*d*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(10/3)) + sqrt(a)*b*e*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + sqrt(a)*b*g*x**10*gamma(10/3)*hy
per((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(13/3)) + a*c*
Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
+ a*f*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x
**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)
) + b*c*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b
*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, Tru
e)) + b*f*Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt
(a + b*x**3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a
 + b*x**3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

```
[Out] int(x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.460 $\int x(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=723

$$\frac{2a^2(7bd-2ag)\sqrt{a+bx^3}}{105b^2}+\frac{54a^2ex\sqrt{a+bx^3}}{935b}+\frac{54a^2fx^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2gx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2(19bc-4af}{1729b^{5/3}\left(\left(1+\sqrt{3}\right.\right.}$$

```
[Out] 2/440895*x*(b*x^3+a)^(3/2)*(20995*g*x^5+23205*f*x^4+25935*e*x^3+29393*d*x^2
+33915*c*x)+2/105*a^2*(-2*a*g+7*b*d)*(b*x^3+a)^(1/2)/b^2+54/935*a^2*e*x*(b*
x^3+a)^(1/2)/b+54/1729*a^2*f*x^2*(b*x^3+a)^(1/2)/b+2/105*a^2*g*x^3*(b*x^3+a
)^(1/2)/b+2/4849845*a*x*(138567*g*x^5+176715*f*x^4+233415*e*x^3+323323*d*x^
2+479655*c*x)*(b*x^3+a)^(1/2)+54/1729*a^2*(-4*a*f+19*b*c)*(b*x^3+a)^(1/2)/b
^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/1729*3^(1/4)*a^(7/3)*(-4*a*f+19*b
*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5
/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)-18/1616615*3^(3/4)*a^(7/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^
(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*(3458*a^(2/3)*b^(1/3)*e+935*(-4*a*f+19*b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*
2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.82, antiderivative size = 723, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}=0.242$, Rules used = {1840, 1850, 1902, 1900, 267, 1892, 224, 1891}

$$-\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(3458a^{2/3}\sqrt[3]{b}e+935\left(1-\sqrt{3}\right)(19bc-4af)\right)}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19bc-4af)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{54a^2\sqrt{a+bx^3}(19bc-4af)}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2\sqrt{a+bx^3}(7bd-2ag)}{105b^2}+\frac{54a^2ex\sqrt{a+bx^3}}{935b}+\frac{54a^2fx^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2gx^3\sqrt{a+bx^3}}{105b}+\frac{2x(a+bx^3)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}+\frac{2ax\sqrt{a+bx^3}\left(479655cx+323323dx^2+233415ex^3+176715fx^4+138567gx^5\right)}{4849845}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a^2*(7*b*d - 2*a*g)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*e*x*Sqrt[a + b*
x^3])/(935*b) + (54*a^2*f*x^2*Sqrt[a + b*x^3])/(1729*b) + (2*a^2*g*x^3*Sqrt
[a + b*x^3])/(105*b) + (54*a^2*(19*b*c - 4*a*f)*Sqrt[a + b*x^3])/(1729*b^(5
/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x*(a + b*x^3)^(3/2)*(33915*c*
x + 29393*d*x^2 + 25935*e*x^3 + 23205*f*x^4 + 20995*g*x^5))/440895 + (2*a*x
```

```
*Sqrt[a + b*x^3]*(479655*c*x + 323323*d*x^2 + 233415*e*x^3 + 176715*f*x^4 +
 138567*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*c - 4
*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(345
8*a^(2/3)*b^(1/3)*e + 935*(1 - Sqrt[3])*(19*b*c - 4*a*f))*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1616615*b^(5/3)*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
```

```
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int x\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{2x(a+bx^3)^{3/2}\left(33915cx+29393dx^2+25935ex^3+2320\right.}{440895}$$

$$= \frac{2x(a+bx^3)^{3/2}\left(33915cx+29393dx^2+25935ex^3+2320\right.}{440895}$$

$$= \frac{2a^2gx^3\sqrt{a+bx^3}}{105b} + \frac{2x(a+bx^3)^{3/2}\left(33915cx+29393dx\right.}{44}$$

$$= \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2gx^3\sqrt{a+bx^3}}{105b} + \frac{2x(a+bx^3)^{3/}}{}$$

$$= \frac{54a^2ex\sqrt{a+bx^3}}{935b} + \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2gx^3\sqrt{a+}}{105b}$$

$$= \frac{54a^2ex\sqrt{a+bx^3}}{935b} + \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2gx^3\sqrt{a+}}{105b}$$

$$= \frac{2a^2(7bd-2ag)\sqrt{a+bx^3}}{105b^2} + \frac{54a^2ex\sqrt{a+bx^3}}{935b} + \frac{54a^2f}{}$$

$$= \frac{2a^2(7bd-2ag)\sqrt{a+bx^3}}{105b^2} + \frac{54a^2ex\sqrt{a+bx^3}}{935b} + \frac{54a^2f}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.74, size = 148, normalized size = 0.20

$$-\frac{\sqrt{a+bx^3}\left(4(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}\,(-2261bd+646ag-5bx(399e+17x(21f+19gx)))+7980a^2bex\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+1785ab(-19bc+4af)x^2\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{67830b^2\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] -1/67830*(Sqrt[a + b*x^3]*(4*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(-2261*b*d +
 646*a*g - 5*b*x*(399*e + 17*x*(21*f + 19*g*x))) + 7980*a^2*b*e*x*Hypergeom
```

```
etric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 1785*a*b*(-19*b*c + 4*a*f)*x^2*Hyp
ergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1382 vs. $2(565) = 1130$.
time = 0.39, size = 1383, normalized size = 1.91

method	result
risch	$-\frac{2(-230945b^3 g\,x^9-255255b^3 f\,x^8-285285b^3 e\,x^7-369512a\,b^2 g\,x^6-323323b^3 d\,x^6-431970a\,b^2 f\,x^5-373065b^3 c\,x^5-518700a\,b^2 e\,x^4-4}{4849845b^2}$
elliptic	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/21*b*x^9*(b*x^3+a)^(1/2)+16/105*a*x^6*(b*x^3+a)^(1/2)+2/105/b*a^2*x^3*
(b*x^3+a)^(1/2)-4/105*a^3/b^2*(b*x^3+a)^(1/2))+f*(2/19*b*x^8*(b*x^3+a)^(1/2
```

```
)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729*a^2*x^2*(b*x^3+a)^(1/2)/b+72/1729*I/
b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+e*(2/17*b*x^7*(b*x^3+a)^
(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935*a^2*x*(b*x^3+a)^(1/2)/b+36/935*I/
b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+2/15
*d*(b*x^3+a)^(5/2)/b+c*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1
/2)-18/91*I*a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)*x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.09, size = 207, normalized size = 0.29

$$-\frac{2\left(280098\,a^{3}\sqrt{b}\,e\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+75735\,(19\,a^{2}bc-4\,a^{3}f)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-(230945\,b^{3}gx^{9}+255255\,b^{3}fx^{8}+285285\,b^{3}ex^{7}+518700\,ab^{2}ex^{4}+46189\,(7\,b^{3}d+8\,ab^{2}g)x^{6}+19635\,(19\,b^{3}c+22\,ab^{2}f)x^{5}+140049\,a^{2}bex+323323\,a^{2}bd-92378\,a^{3}g+46189\,(14\,ab^{2}d+a^{2}bg)x^{3}+2805\,(304\,ab^{2}c+27\,a^{2}bf)x^{2})\sqrt{bx^{3}+a}\right)}{4849845\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] -2/4849845*(280098*a^3*sqrt(b)*e*weierstrassPInverse(0, -4*a/b, x) + 75735*
(19*a^2*b*c - 4*a^3*f)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInver
se(0, -4*a/b, x)) - (230945*b^3*g*x^9 + 255255*b^3*f*x^8 + 285285*b^3*e*x^7
 + 518700*a*b^2*e*x^4 + 46189*(7*b^3*d + 8*a*b^2*g)*x^6 + 19635*(19*b^3*c +
 22*a*b^2*f)*x^5 + 140049*a^2*b*e*x + 323323*a^2*b*d - 92378*a^3*g + 46189*
(14*a*b^2*d + a^2*b*g)*x^3 + 2805*(304*a*b^2*c + 27*a^2*b*f)*x^2)*sqrt(b*x^
3 + a))/b^2
```

Sympy [A]
time = 4.09, size = 525, normalized size = 0.73

$$\frac{a^{\frac{3}{2}}cx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{a^{\frac{3}{2}}ex^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{3}{2}}fx^{5}\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{\sqrt{a}\,bcx^{5}\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{\sqrt{a}\,bex^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt{a}\,bfx^{8}\Gamma\left(\frac{8}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}+ad\left(\begin{cases}\frac{\sqrt{a}\,x^{3}}{3} & \text{for}\: b=0\\ \frac{2(a+bx^{3})^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)+ag\left(\begin{cases}-\frac{4a^{2}\sqrt{a+bx^{3}}}{45b^{2}}+\frac{2ax^{3}\sqrt{a+bx^{3}}}{45b}+\frac{2x^{6}\sqrt{a+bx^{3}}}{15} & \text{for}\: b\neq 0\\ \frac{\sqrt{a}\,x^{6}}{6} & \text{otherwise}\end{cases}\right)+bd\left(\begin{cases}-\frac{4a^{2}\sqrt{a+bx^{3}}}{45b^{2}}+\frac{2ax^{3}\sqrt{a+bx^{3}}}{45b}+\frac{2x^{6}\sqrt{a+bx^{3}}}{15} & \text{for}\: b\neq 0\\ \frac{\sqrt{a}\,x^{6}}{6} & \text{otherwise}\end{cases}\right)+bg\left(\begin{cases}\frac{16a^{3}\sqrt{a+bx^{3}}}{315b^{3}}-\frac{8a^{2}x^{3}\sqrt{a+bx^{3}}}{315b^{2}}+\frac{2ax^{6}\sqrt{a+bx^{3}}}{105b}+\frac{2x^{9}\sqrt{a+bx^{3}}}{21} & \text{for}\: b\neq 0\\ \frac{\sqrt{a}\,x^{9}}{9} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(5/3)) + a**(3/2)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*f*x**5*gamma(5/3)*hype
r((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*b
*c*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(8/3)) + sqrt(a)*b*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**
3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*f*x**8*gamma(8/3)*hyper((-
1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + a*d*Piecewi
se((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + a*g*P
iecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(4
5*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*d
*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/
(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b
*g*Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt(a + b*
x**3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a + b*x*
*3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
[Out] int(x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.461 $\int \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right) dx$

Optimal. Leaf size=694

$$\frac{2a^2e\sqrt{a+bx^3}}{15b}+\frac{54a^2fx\sqrt{a+bx^3}}{935b}+\frac{54a^2gx^2\sqrt{a+bx^3}}{1729b}+\frac{54a^2(19bd-4ag)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2(a+bx^3)^{3/2}}{}$$

```
[Out] 2/692835*(b*x^3+a)^(3/2)*(36465*g*x^5+40755*f*x^4+46189*e*x^3+53295*d*x^2+6
2985*c*x)+2/15*a^2*e*(b*x^3+a)^(1/2)/b+54/935*a^2*f*x*(b*x^3+a)^(1/2)/b+54/
1729*a^2*g*x^2*(b*x^3+a)^(1/2)/b+2/4849845*a*(176715*g*x^5+233415*f*x^4+323
323*e*x^3+479655*d*x^2+793611*c*x)*(b*x^3+a)^(1/2)+54/1729*a^2*(-4*a*g+19*b
*d)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/1729*3^(1/4)
*a^(7/3)*(-4*a*g+19*b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2
*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+18/1616615*3^(3/4)*a^2*(a^(1/3)+b^(1/3)
*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1729*b^(1/3)*(-2*a*f+17*b*c)-935*a^(1/3)*(-4*a*g+19*b*d)
*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1867, 1902, 1900, 267, 1892, 224, 1891}

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19bd-4ag)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{54a^2\sqrt{a+bx^3}(19bd-4ag)}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2e\sqrt{a+bx^3}}{15b}+\frac{54a^2fx\sqrt{a+bx^3}}{935b}+\frac{54a^2gx^2\sqrt{a+bx^3}}{1729b}+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(17bc-2af)-935\left(1-\sqrt{3}\right)\sqrt[3]{a}(19bd-4ag)\right)}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2a\sqrt{a+bx^3}\left(793611cx+479655dx^2+323323ex^3+233415fx^4+176715gx^5\right)}{4849845}+\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a^2*e*Sqrt[a + b*x^3])/(15*b) + (54*a^2*f*x*Sqrt[a + b*x^3])/(935*b) + (
54*a^2*g*x^2*Sqrt[a + b*x^3])/(1729*b) + (54*a^2*(19*b*d - 4*a*g)*Sqrt[a +
b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*(a + b*x^3)
^(3/2)*(62985*c*x + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))
/692835 + (2*a*Sqrt[a + b*x^3]*(793611*c*x + 479655*d*x^2 + 323323*e*x^3 +
233415*f*x^4 + 176715*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/
```

```
3)*(19*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]
]*a^2*(1729*b^(1/3)*(17*b*c - 2*a*f) - 935*(1 - Sqrt[3])*a^(1/3)*(19*b*d -
4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(1616615*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right)\, dx = \frac{2(a+bx^3)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755f\right.}{692835}$$

$$= \frac{2(a+bx^3)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755f\right.}{692835}$$

$$= \frac{54a^2gx^2\sqrt{a+bx^3}}{1729b} + \frac{2(a+bx^3)^{3/2}\left(62985cx+53295dx^2\right.}{69}$$

$$= \frac{54a^2fx\sqrt{a+bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a+bx^3}}{1729b} + \frac{2(a+bx^3)^{3/2}\left(\right.}{}$$

$$= \frac{54a^2fx\sqrt{a+bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a+bx^3}}{1729b} + \frac{2(a+bx^3)^{3/2}\left(\right.}{}$$

$$= \frac{2a^2e\sqrt{a+bx^3}}{15b} + \frac{54a^2fx\sqrt{a+bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a+bx^3}}{1729b}$$

$$= \frac{2a^2e\sqrt{a+bx^3}}{15b} + \frac{54a^2fx\sqrt{a+bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a+bx^3}}{1729b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.32, size = 139, normalized size = 0.20

$$\frac{\sqrt{a+bx^3}\left(4(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}\,(323e+15x(19f+17gx))-570a(-17bc+2af)x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)-255a(-19bd+4ag)x^2\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{9690b\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (Sqrt[a + b*x^3]*(4*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(323*e + 15*x*(19*f +
 17*g*x)) - 570*a*(-17*b*c + 2*a*f)*x*Hypergeometric2F1[-3/2, 1/3, 4/3, -((
b*x^3)/a)] - 255*a*(-19*b*d + 4*a*g)*x^2*Hypergeometric2F1[-3/2, 2/3, 5/3,
-((b*x^3)/a)]))/(9690*b*Sqrt[1 + (b*x^3)/a])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1628 vs. $2(542) = 1084$.
time = 0.38, size = 1629, normalized size = 2.35

method	result	size
elliptic	Expression too large to display	1024
risch	Expression too large to display	1138
default	Expression too large to display	1629

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729*a^2*x^2*
(b*x^3+a)^(1/2)/b+72/1729*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)))+f*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935*a^2*x
*(b*x^3+a)^(1/2)/b+36/935*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)))+2/15*e*(b*x^3+a)^(5/2)/b+d*(2/13*b*x^5*(b*x^3+a)^(1
/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
```

```
3)))^(1/2))))+c*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55
*I*a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 201, normalized size = 0.29

$$\frac{2\left(140049\,(17\,a^2bc-2\,a^3f)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-75735\,(19\,a^2bd-4\,a^3g)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(255255\,b^3gx^8+285285\,b^3fx^7+323323\,b^3ex^6+646646\,ab^2ex^3+19635\,(19\,b^3d+22\,ab^2g)x^5+25935\,(17\,b^3c+20\,ab^2f)x^4+323323\,a^2be+2805\,(304\,ab^2d+27\,a^2bg)x^2+5187\,(238\,ab^2c+27\,a^2bf)x\right)\sqrt{bx^3+a}\right)}{4849845\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] 2/4849845*(140049*(17*a^2*b*c - 2*a^3*f)*sqrt(b)*weierstrassPInverse(0, -4*
a/b, x) - 75735*(19*a^2*b*d - 4*a^3*g)*sqrt(b)*weierstrassZeta(0, -4*a/b, w
eierstrassPInverse(0, -4*a/b, x)) + (255255*b^3*g*x^8 + 285285*b^3*f*x^7 +
323323*b^3*e*x^6 + 646646*a*b^2*e*x^3 + 19635*(19*b^3*d + 22*a*b^2*g)*x^5 +
 25935*(17*b^3*c + 20*a*b^2*f)*x^4 + 323323*a^2*b*e + 2805*(304*a*b^2*d + 2
7*a^2*b*g)*x^2 + 5187*(238*a*b^2*c + 27*a^2*b*f)*x)*sqrt(b*x^3 + a))/b^2
```

Sympy [A]
time = 4.15, size = 444, normalized size = 0.64

$$\frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{3}{2}}dx^2\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{a^{\frac{3}{2}}fx^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{3}{2}}gx^5\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{\sqrt{a}\,bcx^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}\,bdx^5\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{\sqrt{a}\,bfx^7\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt{a}\,bgx^8\Gamma\left(\frac{8}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}+ae\left(\begin{cases}\frac{\sqrt{a}\,x^3}{3} & \text{for}\ b=0\\\frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)+be\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for}\ b\neq 0\\\frac{\sqrt{a}\,x^6}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(3/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + a**(3/2)*f*x**4*gamma(4/3)*hyper((
-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*g*x
```

```
**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(8/3)) + sqrt(a)*b*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp
_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d*x**5*gamma(5/3)*hyper((-1/2, 5
/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*b*f*x**7*ga
mma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/
3)) + sqrt(a)*b*g*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*gamma(11/3)) + a*e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2
*(a + b*x**3)**(3/2)/(9*b), True)) + b*e*Piecewise((-4*a**2*sqrt(a + b*x**3
)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15
, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^3+a\right)^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
[Out] int((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.462 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$

Optimal. Leaf size=676

$$\frac{2a^2f\sqrt{a+bx^3}}{15b}+\frac{54a^2gx\sqrt{a+bx^3}}{935b}+\frac{54a^2e\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2(a+bx^3)^{3/2}\left(12155cx+9945dx^2\right.}{1093}$$

```
[Out] 2/109395*(b*x^3+a)^(3/2)*(6435*g*x^5+7293*f*x^4+8415*e*x^3+9945*d*x^2+12155
*c*x)/x-2/3*a^(3/2)*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))+2/15*a^2*f*(b*x^3+a)
^(1/2)/b+54/935*a^2*g*x*(b*x^3+a)^(1/2)/b+2/255255*a*(12285*g*x^5+17017*f*x
^4+25245*e*x^3+41769*d*x^2+85085*c*x)*(b*x^3+a)^(1/2)/x+54/91*a^2*e*(b*x^3+
a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/91*3^(1/4)*a^(7/3)*e*(a
^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b
*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)+18/85085*3^(3/4)*a^2*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1
/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1547*b*d-1
82*a*g-935*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/
b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {1840, 1846, 272, 65, 214, 1902, 1900, 267, 1892, 224, 1891}

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2}{3}a^{3/2}c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{54a^2e\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2a^2f\sqrt{a+bx^3}}{15b}+\frac{54a^2gx\sqrt{a+bx^3}}{935b}+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(-935\left(1-\sqrt{3}\right)\sqrt[3]{a}\,b^{2/3}e-182ag+1547bd\right)}{85085b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2a\sqrt{a+bx^3}\left(85085cx+41769dx^2+25245ex^3+17017fx^4+12285gx^5\right)}{255255x}+\frac{2(a+bx^3)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

```
[Out] (2*a^2*f*Sqrt[a + b*x^3])/(15*b) + (54*a^2*g*x*Sqrt[a + b*x^3])/(935*b) + (
54*a^2*e*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
+ (2*(a + b*x^3)^(3/2)*(12155*c*x + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 +
6435*g*x^5))/(109395*x) + (2*a*Sqrt[a + b*x^3]*(85085*c*x + 41769*d*x^2 + 2
5245*e*x^3 + 17017*f*x^4 + 12285*g*x^5))/(255255*x) - (2*a^(3/2)*c*ArcTanh[
```

```
Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1547*b*d - 935*(1 -
Sqrt[3])*a^(1/3)*b^(2/3)*e - 182*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(85085*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
```

```
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x}\,dx = \frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+\right.}{109395x}$$

$$= \frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+\right.}{109395x}$$

$$= \frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+\right.}{109395x}$$

$$= \frac{54a^2gx\sqrt{a+bx^3}}{935b} + \frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8\right.}{109395x}$$

$$= \frac{54a^2gx\sqrt{a+bx^3}}{935b} + \frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8\right.}{109395x}$$

$$= \frac{2a^2f\sqrt{a+bx^3}}{15b} + \frac{54a^2gx\sqrt{a+bx^3}}{935b} + \frac{2\left(a+bx^3\right)^{3/2}\left(1215\right.}{}$$

$$= \frac{2a^2f\sqrt{a+bx^3}}{15b} + \frac{54a^2gx\sqrt{a+bx^3}}{935b} + \frac{54a^2e\sqrt{a+}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.14, size = 753, normalized size = 1.11

$$\frac{2\sqrt{a+bx^3}\left(273a^2(187f+81gx)+2ab(170170c+97461dx+67320ex^2+51051fx^3+40950gx^4)+7b^2x^3(12155c+9945dx+33x^2(255e+13x(17f+15gx)))\right)}{765765b} - \frac{2a^{3/2}\left(850850b^{4/3}c\sqrt{\ldots}\ \ldots\right)}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(273*a^2*(187*f + 81*g*x) + 2*a*b*(170170*c + 97461*d*x
+ 67320*e*x^2 + 51051*f*x^3 + 40950*g*x^4) + 7*b^2*x^3*(12155*c + 9945*d*x
```

```
+ 33*x^2*(255*e + 13*x*(17*f + 15*g*x)))))/(765765*b) - (2*a^(3/2)*(85085*b
^(4/3)*c*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*
Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 125307*Sqrt[a]*b*d*((-1)
^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^
(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3)
)*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-
1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 14742*a^(3/2)*g*((-1)^(1/3)*a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)
^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Ellipt
icF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))
]], (-1)^(1/3)] - 75735*Sqrt[2]*a^(5/6)*b^(2/3)*e*((-1)^(1/3)*a^(1/3) - b^(
1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3)
)*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-
1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)
], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b
^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(255255*b^(4/
3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a
 + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1187 vs. 2(520) = 1040.
time = 0.36, size = 1188, normalized size = 1.76

method	result	size
elliptic	Expression too large to display	987
default	Expression too large to display	1188

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935*a^2*x*(b*
x^3+a)^(1/2)/b+36/935*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-
1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)))+2/15*f*(b*x^3+a)^(5/2)/b+e*(2/13*b*x^5*(b*x^3+a)^(1/2)+
32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
```

```
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))))+d*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a
^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+c*(2/9*
b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(
1/2)/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxima"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 457, normalized size = 0.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas"
)
```

```
[Out] [1/1531530*(255255*a^(3/2)*b^2*c*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a
)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 908820*a^2*b^(3/2)*e*weierstrassZ
eta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + 88452*(17*a^2*b*d - 2*a
^3*g)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + 4*(45045*b^3*g*x^7 + 5105
1*b^3*f*x^6 + 58905*b^3*e*x^5 + 134640*a*b^2*e*x^2 + 4095*(17*b^3*d + 20*a*
b^2*g)*x^4 + 340340*a*b^2*c + 51051*a^2*b*f + 17017*(5*b^3*c + 6*a*b^2*f)*x
^3 + 819*(238*a*b^2*d + 27*a^2*b*g)*x)*sqrt(b*x^3 + a))/b^2, 1/765765*(2552
55*sqrt(-a)*a*b^2*c*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 4544
10*a^2*b^(3/2)*e*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b,
x)) + 44226*(17*a^2*b*d - 2*a^3*g)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x
```

```
) + 2*(45045*b^3*g*x^7 + 51051*b^3*f*x^6 + 58905*b^3*e*x^5 + 134640*a*b^2*e
*x^2 + 4095*(17*b^3*d + 20*a*b^2*g)*x^4 + 340340*a*b^2*c + 51051*a^2*b*f +
17017*(5*b^3*c + 6*a*b^2*f)*x^3 + 819*(238*a*b^2*d + 27*a^2*b*g)*x)*sqrt(b*
x^3 + a))/b^2]
```

Sympy [A]
time = 9.74, size = 473, normalized size = 0.70

$$-\frac{2a^{\frac{3}{2}}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3}+\frac{a^{\frac{3}{2}}dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{3}{2}}ex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{a^{\frac{3}{2}}gx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}\,bdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}\,bex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{\sqrt{a}\,bgx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{2a^2c}{3\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2a\sqrt{b}\,cx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}+af\left(\begin{cases}\frac{\sqrt{a}\,x^3}{3} & \text{for}\ b=0\\ \frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)+bc\left(\begin{cases}\frac{\sqrt{a}\,x^3}{3} & \text{for}\ b=0\\ \frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)+bf\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] -2*a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a**(3/2)*d*x*gamma(1/3)
*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**(
3/2)*e*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(5/3)) + a**(3/2)*g*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d*x**4*gamma(4/3)*hyper((
-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*e*
x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(8/3)) + sqrt(a)*b*g*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**2*c/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*
x**3) + 1)) + 2*a*sqrt(b)*c*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + a*f*Piecewi
se((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*c*P
iecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) +
 b*f*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x*
*3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x)
```

```
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x, x)
```

3.463 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$

Optimal. Leaf size=692

$$\frac{2a^2g\sqrt{a+bx^3}}{15b}-\frac{27ac\sqrt{a+bx^3}}{7x}+\frac{27a(13bc+2af)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2a\sqrt{a+bx^3}\,(19305cx+5005dx^2+24}{15015x^2}$$

```
[Out] 2/45045*(b*x^3+a)^(3/2)*(3003*g*x^5+3465*f*x^4+4095*e*x^3+5005*d*x^2+6435*c
*x)/x^2-2/3*a^(3/2)*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))+2/15*a^2*g*(b*x^3+a)
^(1/2)/b-27/7*a*c*(b*x^3+a)^(1/2)/x+2/15015*a*(1001*g*x^5+1485*f*x^4+2457*e
*x^3+5005*d*x^2+19305*c*x)*(b*x^3+a)^(1/2)/x^2+27/91*a*(2*a*f+13*b*c)*(b*x^
3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/182*3^(1/4)*a^(4/3)*(
2*a*f+13*b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)+9/5005*3^(3/4)*a^(4/3)*(a^(1/3)+b^(1/3)*x)*Ellipti
cF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(182*a^(2/3)*b^(1/3)*e-55*(2*a*f+13*b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$,
Rules used = {1840, 1849, 1846, 272, 65, 214, 1900, 267, 1892, 224, 1891}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(182a^{2/3}\sqrt[3]{b}\,e-55\left(1-\sqrt{3}\right)(2af+13bc)\right)}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(2af+13bc)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2}{3}a^{3/2}d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2a^2g\sqrt{a+bx^3}}{15b}+\frac{27a\sqrt{a+bx^3}\,(2af+13bc)}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2a\sqrt{a+bx^3}\,(19305cx+5005dx^2+2457ex^3+1485fx^4+1001gx^5)}{15015x^2}+\frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2}-\frac{27ac\sqrt{a+bx^3}}{7x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

```
[Out] (2*a^2*g*Sqrt[a + b*x^3])/(15*b) - (27*a*c*Sqrt[a + b*x^3])/(7*x) + (27*a*(
13*b*c + 2*a*f)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) + (2*a*Sqrt[a + b*x^3]*(19305*c*x + 5005*d*x^2 + 2457*e*x^3 + 1485*f
*x^4 + 1001*g*x^5))/(15015*x^2) + (2*(a + b*x^3)^(3/2)*(6435*c*x + 5005*d*x
^2 + 4095*e*x^3 + 3465*f*x^4 + 3003*g*x^5))/(45045*x^2) - (2*a^(3/2)*d*ArcT
```

```
anh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13
*b*c + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(4/3
)*(182*a^(2/3)*b^(1/3)*e - 55*(1 - Sqrt[3])*(13*b*c + 2*a*f))*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(2/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+\ldots}{45045x^2}$$

$$= \frac{2a\sqrt{a+bx^3}(19305cx+5005dx^2+2457ex^3+1485fx^4+\ldots}{15015x^2}$$

$$= -\frac{27ac\sqrt{a+bx^3}}{7x} + \frac{2a\sqrt{a+bx^3}(19305cx+5005dx^2+2\ldots}{15015x^2}$$

$$= -\frac{27ac\sqrt{a+bx^3}}{7x} + \frac{2a\sqrt{a+bx^3}(19305cx+5005dx^2+2\ldots}{15015x^2}$$

$$= -\frac{27ac\sqrt{a+bx^3}}{7x} + \frac{2a\sqrt{a+bx^3}(19305cx+5005dx^2+2\ldots}{15015x^2}$$

$$= \frac{2a^2g\sqrt{a+bx^3}}{15b} - \frac{27ac\sqrt{a+bx^3}}{7x} + \frac{2a\sqrt{a+bx^3}(19305c\ldots}{\ldots}$$

$$= \frac{2a^2g\sqrt{a+bx^3}}{15b} - \frac{27ac\sqrt{a+bx^3}}{7x} + \frac{27a(13bc+2af)\ldots}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.26, size = 817, normalized size = 1.18

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
[Out] (Sqrt[a + b*x^3]*(6006*a^2*g*x + 2*b^2*x^3*(6435*c + 7*x*(715*d + 585*e*x +
 495*f*x^2 + 429*g*x^3)) + a*b*(-45045*c + 4*x*(10010*d + 5733*e*x + 33*x^2
```

```
*(120*f + 91*g*x)))))/(45045*b*x) - (a*(10010*Sqrt[a]*b^(2/3)*d*Sqrt[(a^(1/
3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]] + 14742*a*b^(1/3)*e*((-1)^(1/3)*a^(1/3) - b^(1
/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1
/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF
[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]],
 (-1)^(1/3)] - 57915*Sqrt[2]*a^(1/3)*b*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*S
qrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3)
)]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))
*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(
1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)
/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - 8910*Sqrt[2]*a^(4/3)*f
*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^
(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3
*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b
^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[Arc
Sin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1
)^(1/3))])))/(15015*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1316 vs. $2(534) = 1068$.
time = 0.42, size = 1317, normalized size = 1.90

method	result	size
elliptic	Expression too large to display	946
default	Expression too large to display	1317
risch	Expression too large to display	3382

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*g*(b*x^3+a)^(5/2)/b+f*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a
)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^
2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^
2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+e*(2/11*b*
x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)/b*(-a*b^2
```

)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+c*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 424, normalized size = 0.61

$$\left[\frac{15015\,a^{\frac{3}{2}} b d x \log\left(-\frac{b^{2} x^{6} + 8\,a b x^{3} - 4\,{\left(b x^{3} + 2\,a\right)} \sqrt{b x^{3} + a} \sqrt{a} + 8\,a^{2}}{x^{6}}\right) + 88452\,a^{2} \sqrt{b} e x {\rm weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right) - 26730\,{\left(13\,a b c + 2\,a^{2} f\right)} \sqrt{b} x {\rm weierstrassZeta}\left(0, -\frac{4\,a}{b}, {\rm weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right)\right) + 2\,{\left(6006\,b^{2} g x^{7} + 6930\,b^{2} f x^{6} + 8190\,b^{2} e x^{5} + 22932\,a b e x^{2} + 2002\,{\left(5\,b^{2} d + 6\,a b g\right)} x^{4} + 990\,{\left(13\,b^{2} c + 16\,a b f\right)} x^{3} - 45045\,a b c + 2002\,{\left(20\,a b d + 3\,a^{2} g\right)} x\right)} \sqrt{b x^{3} + a}}{90090\,b x}, \frac{15015\,\sqrt{-a} a b d x \arctan\left(\frac{2\,\sqrt{b x^{3} + a} \sqrt{-a}}{b x^{3} + 2\,a}\right) + 44226\,a^{2} \sqrt{b} e x {\rm weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right) - 13365\,{\left(13\,a b c + 2\,a^{2} f\right)} \sqrt{b} x {\rm weierstrassZeta}\left(0, -\frac{4\,a}{b}, {\rm weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right)\right) + {\left(6006\,b^{2} g x^{7} + 6930\,b^{2} f x^{6} + 8190\,b^{2} e x^{5} + 22932\,a b e x^{2} + 2002\,{\left(5\,b^{2} d + 6\,a b g\right)} x^{4} + 990\,{\left(13\,b^{2} c + 16\,a b f\right)} x^{3} - 45045\,a b c + 2002\,{\left(20\,a b d + 3\,a^{2} g\right)} x\right)} \sqrt{b x^{3} + a}}{45045\,b x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] [1/90090*(15015*a^(3/2)*b*d*x*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 88452*a^2*sqrt(b)*e*x*weierstrassPInverse(0, -4*a/b, x) - 26730*(13*a*b*c + 2*a^2*f)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + 2*(6006*b^2*g*x^7 + 6930*b^2*f*x^6 + 8190*b^2*e*x^5 + 22932*a*b*e*x^2 + 2002*(5*b^2*d + 6*a*b*g)*x^4 +
```

```
 990*(13*b^2*c + 16*a*b*f)*x^3 - 45045*a*b*c + 2002*(20*a*b*d + 3*a^2*g)*x)
*sqrt(b*x^3 + a))/(b*x), 1/45045*(15015*sqrt(-a)*a*b*d*x*arctan(2*sqrt(b*x^
3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 44226*a^2*sqrt(b)*e*x*weierstrassPInverse(
0, -4*a/b, x) - 13365*(13*a*b*c + 2*a^2*f)*sqrt(b)*x*weierstrassZeta(0, -4*
a/b, weierstrassPInverse(0, -4*a/b, x)) + (6006*b^2*g*x^7 + 6930*b^2*f*x^6
+ 8190*b^2*e*x^5 + 22932*a*b*e*x^2 + 2002*(5*b^2*d + 6*a*b*g)*x^4 + 990*(13
*b^2*c + 16*a*b*f)*x^3 - 45045*a*b*c + 2002*(20*a*b*d + 3*a^2*g)*x)*sqrt(b*
x^3 + a))/(b*x)]
```

Sympy [A]
time = 5.47, size = 474, normalized size = 0.68

$$\frac{a^{\frac{3}{2}} c \Gamma\left(-\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{b x^{3} e^{i \pi}}{a}\right)}{3 x \Gamma\left(\frac{2}{3}\right)} - \frac{2 a^{\frac{3}{2}} d \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b} x^{\frac{3}{2}}}\right)}{3} + \frac{a^{\frac{3}{2}} e x \Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{b x^{3} e^{i \pi}}{a}\right)}{3 \Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}} f x^{2} \Gamma\left(\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{b x^{3} e^{i \pi}}{a}\right)}{3 \Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a} b c x^{2} \Gamma\left(\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{b x^{3} e^{i \pi}}{a}\right)}{3 \Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a} b e x^{4} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{b x^{3} e^{i \pi}}{a}\right)}{3 \Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a} b f x^{5} \Gamma\left(\frac{5}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{b x^{3} e^{i \pi}}{a}\right)}{3 \Gamma\left(\frac{8}{3}\right)} + \frac{2 a^{2} d}{3 \sqrt{b} x^{\frac{3}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} + \frac{2 a \sqrt{b} d x^{\frac{3}{2}}}{3 \sqrt{\frac{a}{b x^{3}} + 1}} + a g \left(\begin{cases} \frac{\sqrt{a} x^{3}}{3} & \text{for}\: b = 0 \\ \frac{2 \left(a + b x^{3}\right)^{\frac{3}{2}}}{9 b} & \text{otherwise} \end{cases}\right) + b d \left(\begin{cases} \frac{\sqrt{a} x^{3}}{3} & \text{for}\: b = 0 \\ \frac{2 \left(a + b x^{3}\right)^{\frac{3}{2}}}{9 b} & \text{otherwise} \end{cases}\right) + b g \left(\begin{cases} - \frac{4 a^{2} \sqrt{a + b x^{3}}}{45 b^{2}} + \frac{2 a x^{3} \sqrt{a + b x^{3}}}{45 b} + \frac{2 x^{6} \sqrt{a + b x^{3}}}{15} & \text{for}\: b \neq 0 \\ \frac{\sqrt{a} x^{6}}{6} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] a**(3/2)*c*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x*gamma(2/3)) - 2*a**(3/2)*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a**
(3/2)*e*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) + a**(3/2)*f*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**
3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x**2*gamma(2/3)*hyper((-1
/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*e*x*
*4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma
(7/3)) + sqrt(a)*b*f*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(8/3)) + 2*a**2*d/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3
) + 1)) + 2*a*sqrt(b)*d*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + a*g*Piecewise((
sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*d*Piece
wise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*g
*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/
(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x)
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2, x)
```

3.464 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$

Optimal. Leaf size=694

$$\frac{27ac\sqrt{a+bx^3}}{10x^2}-\frac{27ad\sqrt{a+bx^3}}{7x}+\frac{27a(13bd+2ag)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2a\sqrt{a+bx^3}\,(27027cx-19305dx^2-\cdots}{15015x^3\cdots}$$

[Out] 2/45045*(b*x^3+a)^(3/2)*(3465*g*x^5+4095*f*x^4+5005*e*x^3+6435*d*x^2+9009*c*x)/x^3-2/3*a^(3/2)*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))+27/10*a*c*(b*x^3+a)^(1/2)/x^2-27/7*a*d*(b*x^3+a)^(1/2)/x-2/15015*a*(-1485*g*x^5-2457*f*x^4-5005*e*x^3-19305*d*x^2+27027*c*x)*(b*x^3+a)^(1/2)/x^3+27/91*a*(2*a*g+13*b*d)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/182*3^(1/4)*a^(4/3)*(2*a*g+13*b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+9/10010*3^(3/4)*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(91*b^(1/3)*(4*a*f+11*b*c)-110*a^(1/3)*(2*a*g+13*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.59, antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(91\sqrt[3]{b}\,(4af+11bc)-110\left(1-\sqrt{3}\right)\sqrt[3]{a}\,(2ag+13bd)\right)}{10010b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(2ag+13bd)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2}{3}a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{27a\sqrt{a+bx^3}\,(2ag+13bd)}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2a\sqrt{a+bx^3}\,(27027cx-19305dx^2-5005ex^3-2457fx^4-1485gx^5)}{15015x^3}+\frac{2(a+bx^3)^{3/2}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045x^3}+\frac{27ac\sqrt{a+bx^3}}{10x^2}-\frac{27ad\sqrt{a+bx^3}}{7x}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]

[Out] (27*a*c*Sqrt[a + b*x^3])/(10*x^2) - (27*a*d*Sqrt[a + b*x^3])/(7*x) + (27*a*(13*b*d + 2*a*g)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(27027*c*x - 19305*d*x^2 - 5005*e*x^3 - 2457*f*x^4 - 1485*g*x^5))/(15015*x^3) + (2*(a + b*x^3)^(3/2)*(9009*c*x + 6435*d*x^2 + 5005*e*x^3 + 4095*f*x^4 + 3465*g*x^5))/(45045*x^3) - (2*a^(3/2)*e*Ar

```
cTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(
13*b*d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(9
1*b^(1/3)*(11*b*c + 4*a*f) - 110*(1 - Sqrt[3])*a^(1/3)*(13*b*d + 2*a*g))*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10010*b^
(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
```

```
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}\,dx = \frac{2(a+bx^3)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+\right.}{45045x^3}$$

$$= -\frac{2a\sqrt{a+bx^3}\left(27027cx-19305dx^2-5005ex^3-2457fx\right.}{15015x^3}$$

$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{2a\sqrt{a+bx^3}\left(27027cx-19305dx^2-50\right.}{15015x^3}$$

$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} - \frac{2a\sqrt{a+bx^3}\left(27027c\right.}{}$$

$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} - \frac{2a\sqrt{a+bx^3}\left(27027c\right.}{}$$

$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} - \frac{2a\sqrt{a+bx^3}\left(27027c\right.}{}$$

$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} + \frac{27a(13bd+2ag)\sqrt{}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}$$

$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} + \frac{27a(13bd+2ag)\sqrt{}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.61, size = 952, normalized size = 1.37

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^3]*(a*(-45045*c - 90090*d*x + 8*x^2*(10010*e + 9*x*(637*f + 4
40*g*x))) + 4*b*x^3*(9009*c + 5*x*(1287*d + 7*x*(143*e + 117*f*x + 99*g*x^2
)))))/(90090*x^2) - (a*(20020*Sqrt[a]*b^(2/3)*e*Sqrt[(a^(1/3) + (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x
^3]/Sqrt[a]] + 81081*b^(4/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/
3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1
)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1
/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + 294
84*a*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/
((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*
x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 115830*Sqrt[2]*a^(1
/3)*b*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(
1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/
3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6)
 - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - Ellipt
icF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-
1 + (-1)^(1/3))]) - 17820*Sqrt[2]*a^(4/3)*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x
)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1
/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/
3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1
)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)
*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(30030*b^(2/3)*Sqrt
[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^
3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1612 vs. $2(538) = 1076$.
time = 0.41, size = 1613, normalized size = 2.32

method	result	size
elliptic	Expression too large to display	941
default	Expression too large to display	1613
risch	Expression too large to display	3858

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] g*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/
2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
```

```
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+f*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55
*a*x*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)))+c*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2
)-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+e
*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^
3+a)^(1/2)/a^(1/2)))+d*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/2)-9/7*
I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.26, size = 433, normalized size = 0.62

$$\left[\frac{15015\,a^{\frac{3}{2}}bex^{2}\log\left(-\frac{b^{2}x^{6}+8\,abx^{3}-4\,(bx^{3}+2\,a)\sqrt{bx^{3}+a}\,\sqrt{a}+8\,a^{2}}{x^{6}}\right)+22113\,(11\,abc+4\,a^{2}f)\sqrt{b}\,x^{2}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-26730\,(13\,abd+2\,a^{2}g)\sqrt{b}\,x^{2}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(13860\,b^{2}gx^{7}+16380\,b^{2}fx^{6}+20020\,b^{2}ex^{5}+80080\,abex^{2}+1980\,(13\,b^{2}d+16\,abg)x^{4}-90090\,abdx+3276\,(11\,b^{2}c+14\,abf)x^{3}-45045\,abc)\sqrt{bx^{3}+a}}{90090\,bx^{2}},\ \frac{30030\,\sqrt{-a}\,abex^{2}\arctan\left(\frac{2\sqrt{bx^{3}+a}\,\sqrt{-a}}{bx^{3}+2\,a}\right)+22113\,(11\,abc+4\,a^{2}f)\sqrt{b}\,x^{2}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-26730\,(13\,abd+2\,a^{2}g)\sqrt{b}\,x^{2}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(13860\,b^{2}gx^{7}+16380\,b^{2}fx^{6}+20020\,b^{2}ex^{5}+80080\,abex^{2}+1980\,(13\,b^{2}d+16\,abg)x^{4}-90090\,abdx+3276\,(11\,b^{2}c+14\,abf)x^{3}-45045\,abc)\sqrt{bx^{3}+a}}{90090\,bx^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="frica
s")
```

```
[Out] [1/90090*(15015*a^(3/2)*b*e*x^2*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)
*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 22113*(11*a*b*c + 4*a^2*f)*sqrt(b)
*x^2*weierstrassPInverse(0, -4*a/b, x) - 26730*(13*a*b*d + 2*a^2*g)*sqrt(b)
*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (13860
*b^2*g*x^7 + 16380*b^2*f*x^6 + 20020*b^2*e*x^5 + 80080*a*b*e*x^2 + 1980*(13
*b^2*d + 16*a*b*g)*x^4 - 90090*a*b*d*x + 3276*(11*b^2*c + 14*a*b*f)*x^3 - 4
5045*a*b*c)*sqrt(b*x^3 + a))/(b*x^2), 1/90090*(30030*sqrt(-a)*a*b*e*x^2*arc
tan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 22113*(11*a*b*c + 4*a^2*f)*
sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) - 26730*(13*a*b*d + 2*a^2*g)*
sqrt(b)*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) +
 (13860*b^2*g*x^7 + 16380*b^2*f*x^6 + 20020*b^2*e*x^5 + 80080*a*b*e*x^2 + 1
980*(13*b^2*d + 16*a*b*g)*x^4 - 90090*a*b*d*x + 3276*(11*b^2*c + 14*a*b*f)*
x^3 - 45045*a*b*c)*sqrt(b*x^3 + a))/(b*x^2)]
```

Sympy [A]
time = 5.58, size = 462, normalized size = 0.67

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{2}\Gamma\left(\frac{1}{3}\right)}+\frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}-\frac{2a^{\frac{3}{2}}e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3}+\frac{a^{\frac{3}{2}}fx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{3}{2}}gx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt{a}\,bcx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{a}\,bdx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt{a}\,bfx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}\,bgx^{5}\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{2a^{2}e}{3\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx^{3}}+1}}+\frac{2a\sqrt{b}\,ex^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^{3}}+1}}+be\left(\begin{cases}\frac{\sqrt{a}\,x^{3}}{3} & \text{for}\ b=0\\ \frac{2(a+bx^{3})^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**(3/2)*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + a**(3/2)*d*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a**(3/2)*e*asinh(sqrt(a)/(sq
rt(b)*x**(3/2)))/3 + a**(3/2)*f*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**(3/2)*g*x**2*gamma(2/3)*hyper((-
1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x
*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4
/3)) + sqrt(a)*b*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*f*x**4*gamma(4/3)*hyper((-1/2, 4/3)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*g*x**5*gamma
(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) +
 2*a**2*e/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*a*sqrt(b)*e*x**(3/2
)/(3*sqrt(a/(b*x**3) + 1)) + b*e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(
a + b*x**3)**(3/2)/(9*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x)
```

```
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3, x)
```

3.465 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$

Optimal. Leaf size=692

$$\frac{ac\sqrt{a+bx^3}}{x^3}+\frac{27ad\sqrt{a+bx^3}}{10x^2}-\frac{27ae\sqrt{a+bx^3}}{7x}+\frac{27a\sqrt[3]{b}\,e\sqrt{a+bx^3}}{7\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2a\sqrt{a+bx^3}\,(1155cx+2079$$

```
[Out] 2/3465*(b*x^3+a)^(3/2)*(315*g*x^5+385*f*x^4+495*e*x^3+693*d*x^2+1155*c*x)/x
^4-1/3*(2*a*f+3*b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+a*c*(b*x^3+a)
^(1/2)/x^3+27/10*a*d*(b*x^3+a)^(1/2)/x^2-27/7*a*e*(b*x^3+a)^(1/2)/x-2/1155*
a*(-189*g*x^5-385*f*x^4-1485*e*x^3+2079*d*x^2+1155*c*x)*(b*x^3+a)^(1/2)/x^4
+27/7*a*b^(1/3)*e*(b*x^3+a)^(1/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/14*3^(
1/4)*a^(4/3)*b^(1/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)+9/770*3^(3/4)*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^
(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*(77*b*d+28*a*g-110*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2)
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)\left(-110\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+28ag+77bd\right)}{770\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\sqrt[3]{b}\,e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{14\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2(a+bx^3)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}-\frac{2a\sqrt{a+bx^3}\left(1155cx+2079dx^2-1485ex^3-385fx^4-189gx^5\right)}{1155x^4}-\frac{1}{3}\sqrt{a}(2af+3bc)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{ac\sqrt{a+bx^3}}{x^3}+\frac{27ad\sqrt{a+bx^3}}{10x^2}-\frac{27ae\sqrt{a+bx^3}}{7x}+\frac{27a\sqrt[3]{b}\,e\sqrt{a+bx^3}}{7\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

```
[Out] (a*c*Sqrt[a + b*x^3])/x^3 + (27*a*d*Sqrt[a + b*x^3])/(10*x^2) - (27*a*e*Sqr
t[a + b*x^3])/(7*x) + (27*a*b^(1/3)*e*Sqrt[a + b*x^3])/(7*((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(1155*c*x + 2079*d*x^2 - 1485*e*
x^3 - 385*f*x^4 - 189*g*x^5))/(1155*x^4) + (2*(a + b*x^3)^(3/2)*(1155*c*x +
 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g*x^5))/(3465*x^4) - (Sqrt[a]*(3*b
```

```
*c + 2*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt
[3]]*a^(4/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(14*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(77*
b*d - 110*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 28*a*g)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(770*b^(1/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
```

GtQ[p, 0]

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315}{3465x^4}$$

$$= -\frac{2a\sqrt{a+bx^3}(1155cx+2079dx^2-1485ex^3-385fx^4-}{1155x^4}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} - \frac{2a\sqrt{a+bx^3}(1155cx+2079dx^2-1485ex}{1155x^4}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{2a\sqrt{a+bx^3}(1155cx+}{}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{27ae\sqrt{a+bx^3}}{7x} - \frac{2a\sqrt{}}{}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{27ae\sqrt{a+bx^3}}{7x} - \frac{2a\sqrt{}}{}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{27ae\sqrt{a+bx^3}}{7x} - \frac{2a\sqrt{}}{}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{27ae\sqrt{a+bx^3}}{7x} + \frac{}{7((}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{27ae\sqrt{a+bx^3}}{7x} + \frac{}{7((}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.34, size = 813, normalized size = 1.17

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]

[Out] Sqrt[a + b*x^3]*(a*((8*f)/9 - c/(3*x^3) - d/(2*x^2) - e/x + (28*g*x)/55) + b*((2*c)/3 + (2*d*x)/5 + (2*e*x^2)/7 + (2*f*x^3)/9 + (2*g*x^4)/11)) - Sqrt[a]*b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - (2*a^(3/2)*f*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*a*b^(2/3)*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(10*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]) - (54*a^2*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(55*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]) - (27*Sqrt[2]*a^(4/3)*b^(1/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(7*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(534) = 1068$.
time = 0.42, size = 1193, normalized size = 1.72

method	result	size
elliptic	Expression too large to display	920
default	Expression too large to display	1193
risch	Expression too large to display	2513

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x,method=_RETURNVERBOSE)

[Out] g*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/

```
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+c*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^
3+a)^(1/2)-b*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2))+f*(2/9*b*x^3*(b*x^3+
a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))
)+e*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-a*b^2
)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 434, normalized size = 0.63

$$\left[-\frac{53460\,ab^{\frac{3}{2}}ex^{3}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-1155\left(3b^{2}c+2abf\right)\sqrt{a}\,x^{3}\log\left(-\frac{b^{2}x^{6}+8abx^{3}-4\left(bx^{3}+2a\right)\sqrt{bx^{3}+a}\,\sqrt{a}+8a^{2}}{x^{6}}\right)-3402\left(11abd+4a^{2}g\right)\sqrt{b}\,x^{3}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-2\left(1260b^{2}gx^{7}+1540b^{2}fx^{6}+1980b^{2}ex^{5}-6930abex^{2}+252\left(11b^{2}d+14abg\right)x^{4}-3465abdx+1540\left(3b^{2}c+4abf\right)x^{3}-2310abc\right)\sqrt{bx^{3}+a}}{13860\,bx^{3}},-\frac{26730\,ab^{\frac{3}{2}}ex^{3}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-1155\left(3b^{2}c+2abf\right)\sqrt{-a}\,x^{3}\arctan\left(\frac{2\sqrt{bx^{3}+a}\,\sqrt{-a}}{bx^{3}+2a}\right)-1701\left(11abd+4a^{2}g\right)\sqrt{b}\,x^{3}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\left(1260b^{2}gx^{7}+1540b^{2}fx^{6}+1980b^{2}ex^{5}-6930abex^{2}+252\left(11b^{2}d+14abg\right)x^{4}-3465abdx+1540\left(3b^{2}c+4abf\right)x^{3}-2310abc\right)\sqrt{bx^{3}+a}}{6930\,bx^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="frica
s")
```

```
[Out] [-1/13860*(53460*a*b^(3/2)*e*x^3*weierstrassZeta(0, -4*a/b, weierstrassPInv
erse(0, -4*a/b, x)) - 1155*(3*b^2*c + 2*a*b*f)*sqrt(a)*x^3*log(-(b^2*x^6 +
8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 3402*(1
1*a*b*d + 4*a^2*g)*sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) - 2*(1260*
b^2*g*x^7 + 1540*b^2*f*x^6 + 1980*b^2*e*x^5 - 6930*a*b*e*x^2 + 252*(11*b^2*
```

```
d + 14*a*b*g)*x^4 - 3465*a*b*d*x + 1540*(3*b^2*c + 4*a*b*f)*x^3 - 2310*a*b*
c)*sqrt(b*x^3 + a))/(b*x^3), -1/6930*(26730*a*b^(3/2)*e*x^3*weierstrassZeta
(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - 1155*(3*b^2*c + 2*a*b*f)*s
qrt(-a)*x^3*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 1701*(11*a*b
*d + 4*a^2*g)*sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) - (1260*b^2*g*x
^7 + 1540*b^2*f*x^6 + 1980*b^2*e*x^5 - 6930*a*b*e*x^2 + 252*(11*b^2*d + 14*
a*b*g)*x^4 - 3465*a*b*d*x + 1540*(3*b^2*c + 4*a*b*f)*x^3 - 2310*a*b*c)*sqrt
(b*x^3 + a))/(b*x^3)]
```

Sympy [A]
time = 6.73, size = 484, normalized size = 0.70

$$\frac{a^{\frac{3}{2}} d \Gamma\left(-\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 x^{2} \Gamma\left(\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}} e \Gamma\left(-\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 x \Gamma\left(\frac{2}{3}\right)} - \frac{2 a^{\frac{3}{2}} f \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b} x^{\frac{3}{2}}}\right)}{3} + \frac{a^{\frac{3}{2}} g x \Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{4}{3}\right)} - \sqrt{a} b c \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b} x^{\frac{3}{2}}}\right) + \frac{\sqrt{a} b d x \Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a} b e x^{2} \Gamma\left(\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a} b g x^{4} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{7}{3}\right)} + \frac{2 a^{2} f}{3 \sqrt{b} x^{\frac{3}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} - \frac{a \sqrt{b} c \sqrt{\frac{a}{b x^{3}} + 1}}{3 x^{\frac{3}{2}}} + \frac{2 a \sqrt{b} c}{3 x^{\frac{3}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} + \frac{2 a \sqrt{b} f x^{\frac{3}{2}}}{3 \sqrt{\frac{a}{b x^{3}} + 1}} + \frac{2 b^{\frac{3}{2}} c x^{\frac{3}{2}}}{3 \sqrt{\frac{a}{b x^{3}} + 1}} + b f \left(\begin{cases} \frac{\sqrt{a} x^{3}}{3} & \text{for}\: b = 0 \\ \frac{2 \left(a + b x^{3}\right)^{\frac{3}{2}}}{9 b} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a**(3/2)*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + a**(3/2)*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a**(3/2)*f*asinh(sqrt(a)/(sq
rt(b)*x**(3/2)))/3 + a**(3/2)*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) - sqrt(a)*b*c*asinh(sqrt(a)/(sqrt(b)*
x**(3/2))) + sqrt(a)*b*d*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp
_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*e*x**2*gamma(2/3)*hyper((-1/2, 2
/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*g*x**4*ga
mma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)
) + 2*a**2*f/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*c*sqrt(a
/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*c/(3*x**(3/2)*sqrt(a/(b*x**3) + 1
)) + 2*a*sqrt(b)*f*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*c*x**(3/2
)/(3*sqrt(a/(b*x**3) + 1)) + b*f*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(
a + b*x**3)**(3/2)/(9*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x)
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4, x)
```

3.466 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx$

Optimal. Leaf size=741

$$\frac{27ac\sqrt{a+bx^3}}{20x^4}+\frac{ad\sqrt{a+bx^3}}{x^3}+\frac{27ae\sqrt{a+bx^3}}{10x^2}-\frac{27(7bc+8af)\sqrt{a+bx^3}}{56x}+\frac{27\sqrt[3]{b}\,(7bc+8af)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-$$

```
[Out] 2/315*(b*x^3+a)^(3/2)*(35*g*x^5+45*f*x^4+63*e*x^3+105*d*x^2+315*c*x)/x^5-1/
3*(2*a*g+3*b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+27/20*a*c*(b*x^3+a
)^(1/2)/x^4+a*d*(b*x^3+a)^(1/2)/x^3+27/10*a*e*(b*x^3+a)^(1/2)/x^2-27/56*(8*
a*f+7*b*c)*(b*x^3+a)^(1/2)/x-2/105*a*(-35*g*x^5-135*f*x^4+189*e*x^3+105*d*x
^2+189*c*x)*(b*x^3+a)^(1/2)/x^5+27/56*b^(1/3)*(8*a*f+7*b*c)*(b*x^3+a)^(1/2)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/112*3^(1/4)*a^(1/3)*b^(1/3)*(8*a*f+7*b*
c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^
3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)+9/280*3^(3/4)*a^(1/3)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(28*a
^(2/3)*b^(1/3)*e-5*(8*a*f+7*b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A]
time = 0.82, antiderivative size = 741, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(28a^{2/3}\sqrt[3]{b}\,e-5\left(1-\sqrt{3}\right)(8af+7bc)\right)}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(8af+7bc)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2(a+bx^3)^{3/2}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^5}-\frac{2a\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx^4-35gx^5)}{105x^5}-\frac{27\sqrt{a+bx^3}(8af+7bc)}{56x}+\frac{27\sqrt[3]{b}\sqrt{a+bx^3}(8af+7bc)}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{27ac\sqrt{a+bx^3}}{20x^4}-\frac{1}{3}\sqrt{a}(2ag+3bd)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{ad\sqrt{a+bx^3}}{x^3}+\frac{27ae\sqrt{a+bx^3}}{10x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] (27*a*c*Sqrt[a + b*x^3])/(20*x^4) + (a*d*Sqrt[a + b*x^3])/x^3 + (27*a*e*Sqr
t[a + b*x^3])/(10*x^2) - (27*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*x) + (27*
b^(1/3)*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) - (2*a*Sqrt[a + b*x^3]*(189*c*x + 105*d*x^2 + 189*e*x^3 - 135*f*x^4
```

```
- 35*g*x^5))/(105*x^5) + (2*(a + b*x^3)^(3/2)*(315*c*x + 105*d*x^2 + 63*e*x
^3 + 45*f*x^4 + 35*g*x^5))/(315*x^5) - (Sqrt[a]*(3*b*d + 2*a*g)*ArcTanh[Sqr
t[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7
*b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(112*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*b^(1/3
)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 8*a*f))*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(280*Sqrt[(a^(1/3)*(a^(
1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
```

```
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \frac{2(a+bx^3)^{3/2}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^5}$$

$$= -\frac{2a\sqrt{a+bx^3}\,(189cx+105dx^2+189ex^3-135fx^4-35g}{105x^5}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4} - \frac{2a\sqrt{a+bx^3}\,(189cx+105dx^2+189ex^3}{105x^5}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4} + \frac{ad\sqrt{a+bx^3}}{x^3} - \frac{2a\sqrt{a+bx^3}\,(189cx+}{}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4} + \frac{ad\sqrt{a+bx^3}}{x^3} + \frac{27ae\sqrt{a+bx^3}}{10x^2} - \frac{2a\sqrt{}}{}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4} + \frac{ad\sqrt{a+bx^3}}{x^3} + \frac{27ae\sqrt{a+bx^3}}{10x^2} - \frac{27(7}{}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4} + \frac{ad\sqrt{a+bx^3}}{x^3} + \frac{27ae\sqrt{a+bx^3}}{10x^2} - \frac{27(7}{}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4} + \frac{ad\sqrt{a+bx^3}}{x^3} + \frac{27ae\sqrt{a+bx^3}}{10x^2} - \frac{27(7}{}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4} + \frac{ad\sqrt{a+bx^3}}{x^3} + \frac{27ae\sqrt{a+bx^3}}{10x^2} - \frac{27(7}{}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4} + \frac{ad\sqrt{a+bx^3}}{x^3} + \frac{27ae\sqrt{a+bx^3}}{10x^2} - \frac{27(7}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.19, size = 878, normalized size = 1.18

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-70*a*(9*c + 2*x*(6*d + x*(9*e + 2*x*(9*f - 8*g*x)))) + b
*x^3*(-3465*c + 16*x*(105*d + x*(63*e + 5*x*(9*f + 7*g*x))))))/(2520*x^4) -
 Sqrt[a]*b*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - (2*a^(3/2)*g*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/3 - (27*a*b^(2/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/
3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqr
t[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)
])/(10*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sq
rt[a + b*x^3]) - (27*a^(1/3)*b^(4/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqr
t[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*S
qrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*Ellip
ticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(
-1 + (-1)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3
)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(4*Sqrt[2]*Sqrt[(a^(1/3) + (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]) - (27*Sqrt[
2]*a^(4/3)*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/
3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(
-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]
+ EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^
(1/3)/(-1 + (-1)^(1/3))]))/(7*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1341 vs. 2(577) = 1154.
time = 0.40, size = 1342, normalized size = 1.81

method	result

elliptic	$-\frac{ac\sqrt{b\,x^3+a}}{4x^4}-\frac{ad\sqrt{b\,x^3+a}}{3x^3}-\frac{ae\sqrt{b\,x^3+a}}{2x^2}-\frac{\left(af+\frac{11bc}{8}\right)\sqrt{b\,x^3+a}}{x}+\frac{2bg\,x^3\sqrt{b\,x^3+a}}{9}+\frac{2bf\,x^2}{}$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))))+e*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3
+a)^(1/2)-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)))+d*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-b*arctanh((b*x^3
+a)^(1/2)/a^(1/2))*a^(1/2))+g*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1
/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+f*(-a*(b*x^3+a)^(1/2)/x+2
```

```
/7*b*x^2*(b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 384, normalized size = 0.52

$$\left[\frac{6804\,a\sqrt{b}\,e x^4 \mathrm{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + 210\,(3\,bd + 2\,ag)\sqrt{a}\,x^4 \log\left(-\frac{b^2x^6 + 8\,abx^3 - 4\,(bx^3 + 2\,a)\sqrt{bx^3 + a}\,\sqrt{a} + 8\,a^2}{x^6}\right) - 1215\,(7\,bc + 8\,af)\sqrt{b}\,x^4 \mathrm{weierstrassZeta}\left(0, -\frac{4a}{b}, \mathrm{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + (560\,bgx^7 + 720\,bfx^6 + 1008\,bex^5 + 560\,(3\,bd + 4\,ag)x^4 - 1260\,aex^2 - 315\,(11\,bc + 8\,af)x^3 - 840\,adx - 630\,ac)\sqrt{bx^3 + a}}{2520\,x^4}, \frac{6804\,a\sqrt{b}\,e x^4 \mathrm{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + 420\,(3\,bd + 2\,ag)\sqrt{-a}\,x^4 \arctan\left(\frac{2\sqrt{bx^3 + a}\,\sqrt{-a}}{bx^3 + 2\,a}\right) - 1215\,(7\,bc + 8\,af)\sqrt{b}\,x^4 \mathrm{weierstrassZeta}\left(0, -\frac{4a}{b}, \mathrm{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + (560\,bgx^7 + 720\,bfx^6 + 1008\,bex^5 + 560\,(3\,bd + 4\,ag)x^4 - 1260\,aex^2 - 315\,(11\,bc + 8\,af)x^3 - 840\,adx - 630\,ac)\sqrt{bx^3 + a}}{2520\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="frica
s")
```

```
[Out] [1/2520*(6804*a*sqrt(b)*e*x^4*weierstrassPInverse(0, -4*a/b, x) + 210*(3*b*
d + 2*a*g)*sqrt(a)*x^4*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x
^3 + a)*sqrt(a) + 8*a^2)/x^6) - 1215*(7*b*c + 8*a*f)*sqrt(b)*x^4*weierstras
sZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (560*b*g*x^7 + 720*b*
f*x^6 + 1008*b*e*x^5 + 560*(3*b*d + 4*a*g)*x^4 - 1260*a*e*x^2 - 315*(11*b*c
 + 8*a*f)*x^3 - 840*a*d*x - 630*a*c)*sqrt(b*x^3 + a))/x^4, 1/2520*(6804*a*s
qrt(b)*e*x^4*weierstrassPInverse(0, -4*a/b, x) + 420*(3*b*d + 2*a*g)*sqrt(-
a)*x^4*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 1215*(7*b*c + 8*a
*f)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x
)) + (560*b*g*x^7 + 720*b*f*x^6 + 1008*b*e*x^5 + 560*(3*b*d + 4*a*g)*x^4 -
1260*a*e*x^2 - 315*(11*b*c + 8*a*f)*x^3 - 840*a*d*x - 630*a*c)*sqrt(b*x^3 +
 a))/x^4]
```

Sympy [A]

time = 6.71, size = 495, normalized size = 0.67

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}-\frac{2a^{\frac{3}{2}}g\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3}+\frac{\sqrt{a}\,bc\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}-\sqrt{a}\,bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)+\frac{\sqrt{a}\,bex\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{a}\,bfx^2\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{2a^2g}{3\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{a\sqrt{b}\,d\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}+\frac{2a\sqrt{b}\,d}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2a\sqrt{b}\,gx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}+\frac{2b^{\frac{3}{2}}dx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}+bg\left(\begin{cases}\frac{\sqrt{a}\,x^3}{3}&\text{for }b=0\\\frac{2(a+bx^3)^{\frac{3}{2}}}{9b}&\text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a**(3/2)*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**4*gamma(-1/3)) + a**(3/2)*e*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + a**(3/2)*f*gamma(-1/3)*hy
per((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a*
*(3/2)*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*b*c*gamma(-1/3)*hype
r((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a
)*b*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*e*x*gamma(1/3)*hyper((-
1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*f*x
**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(5/3)) + 2*a**2*g/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*d*
sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*d/(3*x**(3/2)*sqrt(a/(b*x**
3) + 1)) + 2*a*sqrt(b)*g*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*d*x
**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + b*g*Piecewise((sqrt(a)*x**3/3, Eq(b, 0))
, (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^5, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x)
```

```
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5, x)
```

3.467 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$

Optimal. Leaf size=689

$$\frac{27bc\sqrt{a+bx^3}}{20x^2}-\frac{27bd\sqrt{a+bx^3}}{8x}+\frac{27\sqrt[3]{b}\,(7bd+8ag)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)(a+$$

```
[Out] -1/60*(12*c/x^5+15*d/x^4+20*e/x^3+30*f/x^2+60*g/x)*(b*x^3+a)^(3/2)-b*e*arct
anh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+27/20*b*c*(b*x^3+a)^(1/2)/x^2-27/8*b*d
*(b*x^3+a)^(1/2)/x-1/140*b*(-180*g*x^5-126*f*x^4-140*e*x^3-315*d*x^2+252*c*
x)*(b*x^3+a)^(1/2)/x^3+27/56*b^(1/3)*(8*a*g+7*b*d)*(b*x^3+a)^(1/2)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))-27/112*3^(1/4)*a^(1/3)*b^(1/3)*(8*a*g+7*b*d)*(a^(1/
3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2
)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+9/2
80*3^(3/4)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1
/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(14*b^(1/3)*(2*a*f+b*c
)-5*a^(1/3)*(8*a*g+7*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*
x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A]

time = 0.62, antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {14, 1839, 1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(14\sqrt[3]{b}\,(2af+bc)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}\,(8ag+7bd)\right)}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(8ag+7bd)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)-\frac{b\sqrt{a+bx^3}\left(252cx-315dx^2-140ex^3-126fx^4-180gx^5\right)}{140x^3}+\frac{27bc\sqrt{a+bx^3}}{20x^2}+\frac{27\sqrt[3]{b}\,\sqrt{a+bx^3}\,(8ag+7bd)}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{27bd\sqrt{a+bx^3}}{8x}-\sqrt{a}\,be\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

```
[Out] (27*b*c*Sqrt[a + b*x^3])/(20*x^2) - (27*b*d*Sqrt[a + b*x^3])/(8*x) + (27*b^
(1/3)*(7*b*d + 8*a*g)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)) - (((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*(a +
 b*x^3)^(3/2))/60 - (b*Sqrt[a + b*x^3]*(252*c*x - 315*d*x^2 - 140*e*x^3 - 1
26*f*x^4 - 180*g*x^5))/(140*x^3) - Sqrt[a]*b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt
```

```
[a]] - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*b*d + 8*a*g)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(112*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(14*b^(1/3)*(b*c + 2*a*f)
- 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d + 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(280*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^6}\,dx = -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\left(a+bx^3\right)^{3/2}-\frac{1}{2}($$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\left(a+bx^3\right)^{3/2}-\frac{b\sqrt{}}{}$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2}-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)($$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2}-\frac{27bd\sqrt{a+bx^3}}{8x}-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}\right.$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2}-\frac{27bd\sqrt{a+bx^3}}{8x}-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}\right.$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2}-\frac{27bd\sqrt{a+bx^3}}{8x}-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}\right.$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2}-\frac{27bd\sqrt{a+bx^3}}{8x}+\frac{27\sqrt[3]{b}\,(7bd+8ag)\sqrt{}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2}-\frac{27bd\sqrt{a+bx^3}}{8x}+\frac{27\sqrt[3]{b}\,(7bd+8ag)\sqrt{}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}$$

Mathematica [C] Result contains complex when optimal does not.

time = 11.65, size = 949, normalized size = 1.38

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

```
[Out] -1/840*(Sqrt[a + b*x^3]*(14*a*(12*c + 5*x*(3*d + 4*e*x + 6*x^2*(f + 2*g*x))
) + b*x^3*(546*c + x*(1155*d - 16*x*(35*e + 3*x*(7*f + 5*g*x))))))/x^5 - (b
^(1/3)*(280*Sqrt[a]*b^(2/3)*e*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 378
*b^(4/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/
((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + 756*a*b^(1/3)*f*((-1)^(
1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1
/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*
a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]], (-1)^(1/3)] - 945*Sqrt[2]*a^(1/3)*b*d*((-1)^(1/3)*a^(1/
3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-
1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-(
(-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]
/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6
) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - 1080*
Sqrt[2]*a^(4/3)*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3
) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3
)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[
(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]
) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1
)^(1/3)/(-1 + (-1)^(1/3))])))/(280*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((
1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1605 vs. 2(535) = 1070.
time = 0.42, size = 1606, normalized size = 2.33

method	result	size
elliptic	Expression too large to display	920
default	Expression too large to display	1606
risch	Expression too large to display	2289

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-1/4*a*(b*x^3+a)^
(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))))+f*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+e*(-1/3*a*(b*
x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-b*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a
^(1/2))+g*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 382, normalized size = 0.55

$$\left[\frac{210\,\sqrt{a}\,b e x^{5} \log\left(-\frac{b^{2} x^{6} + 8\,a b x^{3} - 4\,{\left(b x^{3} + 2\,a\right)} \sqrt{b x^{3} + a} \sqrt{a} + 8\,a^{2}}{x^{6}}\right) + 1134\,{\left(b c + 2\,a f\right)} \sqrt{b} x^{5} {\rm weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right) - 405\,{\left(7\,b d + 8\,a g\right)} \sqrt{b} x^{5} {\rm weierstrassZeta}\left(0, -\frac{4\,a}{b}, {\rm weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right)\right) + {\left(240\,b g x^{7} + 336\,b f x^{6} + 560\,b e x^{5} - 105\,{\left(11\,b d + 8\,a g\right)} x^{4} - 280\,a e x^{2} - 42\,{\left(13\,b c + 10\,a f\right)} x^{3} - 210\,a d x - 168\,a c\right)} \sqrt{b x^{3} + a}}{840\,x^{5}}, \frac{420\,\sqrt{-a}\,b e x^{5} \arctan\left(\frac{2\,\sqrt{b x^{3} + a} \sqrt{-a}}{b x^{3} + 2\,a}\right) + 1134\,{\left(b c + 2\,a f\right)} \sqrt{b} x^{5} {\rm weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right) - 405\,{\left(7\,b d + 8\,a g\right)} \sqrt{b} x^{5} {\rm weierstrassZeta}\left(0, -\frac{4\,a}{b}, {\rm weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right)\right) + {\left(240\,b g x^{7} + 336\,b f x^{6} + 560\,b e x^{5} - 105\,{\left(11\,b d + 8\,a g\right)} x^{4} - 280\,a e x^{2} - 42\,{\left(13\,b c + 10\,a f\right)} x^{3} - 210\,a d x - 168\,a c\right)} \sqrt{b x^{3} + a}}{840\,x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="frica
s")
```

```
[Out] [1/840*(210*sqrt(a)*b*e*x^5*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqr
t(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 1134*(b*c + 2*a*f)*sqrt(b)*x^5*weierst
rassPInverse(0, -4*a/b, x) - 405*(7*b*d + 8*a*g)*sqrt(b)*x^5*weierstrassZet
a(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (240*b*g*x^7 + 336*b*f*x^
6 + 560*b*e*x^5 - 105*(11*b*d + 8*a*g)*x^4 - 280*a*e*x^2 - 42*(13*b*c + 10*
a*f)*x^3 - 210*a*d*x - 168*a*c)*sqrt(b*x^3 + a))/x^5, 1/840*(420*sqrt(-a)*b
*e*x^5*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 1134*(b*c + 2*a*f
)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) - 405*(7*b*d + 8*a*g)*sqrt(
b)*x^5*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (240
*b*g*x^7 + 336*b*f*x^6 + 560*b*e*x^5 - 105*(11*b*d + 8*a*g)*x^4 - 280*a*e*x
^2 - 42*(13*b*c + 10*a*f)*x^3 - 210*a*d*x - 168*a*c)*sqrt(b*x^3 + a))/x^5]
```

Sympy [A]
time = 6.52, size = 476, normalized size = 0.69

$$\frac{a^{\frac{3}{2}} c \Gamma\left(-\frac{5}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 x^{5} \Gamma\left(-\frac{2}{3}\right)} + \frac{a^{\frac{3}{2}} d \Gamma\left(-\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 x^{4} \Gamma\left(-\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}} f \Gamma\left(-\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 x^{2} \Gamma\left(\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}} g \Gamma\left(-\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 x \Gamma\left(\frac{2}{3}\right)} + \frac{\sqrt{a} b c \Gamma\left(-\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 x^{2} \Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a} b d \Gamma\left(-\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 x \Gamma\left(\frac{2}{3}\right)} - \sqrt{a} b e \operatorname{asinh}{\left(\frac{\sqrt{a}}{\sqrt{b} x^{\frac{3}{2}}} \right)} + \frac{\sqrt{a} b f x \Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a} b g x^{2} \Gamma\left(\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{5}{3}\right)} - \frac{a \sqrt{b} e \sqrt{\frac{a}{b x^{3}} + 1}}{3 x^{\frac{3}{2}}} + \frac{2 a \sqrt{b} e}{3 x^{\frac{3}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} + \frac{2 b^{\frac{3}{2}} e x^{\frac{3}{2}}}{3 \sqrt{\frac{a}{b x^{3}} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**6,x)
```

```
[Out] a**(3/2)*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + a**(3/2)*d*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + a**(3/2)*f*gamma(-2/3)*
hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) +
 a**(3/2)*g*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x*gamma(2/3)) + sqrt(a)*b*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b
*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*d*gamma(-1/3)*hype
r((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a
)*b*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*f*x*gamma(1/3)*hyper((-
1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*g*x
```

```
**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(5/3)) - a*sqrt(b)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*e/(3*
x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*e*x**(3/2)/(3*sqrt(a/(b*x**3) +
 1))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x)
```

```
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6, x)
```

3.468 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$

Optimal. Leaf size=692

$$\frac{bc\sqrt{a+bx^3}}{4x^3}+\frac{27bd\sqrt{a+bx^3}}{20x^2}-\frac{27be\sqrt{a+bx^3}}{8x}+\frac{27b^{4/3}e\sqrt{a+bx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right.$$

```
[Out] -1/60*(10*c/x^6+12*d/x^5+15*e/x^4+20*f/x^3+30*g/x^2)*(b*x^3+a)^(3/2)-1/4*b*
(4*a*f+b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+1/4*b*c*(b*x^3+a)^(1/2
)/x^3+27/20*b*d*(b*x^3+a)^(1/2)/x^2-27/8*b*e*(b*x^3+a)^(1/2)/x-1/20*b*(-18*
g*x^5-20*f*x^4-45*e*x^3+36*d*x^2+10*c*x)*(b*x^3+a)^(1/2)/x^4+27/8*b^(4/3)*e
*(b*x^3+a)^(1/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/16*3^(1/4)*a^(1/3)*b^(4
/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b
*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)+9/40*3^(3/4)*b^(2/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1
/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(2*b*d+4*a*
g-5*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+
a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A]
time = 0.69, antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {14, 1839, 1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(-5\left(1-\sqrt{3}\right)\sqrt[3]{a}\,b^{2/3}e+4ag+2bd\right)}{40\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\,b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{27b^{4/3}e\sqrt{a+bx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{b\sqrt{a+bx^3}\left(10cx+36dx^2-45ex^3-20fx^4-18gx^5\right)}{20x^4}-\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)-\frac{b(4af+bc)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{bc\sqrt{a+bx^3}}{4x^3}+\frac{27bd\sqrt{a+bx^3}}{20x^2}-\frac{27be\sqrt{a+bx^3}}{8x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] (b*c*Sqrt[a + b*x^3])/(4*x^3) + (27*b*d*Sqrt[a + b*x^3])/(20*x^2) - (27*b*e
*Sqrt[a + b*x^3])/(8*x) + (27*b^(4/3)*e*Sqrt[a + b*x^3])/(8*((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3
 + (30*g)/x^2)*(a + b*x^3)^(3/2))/60 - (b*Sqrt[a + b*x^3]*(10*c*x + 36*d*x^
2 - 45*e*x^3 - 20*f*x^4 - 18*g*x^5))/(20*x^4) - (b*(b*c + 4*a*f)*ArcTanh[Sq
```

```
rt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)
*b^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3
]])/(16*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d - 5
*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}\,dx = -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\left(a+bx^3\right)^{3/2}-\frac{1}{2}($$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\left(a+bx^3\right)^{3/2}-\frac{b\sqrt{}}{}$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3}-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)(a$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3}+\frac{27bd\sqrt{a+bx^3}}{20x^2}-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}\right.$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3}+\frac{27bd\sqrt{a+bx^3}}{20x^2}-\frac{27be\sqrt{a+bx^3}}{8x}-\frac{1}{60}\left(\right.$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3}+\frac{27bd\sqrt{a+bx^3}}{20x^2}-\frac{27be\sqrt{a+bx^3}}{8x}-\frac{1}{60}\left(\right.$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3}+\frac{27bd\sqrt{a+bx^3}}{20x^2}-\frac{27be\sqrt{a+bx^3}}{8x}-\frac{1}{60}\left(\right.$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3}+\frac{27bd\sqrt{a+bx^3}}{20x^2}-\frac{27be\sqrt{a+bx^3}}{8x}+\frac{}{8\left(\left(1\right.\right.}$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3}+\frac{27bd\sqrt{a+bx^3}}{20x^2}-\frac{27be\sqrt{a+bx^3}}{8x}+\frac{}{8\left(\left(1\right.\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.81, size = 805, normalized size = 1.16

$$-\frac{\sqrt{a+bx^3}\left(bx^3(50c+x(78d+x(165e-80fx-48gx^2)))+a(20c+2x(12d+5x(3e+4fx+6gx^2)))\right)}{120x^6}+\frac{3}{80}b\left(-\frac{20bc\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{80}{3}\sqrt{a}f\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{36b^{2/3}d\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}-\frac{72ag\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}-\frac{90\sqrt{2}\sqrt[3]{a}\sqrt[3]{b}e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}}\left(\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)+F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)\right)}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] -1/120*(Sqrt[a + b*x^3]*(b*x^3*(50*c + x*(78*d + x*(165*e - 80*f*x - 48*g*x
^2))) + a*(20*c + 2*x*(12*d + 5*x*(3*e + 4*f*x + 6*g*x^2)))))/x^6 + (3*b*((
-20*b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (80*Sqrt[a]*f*ArcTa
nh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (36*b^(2/3)*d*((-1)^(1/3)*a^(1/3) - b^(1/3
)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3
)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[Arc
Sin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1
)^(1/3)])/(Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))
]*Sqrt[a + b*x^3]) - (72*a*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3)
 + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(
2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3)
+ (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(b^(1/3)
*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a +
 b*x^3]) - (90*Sqrt[2]*a^(1/3)*b^(1/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*S
qrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]
*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*Ell
ipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)
/(-1 + (-1)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1
/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(Sqrt[(a^(1/3) + (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])))/80
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1195 vs. 2(532) = 1064.
time = 0.54, size = 1196, normalized size = 1.73

method	result

elliptic	$-\frac{ac\sqrt{b\,x^3+a}}{6x^6}-\frac{ad\sqrt{b\,x^3+a}}{5x^5}-\frac{ae\sqrt{b\,x^3+a}}{4x^4}-\frac{\left(af+\frac{5bc}{4}\right)\sqrt{b\,x^3+a}}{3x^3}-\frac{\left(ag+\frac{13bd}{10}\right)\sqrt{b\,x^3+a}}{2x^2}-$ 11
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+e*(-1/4*a*(b*x^3+a)^
(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))))+c*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2
*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+g*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/
5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^
```

```
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)))+f*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-b*ar
ctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="maxim
a")
```

```
[Out] 1/24*(3*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sq
rt(a) - 2*(5*(b*x^3 + a)^(3/2)*b^2 - 3*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^
2 - 2*(b*x^3 + a)*a + a^2))*c + integrate((b*g*x^6 + b*f*x^5 + b*x^4*e + a*
f*x^2 + (b*d + a*g)*x^3 + a*x*e + a*d)*sqrt(b*x^3 + a)/x^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 430, normalized size = 0.62

$$\left[-\frac{810\,ab^{\frac{3}{2}}ex^{6}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-15\,(b^{2}c+4\,abf)\sqrt{a}\,x^{6}\log\left(-\frac{b^{2}x^{6}+8\,abx^{3}-4\,(bx^{3}+2a)\sqrt{bx^{3}+a}\,\sqrt{a}+8a^{2}}{x^{6}}\right)-324\,(abd+2\,a^{2}g)\sqrt{b}\,x^{6}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-2\,(48\,abgx^{7}+80\,abfx^{6}-165\,abex^{5}-30\,a^{2}ex^{2}-6\,(13\,abd+10\,a^{2}g)x^{4}-24\,a^{2}dx-10\,(5\,abc+4\,a^{2}f)x^{3}-20\,a^{2}c)\sqrt{bx^{3}+a}}{240\,ax^{6}},\ -\frac{405\,ab^{\frac{3}{2}}ex^{6}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-15\,(b^{2}c+4\,abf)\sqrt{-a}\,x^{6}\arctan\left(\frac{2\sqrt{bx^{3}+a}\,\sqrt{-a}}{bx^{3}+2a}\right)-162\,(abd+2\,a^{2}g)\sqrt{b}\,x^{6}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-(48\,abgx^{7}+80\,abfx^{6}-165\,abex^{5}-30\,a^{2}ex^{2}-6\,(13\,abd+10\,a^{2}g)x^{4}-24\,a^{2}dx-10\,(5\,abc+4\,a^{2}f)x^{3}-20\,a^{2}c)\sqrt{bx^{3}+a}}{120\,ax^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="frica
s")
```

```
[Out] [-1/240*(810*a*b^(3/2)*e*x^6*weierstrassZeta(0, -4*a/b, weierstrassPInverse
(0, -4*a/b, x)) - 15*(b^2*c + 4*a*b*f)*sqrt(a)*x^6*log(-(b^2*x^6 + 8*a*b*x^
3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 324*(a*b*d + 2*
a^2*g)*sqrt(b)*x^6*weierstrassPInverse(0, -4*a/b, x) - 2*(48*a*b*g*x^7 + 80
*a*b*f*x^6 - 165*a*b*e*x^5 - 30*a^2*e*x^2 - 6*(13*a*b*d + 10*a^2*g)*x^4 - 2
4*a^2*d*x - 10*(5*a*b*c + 4*a^2*f)*x^3 - 20*a^2*c)*sqrt(b*x^3 + a))/(a*x^6)
, -1/120*(405*a*b^(3/2)*e*x^6*weierstrassZeta(0, -4*a/b, weierstrassPInvers
e(0, -4*a/b, x)) - 15*(b^2*c + 4*a*b*f)*sqrt(-a)*x^6*arctan(2*sqrt(b*x^3 +
a)*sqrt(-a)/(b*x^3 + 2*a)) - 162*(a*b*d + 2*a^2*g)*sqrt(b)*x^6*weierstrassP
Inverse(0, -4*a/b, x) - (48*a*b*g*x^7 + 80*a*b*f*x^6 - 165*a*b*e*x^5 - 30*a
^2*e*x^2 - 6*(13*a*b*d + 10*a^2*g)*x^4 - 24*a^2*d*x - 10*(5*a*b*c + 4*a^2*f
)*x^3 - 20*a^2*c)*sqrt(b*x^3 + a))/(a*x^6)]
```

Sympy [A]
time = 8.75, size = 524, normalized size = 0.76

$$\frac{a^{\frac{3}{2}} d\Gamma\left(-\frac{5}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i\pi}}{a}} \right)}{3 x^{5} \Gamma\left(-\frac{2}{3}\right)} + \frac{a^{\frac{3}{2}} e\Gamma\left(-\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i\pi}}{a}} \right)}{3 x^{4} \Gamma\left(-\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}} g\Gamma\left(-\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i\pi}}{a}} \right)}{3 x^{2} \Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a} b d\Gamma\left(-\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i\pi}}{a}} \right)}{3 x^{2} \Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a} b e\Gamma\left(-\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i\pi}}{a}} \right)}{3 x \Gamma\left(\frac{2}{3}\right)} - \sqrt{a} b f \operatorname{asinh}{\left(\frac{\sqrt{a}}{\sqrt{b} x^{\frac{3}{2}}} \right)} + \frac{\sqrt{a} b g x \Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i\pi}}{a}} \right)}{3 \Gamma\left(\frac{4}{3}\right)} - \frac{a^{2} c}{6 \sqrt{b} x^{\frac{15}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} - \frac{a \sqrt{b} c}{4 x^{\frac{9}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} - \frac{a \sqrt{b} f \sqrt{\frac{a}{b x^{3}} + 1}}{3 x^{\frac{3}{2}}} + \frac{2 a \sqrt{b} f}{3 x^{\frac{3}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} - \frac{b^{\frac{3}{2}} c \sqrt{\frac{a}{b x^{3}} + 1}}{3 x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}} c}{12 x^{\frac{3}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} + \frac{2 b^{\frac{3}{2}} f x^{\frac{3}{2}}}{3 \sqrt{\frac{a}{b x^{3}} + 1}} - \frac{b^{2} c \operatorname{asinh}{\left(\frac{\sqrt{a}}{\sqrt{b} x^{\frac{3}{2}}} \right)}}{4 \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**7,x)
```

```
[Out] a**(3/2)*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + a**(3/2)*e*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + a**(3/2)*g*gamma(-2/3)*
hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) +
 sqrt(a)*b*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a)*b*f*asinh(sqrt(a)/(
sqrt(b)*x**(3/2))) + sqrt(a)*b*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) - a**2*c/(6*sqrt(b)*x**(15/2)*sqrt(a
/(b*x**3) + 1)) - a*sqrt(b)*c/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)
*f*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*f/(3*x**(3/2)*sqrt(a/(b*
x**3) + 1)) - b**(3/2)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*c/(12
*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*f*x**(3/2)/(3*sqrt(a/(b*x**3)
+ 1)) - b**2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x)
```

```
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7, x)
```

3.469 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$

Optimal. Leaf size=746

$$\frac{27bc\sqrt{a+bx^3}}{280x^4}+\frac{bd\sqrt{a+bx^3}}{4x^3}+\frac{27be\sqrt{a+bx^3}}{20x^2}-\frac{27b(bc+14af)\sqrt{a+bx^3}}{112ax}+\frac{27b^{4/3}(bc+14af)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

```
[Out] -1/420*(60*c/x^7+70*d/x^6+84*e/x^5+105*f/x^4+140*g/x^3)*(b*x^3+a)^(3/2)-1/4
*b*(4*a*g+b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+27/280*b*c*(b*x^3+a
)^(1/2)/x^4+1/4*b*d*(b*x^3+a)^(1/2)/x^3+27/20*b*e*(b*x^3+a)^(1/2)/x^2-27/11
2*b*(14*a*f+b*c)*(b*x^3+a)^(1/2)/a/x-1/140*b*(-140*g*x^5-315*f*x^4+252*e*x^
3+70*d*x^2+36*c*x)*(b*x^3+a)^(1/2)/x^5+27/112*b^(4/3)*(14*a*f+b*c)*(b*x^3+a
)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/224*3^(1/4)*b^(4/3)*(14*a*f+b*
c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)+9/560*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(28*a
^(2/3)*b^(1/3)*e-5*(14*a*f+b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/
2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.84, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {14, 1839, 1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(14af+bc)\right)}{560a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(14af+bc)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{27b^{4/3}\sqrt{a+bx^3}(14af+bc)}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{b\sqrt{a+bx^3}\left(36cx+70dx^2+252ex^3-315fx^4-140gx^5\right)}{140x^5}-\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)-\frac{27b\sqrt{a+bx^3}(14af+bc)}{112ax}+\frac{27bc\sqrt{a+bx^3}}{280x^4}-\frac{b(4ag+bd)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{bd\sqrt{a+bx^3}}{4x^3}+\frac{27be\sqrt{a+bx^3}}{20x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] (27*b*c*Sqrt[a + b*x^3])/(280*x^4) + (b*d*Sqrt[a + b*x^3])/(4*x^3) + (27*b*
e*Sqrt[a + b*x^3])/(20*x^2) - (27*b*(b*c + 14*a*f)*Sqrt[a + b*x^3])/(112*a*
x) + (27*b^(4/3)*(b*c + 14*a*f)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) - (((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 +
```

```
(140*g)/x^3)*(a + b*x^3)^(3/2))/420 - (b*Sqrt[a + b*x^3]*(36*c*x + 70*d*x^2
 + 252*e*x^3 - 315*f*x^4 - 140*g*x^5))/(140*x^5) - (b*(b*d + 4*a*g)*ArcTanh
[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4
/3)*(b*c + 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b
^(4/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(b*c + 14*a*f))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(560*a^(2/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)(a+bx^3)^{3/2} -$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)(a+bx^3)^{3/2} -$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} - \frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} - \frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\right.$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} + \frac{27be\sqrt{a+bx^3}}{20x^2} - \frac{1}{420}\left(\right.$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} + \frac{27be\sqrt{a+bx^3}}{20x^2} - \frac{27b(bc}{}$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} + \frac{27be\sqrt{a+bx^3}}{20x^2} - \frac{27b(bc}{}$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} + \frac{27be\sqrt{a+bx^3}}{20x^2} - \frac{27b(bc}{}$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} + \frac{27be\sqrt{a+bx^3}}{20x^2} - \frac{27b(bc}{}$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} + \frac{27be\sqrt{a+bx^3}}{20x^2} - \frac{27b(bc}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 11.51, size = 897, normalized size = 1.20

$$-\frac{\sqrt{a+bx^3}\left(405b^2cx^6+2abx^3(255c+7x(50d+x(78e+165fx-80gx^2)))+4a^2(60c+7x(10d+x(12e+5x(3f+4gx))))\right)}{1680ax^7}-\frac{b\left(140\sqrt{a}\,bd\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+560a^{3/2}g\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+756ab^{2/3}e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)-135\sqrt{2}\sqrt[3]{a}b^{4/3}c\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}}\left(-\left(\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)\right)-F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)\right)-1890\sqrt{2}a^{4/3}\sqrt[3]{b}f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}}\left(-\left(\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)\right)-F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)\right)\right)}{560a\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] -1/1680*(Sqrt[a + b*x^3]*(405*b^2*c*x^6 + 2*a*b*x^3*(255*c + 7*x*(50*d + x*
(78*e + 165*f*x - 80*g*x^2))) + 4*a^2*(60*c + 7*x*(10*d + x*(12*e + 5*x*(3*
f + 4*g*x))))))/(a*x^7) - (b*(140*Sqrt[a]*b*d*Sqrt[(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3
]/Sqrt[a]] + 560*a^(3/2)*g*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 756*a*
b^(2/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 +
 (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(
(1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 135*Sqrt[2]*a^(1/3)*b^(4
/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/
3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)
))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) -
 (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - Elliptic
F[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1
+ (-1)^(1/3))]) - 1890*Sqrt[2]*a^(4/3)*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1
/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))
*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1
)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)]
, (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^
(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(560*a*Sqrt[(a
^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1374 vs. 2(580) = 1160.
time = 0.45, size = 1375, normalized size = 1.84

method	result	size
elliptic	Expression too large to display	916
risch	Expression too large to display	1295
default	Expression too large to display	1375

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
```

```
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+f*(-1/4*a*(b*x^3+a)^
(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))))+d*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2
*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+c*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17
/56*b*(b*x^3+a)^(1/2)/x^4-27/112*b^2/a*(b*x^3+a)^(1/2)/x-9/112*I*b^2/a*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+g*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*
(b*x^3+a)^(1/2)-b*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^8, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.39, size = 446, normalized size = 0.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="frica
s")
```

```
[Out] [1/1680*(2268*a*b^(3/2)*e*x^7*weierstrassPInverse(0, -4*a/b, x) + 105*(b^2*
d + 4*a*b*g)*sqrt(a)*x^7*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b
*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 405*(b^2*c + 14*a*b*f)*sqrt(b)*x^7*weiers
trassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (1120*a*b*g*x^7 -
 1092*a*b*e*x^5 - 15*(27*b^2*c + 154*a*b*f)*x^6 - 336*a^2*e*x^2 - 140*(5*a*
b*d + 4*a^2*g)*x^4 - 280*a^2*d*x - 30*(17*a*b*c + 14*a^2*f)*x^3 - 240*a^2*c
)*sqrt(b*x^3 + a))/(a*x^7), 1/1680*(2268*a*b^(3/2)*e*x^7*weierstrassPInvers
e(0, -4*a/b, x) + 210*(b^2*d + 4*a*b*g)*sqrt(-a)*x^7*arctan(2*sqrt(b*x^3 +
a)*sqrt(-a)/(b*x^3 + 2*a)) - 405*(b^2*c + 14*a*b*f)*sqrt(b)*x^7*weierstrass
Zeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (1120*a*b*g*x^7 - 1092
*a*b*e*x^5 - 15*(27*b^2*c + 154*a*b*f)*x^6 - 336*a^2*e*x^2 - 140*(5*a*b*d +
 4*a^2*g)*x^4 - 280*a^2*d*x - 30*(17*a*b*c + 14*a^2*f)*x^3 - 240*a^2*c)*sqr
t(b*x^3 + a))/(a*x^7)]
```

Sympy [**A**]
time = 8.98, size = 536, normalized size = 0.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**8,x)
```

```
[Out] a**(3/2)*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**7*gamma(-4/3)) + a**(3/2)*e*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + a**(3/2)*f*gamma(-4/3)*
hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3))
 + sqrt(a)*b*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*
pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*e*gamma(-2/3)*hyper((-2/3, -1/2), (
1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*f*gamma(-1
/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3))
- sqrt(a)*b*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - a**2*d/(6*sqrt(b)*x**(15/
2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*d/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) -
a*sqrt(b)*g*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*g/(3*x**(3/2)*s
qrt(a/(b*x**3) + 1)) - b**(3/2)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3
/2)*d/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*g*x**(3/2)/(3*sqrt(a/
(b*x**3) + 1)) - b**2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x)
```

```
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8, x)
```

3.470 $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx$

Optimal. Leaf size=705

$$-\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}-\frac{27b^2c\sqrt{a+bx^3}}{320ax^2}-\frac{27b^2d\sqrt{a+bx^3}}{112ax}+\frac{27b^{4/3}(bd\ \cdots}{112a\left(\left(1+\cdots\right.\right.}$$

[Out] -1/840*(105*c/x^8+120*d/x^7+140*e/x^6+168*f/x^5+210*g/x^4)*(b*x^3+a)^(3/2)-1/4*b^2*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/560*b*(63*c/x^5+90*d/x^4+140*e/x^3+252*f/x^2+630*g/x)*(b*x^3+a)^(1/2)-27/320*b^2*c*(b*x^3+a)^(1/2)/a/x^2-27/112*b^2*d*(b*x^3+a)^(1/2)/a/x+27/112*b^(4/3)*(14*a*g+b*d)*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/224*3^(1/4)*b^(4/3)*(14*a*g+b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-9/2240*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*(-16*a*f+b*c)+20*a^(1/3)*(14*a*g+b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.66, antiderivative size = 705, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

[illegible]

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]

[Out] -1/560*(b*((63*c)/x^5 + (90*d)/x^4 + (140*e)/x^3 + (252*f)/x^2 + (630*g)/x)*Sqrt[a + b*x^3]) - (27*b^2*c*Sqrt[a + b*x^3])/(320*a*x^2) - (27*b^2*d*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(b*d + 14*a*g)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210*g)/x^4)*(a + b*x^3)^(3/2))/840 - (b^2*e*ArcTanh

```
[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4
/3)*(b*d + 14*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b
^(4/3)*(7*b^(1/3)*(b*c - 16*a*f) + 20*(1 - Sqrt[3])*a^(1/3)*(b*d + 14*a*g))
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
 + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2240*
a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)(a+bx^3)^{3/}$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3} -$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.74, size = 978, normalized size = 1.39

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] -1/6720*(Sqrt[a + b*x^3]*(81*b^2*x^6*(7*c + 20*d*x) + 4*a*b*x^3*(399*c + 2*
x*(255*d + 7*x*(50*e + 78*f*x + 165*g*x^2))) + 8*a^2*(105*c + 2*x*(60*d + 7
*x*(10*e + 3*x*(4*f + 5*g*x))))))/(a*x^8) - (b^(4/3)*(560*Sqrt[a]*b^(2/3)*e
*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a +
 b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 189*b^(4/3)*c*((-1)^(1/3)*a^(1/3
) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[
((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*E
llipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(
1/3))]], (-1)^(1/3)] + 3024*a*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1
/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[S
qrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/
3)] - 540*Sqrt[2]*a^(1/3)*b*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(
1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*
(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[
ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 +
(-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3
^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - 7560*Sqrt[2]*a^(4/3)*g*((-1)^(1/3
)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3
])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a
^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-
1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))
)/(2240*a*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]
*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1662 vs. $2(549) = 1098$.
time = 0.43, size = 1663, normalized size = 2.36

method	result	size
elliptic	Expression too large to display	949
risch	Expression too large to display	1579
default	Expression too large to display	1663

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] f*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
```

```
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+g*(-1/4*a*(b*x^3+a)^
(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))))+e*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2
*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+c*(-1/8*a*(b*x^3+a)^(1/2)/x^8-19
/80*b*(b*x^3+a)^(1/2)/x^5-27/320*b^2/a*(b*x^3+a)^(1/2)/x^2+9/320*I*b^2/a*3^
(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-1/7*a*(b*x
^3+a)^(1/2)/x^7-17/56*b*(b*x^3+a)^(1/2)/x^4-27/112*b^2/a*(b*x^3+a)^(1/2)/x-
9/112*I*b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^9, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 470, normalized size = 0.67

$$\left[\frac{420\,\sqrt{a}\,b^{2}ex^{8}\log\left(-\frac{b^{2}x^{6}+8\,abx^{3}-4\,(bx^{3}+2a)\sqrt{bx^{3}+a}\,\sqrt{a}+8\,a^{2}}{x^{6}}\right)-567\,(b^{2}c-16\,abf)\sqrt{b}\,x^{8}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-1620\,(b^{2}d+14\,abg)\sqrt{b}\,x^{8}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-\left(2800\,abex^{5}+60\,(27\,b^{2}d+154\,abg)x^{7}+21\,(27\,b^{2}c+208\,abf)x^{6}+1120\,a^{2}ex^{2}+120\,(17\,abd+14\,a^{2}g)x^{4}+960\,a^{2}dx+84\,(19\,abc+16\,a^{2}f)x^{3}+840\,a^{2}c\right)\sqrt{bx^{3}+a}}{6720\,ax^{8}},\ \frac{840\,\sqrt{-a}\,b^{2}ex^{8}\arctan\left(\frac{2\,\sqrt{bx^{3}+a}\,\sqrt{-a}}{bx^{3}+2\,a}\right)-567\,(b^{2}c-16\,abf)\sqrt{b}\,x^{8}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-1620\,(b^{2}d+14\,abg)\sqrt{b}\,x^{8}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-\left(2800\,abex^{5}+60\,(27\,b^{2}d+154\,abg)x^{7}+21\,(27\,b^{2}c+208\,abf)x^{6}+1120\,a^{2}ex^{2}+120\,(17\,abd+14\,a^{2}g)x^{4}+960\,a^{2}dx+84\,(19\,abc+16\,a^{2}f)x^{3}+840\,a^{2}c\right)\sqrt{bx^{3}+a}}{6720\,ax^{8}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="frica
s")
```

```
[Out] [1/6720*(420*sqrt(a)*b^2*e*x^8*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*
sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 567*(b^2*c - 16*a*b*f)*sqrt(b)*x^8*
weierstrassPInverse(0, -4*a/b, x) - 1620*(b^2*d + 14*a*b*g)*sqrt(b)*x^8*wei
erstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (2800*a*b*e*x^
5 + 60*(27*b^2*d + 154*a*b*g)*x^7 + 21*(27*b^2*c + 208*a*b*f)*x^6 + 1120*a^
2*e*x^2 + 120*(17*a*b*d + 14*a^2*g)*x^4 + 960*a^2*d*x + 84*(19*a*b*c + 16*a
^2*f)*x^3 + 840*a^2*c)*sqrt(b*x^3 + a))/(a*x^8), 1/6720*(840*sqrt(-a)*b^2*e
*x^8*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 567*(b^2*c - 16*a*b
*f)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b, x) - 1620*(b^2*d + 14*a*b*g)
*sqrt(b)*x^8*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
- (2800*a*b*e*x^5 + 60*(27*b^2*d + 154*a*b*g)*x^7 + 21*(27*b^2*c + 208*a*b*
f)*x^6 + 1120*a^2*e*x^2 + 120*(17*a*b*d + 14*a^2*g)*x^4 + 960*a^2*d*x + 84*
(19*a*b*c + 16*a^2*f)*x^3 + 840*a^2*c)*sqrt(b*x^3 + a))/(a*x^8)]
```

Sympy [A]
time = 8.30, size = 527, normalized size = 0.75

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{8}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{8}\Gamma\left(-\frac{5}{3}\right)}+\frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)}+\frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{5}\Gamma\left(-\frac{2}{3}\right)}+\frac{a^{\frac{3}{2}}g\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt{a}\,bc\Gamma\left(-\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{5}\Gamma\left(-\frac{2}{3}\right)}+\frac{\sqrt{a}\,bd\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt{a}\,bf\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x^{2}\Gamma\left(\frac{1}{3}\right)}+\frac{\sqrt{a}\,bg\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}-\frac{a^{2}e}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{a\sqrt{b}\,e}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^{3}}+1}}{3x^{\frac{3}{2}}}-\frac{b^{\frac{3}{2}}e}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{b^{2}e\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**9,x)
```

```
[Out] a**(3/2)*c*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**8*gamma(-5/3)) + a**(3/2)*d*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + a**(3/2)*f*gamma(-5/3)*
hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3))
 + a**(3/2)*g*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*p
i)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*c*gamma(-5/3)*hyper((-5/3, -1/2), (-
2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*d*gamma(-
4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-
1/3)) + sqrt(a)*b*f*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*g*gamma(-1/3)*hyper((-1/2, -1/3)
, (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - a**2*e/(6*sqrt(b)*x*
*(15/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*e/(4*x**(9/2)*sqrt(a/(b*x**3) + 1
)) - b**(3/2)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*e/(12*x**(3/2)
```

```
*sqrt(a/(b*x**3) + 1)) - b**2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^9, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x)
```

```
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9, x)
```

3.471 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx$

Optimal. Leaf size=714

$$-\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680}-\frac{b^2c\sqrt{a+bx^3}}{24ax^3}-\frac{27b^2d\sqrt{a+bx^3}}{320ax^2}-\frac{27b^2e\sqrt{a+bx^3}}{112ax}+\frac{}{112a}$$

```
[Out] -1/2520*(280*c/x^9+315*d/x^8+360*e/x^7+420*f/x^6+504*g/x^5)*(b*x^3+a)^(3/2)
+1/24*b^2*(-6*a*f+b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/1680*b*(1
40*c/x^6+189*d/x^5+270*e/x^4+420*f/x^3+756*g/x^2)*(b*x^3+a)^(1/2)-1/24*b^2*
c*(b*x^3+a)^(1/2)/a/x^3-27/320*b^2*d*(b*x^3+a)^(1/2)/a/x^2-27/112*b^2*e*(b*
x^3+a)^(1/2)/a/x+27/112*b^(7/3)*e*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))-27/224*3^(1/4)*b^(7/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^
(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)
*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-9/2240*3^(3/4)*b^(5/3)*(a^(1/3
)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2))),I*3^(1/2)+2*I)*(7*b*d-112*a*g+20*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))
*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.73, antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(20\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-112ag+7bd\right)}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{b^2(bc-6af)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}}+\frac{27b^{7/3}e\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{b^2c\sqrt{a+bx^3}}{24ax^3}-\frac{27b^2d\sqrt{a+bx^3}}{320ax^2}-\frac{27b^2e\sqrt{a+bx^3}}{112ax}-\frac{b\sqrt{a+bx^3}\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)}{1680}-\frac{(a+bx^3)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x]
```

```
[Out] -1/1680*(b*((140*c)/x^6 + (189*d)/x^5 + (270*e)/x^4 + (420*f)/x^3 + (756*g)
/x^2)*Sqrt[a + b*x^3]) - (b^2*c*Sqrt[a + b*x^3])/(24*a*x^3) - (27*b^2*d*Sqr
t[a + b*x^3])/(320*a*x^2) - (27*b^2*e*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(7
/3)*e*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((280
*c)/x^9 + (315*d)/x^8 + (360*e)/x^7 + (420*f)/x^6 + (504*g)/x^5)*(a + b*x^3
```

```
)^(3/2))/2520 + (b^2*(b*c - 6*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^
(3/2)) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) -
(9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(5/3)*(7*b*d + 20*(1 - Sqrt[3])*a^(1/3)*b^(2
/3)*e - 112*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(2240*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx = -\frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)(a+bx^3)^{3/2}}{2520} - \frac{1}{2}(9b)\int$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{\left(\frac{280c}{x^9}+\frac{3}{}\right.}{}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+}}{24ax}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+}}{24ax}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+}}{24ax}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+}}{24ax}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+}}{24ax}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+}}{24ax}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+}}{24ax}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.88, size = 844, normalized size = 1.18

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x]
```

```
[Out] -1/20160*(Sqrt[a + b*x^3]*(3*b^2*x^6*(280*c + 81*x*(7*d + 20*e*x)) + 4*a*b*
x^3*(980*c + 3*x*(399*d + 510*e*x + 28*x^2*(25*f + 39*g*x))) + 8*a^2*(280*c
 + 3*x*(105*d + 4*x*(30*e + 7*x*(5*f + 6*g*x))))))/(a*x^9) + (280*b^3*c*Sqr
t[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x
^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 1680*a*b^2*f*Sqrt[(a^(1/3) + (-1)^(2
/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a +
 b*x^3]/Sqrt[a]] + 567*Sqrt[a]*b^(8/3)*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*S
qrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sq
rt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3
)] - 9072*a^(3/2)*b^(5/3)*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3)
+ b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2
/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 1620*Sqr
t[2]*a^(5/6)*b^(7/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^
(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(
1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt
[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))
] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1
)^(1/3)/(-1 + (-1)^(1/3))]))/(6720*a^(3/2)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1272 vs. 2(554) = 1108.
time = 0.40, size = 1273, normalized size = 1.78

method	result	size
elliptic	Expression too large to display	958
risch	Expression too large to display	1160
default	Expression too large to display	1273

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] g*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+c*(-1/9*a*(b*x^3+a)^
(1/2)/x^9-7/36*b*(b*x^3+a)^(1/2)/x^6-1/24*b^2/a*(b*x^3+a)^(1/2)/x^3+1/24*b^
```

```
3/a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+f*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5
/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))
+d*(-1/8*a*(b*x^3+a)^(1/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320*b^2/a*(b*
x^3+a)^(1/2)/x^2+9/320*I*b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)))+e*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17/56*b*(b*x^3+a)^(1/2)/x^
4-27/112*b^2/a*(b*x^3+a)^(1/2)/x-9/112*I*b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))))
```

<u>**Maxima**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="maxi
ma")
```

```
[Out] -1/144*(3*b^3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/
a^(3/2) + 2*(3*(b*x^3 + a)^(5/2)*b^3 + 8*(b*x^3 + a)^(3/2)*a*b^3 - 3*sqrt(b
*x^3 + a)*a^2*b^3)/((b*x^3 + a)^3*a - 3*(b*x^3 + a)^2*a^2 + 3*(b*x^3 + a)*a
^3 - a^4))*c + integrate((b*g*x^6 + b*f*x^5 + b*x^4*e + a*f*x^2 + (b*d + a*
g)*x^3 + a*x*e + a*d)*sqrt(b*x^3 + a)/x^9, x)
```

<u>**Fricas**</u> [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 525, normalized size = 0.74

$$\left[-\frac{4860\,ab^{\frac{3}{2}}ex^{9}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+210\,(b^{3}c-6\,ab^{2}f)\sqrt{a}\,x^{9}\log\left(\frac{b^{2}x^{6}+8\,abx^{3}-4\,(bx^{3}+2\,a)\sqrt{bx^{3}+a}\,\sqrt{a}+8\,a^{2}}{x^{6}}\right)+1701\,(ab^{2}d-16\,a^{2}bg)\sqrt{b}\,x^{9}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+(4860\,ab^{2}ex^{8}+6120\,a^{2}bex^{5}+63\,(27\,ab^{2}d+208\,a^{2}bg)x^{7}+840\,(ab^{2}c+10\,a^{2}bf)x^{6}+2880\,a^{3}ex^{2}+2520\,a^{3}dx+252\,(19\,a^{2}bd+16\,a^{3}g)x^{4}+2240\,a^{3}c+560\,(7\,a^{2}bc+6\,a^{3}f)x^{3})\sqrt{bx^{3}+a}}{20160\,a^{2}x^{9}},\ -\frac{4860\,ab^{\frac{3}{2}}ex^{9}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+420\,(b^{3}c-6\,ab^{2}f)\sqrt{-a}\,x^{9}\arctan\left(\frac{(bx^{3}+2\,a)\sqrt{bx^{3}+a}\,\sqrt{-a}}{2\,(abx^{3}+a^{2})}\right)+1701\,(ab^{2}d-16\,a^{2}bg)\sqrt{b}\,x^{9}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+(4860\,ab^{2}ex^{8}+6120\,a^{2}bex^{5}+63\,(27\,ab^{2}d+208\,a^{2}bg)x^{7}+840\,(ab^{2}c+10\,a^{2}bf)x^{6}+2880\,a^{3}ex^{2}+2520\,a^{3}dx+252\,(19\,a^{2}bd+16\,a^{3}g)x^{4}+2240\,a^{3}c+560\,(7\,a^{2}bc+6\,a^{3}f)x^{3})\sqrt{bx^{3}+a}}{20160\,a^{2}x^{9}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="fric
as")
```

```
[Out] [-1/20160*(4860*a*b^(5/2)*e*x^9*weierstrassZeta(0, -4*a/b, weierstrassPInve
rse(0, -4*a/b, x)) + 210*(b^3*c - 6*a*b^2*f)*sqrt(a)*x^9*log((b^2*x^6 + 8*a
*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 1701*(a*b^
2*d - 16*a^2*b*g)*sqrt(b)*x^9*weierstrassPInverse(0, -4*a/b, x) + (4860*a*b
^2*e*x^8 + 6120*a^2*b*e*x^5 + 63*(27*a*b^2*d + 208*a^2*b*g)*x^7 + 840*(a*b^
2*c + 10*a^2*b*f)*x^6 + 2880*a^3*e*x^2 + 2520*a^3*d*x + 252*(19*a^2*b*d + 1
6*a^3*g)*x^4 + 2240*a^3*c + 560*(7*a^2*b*c + 6*a^3*f)*x^3)*sqrt(b*x^3 + a))
/(a^2*x^9), -1/20160*(4860*a*b^(5/2)*e*x^9*weierstrassZeta(0, -4*a/b, weier
strassPInverse(0, -4*a/b, x)) + 420*(b^3*c - 6*a*b^2*f)*sqrt(-a)*x^9*arctan
(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) + 1701*(a*b^2*
d - 16*a^2*b*g)*sqrt(b)*x^9*weierstrassPInverse(0, -4*a/b, x) + (4860*a*b^2
*e*x^8 + 6120*a^2*b*e*x^5 + 63*(27*a*b^2*d + 208*a^2*b*g)*x^7 + 840*(a*b^2*
c + 10*a^2*b*f)*x^6 + 2880*a^3*e*x^2 + 2520*a^3*d*x + 252*(19*a^2*b*d + 16*
a^3*g)*x^4 + 2240*a^3*c + 560*(7*a^2*b*c + 6*a^3*f)*x^3)*sqrt(b*x^3 + a))/(
a^2*x^9)]
```

Sympy [A]
time = 14.68, size = 573, normalized size = 0.80

$$\frac{a^{\frac{3}{2}} d\Gamma\left(-\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{8}{3}, -\frac{1}{2} \\ -\frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}} e\Gamma\left(-\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}} g\Gamma\left(-\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}\, bd\Gamma\left(-\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}\, be\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}\, bg\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} - \frac{a^2c}{9\sqrt{b}\, x^{\frac{21}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a^2f}{6\sqrt{b}\, x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{11a\sqrt{b}\, c}{36x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}\, f}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{17b^{\frac{3}{2}}c}{72x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}f}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{5}{2}}c}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^2f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\, x^{\frac{3}{2}}}\right)}{4\sqrt{a}} + \frac{b^3c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\, x^{\frac{3}{2}}}\right)}{24a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**10,x)
```

```
[Out] a**(3/2)*d*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**8*gamma(-5/3)) + a**(3/2)*e*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + a**(3/2)*g*gamma(-5/3)*
hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3))
 + sqrt(a)*b*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*
pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*e*gamma(-4/3)*hyper((-4/3, -1/2), (
-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*g*gamma(
-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1
/3)) - a**2*c/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - a**2*f/(6*sqrt(b
)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*c/(36*x**(15/2)*sqrt(a/(b*
x**3) + 1)) - a*sqrt(b)*f/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - 17*b**(3/2)*c
/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*f*sqrt(a/(b*x**3) + 1)/(3*x*
*(3/2)) - b**(3/2)*f/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(5/2)*c/(24*a*
x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*f*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(
4*sqrt(a)) + b**3*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(24*a**(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="giac
")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^10, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x)
```

```
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10, x)
```

3.472 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx$

Optimal. Leaf size=764

$$-\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680}-\frac{27b^2c\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2d\sqrt{a+bx^3}}{24ax^3}-\frac{27b^2e\sqrt{a+bx^3}}{320ax^2}+\frac{27b^2(\ldots}{}$$

```
[Out] -1/2520*(252*c/x^10+280*d/x^9+315*e/x^8+360*f/x^7+420*g/x^6)*(b*x^3+a)^(3/2
)+1/24*b^2*(-6*a*g+b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/1680*b*(
108*c/x^7+140*d/x^6+189*e/x^5+270*f/x^4+420*g/x^3)*(b*x^3+a)^(1/2)-27/1120*
b^2*c*(b*x^3+a)^(1/2)/a/x^4-1/24*b^2*d*(b*x^3+a)^(1/2)/a/x^3-27/320*b^2*e*(
b*x^3+a)^(1/2)/a/x^2+27/448*b^2*(-4*a*f+b*c)*(b*x^3+a)^(1/2)/a^2/x-27/448*b
^(7/3)*(-4*a*f+b*c)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+27/
896*3^(1/4)*b^(7/3)*(-4*a*f+b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-9/2240*3^(3/4)*b^(7/3)*(a^(1/3)
+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(7*a^(2/3)*b^(1/3)*e-5*(-4*a*f+b*c)*(1-3^(1/2)))*
(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.90, antiderivative size = 764, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(bc-4af)\right)}{2240a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(bc-4af)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{b^2(bd-6ag)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}}-\frac{27b^{7/3}\sqrt{a+bx^3}(bc-4af)}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{27b^2\sqrt{a+bx^3}(bc-4af)}{448a^2x}-\frac{27b^2c\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2d\sqrt{a+bx^3}}{24ax^3}-\frac{27b^2e\sqrt{a+bx^3}}{320ax^2}-\frac{b\sqrt{a+bx^3}\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)}{1680}-\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x]
```

```
[Out] -1/1680*(b*((108*c)/x^7 + (140*d)/x^6 + (189*e)/x^5 + (270*f)/x^4 + (420*g)
/x^3)*Sqrt[a + b*x^3]) - (27*b^2*c*Sqrt[a + b*x^3])/(1120*a*x^4) - (b^2*d*S
qrt[a + b*x^3])/(24*a*x^3) - (27*b^2*e*Sqrt[a + b*x^3])/(320*a*x^2) + (27*b
^2*(b*c - 4*a*f)*Sqrt[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(b*c - 4*a*f)*S
```

```
qrt[a + b*x^3])/(448*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((252*c)/x
^10 + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7 + (420*g)/x^6)*(a + b*x^3)^(3
/2))/2520 + (b^2*(b*d - 6*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3/2
)) + (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(b*c - 4*a*f)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3]) - (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(7/3)*(7*a^(2/3)*b^(1/3)*e - 5*(1 -
Sqrt[3])*(b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(2240*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx = -\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)(a+bx^3)^{3/2}}{2520} - \frac{1}{2}(9b)\int$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{\left(\frac{252c}{x^{10}}+\right.}{}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a}}{1120}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a}}{1120}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a}}{1120}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a}}{1120}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a}}{1120}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a}}{1120}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a}}{1120}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a}}{1120}$$

Mathematica [C] Result contains complex when optimal does not.

time = 11.70, size = 930, normalized size = 1.22

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x]
```

```
[Out] -1/20160*(Sqrt[a + b*x^3]*(-1215*b^3*c*x^9 + 8*a^3*(252*c + 5*x*(56*d + 63*
e*x + 72*f*x^2 + 84*g*x^3)) + 3*a*b^2*x^6*(162*c + x*(280*d + 81*x*(7*e + 2
0*f*x))) + 4*a^2*b*x^3*(828*c + x*(980*d + 3*x*(399*e + 510*f*x + 700*g*x^2
)))))/(a^2*x^10) + (b^2*(280*Sqrt[a]*b*d*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)
*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqr
t[a]] - 1680*a^(3/2)*g*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 567*a*b^(2
/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 +
 (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*
x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 405*Sqrt[2]*a^(1/3)*b^(4/3)*
c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b
^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(
3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*
b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[Ar
cSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-
1)^(1/3))]) + 1620*Sqrt[2]*a^(4/3)*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*
x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(
1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2
/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-
1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3
)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(6720*a^2*Sqrt[(a^
(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1469 vs. 2(598) = 1196.
time = 0.44, size = 1470, normalized size = 1.92

method	result	size
elliptic	Expression too large to display	976
risch	Expression too large to display	1343
default	Expression too large to display	1470

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/9*a*(b*x^3+a)^(1/2)/x^9-7/36*b*(b*x^3+a)^(1/2)/x^6-1/24*b^2/a*(b*x^3+
a)^(1/2)/x^3+1/24*b^3/a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+g*(-1/6*a*(
```

```
b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2*arctanh((b*x^3+a)^(1/
2)/a^(1/2))/a^(1/2))+e*(-1/8*a*(b*x^3+a)^(1/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/
x^5-27/320*b^2/a*(b*x^3+a)^(1/2)/x^2+9/320*I*b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+f*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17/56*
b*(b*x^3+a)^(1/2)/x^4-27/112*b^2/a*(b*x^3+a)^(1/2)/x-9/112*I*b^2/a*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+c*(-1/10*a*(b*x^3+a)^(1/2)/x^10-23/140*b
*(b*x^3+a)^(1/2)/x^7-27/1120*b^2/a*(b*x^3+a)^(1/2)/x^4+27/448*b^3/a^2*(b*x^
3+a)^(1/2)/x+9/448*I*b^3/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b
*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="maxi
ma")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^11, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 559, normalized size = 0.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="fric
as")
```

```
[Out] [-1/20160*(1701*a*b^(5/2)*e*x^10*weierstrassPInverse(0, -4*a/b, x) + 210*(b
^3*d - 6*a*b^2*g)*sqrt(a)*x^10*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*s
qrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 1215*(b^3*c - 4*a*b^2*f)*sqrt(b)*x^1
0*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (1701*a*b
^2*e*x^8 - 1215*(b^3*c - 4*a*b^2*f)*x^9 + 4788*a^2*b*e*x^5 + 840*(a*b^2*d +
 10*a^2*b*g)*x^7 + 18*(27*a*b^2*c + 340*a^2*b*f)*x^6 + 2520*a^3*e*x^2 + 224
0*a^3*d*x + 560*(7*a^2*b*d + 6*a^3*g)*x^4 + 2016*a^3*c + 144*(23*a^2*b*c +
20*a^3*f)*x^3)*sqrt(b*x^3 + a))/(a^2*x^10), -1/20160*(1701*a*b^(5/2)*e*x^10
*weierstrassPInverse(0, -4*a/b, x) + 420*(b^3*d - 6*a*b^2*g)*sqrt(-a)*x^10*
arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) - 1215*(
b^3*c - 4*a*b^2*f)*sqrt(b)*x^10*weierstrassZeta(0, -4*a/b, weierstrassPInve
rse(0, -4*a/b, x)) + (1701*a*b^2*e*x^8 - 1215*(b^3*c - 4*a*b^2*f)*x^9 + 478
8*a^2*b*e*x^5 + 840*(a*b^2*d + 10*a^2*b*g)*x^7 + 18*(27*a*b^2*c + 340*a^2*b
*f)*x^6 + 2520*a^3*e*x^2 + 2240*a^3*d*x + 560*(7*a^2*b*d + 6*a^3*g)*x^4 + 2
016*a^3*c + 144*(23*a^2*b*c + 20*a^3*f)*x^3)*sqrt(b*x^3 + a))/(a^2*x^10)]
```

Sympy [A]
time = 15.16, size = 576, normalized size = 0.75

$$\frac{a^{\frac{3}{2}} c\Gamma\left(-\frac{10}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{10}{3}, -\frac{1}{2} \\ -\frac{7}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i\pi}}{a}} \right)}{3 x^{10} \Gamma\left(-\frac{7}{3}\right)} + \frac{a^{\frac{3}{2}} e\Gamma\left(-\frac{8}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{8}{3}, -\frac{1}{2} \\ -\frac{5}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i\pi}}{a}} \right)}{3 x^{8} \Gamma\left(-\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}} f\Gamma\left(-\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i\pi}}{a}} \right)}{3 x^{7} \Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a} bc\Gamma\left(-\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i\pi}}{a}} \right)}{3 x^{7} \Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a} be\Gamma\left(-\frac{5}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i\pi}}{a}} \right)}{3 x^{5} \Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a} bf\Gamma\left(-\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i\pi}}{a}} \right)}{3 x^{4} \Gamma\left(-\frac{1}{3}\right)} - \frac{a^{2} d}{9 \sqrt{b} x^{\frac{21}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} - \frac{a^{2} g}{6 \sqrt{b} x^{\frac{15}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} - \frac{11 a \sqrt{b} d}{36 x^{\frac{15}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} - \frac{a \sqrt{b} g}{4 x^{\frac{9}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} - \frac{17 b^{\frac{3}{2}} d}{72 x^{\frac{9}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} - \frac{b^{\frac{3}{2}} g \sqrt{\frac{a}{b x^{3}} + 1}}{3 x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}} g}{12 x^{\frac{3}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} - \frac{b^{\frac{5}{2}} d}{24 a x^{\frac{3}{2}} \sqrt{\frac{a}{b x^{3}} + 1}} - \frac{b^{2} g \operatorname{asinh}{\left(\frac{\sqrt{a}}{\sqrt{b} x^{\frac{3}{2}}} \right)}}{4 \sqrt{a}} + \frac{b^{3} d \operatorname{asinh}{\left(\frac{\sqrt{a}}{\sqrt{b} x^{\frac{3}{2}}} \right)}}{24 a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**11,x)
```

```
[Out] a**(3/2)*c*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*x**10*gamma(-7/3)) + a**(3/2)*e*gamma(-8/3)*hyper((-8/3, -1/2), (-5
/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + a**(3/2)*f*gamma(-7/
3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/
3)) + sqrt(a)*b*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar
(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*e*gamma(-5/3)*hyper((-5/3, -1/2)
, (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*f*gam
ma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gam
ma(-1/3)) - a**2*d/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - a**2*g/(6*s
qrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*d/(36*x**(15/2)*sqrt(
a/(b*x**3) + 1)) - a*sqrt(b)*g/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - 17*b**(3
/2)*d/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*g*sqrt(a/(b*x**3) + 1)/
```

```
(3*x**(3/2)) - b**(3/2)*g/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(5/2)*d/(
24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2
)))/(4*sqrt(a)) + b**3*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(24*a**(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="giac
")
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^11, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^{11}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x)
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11, x)
```

3.473 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx$

Optimal. Leaf size=796

$$-\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480}-\frac{27b^2c\sqrt{a+bx^3}}{1760ax^5}-\frac{27b^2d\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2e\sqrt{a+bx^3}}{24ax^3}+\frac{27}{}$$

```
[Out] -1/27720*(2520*c/x^11+2772*d/x^10+3080*e/x^9+3465*f/x^8+3960*g/x^7)*(b*x^3+
a)^(3/2)+1/24*b^3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/18480*b*(945
*c/x^8+1188*d/x^7+1540*e/x^6+2079*f/x^5+2970*g/x^4)*(b*x^3+a)^(1/2)-27/1760
*b^2*c*(b*x^3+a)^(1/2)/a/x^5-27/1120*b^2*d*(b*x^3+a)^(1/2)/a/x^4-1/24*b^2*e
*(b*x^3+a)^(1/2)/a/x^3+27/7040*b^2*(-22*a*f+7*b*c)*(b*x^3+a)^(1/2)/a^2/x^2+
27/448*b^2*(-4*a*g+b*d)*(b*x^3+a)^(1/2)/a^2/x-27/448*b^(7/3)*(-4*a*g+b*d)*(
b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+27/896*3^(1/4)*b^(7/3)*(
-4*a*g+b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)+9/49280*3^(3/4)*b^(7/3)*(a^(1/3)+b^(1/3)*x)*Elliptic
F((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(7*b^(1/3)*(-22*a*f+7*b*c)+110*a^(1/3)*(-4*a*g+b*d)*(1-3^(1/2)))*(1/2
*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^2/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 1.62, antiderivative size = 796, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x]
```

```
[Out] -1/18480*(b*((945*c)/x^8 + (1188*d)/x^7 + (1540*e)/x^6 + (2079*f)/x^5 + (29
70*g)/x^4)*Sqrt[a + b*x^3]) - (27*b^2*c*Sqrt[a + b*x^3])/(1760*a*x^5) - (27
*b^2*d*Sqrt[a + b*x^3])/(1120*a*x^4) - (b^2*e*Sqrt[a + b*x^3])/(24*a*x^3) +
```

```
(27*b^2*(7*b*c - 22*a*f)*Sqrt[a + b*x^3])/(7040*a^2*x^2) + (27*b^2*(b*d -
4*a*g)*Sqrt[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(b*d - 4*a*g)*Sqrt[a + b*
x^3])/(448*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((2520*c)/x^11 + (27
72*d)/x^10 + (3080*e)/x^9 + (3465*f)/x^8 + (3960*g)/x^7)*(a + b*x^3)^(3/2))
/27720 + (b^3*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3/2)) + (27*3^(1/4
)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3
/4)*Sqrt[2 + Sqrt[3]]*b^(7/3)*(7*b^(1/3)*(7*b*c - 22*a*f) + 110*(1 - Sqrt[3
])*a^(1/3)*(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(49280*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx = -\frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)(a+bx^3)^{3/2}}{27720} - \frac{1}{2}(\cdots$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{\left(\frac{252\cdots}{x^{1\cdots}}\right.}{\cdots}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^{\cdots}}{\cdots}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^{\cdots}}{\cdots}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^{\cdots}}{\cdots}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^{\cdots}}{\cdots}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^{\cdots}}{\cdots}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^{\cdots}}{\cdots}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^{\cdots}}{\cdots}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^{\cdots}}{\cdots}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^{\cdots}}{\cdots}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^{\cdots}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.80, size = 1017, normalized size = 1.28

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x]
```

```
[Out] -1/443520*(Sqrt[a + b*x^3]*(-243*b^3*x^9*(49*c + 110*d*x) + 16*a^3*(2520*c
+ 11*x*(252*d + 5*x*(56*e + 9*x*(7*f + 8*g*x)))) + 6*a*b^2*x^6*(1134*c + 11
*x*(162*d + x*(280*e + 81*x*(7*f + 20*g*x)))) + 8*a^2*b*x^3*(7875*c + 11*x*
(828*d + x*(980*e + 9*x*(133*f + 170*g*x))))))/(a^2*x^11) + (b^(7/3)*(6160*
Sqrt[a]*b^(2/3)*e*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a
^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 3969*b^(4/3)*c*
((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3
))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^
(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + 12474*a*b^(1/3)*f*((-1)^(1/3)*a^(1
/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqr
t[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]
*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a
^(1/3))]], (-1)^(1/3)] - 8910*Sqrt[2]*a^(1/3)*b*d*((-1)^(1/3)*a^(1/3) - b^(
1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3)
)*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-
1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)
], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b
^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) + 35640*Sqrt[2]
*a^(4/3)*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1
)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^
(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1
/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - Ell
ipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)
/(-1 + (-1)^(1/3))])))/(147840*a^2*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((
1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1772 vs. 2(628) = 1256.
time = 0.45, size = 1773, normalized size = 2.23

method	result	size
elliptic	Expression too large to display	1006
risch	Expression too large to display	1639
default	Expression too large to display	1773

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x,method=_RETURNVERBOSE)

[Out] c*(-1/11*a*(b*x^3+a)^(1/2)/x^11-25/176*b*(b*x^3+a)^(1/2)/x^8-27/1760*b^2/a*
(b*x^3+a)^(1/2)/x^5+189/7040*b^3/a^2*(b*x^3+a)^(1/2)/x^2-63/7040*I*b^3/a^2*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+e*(-1/9*a*(b
*x^3+a)^(1/2)/x^9-7/36*b*(b*x^3+a)^(1/2)/x^6-1/24*b^2/a*(b*x^3+a)^(1/2)/x^3
+1/24*b^3/a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+f*(-1/8*a*(b*x^3+a)^(1/
2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320*b^2/a*(b*x^3+a)^(1/2)/x^2+9/320*I
*b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(
b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+g*(-1
/7*a*(b*x^3+a)^(1/2)/x^7-17/56*b*(b*x^3+a)^(1/2)/x^4-27/112*b^2/a*(b*x^3+a)
^(1/2)/x-9/112*I*b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b
^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+d*(-1/10*
a*(b*x^3+a)^(1/2)/x^10-23/140*b*(b*x^3+a)^(1/2)/x^7-27/1120*b^2/a*(b*x^3+a)
^(1/2)/x^4+27/448*b^3/a^2*(b*x^3+a)^(1/2)/x+9/448*I*b^3/a^2*3^(1/2)*(-a*b^2
)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
```

```
b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="maxi
ma")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^12, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 606, normalized size = 0.76

$$\left[\frac{4620\sqrt{a}b^3ex^{11}\log\left(\frac{b^2x^6+8abx^3+4(bx^3+2a)\sqrt{bx^3+a}\sqrt{a}+8a^2}{x^6}\right)+1701(7b^3c-22ab^2f)\sqrt{b}x^{11}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+26730(b^3d-4ab^2g)\sqrt{b}x^{11}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-\left(18480ab^2ex^8-26730(b^3d-4ab^2g)x^{10}-1701(7b^3c-22ab^2f)x^9+86240a^2bex^5+396(27ab^2d+340a^2bg)x^7+252(27ab^2c+418a^2bf)x^6+49280a^3ex^2+44352a^3dx+3168(23a^2bd+20a^3g)x^4+40320a^3c+2520(25a^2bc+22a^3f)x^3\right)\sqrt{bx^3+a}}{443520a^2x^{11}},\ -\frac{9240\sqrt{-a}b^3ex^{11}\arctan\left(\frac{(bx^3+2a)\sqrt{bx^3+a}\sqrt{-a}}{2(abx^3+a^2)}\right)-1701(7b^3c-22ab^2f)\sqrt{b}x^{11}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-26730(b^3d-4ab^2g)\sqrt{b}x^{11}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(18480ab^2ex^8-26730(b^3d-4ab^2g)x^{10}-1701(7b^3c-22ab^2f)x^9+86240a^2bex^5+396(27ab^2d+340a^2bg)x^7+252(27ab^2c+418a^2bf)x^6+49280a^3ex^2+44352a^3dx+3168(23a^2bd+20a^3g)x^4+40320a^3c+2520(25a^2bc+22a^3f)x^3\right)\sqrt{bx^3+a}}{443520a^2x^{11}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="fric
as")
```

```
[Out] [1/443520*(4620*sqrt(a)*b^3*e*x^11*log((b^2*x^6 + 8*a*b*x^3 + 4*(b*x^3 + 2*
a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 1701*(7*b^3*c - 22*a*b^2*f)*sqrt
(b)*x^11*weierstrassPInverse(0, -4*a/b, x) + 26730*(b^3*d - 4*a*b^2*g)*sqrt
(b)*x^11*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (1
8480*a*b^2*e*x^8 - 26730*(b^3*d - 4*a*b^2*g)*x^10 - 1701*(7*b^3*c - 22*a*b^
2*f)*x^9 + 86240*a^2*b*e*x^5 + 396*(27*a*b^2*d + 340*a^2*b*g)*x^7 + 252*(27
*a*b^2*c + 418*a^2*b*f)*x^6 + 49280*a^3*e*x^2 + 44352*a^3*d*x + 3168*(23*a^
2*b*d + 20*a^3*g)*x^4 + 40320*a^3*c + 2520*(25*a^2*b*c + 22*a^3*f)*x^3)*sqr
t(b*x^3 + a))/(a^2*x^11), -1/443520*(9240*sqrt(-a)*b^3*e*x^11*arctan(1/2*(b
*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) - 1701*(7*b^3*c - 22*
a*b^2*f)*sqrt(b)*x^11*weierstrassPInverse(0, -4*a/b, x) - 26730*(b^3*d - 4*
a*b^2*g)*sqrt(b)*x^11*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*
a/b, x)) + (18480*a*b^2*e*x^8 - 26730*(b^3*d - 4*a*b^2*g)*x^10 - 1701*(7*b^
3*c - 22*a*b^2*f)*x^9 + 86240*a^2*b*e*x^5 + 396*(27*a*b^2*d + 340*a^2*b*g)*
x^7 + 252*(27*a*b^2*c + 418*a^2*b*f)*x^6 + 49280*a^3*e*x^2 + 44352*a^3*d*x
+ 3168*(23*a^2*b*d + 20*a^3*g)*x^4 + 40320*a^3*c + 2520*(25*a^2*b*c + 22*a^
3*f)*x^3)*sqrt(b*x^3 + a))/(a^2*x^11)]
```

Sympy [A]
time = 13.53, size = 541, normalized size = 0.68

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{11}{3}\right){}_2F_1\left(\begin{matrix}-\frac{11}{3},-\frac{1}{2}\\-\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{11}\Gamma\left(-\frac{8}{3}\right)}+\frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{10}{3},-\frac{1}{2}\\-\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)}+\frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{8}\Gamma\left(-\frac{5}{3}\right)}+\frac{a^{\frac{3}{2}}g\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)}+\frac{\sqrt{a}bc\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{8}\Gamma\left(-\frac{5}{3}\right)}+\frac{\sqrt{a}bd\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)}+\frac{\sqrt{a}bf\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{5}\Gamma\left(-\frac{2}{3}\right)}+\frac{\sqrt{a}bg\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)}-\frac{a^2e}{9\sqrt{b}x^{\frac{21}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{11a\sqrt{b}e}{36x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{17b^{\frac{3}{2}}e}{72x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{b^{\frac{5}{2}}e}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b^3e\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{24a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**12,x)
```

```
[Out] a**(3/2)*c*gamma(-11/3)*hyper((-11/3, -1/2), (-8/3,), b*x**3*exp_polar(I*pi
)/a)/(3*x**11*gamma(-8/3)) + a**(3/2)*d*gamma(-10/3)*hyper((-10/3, -1/2), (
-7/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**10*gamma(-7/3)) + a**(3/2)*f*gamma(
-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(
-5/3)) + a**(3/2)*g*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*c*gamma(-8/3)*hyper((-8/3, -1/
2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + sqrt(a)*b*d*g
amma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*g
amma(-4/3)) + sqrt(a)*b*f*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*g*gamma(-4/3)*hyper((-4/
3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) - a**2*e/
(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*e/(36*x**(15/2)*s
qrt(a/(b*x**3) + 1)) - 17*b**(3/2)*e/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b
**(5/2)*e/(24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + b**3*e*asinh(sqrt(a)/(sqrt
(b)*x**(3/2)))/(24*a**(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="giac
")
```

```
[Out] integrate((g*x^4 + f*x^3 + x^2*e + d*x + c)*(b*x^3 + a)^(3/2)/x^12, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^{12}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x)
```

```
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12, x)
```

3.474 $\int (c+dx+ex^2)(a+bx^3)^p \, dx$

Optimal. Leaf size=102

$$\frac{e(a+bx^3)^{1+p}}{3b(1+p)}+\frac{cx(a+bx^3)^{1+p}\ {}_2F_1\left(1,\frac{4}{3}+p;\frac{4}{3};-\frac{bx^3}{a}\right)}{a}+\frac{dx^2(a+bx^3)^{1+p}\ {}_2F_1\left(1,\frac{5}{3}+p;\frac{5}{3};-\frac{bx^3}{a}\right)}{2a}$$

```
[Out] 1/3*e*(b*x^3+a)^(1+p)/b/(1+p)+c*x*(b*x^3+a)^(1+p)*hypergeom([1, 4/3+p],[4/3
],-b*x^3/a)/a+1/2*d*x^2*(b*x^3+a)^(1+p)*hypergeom([1, 5/3+p],[5/3],-b*x^3/a
)/a
```

Rubi [A]
time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1900, 267, 1907, 252, 251, 372, 371}

$$cx(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};-\frac{bx^3}{a}\right)+\frac{1}{2}dx^2(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)+\frac{e(a+bx^3)^{p+1}}{3b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] (e*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x*(a + b*x^3)^p*Hypergeometric2F
1[1/3, -p, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^p + (d*x^2*(a + b*x^3)^p*Hyp
ergeometric2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(2*(1 + (b*x^3)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx+ex^2)\left(a+bx^3\right)^p\,dx &= e\int x^2\left(a+bx^3\right)^p\,dx+\int (c+dx)\left(a+bx^3\right)^p\,dx\\
&= \frac{e\left(a+bx^3\right)^{1+p}}{3b(1+p)}+\int \left(c\left(a+bx^3\right)^p+dx\left(a+bx^3\right)^p\right)\,dx\\
&= \frac{e\left(a+bx^3\right)^{1+p}}{3b(1+p)}+c\int \left(a+bx^3\right)^p\,dx+d\int x\left(a+bx^3\right)^p\,dx\\
&= \frac{e\left(a+bx^3\right)^{1+p}}{3b(1+p)}+\left(c\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\right)\int \left(1+\frac{bx^3}{a}\right)^p\,dx+\Bigg(d(\\
&= \frac{e\left(a+bx^3\right)^{1+p}}{3b(1+p)}+cx\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}{}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};-\frac{bx^3}{a}\right)+\frac{1}{2}c
\end{aligned}
$$

Mathematica [A]

time = 0.60, size = 114, normalized size = 1.12

$$\frac{(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(2e(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p+6bc(1+p)x\,{}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};-\frac{bx^3}{a}\right)+3bd(1+p)x^2\,{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{6b(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] ((a + b*x^3)^p*(2*e*(a + b*x^3)*(1 + (b*x^3)/a)^p + 6*b*c*(1 + p)*x*Hyperge
ometric2F1[1/3, -p, 4/3, -((b*x^3)/a)] + 3*b*d*(1 + p)*x^2*Hypergeometric2F
1[2/3, -p, 5/3, -((b*x^3)/a)]))/(6*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int\left(e\,x^2+dx+c\right)\left(b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

```
[Out] int((e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d*x + c)*(b*x^3 + a)^p, x)
```

Fricas [F]
time = 0.38, size = 22, normalized size = 0.22

$$\text{integral}\left(\left(ex^2+dx+c\right)\left(bx^3+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)
```

Sympy [A]
time = 29.24, size = 112, normalized size = 1.10

$$\frac{a^pcx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},-p\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^pdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},-p\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+e\left(\begin{cases}\frac{a^px^3}{3} & \text{for}\ b=0\\\frac{\begin{cases}\frac{(a+bx^3)^{p+1}}{p+1} & \text{for}\ p\neq-1\\\log\left(a+bx^3\right) & \text{otherwise}\end{cases}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] a**p*c*x*gamma(1/3)*hyper((1/3, -p), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(4/3)) + a**p*d*x**2*gamma(2/3)*hyper((2/3, -p), (5/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(5/3)) + e*Piecewise((a**p*x**3/3, Eq(b, 0)), (Piecewis
e(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True))/(3*b
), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)*(b*x^3 + a)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^p\,\left(e\,x^2+d\,x+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^p*(c + d*x + e*x^2),x)
```

```
[Out] int((a + b*x^3)^p*(c + d*x + e*x^2), x)
```

3.475 $\int x(c+dx+ex^2)\left(a+bx^3\right)^p\,dx$

Optimal. Leaf size=107

$$\frac{d(a+bx^3)^{1+p}}{3b(1+p)}+\frac{cx^2(a+bx^3)^{1+p}\ {}_2F_1\left(1,\frac{5}{3}+p;\frac{5}{3};-\frac{bx^3}{a}\right)}{2a}+\frac{ex^4(a+bx^3)^{1+p}\ {}_2F_1\left(1,\frac{7}{3}+p;\frac{7}{3};-\frac{bx^3}{a}\right)}{4a}$$

```
[Out] 1/3*d*(b*x^3+a)^(1+p)/b/(1+p)+1/2*c*x^2*(b*x^3+a)^(1+p)*hypergeom([1, 5/3+p
],[5/3],-b*x^3/a)/a+1/4*e*x^4*(b*x^3+a)^(1+p)*hypergeom([1, 7/3+p],[7/3],-b
*x^3/a)/a
```

Rubi [A]
time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {1907, 372, 371, 267}

$$\frac{1}{2}cx^2\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)+\frac{d(a+bx^3)^{p+1}}{3b(p+1)}+\frac{1}{4}ex^4\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] (d*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x^2*(a + b*x^3)^p*Hypergeometric
2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(2*(1 + (b*x^3)/a)^p) + (e*x^4*(a + b*x^3)
^p*Hypergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)])/(4*(1 + (b*x^3)/a)^p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}\int x(c+dx+ex^2)(a+bx^3)^p\,dx &= \int \left(cx(a+bx^3)^p+dx^2(a+bx^3)^p+ex^3(a+bx^3)^p\right)dx\\ &= c\int x(a+bx^3)^p\,dx+d\int x^2(a+bx^3)^p\,dx+e\int x^3(a+bx^3)^p\,dx\\ &= \frac{d(a+bx^3)^{1+p}}{3b(1+p)}+\left(c(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\right)\int x\left(1+\frac{bx^3}{a}\right)^p dx+\Big(\\ &= \frac{d(a+bx^3)^{1+p}}{3b(1+p)}+\frac{1}{2}cx^2(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)+\end{aligned}$$

Mathematica [A]
time = 0.59, size = 116, normalized size = 1.08

$$\frac{(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(4d(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p+6bc(1+p)x^2\,{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)+3be(1+p)x^4\,{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right)\right)}{12b(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
[Out] ((a + b*x^3)^p*(4*d*(a + b*x^3)*(1 + (b*x^3)/a)^p + 6*b*c*(1 + p)*x^2*Hyper
geometric2F1[2/3, -p, 5/3, -((b*x^3)/a)] + 3*b*e*(1 + p)*x^4*Hypergeometric
2F1[4/3, -p, 7/3, -((b*x^3)/a)]))/(12*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x\left(e\,x^2+dx+c\right)\left(b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
[Out] int(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")
[Out] integrate((x^2*e + d*x + c)*(b*x^3 + a)^p*x, x)
```

Fricas [F]
time = 0.39, size = 26, normalized size = 0.24

$$\text{integral}\left(\left(ex^3+dx^2+cx\right)\left(bx^3+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")
[Out] integral((e*x^3 + d*x^2 + c*x)*(b*x^3 + a)^p, x)
```

Sympy [A]
time = 44.05, size = 114, normalized size = 1.07

$$\frac{a^p c x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix}\frac{2}{3},-p\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{a^p e x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{4}{3},-p\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+d\left(\begin{cases}\frac{a^px^3}{3} & \text{for}\ b=0\\ \frac{\begin{cases}\frac{(a+bx^3)^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log\left(a+bx^3\right) & \text{otherwise}\end{cases}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
[Out] a**p*c*x**2*gamma(2/3)*hyper((2/3, -p), (5/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(5/3)) + a**p*e*x**4*gamma(4/3)*hyper((4/3, -p), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(7/3)) + d*Piecewise((a**p*x**3/3, Eq(b, 0)), (Piece
wise(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True))/(
3*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
[Out] integrate((x^2*e + d*x + c)*(b*x^3 + a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(b\,x^3+a\right)^p\left(e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^p*(c + d*x + e*x^2),x)
[Out] int(x*(a + b*x^3)^p*(c + d*x + e*x^2), x)
```

3.476 $\int x^2(c+dx+ex^2)(a+bx^3)^p\,dx$

Optimal. Leaf size=107

$$\frac{c(a+bx^3)^{1+p}}{3b(1+p)}+\frac{dx^4(a+bx^3)^{1+p}\,{}_2F_1\left(1,\frac{7}{3}+p;\frac{7}{3};-\frac{bx^3}{a}\right)}{4a}+\frac{ex^5(a+bx^3)^{1+p}\,{}_2F_1\left(1,\frac{8}{3}+p;\frac{8}{3};-\frac{bx^3}{a}\right)}{5a}$$

```
[Out] 1/3*c*(b*x^3+a)^(1+p)/b/(1+p)+1/4*d*x^4*(b*x^3+a)^(1+p)*hypergeom([1, 7/3+p
],[7/3],-b*x^3/a)/a+1/5*e*x^5*(b*x^3+a)^(1+p)*hypergeom([1, 8/3+p],[8/3],-b
*x^3/a)/a
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.174$, Rules used = {1907, 267, 372, 371}

$$\frac{c(a+bx^3)^{p+1}}{3b(p+1)}+\frac{1}{4}dx^4(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right)+\frac{1}{5}ex^5(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{3},-p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] (c*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (d*x^4*(a + b*x^3)^p*Hypergeometric
2F1[4/3, -p, 7/3, -((b*x^3)/a)])/(4*(1 + (b*x^3)/a)^p) + (e*x^5*(a + b*x^3)
^p*Hypergeometric2F1[5/3, -p, 8/3, -((b*x^3)/a)])/(5*(1 + (b*x^3)/a)^p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx &= \int \left(cx^2\left(a+bx^3\right)^p+dx^3\left(a+bx^3\right)^p+ex^4\left(a+bx^3\right)^p\right)\,dx\\ &= c\int x^2\left(a+bx^3\right)^p\,dx+d\int x^3\left(a+bx^3\right)^p\,dx+e\int x^4\left(a+bx^3\right)^p\,dx\\ &= \frac{c(a+bx^3)^{1+p}}{3b(1+p)}+\left(d\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\right)\int x^3\left(1+\frac{bx^3}{a}\right)^p\,dx+\\ &= \frac{c(a+bx^3)^{1+p}}{3b(1+p)}+\frac{1}{4}dx^4\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.60, size = 116, normalized size = 1.08

$$\frac{(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(20c(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p+15bd(1+p)x^4\,{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right)+12be(1+p)x^5\,{}_2F_1\left(\frac{5}{3},-p;\frac{8}{3};-\frac{bx^3}{a}\right)\right)}{60b(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
[Out] ((a + b*x^3)^p*(20*c*(a + b*x^3)*(1 + (b*x^3)/a)^p + 15*b*d*(1 + p)*x^4*Hyp
ergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)] + 12*b*e*(1 + p)*x^5*Hypergeomet
ric2F1[5/3, -p, 8/3, -((b*x^3)/a)]))/(60*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^2\left(e\,x^2+dx+c\right)\left(b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
[Out] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] 1/3*(b*x^3 + a)^(p + 1)*c/(b*(p + 1)) + integrate((x^4*e + d*x^3)*(b*x^3 +
a)^p, x)
```

Fricas [F]
time = 0.41, size = 28, normalized size = 0.26

$$\text{integral}\Big(\big(ex^4+dx^3+cx^2\big)\big(bx^3+a\big)^p,x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^4 + d*x^3 + c*x^2)*(b*x^3 + a)^p, x)
```

Sympy [A]
time = 61.93, size = 114, normalized size = 1.07

$$\frac{a^pdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},-p\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^pex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},-p\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+c\left(\begin{cases}\frac{a^px^3}{3} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^3)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^3\right) & \text{otherwise}\end{cases}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] a**p*d*x**4*gamma(4/3)*hyper((4/3, -p), (7/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(7/3)) + a**p*e*x**5*gamma(5/3)*hyper((5/3, -p), (8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(8/3)) + c*Piecewise((a**p*x**3/3, Eq(b, 0)), (Piece
wise(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True))/(
3*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d*x + c)*(b*x^3 + a)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(b\,x^3+a\right)}^p\,\left(e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^p*(c + d*x + e*x^2),x)
[Out] int(x^2*(a + b*x^3)^p*(c + d*x + e*x^2), x)
```

3.477 $\int (c+dx+ex^2+fx^3)(a+bx^4)\,dx$

Optimal. Leaf size=68

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*a*f*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x
^7+1/8*b*f*x^8
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.043$, Rules used = {1864}

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6
 + (b*e*x^7)/7 + (b*f*x^8)/8
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (c+dx+ex^2+fx^3)(a+bx^4)\,dx &= \int (ac+adx+aex^2+afx^3+bcx^4+bdx^5+bex^6+bfx^7)\,dx\\ &= acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8\end{aligned}$$

Mathematica [A]
time = 0.00, size = 68, normalized size = 1.00

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6
 + (b*e*x^7)/7 + (b*f*x^8)/8
```

Maple [A]
time = 0.12, size = 55, normalized size = 0.81

method	result	size
gosper	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}ae\,x^3+\frac{1}{4}af\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7+\frac{1}{8}bf\,x^8$	55
default	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}ae\,x^3+\frac{1}{4}af\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7+\frac{1}{8}bf\,x^8$	55
norman	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}ae\,x^3+\frac{1}{4}af\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7+\frac{1}{8}bf\,x^8$	55
risch	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}ae\,x^3+\frac{1}{4}af\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7+\frac{1}{8}bf\,x^8$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*a*f*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x
^7+1/8*b*f*x^8
```

Maxima [A]
time = 0.31, size = 56, normalized size = 0.82

$$\frac{1}{8}\,bfx^8+\frac{1}{7}\,bx^7e+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,afx^4+\frac{1}{3}\,ax^3e+\frac{1}{2}\,adx^2+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*b*f*x^8 + 1/7*b*x^7*e + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*a
*x^3*e + 1/2*a*d*x^2 + a*c*x
```

Fricas [A]
time = 0.37, size = 54, normalized size = 0.79

$$\frac{1}{8}\,bfx^8+\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,afx^4+\frac{1}{3}\,aex^3+\frac{1}{2}\,adx^2+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*a
*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Sympy [A]
time = 0.01, size = 63, normalized size = 0.93

$$acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{afx^4}{4}+\frac{bcx^5}{5}+\frac{bdx^6}{6}+\frac{bex^7}{7}+\frac{bfx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + a*f*x**4/4 + b*c*x**5/5 + b*d*x**6/6 + b*
e*x**7/7 + b*f*x**8/8
```

Giac [A]
time = 0.59, size = 56, normalized size = 0.82

$$\frac{1}{8}bfx^8 + \frac{1}{7}bx^7e + \frac{1}{6}bdx^6 + \frac{1}{5}bcx^5 + \frac{1}{4}afx^4 + \frac{1}{3}ax^3e + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*b*f*x^8 + 1/7*b*x^7*e + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*a
*x^3*e + 1/2*a*d*x^2 + a*c*x
```

Mupad [B]
time = 0.04, size = 54, normalized size = 0.79

$$\frac{b\,f\,x^8}{8} + \frac{b\,e\,x^7}{7} + \frac{b\,d\,x^6}{6} + \frac{b\,c\,x^5}{5} + \frac{a\,f\,x^4}{4} + \frac{a\,e\,x^3}{3} + \frac{a\,d\,x^2}{2} + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^5)/5 + (a*e*x^3)/3 + (b*d*x^6)/6 + (a*f*x^4)/4
 + (b*e*x^7)/7 + (b*f*x^8)/8
```

3.478 $\int x^3(c+dx+ex^2+fx^3)(a+bx^4)\,dx$

Optimal. Leaf size=73

$$\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}afx^7+\frac{1}{8}bcx^8+\frac{1}{9}bdx^9+\frac{1}{10}bex^{10}+\frac{1}{11}bfx^{11}$$

```
[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*a*f*x^7+1/8*b*c*x^8+1/9*b*d*x^9+1/1
0*b*e*x^10+1/11*b*f*x^11
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.038$, Rules used = {1834}

$$\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}afx^7+\frac{1}{8}bcx^8+\frac{1}{9}bdx^9+\frac{1}{10}bex^{10}+\frac{1}{11}bfx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d*
x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^3(c+dx+ex^2+fx^3)(a+bx^4)\,dx &= \int (acx^3+adx^4+aex^5+afx^6+bcx^7+bdx^8+bex^9+bfx^{10})\\&=\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}afx^7+\frac{1}{8}bcx^8+\frac{1}{9}bdx^9+\frac{1}{10}bex^{10}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 73, normalized size = 1.00

$$\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}afx^7+\frac{1}{8}bcx^8+\frac{1}{9}bdx^9+\frac{1}{10}bex^{10}+\frac{1}{11}bfx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d*
x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Maple [A]
time = 0.12, size = 58, normalized size = 0.79

method	result	size
gosper	$\frac{1}{4}ac\,x^{4}+\frac{1}{5}ad\,x^{5}+\frac{1}{6}ae\,x^{6}+\frac{1}{7}af\,x^{7}+\frac{1}{8}bc\,x^{8}+\frac{1}{9}bd\,x^{9}+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
default	$\frac{1}{4}ac\,x^{4}+\frac{1}{5}ad\,x^{5}+\frac{1}{6}ae\,x^{6}+\frac{1}{7}af\,x^{7}+\frac{1}{8}bc\,x^{8}+\frac{1}{9}bd\,x^{9}+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
norman	$\frac{1}{4}ac\,x^{4}+\frac{1}{5}ad\,x^{5}+\frac{1}{6}ae\,x^{6}+\frac{1}{7}af\,x^{7}+\frac{1}{8}bc\,x^{8}+\frac{1}{9}bd\,x^{9}+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
risch	$\frac{1}{4}ac\,x^{4}+\frac{1}{5}ad\,x^{5}+\frac{1}{6}ae\,x^{6}+\frac{1}{7}af\,x^{7}+\frac{1}{8}bc\,x^{8}+\frac{1}{9}bd\,x^{9}+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*a*f*x^7+1/8*b*c*x^8+1/9*b*d*x^9+1/1
0*b*e*x^10+1/11*b*f*x^11
```

Maxima [A]
time = 0.29, size = 59, normalized size = 0.81

$$\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bx^{10}e+\frac{1}{9}\,bdx^{9}+\frac{1}{8}\,bcx^{8}+\frac{1}{7}\,afx^{7}+\frac{1}{6}\,ax^{6}e+\frac{1}{5}\,adx^{5}+\frac{1}{4}\,acx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/11*b*f*x^11 + 1/10*b*x^10*e + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 + 1
/6*a*x^6*e + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

Fricas [A]
time = 0.37, size = 57, normalized size = 0.78

$$\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bex^{10}+\frac{1}{9}\,bdx^{9}+\frac{1}{8}\,bcx^{8}+\frac{1}{7}\,afx^{7}+\frac{1}{6}\,aex^{6}+\frac{1}{5}\,adx^{5}+\frac{1}{4}\,acx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 + 1
/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

Sympy [A]
time = 0.01, size = 66, normalized size = 0.90

$$\frac{acx^{4}}{4}+\frac{adx^{5}}{5}+\frac{aex^{6}}{6}+\frac{afx^{7}}{7}+\frac{bcx^{8}}{8}+\frac{bdx^{9}}{9}+\frac{bex^{10}}{10}+\frac{bfx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

```
[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + a*f*x**7/7 + b*c*x**8/8 + b*d*x**9/9
 + b*e*x**10/10 + b*f*x**11/11
```

Giac [A]
time = 0.54, size = 59, normalized size = 0.81

$$\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bx^{10}e+\frac{1}{9}\,bdx^{9}+\frac{1}{8}\,bcx^{8}+\frac{1}{7}\,afx^{7}+\frac{1}{6}\,ax^{6}e+\frac{1}{5}\,adx^{5}+\frac{1}{4}\,acx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/11*b*f*x^11 + 1/10*b*x^10*e + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 + 1
/6*a*x^6*e + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

Mupad [B]
time = 0.03, size = 57, normalized size = 0.78

$$\frac{b\,f\,x^{11}}{11}+\frac{b\,e\,x^{10}}{10}+\frac{b\,d\,x^9}{9}+\frac{b\,c\,x^8}{8}+\frac{a\,f\,x^7}{7}+\frac{a\,e\,x^6}{6}+\frac{a\,d\,x^5}{5}+\frac{a\,c\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (b*c*x^8)/8 + (a*e*x^6)/6 + (b*d*x^9)/9 + (a*f*
x^7)/7 + (b*e*x^10)/10 + (b*f*x^11)/11
```

3.479 $\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx$

Optimal. Leaf size=109

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{f(a+bx^4)^3}{12b}$$

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/3*a^2*e*x^3+2/5*a*b*c*x^5+1/3*a*b*d*x^6+2/7*a*b*e*x
^7+1/9*b^2*c*x^9+1/10*b^2*d*x^10+1/11*b^2*e*x^11+1/12*f*(b*x^4+a)^3/b
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1596, 1671}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f(a+bx^4)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 +
 (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f*(
a + b*x^4)^3)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = \frac{f(a+bx^4)^3}{12b} + \int \left(c+dx+ex^2\right)\left(a+bx^4\right)^2\,dx$$
$$= \frac{f(a+bx^4)^3}{12b} + \int \left(a^2c+a^2dx+a^2ex^2+2abcx^4+2abdx^5+2abe\right.$$
$$= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{9}b^2cx^9$$

Mathematica [A]
time = 0.00, size = 124, normalized size = 1.14

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b^2fx^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5 +
 (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x
^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

Maple [A]
time = 0.32, size = 103, normalized size = 0.94

method	result
gosper	$\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}x^9b^2c+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}x^5abc+\frac{1}{4}a^2f\,x^4+$
default	$\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}x^9b^2c+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}x^5abc+\frac{1}{4}a^2f\,x^4+$
norman	$\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}x^9b^2c+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}x^5abc+\frac{1}{4}a^2f\,x^4+$
risch	$\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}x^9b^2c+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}x^5abc+\frac{1}{4}a^2f\,x^4+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*x^9*b^2*c+1/4*a*b*f*x^8
+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*x^5*a*b*c+1/4*a^2*f*x^4+1/3*a^2*e*x^3+1/2*
a^2*d*x^2+a^2*c*x
```

Maxima [A]
time = 0.29, size = 105, normalized size = 0.96

$$\frac{1}{12}b^2fx^{12}+\frac{1}{11}b^2x^{11}e+\frac{1}{10}b^2dx^{10}+\frac{1}{9}b^2cx^9+\frac{1}{4}abfx^8+\frac{2}{7}abx^7e+\frac{1}{3}abdx^6+\frac{2}{5}abcx^5+\frac{1}{4}a^2fx^4+\frac{1}{3}a^2x^3e+\frac{1}{2}a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

Fricas [A]
time = 0.37, size = 102, normalized size = 0.94

$$\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Sympy [A]
time = 0.01, size = 121, normalized size = 1.11

$$a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{a^2fx^4}{4}+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{abfx^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}+\frac{b^2fx^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5 +
 a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x**1
0/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

Giac [A]
time = 0.48, size = 105, normalized size = 0.96

$$\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2x^{11}e+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abx^7e+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2x^3e+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

Mupad [B]
time = 0.08, size = 102, normalized size = 0.94

$$\frac{f\,a^2\,x^4}{4}+\frac{e\,a^2\,x^3}{3}+\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{f\,a\,b\,x^8}{4}+\frac{2\,e\,a\,b\,x^7}{7}+\frac{d\,a\,b\,x^6}{3}+\frac{2\,c\,a\,b\,x^5}{5}+\frac{f\,b^2\,x^{12}}{12}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*c*x^9)/9 + (a^2*e*x^3)/3 + (b^2*d*x^10)/10 + (a^2*f*x^
4)/4 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12 + a^2*c*x + (2*a*b*c*x^5)/5 + (a*b
*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4
```

3.480 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx$

Optimal. Leaf size=114

$$\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}+\frac{c(a+bx^4)^3}{12b}$$

```
[Out] 1/5*a^2*d*x^5+1/6*a^2*e*x^6+1/7*a^2*f*x^7+2/9*a*b*d*x^9+1/5*a*b*e*x^10+2/11
*a*b*f*x^11+1/13*b^2*d*x^13+1/14*b^2*e*x^14+1/15*b^2*f*x^15+1/12*c*(b*x^4+a
)^3/b
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1596, 1864}

$$\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{c(a+bx^4)^3}{12b}+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (2*a*b*d*x^9)/9 + (a*b*e*x^
10)/5 + (2*a*b*f*x^11)/11 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15
)/15 + (c*(a + b*x^4)^3)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx = \frac{c(a+bx^4)^3}{12b}+\int \left(a+bx^4\right)^2\left(-cx^3+x^3(c+dx+ex^2+fx^3)\right)$$
$$= \frac{c(a+bx^4)^3}{12b}+\int \left(a^2dx^4+a^2ex^5+a^2fx^6+2abdx^8+2abex^9+\right.$$
$$= \frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}$$

Mathematica [A]
time = 0.00, size = 129, normalized size = 1.13

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{1}{4}abcx^8+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{12}b^2cx^{12}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (a*b*c*x^8)
/4 + (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11 + (b^2*c*x^12)/12
 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15
```

Maple [A]
time = 0.36, size = 106, normalized size = 0.93

method	result
gosper	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}x^5a^2d+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}x^9abd+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c\,x^{12}+$
default	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}x^5a^2d+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}x^9abd+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c\,x^{12}+$
norman	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}x^5a^2d+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}x^9abd+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c\,x^{12}+$
risch	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}x^5a^2d+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}x^9abd+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c\,x^{12}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*c*x^4+1/5*x^5*a^2*d+1/6*a^2*e*x^6+1/7*a^2*f*x^7+1/4*a*b*c*x^8+2/9*x
^9*a*b*d+1/5*a*b*e*x^10+2/11*a*b*f*x^11+1/12*b^2*c*x^12+1/13*b^2*d*x^13+1/1
4*b^2*e*x^14+1/15*b^2*f*x^15
```

Maxima [A]
time = 0.27, size = 108, normalized size = 0.95

$$\frac{1}{15}b^2fx^{15}+\frac{1}{14}b^2x^{14}e+\frac{1}{13}b^2dx^{13}+\frac{1}{12}b^2cx^{12}+\frac{2}{11}abfx^{11}+\frac{1}{5}abx^{10}e+\frac{2}{9}abdx^9+\frac{1}{4}abcx^8+\frac{1}{7}a^2fx^7+\frac{1}{6}a^2x^6e+\frac{1}{5}a^2dx^5+\frac{1}{4}a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*f*x^15 + 1/14*b^2*x^14*e + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/1
1*a*b*f*x^11 + 1/5*a*b*x^10*e + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f*x
^7 + 1/6*a^2*x^6*e + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

Fricas [A]
time = 0.37, size = 105, normalized size = 0.92

$$\frac{1}{15}\,b^2fx^{15}+\frac{1}{14}\,b^2ex^{14}+\frac{1}{13}\,b^2dx^{13}+\frac{1}{12}\,b^2cx^{12}+\frac{2}{11}\,abfx^{11}+\frac{1}{5}\,abex^{10}+\frac{2}{9}\,abdx^9+\frac{1}{4}\,abcx^8+\frac{1}{7}\,a^2fx^7+\frac{1}{6}\,a^2ex^6+\frac{1}{5}\,a^2dx^5+\frac{1}{4}\,a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/1
1*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f*x
^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

Sympy [A]
time = 0.01, size = 124, normalized size = 1.09

$$\frac{a^2cx^4}{4}+\frac{a^2dx^5}{5}+\frac{a^2ex^6}{6}+\frac{a^2fx^7}{7}+\frac{abcx^8}{4}+\frac{2abdx^9}{9}+\frac{abex^{10}}{5}+\frac{2abfx^{11}}{11}+\frac{b^2cx^{12}}{12}+\frac{b^2dx^{13}}{13}+\frac{b^2ex^{14}}{14}+\frac{b^2fx^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + a**2*f*x**7/7 + a*b*c*x**8/
4 + 2*a*b*d*x**9/9 + a*b*e*x**10/5 + 2*a*b*f*x**11/11 + b**2*c*x**12/12 + b
**2*d*x**13/13 + b**2*e*x**14/14 + b**2*f*x**15/15
```

Giac [A]
time = 0.53, size = 108, normalized size = 0.95

$$\frac{1}{15}\,b^2fx^{15}+\frac{1}{14}\,b^2x^{14}e+\frac{1}{13}\,b^2dx^{13}+\frac{1}{12}\,b^2cx^{12}+\frac{2}{11}\,abfx^{11}+\frac{1}{5}\,abx^{10}e+\frac{2}{9}\,abdx^9+\frac{1}{4}\,abcx^8+\frac{1}{7}\,a^2fx^7+\frac{1}{6}\,a^2x^6e+\frac{1}{5}\,a^2dx^5+\frac{1}{4}\,a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*b^2*f*x^15 + 1/14*b^2*x^14*e + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/1
1*a*b*f*x^11 + 1/5*a*b*x^10*e + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f*x
^7 + 1/6*a^2*x^6*e + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

Mupad [B]
time = 0.07, size = 105, normalized size = 0.92

$$\frac{f\,a^2\,x^7}{7}+\frac{e\,a^2\,x^6}{6}+\frac{d\,a^2\,x^5}{5}+\frac{c\,a^2\,x^4}{4}+\frac{2\,f\,a\,b\,x^{11}}{11}+\frac{e\,a\,b\,x^{10}}{5}+\frac{2\,d\,a\,b\,x^9}{9}+\frac{c\,a\,b\,x^8}{4}+\frac{f\,b^2\,x^{15}}{15}+\frac{e\,b^2\,x^{14}}{14}+\frac{d\,b^2\,x^{13}}{13}+\frac{c\,b^2\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (b^2*c*x^12)/12 + (a^2*e*x^6)/6 + (b^2*d*x^
13)/13 + (a^2*f*x^7)/7 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15 + (a*b*c*x^8)/4
+ (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11
```

3.481 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx$

Optimal. Leaf size=151

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}$$

```
[Out] a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+3/5*a^2*b*c*x^5+1/2*a^2*b*d*x^6+3/7*a^2
*b*e*x^7+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/11*a*b^2*e*x^11+1/13*b^3*c*x^1
3+1/14*b^3*d*x^14+1/15*b^3*e*x^15+1/16*f*(b*x^4+a)^4/b
```

Rubi [A]
time = 0.07, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1596, 1671}

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)
/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e
*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a + b
*x^4)^4)/(16*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx = \frac{f(a+bx^4)^4}{16b} + \int \left(c+dx+ex^2\right)\left(a+bx^4\right)^3\,dx$$
$$= \frac{f(a+bx^4)^4}{16b} + \int \left(a^3c+a^3dx+a^3ex^2+3a^2bcx^4+3a^2bdx^5+3a^2\right.$$
$$= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab$$

Mathematica [A]
time = 0.00, size = 180, normalized size = 1.19

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{4}a^3fx^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}a^2bfx^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{4}ab^2fx^{12}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{1}{16}b^3fx^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/5
 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9)/
3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*c*x
^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16
```

Maple [A]
time = 0.31, size = 151, normalized size = 1.00

method	result
gosper	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}a^3f\,x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x^9+$
default	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}a^3f\,x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x^9+$
norman	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}a^3f\,x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x^9+$
risch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}a^3f\,x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x^9+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+1/4*a^3*f*x^4+3/5*a^2*b*c*x^5+1/2*a^2*b
*d*x^6+3/7*a^2*b*e*x^7+3/8*f*a^2*b*x^8+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/
11*a*b^2*e*x^11+1/4*a*b^2*f*x^12+1/13*b^3*c*x^13+1/14*b^3*d*x^14+1/15*b^3*e
*x^15+1/16*b^3*f*x^16
```

Maxima [A]
time = 0.29, size = 154, normalized size = 1.02

$$\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,b^3x^{15}e+\frac{1}{14}\,b^3dx^{14}+\frac{1}{13}\,b^3cx^{13}+\frac{1}{4}\,ab^2fx^{12}+\frac{3}{11}\,ab^2x^{11}e+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bfx^8+\frac{3}{7}\,a^2bx^7e+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3fx^4+\frac{1}{3}\,a^3x^3e+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

Fricas [A]
time = 0.36, size = 150, normalized size = 0.99

$$\frac{1}{16}b^3fx^{16}+\frac{1}{15}b^3ex^{15}+\frac{1}{14}b^3dx^{14}+\frac{1}{13}b^3cx^{13}+\frac{1}{4}ab^2fx^{12}+\frac{3}{11}ab^2ex^{11}+\frac{3}{10}ab^2dx^{10}+\frac{1}{3}ab^2cx^9+\frac{3}{8}a^2bfx^8+\frac{3}{7}a^2bex^7+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{4}a^3fx^4+\frac{1}{3}a^3ex^3+\frac{1}{2}a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Sympy [A]
time = 0.02, size = 180, normalized size = 1.19

$$a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{a^3fx^4}{4}+\frac{3a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{3a^2bfx^8}{8}+\frac{ab^2cx^9}{3}+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{ab^2fx^{12}}{4}+\frac{b^3cx^{13}}{13}+\frac{b^3dx^{14}}{14}+\frac{b^3ex^{15}}{15}+\frac{b^3fx^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5/
5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x**9
/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3*c*
x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16
```

Giac [A]
time = 0.47, size = 154, normalized size = 1.02

$$\frac{1}{16}b^3fx^{16}+\frac{1}{15}b^3x^{15}e+\frac{1}{14}b^3dx^{14}+\frac{1}{13}b^3cx^{13}+\frac{1}{4}ab^2fx^{12}+\frac{3}{11}ab^2x^{11}e+\frac{3}{10}ab^2dx^{10}+\frac{1}{3}ab^2cx^9+\frac{3}{8}a^2bfx^8+\frac{3}{7}a^2bx^7e+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{4}a^3fx^4+\frac{1}{3}a^3x^3e+\frac{1}{2}a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B]
time = 0.16, size = 150, normalized size = 0.99

$$\frac{f a^3 x^4}{4}+\frac{e a^3 x^3}{3}+\frac{d a^3 x^2}{2}+c a^3 x+\frac{3 f a^2 b x^8}{8}+\frac{3 e a^2 b x^7}{7}+\frac{d a^2 b x^6}{2}+\frac{3 c a^2 b x^5}{5}+\frac{f a b^2 x^{12}}{4}+\frac{3 e a b^2 x^{11}}{11}+\frac{3 d a b^2 x^{10}}{10}+\frac{c a b^2 x^9}{3}+\frac{f b^3 x^{16}}{16}+\frac{e b^3 x^{15}}{15}+\frac{d b^3 x^{14}}{14}+\frac{c b^3 x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^3*d*x^2)/2 + (b^3*c*x^13)/13 + (a^3*e*x^3)/3 + (b^3*d*x^14)/14 + (a^3*f*
x^4)/4 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16 + a^3*c*x + (3*a^2*b*c*x^5)/5 +
(a*b^2*c*x^9)/3 + (a^2*b*d*x^6)/2 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^7)/7
 + (3*a*b^2*e*x^11)/11 + (3*a^2*b*f*x^8)/8 + (a*b^2*f*x^12)/4
```

3.482 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx$

Optimal. Leaf size=156

$$\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{1}{17}b^3dx^{17}+$$

```
[Out] 1/5*a^3*d*x^5+1/6*a^3*e*x^6+1/7*a^3*f*x^7+1/3*a^2*b*d*x^9+3/10*a^2*b*e*x^10
+3/11*a^2*b*f*x^11+3/13*a*b^2*d*x^13+3/14*a*b^2*e*x^14+1/5*a*b^2*f*x^15+1/1
7*b^3*d*x^17+1/18*b^3*e*x^18+1/19*b^3*f*x^19+1/16*c*(b*x^4+a)^4/b
```

Rubi [A]
time = 0.07, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1596, 1864}

$$\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{c\left(a+bx^4\right)^4}{16b}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (a^2*b*d*x^9)/3 + (3*a^2*b*
e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/1
4 + (a*b^2*f*x^15)/5 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19
+ (c*(a + b*x^4)^4)/(16*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx=\frac{c\left(a+bx^4\right)^4}{16b}+\int\left(a+bx^4\right)^3\left(-cx^3+x^3\left(c+dx+ex^2+fx^3\right)\right)$$

$$=\frac{c\left(a+bx^4\right)^4}{16b}+\int\left(a^3dx^4+a^3ex^5+a^3fx^6+3a^2bdx^8+3a^2bex^9\right.$$

$$=\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bf$$

Mathematica [A]
time = 0.00, size = 185, normalized size = 1.19

$$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{3}{8}a^2bcx^8+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{1}{4}ab^2cx^{12}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{1}{16}b^3cx^{16}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (3*a^2*b*c*
x^8)/8 + (a^2*b*d*x^9)/3 + (3*a^2*b*e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (a*b
^2*c*x^12)/4 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/14 + (a*b^2*f*x^15)/5
 + (b^3*c*x^16)/16 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19
```

Maple [A]
time = 0.38, size = 154, normalized size = 0.99

method	result
gosper	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}c\,a^2b\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}ac\,b$
default	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}c\,a^2b\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}ac\,b$
norman	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}c\,a^2b\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}ac\,b$
risch	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}c\,a^2b\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}ac\,b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^3*c*x^4+1/5*a^3*d*x^5+1/6*a^3*e*x^6+1/7*a^3*f*x^7+3/8*c*a^2*b*x^8+1/3
*a^2*b*d*x^9+3/10*a^2*b*e*x^10+3/11*a^2*b*f*x^11+1/4*a*c*b^2*x^12+3/13*a*b^
2*d*x^13+3/14*a*b^2*e*x^14+1/5*a*b^2*f*x^15+1/16*b^3*c*x^16+1/17*b^3*d*x^17
+1/18*b^3*e*x^18+1/19*b^3*f*x^19
```

Maxima [A]
time = 0.27, size = 157, normalized size = 1.01

$$\frac{1}{19}b^3fx^{19}+\frac{1}{18}b^3x^{18}e+\frac{1}{17}b^3dx^{17}+\frac{1}{16}b^3cx^{16}+\frac{1}{5}ab^2fx^{15}+\frac{3}{14}ab^2x^{14}e+\frac{3}{13}ab^2dx^{13}+\frac{1}{4}ab^2cx^{12}+\frac{3}{11}a^2bfx^{11}+\frac{3}{10}a^2bx^{10}e+\frac{1}{3}a^2bdx^9+\frac{3}{8}a^2bcx^8+\frac{1}{7}a^3fx^7+\frac{1}{6}a^3x^6e+\frac{1}{5}a^3dx^5+\frac{1}{4}a^3cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/19*b^3*f*x^19 + 1/18*b^3*x^18*e + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/5
*a*b^2*f*x^15 + 3/14*a*b^2*x^14*e + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12 +
3/11*a^2*b*f*x^11 + 3/10*a^2*b*x^10*e + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^8 +
 1/7*a^3*f*x^7 + 1/6*a^3*x^6*e + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

Fricas [A]
time = 0.37, size = 153, normalized size = 0.98

$$\frac{1}{19}b^3fx^{19}+\frac{1}{18}b^3ex^{18}+\frac{1}{17}b^3dx^{17}+\frac{1}{16}b^3cx^{16}+\frac{1}{5}ab^2fx^{15}+\frac{3}{14}ab^2ex^{14}+\frac{3}{13}ab^2dx^{13}+\frac{1}{4}ab^2cx^{12}+\frac{3}{11}a^2bfx^{11}+\frac{3}{10}a^2bex^{10}+\frac{1}{3}a^2bdx^9+\frac{3}{8}a^2bcx^8+\frac{1}{7}a^3fx^7+\frac{1}{6}a^3ex^6+\frac{1}{5}a^3dx^5+\frac{1}{4}a^3cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/5
*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12 +
3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^8 +
 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

Sympy [A]
time = 0.02, size = 184, normalized size = 1.18

$$\frac{a^3cx^4}{4}+\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\frac{a^3fx^7}{7}+\frac{3a^2bcx^8}{8}+\frac{a^2bdx^9}{3}+\frac{3a^2bex^{10}}{10}+\frac{3a^2bfx^{11}}{11}+\frac{ab^2cx^{12}}{4}+\frac{3ab^2dx^{13}}{13}+\frac{3ab^2ex^{14}}{14}+\frac{ab^2fx^{15}}{5}+\frac{b^3cx^{16}}{16}+\frac{b^3dx^{17}}{17}+\frac{b^3ex^{18}}{18}+\frac{b^3fx^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + a**3*f*x**7/7 + 3*a**2*b*c*
x**8/8 + a**2*b*d*x**9/3 + 3*a**2*b*e*x**10/10 + 3*a**2*b*f*x**11/11 + a*b*
*2*c*x**12/4 + 3*a*b**2*d*x**13/13 + 3*a*b**2*e*x**14/14 + a*b**2*f*x**15/5
 + b**3*c*x**16/16 + b**3*d*x**17/17 + b**3*e*x**18/18 + b**3*f*x**19/19
```

Giac [A]
time = 0.63, size = 157, normalized size = 1.01

$$\frac{1}{19}b^3fx^{19}+\frac{1}{18}b^3x^{18}e+\frac{1}{17}b^3dx^{17}+\frac{1}{16}b^3cx^{16}+\frac{1}{5}ab^2fx^{15}+\frac{3}{14}ab^2x^{14}e+\frac{3}{13}ab^2dx^{13}+\frac{1}{4}ab^2cx^{12}+\frac{3}{11}a^2bfx^{11}+\frac{3}{10}a^2bx^{10}e+\frac{1}{3}a^2bdx^9+\frac{3}{8}a^2bcx^8+\frac{1}{7}a^3fx^7+\frac{1}{6}a^3x^6e+\frac{1}{5}a^3dx^5+\frac{1}{4}a^3cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/19*b^3*f*x^19 + 1/18*b^3*x^18*e + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/5
*a*b^2*f*x^15 + 3/14*a*b^2*x^14*e + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12 +
3/11*a^2*b*f*x^11 + 3/10*a^2*b*x^10*e + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^8 +
 1/7*a^3*f*x^7 + 1/6*a^3*x^6*e + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

Mupad [B]
time = 0.16, size = 153, normalized size = 0.98

$$\frac{f\,a^3\,x^7}{7}+\frac{e\,a^3\,x^6}{6}+\frac{d\,a^3\,x^5}{5}+\frac{c\,a^3\,x^4}{4}+\frac{3\,f\,a^2\,b\,x^{11}}{11}+\frac{3\,e\,a^2\,b\,x^{10}}{10}+\frac{d\,a^2\,b\,x^9}{3}+\frac{3\,c\,a^2\,b\,x^8}{8}+\frac{f\,a\,b^2\,x^{15}}{5}+\frac{3\,e\,a\,b^2\,x^{14}}{14}+\frac{3\,d\,a\,b^2\,x^{13}}{13}+\frac{c\,a\,b^2\,x^{12}}{4}+\frac{f\,b^3\,x^{19}}{19}+\frac{e\,b^3\,x^{18}}{18}+\frac{d\,b^3\,x^{17}}{17}+\frac{c\,b^3\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (b^3*c*x^16)/16 + (a^3*e*x^6)/6 + (b^3*d*x^
17)/17 + (a^3*f*x^7)/7 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19 + (3*a^2*b*c*x^8
)/8 + (a*b^2*c*x^12)/4 + (a^2*b*d*x^9)/3 + (3*a*b^2*d*x^13)/13 + (3*a^2*b*e
*x^10)/10 + (3*a*b^2*e*x^14)/14 + (3*a^2*b*f*x^11)/11 + (a*b^2*f*x^15)/5
```

3.483 $\int (c+dx+ex^2+fx^3)(a+bx^4)^4 \, dx$

Optimal. Leaf size=193

$$a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3d$$

```
[Out] a^4*c*x+1/2*a^4*d*x^2+1/3*a^4*e*x^3+4/5*a^3*b*c*x^5+2/3*a^3*b*d*x^6+4/7*a^3
*b*e*x^7+2/3*a^2*b^2*c*x^9+3/5*a^2*b^2*d*x^10+6/11*a^2*b^2*e*x^11+4/13*a*b^
3*c*x^13+2/7*a*b^3*d*x^14+4/15*a*b^3*e*x^15+1/17*b^4*c*x^17+1/18*b^4*d*x^18
+1/19*b^4*e*x^19+1/20*f*(b*x^4+a)^5/b
```

Rubi [A]
time = 0.10, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1596, 1671}

$$a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14}+\frac{4}{15}ab^3ex^{15}+\frac{f(a+bx^4)^5}{20b}+\frac{1}{17}b^4cx^{17}+\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (4*a^3*b*c*x^5)/5 + (2*a^3*b*d*x^
6)/3 + (4*a^3*b*e*x^7)/7 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*
a^2*b^2*e*x^11)/11 + (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3*e*
x^15)/15 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (f*(a + b*
x^4)^5)/(20*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx = \frac{f(a+bx^4)^5}{20b} + \int \left(c+dx+ex^2\right)\left(a+bx^4\right)^4\,dx$$

$$= \frac{f(a+bx^4)^5}{20b} + \int \left(a^4c+a^4dx+a^4ex^2+4a^3bcx^4+4a^3bdx^5+4a^3\right.$$

$$= a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{2}{3}a^2$$

Mathematica [A]
time = 0.00, size = 236, normalized size = 1.22

$$a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{1}{4}a^4fx^4+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{1}{2}a^3bfx^8+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{1}{2}a^2b^2fx^{12}+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14}+\frac{4}{15}ab^3ex^{15}+\frac{1}{4}ab^3fx^{16}+\frac{1}{17}b^4cx^{17}+\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19}+\frac{1}{20}b^4fx^{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (a^4*f*x^4)/4 + (4*a^3*b*c*x^5)/5
 + (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (a^3*b*f*x^8)/2 + (2*a^2*b^2*c*x
^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a^2*b^2*f*x^12)/2 +
 (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3*e*x^15)/15 + (a*b^3*f*
x^16)/4 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (b^4*f*x^20
)/20
```

Maple [A]
time = 0.45, size = 199, normalized size = 1.03

method	result
gosper	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}a^2b^2c\,x^9+$
default	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}a^2b^2c\,x^9+$
norman	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}a^2b^2c\,x^9+$
risch	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}a^2b^2c\,x^9+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*c*x+1/2*a^4*d*x^2+1/3*a^4*e*x^3+1/4*a^4*f*x^4+4/5*a^3*b*c*x^5+2/3*a^3*b
*d*x^6+4/7*a^3*b*e*x^7+1/2*a^3*b*f*x^8+2/3*a^2*b^2*c*x^9+3/5*a^2*b^2*d*x^10
+6/11*a^2*b^2*e*x^11+1/2*f*a^2*b^2*x^12+4/13*a*b^3*c*x^13+2/7*a*b^3*d*x^14+
4/15*a*b^3*e*x^15+1/4*f*a*b^3*x^16+1/17*b^4*c*x^17+1/18*b^4*d*x^18+1/19*b^4
*e*x^19+1/20*f*b^4*x^20
```

Maxima [A]
time = 0.28, size = 203, normalized size = 1.05

$$\frac{1}{20} b^4 f x^{20} + \frac{1}{19} b^4 x^{19} e + \frac{1}{18} b^4 d x^{18} + \frac{1}{17} b^4 c x^{17} + \frac{1}{4} a b^3 f x^{16} + \frac{4}{15} a b^3 x^{15} e + \frac{2}{7} a b^3 d x^{14} + \frac{4}{13} a b^3 c x^{13} + \frac{1}{2} a^2 b^2 f x^{12} + \frac{6}{11} a^2 b^2 x^{11} e + \frac{3}{5} a^2 b^2 d x^{10} + \frac{2}{3} a^2 b^2 c x^9 + \frac{1}{2} a^3 b f x^8 + \frac{4}{7} a^3 b x^7 e + \frac{2}{3} a^3 b d x^6 + \frac{4}{5} a^3 b c x^5 + \frac{1}{4} a^4 f x^4 + \frac{1}{3} a^4 x^3 e + \frac{1}{2} a^4 d x^2 + a^4 c x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/20*b^4*f*x^20 + 1/19*b^4*x^19*e + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/4
*a*b^3*f*x^16 + 4/15*a*b^3*x^15*e + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13 +
1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*x^11*e + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2
*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*x^7*e + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*
x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*x^3*e + 1/2*a^4*d*x^2 + a^4*c*x
```

Fricas [A]
time = 0.38, size = 198, normalized size = 1.03

$$\frac{1}{20} b^4 f x^{20} + \frac{1}{19} b^4 e x^{19} + \frac{1}{18} b^4 d x^{18} + \frac{1}{17} b^4 c x^{17} + \frac{1}{4} a b^3 f x^{16} + \frac{4}{15} a b^3 e x^{15} + \frac{2}{7} a b^3 d x^{14} + \frac{4}{13} a b^3 c x^{13} + \frac{1}{2} a^2 b^2 f x^{12} + \frac{6}{11} a^2 b^2 e x^{11} + \frac{3}{5} a^2 b^2 d x^{10} + \frac{2}{3} a^2 b^2 c x^9 + \frac{1}{2} a^3 b f x^8 + \frac{4}{7} a^3 b e x^7 + \frac{2}{3} a^3 b d x^6 + \frac{4}{5} a^3 b c x^5 + \frac{1}{4} a^4 f x^4 + \frac{1}{3} a^4 e x^3 + \frac{1}{2} a^4 d x^2 + a^4 c x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] 1/20*b^4*f*x^20 + 1/19*b^4*e*x^19 + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/4
*a*b^3*f*x^16 + 4/15*a*b^3*e*x^15 + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13 +
1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2
*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*
x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Sympy [A]
time = 0.02, size = 241, normalized size = 1.25

$$a^4 c x + \frac{a^4 d x^2}{2} + \frac{a^4 e x^3}{3} + \frac{a^4 f x^4}{4} + \frac{4 a^3 b c x^5}{5} + \frac{2 a^3 b d x^6}{3} + \frac{4 a^3 b e x^7}{7} + \frac{a^3 b f x^8}{2} + \frac{2 a^2 b^2 c x^9}{3} + \frac{3 a^2 b^2 d x^{10}}{5} + \frac{6 a^2 b^2 e x^{11}}{11} + \frac{a^2 b^2 f x^{12}}{2} + \frac{4 a b^3 c x^{13}}{13} + \frac{2 a b^3 d x^{14}}{7} + \frac{4 a b^3 e x^{15}}{15} + \frac{a b^3 f x^{16}}{4} + \frac{b^4 c x^{17}}{17} + \frac{b^4 d x^{18}}{18} + \frac{b^4 e x^{19}}{19} + \frac{b^4 f x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**4*f*x**4/4 + 4*a**3*b*c*x**5/
5 + 2*a**3*b*d*x**6/3 + 4*a**3*b*e*x**7/7 + a**3*b*f*x**8/2 + 2*a**2*b**2*c
*x**9/3 + 3*a**2*b**2*d*x**10/5 + 6*a**2*b**2*e*x**11/11 + a**2*b**2*f*x**1
2/2 + 4*a*b**3*c*x**13/13 + 2*a*b**3*d*x**14/7 + 4*a*b**3*e*x**15/15 + a*b*
*3*f*x**16/4 + b**4*c*x**17/17 + b**4*d*x**18/18 + b**4*e*x**19/19 + b**4*f
*x**20/20
```

Giac [A]
time = 0.51, size = 203, normalized size = 1.05

$$\frac{1}{20} b^4 f x^{20} + \frac{1}{19} b^4 x^{19} e + \frac{1}{18} b^4 d x^{18} + \frac{1}{17} b^4 c x^{17} + \frac{1}{4} a b^3 f x^{16} + \frac{4}{15} a b^3 x^{15} e + \frac{2}{7} a b^3 d x^{14} + \frac{4}{13} a b^3 c x^{13} + \frac{1}{2} a^2 b^2 f x^{12} + \frac{6}{11} a^2 b^2 x^{11} e + \frac{3}{5} a^2 b^2 d x^{10} + \frac{2}{3} a^2 b^2 c x^9 + \frac{1}{2} a^3 b f x^8 + \frac{4}{7} a^3 b x^7 e + \frac{2}{3} a^3 b d x^6 + \frac{4}{5} a^3 b c x^5 + \frac{1}{4} a^4 f x^4 + \frac{1}{3} a^4 x^3 e + \frac{1}{2} a^4 d x^2 + a^4 c x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")

[Out] 1/20*b^4*f*x^20 + 1/19*b^4*x^19*e + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/4*a*b^3*f*x^16 + 4/15*a*b^3*x^15*e + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13 + 1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*x^11*e + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*x^7*e + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*x^3*e + 1/2*a^4*d*x^2 + a^4*c*x

Mupad [B]
time = 5.08, size = 198, normalized size = 1.03

$$\frac{f\,a^4\,x^4}{4}+\frac{e\,a^4\,x^3}{3}+\frac{d\,a^4\,x^2}{2}+c\,a^4\,x+\frac{f\,a^3\,b\,x^8}{2}+\frac{4\,e\,a^3\,b\,x^7}{7}+\frac{2\,d\,a^3\,b\,x^6}{3}+\frac{4\,c\,a^3\,b\,x^5}{5}+\frac{f\,a^2\,b^2\,x^{12}}{2}+\frac{6\,e\,a^2\,b^2\,x^{11}}{11}+\frac{3\,d\,a^2\,b^2\,x^{10}}{5}+\frac{2\,c\,a^2\,b^2\,x^9}{3}+\frac{f\,a\,b^3\,x^{16}}{4}+\frac{4\,e\,a\,b^3\,x^{15}}{15}+\frac{2\,d\,a\,b^3\,x^{14}}{7}+\frac{4\,c\,a\,b^3\,x^{13}}{13}+\frac{f\,b^4\,x^{20}}{20}+\frac{e\,b^4\,x^{19}}{19}+\frac{d\,b^4\,x^{18}}{18}+\frac{c\,b^4\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^4)^4*(c + d*x + e*x^2 + f*x^3),x)

[Out] (a^4*d*x^2)/2 + (b^4*c*x^17)/17 + (a^4*e*x^3)/3 + (b^4*d*x^18)/18 + (a^4*f*x^4)/4 + (b^4*e*x^19)/19 + (b^4*f*x^20)/20 + a^4*c*x + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a^2*b^2*f*x^12)/2 + (4*a^3*b*c*x^5)/5 + (4*a*b^3*c*x^13)/13 + (2*a^3*b*d*x^6)/3 + (2*a*b^3*d*x^14)/7 + (4*a^3*b*e*x^7)/7 + (4*a*b^3*e*x^15)/15 + (a^3*b*f*x^8)/2 + (a*b^3*f*x^16)/4

3.484 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx$

Optimal. Leaf size=198

$$\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{4}{17}ab^3dx^{17}$$

```
[Out] 1/5*a^4*d*x^5+1/6*a^4*e*x^6+1/7*a^4*f*x^7+4/9*a^3*b*d*x^9+2/5*a^3*b*e*x^10+
4/11*a^3*b*f*x^11+6/13*a^2*b^2*d*x^13+3/7*a^2*b^2*e*x^14+2/5*a^2*b^2*f*x^15
+4/17*a*b^3*d*x^17+2/9*a*b^3*e*x^18+4/19*a*b^3*f*x^19+1/21*b^4*d*x^21+1/22*
b^4*e*x^22+1/23*b^4*f*x^23+1/20*c*(b*x^4+a)^5/b
```

Rubi [A]

time = 0.10, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1596, 1864}

$$\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{4}{17}ab^3dx^{17}+\frac{2}{9}ab^3ex^{18}+\frac{4}{19}ab^3fx^{19}+\frac{c\left(a+bx^4\right)^5}{20b}+\frac{1}{21}b^4dx^{21}+\frac{1}{22}b^4ex^{22}+\frac{1}{23}b^4fx^{23}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (4*a^3*b*d*x^9)/9 + (2*a^3*
b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^
14)/7 + (2*a^2*b^2*f*x^15)/5 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9 + (
4*a*b^3*f*x^19)/19 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 +
(c*(a + b*x^4)^5)/(20*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{c(a+bx^4)^5}{20b} + \int \left(a+bx^4\right)^4\left(-cx^3+x^3(c+dx+ex^2+fx^3)\right)$$
$$= \frac{c(a+bx^4)^5}{20b} + \int \left(a^4dx^4+a^4ex^5+a^4fx^6+4a^3bdx^8+4a^3bex^9\right.$$
$$= \frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx$$

Mathematica [A]
time = 0.00, size = 241, normalized size = 1.22

$$\frac{1}{4}a^4cx^4+\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{1}{2}a^3bcx^8+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{1}{2}a^2b^2cx^{12}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{1}{4}ab^3cx^{16}+\frac{4}{17}ab^3dx^{17}+\frac{2}{9}ab^3ex^{18}+\frac{4}{19}ab^3fx^{19}+\frac{1}{20}b^4cx^{20}+\frac{1}{21}b^4dx^{21}+\frac{1}{22}b^4ex^{22}+\frac{1}{23}b^4fx^{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] (a^4*c*x^4)/4 + (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (a^3*b*c*x^
8)/2 + (4*a^3*b*d*x^9)/9 + (2*a^3*b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (a^2*
b^2*c*x^12)/2 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f
*x^15)/5 + (a*b^3*c*x^16)/4 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9 + (4
*a*b^3*f*x^19)/19 + (b^4*c*x^20)/20 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (
b^4*f*x^23)/23
```

Maple [A]
time = 0.38, size = 202, normalized size = 1.02

method	result
gosper	$\frac{1}{4}c\,a^4x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}c\,a^3b\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}c\,a^2b$
default	$\frac{1}{4}c\,a^4x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}c\,a^3b\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}c\,a^2b$
norman	$\frac{1}{4}c\,a^4x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}c\,a^3b\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}c\,a^2b$
risch	$\frac{1}{4}c\,a^4x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}c\,a^3b\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}c\,a^2b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c*a^4*x^4+1/5*a^4*d*x^5+1/6*a^4*e*x^6+1/7*a^4*f*x^7+1/2*c*a^3*b*x^8+4/9
*a^3*b*d*x^9+2/5*a^3*b*e*x^10+4/11*a^3*b*f*x^11+1/2*c*a^2*b^2*x^12+6/13*a^2
*b^2*d*x^13+3/7*a^2*b^2*e*x^14+2/5*a^2*b^2*f*x^15+1/4*a*b^3*c*x^16+4/17*a*b
^3*d*x^17+2/9*a*b^3*e*x^18+4/19*a*b^3*f*x^19+1/20*c*b^4*x^20+1/21*b^4*d*x^2
1+1/22*b^4*e*x^22+1/23*b^4*f*x^23
```

Maxima [A]
time = 0.32, size = 206, normalized size = 1.04

$$\frac{1}{23}b^4fx^{23}+\frac{1}{22}b^4x^{22}e+\frac{1}{21}b^4dx^{21}+\frac{1}{20}b^4cx^{20}+\frac{4}{19}ab^3fx^{19}+\frac{2}{9}ab^3x^{18}e+\frac{4}{17}ab^3dx^{17}+\frac{1}{4}ab^3cx^{16}+\frac{2}{5}a^2b^2fx^{15}+\frac{3}{7}a^2b^2x^{14}e+\frac{6}{13}a^2b^2dx^{13}+\frac{1}{2}a^2b^2cx^{12}+\frac{4}{11}a^3bfx^{11}+\frac{2}{5}a^3bx^{10}e+\frac{4}{9}a^3bdx^9+\frac{1}{2}a^3bcx^8+\frac{1}{7}a^4fx^7+\frac{1}{6}a^4x^6e+\frac{1}{5}a^4dx^5+\frac{1}{4}a^4cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/23*b^4*f*x^23 + 1/22*b^4*x^22*e + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/1
9*a*b^3*f*x^19 + 2/9*a*b^3*x^18*e + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16 +
2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*x^14*e + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*b^2
*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*x^10*e + 4/9*a^3*b*d*x^9 + 1/2*a^3*
b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*x^6*e + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^4
```

Fricas [A]
time = 0.38, size = 201, normalized size = 1.02

$$\frac{1}{23}b^4fx^{23}+\frac{1}{22}b^4ex^{22}+\frac{1}{21}b^4dx^{21}+\frac{1}{20}b^4cx^{20}+\frac{4}{19}ab^3fx^{19}+\frac{2}{9}ab^3ex^{18}+\frac{4}{17}ab^3dx^{17}+\frac{1}{4}ab^3cx^{16}+\frac{2}{5}a^2b^2fx^{15}+\frac{3}{7}a^2b^2ex^{14}+\frac{6}{13}a^2b^2dx^{13}+\frac{1}{2}a^2b^2cx^{12}+\frac{4}{11}a^3bfx^{11}+\frac{2}{5}a^3bex^{10}+\frac{4}{9}a^3bdx^9+\frac{1}{2}a^3bcx^8+\frac{1}{7}a^4fx^7+\frac{1}{6}a^4ex^6+\frac{1}{5}a^4dx^5+\frac{1}{4}a^4cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] 1/23*b^4*f*x^23 + 1/22*b^4*e*x^22 + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/1
9*a*b^3*f*x^19 + 2/9*a*b^3*e*x^18 + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16 +
2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*e*x^14 + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*b^2
*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*e*x^10 + 4/9*a^3*b*d*x^9 + 1/2*a^3*
b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*e*x^6 + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^4
```

Sympy [A]
time = 0.02, size = 245, normalized size = 1.24

$$\frac{a^4cx^4}{4}+\frac{a^4dx^5}{5}+\frac{a^4ex^6}{6}+\frac{a^4fx^7}{7}+\frac{a^3bcx^8}{2}+\frac{4a^3bdx^9}{9}+\frac{2a^3bex^{10}}{5}+\frac{4a^3bfx^{11}}{11}+\frac{a^2b^2cx^{12}}{2}+\frac{6a^2b^2dx^{13}}{13}+\frac{3a^2b^2ex^{14}}{7}+\frac{2a^2b^2fx^{15}}{5}+\frac{ab^3cx^{16}}{4}+\frac{4ab^3dx^{17}}{17}+\frac{2ab^3ex^{18}}{9}+\frac{4ab^3fx^{19}}{19}+\frac{b^4cx^{20}}{20}+\frac{b^4dx^{21}}{21}+\frac{b^4ex^{22}}{22}+\frac{b^4fx^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

```
[Out] a**4*c*x**4/4 + a**4*d*x**5/5 + a**4*e*x**6/6 + a**4*f*x**7/7 + a**3*b*c*x*
*8/2 + 4*a**3*b*d*x**9/9 + 2*a**3*b*e*x**10/5 + 4*a**3*b*f*x**11/11 + a**2*
b**2*c*x**12/2 + 6*a**2*b**2*d*x**13/13 + 3*a**2*b**2*e*x**14/7 + 2*a**2*b*
*2*f*x**15/5 + a*b**3*c*x**16/4 + 4*a*b**3*d*x**17/17 + 2*a*b**3*e*x**18/9
+ 4*a*b**3*f*x**19/19 + b**4*c*x**20/20 + b**4*d*x**21/21 + b**4*e*x**22/22
 + b**4*f*x**23/23
```

Giac [A]
time = 0.53, size = 206, normalized size = 1.04

$$\frac{1}{23}b^4fx^{23}+\frac{1}{22}b^4x^{22}e+\frac{1}{21}b^4dx^{21}+\frac{1}{20}b^4cx^{20}+\frac{4}{19}ab^3fx^{19}+\frac{2}{9}ab^3x^{18}e+\frac{4}{17}ab^3dx^{17}+\frac{1}{4}ab^3cx^{16}+\frac{2}{5}a^2b^2fx^{15}+\frac{3}{7}a^2b^2x^{14}e+\frac{6}{13}a^2b^2dx^{13}+\frac{1}{2}a^2b^2cx^{12}+\frac{4}{11}a^3bfx^{11}+\frac{2}{5}a^3bx^{10}e+\frac{4}{9}a^3bdx^9+\frac{1}{2}a^3bcx^8+\frac{1}{7}a^4fx^7+\frac{1}{6}a^4x^6e+\frac{1}{5}a^4dx^5+\frac{1}{4}a^4cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/23*b^4*f*x^23 + 1/22*b^4*x^22*e + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/1
9*a*b^3*f*x^19 + 2/9*a*b^3*x^18*e + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16 +
2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*x^14*e + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*b^2
*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*x^10*e + 4/9*a^3*b*d*x^9 + 1/2*a^3*
b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*x^6*e + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^4
```

Mupad [B]
time = 0.36, size = 201, normalized size = 1.02

$$\frac{f\,a^4\,x^7}{7}+\frac{e\,a^4\,x^6}{6}+\frac{d\,a^4\,x^5}{5}+\frac{c\,a^4\,x^4}{4}+\frac{4\,f\,a^3\,b\,x^{11}}{11}+\frac{2\,e\,a^3\,b\,x^{10}}{5}+\frac{4\,d\,a^3\,b\,x^9}{9}+\frac{c\,a^3\,b\,x^8}{2}+\frac{2\,f\,a^2\,b^2\,x^{15}}{5}+\frac{3\,e\,a^2\,b^2\,x^{14}}{7}+\frac{6\,d\,a^2\,b^2\,x^{13}}{13}+\frac{c\,a^2\,b^2\,x^{12}}{2}+\frac{4\,f\,a\,b^3\,x^{19}}{19}+\frac{2\,e\,a\,b^3\,x^{18}}{9}+\frac{4\,d\,a\,b^3\,x^{17}}{17}+\frac{c\,a\,b^3\,x^{16}}{4}+\frac{f\,b^4\,x^{23}}{23}+\frac{e\,b^4\,x^{22}}{22}+\frac{d\,b^4\,x^{21}}{21}+\frac{c\,b^4\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^4*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^4*c*x^4)/4 + (a^4*d*x^5)/5 + (b^4*c*x^20)/20 + (a^4*e*x^6)/6 + (b^4*d*x^
21)/21 + (a^4*f*x^7)/7 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 + (a^2*b^2*c*x^1
2)/2 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f*x^15)/5
+ (a^3*b*c*x^8)/2 + (a*b^3*c*x^16)/4 + (4*a^3*b*d*x^9)/9 + (4*a*b^3*d*x^17)
/17 + (2*a^3*b*e*x^10)/5 + (2*a*b^3*e*x^18)/9 + (4*a^3*b*f*x^11)/11 + (4*a*
b^3*f*x^19)/19
```

3.485 $\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx$

Optimal. Leaf size=133

$$\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

```
[Out] -1/4*f*ln(-b*x^4+a)/b+1/2*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)+1/
2*arctan(b^(1/4)*x/a^(1/4))*(-e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*arct
anh(b^(1/4)*x/a^(1/4))*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

Rubi [A]
time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1890, 1181, 211, 214, 1262, 649, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{b}\,c-\sqrt{a}\,e\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
 + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^
4])/(4*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx &= \int \left(\frac{c+ex^2}{a-bx^4}+\frac{x(d+fx^2)}{a-bx^4}\right)dx \\
&= \int \frac{c+ex^2}{a-bx^4}\,dx+\int \frac{x(d+fx^2)}{a-bx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}\left(-\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{1}{-\sqrt{a}\,\sqrt{b}\,-bx^2}\,dx+\frac{1}{2}\Big(\\
&= \frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{1}{2}d\text{Subs} \\
&= \frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-}}{2\sqrt{}}
\end{aligned}$$

Mathematica [**A**]

time = 0.04, size = 214, normalized size = 1.61

$$\frac{\left(\sqrt[4]{a}\,\sqrt{b}\,c-a^{3/4}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2ab^{3/4}}-\frac{\left(\sqrt[4]{a}\,\sqrt{b}\,c+\sqrt{a}\,\sqrt[4]{b}\,d+a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)}{4ab^{3/4}}-\frac{\left(-\sqrt[4]{a}\,\sqrt{b}\,c+\sqrt{a}\,\sqrt[4]{b}\,d-a^{3/4}e\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{4ab^{3/4}}+\frac{d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{a}\,\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

```
[Out] ((a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a*b^(3/4))
 - ((a^(1/4)*Sqrt[b]*c + Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/
4)*x])/(4*a*b^(3/4)) - ((-(a^(1/4)*Sqrt[b]*c) + Sqrt[a]*b^(1/4)*d - a^(3/4)
*e)*Log[a^(1/4) + b^(1/4)*x])/(4*a*b^(3/4)) + (d*Log[Sqrt[a] + Sqrt[b]*x^2]
)/(4*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

Maple [A]

time = 0.36, size = 154, normalized size = 1.16

method	result
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(\_R^3 f+\_R^2 e+\_R d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{f\ln(-b\,x^4+}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(
1/4)))+1/4*d/(a*b)^(1/2)*ln((a+x^2*(a*b)^(1/2))/(a-x^2*(a*b)^(1/2)))-1/4*e/
b/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4))))
-1/4*f*ln(-b*x^4+a)/b
```

Maxima [A]

time = 0.50, size = 176, normalized size = 1.32

$$\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{2\,\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{4\,\sqrt{a}\,b}-\frac{\left(\sqrt{b}\,d+\sqrt{a}\,f\right)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{4\,\sqrt{a}\,b}-\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{4\,\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/4*(sqrt(b)*d - sqrt(a)*f)*log(sqrt(b)*
```

```
x^2 + sqrt(a))/(sqrt(a)*b) - 1/4*(sqrt(b)*d + sqrt(a)*f)*log(sqrt(b)*x^2 -
sqrt(a))/(sqrt(a)*b) - 1/4*(sqrt(b)*c + sqrt(a)*e)*log((sqrt(b)*x - sqrt(sq
rt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(b))*sqrt(b))
```

Fricas [**C**] Result contains complex when optimal does not.
time = 5.02, size = 241149, normalized size = 1813.15

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/48*(2*(2*(1/4)^(2/3)*(-I*sqrt(3) + 1)*((sqrt(2)*a*b*sqrt(1/(a*b))*sqrt((
2*a*b*c*e*sqrt(1/(a*b)) + b*c^2 + a*e^2)/(a^2*b^2*sqrt(1/(a*b)))) - sqrt(2)
*(3*a*f*sqrt(1/(a*b)) - d))^2/(a*b) + 3*sqrt(2)*(2*sqrt(2)*a^2*b*f*sqrt(1/(
a*b))*sqrt((2*a*b*c*e*sqrt(1/(a*b)) + b*c^2 + a*e^2)/(a^2*b^2*sqrt(1/(a*b))
)) - 2*sqrt(2)*a*b*d*sqrt((2*a*b*c*e*sqrt(1/(a*b)) + b*c^2 + a*e^2)/(a^2*b^
2*sqrt(1/(a*b)))) + 2*sqrt(2)*(b*c*e*sqrt(1/(a*b)) + d*f)*a - sqrt(2)*(3*a^
2*f^2*sqrt(1/(a*b)) + b*c^2 - (b*d^2*sqrt(1/(a*b)) - e^2)*a))/(a^2*b^2*sqrt
(1/(a*b))))/(9*(2*sqrt(2)*a^2*b*f*sqrt(1/(a*b))*sqrt((2*a*b*c*e*sqrt(1/(a*b
)) + b*c^2 + a*e^2)/(a^2*b^2*sqrt(1/(a*b)))) - 2*sqrt(2)*a*b*d*sqrt((2*a*b*
c*e*sqrt(1/(a*b)) + b*c^2 + a*e^2)/(a^2*b^2*sqrt(1/(a*b)))) + 2*sqrt(2)*(b*
c*e*sqrt(1/(a*b)) + d*f)*a - sqrt(2)*(3*a^2*f^2*sqrt(1/(a*b)) + b*c^2 - (b*
d^2*sqrt(1/(a*b)) - e^2)*a))*(sqrt(2)*a*b*sqrt(1/(a*b))*sqrt((2*a*b*c*e*sqr
t(1/(a*b)) + b*c^2 + a*e^2)/(a^2*b^2*sqrt(1/(a*b)))) - sqrt(2)*(3*a*f*sqrt(
1/(a*b)) - d))/(a^2*b^2)  ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(95) = 190.
time = 0.49, size = 280, normalized size = 2.11

$$-\frac{\sqrt{2}\left(b^{2}c-\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c+\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c-\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b^{2}c-\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{f\log\left(\left|bx^{4}-a\right|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*
sqrt(2)*(b^2*c + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*sqrt(
2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/
(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) - 1/4*f*log(abs(b*x^4 - a))/b
```

Mupad [B]
time = 5.66, size = 1970, normalized size = 14.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x)
```

```
[Out] symsum(log(b^2*c^2*e - b^2*c*d^2 - b^2*d^3*x - a*b*e^3 - a*b*c*f^2 - 16*roo
t(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2
*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^
2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2
*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*
c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)^2*a*b^3*c - 4*ro
ot(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^
2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a
^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^
2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2
*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)*b^3*c^2*x - b^2
*c^2*f*x + 16*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2
 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^
2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*
b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d
^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)
^2*a*b^3*d*x - 4*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*
z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2
*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a
^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*
b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z,
 k)*a*b^2*e^2*x + 2*a*b*d*e*f - 8*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3
- 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2
*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*
a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2
*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*
e^4 - b^3*c^4, z, k)*a*b^2*c*f + 8*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3
 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^
2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16
```

```
*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^
2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b
*e^4 - b^3*c^4, z, k)*a*b^2*d*e + a*b*d*f^2*x - a*b*e^2*f*x + 2*b^2*c*d*e*x
 + 8*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z
 + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f
 - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 +
2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)*a*b^2*d*
f*x)*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z
 + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f
 - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 +
2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k), k, 1, 4
)
```

3.486 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$

Optimal. Leaf size=162

$$-\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{b}\,d+\sqrt{a}\,f\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt{a}\,e\tanh^{-1}}{2b^{3}}$$

```
[Out] -d*x/b-1/2*e*x^2/b-1/3*f*x^3/b-1/4*c*ln(-b*x^4+a)/b+1/2*e*arctanh(x^2*b^(1/
2)/a^(1/2))*a^(1/2)/b^(3/2)+1/2*a^(1/4)*arctan(b^(1/4)*x/a^(1/4))*(-f*a^(1/
2)+d*b^(1/2))/b^(7/4)+1/2*a^(1/4)*arctanh(b^(1/4)*x/a^(1/4))*(f*a^(1/2)+d*b
^(1/2))/b^(7/4)
```

Rubi [A]

time = 0.13, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1845, 1266, 788, 649, 214, 266, 1294, 1181, 211}

$$\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{b}\,d-\sqrt{a}\,f\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}\,f+\sqrt{b}\,d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}}-\frac{c\log\left(a-bx^4\right)}{4b}-\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

```
[Out] -((d*x)/b) - (e*x^2)/(2*b) - (f*x^3)/(3*b) + (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*
f)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]
*f)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (Sqrt[a]*e*ArcTanh[(Sqrt[b]
*x^2)/Sqrt[a]])/(2*b^(3/2)) - (c*Log[a - b*x^4])/(4*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx &= \int \left(\frac{x^3(c+ex^2)}{a-bx^4}+\frac{x^4(d+fx^2)}{a-bx^4}\right)dx \\
&= \int \frac{x^3(c+ex^2)}{a-bx^4}\,dx+\int \frac{x^4(d+fx^2)}{a-bx^4}\,dx \\
&= -\frac{fx^3}{3b}+\frac{1}{2}\text{Subst}\left(\int \frac{x(c+ex)}{a-bx^2}\,dx,x,x^2\right)+\frac{\int \frac{x^2(3af+3bdx^2)}{a-bx^4}\,dx}{3b} \\
&= -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{\int \frac{3abd+3abfx^2}{a-bx^4}\,dx}{3b^2}-\frac{\text{Subst}\left(\int \frac{-ae-bcx}{a-bx^2}\,dx,x,x^2\right)}{2b} \\
&= -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{1}{2}c\text{Subst}\left(\int \frac{x}{a-bx^2}\,dx,x,x^2\right)+\frac{(ae)\text{Subst}\left(\int \frac{1}{a-bx^2}\,d\right.}{2b} \\
&= -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{b}\,d+\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 221, normalized size = 1.36

$$
\frac{-12b^{3/4}dx-6b^{3/4}ex^2-4b^{3/4}fx^3+6\left(\sqrt[4]{a}\sqrt{b}\,d-a^{3/4}f\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-3\left(\sqrt[4]{a}\sqrt{b}\,d+\sqrt{a}\sqrt[4]{b}\,e+a^{3/4}f\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+3\left(\sqrt[4]{a}\sqrt{b}\,d-\sqrt{a}\sqrt[4]{b}\,e+a^{3/4}f\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+3\sqrt{a}\sqrt[4]{b}\,e\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)-3b^{3/4}c\log\left(a-bx^4\right)}{12b^{7/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

```
[Out] (-12*b^(3/4)*d*x - 6*b^(3/4)*e*x^2 - 4*b^(3/4)*f*x^3 + 6*(a^(1/4)*Sqrt[b]*d
 - a^(3/4)*f)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(a^(1/4)*Sqrt[b]*d + Sqrt[a]*
b^(1/4)*e + a^(3/4)*f)*Log[a^(1/4) - b^(1/4)*x] + 3*(a^(1/4)*Sqrt[b]*d - Sq
rt[a]*b^(1/4)*e + a^(3/4)*f)*Log[a^(1/4) + b^(1/4)*x] + 3*Sqrt[a]*b^(1/4)*e
*Log[Sqrt[a] + Sqrt[b]*x^2] - 3*b^(3/4)*c*Log[a - b*x^4])/(12*b^(7/4))
```

Maple [A]
time = 0.35, size = 176, normalized size = 1.09

method	result
risch	$-\frac{f\,x^3}{3b}-\frac{e\,x^2}{2b}-\frac{dx}{b}+\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^4-a\right)}\frac{\left(-\_R^3bc-\_R^2af-\_Rae-ad\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$

default	$-\frac{\frac{1}{3} f\,x^{3}+\frac{1}{2} e\,x^{2}+dx}{b}+\frac{\frac{d\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4}+\frac{ae\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{af\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(1/3*f*x^3+1/2*e*x^2+d*x)+1/b*(1/4*d*(a/b)^(1/4)*(ln((x+(a/b)^(1/4))/(
x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(1/4)))+1/4*a*e/(a*b)^(1/2)*ln((a+x^2*(a*b
)^(1/2))/(a-x^2*(a*b)^(1/2)))-1/4*a*f/b/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4)
)-ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4))))-1/4*c*ln(-b*x^4+a))
```

Maxima [A]
time = 0.49, size = 211, normalized size = 1.30

$$-\frac{2\,fx^{3}+3\,x^{2}e+6\,dx}{6\,b}+\frac{\frac{2\left(a\sqrt{b}\,d-a^{\frac{3}{2}}f\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(\sqrt{a}\,bc-a\sqrt{b}\,e\right)\log\left(\sqrt{b}\,x^{2}+\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(\sqrt{a}\,bc+a\sqrt{b}\,e\right)\log\left(\sqrt{b}\,x^{2}-\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(a\sqrt{b}\,d+a^{\frac{3}{2}}f\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/6*(2*f*x^3 + 3*x^2*e + 6*d*x)/b + 1/4*(2*(a*sqrt(b)*d - a^(3/2)*f)*arcta
n(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
- (sqrt(a)*b*c - a*sqrt(b)*e)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (sqr
t(a)*b*c + a*sqrt(b)*e)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (a*sqrt(b)
*d + a^(3/2)*f)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(s
qrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

Fricas [C] Result contains complex when optimal does not.
time = 3.95, size = 220680, normalized size = 1362.22

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/48*(16*f*x^3 + 24*e*x^2 + 2*(2*(1/4)^(2/3)*(-I*sqrt(3) + 1)*((sqrt(2)*b^
2*sqrt(a/b)*sqrt((2*a*b*d*f*sqrt(a/b) + a*b*d^2 + a^2*f^2)/(b^4*sqrt(a/b)))
 - sqrt(2)*(3*b*c*sqrt(a/b) - a*e))^2/(a*b^3) + 3*sqrt(2)*(2*sqrt(2)*b^3*c*
sqrt(a/b)*sqrt((2*a*b*d*f*sqrt(a/b) + a*b*d^2 + a^2*f^2)/(b^4*sqrt(a/b))) -
 2*sqrt(2)*a*b^2*e*sqrt((2*a*b*d*f*sqrt(a/b) + a*b*d^2 + a^2*f^2)/(b^4*sqrt
(a/b))) + 2*sqrt(2)*(b*d*f*sqrt(a/b) + b*c*e)*a - sqrt(2)*(3*b^2*c^2*sqrt(a
/b) + a^2*f^2 - (b*e^2*sqrt(a/b) - b*d^2)*a))/(b^4*sqrt(a/b)))/(9*(2*sqrt(2
```

```
)*b^3*c*sqrt(a/b)*sqrt((2*a*b*d*f*sqrt(a/b) + a*b*d^2 + a^2*f^2)/(b^4*sqrt(
a/b))) - 2*sqrt(2)*a*b^2*e*sqrt((2*a*b*d*f*sqrt(a/b) + a*b*d^2 + a^2*f^2)/(
b^4*sqrt(a/b))) + 2*sqrt(2)*(b*d*f*sqrt(a/b) + b*c*e)*a - sqrt(2)*(3*b^2*c^
2*sqrt(a/b) + a^2*f^2 - (b*e^2*sqrt(a/b) - b*d^2)*a))*(sqrt(2)*b^2*sqrt(a/b
)*sqrt((2*a*b*d*f*sqrt(a/b) + a*b*d^2 + a^2*f^2)/(b^4*sqrt(a/b))) - sqrt(2)
*(3*b*c*sqrt(a/b) - a*e))/(a*b^5) + sqrt(2)*(sqrt(2)*b^2*sqrt(a/b)*sqrt((2*
a*b*d*f*sqrt(a/b) + a*b*d ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(120) = 240.
time = 0.49, size = 328, normalized size = 2.02

$$-\frac{c\log\left(|bx^4-a|\right)}{4\,b}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}\,b^2e-\left(-ab^3\right)^{\frac{1}{4}}b^2d-\left(-ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}\,b^2e-\left(-ab^3\right)^{\frac{1}{4}}b^2d-\left(-ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2d-\left(-ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,b^4}-\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2d-\left(-ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,b^4}-\frac{2\,b^2fx^3+3\,b^2x^2e+6\,b^2dx}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*c*log(abs(b*x^4 - a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-a*
b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/
b)^(1/4))/(-a/b)^(1/4))/b^4 - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-a*b
^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/b^4 + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(
3/4)*f)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4 - 1/8*sqrt(2)*((
-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) +
sqrt(-a/b))/b^4 - 1/6*(2*b^2*f*x^3 + 3*b^2*x^2*e + 6*b^2*d*x)/b^3
```

Mupad [B]
time = 4.85, size = 846, normalized size = 5.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x)
```

```
[Out] symsum(log(- (a^4*f^3 + a^2*b^2*c^2*d + a^3*b*d*e^2 - a^3*b*d^2*f - 2*a^3*b
*c*e*f)/b^2 - root(256*b^7*z^4 + 256*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*b^
```

4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c*e^2*z + 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 - 4*a*b^2*c^2*d*f + 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*c^2*e^2 + a^2*b*e^4 + b^3*c^4 - a*b^2*d^4 - a^3*f^4, z, k)*(root(256*b^7*z^4 + 256*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*b^4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c*e^2*z + 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 - 4*a*b^2*c^2*d*f + 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*c^2*e^2 + a^2*b*e^4 + b^3*c^4 - a*b^2*d^4 - a^3*f^4, z, k)*(16*a^2*b^2*d - 16*a^2*b^2*e*x) + (8*a^2*b^3*c*d - 8*a^3*b^2*e*f)/b^2 + (x*(4*a^3*b*f^2 + 4*a^2*b^2*d^2 - 8*a^2*b^2*c*e))/b) - (x*(a^3*e^3 + a^3*c*f^2 - 2*a^3*d*e*f + a^2*b*c*d^2 - a^2*b*c^2*e))/b)*root(256*b^7*z^4 + 256*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*b^4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c*e^2*z + 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 - 4*a*b^2*c^2*d*f + 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*c^2*e^2 + a^2*b*e^4 + b^3*c^4 - a*b^2*d^4 - a^3*f^4, z, k), k, 1, 4) - (e*x^2)/(2*b) - (f*x^3)/(3*b) - (d*x)/b

3.487 $\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx$

Optimal. Leaf size=293

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}\right.}{}$$

```
[Out] 1/4*f*ln(b*x^4+a)/b+1/2*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)-1/8*l
n(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+c*b^(1/2))/a^
(3/4)/b^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))
*(-e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(-1+b^(1/4)*x*2^(
1/2)/a^(1/4))*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(1+b^
(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e+\sqrt{b}\,c\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e+\sqrt{b}\,c\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4))
 + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sq
rt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(3/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx &= \int \left(\frac{c+ex^2}{a+bx^4}+\frac{x(d+fx^2)}{a+bx^4}\right)dx\\
&= \int \frac{c+ex^2}{a+bx^4}\,dx+\int \frac{x(d+fx^2)}{a+bx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{a+bx^2}\,dx,x,x^2\right)+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\cdots}{\cdots}\\
&= \frac{1}{2}d\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b}+\frac{\cdots}{\cdots}\\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\cdots}{\cdots}\\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\cdots}{2\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 296, normalized size = 1.01

$$\frac{-2\sqrt[4]{a}\,\sqrt[4]{b}\left(\sqrt{2}\,\sqrt{b}\,c+2\sqrt[4]{a}\,\sqrt[4]{b}\,d+\sqrt{2}\,\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\,\sqrt[4]{b}\left(\sqrt{2}\,\sqrt{b}\,c-2\sqrt[4]{a}\,\sqrt[4]{b}\,d+\sqrt{2}\,\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\sqrt{2}\,\sqrt[4]{b}\left(\sqrt[4]{a}\,\sqrt{b}\,c-a^{3/4}e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+\sqrt{2}\,\sqrt[4]{b}\left(\sqrt[4]{a}\,\sqrt{b}\,c-a^{3/4}e\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+2af\log\left(a+bx^4\right)}{8ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

```
[Out] (-2*a^(1/4)*b^(1/4)*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt
[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(Sqrt[2]
*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*x)/a^(1/4)] - Sqrt[2]*b^(1/4)*(a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(a^(1/4)
*Sqrt[b]*c - a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2] + 2*a*f*Log[a + b*x^4])/(8*a*b)
```

Maple [A]
time = 0.34, size = 240, normalized size = 0.82

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(\_R^3 f+\_R^2 e+\_R d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{e\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^
2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1
/8*e/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2
+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*ar
ctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/4*f*ln(b*x^4+a)/b
```

Maxima [A]
time = 0.50, size = 281, normalized size = 0.96

$$\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}f+bc-\sqrt{a}\,\sqrt{b}\,e\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}f-bc+\sqrt{a}\,\sqrt{b}\,e\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{5}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{3}{4}}e-2\,\sqrt{a}\,bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{5}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{5}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{3}{4}}e+2\,\sqrt{a}\,bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*b^(1/4)*f + b*c - sqrt(a)*sqrt(b)*e)*log(sqrt(
b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/8*sqrt(
2)*(sqrt(2)*a^(3/4)*b^(1/4)*f - b*c + sqrt(a)*sqrt(b)*e)*log(sqrt(b)*x^2 -
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/4*(sqrt(2)*a^(1/
4)*b^(5/4)*c + sqrt(2)*a^(3/4)*b^(3/4)*e - 2*sqrt(a)*b*d)*arctan(1/2*sqrt(2
)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*s
qrt(sqrt(a)*sqrt(b))*b^(5/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(5/4)*c + sqrt(2)*a^
```

```
(3/4)*b^(3/4)*e + 2*sqrt(a)*b*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*
a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5
/4))
```

Fricas [C] Result contains complex when optimal does not.
time = 5.07, size = 254687, normalized size = 869.24

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/3072*(2*(2048*(-I*sqrt(3) + 1)*((sqrt(2)*a*b*sqrt(-1/(a*b))*sqrt(-(2*a*b*
c*e*sqrt(-1/(a*b)) + b*c^2 - a*e^2)/(a^2*b^2*sqrt(-1/(a*b)))) + sqrt(2)*(3*
a*f*sqrt(-1/(a*b)) - d))^2/(a*b) + 3*sqrt(2)*(2*sqrt(2)*a^2*b*f*sqrt(-1/(a*
b))*sqrt(-(2*a*b*c*e*sqrt(-1/(a*b)) + b*c^2 - a*e^2)/(a^2*b^2*sqrt(-1/(a*b)
))) - 2*sqrt(2)*a*b*d*sqrt(-(2*a*b*c*e*sqrt(-1/(a*b)) + b*c^2 - a*e^2)/(a^2
*b^2*sqrt(-1/(a*b)))) + 2*sqrt(2)*(b*c*e*sqrt(-1/(a*b)) - d*f)*a + sqrt(2)*
(3*a^2*f^2*sqrt(-1/(a*b)) - b*c^2 + (b*d^2*sqrt(-1/(a*b)) + e^2)*a))/(a^2*b
^2*sqrt(-1/(a*b)))))/(589824*(2*sqrt(2)*a^2*b*f*sqrt(-1/(a*b))*sqrt(-(2*a*b*
c*e*sqrt(-1/(a*b)) + b*c^2 - a*e^2)/(a^2*b^2*sqrt(-1/(a*b)))) - 2*sqrt(2)*a
*b*d*sqrt(-(2*a*b*c*e*sqrt(-1/(a*b)) + b*c^2 - a*e^2)/(a^2*b^2*sqrt(-1/(a*b
)))) + 2*sqrt(2)*(b*c*e*sqrt(-1/(a*b)) - d*f)*a + sqrt(2)*(3*a^2*f^2*sqrt(-
1/(a*b)) - b*c^2 + (b*d^2*sqrt(-1/(a*b)) + e^2)*a))*(sqrt(2)*a*b*sqrt(-1/(a
*b))*sqrt(-(2*a*b*c*e*sqrt(-1/(a*b)) + b*c^2 - a*e^2)/(a^2*b^2*sqrt(-1/(a*b
)))) + sqrt(2)*(3*a*f*sqr ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 290, normalized size = 0.99

$$\frac{f\log\left(|bx^4+a|\right)}{4\,b} - \frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{ab}\,b^2d - (ab^3)^{\frac{1}{4}}\,b^2c - (ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{ab}\,b^2d - (ab^3)^{\frac{1}{4}}\,b^2c - (ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c - (ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c - (ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*f*log(abs(b*x^4 + a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3
)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^
(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4
))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*
e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b
^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(a*b^3)
```

Mupad [B]
time = 0.93, size = 1952, normalized size = 6.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x)
```

```
[Out] symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - a*b*e^3 - a*b*c*f^2 - 16*roo
t(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2
*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^
2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^2
*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*
c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)^2*a*b^3*c - 4*ro
ot(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^
2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a
^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^
2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2
*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)*b^3*c^2*x + b^2
*c^2*f*x + 16*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2
 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^
2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*
b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d
^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)
^2*a*b^3*d*x + 4*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*
z^2 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2
*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a
^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*
b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z,
 k)*a*b^2*e^2*x + 2*a*b*d*e*f + 8*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3
+ 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2
*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*
a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2
*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*
f^4 + b^3*c^4, z, k)*a*b^2*c*f - 8*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3
 + 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^
```

```
2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16
*a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^
2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3
*f^4 + b^3*c^4, z, k)*a*b^2*d*e + a*b*d*f^2*x - a*b*e^2*f*x - 2*b^2*c*d*e*x
 - 8*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z
 + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f
 + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 +
2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)*a*b^2*d*
f*x)*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z
 + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f
 + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 +
2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k), k, 1, 4
)
```

3.488 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$

Optimal. Leaf size=321

$$\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}-\frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{\sqrt[4]{a}\left(\sqrt{b}\,d+\sqrt{a}\,f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{7/4}}-\frac{\sqrt[4]{a}\left(\sqrt{b}\,d+\sqrt{a}\,f\right.}{2\sqrt{}}$$

[Out] d*x/b+1/2*e*x^2/b+1/3*f*x^3/b+1/4*c*ln(b*x^4+a)/b-1/2*e*arctan(x^2*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)+1/8*a^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-f*a^(1/2)+d*b^(1/2))/b^(7/4)*2^(1/2)-1/8*a^(1/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-f*a^(1/2)+d*b^(1/2))/b^(7/4)*2^(1/2)-1/4*a^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(f*a^(1/2)+d*b^(1/2))/b^(7/4)*2^(1/2)-1/4*a^(1/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(f*a^(1/2)+d*b^(1/2))/b^(7/4)*2^(1/2)

Rubi [A]
time = 0.22, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1845, 1266, 788, 649, 211, 266, 1294, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{a}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,f+\sqrt{b}\,d\right)}{2\sqrt{2}\,b^{7/4}}-\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,f+\sqrt{b}\,d\right)}{2\sqrt{2}\,b^{7/4}}-\frac{\sqrt{a}\,e\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{7/4}}-\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{7/4}}+\frac{c\log\left(a+bx^4\right)}{4b}+\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]

[Out] (d*x)/b + (e*x^2)/(2*b) + (f*x^3)/(3*b) - (Sqrt[a]*e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*b^(3/2)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) + (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) + (c*Log[a + b*x^4])/(4*b)

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx &= \int \left(\frac{x^3(c+ex^2)}{a+bx^4}+\frac{x^4(d+fx^2)}{a+bx^4}\right)dx \\
&= \int \frac{x^3(c+ex^2)}{a+bx^4}\,dx+\int \frac{x^4(d+fx^2)}{a+bx^4}\,dx \\
&= \frac{fx^3}{3b}+\frac{1}{2}\text{Subst}\left(\int \frac{x(c+ex)}{a+bx^2}\,dx,x,x^2\right)-\frac{\int \frac{x^2(3af-3bdx^2)}{a+bx^4}\,dx}{3b} \\
&= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}+\frac{\int \frac{-3abd-3abfx^2}{a+bx^4}\,dx}{3b^2}+\frac{\text{Subst}\left(\int \frac{-ae+bcx}{a+bx^2}\,dx,x,x^2\right)}{2b} \\
&= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}+\frac{1}{2}c\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx,x,x^2\right)-\frac{(ae)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx\right.}{2b} \\
&= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}-\frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{c\log(a+bx^4)}{4b}+\frac{\left(\sqrt[4]{a}\left(\sqrt{b}\,d\right.\right.}{} \\
&= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}-\frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{a}\right.}{4\sqrt{2}\,b} \\
&= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}-\frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{\sqrt[4]{a}\left(\sqrt{b}\,d+\sqrt{a}\,f\right)\tan^{-1}\left(1\right.}{2\sqrt{2}\,b^{7/4}}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 311, normalized size = 0.97

$$\frac{24b^{3/4}dx+12b^{3/4}ex^2+8b^{3/4}fx^3+6\sqrt[4]{a}\left(\sqrt{2}\sqrt{b}\,d+2\sqrt[4]{a}\sqrt[4]{b}\,e+\sqrt{2}\sqrt{a}\,f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-6\sqrt[4]{a}\left(\sqrt{2}\sqrt{b}\,d-2\sqrt[4]{a}\sqrt[4]{b}\,e+\sqrt{2}\sqrt{a}\,f\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-3\sqrt{2}\left(-\sqrt[4]{a}\sqrt{b}\,d+a^{3/4}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+3\sqrt{2}\left(-\sqrt[4]{a}\sqrt{b}\,d+a^{3/4}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+6b^{3/4}c\log(a+bx^4)}{24b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

```
[Out] (24*b^(3/4)*d*x + 12*b^(3/4)*e*x^2 + 8*b^(3/4)*f*x^3 + 6*a^(1/4)*(Sqrt[2]*S
qrt[b]*d + 2*a^(1/4)*b^(1/4)*e + Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)] - 6*a^(1/4)*(Sqrt[2]*Sqrt[b]*d - 2*a^(1/4)*b^(1/4)*e + Sqr
t[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(-(a^(1
/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + 3*Sqrt[2]*(-(a^(1/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 6*b^(3/4)*c*Log[a + b*x^4])/(24*b^(7/4
))
```

Maple [A]
time = 0.35, size = 261, normalized size = 0.81

method	result
risch	$\frac{f x^3}{3b}+\frac{e x^2}{2b}+\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(\_R^3 bc-\_R^2 af-\_R ae-ad\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$\frac{\frac{1}{3}f x^3+\frac{1}{2}e x^2+dx}{b}+\frac{-\frac{d\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8}-\frac{ae\arctan\left(x^2\sqrt{\ldots}\right.}{2\sqrt{ab}}\ldots}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3*f*x^3+1/2*e*x^2+d*x)+1/b*(-1/8*d*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b
)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/2*a*e/(
a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))-1/8*a*f/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(
a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+
2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/4*c*
ln(b*x^4+a))
```

Maxima [A]
time = 0.51, size = 308, normalized size = 0.96

$$\frac{2\,fx^3+3\,x^2e+6\,dx}{6\,b}+\frac{\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}c-abd+a^{\frac{3}{2}}\sqrt{b}\,f\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}c+abd-a^{\frac{3}{2}}\sqrt{b}\,f\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}-\frac{2\left(\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}d+\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{3}{4}}f-2\,a^{\frac{3}{2}}be\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}-\frac{2\left(\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}d+\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{3}{4}}f+2\,a^{\frac{3}{2}}be\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*f*x^3 + 3*x^2*e + 6*d*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*c
 - a*b*d + a^(3/2)*sqrt(b)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x +
 sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*c + a*b*d -
a^(3/2)*sqrt(b)*f)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(
a^(3/4)*b^(5/4)) - 2*(sqrt(2)*a^(5/4)*b^(5/4)*d + sqrt(2)*a^(7/4)*b^(3/4)*f
 - 2*a^(3/2)*b*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) - 2*(sqrt(
2)*a^(5/4)*b^(5/4)*d + sqrt(2)*a^(7/4)*b^(3/4)*f + 2*a^(3/2)*b*e)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))/b
```

Fricas [C] Result contains complex when optimal does not.
time = 3.75, size = 219615, normalized size = 684.16

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/48*(16*f*x^3 + 24*e*x^2 - 12*b*(2*sqrt(-1/4*b*(2*c^3/b^3 - (3*b*c^2 + (e^
2 + 2*d*f)*a)*c/b^4 + (b^2*c^3 + a^2*e*f^2 - (d^2*e - (e^2 + 2*d*f)*c)*a*b)
/b^5)/(e*sqrt(-a/b)) + 1/2*c^2/b^2 - 1/8*(3*b*c^2 + (e^2 + 2*d*f)*a)/b^3 -
1/8*(b*c^2 - (e^2 - 2*d*f)*a)/b^3) + e*sqrt(-a/b)/b - c/b)*log(b^3*c*d^5 +
b^3*c^2*d^3*e + 2*b^3*c^3*d*e^2 - 5*a*b^2*d^3*e^3 + 2*a*b^2*c*d*e^4 + 3*a^2
*b*c*d*f^4 + 3*a^3*e*f^5 + (a*b^2*c^3 + 3*a^2*b*c*e^2)*f^3 + (2*b^6*d*e^2 -
 b^6*d^2*f + a*b^5*f^3)*(2*sqrt(-1/4*b*(2*c^3/b^3 - (3*b*c^2 + (e^2 + 2*d*f
)*a)*c/b^4 + (b^2*c^3 + a^2*e*f^2 - (d^2*e - (e^2 + 2*d*f)*c)*a*b)/b^5)/(e*
sqrt(-a/b)) + 1/2*c^2/b^2 - 1/8*(3*b*c^2 + (e^2 + 2*d*f)*a)/b^3 - 1/8*(b*c^
2 - (e^2 - 2*d*f)*a)/b^3) + e*sqrt(-a/b)/b - c/b)^3 - (4*a*b^2*c*d^3 - 3*a*
b^2*c^2*d*e - 5*a^2*b*d*e^3)*f^2 + (b^5*d^3*e + 6*b^5*c*d*e^2 + 3*a*b^4*d*e
*f^2 + 3*a*b^4*c*f^3 - (3*b^5*c*d^2 + 2*a*b^4*e^3)*f)*(2*sqrt(-1/4*b*(2*c^3
/b^3 - (3*b*c^2 + (e^2 + 2*d*f)*a)*c/b^4 + (b^2*c^3 + a^2*e*f^2 - (d^2*e -
(e^2 + 2*d*f)*c)*a*b)/b^5 ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 308, normalized size = 0.96

$$\frac{c\log\left(|bx^4+a|\right)}{4\,b}+\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{ab}\,b^2e-\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}+\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{ab}\,b^2e-\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4}+\frac{2\,b^2fx^3+3\,b^2x^2e+6\,b^2dx}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*c*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*e - (a*b^3
)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/b^4 + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*e - (a*b^3)^(1/4
)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/b^4 - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*log(x
^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^
2*d - (a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 + 1
/6*(2*b^2*f*x^3 + 3*b^2*x^2*e + 6*b^2*d*x)/b^3
```

Mupad [B]
time = 4.85, size = 838, normalized size = 2.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x)
```

```
[Out] symsum(log((a^4*f^3 + a^2*b^2*c^2*d - a^3*b*d*e^2 + a^3*b*d^2*f + 2*a^3*b*c
*e*f)/b^2 + root(256*b^7*z^4 - 256*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^4*
e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*b^3
*d^2*e*z - 16*a*b^3*c*e^2*z - 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*
f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2
 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)*(root(256*b^7*z^4 - 256
*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^
3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c*e^2*z - 16*b
^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*
d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4
 + b^3*c^4, z, k)*(16*a^2*b^2*d - 16*a^2*b^2*e*x) - (8*a^2*b^3*c*d + 8*a^3*
b^2*e*f)/b^2 + (x*(4*a^3*b*f^2 - 4*a^2*b^2*d^2 + 8*a^2*b^2*c*e))/b) - (x*(a
^3*e^3 + a^3*c*f^2 - 2*a^3*d*e*f - a^2*b*c*d^2 + a^2*b*c^2*e))/b)*root(256*
b^7*z^4 - 256*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^4*e^2*z^2 + 96*b^5*c^2*
z^2 - 32*a*b^3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c
*e^2*z - 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f
 - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*
d^4 + a^3*f^4 + b^3*c^4, z, k), k, 1, 4) + (e*x^2)/(2*b) + (f*x^3)/(3*b) +
(d*x)/b
```

3.489 $\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^2}\,dx$

Optimal. Leaf size=318

$$-\frac{af-bx(c+dx+ex^2)}{4ab\left(a+bx^4\right)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\mathrm{ta}}{8\sqrt{2}}$$

```
[Out] 1/4*(-a*f+b*x*(e*x^2+d*x+c))/a/b/(b*x^4+a)+1/4*d*arctan(x^2*b^(1/2)/a^(1/2)
)/a^(3/2)/b^(1/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(
-e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*x*2
^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^(1/2
)+1/16*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^(7/4)
/b^(3/4)*2^(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*c*b^
(1/2))/a^(7/4)/b^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)}{8\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)}{8\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{d\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{3/4}}-\frac{af-bx(c+dx+ex^2)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

```
[Out] -1/4*(a*f - b*x*(c + d*x + e*x^2))/(a*b*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x
^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c +
Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3
/4)) - ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b
^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
```

```
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx = -\frac{af-bx(c+dx+ex^2)}{4ab(a+bx^4)} - \frac{\int \frac{-3c-2dx-ex^2}{a+bx^4}\,dx}{4a}$$

$$= -\frac{af-bx(c+dx+ex^2)}{4ab(a+bx^4)} - \frac{\int \left(-\frac{2dx}{a+bx^4}+\frac{-3c-ex^2}{a+bx^4}\right)dx}{4a}$$

$$= -\frac{af-bx(c+dx+ex^2)}{4ab(a+bx^4)} - \frac{\int \frac{-3c-ex^2}{a+bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a+bx^4}\,dx}{2a}$$

$$= -\frac{af-bx(c+dx+ex^2)}{4ab(a+bx^4)} + \frac{d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4a} + \frac{\left(\frac{3\sqrt{b}\,c}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{b}}{a+bx}}{8ab}$$

$$= -\frac{af-bx(c+dx+ex^2)}{4ab(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{\left(\frac{3\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}}}{16ab}$$

$$= -\frac{af-bx(c+dx+ex^2)}{4ab(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{}\right.}{16\sqrt{2}\,a^{7/4}b}$$

$$= -\frac{af-bx(c+dx+ex^2)}{4ab(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1-\right.}{8\sqrt{2}\,a^{7/4}b^{3/4}}$$

Mathematica [A]
time = 0.23, size = 315, normalized size = 0.99

$$\frac{-\frac{8a(af-bx(c+x(d+ex)))}{a+bx^4}-2\sqrt[4]{a}\sqrt[4]{b}\left(3\sqrt{2}\sqrt{b}\,c+4\sqrt[4]{a}\sqrt[4]{b}\,d+\sqrt{2}\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\sqrt[4]{b}\left(3\sqrt{2}\sqrt{b}\,c-4\sqrt[4]{a}\sqrt[4]{b}\,d+\sqrt{2}\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\sqrt{2}\sqrt[4]{b}\left(-3\sqrt[4]{a}\sqrt{b}\,c+a^{3/4}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+\sqrt{2}\sqrt[4]{b}\left(3\sqrt[4]{a}\sqrt{b}\,c-a^{3/4}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{32a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a*(a*f - b*x*(c + x*(d + e*x))))/(a + b*x^4) - 2*a^(1/4)*b^(1/4)*(3*Sq
rt[2]*Sqrt[b]*c + 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(3*Sqrt[2]*Sqrt[b]*c - 4*a^(1/4
)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] +
Sqrt[2]*b^(1/4)*(-3*a^(1/4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(3*a^(1/4)*Sqrt[b]*c - a^(
3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^2*b)
```

Maple [A]
time = 0.37, size = 305, normalized size = 0.96

method	result
risch	$\frac{\frac{e x^3}{4a}+\frac{d x^2}{4a}+\frac{cx}{4a}-\frac{f}{4b}}{b x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(\_R^2 e+2\_R d+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(b x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+d\Bigg(\frac{}{4a\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/4*x/a/(b*x^4+a)+3/32/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^
(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2
)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))+d*(1/4*x^2/a/(b*x^4+
a)+1/4/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2)))+e*(1/4*x^3/a/(b*x^4+a)+1/32/a
/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/
b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)))+1/4*f*x^4/a/(b*x^4+a)
```

Maxima [A]
time = 0.48, size = 310, normalized size = 0.97

$$\frac{bx^3e+bdx^2+bcx-af}{4\left(ab^2x^4+a^2b\right)}+\frac{\frac{\sqrt{2}\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e-4\sqrt{a}\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e+4\sqrt{a}\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 + b*c*x - a*f)/(a*b^2*x^4 + a^2*b) + 1/32*(sqrt(2)*(
3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt
```

```
(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2
- sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(
1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e - 4*sqrt(a)*sqrt(b)*d)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*c + sq
rt(2)*a^(3/4)*b^(1/4)*e + 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b
)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)
*sqrt(b))*b^(3/4)))/a
```

Fricas [C] Result contains complex when optimal does not.
time = 3.67, size = 124301, normalized size = 390.88

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9216*(2304*b*e*x^3 + 2304*b*d*x^2 + 2304*b*c*x + 2*(a*b^2*x^4 + a^2*b)*((
-I*sqrt(3) + 1)*((a^2*b*sqrt(-1/(a*b))*sqrt(-(6*a*b*c*e*sqrt(-1/(a*b)) + 9*
b*c^2 - a*e^2)/(a^4*b^2*sqrt(-1/(a*b)))) - 2*d)^2/(a^3*b) - 3*(4*a^2*b*d*sq
rt(-(6*a*b*c*e*sqrt(-1/(a*b)) + 9*b*c^2 - a*e^2)/(a^4*b^2*sqrt(-1/(a*b))))
+ 9*b*c^2 - (2*(2*d^2 + 3*c*e)*b*sqrt(-1/(a*b)) + e^2)*a)/(a^4*b^2*sqrt(-1/
(a*b))))/(-1/24576*(4*a^2*b*d*sqrt(-(6*a*b*c*e*sqrt(-1/(a*b)) + 9*b*c^2 - a
*e^2)/(a^4*b^2*sqrt(-1/(a*b)))) + 9*b*c^2 - (2*(2*d^2 + 3*c*e)*b*sqrt(-1/(a
*b)) + e^2)*a)*(a^2*b*sqrt(-1/(a*b))*sqrt(-(6*a*b*c*e*sqrt(-1/(a*b)) + 9*b*
c^2 - a*e^2)/(a^4*b^2*sqrt(-1/(a*b)))) - 2*d)/(a^5*b^2) + 1/8192*(a^5*b^2*s
qrt(-1/(a*b))*(-(6*a*b*c*e*sqrt(-1/(a*b)) + 9*b*c^2 - a*e^2)/(a^4*b^2*sqrt(
-1/(a*b))))^(3/2) - 4*(d^2*sqrt(-(6*a*b*c*e*sqrt(-1/(a*b)) + 9*b*c^2 - a*e^
2)/(a^4*b^2*sqrt(-1/(a*b)))) - 3*c*e*sqrt(-(6*a*b*c*e*sqrt(-1/(a*b)) + 9*b*
c^2 - a*e^2)/(a^4*b^2*sqrt(-1/(a*b)))))*a^2*b*sqrt(-1/(a*b)) + 18*b*c^2*d*s
qrt(-1/(a*b)) - 2*a*d*e^2 ...
```

Sympy [A]
time = 43.84, size = 517, normalized size = 1.63

$$\operatorname{RootSum}\left(65536t^4a^7b^3 + t^2\cdot\left(3072a^4b^2ce + 2048a^4b^2d^2\right) + t\left(128a^3bde^2 - 1152a^2b^2c^2d\right) + a^2e^4 + 18abc^2e^2 - 48abcd^2e + 16abd^4 + 81b^2c^4, \left(t \mapsto t\log\left(x + \frac{4096t^3a^7b^2e^3 - 36864t^3a^6b^3c^2e + 98304t^3a^6b^3cd^2 + 4608t^2a^5b^2cde^2 - 4096t^2a^5b^2d^3e + 13824t^2a^4b^3c^3d + 144ta^4bce^4 + 192ta^4bd^2e^3 - 1728ta^3b^2c^3e^2 + 5184ta^3b^2c^2d^2e + 1536ta^3b^2cd^4 + 3888ta^2b^3c^5 + 6a^3de^5 + 120a^2bcd^3e^2 - 64a^2bd^5e + 810ab^2c^4de - 1080ab^2c^3d^3}{a^3e^6 - 9a^2bc^2e^4 + 96a^2bcd^2e^3 - 64a^2bd^4e^2 - 81ab^2c^4e^2 + 864ab^2c^3d^2e - 576ab^2c^2d^4 + 729b^3c^6}\right)\right)\right) + \frac{-af + bcx + bdx^2 + bex^3}{4a^2b + 4ab^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2*
d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a*b
*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, Lambda(_t, _t*lo
g(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304*_t*
*3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*
d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t*a**4*
```

```
b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e +
 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 + 120*a*
*2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c**
3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 - 64*a**2*b
*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2
*d**4 + 729*b**3*c**6)))) + (-a*f + b*c*x + b*d*x**2 + b*e*x**3)/(4*a**2*b
+ 4*a*b**2*x**4)
```

Giac [A]
time = 0.60, size = 316, normalized size = 0.99

$$\frac{bx^3e+bdx^2+bcx-af}{4\,(bx^4+a)ab}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2d+3\,(ab^3)^{\frac14}b^2c+(ab^3)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2d+3\,(ab^3)^{\frac14}b^2c+(ab^3)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac14}b^2c-(ab^3)^{\frac34}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}-\frac{\sqrt{2}\left(3\,(ab^3)^{\frac14}b^2c-(ab^3)^{\frac34}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 + b*c*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*s
qrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)
*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arct
an(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sq
rt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)
^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

Mupad [B]
time = 0.36, size = 478, normalized size = 1.50

$$\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(65536\,a^7b^3z^4+3072\,a^4b^2c\,e\,z^2+2048\,a^4b^2d^2z^2-1152\,a^2b^2c^2d\,z+128\,a^3b\,d\,e^2z-48\,a\,b\,c\,d^2e+18\,a\,b\,c^2e^2+16\,a\,b\,d^4+81\,b^2c^4+a^2e^4,z,k\right)\left(\mathrm{root}\left(65536\,a^7b^3z^4+3072\,a^4b^2c\,e\,z^2+2048\,a^4b^2d^2z^2-1152\,a^2b^2c^2d\,z+128\,a^3b\,d\,e^2z-48\,a\,b\,c\,d^2e+18\,a\,b\,c^2e^2+16\,a\,b\,d^4+81\,b^2c^4+a^2e^4,z,k\right)\left(12\,b^3c-8\,b^3d\,x\right)+\frac{x\left(36\,a\,b^3c^2-4\,a^2b^2e^2\right)}{16\,a^3}+\frac{b^2d\,e}{a}\right)-\frac{9\,b^2c^2e-12\,b^2c\,d^2+a\,b\,e^3}{64\,a^3}+\frac{x\left(2\,b^2d^3-3\,b^2c\,d\,e\right)}{16\,a^3}\right)\mathrm{root}\left(65536\,a^7b^3z^4+3072\,a^4b^2c\,e\,z^2+2048\,a^4b^2d^2z^2-1152\,a^2b^2c^2d\,z+128\,a^3b\,d\,e^2z-48\,a\,b\,c\,d^2e+18\,a\,b\,c^2e^2+16\,a\,b\,d^4+81\,b^2c^4+a^2e^4,z,k\right)\right)+\frac{\frac{d\,x^2}{4\,a}-\frac{f}{4\,b}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log((x*(2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3) - (9*b^2*c^2*e - 12*b^2*c
*d^2 + a*b*e^3)/(64*a^3) - root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 +
2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*
d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)*(root(655
36*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2
*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4
 + 81*b^2*c^4 + a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 -
4*a^2*b^2*e^2))/(16*a^3) + (b^2*d*e)/a))*root(65536*a^7*b^3*z^4 + 3072*a^4*
b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2
*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z
, k), k, 1, 4) + ((d*x^2)/(4*a) - f/(4*b) + (e*x^3)/(4*a) + (c*x)/(4*a))/(a
 + b*x^4)
```

3.490 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2} dx$

Optimal. Leaf size=310

$$-\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{3/2}}-\frac{\left(\sqrt{b}\,d+3\sqrt{a}\,f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\left(\sqrt{b}\,d+3\sqrt{a}\,f\right)\tan}{8\sqrt{2}\,a^{3}}$$

```
[Out] 1/4*(-f*x^3-e*x^2-d*x-c)/b/(b*x^4+a)+1/4*e*arctan(x^2*b^(1/2)/a^(1/2))/b^(3
/2)/a^(1/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-3*f*a
^(1/2)+d*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*x*2^(1/2)
+a^(1/2)+x^2*b^(1/2))*(-3*f*a^(1/2)+d*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)+1/16
*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(3*f*a^(1/2)+d*b^(1/2))/a^(3/4)/b^(7/
4)*2^(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(3*f*a^(1/2)+d*b^(1/2))
/a^(3/4)/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1837, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(3\sqrt{a}\,f+\sqrt{b}\,d\right)}{8\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{a}\,f+\sqrt{b}\,d\right)}{8\sqrt{2}\,a^{3/4}b^{7/4}}-\frac{\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{e\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{3/2}}-\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

```
[Out] -1/4*(c + d*x + e*x^2 + f*x^3)/(b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sq
rt[a]])/(4*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*d + 3*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*d + 3*Sqrt[a
]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/4)) -
 ((Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/(16*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(3/4)*b^(7/4)
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
```

```
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = -\frac{c+dx+ex^2+fx^3}{4b\,(a+bx^4)} + \frac{\int \frac{d+2ex+3fx^2}{a+bx^4}\,dx}{4b}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\,(a+bx^4)} + \frac{\int \left(\frac{2ex}{a+bx^4} + \frac{d+3fx^2}{a+bx^4}\right)dx}{4b}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\,(a+bx^4)} + \frac{\int \frac{d+3fx^2}{a+bx^4}\,dx}{4b} + \frac{e\int \frac{x}{a+bx^4}\,dx}{2b}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\,(a+bx^4)} + \frac{e\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4b} + \frac{\left(\frac{\sqrt{b}\,d}{\sqrt{a}} - 3f\right)\int \frac{\sqrt{a}\,\sqrt{\ldots}}{a+\ldots}}{8b^2}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\,(a+bx^4)} + \frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{3/2}} + \frac{\left(\frac{\sqrt{b}\,d}{\sqrt{a}} + 3f\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\,\sqrt[4]{a}\ldots}{\sqrt[4]{b}}}}{16b^2}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\,(a+bx^4)} + \frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{3/2}} - \frac{\left(\sqrt{b}\,d - 3\sqrt{a}\,f\right)\log\left(\sqrt{a} - \ldots\right.}{16\sqrt{2}\,a^{3/4}\ldots}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\,(a+bx^4)} + \frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{3/2}} - \frac{\left(\sqrt{b}\,d + 3\sqrt{a}\,f\right)\tan^{-1}\left(1 - \ldots\right.}{8\sqrt{2}\,a^{3/4}b^{7/4}}$$

Mathematica [A]
time = 0.18, size = 294, normalized size = 0.95

$$\frac{-\frac{8b^{3/4}(c+x(d+x(e+fx)))}{a+bx^4} - \frac{2\left(\sqrt{2}\,\sqrt{b}\,d+4\sqrt[4]{a}\,\sqrt[4]{b}\,e+3\sqrt{2}\,\sqrt{a}\,f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{2\left(\sqrt{2}\,\sqrt{b}\,d-4\sqrt[4]{a}\,\sqrt[4]{b}\,e+3\sqrt{2}\,\sqrt{a}\,f\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{\sqrt{2}\left(-\sqrt{b}\,d+3\sqrt{a}\,f\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{3/4}} + \frac{\sqrt{2}\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{3/4}}}{32b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

```
[Out] ((-8*b^(3/4)*(c + x*(d + x*(e + f*x))))/(a + b*x^4) - (2*(Sqrt[2]*Sqrt[b]*d
 + 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/a^(3/4) + (2*(Sqrt[2]*Sqrt[b]*d - 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[
2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[2]*(
-(Sqrt[b]*d) + 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/a^(3/4) + (Sqrt[2]*(Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4))/(32*b^(7/4))
```

Maple [A]
time = 0.37, size = 273, normalized size = 0.88

method	result
risch	$\frac{-\frac{f\,x^{3}}{4b}-\frac{e\,x^{2}}{4b}-\frac{dx}{4b}-\frac{c}{4b}}{b\,x^{4}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{4}+a\right)}\frac{\left(3f\,\_R^{2}+2e\,\_R+d\right)\ln\left(x-\_R\right)}{\_R^{3}}}{16b^{2}}$
default	$\frac{-\frac{f\,x^{3}}{4b}-\frac{e\,x^{2}}{4b}-\frac{dx}{4b}-\frac{c}{4b}}{b\,x^{4}+a}+\frac{\frac{d\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{e\arctan\left(x^{2}\ldots\right.}{\sqrt{ab}}}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*f*x^3/b-1/4*e*x^2/b-1/4*d*x/b-1/4*c/b)/(b*x^4+a)+1/4/b*(1/8*d*(a/b)^(
1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)
*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)
/(a/b)^(1/4)*x-1))+e/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+3/8*f/b/(a/b)^(1/4
)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^
(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b
)^(1/4)*x-1)))
```

Maxima [A]
time = 0.49, size = 297, normalized size = 0.96

$$-\frac{fx^{3}+x^{2}e+dx+c}{4\left(b^{2}x^{4}+ab\right)}+\frac{\frac{\sqrt{2}\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}d+3\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}f-4\sqrt{a}\sqrt{b}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}d+3\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}f+4\sqrt{a}\sqrt{b}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(f*x^3 + x^2*e + d*x + c)/(b^2*x^4 + a*b) + 1/32*(sqrt(2)*(sqrt(b)*d -
 3*sqrt(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(3/4)) - sqrt(2)*(sqrt(b)*d - 3*sqrt(a)*f)*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(sqrt(2)*a^(1/4)*b^(3/4)*
d + 3*sqrt(2)*a^(3/4)*b^(1/4)*f - 4*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(
```

```
2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt
(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(sqrt(2)*a^(1/4)*b^(3/4)*d + 3*sqrt(2)*a^(3/
4)*b^(1/4)*f + 4*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b
^(3/4)))/b
```

Fricas [C] Result contains complex when optimal does not.
time = 3.71, size = 122993, normalized size = 396.75

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] -1/9216*(2304*f*x^3 + 2304*e*x^2 - 2*(b^2*x^4 + a*b)*((-I*sqrt(3) + 1)*((a*
b^2*sqrt(-1/(a*b))*sqrt(-(6*a*b*d*f*sqrt(-1/(a*b)) + b*d^2 - 9*a*f^2)/(a^2*
b^4*sqrt(-1/(a*b)))) - 2*e)^2/(a*b^3) - 3*(b*d^2 + (4*b^2*e*sqrt(-(6*a*b*d*
f*sqrt(-1/(a*b)) + b*d^2 - 9*a*f^2)/(a^2*b^4*sqrt(-1/(a*b)))) - 2*(2*e^2 +
3*d*f)*b*sqrt(-1/(a*b)) - 9*f^2)*a)/(a^2*b^4*sqrt(-1/(a*b))))/(-1/24576*(a*
b^2*sqrt(-1/(a*b))*sqrt(-(6*a*b*d*f*sqrt(-1/(a*b)) + b*d^2 - 9*a*f^2)/(a^2*
b^4*sqrt(-1/(a*b)))) - 2*e)*(b*d^2 + (4*b^2*e*sqrt(-(6*a*b*d*f*sqrt(-1/(a*b
)) + b*d^2 - 9*a*f^2)/(a^2*b^4*sqrt(-1/(a*b)))) - 2*(2*e^2 + 3*d*f)*b*sqrt(
-1/(a*b)) - 9*f^2)*a)/(a^2*b^5) + 1/8192*(a^2*b^5*sqrt(-1/(a*b))*(-(6*a*b*d
*f*sqrt(-1/(a*b)) + b*d^2 - 9*a*f^2)/(a^2*b^4*sqrt(-1/(a*b))))^(3/2) + 2*b*
d^2*e*sqrt(-1/(a*b)) - 8*e^3 + 12*d*e*f - 2*(2*(e^2*sqrt(-(6*a*b*d*f*sqrt(-
1/(a*b)) + b*d^2 - 9*a*f^2)/(a^2*b^4*sqrt(-1/(a*b)))) - 3*d*f*sqrt(-(6*a*b*
d*f*sqrt(-1/(a*b)) + b*d^2 - 9*a*f^2)/(a^2*b^4*sqrt(-1/(a*b)))))*b^2*sqrt(-
1/(a*b)) + 9*e*f^2*sqrt(- ...
```

Sympy [A]
time = 196.05, size = 510, normalized size = 1.65

$$\operatorname{RootSum}\left(65536t^4a^3b^7 + t^2\cdot\left(3072a^2b^4df + 2048a^2b^4e^2\right) + t\left(1152a^2b^2ef^2 - 128ab^3d^2e\right) + 81a^2f^4 + 18abd^2f^2 - 48abde^2f + 16abe^4 + b^2d^4, \left(t\mapsto t\log\left(x + \frac{110592t^3a^4b^5f^3 - 12288t^3a^3b^6d^2f + 32768t^3a^3b^6de^2 + 13824t^2a^3b^4def^2 - 12288t^2a^3b^4e^3f + 512t^2a^2b^5d^3e + 3888ta^3b^2df^4 + 5184ta^3b^2e^2f^3 - 576ta^2b^3d^3f^2 + 1728ta^2b^3d^2e^2f + 512ta^2b^3de^4 + 16tab^4d^5 + 1458a^3ef^5 + 360a^2bde^3f^2 - 192a^2be^5f + 30ab^2d^4ef - 40ab^2d^3e^3}{729a^3f^6 - 81a^2bd^2f^4 + 864a^2bde^2f^3 - 576a^2be^4f^2 - 9ab^2d^4f^2 + 96ab^2d^3e^2f - 64ab^2d^2e^4 + b^3d^6}\right)\right)\right) + \frac{-c - dx - ex^2 - fx^3}{4ab + 4b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**3*b**7 + _t**2*(3072*a**2*b**4*d*f + 2048*a**2*b**4*
e**2) + _t*(1152*a**2*b**2*e*f**2 - 128*a*b**3*d**2*e) + 81*a**2*f**4 + 18*
a*b*d**2*f**2 - 48*a*b*d*e**2*f + 16*a*b*e**4 + b**2*d**4, Lambda(_t, _t*lo
g(x + (110592*_t**3*a**4*b**5*f**3 - 12288*_t**3*a**3*b**6*d**2*f + 32768*_
t**3*a**3*b**6*d*e**2 + 13824*_t**2*a**3*b**4*d*e*f**2 - 12288*_t**2*a**3*b
**4*e**3*f + 512*_t**2*a**2*b**5*d**3*e + 3888*_t*a**3*b**2*d*f**4 + 5184*_
t*a**3*b**2*e**2*f**3 - 576*_t*a**2*b**3*d**3*f**2 + 1728*_t*a**2*b**3*d**2
*e**2*f + 512*_t*a**2*b**3*d*e**4 + 16*_t*a*b**4*d**5 + 1458*a**3*e*f**5 +
360*a**2*b*d*e**3*f**2 - 192*a**2*b*e**5*f + 30*a*b**2*d**4*e*f - 40*a*b**2
```

```
*d**3*e**3)/(729*a**3*f**6 - 81*a**2*b*d**2*f**4 + 864*a**2*b*d*e**2*f**3 -
 576*a**2*b*e**4*f**2 - 9*a*b**2*d**4*f**2 + 96*a*b**2*d**3*e**2*f - 64*a*b
**2*d**2*e**4 + b**3*d**6)))) + (-c - d*x - e*x**2 - f*x**3)/(4*a*b + 4*b**
2*x**4)
```

Giac [A]
time = 0.53, size = 303, normalized size = 0.98

$$-\frac{fx^3+x^2e+dx+c}{4\,(bx^4+a)b}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2e+(ab^3)^{\frac{1}{4}}b^2d+3\,(ab^3)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2e+(ab^3)^{\frac{1}{4}}b^2d+3\,(ab^3)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4}+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2d-3\,(ab^3)^{\frac{3}{4}}f\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^4}-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2d-3\,(ab^3)^{\frac{3}{4}}f\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(f*x^3 + x^2*e + d*x + c)/((b*x^4 + a)*b) + 1/16*sqrt(2)*(2*sqrt(2)*sq
rt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)
*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/16*sqrt(2)*(2*sqrt(2)
*sqrt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/32*sqrt(2)*((a*b^3
)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(a*b^4) - 1/32*sqrt(2)*((a*b^3)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f)*log(x
^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)
```

Mupad [B]
time = 5.10, size = 559, normalized size = 1.80

$$\left(\sum_{k=1}^{4}\ln\left(\frac{x\,(2\,e^3-3\,d\,e\,f)}{16\,b}-\frac{3\,b\,d^2\,f-4\,b\,d\,e^2+27\,a\,f^3}{64\,b^2}-\mathrm{root}\left(65536\,a^3\,b^7\,z^4+3072\,a^2\,b^4\,d\,f\,z^2+2048\,a^2\,b^4\,e^2\,z^2+1152\,a^2\,b^2\,e\,f^2\,z-128\,a\,b^3\,d^2\,e\,z-48\,a\,b\,d\,e^2\,f+18\,a\,b\,d^2\,f^2+16\,a\,b\,e^4+81\,a^2\,f^4+b^2\,d^4,z,k\right)\left(3\,a\,e\,f+\frac{b\,d^2\,x}{4}-\frac{9\,a\,f^2\,x}{4}+\mathrm{root}\left(65536\,a^3\,b^7\,z^4+3072\,a^2\,b^4\,d\,f\,z^2+2048\,a^2\,b^4\,e^2\,z^2+1152\,a^2\,b^2\,e\,f^2\,z-128\,a\,b^3\,d^2\,e\,z-48\,a\,b\,d\,e^2\,f+18\,a\,b\,d^2\,f^2+16\,a\,b\,e^4+81\,a^2\,f^4+b^2\,d^4,z,k\right)a\,b^2\,d\,4-\mathrm{root}\left(65536\,a^3\,b^7\,z^4+3072\,a^2\,b^4\,d\,f\,z^2+2048\,a^2\,b^4\,e^2\,z^2+1152\,a^2\,b^2\,e\,f^2\,z-128\,a\,b^3\,d^2\,e\,z-48\,a\,b\,d\,e^2\,f+18\,a\,b\,d^2\,f^2+16\,a\,b\,e^4+81\,a^2\,f^4+b^2\,d^4,z,k\right)a\,b^2\,e\,x\,8\right)\right)\mathrm{root}\left(65536\,a^3\,b^7\,z^4+3072\,a^2\,b^4\,d\,f\,z^2+2048\,a^2\,b^4\,e^2\,z^2+1152\,a^2\,b^2\,e\,f^2\,z-128\,a\,b^3\,d^2\,e\,z-48\,a\,b\,d\,e^2\,f+18\,a\,b\,d^2\,f^2+16\,a\,b\,e^4+81\,a^2\,f^4+b^2\,d^4,z,k\right)\right)-\frac{\frac{c}{4\,b}+\frac{e\,x^2}{4\,b}+\frac{f\,x^3}{4\,b}+\frac{d\,x}{4\,b}}{b\,x^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x)
```

```
[Out] symsum(log((x*(2*e^3 - 3*d*e*f))/(16*b) - (27*a*f^3 - 4*b*d*e^2 + 3*b*d^2*f
)/(64*b^2) - root(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^2*b^4*e
^2*z^2 + 1152*a^2*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f + 18*a*b
*d^2*f^2 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*(3*a*e*f + (b*d^2*x)/4
- (9*a*f^2*x)/4 + 4*root(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^
2*b^4*e^2*z^2 + 1152*a^2*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f +
 18*a*b*d^2*f^2 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*a*b^2*d - 8*root
(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^2*b^4*e^2*z^2 + 1152*a^2
*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f + 18*a*b*d^2*f^2 + 16*a*b
*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*a*b^2*e*x))*root(65536*a^3*b^7*z^4 + 307
2*a^2*b^4*d*f*z^2 + 2048*a^2*b^4*e^2*z^2 + 1152*a^2*b^2*e*f^2*z - 128*a*b^3
*d^2*e*z - 48*a*b*d*e^2*f + 18*a*b*d^2*f^2 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*
d^4, z, k), k, 1, 4) - (c/(4*b) + (e*x^2)/(4*b) + (f*x^3)/(4*b) + (d*x)/(4*
b))/(a + b*x^4)
```

3.491 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3} \, dx$

Optimal. Leaf size=351

$$\frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)} - \frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21\sqrt{b}\,c+5\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}b^{3/4}}$$

```
[Out] 1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(b*x^4+a)+1/8*(-a*f+b*x*(e*x^2+d*x+c))/a/b/(
b*x^4+a)^2+3/16*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)-1/256*ln(-a^(
1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(
11/4)/b^(3/4)*2^(1/2)+1/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2
))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)*2^(1/2)+1/128*arctan(-1+b^(
1/4)*x*2^(1/2)/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)*2^(1/2)
+1/128*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11
/4)/b^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = $\{1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642\}$

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)}{64\sqrt{2}\,a^{11/4}b^{3/4}} + \frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)}{64\sqrt{2}\,a^{11/4}b^{3/4}} + \frac{3d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}b^{3/4}} + \frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}b^{3/4}} + \frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)} - \frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*
x^2))/(8*a*b*(a + b*x^4)^2) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/
2)*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*
Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(
3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^3}\,dx = -\frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2} - \frac{\int \frac{-7c-6dx-5ex^2}{\left(a+bx^4\right)^2}\,dx}{8a}$$

$$= \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} - \frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2} + \frac{\int \frac{21c+12dx+5ex^2}{a+bx^4}\,dx}{32a^2}$$

$$= \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} - \frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2} + \frac{\int \left(\frac{12dx}{a+bx^4}+\frac{21c+5ex^2}{a+bx^4}\right)dx}{32a^2}$$

$$= \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} - \frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2} + \frac{\int \frac{21c+5ex^2}{a+bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a+bx^4}\,d\cdots}{8a^2}$$

$$= \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} - \frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2} + \frac{(3d)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{16a^2} \cdots$$

$$= \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} - \frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{b}}{\sqrt{a}}\cdots\right.}{\cdots}$$

$$= \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} - \frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{b}}{\sqrt{a}}\cdots\right.}{\cdots}$$

$$= \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} - \frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21\sqrt{b}\cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.21, size = 347, normalized size = 0.99

$$\frac{\frac{8ax(7c+x(6d+5ex))}{a+bx^4} - \frac{32a^2(af-bx(c+x(d+ex)))}{b\left(a+bx^4\right)^2} - \frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}\,c+24\sqrt[4]{a}\sqrt[4]{b}\,d+5\sqrt{2}\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}\,c-24\sqrt[4]{a}\sqrt[4]{b}\,d+5\sqrt{2}\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{\sqrt{2}\left(-21\sqrt[4]{a}\sqrt{b}\,c+5a^{3/4}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{b^{3/4}} + \frac{\sqrt{2}\left(21\sqrt[4]{a}\sqrt{b}\,c-5a^{3/4}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{b^{3/4}}}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a*x*(7*c + x*(6*d + 5*e*x)))/(a + b*x^4) - (32*a^2*(a*f - b*x*(c + x*(d
 + e*x))))/(b*(a + b*x^4)^2) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1/4
)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/b^(3/4) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*Sqrt
[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*
(-21*a^(1/4)*Sqrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
```

```
*x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4)*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(256*a^3)
```

Maple [A]
time = 0.36, size = 391, normalized size = 1.11

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{7bc\,x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}-\frac{f}{8b}}{\left(b\,x^4+a\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^4+a\right)}\frac{\left(5\_R^2e+12\_Rd+21c\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2b}$
default	$c\left(\frac{x}{8a\left(b\,x^4+a\right)^2}+\frac{\frac{7x}{32a\left(b\,x^4+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/8*x/a/(b*x^4+a)^2+7/8/a*(1/4*x/a/(b*x^4+a)+3/32/a^2*(a/b)^(1/4)*2^(1/2
)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a
/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*
x-1))))+d*(1/8*x^2/a/(b*x^4+a)^2+3/4/a*(1/4*x^2/a/(b*x^4+a)+1/4/a/(a*b)^(1/
2)*arctan(x^2*(b/a)^(1/2))))+e*(1/8*x^3/a/(b*x^4+a)^2+5/8/a*(1/4*x^3/a/(b*x
^4+a)+1/32/a/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/
2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x
+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))))+f*(1/8*x^4/a/(b*x^4+a)^2+1/8/a^2*x
^4/(b*x^4+a))
```

Maxima [A]
time = 0.50, size = 361, normalized size = 1.03

$$\frac{5\,b^2x^7e+6\,b^2dx^6+7\,b^2cx^5+9\,abx^3e+10\,abdx^2+11\,abcx-4\,a^2f}{32\left(a^2b^3x^8+2\,a^3b^2x^4+a^4b\right)}+\frac{\frac{\sqrt{2}\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(21\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+5\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e-24\sqrt{a}\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(21\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+5\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e+24\sqrt{a}\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{256\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 + 9*a*b*x^3*e + 10*a*b*d*x^2
+ 11*a*b*c*x - 4*a^2*f)/(a^2*b^3*x^8 + 2*a^3*b^2*x^4 + a^4*b) + 1/256*(sqrt
(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*
x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*log(s
qrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21
```

```
*sqrt(2)*a^(1/4)*b^(3/4)*c + 5*sqrt(2)*a^(3/4)*b^(1/4)*e - 24*sqrt(a)*sqrt(
b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(
a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4
)*b^(3/4)*c + 5*sqrt(2)*a^(3/4)*b^(1/4)*e + 24*sqrt(a)*sqrt(b)*d)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a^2
```

Fricas [C] Result contains complex when optimal does not.
time = 7.94, size = 124838, normalized size = 355.66

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/589824*(92160*b^2*e*x^7 + 110592*b^2*d*x^6 + 129024*b^2*c*x^5 + 165888*a*
b*e*x^3 + 184320*a*b*d*x^2 + 202752*a*b*c*x - 73728*a^2*f + 2*(a^2*b^3*x^8
+ 2*a^3*b^2*x^4 + a^4*b)*((-I*sqrt(3) + 1)*((a^3*b*sqrt(-1/(a*b))*sqrt(-(21
0*a*b*c*e*sqrt(-1/(a*b)) + 441*b*c^2 - 25*a*e^2)/(a^6*b^2*sqrt(-1/(a*b))))
- 12*d)^2/(a^5*b) - 3*(24*a^3*b*d*sqrt(-(210*a*b*c*e*sqrt(-1/(a*b)) + 441*b
*c^2 - 25*a*e^2)/(a^6*b^2*sqrt(-1/(a*b)))) + 441*b*c^2 - (6*(24*d^2 + 35*c*
e)*b*sqrt(-1/(a*b)) + 25*e^2)*a)/(a^6*b^2*sqrt(-1/(a*b)))))/(-1/12582912*(24
*a^3*b*d*sqrt(-(210*a*b*c*e*sqrt(-1/(a*b)) + 441*b*c^2 - 25*a*e^2)/(a^6*b^2
*sqrt(-1/(a*b)))) + 441*b*c^2 - (6*(24*d^2 + 35*c*e)*b*sqrt(-1/(a*b)) + 25*
e^2)*a)*(a^3*b*sqrt(-1/(a*b))*sqrt(-(210*a*b*c*e*sqrt(-1/(a*b)) + 441*b*c^2
 - 25*a*e^2)/(a^6*b^2*sqrt(-1/(a*b)))) - 12*d)/(a^8*b^2) + 1/4194304*(a^8*b
^2*sqrt(-1/(a*b))*(-(210*a*b*c*e*sqrt(-1/(a*b)) + 441*b*c^2 - 25*a*e^2)/(a^
6*b^2*sqrt(-1/(a*b))))^(3/2) - 12*(12*d^2*sqrt(-(210*a*b*c*e*sqrt(-1/(a*b))
 + 441*b*c^2 - 25*a*e^2)/ ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.64, size = 354, normalized size = 1.01

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^2d+21\left(ab^3\right)^{\frac{1}{4}}b^2c+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^2d+21\left(ab^3\right)^{\frac{1}{4}}b^2c+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}-\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}+\frac{5\,b^2x^7e+6\,b^2dx^6+7\,b^2cx^5+9\,abx^3e+10\,abdx^2+11\,abcx-4\,a^2f}{32\left(bx^4+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b
^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*
c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)
*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*
(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 +
9*a*b*x^3*e + 10*a*b*d*x^2 + 11*a*b*c*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

Mupad [B]
time = 5.20, size = 832, normalized size = 2.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x)
```

```
[Out] symsum(log(-(b*(125*a*e^3 - 3024*b*c*d^2 + 2205*b*c^2*e - 1728*b*d^3*x + 34
4064*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^
2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^
2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z,
k)^2*a^5*b^2*c - 3200*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2
 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z
 - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 19
4481*b^2*c^4, z, k)*a^3*b*e^2*x + 2520*b*c*d*e*x + 56448*root(268435456*a^1
1*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3
*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2
 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^2*b^2*c^2*x - 1966
08*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*
d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*
e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)
^2*a^5*b^2*d*x + 15360*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^
2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*
z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 1
94481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435456*a^11*b^3*z^4 +
 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*
z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*
b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k), k, 1, 4) + ((5*d*x^2)/(16*a) -
 f/(8*b) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) + (7*b*c*x^5)/(32*a^2) + (3*b
*d*x^6)/(16*a^2) + (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4)
```

3.492 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=340

$$-\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x(d+2ex+3fx^2)}{32ab\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}-\frac{3\left(\sqrt{b}\,d+\sqrt{a}\,f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{7/4}b^{7/4}}+$$

```
[Out] 1/8*(-f*x^3-e*x^2-d*x-c)/b/(b*x^4+a)^2+1/32*x*(3*f*x^2+2*e*x+d)/a/b/(b*x^4+
a)+1/16*e*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-3/256*ln(-a^(1/4)*b^(
1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-f*a^(1/2)+d*b^(1/2))/a^(7/4)/b^(7/4)*
2^(1/2)+3/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-f*a^(1/2)
+d*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+3/128*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/
4))*(f*a^(1/2)+d*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+3/128*arctan(1+b^(1/4)*x*
2^(1/2)/a^(1/4))*(f*a^(1/2)+d*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$,
Rules used = {1837, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,f+\sqrt{b}\,d\right)}{64\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,f+\sqrt{b}\,d\right)}{64\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{e\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}-\frac{3\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{3\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{7/4}b^{7/4}}-\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x(d+2ex+3fx^2)}{32ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```

```
[Out] -1/8*(c + d*x + e*x^2 + f*x^3)/(b*(a + b*x^4)^2) + (x*(d + 2*e*x + 3*f*x^2)
)/(32*a*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(3/2)*b^(3
/2)) - (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/
(64*Sqrt[2]*a^(7/4)*b^(7/4)) + (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(7/4)*b^(7/4)) - (3*(Sqrt[b]*d - Sqrt
[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]
*a^(7/4)*b^(7/4)) + (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(7/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
```

```
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = -\frac{c+dx+ex^2+fx^3}{8b\,(a+bx^4)^2}+\frac{\int \frac{d+2ex+3fx^2}{(a+bx^4)^2}\,dx}{8b}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\,(a+bx^4)^2}+\frac{x(d+2ex+3fx^2)}{32ab\,(a+bx^4)}-\frac{\int \frac{-3d-4ex-3fx^2}{a+bx^4}\,dx}{32ab}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\,(a+bx^4)^2}+\frac{x(d+2ex+3fx^2)}{32ab\,(a+bx^4)}-\frac{\int \left(-\frac{4ex}{a+bx^4}+\frac{-3d-3fx^2}{a+bx^4}\right)dx}{32ab}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\,(a+bx^4)^2}+\frac{x(d+2ex+3fx^2)}{32ab\,(a+bx^4)}-\frac{\int \frac{-3d-3fx^2}{a+bx^4}\,dx}{32ab}+\frac{e\int \frac{x}{a+bx^4}\,dx}{8ab}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\,(a+bx^4)^2}+\frac{x(d+2ex+3fx^2)}{32ab\,(a+bx^4)}+\frac{e\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{16ab}+$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\,(a+bx^4)^2}+\frac{x(d+2ex+3fx^2)}{32ab\,(a+bx^4)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}+\frac{\left(3\left(\frac{\sqrt{b}}{\sqrt{a}}\right.\right.}{}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\,(a+bx^4)^2}+\frac{x(d+2ex+3fx^2)}{32ab\,(a+bx^4)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}-\frac{3\left(\sqrt{b}\,d\right.}{}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\,(a+bx^4)^2}+\frac{x(d+2ex+3fx^2)}{32ab\,(a+bx^4)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}-\frac{3\left(\sqrt{b}\,d\right.}{}$$

Mathematica [A]
time = 0.22, size = 329, normalized size = 0.97

$$\frac{\frac{8b^{3/4}x(d+x(2e+3fx))}{a(a+bx^4)}-\frac{32b^{3/4}(c+x(d+x(e+fx)))}{(a+bx^4)^2}-\frac{2\left(3\sqrt{2}\sqrt{b}\,d+8\sqrt[4]{a}\sqrt[4]{b}\,e+3\sqrt{2}\sqrt{a}\,f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{2\left(3\sqrt{2}\sqrt{b}\,d-8\sqrt[4]{a}\sqrt[4]{b}\,e+3\sqrt{2}\sqrt{a}\,f\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{3\sqrt{2}\left(-\sqrt{b}\,d+\sqrt{a}\,f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}}+\frac{3\sqrt{2}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}}}{256b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```

```
[Out] ((8*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)) - (32*b^(3/4)*(c + x*(
d + x*(e + f*x))))/(a + b*x^4)^2 - (2*(3*Sqrt[2]*Sqrt[b]*d + 8*a^(1/4)*b^(1
/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/
4) + (2*(3*Sqrt[2]*Sqrt[b]*d - 8*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (3*Sqrt[2]*(-(Sqrt[b]*d)
+ Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4
```

```
) + (3*Sqrt[2]*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/a^(7/4))/(256*b^(7/4))
```

Maple [A]

time = 0.35, size = 304, normalized size = 0.89

method	result
risch	$\frac{\frac{3 f x^{7}}{32 a}+\frac{e x^{6}}{16 a}+\frac{d x^{5}}{32 a}-\frac{f x^{3}}{32 b}-\frac{e x^{2}}{16 b}-\frac{3 d x}{32 b}-\frac{c}{8 b}}{\left(b x^{4}+a \right)^{2}}+\frac{\sum_{\_R =\mathit{RootOf}\left(b \_Z^{4}+a \right)}\frac{\left(3 f \_R^{2}+4 e \_R +3 d \right) \ln \left(x -\_R \right)}{\_R^{3}}}{128 a \,b^{2}}$
default	$\frac{\frac{3 f x^{7}}{32 a}+\frac{e x^{6}}{16 a}+\frac{d x^{5}}{32 a}-\frac{f x^{3}}{32 b}-\frac{e x^{2}}{16 b}-\frac{3 d x}{32 b}-\frac{c}{8 b}}{\left(b x^{4}+a \right)^{2}}+\frac{\frac{3 d \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{8 a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (3/32*f/a*x^7+1/16/a*e*x^6+1/32*d/a*x^5-1/32*f*x^3/b-1/16*e*x^2/b-3/32*d*x/
b-1/8*c/b)/(b*x^4+a)^2+1/32/b/a*(3/8*d*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)
^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*ar
ctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+2*e/(a*b)^
(1/2)*arctan(x^2*(b/a)^(1/2))+3/8*f/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1
/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arcta
n(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]

time = 0.49, size = 347, normalized size = 1.02

$$\frac{3 \, b f x^{7} + 2 \, b x^{6} e + b d x^{5} - a f x^{3} - 2 \, a x^{2} e - 3 \, a d x - 4 \, a c}{32 \, {\left(a b^{3} x^{8} + 2 \, a^{2} b^{2} x^{4} + a^{3} b\right)}} + \frac{\frac{3 \, \sqrt{2} {\left(\sqrt{b} d - \sqrt{a} f\right)} \log\left(\sqrt{b} x^{2} + \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{3}{4}}} - \frac{3 \, \sqrt{2} {\left(\sqrt{b} d - \sqrt{a} f\right)} \log\left(\sqrt{b} x^{2} - \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{3}{4}}} + \frac{2 \, {\left(3 \, \sqrt{2} a^{\frac{1}{4}} b^{\frac{3}{4}} d + 3 \, \sqrt{2} a^{\frac{3}{4}} b^{\frac{1}{4}} f - 8 \, \sqrt{a} \sqrt{b} e\right)} \arctan\left(\frac{\sqrt{2} {\left(2 \, \sqrt{b} x + \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}\right)}}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{a^{\frac{3}{4}} \sqrt{\sqrt{a} \sqrt{b}} b^{\frac{3}{4}}} + \frac{2 \, {\left(3 \, \sqrt{2} a^{\frac{1}{4}} b^{\frac{3}{4}} d + 3 \, \sqrt{2} a^{\frac{3}{4}} b^{\frac{1}{4}} f + 8 \, \sqrt{a} \sqrt{b} e\right)} \arctan\left(\frac{\sqrt{2} {\left(2 \, \sqrt{b} x - \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}\right)}}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{a^{\frac{3}{4}} \sqrt{\sqrt{a} \sqrt{b}} b^{\frac{3}{4}}}}{256 \, a b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(3*b*f*x^7 + 2*b*x^6*e + b*d*x^5 - a*f*x^3 - 2*a*x^2*e - 3*a*d*x - 4*a
*c)/(a*b^3*x^8 + 2*a^2*b^2*x^4 + a^3*b) + 1/256*(3*sqrt(2)*(sqrt(b)*d - sqr
t(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(
3/4)) - 3*sqrt(2)*(sqrt(b)*d - sqrt(a)*f)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*d +
3*sqrt(2)*a^(3/4)*b^(1/4)*f - 8*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(2*sq
rt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqr
t(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*d + 3*sqrt(2)*a^(3/4)
*b^(1/4)*f + 8*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)
```

```
*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(
3/4)))/(a*b)
```

Fricas [C] Result contains complex when optimal does not.
time = 7.96, size = 124542, normalized size = 366.30

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/589824*(55296*b*f*x^7 + 36864*b*e*x^6 + 18432*b*d*x^5 - 18432*a*f*x^3 - 3
6864*a*e*x^2 - 55296*a*d*x + 2*(a*b^3*x^8 + 2*a^2*b^2*x^4 + a^3*b)*((-I*sqr
t(3) + 1)*((3*a^2*b^2*sqrt(-1/(a*b))*sqrt(-(2*a*b*d*f*sqrt(-1/(a*b)) + b*d^
2 - a*f^2)/(a^4*b^4*sqrt(-1/(a*b)))) - 4*e)^2/(a^3*b^3) - 3*(24*a^2*b^2*e*s
qrt(-(2*a*b*d*f*sqrt(-1/(a*b)) + b*d^2 - a*f^2)/(a^4*b^4*sqrt(-1/(a*b)))) +
 9*b*d^2 - (2*(8*e^2 + 9*d*f)*b*sqrt(-1/(a*b)) + 9*f^2)*a)/(a^4*b^4*sqrt(-1
/(a*b))))/(-1/12582912*(24*a^2*b^2*e*sqrt(-(2*a*b*d*f*sqrt(-1/(a*b)) + b*d^
2 - a*f^2)/(a^4*b^4*sqrt(-1/(a*b)))) + 9*b*d^2 - (2*(8*e^2 + 9*d*f)*b*sqrt(
-1/(a*b)) + 9*f^2)*a)*(3*a^2*b^2*sqrt(-1/(a*b))*sqrt(-(2*a*b*d*f*sqrt(-1/(a
*b)) + b*d^2 - a*f^2)/(a^4*b^4*sqrt(-1/(a*b)))) - 4*e)/(a^5*b^5) + 1/419430
4*(27*a^5*b^5*sqrt(-1/(a*b))*(-(2*a*b*d*f*sqrt(-1/(a*b)) + b*d^2 - a*f^2)/(
a^4*b^4*sqrt(-1/(a*b))))^(3/2) - 12*(4*e^2*sqrt(-(2*a*b*d*f*sqrt(-1/(a*b))
+ b*d^2 - a*f^2)/(a^4*b^4*sqrt(-1/(a*b)))) - 9*d*f*sqrt(-(2*a*b*d*f*sqrt(-1
/(a*b)) + b*d^2 - a*f^2)/ ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.73, size = 338, normalized size = 0.99

$$\frac{3\,bfx^7+2\,bx^6e+bdx^5-afx^3-2\,ax^2e-3\,adx-4\,ac}{32\,(bx^4+a)^2ab}+\frac{\sqrt{2}\left(4\,\sqrt{2}\,\sqrt{ab}\,b^2e+3\,(ab^3)^{\frac{1}{4}}\,b^2d+3\,(ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4}+\frac{\sqrt{2}\left(4\,\sqrt{2}\,\sqrt{ab}\,b^2e+3\,(ab^3)^{\frac{1}{4}}\,b^2d+3\,(ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4}+\frac{3\,\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2d-(ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}-\frac{3\,\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2d-(ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(3*b*f*x^7 + 2*b*x^6*e + b*d*x^5 - a*f*x^3 - 2*a*x^2*e - 3*a*d*x - 4*a
*c)/((b*x^4 + a)^2*a*b) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^2*e + 3*(a*b
```

```
^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^2*e
+ 3*(a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/256*sqrt(2)*((a*b^3)^(1/4)*b^2*
d - (a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
 - 3/256*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
```

Mupad [B]
time = 0.40, size = 521, normalized size = 1.53

$$\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(268435456\,a^7\,b^7\,z^4+589824\,a^4\,b^4\,d\,f\,z^2+524288\,a^4\,b^4\,e^2\,z^2+18432\,a^3\,b^2\,e\,f^2\,z-18432\,a^2\,b^3\,d^2\,e\,z-576\,a\,b\,d\,e^2\,f+162\,a\,b\,d^2\,f^2+256\,a\,b\,e^4+81\,a^2\,f^4+81\,b^2\,d^4,z,k\right)\left(\mathrm{root}\left(268435456\,a^7\,b^7\,z^4+589824\,a^4\,b^4\,d\,f\,z^2+524288\,a^4\,b^4\,e^2\,z^2+18432\,a^3\,b^2\,e\,f^2\,z-18432\,a^2\,b^3\,d^2\,e\,z-576\,a\,b\,d\,e^2\,f+162\,a\,b\,d^2\,f^2+256\,a\,b\,e^4+81\,a^2\,f^4+81\,b^2\,d^4,z,k\right)\left(\frac{3\,b^2\,d}{2}-2\,b^2\,e\,x\right)+\frac{3\,e\,f}{32\,a}+\frac{x\,\left(144\,a\,b^2\,d^2-144\,a^2\,b\,f^2\right)}{4096\,a^3\,b}\right)-\frac{3\,\left(9\,b\,d^2\,f-16\,b\,d\,e^2+9\,a\,f^3\right)}{32768\,a^3\,b^2}+\frac{x\,\left(8\,e^3-9\,d\,e\,f\right)}{4096\,a^3\,b}\right)\,\mathrm{root}\left(268435456\,a^7\,b^7\,z^4+589824\,a^4\,b^4\,d\,f\,z^2+524288\,a^4\,b^4\,e^2\,z^2+18432\,a^3\,b^2\,e\,f^2\,z-18432\,a^2\,b^3\,d^2\,e\,z-576\,a\,b\,d\,e^2\,f+162\,a\,b\,d^2\,f^2+256\,a\,b\,e^4+81\,a^2\,f^4+81\,b^2\,d^4,z,k\right)\right)-\frac{\frac{c}{8\,b}-\frac{d\,x^5}{32\,a}-\frac{e\,x^6}{16\,a}+\frac{e\,x^2}{16\,b}-\frac{3\,f\,x^7}{32\,a}+\frac{f\,x^3}{32\,b}+\frac{3\,d\,x}{32\,b}}{a^2+2\,a\,b\,x^4+b^2\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x)
```

```
[Out] symsum(log((x*(8*e^3 - 9*d*e*f))/(4096*a^3*b) - (3*(9*a*f^3 - 16*b*d*e^2 +
9*b*d^2*f))/(32768*a^3*b^2) - root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4*d
*f*z^2 + 524288*a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*d^2
*e*z - 576*a*b*d*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 81*b^
2*d^4, z, k)*(root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4*d*f*z^2 + 524288*
a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*d^2*e*z - 576*a*b*d
*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 81*b^2*d^4, z, k)*((3
*b^2*d)/2 - 2*b^2*e*x) + (3*e*f)/(32*a) + (x*(144*a*b^2*d^2 - 144*a^2*b*f^2
))/(4096*a^3*b)))*root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4*d*f*z^2 + 524
288*a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*d^2*e*z - 576*a
*b*d*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 81*b^2*d^4, z, k)
, k, 1, 4) - (c/(8*b) - (d*x^5)/(32*a) - (e*x^6)/(16*a) + (e*x^2)/(16*b) -
(3*f*x^7)/(32*a) + (f*x^3)/(32*b) + (3*d*x)/(32*b))/(a^2 + b^2*x^8 + 2*a*b*
x^4)
```

3.493 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=382

$$\frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)}-\frac{af-bx(c+dx+ex^2)}{12ab\left(a+bx^4\right)^3}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}-\frac{\left(77\sqrt{b}\,c+15\right.}{}$$

```
[Out] 1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(b*x^4+a)^2+1/384*x*(45*e*x^2+60*d*x+77*c)
/a^3/(b*x^4+a)+1/12*(-a*f+b*x*(e*x^2+d*x+c))/a/b/(b*x^4+a)^3+5/32*d*arctan(
x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*b^(1/2))*(-15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1
/1024*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-15*e*a^(1/2)+77*c
*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4
))*(15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1/512*arctan(1+b^(1
/4)*x*2^(1/2)/a^(1/4))*(15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)}{256\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)}{256\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{5d\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}-\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)}+\frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2}-\frac{af-bx(c+dx+ex^2)}{12ab\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```

```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x +
45*e*x^2))/(384*a^3*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*x^2))/(12*a*b*(a
 + b*x^4)^3) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - (
(77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256
*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c -
 15*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512
*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx = -\frac{af-bx(c+dx+ex^2)}{12ab\,(a+bx^4)^3} - \frac{\int \frac{-11c-10dx-9ex^2}{(a+bx^4)^3}\,dx}{12a}$$

$$= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a+bx^4)^2} - \frac{af-bx(c+dx+ex^2)}{12ab\,(a+bx^4)^3} + \frac{\int \frac{77c+60dx+45ex^2}{(a+bx^4)^2}\,dx}{96a^2}$$

$$= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a+bx^4)} - \frac{af-bx(c+dx+ex^2)}{12ab\,(a+bx^4)^3} -$$

$$= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a+bx^4)} - \frac{af-bx(c+dx+ex^2)}{12ab\,(a+bx^4)^3} -$$

$$= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a+bx^4)} - \frac{af-bx(c+dx+ex^2)}{12ab\,(a+bx^4)^3} -$$

$$= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a+bx^4)} - \frac{af-bx(c+dx+ex^2)}{12ab\,(a+bx^4)^3} +$$

$$= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a+bx^4)} - \frac{af-bx(c+dx+ex^2)}{12ab\,(a+bx^4)^3} +$$

$$= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a+bx^4)} - \frac{af-bx(c+dx+ex^2)}{12ab\,(a+bx^4)^3} +$$

$$= \frac{x(11c+10dx+9ex^2)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a+bx^4)} - \frac{af-bx(c+dx+ex^2)}{12ab\,(a+bx^4)^3} +$$

Mathematica [A]
time = 0.25, size = 379, normalized size = 0.99

$$\frac{\frac{8ax(77c+15x(4d+3ex))}{a+bx^4} + \frac{32a^2x(11c+x(10d+9ex))}{(a+bx^4)^2} - \frac{256a^3(af-bx(c+x(d+ex)))}{b(a+bx^4)^3} - \frac{6\sqrt[4]{a}\left(77\sqrt{2}\sqrt{b}\,c+80\sqrt[4]{a}\sqrt[4]{b}\,d+15\sqrt{2}\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{6\sqrt[4]{a}\left(77\sqrt{2}\sqrt{b}\,c-80\sqrt[4]{a}\sqrt[4]{b}\,d+15\sqrt{2}\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{3\sqrt{2}\left(-77\sqrt[4]{a}\sqrt{b}\,c+15a^{3/4}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{b^{3/4}} + \frac{3\sqrt{2}\left(77\sqrt[4]{a}\sqrt{b}\,c-15a^{3/4}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{b^{3/4}}}{3072a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10*
d + 9*e*x)))/(a + b*x^4)^2 - (256*a^3*(a*f - b*x*(c + x*(d + e*x))))/(b*(a
+ b*x^4)^3) - (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)*b^(1/4)*d + 15*
```

```
Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (6*a^
(1/4)*(77*Sqrt[2]*Sqrt[b]*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^(1/4)*
Sqrt[b]*c + 15*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/b^(3/4) + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(3072*a^4)
```

Maple [A]

time = 0.36, size = 340, normalized size = 0.89

method	result
risch	$\frac{\frac{15 e\,b^{2} x^{11}}{128 a^{3}}+\frac{5 d\,b^{2} x^{10}}{32 a^{3}}+\frac{77 c\,b^{2} x^{9}}{384 a^{3}}+\frac{21 b e\,x^{7}}{64 a^{2}}+\frac{5 b d\,x^{6}}{12 a^{2}}+\frac{33 b c\,x^{5}}{64 a^{2}}+\frac{113 e\,x^{3}}{384 a}+\frac{11 d\,x^{2}}{32 a}+\frac{51 c x}{128 a}-\frac{f}{12 b}}{\left(b\,x^{4}+a\right)^{3}}+\frac{\sum_{\_R =\mathrm{RootOf}\left(b\_Z^{4}+a\right)}\frac{\left(15\_R^{2} e+40\_R d \cdots\right.}{\_R \cdots}}{512 a^{3} b}$
default	$\frac{\frac{15 e\,b^{2} x^{11}}{128 a^{3}}+\frac{5 d\,b^{2} x^{10}}{32 a^{3}}+\frac{77 c\,b^{2} x^{9}}{384 a^{3}}+\frac{21 b e\,x^{7}}{64 a^{2}}+\frac{5 b d\,x^{6}}{12 a^{2}}+\frac{33 b c\,x^{5}}{64 a^{2}}+\frac{113 e\,x^{3}}{384 a}+\frac{11 d\,x^{2}}{32 a}+\frac{51 c x}{128 a}-\frac{f}{12 b}}{\left(b\,x^{4}+a\right)^{3}}+\frac{77 c \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right) \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32/a^3*d*b^2*x^10+77/384*c/a^3*b^2*x^9+21/64*b*e/a
^2*x^7+5/12/a^2*b*d*x^6+33/64*b*c/a^2*x^5+113/384/a*e*x^3+11/32*d/a*x^2+51/
128/a*c*x-1/12*f/b)/(b*x^4+a)^3+1/128/a^3*(77/8*c*(a/b)^(1/4)/a*2^(1/2)*(ln
((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
+20*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+15/8*e/b/(a/b)^(1/4)*2^(1/2)*(ln(
(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1
/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]

time = 0.51, size = 409, normalized size = 1.07

$$\frac{45\,b^{3}x^{11}e+60\,b^{3}dx^{10}+77\,b^{3}cx^{9}+126\,ab^{2}x^{7}e+160\,ab^{2}dx^{6}+198\,ab^{2}cx^{5}+113\,a^{2}bx^{3}e+132\,a^{2}bdx^{2}+153\,a^{2}bcx-32\,a^{3}f}{384\left(a^{3}b^{4}x^{12}+3\,a^{4}b^{3}x^{8}+3\,a^{5}b^{2}x^{4}+a^{6}b\right)}+\frac{\frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+15\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-80\,\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+15\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e+80\,\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{1024\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*b^3*c*x^9 + 126*a*b^2*x^7*e + 160
*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 + 153*a^
2*b*c*x - 32*a^3*f)/(a^3*b^4*x^12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b)
+ 1/1024*(sqrt(2)*(77*sqrt(b)*c - 15*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*sqrt(b)*c - 15*
```

```
sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*
b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e -
 80*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^
(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(
77*sqrt(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e + 80*sqrt(a)*sq
rt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sq
rt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a^3
```

Fricas [C] Result contains complex when optimal does not.
time = 16.51, size = 125011, normalized size = 327.25

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] 1/9437184*(1105920*b^3*e*x^11 + 1474560*b^3*d*x^10 + 1892352*b^3*c*x^9 + 30
96576*a*b^2*e*x^7 + 3932160*a*b^2*d*x^6 + 4866048*a*b^2*c*x^5 + 2777088*a^2
*b*e*x^3 + 3244032*a^2*b*d*x^2 + 3760128*a^2*b*c*x - 786432*a^3*f + 2*(a^3*
b^4*x^12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b)*((-I*sqrt(3) + 1)*((a^4*b
*sqrt(-1/(a*b))*sqrt(-(2310*a*b*c*e*sqrt(-1/(a*b)) + 5929*b*c^2 - 225*a*e^2
)/(a^8*b^2*sqrt(-1/(a*b)))) - 40*d)^2/(a^7*b) - 3*(80*a^4*b*d*sqrt(-(2310*a
*b*c*e*sqrt(-1/(a*b)) + 5929*b*c^2 - 225*a*e^2)/(a^8*b^2*sqrt(-1/(a*b)))) +
 5929*b*c^2 - 5*(2*(160*d^2 + 231*c*e)*b*sqrt(-1/(a*b)) + 45*e^2)*a)/(a^8*b
^2*sqrt(-1/(a*b)))))/(-1/805306368*(80*a^4*b*d*sqrt(-(2310*a*b*c*e*sqrt(-1/(
a*b)) + 5929*b*c^2 - 225*a*e^2)/(a^8*b^2*sqrt(-1/(a*b)))) + 5929*b*c^2 - 5*
(2*(160*d^2 + 231*c*e)*b*sqrt(-1/(a*b)) + 45*e^2)*a)*(a^4*b*sqrt(-1/(a*b))*
sqrt(-(2310*a*b*c*e*sqrt(-1/(a*b)) + 5929*b*c^2 - 225*a*e^2)/(a^8*b^2*sqrt(
-1/(a*b)))) - 40*d)/(a^11*b^2) + 1/268435456*(a^11*b^2*sqrt(-1/(a*b))*(-(23
10*a*b*c*e*sqrt(-1/(a*b)) ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.66, size = 391, normalized size = 1.02

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d + 77\,(ab^3)^{\frac{1}{4}}\,b^2c + 15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d + 77\,(ab^3)^{\frac{1}{4}}\,b^2c + 15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3} + \frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c - 15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2 + \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3} - \frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c - 15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2 - \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3} + \frac{45\,b^3x^{11}e + 60\,b^3dx^{10} + 77\,b^3cx^9 + 126\,ab^2x^7e + 160\,ab^2dx^6 + 198\,ab^2cx^5 + 113\,a^2bx^3e + 132\,a^2bdx^2 + 153\,a^2bcx - 32\,a^3f}{384\,(bx^4+a)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a*
b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2
*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqr
t(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b
)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*
b^3*c*x^9 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 113*a^2*b
*x^3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

Mupad [B]
time = 5.25, size = 879, normalized size = 2.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x)
```

```
[Out] symsum(log(-(b*(3375*a*e^3 - 123200*b*c*d^2 + 88935*b*c^2*e - 64000*b*d^3*x
 + 20185088*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 83
8860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*
z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*
b^2*c^4 + 50625*a^2*e^4, z, k)^2*a^7*b^2*c - 115200*root(68719476736*a^15*b
^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680
*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a
*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)*a^4*
b*e^2*x + 92400*b*c*d*e*x + 3035648*root(68719476736*a^15*b^3*z^4 + 1211105
280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z
 + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 256
0000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)*a^3*b^2*c^2*x - 1048
5760*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800
*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 739
2000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4
 + 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x + 614400*root(68719476736*a^15*b^3*z^
4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*
b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)*a^4*b*d*e
))/(2097152*a^9))*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^
2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*
d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 351
53041*b^2*c^4 + 50625*a^2*e^4, z, k), k, 1, 4) + ((11*d*x^2)/(32*a) - f/(12
*b) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9)/(384*a^3) + (
5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (33*b*c*x^5)/(64*a^2)
```

```
+ (5*b*d*x^6)/(12*a^2) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x
^4 + 3*a*b^2*x^8)
```

3.494 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=380

$$-\frac{c+dx+ex^2+fx^3}{12b\,(a+bx^4)^3}+\frac{x(d+2ex+3fx^2)}{96ab\,(a+bx^4)^2}+\frac{x(7d+12ex+15fx^2)}{384a^2b\,(a+bx^4)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}}-\frac{\left(7\sqrt{b}\,d+5\sqrt{a}\,f\right)}{256\sqrt{2}}$$

```
[Out] 1/12*(-f*x^3-e*x^2-d*x-c)/b/(b*x^4+a)^3+1/96*x*(3*f*x^2+2*e*x+d)/a/b/(b*x^4
+a)^2+1/384*x*(15*f*x^2+12*e*x+7*d)/a^2/b/(b*x^4+a)+1/32*e*arctan(x^2*b^(1/
2)/a^(1/2))/a^(5/2)/b^(3/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*b^(1/2))*(-5*f*a^(1/2)+7*d*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/1024*ln(a^
(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*f*a^(1/2)+7*d*b^(1/2))/a^(
11/4)/b^(7/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*f*a^(1/
2)+7*d*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/512*arctan(1+b^(1/4)*x*2^(1/2)/a
^(1/4))*(5*f*a^(1/2)+7*d*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1837, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\,f+7\sqrt{b}\,d\right)}{256\sqrt{2}\,a^{11/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\,f+7\sqrt{b}\,d\right)}{256\sqrt{2}\,a^{11/4}b^{7/4}}+\frac{e\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}}-\frac{\left(7\sqrt{b}\,d-5\sqrt{a}\,f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{11/4}b^{7/4}}+\frac{\left(7\sqrt{b}\,d-5\sqrt{a}\,f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{11/4}b^{7/4}}+\frac{x(7d+12ex+15fx^2)}{384a^2b\,(a+bx^4)}-\frac{c+dx+ex^2+fx^3}{12b\,(a+bx^4)^3}+\frac{x(d+2ex+3fx^2)}{96ab\,(a+bx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

```
[Out] -1/12*(c + d*x + e*x^2 + f*x^3)/(b*(a + b*x^4)^3) + (x*(d + 2*e*x + 3*f*x^2
))/(96*a*b*(a + b*x^4)^2) + (x*(7*d + 12*e*x + 15*f*x^2))/(384*a^2*b*(a + b
*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(5/2)*b^(3/2)) - ((7*Sqrt[
b]*d + 5*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a
^(11/4)*b^(7/4)) + ((7*Sqrt[b]*d + 5*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(256*Sqrt[2]*a^(11/4)*b^(7/4)) - ((7*Sqrt[b]*d - 5*Sqrt[a]*f)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(11
/4)*b^(7/4)) + ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx = -\frac{c+dx+ex^2+fx^3}{12b\,(a+bx^4)^3} + \frac{\int \frac{d+2ex+3fx^2}{(a+bx^4)^3}\,dx}{12b}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\,(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab\,(a+bx^4)^2} - \frac{\int \frac{-7d-12ex-15fx^2}{(a+bx^4)^2}\,dx}{96ab}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\,(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab\,(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b\,(a+bx^4)} + \int$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\,(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab\,(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b\,(a+bx^4)} + \int$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\,(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab\,(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b\,(a+bx^4)} + \int$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\,(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab\,(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b\,(a+bx^4)} + \frac{eS}{}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\,(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab\,(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b\,(a+bx^4)} + \frac{e}{}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\,(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab\,(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b\,(a+bx^4)} + \frac{e}{}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\,(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab\,(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b\,(a+bx^4)} + \frac{e}{}$$

Mathematica [A]
time = 0.27, size = 366, normalized size = 0.96

$$\frac{\frac{32b^{3/4}x(d+x(2e+3fx))}{a(a+bx^4)^2} + \frac{8b^{3/4}x(7d+3x(4e+5fx))}{a^2(a+bx^4)} - \frac{256b^{3/4}(c+x(d+x(e+fx)))}{(a+bx^4)^3} - \frac{6\left(7\sqrt{2}\sqrt{b}\,d+16\sqrt[4]{a}\sqrt[4]{b}\,e+5\sqrt{2}\sqrt{a}\,f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{11/4}} + \frac{6\left(7\sqrt{2}\sqrt{b}\,d-16\sqrt[4]{a}\sqrt[4]{b}\,e+5\sqrt{2}\sqrt{a}\,f\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{11/4}} + \frac{3\sqrt{2}\left(-7\sqrt{b}\,d+5\sqrt{a}\,f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{11/4}} + \frac{3\sqrt{2}\left(7\sqrt{b}\,d-5\sqrt{a}\,f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{11/4}}}{3072b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

```
[Out] ((32*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)^2) + (8*b^(3/4)*x*(7*d
 + 3*x*(4*e + 5*f*x)))/(a^2*(a + b*x^4)) - (256*b^(3/4)*(c + x*(d + x*(e +
f*x))))/(a + b*x^4)^3 - (6*(7*Sqrt[2]*Sqrt[b]*d + 16*a^(1/4)*b^(1/4)*e + 5*
```

```
Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*(
7*Sqrt[2]*Sqrt[b]*d - 16*a^(1/4)*b^(1/4)*e + 5*Sqrt[2]*Sqrt[a]*f)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(11/4) + (3*Sqrt[2]*(-7*Sqrt[b]*d + 5*Sqr
t[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(11/4) +
(3*Sqrt[2]*(7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/a^(11/4))/(3072*b^(7/4))
```

Maple [A]

time = 0.37, size = 334, normalized size = 0.88

method	result
risch	$\frac{\frac{5bf\,x^{11}}{128a^2}+\frac{be\,x^{10}}{32a^2}+\frac{7bd\,x^9}{384a^2}+\frac{7f\,x^7}{64a}+\frac{e\,x^6}{12a}+\frac{3d\,x^5}{64a}-\frac{5f\,x^3}{384b}-\frac{e\,x^2}{32b}-\frac{7dx}{128b}-\frac{c}{12b}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^4+a)}\frac{(5f\_R^2+8e\_R+7d)\ln(x-\_R)}{\_R^3}}{512a^2b^2}$
default	$\frac{\frac{5bf\,x^{11}}{128a^2}+\frac{be\,x^{10}}{32a^2}+\frac{7bd\,x^9}{384a^2}+\frac{7f\,x^7}{64a}+\frac{e\,x^6}{12a}+\frac{3d\,x^5}{64a}-\frac{5f\,x^3}{384b}-\frac{e\,x^2}{32b}-\frac{7dx}{128b}-\frac{c}{12b}}{(b\,x^4+a)^3}+\frac{\frac{7d\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\ldots\right.\right.}{8a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (5/128/a^2*b*f*x^11+1/32*b*e/a^2*x^10+7/384/a^2*b*d*x^9+7/64*f/a*x^7+1/12/a
*e*x^6+3/64*d/a*x^5-5/384*f*x^3/b-1/32*e*x^2/b-7/128*d*x/b-1/12*c/b)/(b*x^4
+a)^3+1/128/a^2/b*(7/8*d*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/
2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(
a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+4*e/(a*b)^(1/2)*arctan(x
^2*(b/a)^(1/2))+5/8*f/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+
(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b
)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]

time = 0.50, size = 401, normalized size = 1.06

$$\frac{15\,b^2fx^{11}+12\,b^2x^{10}e+7\,b^2dx^9+42\,abfx^7+32\,abx^6e+18\,abdx^5-5\,a^2fx^3-12\,a^2x^2e-21\,a^2dx-32\,a^2c}{384\,(a^2b^4x^{12}+3\,a^3b^3x^8+3\,a^4b^2x^4+a^5b)}+\frac{\frac{\sqrt{2}\left(7\sqrt{b}\,d-5\sqrt{a}\,f\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(7\sqrt{b}\,d-5\sqrt{a}\,f\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(7\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}d+5\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}f-16\sqrt{a}\sqrt{b}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(7\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}d+5\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}f+16\sqrt{a}\sqrt{b}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{1024\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/384*(15*b^2*f*x^11 + 12*b^2*x^10*e + 7*b^2*d*x^9 + 42*a*b*f*x^7 + 32*a*b*
x^6*e + 18*a*b*d*x^5 - 5*a^2*f*x^3 - 12*a^2*x^2*e - 21*a^2*d*x - 32*a^2*c)/
(a^2*b^4*x^12 + 3*a^3*b^3*x^8 + 3*a^4*b^2*x^4 + a^5*b) + 1/1024*(sqrt(2)*(7
*sqrt(b)*d - 5*sqrt(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqr
t(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(7*sqrt(b)*d - 5*sqrt(a)*f)*log(sqrt(b)*x
```

```
^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(7*sqrt(2)*
a^(1/4)*b^(3/4)*d + 5*sqrt(2)*a^(3/4)*b^(1/4)*f - 16*sqrt(a)*sqrt(b)*e)*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*b^(3/4)*
d + 5*sqrt(2)*a^(3/4)*b^(1/4)*f + 16*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*
(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqr
t(sqrt(a)*sqrt(b))*b^(3/4)))/(a^2*b)
```

Fricas [C] Result contains complex when optimal does not.
time = 17.04, size = 125996, normalized size = 331.57

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] 1/9437184*(368640*b^2*f*x^11 + 294912*b^2*e*x^10 + 172032*b^2*d*x^9 + 10321
92*a*b*f*x^7 + 786432*a*b*e*x^6 + 442368*a*b*d*x^5 - 122880*a^2*f*x^3 - 294
912*a^2*e*x^2 - 516096*a^2*d*x - 786432*a^2*c + 2*(a^2*b^4*x^12 + 3*a^3*b^3
*x^8 + 3*a^4*b^2*x^4 + a^5*b)*((-I*sqrt(3) + 1)*((a^3*b^2*sqrt(-1/(a*b))*sq
rt(-(70*a*b*d*f*sqrt(-1/(a*b)) + 49*b*d^2 - 25*a*f^2)/(a^6*b^4*sqrt(-1/(a*b
)))) - 8*e)^2/(a^5*b^3) - 3*(16*a^3*b^2*e*sqrt(-(70*a*b*d*f*sqrt(-1/(a*b))
+ 49*b*d^2 - 25*a*f^2)/(a^6*b^4*sqrt(-1/(a*b)))) + 49*b*d^2 - (2*(32*e^2 +
35*d*f)*b*sqrt(-1/(a*b)) + 25*f^2)*a)/(a^6*b^4*sqrt(-1/(a*b))))/(-1/8053063
68*(16*a^3*b^2*e*sqrt(-(70*a*b*d*f*sqrt(-1/(a*b)) + 49*b*d^2 - 25*a*f^2)/(a
^6*b^4*sqrt(-1/(a*b)))) + 49*b*d^2 - (2*(32*e^2 + 35*d*f)*b*sqrt(-1/(a*b))
+ 25*f^2)*a)*(a^3*b^2*sqrt(-1/(a*b))*sqrt(-(70*a*b*d*f*sqrt(-1/(a*b)) + 49*
b*d^2 - 25*a*f^2)/(a^6*b^4*sqrt(-1/(a*b)))) - 8*e)/(a^8*b^5) + 1/268435456*
(a^8*b^5*sqrt(-1/(a*b))*(-(70*a*b*d*f*sqrt(-1/(a*b)) + 49*b*d^2 - 25*a*f^2)
/(a^6*b^4*sqrt(-1/(a*b))) ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 380, normalized size = 1.00

$$\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{ab}\,b^{2}e+7\left(ab^{3}\right)^{\frac{1}{4}}b^{2}d+5\left(ab^{3}\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^{3}b^{4}}+\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{ab}\,b^{2}e+7\left(ab^{3}\right)^{\frac{1}{4}}b^{2}d+5\left(ab^{3}\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^{3}b^{4}}+\frac{\sqrt{2}\left(7\left(ab^{3}\right)^{\frac{1}{4}}b^{2}d-5\left(ab^{3}\right)^{\frac{3}{4}}f\right)\log\left(x^{2}+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^{3}b^{4}}-\frac{\sqrt{2}\left(7\left(ab^{3}\right)^{\frac{1}{4}}b^{2}d-5\left(ab^{3}\right)^{\frac{3}{4}}f\right)\log\left(x^{2}-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^{3}b^{4}}+\frac{15\,b^{2}fx^{11}+12\,b^{2}x^{10}e+7\,b^{2}dx^{9}+42\,abfx^{7}+32\,abx^{6}e+18\,abdx^{5}-5\,a^{2}fx^{3}-12\,a^{2}x^{2}e-21\,a^{2}dx-32\,a^{2}c}{384\left(bx^{4}+a\right)^{3}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d + 5*(a*b^3
)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3
*b^4) + 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d +
5*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(a^3*b^4) + 1/1024*sqrt(2)*(7*(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*
log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) - 1/1024*sqrt(2)*(7*
(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(a^3*b^4) + 1/384*(15*b^2*f*x^11 + 12*b^2*x^10*e + 7*b^2*d*x^9 +
 42*a*b*f*x^7 + 32*a*b*x^6*e + 18*a*b*d*x^5 - 5*a^2*f*x^3 - 12*a^2*x^2*e -
21*a^2*d*x - 32*a^2*c)/((b*x^4 + a)^3*a^2*b)
```

Mupad [B]
time = 0.48, size = 888, normalized size = 2.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x)
```

```
[Out] ((3*d*x^5)/(64*a) - c/(12*b) + (e*x^6)/(12*a) - (e*x^2)/(32*b) + (7*f*x^7)/
(64*a) - (5*f*x^3)/(384*b) - (7*d*x)/(128*b) + (7*b*d*x^9)/(384*a^2) + (b*e
*x^10)/(32*a^2) + (5*b*f*x^11)/(128*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3
*a*b^2*x^8) + symsum(log(-(125*a*f^3 - 448*b*d*e^2 + 245*b*d^2*f - 512*b*e^
3*x + 1835008*root(68719476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33
554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z -
8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^2
*d^4, z, k)^2*a^5*b^4*d + 560*b*d*e*f*x + 25088*root(68719476736*a^11*b^7*z
^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e
*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 409
6*a*b*e^4 + 625*a^2*f^4 + 2401*b^2*d^4, z, k)*a^2*b^3*d^2*x - 2097152*root(
68719476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*
z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f +
2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^2*d^4, z, k)^2*a^5*b
^4*e*x - 12800*root(68719476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 3
3554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z -
 8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^
2*d^4, z, k)*a^3*b^2*f^2*x + 40960*root(68719476736*a^11*b^7*z^4 + 36700160
*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 8028
16*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 6
25*a^2*f^4 + 2401*b^2*d^4, z, k)*a^3*b^2*e*f)/(2097152*a^6*b^2))*root(68719
476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 +
 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*
a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^2*d^4, z, k), k, 1, 4)
```

3.495 $\int x^4(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

Optimal. Leaf size=418

$$\frac{2acx\sqrt{a+bx^4}}{21b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}-\frac{2a^2ex\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+bx^4}-$$

```
[Out] 1/10*f*x^4*(b*x^4+a)^(3/2)/b-1/120*(-15*b*d*x^2+8*a*f)*(b*x^4+a)^(3/2)/b^2-
1/16*a^2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+2/21*a*c*x*(b*x^4+a
)^(1/2)/b-1/16*a*d*x^2*(b*x^4+a)^(1/2)/b+2/45*a*e*x^3*(b*x^4+a)^(1/2)/b+1/6
3*x^5*(7*e*x^2+9*c)*(b*x^4+a)^(1/2)-2/15*a^2*e*x*(b*x^4+a)^(1/2)/b^(3/2)/(a
^(1/2)+x^2*b^(1/2))+2/15*a^(9/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4
))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^
(1/2)/b^(7/4)/(b*x^4+a)^(1/2)-1/105*a^(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)
*x/a^(1/4))),1/2*2^(1/2))*(7*e*a^(1/2)+5*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*(
(b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1847, 1288, 1294, 1212, 226, 1210, 1266, 847, 794, 201, 223, 212}

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(7\sqrt{a}\,e+5\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}-\frac{a^2d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2ex\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{\left(a+bx^4\right)^{3/2}\left(8af-15bdx^2\right)}{120b^2}+\frac{1}{63}x^5\sqrt{a+bx^4}\left(9c+7ex^2\right)+\frac{2acx\sqrt{a+bx^4}}{21b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}+\frac{fx^4\left(a+bx^4\right)^{3/2}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (2*a*c*x*Sqrt[a + b*x^4])/(21*b) - (a*d*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a*
e*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*e*x*Sqrt[a + b*x^4])/(15*b^(3/2)*(Sq
rt[a] + Sqrt[b]*x^2)) + (x^5*(9*c + 7*e*x^2)*Sqrt[a + b*x^4])/63 + (f*x^4*(
a + b*x^4)^(3/2))/(10*b) - ((8*a*f - 15*b*d*x^2)*(a + b*x^4)^(3/2))/(120*b^
2) - (a^2*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) + (2*a^(9/
4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^4]) -
 (a^(7/4)*(5*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(105*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int x^4\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx = \int \left(x^4\left(c+ex^2\right)\sqrt{a+bx^4}+x^5\left(d+fx^2\right)\sqrt{a+bx^4}\right)dx$$

$$= \int x^4\left(c+ex^2\right)\sqrt{a+bx^4}\,dx+\int x^5\left(d+fx^2\right)\sqrt{a+bx^4}\,dx$$

$$= \frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+bx^4}+\frac{1}{2}\text{Subst}\left(\int x^2(d+fx)\sqrt{a+bx^2}\right.$$

$$= \frac{2aex^3\sqrt{a+bx^4}}{45b}+\frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+bx^4}+\frac{fx^4(a+bx^4)^{3/}}{10b}$$

$$= \frac{2acx\sqrt{a+bx^4}}{21b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}+\frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+bx^4}$$

$$= \frac{2acx\sqrt{a+bx^4}}{21b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}+\frac{1}{63}x^5\left(9c\right.$$

$$= \frac{2acx\sqrt{a+bx^4}}{21b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}-\frac{2a^2e}{15b^{3/2}\left(\right.}$$

$$= \frac{2acx\sqrt{a+bx^4}}{21b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}-\frac{2a^2e}{15b^{3/2}\left(\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.45, size = 202, normalized size = 0.48

$$\frac{\sqrt{a+bx^4}\left(720bcx(a+bx^4)+560bex^3(a+bx^4)+315bdx^2(a+2bx^4)+168f(a+bx^4)(-2a+3bx^4)-\frac{315a^{3/2}\sqrt{b}\,d\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{1+\frac{bx^4}{a}}}-\frac{720abcx\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}-\frac{560abex^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)}{5040b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*(720*b*c*x*(a + b*x^4) + 560*b*e*x^3*(a + b*x^4) + 315*b*d
*x^2*(a + 2*b*x^4) + 168*f*(a + b*x^4)*(-2*a + 3*b*x^4) - (315*a^(3/2)*Sqrt
[b]*d*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a] - (720*a*b*c*x*Hy
pergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] - (560*a
```

*b*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/(5040*b^2)

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 331, normalized size = 0.79

method	result
default	$-\frac{f\left(b\,x^{4}+a\right)^{\frac{3}{2}}\left(-3b\,x^{4}+2a\right)}{30b^{2}}+e\left(\frac{x^{7}\sqrt{b\,x^{4}+a}}{9}+\frac{2a\,x^{3}\sqrt{b\,x^{4}+a}}{45b}-\frac{2ia^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\right.}{15b^{\frac{3}{2}}\sqrt{\frac{i}{}}}\right.$
risch	$-\frac{\left(-504b^{2}f\,x^{8}-560b^{2}e\,x^{7}-630b^{2}d\,x^{6}-720b^{2}c\,x^{5}-168abf\,x^{4}-224abe\,x^{3}-315abd\,x^{2}-480abcx+336a^{2}f\right)\sqrt{b\,x^{4}+a}}{5040b^{2}}-\frac{2ia^{\frac{5}{2}}e}{}$
elliptic	$\frac{f\,x^{8}\sqrt{b\,x^{4}+a}}{10}+\frac{e\,x^{7}\sqrt{b\,x^{4}+a}}{9}+\frac{d\,x^{6}\sqrt{b\,x^{4}+a}}{8}+\frac{c\,x^{5}\sqrt{b\,x^{4}+a}}{7}+\frac{af\,x^{4}\sqrt{b\,x^{4}+a}}{30b}+\frac{2ae\,x^{3}\sqrt{b\,x}}{45b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*f*(b*x^4+a)^(3/2)*(-3*b*x^4+2*a)/b^2+e*(1/9*x^7*(b*x^4+a)^(1/2)+2/45*
a/b*x^3*(b*x^4+a)^(1/2)-2/15*I/b^(3/2)*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1
-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)))+d*(1/8*x^2*(b*x^4+a)^(3/2)/b-1/16*a/b*x^2*(b*x^4+a)^(1/2)-1/16
*a^2/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+c*(1/7*x^5*(b*x^4+a)^(1/2)+2/
21*a/b*x*(b*x^4+a)^(1/2)-2/21/b*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elliptic
F(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)*x^4, x)
```

Fricas [A]
time = 0.12, size = 214, normalized size = 0.51

$$-\frac{1344\,a^{2}\sqrt{b}\,ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1) - 315\,a^{2}\sqrt{b}\,dx\log\left(-2\,bx^{4} + 2\,\sqrt{bx^{4}+a}\,\sqrt{b}\,x^{2} - a\right) + 192\,(5\,abc - 7\,a^{2}e)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1) - 2\,(504\,b^{2}fx^{9} + 560\,b^{2}ex^{8} + 630\,b^{2}dx^{7} + 720\,b^{2}cx^{6} + 168\,abfx^{5} + 224\,abex^{4} + 315\,abdx^{3} + 480\,abcx^{2} - 336\,a^{2}fx - 672\,a^{2}e)\sqrt{bx^{4}+a}}{10080\,b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/10080*(1344*a^2*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/
x), -1) - 315*a^2*sqrt(b)*d*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2
- a) + 192*(5*a*b*c - 7*a^2*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a
/b)^(1/4)/x), -1) - 2*(504*b^2*f*x^9 + 560*b^2*e*x^8 + 630*b^2*d*x^7 + 720*
b^2*c*x^6 + 168*a*b*f*x^5 + 224*a*b*e*x^4 + 315*a*b*d*x^3 + 480*a*b*c*x^2 -
 336*a^2*f*x - 672*a^2*e)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A]
time = 3.71, size = 252, normalized size = 0.60

$$\frac{a^{\frac{3}{2}}dx^{2}}{16b\sqrt{1+\frac{bx^{4}}{a}}} + \frac{\sqrt{a}\,cx^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}\,dx^{6}}{16\sqrt{1+\frac{bx^{4}}{a}}} + \frac{\sqrt{a}\,ex^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} - \frac{a^{2}d\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + f\left(\begin{cases}-\frac{a^{2}\sqrt{a+bx^{4}}}{15b^{2}} + \frac{ax^{4}\sqrt{a+bx^{4}}}{30b} + \frac{x^{8}\sqrt{a+bx^{4}}}{10} & \text{for}\ b\neq 0\\\frac{\sqrt{a}\,x^{8}}{8} & \text{otherwise}\end{cases}\right) + \frac{bdx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^{4}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] a**(3/2)*d*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*c*x**5*gamma(5/4)*hyper
((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*
d*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*e*x**7*gamma(7/4)*hyper((-1/2, 7/4
), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) - a**2*d*asinh(sqrt(b
)*x**2/sqrt(a))/(16*b**(3/2)) + f*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**
2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)),
(sqrt(a)*x**8/8, True)) + b*d*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
[Out] int(x^4*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.496 $\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

Optimal. Leaf size=394

$$\frac{2adx\sqrt{a+bx^4}}{21b}-\frac{aex^2\sqrt{a+bx^4}}{16b}+\frac{2afx^3\sqrt{a+bx^4}}{45b}-\frac{2a^2fx\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{63}x^5\left(9d+7fx^2\right)\sqrt{a+bx^4}+$$

```
[Out] 1/24*(3*e*x^2+4*c)*(b*x^4+a)^(3/2)/b-1/16*a^2*e*arctanh(x^2*b^(1/2)/(b*x^4+
a)^(1/2))/b^(3/2)+2/21*a*d*x*(b*x^4+a)^(1/2)/b-1/16*a*e*x^2*(b*x^4+a)^(1/2)
/b+2/45*a*f*x^3*(b*x^4+a)^(1/2)/b+1/63*x^5*(7*f*x^2+9*d)*(b*x^4+a)^(1/2)-2/
15*a^2*f*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))+2/15*a^(9/4)*f*(co
s(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*El
lipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))
*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)-1/105*a^
(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(
1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(7*f*a^(1/2)
+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/
2)/b^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1847, 1266, 794, 201, 223, 212, 1288, 1294, 1212, 226, 1210}

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(7\sqrt{a}\,f+5\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}-\frac{a^2e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2fx\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{\left(a+bx^4\right)^{3/2}\left(4c+3ex^2\right)}{24b}+\frac{1}{63}x^5\sqrt{a+bx^4}\left(9d+7fx^2\right)+\frac{2adx\sqrt{a+bx^4}}{21b}-\frac{aex^2\sqrt{a+bx^4}}{16b}+\frac{2afx^3\sqrt{a+bx^4}}{45b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (2*a*d*x*Sqrt[a + b*x^4])/(21*b) - (a*e*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a*
f*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*f*x*Sqrt[a + b*x^4])/(15*b^(3/2)*(Sq
rt[a] + Sqrt[b]*x^2)) + (x^5*(9*d + 7*f*x^2)*Sqrt[a + b*x^4])/63 + ((4*c +
3*e*x^2)*(a + b*x^4)^(3/2))/(24*b) - (a^2*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/(16*b^(3/2)) + (2*a^(9/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(15*b^(7/4)*Sqrt[a + b*x^4]) - (a^(7/4)*(5*Sqrt[b]*d + 7*Sqrt[a]*f)*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \int \left(x^3(c+ex^2)\sqrt{a+bx^4}+x^4(d+fx^2)\sqrt{a+bx^4}\right)dx$$

$$= \int x^3(c+ex^2)\sqrt{a+bx^4}\,dx + \int x^4(d+fx^2)\sqrt{a+bx^4}\,dx$$

$$= \frac{1}{63}x^5(9d+7fx^2)\sqrt{a+bx^4} + \frac{1}{2}\text{Subst}\left(\int x(c+ex)\sqrt{a+bx^2}\right.$$

$$= \frac{2afx^3\sqrt{a+bx^4}}{45b} + \frac{1}{63}x^5(9d+7fx^2)\sqrt{a+bx^4} + \frac{(4c+3ex^2)}{24}$$

$$= \frac{2adx\sqrt{a+bx^4}}{21b} - \frac{aex^2\sqrt{a+bx^4}}{16b} + \frac{2afx^3\sqrt{a+bx^4}}{45b} + \frac{1}{63}x^5($$

$$= \frac{2adx\sqrt{a+bx^4}}{21b} - \frac{aex^2\sqrt{a+bx^4}}{16b} + \frac{2afx^3\sqrt{a+bx^4}}{45b} + \frac{1}{63}x^5($$

$$= \frac{2adx\sqrt{a+bx^4}}{21b} - \frac{aex^2\sqrt{a+bx^4}}{16b} + \frac{2afx^3\sqrt{a+bx^4}}{45b} - \frac{2a^2}{15b^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.42, size = 215, normalized size = 0.55

$$\frac{\sqrt{a+bx^4}\left(168\sqrt{b}\,c(a+bx^4)+144\sqrt{b}\,dx(a+bx^4)+112\sqrt{b}\,fx^3(a+bx^4)+63e\left(\sqrt{b}\,x^2(a+2bx^4)-\frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)-\frac{144a\sqrt{b}\,dx\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}-\frac{112a\sqrt{b}\,fx^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)}{1008b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*(168*Sqrt[b]*c*(a + b*x^4) + 144*Sqrt[b]*d*x*(a + b*x^4) +
 112*Sqrt[b]*f*x^3*(a + b*x^4) + 63*e*(Sqrt[b]*x^2*(a + 2*b*x^4) - (a^(3/2)
*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]) - (144*a*Sqrt[b]*d*x*
Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] - (112
*a*Sqrt[b]*f*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 +
(b*x^4)/a]))/(1008*b^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 321, normalized size = 0.81

method	result
default	$f\left(\frac{x^{7}\sqrt{b\,x^{4}+a}}{9}+\frac{2a\,x^{3}\sqrt{b\,x^{4}+a}}{45b}-\frac{2ia^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{Ellipt}\right.}{15b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.$
risch	$\frac{\left(560bf\,x^{7}+630be\,x^{6}+720bd\,x^{5}+840bc\,x^{4}+224af\,x^{3}+315ae\,x^{2}+480adx+840ac\right)\sqrt{b\,x^{4}+a}}{5040b}-\frac{2ia^{\frac{5}{2}}f\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+}}{15b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{}}{\sqrt{}}}}$
elliptic	$\frac{f\,x^{7}\sqrt{b\,x^{4}+a}}{9}+\frac{e\,x^{6}\sqrt{b\,x^{4}+a}}{8}+\frac{d\,x^{5}\sqrt{b\,x^{4}+a}}{7}+\frac{x^{4}c\sqrt{b\,x^{4}+a}}{6}+\frac{2af\,x^{3}\sqrt{b\,x^{4}+a}}{45b}+\frac{ae\,x^{2}\sqrt{b\,x^{4}}}{16b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/9*x^7*(b*x^4+a)^(1/2)+2/45*a/b*x^3*(b*x^4+a)^(1/2)-2/15*I/b^(3/2)*a^(5
/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+e*(1/8*x^2*(b*x^4+a)^(3/2)/b-1
/16*a/b*x^2*(b*x^4+a)^(1/2)-1/16*a^2/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)
))+d*(1/7*x^5*(b*x^4+a)^(1/2)+2/21*a/b*x*(b*x^4+a)^(1/2)-2/21/b*a^2/(I/a^(1
/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/6*c/b*(
b*x^4+a)^(3/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^4 + a)^(3/2)*c/b + integrate((f*x^6 + x^5*e + d*x^4)*sqrt(b*x^4 +
a), x)
```

Fricas [A]

time = 0.12, size = 205, normalized size = 0.52

$$-\frac{1344\,a^{2}\sqrt{b}\,fx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)-315\,a^{2}\sqrt{b}\,ex\log\left(-2\,bx^{4}+2\,\sqrt{bx^{4}+a}\,\sqrt{b}\,x^{2}-a\right)+192\,(5\,abd-7\,a^{2}f)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)-2\,(560\,b^{2}fx^{8}+630\,b^{2}ex^{7}+720\,b^{2}dx^{6}+840\,b^{2}cx^{5}+224\,abfx^{4}+315\,abex^{3}+480\,abdx^{2}+840\,abcx-672\,a^{2}f)\sqrt{bx^{4}+a}}{10080\,b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/10080*(1344*a^2*sqrt(b)*f*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/
x), -1) - 315*a^2*sqrt(b)*e*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2
- a) + 192*(5*a*b*d - 7*a^2*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a
/b)^(1/4)/x), -1) - 2*(560*b^2*f*x^8 + 630*b^2*e*x^7 + 720*b^2*d*x^6 + 840*
b^2*c*x^5 + 224*a*b*f*x^4 + 315*a*b*e*x^3 + 480*a*b*d*x^2 + 840*a*b*c*x - 6
72*a^2*f)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A]
time = 3.67, size = 212, normalized size = 0.54

$$\frac{a^{\frac{3}{2}}ex^2}{16b\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}\,dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}\,ex^6}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}\,fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}-\frac{a^2e\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+c\left(\begin{cases}\frac{\sqrt{a}\,x^4}{4} & \text{for}\ b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+\frac{bex^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] a**(3/2)*e*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*d*x**5*gamma(5/4)*hyper
((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*
e*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*f*x**7*gamma(7/4)*hyper((-1/2, 7/4
), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) - a**2*e*asinh(sqrt(b
)*x**2/sqrt(a))/(16*b**(3/2)) + c*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a
 + b*x**4)**(3/2)/(6*b), True)) + b*e*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] int(x^3*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.497 $\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

Optimal. Leaf size=369

$$\frac{2aex\sqrt{a+bx^4}}{21b}-\frac{afx^2\sqrt{a+bx^4}}{16b}+\frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{35}x^3\left(7c+5ex^2\right)\sqrt{a+bx^4}+\frac{(4d+3fx^2)\,(a+bx}{24b}$$

```
[Out] 1/24*(3*f*x^2+4*d)*(b*x^4+a)^(3/2)/b-1/16*a^2*f*arctanh(x^2*b^(1/2)/(b*x^4+
a)^(1/2))/b^(3/2)+2/21*a*e*x*(b*x^4+a)^(1/2)/b-1/16*a*f*x^2*(b*x^4+a)^(1/2)
/b+1/35*x^3*(5*e*x^2+7*c)*(b*x^4+a)^(1/2)+2/5*a*c*x*(b*x^4+a)^(1/2)/b^(1/2)
/(a^(1/2)+x^2*b^(1/2))-2/5*a^(5/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(
1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2
)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+1/105*a^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/
4)*x/a^(1/4))),1/2*2^(1/2))*(-5*e*a^(1/2)+21*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2
))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1847, 1288, 1294, 1212, 226, 1210, 1266, 794, 201, 223, 212}

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}-\frac{a^2f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}+\frac{1}{35}x^3\sqrt{a+bx^4}\left(7c+5ex^2\right)+\frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{\left(a+bx^4\right)^{3/2}\left(4d+3fx^2\right)}{24b}+\frac{2aex\sqrt{a+bx^4}}{21b}-\frac{afx^2\sqrt{a+bx^4}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (2*a*e*x*Sqrt[a + b*x^4])/(21*b) - (a*f*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a*
c*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^3*(7*c + 5*e*
x^2)*Sqrt[a + b*x^4])/35 + ((4*d + 3*f*x^2)*(a + b*x^4)^(3/2))/(24*b) - (a^
2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) - (2*a^(5/4)*c*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)
*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqr
t[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*
b^(5/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \int \left(x^2(c+ex^2)\sqrt{a+bx^4}+x^3(d+fx^2)\sqrt{a+bx^4}\right)dx$$
$$= \int x^2(c+ex^2)\sqrt{a+bx^4}\,dx + \int x^3(d+fx^2)\sqrt{a+bx^4}\,dx$$
$$= \frac{1}{35}x^3(7c+5ex^2)\sqrt{a+bx^4} + \frac{1}{2}\text{Subst}\left(\int x(d+fx)\sqrt{a+bx^2}\right.$$
$$= \frac{2aex\sqrt{a+bx^4}}{21b} + \frac{1}{35}x^3(7c+5ex^2)\sqrt{a+bx^4} + \frac{(4d+3fx^2)(a}{24b}$$
$$= \frac{2aex\sqrt{a+bx^4}}{21b} - \frac{afx^2\sqrt{a+bx^4}}{16b} + \frac{1}{35}x^3(7c+5ex^2)\sqrt{a+bx$$
$$= \frac{2aex\sqrt{a+bx^4}}{21b} - \frac{afx^2\sqrt{a+bx^4}}{16b} + \frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + $$
$$= \frac{2aex\sqrt{a+bx^4}}{21b} - \frac{afx^2\sqrt{a+bx^4}}{16b} + \frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.48, size = 182, normalized size = 0.49

$$\frac{1}{336}\sqrt{a+bx^4}\left(\frac{56d(a+bx^4)}{b}+\frac{48ex(a+bx^4)}{b}+\frac{21fx^2(a+2bx^4)}{b}-\frac{21a^{3/2}f\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{b^{3/2}\sqrt{1+\frac{bx^4}{a}}}-\frac{48aex\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{b\sqrt{1+\frac{bx^4}{a}}}+\frac{112cx^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*((56*d*(a + b*x^4))/b + (48*e*x*(a + b*x^4))/b + (21*f*x^2
*(a + 2*b*x^4))/b - (21*a^(3/2)*f*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(b^(3/2)*
Sqrt[1 + (b*x^4)/a]) - (48*a*e*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4
)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (112*c*x^3*Hypergeometric2F1[-1/2, 3/4, 7/
4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/336
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 303, normalized size = 0.82

method	result
default	$f\left(\frac{x^{2}(b\,x^{4}+a)^{\frac{3}{2}}}{8b}-\frac{a\,x^{2}\sqrt{b\,x^{4}+a}}{16b}-\frac{a^{2}\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{16b^{\frac{3}{2}}}\right)+e\left(\frac{x^{5}\sqrt{b\,x^{4}+a}}{7}+\frac{2ax\sqrt{b\,x^{4}+a}}{21b}-\right.$
elliptic	$\frac{f\,x^{6}\sqrt{b\,x^{4}+a}}{8}+\frac{e\,x^{5}\sqrt{b\,x^{4}+a}}{7}+\frac{d\,x^{4}\sqrt{b\,x^{4}+a}}{6}+\frac{c\,x^{3}\sqrt{b\,x^{4}+a}}{5}+\frac{af\,x^{2}\sqrt{b\,x^{4}+a}}{16b}+\frac{2aex\sqrt{b\,x^{4}-}}{21b}$
risch	$\frac{(210bf\,x^{6}+240be\,x^{5}+280bd\,x^{4}+336bc\,x^{3}+105x^{2}af+160aex+280ad)\sqrt{b\,x^{4}+a}}{1680b}+\frac{2ia^{\frac{3}{2}}c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{5\sqrt{b}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/8*x^2*(b*x^4+a)^(3/2)/b-1/16*a/b*x^2*(b*x^4+a)^(1/2)-1/16*a^2/b^(3/2)*
ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+e*(1/7*x^5*(b*x^4+a)^(1/2)+2/21*a/b*x*(b*x
^4+a)^(1/2)-2/21/b*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^
(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I))+1/6*d/b*(b*x^4+a)^(3/2)+c*(1/5*x^3*(b*x^4+a)^(1/2)+2/5
*I*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)*x^2, x)
```

Fricas [A]
time = 0.12, size = 193, normalized size = 0.52

$$\frac{1344\,ab^{\frac{3}{2}}cx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+105\,a^{2}\sqrt{b}\,fx\log\left(-2\,bx^{4}+2\,\sqrt{bx^{4}+a}\,\sqrt{b}\,x^{2}-a\right)-64\,(21\,abc+5\,abe)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+2\,(210\,b^{2}fx^{7}+240\,b^{2}ex^{6}+280\,b^{2}dx^{5}+336\,b^{2}cx^{4}+105\,abfx^{3}+160\,abex^{2}+280\,abdx+672\,abc)\sqrt{bx^{4}+a}}{3360\,b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] 1/3360*(1344*a*b^(3/2)*c*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) + 105*a^2*sqrt(b)*f*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) - 64*(21*a*b*c + 5*a*b*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 2*(210*b^2*f*x^7 + 240*b^2*e*x^6 + 280*b^2*d*x^5 + 336*b^2*c*x^4 + 105*a*b*f*x^3 + 160*a*b*e*x^2 + 280*a*b*d*x + 672*a*b*c)*sqrt(b*x^4 + a))/(b^2*x)

Sympy [A]
time = 3.58, size = 212, normalized size = 0.57

$$\frac{a^{\frac{3}{2}} f x^{2}}{16 b \sqrt{1 + \frac{b x^{4}}{a}}} + \frac{\sqrt{a} c x^{3} \Gamma\left(\frac{3}{4}\right) {{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| {\frac{b x^{4} e^{i \pi}}{a}} \right)}}{4 \Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a} e x^{5} \Gamma\left(\frac{5}{4}\right) {{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| {\frac{b x^{4} e^{i \pi}}{a}} \right)}}{4 \Gamma\left(\frac{9}{4}\right)} + \frac{3 \sqrt{a} f x^{6}}{16 \sqrt{1 + \frac{b x^{4}}{a}}} - \frac{a^{2} f \operatorname{asinh}{\left(\frac{\sqrt{b} x^{2}}{\sqrt{a}} \right)}}{16 b^{\frac{3}{2}}} + d \left(\begin{cases} \frac{\sqrt{a} x^{4}}{4} & \text{for}\: b = 0 \\ \frac{\left(a + b x^{4}\right)^{\frac{3}{2}}}{6 b} & \text{otherwise} \end{cases}\right) + \frac{b f x^{10}}{8 \sqrt{a} \sqrt{1 + \frac{b x^{4}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)

[Out] a**(3/2)*f*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*c*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*f*x**6/(16*sqrt(1 + b*x**4/a)) - a**2*f*asinh(sqrt(b)*x**2/sqrt(a))/(16*b**(3/2)) + d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*f*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)*x^2, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)

[Out] int(x^2*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)

3.498 $\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

Optimal. Leaf size=354

$$\frac{2afx\sqrt{a+bx^4}}{21b}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+bx^4}+\frac{e(a+bx^4)^{3/2}}{6b}+\frac{act}{}$$

```
[Out] 1/6*e*(b*x^4+a)^(3/2)/b+1/4*a*c*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2
)+2/21*a*f*x*(b*x^4+a)^(1/2)/b+1/4*c*x^2*(b*x^4+a)^(1/2)+1/35*x^3*(5*f*x^2+
7*d)*(b*x^4+a)^(1/2)+2/5*a*d*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2)
)-2/5*a^(5/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^
4+a)^(1/2)+1/105*a^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(-5*f*a^(1/2)+21*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2
)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1847, 1262, 655, 201, 223, 212, 1288, 1294, 1212, 226, 1210}

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(21\sqrt{b}\,d-5\sqrt{a}\,f\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{ac\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{1}{35}x^3\sqrt{a+bx^4}\left(7d+5fx^2\right)+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{e(a+bx^4)^{3/2}}{6b}+\frac{2afx\sqrt{a+bx^4}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (2*a*f*x*Sqrt[a + b*x^4])/(21*b) + (c*x^2*Sqrt[a + b*x^4])/4 + (2*a*d*x*Sqr
t[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^3*(7*d + 5*f*x^2)*Sq
rt[a + b*x^4])/35 + (e*(a + b*x^4)^(3/2))/(6*b) + (a*c*ArcTanh[(Sqrt[b]*x^2
)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (2*a^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)*(21*Sqrt[b]*d - 5*Sqrt[a
]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int x\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int\left(x\left(c+ex^2\right)\sqrt{a+bx^4}+x^2\left(d+fx^2\right)\sqrt{a+bx^4}\right)dx$$

$$=\int x\left(c+ex^2\right)\sqrt{a+bx^4}\,dx+\int x^2\left(d+fx^2\right)\sqrt{a+bx^4}\,dx$$

$$=\frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+bx^4}+\frac{1}{2}\text{Subst}\left(\int(c+ex)\sqrt{a+bx^2}\,dx\right.$$

$$=\frac{2afx\sqrt{a+bx^4}}{21b}+\frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+bx^4}+\frac{e\left(a+bx^4\right)^{3/2}}{6b}$$

$$=\frac{2afx\sqrt{a+bx^4}}{21b}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+bx^4}$$

$$=\frac{2afx\sqrt{a+bx^4}}{21b}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{35}$$

$$=\frac{2afx\sqrt{a+bx^4}}{21b}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{35}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.13, size = 211, normalized size = 0.60

$$\frac{\sqrt{a+bx^4}\left(14ae\sqrt{1+\frac{bx^4}{a}}+12afx\sqrt{1+\frac{bx^4}{a}}+21bcx^2\sqrt{1+\frac{bx^4}{a}}+14bex^4\sqrt{1+\frac{bx^4}{a}}+12bfx^5\sqrt{1+\frac{bx^4}{a}}+21\sqrt{a}\,\sqrt{b}\,c\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)-12afx\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)+28bdx^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)\right)}{84b\sqrt{1+\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*(14*a*e*Sqrt[1 + (b*x^4)/a] + 12*a*f*x*Sqrt[1 + (b*x^4)/a]
 + 21*b*c*x^2*Sqrt[1 + (b*x^4)/a] + 14*b*e*x^4*Sqrt[1 + (b*x^4)/a] + 12*b*f
*x^5*Sqrt[1 + (b*x^4)/a] + 21*Sqrt[a]*Sqrt[b]*c*ArcSinh[(Sqrt[b]*x^2)/Sqrt[
a]] - 12*a*f*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)] + 28*b*d*x^3
*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]))/(84*b*Sqrt[1 + (b*x^4)/a
])
```

Maple [C] Result contains complex when optimal does not.
time = 0.44, size = 280, normalized size = 0.79

method	result
default	$f\left(\frac{x^5\sqrt{b\,x^4+a}}{7}+\frac{2ax\sqrt{b\,x^4+a}}{21b}-\frac{2a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{e(b\,x}{}$
elliptic	$\frac{f\,x^5\sqrt{b\,x^4+a}}{7}+\frac{e\,x^4\sqrt{b\,x^4+a}}{6}+\frac{d\,x^3\sqrt{b\,x^4+a}}{5}+\frac{c\,x^2\sqrt{b\,x^4+a}}{4}+\frac{2afx\sqrt{b\,x^4+a}}{21b}+\frac{ae\sqrt{b\,x^4+a}}{6b}$
risch	$\frac{(60bf\,x^5+70be\,x^4+84bd\,x^3+105c\,x^2b+40afx+70ae)\sqrt{b\,x^4+a}}{420b}+\frac{2ia^{\frac{3}{2}}d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\right.}{5\sqrt{b}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/7*x^5*(b*x^4+a)^(1/2)+2/21*a/b*x*(b*x^4+a)^(1/2)-2/21/b*a^2/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/6*e*(b*x^4
+a)^(3/2)/b+d*(1/5*x^3*(b*x^4+a)^(1/2)+2/5*I*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1
/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a
^(1/2)*b^(1/2))^(1/2),I)))+c*(1/4*x^2*(b*x^4+a)^(1/2)+1/4*a/b^(1/2)*ln(x^2*
b^(1/2)+(b*x^4+a)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(a*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a)/x^2
))/sqrt(b) + 2*sqrt(b*x^4 + a)*a/((b - (b*x^4 + a)/x^4)*x^2))*c + integrate
(sqrt(b*x^4 + a)*(f*x^4 + x^3*e + d*x^2), x)
```

Fricas [A]
time = 0.13, size = 170, normalized size = 0.48

$$\frac{336\,a\sqrt{b}\,dx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+105\,a\sqrt{b}\,cx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\,\sqrt{b}\,x^2-a\right)-16\,(21\,ad+5\,af)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+2\,(60\,bfx^6+70\,bex^5+84\,bdx^4+105\,bcx^3+40\,afx^2+70\,aex+168\,ad)\sqrt{bx^4+a}}{840\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/840*(336*a*sqrt(b)*d*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1
) + 105*a*sqrt(b)*c*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) - 1
6*(21*a*d + 5*a*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x)
, -1) + 2*(60*b*f*x^6 + 70*b*e*x^5 + 84*b*d*x^4 + 105*b*c*x^3 + 40*a*f*x^2
+ 70*a*e*x + 168*a*d)*sqrt(b*x^4 + a))/(b*x)
```

Sympy [A]
time = 2.46, size = 158, normalized size = 0.45

$$\frac{\sqrt{a}\,cx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{\sqrt{a}\,dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}\,fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{ac\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4\sqrt{b}}+e\left(\begin{cases}\frac{\sqrt{a}\,x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*d*x**3*gamma(3/4)*hyper((-1/2
, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*f*x**5*g
amma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4
)) + a*c*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + e*Piecewise((sqrt(a)*x**
4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] int(x*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.499 $\int (c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

Optimal. Leaf size=331

$$\frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+bx^4}+\frac{f\left(a+bx^4\right)^{3/2}}{6b}+\frac{ad\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right.}{4\sqrt{b}}$$

```
[Out] 1/6*f*(b*x^4+a)^(3/2)/b+1/4*a*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2
)+1/4*d*x^2*(b*x^4+a)^(1/2)+1/15*x*(3*e*x^2+5*c)*(b*x^4+a)^(1/2)+2/5*a*e*x*
(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-2/5*a^(5/4)*e*(cos(2*arctan(b
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(
2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/
(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+1/15*a^(3/4)*(cos(2*
arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellipt
icF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*e*a^(1/2)+5*c*b^(1/2))
*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b
*x^4+a)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {1899, 1191, 1212, 226, 1210, 1262, 655, 201, 223, 212}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(3\sqrt{a}\,e+5\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}+\frac{1}{15}x\sqrt{a+bx^4}\left(5c+3ex^2\right)+\frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{ad\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{f\left(a+bx^4\right)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (d*x^2*Sqrt[a + b*x^4])/4 + (2*a*e*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] +
 Sqrt[b]*x^2)) + (x*(5*c + 3*e*x^2)*Sqrt[a + b*x^4])/15 + (f*(a + b*x^4)^(3
/2))/(6*b) + (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (2*
a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4
]) + (a^(3/4)*(5*Sqrt[b]*c + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int \left(\left(c+ex^2\right)\sqrt{a+bx^4}+x\left(d+fx^2\right)\sqrt{a+bx^4}\right)dx$$

$$=\int \left(c+ex^2\right)\sqrt{a+bx^4}\,dx+\int x\left(d+fx^2\right)\sqrt{a+bx^4}\,dx$$

$$=\frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+bx^4}+\frac{1}{15}\int \frac{10ac+6aex^2}{\sqrt{a+bx^4}}\,dx+\frac{1}{2}\text{Subst}\Bigg(\int$$

$$=\frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+bx^4}+\frac{f\left(a+bx^4\right)^{3/2}}{6b}+\frac{1}{2}d\text{Subst}\Bigg(\int \sqrt{a+b}$$

$$=\frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+b}$$

$$=\frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+b}$$

$$=\frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.10, size = 171, normalized size = 0.52

$$\frac{\sqrt{a+bx^4}\left(2af\sqrt{1+\frac{bx^4}{a}}+3bdx^2\sqrt{1+\frac{bx^4}{a}}+2bfx^4\sqrt{1+\frac{bx^4}{a}}+3\sqrt{a}\,\sqrt{b}\,d\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)+12bcx\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)+4bex^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)\right)}{12b\sqrt{1+\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*(2*a*f*Sqrt[1 + (b*x^4)/a] + 3*b*d*x^2*Sqrt[1 + (b*x^4)/a]
 + 2*b*f*x^4*Sqrt[1 + (b*x^4)/a] + 3*Sqrt[a]*Sqrt[b]*d*ArcSinh[(Sqrt[b]*x^2
)/Sqrt[a]] + 12*b*c*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)] + 4*b
*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]))/(12*b*Sqrt[1 + (b*
x^4)/a])
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 257, normalized size = 0.78

method	result
default	$\frac{f(bx^4+a)^{\frac{3}{2}}}{6b}+e\left(\frac{x^3\sqrt{bx^4+a}}{5}+\frac{2ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(\ldots\right.\right.}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{bx^4+a}\,\sqrt{b}}\right.$
elliptic	$\frac{fx^4\sqrt{bx^4+a}}{6}+\frac{ex^3\sqrt{bx^4+a}}{5}+\frac{dx^2\sqrt{bx^4+a}}{4}+\frac{cx\sqrt{bx^4+a}}{3}+\frac{af\sqrt{bx^4+a}}{6b}+\frac{2ac\sqrt{1-\frac{i\sqrt{\ldots}}{\ldots}}}{\ldots}$
risch	$\frac{(10bfx^4+12bex^3+15bdx^2+20bcx+10af)\sqrt{bx^4+a}}{60b}+\frac{2ia^{\frac{3}{2}}e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ldots\right.}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{bx^4+a}\,\sqrt{b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*f*(b*x^4+a)^(3/2)/b+e*(1/5*x^3*(b*x^4+a)^(1/2)+2/5*I*a^(3/2)/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Elli
pticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+d*(1/4*x^2*(b*x^4+a)^(1/2)+1/4*a/b^(
1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+c*(1/3*x*(b*x^4+a)^(1/2)+2/3*a/(I/a^(
```

```
1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^
2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c), x)
```

Fricas [A]
time = 0.12, size = 163, normalized size = 0.49

$$\frac{48\,a\sqrt{b}\,ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+15\,a\sqrt{b}\,dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\,\sqrt{b}\,x^2-a\right)+16\,(5\,bc-3\,ae)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+2\,(10\,bfx^5+12\,bex^4+15\,bdx^3+20\,bcx^2+10\,afx+24\,ae)\sqrt{bx^4+a}}{120\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/120*(48*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1)
 + 15*a*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 16*
(5*b*c - 3*a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -
1) + 2*(10*b*f*x^5 + 12*b*e*x^4 + 15*b*d*x^3 + 20*b*c*x^2 + 10*a*f*x + 24*a
*e)*sqrt(b*x^4 + a))/(b*x)
```

Sympy [A]
time = 2.41, size = 156, normalized size = 0.47

$$\frac{\sqrt{a}\,cx\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}\,dx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{\sqrt{a}\,ex^3\Gamma\left(\frac{3}{4}\right)\,{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4\sqrt{b}}+f\left(\begin{cases}\frac{\sqrt{a}\,x^4}{4} & \text{for}\ b=0\\\frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)
/(4*gamma(5/4)) + sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*e*x**3*gamm
a(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4))
+ a*d*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + f*Piecewise((sqrt(a)*x**4/4
, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] int((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.500 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx$

Optimal. Leaf size=345

$$\frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{4}\left(2c+ex^2\right)\sqrt{a+bx^4}+\frac{1}{15}x\left(5d+3fx^2\right)\sqrt{a+bx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}-\frac{1}{2}\sqrt{}$$

```
[Out] -1/2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+1/4*a*e*arctanh(x^2*b^(1/2)
/(b*x^4+a)^(1/2))/b^(1/2)+1/4*(e*x^2+2*c)*(b*x^4+a)^(1/2)+1/15*x*(3*f*x^2+5
*d)*(b*x^4+a)^(1/2)+2/5*a*f*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))
-2/5*a^(5/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^
(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4
+a)^(1/2)+1/15*a^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1
/2))*(3*f*a^(1/2)+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^
2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {1847, 1266, 829, 858, 223, 212, 272, 65, 214, 1191, 1212, 226, 1210}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(3\sqrt{a}\,f+5\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}+\frac{1}{4}\sqrt{a+bx^4}\left(2c+ex^2\right)-\frac{1}{2}\sqrt{a}\,c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{1}{15}x\sqrt{a+bx^4}\left(5d+3fx^2\right)+\frac{ae\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]
```

```
[Out] (2*a*f*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*x
^2)*Sqrt[a + b*x^4])/4 + (x*(5*d + 3*f*x^2)*Sqrt[a + b*x^4])/15 + (a*e*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*c*ArcTanh[Sqrt[a
 + b*x^4]/Sqrt[a]])/2 - (2*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(3/4)*(5*Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx &= \int \left(\frac{(c+ex^2)\sqrt{a+bx^4}}{x}+(d+fx^2)\sqrt{a+bx^4}\right)dx \\
&= \int \frac{(c+ex^2)\sqrt{a+bx^4}}{x}\,dx+\int (d+fx^2)\sqrt{a+bx^4}\,dx \\
&= \frac{1}{15}x(5d+3fx^2)\sqrt{a+bx^4}+\frac{1}{15}\int \frac{10ad+6afx^2}{\sqrt{a+bx^4}}\,dx+\frac{1}{2}\text{Subst}\Bigg(\\
&= \frac{1}{4}(2c+ex^2)\sqrt{a+bx^4}+\frac{1}{15}x(5d+3fx^2)\sqrt{a+bx^4}+\frac{\text{Subst}\Big(\int}{} \\
&= \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{4}(2c+ex^2)\sqrt{a+bx^4}+\frac{1}{15}x(5d+3fx \\
&= \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{4}(2c+ex^2)\sqrt{a+bx^4}+\frac{1}{15}x(5d+3fx \\
&= \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{4}(2c+ex^2)\sqrt{a+bx^4}+\frac{1}{15}x(5d+3fx \\
&= \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{4}(2c+ex^2)\sqrt{a+bx^4}+\frac{1}{15}x(5d+3fx
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.85, size = 280, normalized size = 0.81

$$
\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(15ae\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)+\sqrt{b}\left((a+bx^4)(30c+x(20d+3x(5e+4fx)))-30\sqrt{a}\,c\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)\right)+24a^{3/2}f\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-8a\left(5i\sqrt{b}\,d+3\sqrt{a}\,f\right)\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{60\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b}\sqrt{a+bx^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]
```

[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(15*a*e*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] + Sqrt[b]*((a + b*x^4)*(30*c + x*(20*d + 3*x*(5*e + 4*f*x))) - 30*Sqrt[a]*c*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 24*a^(3/2)*f*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 8*a*((5*I)*Sqrt[b]*d + 3*Sqrt[a]*f)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(60*Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[b]*Sqrt[a + b*x^4])

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 284, normalized size = 0.82

method	result
elliptic	$\frac{f x^3 \sqrt{b x^4+a}}{5}+\frac{e x^2 \sqrt{b x^4+a}}{4}+\frac{d x \sqrt{b x^4+a}}{3}+\frac{c \sqrt{b x^4+a}}{2}+\frac{2 a d \sqrt{1-\frac{i \sqrt{b}\, x^2}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^2}{\sqrt{a}}}}{3 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b x^4}}$
default	$f \left(\frac{x^3 \sqrt{b x^4+a}}{5}+\frac{2 i a^{\frac{3}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^2}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^2}{\sqrt{a}}}\, \left(\mathrm{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\mathrm{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{5 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b x^4+a}\, \sqrt{b}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x,method=_RETURNVERBOSE)

[Out] f*(1/5*x^3*(b*x^4+a)^(1/2)+2/5*I*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+e*(1/4*x^2*(b*x^4+a)^(1/2)+1/4*a/b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+d*(1/3*x*(b*x^4+a)^(1/2)+2/3*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+c*(1/2*(b*x^4+a)^(1/2)-1/2*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x, x)

Fricas [F]
time = 0.25, size = 30, normalized size = 0.09

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,\left(fx^3+ex^2+dx+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.59, size = 204, normalized size = 0.59

$$-\frac{\sqrt{a}\,c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{2}+\frac{\sqrt{a}\,dx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}\,ex^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{\sqrt{a}\,fx^3\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ac}{2\sqrt{b}\,x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{ae\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4\sqrt{b}}+\frac{\sqrt{b}\,cx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x,x)
```

```
[Out] -sqrt(a)*c*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*d*x*gamma(1/4)*hyper((
-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*e*x*
*2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4
,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a*c/(2*sqrt(b)*x**2*sqrt(a/(
b*x**4) + 1)) + a*e*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + sqrt(b)*c*x**
2/(2*sqrt(a/(b*x**4) + 1))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x,x)
```

```
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x, x)
```

3.501 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx$

Optimal. Leaf size=341

$$\frac{2\sqrt{b}\,cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2}-\frac{(3c-ex^2)\sqrt{a+bx^4}}{3x}+\frac{1}{4}(2d+fx^2)\sqrt{a+bx^4}+\frac{af\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}-\frac{1}{2}\sqrt{a}\,d\tanh$$

```
[Out] -1/2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+1/4*a*f*arctanh(x^2*b^(1/2)
/(b*x^4+a)^(1/2))/b^(1/2)-1/3*(-e*x^2+3*c)*(b*x^4+a)^(1/2)/x+1/4*(f*x^2+2*d
)*(b*x^4+a)^(1/2)+2*c*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-2*a^(
1/4)*b^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^
(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2
)+1/3*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/
4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(e*a
^(1/2)+3*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^
2)^(1/2)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$,
Rules used = {1847, 1286, 1212, 226, 1210, 1266, 829, 858, 223, 212, 272, 65, 214}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}\,(3c-ex^2)}{3x}+\frac{2\sqrt{b}\,cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2}+\frac{1}{4}\sqrt{a+bx^4}\,(2d+fx^2)-\frac{1}{2}\sqrt{a}\,d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{af\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]
```

```
[Out] (2*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((3*c - e*x^2)*Sq
rt[a + b*x^4])/(3*x) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/4 + (a*f*ArcTanh[(Sq
rt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^
4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \int \left(\frac{(c+ex^2)\sqrt{a+bx^4}}{x^2}+\frac{(d+fx^2)\sqrt{a+bx^4}}{x}\right)dx$$

$$= \int \frac{(c+ex^2)\sqrt{a+bx^4}}{x^2}\,dx + \int \frac{(d+fx^2)\sqrt{a+bx^4}}{x}\,dx$$

$$= -\frac{(3c-ex^2)\sqrt{a+bx^4}}{3x} + \frac{1}{2}\text{Subst}\left(\int \frac{(d+fx)\sqrt{a+bx^2}}{x}\,dx, x, x$$

$$= -\frac{(3c-ex^2)\sqrt{a+bx^4}}{3x} + \frac{1}{4}(2d+fx^2)\sqrt{a+bx^4} + \frac{\text{Subst}\left(\int \frac{2c}{x\sqrt{}}\right.}{}$$

$$= \frac{2\sqrt{b}\,cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2} - \frac{(3c-ex^2)\sqrt{a+bx^4}}{3x} + \frac{1}{4}(2d+fx^2)\sqrt{a+}$$

$$= \frac{2\sqrt{b}\,cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2} - \frac{(3c-ex^2)\sqrt{a+bx^4}}{3x} + \frac{1}{4}(2d+fx^2)\sqrt{a+}$$

$$= \frac{2\sqrt{b}\,cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2} - \frac{(3c-ex^2)\sqrt{a+bx^4}}{3x} + \frac{1}{4}(2d+fx^2)\sqrt{a+}$$

$$= \frac{2\sqrt{b}\,cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2} - \frac{(3c-ex^2)\sqrt{a+bx^4}}{3x} + \frac{1}{4}(2d+fx^2)\sqrt{a+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.67, size = 267, normalized size = 0.78

$$\frac{1}{12}\left(\frac{\sqrt{a+bx^4}\,(-12c+x(6d+x(4e+3fx)))}{x}+\frac{3af\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{\sqrt{b}}-6\sqrt{a}\,d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{24ia\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,c\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{a+bx^4}}+\frac{8\sqrt{b}\left(-3i\sqrt{b}\,c+\sqrt{a}\,e\right)\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\left(\frac{i\sqrt{b}}{\sqrt{a}}\right)^{3/2}\sqrt{a+bx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]
```

```
[Out] ((Sqrt[a + b*x^4]*(-12*c + x*(6*d + x*(4*e + 3*f*x))))/x + (3*a*f*ArcTanh[(
Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/Sqrt[b] - 6*Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^4
```

```
]/Sqrt[a]] - ((24*I)*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*c*Sqrt[1 + (b*x^4)/a]*Elli
pticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/Sqrt[a + b*x^4] + (8*Sqr
t[b]*((-3*I)*Sqrt[b]*c + Sqrt[a]*e)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh
[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(((I*Sqrt[b])/Sqrt[a])^(3/2)*Sqrt[a + b
*x^4]))/12
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 284, normalized size = 0.83

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{x}+\frac{f\,x^2\sqrt{b\,x^4+a}}{4}+\frac{e x\sqrt{b\,x^4+a}}{3}+\frac{d\sqrt{b\,x^4+a}}{2}+\frac{2ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4}}$
risch	$-\frac{c\sqrt{b\,x^4+a}}{x}+\frac{f\,x^2\sqrt{b\,x^4+a}}{4}+\frac{af\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{4\sqrt{b}}+\frac{e x\sqrt{b\,x^4+a}}{3}+\frac{2ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{3\sqrt{}}$
default	$f\left(\frac{x^2\sqrt{b\,x^4+a}}{4}+\frac{a\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{4\sqrt{b}}\right)+e\left(\frac{x\sqrt{b\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/4*x^2*(b*x^4+a)^(1/2)+1/4*a/b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+e
*(1/3*x*(b*x^4+a)^(1/2)+2/3*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*b^(1/2))^(1/2),I))+d*(1/2*(b*x^4+a)^(1/2)-1/2*a^(1/2)*ln((2*a+2*a^
(1/2)*(b*x^4+a)^(1/2))/x^2))+c*(-1/x*(b*x^4+a)^(1/2)+2*I*b^(1/2)*a^(1/2)/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Elli
pticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^2, x)
```

Fricas [F]
time = 0.24, size = 30, normalized size = 0.09

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,(fx^3+ex^2+dx+c)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.18, size = 206, normalized size = 0.60

$$\frac{\sqrt{a}\, c\Gamma\left(-\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| {\frac{b x^{4} e^{i\pi}}{a}} \right)}{4 x \Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{a}\, d \operatorname{asinh}{\left(\frac{\sqrt{a}}{\sqrt{b} x^{2}} \right)}}{2} + \frac{\sqrt{a}\, e x \Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| {\frac{b x^{4} e^{i\pi}}{a}} \right)}{4 \Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}\, f x^{2} \sqrt{1 + \frac{b x^{4}}{a}}}{4} + \frac{a d}{2 \sqrt{b} x^{2} \sqrt{\frac{a}{b x^{4}} + 1}} + \frac{a f \operatorname{asinh}{\left(\frac{\sqrt{b} x^{2}}{\sqrt{a}} \right)}}{4 \sqrt{b}} + \frac{\sqrt{b}\, d x^{2}}{2 \sqrt{\frac{a}{b x^{4}} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**2,x)
[Out] sqrt(a)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x*gamma(3/4)) - sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*e*x
*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5
/4)) + sqrt(a)*f*x**2*sqrt(1 + b*x**4/a)/4 + a*d/(2*sqrt(b)*x**2*sqrt(a/(b*
x**4) + 1)) + a*f*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + sqrt(b)*d*x**2/
(2*sqrt(a/(b*x**4) + 1))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^2,x)
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^2, x)
```

3.502 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx$

Optimal. Leaf size=342

$$\frac{2\sqrt{b}\,dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2}-\frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2}-\frac{(3d-fx^2)\sqrt{a+bx^4}}{3x}+\frac{1}{2}\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}\sqrt{a}\,e\tan$$

```
[Out] -1/2*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+1/2*c*arctanh(x^2*b^(1/2)/(
b*x^4+a)^(1/2))*b^(1/2)-1/2*(-e*x^2+c)*(b*x^4+a)^(1/2)/x^2-1/3*(-f*x^2+3*d)
*(b*x^4+a)^(1/2)/x+2*d*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-2*a^
(1/4)*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/
2)+1/3*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(f*
a^(1/2)+3*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))
^2)^(1/2)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {1847, 1266, 827, 858, 223, 212, 272, 65, 214, 1286, 1212, 226, 1210}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\sqrt{a}\,f+3\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}\,(c-ex^2)}{2x^2}+\frac{1}{2}\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-\frac{\sqrt{a+bx^4}\,(3d-fx^2)}{3x}+\frac{2\sqrt{b}\,dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2}-\frac{1}{2}\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3,x]
```

```
[Out] (2*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((c - e*x^2)*Sqrt
[a + b*x^4])/(2*x^2) - ((3*d - f*x^2)*Sqrt[a + b*x^4])/(3*x) + (Sqrt[b]*c*A
rcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (Sqrt[a]*e*ArcTanh[Sqrt[a + b*x^
4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx = \int \left(\frac{(c+ex^2)\sqrt{a+bx^4}}{x^3} + \frac{(d+fx^2)\sqrt{a+bx^4}}{x^2}\right) dx$$

$$= \int \frac{(c+ex^2)\sqrt{a+bx^4}}{x^3}\,dx + \int \frac{(d+fx^2)\sqrt{a+bx^4}}{x^2}\,dx$$

$$= -\frac{(3d-fx^2)\sqrt{a+bx^4}}{3x} + \frac{1}{2}\text{Subst}\left(\int \frac{(c+ex)\sqrt{a+bx^2}}{x^2}\,dx, x, x\right.$$

$$= -\frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2} - \frac{(3d-fx^2)\sqrt{a+bx^4}}{3x} - \frac{1}{4}\text{Subst}\left(\int \frac{-2}{x\sqrt{}}\right.$$

$$= \frac{2\sqrt{b}\,dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2} - \frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2} - \frac{(3d-fx^2)\sqrt{a+bx^4}}{3x}$$

$$= \frac{2\sqrt{b}\,dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2} - \frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2} - \frac{(3d-fx^2)\sqrt{a+bx^4}}{3x}$$

$$= \frac{2\sqrt{b}\,dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2} - \frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2} - \frac{(3d-fx^2)\sqrt{a+bx^4}}{3x}$$

$$= \frac{2\sqrt{b}\,dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2} - \frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2} - \frac{(3d-fx^2)\sqrt{a+bx^4}}{3x}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.48, size = 296, normalized size = 0.87

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((a+bx^4)(-3c+x(-6d+3ex+2fx^2))+3\sqrt{b}\,cx^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-3\sqrt{a}\,ex^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+12\sqrt{a}\sqrt{b}\,dx^2\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-4i\sqrt{a}\left(-3i\sqrt{b}\,d+\sqrt{a}\,f\right)x^2\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{6\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(-3*c + x*(-6*d + 3*e*x + 2*f*x^2))
 + 3*Sqrt[b]*c*x^2*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] -
 3*Sqrt[a]*e*x^2*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) + 12*Sqr
```

```
t[a]*Sqrt[b]*d*x^2*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])
/Sqrt[a]]*x], -1] - (4*I)*Sqrt[a]*((-3*I)*Sqrt[b]*d + Sqrt[a]*f)*x^2*Sqrt[1
 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(6*Sqr
t[(I*Sqrt[b])/Sqrt[a]]*x^2*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 304, normalized size = 0.89

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(2dx+c)}{2x^{2}}+\frac{fx\sqrt{b\,x^{4}+a}}{3}+\frac{2af\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{e\sqrt{}}{}$
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{2x^{2}}-\frac{d\sqrt{b\,x^{4}+a}}{x}+\frac{fx\sqrt{b\,x^{4}+a}}{3}+\frac{e\sqrt{b\,x^{4}+a}}{2}+\frac{2af\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{El}}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+}}$
default	$f\left(\frac{x\sqrt{b\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+c\left(-\frac{(b\,x^{4}+a)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\,x^{2}\sqrt{b}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/3*x*(b*x^4+a)^(1/2)+2/3*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(
I/a^(1/2)*b^(1/2))^(1/2),I))+c*(-1/2/a/x^2*(b*x^4+a)^(3/2)+1/2*b/a*x^2*(b*x
^4+a)^(1/2)+1/2*b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+e*(1/2*(b*x^4+a)^(
1/2)-1/2*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))+d*(-1/x*(b*x^4+a)
^(1/2)+2*I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a
^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^3, x)
```

Fricas [F]
time = 0.23, size = 30, normalized size = 0.09

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,(fx^3+ex^2+dx+c)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.00, size = 230, normalized size = 0.67

$$-\frac{\sqrt{a}\,c}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}\,d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt{a}\,e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{2}+\frac{\sqrt{a}\,fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{ae}{2\sqrt{b}\,x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{\sqrt{b}\,c\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}+\frac{\sqrt{b}\,ex^2}{2\sqrt{\frac{a}{bx^4}+1}}-\frac{bcx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**3,x)
```

```
[Out] -sqrt(a)*c/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*d*gamma(-1/4)*hyper((-1/2,
 -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - sqrt(a)*e*asin
h(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*f*x*gamma(1/4)*hyper((-1/2, 1/4), (5/
4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a*e/(2*sqrt(b)*x**2*sqrt(a/
(b*x**4) + 1)) + sqrt(b)*c*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)*e*x**2/(
2*sqrt(a/(b*x**4) + 1)) - b*c*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^3,x)
```

```
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^3, x)
```

3.503 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx$

Optimal. Leaf size=357

$$-\frac{2e\sqrt{a+bx^4}}{x}+\frac{2\sqrt{b}\,ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2}-\frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3}-\frac{(d-fx^2)\sqrt{a+bx^4}}{2x^2}+\frac{1}{2}\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right.$$

```
[Out] -1/2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+1/2*d*arctanh(x^2*b^(1/2)/(
b*x^4+a)^(1/2))*b^(1/2)-2*e*(b*x^4+a)^(1/2)/x-1/3*(-3*e*x^2+c)*(b*x^4+a)^(1
/2)/x^3-1/2*(-f*x^2+d)*(b*x^4+a)^(1/2)/x^2+2*e*x*b^(1/2)*(b*x^4+a)^(1/2)/(a
^(1/2)+x^2*b^(1/2))-2*a^(1/4)*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2
)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a
^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2)
)^2)^(1/2)/(b*x^4+a)^(1/2)+1/3*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(3*e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+
a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1847, 1286, 1296, 1212, 226, 1210, 1266, 827, 858, 223, 212, 272, 65, 214}

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(3\sqrt{a}\,e+\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{b}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}(c-3ex^2)}{3x^3}-\frac{\sqrt{a+bx^4}(d-fx^2)}{2x^2}+\frac{1}{2}\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-\frac{2e\sqrt{a+bx^4}}{x}+\frac{2\sqrt{b}\,ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2}-\frac{1}{2}\sqrt{a}\,f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4,x]
```

```
[Out] (-2*e*Sqrt[a + b*x^4])/x + (2*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[
b]*x^2) - ((c - 3*e*x^2)*Sqrt[a + b*x^4])/(3*x^3) - ((d - f*x^2)*Sqrt[a + b
*x^4])/(2*x^2) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (Sq
rt[a]*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*e*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*c + 3*S
qrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a + b*x^4
])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
```

```
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx = \int \left(\frac{(c+ex^2)\sqrt{a+bx^4}}{x^4}+\frac{(d+fx^2)\sqrt{a+bx^4}}{x^3}\right)dx$$

$$= \int \frac{(c+ex^2)\sqrt{a+bx^4}}{x^4}\,dx + \int \frac{(d+fx^2)\sqrt{a+bx^4}}{x^3}\,dx$$

$$= -\frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3} + \frac{1}{2}\text{Subst}\left(\int \frac{(d+fx)\sqrt{a+bx^2}}{x^2}\,dx, x, x\right.$$

$$= -\frac{2e\sqrt{a+bx^4}}{x} - \frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3} - \frac{(d-fx^2)\sqrt{a+bx^4}}{2x^2} -$$

$$= -\frac{2e\sqrt{a+bx^4}}{x} - \frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3} - \frac{(d-fx^2)\sqrt{a+bx^4}}{2x^2} +$$

$$= -\frac{2e\sqrt{a+bx^4}}{x} + \frac{2\sqrt{b}\,ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2} - \frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3} -$$

$$= -\frac{2e\sqrt{a+bx^4}}{x} + \frac{2\sqrt{b}\,ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2} - \frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3} -$$

$$= -\frac{2e\sqrt{a+bx^4}}{x} + \frac{2\sqrt{b}\,ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2} - \frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.50, size = 295, normalized size = 0.83

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((a+bx^4)(2c+3x(d+2ex-fx^2))-3\sqrt{b}\,dx^3\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)+3\sqrt{a}\,fx^3\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+12\sqrt{a}\,\sqrt{b}\,ex^3\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-4\sqrt{b}\left(i\sqrt{b}\,c+3\sqrt{a}\,e\right)x^3\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{6\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(2*c + 3*x*(d + 2*e*x - f*x^2)) -
 3*Sqrt[b]*d*x^3*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] + 3
*Sqrt[a]*f*x^3*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 12*Sqrt
[a]*Sqrt[b]*e*x^3*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/
Sqrt[a]]*x], -1] - 4*Sqrt[b]*(I*Sqrt[b]*c + 3*Sqrt[a]*e)*x^3*Sqrt[1 + (b*x^
4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(6*Sqrt[(I*Sqr
t[b])/Sqrt[a]]*x^3*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.39, size = 306, normalized size = 0.86

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(6e\,x^{2}+3dx+2c)}{6x^{3}}+\frac{f\sqrt{b\,x^{4}+a}}{2}+\frac{2i\sqrt{b}\,e\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{3x^{3}}-\frac{d\sqrt{b\,x^{4}+a}}{2x^{2}}-\frac{e\sqrt{b\,x^{4}+a}}{x}+\frac{f\sqrt{b\,x^{4}+a}}{2}+\frac{2bc\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{Ellip}}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$
default	$d\left(-\frac{(b\,x^{4}+a)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\,x^{2}\sqrt{b\,x^{4}+a}}{2a}+\frac{\sqrt{b}\,\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{2}\right)+c\left(-\frac{\sqrt{b\,x^{4}+a}}{3x^{3}}+\frac{2b\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/2/a/x^2*(b*x^4+a)^(3/2)+1/2*b/a*x^2*(b*x^4+a)^(1/2)+1/2*b^(1/2)*ln(x^
2*b^(1/2)+(b*x^4+a)^(1/2)))+c*(-1/3/x^3*(b*x^4+a)^(1/2)+2/3*b/(I/a^(1/2)*b^
(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2
)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+f*(1/2*(b*x^4+a
)^(1/2)-1/2*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))+e*(-1/x*(b*x^4
+a)^(1/2)+2*I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(
I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^4, x)
```

Fricas [F]
time = 0.23, size = 30, normalized size = 0.08

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,\left(fx^3+ex^2+dx+c\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="fricas")
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.04, size = 235, normalized size = 0.66

$$\frac{\sqrt{a}\,c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{a}\,d}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}\,e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt{a}\,f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{2}+\frac{af}{2\sqrt{b}\,x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{\sqrt{b}\,d\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}+\frac{\sqrt{b}\,fx^2}{2\sqrt{\frac{a}{bx^4}+1}}-\frac{bdx^2}{2\sqrt{a}\,\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**4,x)
[Out] sqrt(a)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x**3*gamma(1/4)) - sqrt(a)*d/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*e*ga
mma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(
3/4)) - sqrt(a)*f*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a*f/(2*sqrt(b)*x**2*sqr
t(a/(b*x**4) + 1)) + sqrt(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)*f*x*
*2/(2*sqrt(a/(b*x**4) + 1)) - b*d*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^4,x)
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^4, x)
```

3.504 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx$

Optimal. Leaf size=329

$$-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}+\frac{2\sqrt{b}\,fx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2}+\frac{1}{2}\sqrt{b}\,e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-\frac{bc\tanh^{-1}\left(\sqrt{\right.}}{4\sqrt{a}}$$

```
[Out] -1/4*b*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)+1/2*e*arctanh(x^2*b^(1/2)
/(b*x^4+a)^(1/2))*b^(1/2)-1/12*(3*c/x^4+4*d/x^3+6*e/x^2+12*f/x)*(b*x^4+a)^(
1/2)+2*f*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-2*a^(1/4)*b^(1/4)*
f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)
))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(
1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+1/3*b^(1/4)
*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4))
)*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*f*a^(1/2)+d*b^
(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1
/4)/(b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {14, 1839, 1846, 272, 65, 214, 1899, 281, 223, 212, 1212, 226, 1210}

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(3\sqrt{a}\,f+\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{b}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)-\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{b}\,e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)+\frac{2\sqrt{b}\,fx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]
```

```
[Out] -1/12*(((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*Sqrt[a + b*x^4]) + (2
*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + (Sqrt[b]*e*ArcTanh[
(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/
(4*Sqrt[a]) - (2*a^(1/4)*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
Sqrt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
```

```
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4} - (2b)\int \frac{-\frac{c}{4}-\frac{dx}{3}-\frac{ex^2}{2}}{x\sqrt{a+bx^4}}$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4} - (2b)\int \frac{-\frac{d}{3}-\frac{ex}{2}-fx^2}{\sqrt{a+bx^4}}$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4} - (2b)\int \left(-\frac{ex}{2\sqrt{a+bx^4}}\right.$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4} - (2b)\int \frac{-\frac{d}{3}-fx^2}{\sqrt{a+bx^4}}\,dx -$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4} + \frac{2\sqrt{b}\,fx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2} -$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4} + \frac{2\sqrt{b}\,fx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.37, size = 267, normalized size = 0.81

$$\frac{1}{12}\left(-\frac{\sqrt{a+bx^4}\,(3c+4dx+6x^2(e+2fx))}{x^4}+6\sqrt{b}\,e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-\frac{3bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{24ia\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,f\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{a+bx^4}}-\frac{8\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\,d-3i\sqrt{a}\,f\right)\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{a+bx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]
```

```
[Out] (-((Sqrt[a + b*x^4]*(3*c + 4*d*x + 6*x^2*(e + 2*f*x)))/x^4) + 6*Sqrt[b]*e*A
rcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - (3*b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt
[a]])/Sqrt[a] - ((24*I)*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*f*Sqrt[1 + (b*x^4)/a]*E
llipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/Sqrt[a + b*x^4] - (8*
Sqrt[a]*Sqrt[(I*Sqrt[b])/Sqrt[a]]*(Sqrt[b]*d - (3*I)*Sqrt[a]*f)*Sqrt[1 + (b
*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/Sqrt[a + b*
x^4])/12
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 328, normalized size = 1.00

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{4x^4}-\frac{d\sqrt{b\,x^4+a}}{3x^3}-\frac{e\sqrt{b\,x^4+a}}{2x^2}-\frac{f\sqrt{b\,x^4+a}}{x}+\frac{2bd\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{Ellip}}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$
risch	$-\frac{\sqrt{b\,x^4+a}\,(12f\,x^3+6e\,x^2+4dx+3c)}{12x^4}+\frac{2i\sqrt{b}\,f\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}-$
default	$c\left(-\frac{(b\,x^4+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{4\sqrt{a}}+\frac{b\sqrt{b\,x^4+a}}{4a}\right)+e\left(-\frac{(b\,x^4+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\,x^2\sqrt{b\,x^4+a}}{2a}+\frac{\sqrt{b}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/4/a/x^4*(b*x^4+a)^(3/2)-1/4*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/
2))/x^2)+1/4*b/a*(b*x^4+a)^(1/2))+e*(-1/2/a/x^2*(b*x^4+a)^(3/2)+1/2*b/a*x^2
*(b*x^4+a)^(1/2)+1/2*b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+d*(-1/3/x^3*(
b*x^4+a)^(1/2)+2/3*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I))+f*(-1/x*(b*x^4+a)^(1/2)+2*I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/
2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I
/a^(1/2)*b^(1/2))^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(b*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/sqrt(a)
 - 2*sqrt(b*x^4 + a)/x^4)*c + integrate(sqrt(b*x^4 + a)*(f*x^2 + x*e + d)/x
^4, x)
```

Fricas [F]

time = 0.25, size = 30, normalized size = 0.09

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,(fx^3+ex^2+dx+c)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.14, size = 211, normalized size = 0.64

$$\frac{\sqrt{a}\,d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{a}\,e}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}\,f\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt{b}\,c\sqrt{\frac{a}{bx^4}+1}}{4x^2}+\frac{\sqrt{b}\,e\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}-\frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{4\sqrt{a}}-\frac{bex^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**5,x)
```

```
[Out] sqrt(a)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x**3*gamma(1/4)) - sqrt(a)*e/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*f*ga
mma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(
3/4)) - sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*x**2) + sqrt(b)*e*asinh(sqrt(b)*x
**2/sqrt(a))/2 - b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) - b*e*x**2/(
2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^5,x)
```

```
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^5, x)
```

3.505 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx$

Optimal. Leaf size=360

$$-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}cx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{2}\sqrt{b}\,f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx}}\right.$$

```
[Out] -1/4*b*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)+1/2*f*arctanh(x^2*b^(1/2)
/(b*x^4+a)^(1/2))*b^(1/2)-1/60*(12*c/x^5+15*d/x^4+20*e/x^3+30*f/x^2)*(b*x^4
+a)^(1/2)-2/5*b*c*(b*x^4+a)^(1/2)/a/x+2/5*b^(3/2)*c*x*(b*x^4+a)^(1/2)/a/(a^
(1/2)+x^2*b^(1/2))-2/5*b^(5/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1
/2)/a^(3/4)/(b*x^4+a)^(1/2)+1/15*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/
a^(1/4))),1/2*2^(1/2))*(5*e*a^(1/2)+3*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*
x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {14, 1839, 1847, 1296, 1212, 226, 1210, 1266, 858, 223, 212, 272, 65, 214}

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(5\sqrt{a}\,e+3\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}+\frac{2b^{3/2}cx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)-\frac{2bc\sqrt{a+bx^4}}{5ax}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{b}\,f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]
```

```
[Out] -1/60*(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*Sqrt[a + b*x^4])
 - (2*b*c*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*a*(
Sqrt[a] + Sqrt[b]*x^2)) + (Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]
)/2 - (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*c*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)
*(3*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^
(3/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx = -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4} - (2b)\int \frac{-\frac{c}{5}-\frac{dx}{4}-}{x^2\sqrt{a+}}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4} - (2b)\int \left(\frac{-\frac{c}{5}-\frac{ex}{3}}{x^2\sqrt{a+b}}\right.$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4} - (2b)\int \frac{-\frac{c}{5}-\frac{ex^2}{3}}{x^2\sqrt{a+bx^4}}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4} - \frac{2bc\sqrt{a+bx^4}}{5ax} - bS$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4} - \frac{2bc\sqrt{a+bx^4}}{5ax} - \frac{(2}{}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4} - \frac{2bc\sqrt{a+bx^4}}{5ax} + \frac{2}{5a}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4} - \frac{2bc\sqrt{a+bx^4}}{5ax} + \frac{2}{5a}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4} - \frac{2bc\sqrt{a+bx^4}}{5ax} + \frac{2}{5a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.50, size = 314, normalized size = 0.87

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((a+bx^4)(12ac+24bcx^4+5ax(3d+4ex+6fx^2))-30a\sqrt{b}fx^5\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+15\sqrt{a}bdx^5\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+24\sqrt{a}b^{3/2}cx^5\sqrt{1+\frac{bx^4}{a}}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-8i\sqrt{a}b\left(-3i\sqrt{b}c+5\sqrt{a}e\right)x^5\sqrt{1+\frac{bx^4}{a}}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{60a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x^5\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(12*a*c + 24*b*c*x^4 + 5*a*x*(3*d
 + 4*e*x + 6*f*x^2)) - 30*a*Sqrt[b]*f*x^5*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*
```

```
x^2)/Sqrt[a + b*x^4]] + 15*Sqrt[a]*b*d*x^5*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a +
 b*x^4]/Sqrt[a]])) + 24*Sqrt[a]*b^(3/2)*c*x^5*Sqrt[1 + (b*x^4)/a]*EllipticE
[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - (8*I)*Sqrt[a]*b*((-3*I)*Sqrt
[b]*c + 5*Sqrt[a]*e)*x^5*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sq
rt[b])/Sqrt[a]]*x], -1])/(60*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^5*Sqrt[a + b*x^4
])
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 346, normalized size = 0.96

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{5x^5}-\frac{d\sqrt{b\,x^4+a}}{4x^4}-\frac{e\sqrt{b\,x^4+a}}{3x^3}-\frac{f\sqrt{b\,x^4+a}}{2x^2}-\frac{2bc\sqrt{b\,x^4+a}}{5ax}+\frac{2be\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \sqrt{\ }}{3\sqrt{\ }}$
risch	$-\frac{\sqrt{b\,x^4+a}\,(24bc\,x^4+30af\,x^3+20ae\,x^2+15adx+12ac)}{60x^5a}+\frac{2ib^{\frac{3}{2}}c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.}{5\sqrt{a}\ \sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ \sqrt{b\,x^4+a}}$
default	$c\left(-\frac{\sqrt{b\,x^4+a}}{5x^5}-\frac{2b\sqrt{b\,x^4+a}}{5ax}+\frac{2ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\right.}{5\sqrt{a}\ \sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ \sqrt{b\,x^4+a}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/5/x^5*(b*x^4+a)^(1/2)-2/5*b/a*(b*x^4+a)^(1/2)/x+2/5*I*b^(3/2)/a^(1/2)
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-E
llipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+d*(-1/4/a/x^4*(b*x^4+a)^(3/2)-1/4
*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/4*b/a*(b*x^4+a)^(1/2))
+f*(-1/2/a/x^2*(b*x^4+a)^(3/2)+1/2*b/a*x^2*(b*x^4+a)^(1/2)+1/2*b^(1/2)*ln(x
^2*b^(1/2)+(b*x^4+a)^(1/2)))+e*(-1/3/x^3*(b*x^4+a)^(1/2)+2/3*b/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/
2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^6, x)
```

Fricas [F]
time = 0.25, size = 30, normalized size = 0.08

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,(fx^3+ex^2+dx+c)}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.26, size = 216, normalized size = 0.60

$$\frac{\sqrt{a}\,c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}+\frac{\sqrt{a}\,e\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{a}\,f}{2x^2\sqrt{1+\frac{bx^4}{a}}}-\frac{\sqrt{b}\,d\sqrt{\frac{a}{bx^4}+1}}{4x^2}+\frac{\sqrt{b}\,f\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}-\frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{4\sqrt{a}}-\frac{bfx^2}{2\sqrt{a}\,\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**6,x)
```

```
[Out] sqrt(a)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**5*gamma(-1/4)) + sqrt(a)*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,),
b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*f/(2*x**2*sqrt(1 +
b*x**4/a)) - sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*x**2) + sqrt(b)*f*asinh(sqrt
(b)*x**2/sqrt(a))/2 - b*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) - b*f*x
**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^6,x)
```

```
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^6, x)
```

3.506 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx$

Optimal. Leaf size=352

$$-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{6ax^2}-\frac{2bd\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}dx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{be\tanh^{-1}}{4}$$

```
[Out] -1/4*b*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/60*(10*c/x^6+12*d/x^5+1
5*e/x^4+20*f/x^3)*(b*x^4+a)^(1/2)-1/6*b*c*(b*x^4+a)^(1/2)/a/x^2-2/5*b*d*(b*
x^4+a)^(1/2)/a/x+2/5*b^(3/2)*d*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-2/
5*b^(5/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)
*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/
2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)
^(1/2)+1/15*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(5*f*a^(1/2)+3*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b
^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {14, 1839, 1847, 1266, 821, 272, 65, 214, 1296, 1212, 226, 1210}

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(5\sqrt{a}\,f+3\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}+\frac{2b^{3/2}dx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)-\frac{bc\sqrt{a+bx^4}}{6ax^2}-\frac{2bd\sqrt{a+bx^4}}{5ax}-\frac{be\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]
```

```
[Out] -1/60*(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*Sqrt[a + b*x^4])
 - (b*c*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*d*Sqrt[a + b*x^4])/(5*a*x) + (2*b
^(3/2)*d*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*e*ArcTanh[Sq
rt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*d + 5*Sqr
t[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
```

```
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx = -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{6}-\frac{dx}{5}-}{x^3\sqrt{a+}}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-(2b)\int \left(\frac{-\frac{c}{6}-\frac{ex}{4}}{x^3\sqrt{a+b}}\right.$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{6}-\frac{ex^2}{4}}{x^3\sqrt{a+bx^4}}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{2bd\sqrt{a+bx^4}}{5ax}-bS$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{6ax^2}-\frac{2bc}{}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{6ax^2}-\frac{2bc}{}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{6ax^2}-\frac{2bc}{}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{6ax^2}-\frac{2bc}{}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 10.39, size = 277, normalized size = 0.79

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((a+bx^4)\left(10ac+2bx^4(5c+12dx)+ax(12d+5x(3e+4fx))\right)+15\sqrt{a}\,bex^6\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+24\sqrt{a}\,b^{3/2}dx^6\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-8i\sqrt{a}\,b\left(-3i\sqrt{b}\,d+5\sqrt{a}\,f\right)x^6\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{60a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^6\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(10*a*c + 2*b*x^4*(5*c + 12*d*x)
+ a*x*(12*d + 5*x*(3*e + 4*f*x))) + 15*Sqrt[a]*b*e*x^6*Sqrt[a + b*x^4]*ArcT
```

```
anh[Sqrt[a + b*x^4]/Sqrt[a]])) + 24*Sqrt[a]*b^(3/2)*d*x^6*Sqrt[1 + (b*x^4)/
a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - (8*I)*Sqrt[a]*b*
((-3*I)*Sqrt[b]*d + 5*Sqrt[a]*f)*x^6*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSin
h[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(60*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^6*Sq
rt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 303, normalized size = 0.86

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{6x^6}-\frac{d\sqrt{b\,x^4+a}}{5x^5}-\frac{e\sqrt{b\,x^4+a}}{4x^4}-\frac{f\sqrt{b\,x^4+a}}{3x^3}-\frac{bc\sqrt{b\,x^4+a}}{6a\,x^2}-\frac{2bd\sqrt{b\,x^4+a}}{5ax}+\frac{2bf\cdots}{\cdots}$
default	$d\left(-\frac{\sqrt{b\,x^4+a}}{5x^5}-\frac{2b\sqrt{b\,x^4+a}}{5ax}+\frac{2ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\cdots\right.}{5\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,(24bd\,x^5+10bc\,x^4+20af\,x^3+15ae\,x^2+12adx+10ac)}{60x^6a}+\frac{2ib^{\frac{3}{2}}d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\cdots\right.}{5\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/5/x^5*(b*x^4+a)^(1/2)-2/5*b/a*(b*x^4+a)^(1/2)/x+2/5*I*b^(3/2)/a^(1/2)
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-E
llipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+e*(-1/4/a/x^4*(b*x^4+a)^(3/2)-1/4
*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/4*b/a*(b*x^4+a)^(1/2))
-1/6*c*(b*x^4+a)^(3/2)/a/x^6+f*(-1/3/x^3*(b*x^4+a)^(1/2)+2/3*b/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/
2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(b*x^4 + a)^(3/2)*c/(a*x^6) + integrate(sqrt(b*x^4 + a)*(f*x^2 + x*e +
 d)/x^6, x)
```

Fricas [A]
time = 0.12, size = 166, normalized size = 0.47

$$-\frac{48\sqrt{a}\,bdx^6\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-15\sqrt{a}\,bex^6\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\,\sqrt{a}+2a}{x^4}\right)-16\,(3\,bd-5\,af)\sqrt{a}\,x^6\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)+2\,(24\,bdx^5+10\,bcx^4+20\,afx^3+15\,aex^2+12\,adx+10\,ac)\sqrt{bx^4+a}}{120\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/120*(48*sqrt(a)*b*d*x^6*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)),
-1) - 15*sqrt(a)*b*e*x^6*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4
) - 16*(3*b*d - 5*a*f)*sqrt(a)*x^6*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^
(1/4)), -1) + 2*(24*b*d*x^5 + 10*b*c*x^4 + 20*a*f*x^3 + 15*a*e*x^2 + 12*a*d
*x + 10*a*c)*sqrt(b*x^4 + a))/(a*x^6)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.98, size = 189, normalized size = 0.54

$$\frac{\sqrt{a}\,d\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}+\frac{\sqrt{a}\,f\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{b}\,c\sqrt{\frac{a}{bx^4}+1}}{6x^4}-\frac{\sqrt{b}\,e\sqrt{\frac{a}{bx^4}+1}}{4x^2}-\frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{6a}-\frac{be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**7,x)
```

```
[Out] sqrt(a)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**5*gamma(-1/4)) + sqrt(a)*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,),
b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(b)*c*sqrt(a/(b*x**4) +
 1)/(6*x**4) - sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*c*sqrt(a/
(b*x**4) + 1)/(6*a) - b*e*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^7,x)
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^7, x)
```

3.507 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx$

Optimal. Leaf size=375

$$-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-\frac{bd\sqrt{a+bx^4}}{6ax^2}-\frac{2be\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}ex\sqrt{a}}{5a\left(\sqrt{a}+\right.}$$

```
[Out] -1/4*b*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/420*(60*c/x^7+70*d/x^6+
84*e/x^5+105*f/x^4)*(b*x^4+a)^(1/2)-2/21*b*c*(b*x^4+a)^(1/2)/a/x^3-1/6*b*d*
(b*x^4+a)^(1/2)/a/x^2-2/5*b*e*(b*x^4+a)^(1/2)/a/x+2/5*b^(3/2)*e*x*(b*x^4+a)
^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-2/5*b^(5/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4
)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^
(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)-1/105*b^(5/4)*(cos(2*arctan(b^(1/4)
*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arct
an(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-21*e*a^(1/2)+5*c*b^(1/2))*(a^(1/2)+x^
2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^4+a)^(1/2
)
```

Rubi [A]
time = 0.24, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {14, 1839, 1847, 1296, 1212, 226, 1210, 1266, 821, 272, 65, 214}

$$-\frac{b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-21\sqrt{a}e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}+\frac{2b^{3/2}ex\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{420}\sqrt{a+bx^4}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-\frac{bd\sqrt{a+bx^4}}{6ax^2}-\frac{2be\sqrt{a+bx^4}}{5ax}-\frac{bf\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8,x]
```

```
[Out] -1/420*(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*Sqrt[a + b*x^4
]) - (2*b*c*Sqrt[a + b*x^4])/(21*a*x^3) - (b*d*Sqrt[a + b*x^4])/(6*a*x^2) -
 (2*b*e*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*e*x*Sqrt[a + b*x^4])/(5*a*(Sq
rt[a] + Sqrt[b]*x^2)) - (b*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a])
- (2*b^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a +
b*x^4]) - (b^(5/4)*(5*Sqrt[b]*c - 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(105*a^(5/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
```

```
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4} - (2b)\int \frac{-\frac{c}{7}-\frac{dx}{6}-}{x^4\sqrt{a+}}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4} - (2b)\int \left(\frac{-\frac{c}{7}-}{x^4\sqrt{a+}}\right.$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4} - (2b)\int \frac{-\frac{c}{7}-\frac{ex^2}{5}}{x^4\sqrt{a+bx}}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4} - \frac{2bc\sqrt{a+bx^4}}{21ax^3} - b$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4} - \frac{2bc\sqrt{a+bx^4}}{21ax^3} - b$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4} - \frac{2bc\sqrt{a+bx^4}}{21ax^3} - b$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4} - \frac{2bc\sqrt{a+bx^4}}{21ax^3} - b$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4} - \frac{2bc\sqrt{a+bx^4}}{21ax^3} - b$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 10.34, size = 283, normalized size = 0.75

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(2bx^4(20c+7x(5d+12ex))+a(60c+7x(10d+3x(4e+5fx)))\right)+105\sqrt{a}\,bfx^7\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+168\sqrt{a}\,b^{3/2}ex^7\sqrt{1+\frac{bx^4}{a}}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-8b^{3/2}\left(-5i\sqrt{b}\,c+21\sqrt{a}\,e\right)x^7\sqrt{1+\frac{bx^4}{a}}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{420a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x^7\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(2*b*x^4*(20*c + 7*x*(5*d + 12*e*
x)) + a*(60*c + 7*x*(10*d + 3*x*(4*e + 5*f*x)))) + 105*Sqrt[a]*b*f*x^7*Sqrt
[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 168*Sqrt[a]*b^(3/2)*e*x^7*
Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] -
```

```
8*b^(3/2)*((-5*I)*Sqrt[b]*c + 21*Sqrt[a]*e)*x^7*Sqrt[1 + (b*x^4)/a]*Ellipt
icF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(420*a*Sqrt[(I*Sqrt[b])/Sq
rt[a]]*x^7*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 326, normalized size = 0.87

method	result
elliptic	$-\frac{c\sqrt{b x^4+a}}{7x^7}-\frac{d\sqrt{b x^4+a}}{6x^6}-\frac{e\sqrt{b x^4+a}}{5x^5}-\frac{f\sqrt{b x^4+a}}{4x^4}-\frac{2bc\sqrt{b x^4+a}}{21a x^3}-\frac{bd\sqrt{b x^4+a}}{6a x^2}-2b\cdots$
default	$e\left(-\frac{\sqrt{b x^4+a}}{5x^5}-\frac{2b\sqrt{b x^4+a}}{5ax}+\frac{2ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{Elliptic}\cdots\right.}{5\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b x^4+a}}\cdots\right.$
risch	$-\frac{\sqrt{b x^4+a}\,(168be x^6+70bd x^5+40bc x^4+105af x^3+84ae x^2+70adx+60ac)}{420x^7a}+\frac{2ib^{\frac{3}{2}}e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\cdots}{5\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b x\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/5/x^5*(b*x^4+a)^(1/2)-2/5*b/a*(b*x^4+a)^(1/2)/x+2/5*I*b^(3/2)/a^(1/2)
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-E
llipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+f*(-1/4/a/x^4*(b*x^4+a)^(3/2)-1/4
*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/4*b/a*(b*x^4+a)^(1/2))
-1/6*d*(b*x^4+a)^(3/2)/a/x^6+c*(-1/7/x^7*(b*x^4+a)^(1/2)-2/21*b/a*(b*x^4+a)
^(1/2)/x^3-2/21*b^2/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^8, x)
```

Fricas [A]
time = 0.11, size = 173, normalized size = 0.46

$$-\frac{336\,\sqrt{a}\,bex^{7}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1) - 105\,\sqrt{a}\,bfx^{7}\log\left(-\frac{bx^{4}-2\,\sqrt{bx^{4}+a}\,\sqrt{a}+2\,a}{x^{4}}\right) - 16\,(5\,bc+21\,be)\sqrt{a}\,x^{7}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1) + 2\,(168\,bex^{6}+70\,bdx^{5}+40\,bcx^{4}+105\,afx^{3}+84\,aex^{2}+70\,adx+60\,ac)\sqrt{bx^{4}+a}}{840\,ax^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/840*(336*sqrt(a)*b*e*x^7*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)),
 -1) - 105*sqrt(a)*b*f*x^7*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x
^4) - 16*(5*b*c + 21*b*e)*sqrt(a)*x^7*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/
a)^(1/4)), -1) + 2*(168*b*e*x^6 + 70*b*d*x^5 + 40*b*c*x^4 + 105*a*f*x^3 + 8
4*a*e*x^2 + 70*a*d*x + 60*a*c)*sqrt(b*x^4 + a))/(a*x^7)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.09, size = 192, normalized size = 0.51

$$\frac{\sqrt{a}\,c\Gamma\left(-\frac{7}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}\,e\Gamma\left(-\frac{5}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4x^{5}\Gamma\left(-\frac{1}{4}\right)}-\frac{\sqrt{b}\,d\sqrt{\frac{a}{bx^{4}}+1}}{6x^{4}}-\frac{\sqrt{b}\,f\sqrt{\frac{a}{bx^{4}}+1}}{4x^{2}}-\frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^{4}}+1}}{6a}-\frac{bf\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{2}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**8,x)
```

```
[Out] sqrt(a)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**7*gamma(-3/4)) + sqrt(a)*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - sqrt(b)*d*sqrt(a/(b*x**4)
 + 1)/(6*x**4) - sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*d*sqrt(
a/(b*x**4) + 1)/(6*a) - b*f*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^8} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^8,x)
```

```
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^8, x)
```

3.508 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx$

Optimal. Leaf size=400

$$-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}-\frac{be\sqrt{a+bx^4}}{6ax^2}-\frac{2bf\sqrt{a+}}{5ax}$$

```
[Out] 1/16*b^2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/840*(105*c/x^8+120*d/
x^7+140*e/x^6+168*f/x^5)*(b*x^4+a)^(1/2)-1/16*b*c*(b*x^4+a)^(1/2)/a/x^4-2/2
1*b*d*(b*x^4+a)^(1/2)/a/x^3-1/6*b*e*(b*x^4+a)^(1/2)/a/x^2-2/5*b*f*(b*x^4+a)
^(1/2)/a/x+2/5*b^(3/2)*f*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-2/5*b^(5
/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(
1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2
*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
-1/105*b^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-2
1*f*a^(1/2)+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1
/2))^2)^(1/2)/a^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {14, 1839, 1847, 1266, 849, 821, 272, 65, 214, 1296, 1212, 226, 1210}

$$-\frac{b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}d-21\sqrt{a}f\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}+\frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{2b^{3/2}fx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{840}\sqrt{a+bx^4}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}-\frac{be\sqrt{a+bx^4}}{6ax^2}-\frac{2bf\sqrt{a+bx^4}}{5ax}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]
```

```
[Out] -1/840*(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*Sqrt[a + b*
x^4]) - (b*c*Sqrt[a + b*x^4])/(16*a*x^4) - (2*b*d*Sqrt[a + b*x^4])/(21*a*x^
3) - (b*e*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*f*Sqrt[a + b*x^4])/(5*a*x) + (2
*b^(3/2)*f*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) + (b^2*c*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^(3/2)) - (2*b^(5/4)*f*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) - (b^(5/4)*(5*Sqrt[b]*d -
21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(5/4)*Sqrt[a +
 b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx = -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4} - (2b)\int \frac{-\frac{c}{8}-\cdots}{x^5\cdots}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4} - (2b)\int \left(\frac{-\frac{c}{8}\cdots}{x^5\sqrt{a\cdots}}\right.$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4} - (2b)\int \frac{-\frac{c}{8}-\cdots}{x^5\sqrt{a-\cdots}}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4} - \frac{2bd\sqrt{a+bx^4}}{21ax^3}\cdots$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4} - \frac{bc\sqrt{a+bx^4}}{16ax^4} - \cdots$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4} - \frac{bc\sqrt{a+bx^4}}{16ax^4} - \cdots$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4} - \frac{bc\sqrt{a+bx^4}}{16ax^4} - \cdots$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4} - \frac{bc\sqrt{a+bx^4}}{16ax^4} - \cdots$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4} - \frac{bc\sqrt{a+bx^4}}{16ax^4} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.39, size = 293, normalized size = 0.73

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\sqrt{a}\,(a+bx^4)\left(bx^4(105c+8x(20d+35ex+84fx^2))+a(210c+8x(30d+7x(5e+6fx)))\right)+105b^2cx^8\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+672ab^{3/2}fx^8\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-32\sqrt{a}\,b^{3/2}\left(-5i\sqrt{b}\,d+21\sqrt{a}\,f\right)x^8\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{1680a^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^8\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-(Sqrt[a]*(a + b*x^4)*(b*x^4*(105*c + 8*x*(20*d
 + 35*e*x + 84*f*x^2)) + a*(210*c + 8*x*(30*d + 7*x*(5*e + 6*f*x))))) + 105
*b^2*c*x^8*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) + 672*a*b^(3/2
)*f*x^8*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x
], -1] - 32*Sqrt[a]*b^(3/2)*((-5*I)*Sqrt[b]*d + 21*Sqrt[a]*f)*x^8*Sqrt[1 +
(b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(1680*a^(
3/2)*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^8*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 348, normalized size = 0.87

method	result
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{8x^{8}}-\frac{d\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{e\sqrt{b\,x^{4}+a}}{6x^{6}}-\frac{f\sqrt{b\,x^{4}+a}}{5x^{5}}-\frac{bc\sqrt{b\,x^{4}+a}}{16a\,x^{4}}-\frac{2bd\sqrt{b\,x^{4}+a}}{21a\,x^{3}}-\frac{be}{}$
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(672bf\,x^{7}+280be\,x^{6}+160bd\,x^{5}+105bc\,x^{4}+336af\,x^{3}+280ae\,x^{2}+240adx+210ac)}{1680x^{8}a}+\frac{2ib^{\frac{3}{2}}f\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{}}{5\sqrt{a}\,\sqrt{}}$
default	$f\left(-\frac{\sqrt{b\,x^{4}+a}}{5x^{5}}-\frac{2b\sqrt{b\,x^{4}+a}}{5ax}+\frac{2ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{Ellipti}\right.}{5\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] f*(-1/5/x^5*(b*x^4+a)^(1/2)-2/5*b/a*(b*x^4+a)^(1/2)/x+2/5*I*b^(3/2)/a^(1/2)
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-E
llipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))-1/6*e*(b*x^4+a)^(3/2)/a/x^6+c*(-1
/8/a/x^8*(b*x^4+a)^(3/2)+1/16*b/a^2/x^4*(b*x^4+a)^(3/2)+1/16*b^2/a^(3/2)*ln
((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/16*b^2/a^2*(b*x^4+a)^(1/2))+d*(-1/7
/x^7*(b*x^4+a)^(1/2)-2/21*b/a*(b*x^4+a)^(1/2)/x^3-2/21*b^2/a/(I/a^(1/2)*b^(
1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)
/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/32*(b^2*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^(
3/2) + 2*((b*x^4 + a)^(3/2)*b^2 + sqrt(b*x^4 + a)*a*b^2)/((b*x^4 + a)^2*a -
 2*(b*x^4 + a)*a^2 + a^3))*c + integrate(sqrt(b*x^4 + a)*(f*x^2 + x*e + d)/
x^8, x)
```

Fricas [A]
time = 0.12, size = 196, normalized size = 0.49

$$-\frac{1344\,a^{\frac{3}{2}}bfx^{8}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-105\,\sqrt{a}\,b^{2}cx^{8}\log\left(-\frac{bx^{4}+2\,\sqrt{bx^{4}+a}\,\sqrt{a}+2\,a}{x^{4}}\right)-64\,(5\,abd+21\,abf)\sqrt{a}\,x^{8}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)+2\,(672\,abfx^{7}+280\,abex^{6}+160\,abdx^{5}+105\,abcx^{4}+336\,a^{2}fx^{3}+280\,a^{2}ex^{2}+240\,a^{2}dx+210\,a^{2}c)\sqrt{bx^{4}+a}}{3360\,a^{2}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/3360*(1344*a^(3/2)*b*f*x^8*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)
), -1) - 105*sqrt(a)*b^2*c*x^8*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) + 2*
a)/x^4) - 64*(5*a*b*d + 21*a*b*f)*sqrt(a)*x^8*(-b/a)^(3/4)*elliptic_f(arcsi
n(x*(-b/a)^(1/4)), -1) + 2*(672*a*b*f*x^7 + 280*a*b*e*x^6 + 160*a*b*d*x^5 +
 105*a*b*c*x^4 + 336*a^2*f*x^3 + 280*a^2*e*x^2 + 240*a^2*d*x + 210*a^2*c)*s
qrt(b*x^4 + a))/(a^2*x^8)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.30, size = 246, normalized size = 0.62

$$\frac{\sqrt{a}\,d\Gamma\left(-\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}\,f\Gamma\left(-\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4x^{5}\Gamma\left(-\frac{1}{4}\right)}-\frac{ac}{8\sqrt{b}\,x^{10}\sqrt{\frac{a}{bx^{4}}+1}}-\frac{3\sqrt{b}\,c}{16x^{6}\sqrt{\frac{a}{bx^{4}}+1}}-\frac{\sqrt{b}\,e\sqrt{\frac{a}{bx^{4}}+1}}{6x^{4}}-\frac{b^{\frac{3}{2}}c}{16ax^{2}\sqrt{\frac{a}{bx^{4}}+1}}-\frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^{4}}+1}}{6a}+\frac{b^{2}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{2}}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**9,x)
```

```
[Out] sqrt(a)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**7*gamma(-3/4)) + sqrt(a)*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - a*c/(8*sqrt(b)*x**10*sqrt
(a/(b*x**4) + 1)) - 3*sqrt(b)*c/(16*x**6*sqrt(a/(b*x**4) + 1)) - sqrt(b)*e*
sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*c/(16*a*x**2*sqrt(a/(b*x**4) + 1))
 - b**(3/2)*e*sqrt(a/(b*x**4) + 1)/(6*a) + b**2*c*asinh(sqrt(a)/(sqrt(b)*x*
*2))/(16*a**(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^9, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^9,x)
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^9, x)
```

3.509 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx$

Optimal. Leaf size=425

$$-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4}-\frac{2be\sqrt{a+bx^4}}{21ax^3}-\frac{bf\sqrt{a+bx^4}}{6ax^2}+$$

```
[Out] 1/16*b^2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/504*(56*c/x^9+63*d/x^
8+72*e/x^7+84*f/x^6)*(b*x^4+a)^(1/2)-2/45*b*c*(b*x^4+a)^(1/2)/a/x^5-1/16*b*
d*(b*x^4+a)^(1/2)/a/x^4-2/21*b*e*(b*x^4+a)^(1/2)/a/x^3-1/6*b*f*(b*x^4+a)^(1
/2)/a/x^2+2/15*b^2*c*(b*x^4+a)^(1/2)/a^2/x-2/15*b^(5/2)*c*x*(b*x^4+a)^(1/2)
/a^2/(a^(1/2)+x^2*b^(1/2))+2/15*b^(9/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/
2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)-1/105*b^(7/4)*(cos(2*arctan(b^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(
b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(5*e*a^(1/2)+7*c*b^(1/2))*(a^(1/2)+x^2*b^(
1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$,
Rules used = {14, 1839, 1847, 1296, 1212, 226, 1210, 1266, 849, 821, 272, 65, 214}

$$-\frac{b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(5\sqrt{a}\,e+7\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{7/4}\sqrt{a+bx^4}}+\frac{2b^{9/4}c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^4}}+\frac{b^2d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}-\frac{2b^{5/2}cx\sqrt{a+bx^4}}{15a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{2b^2c\sqrt{a+bx^4}}{15a^2x}-\frac{1}{504}\sqrt{a+bx^4}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4}-\frac{2be\sqrt{a+bx^4}}{21ax^3}-\frac{bf\sqrt{a+bx^4}}{6ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^10,x]
```

```
[Out] -1/504*(((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*Sqrt[a + b*x^4]
) - (2*b*c*Sqrt[a + b*x^4])/(45*a*x^5) - (b*d*Sqrt[a + b*x^4])/(16*a*x^4) -
 (2*b*e*Sqrt[a + b*x^4])/(21*a*x^3) - (b*f*Sqrt[a + b*x^4])/(6*a*x^2) + (2*
b^2*c*Sqrt[a + b*x^4])/(15*a^2*x) - (2*b^(5/2)*c*x*Sqrt[a + b*x^4])/(15*a^2
*(Sqrt[a] + Sqrt[b]*x^2)) + (b^2*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^
(3/2)) + (2*b^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(7/4)*S
qrt[a + b*x^4]) - (b^(7/4)*(7*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(105*a^(7/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx = -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{9}-\frac{dx}{8}-}{x^6\sqrt{a}}$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-(2b)\int \left(\frac{-\frac{c}{9}-}{x^6\sqrt{a+}}\right.$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{9}-\frac{ex^2}{7}}{x^6\sqrt{a+bx}}$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-b$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.38, size = 305, normalized size = 0.72

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\left((a+bx^4)\left(-672b^2cx^8+10a^2(56c+63dx+72ex^2+84fx^3)+abx^4(224c+15x(21d+8x(4e+7fx)))\right)\right)+315\sqrt{a}\,b^2dx^9\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)-672\sqrt{a}\,b^{5/2}cx^9\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)+96\sqrt{a}\,b^2\left(7\sqrt{b}\,c+5i\sqrt{a}\,e\right)x^9\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{5040a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^9\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^10,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-((a + b*x^4)*(-672*b^2*c*x^8 + 10*a^2*(56*c +
63*d*x + 72*e*x^2 + 84*f*x^3) + a*b*x^4*(224*c + 15*x*(21*d + 8*x*(4*e + 7*
f*x))))) + 315*Sqrt[a]*b^2*d*x^9*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sq
rt[a]]) - 672*Sqrt[a]*b^(5/2)*c*x^9*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh
[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] + 96*Sqrt[a]*b^2*(7*Sqrt[b]*c + (5*I)*Sq
rt[a]*e)*x^9*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[
a]]*x], -1])/(5040*a^2*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^9*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 368, normalized size = 0.87

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{9x^9}-\frac{d\sqrt{b\,x^4+a}}{8x^8}-\frac{e\sqrt{b\,x^4+a}}{7x^7}-\frac{f\sqrt{b\,x^4+a}}{6x^6}-\frac{2bc\sqrt{b\,x^4+a}}{45a\,x^5}-\frac{bd\sqrt{b\,x^4+a}}{16a\,x^4}-\frac{2be}{}$
risch	$-\frac{\sqrt{b\,x^4+a}\,(-672b^2c\,x^8+840abf\,x^7+480abe\,x^6+315abd\,x^5+224abc\,x^4+840a^2f\,x^3+720a^2e\,x^2+630a^2dx+560a^2c)}{5040x^9a^2}-\frac{2ib^{\frac{5}{2}}c\sqrt{}}{}$
default	$c\left(-\frac{\sqrt{b\,x^4+a}}{9x^9}-\frac{2b\sqrt{b\,x^4+a}}{45a\,x^5}+\frac{2b^2\sqrt{b\,x^4+a}}{15a^2x}-\frac{2ib^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{}\right.\right.}{15a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/9/x^9*(b*x^4+a)^(1/2)-2/45*b/a*(b*x^4+a)^(1/2)/x^5+2/15*b^2/a^2*(b*x^
4+a)^(1/2)/x-2/15*I*b^(5/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(Ellipti
cF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
)-1/6*f*(b*x^4+a)^(3/2)/a/x^6+d*(-1/8/a/x^8*(b*x^4+a)^(3/2)+1/16*b/a^2/x^4*
(b*x^4+a)^(3/2)+1/16*b^2/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/
16*b^2/a^2*(b*x^4+a)^(1/2))+e*(-1/7/x^7*(b*x^4+a)^(1/2)-2/21*b/a*(b*x^4+a)^
(1/2)/x^3-2/21*b^2/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^10, x)
```

Fricas [A]
time = 0.13, size = 208, normalized size = 0.49

$$\frac{1344\sqrt{a}\,b^2cx^9\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)+315\sqrt{a}\,b^2dx^9\log\left(-\frac{bx^4+2\sqrt{bx^4+a}\,\sqrt{a}+2a}{x^4}\right)-192\left(7b^2c-5abe\right)\sqrt{a}\,x^9\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)+2\left(672b^2cx^8-840abfx^7-480abex^6-315abdx^5-224abcx^4-840a^2fx^3-720a^2ex^2-630a^2dx-560a^2c\right)\sqrt{bx^4+a}}{10080a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/10080*(1344*sqrt(a)*b^2*c*x^9*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/
4)), -1) + 315*sqrt(a)*b^2*d*x^9*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) +
2*a)/x^4) - 192*(7*b^2*c - 5*a*b*e)*sqrt(a)*x^9*(-b/a)^(3/4)*elliptic_f(arc
sin(x*(-b/a)^(1/4)), -1) + 2*(672*b^2*c*x^8 - 840*a*b*f*x^7 - 480*a*b*e*x^6
 - 315*a*b*d*x^5 - 224*a*b*c*x^4 - 840*a^2*f*x^3 - 720*a^2*e*x^2 - 630*a^2*
d*x - 560*a^2*c)*sqrt(b*x^4 + a))/(a^2*x^9)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.47, size = 246, normalized size = 0.58

$$\frac{\sqrt{a}\,c\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)}+\frac{\sqrt{a}\,e\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}-\frac{ad}{8\sqrt{b}\,x^{10}\sqrt{\frac{a}{bx^4}+1}}-\frac{3\sqrt{b}\,d}{16x^6\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}\,f\sqrt{\frac{a}{bx^4}+1}}{6x^4}-\frac{b^{\frac{3}{2}}d}{16ax^2\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{6a}+\frac{b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**10,x)
```

```
[Out] sqrt(a)*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**9*gamma(-5/4)) + sqrt(a)*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) - a*d/(8*sqrt(b)*x**10*sqrt
(a/(b*x**4) + 1)) - 3*sqrt(b)*d/(16*x**6*sqrt(a/(b*x**4) + 1)) - sqrt(b)*f*
sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*d/(16*a*x**2*sqrt(a/(b*x**4) + 1))
 - b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(6*a) + b**2*d*asinh(sqrt(a)/(sqrt(b)*x*
*2))/(16*a**(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + x^2*e + d*x + c)/x^10, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^10,x)
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^10, x)
```

3.510 $\int x^4(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

Optimal. Leaf size=476

$$\frac{4a^2cx\sqrt{a+bx^4}}{77b}-\frac{a^2dx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}-\frac{4a^3ex\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{2ax^5(117c+77ex^2)\sqrt{a+}}{3003}$$

```
[Out] -1/48*a*d*x^2*(b*x^4+a)^(3/2)/b+1/143*x^5*(11*e*x^2+13*c)*(b*x^4+a)^(3/2)+1
/14*f*x^4*(b*x^4+a)^(5/2)/b-1/420*(-35*b*d*x^2+12*a*f)*(b*x^4+a)^(5/2)/b^2-
1/32*a^3*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+4/77*a^2*c*x*(b*x^4
+a)^(1/2)/b-1/32*a^2*d*x^2*(b*x^4+a)^(1/2)/b+4/195*a^2*e*x^3*(b*x^4+a)^(1/2
)/b+2/3003*a*x^5*(77*e*x^2+117*c)*(b*x^4+a)^(1/2)-4/65*a^3*e*x*(b*x^4+a)^(1
/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))+4/65*a^(13/4)*e*(cos(2*arctan(b^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b
^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x
^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)-2/5005*a^(11/4)*(cos(2*arctan(
b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin
(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(77*e*a^(1/2)+65*c*b^(1/2))*(a^(
1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+
a)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {1847, 1288, 1294, 1212, 226, 1210, 1266, 847, 794, 201, 223, 212}

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(77\sqrt{a}\,e+65\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}+\frac{4a^{13/4}e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}}-\frac{a^3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^3ex\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{4a^2cx\sqrt{a+bx^4}}{77b}-\frac{a^2dx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}-\frac{(a+bx^4)^{5/2}(12af-35bdx^2)}{420b^2}+\frac{1}{143}x^5\left(a+bx^4\right)^{3/2}\left(13c+11ex^2\right)+\frac{2ax^5\sqrt{a+bx^4}\,(117c+77ex^2)}{3003}-\frac{adx^2(a+bx^4)^{3/2}}{48b}+\frac{fx^4(a+bx^4)^{5/2}}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (4*a^2*c*x*Sqrt[a + b*x^4])/(77*b) - (a^2*d*x^2*Sqrt[a + b*x^4])/(32*b) + (
4*a^2*e*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*e*x*Sqrt[a + b*x^4])/(65*b^(3
/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*c + 77*e*x^2)*Sqrt[a + b*x^4])
/3003 - (a*d*x^2*(a + b*x^4)^(3/2))/(48*b) + (x^5*(13*c + 11*e*x^2)*(a + b*
x^4)^(3/2))/143 + (f*x^4*(a + b*x^4)^(5/2))/(14*b) - ((12*a*f - 35*b*d*x^2)
*(a + b*x^4)^(5/2))/(420*b^2) - (a^3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4
]])/(32*b^(3/2)) + (4*a^(13/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6
5*b^(7/4)*Sqrt[a + b*x^4]) - (2*a^(11/4)*(65*Sqrt[b]*c + 77*Sqrt[a]*e)*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int x^4\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx = \int \left(x^4\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}+x^5\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)\,dx$$

$$= \int x^4\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}\,dx + \int x^5\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\,d$$

$$= \frac{1}{143}x^5\left(13c+11ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{1}{2}\text{Subst}\left(\int x^2(d+fx)\,(a\right.$$

$$= \frac{2ax^5(117c+77ex^2)\sqrt{a+bx^4}}{3003}+\frac{1}{143}x^5\left(13c+11ex^2\right)\left(a+bx\right.$$

$$= \frac{4a^2ex^3\sqrt{a+bx^4}}{195b}+\frac{2ax^5(117c+77ex^2)\sqrt{a+bx^4}}{3003}+\frac{1}{143}x^5(1$$

$$= \frac{4a^2cx\sqrt{a+bx^4}}{77b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}+\frac{2ax^5(117c+77ex^2)\sqrt{}}{3003}$$

$$= \frac{4a^2cx\sqrt{a+bx^4}}{77b}-\frac{a^2dx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}+\frac{2a}{}$$

$$= \frac{4a^2cx\sqrt{a+bx^4}}{77b}-\frac{a^2dx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}-\frac{}{65b}$$

$$= \frac{4a^2cx\sqrt{a+bx^4}}{77b}-\frac{a^2dx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}-\frac{}{65b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.54, size = 225, normalized size = 0.47

$$\frac{\sqrt{a+bx^4}\left(43680bcx(a+bx^4)^2+36960bex^3(a+bx^4)^2+6864f(a+bx^4)^2(-2a+5bx^4)+5005bdx^2(3a^2+14abx^4+8b^2x^8)-\frac{15015a^{5/2}\sqrt{b}\,d\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{1+\frac{bx^4}{a}}}-\frac{43680a^2bcx\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}-\frac{36960a^2bex^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)}{480480b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*(43680*b*c*x*(a + b*x^4)^2 + 36960*b*e*x^3*(a + b*x^4)^2 +
 6864*f*(a + b*x^4)^2*(-2*a + 5*b*x^4) + 5005*b*d*x^2*(3*a^2 + 14*a*b*x^4 +
```

```
8*b^2*x^8) - (15015*a^(5/2)*Sqrt[b]*d*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt
[1 + (b*x^4)/a] - (43680*a^2*b*c*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x
^4)/a)])/Sqrt[1 + (b*x^4)/a] - (36960*a^2*b*e*x^3*Hypergeometric2F1[-3/2, 3
/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/(480480*b^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 400, normalized size = 0.84

method	result
risch	$-\frac{\left(-34320 f\,x^{12} b^{3}-36960 b^{3} e\,x^{11}-40040 b^{3} d\,x^{10}-43680 b^{3} c\,x^{9}-54912 a\,b^{2} f\,x^{8}-61600 a\,b^{2} e\,x^{7}-70070 a\,b^{2} d\,x^{6}-81120 a\,b^{2} c\,x^{5}-6864 a\right.}{480480 b^{2}}$
default	$-\frac{f\sqrt{b\,x^{4}+a}\,\left(-5 b\,x^{4}+2 a\right)\left(b^{2} x^{8}+2 a b\,x^{4}+a^{2}\right)}{70 b^{2}}+e\left(\frac{b\,x^{11}\sqrt{b\,x^{4}+a}}{13}+\frac{5 a\,x^{7}\sqrt{b\,x^{4}+a}}{39}+\frac{4 a^{2} x^{3}\sqrt{b\,x^{4}+a}}{195 b}\right.$
elliptic	$\frac{b f\,x^{12}\sqrt{b\,x^{4}+a}}{14}+\frac{b e\,x^{11}\sqrt{b\,x^{4}+a}}{13}+\frac{b d\,x^{10}\sqrt{b\,x^{4}+a}}{12}+\frac{b c\,x^{9}\sqrt{b\,x^{4}+a}}{11}+\frac{4 a f\,x^{8}\sqrt{b\,x^{4}+a}}{35}+\frac{5 a e\,x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/70*f*(b*x^4+a)^(1/2)*(-5*b*x^4+2*a)*(b^2*x^8+2*a*b*x^4+a^2)/b^2+e*(1/13*
b*x^11*(b*x^4+a)^(1/2)+5/39*a*x^7*(b*x^4+a)^(1/2)+4/195/b*a^2*x^3*(b*x^4+a)
^(1/2)-4/65*I/b^(3/2)*a^(7/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(
I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+d*(1
/12*b*x^10*(b*x^4+a)^(1/2)+7/48*a*x^6*(b*x^4+a)^(1/2)+1/32/b*a^2*x^2*(b*x^4
+a)^(1/2)-1/32/b^(3/2)*a^3*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+c*(1/11*b*x^9*(
b*x^4+a)^(1/2)+13/77*a*x^5*(b*x^4+a)^(1/2)+4/77/b*a^2*x*(b*x^4+a)^(1/2)-4/7
7/b*a^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2
),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)*x^4, x)
```

Fricas [A]
time = 0.15, size = 264, normalized size = 0.55

$$-\frac{59136\,a^{3}\sqrt{b}\,ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1) - 15015\,a^{3}\sqrt{b}\,dx\log\left(-2\,bx^{4}+2\,\sqrt{bx^{4}+a}\,\sqrt{b}\,x^{2}-a\right) + 768\,(65\,a^{2}bc-77\,a^{3}e)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1) - 2\,(34320\,b^{3}fx^{13}+36960\,b^{3}ex^{12}+40040\,b^{3}dx^{11}+43680\,b^{3}cx^{10}+54912\,ab^{2}fx^{9}+61600\,ab^{2}ex^{8}+70070\,ab^{2}dx^{7}+81120\,ab^{2}cx^{6}+6864\,a^{2}bfx^{5}+9856\,a^{2}bex^{4}+15015\,a^{2}bdx^{3}+24960\,a^{2}bcx^{2}-13728\,a^{3}fx-29568\,a^{3}e)\sqrt{bx^{4}+a}}{960960\,b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/960960*(59136*a^3*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4
)/x), -1) - 15015*a^3*sqrt(b)*d*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*
x^2 - a) + 768*(65*a^2*b*c - 77*a^3*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(ar
csin((-a/b)^(1/4)/x), -1) - 2*(34320*b^3*f*x^13 + 36960*b^3*e*x^12 + 40040*
b^3*d*x^11 + 43680*b^3*c*x^10 + 54912*a*b^2*f*x^9 + 61600*a*b^2*e*x^8 + 700
70*a*b^2*d*x^7 + 81120*a*b^2*c*x^6 + 6864*a^2*b*f*x^5 + 9856*a^2*b*e*x^4 +
15015*a^2*b*d*x^3 + 24960*a^2*b*c*x^2 - 13728*a^3*f*x - 29568*a^3*e)*sqrt(b
*x^4 + a))/(b^2*x)
```

Sympy [A]
time = 10.23, size = 462, normalized size = 0.97

$$\frac{a^{\frac{5}{2}}dx^{2}}{32b\sqrt{1+\frac{bx^{4}}{a}}} + \frac{a^{\frac{3}{2}}cx^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{17a^{\frac{3}{2}}dx^{6}}{96\sqrt{1+\frac{bx^{4}}{a}}} + \frac{a^{\frac{3}{2}}ex^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{a}\,bcx^{9}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)} + \frac{11\sqrt{a}\,bdx^{10}}{48\sqrt{1+\frac{bx^{4}}{a}}} + \frac{\sqrt{a}\,bex^{11}\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)} - \frac{a^{3}d\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}} + af\left(\begin{cases}-\frac{a^{2}\sqrt{a+bx^{4}}}{15b^{2}}+\frac{ax^{4}\sqrt{a+bx^{4}}}{30b}+\frac{x^{8}\sqrt{a+bx^{4}}}{10} & \text{for}\: b\neq 0\\ \frac{\sqrt{a}\,x^{8}}{8} & \text{otherwise}\end{cases}\right) + bf\left(\begin{cases}\frac{4a^{3}\sqrt{a+bx^{4}}}{105b^{3}}-\frac{2a^{2}x^{4}\sqrt{a+bx^{4}}}{105b^{2}}+\frac{ax^{8}\sqrt{a+bx^{4}}}{70b}+\frac{x^{12}\sqrt{a+bx^{4}}}{14} & \text{for}\: b\neq 0\\ \frac{\sqrt{a}\,x^{12}}{12} & \text{otherwise}\end{cases}\right) + \frac{b^{2}dx^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^{4}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(5/2)*d*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*c*x**5*gamma(5/4)*hype
r((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 17*a**(3/
2)*d*x**6/(96*sqrt(1 + b*x**4/a)) + a**(3/2)*e*x**7*gamma(7/4)*hyper((-1/2,
 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*c*x**
9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(13/4)) + 11*sqrt(a)*b*d*x**10/(48*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*x**11*
gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(15/4)) - a**3*d*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*f*Piecewise(
(-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*s
qrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b*f*Piecewise((4*a
**3*sqrt(a + b*x**4)/(105*b**3) - 2*a**2*x**4*sqrt(a + b*x**4)/(105*b**2) +
 a*x**8*sqrt(a + b*x**4)/(70*b) + x**12*sqrt(a + b*x**4)/14, Ne(b, 0)), (sq
rt(a)*x**12/12, True)) + b**2*d*x**14/(12*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(b\,x^4+a\right)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] int(x^4*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.511 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

Optimal. Leaf size=452

$$\frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}-\frac{4a^3fx\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{2ax^5(117d+77fx^2)\sqrt{a+}$$

```
[Out] -1/48*a*e*x^2*(b*x^4+a)^(3/2)/b+1/143*x^5*(11*f*x^2+13*d)*(b*x^4+a)^(3/2)+1
/60*(5*e*x^2+6*c)*(b*x^4+a)^(5/2)/b-1/32*a^3*e*arctanh(x^2*b^(1/2)/(b*x^4+a
)^(1/2))/b^(3/2)+4/77*a^2*d*x*(b*x^4+a)^(1/2)/b-1/32*a^2*e*x^2*(b*x^4+a)^(1
/2)/b+4/195*a^2*f*x^3*(b*x^4+a)^(1/2)/b+2/3003*a*x^5*(77*f*x^2+117*d)*(b*x^
4+a)^(1/2)-4/65*a^3*f*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))+4/65*
a^(13/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2
)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^
(1/2)-2/5005*a^(11/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/
2))*(77*f*a^(1/2)+65*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x
^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1847, 1266, 794, 201, 223, 212, 1288, 1294, 1212, 226, 1210}

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(77\sqrt{a}\,f+65\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}+\frac{4a^{13/4}f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}}-\frac{a^3e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^3fx\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}+\frac{(a+bx^4)^{5/2}(6c+5ex^2)}{60b}+\frac{1}{143}x^5(a+bx^4)^{3/2}(13d+11fx^2)+\frac{2ax^5\sqrt{a+bx^4}(117d+77fx^2)}{3003}-\frac{aex^2(a+bx^4)^{3/2}}{48b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (4*a^2*d*x*Sqrt[a + b*x^4])/(77*b) - (a^2*e*x^2*Sqrt[a + b*x^4])/(32*b) + (
4*a^2*f*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*f*x*Sqrt[a + b*x^4])/(65*b^(3
/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*d + 77*f*x^2)*Sqrt[a + b*x^4])
/3003 - (a*e*x^2*(a + b*x^4)^(3/2))/(48*b) + (x^5*(13*d + 11*f*x^2)*(a + b*
x^4)^(3/2))/143 + ((6*c + 5*e*x^2)*(a + b*x^4)^(5/2))/(60*b) - (a^3*e*ArcTa
nh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3/2)) + (4*a^(13/4)*f*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a + b*x^4]) - (2*a^(11/4)*(65
*Sqrt[b]*d + 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*b^
(7/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx &= \int \left(x^3\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}+x^4\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)\,dx \\
&= \int x^3\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}\,dx+\int x^4\left(d+fx^2\right)\left(a+bx^4\right)^{3/2} \\
&= \frac{1}{143}x^5\left(13d+11fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{1}{2}\text{Subst}\left(\int x(c+ex)\,(a \right. \\
&= \frac{2ax^5(117d+77fx^2)\sqrt{a+bx^4}}{3003}+\frac{1}{143}x^5\left(13d+11fx^2\right)(a+ \\
&= \frac{4a^2fx^3\sqrt{a+bx^4}}{195b}+\frac{2ax^5(117d+77fx^2)\sqrt{a+bx^4}}{3003}-\frac{aex^2(}{} \\
&= \frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}+\frac{2}{} \\
&= \frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}+\frac{2}{} \\
&= \frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}-\frac{}{6}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.45, size = 238, normalized size = 0.53

$$
\frac{\sqrt{a+bx^4}\left(6864\sqrt{b}\,c(a+bx^4)^2+6240\sqrt{b}\,dx(a+bx^4)^2+5280\sqrt{b}\,fx^3(a+bx^4)^2+715e\left(\sqrt{b}\,x^2(3a^2+14abx^4+8b^2x^8)-\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)-\frac{6240a^2\sqrt{b}\,dx\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}-\frac{5280a^2\sqrt{b}\,fx^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)}{68640b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*(6864*Sqrt[b]*c*(a + b*x^4)^2 + 6240*Sqrt[b]*d*x*(a + b*x^
4)^2 + 5280*Sqrt[b]*f*x^3*(a + b*x^4)^2 + 715*e*(Sqrt[b]*x^2*(3*a^2 + 14*a*
b*x^4 + 8*b^2*x^8) - (3*a^(5/2)*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b
*x^4)/a]) - (6240*a^2*Sqrt[b]*d*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^
4)/a)])/Sqrt[1 + (b*x^4)/a] - (5280*a^2*Sqrt[b]*f*x^3*Hypergeometric2F1[-3/
2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/(68640*b^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 372, normalized size = 0.82

method	result
default	$f\left(\frac{b\,x^{11}\sqrt{b\,x^{4}+a}}{13}+\frac{5a\,x^{7}\sqrt{b\,x^{4}+a}}{39}+\frac{4a^{2}x^{3}\sqrt{b\,x^{4}+a}}{195b}-\frac{4ia^{\frac{7}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\text{Ellipti}\right.}{65b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right.$
risch	$\frac{(36960b^{2}f\,x^{11}+40040b^{2}e\,x^{10}+43680b^{2}d\,x^{9}+48048b^{2}c\,x^{8}+61600abf\,x^{7}+70070abe\,x^{6}+81120abd\,x^{5}+96096abc\,x^{4}+9856a^{2}f\,x^{3}+15015}{480480b}$
elliptic	$\frac{bf\,x^{11}\sqrt{b\,x^{4}+a}}{13}+\frac{be\,x^{10}\sqrt{b\,x^{4}+a}}{12}+\frac{bd\,x^{9}\sqrt{b\,x^{4}+a}}{11}+\frac{bc\,x^{8}\sqrt{b\,x^{4}+a}}{10}+\frac{5af\,x^{7}\sqrt{b\,x^{4}+a}}{39}+\frac{7ae\,x^{6}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/13*b*x^11*(b*x^4+a)^(1/2)+5/39*a*x^7*(b*x^4+a)^(1/2)+4/195/b*a^2*x^3*(
b*x^4+a)^(1/2)-4/65*I/b^(3/2)*a^(7/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(Ellip
ticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)))+e*(1/12*b*x^10*(b*x^4+a)^(1/2)+7/48*a*x^6*(b*x^4+a)^(1/2)+1/32/b*a^2*x^
2*(b*x^4+a)^(1/2)-1/32/b^(3/2)*a^3*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+d*(1/11
*b*x^9*(b*x^4+a)^(1/2)+13/77*a*x^5*(b*x^4+a)^(1/2)+4/77/b*a^2*x*(b*x^4+a)^(
1/2)-4/77/b*a^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(
1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/
2))^(1/2),I))+1/10*c/b*(b*x^4+a)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^4 + a)^(5/2)*c/b + integrate((b*f*x^10 + b*x^9*e + b*d*x^8 + a*f*
x^6 + a*x^5*e + a*d*x^4)*sqrt(b*x^4 + a), x)
```

Fricas [A]
time = 0.12, size = 255, normalized size = 0.56

$$-\frac{59136\,a^3\sqrt{b}\,fx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1) - 15015\,a^3\sqrt{b}\,ex\log\left(-2\,bx^4 + 2\,\sqrt{bx^4+a}\,\sqrt{b}\,x^2 - a\right) + 768\,(65\,a^2bd - 77\,a^3f)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1) - 2\,(36960\,b^3fx^{12} + 40040\,b^3ex^{11} + 43680\,b^3dx^{10} + 48048\,b^3cx^9 + 61600\,ab^2fx^8 + 70070\,ab^2ex^7 + 81120\,ab^2dx^6 + 96096\,ab^2cx^5 + 9856\,a^2bfx^4 + 15015\,a^2bex^3 + 24960\,a^2bdx^2 + 48048\,a^2bcx - 29568\,a^3f)\sqrt{bx^4+a}}{960960\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/960960*(59136*a^3*sqrt(b)*f*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4
)/x), -1) - 15015*a^3*sqrt(b)*e*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*
x^2 - a) + 768*(65*a^2*b*d - 77*a^3*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(ar
csin((-a/b)^(1/4)/x), -1) - 2*(36960*b^3*f*x^12 + 40040*b^3*e*x^11 + 43680*
b^3*d*x^10 + 48048*b^3*c*x^9 + 61600*a*b^2*f*x^8 + 70070*a*b^2*e*x^7 + 8112
0*a*b^2*d*x^6 + 96096*a*b^2*c*x^5 + 9856*a^2*b*f*x^4 + 15015*a^2*b*e*x^3 +
24960*a^2*b*d*x^2 + 48048*a^2*b*c*x - 29568*a^3*f)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A]
time = 9.94, size = 398, normalized size = 0.88

$$\frac{a^{\frac{5}{2}}ex^2}{32b\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{17a^{\frac{3}{2}}ex^6}{96\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{a}\,bdx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)} + \frac{11\sqrt{a}\,bex^{10}}{48\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}\,bfx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)} - \frac{a^3e\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}} + ac\left(\begin{cases}\frac{\sqrt{a}\,x^4}{4} & \text{for}\: b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + bc\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2} + \frac{ax^4\sqrt{a+bx^4}}{30b} + \frac{x^8\sqrt{a+bx^4}}{10} & \text{for}\: b\neq 0\\ \frac{\sqrt{a}\,x^8}{8} & \text{otherwise}\end{cases}\right) + \frac{b^2ex^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(5/2)*e*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*d*x**5*gamma(5/4)*hype
r((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 17*a**(3/
2)*e*x**6/(96*sqrt(1 + b*x**4/a)) + a**(3/2)*f*x**7*gamma(7/4)*hyper((-1/2,
 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*d*x**
9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(13/4)) + 11*sqrt(a)*b*e*x**10/(48*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*x**11*
gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(15/4)) - a**3*e*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*c*Piecewise(
(sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*c*Piecew
ise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x*
*8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*e*x**14/(
12*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(b x^4 + a\right)^{3/2} \left(f x^3 + e x^2 + d x + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
[Out] int(x^3*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.512 $\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

Optimal. Leaf size=427

$$\frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{2ax^3(77c+45ex^2)\sqrt{a+bx^4}}{1155}-\frac{afx^2(a+bx^4}{48b}$$

```
[Out] -1/48*a*f*x^2*(b*x^4+a)^(3/2)/b+1/99*x^3*(9*e*x^2+11*c)*(b*x^4+a)^(3/2)+1/6
0*(5*f*x^2+6*d)*(b*x^4+a)^(5/2)/b-1/32*a^3*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^
(1/2))/b^(3/2)+4/77*a^2*e*x*(b*x^4+a)^(1/2)/b-1/32*a^2*f*x^2*(b*x^4+a)^(1/2
)/b+2/1155*a*x^3*(45*e*x^2+77*c)*(b*x^4+a)^(1/2)+4/15*a^2*c*x*(b*x^4+a)^(1/
2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-4/15*a^(9/4)*c*(cos(2*arctan(b^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(
1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2
*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+2/1155*a^(9/4)*(cos(2*arctan(b^(
1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*
arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-15*e*a^(1/2)+77*c*b^(1/2))*(a^(1/
2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)
^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1847, 1288, 1294, 1212, 226, 1210, 1266, 794, 201, 223, 212}

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{a^3f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}+\frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{2ax^3\sqrt{a+bx^4}\,(77c+45ex^2)}{1155}+\frac{1}{99}x^3\left(a+bx^4\right)^{3/2}\left(11c+9ex^2\right)+\frac{\left(a+bx^4\right)^{5/2}\left(6d+5fx^2\right)}{60b}-\frac{afx^2(a+bx^4)^{3/2}}{48b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (4*a^2*e*x*Sqrt[a + b*x^4])/(77*b) - (a^2*f*x^2*Sqrt[a + b*x^4])/(32*b) + (
4*a^2*c*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^3*
(77*c + 45*e*x^2)*Sqrt[a + b*x^4])/1155 - (a*f*x^2*(a + b*x^4)^(3/2))/(48*b
) + (x^3*(11*c + 9*e*x^2)*(a + b*x^4)^(3/2))/99 + ((6*d + 5*f*x^2)*(a + b*x
^4)^(5/2))/(60*b) - (a^3*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3
/2)) - (4*a^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqr
t[a + b*x^4]) + (2*a^(9/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(1155*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx &= \int \left(x^2\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}+x^3\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)\,dx \\
&= \int x^2\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}\,dx+\int x^3\left(d+fx^2\right)\left(a+bx^4\right)^{3/2} \\
&= \frac{1}{99}x^3\left(11c+9ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{1}{2}\text{Subst}\left(\int x(d+fx)\left(a+b\right.\right. \\
&= \frac{2ax^3\left(77c+45ex^2\right)\sqrt{a+bx^4}}{1155}+\frac{1}{99}x^3\left(11c+9ex^2\right)\left(a+bx^4\right)^{3/} \\
&= \frac{4a^2ex\sqrt{a+bx^4}}{77b}+\frac{2ax^3\left(77c+45ex^2\right)\sqrt{a+bx^4}}{1155}-\frac{afx^2(a+}{48b} \\
&= \frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{2ax^3\left(77c+45ex^2\right)\sqrt{a}}{1155} \\
&= \frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} \\
&= \frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.54, size = 205, normalized size = 0.48

$$
\frac{\sqrt{a+bx^4}\left(\frac{528d\left(a+bx^4\right)^2}{b}+\frac{480ex\left(a+bx^4\right)^2}{b}+\frac{55f\left(\sqrt{b}\,x^2\left(3a^2+14abx^4+8b^2x^8\right)-\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)}{b^{3/2}}-\frac{480a^2ex\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{b\sqrt{1+\frac{bx^4}{a}}}+\frac{1760acx^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)}{5280}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*((528*d*(a + b*x^4)^2)/b + (480*e*x*(a + b*x^4)^2)/b + (55
*f*(Sqrt[b]*x^2*(3*a^2 + 14*a*b*x^4 + 8*b^2*x^8) - (3*a^(5/2)*ArcSinh[(Sqrt
```

```
[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]))/b^(3/2) - (480*a^2*e*x*Hypergeomet
ric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (1760*a*c*x
^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/5
280
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 352, normalized size = 0.82

method	result
default	$f\left(\frac{b\,x^{10}\sqrt{b\,x^{4}+a}}{12}+\frac{7a\,x^{6}\sqrt{b\,x^{4}+a}}{48}+\frac{a^{2}x^{2}\sqrt{b\,x^{4}+a}}{32b}-\frac{a^{3}\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{32b^{\frac{3}{2}}}\right)+e\left(\frac{b\,x^{9}\sqrt{b}}{1}\right.$
risch	$\frac{(9240b^{2}f\,x^{10}+10080b^{2}e\,x^{9}+11088b^{2}d\,x^{8}+12320b^{2}c\,x^{7}+16170abf\,x^{6}+18720abe\,x^{5}+22176abd\,x^{4}+27104abc\,x^{3}+3465x^{2}a^{2}f+5760a}{110880b}$
elliptic	$\frac{bf\,x^{10}\sqrt{b\,x^{4}+a}}{12}+\frac{be\,x^{9}\sqrt{b\,x^{4}+a}}{11}+\frac{bd\,x^{8}\sqrt{b\,x^{4}+a}}{10}+\frac{bc\,x^{7}\sqrt{b\,x^{4}+a}}{9}+\frac{7af\,x^{6}\sqrt{b\,x^{4}+a}}{48}+\frac{13ae\,x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/12*b*x^10*(b*x^4+a)^(1/2)+7/48*a*x^6*(b*x^4+a)^(1/2)+1/32/b*a^2*x^2*(b
*x^4+a)^(1/2)-1/32/b^(3/2)*a^3*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+e*(1/11*b*x
^9*(b*x^4+a)^(1/2)+13/77*a*x^5*(b*x^4+a)^(1/2)+4/77/b*a^2*x*(b*x^4+a)^(1/2)
-4/77/b*a^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^
(1/2),I))+1/10*d/b*(b*x^4+a)^(5/2)+c*(1/9*b*x^7*(b*x^4+a)^(1/2)+11/45*a*x^3
*(b*x^4+a)^(1/2)+4/15*I*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)*x^2, x)
```

Fricas [A]
time = 0.12, size = 247, normalized size = 0.58

$$\frac{59136\,a^{2}b^{\frac{3}{2}}cx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+3465\,a^{3}\sqrt{b}\,fx\log\left(-2\,bx^{4}+2\,\sqrt{bx^{4}+a}\,\sqrt{b}\,x^{2}-a\right)-768\,(77\,a^{2}bc+15\,a^{2}be)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+2\,(9240\,b^{3}fx^{11}+10080\,b^{3}ex^{10}+11088\,b^{3}dx^{9}+12320\,b^{3}cx^{8}+16170\,ab^{2}fx^{7}+18720\,ab^{2}ex^{6}+22176\,ab^{2}dx^{5}+27104\,ab^{2}cx^{4}+3465\,a^{2}bfx^{3}+5760\,a^{2}bex^{2}+11088\,a^{2}bdx+29568\,a^{2}bc)\sqrt{bx^{4}+a}}{221760\,b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/221760*(59136*a^2*b^(3/2)*c*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)
/x), -1) + 3465*a^3*sqrt(b)*f*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^
2 - a) - 768*(77*a^2*b*c + 15*a^2*b*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(ar
csin((-a/b)^(1/4)/x), -1) + 2*(9240*b^3*f*x^11 + 10080*b^3*e*x^10 + 11088*b
^3*d*x^9 + 12320*b^3*c*x^8 + 16170*a*b^2*f*x^7 + 18720*a*b^2*e*x^6 + 22176*
a*b^2*d*x^5 + 27104*a*b^2*c*x^4 + 3465*a^2*b*f*x^3 + 5760*a^2*b*e*x^2 + 110
88*a^2*b*d*x + 29568*a^2*b*c)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A]
time = 9.76, size = 398, normalized size = 0.93

$$\frac{a^{\frac{5}{2}}fx^{2}}{32b\sqrt{1+\frac{bx^{4}}{a}}}+\frac{a^{\frac{3}{2}}cx^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}ex^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{17a^{\frac{3}{2}}fx^{6}}{96\sqrt{1+\frac{bx^{4}}{a}}}+\frac{\sqrt{a}\,bcx^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}\,bex^{9}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{11\sqrt{a}\,bfx^{10}}{48\sqrt{1+\frac{bx^{4}}{a}}}-\frac{a^{3}f\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}}+ad\left(\begin{cases}\frac{\sqrt{a}\,x^{4}}{4} & \text{for}\ b=0\\ \frac{\left(a+bx^{4}\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+bd\left(\begin{cases}-\frac{a^{2}\sqrt{a+bx^{4}}}{15b^{2}}+\frac{ax^{4}\sqrt{a+bx^{4}}}{30b}+\frac{x^{8}\sqrt{a+bx^{4}}}{10} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^{8}}{8} & \text{otherwise}\end{cases}\right)+\frac{b^{2}fx^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^{4}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(5/2)*f*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*c*x**3*gamma(3/4)*hype
r((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**(3/2)*
e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(9/4)) + 17*a**(3/2)*f*x**6/(96*sqrt(1 + b*x**4/a)) + sqrt(a)*b*c*x**7*
gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(1
1/4)) + sqrt(a)*b*e*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_
polar(I*pi)/a)/(4*gamma(13/4)) + 11*sqrt(a)*b*f*x**10/(48*sqrt(1 + b*x**4/a
)) - a**3*f*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*d*Piecewise((sqrt
(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*d*Piecewise((
-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sq
rt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*f*x**14/(12*sq
rt(a)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)*x^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(b x^4 + a\right)^{3/2} \left(f x^3 + e x^2 + d x + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
[Out] int(x^2*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.513 $\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

Optimal. Leaf size=409

$$\frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{2ax^3(77d+45fx^2)\sqrt{a+bx^4}}{1155}+\frac{1}{8}cx^2\left(a+bx\right.$$

```
[Out] 1/8*c*x^2*(b*x^4+a)^(3/2)+1/99*x^3*(9*f*x^2+11*d)*(b*x^4+a)^(3/2)+1/10*e*(b
*x^4+a)^(5/2)/b+3/16*a^2*c*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+4/7
7*a^2*f*x*(b*x^4+a)^(1/2)/b+3/16*a*c*x^2*(b*x^4+a)^(1/2)+2/1155*a*x^3*(45*f
*x^2+77*d)*(b*x^4+a)^(1/2)+4/15*a^2*d*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^
2*b^(1/2))-4/15*a^(9/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2
^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(
3/4)/(b*x^4+a)^(1/2)+2/1155*a^(9/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(-15*f*a^(1/2)+77*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^
4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1847, 1262, 655, 201, 223, 212, 1288, 1294, 1212, 226, 1210}

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(77\sqrt{b}\,d-15\sqrt{a}\,f\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{3a^2c\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{1}{8}cx^2\left(a+bx^4\right)^{3/2}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{1}{99}x^3\left(a+bx^4\right)^{3/2}\left(11d+9fx^2\right)+\frac{2ax^3\sqrt{a+bx^4}\left(77d+45fx^2\right)}{1155}+\frac{e\left(a+bx^4\right)^{5/2}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(77*b) + (3*a*c*x^2*Sqrt[a + b*x^4])/16 + (4*a^
2*d*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^3*(77*
d + 45*f*x^2)*Sqrt[a + b*x^4])/1155 + (c*x^2*(a + b*x^4)^(3/2))/8 + (x^3*(1
1*d + 9*f*x^2)*(a + b*x^4)^(3/2))/99 + (e*(a + b*x^4)^(5/2))/(10*b) + (3*a^
2*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)*d*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(9
/4)*(77*Sqrt[b]*d - 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
1155*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, c, d, e, p, q}, x]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int x\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx &= \int \left(x\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}+x^2\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)dx \\
&= \int x\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}dx+\int x^2\left(d+fx^2\right)\left(a+bx^4\right)^{3/2} d \\
&= \frac{1}{99}x^3\left(11d+9fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{1}{2}\text{Subst}\left(\int (c+ex)\left(a+bx\right.\right. \\
&= \frac{2ax^3(77d+45fx^2)\sqrt{a+bx^4}}{1155}+\frac{1}{99}x^3\left(11d+9fx^2\right)\left(a+bx^4\right) \\
&= \frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{2ax^3(77d+45fx^2)\sqrt{a+bx^4}}{1155}+\frac{1}{8}cx^2(a+ \\
&= \frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{2ax^3(77d+45fx^2)\sqrt{}}{1155} \\
&= \frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right.} \\
&= \frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right.}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.43, size = 196, normalized size = 0.48

$$\frac{\sqrt{a+bx^4}\left(\frac{264e(a+bx^4)^2}{b}+\frac{240fx(a+bx^4)^2}{b}+165c\left(5ax^2+2bx^6+\frac{3a^{5/2}\sqrt{1+\frac{bx^4}{a}}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}(a+bx^4)}\right)-\frac{240a^2fx\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{b\sqrt{1+\frac{bx^4}{a}}}+\frac{880adx^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)}{2640}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*((264*e*(a + b*x^4)^2)/b + (240*f*x*(a + b*x^4)^2)/b + 165
*c*(5*a*x^2 + 2*b*x^6 + (3*a^(5/2)*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2
)/Sqrt[a]])/(Sqrt[b]*(a + b*x^4))) - (240*a^2*f*x*Hypergeometric2F1[-3/2, 1
```

/4, 5/4, -((b*x^4)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (880*a*d*x^3*Hypergeometr
ic2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/2640

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 332, normalized size = 0.81

method	result
default	$f\left(\frac{b\,x^{9}\sqrt{b\,x^{4}+a}}{11}+\frac{13a\,x^{5}\sqrt{b\,x^{4}+a}}{77}+\frac{4a^{2}x\sqrt{b\,x^{4}+a}}{77b}-\frac{4a^{3}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\right.}{77b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.$
risch	$\frac{\left(5040f\,x^{9}b^{2}+5544b^{2}e\,x^{8}+6160b^{2}d\,x^{7}+6930b^{2}c\,x^{6}+9360abf\,x^{5}+11088abe\,x^{4}+13552abd\,x^{3}+17325abc\,x^{2}+2880a^{2}fx+5544a^{2}e\right)\sqrt{b\,x}}{55440b}$
elliptic	$\frac{bf\,x^{9}\sqrt{b\,x^{4}+a}}{11}+\frac{be\,x^{8}\sqrt{b\,x^{4}+a}}{10}+\frac{bd\,x^{7}\sqrt{b\,x^{4}+a}}{9}+\frac{bc\,x^{6}\sqrt{b\,x^{4}+a}}{8}+\frac{13af\,x^{5}\sqrt{b\,x^{4}+a}}{77}+\frac{ae\,x^{4}\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/11*b*x^9*(b*x^4+a)^(1/2)+13/77*a*x^5*(b*x^4+a)^(1/2)+4/77/b*a^2*x*(b*x
^4+a)^(1/2)-4/77/b*a^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^
(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I))+1/10*e*(b*x^4+a)^(5/2)/b+d*(1/9*b*x^7*(b*x^4+a)^(1/2)+
11/45*a*x^3*(b*x^4+a)^(1/2)+4/15*I*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a
^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b
^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)))+c*(1/8*b*x^6*(b*x^4+a)^(1/2)+5/16*a*x^2*(b*x^4+a)^(1/2)+3/
16*a^2*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/32*(3*a^2*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a
)/x^2))/sqrt(b) + 2*(3*sqrt(b*x^4 + a)*a^2*b/x^2 - 5*(b*x^4 + a)^(3/2)*a^2/
```

```
x^6)/(b^2 - 2*(b*x^4 + a)*b/x^4 + (b*x^4 + a)^2/x^8))*c + integrate((b*f*x^
8 + b*x^7*e + b*d*x^6 + a*f*x^4 + a*x^3*e + a*d*x^2)*sqrt(b*x^4 + a), x)
```

Fricas [A]
time = 0.13, size = 224, normalized size = 0.55

$$\frac{29568\,a^{2}\sqrt{b}\,dx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+10395\,a^{2}\sqrt{b}\,cx\log\left(-2\,bx^{4}-2\,\sqrt{bx^{4}+a}\,\sqrt{b}\,x^{2}-a\right)-384\,(77\,a^{2}d+15\,a^{2}f)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+2\,(5040\,b^{2}fx^{10}+5544\,b^{2}ex^{9}+6160\,b^{2}dx^{8}+6930\,b^{2}cx^{7}+9360\,abfx^{6}+11088\,abex^{5}+13552\,abdx^{4}+17325\,abcx^{3}+2880\,a^{2}fx^{2}+5544\,a^{2}ex+14784\,a^{2}d)\sqrt{bx^{4}+a}}{110880\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/110880*(29568*a^2*sqrt(b)*d*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)
/x), -1) + 10395*a^2*sqrt(b)*c*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x
^2 - a) - 384*(77*a^2*d + 15*a^2*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsi
n((-a/b)^(1/4)/x), -1) + 2*(5040*b^2*f*x^10 + 5544*b^2*e*x^9 + 6160*b^2*d*x
^8 + 6930*b^2*c*x^7 + 9360*a*b*f*x^6 + 11088*a*b*e*x^5 + 13552*a*b*d*x^4 +
17325*a*b*c*x^3 + 2880*a^2*f*x^2 + 5544*a^2*e*x + 14784*a^2*d)*sqrt(b*x^4 +
 a))/(b*x)
```

Sympy [A]
time = 5.58, size = 396, normalized size = 0.97

$$\frac{a^{\frac{3}{2}}cx^{2}\sqrt{1+\frac{bx^{4}}{a}}}{4}+\frac{a^{\frac{3}{2}}cx^{2}}{16\sqrt{1+\frac{bx^{4}}{a}}}+\frac{a^{\frac{3}{2}}dx^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}fx^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}\,bcx^{6}}{16\sqrt{1+\frac{bx^{4}}{a}}}+\frac{\sqrt{a}\,bdx^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}\,bfx^{9}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{3a^{2}c\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}\right)}{16\sqrt{b}}+ae\left(\begin{cases}\frac{\sqrt{a}\,x^{4}}{4}&\text{for}\;b=0\\\frac{\left(a+bx^{4}\right)^{\frac{3}{2}}}{6b}&\text{otherwise}\end{cases}\right)+be\left(\begin{cases}-\frac{a^{2}\sqrt{a+bx^{4}}}{15b^{2}}+\frac{ax^{4}\sqrt{a+bx^{4}}}{30b}+\frac{x^{8}\sqrt{a+bx^{4}}}{10}&\text{for}\;b\neq0\\\frac{\sqrt{a}\,x^{8}}{8}&\text{otherwise}\end{cases}\right)+\frac{b^{2}cx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^{4}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(3/2)*c*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*c*x**2/(16*sqrt(1 + b*x**4/
a)) + a**(3/2)*d*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*gamma(7/4)) + a**(3/2)*f*x**5*gamma(5/4)*hyper((-1/2, 5/4), (
9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*c*x**6/(16*sq
rt(1 + b*x**4/a)) + sqrt(a)*b*d*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*f*x**9*gamma(9/4)*hy
per((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4)) + 3*a**
2*c*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*e*Piecewise((sqrt(a)*x**4/
4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*e*Piecewise((-a**2*sqr
t(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*
x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*c*x**10/(8*sqrt(a)*sqrt
(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)*x, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(b\,x^4+a\right)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] int(x*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.514 $\int (c + dx + ex^2 + fx^3) (a + bx^4)^{3/2} dx$

Optimal. Leaf size=382

$$\frac{3}{16}adx^2\sqrt{a+bx^4} + \frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2}{105}ax(15c+7ex^2)\sqrt{a+bx^4} + \frac{1}{8}dx^2(a+bx^4)^{3/2} + \frac{1}{63}x(9c+$$

```
[Out] 1/8*d*x^2*(b*x^4+a)^(3/2)+1/63*x*(7*e*x^2+9*c)*(b*x^4+a)^(3/2)+1/10*f*(b*x^
4+a)^(5/2)/b+3/16*a^2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+3/16*a
*d*x^2*(b*x^4+a)^(1/2)+2/105*a*x*(7*e*x^2+15*c)*(b*x^4+a)^(1/2)+4/15*a^2*e*
x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-4/15*a^(9/4)*e*(cos(2*arcta
n(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(s
in(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+
a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+2/105*a^(7/4)*(co
s(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*El
lipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(7*e*a^(1/2)+15*c*b^(
1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/
4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {1899, 1191, 1212, 226, 1210, 1262, 655, 201, 223, 212}

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+15\sqrt{b}c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}} - \frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} + \frac{3a^2d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} + \frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{63}x(a+bx^4)^{3/2}(9c+7ex^2) + \frac{2}{105}ax\sqrt{a+bx^4}(15c+7ex^2) + \frac{1}{8}dx^2(a+bx^4)^{3/2} + \frac{3}{16}adx^2\sqrt{a+bx^4} + \frac{f(a+bx^4)^{5/2}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (3*a*d*x^2*Sqrt[a + b*x^4])/16 + (4*a^2*e*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(S
qrt[a] + Sqrt[b]*x^2)) + (2*a*x*(15*c + 7*e*x^2)*Sqrt[a + b*x^4])/105 + (d*
x^2*(a + b*x^4)^(3/2))/8 + (x*(9*c + 7*e*x^2)*(a + b*x^4)^(3/2))/63 + (f*(a
 + b*x^4)^(5/2))/(10*b) + (3*a^2*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/
(16*Sqrt[b]) - (4*a^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(
3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4)*(15*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
```

```
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx = \int \left(\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}+x\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)dx$$

$$= \int \left(c+ex^2\right)\left(a+bx^4\right)^{3/2}\,dx + \int x\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\,dx$$

$$= \frac{1}{63}x\left(9c+7ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{1}{21}\int \left(18ac+14aex^2\right)\sqrt{a+bx^4}$$

$$= \frac{2}{105}ax\left(15c+7ex^2\right)\sqrt{a+bx^4} + \frac{1}{63}x\left(9c+7ex^2\right)\left(a+bx^4\right)^{3/2} +$$

$$= \frac{2}{105}ax\left(15c+7ex^2\right)\sqrt{a+bx^4} + \frac{1}{8}dx^2\left(a+bx^4\right)^{3/2} + \frac{1}{63}x(9c+7$$

$$= \frac{3}{16}adx^2\sqrt{a+bx^4} + \frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2}{105}ax(15c+7ex$$

$$= \frac{3}{16}adx^2\sqrt{a+bx^4} + \frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2}{105}ax(15c+7ex$$

$$= \frac{3}{16}adx^2\sqrt{a+bx^4} + \frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2}{105}ax(15c+7ex$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.39, size = 175, normalized size = 0.46

$$\frac{1}{240}\sqrt{a+bx^4}\left(\frac{24f(a+bx^4)^2}{b}+15d\left(5ax^2+2bx^6+\frac{3a^{5/2}\sqrt{1+\frac{bx^4}{a}}\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{\sqrt{b}\,(a+bx^4)}\right)+\frac{240acx\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}+\frac{80aex^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*((24*f*(a + b*x^4)^2)/b + 15*d*(5*a*x^2 + 2*b*x^6 + (3*a^(
5/2)*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(a + b*x^
```

```
4))) + (240*a*c*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 +
 (b*x^4)/a] + (80*a*e*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/
Sqrt[1 + (b*x^4)/a]))/240
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 309, normalized size = 0.81

method	result
default	$\frac{f(bx^4+a)^{\frac{5}{2}}}{10b}+e\left(\frac{bx^7\sqrt{bx^4+a}}{9}+\frac{11ax^3\sqrt{bx^4+a}}{45}+\frac{4ia^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\right.\right.}{15\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+}}\right.$
risch	$\frac{(504b^2fx^8+560b^2ex^7+630b^2dx^6+720b^2cx^5+1008abfx^4+1232abex^3+1575abdx^2+2160abcx+504a^2f)\sqrt{bx^4+a}}{5040b}+\frac{4ia^{\frac{5}{2}}e}{}$
elliptic	$\frac{bfx^8\sqrt{bx^4+a}}{10}+\frac{bex^7\sqrt{bx^4+a}}{9}+\frac{bdx^6\sqrt{bx^4+a}}{8}+\frac{bcx^5\sqrt{bx^4+a}}{7}+\frac{afx^4\sqrt{bx^4+a}}{5}+\frac{11aex^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*f*(b*x^4+a)^(5/2)/b+e*(1/9*b*x^7*(b*x^4+a)^(1/2)+11/45*a*x^3*(b*x^4+a)
^(1/2)+4/15*I*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(
I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+d*(1
/8*b*x^6*(b*x^4+a)^(1/2)+5/16*a*x^2*(b*x^4+a)^(1/2)+3/16*a^2*ln(x^2*b^(1/2)
+(b*x^4+a)^(1/2))/b^(1/2))+c*(1/7*b*x^5*(b*x^4+a)^(1/2)+3/7*a*x*(b*x^4+a)^(
1/2)+4/7*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))
^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c), x)
```

Fricas [A]
time = 0.12, size = 214, normalized size = 0.56

$$\frac{2688\,a^{2}\sqrt{b}\,ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)+945\,a^{2}\sqrt{b}\,dx\log\left(-2\,bx^{4}-2\,\sqrt{bx^{4}+a}\,\sqrt{b}\,x^{2}-a\right)+384\,(15\,abc-7\,a^{2}e)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)+2\,(504\,b^{2}fx^{9}+560\,b^{2}ex^{8}+630\,b^{2}dx^{7}+720\,b^{2}cx^{6}+1008\,abfx^{5}+1232\,abex^{4}+1575\,abdx^{3}+2160\,abcx^{2}+504\,a^{2}fx+1344\,a^{2}e)\sqrt{bx^{4}+a}}{10080\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10080*(2688*a^2*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x
), -1) + 945*a^2*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 -
 a) + 384*(15*a*b*c - 7*a^2*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a
/b)^(1/4)/x), -1) + 2*(504*b^2*f*x^9 + 560*b^2*e*x^8 + 630*b^2*d*x^7 + 720*
b^2*c*x^6 + 1008*a*b*f*x^5 + 1232*a*b*e*x^4 + 1575*a*b*d*x^3 + 2160*a*b*c*x
^2 + 504*a^2*f*x + 1344*a^2*e)*sqrt(b*x^4 + a))/(b*x)
```

Sympy [A]
time = 5.42, size = 394, normalized size = 1.03

$$\frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{\frac{3}{2}}dx^{2}\sqrt{1+\frac{bx^{4}}{a}}}{4}+\frac{a^{\frac{3}{2}}dx^{2}}{16\sqrt{1+\frac{bx^{4}}{a}}}+\frac{a^{\frac{3}{2}}ex^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}\,bcx^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}\,bdx^{6}}{16\sqrt{1+\frac{bx^{4}}{a}}}+\frac{\sqrt{a}\,bex^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{3a^{2}d\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}\right)}{16\sqrt{b}}+af\left(\begin{cases}\frac{\sqrt{a}\,x^{4}}{4}&\text{for}\ b=0\\\frac{\left(a+bx^{4}\right)^{\frac{3}{2}}}{6b}&\text{otherwise}\end{cases}\right)+bf\left(\begin{cases}-\frac{a^{2}\sqrt{a+bx^{4}}}{15b^{2}}+\frac{ax^{4}\sqrt{a+bx^{4}}}{30b}+\frac{x^{8}\sqrt{a+bx^{4}}}{10}&\text{for}\ b\neq 0\\\frac{\sqrt{a}\,x^{8}}{8}&\text{otherwise}\end{cases}\right)+\frac{b^{2}dx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^{4}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(3/2)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a
)/(4*gamma(5/4)) + a**(3/2)*d*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*d*x**2/(
16*sqrt(1 + b*x**4/a)) + a**(3/2)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4
,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*b*c*x**5*gamma(5/4)*
hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqr
t(a)*b*d*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*x**7*gamma(7/4)*hyper((
-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + 3*a**2*d*a
sinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*f*Piecewise((sqrt(a)*x**4/4, Eq
(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*f*Piecewise((-a**2*sqrt(a +
 b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)
/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*d*x**10/(8*sqrt(a)*sqrt(1 +
b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (b\,x^4+a)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
[Out] int((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.515 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=403

$$\frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{16}a(8c+3ex^2)\sqrt{a+bx^4}+\frac{2}{105}ax(15d+7fx^2)\sqrt{a+bx^4}+\frac{1}{24}(4c+3ex^2)(a+b$$

```
[Out] 1/24*(3*e*x^2+4*c)*(b*x^4+a)^(3/2)+1/63*x*(7*f*x^2+9*d)*(b*x^4+a)^(3/2)-1/2
*a^(3/2)*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))+3/16*a^2*e*arctanh(x^2*b^(1/2)/
(b*x^4+a)^(1/2))/b^(1/2)+1/16*a*(3*e*x^2+8*c)*(b*x^4+a)^(1/2)+2/105*a*x*(7*
f*x^2+15*d)*(b*x^4+a)^(1/2)+4/15*a^2*f*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x
^2*b^(1/2))-4/15*a^(9/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*
2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^
(3/4)/(b*x^4+a)^(1/2)+2/105*a^(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(7*f*a^(1/2)+15*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+
a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {1847, 1266, 829, 858, 223, 212, 272, 65, 214, 1191, 1212, 226, 1210}

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(7\sqrt{a}\,f+15\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3a^2e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{16}a\sqrt{a+bx^4}\,(8c+3ex^2)+\frac{1}{24}(a+bx^4)^{3/2}(4c+3ex^2)+\frac{2}{105}ax\sqrt{a+bx^4}\,(15d+7fx^2)+\frac{1}{63}x(a+bx^4)^{3/2}(9d+7fx^2)$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]
```

```
[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (a*(8*c
+ 3*e*x^2)*Sqrt[a + b*x^4])/16 + (2*a*x*(15*d + 7*f*x^2)*Sqrt[a + b*x^4])/1
05 + ((4*c + 3*e*x^2)*(a + b*x^4)^(3/2))/24 + (x*(9*d + 7*f*x^2)*(a + b*x^4
)^(3/2))/63 + (3*a^2*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b])
 - (a^(3/2)*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (4*a^(9/4)*f*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4)*(15
*Sqrt[b]*d + 7*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3
/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx = \int \left(\frac{(c+ex^2)(a+bx^4)^{3/2}}{x} + (d+fx^2)(a+bx^4)^{3/2}\right) dx$$

$$= \int \frac{(c+ex^2)(a+bx^4)^{3/2}}{x}\,dx + \int (d+fx^2)(a+bx^4)^{3/2}\,dx$$

$$= \frac{1}{63}x(9d+7fx^2)(a+bx^4)^{3/2} + \frac{1}{21}\int (18ad+14afx^2)\sqrt{a+bx$$

$$= \frac{2}{105}ax(15d+7fx^2)\sqrt{a+bx^4} + \frac{1}{24}(4c+3ex^2)(a+bx^4)^{3/2} +$$

$$= \frac{1}{16}a(8c+3ex^2)\sqrt{a+bx^4} + \frac{2}{105}ax(15d+7fx^2)\sqrt{a+bx^4} +$$

$$= \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{16}a(8c+3ex^2)\sqrt{a+bx^4} + \frac{2}{105}ax($$

$$= \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{16}a(8c+3ex^2)\sqrt{a+bx^4} + \frac{2}{105}ax($$

$$= \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{16}a(8c+3ex^2)\sqrt{a+bx^4} + \frac{2}{105}ax($$

$$= \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{16}a(8c+3ex^2)\sqrt{a+bx^4} + \frac{2}{105}ax($$

Mathematica [C] Result contains complex when optimal does not.
time = 10.46, size = 319, normalized size = 0.79

$$\frac{\sqrt{a+bx^4}\,(10bx^4(84c+x(72d+7x(9e+8fx)))+a(3360c+x(2160d+7x(225e+176fx))))}{5040} + \frac{3a^2e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} - \frac{1}{2}a^{3/2}c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) + \frac{4ia^2f\sqrt{1+\frac{bx^4}{a}}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)\right)}{15\left(\frac{i\sqrt{b}}{\sqrt{a}}\right)^{3/2}\sqrt{a+bx^4}} - \frac{4ia^2d\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]
```

```
[Out] (Sqrt[a + b*x^4]*(10*b*x^4*(84*c + x*(72*d + 7*x*(9*e + 8*f*x))) + a*(3360*
c + x*(2160*d + 7*x*(225*e + 176*f*x)))))/5040 + (3*a^2*e*ArcTanh[(Sqrt[b]*
x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (a^(3/2)*c*ArcTanh[Sqrt[a + b*x^4]/Sq
rt[a]])/2 + (((4*I)/15)*a^2*f*Sqrt[1 + (b*x^4)/a]*(EllipticE[I*ArcSinh[Sqrt
[(I*Sqrt[b])/Sqrt[a]]*x], -1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a
]]*x], -1]))/(((I*Sqrt[b])/Sqrt[a])^(3/2)*Sqrt[a + b*x^4]) - (((4*I)/7)*a^2
*d*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1
])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 352, normalized size = 0.87

method	result
elliptic	$\frac{b f x^{7} \sqrt{b x^{4}+a}}{9}+\frac{b e x^{6} \sqrt{b x^{4}+a}}{8}+\frac{b d x^{5} \sqrt{b x^{4}+a}}{7}+\frac{b c x^{4} \sqrt{b x^{4}+a}}{6}+\frac{11 a f x^{3} \sqrt{b x^{4}+a}}{45}+\frac{5 a e x^{2} \sqrt{\cdots}}{\cdots}$
default	$f \left(\frac{b x^{7} \sqrt{b x^{4}+a}}{9}+\frac{11 a x^{3} \sqrt{b x^{4}+a}}{45}+\frac{4 i a^{\frac{5}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\mathrm{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\mathrm{Ell}\cdots\right.}{15 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b x^{4}+a}\, \sqrt{b}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/9*b*x^7*(b*x^4+a)^(1/2)+11/45*a*x^3*(b*x^4+a)^(1/2)+4/15*I*a^(5/2)/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+e*(1/8*b*x^6*(b*x^4+a)^(1/2)+
5/16*a*x^2*(b*x^4+a)^(1/2)+3/16*a^2*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)
)+d*(1/7*b*x^5*(b*x^4+a)^(1/2)+3/7*a*x*(b*x^4+a)^(1/2)+4/7*a^2/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/
2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+c*(1/6*b*x^4*(
b*x^4+a)^(1/2)+2/3*a*(b*x^4+a)^(1/2)-1/2*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a
)^(1/2))/x^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x, x)
```

Fricas [F]
time = 0.25, size = 59, normalized size = 0.15

$$\text{integral}\left(\frac{(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac)\sqrt{bx^4+a}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x, x)
```

Sympy [A]
time = 14.00, size = 405, normalized size = 1.00

$$-\frac{a^{\frac{3}{2}}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{2}+\frac{a^{\frac{3}{2}}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{\frac{3}{2}}ex^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{a^{\frac{3}{2}}ex^2}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}\,bdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}\,bex^6}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}\,bfx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{a^2c}{2\sqrt{b}\,x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{3a^2e\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{a\sqrt{b}\,cx^2}{2\sqrt{\frac{a}{bx^4}+1}}+bc\left(\begin{cases}\frac{\sqrt{a}\,x^4}{4} & \text{for}\ b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+\frac{b^2ex^{10}}{8\sqrt{a}\,\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x,x)
```

```
[Out] -a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*d*x*gamma(1/4)*hyper
((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a**(3/2)*e
*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*e*x**2/(16*sqrt(1 + b*x**4/a)) + a**(
3/2)*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)
/(4*gamma(7/4)) + sqrt(a)*b*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*
x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*e*x**6/(16*sqrt(1 + b*
x**4/a)) + sqrt(a)*b*f*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*gamma(11/4)) + a**2*c/(2*sqrt(b)*x**2*sqrt(a/(b*x**4)
+ 1)) + 3*a**2*e*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*sqrt(b)*c*x**
2/(2*sqrt(a/(b*x**4) + 1)) + b*c*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a
+ b*x**4)**(3/2)/(6*b), True)) + b**2*e*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4+a\right)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x, x)
```

3.516 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=404

$$\frac{12a\sqrt{b}\,cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{2}{35}x\left(5ae+21bcx^2\right)\sqrt{a+bx^4}+\frac{1}{16}a\left(8d+3fx^2\right)\sqrt{a+bx^4}-\frac{(7c-ex^2)\left(a+bx^4\right)^{3/2}}{7x}$$

```
[Out] -1/7*(-e*x^2+7*c)*(b*x^4+a)^(3/2)/x+1/24*(3*f*x^2+4*d)*(b*x^4+a)^(3/2)-1/2*
a^(3/2)*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))+3/16*a^2*f*arctanh(x^2*b^(1/2)/(
b*x^4+a)^(1/2))/b^(1/2)+2/35*x*(21*b*c*x^2+5*a*e)*(b*x^4+a)^(1/2)+1/16*a*(3
*f*x^2+8*d)*(b*x^4+a)^(1/2)+12/5*a*c*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2
*b^(1/2))-12/5*a^(5/4)*b^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1
/2)/(b*x^4+a)^(1/2)+2/35*a^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(5*e*a^(1/2)+21*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/
(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1847, 1286, 1191, 1212, 226, 1210, 1266, 829, 858, 223, 212, 272, 65, 214}

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\,\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3a^2f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}-\frac{\left(a+bx^4\right)^{3/2}\left(7c-ex^2\right)}{7x}+\frac{2}{35}x\sqrt{a+bx^4}\left(5ae+21bcx^2\right)+\frac{12a\sqrt{b}\,cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{16}a\sqrt{a+bx^4}\left(8d+3fx^2\right)+\frac{1}{24}\left(a+bx^4\right)^{3/2}\left(4d+3fx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2,x]
```

```
[Out] (12*a*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + (2*x*(5*a*
e + 21*b*c*x^2)*Sqrt[a + b*x^4])/35 + (a*(8*d + 3*f*x^2)*Sqrt[a + b*x^4])/1
6 - ((7*c - e*x^2)*(a + b*x^4)^(3/2))/(7*x) + ((4*d + 3*f*x^2)*(a + b*x^4)^
(3/2))/24 + (3*a^2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) -
 (a^(3/2)*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*c*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*(21*S
qrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*b^(1/4)
*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^2}\,dx = \int \left(\frac{(c+ex^2)\,(a+bx^4)^{3/2}}{x^2}+\frac{(d+fx^2)\,(a+bx^4)^{3/2}}{x}\right)dx$$

$$= \int \frac{(c+ex^2)\,(a+bx^4)^{3/2}}{x^2}\,dx+\int \frac{(d+fx^2)\,(a+bx^4)^{3/2}}{x}\,dx$$

$$= -\frac{(7c-ex^2)\,(a+bx^4)^{3/2}}{7x}+\frac{1}{2}\mathrm{Subst}\left(\int \frac{(d+fx)\,(a+bx^2)^{3/2}}{x}\,dx,\right.$$

$$= \frac{2}{35}x\left(5ae+21bcx^2\right)\sqrt{a+bx^4}-\frac{(7c-ex^2)\,(a+bx^4)^{3/2}}{7x}+\frac{1}{24}\left(4d\right.$$

$$= \frac{2}{35}x\left(5ae+21bcx^2\right)\sqrt{a+bx^4}+\frac{1}{16}a\left(8d+3fx^2\right)\sqrt{a+bx^4}-\frac{(7}{}$$

$$= \frac{12a\sqrt{b}\,cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{2}{35}x\left(5ae+21bcx^2\right)\sqrt{a+bx^4}+\frac{1}{16}a\left(8d\right.$$

$$= \frac{12a\sqrt{b}\,cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{2}{35}x\left(5ae+21bcx^2\right)\sqrt{a+bx^4}+\frac{1}{16}a\left(8d\right.$$

$$= \frac{12a\sqrt{b}\,cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{2}{35}x\left(5ae+21bcx^2\right)\sqrt{a+bx^4}+\frac{1}{16}a\left(8d\right.$$

$$= \frac{12a\sqrt{b}\,cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{2}{35}x\left(5ae+21bcx^2\right)\sqrt{a+bx^4}+\frac{1}{16}a\left(8d\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.49, size = 328, normalized size = 0.81

$$\sqrt{a+bx^4}\left(a\left(\frac{2d}{3}-\frac{c}{x}+\frac{3ex}{7}+\frac{5fx^2}{16}\right)+b\left(\frac{cx^3}{5}+\frac{dx^4}{6}+\frac{ex^5}{7}+\frac{fx^6}{8}\right)\right)+\frac{3a^2f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}-\frac{1}{2}a^{3/2}d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{12iabc\sqrt{1+\frac{bx^4}{a}}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)\right)}{5\left(\frac{i\sqrt{b}}{\sqrt{a}}\right)^{3/2}\sqrt{a+bx^4}}-\frac{4ia^2e\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2,x]
```

```
[Out] Sqrt[a + b*x^4]*(a*((2*d)/3 - c/x + (3*e*x)/7 + (5*f*x^2)/16) + b*((c*x^3)/
5 + (d*x^4)/6 + (e*x^5)/7 + (f*x^6)/8)) + (3*a^2*f*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a + b*x^4]])/(16*Sqrt[b]) - (a^(3/2)*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])
/2 + (((12*I)/5)*a*b*c*Sqrt[1 + (b*x^4)/a]*(EllipticE[I*ArcSinh[Sqrt[(I*Sqr
t[b])/Sqrt[a]]*x], -1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x],
-1]))/(((I*Sqrt[b])/Sqrt[a])^(3/2)*Sqrt[a + b*x^4]) - (((4*I)/7)*a^2*e*Sqrt
[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(Sqr
t[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.45, size = 352, normalized size = 0.87

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{x}+\frac{bf\,x^6\sqrt{b\,x^4+a}}{8}+\frac{be\,x^5\sqrt{b\,x^4+a}}{7}+\frac{bd\,x^4\sqrt{b\,x^4+a}}{6}+\frac{bc\,x^3\sqrt{b\,x^4+a}}{5}+\frac{5fa\,x^2\sqrt{}}{}$
default	$f\left(\frac{b\,x^6\sqrt{b\,x^4+a}}{8}+\frac{5a\,x^2\sqrt{b\,x^4+a}}{16}+\frac{3a^2\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{16\sqrt{b}}\right)+e\left(\frac{b\,x^5\sqrt{b\,x^4+a}}{7}+\frac{3ax\sqrt{b\,x}}{7}\right.$
risch	$-\frac{ac\sqrt{b\,x^4+a}}{x}+\frac{bf\,x^6\sqrt{b\,x^4+a}}{8}+\frac{5fa\,x^2\sqrt{b\,x^4+a}}{16}+\frac{3a^2f\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{16\sqrt{b}}+\frac{be\,x^5\sqrt{b\,x^4+}}{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/8*b*x^6*(b*x^4+a)^(1/2)+5/16*a*x^2*(b*x^4+a)^(1/2)+3/16*a^2*ln(x^2*b^(
1/2)+(b*x^4+a)^(1/2))/b^(1/2))+e*(1/7*b*x^5*(b*x^4+a)^(1/2)+3/7*a*x*(b*x^4+
a)^(1/2)+4/7*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*
(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1
```

```
/2))^(1/2),I))+d*(1/6*b*x^4*(b*x^4+a)^(1/2)+2/3*a*(b*x^4+a)^(1/2)-1/2*a^(3/
2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))+c*(-a*(b*x^4+a)^(1/2)/x+1/5*(b*
x^4+a)^(1/2)*b*x^3+12/5*I*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(E
llipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/
2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^2, x)
```

Fricas [F]
time = 0.24, size = 59, normalized size = 0.15

$$\text{integral}\left(\frac{(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac)\sqrt{bx^4+a}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^2, x)
```

Sympy [A]
time = 6.53, size = 406, normalized size = 1.00

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{a^{\frac{3}{2}}d\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{2}+\frac{a^{\frac{3}{2}}ex\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{\frac{3}{2}}fx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{a^{\frac{3}{2}}fx^2}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}\,bcx^3\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}\,bex^5\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}\,bfx^6}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{a^2d}{2\sqrt{b}\,x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{3a^2f\,\text{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{a\sqrt{b}\,dx^2}{2\sqrt{\frac{a}{bx^4}+1}}+bd\left(\begin{cases}\frac{\sqrt{a}\,x^4}{4} & \text{for}\ b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+\frac{b^2fx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**2,x)
```

```
[Out] a**(3/2)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x*gamma(3/4)) - a**(3/2)*d*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*
e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamm
a(5/4)) + a**(3/2)*f*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*f*x**2/(16*sqrt(1
 + b*x**4/a)) + sqrt(a)*b*c*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*b*e*x**5*gamma(5/4)*hyper((-
1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*f
*x**6/(16*sqrt(1 + b*x**4/a)) + a**2*d/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1)
```

```
) + 3*a**2*f*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*sqrt(b)*d*x**2/(2
*sqrt(a/(b*x**4) + 1)) + b*d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*
x**4)**(3/2)/(6*b), True)) + b**2*f*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^4+a\right)}^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^2,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^2, x)
```

3.517 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=406

$$\frac{12a\sqrt{b}\,dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{4}\left(2ae+3bcx^2\right)\sqrt{a+bx^4}+\frac{2}{35}x\left(5af+21bdx^2\right)\sqrt{a+bx^4}-\frac{\left(3c-ex^2\right)\left(a+bx^4\right)^{3/2}}{6x^2}-$$

```
[Out] -1/6*(-e*x^2+3*c)*(b*x^4+a)^(3/2)/x^2-1/7*(-f*x^2+7*d)*(b*x^4+a)^(3/2)/x-1/
2*a^(3/2)*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))+3/4*a*c*arctanh(x^2*b^(1/2)/(b
*x^4+a)^(1/2))*b^(1/2)+1/4*(3*b*c*x^2+2*a*e)*(b*x^4+a)^(1/2)+2/35*x*(21*b*d
*x^2+5*a*f)*(b*x^4+a)^(1/2)+12/5*a*d*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2
*b^(1/2))-12/5*a^(5/4)*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1
/2)/(b*x^4+a)^(1/2)+2/35*a^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(5*f*a^(1/2)+21*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/
(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1847, 1266, 827, 829, 858, 223, 212, 272, 65, 214, 1286, 1191, 1212, 226, 1210}

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(5\sqrt{a}\,f+21\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{\left(a+bx^4\right)^{3/2}\left(3c-ex^2\right)}{6x^2}+\frac{1}{4}\sqrt{a+bx^4}\left(2ae+3bcx^2\right)+\frac{3}{4}a\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-\frac{\left(a+bx^4\right)^{3/2}\left(7d-fx^2\right)}{7x}+\frac{2}{35}x\sqrt{a+bx^4}\left(5af+21bdx^2\right)+\frac{12a\sqrt{b}\,dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^3,x]
```

```
[Out] (12*a*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*a*e +
3*b*c*x^2)*Sqrt[a + b*x^4])/4 + (2*x*(5*a*f + 21*b*d*x^2)*Sqrt[a + b*x^4])/
35 - ((3*c - e*x^2)*(a + b*x^4)^(3/2))/(6*x^2) - ((7*d - f*x^2)*(a + b*x^4)
^(3/2))/(7*x) + (3*a*Sqrt[b]*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 -
(a^(3/2)*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*d*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*(21*Sq
rt[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*b^(1/4)*
Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3}\,dx = \int \left(\frac{(c+ex^2)(a+bx^4)^{3/2}}{x^3} + \frac{(d+fx^2)(a+bx^4)^{3/2}}{x^2}\right) dx$$

$$= \int \frac{(c+ex^2)(a+bx^4)^{3/2}}{x^3}\,dx + \int \frac{(d+fx^2)(a+bx^4)^{3/2}}{x^2}\,dx$$

$$= -\frac{(7d-fx^2)(a+bx^4)^{3/2}}{7x} + \frac{1}{2}\text{Subst}\left(\int \frac{(c+ex)(a+bx^2)^{3/2}}{x^2}\,dx,\right.$$

$$= \frac{2}{35}x(5af+21bdx^2)\sqrt{a+bx^4} - \frac{(3c-ex^2)(a+bx^4)^{3/2}}{6x^2} - \frac{(7d-}{}$$

$$= \frac{1}{4}(2ae+3bcx^2)\sqrt{a+bx^4} + \frac{2}{35}x(5af+21bdx^2)\sqrt{a+bx^4} - \frac{(3}{}$$

$$= \frac{12a\sqrt{b}\,dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{1}{4}(2ae+3bcx^2)\sqrt{a+bx^4} + \frac{2}{35}x(5af+$$

$$= \frac{12a\sqrt{b}\,dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{1}{4}(2ae+3bcx^2)\sqrt{a+bx^4} + \frac{2}{35}x(5af+$$

$$= \frac{12a\sqrt{b}\,dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{1}{4}(2ae+3bcx^2)\sqrt{a+bx^4} + \frac{2}{35}x(5af+$$

$$= \frac{12a\sqrt{b}\,dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{1}{4}(2ae+3bcx^2)\sqrt{a+bx^4} + \frac{2}{35}x(5af+$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.55, size = 326, normalized size = 0.80

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((a+bx^4)\left(-210ac+bx^4(105c+84dx+70ex^2+60fx^3)+20ax(-21d+x(14e+9fx))\right)+315a\sqrt{b}\,cx^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-210a^{3/2}ex^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+1008a^{3/2}\sqrt{b}\,dx^2\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-48ia^{3/2}\left(-21i\sqrt{b}\,d+5\sqrt{a}\,f\right)x^2\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{420\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(-210*a*c + b*x^4*(105*c + 84*d*x +
 70*e*x^2 + 60*f*x^3) + 20*a*x*(-21*d + x*(14*e + 9*f*x))) + 315*a*Sqrt[b]*
c*x^2*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 210*a^(3/2)*
e*x^2*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) + 1008*a^(3/2)*Sqrt
[b]*d*x^2*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]
*x], -1] - (48*I)*a^(3/2)*((-21*I)*Sqrt[b]*d + 5*Sqrt[a]*f)*x^2*Sqrt[1 + (b
*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(420*Sqrt[(
I*Sqrt[b])/Sqrt[a]]*x^2*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 350, normalized size = 0.86

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{2x^2}-\frac{ad\sqrt{b\,x^4+a}}{x}+\frac{bf\,x^5\sqrt{b\,x^4+a}}{7}+\frac{be\,x^4\sqrt{b\,x^4+a}}{6}+\frac{bd\,x^3\sqrt{b\,x^4+a}}{5}+\frac{bc\,x^2\sqrt{b\,x}}{4}$
default	$f\left(\frac{b\,x^5\sqrt{b\,x^4+a}}{7}+\frac{3ax\sqrt{b\,x^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+c$
risch	$-\frac{a\sqrt{b\,x^4+a}\,(2dx+c)}{2x^2}+\frac{bf\,x^5\sqrt{b\,x^4+a}}{7}+\frac{3fax\sqrt{b\,x^4+a}}{7}+\frac{4a^2f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{Ellip}}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/7*b*x^5*(b*x^4+a)^(1/2)+3/7*a*x*(b*x^4+a)^(1/2)+4/7*a^2/(I/a^(1/2)*b^(
1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)
/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+c*(1/4*b*x^2*(b*
x^4+a)^(1/2)+3/4*a*b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/2*a/x^2*(b*x^4
+a)^(1/2))+e*(1/6*b*x^4*(b*x^4+a)^(1/2)+2/3*a*(b*x^4+a)^(1/2)-1/2*a^(3/2)*l
n((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))+d*(-a*(b*x^4+a)^(1/2)/x+1/5*(b*x^4+
a)^(1/2)*b*x^3+12/5*I*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(Ellip
ticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^3, x)
```

Fricas [F]
time = 0.25, size = 59, normalized size = 0.15

$$\text{integral}\left(\frac{(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^3, x)
```

Sympy [A]
time = 5.17, size = 377, normalized size = 0.93

$$-\frac{a^{\frac{3}{2}}c}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, -\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^{\frac{3}{2}}e\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{2} + \frac{a^{\frac{3}{2}}fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}\,bcx^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}\,bcx^2}{2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}\,bdx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}\,bfx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{a^2e}{2\sqrt{b}\,x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a\sqrt{b}\,c\,\text{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4} + \frac{a\sqrt{b}\,ex^2}{2\sqrt{\frac{a}{bx^4}+1}} + be\left(\begin{cases}\frac{\sqrt{a}\,x^4}{4} & \text{for}\ b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**3,x)
```

```
[Out] -a**(3/2)*c/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*d*gamma(-1/4)*hyper((-1/
2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**(3/2)*e*a
sinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*f*x*gamma(1/4)*hyper((-1/2, 1/4),
 (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*c*x**2*sqrt(1
 + b*x**4/a)/4 - sqrt(a)*b*c*x**2/(2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*d*x**3
*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7
/4)) + sqrt(a)*b*f*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*gamma(9/4)) + a**2*e/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1))
+ 3*a*sqrt(b)*c*asinh(sqrt(b)*x**2/sqrt(a))/4 + a*sqrt(b)*e*x**2/(2*sqrt(a/
(b*x**4) + 1)) + b*e*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(
3/2)/(6*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^3,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^3, x)
```

3.518 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=408

$$\frac{12a\sqrt{b}\,ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2(9ae-5bcx^2)\sqrt{a+bx^4}}{15x}+\frac{1}{4}(2af+3bdx^2)\sqrt{a+bx^4}-\frac{(5c-3ex^2)(a+bx^4)^{3/2}}{15x^3}-\frac{(3}{}$$

```
[Out] -1/15*(-3*e*x^2+5*c)*(b*x^4+a)^(3/2)/x^3-1/6*(-f*x^2+3*d)*(b*x^4+a)^(3/2)/x
^2-1/2*a^(3/2)*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))+3/4*a*d*arctanh(x^2*b^(1/
2)/(b*x^4+a)^(1/2))*b^(1/2)-2/15*(-5*b*c*x^2+9*a*e)*(b*x^4+a)^(1/2)/x+1/4*(
3*b*d*x^2+2*a*f)*(b*x^4+a)^(1/2)+12/5*a*e*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2
)+x^2*b^(1/2))-12/5*a^(5/4)*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^
2)^(1/2)/(b*x^4+a)^(1/2)+2/15*a^(3/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/
4)*x/a^(1/4))),1/2*2^(1/2))*(9*e*a^(1/2)+5*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))
*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1847, 1286, 1212, 226, 1210, 1266, 827, 829, 858, 223, 212, 272, 65, 214}

$$\frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(9\sqrt{a}\,e+5\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{2\sqrt{a+bx^4}\,(9ae-5bcx^2)}{15x}-\frac{(a+bx^4)^{3/2}(5c-3ex^2)}{15x^3}-\frac{(a+bx^4)^{3/2}(3d-fx^2)}{6x^2}+\frac{1}{4}\sqrt{a+bx^4}\,(2af+3bdx^2)+\frac{3}{4}a\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)+\frac{12a\sqrt{b}\,ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4,x]
```

```
[Out] (12*a*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*(9*a*e
 - 5*b*c*x^2)*Sqrt[a + b*x^4])/(15*x) + ((2*a*f + 3*b*d*x^2)*Sqrt[a + b*x^4]
)/4 - ((5*c - 3*e*x^2)*(a + b*x^4)^(3/2))/(15*x^3) - ((3*d - f*x^2)*(a + b*
x^4)^(3/2))/(6*x^2) + (3*a*Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]
)/4 - (a^(3/2)*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*
e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(3/4)*
b^(1/4)*(5*Sqrt[b]*c + 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4
)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])
/(15*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
```

```
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^4}\,dx = \int \left(\frac{(c+ex^2)\,(a+bx^4)^{3/2}}{x^4} + \frac{(d+fx^2)\,(a+bx^4)^{3/2}}{x^3}\right) dx$$

$$= \int \frac{(c+ex^2)\,(a+bx^4)^{3/2}}{x^4}\,dx + \int \frac{(d+fx^2)\,(a+bx^4)^{3/2}}{x^3}\,dx$$

$$= -\frac{(5c-3ex^2)\,(a+bx^4)^{3/2}}{15x^3} - \frac{2}{5}\int \frac{(-3ae-5bcx^2)\,\sqrt{a+bx^4}}{x^2}\,dx +$$

$$= -\frac{2(9ae-5bcx^2)\,\sqrt{a+bx^4}}{15x} - \frac{(5c-3ex^2)\,(a+bx^4)^{3/2}}{15x^3} - \frac{(3d-f}{}$$

$$= -\frac{2(9ae-5bcx^2)\,\sqrt{a+bx^4}}{15x} + \frac{1}{4}(2af+3bdx^2)\,\sqrt{a+bx^4} - \frac{(5c-}{}$$

$$= \frac{12a\sqrt{b}\,ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{2(9ae-5bcx^2)\,\sqrt{a+bx^4}}{15x} + \frac{1}{4}(2af+3bd$$

$$= \frac{12a\sqrt{b}\,ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{2(9ae-5bcx^2)\,\sqrt{a+bx^4}}{15x} + \frac{1}{4}(2af+3bd$$

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$$= \frac{12a\sqrt{b}\,ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{2(9ae-5bcx^2)\,\sqrt{a+bx^4}}{15x} + \frac{1}{4}(2af+3bd$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 10.55, size = 327, normalized size = 0.80

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((a+bx^4)\,(-10a(2c+x(3d+6ex-4fx^2))+bx^4(20c+x(15d+2x(6e+5fx))))+45a\sqrt{b}\,dx^3\sqrt{a+bx^4}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-30a^{3/2}fx^3\sqrt{a+bx^4}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+144a^{3/2}\sqrt{b}\,ex^3\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-16a\sqrt{b}\left(5i\sqrt{b}\,c+9\sqrt{a}\,e\right)x^3\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{60\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(-10*a*(2*c + x*(3*d + 6*e*x - 4*f*
x^2)) + b*x^4*(20*c + x*(15*d + 2*x*(6*e + 5*f*x)))) + 45*a*Sqrt[b]*d*x^3*S
qrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 30*a^(3/2)*f*x^3*Sq
rt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) + 144*a^(3/2)*Sqrt[b]*e*x^3
*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1]
- 16*a*Sqrt[b]*((5*I)*Sqrt[b]*c + 9*Sqrt[a]*e)*x^3*Sqrt[1 + (b*x^4)/a]*Elli
pticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(60*Sqrt[(I*Sqrt[b])/Sqr
t[a]]*x^3*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 349, normalized size = 0.86

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{3x^3}-\frac{ad\sqrt{b\,x^4+a}}{2x^2}-\frac{ae\sqrt{b\,x^4+a}}{x}+\frac{bf\,x^4\sqrt{b\,x^4+a}}{6}+\frac{be\,x^3\sqrt{b\,x^4+a}}{5}+\frac{bd\,x^2\sqrt{b\,x^4}}{4}$
default	$d\left(\frac{b\,x^2\sqrt{b\,x^4+a}}{4}+\frac{3a\sqrt{b}\,\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{4}-\frac{a\sqrt{b\,x^4+a}}{2x^2}\right)+c\left(-\frac{a\sqrt{b\,x^4+a}}{3x^3}+\frac{bx\sqrt{b\,x^4}}{3}\right.$
risch	$-\frac{a\sqrt{b\,x^4+a}\,(6e\,x^2+3dx+2c)}{6x^3}-\frac{f\sqrt{b\,x^4+a}\,(-b\,x^4+2a)}{6}+\frac{be\,x^3\sqrt{b\,x^4+a}}{5}-\frac{3i\sqrt{b}\,e\,a^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{5\sqrt{\frac{i}{}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d*(1/4*b*x^2*(b*x^4+a)^(1/2)+3/4*a*b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-
1/2*a/x^2*(b*x^4+a)^(1/2))+c*(-1/3*a*(b*x^4+a)^(1/2)/x^3+1/3*b*x*(b*x^4+a)^
(1/2)+4/3*a*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I))+f*(1/6*b*x^4*(b*x^4+a)^(1/2)+2/3*a*(b*x^4+a)^(1/2)-1/2*a^(3/2)*
ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))+e*(-a*(b*x^4+a)^(1/2)/x+1/5*(b*x^4
+a)^(1/2)*b*x^3+12/5*I*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/
2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(Elli
pticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^4, x)
```

Fricas [F]
time = 0.28, size = 59, normalized size = 0.14

$$\text{integral}\left(\frac{(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac)\sqrt{bx^4+a}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^4, x)
```

Sympy [A]
time = 5.20, size = 381, normalized size = 0.93

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}-\frac{a^{\frac{3}{2}}d}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{a^{\frac{3}{2}}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{\sqrt{a}bcx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}bdx^2\sqrt{1+\frac{bx^4}{a}}}{4}-\frac{\sqrt{a}bdx^2}{2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{a^2f}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{3a\sqrt{b}d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4}+\frac{a\sqrt{b}fx^2}{2\sqrt{\frac{a}{bx^4}+1}}+bf\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\\frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**4,x)
```

```
[Out] a**(3/2)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**3*gamma(1/4)) - a**(3/2)*d/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*e
*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gam
ma(3/4)) - a**(3/2)*f*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*b*c*x*gamma
(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) +
 sqrt(a)*b*d*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*d*x**2/(2*sqrt(1 + b*x**
4/a)) + sqrt(a)*b*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(7/4)) + a**2*f/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1))
 + 3*a*sqrt(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/4 + a*sqrt(b)*f*x**2/(2*sqrt(a
/(b*x**4) + 1)) + b*f*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**
(3/2)/(6*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4+a\right)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^4,x)
```

```
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^4, x)
```

3.519 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=386

$$\frac{12a\sqrt{b}\,fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{3}{4}b(c+ex^2)\sqrt{a+bx^4}+\frac{2}{15}bx(5d+9fx^2)\sqrt{a+bx^4}-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)(a$$

```
[Out] -1/12*(3*c/x^4+4*d/x^3+6*e/x^2+12*f/x)*(b*x^4+a)^(3/2)-3/4*b*c*arctanh((b*x
^4+a)^(1/2)/a^(1/2))*a^(1/2)+3/4*a*e*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))*b
^(1/2)+3/4*b*(e*x^2+c)*(b*x^4+a)^(1/2)+2/15*b*x*(9*f*x^2+5*d)*(b*x^4+a)^(1/
2)+12/5*a*f*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5*a^(5/4)*b^
(1/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a
^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x
^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+2/15*
a^(3/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(9
*f*a^(1/2)+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/
2))^2)^(1/2)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {14, 1839, 1847, 1266, 829, 858, 223, 212, 272, 65, 214, 1191, 1212, 226, 1210}

$$\frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(9\sqrt{a}\,f+5\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{12}(a+bx^4)^{3/2}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)+\frac{3}{4}b\sqrt{a+bx^4}\,(c+ex^2)-\frac{3}{4}\sqrt{a}\,bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{2}{15}bx\sqrt{a+bx^4}\,(5d+9fx^2)+\frac{3}{4}a\sqrt{b}\,e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)+\frac{12a\sqrt{b}\,fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^5,x]
```

```
[Out] (12*a*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + (3*b*(c +
e*x^2)*Sqrt[a + b*x^4])/4 + (2*b*x*(5*d + 9*f*x^2)*Sqrt[a + b*x^4])/15 - ((
(3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*(a + b*x^4)^(3/2))/12 + (3*a*
Sqrt[b]*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (3*Sqrt[a]*b*c*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(5/4)*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(3/4)*b^(1/4)*(5*Sqrt[b]*d +
9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
```

```
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
```

```
0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}dx = -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)(a+bx^4)^{3/2}-(6b)\int \frac{\left(-\frac{c}{4}-\frac{dx}{3}-\right.}{}$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)(a+bx^4)^{3/2}-(6b)\int \left(\frac{\left(-\frac{c}{4}-\frac{ex^2}{2}\right.}{}\right.$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)(a+bx^4)^{3/2}-(6b)\int \frac{\left(-\frac{c}{4}-\frac{ex^2}{2}\right)}{x}$$

$$= \frac{2}{15}bx(5d+9fx^2)\sqrt{a+bx^4}-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)(a+b$$

$$= \frac{3}{4}b(c+ex^2)\sqrt{a+bx^4}+\frac{2}{15}bx(5d+9fx^2)\sqrt{a+bx^4}-\frac{1}{12}\left(\frac{3c}{x^4}\right.$$

$$= \frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3}{4}b(c+ex^2)\sqrt{a+bx^4}+\frac{2}{15}bx(5d+9fx$$

$$= \frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3}{4}b(c+ex^2)\sqrt{a+bx^4}+\frac{2}{15}bx(5d+9fx$$

$$= \frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3}{4}b(c+ex^2)\sqrt{a+bx^4}+\frac{2}{15}bx(5d+9fx$$

$$= \frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3}{4}b(c+ex^2)\sqrt{a+bx^4}+\frac{2}{15}bx(5d+9fx$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.53, size = 329, normalized size = 0.85

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\left((a+bx^4)\left(5a(3c+4dx+6x^2(e+2fx))-bx^4(30c+x(20d+3x(5e+4fx)))\right)\right)+45a\sqrt{b}ex^4\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-45\sqrt{a}bcx^4\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+144a^{3/2}\sqrt{b}fx^4\sqrt{1+\frac{bx^4}{a}}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-16a\sqrt{b}\left(5i\sqrt{b}d+9\sqrt{a}f\right)x^4\sqrt{1+\frac{bx^4}{a}}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{60\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x^4\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-((a + b*x^4)*(5*a*(3*c + 4*d*x + 6*x^2*(e + 2*
f*x)) - b*x^4*(30*c + x*(20*d + 3*x*(5*e + 4*f*x))))) + 45*a*Sqrt[b]*e*x^4*
Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 45*Sqrt[a]*b*c*x^4
*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) + 144*a^(3/2)*Sqrt[b]*f*
x^4*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -
1] - 16*a*Sqrt[b]*((5*I)*Sqrt[b]*d + 9*Sqrt[a]*f)*x^4*Sqrt[1 + (b*x^4)/a]*E
llipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(60*Sqrt[(I*Sqrt[b])/
Sqrt[a]]*x^4*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.43, size = 350, normalized size = 0.91

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{4x^4}-\frac{ad\sqrt{b\,x^4+a}}{3x^3}-\frac{ae\sqrt{b\,x^4+a}}{2x^2}-\frac{af\sqrt{b\,x^4+a}}{x}+\frac{bf\,x^3\sqrt{b\,x^4+a}}{5}+\frac{be\,x^2\sqrt{b\,x^4+}}{4}$
default	$c\left(\frac{b\sqrt{b\,x^4+a}}{2}-\frac{3\sqrt{a}\,b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{4}-\frac{a\sqrt{b\,x^4+a}}{4x^4}\right)+e\left(\frac{b\,x^2\sqrt{b\,x^4+a}}{4}+\frac{3a\sqrt{b}\,\ln}{}\right.$
risch	$-\frac{a\sqrt{b\,x^4+a}\,(12f\,x^3+6e\,x^2+4dx+3c)}{12x^4}+\frac{bf\,x^3\sqrt{b\,x^4+a}}{5}+\frac{12i\sqrt{b}\,f\,a^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{Elli}}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/2*b*(b*x^4+a)^(1/2)-3/4*a^(1/2)*b*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x
^2)-1/4*a*(b*x^4+a)^(1/2)/x^4)+e*(1/4*b*x^2*(b*x^4+a)^(1/2)+3/4*a*b^(1/2)*l
n(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/2*a/x^2*(b*x^4+a)^(1/2))+d*(-1/3*a*(b*x^4+
a)^(1/2)/x^3+1/3*b*x*(b*x^4+a)^(1/2)+4/3*a*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)
*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+f*(-a*(b*x^4+a)^(1/2)/x+1/5*(b*x
^4+a)^(1/2)*b*x^3+12/5*I*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(
1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(El
lipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(3*sqrt(a)*b*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)
)) + 4*sqrt(b*x^4 + a)*b - 2*sqrt(b*x^4 + a)*a/x^4)*c + integrate((b*f*x^6
+ b*x^5*e + b*d*x^4 + a*f*x^2 + a*x*e + a*d)*sqrt(b*x^4 + a)/x^4, x)
```

Fricas [**F**]
time = 0.27, size = 59, normalized size = 0.15

$$\text{integral}\left(\frac{(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^5, x)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 5.74, size = 379, normalized size = 0.98

$$\frac{a^{\frac{3}{2}} d\Gamma\left(-\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| {\frac{bx^{4} e^{i\pi}}{a}} \right)}{4x^{3}\Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}} e}{2x^{2}\sqrt{1 + \frac{bx^{4}}{a}}} + \frac{a^{\frac{3}{2}} f\Gamma\left(-\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| {\frac{bx^{4} e^{i\pi}}{a}} \right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}\, bc \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\, x^{2}}\right)}{4} + \frac{\sqrt{a}\, bdx\Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| {\frac{bx^{4} e^{i\pi}}{a}} \right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}\, bex^{2}\sqrt{1 + \frac{bx^{4}}{a}}}{4} - \frac{\sqrt{a}\, bex^{2}}{2\sqrt{1 + \frac{bx^{4}}{a}}} + \frac{\sqrt{a}\, bfx^{3}\Gamma\left(\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| {\frac{bx^{4} e^{i\pi}}{a}} \right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{a\sqrt{b}\, c\sqrt{\frac{a}{bx^{4}} + 1}}{4x^{2}} + \frac{a\sqrt{b}\, c}{2x^{2}\sqrt{\frac{a}{bx^{4}} + 1}} + \frac{3a\sqrt{b}\, e \operatorname{asinh}\left(\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}\right)}{4} + \frac{b^{\frac{3}{2}} cx^{2}}{2\sqrt{\frac{a}{bx^{4}} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**5,x)
```

```
[Out] a**(3/2)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**3*gamma(1/4)) - a**(3/2)*e/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*f
*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gam
ma(3/4)) - 3*sqrt(a)*b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/4 + sqrt(a)*b*d*x*ga
mma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)
) + sqrt(a)*b*e*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*e*x**2/(2*sqrt(1 + b*
x**4/a)) + sqrt(a)*b*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*gamma(7/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*x**2)
 + a*sqrt(b)*c/(2*x**2*sqrt(a/(b*x**4) + 1)) + 3*a*sqrt(b)*e*asinh(sqrt(b)*
x**2/sqrt(a))/4 + b**(3/2)*c*x**2/(2*sqrt(a/(b*x**4) + 1))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^4+a\right)}^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^5,x)
```

```
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^5, x)
```

3.520 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6} dx$

Optimal. Leaf size=387

$$\frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x}+\frac{3}{4}b(d+fx^2)\sqrt{a+bx^4}-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)(a -$$

```
[Out] -1/60*(12*c/x^5+15*d/x^4+20*e/x^3+30*f/x^2)*(b*x^4+a)^(3/2)-3/4*b*d*arctanh
((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+3/4*a*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/
2))*b^(1/2)-2/15*b*(-5*e*x^2+9*c)*(b*x^4+a)^(1/2)/x+3/4*b*(f*x^2+d)*(b*x^4+
a)^(1/2)+12/5*b^(3/2)*c*x*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5*a^(1/4
)*b^(5/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)
*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/
2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+2
/15*a^(1/4)*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(5*e*a^(1/2)+9*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b
^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {14, 1839, 1847, 1286, 1212, 226, 1210, 1266, 829, 858, 223, 212, 272, 65, 214}

$$\frac{2\sqrt[4]{a}\,b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(5\sqrt{a}\,e+9\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}-\frac{12\sqrt[4]{a}\,b^{5/4}c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}+\frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{1}{60}(a+bx^4)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)-\frac{2b\sqrt{a+bx^4}(9c-5ex^2)}{15x}+\frac{3}{4}b\sqrt{a+bx^4}(d+fx^2)-\frac{3}{4}\sqrt{a}\,bd\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3}{4}a\sqrt{b}\,f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6,x]
```

```
[Out] (12*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*b*(9*c -
5*e*x^2)*Sqrt[a + b*x^4])/(15*x) + (3*b*(d + f*x^2)*Sqrt[a + b*x^4])/4 - ((
(12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*(a + b*x^4)^(3/2))/60 +
(3*a*Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (3*Sqrt[a]*b*d*A
rcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(1/4)*b^(5/4)*c*(Sqrt[a] + Sqrt[
b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(
1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(9*Sqrt[b]
*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[
b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a + b*x^
4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
```

```
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^6}\,dx = -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)(a+bx^4)^{3/2}-(6b)\int \frac{\left(-\frac{c}{5}-\frac{dx}{4}\right.}{}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)(a+bx^4)^{3/2}-(6b)\int \left(\frac{\left(-\frac{c}{5}-\right.}{}\right.$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)(a+bx^4)^{3/2}-(6b)\int \frac{\left(-\frac{c}{5}-\frac{ex}{3}\right.}{}$$

$$= -\frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x}-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)(a+$$

$$= -\frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x}+\frac{3}{4}b(d+fx^2)\sqrt{a+bx^4}-\frac{1}{60}\left(\frac{12c}{x^5}\right.$$

$$= \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x}+\frac{3}{4}b(d+fx^2)\sqrt{}$$

$$= \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x}+\frac{3}{4}b(d+fx^2)\sqrt{}$$

$$= \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x}+\frac{3}{4}b(d+fx^2)\sqrt{}$$

$$= \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x}+\frac{3}{4}b(d+fx^2)\sqrt{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.52, size = 331, normalized size = 0.86

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((a+bx^4)(12ac+84bcx^4+5ax(3d+4ex+6fx^2)-5bx^5(6d+x(4e+3fx)))-45a\sqrt{b}\,fx^5\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)+45\sqrt{a}\,bdx^5\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+144\sqrt{a}\,b^{3/2}cx^5\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-16i\sqrt{a}\,b\left(-9i\sqrt{b}\,c+5\sqrt{a}\,e\right)x^5\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{60\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^5\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(12*a*c + 84*b*c*x^4 + 5*a*x*(3*d
 + 4*e*x + 6*f*x^2) - 5*b*x^5*(6*d + x*(4*e + 3*f*x))) - 45*a*Sqrt[b]*f*x^5
*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] + 45*Sqrt[a]*b*d*x^
5*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 144*Sqrt[a]*b^(3/2)*
c*x^5*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x],
 -1] - (16*I)*Sqrt[a]*b*((-9*I)*Sqrt[b]*c + 5*Sqrt[a]*e)*x^5*Sqrt[1 + (b*x^
4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(60*Sqrt[(I*Sq
rt[b])/Sqrt[a]]*x^5*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.46, size = 350, normalized size = 0.90

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{5x^5}-\frac{ad\sqrt{b\,x^4+a}}{4x^4}-\frac{ae\sqrt{b\,x^4+a}}{3x^3}-\frac{af\sqrt{b\,x^4+a}}{2x^2}-\frac{7bc\sqrt{b\,x^4+a}}{5x}+\frac{bf\,x^2\sqrt{b\,x^4+a}}{4}$
default	$c\left(-\frac{a\sqrt{b\,x^4+a}}{5x^5}-\frac{7b\sqrt{b\,x^4+a}}{5x}+\frac{12ib^{\frac{3}{2}}\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right.}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,\left(84bc\,x^4+30af\,x^3+20ae\,x^2+15adx+12ac\right)}{60x^5}+\frac{bf\,x^2\sqrt{b\,x^4+a}}{4}+\frac{3\sqrt{b}\,af\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/5*a*(b*x^4+a)^(1/2)/x^5-7/5*b*(b*x^4+a)^(1/2)/x+12/5*I*b^(3/2)*a^(1/2
)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-
EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+d*(1/2*b*(b*x^4+a)^(1/2)-3/4*a^(
1/2)*b*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/4*a*(b*x^4+a)^(1/2)/x^4)+f
*(1/4*b*x^2*(b*x^4+a)^(1/2)+3/4*a*b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1
/2*a/x^2*(b*x^4+a)^(1/2))+e*(-1/3*a*(b*x^4+a)^(1/2)/x^3+1/3*b*x*(b*x^4+a)^(
1/2)+4/3*a*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))
^(1/2),I))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^6, x)
```

Fricas [F]
time = 0.26, size = 59, normalized size = 0.15

$$\mathrm{integral}\left(\frac{(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^6, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.85, size = 386, normalized size = 1.00

$$\frac{a^{\frac{3}{2}} c \Gamma\left(-\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| {\frac{bx^{4} e^{i\pi}}{a}} \right)}{4 x^{5} \Gamma\left(-\frac{1}{4}\right)} + \frac{a^{\frac{3}{2}} e \Gamma\left(-\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| {\frac{bx^{4} e^{i\pi}}{a}} \right)}{4 x^{3} \Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}} f}{2 x^{2} \sqrt{1 + \frac{bx^{4}}{a}}} + \frac{\sqrt{a} b c \Gamma\left(-\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| {\frac{bx^{4} e^{i\pi}}{a}} \right)}{4 x \Gamma\left(\frac{3}{4}\right)} - \frac{3 \sqrt{a} b d \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b} x^{2}}\right)}{4} + \frac{\sqrt{a} b e x \Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| {\frac{bx^{4} e^{i\pi}}{a}} \right)}{4 \Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a} b f x^{2} \sqrt{1 + \frac{bx^{4}}{a}}}{4} - \frac{\sqrt{a} b f x^{2}}{2 \sqrt{1 + \frac{bx^{4}}{a}}} - \frac{a \sqrt{b} d \sqrt{\frac{a}{bx^{4}} + 1}}{4 x^{2}} + \frac{a \sqrt{b} d}{2 x^{2} \sqrt{\frac{a}{bx^{4}} + 1}} + \frac{3 a \sqrt{b} f \operatorname{asinh}\left(\frac{\sqrt{b} x^{2}}{\sqrt{a}}\right)}{4} + \frac{b^{\frac{3}{2}} d x^{2}}{2 \sqrt{\frac{a}{bx^{4}} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**6,x)
```

```
[Out] a**(3/2)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**5*gamma(-1/4)) + a**(3/2)*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,)
, b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - a**(3/2)*f/(2*x**2*sqrt(1
 + b*x**4/a)) + sqrt(a)*b*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*d*asinh(sqrt(a)/(sqrt(b)*
x**2))/4 + sqrt(a)*b*e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*f*x**2*sqrt(1 + b*x**4/a)/4 - sqrt
(a)*b*f*x**2/(2*sqrt(1 + b*x**4/a)) - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*x
**2) + a*sqrt(b)*d/(2*x**2*sqrt(a/(b*x**4) + 1)) + 3*a*sqrt(b)*f*asinh(sqrt
(b)*x**2/sqrt(a))/4 + b**(3/2)*d*x**2/(2*sqrt(a/(b*x**4) + 1))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^6,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^6, x)
```

3.521 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7} \, dx$

Optimal. Leaf size=392

$$\frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2}-\frac{2b(9d-5fx^2)\sqrt{a+bx^4}}{15x}-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)(a$$

```
[Out] -1/60*(10*c/x^6+12*d/x^5+15*e/x^4+20*f/x^3)*(b*x^4+a)^(3/2)+1/2*b^(3/2)*c*a
rctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))-3/4*b*e*arctanh((b*x^4+a)^(1/2)/a^(1/2)
)*a^(1/2)-1/4*b*(-3*e*x^2+2*c)*(b*x^4+a)^(1/2)/x^2-2/15*b*(-5*f*x^2+9*d)*(b
*x^4+a)^(1/2)/x+12/5*b^(3/2)*d*x*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5
*a^(1/4)*b^(5/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(
b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))
*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^
(1/2)+2/15*a^(1/4)*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*
2^(1/2))*(5*f*a^(1/2)+9*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2
)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {14, 1839, 1847, 1266, 827, 858, 223, 212, 272, 65, 214, 1286, 1212, 226, 1210}

$$\frac{2\sqrt[4]{a}\,b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(5\sqrt{a}\,f+9\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}-\frac{12\sqrt[4]{a}\,b^{5/4}d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}+\frac{1}{2}b^{3/2}c\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)+\frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{1}{60}(a+bx^4)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)-\frac{b\sqrt{a+bx^4}\,(2c-3ex^2)}{4x^2}-\frac{2b\sqrt{a+bx^4}\,(9d-5fx^2)}{15x}-\frac{3}{4}\sqrt{a}\,be\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^7,x]
```

```
[Out] (12*b^(3/2)*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (b*(2*c - 3*
e*x^2)*Sqrt[a + b*x^4])/(4*x^2) - (2*b*(9*d - 5*f*x^2)*Sqrt[a + b*x^4])/(15
*x) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*(a + b*x^4)^(3/2
))/60 + (b^(3/2)*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (3*Sqrt[a]*b
*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(1/4)*b^(5/4)*d*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(9*Sqr
t[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a +
b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
```

```
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
```

```
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^7}\,dx = -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)(a+bx^4)^{3/2}-(6b)\int \frac{\left(-\frac{c}{6}-\frac{dx}{5}\right.}{}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)(a+bx^4)^{3/2}-(6b)\int \left(\frac{\left(-\frac{c}{6}-\right.}{}\right.$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)(a+bx^4)^{3/2}-(6b)\int \frac{\left(-\frac{c}{6}-\frac{ex}{4}\right.}{}$$

$$= -\frac{2b(9d-5fx^2)\sqrt{a+bx^4}}{15x}-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)(a+$$

$$= -\frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2}-\frac{2b(9d-5fx^2)\sqrt{a+bx^4}}{15x}-\frac{1}{60}\left(\frac{10c}{x^6}\right.$$

$$= \frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2}-\frac{2b(9d-5fx^2)\sqrt{}}{15x}$$

$$= \frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2}-\frac{2b(9d-5fx^2)\sqrt{}}{15x}$$

$$= \frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2}-\frac{2b(9d-5fx^2)\sqrt{}}{15x}$$

$$= \frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2}-\frac{2b(9d-5fx^2)\sqrt{}}{15x}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.55, size = 331, normalized size = 0.84

$$-\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((a+bx^4)\left(2bx^4(20c+x(42d-5x(3e+2fx)))+a(10c+x(12d+5x(3e+4fx)))\right)-30b^{3/2}cx^6\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)+45\sqrt{a}\,bex^6\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+144\sqrt{a}\,b^{3/2}dx^6\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-16i\sqrt{a}\,b\left(-9i\sqrt{b}\,d+5\sqrt{a}\,f\right)x^6\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{60\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^6\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^7,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(2*b*x^4*(20*c + x*(42*d - 5*x*(3
*e + 2*f*x))) + a*(10*c + x*(12*d + 5*x*(3*e + 4*f*x)))) - 30*b^(3/2)*c*x^6
*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] + 45*Sqrt[a]*b*e*x^
6*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 144*Sqrt[a]*b^(3/2)*
d*x^6*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x],
 -1] - (16*I)*Sqrt[a]*b*((-9*I)*Sqrt[b]*d + 5*Sqrt[a]*f)*x^6*Sqrt[1 + (b*x^
4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(60*Sqrt[(I*Sq
rt[b])/Sqrt[a]]*x^6*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.44, size = 349, normalized size = 0.89

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{6x^6}-\frac{ad\sqrt{b\,x^4+a}}{5x^5}-\frac{ae\sqrt{b\,x^4+a}}{4x^4}-\frac{af\sqrt{b\,x^4+a}}{3x^3}-\frac{2bc\sqrt{b\,x^4+a}}{3x^2}-\frac{7bd\sqrt{b\,x^4+a}}{5x}$
default	$d\left(-\frac{a\sqrt{b\,x^4+a}}{5x^5}-\frac{7b\sqrt{b\,x^4+a}}{5x}+\frac{12ib^{\frac{3}{2}}\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right.}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,(84bd\,x^5+40bc\,x^4+20af\,x^3+15ae\,x^2+12adx+10ac)}{60x^6}+\frac{bfx\sqrt{b\,x^4+a}}{3}+\frac{4bfa\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+}}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/5*a*(b*x^4+a)^(1/2)/x^5-7/5*b*(b*x^4+a)^(1/2)/x+12/5*I*b^(3/2)*a^(1/2
)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-
EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+e*(1/2*b*(b*x^4+a)^(1/2)-3/4*a^(
1/2)*b*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/4*a*(b*x^4+a)^(1/2)/x^4)+c
*(1/2*b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/6*a/x^6*(b*x^4+a)^(1/2)-2/3
*b/x^2*(b*x^4+a)^(1/2))+f*(-1/3*a*(b*x^4+a)^(1/2)/x^3+1/3*b*x*(b*x^4+a)^(1/
2)+4/3*a*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^(3/2)*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4
 + a)/x^2)) + 6*sqrt(b*x^4 + a)*b/x^2 + 2*(b*x^4 + a)^(3/2)/x^6)*c + integr
ate((b*f*x^6 + b*x^5*e + b*d*x^4 + a*f*x^2 + a*x*e + a*d)*sqrt(b*x^4 + a)/x
^6, x)
```

Fricas [F]
time = 0.28, size = 59, normalized size = 0.15

$$\text{integral}\left(\frac{(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^7, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.50, size = 406, normalized size = 1.04

$$\frac{a^{\frac{3}{2}} d\Gamma\left(-\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| {\frac{bx^4 e^{i\pi}}{a}} \right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{a^{\frac{3}{2}} f\Gamma\left(-\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| {\frac{bx^4 e^{i\pi}}{a}} \right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}\, bc}{2x^2\sqrt{1 + \frac{bx^4}{a}}} + \frac{\sqrt{a}\, bd\Gamma\left(-\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| {\frac{bx^4 e^{i\pi}}{a}} \right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}\, be \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\, x^2}\right)}{4} + \frac{\sqrt{a}\, bfx\Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| {\frac{bx^4 e^{i\pi}}{a}} \right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{a\sqrt{b}\, c\sqrt{\frac{a}{bx^4} + 1}}{6x^4} - \frac{a\sqrt{b}\, e\sqrt{\frac{a}{bx^4} + 1}}{4x^2} + \frac{a\sqrt{b}\, e}{2x^2\sqrt{\frac{a}{bx^4} + 1}} - \frac{b^{\frac{3}{2}} c\sqrt{\frac{a}{bx^4} + 1}}{6} + \frac{b^{\frac{3}{2}} c \operatorname{asinh}\left(\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right)}{2} + \frac{b^{\frac{3}{2}} ex^2}{2\sqrt{\frac{a}{bx^4} + 1}} - \frac{b^2 cx^2}{2\sqrt{a}\sqrt{1 + \frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**7,x)
```

```
[Out] a**(3/2)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**5*gamma(-1/4)) + a**(3/2)*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,)
, b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*c/(2*x**2*sqrt(
1 + b*x**4/a)) + sqrt(a)*b*d*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4
*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*e*asinh(sqrt(a)/(sqrt(b)
*x**2))/4 + sqrt(a)*b*f*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_
polar(I*pi)/a)/(4*gamma(5/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(6*x**4) -
 a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*e/(2*x**2*sqrt(a/(b*
x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*c*asinh(sqrt(b)*
x**2/sqrt(a))/2 + b**(3/2)*e*x**2/(2*sqrt(a/(b*x**4) + 1)) - b**2*c*x**2/(2
*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^4+a\right)}^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^7,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^7, x)
```

3.522 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=412

$$-\frac{12be\sqrt{a+bx^4}}{5x}+\frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2b(5c-21ex^2)\sqrt{a+bx^4}}{35x^3}-\frac{b(2d-3fx^2)\sqrt{a+bx^4}}{4x^2}-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{7}{x}\right.$$

[Out] -1/420*(60*c/x^7+70*d/x^6+84*e/x^5+105*f/x^4)*(b*x^4+a)^(3/2)+1/2*b^(3/2)*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))-3/4*b*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)-12/5*b*e*(b*x^4+a)^(1/2)/x-2/35*b*(-21*e*x^2+5*c)*(b*x^4+a)^(1/2)/x^3-1/4*b*(-3*f*x^2+2*d)*(b*x^4+a)^(1/2)/x^2+12/5*b^(3/2)*e*x*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5*a^(1/4)*b^(5/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+2/35*b^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(21*e*a^(1/2)+5*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^4+a)^(1/2)

Rubi [A]
time = 0.27, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 16, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {14, 1839, 1847, 1286, 1296, 1212, 226, 1210, 1266, 827, 858, 223, 212, 272, 65, 214}

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(21\sqrt{a}\,e+5\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{12\sqrt[4]{a}\,b^{5/4}e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}+\frac{1}{2}b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)+\frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{1}{420}(a+bx^4)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)-\frac{2b\sqrt{a+bx^4}(5c-21ex^2)}{35x^3}-\frac{b\sqrt{a+bx^4}(2d-3fx^2)}{4x^2}-\frac{12be\sqrt{a+bx^4}}{5x}-\frac{3}{4}\sqrt{a}\,bf\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^8,x]

[Out] (-12*b*e*Sqrt[a + b*x^4])/(5*x) + (12*b^(3/2)*e*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*b*(5*c - 21*e*x^2)*Sqrt[a + b*x^4])/(35*x^3) - (b*(2*d - 3*f*x^2)*Sqrt[a + b*x^4])/(4*x^2) - (((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*(a + b*x^4)^(3/2))/420 + (b^(3/2)*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (3*Sqrt[a]*b*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(1/4)*b^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*b^(5/4)*(5*Sqrt[b]*c + 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*a^(1/4)*Sqrt[a + b*x^4])

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
```

```
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
```

```
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^8}\,dx = -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2}-(6b)\int \frac{\left(-\frac{c}{7}-\right.}{}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2}-(6b)\int \left(\frac{\left(-\frac{c}{7}\right.}{}\right.$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2}-(6b)\int \frac{\left(-\frac{c}{7}-\right.}{}$$

$$= -\frac{2b(5c-21ex^2)\sqrt{a+bx^4}}{35x^3}-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(\right.$$

$$= -\frac{12be\sqrt{a+bx^4}}{5x}-\frac{2b(5c-21ex^2)\sqrt{a+bx^4}}{35x^3}-\frac{b(2d-3fx^2)\sqrt{a}}{4x^2}$$

$$= -\frac{12be\sqrt{a+bx^4}}{5x}-\frac{2b(5c-21ex^2)\sqrt{a+bx^4}}{35x^3}-\frac{b(2d-3fx^2)\sqrt{a}}{4x^2}$$

$$= -\frac{12be\sqrt{a+bx^4}}{5x}+\frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2b(5c-21ex^2)\sqrt{a+b}}{35x^3}$$

$$= -\frac{12be\sqrt{a+bx^4}}{5x}+\frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2b(5c-21ex^2)\sqrt{a+b}}{35x^3}$$

$$= -\frac{12be\sqrt{a+bx^4}}{5x}+\frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2b(5c-21ex^2)\sqrt{a+b}}{35x^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.57, size = 330, normalized size = 0.80

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(2bx^4(90c+7x(20d+3x(14e-5fx)))+a(60c+7x(10d+3x(4e+5fx)))\right)-210b^{3/2}dx^7\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)+315\sqrt{a}\,bfx^7\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+1008\sqrt{a}\,b^{3/2}ex^7\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-48b^{3/2}\left(5i\sqrt{b}\,c+21\sqrt{a}\,e\right)x^7\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{420\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^7\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^8,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(2*b*x^4*(90*c + 7*x*(20*d + 3*x*
(14*e - 5*f*x))) + a*(60*c + 7*x*(10*d + 3*x*(4*e + 5*f*x)))) - 210*b^(3/2)
*d*x^7*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] + 315*Sqrt[a]
*b*f*x^7*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 1008*Sqrt[a]*
b^(3/2)*e*x^7*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt
[a]]*x], -1] - 48*b^(3/2)*((5*I)*Sqrt[b]*c + 21*Sqrt[a]*e)*x^7*Sqrt[1 + (b*
x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(420*Sqrt[(I
*Sqrt[b])/Sqrt[a]]*x^7*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 352, normalized size = 0.85

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{7x^7}-\frac{ad\sqrt{b\,x^4+a}}{6x^6}-\frac{ae\sqrt{b\,x^4+a}}{5x^5}-\frac{af\sqrt{b\,x^4+a}}{4x^4}-\frac{3bc\sqrt{b\,x^4+a}}{7x^3}-\frac{2bd\sqrt{b\,x^4+a}}{3x^2}$
default	$e\left(-\frac{a\sqrt{b\,x^4+a}}{5x^5}-\frac{7b\sqrt{b\,x^4+a}}{5x}+\frac{12ib^{\frac{3}{2}}\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\cdots\right.}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,(588be\,x^6+280bd\,x^5+180bc\,x^4+105af\,x^3+84ae\,x^2+70adx+60ac)}{420x^7}+\frac{bf\sqrt{b\,x^4+a}}{2}+\frac{12ib^{\frac{3}{2}}e\sqrt{a}\,\sqrt{1-\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/5*a*(b*x^4+a)^(1/2)/x^5-7/5*b*(b*x^4+a)^(1/2)/x+12/5*I*b^(3/2)*a^(1/2
)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-
EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+f*(1/2*b*(b*x^4+a)^(1/2)-3/4*a^(
1/2)*b*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/4*a*(b*x^4+a)^(1/2)/x^4)+d
*(1/2*b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/6*a/x^6*(b*x^4+a)^(1/2)-2/3
*b/x^2*(b*x^4+a)^(1/2))+c*(-1/7*a*(b*x^4+a)^(1/2)/x^7-3/7*b*(b*x^4+a)^(1/2)
/x^3+4/7*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))
^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^8, x)
```

Fricas [F]
time = 0.27, size = 59, normalized size = 0.14

$$\text{integral}\left(\frac{(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^8, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.71, size = 415, normalized size = 1.01

$$\frac{a^{\frac{3}{2}} c \Gamma\left(-\frac{7}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{7}{4}, -\frac{1}{2} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{a^{\frac{3}{2}} e \Gamma\left(-\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a} bc \Gamma\left(-\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a} bd}{2x^2\sqrt{1 + \frac{bx^4}{a}}} + \frac{\sqrt{a} be \Gamma\left(-\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a} bf \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b} x^2}\right)}{4} - \frac{a\sqrt{b} d\sqrt{\frac{a}{bx^4} + 1}}{6x^4} - \frac{a\sqrt{b} f\sqrt{\frac{a}{bx^4} + 1}}{4x^2} + \frac{a\sqrt{b} f}{2x^2\sqrt{\frac{a}{bx^4} + 1}} - \frac{b^{\frac{3}{2}} d\sqrt{\frac{a}{bx^4} + 1}}{6} + \frac{b^{\frac{3}{2}} d \operatorname{asinh}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right)}{2} + \frac{b^{\frac{3}{2}} fx^2}{2\sqrt{\frac{a}{bx^4} + 1}} - \frac{b^2 dx^2}{2\sqrt{a}\sqrt{1 + \frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**8,x)
```

```
[Out] a**(3/2)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**7*gamma(-3/4)) + a**(3/2)*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*c*gamma(-3/4)
*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4))
- sqrt(a)*b*d/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*gamma(-1/4)*hyper((
-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)
*b*f*asinh(sqrt(a)/(sqrt(b)*x**2))/4 - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(6*
x**4) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*f/(2*x**2*sqr
t(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*d*asinh(s
qrt(b)*x**2/sqrt(a))/2 + b**(3/2)*f*x**2/(2*sqrt(a/(b*x**4) + 1)) - b**2*d*
x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^8, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^8,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^8, x)
```

3.523 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=377

$$-\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}+\frac{12b^{3/2}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right.$$

```
[Out] -1/840*(105*c/x^8+120*d/x^7+140*e/x^6+168*f/x^5)*(b*x^4+a)^(3/2)+1/2*b^(3/2
)*e*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))-3/16*b^2*c*arctanh((b*x^4+a)^(1/2)
/a^(1/2))/a^(1/2)-1/560*b*(105*c/x^4+160*d/x^3+280*e/x^2+672*f/x)*(b*x^4+a)
^(1/2)+12/5*b^(3/2)*f*x*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5*a^(1/4)*
b^(5/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x
/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)
+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+2/3
5*b^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x
/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(21*f*a^
(1/2)+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2
)^(1/2)/a^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {14, 1839, 1846, 272, 65, 214, 1899, 281, 223, 212, 1212, 226, 1210}

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(21\sqrt{a}\,f+5\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{12\sqrt[4]{a}\,b^{5/4}f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}+\frac{1}{2}b^{3/2}e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)+\frac{12b^{3/2}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{1}{560}b\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)-\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^9,x]
```

```
[Out] -1/560*(b*((105*c)/x^4 + (160*d)/x^3 + (280*e)/x^2 + (672*f)/x)*Sqrt[a + b*
x^4]) + (12*b^(3/2)*f*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (((1
05*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*(a + b*x^4)^(3/2))/840
 + (b^(3/2)*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (3*b^2*c*ArcTanh[
Sqrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (12*a^(1/4)*b^(5/4)*f*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*b^(5/4)*(5*Sqrt[b]*d +
 21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*a^(1/4)*Sqrt[a +
 b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)(a+bx^4)^{3/2}-(6b)\int \frac{(\cdots}{\cdots}$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}-\frac{1}{840}\left(\frac{105c}{x^8}\cdots\right.$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}-\frac{1}{840}\left(\frac{105c}{x^8}\cdots\right.$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}-\frac{1}{840}\left(\frac{105c}{x^8}\cdots\right.$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}-\frac{1}{840}\left(\frac{105c}{x^8}\cdots\right.$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}-\frac{1}{840}\left(\frac{105c}{x^8}\cdots\right.$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}+\frac{12b^{3/2}fx\sqrt{\cdots}}{5\left(\sqrt{a}+\cdots\right.}$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}+\frac{12b^{3/2}fx\sqrt{\cdots}}{5\left(\sqrt{a}+\cdots\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.77, size = 309, normalized size = 0.82

$$-\frac{\sqrt{a+bx^4}\,(bx^4(525c+16x(45d+70ex+147fx^2))+a(210c+8x(30d+7x(5e+6fx))))}{1680x^8}+\frac{1}{2}b^{3/2}e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{12ia\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,bf\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{5\sqrt{a+bx^4}}-\frac{4b^{3/2}\left(5i\sqrt{b}\,d+21\sqrt{a}\,f\right)\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{35\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^9,x]
```

```
[Out] -1/1680*(Sqrt[a + b*x^4]*(b*x^4*(525*c + 16*x*(45*d + 70*e*x + 147*f*x^2))
+ a*(210*c + 8*x*(30*d + 7*x*(5*e + 6*f*x)))))/x^8 + (b^(3/2)*e*ArcTanh[(Sq
rt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (3*b^2*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])
/(16*Sqrt[a]) - (((12*I)/5)*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b*f*Sqrt[1 + (b*x^4
)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/Sqrt[a + b*x^4]
 - (4*b^(3/2)*((5*I)*Sqrt[b]*d + 21*Sqrt[a]*f)*Sqrt[1 + (b*x^4)/a]*Elliptic
```

```
F[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(35*Sqrt[(I*Sqrt[b])/Sqrt[a]
]*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 357, normalized size = 0.95

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{8x^8}-\frac{ad\sqrt{b\,x^4+a}}{7x^7}-\frac{ae\sqrt{b\,x^4+a}}{6x^6}-\frac{af\sqrt{b\,x^4+a}}{5x^5}-\frac{5bc\sqrt{b\,x^4+a}}{16x^4}-\frac{3bd\sqrt{b\,x^4+a}}{7x^3}-$
risch	$-\frac{\sqrt{b\,x^4+a}\,(2352bf\,x^7+1120be\,x^6+720bd\,x^5+525bc\,x^4+336af\,x^3+280ae\,x^2+240adx+210ac)}{1680x^8}+\frac{12ib^{\frac{3}{2}}f\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}}{5}$
default	$f\left(-\frac{a\sqrt{b\,x^4+a}}{5x^5}-\frac{7b\sqrt{b\,x^4+a}}{5x}+\frac{12ib^{\frac{3}{2}}\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\right.}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] f*(-1/5*a*(b*x^4+a)^(1/2)/x^5-7/5*b*(b*x^4+a)^(1/2)/x+12/5*I*b^(3/2)*a^(1/2
)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-
EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+e*(1/2*b^(3/2)*ln(x^2*b^(1/2)+(b
*x^4+a)^(1/2))-1/6*a/x^6*(b*x^4+a)^(1/2)-2/3*b/x^2*(b*x^4+a)^(1/2))+c*(-1/8
*a/x^8*(b*x^4+a)^(1/2)-5/16*b/x^4*(b*x^4+a)^(1/2)-3/16*b^2/a^(1/2)*ln((2*a+
2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))+d*(-1/7*a*(b*x^4+a)^(1/2)/x^7-3/7*b*(b*x^4
+a)^(1/2)/x^3+4/7*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/32*(3*b^2*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/sq
rt(a) - 2*(5*(b*x^4 + a)^(3/2)*b^2 - 3*sqrt(b*x^4 + a)*a*b^2)/((b*x^4 + a)^
2 - 2*(b*x^4 + a)*a + a^2))*c + integrate((b*f*x^6 + b*x^5*e + b*d*x^4 + a*
f*x^2 + a*x*e + a*d)*sqrt(b*x^4 + a)/x^8, x)
```

Fricas [F]
time = 0.27, size = 59, normalized size = 0.16

$$\text{integral}\left(\frac{(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^9, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.11, size = 444, normalized size = 1.18

$$\frac{a^{\frac{3}{2}} d\Gamma\left(-\frac{7}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{7}{4}, -\frac{1}{2} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{a^{\frac{3}{2}} f\Gamma\left(-\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}\, bd\Gamma\left(-\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}\, be}{2x^2\sqrt{1 + \frac{bx^4}{a}}} + \frac{\sqrt{a}\, bf\Gamma\left(-\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^2 c}{8\sqrt{b}\, x^{10}\sqrt{\frac{a}{bx^4} + 1}} - \frac{3a\sqrt{b}\, c}{16x^6\sqrt{\frac{a}{bx^4} + 1}} - \frac{a\sqrt{b}\, e\sqrt{\frac{a}{bx^4} + 1}}{6x^4} - \frac{b^{\frac{3}{2}} c\sqrt{\frac{a}{bx^4} + 1}}{4x^2} - \frac{b^{\frac{3}{2}} c}{16x^2\sqrt{\frac{a}{bx^4} + 1}} - \frac{b^{\frac{3}{2}} e\sqrt{\frac{a}{bx^4} + 1}}{6} + \frac{b^{\frac{3}{2}} e \operatorname{asinh}\left(\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right)}{2} - \frac{3b^2 c \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\, x^2}\right)}{16\sqrt{a}} - \frac{b^2 e x^2}{2\sqrt{a}\sqrt{1 + \frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**9,x)
```

```
[Out] a**(3/2)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**7*gamma(-3/4)) + a**(3/2)*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*d*gamma(-3/4)
*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4))
- sqrt(a)*b*e/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*gamma(-1/4)*hyper((
-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**2*c/(8
*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*c/(16*x**6*sqrt(a/(b*x**
4) + 1)) - a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*c*sqrt(a/(b
*x**4) + 1)/(4*x**2) - b**(3/2)*c/(16*x**2*sqrt(a/(b*x**4) + 1)) - b**(3/2)
*e*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*e*asinh(sqrt(b)*x**2/sqrt(a))/2 - 3*b*
*2*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)) - b**2*e*x**2/(2*sqrt(a)*sq
rt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^9, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^9,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^9, x)
```

3.524 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=405

$$-\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{4b^2c\sqrt{a+bx^4}}{15ax}+\frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\right.$$

```
[Out] -1/504*(56*c/x^9+63*d/x^8+72*e/x^7+84*f/x^6)*(b*x^4+a)^(3/2)+1/2*b^(3/2)*f*
arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))-3/16*b^2*d*arctanh((b*x^4+a)^(1/2)/a^(
1/2))/a^(1/2)-1/1680*b*(224*c/x^5+315*d/x^4+480*e/x^3+840*f/x^2)*(b*x^4+a)^
(1/2)-4/15*b^2*c*(b*x^4+a)^(1/2)/a/x+4/15*b^(5/2)*c*x*(b*x^4+a)^(1/2)/a/(a^
(1/2)+x^2*b^(1/2))-4/15*b^(9/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(
1/2)/a^(3/4)/(b*x^4+a)^(1/2)+2/105*b^(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(15*e*a^(1/2)+7*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*(
(b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {14, 1839, 1847, 1296, 1212, 226, 1210, 1266, 858, 223, 212, 272, 65, 214}

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(15\sqrt{a}\,e+7\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}+\frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{2}b^{3/2}f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-\frac{4b^2c\sqrt{a+bx^4}}{15ax}-\frac{3b^2d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{b\sqrt{a+bx^4}\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)}{1680}-\frac{1}{504}\left(a+bx^4\right)^{3/2}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^10,x]
```

```
[Out] -1/1680*(b*((224*c)/x^5 + (315*d)/x^4 + (480*e)/x^3 + (840*f)/x^2)*Sqrt[a +
 b*x^4]) - (4*b^2*c*Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2)*c*x*Sqrt[a + b*x
^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - (((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^
7 + (84*f)/x^6)*(a + b*x^4)^(3/2))/504 + (b^(3/2)*f*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a + b*x^4]])/2 - (3*b^2*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]
) - (4*b^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b
]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a
 + b*x^4]) + (2*b^(7/4)*(7*Sqrt[b]*c + 15*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(105*a^(3/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{10}}\,dx = -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\left(a+bx^4\right)^{3/2}-(6b)\int \frac{\left(-\frac{c}{9}-\cdots\right.}{\cdots}$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72\cdots}{x^7}\cdots\right.$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72\cdots}{x^7}\cdots\right.$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72\cdots}{x^7}\cdots\right.$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{4b^2c\sqrt{a+bx^4}}{15ax}-\frac{1}{504}\cdots$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{4b^2c\sqrt{a+bx^4}}{15ax}-\frac{1}{504}\cdots$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{4b^2c\sqrt{a+bx^4}}{15ax}+\frac{4b^{5\cdots}}{15a\cdots}\cdots$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{4b^2c\sqrt{a+bx^4}}{15ax}+\frac{4b^{5\cdots}}{15a\cdots}\cdots$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{4b^2c\sqrt{a+bx^4}}{15ax}+\frac{4b^{5\cdots}}{15a\cdots}\cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.63, size = 351, normalized size = 0.87

$$-\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(1344b^2cx^8+10a^2(56c+63dx+72ex^2+84fx^3)+abx^4(1232c+15x(105d+16x(9e+14fx)))\right)-2520ab^{3/2}fx^9\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)+945\sqrt{a}\,b^2dx^9\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+1344\sqrt{a}\,b^{5/2}cx^9\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-192i\sqrt{a}\,b^2\left(-7i\sqrt{b}\,c+15\sqrt{a}\,e\right)x^9\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{5040a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^9\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^10,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(1344*b^2*c*x^8 + 10*a^2*(56*c +
63*d*x + 72*e*x^2 + 84*f*x^3) + a*b*x^4*(1232*c + 15*x*(105*d + 16*x*(9*e +
 14*f*x)))) - 2520*a*b^(3/2)*f*x^9*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a + b*x^4]] + 945*Sqrt[a]*b^2*d*x^9*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x
^4]/Sqrt[a]])) + 1344*Sqrt[a]*b^(5/2)*c*x^9*Sqrt[1 + (b*x^4)/a]*EllipticE[I
*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - (192*I)*Sqrt[a]*b^2*((-7*I)*Sq
rt[b]*c + 15*Sqrt[a]*e)*x^9*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I
*Sqrt[b])/Sqrt[a]]*x], -1])/(5040*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^9*Sqrt[a +
b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 377, normalized size = 0.93

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{9x^9}-\frac{ad\sqrt{b\,x^4+a}}{8x^8}-\frac{ae\sqrt{b\,x^4+a}}{7x^7}-\frac{af\sqrt{b\,x^4+a}}{6x^6}-\frac{11bc\sqrt{b\,x^4+a}}{45x^5}-\frac{5bd\sqrt{b\,x^4+a}}{16x^4}$
default	$c\left(-\frac{a\sqrt{b\,x^4+a}}{9x^9}-\frac{11b\sqrt{b\,x^4+a}}{45x^5}-\frac{4b^2\sqrt{b\,x^4+a}}{15ax}+\frac{4ib^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(\right.\right.}{15\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{}}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,\left(1344b^2c\,x^8+3360abf\,x^7+2160abe\,x^6+1575abd\,x^5+1232abc\,x^4+840a^2f\,x^3+720a^2e\,x^2+630a^2dx+560a^2c\right)}{5040x^9a}+-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/9*a*(b*x^4+a)^(1/2)/x^9-11/45*b*(b*x^4+a)^(1/2)/x^5-4/15*b^2/a*(b*x^4
+a)^(1/2)/x+4/15*I*b^(5/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(Elliptic
F(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
+f*(1/2*b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/6*a/x^6*(b*x^4+a)^(1/2)-2
/3*b/x^2*(b*x^4+a)^(1/2))+d*(-1/8*a/x^8*(b*x^4+a)^(1/2)-5/16*b/x^4*(b*x^4+a
)^(1/2)-3/16*b^2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))+e*(-1/7*a
*(b*x^4+a)^(1/2)/x^7-3/7*b*(b*x^4+a)^(1/2)/x^3+4/7*b^2/(I/a^(1/2)*b^(1/2))^
(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^
4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^10, x)
```

Fricas [F]
time = 0.27, size = 59, normalized size = 0.15

$$\text{integral}\left(\frac{(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^10, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.39, size = 449, normalized size = 1.11

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}\,bc\Gamma\left(-\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}\,be\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}\,bf}{2x^2\sqrt{1+\frac{bx^4}{a}}} - \frac{a^2d}{8\sqrt{b}\,x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{3a\sqrt{b}\,d}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}\,f\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}d}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}f\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2} - \frac{3b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{16\sqrt{a}} - \frac{b^2fx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**10,x)
```

```
[Out] a**(3/2)*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**9*gamma(-5/4)) + a**(3/2)*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*c*gamma(-5/4)
*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)
) + sqrt(a)*b*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*
pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*f/(2*x**2*sqrt(1 + b*x**4/a)) - a**2
*d/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*d/(16*x**6*sqrt(a/(
b*x**4) + 1)) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*d*sqrt
(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*d/(16*x**2*sqrt(a/(b*x**4) + 1)) - b**
(3/2)*f*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*f*asinh(sqrt(b)*x**2/sqrt(a))/2 -
 3*b**2*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)) - b**2*f*x**2/(2*sqrt(
a)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^10, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^10,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^10, x)
```

3.525 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=399

$$-\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680}-\frac{b^2c\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2d\sqrt{a+bx^4}}{15ax}+\frac{4b^{5/2}dx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{\left(\frac{252c}{x^{10}}+\frac{280}{x^9}\right.}{}$$

```
[Out] -1/2520*(252*c/x^10+280*d/x^9+315*e/x^8+360*f/x^7)*(b*x^4+a)^(3/2)-3/16*b^2
*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/1680*b*(168*c/x^6+224*d/x^5+3
15*e/x^4+480*f/x^3)*(b*x^4+a)^(1/2)-1/10*b^2*c*(b*x^4+a)^(1/2)/a/x^2-4/15*b
^2*d*(b*x^4+a)^(1/2)/a/x+4/15*b^(5/2)*d*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^
(1/2))-4/15*b^(9/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)
/(b*x^4+a)^(1/2)+2/105*b^(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),
1/2*2^(1/2))*(15*f*a^(1/2)+7*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a
^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {14, 1839, 1847, 1266, 821, 272, 65, 214, 1296, 1212, 226, 1210}

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(15\sqrt{a}\,f+7\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}+\frac{4b^{5/2}dx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{b^2c\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2d\sqrt{a+bx^4}}{15ax}-\frac{3b^2e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{b\sqrt{a+bx^4}\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)}{1680}-\frac{(a+bx^4)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^11,x]
```

```
[Out] -1/1680*(b*((168*c)/x^6 + (224*d)/x^5 + (315*e)/x^4 + (480*f)/x^3)*Sqrt[a +
 b*x^4]) - (b^2*c*Sqrt[a + b*x^4])/(10*a*x^2) - (4*b^2*d*Sqrt[a + b*x^4])/(
15*a*x) + (4*b^(5/2)*d*x*Sqrt[a + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) -
(((252*c)/x^10 + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7)*(a + b*x^4)^(3/2)
)/2520 - (3*b^2*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (4*b^(9/
4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]) +
 (2*b^(7/4)*(7*Sqrt[b]*d + 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
1/2])/(105*a^(3/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{11}}\,dx = -\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)(a+bx^4)^{3/2}}{2520} - (6b)\int \frac{\left(-\frac{c}{10}-\frac{dx}{9}-\right.}{}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\right.}{252}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\right.}{252}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\right.}{252}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{4b^2d\sqrt{a+bx^4}}{15ax} - $$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2}{}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2}{}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2}{}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.43, size = 314, normalized size = 0.79

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((a+bx^4)(168b^2x^8(3c+8dx)+a^2(504c+10x(56d+9x(7e+8fx)))+abx^4(1008c+x(1232d+45x(35e+48fx))))+945\sqrt{a}\,b^2ex^{10}\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+1344\sqrt{a}\,b^{5/2}dx^{10}\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-192i\sqrt{a}\,b^2\left(-7i\sqrt{b}\,d+15\sqrt{a}\,f\right)x^{10}\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{5040a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^{10}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^11,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(168*b^2*x^8*(3*c + 8*d*x) + a^2*
(504*c + 10*x*(56*d + 9*x*(7*e + 8*f*x))) + a*b*x^4*(1008*c + x*(1232*d + 4
5*x*(35*e + 48*f*x)))) + 945*Sqrt[a]*b^2*e*x^10*Sqrt[a + b*x^4]*ArcTanh[Sqr
t[a + b*x^4]/Sqrt[a]])) + 1344*Sqrt[a]*b^(5/2)*d*x^10*Sqrt[1 + (b*x^4)/a]*E
llipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - (192*I)*Sqrt[a]*b^2*
((-7*I)*Sqrt[b]*d + 15*Sqrt[a]*f)*x^10*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcS
inh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(5040*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^
10*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 357, normalized size = 0.89

method	result
default	$d\left(-\frac{a\sqrt{b\,x^4+a}}{9x^9}-\frac{11b\sqrt{b\,x^4+a}}{45x^5}-\frac{4b^2\sqrt{b\,x^4+a}}{15ax}+\frac{4ib^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\right.\right.}{15\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b}}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,\left(1344b^2d\,x^9+504b^2c\,x^8+2160abf\,x^7+1575abe\,x^6+1232abd\,x^5+1008abc\,x^4+720a^2f\,x^3+630a^2e\,x^2+560a^2dx+504a^2\right.}{5040x^{10}a}$
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{10x^{10}}-\frac{ad\sqrt{b\,x^4+a}}{9x^9}-\frac{ae\sqrt{b\,x^4+a}}{8x^8}-\frac{af\sqrt{b\,x^4+a}}{7x^7}-\frac{bc\sqrt{b\,x^4+a}}{5x^6}-\frac{11bd\sqrt{b\,x^4+a}}{45x^5}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/9*a*(b*x^4+a)^(1/2)/x^9-11/45*b*(b*x^4+a)^(1/2)/x^5-4/15*b^2/a*(b*x^4
+a)^(1/2)/x+4/15*I*b^(5/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(Elliptic
F(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
+e*(-1/8*a/x^8*(b*x^4+a)^(1/2)-5/16*b/x^4*(b*x^4+a)^(1/2)-3/16*b^2/a^(1/2)*
ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))+f*(-1/7*a*(b*x^4+a)^(1/2)/x^7-3/7*
b*(b*x^4+a)^(1/2)/x^3+4/7*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*b^(1/2))^(1/2),I))-1/10*c*(b^2*x^8+2*a*b*x^4+a^2)/a/x^10*(b*x^4+a)
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/10*(b*x^4 + a)^(5/2)*c/(a*x^10) + integrate((b*f*x^6 + b*x^5*e + b*d*x^4
 + a*f*x^2 + a*x*e + a*d)*sqrt(b*x^4 + a)/x^10, x)
```

Fricas [A]
time = 0.12, size = 217, normalized size = 0.54

$$-\frac{2688\,\sqrt{a}\,b^{2}dx^{10}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1)-945\,\sqrt{a}\,b^{2}ex^{10}\log\left(-\frac{bx^{4}-2\,\sqrt{bx^{4}+a}\,\sqrt{a}+2\,a}{x^{4}}\right)-384\,(7\,b^{2}d-15\,abf)\sqrt{a}\,x^{10}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1)+2\,(1344\,b^{2}dx^{9}+504\,b^{2}cx^{8}+2160\,abfx^{7}+1575\,abex^{6}+1232\,abdx^{5}+1008\,abcx^{4}+720\,a^{2}fx^{3}+630\,a^{2}ex^{2}+560\,a^{2}dx+504\,a^{2}c)\sqrt{bx^{4}+a}}{10080\,ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/10080*(2688*sqrt(a)*b^2*d*x^10*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(
1/4)), -1) - 945*sqrt(a)*b^2*e*x^10*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a)
 + 2*a)/x^4) - 384*(7*b^2*d - 15*a*b*f)*sqrt(a)*x^10*(-b/a)^(3/4)*elliptic_
f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(1344*b^2*d*x^9 + 504*b^2*c*x^8 + 2160*a*
b*f*x^7 + 1575*a*b*e*x^6 + 1232*a*b*d*x^5 + 1008*a*b*c*x^4 + 720*a^2*f*x^3
+ 630*a^2*e*x^2 + 560*a^2*d*x + 504*a^2*c)*sqrt(b*x^4 + a))/(a*x^10)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.31, size = 398, normalized size = 1.00

$$\frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4x^{9}\Gamma\left(-\frac{5}{4}\right)}+\frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}\,bd\Gamma\left(-\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4x^{5}\Gamma\left(-\frac{1}{4}\right)}+\frac{\sqrt{a}\,bf\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4x^{3}\Gamma\left(\frac{1}{4}\right)}-\frac{a^{2}e}{8\sqrt{b}\,x^{10}\sqrt{\frac{a}{bx^{4}}+1}}-\frac{a\sqrt{b}\,c\sqrt{\frac{a}{bx^{4}}+1}}{10x^{8}}-\frac{3a\sqrt{b}\,e}{16x^{6}\sqrt{\frac{a}{bx^{4}}+1}}-\frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^{4}}+1}}{5x^{4}}-\frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^{4}}+1}}{4x^{2}}-\frac{b^{\frac{3}{2}}e}{16x^{2}\sqrt{\frac{a}{bx^{4}}+1}}-\frac{b^{\frac{5}{2}}c\sqrt{\frac{a}{bx^{4}}+1}}{10a}-\frac{3b^{2}e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{2}}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**11,x)
```

```
[Out] a**(3/2)*d*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**9*gamma(-5/4)) + a**(3/2)*f*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*d*gamma(-5/4)
*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)
) + sqrt(a)*b*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*
pi)/a)/(4*x**3*gamma(1/4)) - a**2*e/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1))
- a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*e/(16*x**6*sqrt(
a/(b*x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(3/2)*e*sq
rt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*e/(16*x**2*sqrt(a/(b*x**4) + 1)) - b
**(5/2)*c*sqrt(a/(b*x**4) + 1)/(10*a) - 3*b**2*e*asinh(sqrt(a)/(sqrt(b)*x**
2))/(16*sqrt(a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^11, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4+a\right)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^11,x)
```

```
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^11, x)
```

3.526 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=424

$$-\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480}-\frac{4b^2c\sqrt{a+bx^4}}{77ax^3}-\frac{b^2d\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2e\sqrt{a+bx^4}}{15ax}+\frac{4b^{5/2}ex\sqrt{a}}{15a\left(\sqrt{a}+\right.}$$

```
[Out] -1/3960*(360*c/x^11+396*d/x^10+440*e/x^9+495*f/x^8)*(b*x^4+a)^(3/2)-3/16*b^
2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/18480*b*(1440*c/x^7+1848*d/x
^6+2464*e/x^5+3465*f/x^4)*(b*x^4+a)^(1/2)-4/77*b^2*c*(b*x^4+a)^(1/2)/a/x^3-
1/10*b^2*d*(b*x^4+a)^(1/2)/a/x^2-4/15*b^2*e*(b*x^4+a)^(1/2)/a/x+4/15*b^(5/2
)*e*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-4/15*b^(9/4)*e*(cos(2*arctan(
b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin
(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)
/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)-2/1155*b^(9/4)*(cos
(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ell
ipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-77*e*a^(1/2)+15*c*b^
(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5
/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {14, 1839, 1847, 1296, 1212, 226, 1210, 1266, 821, 272, 65, 214}

$$-\frac{2b^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}c-77\sqrt{a}e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}+\frac{4b^{5/2}ex\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{4b^2c\sqrt{a+bx^4}}{77ax^3}-\frac{b^2d\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2e\sqrt{a+bx^4}}{15ax}-\frac{3b^2f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{b\sqrt{a+bx^4}\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)}{18480}-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^12,x]
```

```
[Out] -1/18480*(b*((1440*c)/x^7 + (1848*d)/x^6 + (2464*e)/x^5 + (3465*f)/x^4)*Sqr
t[a + b*x^4]) - (4*b^2*c*Sqrt[a + b*x^4])/(77*a*x^3) - (b^2*d*Sqrt[a + b*x^
4])/(10*a*x^2) - (4*b^2*e*Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2)*e*x*Sqrt[a
 + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - (((360*c)/x^11 + (396*d)/x^10 +
 (440*e)/x^9 + (495*f)/x^8)*(a + b*x^4)^(3/2))/3960 - (3*b^2*f*ArcTanh[Sqrt
[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (4*b^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]) - (2*b^(9/4)*(15*Sqrt[b]*c - 77
*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(1155*a^(5/4)*Sqrt[a +
b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{12}}\,dx = -\frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)\left(a+bx^4\right)^{3/2}}{3960} - (6b)\int \frac{\left(-\frac{c}{11}-\frac{dx}{10}-\cdots\right.}{\cdots}$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}\cdots\right.}{39\cdots}$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}\cdots\right.}{39\cdots}$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}\cdots\right.}{39\cdots}$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \cdots$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \cdots$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \cdots$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \cdots$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.43, size = 317, normalized size = 0.75

$$-\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((a+bx^4)\left(24b^2x^8(120c+77x(3d+8ex))+abx^4(9360c+77x(144d+176ex+225fx^2))+14a^2(360c+11x(36d+5x(8e+9fx)))\right)+10395\sqrt{a}\,b^2fx^{11}\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+14784\sqrt{a}\,b^{5/2}ex^{11}\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-192b^{5/2}\left(-15i\sqrt{b}\,c+77\sqrt{a}\,e\right)x^{11}\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{55440a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^{11}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^12,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(24*b^2*x^8*(120*c + 77*x*(3*d +
8*e*x)) + a*b*x^4*(9360*c + 77*x*(144*d + 176*e*x + 225*f*x^2)) + 14*a^2*(3
```

```
60*c + 11*x*(36*d + 5*x*(8*e + 9*f*x)))) + 10395*Sqrt[a]*b^2*f*x^11*Sqrt[a
+ b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 14784*Sqrt[a]*b^(5/2)*e*x^11*
Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] -
 192*b^(5/2)*((-15*I)*Sqrt[b]*c + 77*Sqrt[a]*e)*x^11*Sqrt[1 + (b*x^4)/a]*El
lipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(55440*a*Sqrt[(I*Sqrt[
b])/Sqrt[a]]*x^11*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.44, size = 380, normalized size = 0.90

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(14784b^2e\,x^{10}+5544b^2d\,x^9+2880b^2c\,x^8+17325abf\,x^7+13552abe\,x^6+11088abd\,x^5+9360abc\,x^4+6930a^2f\,x^3+6160}{55440x^{11}a}$
default	$c\left(-\frac{a\sqrt{b\,x^4+a}}{11x^{11}}-\frac{13b\sqrt{b\,x^4+a}}{77x^7}-\frac{4b^2\sqrt{b\,x^4+a}}{77a\,x^3}-\frac{4b^3\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\right.}{77a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{11x^{11}}-\frac{ad\sqrt{b\,x^4+a}}{10x^{10}}-\frac{ae\sqrt{b\,x^4+a}}{9x^9}-\frac{af\sqrt{b\,x^4+a}}{8x^8}-\frac{13bc\sqrt{b\,x^4+a}}{77x^7}-\frac{bd\sqrt{b\,x^4+a}}{5x^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/11*a*(b*x^4+a)^(1/2)/x^11-13/77*b*(b*x^4+a)^(1/2)/x^7-4/77*b^2/a*(b*x
^4+a)^(1/2)/x^3-4/77*b^3/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*b^(1/2))^(1/2),I))+e*(-1/9*a*(b*x^4+a)^(1/2)/x^9-11/45*b*(b*x^4+a)^(1
/2)/x^5-4/15*b^2/a*(b*x^4+a)^(1/2)/x+4/15*I*b^(5/2)/a^(1/2)/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/
(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^
(1/2)*b^(1/2))^(1/2),I)))+f*(-1/8*a/x^8*(b*x^4+a)^(1/2)-5/16*b/x^4*(b*x^4+a
)^(1/2)-3/16*b^2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))-1/10*d*(b
^2*x^8+2*a*b*x^4+a^2)/a/x^10*(b*x^4+a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^12, x)
```

Fricas [**A**]
time = 0.12, size = 227, normalized size = 0.54

$$-\frac{29568\,\sqrt{a}\,b^2 e x^{11}\left(-\frac{b}{a}\right)^{\frac{3}{4}} E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1) - 10395\,\sqrt{a}\,b^2 f x^{11}\log\left(-\frac{bx^4-2\,\sqrt{bx^4+a}\,\sqrt{a}+2\,a}{x^4}\right) - 384\,(15\,b^2c+77\,b^2e)\sqrt{a}\,x^{11}\left(-\frac{b}{a}\right)^{\frac{3}{4}} F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1) + 2\,(14784\,b^2ex^{10}+5544\,b^2dx^9+2880\,b^2cx^8+17325\,abfx^7+13552\,abex^6+11088\,abdx^5+9360\,abcx^4+6930\,a^2fx^3+6160\,a^2ex^2+5544\,a^2dx+5040\,a^2c)\sqrt{bx^4+a}}{110880\,ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/110880*(29568*sqrt(a)*b^2*e*x^11*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)
^(1/4)), -1) - 10395*sqrt(a)*b^2*f*x^11*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqr
t(a) + 2*a)/x^4) - 384*(15*b^2*c + 77*b^2*e)*sqrt(a)*x^11*(-b/a)^(3/4)*elli
ptic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(14784*b^2*e*x^10 + 5544*b^2*d*x^9 +
 2880*b^2*c*x^8 + 17325*a*b*f*x^7 + 13552*a*b*e*x^6 + 11088*a*b*d*x^5 + 936
0*a*b*c*x^4 + 6930*a^2*f*x^3 + 6160*a^2*e*x^2 + 5544*a^2*d*x + 5040*a^2*c)*
sqrt(b*x^4 + a))/(a*x^11)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 7.65, size = 401, normalized size = 0.95

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)}+\frac{\sqrt{a}\,bc\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}\,be\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}-\frac{a^2f}{8\sqrt{b}\,x^{10}\sqrt{\frac{a}{bx^4}+1}}-\frac{a\sqrt{b}\,d\sqrt{\frac{a}{bx^4}+1}}{10x^8}-\frac{3a\sqrt{b}\,f}{16x^6\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{5x^4}-\frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{4x^2}-\frac{b^{\frac{3}{2}}f}{16x^2\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{5}{2}}d\sqrt{\frac{a}{bx^4}+1}}{10a}-\frac{3b^2f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**12,x)
```

```
[Out] a**(3/2)*c*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*pi
)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*e*gamma(-9/4)*hyper((-9/4, -1/2), (-5
/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*c*gamma(-7
/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3
/4)) + sqrt(a)*b*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*x**5*gamma(-1/4)) - a**2*f/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) +
 1)) - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*f/(16*x**6*
sqrt(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(3/2)
*f*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*f/(16*x**2*sqrt(a/(b*x**4) + 1)
) - b**(5/2)*d*sqrt(a/(b*x**4) + 1)/(10*a) - 3*b**2*f*asinh(sqrt(a)/(sqrt(b
)*x**2))/(16*sqrt(a))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^12, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^4+a\right)}^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^{12}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^12,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^12, x)
```

3.527 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=449

$$-\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480}-\frac{b^2c\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2d\sqrt{a+bx^4}}{77ax^3}-\frac{b^2e\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2f\sqrt{a+bx^4}}{15ax}$$

```
[Out] -1/1980*(165*c/x^12+180*d/x^11+198*e/x^10+220*f/x^9)*(b*x^4+a)^(3/2)+1/32*b
^3*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/18480*b*(1155*c/x^8+1440*d/
x^7+1848*e/x^6+2464*f/x^5)*(b*x^4+a)^(1/2)-1/32*b^2*c*(b*x^4+a)^(1/2)/a/x^4
-4/77*b^2*d*(b*x^4+a)^(1/2)/a/x^3-1/10*b^2*e*(b*x^4+a)^(1/2)/a/x^2-4/15*b^2
*f*(b*x^4+a)^(1/2)/a/x+4/15*b^(5/2)*f*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1
/2))-4/15*b^(9/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(
b*x^4+a)^(1/2)-2/1155*b^(9/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1
/2*2^(1/2))*(-77*f*a^(1/2)+15*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(
a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {14, 1839, 1847, 1266, 849, 821, 272, 65, 214, 1296, 1212, 226, 1210}

$$-\frac{2b^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}d-77\sqrt{a}f\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}+\frac{b^3c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}}+\frac{4b^{5/2}fx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{b^2c\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2d\sqrt{a+bx^4}}{77ax^3}-\frac{b^2e\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2f\sqrt{a+bx^4}}{15ax}-\frac{(a+bx^4)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}-\frac{b\sqrt{a+bx^4}\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)}{18480}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^13,x]
```

```
[Out] -1/18480*(b*((1155*c)/x^8 + (1440*d)/x^7 + (1848*e)/x^6 + (2464*f)/x^5)*Sqr
t[a + b*x^4]) - (b^2*c*Sqrt[a + b*x^4])/(32*a*x^4) - (4*b^2*d*Sqrt[a + b*x^
4])/(77*a*x^3) - (b^2*e*Sqrt[a + b*x^4])/(10*a*x^2) - (4*b^2*f*Sqrt[a + b*x
^4])/(15*a*x) + (4*b^(5/2)*f*x*Sqrt[a + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^
2)) - (((165*c)/x^12 + (180*d)/x^11 + (198*e)/x^10 + (220*f)/x^9)*(a + b*x^
4)^(3/2))/1980 + (b^3*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(32*a^(3/2)) - (4
*b^(9/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x
^4]) - (2*b^(9/4)*(15*Sqrt[b]*d - 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(1155*a^(5/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx = -\frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)(a+bx^4)^{3/2}}{1980} - (6b)\int \frac{\left(-\frac{c}{12}-\frac{dx}{11}-\cdots\right.}{\cdots}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198\cdots}{x^{1\cdots}}\right.}{\cdots}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198\cdots}{x^{1\cdots}}\right.}{\cdots}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198\cdots}{x^{1\cdots}}\right.}{\cdots}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2d\sqrt{a+bx^4}}{77ax^3} - \cdots$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \cdots$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \cdots$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \cdots$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \cdots$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.47, size = 328, normalized size = 0.73

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\sqrt{a}(a+bx^4)(56a^2(165c+2x(90d+99ex+110fx^2))+3b^2x^8(1155c+16x(120d+77x(3e+8fx)))+2abx^4(8085c+16x(585d+77x(9e+11fx))))+3465b^3cx^{12}\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+29568ab^{5/2}fx^{12}\sqrt{1+\frac{bx^4}{a}}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-384\sqrt{a}b^{5/2}\left(-15i\sqrt{b}d+77\sqrt{a}f\right)x^{12}\sqrt{1+\frac{bx^4}{a}}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{110880a^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x^{12}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^13,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-(Sqrt[a]*(a + b*x^4)*(56*a^2*(165*c + 2*x*(90*
d + 99*e*x + 110*f*x^2)) + 3*b^2*x^8*(1155*c + 16*x*(120*d + 77*x*(3*e + 8*
f*x))) + 2*a*b*x^4*(8085*c + 16*x*(585*d + 77*x*(9*e + 11*f*x))))) + 3465*b
^3*c*x^12*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) + 29568*a*b^(5/
2)*f*x^12*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]
*x], -1] - 384*Sqrt[a]*b^(5/2)*((-15*I)*Sqrt[b]*d + 77*Sqrt[a]*f)*x^12*Sqrt
[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(110
880*a^(3/2)*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^12*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.44, size = 400, normalized size = 0.89

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(29568 b^{2} f\,x^{11}+11088 b^{2} e\,x^{10}+5760 b^{2} d\,x^{9}+3465 b^{2} c\,x^{8}+27104 a b f\,x^{7}+22176 a b e\,x^{6}+18720 a b d\,x^{5}+16170 a b c\,x^{4}+123\right.}{110880 x^{12} a}$
default	$d\left(-\frac{a\sqrt{b\,x^{4}+a}}{11 x^{11}}-\frac{13 b\sqrt{b\,x^{4}+a}}{77 x^{7}}-\frac{4 b^{2}\sqrt{b\,x^{4}+a}}{77 a\,x^{3}}-\frac{4 b^{3}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\;\mathrm{EllipticF}\left(x\sqrt{\right.}{77 a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.$
elliptic	$-\frac{a c\sqrt{b\,x^{4}+a}}{12 x^{12}}-\frac{a d\sqrt{b\,x^{4}+a}}{11 x^{11}}-\frac{a e\sqrt{b\,x^{4}+a}}{10 x^{10}}-\frac{a f\sqrt{b\,x^{4}+a}}{9 x^{9}}-\frac{7 b c\sqrt{b\,x^{4}+a}}{48 x^{8}}-\frac{13 b d\sqrt{b\,x^{4}+a}}{77 x^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/11*a*(b*x^4+a)^(1/2)/x^11-13/77*b*(b*x^4+a)^(1/2)/x^7-4/77*b^2/a*(b*x
^4+a)^(1/2)/x^3-4/77*b^3/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*b^(1/2))^(1/2),I))+f*(-1/9*a*(b*x^4+a)^(1/2)/x^9-11/45*b*(b*x^4+a)^(1
/2)/x^5-4/15*b^2/a*(b*x^4+a)^(1/2)/x+4/15*I*b^(5/2)/a^(1/2)/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/
(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^
(1/2)*b^(1/2))^(1/2),I)))+c*(-7/48*b/x^8*(b*x^4+a)^(1/2)-1/32/a*b^2/x^4*(b*
x^4+a)^(1/2)+1/32/a^(3/2)*b^3*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/12*
a/x^12*(b*x^4+a)^(1/2))-1/10*e*(b^2*x^8+2*a*b*x^4+a^2)/a/x^10*(b*x^4+a)^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] -1/192*(3*b^3*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/
a^(3/2) + 2*(3*(b*x^4 + a)^(5/2)*b^3 + 8*(b*x^4 + a)^(3/2)*a*b^3 - 3*sqrt(b
*x^4 + a)*a^2*b^3)/((b*x^4 + a)^3*a - 3*(b*x^4 + a)^2*a^2 + 3*(b*x^4 + a)*a
^3 - a^4))*c + integrate((b*f*x^6 + b*x^5*e + b*d*x^4 + a*f*x^2 + a*x*e + a
*d)*sqrt(b*x^4 + a)/x^12, x)
```

Fricas [A]
time = 0.12, size = 250, normalized size = 0.56

$$-\frac{59136\,a^{\frac{3}{2}}b^{2}fx^{12}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1)-3465\,\sqrt{a}\,b^{3}cx^{12}\log\left(-\frac{bx^{4}+2\,\sqrt{bx^{4}+a}\,\sqrt{a}+2\,a}{x^{4}}\right)-768\,(15\,ab^{2}d+77\,ab^{2}f)\sqrt{a}\,x^{12}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1)+2\,(29568\,ab^{2}fx^{11}+11088\,ab^{2}ex^{10}+5760\,ab^{2}dx^{9}+3465\,ab^{2}cx^{8}+27104\,a^{2}bfx^{7}+22176\,a^{2}bex^{6}+18720\,a^{2}bdx^{5}+16170\,a^{2}bcx^{4}+12320\,a^{3}fx^{3}+11088\,a^{3}ex^{2}+10080\,a^{3}dx+9240\,a^{3}c)\sqrt{bx^{4}+a}}{221760\,a^{2}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/221760*(59136*a^(3/2)*b^2*f*x^12*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)
^(1/4)), -1) - 3465*sqrt(a)*b^3*c*x^12*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt
(a) + 2*a)/x^4) - 768*(15*a*b^2*d + 77*a*b^2*f)*sqrt(a)*x^12*(-b/a)^(3/4)*e
lliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(29568*a*b^2*f*x^11 + 11088*a*b^2
*e*x^10 + 5760*a*b^2*d*x^9 + 3465*a*b^2*c*x^8 + 27104*a^2*b*f*x^7 + 22176*a
^2*b*e*x^6 + 18720*a^2*b*d*x^5 + 16170*a^2*b*c*x^4 + 12320*a^3*f*x^3 + 1108
8*a^3*e*x^2 + 10080*a^3*d*x + 9240*a^3*c)*sqrt(b*x^4 + a))/(a^2*x^12)
```

Sympy [C] Result contains complex when optimal does not.
time = 12.06, size = 403, normalized size = 0.90

$$\frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{11}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\,\frac{bx^{4}e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{9}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\,\frac{bx^{4}e^{i\pi}}{a}\right)}{4x^{9}\Gamma\left(-\frac{5}{4}\right)}+\frac{\sqrt{a}\,bd\Gamma\left(-\frac{7}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\,\frac{bx^{4}e^{i\pi}}{a}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}\,bf\Gamma\left(-\frac{5}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\,\frac{bx^{4}e^{i\pi}}{a}\right)}{4x^{5}\Gamma\left(-\frac{1}{4}\right)}-\frac{a^{2}c}{12\sqrt{b}\,x^{14}\sqrt{\frac{a}{bx^{4}}+1}}-\frac{11a\sqrt{b}\,c}{48x^{10}\sqrt{\frac{a}{bx^{4}}+1}}-\frac{a\sqrt{b}\,e\sqrt{\frac{a}{bx^{4}}+1}}{10x^{8}}-\frac{17b^{\frac{3}{2}}c}{96x^{6}\sqrt{\frac{a}{bx^{4}}+1}}-\frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^{4}}+1}}{5x^{4}}-\frac{b^{\frac{5}{2}}c}{32ax^{2}\sqrt{\frac{a}{bx^{4}}+1}}-\frac{b^{\frac{5}{2}}e\sqrt{\frac{a}{bx^{4}}+1}}{10a}+\frac{b^{3}c\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{2}}\right)}{32a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**13,x)
```

```
[Out] a**(3/2)*d*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*pi
)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*f*gamma(-9/4)*hyper((-9/4, -1/2), (-5
/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*d*gamma(-7
/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3
/4)) + sqrt(a)*b*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*x**5*gamma(-1/4)) - a**2*c/(12*sqrt(b)*x**14*sqrt(a/(b*x**4)
+ 1)) - 11*a*sqrt(b)*c/(48*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*e*sqrt(a
```

```
/(b*x**4) + 1)/(10*x**8) - 17*b**(3/2)*c/(96*x**6*sqrt(a/(b*x**4) + 1)) - b
**(3/2)*e*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(5/2)*c/(32*a*x**2*sqrt(a/(b*x
**4) + 1)) - b**(5/2)*e*sqrt(a/(b*x**4) + 1)/(10*a) + b**3*c*asinh(sqrt(a)/
(sqrt(b)*x**2))/(32*a**(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^13, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^4+a\right)}^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^{13}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^13,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^13, x)
```

3.528 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=474

$$-\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240}-\frac{4b^2c\sqrt{a+bx^4}}{195ax^5}-\frac{b^2d\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2e\sqrt{a+bx^4}}{77ax^3}-\frac{b^2f\sqrt{a+}}{10a}$$

```
[Out] -1/8580*(660*c/x^13+715*d/x^12+780*e/x^11+858*f/x^10)*(b*x^4+a)^(3/2)+1/32*
b^3*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/240240*b*(12320*c/x^9+1501
5*d/x^8+18720*e/x^7+24024*f/x^6)*(b*x^4+a)^(1/2)-4/195*b^2*c*(b*x^4+a)^(1/2
)/a/x^5-1/32*b^2*d*(b*x^4+a)^(1/2)/a/x^4-4/77*b^2*e*(b*x^4+a)^(1/2)/a/x^3-1
/10*b^2*f*(b*x^4+a)^(1/2)/a/x^2+4/65*b^3*c*(b*x^4+a)^(1/2)/a^2/x-4/65*b^(7/
2)*c*x*(b*x^4+a)^(1/2)/a^2/(a^(1/2)+x^2*b^(1/2))+4/65*b^(13/4)*c*(cos(2*arc
tan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE
(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^
4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)-2/5005*b^(11/4)
*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4))
)*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(65*e*a^(1/2)+77*
c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/
a^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {14, 1839, 1847, 1296, 1212, 226, 1210, 1266, 849, 821, 272, 65, 214}

$$-\frac{2b^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(65\sqrt{a}e+77\sqrt{b}c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005a^{7/4}\sqrt{a+bx^4}}+\frac{4b^{13/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65a^{7/4}\sqrt{a+bx^4}}+\frac{b^3d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}}-\frac{4b^{7/2}cx\sqrt{a+bx^4}}{65a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4b^3c\sqrt{a+bx^4}}{65a^2x}-\frac{4b^2c\sqrt{a+bx^4}}{195ax^5}-\frac{b^2d\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2e\sqrt{a+bx^4}}{77ax^3}-\frac{b^2f\sqrt{a+bx^4}}{10ax^2}-\frac{(a+bx^4)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}-\frac{b\sqrt{a+bx^4}\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)}{240240}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^14,x]
```

```
[Out] -1/240240*(b*((12320*c)/x^9 + (15015*d)/x^8 + (18720*e)/x^7 + (24024*f)/x^6
)*Sqrt[a + b*x^4]) - (4*b^2*c*Sqrt[a + b*x^4])/(195*a*x^5) - (b^2*d*Sqrt[a
+ b*x^4])/(32*a*x^4) - (4*b^2*e*Sqrt[a + b*x^4])/(77*a*x^3) - (b^2*f*Sqrt[a
 + b*x^4])/(10*a*x^2) + (4*b^3*c*Sqrt[a + b*x^4])/(65*a^2*x) - (4*b^(7/2)*c
*x*Sqrt[a + b*x^4])/(65*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (((660*c)/x^13 + (71
5*d)/x^12 + (780*e)/x^11 + (858*f)/x^10)*(a + b*x^4)^(3/2))/8580 + (b^3*d*A
rcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(32*a^(3/2)) + (4*b^(13/4)*c*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(65*a^(7/4)*Sqrt[a + b*x^4]) - (2*b^(11/4)*(77*S
qrt[b]*c + 65*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
```

```
+ Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*a^(7
/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
```

```
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
```

```
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{14}}\,dx = -\frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)\left(a+bx^4\right)^{3/2}}{8580} - (6b)\int \frac{\left(-\frac{c}{13}-\frac{dx}{12}-\right.}{}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\right.}{}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\right.}{}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\right.}{}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.43, size = 339, normalized size = 0.72

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\left((a+bx^4)\left(-29568b^3cx^{12}+56a^3(660c+13x(55d+60ex+66fx^2))+ab^2x^8(9856c+39x(385d+16x(40e+77fx)))+2a^2bx^4(30800c+13x(2695d+48x(65e+77fx)))\right)\right)+15015\sqrt{a}\,b^3dx^{13}\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)-29568\sqrt{a}\,b^{7/2}cx^{13}\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)+384\sqrt{a}\,b^3\left(77\sqrt{b}\,c+65i\sqrt{a}\,e\right)x^{13}\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{480480a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^{13}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^14,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-((a + b*x^4)*(-29568*b^3*c*x^12 + 56*a^3*(660*
c + 13*x*(55*d + 60*e*x + 66*f*x^2)) + a*b^2*x^8*(9856*c + 39*x*(385*d + 16
*x*(40*e + 77*f*x))) + 2*a^2*b*x^4*(30800*c + 13*x*(2695*d + 48*x*(65*e + 7
7*f*x))))) + 15015*Sqrt[a]*b^3*d*x^13*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^
4]/Sqrt[a]]) - 29568*Sqrt[a]*b^(7/2)*c*x^13*Sqrt[1 + (b*x^4)/a]*EllipticE[I
*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] + 384*Sqrt[a]*b^3*(77*Sqrt[b]*c
+ (65*I)*Sqrt[a]*e)*x^13*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sq
rt[b])/Sqrt[a]]*x], -1])/(480480*a^2*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^13*Sqrt[a
+ b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.46, size = 420, normalized size = 0.89

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,\left(-29568b^3c\,x^{12}+48048a\,b^2f\,x^{11}+24960a\,b^2e\,x^{10}+15015a\,b^2d\,x^9+9856a\,b^2c\,x^8+96096a^2bf\,x^7+81120a^2be\,x^6+70\right.}{480480x^{13}a^2}$
default	$e\left(-\frac{a\sqrt{b\,x^4+a}}{11x^{11}}-\frac{13b\sqrt{b\,x^4+a}}{77x^7}-\frac{4b^2\sqrt{b\,x^4+a}}{77a\,x^3}-\frac{4b^3\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\right.}{77a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{13x^{13}}-\frac{ad\sqrt{b\,x^4+a}}{12x^{12}}-\frac{ae\sqrt{b\,x^4+a}}{11x^{11}}-\frac{af\sqrt{b\,x^4+a}}{10x^{10}}-\frac{5bc\sqrt{b\,x^4+a}}{39x^9}-\frac{7bd\sqrt{b\,x^4+a}}{48x^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/11*a*(b*x^4+a)^(1/2)/x^11-13/77*b*(b*x^4+a)^(1/2)/x^7-4/77*b^2/a*(b*x
^4+a)^(1/2)/x^3-4/77*b^3/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*b^(1/2))^(1/2),I))+d*(-7/48*b/x^8*(b*x^4+a)^(1/2)-1/32/a*b^2/x^4*(b*x
^4+a)^(1/2)+1/32/a^(3/2)*b^3*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/12*a
```

```
/x^12*(b*x^4+a)^(1/2))+c*(-1/13*a*(b*x^4+a)^(1/2)/x^13-5/39*b*(b*x^4+a)^(1/
2)/x^9-4/195*b^2/a*(b*x^4+a)^(1/2)/x^5+4/65*b^3/a^2*(b*x^4+a)^(1/2)/x-4/65*
I*b^(7/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)
*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^
(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))-1/10*f*(b^2*x^8+
2*a*b*x^4+a^2)/a/x^10*(b*x^4+a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^14, x)
```

Fricas [A]
time = 0.13, size = 258, normalized size = 0.54

$$\frac{59136\sqrt{a}\,b^3cx^{13}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1\right)+15015\sqrt{a}\,b^3dx^{13}\log\left(-\frac{bx^4+2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-768\left(77b^3c-65ab^2e\right)\sqrt{a}\,x^{13}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1\right)+2\left(29568b^3cx^{12}-48048ab^2fx^{11}-24960ab^2ex^{10}-15015ab^2dx^9-9856ab^2cx^8-96096a^2bfx^7-81120a^2bex^6-70070a^2bdx^5-61600a^2bcx^4-48048a^3fx^3-43680a^3ex^2-40040a^3dx-36960a^3c\right)\sqrt{bx^4+a}}{960960a^2x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/960960*(59136*sqrt(a)*b^3*c*x^13*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^
(1/4)), -1) + 15015*sqrt(a)*b^3*d*x^13*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt
(a) + 2*a)/x^4) - 768*(77*b^3*c - 65*a*b^2*e)*sqrt(a)*x^13*(-b/a)^(3/4)*ell
iptic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(29568*b^3*c*x^12 - 48048*a*b^2*f*x
^11 - 24960*a*b^2*e*x^10 - 15015*a*b^2*d*x^9 - 9856*a*b^2*c*x^8 - 96096*a^2
*b*f*x^7 - 81120*a^2*b*e*x^6 - 70070*a^2*b*d*x^5 - 61600*a^2*b*c*x^4 - 4804
8*a^3*f*x^3 - 43680*a^3*e*x^2 - 40040*a^3*d*x - 36960*a^3*c)*sqrt(b*x^4 + a
))/(a^2*x^13)
```

Sympy [C] Result contains complex when optimal does not.
time = 12.57, size = 403, normalized size = 0.85

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{13}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{13}{4},-\frac{1}{2}\\-\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{13}\Gamma\left(-\frac{9}{4}\right)}+\frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)}+\frac{\sqrt{a}\,bc\Gamma\left(-\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{9}\Gamma\left(-\frac{5}{4}\right)}+\frac{\sqrt{a}\,be\Gamma\left(-\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)}-\frac{a^2d}{12\sqrt{b}\,x^{14}\sqrt{\frac{a}{bx^4}+1}}-\frac{11a\sqrt{b}\,d}{48x^{10}\sqrt{\frac{a}{bx^4}+1}}-\frac{a\sqrt{b}\,f\sqrt{\frac{a}{bx^4}+1}}{10x^8}-\frac{17b^{\frac{3}{2}}d}{96x^6\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{5x^4}-\frac{b^{\frac{5}{2}}d}{32ax^2\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{5}{2}}f\sqrt{\frac{a}{bx^4}+1}}{10a}+\frac{b^3d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{32a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**14,x)
```

```
[Out] a**(3/2)*c*gamma(-13/4)*hyper((-13/4, -1/2), (-9/4,), b*x**4*exp_polar(I*pi
)/a)/(4*x**13*gamma(-9/4)) + a**(3/2)*e*gamma(-11/4)*hyper((-11/4, -1/2), (
-7/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**11*gamma(-7/4)) + sqrt(a)*b*c*gamma
(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma
```

```
(-5/4)) + sqrt(a)*b*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*x**7*gamma(-3/4)) - a**2*d/(12*sqrt(b)*x**14*sqrt(a/(b*x**
4) + 1)) - 11*a*sqrt(b)*d/(48*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*f*sqr
t(a/(b*x**4) + 1)/(10*x**8) - 17*b**(3/2)*d/(96*x**6*sqrt(a/(b*x**4) + 1))
- b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(5/2)*d/(32*a*x**2*sqrt(a/(
b*x**4) + 1)) - b**(5/2)*f*sqrt(a/(b*x**4) + 1)/(10*a) + b**3*d*asinh(sqrt(
a)/(sqrt(b)*x**2))/(32*a**(3/2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + x^2*e + d*x + c)/x^14, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^4+a\right)}^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^14,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^14, x)
```

3.529 $\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=361

$$\frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}-\frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{(4af-3bdx^2)\sqrt{a+bx^4}}{12b^2}-\frac{ad\tanh^{-1}}{}$$

```
[Out] -1/4*a*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/3*c*x*(b*x^4+a)^(1/
2)/b+1/5*e*x^3*(b*x^4+a)^(1/2)/b+1/6*f*x^4*(b*x^4+a)^(1/2)/b-1/12*(-3*b*d*x
^2+4*a*f)*(b*x^4+a)^(1/2)/b^2-3/5*a*e*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^
2*b^(1/2))+3/5*a^(5/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7
/4)/(b*x^4+a)^(1/2)-1/30*a^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(9*e*a^(1/2)+5*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(
a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {1847, 1294, 1212, 226, 1210, 1266, 847, 794, 223, 212}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(9\sqrt{a}\,e+5\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}-\frac{ad\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}-\frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{\sqrt{a+bx^4}\,(4af-3bdx^2)}{12b^2}+\frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (c*x*Sqrt[a + b*x^4])/(3*b) + (e*x^3*Sqrt[a + b*x^4])/(5*b) + (f*x^4*Sqrt[a
 + b*x^4])/(6*b) - (3*a*e*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*
x^2)) - ((4*a*f - 3*b*d*x^2)*Sqrt[a + b*x^4])/(12*b^2) - (a*d*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4)*(5*Sqrt[b]*c + 9*
Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x
^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx &= \int \left(\frac{x^4(c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^5(d+fx^2)}{\sqrt{a+bx^4}}\right)dx \\
&= \int \frac{x^4(c+ex^2)}{\sqrt{a+bx^4}}\,dx+\int \frac{x^5(d+fx^2)}{\sqrt{a+bx^4}}\,dx \\
&= \frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{1}{2}\text{Subst}\left(\int \frac{x^2(d+fx)}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{x^2(3ae-5bcx^2)}{\sqrt{a+bx^4}}\,dx}{5b} \\
&= \frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}+\frac{\int \frac{-5abc-9abex^2}{\sqrt{a+bx^4}}\,dx}{15b^2}+\text{Sub} \\
&= \frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}-\frac{(4af-3bdx^2)\sqrt{a+bx}}{12b^2} \\
&= \frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}-\frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} \\
&= \frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}-\frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.12, size = 212, normalized size = 0.59

$$
\frac{-20a^2f+20abcx+15abdx^2+12abex^3-10abfx^4+20b^2cx^5+15b^2dx^6+12b^2ex^7+10b^2fx^8-15a\sqrt{b}\,d\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-20abcx\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)-12abex^3\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{60b^2\sqrt{a+bx^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (-20*a^2*f + 20*a*b*c*x + 15*a*b*d*x^2 + 12*a*b*e*x^3 - 10*a*b*f*x^4 + 20*b
^2*c*x^5 + 15*b^2*d*x^6 + 12*b^2*e*x^7 + 10*b^2*f*x^8 - 15*a*Sqrt[b]*d*Sqrt
[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*a*b*c*x*Sqrt[1 + (b
*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*a*b*e*x^3*Sqrt
[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^2*Sqr
t[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 279, normalized size = 0.77

method	result
default	$-\frac{f\sqrt{b\,x^{4}+a}\,\left(-b\,x^{4}+2a\right)}{6b^{2}}+e\left(\frac{x^{3}\sqrt{b\,x^{4}+a}}{5b}-\frac{3ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{5b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.$
elliptic	$\frac{f\,x^{4}\sqrt{b\,x^{4}+a}}{6b}+\frac{e\,x^{3}\sqrt{b\,x^{4}+a}}{5b}+\frac{d\,x^{2}\sqrt{b\,x^{4}+a}}{4b}+\frac{cx\sqrt{b\,x^{4}+a}}{3b}-\frac{af\sqrt{b\,x^{4}+a}}{3b^{2}}-\frac{ac\sqrt{1-\frac{i\sqrt{b}\,x}{\sqrt{a}}}}{}$
risch	$-\frac{\left(-10bf\,x^{4}-12be\,x^{3}-15bd\,x^{2}-20bcx+20af\right)\sqrt{b\,x^{4}+a}}{60b^{2}}-\frac{3ia^{\frac{3}{2}}e\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{}}{\sqrt{a}}}\right.}{5b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*f*(b*x^4+a)^(1/2)*(-b*x^4+2*a)/b^2+e*(1/5*x^3*(b*x^4+a)^(1/2)/b-3/5*I*
a^(3/2)/b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(
1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1
/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+d*(1/4*x^2*(b*x^4+
a)^(1/2)/b-1/4*a/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+c*(1/3*x*(b*x^4+a
)^(1/2)/b-1/3*a/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)
*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(
1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x^4/sqrt(b*x^4 + a), x)
```

Fricas [A]
time = 0.13, size = 163, normalized size = 0.45

$$-\frac{72\,a\sqrt{b}\,ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-15\,a\sqrt{b}\,dx\log\left(-2\,bx^{4}+2\sqrt{bx^{4}+a}\,\sqrt{b}\,x^{2}-a\right)+8\,(5\,bc-9\,ae)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-2\,(10\,bfx^{5}+12\,bex^{4}+15\,bdx^{3}+20\,bcx^{2}-20\,afx-36\,ae)\sqrt{bx^{4}+a}}{120\,b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(72*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1
) - 15*a*sqrt(b)*d*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 8*
(5*b*c - 9*a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -
1) - 2*(10*b*f*x^5 + 12*b*e*x^4 + 15*b*d*x^3 + 20*b*c*x^2 - 20*a*f*x - 36*a
*e)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A]
time = 3.04, size = 177, normalized size = 0.49

$$\frac{\sqrt{a}\,dx^2\sqrt{1+\frac{bx^4}{a}}}{4b}-\frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+f\left(\begin{cases}-\frac{a\sqrt{a+bx^4}}{3b^2}+\frac{x^4\sqrt{a+bx^4}}{6b} & \text{for } b\neq 0\\ \frac{x^8}{8\sqrt{a}} & \text{otherwise}\end{cases}\right)+\frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*d*asinh(sqrt(b)*x**2/sqrt(a))/(
4*b**(3/2)) + f*Piecewise((-a*sqrt(a + b*x**4)/(3*b**2) + x**4*sqrt(a + b*x
**4)/(6*b), Ne(b, 0)), (x**8/(8*sqrt(a)), True)) + c*x**5*gamma(5/4)*hyper(
(1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + e*x*
*7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x^4/sqrt(b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
```

```
[Out] int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

3.530 $\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=336

$$\frac{dx\sqrt{a+bx^4}}{3b}+\frac{fx^3\sqrt{a+bx^4}}{5b}-\frac{3afx\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{(2c+ex^2)\sqrt{a+bx^4}}{4b}-\frac{ae\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}+\frac{3a}{}$$

```
[Out] -1/4*a*e*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/3*d*x*(b*x^4+a)^(1/
2)/b+1/5*f*x^3*(b*x^4+a)^(1/2)/b+1/4*(e*x^2+2*c)*(b*x^4+a)^(1/2)/b-3/5*a*f*
x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))+3/5*a^(5/4)*f*(cos(2*arctan
(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(si
n(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a
)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)-1/30*a^(3/4)*(cos(
2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Elli
pticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(9*f*a^(1/2)+5*d*b^(1/2
))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/
(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1847, 1266, 794, 223, 212, 1294, 1212, 226, 1210}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(9\sqrt{a}\,f+5\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}-\frac{ae\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}-\frac{3afx\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{\sqrt{a+bx^4}\,(2c+ex^2)}{4b}+\frac{dx\sqrt{a+bx^4}}{3b}+\frac{fx^3\sqrt{a+bx^4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (d*x*Sqrt[a + b*x^4])/(3*b) + (f*x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*f*x*Sqrt
[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*x^2)*Sqrt[a +
b*x^4])/(4*b) - (a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) +
(3*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*
x^4]) - (a^(3/4)*(5*Sqrt[b]*d + 9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(
a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
```

```
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \left(\frac{x^3(c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^4(d+fx^2)}{\sqrt{a+bx^4}}\right)dx$$

$$= \int \frac{x^3(c+ex^2)}{\sqrt{a+bx^4}}\,dx + \int \frac{x^4(d+fx^2)}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{fx^3\sqrt{a+bx^4}}{5b} + \frac{1}{2}\text{Subst}\left(\int \frac{x(c+ex)}{\sqrt{a+bx^2}}\,dx, x, x^2\right) - \frac{\int \frac{x^2(3af-5bdx^2)}{\sqrt{a+bx^4}}\,dx}{5b}$$

$$= \frac{dx\sqrt{a+bx^4}}{3b} + \frac{fx^3\sqrt{a+bx^4}}{5b} + \frac{(2c+ex^2)\sqrt{a+bx^4}}{4b} + \frac{\int \frac{-5abd-9abfx^2}{\sqrt{a+bx^4}}\,dx}{15b^2}$$

$$= \frac{dx\sqrt{a+bx^4}}{3b} + \frac{fx^3\sqrt{a+bx^4}}{5b} + \frac{(2c+ex^2)\sqrt{a+bx^4}}{4b} - \frac{(ae)\text{Subst}\left(\int \frac{1}{1-b\ldots}\right.}{}$$

$$= \frac{dx\sqrt{a+bx^4}}{3b} + \frac{fx^3\sqrt{a+bx^4}}{5b} - \frac{3afx\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{(2c+ex^2)\sqrt{a+\ldots}}{4b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 212, normalized size = 0.63

$$\frac{30\sqrt{b}\,c(a+bx^4)+20\sqrt{b}\,dx(a+bx^4)+15\sqrt{b}\,ex^2(a+bx^4)+12\sqrt{b}\,fx^3(a+bx^4)-15ae\sqrt{a+bx^4}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-20a\sqrt{b}\,dx\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)-12a\sqrt{b}\,fx^3\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{60b^{3/2}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (30*Sqrt[b]*c*(a + b*x^4) + 20*Sqrt[b]*d*x*(a + b*x^4) + 15*Sqrt[b]*e*x^2*(
a + b*x^4) + 12*Sqrt[b]*f*x^3*(a + b*x^4) - 15*a*e*Sqrt[a + b*x^4]*ArcTanh[
(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*a*Sqrt[b]*d*x*Sqrt[1 + (b*x^4)/a]*Hyper
geometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*a*Sqrt[b]*f*x^3*Sqrt[1 + (b*
x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^(3/2)*Sqrt[a
+ b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 269, normalized size = 0.80

method	result
default	$f\left(\frac{x^3\sqrt{b\,x^4+a}}{5b}-\frac{3ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right.}{5b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$
elliptic	$\frac{f\,x^3\sqrt{b\,x^4+a}}{5b}+\frac{e\,x^2\sqrt{b\,x^4+a}}{4b}+\frac{dx\sqrt{b\,x^4+a}}{3b}+\frac{c\sqrt{b\,x^4+a}}{2b}-\frac{ad\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4}}$
risch	$\frac{(12f\,x^3+15e\,x^2+20dx+30c)\sqrt{b\,x^4+a}}{60b}-\frac{3ia^{\frac{3}{2}}f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{5b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{3ia^{\frac{3}{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/5*x^3*(b*x^4+a)^(1/2)/b-3/5*I*a^(3/2)/b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2
)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)
^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)))+e*(1/4*x^2*(b*x^4+a)^(1/2)/b-1/4*a/b^(3/2)*ln(x^2*b^(1/2)+
(b*x^4+a)^(1/2)))+d*(1/3*x*(b*x^4+a)^(1/2)/b-1/3*a/b/(I/a^(1/2)*b^(1/2))^(1
/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*c*(b*x^4+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*c/b + integrate((f*x^6 + x^5*e + d*x^4)/sqrt(b*x^4 + a)
, x)
```

Fricas [A]
time = 0.12, size = 156, normalized size = 0.46

$$-\frac{72\,a\sqrt{b}\,fx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-15\,a\sqrt{b}\,ex\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\,\sqrt{b}\,x^2-a\right)+8\,(5\,bd-9\,af)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-2\,(12\,bfx^4+15\,bex^3+20\,bdx^2+30\,bcx-36\,af)\sqrt{bx^4+a}}{120\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(72*a*sqrt(b)*f*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1
) - 15*a*sqrt(b)*e*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 8*
(5*b*d - 9*a*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -
1) - 2*(12*b*f*x^4 + 15*b*e*x^3 + 20*b*d*x^2 + 30*b*c*x - 36*a*f)*sqrt(b*x^
4 + a))/(b^2*x)
```

Sympy [A]
time = 2.94, size = 156, normalized size = 0.46

$$\frac{\sqrt{a}\,ex^2\sqrt{1+\frac{bx^4}{a}}}{4b}-\frac{ae\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+c\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for}\ b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*e*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*e*asinh(sqrt(b)*x**2/sqrt(a))/(
4*b**(3/2)) + c*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(
2*b), True)) + d*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar
(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + f*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/
4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x^3/sqrt(b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{\sqrt{b\,x^4+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
[Out] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

3.531 $\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=308

$$\frac{ex\sqrt{a+bx^4}}{3b}+\frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{(2d+fx^2)\sqrt{a+bx^4}}{4b}-\frac{af\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}-\frac{\sqrt[4]{a}\,c\left(\sqrt{a}+\sqrt{b}\,x\right.}{}$$

```
[Out] -1/4*a*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/3*e*x*(b*x^4+a)^(1/
2)/b+1/4*(f*x^2+2*d)*(b*x^4+a)^(1/2)/b+c*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)
+x^2*b^(1/2))-a^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(
1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/
4)/(b*x^4+a)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),
1/2*2^(1/2))*(-e*a^(1/2)+3*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(
1/2)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1847, 1294, 1212, 226, 1210, 1266, 794, 223, 212}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}-\frac{af\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}+\frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{\sqrt{a+bx^4}\,(2d+fx^2)}{4b}+\frac{ex\sqrt{a+bx^4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (e*x*Sqrt[a + b*x^4])/(3*b) + (c*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqr
t[b]*x^2)) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/(4*b) - (a*f*ArcTanh[(Sqrt[b]*
x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*c - Sqrt[a]*e)
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
```

```
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx &= \int \left(\frac{x^2(c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^3(d+fx^2)}{\sqrt{a+bx^4}}\right) dx \\
&= \int \frac{x^2(c+ex^2)}{\sqrt{a+bx^4}}\,dx+\int \frac{x^3(d+fx^2)}{\sqrt{a+bx^4}}\,dx \\
&= \frac{ex\sqrt{a+bx^4}}{3b}+\frac{1}{2}\text{Subst}\left(\int \frac{x(d+fx)}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{ae-3bcx^2}{\sqrt{a+bx^4}}\,dx}{3b} \\
&= \frac{ex\sqrt{a+bx^4}}{3b}+\frac{(2d+fx^2)\sqrt{a+bx^4}}{4b}-\frac{\left(\sqrt{a}\,c\right)\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}}+\frac{\left(\sqrt{a}\,\left(\right.\right.}{} \\
&= \frac{ex\sqrt{a+bx^4}}{3b}+\frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{(2d+fx^2)\sqrt{a+bx^4}}{4b}-\frac{\sqrt[4]{a}\,c\left(\sqrt{a}\right.}{} \\
&= \frac{ex\sqrt{a+bx^4}}{3b}+\frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{(2d+fx^2)\sqrt{a+bx^4}}{4b}-\frac{af\tanh^{-1}}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.13, size = 193, normalized size = 0.63

$$\frac{6\sqrt{b}\,d(a+bx^4)+4\sqrt{b}\,ex(a+bx^4)+3\sqrt{b}\,fx^2(a+bx^4)-3af\sqrt{a+bx^4}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-4a\sqrt{b}\,ex\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+4b^{3/2}cx^3\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{12b^{3/2}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (6*Sqrt[b]*d*(a + b*x^4) + 4*Sqrt[b]*e*x*(a + b*x^4) + 3*Sqrt[b]*f*x^2*(a +
 b*x^4) - 3*a*f*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 4*
a*Sqrt[b]*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4
)/a)] + 4*b^(3/2)*c*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4
, -((b*x^4)/a)])/(12*b^(3/2)*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 250, normalized size = 0.81

method	result
default	$f\left(\frac{x^{2}\sqrt{b\,x^{4}+a}}{4b}-\frac{a\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{4b^{\frac{3}{2}}}\right)+e\left(\frac{x\sqrt{b\,x^{4}+a}}{3b}-\frac{a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x}}\right.$
elliptic	$\frac{f\,x^{2}\sqrt{b\,x^{4}+a}}{4b}+\frac{e x\sqrt{b\,x^{4}+a}}{3b}+\frac{d\sqrt{b\,x^{4}+a}}{2b}-\frac{a e\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$
risch	$\frac{\left(3f\,x^{2}+4 e x+6d\right)\sqrt{b\,x^{4}+a}}{12b}+\frac{i c\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{b}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{i c\sqrt{a}\,\sqrt{1}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/4*x^2*(b*x^4+a)^(1/2)/b-1/4*a/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))
+e*(1/3*x*(b*x^4+a)^(1/2)/b-1/3*a/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elliptic
F(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*d*(b*x^4+a)^(1/2)/b+I*c*a^(1/2)/(I/a^
(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x
^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x^2/sqrt(b*x^4 + a), x)
```

Fricas [A]
time = 0.13, size = 147, normalized size = 0.48

$$\frac{24\,b^{\frac{3}{2}}cx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+3\,a\sqrt{b}\,fx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\,\sqrt{b}\,x^2-a\right)-8\,(3\,bc+be)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+2\,(3\,bfx^3+4\,bex^2+6\,bdx+12\,bc)\sqrt{bx^4+a}}{24\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
[Out] 1/24*(24*b^(3/2)*c*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) +
3*a*sqrt(b)*f*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) - 8*(3*b*
c + b*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 2*
(3*b*f*x^3 + 4*b*e*x^2 + 6*b*d*x + 12*b*c)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A]
time = 2.81, size = 156, normalized size = 0.51

$$\frac{\sqrt{a}\,fx^2\sqrt{1+\frac{bx^4}{a}}}{4b}-\frac{af\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+d\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
[Out] sqrt(a)*f*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*f*asinh(sqrt(b)*x**2/sqrt(a))/(
4*b**(3/2)) + d*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(
2*b), True)) + c*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar
(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + e*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4
,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x^2/sqrt(b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
[Out] int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

3.532 $\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=299

$$\frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}+\frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt[4]{a}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{}{\left(\sqrt{a}\right.}}}{b^{3/}}$$

```
[Out] 1/2*c*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+1/2*e*(b*x^4+a)^(1/2)/b+
1/3*f*x*(b*x^4+a)^(1/2)/b+d*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))
-a^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2
)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^
(1/2)+1/6*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b
^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*
(-f*a^(1/2)+3*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1
/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1847, 1262, 655, 223, 212, 1294, 1212, 226, 1210}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(3\sqrt{b}\,d-\sqrt{a}\,f\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (e*Sqrt[a + b*x^4])/(2*b) + (f*x*Sqrt[a + b*x^4])/(3*b) + (d*x*Sqrt[a + b*x
^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \left(\frac{x(c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^2(d+fx^2)}{\sqrt{a+bx^4}}\right)dx$$

$$= \int \frac{x(c+ex^2)}{\sqrt{a+bx^4}}\,dx + \int \frac{x^2(d+fx^2)}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{fx\sqrt{a+bx^4}}{3b} + \frac{1}{2}\text{Subst}\left(\int \frac{c+ex}{\sqrt{a+bx^2}}\,dx,x,x^2\right) - \frac{\int \frac{af-3bdx^2}{\sqrt{a+bx^4}}\,dx}{3b}$$

$$= \frac{e\sqrt{a+bx^4}}{2b} + \frac{fx\sqrt{a+bx^4}}{3b} + \frac{1}{2}c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right) - \frac{(\sqrt{a}\ a}{}$$

$$= \frac{e\sqrt{a+bx^4}}{2b} + \frac{fx\sqrt{a+bx^4}}{3b} + \frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{\sqrt[4]{a}\,d\left(\sqrt{a}+\sqrt{b}\,x\right.}{}$$

$$= \frac{e\sqrt{a+bx^4}}{2b} + \frac{fx\sqrt{a+bx^4}}{3b} + \frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx}}\right.}{2\sqrt{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 160, normalized size = 0.54

$$\frac{3ae+2afx+3bex^4+2bfx^5+3\sqrt{b}\,c\sqrt{a+bx^4}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-2afx\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+2bdx^3\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{6b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

[Out] (3*a*e + 2*a*f*x + 3*b*e*x^4 + 2*b*f*x^5 + 3*Sqrt[b]*c*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 2*a*f*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*d*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(6*b*Sqrt[a + b*x^4])

Maple [C] Result contains complex when optimal does not.
time = 0.36, size = 230, normalized size = 0.77

method	result
default	$f\left(\frac{x\sqrt{b\,x^4+a}}{3b}-\frac{a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{e\sqrt{b\,x^4+a}}{2b}+\frac{id\sqrt{a}\ \ldots}{\ldots}$
elliptic	$\frac{fx\sqrt{b\,x^4+a}}{3b}+\frac{e\sqrt{b\,x^4+a}}{2b}-\frac{af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{c\ln\left(2x^2\sqrt{b}+\ldots\right.}{2\ldots}$
risch	$\frac{(2fx+3e)\sqrt{b\,x^4+a}}{6b}+\frac{id\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{b}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{id\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] f*(1/3*x*(b*x^4+a)^(1/2)/b-1/3*a/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*e*(b*x^4+a)^(1/2)/b+I*d*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*c*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] -1/4*c*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a)/x^2))/sqrt(b) + integrate((f*x^4 + x^3*e + d*x^2)/sqrt(b*x^4 + a), x)

Fricas [A]
time = 0.11, size = 133, normalized size = 0.44

$$\frac{12\,\sqrt{b}\,dx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)-4\,\sqrt{b}\,(3\,d+f)x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+3\,\sqrt{b}\,cx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\,\sqrt{b}\,x^2-a\right)+2\,\sqrt{bx^4+a}\,(2\,fx^2+3\,ex+6\,d)}{12\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(b)*d*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) -
4*sqrt(b)*(3*d + f)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) +
 3*sqrt(b)*c*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*sqrt(b
*x^4 + a)*(2*f*x^2 + 3*e*x + 6*d))/(b*x)
```

Sympy [A]
time = 2.15, size = 129, normalized size = 0.43

$$e\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for}\ b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{c\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True)) +
 c*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + d*x**3*gamma(3/4)*hyper((1/2,
3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + f*x**5*gam
ma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamm
a(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x/sqrt(b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
```

```
[Out] int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

3.533 $\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=276

$$\frac{f\sqrt{a+bx^4}}{2b}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E}{b^{3/4}\sqrt{a+bx^4}}$$

[Out] 1/2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+1/2*f*(b*x^4+a)^(1/2)/b+e*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-a^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(e+c*b^(1/2)/a^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.296$, Rules used = {1899, 1212, 226, 1210, 1262, 655, 223, 212}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{f\sqrt{a+bx^4}}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4],x]

[Out] (f*Sqrt[a + b*x^4])/(2*b) + (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = \int \left(\frac{c+ex^2}{\sqrt{a+bx^4}} + \frac{x(d+fx^2)}{\sqrt{a+bx^4}}\right) dx$$

$$= \int \frac{c+ex^2}{\sqrt{a+bx^4}}\,dx + \int \frac{x(d+fx^2)}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{\sqrt{a+bx^2}}\,dx, x, x^2\right) - \frac{(\sqrt{a}\,e)\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}} + \left(c+\frac{\sqrt{a}\,e}{\sqrt{b}}\right)\int$$

$$= \frac{f\sqrt{a+bx^4}}{2b} + \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right.}}}{b^{3/4}\sqrt{a+bx^4}}$$

$$= \frac{f\sqrt{a+bx^4}}{2b} + \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right.}}}{b^{3/4}\sqrt{a+bx^4}}$$

$$= \frac{f\sqrt{a+bx^4}}{2b} + \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 150, normalized size = 0.54

$$\frac{f\sqrt{a+bx^4}}{2b} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1+\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}} + \frac{ex^3\sqrt{1+\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4],x]
```

```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*S
qrt[b]) + (c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^
4)/a)])/Sqrt[a + b*x^4] + (e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2,
 3/4, 7/4, -((b*x^4)/a)])/(3*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 208, normalized size = 0.75

method	result
default	$\frac{f\sqrt{b\,x^{4}+a}}{2b}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}$
risch	$\frac{f\sqrt{b\,x^{4}+a}}{2b}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}$
elliptic	$\frac{f\sqrt{b\,x^{4}+a}}{2b}+\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{d\ln\left(2x^{2}\sqrt{b}+2\sqrt{b\,x^{4}+a}\right)}{2\sqrt{b}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*(b*x^4+a)^(1/2)/b+I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*
(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(
1/2),I))+1/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+c/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*
x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/sqrt(b*x^4 + a), x)
```

Fricas [A]

time = 0.12, size = 135, normalized size = 0.49

$$\frac{4\,a\sqrt{b}\,ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+a\sqrt{b}\,dx\log\left(-2\,bx^{4}-2\,\sqrt{bx^{4}+a}\,\sqrt{b}\,x^{2}-a\right)+4\,(bc-ae)\sqrt{b}\,x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+2\,\sqrt{bx^{4}+a}\,(afx+2\,ae)}{4\,abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) +
a*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 4*(b*c -
a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 2*sqrt
(b*x^4 + a)*(a*f*x + 2*a*e))/(a*b*x)
```

Sympy [A]
time = 1.54, size = 128, normalized size = 0.46

$$f\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{d\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True)) +
 d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(
3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7
/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/sqrt(b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^3+e\,x^2+d\,x+c}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(1/2),x)
```

```
[Out] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(1/2), x)
```

3.534 $\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=285

$$\frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt[4]{a}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+}{\left(\sqrt{a}+\right.}}}{b^{3/4}\sqrt{}}$$

```
[Out] -1/2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)+1/2*e*arctanh(x^2*b^(1/2)/(
b*x^4+a)^(1/2))/b^(1/2)+f*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-a
^(1/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/
a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1
/2)+1/2*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*b^(1/2))*(f+d*b^(1/2)/a^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^
2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1846, 272, 65, 214, 1899, 281, 223, 212, 1212, 226, 1210}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\frac{\sqrt{b}\,d}{\sqrt{a}}+f\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x*Sqrt[a + b*x^4]),x]
```

```
[Out] (f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]
)/(2*Sqrt[a]) - (a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)
*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx = c\int \frac{1}{x\sqrt{a+bx^4}}\,dx + \int \frac{d+ex+fx^2}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{1}{4}c\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^4\right) + \int \left(\frac{ex}{\sqrt{a+bx^4}} + \frac{d+fx^2}{\sqrt{a+bx^4}}\right)dx$$

$$= \frac{c\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{2b} + e\int \frac{x}{\sqrt{a+bx^4}}\,dx + \int \frac{d+fx^2}{\sqrt{a+bx^4}}\,dx$$

$$= -\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{1}{2}e\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right) - \frac{\left(\sqrt{a}\,f\right)\int \frac{1}{\sqrt{}}}{\sqrt{}}$$

$$= \frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt[4]{a}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{}{\left(\right.}}}{b}$$

$$= \frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.34, size = 235, normalized size = 0.82

$$-\frac{i\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}\left(\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)-\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+2af\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-2\sqrt{a}\left(i\sqrt{b}\,d+\sqrt{a}\,f\right)\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)\right)}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x*Sqrt[a + b*x^4]),x]
```

```
[Out] ((-1/2*I)*Sqrt[(I*Sqrt[b])/Sqrt[a]]*(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*x
^4]*(Sqrt[a]*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - Sqrt[b]*c*ArcTanh[S
qrt[a + b*x^4]/Sqrt[a]]) + 2*a*f*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sq
rt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 2*Sqrt[a]*(I*Sqrt[b]*d + Sqrt[a]*f)*Sqrt[
1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1]))/(b*S
qrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.35, size = 222, normalized size = 0.78

method	result
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{e\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{if\sqrt{a}\,\sqrt{1-\frac{i\sqrt{}}{\sqrt{}}}\ldots}{\ldots}$
default	$\frac{if\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}+\frac{e\ln\left(x^2\sqrt{b}+\sqrt{b}\ldots\right.}{2\sqrt{b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*e*ln(x^2*b^
(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/
2))/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x^3 + x^2*e + d*x + c)/(sqrt(b*x^4 + a)*x), x)
```

Fricas [F]
time = 0.23, size = 38, normalized size = 0.13

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,(fx^3+ex^2+dx+c)}{bx^5+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^5 + a*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.47, size = 126, normalized size = 0.44

$$\frac{e\,\text{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}}-\frac{c\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{2\sqrt{a}}+\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(1/2),x)
[Out] e*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - c*asinh(sqrt(a)/(sqrt(b)*x**2))
/(2*sqrt(a)) + d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*
pi)/a)/(4*sqrt(a)*gamma(5/4)) + f*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^3 + x^2*e + d*x + c)/(sqrt(b*x^4 + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^3+e\,x^2+d\,x+c}{x\,\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(1/2)),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(1/2)), x)
```

3.535 $\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=309

$$-\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right.}{}$$

```
[Out] -1/2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)+1/2*f*arctanh(x^2*b^(1/2)/(
b*x^4+a)^(1/2))/b^(1/2)-c*(b*x^4+a)^(1/2)/a/x+c*x*b^(1/2)*(b*x^4+a)^(1/2)/a
/(a^(1/2)+x^2*b^(1/2))-b^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1
/2)/a^(3/4)/(b*x^4+a)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4)))
,1/2*2^(1/2))*(e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/
2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1847, 1296, 1212, 226, 1210, 1266, 858, 223, 212, 272, 65, 214}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\sqrt{a}\,e+\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\,\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]),x]
```

```
[Out] -((c*Sqrt[a + b*x^4])/(a*x)) + (Sqrt[b]*c*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] +
Sqrt[b]*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (d
*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + Sqrt[a]*
e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^
4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx &= \int \left(\frac{c+ex^2}{x^2\sqrt{a+bx^4}}+\frac{d+fx^2}{x\sqrt{a+bx^4}}\right)dx \\
&= \int \frac{c+ex^2}{x^2\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{x\sqrt{a+bx^4}}\,dx \\
&= -\frac{c\sqrt{a+bx^4}}{ax}+\frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{-ae-bcx^2}{\sqrt{a+bx^4}}\,dx}{a} \\
&= -\frac{c\sqrt{a+bx^4}}{ax}-\frac{\left(\sqrt{b}\,c\right)\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{a}}+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right) \\
&= -\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}}{a^{3/4}\sqrt{a+bx^4}} \\
&= -\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\right.}{} \\
&= -\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{2\sqrt{a}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.32, size = 250, normalized size = 0.81

$$
\frac{1}{2}\left(-\frac{2c\sqrt{a+bx^4}}{ax}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{\sqrt{b}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{\sqrt{a}}\right)-\frac{i\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,c\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{a+bx^4}}-\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-i\sqrt{b}\,c+\sqrt{a}\,e\right)\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{b}\sqrt{a+bx^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]),x]
```

```
[Out] ((-2*c*Sqrt[a + b*x^4])/(a*x) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/
Sqrt[b] - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/Sqrt[a])/2 - (I*Sqrt[(I*Sqrt
[b])/Sqrt[a]]*c*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sq
rt[a]]*x], -1])/Sqrt[a + b*x^4] - (Sqrt[(I*Sqrt[b])/Sqrt[a]]*((-I)*Sqrt[b]*
c + Sqrt[a]*e)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqr
t[a]]*x], -1])/(Sqrt[b]*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 241, normalized size = 0.78

method	result
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{ax}+\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{f\ln\left(2x^{2}\sqrt{b}+2\sqrt{b\,x^{4}+a}\right)}{2\sqrt{b}}$
default	$\frac{f\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{2\sqrt{b}}+\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{d\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b}}{x^{2}}\right.}{2\sqrt{a}}$
risch	$-\frac{c\sqrt{b\,x^{4}+a}}{ax}+\frac{i\sqrt{b}\,c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{i\sqrt{b}\,c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+e/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*d/a^(1/2)*ln((2*a+2*a^(1/2
)*(b*x^4+a)^(1/2))/x^2)+c*(-(b*x^4+a)^(1/2)/a/x+I*b^(1/2)/a^(1/2)/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^
(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)
```

Fricas [F]
time = 0.23, size = 40, normalized size = 0.13

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\,\left(fx^3+ex^2+dx+c\right)}{bx^6+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^6 + a*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.87, size = 128, normalized size = 0.41

$$\frac{f\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,x\Gamma\left(\frac{3}{4}\right)}-\frac{d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{2\sqrt{a}}+\frac{ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(1/2),x)
[Out] f*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*gamma(-1/4)*hyper((-1/4, 1/2)
, (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - d*asinh(sqrt
(a)/(sqrt(b)*x**2))/(2*sqrt(a)) + e*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,),
b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^3 + x^2*e + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^2\,\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(1/2)),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(1/2)), x)
```

3.536 $\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=300

$$-\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a}{\left(\sqrt{a}\right.}}}{a^{3/4}\sqrt{}}$$

```
[Out] -1/2*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/2*c*(b*x^4+a)^(1/2)/a/x^2
-d*(b*x^4+a)^(1/2)/a/x+d*x*b^(1/2)*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-
b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x
/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)
+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(
1/2)+1/2*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/
a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(f*a^(1/2
)+d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2
)/a^(3/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1847, 1266, 821, 272, 65, 214, 1296, 1212, 226, 1210}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\sqrt{a}\,f+\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\,\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^3*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/2*(c*Sqrt[a + b*x^4])/(a*x^2) - (d*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*d*x
*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (e*ArcTanh[Sqrt[a + b*x^4]/
Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx &= \int \left(\frac{c+ex^2}{x^3\sqrt{a+bx^4}}+\frac{d+fx^2}{x^2\sqrt{a+bx^4}}\right)dx \\
&= \int \frac{c+ex^2}{x^3\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{x^2\sqrt{a+bx^4}}\,dx \\
&= -\frac{d\sqrt{a+bx^4}}{ax}+\frac{1}{2}\text{Subst}\left(\int \frac{c+ex}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{-af-bdx^2}{\sqrt{a+bx^4}}\,dx}{a} \\
&= -\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}-\frac{\left(\sqrt{b}\,d\right)\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{a}}+\frac{1}{2}e\text{Subst}\left(\int \frac{1}{x\sqrt{a+}}\right. \\
&= -\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{}{(}}}{} \\
&= -\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{}{(}}}{} \\
&= -\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.24, size = 242, normalized size = 0.81

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((c+2dx)\left(a+bx^4\right)+\sqrt{a}\,ex^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+2\sqrt{a}\sqrt{b}\,dx^2\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-2i\sqrt{a}\left(-i\sqrt{b}\,d+\sqrt{a}\,f\right)x^2\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*Sqrt[a + b*x^4]),x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((c + 2*d*x)*(a + b*x^4) + Sqrt[a]*e*x^2*Sqrt[
a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 2*Sqrt[a]*Sqrt[b]*d*x^2*Sqr
t[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - (2
*I)*Sqrt[a]*((-I)*Sqrt[b]*d + Sqrt[a]*f)*x^2*Sqrt[1 + (b*x^4)/a]*EllipticF[
```

```
I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(2*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]
*x^2*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 235, normalized size = 0.78

method	result
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{2a\,x^{2}}-\frac{d\sqrt{b\,x^{4}+a}}{ax}+\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{i\sqrt{b}\,d\sqrt{1}}{}$
default	$\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{c\sqrt{b\,x^{4}+a}}{2a\,x^{2}}-\frac{e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{2\sqrt{a}}+$
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(2dx+c)}{2a\,x^{2}}+\frac{i\sqrt{b}\,d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{i\sqrt{b}\,d\sqrt{1-\frac{i\sqrt{}}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1
/2*c*(b*x^4+a)^(1/2)/a/x^2-1/2*e/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))
/x^2)+d*(-(b*x^4+a)^(1/2)/a/x+I*b^(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2
))^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)
```

Fricas [A]
time = 0.12, size = 139, normalized size = 0.46

$$-\frac{4\sqrt{a}\,bdx^{2}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1)-\sqrt{a}\,bex^{2}\log\left(-\frac{bx^{4}-2\sqrt{bx^{4}+a}\,\sqrt{a}+2a}{x^{4}}\right)-4\,(bd-af)\sqrt{a}\,x^{2}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1)+2\sqrt{bx^{4}+a}\,(2\,bdx+bc)}{4\,abx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(a)*b*d*x^2*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1)
 - sqrt(a)*b*e*x^2*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) - 4*
(b*d - a*f)*sqrt(a)*x^2*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1)
 + 2*sqrt(b*x^4 + a)*(2*b*d*x + b*c))/(a*b*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.79, size = 126, normalized size = 0.42

$$-\frac{\sqrt{b}\,c\sqrt{\frac{a}{bx^4}+1}}{2a}+\frac{d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,x\Gamma\left(\frac{3}{4}\right)}-\frac{e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{2\sqrt{a}}+\frac{fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(2*a) + d*gamma(-1/4)*hyper((-1/4, 1/2), (3
/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - e*asinh(sqrt(a)/
(sqrt(b)*x**2))/(2*sqrt(a)) + f*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)
```

Mupad [B]
time = 5.85, size = 118, normalized size = 0.39

$$\frac{f\,x\,\sqrt{\frac{b\,x^4}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\;\frac{5}{4};\;-\frac{b\,x^4}{a}\right)}{\sqrt{b\,x^4+a}}-\frac{c\,\sqrt{b\,x^4+a}}{2\,a\,x^2}-\frac{d\,\sqrt{\frac{a}{b\,x^4}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\;\frac{7}{4};\;-\frac{a}{b\,x^4}\right)}{3\,x\,\sqrt{b\,x^4+a}}-\frac{e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(1/2)),x)
```

```
[Out] (f*x*((b*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(b*x^4)/a))/(a + b*x
^4)^(1/2) - (c*(a + b*x^4)^(1/2))/(2*a*x^2) - (d*(a/(b*x^4) + 1)^(1/2)*hype
rgeom([1/2, 3/4], 7/4, -a/(b*x^4)))/(3*x*(a + b*x^4)^(1/2)) - (e*atanh((a +
 b*x^4)^(1/2)/a^(1/2)))/(2*a^(1/2))
```

3.537 $\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=323

$$-\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt[4]{b}\,e\left(\sqrt{a}+\sqrt{b}\right.}{}$$

```
[Out] -1/2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/3*c*(b*x^4+a)^(1/2)/a/x^3
-1/2*d*(b*x^4+a)^(1/2)/a/x^2-e*(b*x^4+a)^(1/2)/a/x+e*x*b^(1/2)*(b*x^4+a)^(1
/2)/a/(a^(1/2)+x^2*b^(1/2))-b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^
2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)-1/6*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4
)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(-3*e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*(
(b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1847, 1296, 1212, 226, 1210, 1266, 821, 272, 65, 214}

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\sqrt{b}\,c-3\sqrt{a}\,e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^4*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/3*(c*Sqrt[a + b*x^4])/(a*x^3) - (d*Sqrt[a + b*x^4])/(2*a*x^2) - (e*Sqrt[
a + b*x^4])/(a*x) + (Sqrt[b]*e*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2
)) - (f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*e*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[
b]*c - 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt
[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx &= \int \left(\frac{c+ex^2}{x^4\sqrt{a+bx^4}}+\frac{d+fx^2}{x^3\sqrt{a+bx^4}}\right)dx \\
&= \int \frac{c+ex^2}{x^4\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{x^3\sqrt{a+bx^4}}\,dx \\
&= -\frac{c\sqrt{a+bx^4}}{3ax^3}+\frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{-3ae+bcx^2}{x^2\sqrt{a+bx^4}}\,dx}{3a} \\
&= -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\int \frac{-abc+3abex^2}{\sqrt{a+bx^4}}\,dx}{3a^2}+\frac{1}{2}f\text{Subst}\left(\int \cdots\right. \\
&= -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}-\frac{\left(\sqrt{b}\,e\right)\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{a}}-\frac{\left(\sqrt{b}\cdots\right.}{\cdots} \\
&= -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{\sqrt[4]{b}\,e\left(\sqrt{a}\cdots\right.}{\cdots} \\
&= -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{f\tanh^{-1}\left(\cdots\right.}{2\sqrt{\cdots}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.28, size = 249, normalized size = 0.77

$$
\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((a+bx^4)(2c+3x(d+2ex))+3\sqrt{a}\,fx^3\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+6\sqrt{a}\,\sqrt{b}\,ex^3\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-2\sqrt{b}\left(-i\sqrt{b}\,c+3\sqrt{a}\,e\right)x^3\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{6a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^3\sqrt{a+bx^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^4*Sqrt[a + b*x^4]),x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(2*c + 3*x*(d + 2*e*x)) + 3*Sqrt[
a]*f*x^3*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 6*Sqrt[a]*Sqr
t[b]*e*x^3*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]
]*x], -1] - 2*Sqrt[b]*((-I)*Sqrt[b]*c + 3*Sqrt[a]*e)*x^3*Sqrt[1 + (b*x^4)/a
]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(6*a*Sqrt[(I*Sqrt[
b])/Sqrt[a]]*x^3*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 259, normalized size = 0.80

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{d\sqrt{b\,x^4+a}}{2a\,x^2}-\frac{e\sqrt{b\,x^4+a}}{ax}-\frac{bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\right.}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$-\frac{d\sqrt{b\,x^4+a}}{2a\,x^2}+c\left(-\frac{\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)-\frac{f\ln\left(\right.}{}$
risch	$-\frac{\sqrt{b\,x^4+a}\,(6e\,x^2+3dx+2c)}{6a\,x^3}+\frac{i\sqrt{b}\,e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{i\sqrt{b}\,e\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*(b*x^4+a)^(1/2)/a/x^2+c*(-1/3*(b*x^4+a)^(1/2)/a/x^3-1/3*b/a/(I/a^(1/
2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)
^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*f/a^(1
/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+e*(-(b*x^4+a)^(1/2)/a/x+I*b^(1/
2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^
(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)
```

Fricas [A]
time = 0.12, size = 136, normalized size = 0.42

$$-\frac{12\sqrt{a}\,ex^3\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1)-4\sqrt{a}\,(c+3e)x^3\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1)-3\sqrt{a}\,fx^3\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\,\sqrt{a}+2a}{x^4}\right)+2\sqrt{bx^4+a}\,(6ex^2+3dx+2c)}{12ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(a)*e*x^3*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1)
 - 4*sqrt(a)*(c + 3*e)*x^3*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)),
-1) - 3*sqrt(a)*f*x^3*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) +
 2*sqrt(b*x^4 + a)*(6*e*x^2 + 3*d*x + 2*c))/(a*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.92, size = 131, normalized size = 0.41

$$-\frac{\sqrt{b}\,d\sqrt{\frac{a}{bx^4}+1}}{2a}+\frac{c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,x^3\Gamma\left(\frac{1}{4}\right)}+\frac{e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,x\Gamma\left(\frac{3}{4}\right)}-\frac{f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(2*a) + c*gamma(-3/4)*hyper((-3/4, 1/2), (1
/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + e*gamma(-1/4)
*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/
4)) - f*asinh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^4\,\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(1/2)),x)
```

```
[Out] int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(1/2)), x)
```

3.538 $\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=346

$$-\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\cdots$$

```
[Out] 1/4*b*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/4*c*(b*x^4+a)^(1/2)/a/x^
4-1/3*d*(b*x^4+a)^(1/2)/a/x^3-1/2*e*(b*x^4+a)^(1/2)/a/x^2-f*(b*x^4+a)^(1/2)
/a/x+f*x*b^(1/2)*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-b^(1/4)*f*(cos(2*a
rctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellipti
cE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*
x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)-1/6*b^(1/4)*(
cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*
EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-3*f*a^(1/2)+d*b^(
1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/
4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1847, 1266, 849, 821, 272, 65, 214, 1296, 1212, 226, 1210}

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^5*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/4*(c*Sqrt[a + b*x^4])/(a*x^4) - (d*Sqrt[a + b*x^4])/(3*a*x^3) - (e*Sqrt[
a + b*x^4])/(2*a*x^2) - (f*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*f*x*Sqrt[a + b
*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])
/(4*a^(3/2)) - (b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*
Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]*d - 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx &= \int \left(\frac{c+ex^2}{x^5\sqrt{a+bx^4}}+\frac{d+fx^2}{x^4\sqrt{a+bx^4}}\right)dx \\
&= \int \frac{c+ex^2}{x^5\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{x^4\sqrt{a+bx^4}}\,dx \\
&= -\frac{d\sqrt{a+bx^4}}{3ax^3}+\frac{1}{2}\text{Subst}\left(\int \frac{c+ex}{x^3\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{-3af+bdx^2}{x^2\sqrt{a+bx^4}}\,dx}{3a} \\
&= -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\int \frac{-abd+3abfx^2}{\sqrt{a+bx^4}}\,dx}{3a^2}-\frac{\text{Subst}\left(\int \frac{-}{x^2\sqrt{}}\right.}{} \\
&= -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}-\frac{(bc)\text{Subst}\left(\int \frac{}{x\sqrt{a}}\right.}{4} \\
&= -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)} \\
&= -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)} \\
&= -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}\,fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.34, size = 259, normalized size = 0.75

$$
\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\sqrt{a}\,(a+bx^4)\,(3c+4dx+6x^2(e+2fx))+3bcx^4\sqrt{a+bx^4}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+12a\sqrt{b}\,fx^4\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-4\sqrt{a}\,\sqrt{b}\left(-i\sqrt{b}\,d+3\sqrt{a}\,f\right)x^4\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{12a^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^4\sqrt{a+bx^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^5*Sqrt[a + b*x^4]),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-(Sqrt[a]*(a + b*x^4)*(3*c + 4*d*x + 6*x^2*(e +
 2*f*x))) + 3*b*c*x^4*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) + 1
```

```
2*a*Sqrt[b]*f*x^4*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/
Sqrt[a]]*x], -1] - 4*Sqrt[a]*Sqrt[b]*((-I)*Sqrt[b]*d + 3*Sqrt[a]*f)*x^4*Sqr
t[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(12
*a^(3/2)*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^4*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 279, normalized size = 0.81

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{4a\,x^4}-\frac{d\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{e\sqrt{b\,x^4+a}}{2a\,x^2}-\frac{f\sqrt{b\,x^4+a}}{ax}-\frac{bd\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{Elli}}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+}}$
default	$c\left(-\frac{\sqrt{b\,x^4+a}}{4a\,x^4}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{4a^{\frac{3}{2}}}\right)-\frac{e\sqrt{b\,x^4+a}}{2a\,x^2}+d\left(-\frac{\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{b\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}}{}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,(12f\,x^3+6e\,x^2+4dx+3c)}{12a\,x^4}+\frac{i\sqrt{b}\,f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{i\surd}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/4*(b*x^4+a)^(1/2)/a/x^4+1/4*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/
2))/x^2))-1/2*e*(b*x^4+a)^(1/2)/a/x^2+d*(-1/3*(b*x^4+a)^(1/2)/a/x^3-1/3*b/a
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+f
*(-(b*x^4+a)^(1/2)/a/x+I*b^(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(
1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(El
lipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*c*(2*sqrt(b*x^4 + a)*b/((b*x^4 + a)*a - a^2) + b*log((sqrt(b*x^4 + a)
 - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^(3/2)) + integrate((f*x^2 + x*e +
 d)/(sqrt(b*x^4 + a)*x^4), x)
```

Fricas [A]
time = 0.11, size = 150, normalized size = 0.43

$$-\frac{24\,a^{\frac{3}{2}}fx^4\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-3\,\sqrt{a}\,bcx^4\log\left(-\frac{bx^4+2\,\sqrt{bx^4+a}\,\sqrt{a}+2\,a}{x^4}\right)-8\,(ad+3\,af)\sqrt{a}\,x^4\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)+2\,(12\,afx^3+6\,aex^2+4\,adx+3\,ac)\sqrt{bx^4+a}}{24\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(24*a^(3/2)*f*x^4*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1)
 - 3*sqrt(a)*b*c*x^4*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) -
8*(a*d + 3*a*f)*sqrt(a)*x^4*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)),
 -1) + 2*(12*a*f*x^3 + 6*a*e*x^2 + 4*a*d*x + 3*a*c)*sqrt(b*x^4 + a))/(a^2*x
^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.67, size = 158, normalized size = 0.46

$$-\frac{\sqrt{b}\,c\sqrt{\frac{a}{bx^4}+1}}{4ax^2}-\frac{\sqrt{b}\,e\sqrt{\frac{a}{bx^4}+1}}{2a}+\frac{d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,x^3\Gamma\left(\frac{1}{4}\right)}+\frac{f\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,x\Gamma\left(\frac{3}{4}\right)}+\frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**5/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*e*sqrt(a/(b*x**4) + 1)
/(2*a) + d*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)
/(4*sqrt(a)*x**3*gamma(1/4)) + f*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x
**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) + b*c*asinh(sqrt(a)/(sqrt(b
)*x**2))/(4*a**(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/(sqrt(b*x^4 + a)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^5\,\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^5*(a + b*x^4)^(1/2)),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(x^5*(a + b*x^4)^(1/2)), x)
```

3.539 $\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=377

$$-\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}-\frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{bd\tanh^{-1}\left(\right.}{4a}$$

```
[Out] 1/4*b*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/5*c*(b*x^4+a)^(1/2)/a/x^
5-1/4*d*(b*x^4+a)^(1/2)/a/x^4-1/3*e*(b*x^4+a)^(1/2)/a/x^3-1/2*f*(b*x^4+a)^(
1/2)/a/x^2+3/5*b*c*(b*x^4+a)^(1/2)/a^2/x-3/5*b^(3/2)*c*x*(b*x^4+a)^(1/2)/a^
2/(a^(1/2)+x^2*b^(1/2))+3/5*b^(5/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^
2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)-1/30*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/
4)*x/a^(1/4))),1/2*2^(1/2))*(5*e*a^(1/2)+9*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))
*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1847, 1296, 1212, 226, 1210, 1266, 849, 821, 272, 65, 214}

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(5\sqrt{a}\,e+9\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}}+\frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}-\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^6*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/5*(c*Sqrt[a + b*x^4])/(a*x^5) - (d*Sqrt[a + b*x^4])/(4*a*x^4) - (e*Sqrt[
a + b*x^4])/(3*a*x^3) - (f*Sqrt[a + b*x^4])/(2*a*x^2) + (3*b*c*Sqrt[a + b*x
^4])/(5*a^2*x) - (3*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*a^2*(Sqrt[a] + Sqrt[b]*
x^2)) + (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) + (3*b^(5/4)*c*(
Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
E[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a + b*x^4]) - (b^(3/
4)*(9*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sq
rt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*
a^(7/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx &= \int \left(\frac{c+ex^2}{x^6\sqrt{a+bx^4}}+\frac{d+fx^2}{x^5\sqrt{a+bx^4}}\right)dx \\
&= \int \frac{c+ex^2}{x^6\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{x^5\sqrt{a+bx^4}}\,dx \\
&= -\frac{c\sqrt{a+bx^4}}{5ax^5}+\frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{x^3\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{-5ae+3bcx^2}{x^4\sqrt{a+bx^4}}\,dx}{5a} \\
&= -\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}+\frac{\int \frac{-9abc-5abex^2}{x^2\sqrt{a+bx^4}}\,dx}{15a^2}-\frac{\text{Subst}\left(\int \cdots\right.}{\cdots} \\
&= -\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}-\cdots \\
&= -\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}+\cdots \\
&= -\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}-\cdots \\
&= -\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}-\cdots
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.34, size = 268, normalized size = 0.71

$$
\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\left(\left(a+bx^4\right)\left(12ac-36bcx^4+5ax\left(3d+4ex+6fx^2\right)\right)\right)+15\sqrt{a}\,bdx^5\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)-36\sqrt{a}\,b^{3/2}cx^5\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)+4\sqrt{a}\,b\left(9\sqrt{b}\,c+5i\sqrt{a}\,e\right)x^5\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{60a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^5\sqrt{a+bx^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^6*Sqrt[a + b*x^4]),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-((a + b*x^4)*(12*a*c - 36*b*c*x^4 + 5*a*x*(3*d
 + 4*e*x + 6*f*x^2))) + 15*Sqrt[a]*b*d*x^5*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a +
 b*x^4]/Sqrt[a]]) - 36*Sqrt[a]*b^(3/2)*c*x^5*Sqrt[1 + (b*x^4)/a]*EllipticE[
I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] + 4*Sqrt[a]*b*(9*Sqrt[b]*c + (5
```

*I)*Sqrt[a]*e)*x^5*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])
/Sqrt[a]]*x], -1])/(60*a^2*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^5*Sqrt[a + b*x^4])

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 297, normalized size = 0.79

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{5a\,x^5}-\frac{d\sqrt{b\,x^4+a}}{4a\,x^4}-\frac{e\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{f\sqrt{b\,x^4+a}}{2a\,x^2}+\frac{3bc\sqrt{b\,x^4+a}}{5a^2x}-\frac{be\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{\cdots}}{3a\sqrt{\cdots}}$ …
default	$c\left(-\frac{\sqrt{b\,x^4+a}}{5a\,x^5}+\frac{3b\sqrt{b\,x^4+a}}{5a^2x}-\frac{3ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\cdots\right.}{5a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$ …
risch	$-\frac{\sqrt{b\,x^4+a}\,\left(-36bc\,x^4+30af\,x^3+20ae\,x^2+15adx+12ac\right)}{60a^2x^5}-\frac{3ib^{\frac{3}{2}}c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\cdots}\right.}{5a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/5*(b*x^4+a)^(1/2)/a/x^5+3/5*b*(b*x^4+a)^(1/2)/a^2/x-3/5*I*b^(3/2)/a^(
3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+d*(-1/4*(b*x^4+a)^(1/2)/a/x^4
+1/4*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))-1/2*f*(b*x^4+a)^(1/
2)/a/x^2+e*(-1/3*(b*x^4+a)^(1/2)/a/x^3-1/3*b/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1
-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)
```

Fricas [A]
time = 0.13, size = 159, normalized size = 0.42

$$\frac{72\,\sqrt{a}\,bcx^5\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)+15\,\sqrt{a}\,bdx^5\log\left(-\frac{bx^4+2\,\sqrt{bx^4+a}\,\sqrt{a}+2\,a}{x^4}\right)-8\,(9\,bc-5\,ae)\sqrt{a}\,x^5\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)+2\,(36\,bcx^4-30\,afx^3-20\,aex^2-15\,adx-12\,ac)\sqrt{bx^4+a}}{120\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/120*(72*sqrt(a)*b*c*x^5*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -
1) + 15*sqrt(a)*b*d*x^5*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4)
 - 8*(9*b*c - 5*a*e)*sqrt(a)*x^5*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1
/4)), -1) + 2*(36*b*c*x^4 - 30*a*f*x^3 - 20*a*e*x^2 - 15*a*d*x - 12*a*c)*sq
rt(b*x^4 + a))/(a^2*x^5)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.73, size = 163, normalized size = 0.43

$$-\frac{\sqrt{b}\,d\sqrt{\frac{a}{bx^4}+1}}{4ax^2}-\frac{\sqrt{b}\,f\sqrt{\frac{a}{bx^4}+1}}{2a}+\frac{c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,x^5\Gamma\left(-\frac{1}{4}\right)}+\frac{e\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,x^3\Gamma\left(\frac{1}{4}\right)}+\frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**6/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*f*sqrt(a/(b*x**4) + 1)
/(2*a) + c*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*sqrt(a)*x**5*gamma(-1/4)) + e*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b
*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + b*d*asinh(sqrt(a)/(s
qrt(b)*x**2))/(4*a**(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^6\,\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^6*(a + b*x^4)^(1/2)),x)
```

```
[Out] int((c + d*x + e*x^2 + f*x^3)/(x^6*(a + b*x^4)^(1/2)), x)
```

3.540 $\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=365

$$\frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}}+\frac{d\sqrt{a+bx^4}}{b^2}+\frac{ex\sqrt{a+bx^4}}{3b^2}+\frac{fx^2\sqrt{a+bx^4}}{4b^2}+\frac{3cx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{3af\tanh^{-1}}{}$$

```
[Out] -3/4*a*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(5/2)+1/2*x*(-b*d*x^3-b*c*x
^2+a*f*x+a*e)/b^2/(b*x^4+a)^(1/2)+d*(b*x^4+a)^(1/2)/b^2+1/3*e*x*(b*x^4+a)^(
1/2)/b^2+1/4*f*x^2*(b*x^4+a)^(1/2)/b^2+3/2*c*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(
1/2)+x^2*b^(1/2))-3/2*a^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4)))
,1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/
2)/b^(7/4)/(b*x^4+a)^(1/2)+1/12*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2
)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a
^(1/4))),1/2*2^(1/2))*(-5*e*a^(1/2)+9*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*
x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$,
Rules used = {1842, 1899, 1902, 1212, 226, 1210, 1833, 1829, 655, 223, 212}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(9\sqrt{b}\,c-5\sqrt{a}\,e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{a}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}+\frac{3cx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{3af\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{5/2}}+\frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}}+\frac{d\sqrt{a+bx^4}}{b^2}+\frac{ex\sqrt{a+bx^4}}{3b^2}+\frac{fx^2\sqrt{a+bx^4}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (d*Sqrt[a +
 b*x^4])/b^2 + (e*x*Sqrt[a + b*x^4])/(3*b^2) + (f*x^2*Sqrt[a + b*x^4])/(4*b
^2) + (3*c*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) - (3*a*f*
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(5/2)) - (3*a^(1/4)*c*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*S
qrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)
*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
```

```
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}} - \frac{\int \frac{a^2be+2a^2bfx-3ab^2cx^2-4ab^2dx^3-2ab^2ex^4-2ab^2fx^5}{\sqrt{a+bx^4}}\,d\cdots}{2ab^3}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{a^2be-3ab^2cx^2-2ab^2ex^4}{\sqrt{a+bx^4}} + \frac{x(2a^2bf-4ab^2dx^2-2ab^2\cdots}{\sqrt{a+bx^4}}\cdots\right.}{2ab^3}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}} - \frac{\int \frac{a^2be-3ab^2cx^2-2ab^2ex^4}{\sqrt{a+bx^4}}\,dx}{2ab^3} - \frac{\int \frac{x(2a^2bf-4ab^2dx^2-\cdots}{\sqrt{a+bx^4}\cdots}}{2ab^3}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}} + \frac{ex\sqrt{a+bx^4}}{3b^2} - \frac{\int \frac{5a^2b^2e-9ab^3cx^2}{\sqrt{a+bx^4}}\,dx}{6ab^4} - \frac{\mathrm{Subst}(\cdots}{\cdots}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}} + \frac{ex\sqrt{a+bx^4}}{3b^2} + \frac{fx^2\sqrt{a+bx^4}}{4b^2} - \frac{\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{b^2} + \frac{ex\sqrt{a+bx^4}}{3b^2} + \frac{fx^2\sqrt{a+bx\cdots}}{4b^2}\cdots$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{b^2} + \frac{ex\sqrt{a+bx^4}}{3b^2} + \frac{fx^2\sqrt{a+bx\cdots}}{4b^2}\cdots$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{b^2} + \frac{ex\sqrt{a+bx^4}}{3b^2} + \frac{fx^2\sqrt{a+bx\cdots}}{4b^2}\cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.13, size = 220, normalized size = 0.60

$$\frac{12a\sqrt{b}\,d+10a\sqrt{b}\,ex+9a\sqrt{b}\,fx^2+12b^{3/2}cx^3+6b^{3/2}dx^4+4b^{3/2}ex^5+3b^{3/2}fx^6-9a^{3/2}f\sqrt{1+\frac{bx^4}{a}}\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)-10a\sqrt{b}\,ex\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)-12b^{3/2}cx^3\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)}{12b^{5/2}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (12*a*Sqrt[b]*d + 10*a*Sqrt[b]*e*x + 9*a*Sqrt[b]*f*x^2 + 12*b^(3/2)*c*x^3 +
 6*b^(3/2)*d*x^4 + 4*b^(3/2)*e*x^5 + 3*b^(3/2)*f*x^6 - 9*a^(3/2)*f*Sqrt[1 +
 (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - 10*a*Sqrt[b]*e*x*Sqrt[1 + (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*b^(3/2)*c*x^3*Sq
rt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(12*b^(5/
2)*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 320, normalized size = 0.88

method	result
elliptic	$-\frac{2b\left(\frac{c x^3}{4b^2}-\frac{a f x^2}{4b^3}-\frac{a e x}{4b^3}-\frac{a d}{4b^3}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right) b}}+\frac{f x^2\sqrt{b x^4+a}}{4b^2}+\frac{e x\sqrt{b x^4+a}}{3b^2}+\frac{d\sqrt{b x^4+a}}{2b^2}-\frac{5ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\cdots}}{6b^2\sqrt{\frac{i\sqrt{\cdots}}{\sqrt{\cdots}}}}$
default	$f\left(\frac{x^6}{4b\sqrt{b x^4+a}}+\frac{3a x^2}{4b^2\sqrt{b x^4+a}}-\frac{3a\ln\left(x^2\sqrt{b}+\sqrt{b x^4+a}\right)}{4b^{\frac{5}{2}}}\right)+e\left(\frac{a x}{2b^2\sqrt{\left(x^4+\frac{a}{b}\right) b}}+\frac{x\sqrt{b x^4+\cdots}}{3b^2}\cdots\right.$
risch	$\frac{\left(3f x^2+4ex+6d\right)\sqrt{b x^4+a}}{12b^2}-\frac{c x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right) b}}+\frac{3ic\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\cdots\right.}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b x^4+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/4*x^6/b/(b*x^4+a)^(1/2)+3/4*a/b^2*x^2/(b*x^4+a)^(1/2)-3/4*a/b^(5/2)*ln
(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+e*(1/2/b^2*a*x/((x^4+a/b)*b)^(1/2)+1/3*x*(b*
x^4+a)^(1/2)/b^2-5/6*a/b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*b^(1/2))^(1/2),I))+1/2*d*(b*x^4+2*a)/(b*x^4+a)^(1/2)/b^2+c*(-1/2/b*x^
3/((x^4+a/b)*b)^(1/2)+3/2*I/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/
a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*
(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)
```

Fricas [A]
time = 0.12, size = 242, normalized size = 0.66

$$\frac{36\,(b^2cx^5+abcx)\sqrt{b}\,\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-4\,((9\,b^2c+5\,b^2e)x^5+(9\,abc+5\,abe)x)\sqrt{b}\,\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+9\,(abfx^5+a^2fx)\sqrt{b}\,\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\,\sqrt{b}\,x^2-a\right)+2\,(3\,b^2fx^7+4\,b^2ex^6+6\,b^2dx^5+12\,b^2cx^4+9\,abfx^3+10\,abex^2+12\,abdx+18\,abc)\sqrt{bx^4+a}}{24\,(b^4x^5+ab^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(36*(b^2*c*x^5 + a*b*c*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b
)^(1/4)/x), -1) - 4*((9*b^2*c + 5*b^2*e)*x^5 + (9*a*b*c + 5*a*b*e)*x)*sqrt(
b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 9*(a*b*f*x^5 + a^2
*f*x)*sqrt(b)*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(3*b^2*
f*x^7 + 4*b^2*e*x^6 + 6*b^2*d*x^5 + 12*b^2*c*x^4 + 9*a*b*f*x^3 + 10*a*b*e*x
^2 + 12*a*b*d*x + 18*a*b*c)*sqrt(b*x^4 + a))/(b^4*x^5 + a*b^3*x)
```

Sympy [A]
time = 12.62, size = 202, normalized size = 0.55

$$d\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}}+\frac{x^4}{2b\sqrt{a+bx^4}} & \text{for}\ b\neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+f\left(\frac{3\sqrt{a}\,x^2}{4b^2\sqrt{1+\frac{bx^4}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{x^6}{4\sqrt{a}\,b\sqrt{1+\frac{bx^4}{a}}}\right)+\frac{cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}+\frac{ex^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] d*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x**4)), Ne(b,
 0)), (x**8/(8*a**(3/2)), True)) + f*(3*sqrt(a)*x**2/(4*b**2*sqrt(1 + b*x**
4/a)) - 3*a*asinh(sqrt(b)*x**2/sqrt(a))/(4*b**(5/2)) + x**6/(4*sqrt(a)*b*sq
rt(1 + b*x**4/a))) + c*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4)) + e*x**9*gamma(9/4)*hyper((3/2, 9
/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{{\left(b\,x^4+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
[Out] int((x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.541 $\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} \, dx$

Optimal. Leaf size=343

$$\frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}}+\frac{e\sqrt{a+bx^4}}{b^2}+\frac{fx\sqrt{a+bx^4}}{3b^2}+\frac{3dx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}-$$

```
[Out] 1/2*c*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/2*x*(-b*e*x^3-b*d*x^2-
b*c*x+a*f)/b^2/(b*x^4+a)^(1/2)+e*(b*x^4+a)^(1/2)/b^2+1/3*f*x*(b*x^4+a)^(1/2
)/b^2+3/2*d*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))-3/2*a^(1/4)*d*(
cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*
EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2
))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)+1/12*a
^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^
(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-5*f*a^(1/
2)+9*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(
1/2)/b^(9/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1842, 1899, 1262, 655, 223, 212, 1902, 1212, 226, 1210}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(9\sqrt{b}\,d-5\sqrt{a}\,f\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{a}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}}+\frac{e\sqrt{a+bx^4}}{b^2}+\frac{fx\sqrt{a+bx^4}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (e*Sqrt[a +
 b*x^4])/b^2 + (f*x*Sqrt[a + b*x^4])/(3*b^2) + (3*d*x*Sqrt[a + b*x^4])/(2*b
^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]
)/(2*b^(3/2)) - (3*a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(
7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*Sqrt[b]*d - 5*Sqrt[a]*f)*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
```

```
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} - \frac{\int \frac{a^2f-2abcx-3abdx^2-4abex^3-2abfx^4}{\sqrt{a+bx^4}}\,dx}{2ab^2}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{x(-2abc-4abex^2)}{\sqrt{a+bx^4}} + \frac{a^2f-3abdx^2-2abfx^4}{\sqrt{a+bx^4}}\right)dx}{2ab^2}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} - \frac{\int \frac{x(-2abc-4abex^2)}{\sqrt{a+bx^4}}\,dx}{2ab^2} - \frac{\int \frac{a^2f-3abdx^2-2abfx^4}{\sqrt{a+bx^4}}\,dx}{2ab^2}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} + \frac{fx\sqrt{a+bx^4}}{3b^2} - \frac{\int \frac{5a^2bf-9ab^2dx^2}{\sqrt{a+bx^4}}\,dx}{6ab^3} - \frac{\text{Subst}\Big(}{}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{b^2} + \frac{fx\sqrt{a+bx^4}}{3b^2} + \frac{c\text{Subst}\Big(\int \frac{}{\sqrt{a}}}{}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{b^2} + \frac{fx\sqrt{a+bx^4}}{3b^2} + \frac{3dx\sqrt{a+}}{2b^{3/2}\left(\sqrt{a}+\right.}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{b^2} + \frac{fx\sqrt{a+bx^4}}{3b^2} + \frac{3dx\sqrt{a+}}{2b^{3/2}\left(\sqrt{a}+\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 176, normalized size = 0.51

$$\frac{6ae+5afx-3bcx^2+6bdx^3+3bex^4+2bfx^5+3\sqrt{a}\sqrt{b}\,c\sqrt{1+\frac{bx^4}{a}}\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)-5afx\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)-6bdx^3\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)}{6b^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (6*a*e + 5*a*f*x - 3*b*c*x^2 + 6*b*d*x^3 + 3*b*e*x^4 + 2*b*f*x^5 + 3*Sqrt[a
]*Sqrt[b]*c*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - 5*a*f*x*Sq
rt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 6*b*d*x^
3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*b^
2*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 301, normalized size = 0.88

method	result
elliptic	$-\frac{2b\left(\frac{d\,x^3}{4b^2}+\frac{x^2c}{4b^2}-\frac{afx}{4b^3}-\frac{ae}{4b^3}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{fx\sqrt{b\,x^4+a}}{3b^2}+\frac{e\sqrt{b\,x^4+a}}{2b^2}-\frac{5af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\Big(}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$f\left(\frac{ax}{2b^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{x\sqrt{b\,x^4+a}}{3b^2}-\frac{5a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{}{2}$
risch	$\frac{(2fx+3e)\sqrt{b\,x^4+a}}{6b^2}-\frac{d\,x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3id\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/2/b^2*a*x/((x^4+a/b)*b)^(1/2)+1/3*x*(b*x^4+a)^(1/2)/b^2-5/6*a/b^2/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*
x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*e*
(b*x^4+2*a)/(b*x^4+a)^(1/2)/b^2+d*(-1/2/b*x^3/((x^4+a/b)*b)^(1/2)+3/2*I/b^(
3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+c*(-1/2*x^2/b/(b*x^4+
a)^(1/2)+1/2/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*c*(2*x^2/(sqrt(b*x^4 + a)*b) + log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(s
qrt(b) + sqrt(b*x^4 + a)/x^2))/b^(3/2)) + integrate((f*x^8 + x^7*e + d*x^6)
/(b*x^4 + a)^(3/2), x)
```

Fricas [A]
time = 0.12, size = 211, normalized size = 0.62

$$\frac{18\,(bdx^5+adx)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)-2\,((9\,bd+5\,bf)x^5+(9\,ad+5\,af)x)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+3\,(bcx^5+acx)\sqrt{b}\,\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\,\sqrt{b}\,x^2-a\right)+2\,(2\,bfx^6+3\,bex^5+6\,bdx^4-3\,bcx^3+5\,afx^2+6\,aex+9\,ad)\sqrt{bx^4+a}}{12\,(b^3x^5+ab^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(18*(b*d*x^5 + a*d*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1
/4)/x), -1) - 2*((9*b*d + 5*b*f)*x^5 + (9*a*d + 5*a*f)*x)*sqrt(b)*(-a/b)^(3
/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 3*(b*c*x^5 + a*c*x)*sqrt(b)*lo
g(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(2*b*f*x^6 + 3*b*e*x^5
+ 6*b*d*x^4 - 3*b*c*x^3 + 5*a*f*x^2 + 6*a*e*x + 9*a*d)*sqrt(b*x^4 + a))/(b^
3*x^5 + a*b^2*x)
```

Sympy [A]
time = 10.82, size = 172, normalized size = 0.50

$$c\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^{2}}{2\sqrt{a}\,b\sqrt{1+\frac{bx^{4}}{a}}}\right) + e\left(\begin{cases}\frac{a}{b^{2}\sqrt{a+bx^{4}}} + \frac{x^{4}}{2b\sqrt{a+bx^{4}}} & \text{for } b\neq 0\\ \frac{x^{8}}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + \frac{dx^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{fx^{9}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] c*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b*
x**4/a))) + e*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x
**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + d*x**7*gamma(7/4)*hyper((3/2
, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4)) + f*x**
9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
2)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x^5/(b*x^4 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{{\left(b\,x^4+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

```
[Out] int((x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.542 $\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=314

$$-\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}}+\frac{f\sqrt{a+bx^4}}{b^2}+\frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}-\frac{3\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{\right.}}{}$$

```
[Out] 1/2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)-1/2*x*(f*x^3+e*x^2+d*x+c
)/b/(b*x^4+a)^(1/2)+f*(b*x^4+a)^(1/2)/b^2+3/2*e*x*(b*x^4+a)^(1/2)/b^(3/2)/(
a^(1/2)+x^2*b^(1/2))-3/2*a^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4
))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^
(1/2)/b^(7/4)/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))*(3*e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a
^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1842, 1899, 1212, 226, 1210, 1262, 655, 223, 212}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(3\sqrt{a}\,e+\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\,b^{7/4}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{f\sqrt{a+bx^4}}{b^2}-\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/2*(x*(c + d*x + e*x^2 + f*x^3))/(b*Sqrt[a + b*x^4]) + (f*Sqrt[a + b*x^4]
)/b^2 + (3*e*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (d*Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) - (3*a^(1/4)*e*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + 3
*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(7/4)*Sqrt
[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

```
&& LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} - \frac{\int \frac{-abc-2abdx-3abex^2-4abfx^3}{\sqrt{a+bx^4}}\,dx}{2ab^2}$$

$$= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} - \frac{\int \left(\frac{-abc-3abex^2}{\sqrt{a+bx^4}} + \frac{x(-2abd-4abfx^2)}{\sqrt{a+bx^4}}\right)dx}{2ab^2}$$

$$= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} - \frac{\int \frac{-abc-3abex^2}{\sqrt{a+bx^4}}\,dx}{2ab^2} - \frac{\int \frac{x(-2abd-4abfx^2)}{\sqrt{a+bx^4}}\,dx}{2ab^2}$$

$$= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} - \frac{\text{Subst}\left(\int \frac{-2abd-4abfx}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{4ab^2} - \frac{(3\sqrt{a}\,e)\int \cdots}{2b\cdots}$$

$$= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{b^2} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{3\sqrt[4]{a}\,e\cdots}{\cdots}$$

$$= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{b^2} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{3\sqrt[4]{a}\,e\cdots}{\cdots}$$

$$= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{b^2} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{d\tanh\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.11, size = 166, normalized size = 0.53

$$\frac{2af - bcx - bdx^2 + 2bex^3 + bfx^4 + \sqrt{a}\,\sqrt{b}\,d\sqrt{1+\frac{bx^4}{a}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right) + bcx\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right) - 2bex^3\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)}{2b^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (2*a*f - b*c*x - b*d*x^2 + 2*b*e*x^3 + b*f*x^4 + Sqrt[a]*Sqrt[b]*d*Sqrt[1 +
 (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + b*c*x*Sqrt[1 + (b*x^4)/a]*Hype
rgeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 2*b*e*x^3*Sqrt[1 + (b*x^4)/a]*
Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(2*b^2*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 284, normalized size = 0.90

method	result
elliptic	$-\frac{2b\left(\frac{e\,x^3}{4b^2}+\frac{d\,x^2}{4b^2}+\frac{cx}{4b^2}-\frac{af}{4b^3}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}} + \frac{f\sqrt{b\,x^4+a}}{2b^2} + \frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}} + \frac{d\ln\left(\cdots\right.}{\cdots}$
default	$\frac{f\left(b\,x^4+2a\right)}{2\sqrt{b\,x^4+a}\,b^2} + e\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}} + \frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\cdots\right.}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\cdots\right.$
risch	$\frac{f\sqrt{b\,x^4+a}}{2b^2} - \frac{e\,x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}} + \frac{3ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}} - \frac{3ie\sqrt{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*(b*x^4+2*a)/(b*x^4+a)^(1/2)/b^2+e*(-1/2/b*x^3/((x^4+a/b)*b)^(1/2)+3/2
*I/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+d*(-1/2*x^2/b/(
b*x^4+a)^(1/2)+1/2/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+c*(-1/2/b*x/((x
^4+a/b)*b)^(1/2)+1/2/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^
(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)
```

Fricas [A]
time = 0.12, size = 223, normalized size = 0.71

$$\frac{6\,(abex^{5}+a^{2}ex)\sqrt{b}\,\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+2\,((b^{2}c-3\,abe)x^{5}+(abc-3\,a^{2}e)x)\sqrt{b}\,\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+(abdx^{5}+a^{2}dx)\sqrt{b}\,\log\left(-2\,bx^{4}-2\,\sqrt{bx^{4}+a}\,\sqrt{b}\,x^{2}-a\right)+2\,(abfx^{5}+2\,abex^{4}-abdx^{3}-abcx^{2}+2\,a^{2}fx+3\,a^{2}e)\sqrt{bx^{4}+a}}{4\,(ab^{3}x^{5}+a^{2}b^{2}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(6*(a*b*e*x^5 + a^2*e*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^
(1/4)/x), -1) + 2*((b^2*c - 3*a*b*e)*x^5 + (a*b*c - 3*a^2*e)*x)*sqrt(b)*(-a
/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + (a*b*d*x^5 + a^2*d*x)*sq
rt(b)*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(a*b*f*x^5 + 2*
a*b*e*x^4 - a*b*d*x^3 - a*b*c*x^2 + 2*a^2*f*x + 3*a^2*e)*sqrt(b*x^4 + a))/(
a*b^3*x^5 + a^2*b^2*x)
```

Sympy [A]
time = 8.95, size = 172, normalized size = 0.55

$$d\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{x^{2}}{2\sqrt{a}\,b\sqrt{1+\frac{bx^{4}}{a}}}\right)+f\left(\begin{cases}\frac{a}{b^{2}\sqrt{a+bx^{4}}}+\frac{x^{4}}{2b\sqrt{a+bx^{4}}} & \text{for } b\neq 0\\ \frac{x^{8}}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{ex^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] d*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b*
x**4/a))) + f*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x
**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + c*x**5*gamma(5/4)*hyper((5/4
, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**7*
gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)
*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{{\left(b\,x^4+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

```
[Out] int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.543 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=302

$$\frac{-c-dx-ex^2-fx^3}{2b\sqrt{a+bx^4}}+\frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}-\frac{3\sqrt[4]{a}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a\ -}{\left(\sqrt{a}\ +\right.}}}{2b^{7/4}\sqrt{}}$$

```
[Out] 1/2*e*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/2*(-f*x^3-e*x^2-d*x-c)
/b/(b*x^4+a)^(1/2)+3/2*f*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))-3/
2*a^(1/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)
*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/
2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)
^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*f*a^
(1/2)+d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^
(1/2)/a^(1/4)/b^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 297, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1837, 1899, 281, 223, 212, 1212, 226, 1210}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(3\sqrt{a}\,f+\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\,b^{7/4}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{a}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/2*(c + d*x + e*x^2 + f*x^3)/(b*Sqrt[a + b*x^4]) + (3*f*x*Sqrt[a + b*x^4]
)/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b
*x^4]])/(2*b^(3/2)) - (3*a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(2*b^(7/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx &= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{\int \frac{d+2ex+3fx^2}{\sqrt{a+bx^4}}\,dx}{2b}\\
&= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{\int \left(\frac{2ex}{\sqrt{a+bx^4}}+\frac{d+3fx^2}{\sqrt{a+bx^4}}\right)dx}{2b}\\
&= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{\int \frac{d+3fx^2}{\sqrt{a+bx^4}}\,dx}{2b}+\frac{e\int \frac{x}{\sqrt{a+bx^4}}\,dx}{b}\\
&= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{e\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{2b}-\frac{\left(3\sqrt{a}\,f\right)\int \frac{1-\cdots}{\sqrt{\cdots}}}{2b^{3/\cdots}}\\
&= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{3\sqrt[4]{a}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\cdots}{\cdots}\\
&= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}-\cdots
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.10, size = 181, normalized size = 0.60

$$
\frac{-\sqrt{b}\,c-\sqrt{b}\,dx-\sqrt{b}\,ex^2+2\sqrt{b}\,fx^3+\sqrt{a}\,e\sqrt{1+\frac{bx^4}{a}}\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)+\sqrt{b}\,dx\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)-2\sqrt{b}\,fx^3\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)}{2b^{3/2}\sqrt{a+bx^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[b]*c) - Sqrt[b]*d*x - Sqrt[b]*e*x^2 + 2*Sqrt[b]*f*x^3 + Sqrt[a]*e*S
qrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + Sqrt[b]*d*x*Sqrt[1 + (b
*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 2*Sqrt[b]*f*x^3*S
qrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(2*b^(3/
2)*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 275, normalized size = 0.91

method	result
elliptic	$-\frac{2b\left(\frac{f\,x^3}{4b^2}+\frac{e\,x^2}{4b^2}+\frac{dx}{4b^2}+\frac{c}{4b^2}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{e\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4 \cdots\right.}{2b^{\frac{3}{2}}}$
default	$f\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} \cdots\right.\right.}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(-1/2/b*x^3/((x^4+a/b)*b)^(1/2)+3/2*I/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)))+e*(-1/2*x^2/b/(b*x^4+a)^(1/2)+1/2/b^(3/2)*ln(x^2*b^(1
/2)+(b*x^4+a)^(1/2)))+d*(-1/2/b*x/((x^4+a/b)*b)^(1/2)+1/2/b/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/
(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*c/b/(b*x^4+a)
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*c/(sqrt(b*x^4 + a)*b) + integrate((f*x^6 + x^5*e + d*x^4)/(b*x^4 + a)^
(3/2), x)
```

Fricas [A]
time = 0.12, size = 215, normalized size = 0.71

$$\frac{6\,(abfx^5+a^2fx)\sqrt{b}\,\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+2\,((b^2d-3\,abf)x^5+(abd-3\,a^2f)x)\sqrt{b}\,\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+(abex^5+a^2ex)\sqrt{b}\,\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\,\sqrt{b}\,x^2-a\right)+2\,(2\,abfx^4-abex^3-abdx^2-abcx+3\,a^2f)\sqrt{bx^4+a}}{4\,(ab^3x^5+a^2b^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(6*(a*b*f*x^5 + a^2*f*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^
(1/4)/x), -1) + 2*((b^2*d - 3*a*b*f)*x^5 + (a*b*d - 3*a^2*f)*x)*sqrt(b)*(-a
```

```
/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + (a*b*e*x^5 + a^2*e*x)*sq
rt(b)*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(2*a*b*f*x^4 -
a*b*e*x^3 - a*b*d*x^2 - a*b*c*x + 3*a^2*f)*sqrt(b*x^4 + a))/(a*b^3*x^5 + a^
2*b^2*x)
```

Sympy [A]
time = 7.91, size = 156, normalized size = 0.52

$$c\left(\begin{cases}-\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+e\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{x^2}{2\sqrt{a}\,b\sqrt{1+\frac{bx^4}{a}}}\right)+\frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
[Out] c*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + e*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1
 + b*x**4/a))) + d*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + f*x**7*gamma(7/4)*hyper((3/2, 7/4), (
11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x^3/(b*x^4 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{{\left(b\,x^4+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
[Out] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.544 $\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=333

$$-\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}}-\frac{d\sqrt{a+bx^4}}{2ab}-\frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{c\left(\sqrt{a}+\sqrt{\ }\right.}{}$$

```
[Out] 1/2*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)-1/2*x*(-b*d*x^3-b*c*x^2+
a*f*x+a*e)/a/b/(b*x^4+a)^(1/2)-1/2*d*(b*x^4+a)^(1/2)/a/b-1/2*c*x*(b*x^4+a)^
(1/2)/a/b^(1/2)/(a^(1/2)+x^2*b^(1/2))+1/2*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)
*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(
1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)-1/4*(cos(2*arctan(b^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(
b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2
))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/b^(5/4)/(b*x^4+a)^(1/2
)
```

Rubi [A]
time = 0.18, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1842, 1899, 1212, 226, 1210, 1262, 655, 223, 212}

$$-\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\sqrt{b}\,c-\sqrt{a}\,e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}+\frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}-\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}}-\frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{d\sqrt{a+bx^4}}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/2*(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(a*b*Sqrt[a + b*x^4]) - (d*Sqrt[
a + b*x^4])/(2*a*b) - (c*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]
*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) + (c*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) - ((S
qrt[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(3/4)*b^
(5/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
```

```
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{\int \frac{-abe-2abfx+b^2cx^2+2b^2dx^3}{\sqrt{a+bx^4}}\,dx}{2ab^2}$$

$$= -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{\int \left(\frac{-abe+b^2cx^2}{\sqrt{a+bx^4}} + \frac{x(-2abf+2b^2dx^2)}{\sqrt{a+bx^4}}\right)dx}{2ab^2}$$

$$= -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{\int \frac{-abe+b^2cx^2}{\sqrt{a+bx^4}}\,dx}{2ab^2} - \frac{\int \frac{x(-2abf+2b^2dx^2)}{\sqrt{a+bx^4}}\,dx}{2ab^2}$$

$$= -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{\text{Subst}\left(\int \frac{-2abf+2b^2dx}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{4ab^2} + \frac{c\int \frac{1-\sqrt{}}{\sqrt{a+}}}{2\sqrt{a}}$$

$$= -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{d\sqrt{a+bx^4}}{2ab} - \frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{c(}{}$$

$$= -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{d\sqrt{a+bx^4}}{2ab} - \frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{c(}{}$$

$$= -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{d\sqrt{a+bx^4}}{2ab} - \frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{f}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.15, size = 165, normalized size = 0.50

$$\frac{-3a\sqrt{b}\,(d+x(e+fx))+3a^{3/2}f\sqrt{1+\frac{bx^4}{a}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)+3a\sqrt{b}\,ex\sqrt{1+\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+2b^{3/2}cx^3\sqrt{1+\frac{bx^4}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)}{6ab^{3/2}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (-3*a*Sqrt[b]*(d + x*(e + f*x)) + 3*a^(3/2)*f*Sqrt[1 + (b*x^4)/a]*ArcSinh[(
Sqrt[b]*x^2)/Sqrt[a]] + 3*a*Sqrt[b]*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b^(3/2)*c*x^3*Sqrt[1 + (b*x^4)/a]*Hyper
geometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b^(3/2)*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 275, normalized size = 0.83

method	result
elliptic	$-\frac{2b\left(-\frac{c\,x^3}{4ba}+\frac{f\,x^2}{4b^2}+\frac{ex}{4b^2}+\frac{d}{4b^2}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{f\ln\left(2x^2\sqrt{b}+2\sqrt{b}\right.}{2b^{\frac{3}{2}}}$
default	$f\left(-\frac{x^2}{2b\sqrt{b\,x^4+a}}+\frac{\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{2b^{\frac{3}{2}}}\right)+e\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{}}{}}}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(-1/2*x^2/b/(b*x^4+a)^(1/2)+1/2/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))+
e*(-1/2/b*x/((x^4+a/b)*b)^(1/2)+1/2/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellipt
icF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*d/b/(b*x^4+a)^(1/2)+c*(1/2/a*x^3/((
x^4+a/b)*b)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)
```

Fricas [A]
time = 0.11, size = 201, normalized size = 0.60

$$-\frac{2\left(b^{2}cx^{5}+abcx\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-2\left(\left(b^{2}c+b^{2}e\right)x^{5}+\left(abc+abe\right)x\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-\left(abfx^{5}+a^{2}fx\right)\sqrt{b}\log\left(-2\,bx^{4}-2\,\sqrt{bx^{4}+a}\,\sqrt{b}\,x^{2}-a\right)+2\left(abfx^{3}+abex^{2}+abdx+abc\right)\sqrt{bx^{4}+a}}{4\left(ab^{3}x^{5}+a^{2}b^{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2*c*x^5 + a*b*c*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)
^(1/4)/x), -1) - 2*((b^2*c + b^2*e)*x^5 + (a*b*c + a*b*e)*x)*sqrt(b)*(-a/b)
^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) - (a*b*f*x^5 + a^2*f*x)*sqrt(
b)*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(a*b*f*x^3 + a*b*e
*x^2 + a*b*d*x + a*b*c)*sqrt(b*x^4 + a))/(a*b^3*x^5 + a^2*b^2*x)
```

Sympy [A]
time = 7.32, size = 156, normalized size = 0.47

$$d\left(\begin{cases}-\frac{1}{2b\sqrt{a+bx^{4}}} & \text{for}\: b\neq 0\\ \frac{x^{4}}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+f\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{x^{2}}{2\sqrt{a}\,b\sqrt{1+\frac{bx^{4}}{a}}}\right)+\frac{cx^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{ex^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] d*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + f*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1
 + b*x**4/a))) + c*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + e*x**5*gamma(5/4)*hyper((5/4, 3/2), (
9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{{\left(b\,x^4+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
[Out] int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.545 $\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=303

$$-\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}}-\frac{e\sqrt{a+bx^4}}{2ab}-\frac{dx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}}{2a^{3/4}b^{3/4}\sqrt{a+b}}$$

```
[Out] -1/2*x*(-b*e*x^3-b*d*x^2-b*c*x+a*f)/a/b/(b*x^4+a)^(1/2)-1/2*e*(b*x^4+a)^(1/
2)/a/b-1/2*d*x*(b*x^4+a)^(1/2)/a/b^(1/2)/(a^(1/2)+x^2*b^(1/2))+1/2*d*(cos(2
*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellip
ticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((
b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)-1/4
*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4))
)*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-f*a^(1/2)+d*b^(
1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/
4)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1842, 1899, 267, 1212, 226, 1210}

$$-\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}+\frac{d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}-\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}}-\frac{dx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{e\sqrt{a+bx^4}}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/2*(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(a*b*Sqrt[a + b*x^4]) - (e*Sqrt[
a + b*x^4])/(2*a*b) - (d*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]
*x^2)) + (d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt
[a + b*x^4]) - ((Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1
/2])/(4*a^(3/4)*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = -\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}} - \frac{\int \frac{-af+bdx^2+2bex^3}{\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= -\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}} - \frac{\int \left(\frac{2bex^3}{\sqrt{a+bx^4}} + \frac{-af+bdx^2}{\sqrt{a+bx^4}}\right)dx}{2ab}$$

$$= -\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}} - \frac{\int \frac{-af+bdx^2}{\sqrt{a+bx^4}}\,dx}{2ab} - \frac{e\int \frac{x^3}{\sqrt{a+bx^4}}\,dx}{a}$$

$$= -\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}} - \frac{e\sqrt{a+bx^4}}{2ab} + \frac{d\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2\sqrt{a}\sqrt{b}} - \frac{\left(\frac{\sqrt{b}d}{\sqrt{a}} - \cdots\right.}{\cdots}$$

$$= -\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}} - \frac{e\sqrt{a+bx^4}}{2ab} - \frac{dx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{d\left(\cdots\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 116, normalized size = 0.38

$$\frac{-3ae-3afx+3bcx^2+3afx\sqrt{1+\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+2bdx^3\sqrt{1+\frac{bx^4}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)}{6ab\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (-3*a*e - 3*a*f*x + 3*b*c*x^2 + 3*a*f*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*d*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeomet
ric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 250, normalized size = 0.83

method	result

elliptic	$-\frac{2b\left(-\frac{d\,x^{3}}{4ab}-\frac{c\,x^{2}}{4ba}+\frac{fx}{4b^{2}}+\frac{e}{4b^{2}}\right)}{\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{id\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{}$
default	$f\left(-\frac{x}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)-\frac{e}{2b\sqrt{b\,x^{4}+a}}+d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(-1/2/b*x/((x^4+a/b)*b)^(1/2)+1/2/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellipt
icF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*e/b/(b*x^4+a)^(1/2)+d*(1/2/a*x^3/((
x^4+a/b)*b)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)))+1/2*c*x^2/a/(b*x^4+a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2/(sqrt(b*x^4 + a)*a) + integrate((f*x^4 + x^3*e + d*x^2)/(b*x^4 +
a)^(3/2), x)
```

Fricas [A]
time = 0.09, size = 147, normalized size = 0.49

$$\frac{(b^2dx^4+abd)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-((b^2d+abf)x^4+abd+a^2f)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)+(b^2dx^3+b^2cx^2-abfx-abe)\sqrt{bx^4+a}}{2\,(ab^3x^4+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((b^2*d*x^4 + a*b*d)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1
/4)), -1) - ((b^2*d + a*b*f)*x^4 + a*b*d + a^2*f)*sqrt(a)*(-b/a)^(3/4)*elli
ptic_f(arcsin(x*(-b/a)^(1/4)), -1) + (b^2*d*x^3 + b^2*c*x^2 - a*b*f*x - a*b
*e)*sqrt(b*x^4 + a))/(a*b^3*x^4 + a^2*b^2)
```

Sympy [A]
time = 6.82, size = 133, normalized size = 0.44

$$e\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}} + \frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] e*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + c*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + d*x**3*gamma(3/4)*hyper((3/4,
 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + f*x**5*g
amma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*g
amma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*x/(b*x^4 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{{\left(b\,x^4+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

```
[Out] int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.546 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=275

$$-\frac{ex\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}+\frac{e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\ldots}{\sqrt[4]{\ldots}}\right.\right.}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] 1/2*(-a*f+b*x*(e*x^2+d*x+c))/a/b/(b*x^4+a)^(1/2)-1/2*e*x*(b*x^4+a)^(1/2)/a/
b^(1/2)/(a^(1/2)+x^2*b^(1/2))+1/2*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)
^(1/2)/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(-e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x
^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1868, 1212, 226, 1210}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\sqrt{b}\,c-\sqrt{a}\,e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}+\frac{e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}-\frac{ex\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/2*(e*x*Sqrt[a + b*x^4])/(a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) - (a*f - b*x
*(c + d*x + e*x^2))/(2*a*b*Sqrt[a + b*x^4]) + (e*(Sqrt[a] + Sqrt[b]*x^2)*Sq
rt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^
(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c - Sqrt[a]*e
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4)*Sqrt[a + b*x^4
])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = -\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}} - \frac{\int \frac{-c+ex^2}{\sqrt{a+bx^4}}\,dx}{2a}$$

$$= -\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}} + \frac{e\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2\sqrt{a}\sqrt{b}} + \frac{\left(c-\frac{\sqrt{a}\,e}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2a}$$

$$= -\frac{ex\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}} + \frac{e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{\cdots}{\left(\sqrt{a}\cdots\right.}}}{2a^3\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 116, normalized size = 0.42

$$\frac{-3af+3bcx+3bdx^2+3bcx\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+2bex^3\sqrt{1+\frac{bx^4}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)}{6ab\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x]
```

```
[Out] (-3*a*f + 3*b*c*x + 3*b*d*x^2 + 3*b*c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeomet
ric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 250, normalized size = 0.91

method	result
elliptic	$-\frac{2b\left(-\frac{e\,x^3}{4ba}-\frac{d\,x^2}{4ab}-\frac{cx}{4ba}+\frac{f}{4b^2}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{ie\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{}$
default	$-\frac{f}{2b\sqrt{b\,x^4+a}}+e\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{Ellip}\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f/b/(b*x^4+a)^(1/2)+e*(1/2/a*x^3/((x^4+a/b)*b)^(1/2)-1/2*I/a^(1/2)/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+1/2*d*x^2/a/(b*x^4+a)^(1/2)+c
*(1/2/a*x/((x^4+a/b)*b)^(1/2)+1/2/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elliptic
F(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/(b*x^4 + a)^(3/2), x)
```

Fricas [A]
time = 0.10, size = 129, normalized size = 0.47

$$\frac{(bex^4+ae)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1)-((bc+be)x^4+ac+ae)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1)+(bex^3+bdx^2+bcx-af)\sqrt{bx^4+a}}{2\,(ab^2x^4+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((b*e*x^4 + a*e)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4))
, -1) - ((b*c + b*e)*x^4 + a*c + a*e)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsi
n(x*(-b/a)^(1/4)), -1) + (b*e*x^3 + b*d*x^2 + b*c*x - a*f)*sqrt(b*x^4 + a))
/(a*b^2*x^4 + a^2*b)
```

Sympy [A]
time = 6.29, size = 131, normalized size = 0.48

$$f\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{dx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + c*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(3/2)*gamma(5/4)) + d*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + e*x**3*gamm
a(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamm
a(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/(b*x^4 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^3+e\,x^2+d\,x+c}{{\left(b\,x^4+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x)
```

```
[Out] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2), x)
```

3.547 $\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=323

$$\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}+\frac{c\sqrt{a+bx^4}}{2a^2}-\frac{fx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{f\left(\sqrt{a}+\sqrt{b}\ldots\right.}{}$$

```
[Out] -1/2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)+1/2*x*(-b*c*x^3+a*f*x^2+a*e
*x+a*d)/a^2/(b*x^4+a)^(1/2)+1/2*c*(b*x^4+a)^(1/2)/a^2-1/2*f*x*(b*x^4+a)^(1/
2)/a/b^(1/2)/(a^(1/2)+x^2*b^(1/2))+1/2*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2
))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(
1/4)*x/a^(1/4))),1/2*2^(1/2))*(-f*a^(1/2)+d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*
((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1843, 1846, 272, 65, 214, 1899, 267, 1212, 226, 1210}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}+\frac{f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}+\frac{c\sqrt{a+bx^4}}{2a^2}-\frac{fx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x]
```

```
[Out] (x*(a*d + a*e*x + a*f*x^2 - b*c*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (c*Sqrt[a +
 b*x^4])/(2*a^2) - (f*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^
2)) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) + (f*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d
- Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4)*Sqr
t[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
```

```
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx = \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-bdx+bfx^3-\frac{2b^2cx^4}{a}}{x\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-bd+bfx^2-\frac{2b^2cx^3}{a}}{\sqrt{a+bx^4}}\,dx}{2ab} + \frac{c\int \frac{1}{x\sqrt{a+bx^4}}\,dx}{a}$$

$$= \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \left(-\frac{2b^2cx^3}{a\sqrt{a+bx^4}}+\frac{-bd+bfx^2}{\sqrt{a+bx^4}}\right)dx}{2ab} + \frac{c\text{Subst}\left(\right.}{}$$

$$= \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-bd+bfx^2}{\sqrt{a+bx^4}}\,dx}{2ab} + \frac{c\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\right.}{2ab}$$

$$= \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} + \frac{c\sqrt{a+bx^4}}{2a^2} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{f\int \frac{1}{\sqrt{}}}{2\sqrt{}}$$

$$= \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} + \frac{c\sqrt{a+bx^4}}{2a^2} - \frac{fx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{c\tanh}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.08, size = 225, normalized size = 0.70

$$\frac{\sqrt{a}\,b(c+x(d+x(e+fx)))-bc\sqrt{a+bx^4}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+ia^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,f\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)+\frac{b\left(\sqrt{b}\,d+i\sqrt{a}\,f\right)\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\left(\frac{i\sqrt{b}}{\sqrt{a}}\right)^{3/2}}}{2a^{3/2}b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*b*(c + x*(d + x*(e + f*x))) - b*c*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a +
 b*x^4]/Sqrt[a]] + I*a^(3/2)*Sqrt[(I*Sqrt[b])/Sqrt[a]]*f*Sqrt[1 + (b*x^4)/a
]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] + (b*(Sqrt[b]*d + I
*Sqrt[a]*f)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a
]]*x], -1])/((I*Sqrt[b])/Sqrt[a])^(3/2))/(2*a^(3/2)*b*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 280, normalized size = 0.87

method	result
elliptic	$-\frac{2b\left(-\frac{f\,x^3}{4ab}-\frac{x^2 e}{4ab}-\frac{dx}{4ab}-\frac{c}{4ba}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{if\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}$
default	$f\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/2/a*x^3/((x^4+a/b)*b)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1
-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)))+1/2*e*x^2/a/(b*x^4+a)^(1/2)+d*(1/2/a*x/((x^4+a/b)*b)^(
1/2)+1/2/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I))+c*(1/2/a/(b*x^4+a)^(1/2)-1/2/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(
1/2))/x^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/((b*x^4 + a)^(3/2)*x), x)
```

Fricas [A]
time = 0.13, size = 191, normalized size = 0.59

$$\frac{2\left(abfx^4+a^2f\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1\right)-2\left(\left(abd+abf\right)x^4+a^2d+a^2f\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1\right)+\left(b^2cx^4+abc\right)\sqrt{a}\,\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\,\sqrt{a}+2a}{x^4}\right)+2\left(abfx^3+abex^2+abdx+abc\right)\sqrt{bx^4+a}}{4\left(a^2b^2x^4+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*b*f*x^4 + a^2*f)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^
(1/4)), -1) - 2*((a*b*d + a*b*f)*x^4 + a^2*d + a^2*f)*sqrt(a)*(-b/a)^(3/4)*
elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + (b^2*c*x^4 + a*b*c)*sqrt(a)*log(-(
b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(a*b*f*x^3 + a*b*e*x^2 +
a*b*d*x + a*b*c)*sqrt(b*x^4 + a))/(a^2*b^2*x^4 + a^3*b)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.28, size = 289, normalized size = 0.89

$$c\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right)+\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}+\frac{fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(3/2),x)
```

```
[Out] c*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b*
x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a) +
1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2)
 + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/
2) + 4*a**(7/2)*b*x**4)) + d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*
x**4/a)) + f*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*p
i)/a)/(4*a**(3/2)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/((b*x^4 + a)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^3+e\,x^2+d\,x+c}{x\,{\left(b\,x^4+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)), x)
```

3.548 $\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}} \, dx$

Optimal. Leaf size=344

$$\frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}}+\frac{d\sqrt{a+bx^4}}{2a^2}-\frac{c\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}\,cx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}-$$

```
[Out] -1/2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)+1/2*x*(-b*d*x^3-b*c*x^2+a*f
*x+a*e)/a^2/(b*x^4+a)^(1/2)+1/2*d*(b*x^4+a)^(1/2)/a^2-c*(b*x^4+a)^(1/2)/a^2
/x+3/2*c*x*b^(1/2)*(b*x^4+a)^(1/2)/a^2/(a^(1/2)+x^2*b^(1/2))-3/2*b^(1/4)*c*
(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))
*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/
2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)+1/4*(
cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*
EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(e*a^(1/2)+3*c*b^(1
/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4
)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1843, 1847, 1849, 1598, 1212, 226, 1210, 21, 272, 52, 65, 214}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}\,cx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{d\sqrt{a+bx^4}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x]
```

```
[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (d*Sqrt[a +
 b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*c*x*Sqrt[a + b*
x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]
)/(2*a^(3/2)) - (3*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(
7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx = \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-2bdx-bex^2-\frac{b^2cx^4}{a}-\frac{2b^2dx^5}{a}}{x^2\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{-2bc-bex^2-\frac{b^2cx^4}{a}}{x^2\sqrt{a+bx^4}}+\frac{-2bd-\frac{2b^2dx^4}{a}}{x\sqrt{a+bx^4}}\right)dx}{2ab}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-bex^2-\frac{b^2cx^4}{a}}{x^2\sqrt{a+bx^4}}\,dx}{2ab} - \frac{\int \frac{-2bd-\frac{2b^2dx^4}{a}}{x\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{a^2x} + \frac{\int \frac{2abex+6b^2cx^3}{x\sqrt{a+bx^4}}\,dx}{4a^2b} + \frac{d\int \frac{\sqrt{a+bx^4}}{x}}{a^2}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{a^2x} + \frac{\int \frac{2abe+6b^2cx^2}{\sqrt{a+bx^4}}\,dx}{4a^2b} + \frac{d\text{Subst}\left(\int \frac{\sqrt{a}}{}\right.}{4}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{a^2x} - \frac{\left(3\sqrt{b}\,c\right)\int \frac{1-\frac{\sqrt{b}}{\sqrt{a}}}{\sqrt{a+b}}}{2a^{3/2}}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}\,cx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}\,cx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.30, size = 245, normalized size = 0.71

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-2ac-3bcx^4+ax(d+x(e+fx))-\sqrt{a}\,dx\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+3\sqrt{a}\sqrt{b}\,cx\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-i\sqrt{a}\left(-3i\sqrt{b}\,c+\sqrt{a}\,e\right)x\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{2a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-2*a*c - 3*b*c*x^4 + a*x*(d + x*(e + f*x)) - Sq
rt[a]*d*x*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) + 3*Sqrt[a]*Sqr
```

```
t[b]*c*x*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*
x], -1] - I*Sqrt[a]*((-3*I)*Sqrt[b]*c + Sqrt[a]*e)*x*Sqrt[1 + (b*x^4)/a]*El
lipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(2*a^2*Sqrt[(I*Sqrt[b]
)/Sqrt[a]]*x*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 298, normalized size = 0.87

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{a^2 x}-\frac{2b\left(\frac{c\,x^3}{4a^2}-\frac{x^2 f}{4ab}-\frac{xe}{4ab}-\frac{d}{4ab}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{3i}{}$
default	$\frac{f\,x^2}{2a\sqrt{b\,x^4+a}}+e\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\Bigg($
risch	$-\frac{c\sqrt{b\,x^4+a}}{a^2 x}-\frac{bc\,x^3}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{b}\,c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2/a/(b*x^4+a)^(1/2)+e*(1/2/a*x/((x^4+a/b)*b)^(1/2)+1/2/a/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+d*(1/2/a/(b*
x^4+a)^(1/2)-1/2/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))+c*(-1/2*b
*x^3/a^2/((x^4+a/b)*b)^(1/2)-(b*x^4+a)^(1/2)/a^2/x+3/2*I*b^(1/2)/a^(3/2)/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Elli
pticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

Fricas [A]
time = 0.12, size = 215, normalized size = 0.62

$$-\frac{6\,(b^2cx^5+abcx)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1)-2\,((3\,b^2c-abe)x^5+(3\,abc-a^2e)x)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1)-(b^2dx^5+abdx)\sqrt{a}\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)+2\,(3\,b^2cx^4-abfx^3-abex^2-abdx+2\,abc)\sqrt{bx^4+a}}{4\,(a^2b^2x^5+a^3bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(6*(b^2*c*x^5 + a*b*c*x)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/
a)^(1/4)), -1) - 2*((3*b^2*c - a*b*e)*x^5 + (3*a*b*c - a^2*e)*x)*sqrt(a)*(-
b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - (b^2*d*x^5 + a*b*d*x)*s
qrt(a)*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(3*b^2*c*x^4
 - a*b*f*x^3 - a*b*e*x^2 - a*b*d*x + 2*a*b*c)*sqrt(b*x^4 + a))/(a^2*b^2*x^5
 + a^3*b*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.67, size = 291, normalized size = 0.85

$$d\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right)+\frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}+\frac{ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{fx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(3/2),x)
```

```
[Out] d*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b*
x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a) +
1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2)
 + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/
2) + 4*a**(7/2)*b*x**4)) + c*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + e*x*gamma(1/4)*hyper((1/4, 3
/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + f*x**2/(2*
a**(3/2)*sqrt(1 + b*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

Mupad [B]
time = 5.94, size = 133, normalized size = 0.39

$$\frac{d}{2\,a\,\sqrt{b\,x^4+a}}-\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}+\frac{f\,x^2}{2\,a\,\sqrt{b\,x^4+a}}-\frac{c\,\left(\frac{a}{b\,x^4}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a}{b\,x^4}\right)}{7\,x\,\left(b\,x^4+a\right)^{3/2}}+\frac{e\,x\,\left(\frac{b\,x^4}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(b\,x^4+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x)
```

```
[Out] d/(2*a*(a + b*x^4)^(1/2)) - (d*atanh((a + b*x^4)^(1/2)/a^(1/2)))/(2*a^(3/2)
) + (f*x^2)/(2*a*(a + b*x^4)^(1/2)) - (c*(a/(b*x^4) + 1)^(3/2)*hypergeom([3
/2, 7/4], 11/4, -a/(b*x^4)))/(7*x*(a + b*x^4)^(3/2)) + (e*x*((b*x^4)/a + 1)
^(3/2)*hypergeom([1/4, 3/2], 5/4, -(b*x^4)/a))/(a + b*x^4)^(3/2)
```

3.549 $\int \frac{c+dx+ex^2+fx^3}{x^3\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=367

$$\frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}}+\frac{e\sqrt{a+bx^4}}{2a^2}-\frac{c\sqrt{a+bx^4}}{2a^2x^2}-\frac{d\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}\,dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{e\tanh^{-1}\left(\frac{\sqrt{a+}}{}\right.}{2a^{3/2}}$$

```
[Out] -1/2*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)+1/2*x*(-b*e*x^3-b*d*x^2-b*c
*x+a*f)/a^2/(b*x^4+a)^(1/2)+1/2*e*(b*x^4+a)^(1/2)/a^2-1/2*c*(b*x^4+a)^(1/2)
/a^2/x^2-d*(b*x^4+a)^(1/2)/a^2/x+3/2*d*x*b^(1/2)*(b*x^4+a)^(1/2)/a^2/(a^(1/
2)+x^2*b^(1/2))-3/2*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1
/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)
/a^(7/4)/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/
2*2^(1/2))*(f*a^(1/2)+3*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2
)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1843, 1847, 1849, 1598, 1212, 226, 1210, 21, 272, 52, 65, 214}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(\sqrt{a}\,f+3\sqrt{b}\,d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\,\sqrt{a+bx^4}}-\frac{3\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}}-\frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{2a^2x^2}-\frac{d\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}\,dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{e\sqrt{a+bx^4}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x]
```

```
[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (e*Sqrt[a +
 b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(2*a^2*x^2) - (d*Sqrt[a + b*x^4])/(a
^2*x) + (3*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (
e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) - (3*b^(1/4)*d*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*d + S
qrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a
 + b*x^4])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x^3\left(a+bx^4\right)^{3/2}}\,dx = \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-2bdx-2bex^2-bfx^3-\frac{b^2dx^5}{a}-\frac{2b^2ex^6}{a}}{x^3\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{-2bd-bfx^2-\frac{b^2dx^4}{a}}{x^2\sqrt{a+bx^4}} + \frac{-2bc-2bex^2-\frac{2b^2ex^6}{a}}{x^3\sqrt{a+bx^4}}\right)dx}{2ab}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bd-bfx^2-\frac{b^2dx^4}{a}}{x^2\sqrt{a+bx^4}}\,dx}{2ab} - \frac{\int \frac{-2bc-2bex^2-\frac{2b^2ex^6}{a}}{x^3\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{\int \frac{8abex+8b^2ex^5}{x^2\sqrt{a+bx^4}}\,dx}{8a^2b} \cdots$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{\int \frac{8abe+8b^2ex^4}{x\sqrt{a+bx^4}}\,dx}{8a^2b} \cdots$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} - \frac{\left(3\sqrt{b}\,d\right)\int \frac{1-\sqrt{\cdots}}{\sqrt{a+\cdots}}}{2a^{3/2}} \cdots$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}\,dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}\,x^2 \cdots\right.} \cdots$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{3\cdots}{2a^2\cdots}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{3\cdots}{2a^2\cdots}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{3\cdots}{2a^2\cdots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.30, size = 259, normalized size = 0.71

$$-\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(bx^4(2c+3dx)+a(c+2dx-x^2(e+fx))+\sqrt{a}\,ex^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+3\sqrt{a}\sqrt{b}\,dx^2\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-i\sqrt{a}\left(-3i\sqrt{b}\,d+\sqrt{a}\,f\right)x^2\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{2a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*(b*x^4*(2*c + 3*d*x) + a*(c + 2*d*x - x^2*(e +
 f*x)) + Sqrt[a]*e*x^2*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) +
 3*Sqrt[a]*Sqrt[b]*d*x^2*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sq
rt[b])/Sqrt[a]]*x], -1] - I*Sqrt[a]*((-3*I)*Sqrt[b]*d + Sqrt[a]*f)*x^2*Sqrt
[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(2*a
^2*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^2*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 306, normalized size = 0.83

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{2a^2x^2}-\frac{d\sqrt{b\,x^4+a}}{a^2x}-\frac{2b\left(\frac{d\,x^3}{4a^2}+\frac{c\,x^2}{4a^2}-\frac{xf}{4ab}-\frac{e}{4ba}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\cdots}\right.}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$
default	$f\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)-\frac{c(2b\,x^4+a)}{2x^2\sqrt{b\,x^4+a}\,a^2}+e$
risch	$-\frac{\sqrt{b\,x^4+a}\,(2dx+c)}{2a^2x^2}-\frac{bd\,x^3}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{b}\,d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right.}{2a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/2/a*x/((x^4+a/b)*b)^(1/2)+1/2/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)
*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellipti
cF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*c/x^2*(2*b*x^4+a)/(b*x^4+a)^(1/2)/a^
2+e*(1/2/a/(b*x^4+a)^(1/2)-1/2/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x
^2))+d*(-1/2*b*x^3/a^2/((x^4+a/b)*b)^(1/2)-(b*x^4+a)^(1/2)/a^2/x+3/2*I*b^(1
/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
```

```
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))
^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)
```

Fricas [A]
time = 0.13, size = 231, normalized size = 0.63

$$-\frac{6\,(b^2dx^6+abdx^2)\sqrt{a}\,\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-2\,((3\,b^2d-abf)x^6+(3\,abd-a^2f)x^2)\sqrt{a}\,\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-(b^2ex^6+abex^2)\sqrt{a}\,\log\left(-\frac{bx^4-2\,\sqrt{bx^4+a}\,\sqrt{a}+2\,a}{x^4}\right)+2\,(3\,b^2dx^5+2\,b^2cx^4-abfx^3-abex^2+2\,abdx+abc)\sqrt{bx^4+a}}{4\,(a^2b^2x^6+a^3bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(6*(b^2*d*x^6 + a*b*d*x^2)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-
b/a)^(1/4)), -1) - 2*((3*b^2*d - a*b*f)*x^6 + (3*a*b*d - a^2*f)*x^2)*sqrt(a
)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - (b^2*e*x^6 + a*b*e*
x^2)*sqrt(a)*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(3*b^2
*d*x^5 + 2*b^2*c*x^4 - a*b*f*x^3 - a*b*e*x^2 + 2*a*b*d*x + a*b*c)*sqrt(b*x^
4 + a))/(a^2*b^2*x^6 + a^3*b*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.06, size = 316, normalized size = 0.86

$$c\left(-\frac{1}{2a\sqrt{b}\,x^4\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}\right)+e\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right)+\frac{d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}+\frac{fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(3/2),x)
```

```
[Out] c*(-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqrt(a/(b*x**
4) + 1))) + e*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) +
 a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 +
b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)
/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) +
1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)) + d*gamma(-1/4)*hyper((-1/4, 3/2), (3/
4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + f*x*gamma(1/4)*h
yper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((f*x^3 + x^2*e + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)
```

Mupad [**B**]
time = 6.08, size = 147, normalized size = 0.40

$$\frac{e}{2\,a\,\sqrt{b\,x^4+a}}-\frac{e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{2\,c\,(b\,x^4+a)-a\,c}{2\,a^2\,x^2\,\sqrt{b\,x^4+a}}-\frac{d\,\left(\frac{a}{b\,x^4}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a}{b\,x^4}\right)}{7\,x\,(b\,x^4+a)^{3/2}}+\frac{f\,x\,\left(\frac{b\,x^4}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{(b\,x^4+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x)
[Out] e/(2*a*(a + b*x^4)^(1/2)) - (e*atanh((a + b*x^4)^(1/2)/a^(1/2)))/(2*a^(3/2)
) - (2*c*(a + b*x^4) - a*c)/(2*a^2*x^2*(a + b*x^4)^(1/2)) - (d*(a/(b*x^4) +
 1)^(3/2)*hypergeom([3/2, 7/4], 11/4, -a/(b*x^4)))/(7*x*(a + b*x^4)^(3/2))
+ (f*x*((b*x^4)/a + 1)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(b*x^4)/a))/(a + b
*x^4)^(3/2)
```

3.550 $\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=387

$$-\frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}}+\frac{f\sqrt{a+bx^4}}{2a^2}-\frac{c\sqrt{a+bx^4}}{3a^2x^3}-\frac{d\sqrt{a+bx^4}}{2a^2x^2}-\frac{e\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}\,ex\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}$$

```
[Out] -1/2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*x*(b*f*x^3+b*e*x^2+b*d*
x+b*c)/a^2/(b*x^4+a)^(1/2)+1/2*f*(b*x^4+a)^(1/2)/a^2-1/3*c*(b*x^4+a)^(1/2)/
a^2/x^3-1/2*d*(b*x^4+a)^(1/2)/a^2/x^2-e*(b*x^4+a)^(1/2)/a^2/x+3/2*e*x*b^(1/
2)*(b*x^4+a)^(1/2)/a^2/(a^(1/2)+x^2*b^(1/2))-3/2*b^(1/4)*e*(cos(2*arctan(b^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2
*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(
a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)-1/12*b^(1/4)*(cos(2*a
rctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellipti
cF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-9*e*a^(1/2)+5*c*b^(1/2))
*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(9/4)/(b
*x^4+a)^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {1843, 1847, 1849, 1599, 1598, 1212, 226, 1210, 21, 272, 52, 65, 214}

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\left(5\sqrt{b}\,c-9\sqrt{a}\,e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{b}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}}-\frac{f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{3a^2x^3}-\frac{d\sqrt{a+bx^4}}{2a^2x^2}-\frac{e\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}\,ex\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{f\sqrt{a+bx^4}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x]
```

```
[Out] -1/2*(x*(b*c + b*d*x + b*e*x^2 + b*f*x^3))/(a^2*Sqrt[a + b*x^4]) + (f*Sqrt[
a + b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(3*a^2*x^3) - (d*Sqrt[a + b*x^4])
/(2*a^2*x^2) - (e*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*e*x*Sqrt[a + b*x^4]
)/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2
*a^(3/2)) - (3*b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
+ Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)
*Sqrt[a + b*x^4]) - (b^(1/4)*(5*Sqrt[b]*c - 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(12*a^(9/4)*Sqrt[a + b*x^4])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx = -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-2bdx-2bex^2-2bfx^3+\frac{b^2cx^4}{a}-\frac{b^2ex^6}{a}-\frac{2b^2fx^7}{a}}{x^4\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{-2bc-2bex^2+\frac{b^2cx^4}{a}-\frac{b^2ex^6}{a}}{x^4\sqrt{a+bx^4}}+\frac{-2bd-2bfx^2-\frac{2b^2fx^6}{a}}{x^3\sqrt{a+bx^4}}\right)d}{2ab}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-2bex^2+\frac{b^2cx^4}{a}-\frac{b^2ex^6}{a}}{x^4\sqrt{a+bx^4}}\,dx}{2ab} - \frac{\int \frac{-2bd-2bfx^2-\frac{2b^2fx^6}{a}}{x^3\sqrt{a+bx^4}}}{2ab}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} + \frac{\int \frac{12abex-10b^2cx^3+6b^2}{x^3\sqrt{a+bx^4}}}{12a^2b}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} + \frac{\int \frac{12abe-10b^2cx^2+6b^2e}{x^2\sqrt{a+bx^4}}}{12a^2b}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - \frac{e\sqrt{a+bx^4}}{a^2x} - \int 2$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - \frac{e\sqrt{a+bx^4}}{a^2x} - \int 2$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - e\sqrt{}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - e\sqrt{}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - e\sqrt{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.31, size = 267, normalized size = 0.69

$$-\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(2ac+bx^4(5c+6dx+9ex^2)+3ax(d+x(2e-fx))+3\sqrt{a}\,fx^3\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+9\sqrt{a}\,\sqrt{b}\,ex^3\sqrt{1+\frac{bx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)-\sqrt{b}\left(-5i\sqrt{b}\,c+9\sqrt{a}\,e\right)x^3\sqrt{1+\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{6a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*(2*a*c + b*x^4*(5*c + 6*d*x + 9*e*x^2) + 3*a*x
*(d + x*(2*e - f*x)) + 3*Sqrt[a]*f*x^3*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x
^4]/Sqrt[a]])) + 9*Sqrt[a]*Sqrt[b]*e*x^3*Sqrt[1 + (b*x^4)/a]*EllipticE[I*Ar
cSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - Sqrt[b]*((-5*I)*Sqrt[b]*c + 9*Sqr
t[a]*e)*x^3*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a
]]*x], -1])/(6*a^2*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^3*Sqrt[a + b*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.43, size = 325, normalized size = 0.84

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{3a^2x^3}-\frac{d\sqrt{b\,x^4+a}}{2a^2x^2}-\frac{e\sqrt{b\,x^4+a}}{a^2x}-\frac{2b\left(\frac{e\,x^3}{4a^2}+\frac{x^2d}{4a^2}+\frac{cx}{4a^2}-\frac{f}{4ab}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{5bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{\cdots}}{\cdots}}}{6a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\cdots}$
default	$-\frac{d(2b\,x^4+a)}{2x^2\sqrt{b\,x^4+a}\,a^2}+c\left(-\frac{\sqrt{b\,x^4+a}}{3a^2x^3}-\frac{bx}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{5b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{Elliptic}\cdots}{6a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}\cdots}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,(6e\,x^2+3dx+2c)}{6a^2x^3}-\frac{be\,x^3}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{b}\,e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\cdots}\right.}{2a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d/x^2*(2*b*x^4+a)/(b*x^4+a)^(1/2)/a^2+c*(-1/3*(b*x^4+a)^(1/2)/a^2/x^3-
1/2*b*x/a^2/((x^4+a/b)*b)^(1/2)-5/6*b/a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*El
lipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+f*(1/2/a/(b*x^4+a)^(1/2)-1/2/a^(3/2
)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))+e*(-1/2*b*x^3/a^2/((x^4+a/b)*b)^
(1/2)-(b*x^4+a)^(1/2)/a^2/x+3/2*I*b^(1/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^
(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1
/2))^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)/((b*x^4 + a)^(3/2)*x^4), x)
```

Fricas [A]
time = 0.11, size = 216, normalized size = 0.56

$$-\frac{18\,(bex^7+aex^3)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1)-2\,((5\,bc+9\,be)x^7+(5\,ac+9\,ae)x^3)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1)-3\,(bfx^7+afx^3)\sqrt{a}\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)+2\,(9\,bex^6+6\,bdx^5+5\,bcx^4-3\,afx^3+6\,aex^2+3\,adx+2\,ac)\sqrt{bx^4+a}}{12\,(a^2bx^7+a^3x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(18*(b*e*x^7 + a*e*x^3)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/
a)^(1/4)), -1) - 2*((5*b*c + 9*b*e)*x^7 + (5*a*c + 9*a*e)*x^3)*sqrt(a)*(-b/
a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - 3*(b*f*x^7 + a*f*x^3)*sqr
t(a)*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(9*b*e*x^6 + 6
*b*d*x^5 + 5*b*c*x^4 - 3*a*f*x^3 + 6*a*e*x^2 + 3*a*d*x + 2*a*c)*sqrt(b*x^4
+ a))/(a^2*b*x^7 + a^3*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 11.34, size = 321, normalized size = 0.83

$$d\left(-\frac{1}{2a\sqrt{b}\,x^4\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}\right)+f\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right)+\frac{c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{3}{2}\\\frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^3\Gamma\left(\frac{1}{4}\right)}+\frac{e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{2}\\\frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(3/2),x)
```

```
[Out] d*(-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqrt(a/(b*x**
4) + 1))) + f*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) +
 a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 +
b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)
/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) +
1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)) + c*gamma(-3/4)*hyper((-3/4, 3/2), (1/
4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x**3*gamma(1/4)) + e*gamma(-1/4)
*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3
/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((f*x^3 + x^2*e + d*x + c)/((b*x^4 + a)^(3/2)*x^4), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^4\,{\left(b\,x^4+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)), x)
```

3.551 $\int (gx)^m (c+dx+ex^2+fx^3) (a+bx^4)^p \, dx$

Optimal. Leaf size=269

$$\frac{c(gx)^{1+m} (a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{4},-p;\frac{5+m}{4};-\frac{bx^4}{a}\right)}{g(1+m)}+\frac{d(gx)^{2+m} (a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{2+m}{4},-p;\right.}{g^2(2+m)}$$

```
[Out] c*(g*x)^(1+m)*(b*x^4+a)^p*hypergeom([-p, 1/4+1/4*m],[5/4+1/4*m],-b*x^4/a)/g
/(1+m)/((1+b*x^4/a)^p)+d*(g*x)^(2+m)*(b*x^4+a)^p*hypergeom([-p, 1/2+1/4*m],
[3/2+1/4*m],-b*x^4/a)/g^2/(2+m)/((1+b*x^4/a)^p)+e*(g*x)^(3+m)*(b*x^4+a)^p*h
ypergeom([-p, 3/4+1/4*m],[7/4+1/4*m],-b*x^4/a)/g^3/(3+m)/((1+b*x^4/a)^p)+f*
(g*x)^(4+m)*(b*x^4+a)^p*hypergeom([-p, 1+1/4*m],[2+1/4*m],-b*x^4/a)/g^4/(4+
m)/((1+b*x^4/a)^p)
```

Rubi [A]
time = 0.17, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1847, 1350, 372, 371}

$$\frac{c(gx)^{m+1} (a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+1}{4},-p;\frac{m+5}{4};-\frac{bx^4}{a}\right)}{g(m+1)}+\frac{d(gx)^{m+2} (a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+2}{4},-p;\frac{m+6}{4};-\frac{bx^4}{a}\right)}{g^2(m+2)}+\frac{e(gx)^{m+3} (a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+3}{4},-p;\frac{m+7}{4};-\frac{bx^4}{a}\right)}{g^3(m+3)}+\frac{f(gx)^{m+4} (a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+4}{4},-p;\frac{m+8}{4};-\frac{bx^4}{a}\right)}{g^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (c*(g*x)^(1 + m)*(a + b*x^4)^p*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4,
-((b*x^4)/a)])/(g*(1 + m)*(1 + (b*x^4)/a)^p) + (d*(g*x)^(2 + m)*(a + b*x^4)
^p*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4, -((b*x^4)/a)])/(g^2*(2 + m)*
(1 + (b*x^4)/a)^p) + (e*(g*x)^(3 + m)*(a + b*x^4)^p*Hypergeometric2F1[(3 +
m)/4, -p, (7 + m)/4, -((b*x^4)/a)])/(g^3*(3 + m)*(1 + (b*x^4)/a)^p) + (f*(g
*x)^(4 + m)*(a + b*x^4)^p*Hypergeometric2F1[(4 + m)/4, -p, (8 + m)/4, -((b*
x^4)/a)])/(g^4*(4 + m)*(1 + (b*x^4)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1350

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int (gx)^m \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx &= \int \left(\left(gx\right)^m\left(c+ex^2\right)\left(a+bx^4\right)^p+\frac{(gx)^{1+m}\left(d+fx^2\right)\left(a+b\right.}{g}\right.\\
&= \frac{\int (gx)^{1+m}\left(d+fx^2\right)\left(a+bx^4\right)^p\,dx}{g}+\int (gx)^m\left(c+ex^2\right)\left(a\right.\\
&= \frac{\int \left(d(gx)^{1+m}\left(a+bx^4\right)^p+\frac{f(gx)^{3+m}\left(a+bx^4\right)^p}{g^2}\right)dx}{g}+\int \left(c(gx)\right.\\
&= c\int (gx)^m\left(a+bx^4\right)^p\,dx+\frac{f\int (gx)^{3+m}\left(a+bx^4\right)^p\,dx}{g^3}+\frac{e\int}{}\\
&= \left(c\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int (gx)^m\left(1+\frac{bx^4}{a}\right)^p\,dx+\\
&= \frac{c(gx)^{1+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{4},-p;\frac{5+m}{4};-\frac{bx^4}{a}\right)}{g(1+m)}
\end{aligned}$$

Mathematica [A]
time = 0.27, size = 174, normalized size = 0.65

$$x(gx)^m\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(\frac{fx^3\,{}_2F_1\left(1+\frac{m}{4},-p;2+\frac{m}{4};-\frac{bx^4}{a}\right)}{4+m}+\frac{c\,{}_2F_1\left(\frac{1+m}{4},-p;\frac{5+m}{4};-\frac{bx^4}{a}\right)}{1+m}+x\left(\frac{d\,{}_2F_1\left(\frac{2+m}{4},-p;\frac{6+m}{4};-\frac{bx^4}{a}\right)}{2+m}+\frac{ex\,{}_2F_1\left(\frac{3+m}{4},-p;\frac{7+m}{4};-\frac{bx^4}{a}\right)}{3+m}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (x*(g*x)^m*(a + b*x^4)^p*((f*x^3*Hypergeometric2F1[1 + m/4, -p, 2 + m/4, -(
(b*x^4)/a)])/(4 + m) + (c*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4, -((b*
x^4)/a)])/(1 + m) + x*((d*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4, -((b*
x^4)/a)])/(2 + m) + (e*x*Hypergeometric2F1[(3 + m)/4, -p, (7 + m)/4, -((b*x
^4)/a)])/(3 + m))))/(1 + (b*x^4)/a)^p
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (gx)^m \left(f\,x^3+e\,x^2+dx+c\right) \left(b\,x^4+a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

```
[Out] int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

Fricas [F]
time = 0.39, size = 32, normalized size = 0.12

$$\text{integral}\left(\left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p(gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
[Out] integrate((f*x^3 + x^2*e + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g\,x)^m\,\left(b\,x^4+a\right)^p\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3),x)
[Out] int((g*x)^m*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3), x)
```

3.552 $\int (c + dx + ex^2 + fx^3) (a + bx^4)^p \, dx$

Optimal. Leaf size=143

$$\frac{f(a+bx^4)^{1+p}}{4b(1+p)}+\frac{cx(a+bx^4)^{1+p}\ {}_2F_1\left(1,\frac{5}{4}+p;\frac{5}{4};-\frac{bx^4}{a}\right)}{a}+\frac{dx^2(a+bx^4)^{1+p}\ {}_2F_1\left(1,\frac{3}{2}+p;\frac{3}{2};-\frac{bx^4}{a}\right)}{2a}+\frac{ex^3(a+bx$$

```
[Out] 1/4*f*(b*x^4+a)^(1+p)/b/(1+p)+c*x*(b*x^4+a)^(1+p)*hypergeom([1, 5/4+p],[5/4
],-b*x^4/a)/a+1/2*d*x^2*(b*x^4+a)^(1+p)*hypergeom([1, 3/2+p],[3/2],-b*x^4/a
)/a+1/3*e*x^3*(b*x^4+a)^(1+p)*hypergeom([1, 7/4+p],[7/4],-b*x^4/a)/a
```

Rubi [A]

time = 0.10, antiderivative size = 170, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1899, 1218, 252, 251, 372, 371, 1262, 655}

$$cx\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{1}{2}dx^2\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+\frac{f\left(a+bx^4\right)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (f*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (c*x*(a + b*x^4)^p*Hypergeometric2F
1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (d*x^2*(a + b*x^4)^p*Hyp
ergeometric2F1[1/2, -p, 3/2, -((b*x^4)/a)])/(2*(1 + (b*x^4)/a)^p) + (e*x^3*
(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)])/(3*(1 + (b*x^4
)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1218

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx=\int \left(\left(c+ex^2\right)\left(a+bx^4\right)^p+x\left(d+fx^2\right)\left(a+bx^4\right)^p\right)\,dx$$
$$=\int \left(c+ex^2\right)\left(a+bx^4\right)^p\,dx+\int x\left(d+fx^2\right)\left(a+bx^4\right)^p\,dx$$
$$=\frac{1}{2}\text{Subst}\left(\int (d+fx)\left(a+bx^2\right)^p\,dx,x,x^2\right)+\int \left(c\left(a+bx^4\right)^p+ex^2\right.$$
$$=\frac{f\left(a+bx^4\right)^{1+p}}{4b(1+p)}+c\int \left(a+bx^4\right)^p\,dx+\frac{1}{2}d\text{Subst}\left(\int \left(a+bx^2\right)^p\,dx,\right.$$
$$=\frac{f\left(a+bx^4\right)^{1+p}}{4b(1+p)}+\left(c\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \left(1+\frac{bx^4}{a}\right)^p\,dx$$
$$=\frac{f\left(a+bx^4\right)^{1+p}}{4b(1+p)}+cx\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

Mathematica [A]
time = 0.66, size = 147, normalized size = 1.03

$$\frac{1}{12}\left(a+bx^4\right)^p\left(\frac{3f\left(a+bx^4\right)}{b(1+p)}+12cx\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+6dx^2\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^4}{a}\right)+4ex^3\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] ((a + b*x^4)^p*((3*f*(a + b*x^4))/(b*(1 + p)) + (12*c*x*Hypergeometric2F1[1
/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (6*d*x^2*Hypergeometric2F1[
1/2, -p, 3/2, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (4*e*x^3*Hypergeometric2F1
[3/4, -p, 7/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p))/12
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(f\,x^3+e\,x^2+dx+c\right)\left(b\,x^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

```
[Out] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*(b*x^4 + a)^p, x)
```

Fricas [F]
time = 0.37, size = 27, normalized size = 0.19

$$\mathrm{integral}\left(\left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

Sympy [A]
time = 20.50, size = 141, normalized size = 0.99

$$\frac{a^pcx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^pdx^2{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2}+\frac{a^pex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+f\left(\begin{cases}\frac{a^px^4}{4} & \text{for}\ b=0\\ \frac{\begin{cases}\frac{(a+bx^4)^{p+1}}{p+1} & \text{for}\ p\neq-1\\ \log\left(a+bx^4\right) & \text{otherwise}\end{cases}}{4b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

```
[Out] a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(5/4)) + a**p*d*x**2*hyper((1/2, -p), (3/2,), b*x**4*exp_polar(I*pi)/a)
/2 + a**p*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(7/4)) + f*Piecewise((a**p*x**4/4, Eq(b, 0)), (Piecewise(((a +
b*x**4)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**4), True))/(4*b), True)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*(b*x^4 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^4+a\right)}^p\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3),x)
[Out] int((a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3), x)
```

3.553 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx$

Optimal. Leaf size=175

$$\frac{c(a+bx^4)^{1+p}}{4b(1+p)}+\frac{1}{5}dx^5\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)+\frac{1}{6}ex^6\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2}\right.$$

```
[Out] 1/4*c*(b*x^4+a)^(1+p)/b/(1+p)+1/5*d*x^5*(b*x^4+a)^p*hypergeom([5/4, -p],[9/
4],-b*x^4/a)/((1+b*x^4/a)^p)+1/6*e*x^6*(b*x^4+a)^p*hypergeom([3/2, -p],[5/2
],-b*x^4/a)/((1+b*x^4/a)^p)+1/7*f*x^7*(b*x^4+a)^p*hypergeom([7/4, -p],[11/4
],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A]

time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1847, 1266, 778, 267, 372, 371, 1350}

$$\frac{c(a+bx^4)^{p+1}}{4b(p+1)}+\frac{1}{5}dx^5\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)+\frac{1}{6}ex^6\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^4}{a}\right)+\frac{1}{7}fx^7\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (c*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (d*x^5*(a + b*x^4)^p*Hypergeometric
2F1[5/4, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p) + (e*x^6*(a + b*x^4)
^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^4)/a)])/(6*(1 + (b*x^4)/a)^p) + (
f*x^7*(a + b*x^4)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^4)/a)])/(7*(1 +
 (b*x^4)/a)^p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1350

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx &= \int \left(x^3\left(c+ex^2\right)\left(a+bx^4\right)^p+x^4\left(d+fx^2\right)\left(a+bx^4\right)^p\right)dx \\
&= \int x^3\left(c+ex^2\right)\left(a+bx^4\right)^p\,dx+\int x^4\left(d+fx^2\right)\left(a+bx^4\right)^p\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int x(c+ex)\left(a+bx^2\right)^p\,dx,x,x^2\right)+\int \left(dx^4\left(a+bx^4\right)^p \right. \\
&= \frac{1}{2}c\text{Subst}\left(\int x\left(a+bx^2\right)^p\,dx,x,x^2\right)+d\int x^4\left(a+bx^4\right)^p\,dx+\frac{1}{2}e \\
&= \frac{c\left(a+bx^4\right)^{1+p}}{4b(1+p)}+\left(d\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^4}{a}\right)^p \\
&= \frac{c\left(a+bx^4\right)^{1+p}}{4b(1+p)}+\frac{1}{5}dx^5\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\right.
\end{aligned}$$

Mathematica [A]
time = 0.67, size = 145, normalized size = 0.83

$$\frac{(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(105c(a+bx^4)\left(1+\frac{bx^4}{a}\right)^p+84bd(1+p)x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)+70be(1+p)x^6\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^4}{a}\right)+60bf(1+p)x^7\,{}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-\frac{bx^4}{a}\right)\right)}{420b(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] ((a + b*x^4)^p*(105*c*(a + b*x^4)*(1 + (b*x^4)/a)^p + 84*b*d*(1 + p)*x^5*Hy
pergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 70*b*e*(1 + p)*x^6*Hypergeome
tric2F1[3/2, -p, 5/2, -((b*x^4)/a)] + 60*b*f*(1 + p)*x^7*Hypergeometric2F1[
7/4, -p, 11/4, -((b*x^4)/a)]))/(420*b*(1 + p)*(1 + (b*x^4)/a)^p)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^3\left(f\,x^3+e\,x^2+dx+c\right)\left(b\,x^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

```
[Out] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^4 + a)^(p + 1)*c/(b*(p + 1)) + integrate((f*x^6 + x^5*e + d*x^4)*(
b*x^4 + a)^p, x)
```

Fricas [F]
time = 0.38, size = 33, normalized size = 0.19

$$\text{integral}\left(\left(fx^6+ex^5+dx^4+cx^3\right)\left(bx^4+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)*(b*x^4 + a)^p, x)
```

Sympy [A]
time = 47.45, size = 143, normalized size = 0.82

$$\frac{a^p d x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, -p \\ \frac{9}{4} \end{matrix}\middle| \frac{b x^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{a^p e x^6 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{b x^4 e^{i\pi}}{a}\right)}{6} + \frac{a^p f x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} \frac{7}{4}, -p \\ \frac{11}{4} \end{matrix}\middle| \frac{b x^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + c\left(\begin{cases} \frac{a^p x^4}{4} & \text{for}\ b = 0 \\ \frac{\begin{cases} \frac{(a+bx^4)^{p+1}}{p+1} & \text{for}\ p \neq -1 \\ \log\left(a + bx^4\right) & \text{otherwise} \end{cases}}{4b} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

```
[Out] a**p*d*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(9/4)) + a**p*e*x**6*hyper((3/2, -p), (5/2,), b*x**4*exp_polar(I*pi)
/a)/6 + a**p*f*x**7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I
*pi)/a)/(4*gamma(11/4)) + c*Piecewise((a**p*x**4/4, Eq(b, 0)), (Piecewise((
(a + b*x**4)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**4), True))/(4*b),
True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + x^2*e + d*x + c)*(b*x^4 + a)^p*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(b\,x^4+a\right)^p \left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] int(x^3*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3), x)
```

3.554 $\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

```
[Out] -ln(1-x)
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1600, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

```
[Out] -Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx &= \int \frac{1}{1-x}\,dx\\ &= -\log(1-x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

```
[Out] -Log[1 - x]
```

Maple [A]
time = 0.39, size = 7, normalized size = 0.88

method	result
default	$-\ln \left(x -1\right)$
norman	$-\ln \left(x -1\right)$
risch	$-\ln \left(x -1\right)$
meijerg	$-\frac{\ln \left(-x^{5}+1\right)}{5}-\frac{x^{4} \left(\ln \left(1-\left(x^{5}\right)^{\frac{1}{5}}\right)+\cos \left(\frac{2 \pi}{5}\right) \ln \left(1-2 \cos \left(\frac{2 \pi}{5}\right) \left(x^{5}\right)^{\frac{1}{5}}+\left(x^{5}\right)^{\frac{2}{5}}\right)+2 \sin \left(\frac{2 \pi}{5}\right) \arctan \left(\frac{\sin \left(\frac{2 \pi}{5}\right) \left(x^{5}\right)^{\frac{1}{5}}}{1-\cos \left(\frac{2 \pi}{5}\right) \left(x^{5}\right)^{\frac{1}{5}}}\right)-\cos \left(\frac{\pi}{5}\right) \ln}{5 \left(x^{5}\right)^{\frac{4}{5}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3+x^2+x+1)/(-x^5+1),x,method=_RETURNVERBOSE)
[Out] -ln(x-1)
```

Maxima [A]
time = 0.29, size = 6, normalized size = 0.75

$$-\log \left(x -1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="maxima")
[Out] -log(x - 1)
```

Fricas [A]
time = 0.36, size = 6, normalized size = 0.75

$$-\log \left(x -1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="fricas")
[Out] -log(x - 1)
```

Sympy [A]
time = 0.01, size = 5, normalized size = 0.62

$$-\log \left(x -1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3+x**2+x+1)/(-x**5+1),x)
```

```
[Out] -log(x - 1)
```

Giac [A]
time = 0.82, size = 7, normalized size = 0.88

$$-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="giac")
```

```
[Out] -log(abs(x - 1))
```

Mupad [B]
time = 0.02, size = 6, normalized size = 0.75

$$-\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2 + x^3 + x^4 + 1)/(x^5 - 1),x)
```

```
[Out] -log(x - 1)
```

3.555 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\log(3+2x)$$

[Out] 1/2*ln(3+2*x)

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1600, 31}

$$\frac{1}{2}\log(2x+3)$$

Antiderivative was successfully verified.

[In] Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6),x]

[Out] Log[3 + 2*x]/2

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx &= \int \frac{1}{3+2x}\,dx\\ &= \frac{1}{2}\log(3+2x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{2}\log(3+2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)
,x]
```

```
[Out] Log[3 + 2*x]/2
```

Maple [A]
time = 0.44, size = 9, normalized size = 0.90

method	result
default	$\frac{\ln(2x+3)}{2}$
norman	$\frac{\ln(2x+3)}{2}$
risch	$\frac{\ln(2x+3)}{2}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\right.}{12(x^6)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/2*ln(2*x+3)
```

Maxima [A]
time = 0.27, size = 8, normalized size = 0.80

$$\frac{1}{2}\log\left(2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algori
thm="maxima")
```

```
[Out] 1/2*log(2*x + 3)
```

Fricas [A]
time = 0.37, size = 8, normalized size = 0.80

$$\frac{1}{2}\log\left(2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algori
thm="fricas")
```

```
[Out] 1/2*log(2*x + 3)
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.70

$$\frac{\log{(2x+3)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729),x)
```

```
[Out] log(2*x + 3)/2
```

Giac [A]
time = 0.67, size = 9, normalized size = 0.90

$$\frac{1}{2}\,\log\left(|2\,x+3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algori
thm="giac")
```

```
[Out] 1/2*log(abs(2*x + 3))
```

Mupad [B]
time = 0.06, size = 6, normalized size = 0.60

$$\frac{\ln\left(x+\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((162*x - 108*x^2 + 72*x^3 - 48*x^4 + 32*x^5 - 243)/(64*x^6 - 729),x)
```

```
[Out] log(x + 3/2)/2
```

3.556 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6} dx$

Optimal. Leaf size=10

$$-\frac{1}{2}\log(3-2x)$$

[Out] -1/2*ln(3-2*x)

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1600, 31}

$$-\frac{1}{2}\log(3-2x)$$

Antiderivative was successfully verified.

[In] Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6),x]

[Out] -1/2*Log[3 - 2*x]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx &= \int \frac{1}{3-2x}\,dx\\ &= -\frac{1}{2}\log(3-2x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{2}\log(3-2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)
,x]
```

```
[Out] -1/2*Log[3 - 2*x]
```

Maple [A]
time = 0.38, size = 9, normalized size = 0.90

method	result
default	$-\frac{\ln(-3+2x)}{2}$
norman	$-\frac{\ln(-3+2x)}{2}$
risch	$-\frac{\ln(-3+2x)}{2}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\cdots\right)}{12\left(x^{6}\right)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/2*ln(-3+2*x)
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.80

$$-\frac{1}{2}\,\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorit
hm="maxima")
```

```
[Out] -1/2*log(2*x - 3)
```

Fricas [A]
time = 0.35, size = 8, normalized size = 0.80

$$-\frac{1}{2}\,\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorit
hm="fricas")
```

```
[Out] -1/2*log(2*x - 3)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.80

$$-\frac{\log(2x-3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729),x)
```

```
[Out] -log(2*x - 3)/2
```

Giac [A]
time = 0.75, size = 9, normalized size = 0.90

$$-\frac{1}{2}\log(|2x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorit
hm="giac")
```

```
[Out] -1/2*log(abs(2*x - 3))
```

Mupad [B]
time = 4.99, size = 6, normalized size = 0.60

$$-\frac{\ln\left(x-\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5 + 243)/(64*x^6 - 729),x)
```

```
[Out] -log(x - 3/2)/2
```

3.557 $\int \frac{81+36x^2+16x^4}{729-64x^6} dx$

Optimal. Leaf size=10

$$\frac{1}{6}\tanh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 1/6*arctanh(2/3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1600, 212}

$$\frac{1}{6}\tanh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] ArcTanh[(2*x)/3]/6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx &= \int \frac{1}{9-4x^2}\,dx\\ &= \frac{1}{6}\tanh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 0.00, size = 21, normalized size = 2.10

$$-\frac{1}{12}\log(3-2x)+\frac{1}{12}\log(3+2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] -1/12*Log[3 - 2*x] + Log[3 + 2*x]/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.36, size = 18, normalized size = 1.80

method	result
default	$\frac{\ln(2x+3)}{12}-\frac{\ln(-3+2x)}{12}$
norman	$\frac{\ln(2x+3)}{12}-\frac{\ln(-3+2x)}{12}$
risch	$\frac{\ln(2x+3)}{12}-\frac{\ln(-3+2x)}{12}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-}{36\left(x^{6}\right)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((16*x^4+36*x^2+81)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*ln(2*x+3)-1/12*ln(-3+2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.28, size = 17, normalized size = 1.70

$$\frac{1}{12}\,\log\left(2\,x+3\right)-\frac{1}{12}\,\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.35, size = 17, normalized size = 1.70

$$\frac{1}{12}\,\log\left(2\,x+3\right)-\frac{1}{12}\,\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.
time = 0.03, size = 15, normalized size = 1.50

$$-\frac{\log\left(x-\frac{3}{2}\right)}{12}+\frac{\log\left(x+\frac{3}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x**4+36*x**2+81)/(-64*x**6+729),x)
[Out] -log(x - 3/2)/12 + log(x + 3/2)/12
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.
time = 0.99, size = 15, normalized size = 1.50

$$\frac{1}{12}\log\left(\left|x+\frac{3}{2}\right|\right)-\frac{1}{12}\log\left(\left|x-\frac{3}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="giac")
[Out] 1/12*log(abs(x + 3/2)) - 1/12*log(abs(x - 3/2))
```

Mupad [B]
time = 0.10, size = 6, normalized size = 0.60

$$\frac{\operatorname{atanh}\left(\frac{2\,x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(36*x^2 + 16*x^4 + 81)/(64*x^6 - 729),x)
[Out] atanh((2*x)/3)/6
```

3.558 $\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx$

Optimal. Leaf size=24

$$-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/9*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1600, 632, 210}

$$-\frac{\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] -1/3*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx = \int \frac{1}{9-6x+4x^2}\,dx$$
$$= -\left(2\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, -6+8x\right)\right)$$
$$= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])
```

Maple [A]
time = 0.37, size = 17, normalized size = 0.71

method	result
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{9}$
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{(-3+4x)\sqrt{3}}{9}\right)}{9}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{}\right.}{36\left(x^6\right)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x,method=_RETURNVERBOSE)
[Out] 1/9*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A]
time = 0.49, size = 16, normalized size = 0.67

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="maxima")
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

Fricas [A]
time = 0.35, size = 16, normalized size = 0.67

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="fricas")
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

Sympy [A]
time = 0.04, size = 24, normalized size = 1.00

$$\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729),x)
[Out] sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/9
```

Giac [A]
time = 0.88, size = 16, normalized size = 0.67

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="giac")
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.67

$$\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,(4\,x-3)}{9}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(54*x - 24*x^3 - 16*x^4 + 81)/(64*x^6 - 729),x)
[Out] (3^(1/2)*atan((3^(1/2)*(4*x - 3))/9))/9
```

3.559 $\int \frac{3-2x}{729-64x^6} dx$

Optimal. Leaf size=50

$$\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{162\sqrt{3}}+\frac{1}{486}\log(3+2x)-\frac{1}{972}\log\left(9-6x+4x^2\right)$$

```
[Out] 1/486*ln(3+2*x)-1/972*ln(4*x^2-6*x+9)+1/486*arctan(1/9*(3+4*x)*3^(1/2))*3^(
1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1600, 2083, 642, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6*
x + 4*x^2]/972
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{3-2x}{729-64x^6}\,dx &= \int \frac{1}{243+162x+108x^2+72x^3+48x^4+32x^5}\,dx\\
&= \int \left(\frac{1}{243(3+2x)}+\frac{3-4x}{486\,(9-6x+4x^2)}+\frac{1}{54\,(9+6x+4x^2)}\right)dx\\
&= \frac{1}{486}\log(3+2x)+\frac{1}{486}\int \frac{3-4x}{9-6x+4x^2}\,dx+\frac{1}{54}\int \frac{1}{9+6x+4x^2}\,dx\\
&= \frac{1}{486}\log(3+2x)-\frac{1}{972}\log\left(9-6x+4x^2\right)-\frac{1}{27}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,6+8x\right)\\
&= \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{162\sqrt{3}}+\frac{1}{486}\log(3+2x)-\frac{1}{972}\log\left(9-6x+4x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{162\sqrt{3}}+\frac{1}{486}\log(3+2x)-\frac{1}{972}\log\left(9-6x+4x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6*
x + 4*x^2]/972
```

Maple [A]
time = 0.37, size = 39, normalized size = 0.78

method	result

default	$-\frac{\ln\left(4x^{2}-6x+9\right)}{972}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{486}+\frac{\ln\left(2x+3\right)}{486}$
risch	$\frac{\ln\left(2x+3\right)}{486}-\frac{\ln\left(4x^{2}-6x+9\right)}{972}+\frac{\arctan\left(\frac{(3+4x)\sqrt{3}}{9}\right)\sqrt{3}}{486}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\right.}{972\left(x^{6}\right)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)/(-64*x^6+729),x,method=_RETURNVERBOSE)
[Out] -1/972*ln(4*x^2-6*x+9)+1/486*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/486*ln(
2*x+3)
```

Maxima [A]
time = 0.48, size = 38, normalized size = 0.76

$$\frac{1}{486}\,\sqrt{3}\,\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x+3)\right)-\frac{1}{972}\,\log\left(4\,x^{2}-6\,x+9\right)+\frac{1}{486}\,\log\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="maxima")
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
1/486*log(2*x + 3)
```

Fricas [A]
time = 0.36, size = 38, normalized size = 0.76

$$\frac{1}{486}\,\sqrt{3}\,\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x+3)\right)-\frac{1}{972}\,\log\left(4\,x^{2}-6\,x+9\right)+\frac{1}{486}\,\log\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="fricas")
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
1/486*log(2*x + 3)
```

Sympy [A]
time = 0.07, size = 46, normalized size = 0.92

$$\frac{\log\left(x+\frac{3}{2}\right)}{486}-\frac{\log\left(4x^{2}-6x+9\right)}{972}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}+\frac{\sqrt{3}}{3}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x**6+729),x)
```

```
[Out] log(x + 3/2)/486 - log(4*x**2 - 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9 +
 sqrt(3)/3)/486
```

Giac [A]
time = 1.09, size = 39, normalized size = 0.78

$$\frac{1}{486}\sqrt{3}\,\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x+3)\right)-\frac{1}{972}\,\log\left(4\,x^2-6\,x+9\right)+\frac{1}{486}\,\log\left(|2\,x+3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
1/486*log(abs(2*x + 3))
```

Mupad [B]
time = 0.13, size = 49, normalized size = 0.98

$$\frac{\ln\left(x+\frac{3}{2}\right)}{486}-\frac{\ln\left(x^2-\frac{3\,x}{2}+\frac{9}{4}\right)}{972}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{1327104\,\left(\frac{x}{884736}+\frac{1}{884736}\right)}-\frac{\sqrt{3}\,x}{7962624\,\left(\frac{x}{884736}+\frac{1}{884736}\right)}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3)/(64*x^6 - 729),x)
```

```
[Out] log(x + 3/2)/486 - log(x^2 - (3*x)/2 + 9/4)/972 - (3^(1/2)*atan(3^(1/2)/(13
27104*(x/884736 + 1/884736)) - (3^(1/2)*x)/(7962624*(x/884736 + 1/884736)))
)/486
```

3.560 $\int \frac{3+2x}{729-64x^6} \, dx$

Optimal. Leaf size=50

$$-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}-\frac{1}{486}\log(3-2x)+\frac{1}{972}\log\left(9+6x+4x^2\right)$$

```
[Out] -1/486*ln(3-2*x)+1/972*ln(4*x^2+6*x+9)-1/486*arctan(1/9*(3-4*x)*3^(1/2))*3^
(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1600, 2083, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}+\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log(3-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(729 - 64*x^6),x]
```

```
[Out] -1/162*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] - Log[3 - 2*x]/486 + Log[9 + 6
*x + 4*x^2]/972
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{3+2x}{729-64x^6}\,dx &= \int \frac{1}{243-162x+108x^2-72x^3+48x^4-32x^5}\,dx\\
&= \int \left(-\frac{1}{243(-3+2x)}+\frac{1}{54\,(9-6x+4x^2)}+\frac{3+4x}{486\,(9+6x+4x^2)}\right)dx\\
&= -\frac{1}{486}\log(3-2x)+\frac{1}{486}\int \frac{3+4x}{9+6x+4x^2}\,dx+\frac{1}{54}\int \frac{1}{9-6x+4x^2}\,dx\\
&= -\frac{1}{486}\log(3-2x)+\frac{1}{972}\log\left(9+6x+4x^2\right)-\frac{1}{27}\mathrm{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)\\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}-\frac{1}{486}\log(3-2x)+\frac{1}{972}\log\left(9+6x+4x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.92

$$\frac{1}{972}\left(2\sqrt{3}\,\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right)-2\log(3-2x)+\log\left(9+6x+4x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(729 - 64*x^6),x]
[Out] (2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 2*Log[3 - 2*x] + Log[9 + 6*x +
4*x^2])/972
```

Maple [A]
time = 0.37, size = 39, normalized size = 0.78

method	result
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{486}+\frac{\ln(4x^2+6x+9)}{972}-\frac{\ln(-3+2x)}{486}$

risch	$-\frac{\ln(-3+2x)}{486}+\frac{\sqrt{3}\,\arctan\left(\frac{(-3+4x)\sqrt{3}}{9}\right)}{486}+\frac{\ln(4x^2+6x+9)}{972}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\right.}{972(x^6)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+3)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

```
[Out] 1/486*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/972*ln(4*x^2+6*x+9)-1/486*ln(-
3+2*x)
```

Maxima [A]
time = 0.50, size = 38, normalized size = 0.76

$$\frac{1}{486}\sqrt{3}\,\arctan\left(\frac{1}{9}\sqrt{3}\,(4x-3)\right)+\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
1/486*log(2*x - 3)
```

Fricas [A]
time = 0.39, size = 38, normalized size = 0.76

$$\frac{1}{486}\sqrt{3}\,\arctan\left(\frac{1}{9}\sqrt{3}\,(4x-3)\right)+\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
1/486*log(2*x - 3)
```

Sympy [A]
time = 0.07, size = 46, normalized size = 0.92

$$-\frac{\log\left(x-\frac{3}{2}\right)}{486}+\frac{\log\left(4x^2+6x+9\right)}{972}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/486 + log(4*x**2 + 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9
- sqrt(3)/3)/486
```

Giac [A]
time = 0.80, size = 39, normalized size = 0.78

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
1/486*log(abs(2*x - 3))
```

Mupad [B]
time = 4.99, size = 48, normalized size = 0.96

$$\frac{\ln\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{972}-\frac{\ln\left(x-\frac{3}{2}\right)}{486}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{1327104\left(\frac{x}{884736}-\frac{1}{884736}\right)}+\frac{\sqrt{3}\,x}{7962624\left(\frac{x}{884736}-\frac{1}{884736}\right)}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + 3)/(64*x^6 - 729),x)
```

```
[Out] log((3*x)/2 + x^2 + 9/4)/972 - log(x - 3/2)/486 - (3^(1/2)*atan(3^(1/2)/(13
27104*(x/884736 - 1/884736)) + (3^(1/2)*x)/(7962624*(x/884736 - 1/884736)))
)/486
```

3.561 $\int \frac{9-6x+4x^2}{729-64x^6} \, dx$

Optimal. Leaf size=60

$$\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{54\sqrt{3}} - \frac{1}{324}\log(3-2x) + \frac{1}{108}\log(3+2x) - \frac{1}{324}\log\left(9+6x+4x^2\right)$$

```
[Out] -1/324*ln(3-2*x)+1/108*ln(3+2*x)-1/324*ln(4*x^2+6*x+9)+1/162*arctan(1/9*(3+
4*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1600, 2083, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{4x+3}{3\sqrt{3}}\right)}{54\sqrt{3}} - \frac{1}{324}\log\left(4x^2+6x+9\right) - \frac{1}{324}\log(3-2x) + \frac{1}{108}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/324 + Log[3 + 2*x
]/108 - Log[9 + 6*x + 4*x^2]/324
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{9-6x+4x^2}{729-64x^6}\,dx &= \int \frac{1}{81+54x-24x^3-16x^4}\,dx \\
&= \int \left(-\frac{1}{162(-3+2x)}+\frac{1}{54(3+2x)}+\frac{3-2x}{81\left(9+6x+4x^2\right)}\right)dx \\
&= -\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)+\frac{1}{81}\int \frac{3-2x}{9+6x+4x^2}\,dx \\
&= -\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)-\frac{1}{324}\int \frac{6+8x}{9+6x+4x^2}\,dx+\frac{1}{18}\int \frac{1}{9+6x+4x^2}\,\mathrm{d} \\
&= -\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)-\frac{1}{324}\log\left(9+6x+4x^2\right)-\frac{1}{9}\mathrm{Subst}\left(\int \frac{1}{-108-} \right. \\
&= \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{54\sqrt{3}}-\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)-\frac{1}{324}\log\left(9+6x+4x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 0.93

$$
\frac{1}{324}\left(2\sqrt{3}\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)-\log(3-2x)+3\log(3+2x)-\log\left(9+6x+4x^2\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

[Out] (2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - Log[3 - 2*x] + 3*Log[3 + 2*x] - Log[9 + 6*x + 4*x^2])/324

Maple [A]
time = 0.37, size = 47, normalized size = 0.78

method	result
default	$-\frac{\ln(4x^2+6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{162}+\frac{\ln(2x+3)}{108}-\frac{\ln(-3+2x)}{324}$
risch	$-\frac{\ln(-3+2x)}{324}+\frac{\ln(2x+3)}{108}-\frac{\ln(4x^2+6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{162}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\cdots\right.}{324(x^6)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2-6*x+9)/(-64*x^6+729),x,method=_RETURNVERBOSE)

[Out] -1/324*ln(4*x^2+6*x+9)+1/162*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/108*ln(2*x+3)-1/324*ln(-3+2*x)

Maxima [A]
time = 0.49, size = 46, normalized size = 0.77

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{324}\log\left(4x^2+6x+9\right)+\frac{1}{108}\log(2x+3)-\frac{1}{324}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="maxima")

[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) + 1/108*log(2*x + 3) - 1/324*log(2*x - 3)

Fricas [A]
time = 0.37, size = 46, normalized size = 0.77

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{324}\log\left(4x^2+6x+9\right)+\frac{1}{108}\log(2x+3)-\frac{1}{324}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="fricas")

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
1/108*log(2*x + 3) - 1/324*log(2*x - 3)
```

Sympy [A]
time = 0.08, size = 56, normalized size = 0.93

$$-\frac{\log\left(x-\frac{3}{2}\right)}{324}+\frac{\log\left(x+\frac{3}{2}\right)}{108}-\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{324}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}+\frac{\sqrt{3}}{3}\right)}{162}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-6*x+9)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/324 + log(x + 3/2)/108 - log(x**2 + 3*x/2 + 9/4)/324 + sqrt(3
)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/162
```

Giac [A]
time = 0.88, size = 48, normalized size = 0.80

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x+3)\right)-\frac{1}{324}\log\left(4\,x^2+6\,x+9\right)+\frac{1}{108}\log\left(|2\,x+3|\right)-\frac{1}{324}\log\left(|2\,x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
1/108*log(abs(2*x + 3)) - 1/324*log(abs(2*x - 3))
```

Mupad [B]
time = 5.01, size = 52, normalized size = 0.87

$$\frac{\ln\left(x+\frac{3}{2}\right)}{108}-\frac{\ln\left(x-\frac{3}{2}\right)}{324}-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x^2 - 6*x + 9)/(64*x^6 - 729),x)
```

```
[Out] log(x + 3/2)/108 - log(x - 3/2)/324 - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/
2)*1i)/324 + 1/324) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/324 - 1/3
24)
```

3.562 $\int \frac{9+6x+4x^2}{729-64x^6} \, dx$

Optimal. Leaf size=60

$$-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{324}\log\left(9-6x+4x^2\right)$$

```
[Out] -1/108*ln(3-2*x)+1/324*ln(3+2*x)+1/324*ln(4*x^2-6*x+9)-1/162*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1600, 2083, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}+\frac{1}{324}\log\left(4x^2-6x+9\right)-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] -1/54*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] - Log[3 - 2*x]/108 + Log[3 + 2*x]/324 + Log[9 - 6*x + 4*x^2]/324
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{9+6x+4x^2}{729-64x^6}\,dx &= \int \frac{1}{81-54x+24x^3-16x^4}\,dx \\
&= \int \left(-\frac{1}{54(-3+2x)}+\frac{1}{162(3+2x)}+\frac{3+2x}{81\,(9-6x+4x^2)}\right)dx \\
&= -\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{81}\int \frac{3+2x}{9-6x+4x^2}\,dx \\
&= -\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{324}\int \frac{-6+8x}{9-6x+4x^2}\,dx+\frac{1}{18}\int \frac{1}{9-6x+4x^2}\,\mathrm{c} \\
&= -\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{324}\log\left(9-6x+4x^2\right)-\frac{1}{9}\mathrm{Subst}\left(\int \frac{1}{-108-\ldots}\right. \\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{324}\log\left(9-6x+4x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 0.87

$$\frac{1}{324}\left(2\sqrt{3}\,\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right)-3\log(3-2x)+\log(3+2x)+\log\left(9-6x+4x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

[Out] (2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + Log[3 + 2*x] + Log[9 - 6*x + 4*x^2])/324

Maple [A]
time = 0.36, size = 47, normalized size = 0.78

method	result
default	$\frac{\ln(4x^2-6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{162}+\frac{\ln(2x+3)}{324}-\frac{\ln(-3+2x)}{108}$
risch	$-\frac{\ln(-3+2x)}{108}+\frac{\ln(4x^2-6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x-\frac{3}{2}\right)\sqrt{3}}{9}\right)}{162}+\frac{\ln(2x+3)}{324}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\right.}{324(x^6)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2+6*x+9)/(-64*x^6+729),x,method=_RETURNVERBOSE)

[Out] 1/324*ln(4*x^2-6*x+9)+1/162*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/324*ln(2*x+3)-1/108*ln(-3+2*x)

Maxima [A]
time = 0.49, size = 46, normalized size = 0.77

$$\frac{1}{162}\sqrt{3}\,\arctan\left(\frac{1}{9}\sqrt{3}\,(4x-3)\right)+\frac{1}{324}\log\left(4x^2-6x+9\right)+\frac{1}{324}\log(2x+3)-\frac{1}{108}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="maxima")

[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) + 1/324*log(2*x + 3) - 1/108*log(2*x - 3)

Fricas [A]
time = 0.36, size = 46, normalized size = 0.77

$$\frac{1}{162}\sqrt{3}\,\arctan\left(\frac{1}{9}\sqrt{3}\,(4x-3)\right)+\frac{1}{324}\log\left(4x^2-6x+9\right)+\frac{1}{324}\log(2x+3)-\frac{1}{108}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="fricas")

[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) + 1/324*log(2*x + 3) - 1/108*log(2*x - 3)

Sympy [A]
time = 0.09, size = 56, normalized size = 0.93

$$-\frac{\log\left(x-\frac{3}{2}\right)}{108}+\frac{\log\left(x+\frac{3}{2}\right)}{324}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{324}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{162}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**2+6*x+9)/(-64*x**6+729),x)

[Out] -log(x - 3/2)/108 + log(x + 3/2)/324 + log(x**2 - 3*x/2 + 9/4)/324 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/162

Giac [A]
time = 0.97, size = 48, normalized size = 0.80

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{324}\log\left(4x^2-6x+9\right)+\frac{1}{324}\log\left(|2x+3|\right)-\frac{1}{108}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="giac")

[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) + 1/324*log(abs(2*x + 3)) - 1/108*log(abs(2*x - 3))

Mupad [B]
time = 4.98, size = 52, normalized size = 0.87

$$\frac{\ln\left(x+\frac{3}{2}\right)}{324}-\frac{\ln\left(x-\frac{3}{2}\right)}{108}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(6*x + 4*x^2 + 9)/(64*x^6 - 729),x)

[Out] log(x + 3/2)/324 - log(x - 3/2)/108 - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/324 - 1/324) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/324 + 1/324)

3.563 $\int \frac{27-8x^3}{729-64x^6} \, dx$

Optimal. Leaf size=50

$$-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}+\frac{1}{54}\log(3+2x)-\frac{1}{108}\log\left(9-6x+4x^2\right)$$

```
[Out] 1/54*ln(3+2*x)-1/108*ln(4*x^2-6*x+9)-1/54*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {26, 206, 31, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}-\frac{1}{108}\log\left(4x^2-6x+9\right)+\frac{1}{54}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(27 - 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] -1/18*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] + Log[3 + 2*x]/54 - Log[9 - 6*x
 + 4*x^2]/108
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{27-8x^3}{729-64x^6}\,dx &= \int \frac{1}{27+8x^3}\,dx \\
&= \frac{1}{27}\int \frac{1}{3+2x}\,dx + \frac{1}{27}\int \frac{6-2x}{9-6x+4x^2}\,dx \\
&= \frac{1}{54}\log(3+2x) - \frac{1}{108}\int \frac{-6+8x}{9-6x+4x^2}\,dx + \frac{1}{6}\int \frac{1}{9-6x+4x^2}\,dx \\
&= \frac{1}{54}\log(3+2x) - \frac{1}{108}\log\left(9-6x+4x^2\right) - \frac{1}{3}\mathrm{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, -6+8x\right) \\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}} + \frac{1}{54}\log(3+2x) - \frac{1}{108}\log\left(9-6x+4x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 50, normalized size = 1.00

$$
\frac{\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right)}{18\sqrt{3}} + \frac{1}{54}\log(3+2x) - \frac{1}{108}\log\left(9-6x+4x^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 - 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - Log[9 - 6*x
 + 4*x^2]/108
```

Maple [A]
time = 0.36, size = 39, normalized size = 0.78

method	result
default	$-\frac{\ln(4x^2-6x+9)}{108}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{54}+\frac{\ln(2x+3)}{54}$
risch	$-\frac{\ln(16x^2-24x+36)}{108}+\frac{\sqrt{3}\,\arctan\left(\frac{(-3+4x)\sqrt{3}}{9}\right)}{54}+\frac{\ln(2x+3)}{54}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\right.}{108(x^6)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-8*x^3+27)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

```
[Out] -1/108*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/54*ln(2*
x+3)
```

Maxima [A]
time = 0.50, size = 38, normalized size = 0.76

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)-\frac{1}{108}\log\left(4\,x^2-6\,x+9\right)+\frac{1}{54}\log\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) + 1
/54*log(2*x + 3)
```

Fricas [A]
time = 0.39, size = 38, normalized size = 0.76

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)-\frac{1}{108}\log\left(4\,x^2-6\,x+9\right)+\frac{1}{54}\log\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) + 1
/54*log(2*x + 3)
```

Sympy [A]
time = 0.05, size = 48, normalized size = 0.96

$$\frac{\log\left(x+\frac{3}{2}\right)}{54}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{108}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x**3+27)/(-64*x**6+729),x)
```

```
[Out] log(x + 3/2)/54 - log(x**2 - 3*x/2 + 9/4)/108 + sqrt(3)*atan(4*sqrt(3)*x/9
- sqrt(3)/3)/54
```

Giac [A]
time = 0.68, size = 35, normalized size = 0.70

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)-\frac{1}{108}\log\left(x^2-\frac{3}{2}\,x+\frac{9}{4}\right)+\frac{1}{54}\log\left(\left|x+\frac{3}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(x^2 - 3/2*x + 9/4) +
 1/54*log(abs(x + 3/2))
```

Mupad [B]
time = 0.09, size = 46, normalized size = 0.92

$$\frac{\ln\left(x+\frac{3}{2}\right)}{54}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3 - 27)/(64*x^6 - 729),x)
```

```
[Out] log(x + 3/2)/54 - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 + 1/108)
+ log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 - 1/108)
```

3.564 $\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx$

Optimal. Leaf size=50

$$-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}-\frac{1}{18}\log(3-2x)+\frac{1}{36}\log\left(9-6x+4x^2\right)$$

```
[Out] -1/18*ln(3-2*x)+1/36*ln(4*x^2-6*x+9)-1/54*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1600, 2083, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(3-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] -1/18*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] - Log[3 - 2*x]/18 + Log[9 - 6*x
 + 4*x^2]/36
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx &= \int \frac{1}{27-36x+24x^2-8x^3}\,dx \\
&= \int \left(-\frac{1}{9(-3+2x)}+\frac{2x}{9\left(9-6x+4x^2\right)}\right)dx \\
&= -\frac{1}{18}\log(3-2x)+\frac{2}{9}\int \frac{x}{9-6x+4x^2}\,dx \\
&= -\frac{1}{18}\log(3-2x)+\frac{1}{36}\int \frac{-6+8x}{9-6x+4x^2}\,dx+\frac{1}{6}\int \frac{1}{9-6x+4x^2}\,dx \\
&= -\frac{1}{18}\log(3-2x)+\frac{1}{36}\log\left(9-6x+4x^2\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6
\end{aligned}$$

$$= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}-\frac{1}{18}\log(3-2x)+\frac{1}{36}\log\left(9-6x+4x^2\right)$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right)}{18\sqrt{3}}-\frac{1}{18}\log(3-2x)+\frac{1}{36}\log\left(9-6x+4x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + Log[9 - 6*x + 4*x^2]/36

Maple [A]
time = 0.50, size = 39, normalized size = 0.78

method	result
default	$\frac{\ln(4x^2-6x+9)}{36}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{54}-\frac{\ln(-3+2x)}{18}$
risch	$-\frac{\ln(-3+2x)}{18}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x-\frac{3}{2}\right)\sqrt{3}}{9}\right)}{54}+\frac{\ln(4x^2-6x+9)}{36}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\right.}{108(x^6)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x,method=_RETURNVERBOSE)

[Out] 1/36*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-1/18*ln(-3+2*x)

Maxima [A]
time = 0.50, size = 38, normalized size = 0.76

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4x-3)\right)+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="maxima")

[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/18*log(2*x - 3)

Fricas [A]
time = 0.37, size = 38, normalized size = 0.76

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4x-3)\right)+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="fricas")

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/
18*log(2*x - 3)
```

Sympy [A]
time = 0.06, size = 48, normalized size = 0.96

$$-\frac{\log\left(x-\frac{3}{2}\right)}{18}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{36}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/18 + log(x**2 - 3*x/2 + 9/4)/36 + sqrt(3)*atan(4*sqrt(3)*x/9
- sqrt(3)/3)/54
```

Giac [A]
time = 0.81, size = 39, normalized size = 0.78

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)+\frac{1}{36}\log\left(4\,x^2-6\,x+9\right)-\frac{1}{18}\log\left(|2\,x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/
18*log(abs(2*x - 3))
```

Mupad [B]
time = 0.10, size = 46, normalized size = 0.92

$$-\frac{\ln\left(x-\frac{3}{2}\right)}{18}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(-\frac{1}{36}+\frac{\sqrt{3}\ 1\mathrm{i}}{108}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(\frac{1}{36}+\frac{\sqrt{3}\ 1\mathrm{i}}{108}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(36*x + 24*x^2 + 8*x^3 + 27)/(64*x^6 - 729),x)
```

```
[Out] log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 + 1/36) - log(x - (3^(1/2)*
3i)/4 - 3/4)*((3^(1/2)*1i)/108 - 1/36) - log(x - 3/2)/18
```

3.565 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{1}{2916(3+2x)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}-\frac{\log(3-2x)}{17496}+\frac{5\log(3+2x)}{17496}-\frac{\log(9-6x+4x^2)}{17496}-\frac{\log(9}{}$$

```
[Out] -1/2916/(3+2*x)-1/17496*ln(3-2*x)+5/17496*ln(3+2*x)-1/17496*ln(4*x^2-6*x+9)
-1/17496*ln(4*x^2+6*x+9)-1/26244*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/8748
*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1600, 2099, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{4x+3}{3\sqrt{3}}\right)}{2916\sqrt{3}}-\frac{\log(4x^2-6x+9)}{17496}-\frac{\log(4x^2+6x+9)}{17496}-\frac{1}{2916(2x+3)}-\frac{\log(3-2x)}{17496}+\frac{5\log(2x+3)}{17496}$$

Antiderivative was successfully verified.

```
[In] Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)^2,x]
```

```
[Out] -1/2916*1/(3 + 2*x) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) + ArcTan
[(3 + 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) - Log[3 - 2*x]/17496 + (5*Log[3 + 2*
x])/17496 - Log[9 - 6*x + 4*x^2]/17496 - Log[9 + 6*x + 4*x^2]/17496
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{(3+2x)^2\left(243-162x+108x^2-72x^3+48x^4-32x^5\right.}$$

$$= \int \left(-\frac{1}{8748(-3+2x)}+\frac{1}{1458(3+2x)^2}+\frac{5}{8748(3+2x)}\right.$$

$$= -\frac{1}{2916(3+2x)}-\frac{\log(3-2x)}{17496}+\frac{5\log(3+2x)}{17496}+\frac{\int \frac{3-}{9-6x}}{43}$$

$$= -\frac{1}{2916(3+2x)}-\frac{\log(3-2x)}{17496}+\frac{5\log(3+2x)}{17496}-\frac{\int \frac{-6+}{9-6x}}{17}$$

$$= -\frac{1}{2916(3+2x)}-\frac{\log(3-2x)}{17496}+\frac{5\log(3+2x)}{17496}-\frac{\log(9}{}$$

$$= -\frac{1}{2916(3+2x)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}-$$

Mathematica [A]
time = 0.06, size = 100, normalized size = 0.91

$$\frac{-\frac{18}{3+2x}+2\sqrt{3}\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right)+6\sqrt{3}\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)-3\log(3-2x)+15\log(3+2x)-3\log\left(9-6x+4x^2\right)-3\log\left(9+6x+4x^2\right)}{52488}$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)
^2,x]
```

```
[Out] (-18/(3 + 2*x) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqrt[3]*ArcTa
n[(3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + 15*Log[3 + 2*x] - 3*Log[9 - 6*x
 + 4*x^2] - 3*Log[9 + 6*x + 4*x^2])/52488
```

Maple [A]
time = 0.39, size = 85, normalized size = 0.77

method	result
risch	$-\frac{1}{5832\left(x+\frac{3}{2}\right)}+\frac{\sqrt{3}\,\arctan\left(\frac{(-3+4x)\sqrt{3}}{9}\right)}{26244}-\frac{\ln\left(16x^{2}-24x+36\right)}{17496}-\frac{\ln(-3+2x)}{17496}+\frac{5\ln(2x+3)}{17496}-\frac{\ln\left(4x^{2}+6x+9\right)}{17496}+$
default	$-\frac{\ln\left(4x^{2}-6x+9\right)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{26244}-\frac{\ln\left(4x^{2}+6x+9\right)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{8748}-\frac{1}{2916(2x+3)}+\frac{5\ln(2}{17}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^{6}}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\right.}{}\right.}{6\left(x^{6}\right)^{\frac{1}{6}}}\right.}{8748}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x,method=_RET
URNVERBOSE)
```

```
[Out] -1/17496*ln(4*x^2-6*x+9)+1/26244*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-1/174
96*ln(4*x^2+6*x+9)+1/8748*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/2916/(2*x+
3)+5/17496*ln(2*x+3)-1/17496*ln(-3+2*x)
```

Maxima [A]
time = 0.52, size = 84, normalized size = 0.76

$$\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916(2x+3)}-\frac{1}{17496}\log\left(4x^{2}+6x+9\right)-\frac{1}{17496}\log\left(4x^{2}-6x+9\right)+\frac{5}{17496}\log(2x+3)-\frac{1}{17496}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algo
rithm="maxima")
```

```
[Out] 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/174
96*log(4*x^2 - 6*x + 9) + 5/17496*log(2*x + 3) - 1/17496*log(2*x - 3)
```

Fricas [A]
time = 0.38, size = 115, normalized size = 1.05

$$\frac{6\sqrt{3}\,(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}\,(4x+3)\right)+2\sqrt{3}\,(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}\,(4x-3)\right)-3\,(2x+3)\log\left(4x^2+6x+9\right)-3\,(2x+3)\log\left(4x^2-6x+9\right)+15\,(2x+3)\log\left(2x+3\right)-3\,(2x+3)\log\left(2x-3\right)-18}{52488\,(2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algo
rithm="fricas")
```

```
[Out] 1/52488*(6*sqrt(3)*(2*x + 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(2*x
 + 3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 3*(2*x + 3)*log(4*x^2 + 6*x + 9) - 3*
(2*x + 3)*log(4*x^2 - 6*x + 9) + 15*(2*x + 3)*log(2*x + 3) - 3*(2*x + 3)*lo
g(2*x - 3) - 18)/(2*x + 3)
```

Sympy [A]
time = 0.20, size = 105, normalized size = 0.95

$$-\frac{\log\left(x-\frac{3}{2}\right)}{17496}+\frac{5\log\left(x+\frac{3}{2}\right)}{17496}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{17496}-\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{26244}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}+\frac{\sqrt{3}}{3}\right)}{8748}-\frac{1}{5832x+8748}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729)**2,x
)
```

```
[Out] -log(x - 3/2)/17496 + 5*log(x + 3/2)/17496 - log(x**2 - 3*x/2 + 9/4)/17496
- log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/2
6244 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/8748 - 1/(5832*x + 8748)
```

Giac [A]
time = 0.83, size = 86, normalized size = 0.78

$$\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4x+3)\right)+\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4x-3)\right)-\frac{1}{2916\,(2x+3)}-\frac{1}{17496}\log\left(4x^2+6x+9\right)-\frac{1}{17496}\log\left(4x^2-6x+9\right)+\frac{5}{17496}\log\left(|2x+3|\right)-\frac{1}{17496}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algo
rithm="giac")
```

```
[Out] 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/174
96*log(4*x^2 - 6*x + 9) + 5/17496*log(abs(2*x + 3)) - 1/17496*log(abs(2*x -
 3))
```

<u>Mupad</u> [B]
time = 5.10, size = 100, normalized size = 0.91

$$\frac{5\ln\left(x+\frac{3}{2}\right)}{17496}-\frac{\ln\left(x-\frac{3}{2}\right)}{17496}-\frac{1}{5832\left(x+\frac{3}{2}\right)}-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\ 1\mathrm{i}}{17496}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\ 1\mathrm{i}}{17496}\right)-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\ 1\mathrm{i}}{52488}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\ 1\mathrm{i}}{52488}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(162*x - 108*x^2 + 72*x^3 - 48*x^4 + 32*x^5 - 243)/(64*x^6 - 729)^2,x)
```

```
[Out] (5*log(x + 3/2))/17496 - log(x - 3/2)/17496 - 1/(5832*(x + 3/2)) - log(x -
(3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/17496 + 1/17496) + log(x + (3^(1/2)*3i)
/4 + 3/4)*((3^(1/2)*1i)/17496 - 1/17496) - log(x - (3^(1/2)*3i)/4 - 3/4)*((
3^(1/2)*1i)/52488 + 1/17496) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/
52488 - 1/17496)
```

3.566 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2} dx$

Optimal. Leaf size=110

$$\frac{1}{2916(3-2x)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}-\frac{5\log(3-2x)}{17496}+\frac{\log(3+2x)}{17496}+\frac{\log(9-6x+4x^2)}{17496}+\frac{\log(9+$$

```
[Out] 1/2916/(3-2*x)-5/17496*ln(3-2*x)+1/17496*ln(3+2*x)+1/17496*ln(4*x^2-6*x+9)+
1/17496*ln(4*x^2+6*x+9)-1/8748*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/26244*
arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1600, 2099, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{4x+3}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\log(4x^2-6x+9)}{17496}+\frac{\log(4x^2+6x+9)}{17496}+\frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(2x+3)}{17496}$$

Antiderivative was successfully verified.

```
[In] Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(2916*(3 - 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) + ArcTan[
(3 + 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) - (5*Log[3 - 2*x])/17496 + Log[3 + 2*
x]/17496 + Log[9 - 6*x + 4*x^2]/17496 + Log[9 + 6*x + 4*x^2]/17496
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{\left(729-64x^6\right)^2}\,dx &= \int \frac{1}{(3-2x)^2\left(243+162x+108x^2+72x^3+48x^4+32x\right.}\\
&= \int \left(\frac{1}{1458(-3+2x)^2}-\frac{5}{8748(-3+2x)}+\frac{1}{8748(3+2x)}\right.\\
&= \frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(3+2x)}{17496}+\frac{\int\frac{3+2}{9-6x+}}{43}\\
&= \frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(3+2x)}{17496}+\frac{\int\frac{-6+}{9-6x+}}{174}\\
&= \frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(3+2x)}{17496}+\frac{\log\left(9-\right.}{1}\\
&= \frac{1}{2916(3-2x)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}-\frac{5}{}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 97, normalized size = 0.88

$$\frac{6\sqrt{3}\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)+3\left(\frac{6}{3-2x}-5\log(3-2x)+\log(3+2x)+\log\left(9-6x+4x^2\right)+\log\left(9+6x+4x^2\right)\right)}{52488}$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)
^2,x]
```

```
[Out] (6*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*S
qrt[3])] + 3*(6/(3 - 2*x) - 5*Log[3 - 2*x] + Log[3 + 2*x] + Log[9 - 6*x + 4
*x^2] + Log[9 + 6*x + 4*x^2]))/52488
```

Maple [A]
time = 0.38, size = 85, normalized size = 0.77

method	result
risch	$-\frac{1}{5832\left(x-\frac{3}{2}\right)}-\frac{5\ln(-3+2x)}{17496}+\frac{\ln(2x+3)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{26244}+\frac{\ln(4x^2+6x+9)}{17496}+\frac{\ln(4x^2-6x+9)}{17496}+\frac{\sqrt{3}\ \mathrm{a}}{}$
default	$\frac{\ln(4x^2-6x+9)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{8748}+\frac{\ln(4x^2+6x+9)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{26244}+\frac{\ln(2x+3)}{17496}-\frac{1}{2916(-3+2x}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(}{}\right.}{}\right.}{6\left(x^6\right)^{\frac{1}{6}}}\right.}{8748}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/17496*ln(4*x^2-6*x+9)+1/8748*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/17496
*ln(4*x^2+6*x+9)+1/26244*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/17496*ln(2*
x+3)-1/2916/(-3+2*x)-5/17496*ln(-3+2*x)
```

Maxima [A]
time = 0.52, size = 84, normalized size = 0.76

$$\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916(2x-3)}+\frac{1}{17496}\log\left(4x^2+6x+9\right)+\frac{1}{17496}\log\left(4x^2-6x+9\right)+\frac{1}{17496}\log(2x+3)-\frac{5}{17496}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algor
ithm="maxima")
```

```
[Out] 1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/174
96*log(4*x^2 - 6*x + 9) + 1/17496*log(2*x + 3) - 5/17496*log(2*x - 3)
```

Fricas [A]
time = 0.40, size = 115, normalized size = 1.05

$$\frac{2\sqrt{3}\,(2\,x-3)\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x+3)\right)+6\sqrt{3}\,(2\,x-3)\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)+3\,(2\,x-3)\log\left(4\,x^2+6\,x+9\right)+3\,(2\,x-3)\log\left(4\,x^2-6\,x+9\right)+3\,(2\,x-3)\log\left(2\,x+3\right)-15\,(2\,x-3)\log\left(2\,x-3\right)-18}{52488\,(2\,x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algor
ithm="fricas")
```

```
[Out] 1/52488*(2*sqrt(3)*(2*x - 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 6*sqrt(3)*(2*x
 - 3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(2*x - 3)*log(4*x^2 + 6*x + 9) + 3*
(2*x - 3)*log(4*x^2 - 6*x + 9) + 3*(2*x - 3)*log(2*x + 3) - 15*(2*x - 3)*lo
g(2*x - 3) - 18)/(2*x - 3)
```

Sympy [A]
time = 0.20, size = 105, normalized size = 0.95

$$-\frac{5\log\left(x-\frac{3}{2}\right)}{17496}+\frac{\log\left(x+\frac{3}{2}\right)}{17496}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{8748}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}+\frac{\sqrt{3}}{3}\right)}{26244}-\frac{1}{5832x-8748}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729)**2,x)
```

```
[Out] -5*log(x - 3/2)/17496 + log(x + 3/2)/17496 + log(x**2 - 3*x/2 + 9/4)/17496
+ log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/8
748 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/26244 - 1/(5832*x - 8748)
```

Giac [A]
time = 0.82, size = 86, normalized size = 0.78

$$\frac{1}{26244}\sqrt{3}\,\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x+3)\right)+\frac{1}{8748}\sqrt{3}\,\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)-\frac{1}{2916\,(2\,x-3)}+\frac{1}{17496}\log\left(4\,x^2+6\,x+9\right)+\frac{1}{17496}\log\left(4\,x^2-6\,x+9\right)+\frac{1}{17496}\log\left(|2\,x+3|\right)-\frac{5}{17496}\log\left(|2\,x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algor
ithm="giac")
```

```
[Out] 1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/174
96*log(4*x^2 - 6*x + 9) + 1/17496*log(abs(2*x + 3)) - 5/17496*log(abs(2*x -
 3))
```

Mupad [B]
time = 0.19, size = 100, normalized size = 0.91

$$\frac{\ln\left(x+\frac{3}{2}\right)}{17496}-\frac{5\,\ln\left(x-\frac{3}{2}\right)}{17496}-\frac{1}{5832\,\left(x-\frac{3}{2}\right)}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3{}\mathrm{i}}{4}\right)\,\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1{}\mathrm{i}}{17496}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3{}\mathrm{i}}{4}\right)\,\left(\frac{1}{17496}+\frac{\sqrt{3}\,1{}\mathrm{i}}{17496}\right)-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3{}\mathrm{i}}{4}\right)\,\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1{}\mathrm{i}}{52488}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3{}\mathrm{i}}{4}\right)\,\left(\frac{1}{17496}+\frac{\sqrt{3}\,1{}\mathrm{i}}{52488}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5 + 243)/(64*x^6 - 729)^2,x)

[Out] log(x + 3/2)/17496 - (5*log(x - 3/2))/17496 - 1/(5832*(x - 3/2)) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/17496 - 1/17496) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/17496 + 1/17496) - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/52488 - 1/17496) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/52488 + 1/17496)

3.567 $\int \frac{81+36x^2+16x^4}{\left(729-64x^6\right)^2} \, dx$

Optimal. Leaf size=81

$$\frac{1}{17496(3-2x)}-\frac{1}{17496(3+2x)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}$$

```
[Out] 1/17496/(3-2*x)-1/17496/(3+2*x)+1/8748*arctanh(2/3*x)-1/39366*arctan(1/9*(3
-4*x)*3^(1/2))*3^(1/2)+1/39366*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1600, 1184, 213, 632, 210}

$$-\frac{\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{4x+3}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{1}{17496(3-2x)}-\frac{1}{17496(2x+3)}+\frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}$$

Antiderivative was successfully verified.

```
[In] Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(17496*(3 - 2*x)) - 1/(17496*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(
13122*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(13122*Sqrt[3]) + ArcTanh[(2
*x)/3]/8748
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx &= \int \frac{1}{(9-4x^2)^2\,(81+36x^2+16x^4)}\,dx \\
&= \int \left(\frac{1}{8748(-3+2x)^2}+\frac{1}{8748(3+2x)^2}-\frac{1}{1458\,(-9+4x^2)}+\frac{1}{4374\,(9-6x+4x^2)}\right. \\
&= \frac{1}{17496(3-2x)}-\frac{1}{17496(3+2x)}+\frac{\int\frac{1}{9-6x+4x^2}\,dx}{4374}+\frac{\int\frac{1}{9+6x+4x^2}\,dx}{4374}-\frac{\int\frac{1}{-9+4x^2}\,dx}{1458} \\
&= \frac{1}{17496(3-2x)}-\frac{1}{17496(3+2x)}+\frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}-\frac{\mathrm{Subst}\left(\int\frac{1}{-108-x^2}\,dx,x,-6+8x\right)}{2187} \\
&= \frac{1}{17496(3-2x)}-\frac{1}{17496(3+2x)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 122, normalized size = 1.51

$$\frac{\frac{36x}{9-4x^2}+3\sqrt{3}\,\tan^{-1}\left(\frac{1}{3}\left(-i+\sqrt{3}\right)x\right)+4i\sqrt{3}\,\tanh^{-1}\left(\frac{1}{3}\left(1-i\sqrt{3}\right)x\right)+\left(-3+\frac{2}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}\right)\tanh^{-1}\left(\frac{1}{3}\left(x+i\sqrt{3}\,x\right)\right)-9\log(3-2x)+9\log(3+2x)}{157464}$$

Antiderivative was successfully verified.

```
[In] Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] ((36*x)/(9 - 4*x^2) + 3*Sqrt[3]*ArcTan[((-I + Sqrt[3])*x)/3] + (4*I)*Sqrt[3
]*ArcTanh[((1 - I*Sqrt[3])*x)/3] + (-3 + 2/Sqrt[(1 + I*Sqrt[3])/6])*ArcTanh
[(x + I*Sqrt[3]*x)/3] - 9*Log[3 - 2*x] + 9*Log[3 + 2*x])/157464
```

Maple [A]
time = 0.38, size = 68, normalized size = 0.84

method	result
risch	$-\frac{x}{17496\left(x^2-\frac{9}{4}\right)}+\frac{\ln(2x+3)}{17496}-\frac{\ln(-3+2x)}{17496}+\frac{\sqrt{3}\arctan\left(\frac{2\sqrt{3}\,x}{9}\right)}{39366}+\frac{\sqrt{3}\arctan\left(\frac{8\sqrt{3}\,x^3}{81}+\frac{4\sqrt{3}\,x}{9}\right)}{39366}$
default	$\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{39366}+\frac{\sqrt{3}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{39366}-\frac{1}{17496(2x+3)}+\frac{\ln(2x+3)}{17496}-\frac{1}{17496(-3+2x)}-\frac{\ln(-3+2x)}{17496}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\cdots\right.}{\cdots}\right)}{6(x^6)^{\frac{1}{6}}}\right)}{26244}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/39366*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/39366*3^(1/2)*arctan(1/18*(8
*x+6)*3^(1/2))-1/17496/(2*x+3)+1/17496*ln(2*x+3)-1/17496/(-3+2*x)-1/17496*l
n(-3+2*x)
```

Maxima [A]
time = 0.53, size = 61, normalized size = 0.75

$$\frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4x+3)\right)+\frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4x-3)\right)-\frac{x}{4374\,(4x^2-9)}+\frac{1}{17496}\log(2x+3)-\frac{1}{17496}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(2*x + 3) - 1/17496*
log(2*x - 3)
```

Fricas [A]
time = 0.36, size = 91, normalized size = 1.12

$$\frac{4\sqrt{3}\,(4x^2-9)\arctan\left(\frac{4}{81}\sqrt{3}\,(2x^3+9x)\right)+4\sqrt{3}\,(4x^2-9)\arctan\left(\frac{2}{9}\sqrt{3}\,x\right)+9\,(4x^2-9)\log(2x+3)-9\,(4x^2-9)\log(2x-3)-36x}{157464\,(4x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/157464*(4*sqrt(3)*(4*x^2 - 9)*arctan(4/81*sqrt(3)*(2*x^3 + 9*x)) + 4*sqrt
(3)*(4*x^2 - 9)*arctan(2/9*sqrt(3)*x) + 9*(4*x^2 - 9)*log(2*x + 3) - 9*(4*x
^2 - 9)*log(2*x - 3) - 36*x)/(4*x^2 - 9)
```

Sympy [A]
time = 0.08, size = 70, normalized size = 0.86

$$-\frac{x}{17496x^2-39366}+\frac{\sqrt{3}\cdot\left(2\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{9}\right)+2\operatorname{atan}\left(\frac{8\sqrt{3}\,x^3}{81}+\frac{4\sqrt{3}\,x}{9}\right)\right)}{78732}-\frac{\log\left(x-\frac{3}{2}\right)}{17496}+\frac{\log\left(x+\frac{3}{2}\right)}{17496}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x**4+36*x**2+81)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(17496*x**2 - 39366) + sqrt(3)*(2*atan(2*sqrt(3)*x/9) + 2*atan(8*sqrt(3)
*x**3/81 + 4*sqrt(3)*x/9))/78732 - log(x - 3/2)/17496 + log(x + 3/2)/17496
```

Giac [A]
time = 0.70, size = 63, normalized size = 0.78

$$\frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374(4x^2-9)}+\frac{1}{17496}\log\left(|2x+3|\right)-\frac{1}{17496}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(abs(2*x + 3)) - 1/1
7496*log(abs(2*x - 3))
```

Mupad [B]
time = 4.92, size = 52, normalized size = 0.64

$$\frac{\operatorname{atanh}\left(\frac{2x}{3}\right)}{8748}+\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{8\sqrt{3}\,x^3}{81}+\frac{4\sqrt{3}\,x}{9}\right)+2\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{9}\right)\right)}{78732}-\frac{x}{17496\left(x^2-\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((36*x^2 + 16*x^4 + 81)/(64*x^6 - 729)^2,x)
```

```
[Out] atanh((2*x)/3)/8748 + (3^(1/2)*(2*atan((4*3^(1/2)*x)/9 + (8*3^(1/2)*x^3)/81
) + 2*atan((2*3^(1/2)*x)/9)))/78732 - x/(17496*(x^2 - 9/4))
```

3.568 $\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2} dx$

Optimal. Leaf size=92

$$\frac{x}{4374(9-6x+4x^2)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}-\frac{\log(3-2x)}{26244}+\frac{\log(3+2x)}{78732}-\frac{\log(9-6x+4x^2)}{157464}+\frac{\log(9+6x+4x^2)}{52488}$$

```
[Out] 1/4374*x/(4*x^2-6*x+9)-1/26244*ln(3-2*x)+1/78732*ln(3+2*x)-1/157464*ln(4*x^
2-6*x+9)+1/52488*ln(4*x^2+6*x+9)-1/13122*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2
)
```

Rubi [A]
time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1600, 2099, 652, 632, 210, 648, 642}

$$-\frac{\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}+\frac{x}{4374(4x^2-6x+9)}-\frac{\log(4x^2-6x+9)}{157464}+\frac{\log(4x^2+6x+9)}{52488}-\frac{\log(3-2x)}{26244}+\frac{\log(2x+3)}{78732}$$

Antiderivative was successfully verified.

```
[In] Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] x/(4374*(9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(4374*Sqrt[3]) -
 Log[3 - 2*x]/26244 + Log[3 + 2*x]/78732 - Log[9 - 6*x + 4*x^2]/157464 + Lo
g[9 + 6*x + 4*x^2]/52488
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \int \frac{1}{(9-6x+4x^2)^2(81+54x-24x^3-16x^4)}\,dx$$

$$= \int \left(-\frac{1}{13122(-3+2x)} + \frac{1}{39366(3+2x)} + \frac{3-x}{729(9-6x+4x^2)^2} + \frac{3}{78732(9}\right.$$

$$= -\frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} + \frac{\int \frac{39-4x}{9-6x+4x^2}\,dx}{78732} + \frac{\int \frac{3+4x}{9+6x+4x^2}\,dx}{26244} + \frac{1}{729}\int \frac{}{(9}$$

$$= \frac{x}{4374(9-6x+4x^2)} - \frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} + \frac{\log(9+6x+4x^2)}{52488} -$$

$$= \frac{x}{4374(9-6x+4x^2)} - \frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} - \frac{\log(9-6x+4x^2)}{157464} +$$

$$= \frac{x}{4374(9-6x+4x^2)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}} - \frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} - \log$$

Mathematica [A]
time = 0.02, size = 84, normalized size = 0.91

$$\frac{\frac{36x}{9-6x+4x^2}+12\sqrt{3}\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right)-6\log(3-2x)+2\log(3+2x)-\log\left(9-6x+4x^2\right)+3\log\left(9+6x+4x^2\right)}{157464}$$

Antiderivative was successfully verified.

```
[In] Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] ((36*x)/(9 - 6*x + 4*x^2) + 12*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*L
og[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2
])/157464
```

Maple [A]
time = 0.38, size = 73, normalized size = 0.79

method	result
default	$\frac{x}{17496x^2-26244x+39366}-\frac{\ln(4x^2-6x+9)}{157464}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{13122}+\frac{\ln(4x^2+6x+9)}{52488}+\frac{\ln(2x+3)}{78732}-\frac{\ln(-3+2x)}{26244}$
risch	$\frac{x}{17496x^2-26244x+39366}+\frac{\ln(2x+3)}{78732}-\frac{\ln(64x^2-96x+144)}{157464}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{13122}-\frac{\ln(-3+2x)}{26244}+\frac{\ln(4x^2+6x+9}{52488}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\cdots\right.}{\cdots}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{26244}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/17496*x/(x^2-3/2*x+9/4)-1/157464*ln(4*x^2-6*x+9)+1/13122*3^(1/2)*arctan(1
/18*(8*x-6)*3^(1/2))+1/52488*ln(4*x^2+6*x+9)+1/78732*ln(2*x+3)-1/26244*ln(-
3+2*x)
```

Maxima [A]
time = 0.52, size = 74, normalized size = 0.80

$$\frac{1}{13122}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374(4x^2-6x+9)}+\frac{1}{52488}\log\left(4x^2+6x+9\right)-\frac{1}{157464}\log\left(4x^2-6x+9\right)+\frac{1}{78732}\log\left(2x+3\right)-\frac{1}{26244}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="maxima")

[Out] 1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 - 6*x + 9) + 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6*x + 9) + 1/78732*log(2*x + 3) - 1/26244*log(2*x - 3)

Fricas [A]
time = 0.37, size = 126, normalized size = 1.37

$$\frac{12\sqrt{3}\,(4x^2-6x+9)\arctan\left(\frac{1}{9}\sqrt{3}\,(4x-3)\right)+3\,(4x^2-6x+9)\log(4x^2+6x+9)-(4x^2-6x+9)\log(4x^2-6x+9)+2\,(4x^2-6x+9)\log(2x+3)-6\,(4x^2-6x+9)\log(2x-3)+36x}{157464\,(4x^2-6x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="fricas")

[Out] 1/157464*(12*sqrt(3)*(4*x^2 - 6*x + 9)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(4*x^2 - 6*x + 9)*log(4*x^2 + 6*x + 9) - (4*x^2 - 6*x + 9)*log(4*x^2 - 6*x + 9) + 2*(4*x^2 - 6*x + 9)*log(2*x + 3) - 6*(4*x^2 - 6*x + 9)*log(2*x - 3) + 36*x)/(4*x^2 - 6*x + 9)

Sympy [A]
time = 0.14, size = 82, normalized size = 0.89

$$\frac{x}{17496x^2-26244x+39366}-\frac{\log\left(x-\frac{3}{2}\right)}{26244}+\frac{\log\left(x+\frac{3}{2}\right)}{78732}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{157464}+\frac{\log\left(4x^2+6x+9\right)}{52488}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{13122}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729)**2,x)

[Out] x/(17496*x**2 - 26244*x + 39366) - log(x - 3/2)/26244 + log(x + 3/2)/78732 - log(x**2 - 3*x/2 + 9/4)/157464 + log(4*x**2 + 6*x + 9)/52488 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/13122

Giac [A]
time = 0.79, size = 76, normalized size = 0.83

$$\frac{1}{13122}\sqrt{3}\ \arctan\left(\frac{1}{9}\sqrt{3}\,(4x-3)\right)+\frac{x}{4374\,(4x^2-6x+9)}+\frac{1}{52488}\log\left(4x^2+6x+9\right)-\frac{1}{157464}\log\left(4x^2-6x+9\right)+\frac{1}{78732}\log\left(|2x+3|\right)-\frac{1}{26244}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="giac")

[Out] 1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 - 6*x + 9) + 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6*x + 9) + 1/78732*log(abs(2*x + 3)) - 1/26244*log(abs(2*x - 3))

Mupad [B]
time = 0.12, size = 77, normalized size = 0.84

$$\frac{\ln\left(x+\frac{3}{2}\right)}{78732}-\frac{\ln\left(x-\frac{3}{2}\right)}{26244}+\frac{\ln\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{52488}+\frac{x}{17496\,\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\,\left(\frac{1}{157464}+\frac{\sqrt{3}\ 1\mathrm{i}}{26244}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\,\left(-\frac{1}{157464}+\frac{\sqrt{3}\ 1\mathrm{i}}{26244}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((54*x - 24*x^3 - 16*x^4 + 81)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/78732 - log(x - 3/2)/26244 + log((3*x)/2 + x^2 + 9/4)/52488 +
x/(17496*(x^2 - (3*x)/2 + 9/4)) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1
i)/26244 + 1/157464) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/26244 -
1/157464)
```

3.569 $\int \frac{3-2x}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=148

$$-\frac{1}{708588(3+2x)}+\frac{3-x}{708588\left(9-6x+4x^2\right)}+\frac{x}{236196\left(9+6x+4x^2\right)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{\log(3\ldots}{425\ldots}$$

```
[Out] -1/708588/(3+2*x)+1/708588*(3-x)/(4*x^2-6*x+9)+1/236196*x/(4*x^2+6*x+9)-1/4
251528*ln(3-2*x)+1/472392*ln(3+2*x)-1/944784*ln(4*x^2-6*x+9)+1/8503056*ln(4
*x^2+6*x+9)-1/4251528*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/472392*arctan(1
/9*(3+4*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1600, 2099, 652, 632, 210, 648, 642}

$$-\frac{\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{3-x}{708588\left(4x^2-6x+9\right)}+\frac{x}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{8503056}-\frac{1}{708588(2x+3)}-\frac{\log(3-2x)}{4251528}+\frac{\log(2x+3)}{472392}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] -1/708588*1/(3 + 2*x) + (3 - x)/(708588*(9 - 6*x + 4*x^2)) + x/(236196*(9 +
 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) + ArcTan[(
3 + 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) - Log[3 - 2*x]/4251528 + Log[3 + 2*x
]/472392 - Log[9 - 6*x + 4*x^2]/944784 + Log[9 + 6*x + 4*x^2]/8503056
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{3-2x}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{(3-2x)\left(243+162x+108x^2+72x^3+48x^4+32x^5\right)^2}\,dx$$

$$= \int \left(-\frac{1}{2125764(-3+2x)} + \frac{1}{354294(3+2x)^2} + \frac{1}{236196(3+2x)} - \frac{x}{39366\,(9-6x+\right.$$

$$= -\frac{1}{708588(3+2x)} - \frac{\log(3-2x)}{4251528} + \frac{\log(3+2x)}{472392} + \frac{\int \frac{33+2x}{9+6x+4x^2}\,dx}{2125764} + \frac{\int \frac{7-6x}{9-6x+4x^2}\,dx}{708588} -$$

$$= -\frac{1}{708588(3+2x)} + \frac{3-x}{708588\left(9-6x+4x^2\right)} + \frac{x}{236196\left(9+6x+4x^2\right)} - \frac{\log(3-2x)}{4251528}$$

$$= -\frac{1}{708588(3+2x)} + \frac{3-x}{708588\left(9-6x+4x^2\right)} + \frac{x}{236196\left(9+6x+4x^2\right)} - \frac{\log(3-2x)}{4251528}$$

$$= -\frac{1}{708588(3+2x)} + \frac{3-x}{708588\left(9-6x+4x^2\right)} + \frac{x}{236196\left(9+6x+4x^2\right)} - \frac{\tan^{-1}\left(\frac{3-4}{3\sqrt{3}}\right.}{1417176\sqrt{}}$$

Mathematica [A]

time = 0.04, size = 119, normalized size = 0.80

$$\frac{\frac{1944x}{243+162x+108x^2+72x^3+48x^4+32x^5}+2\sqrt{3}\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right)+18\sqrt{3}\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)-2\log(3-2x)+18\log(3+2x)-9\log\left(9-6x+4x^2\right)+\log\left(9+6x+4x^2\right)}{8503056}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] ((1944*x)/(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5) + 2*Sqrt[3]*Ar
cTan[(-3 + 4*x)/(3*Sqrt[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 2
*Log[3 - 2*x] + 18*Log[3 + 2*x] - 9*Log[9 - 6*x + 4*x^2] + Log[9 + 6*x + 4*
x^2])/8503056
```

Maple [A]

time = 0.47, size = 115, normalized size = 0.78

method	result
risch	$\frac{x}{139968x^5+209952x^4+314928x^3+472392x^2+708588x+1062882}+\frac{\ln(2x+3)}{472392}+\frac{\ln\left(36x^2+54x+81\right)}{8503056}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(6x+\frac{9}{2}\right)\sqrt{3}}{27}\cdots\right)}{472392}\cdots$
default	$-\frac{\frac{x}{4}-\frac{3}{4}}{708588\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)}-\frac{\ln\left(4x^2-6x+9\right)}{944784}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{4251528}+\frac{x}{944784x^2+1417176x+2125764}+\frac{\ln\left(4x^2+6x+9\right)}{8503056}\cdots$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{6\left(x^6\right)^{\frac{1}{6}}}\cdots\right)}{708588}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/708588*(1/4*x-3/4)/(x^2-3/2*x+9/4)-1/944784*ln(4*x^2-6*x+9)+1/4251528*3^
(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/944784*x/(x^2+3/2*x+9/4)+1/8503056*ln(
4*x^2+6*x+9)+1/472392*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/708588/(2*x+3)
+1/472392*ln(2*x+3)-1/4251528*ln(-3+2*x)
```

Maxima [A]

time = 0.51, size = 105, normalized size = 0.71

$$\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374\left(32x^5+48x^4+72x^3+108x^2+162x+243\right)}+\frac{1}{8503056}\log\left(4x^2+6x+9\right)-\frac{1}{944784}\log\left(4x^2-6x+9\right)+\frac{1}{472392}\log(2x+3)-\frac{1}{4251528}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*
x + 243) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) +
 1/472392*log(2*x + 3) - 1/4251528*log(2*x - 3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(116) = 232$.
time = 0.41, size = 256, normalized size = 1.73

$$\frac{18\sqrt{3}(32x^5+48x^4+72x^3+108x^2+162x+243)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+2\sqrt{3}(32x^5+48x^4+72x^3+108x^2+162x+243)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+(32x^5+48x^4+72x^3+108x^2+162x+243)\log\left(4x^2+6x+9\right)-9(32x^5+48x^4+72x^3+108x^2+162x+243)\log\left(4x^2-6x+9\right)+18(32x^5+48x^4+72x^3+108x^2+162x+243)\log\left(2x+3\right)-2(32x^5+48x^4+72x^3+108x^2+162x+243)\log\left(2x-3\right)+1944x}{8503056(32x^5+48x^4+72x^3+108x^2+162x+243)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/8503056*(18*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*ar
ctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2
 + 162*x + 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + (32*x^5 + 48*x^4 + 72*x^3 +
 108*x^2 + 162*x + 243)*log(4*x^2 + 6*x + 9) - 9*(32*x^5 + 48*x^4 + 72*x^3
+ 108*x^2 + 162*x + 243)*log(4*x^2 - 6*x + 9) + 18*(32*x^5 + 48*x^4 + 72*x^
3 + 108*x^2 + 162*x + 243)*log(2*x + 3) - 2*(32*x^5 + 48*x^4 + 72*x^3 + 108
*x^2 + 162*x + 243)*log(2*x - 3) + 1944*x)/(32*x^5 + 48*x^4 + 72*x^3 + 108*
x^2 + 162*x + 243)
```

Sympy [A]
time = 0.32, size = 124, normalized size = 0.84

$$\frac{x}{139968x^5+209952x^4+314928x^3+472392x^2+708588x+1062882}-\frac{\log\left(x-\frac{3}{2}\right)}{4251528}+\frac{\log\left(x+\frac{3}{2}\right)}{472392}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{8503056}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{4251528}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}\,x}{9}+\frac{\sqrt{3}}{3}\right)}{472392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x**6+729)**2,x)
```

```
[Out] x/(139968*x**5 + 209952*x**4 + 314928*x**3 + 472392*x**2 + 708588*x + 10628
82) - log(x - 3/2)/4251528 + log(x + 3/2)/472392 - log(x**2 - 3*x/2 + 9/4)/
944784 + log(x**2 + 3*x/2 + 9/4)/8503056 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqr
t(3)/3)/4251528 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392
```

Giac [A]
time = 1.47, size = 111, normalized size = 0.75

$$\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374(4x^2+6x+9)(4x^2-6x+9)(2x+3)}+\frac{1}{8503056}\log\left(4x^2+6x+9\right)-\frac{1}{944784}\log\left(4x^2-6x+9\right)+\frac{1}{472392}\log\left(|2x+3|\right)-\frac{1}{4251528}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) + 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*x
+ 3)) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/
472392*log(abs(2*x + 3)) - 1/4251528*log(abs(2*x - 3))
```

Mupad [B]
time = 0.19, size = 120, normalized size = 0.81

$$\frac{\ln\left(x+\frac{3}{2}\right)}{472392}-\frac{\ln\left(x-\frac{3}{2}\right)}{4251528}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\ 1\mathrm{i}}{8503056}\right)-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(-\frac{1}{8503056}+\frac{\sqrt{3}\ 1\mathrm{i}}{944784}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\ 1\mathrm{i}}{8503056}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(\frac{1}{8503056}+\frac{\sqrt{3}\ 1\mathrm{i}}{944784}\right)+\frac{x}{139968\left(x^5+\frac{3x^4}{2}+\frac{9x^3}{4}+\frac{27x^2}{8}+\frac{81x}{16}+\frac{243}{32}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 3)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/472392 - log(x - 3/2)/4251528 - log(x - (3^(1/2)*3i)/4 - 3/4)*
((3^(1/2)*1i)/8503056 + 1/944784) - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)
*1i)/944784 - 1/8503056) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/8503
056 - 1/944784) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 + 1/85
03056) + x/(139968*((81*x)/16 + (27*x^2)/8 + (9*x^3)/4 + (3*x^4)/2 + x^5 +
243/32))
```

3.570 $\int \frac{3+2x}{\left(729-64x^6\right)^2} dx$

Optimal. Leaf size=146

$$\frac{1}{708588(3-2x)}+\frac{x}{236196\left(9-6x+4x^2\right)}-\frac{3+x}{708588\left(9+6x+4x^2\right)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}-\frac{\log(3}{472}$$

```
[Out] 1/708588/(3-2*x)+1/236196*x/(4*x^2-6*x+9)+1/708588*(-3-x)/(4*x^2+6*x+9)-1/4
72392*ln(3-2*x)+1/4251528*ln(3+2*x)-1/8503056*ln(4*x^2-6*x+9)+1/944784*ln(4
*x^2+6*x+9)-1/472392*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/4251528*arctan(1
/9*(3+4*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1600, 2099, 652, 632, 210, 648, 642}

$$-\frac{\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{4x+3}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{x}{236196\left(4x^2-6x+9\right)}-\frac{x+3}{708588\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{8503056}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{708588(3-2x)}-\frac{\log(3-2x)}{472392}+\frac{\log(2x+3)}{4251528}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(708588*(3 - 2*x)) + x/(236196*(9 - 6*x + 4*x^2)) - (3 + x)/(708588*(9 +
6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) + ArcTan[(3
+ 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) - Log[3 - 2*x]/472392 + Log[3 + 2*x]/
4251528 - Log[9 - 6*x + 4*x^2]/8503056 + Log[9 + 6*x + 4*x^2]/944784
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3+2x}{\left(729-64x^6\right)^2}\,dx &= \int \frac{1}{(3+2x)\left(243-162x+108x^2-72x^3+48x^4-32x^5\right)^2}\,dx \\
&= \int \left(\frac{1}{354294(-3+2x)^2}-\frac{1}{236196(-3+2x)}+\frac{1}{2125764(3+2x)}+\frac{3-x}{39366\left(9-6x+4\right.}\right. \\
&= \frac{1}{708588(3-2x)}-\frac{\log(3-2x)}{472392}+\frac{\log(3+2x)}{4251528}+\frac{\int\frac{33-2x}{9-6x+4x^2}\,dx}{2125764}+\frac{\int\frac{7+6x}{9+6x+4x^2}\,dx}{708588}+\frac{\int}{} \\
&= \frac{1}{708588(3-2x)}+\frac{x}{236196\left(9-6x+4x^2\right)}-\frac{3+x}{708588\left(9+6x+4x^2\right)}-\frac{\log(3-2x)}{472392}+ \\
&= \frac{1}{708588(3-2x)}+\frac{x}{236196\left(9-6x+4x^2\right)}-\frac{3+x}{708588\left(9+6x+4x^2\right)}-\frac{\log(3-2x)}{472392}+ \\
&= \frac{1}{708588(3-2x)}+\frac{x}{236196\left(9-6x+4x^2\right)}-\frac{3+x}{708588\left(9+6x+4x^2\right)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 121, normalized size = 0.83

$$\frac{\frac{1944x}{243-162x+108x^2-72x^3+48x^4-32x^5}+18\sqrt{3}\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)-18\log(3-2x)+2\log(3+2x)-\log\left(9-6x+4x^2\right)+9\log\left(9+6x+4x^2\right)}{8503056}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] ((1944*x)/(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5) + 18*Sqrt[3]*A
rcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 1
8*Log[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 9*Log[9 + 6*x + 4*
x^2])/8503056
```

Maple [A]
time = 0.40, size = 115, normalized size = 0.79

method	result
risch	$-\frac{x}{139968\left(x^5-\frac{3}{2}x^4+\frac{9}{4}x^3-\frac{27}{8}x^2+\frac{81}{16}x-\frac{243}{32}\right)}+\frac{\ln(16x^2+24x+36)}{944784}+\frac{\arctan\left(\frac{(3+4x)\sqrt{3}}{9}\right)\sqrt{3}}{4251528}+\frac{\ln(2x+3)}{4251528}-\frac{\ln(36x^2-54}{850305}$
default	$\frac{x}{944784x^2-1417176x+2125764}-\frac{\ln(4x^2-6x+9)}{8503056}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392}+\frac{-\frac{x}{4}-\frac{3}{4}}{708588x^2+1062882x+1594323}+\frac{\ln(4x^2}{94}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\right.}{}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{708588}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+3)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/944784*x/(x^2-3/2*x+9/4)-1/8503056*ln(4*x^2-6*x+9)+1/472392*3^(1/2)*arcta
n(1/18*(8*x-6)*3^(1/2))+1/708588*(-1/4*x-3/4)/(x^2+3/2*x+9/4)+1/944784*ln(4
*x^2+6*x+9)+1/4251528*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/4251528*ln(2*x
+3)-1/708588/(-3+2*x)-1/472392*ln(-3+2*x)
```

Maxima [A]
time = 0.52, size = 105, normalized size = 0.72

$$\frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(32x^5-48x^4+72x^3-108x^2+162x-243\right)}+\frac{1}{944784}\log\left(4x^2+6x+9\right)-\frac{1}{8503056}\log\left(4x^2-6x+9\right)+\frac{1}{4251528}\log(2x+3)-\frac{1}{472392}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*
x - 243) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9) +
 1/4251528*log(2*x + 3) - 1/472392*log(2*x - 3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(116) = 232$.
time = 0.42, size = 257, normalized size = 1.76

$$\frac{2\sqrt{3}\,(32x^5-48x^4+72x^3-108x^2+162x-243)\arctan\left(\frac{1}{9}\sqrt{3}\,(4x+3)\right)+18\sqrt{3}\,(32x^5-48x^4+72x^3-108x^2+162x-243)\arctan\left(\frac{1}{9}\sqrt{3}\,(4x-3)\right)+9\,(32x^5-48x^4+72x^3-108x^2+162x-243)\log\left(4x^2+6x+9\right)-(32x^5-48x^4+72x^3-108x^2+162x-243)\log\left(4x^2-6x+9\right)+2\,(32x^5-48x^4+72x^3-108x^2+162x-243)\log\left(2x+3\right)-18\,(32x^5-48x^4+72x^3-108x^2+162x-243)\log\left(2x-3\right)-1944x}{8503056\,(32x^5-48x^4+72x^3-108x^2+162x-243)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/8503056*(2*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*arc
tan(1/9*sqrt(3)*(4*x + 3)) + 18*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2
 + 162*x - 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + 9*(32*x^5 - 48*x^4 + 72*x^3
 - 108*x^2 + 162*x - 243)*log(4*x^2 + 6*x + 9) - (32*x^5 - 48*x^4 + 72*x^3
- 108*x^2 + 162*x - 243)*log(4*x^2 - 6*x + 9) + 2*(32*x^5 - 48*x^4 + 72*x^3
 - 108*x^2 + 162*x - 243)*log(2*x + 3) - 18*(32*x^5 - 48*x^4 + 72*x^3 - 108
*x^2 + 162*x - 243)*log(2*x - 3) - 1944*x)/(32*x^5 - 48*x^4 + 72*x^3 - 108*
x^2 + 162*x - 243)
```

Sympy [A]
time = 0.31, size = 124, normalized size = 0.85

$$-\frac{x}{139968x^5-209952x^4+314928x^3-472392x^2+708588x-1062882}-\frac{\log\left(x-\frac{3}{2}\right)}{472392}+\frac{\log\left(x+\frac{3}{2}\right)}{4251528}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{8503056}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{472392}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}+\frac{\sqrt{3}}{3}\right)}{4251528}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(139968*x**5 - 209952*x**4 + 314928*x**3 - 472392*x**2 + 708588*x - 1062
882) - log(x - 3/2)/472392 + log(x + 3/2)/4251528 - log(x**2 - 3*x/2 + 9/4)
/8503056 + log(x**2 + 3*x/2 + 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 - sq
rt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/4251528
```

Giac [A]
time = 2.11, size = 111, normalized size = 0.76

$$\frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4x+3)\right)+\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4x-3)\right)-\frac{x}{4374\,(4x^2+6x+9)(4x^2-6x+9)(2x-3)}+\frac{1}{944784}\log\left(4x^2+6x+9\right)-\frac{1}{8503056}\log\left(4x^2-6x+9\right)+\frac{1}{4251528}\log\left(|2x+3|\right)-\frac{1}{472392}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*x
- 3)) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9) + 1/
4251528*log(abs(2*x + 3)) - 1/472392*log(abs(2*x - 3))
```

Mupad [B]
time = 5.09, size = 121, normalized size = 0.83

$$\frac{\ln\left(x+\frac{3}{2}\right)}{4251528}-\frac{\ln\left(x-\frac{3}{2}\right)}{472392}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right)-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right)-\frac{x}{139968\,\left(x^5-\frac{3\,x^4}{2}+\frac{9\,x^3}{4}-\frac{27\,x^2}{8}+\frac{81\,x}{16}-\frac{243}{32}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/4251528 - log(x - 3/2)/472392 - log(x - (3^(1/2)*3i)/4 - 3/4)*
((3^(1/2)*1i)/944784 + 1/8503056) - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)
*1i)/8503056 - 1/944784) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/9447
84 - 1/8503056) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/8503056 + 1/9
44784) - x/(139968*((81*x)/16 - (27*x^2)/8 + (9*x^3)/4 - (3*x^4)/2 + x^5 -
243/32))
```

3.571 $\int \frac{9-6x+4x^2}{\left(729-64x^6\right)^2}\,dx$

Optimal. Leaf size=142

$$\frac{1}{472392(3-2x)}-\frac{1}{157464(3+2x)}+\frac{3+4x}{236196\left(9+6x+4x^2\right)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}-\frac{\log(3-2x)}{354294}+$$

```
[Out] 1/472392/(3-2*x)-1/157464/(3+2*x)+1/236196*(3+4*x)/(4*x^2+6*x+9)-1/354294*l
n(3-2*x)+1/118098*ln(3+2*x)-1/944784*ln(4*x^2-6*x+9)-5/2834352*ln(4*x^2+6*x
+9)-1/1417176*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/157464*arctan(1/9*(3+4*
x)*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1600, 2099, 648, 632, 210, 642, 628}

$$-\frac{\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{4x+3}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}-\frac{5\log\left(4x^2+6x+9\right)}{2834352}+\frac{1}{472392(3-2x)}-\frac{1}{157464(2x+3)}-\frac{\log(3-2x)}{354294}+\frac{\log(2x+3)}{118098}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(472392*(3 - 2*x)) - 1/(157464*(3 + 2*x)) + (3 + 4*x)/(236196*(9 + 6*x +
4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]) + ArcTan[(3 + 4*x)
/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/354294 + Log[3 + 2*x]/118098 -
 Log[9 - 6*x + 4*x^2]/944784 - (5*Log[9 + 6*x + 4*x^2])/2834352
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = \int \frac{1}{(9-6x+4x^2)\,(81+54x-24x^3-16x^4)^2}\,dx$$

$$= \int \left(\frac{1}{236196(-3+2x)^2} - \frac{1}{177147(-3+2x)} + \frac{1}{78732(3+2x)^2} + \frac{1}{59049(3+2x)} +$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} - \frac{\log(3-2x)}{354294} + \frac{\log(3+2x)}{118098} + \frac{\int \frac{21-10x}{9+6x+4x^2}\,dx}{708588} +$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} + \frac{3+4x}{236196\,(9+6x+4x^2)} - \frac{\log(3-2x)}{354294} + \frac{\log(3}{118}$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} + \frac{3+4x}{236196\,(9+6x+4x^2)} - \frac{\log(3-2x)}{354294} + \frac{\log(3}{118}$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} + \frac{3+4x}{236196\,(9+6x+4x^2)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}} + \frac{\mathrm{ta}}{}$$

Mathematica [A]
time = 0.04, size = 111, normalized size = 0.78

$$\frac{\frac{648x}{81+54x-24x^3-16x^4}+2\sqrt{3}\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right)+18\sqrt{3}\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)-8\log(3-2x)+24\log(3+2x)-3\log\left(9-6x+4x^2\right)-5\log\left(9+6x+4x^2\right)}{2834352}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] ((648*x)/(81 + 54*x - 24*x^3 - 16*x^4) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqr
t[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 8*Log[3 - 2*x] + 24*Log
[3 + 2*x] - 3*Log[9 - 6*x + 4*x^2] - 5*Log[9 + 6*x + 4*x^2])/2834352
```

Maple [A]
time = 0.40, size = 111, normalized size = 0.78

method	result
risch	$-\frac{x}{69984\left(x^4+\frac{3}{2}x^3-\frac{27}{8}x-\frac{81}{16}\right)}+\frac{\ln(2x+3)}{118098}-\frac{5\ln(36x^2+54x+81)}{2834352}+\frac{\sqrt{3}\arctan\left(\frac{2\left(6x+\frac{9}{2}\right)\sqrt{3}}{27}\right)}{157464}-\frac{\ln(-3+2x)}{354294}+\frac{\sqrt{3}\ \text{ar}}{}$
default	$-\frac{\ln(4x^2-6x+9)}{944784}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{1417176}-\frac{-3x-\frac{9}{4}}{708588\left(x^2+\frac{3}{2}x+\frac{9}{4}\right)}-\frac{5\ln(4x^2+6x+9)}{2834352}+\frac{\sqrt{3}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{157464}-$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(}{}\right.}{}\right.}{6\left(x^6\right)^{\frac{1}{6}}}\right.}{236196}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-6*x+9)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/944784*ln(4*x^2-6*x+9)+1/1417176*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-1/
708588*(-3*x-9/4)/(x^2+3/2*x+9/4)-5/2834352*ln(4*x^2+6*x+9)+1/157464*3^(1/2
)*arctan(1/18*(8*x+6)*3^(1/2))-1/157464/(2*x+3)+1/118098*ln(2*x+3)-1/472392
/(-3+2*x)-1/354294*ln(-3+2*x)
```

Maxima [A]
time = 0.49, size = 95, normalized size = 0.67

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(16x^4+24x^3-54x-81\right)}-\frac{5}{2834352}\log\left(4x^2+6x+9\right)-\frac{1}{944784}\log\left(4x^2-6x+9\right)+\frac{1}{118098}\log(2x+3)-\frac{1}{354294}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 + 24*x^3 - 54*x - 81) - 5/2834352*
log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/118098*log(2*x + 3
) - 1/354294*log(2*x - 3)
```

Fricas [A]
time = 0.36, size = 187, normalized size = 1.32

$$\frac{18\sqrt{3}\left(16x^4+24x^3-54x-81\right)\arctan\left(\frac{1}{9}\sqrt{3}\left(4x+3\right)\right)+2\sqrt{3}\left(16x^4+24x^3-54x-81\right)\arctan\left(\frac{1}{9}\sqrt{3}\left(4x-3\right)\right)-5\left(16x^4+24x^3-54x-81\right)\log\left(4x^2+6x+9\right)-3\left(16x^4+24x^3-54x-81\right)\log\left(4x^2-6x+9\right)+24\left(16x^4+24x^3-54x-81\right)\log\left(2x+3\right)-8\left(16x^4+24x^3-54x-81\right)\log\left(2x-3\right)-648x}{2834352\left(16x^4+24x^3-54x-81\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/2834352*(18*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
 + 3)) + 2*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x -
3)) - 5*(16*x^4 + 24*x^3 - 54*x - 81)*log(4*x^2 + 6*x + 9) - 3*(16*x^4 + 24
*x^3 - 54*x - 81)*log(4*x^2 - 6*x + 9) + 24*(16*x^4 + 24*x^3 - 54*x - 81)*l
og(2*x + 3) - 8*(16*x^4 + 24*x^3 - 54*x - 81)*log(2*x - 3) - 648*x)/(16*x^4
 + 24*x^3 - 54*x - 81)
```

Sympy [A]
time = 0.28, size = 116, normalized size = 0.82

$$-\frac{x}{69984x^4+104976x^3-236196x-354294}-\frac{\log\left(x-\frac{3}{2}\right)}{354294}+\frac{\log\left(x+\frac{3}{2}\right)}{118098}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}-\frac{5\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{2834352}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{1417176}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}\,x}{9}+\frac{\sqrt{3}}{3}\right)}{157464}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-6*x+9)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(69984*x**4 + 104976*x**3 - 236196*x - 354294) - log(x - 3/2)/354294 + l
og(x + 3/2)/118098 - log(x**2 - 3*x/2 + 9/4)/944784 - 5*log(x**2 + 3*x/2 +
9/4)/2834352 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/1417176 + sqrt(3)*at
an(4*sqrt(3)*x/9 + sqrt(3)/3)/157464
```

Giac [A]
time = 1.31, size = 106, normalized size = 0.75

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(4x+3\right)\right)+\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(4x-3\right)\right)-\frac{x}{4374\left(4x^2+6x+9\right)(2x+3)(2x-3)}-\frac{5}{2834352}\log\left(4x^2+6x+9\right)-\frac{1}{944784}\log\left(4x^2-6x+9\right)+\frac{1}{118098}\log\left(|2x+3|\right)-\frac{1}{354294}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(2*x + 3)*(2*x - 3)) -
```

```
5/2834352*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/118098*l
og(abs(2*x + 3)) - 1/354294*log(abs(2*x - 3))
```

Mupad [B]
time = 5.08, size = 110, normalized size = 0.77

$$\frac{\ln\left(x+\frac{3}{2}\right)}{118098}-\frac{\ln\left(x-\frac{3}{2}\right)}{354294}-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(\frac{5}{2834352}+\frac{\sqrt{3}\ 1\mathrm{i}}{314928}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(-\frac{5}{2834352}+\frac{\sqrt{3}\ 1\mathrm{i}}{314928}\right)-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\ 1\mathrm{i}}{2834352}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\ 3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\ 1\mathrm{i}}{2834352}\right)+\frac{x}{69984\left(-x^4-\frac{3x^3}{2}+\frac{27x}{8}+\frac{81}{16}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 6*x + 9)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/118098 - log(x - 3/2)/354294 - log(x - (3^(1/2)*3i)/4 + 3/4)*(
(3^(1/2)*1i)/314928 + 5/2834352) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*
1i)/314928 - 5/2834352) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/28343
52 + 1/944784) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/2834352 - 1/94
4784) + x/(69984*((27*x)/8 - (3*x^3)/2 - x^4 + 81/16))
```

3.572 $\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=142

$$\frac{1}{157464(3-2x)}-\frac{1}{472392(3+2x)}-\frac{3-4x}{236196\,(9-6x+4x^2)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}-\frac{\log(3-2x)}{118098}$$

```
[Out] 1/157464/(3-2*x)-1/472392/(3+2*x)+1/236196*(-3+4*x)/(4*x^2-6*x+9)-1/118098*
ln(3-2*x)+1/354294*ln(3+2*x)+5/2834352*ln(4*x^2-6*x+9)+1/944784*ln(4*x^2+6*
x+9)-1/157464*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/1417176*arctan(1/9*(3+4
*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1600, 2099, 628, 632, 210, 648, 642}

$$-\frac{\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{4x+3}{3\sqrt{3}}\right)}{472392\sqrt{3}}-\frac{3-4x}{236196\,(4x^2-6x+9)}+\frac{5\log(4x^2-6x+9)}{2834352}+\frac{\log(4x^2+6x+9)}{944784}+\frac{1}{157464(3-2x)}-\frac{1}{472392(2x+3)}-\frac{\log(3-2x)}{118098}+\frac{\log(2x+3)}{354294}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(157464*(3 - 2*x)) - 1/(472392*(3 + 2*x)) - (3 - 4*x)/(236196*(9 - 6*x +
4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) + ArcTan[(3 + 4*x)/
(3*Sqrt[3])]/(472392*Sqrt[3]) - Log[3 - 2*x]/118098 + Log[3 + 2*x]/354294 +
 (5*Log[9 - 6*x + 4*x^2])/2834352 + Log[9 + 6*x + 4*x^2]/944784
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{9+6x+4x^2}{\left(729-64x^6\right)^2}\,dx &= \int \frac{1}{\left(9+6x+4x^2\right)\left(81-54x+24x^3-16x^4\right)^2}\,dx\\
&= \int \left(\frac{1}{78732(-3+2x)^2}-\frac{1}{59049(-3+2x)}+\frac{1}{236196(3+2x)^2}+\frac{1}{177147(3+2x)}+\frac{}{43}\right.\\
&= \frac{1}{157464(3-2x)}-\frac{1}{472392(3+2x)}-\frac{\log(3-2x)}{118098}+\frac{\log(3+2x)}{354294}+\frac{\int \frac{21+10x}{9-6x+4x^2}\,dx}{708588}+\frac{\int}{}\\
&= \frac{1}{157464(3-2x)}-\frac{1}{472392(3+2x)}-\frac{3-4x}{236196\left(9-6x+4x^2\right)}-\frac{\log(3-2x)}{118098}+\frac{\log(3+}{3542}\\
&= \frac{1}{157464(3-2x)}-\frac{1}{472392(3+2x)}-\frac{3-4x}{236196\left(9-6x+4x^2\right)}-\frac{\log(3-2x)}{118098}+\frac{\log(3+}{3542}\\
&= \frac{1}{157464(3-2x)}-\frac{1}{472392(3+2x)}-\frac{3-4x}{236196\left(9-6x+4x^2\right)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{\tan}{4}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 111, normalized size = 0.78

$$\frac{\frac{648x}{81-54x+24x^3-16x^4}+18\sqrt{3}\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)-24\log(3-2x)+8\log(3+2x)+5\log(9-6x+4x^2)+3\log(9+6x+4x^2)}{2834352}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] ((648*x)/(81 - 54*x + 24*x^3 - 16*x^4) + 18*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sq
rt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 24*Log[3 - 2*x] + 8*Log
[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/2834352
```

Maple [A]

time = 0.40, size = 111, normalized size = 0.78

method	result
risch	$-\frac{x}{69984\left(x^4-\frac{3}{2}x^3+\frac{27}{8}x-\frac{81}{16}\right)}+\frac{\ln(2x+3)}{354294}-\frac{\ln(-3+2x)}{118098}+\frac{5\ln(36x^2-54x+81)}{2834352}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(6x-\frac{9}{2}\right)\sqrt{3}}{27}\right)}{157464}+\frac{\sqrt{3}}{}$
default	$\frac{3x-\frac{9}{4}}{708588x^2-1062882x+1594323}+\frac{5\ln(4x^2-6x+9)}{2834352}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{157464}+\frac{\ln(4x^2+6x+9)}{944784}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)}{1}\right.}{1417176}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\right.}{}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{236196}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+6*x+9)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/708588*(3*x-9/4)/(x^2-3/2*x+9/4)+5/2834352*ln(4*x^2-6*x+9)+1/157464*3^(1/
2)*arctan(1/18*(8*x-6)*3^(1/2))+1/944784*ln(4*x^2+6*x+9)+1/1417176*3^(1/2)*
arctan(1/18*(8*x+6)*3^(1/2))-1/472392/(2*x+3)+1/354294*ln(2*x+3)-1/157464/(
-3+2*x)-1/118098*ln(-3+2*x)
```

Maxima [A]

time = 0.50, size = 95, normalized size = 0.67

$$\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374(16x^4-24x^3+54x-81)}+\frac{1}{944784}\log(4x^2+6x+9)+\frac{5}{2834352}\log(4x^2-6x+9)+\frac{1}{354294}\log(2x+3)-\frac{1}{118098}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 - 24*x^3 + 54*x - 81) + 1/944784*l
og(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/354294*log(2*x + 3
) - 1/118098*log(2*x - 3)
```

Fricas [A]
time = 0.36, size = 187, normalized size = 1.32

$$\frac{2\sqrt{3}\,(16\,x^4-24\,x^3+54\,x-81)\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x+3)\right)+18\sqrt{3}\,(16\,x^4-24\,x^3+54\,x-81)\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)+3\,(16\,x^4-24\,x^3+54\,x-81)\log\left(4\,x^2+6\,x+9\right)+5\,(16\,x^4-24\,x^3+54\,x-81)\log\left(4\,x^2-6\,x+9\right)+8\,(16\,x^4-24\,x^3+54\,x-81)\log\left(2\,x+3\right)-24\,(16\,x^4-24\,x^3+54\,x-81)\log\left(2\,x-3\right)-648\,x}{2834352\,(16\,x^4-24\,x^3+54\,x-81)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/2834352*(2*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
+ 3)) + 18*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x -
3)) + 3*(16*x^4 - 24*x^3 + 54*x - 81)*log(4*x^2 + 6*x + 9) + 5*(16*x^4 - 24
*x^3 + 54*x - 81)*log(4*x^2 - 6*x + 9) + 8*(16*x^4 - 24*x^3 + 54*x - 81)*lo
g(2*x + 3) - 24*(16*x^4 - 24*x^3 + 54*x - 81)*log(2*x - 3) - 648*x)/(16*x^4
 - 24*x^3 + 54*x - 81)
```

Sympy [A]
time = 0.28, size = 116, normalized size = 0.82

$$-\frac{x}{69984x^4-104976x^3+236196x-354294}-\frac{\log\left(x-\frac{3}{2}\right)}{118098}+\frac{\log\left(x+\frac{3}{2}\right)}{354294}+\frac{5\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{2834352}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{157464}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}+\frac{\sqrt{3}}{3}\right)}{1417176}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+6*x+9)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(69984*x**4 - 104976*x**3 + 236196*x - 354294) - log(x - 3/2)/118098 + l
og(x + 3/2)/354294 + 5*log(x**2 - 3*x/2 + 9/4)/2834352 + log(x**2 + 3*x/2 +
 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/157464 + sqrt(3)*ata
n(4*sqrt(3)*x/9 + sqrt(3)/3)/1417176
```

Giac [A]
time = 1.46, size = 106, normalized size = 0.75

$$\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x+3)\right)+\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)-\frac{x}{4374\,(4\,x^2-6\,x+9)(2\,x+3)(2\,x-3)}+\frac{1}{944784}\log\left(4\,x^2+6\,x+9\right)+\frac{5}{2834352}\log\left(4\,x^2-6\,x+9\right)+\frac{1}{354294}\log\left(|2\,x+3|\right)-\frac{1}{118098}\log\left(|2\,x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x + 3)*(2*x - 3)) +
```

```
1/944784*log(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/354294*l
og(abs(2*x + 3)) - 1/118098*log(abs(2*x - 3))
```

Mupad [B]
time = 0.19, size = 111, normalized size = 0.78

$$\frac{\ln\left(x+\frac{3}{2}\right)}{354294}-\frac{\ln\left(x-\frac{3}{2}\right)}{118098}-\frac{x}{69984\,\left(x^4-\frac{3\,x^3}{2}+\frac{27\,x}{8}-\frac{81}{16}\right)}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3{}\mathrm{i}}{4}\right)\,\left(-\frac{5}{2834352}+\frac{\sqrt{3}\,1{}\mathrm{i}}{314928}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3{}\mathrm{i}}{4}\right)\,\left(\frac{5}{2834352}+\frac{\sqrt{3}\,1{}\mathrm{i}}{314928}\right)-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3{}\mathrm{i}}{4}\right)\,\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2834352}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3{}\mathrm{i}}{4}\right)\,\left(\frac{1}{944784}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2834352}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x + 4*x^2 + 9)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/354294 - log(x - 3/2)/118098 - x/(69984*((27*x)/8 - (3*x^3)/2
+ x^4 - 81/16)) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/314928 - 5/28
34352) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/314928 + 5/2834352) -
log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/2834352 - 1/944784) + log(x + (
3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/2834352 + 1/944784)
```

3.573 $\int \frac{27-8x^3}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=113

$$\frac{x}{4374(27+8x^3)}-\frac{7\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}-\frac{\log(3-2x)}{157464}+\frac{7\log(3+2x)}{472392}-\frac{7\log(9-6x+4x^2)}{944784}+\frac{\log}{}$$

```
[Out] 1/4374*x/(8*x^3+27)-1/157464*ln(3-2*x)+7/472392*ln(3+2*x)-7/944784*ln(4*x^2
-6*x+9)+1/314928*ln(4*x^2+6*x+9)-7/472392*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/
2)+1/157464*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1418, 425, 536, 206, 31, 648, 632, 210, 642}

$$-\frac{7\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{x}{4374(8x^3+27)}-\frac{7\log(4x^2-6x+9)}{944784}+\frac{\log(4x^2+6x+9)}{314928}-\frac{\log(3-2x)}{157464}+\frac{7\log(2x+3)}{472392}$$

Antiderivative was successfully verified.

```
[In] Int[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] x/(4374*(27 + 8*x^3)) - (7*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3])
+ ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/157464 + (7*
Log[3 + 2*x])/472392 - (7*Log[9 - 6*x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^
2]/314928
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1418

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \int \frac{1}{(27-8x^3)\,(27+8x^3)^2}\,dx$$
$$= \frac{x}{4374\,(27+8x^3)} - \frac{\int \frac{-1080+128x^3}{(27-8x^3)(27+8x^3)}\,dx}{34992}$$
$$= \frac{x}{4374\,(27+8x^3)} + \frac{\int \frac{1}{27-8x^3}\,dx}{2916} + \frac{7\int \frac{1}{27+8x^3}\,dx}{8748}$$
$$= \frac{x}{4374\,(27+8x^3)} + \frac{\int \frac{1}{3-2x}\,dx}{78732} + \frac{\int \frac{6+2x}{9+6x+4x^2}\,dx}{78732} + \frac{7\int \frac{1}{3+2x}\,dx}{236196} + \frac{7\int \frac{6-2x}{9-6x+4x^2}\,dx}{236196}$$
$$= \frac{x}{4374\,(27+8x^3)} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392} + \frac{\int \frac{6+8x}{9+6x+4x^2}\,dx}{314928} - \frac{7\int \frac{-6+8x}{9-6x+4x^2}\,dx}{944784} +$$
$$= \frac{x}{4374\,(27+8x^3)} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392} - \frac{7\log(9-6x+4x^2)}{944784} + \frac{\log(9+6x}{31492}$$
$$= \frac{x}{4374\,(27+8x^3)} - \frac{7\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392}$$

Mathematica [A]
time = 0.04, size = 103, normalized size = 0.91

$$\frac{\frac{216x}{27+8x^3} + 14\sqrt{3}\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right) + 6\sqrt{3}\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right) - 6\log(3-2x) + 14\log(3+2x) - 7\log(9-6x+4x^2) + 3\log(9+6x+4x^2)}{944784}$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] ((216*x)/(27 + 8*x^3) + 14*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqrt[
3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 6*Log[3 - 2*x] + 14*Log[3 + 2*x] - 7*Log
[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/944784
```

Maple [A]
time = 0.39, size = 102, normalized size = 0.90

method	result
risch	$\frac{x}{34992x^3+118098} - \frac{\ln(-3+2x)}{157464} - \frac{7\ln(4x^2-6x+9)}{944784} + \frac{7\sqrt{3}\,\arctan\left(\frac{2\left(2x-\frac{3}{2}\right)\sqrt{3}}{9}\right)}{472392} + \frac{7\ln(2x+3)}{472392} + \frac{\ln(4x^2+6x+9)}{314928} + -$
default	$-\frac{-\frac{3x}{4}-\frac{9}{8}}{118098\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)} - \frac{7\ln(4x^2-6x+9)}{944784} + \frac{7\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392} + \frac{\ln(4x^2+6x+9)}{314928} + \frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{157464}$

meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\right.}{}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{78732}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-8*x^3+27)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/118098*(-3/4*x-9/8)/(x^2-3/2*x+9/4)-7/944784*ln(4*x^2-6*x+9)+7/472392*3^
(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/314928*ln(4*x^2+6*x+9)+1/157464*3^(1/2
)*arctan(1/18*(8*x+6)*3^(1/2))-1/78732/(2*x+3)+7/472392*ln(2*x+3)-1/157464*
ln(-3+2*x)
```

Maxima [A]
time = 0.52, size = 87, normalized size = 0.77

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{7}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374(8x^3+27)}+\frac{1}{314928}\log(4x^2+6x+9)-\frac{7}{944784}\log(4x^2-6x+9)+\frac{7}{472392}\log(2x+3)-\frac{1}{157464}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1/
9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x + 9
) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(2*x + 3) - 1/157464*log(2*
x - 3)
```

Fricas [A]
time = 0.37, size = 131, normalized size = 1.16

$$\frac{6\sqrt{3}(8x^3+27)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+14\sqrt{3}(8x^3+27)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3(8x^3+27)\log(4x^2+6x+9)-7(8x^3+27)\log(4x^2-6x+9)+14(8x^3+27)\log(2x+3)-6(8x^3+27)\log(2x-3)+216x}{944784(8x^3+27)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/944784*(6*sqrt(3)*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x + 3)) + 14*sqrt(3)
*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(8*x^3 + 27)*log(4*x^2 + 6*
x + 9) - 7*(8*x^3 + 27)*log(4*x^2 - 6*x + 9) + 14*(8*x^3 + 27)*log(2*x + 3)
 - 6*(8*x^3 + 27)*log(2*x - 3) + 216*x)/(8*x^3 + 27)
```

Sympy [A]
time = 0.22, size = 110, normalized size = 0.97

$$\frac{x}{34992x^3+118098}-\frac{\log\left(x-\frac{3}{2}\right)}{157464}+\frac{7\log\left(x+\frac{3}{2}\right)}{472392}-\frac{7\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{314928}+\frac{7\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{472392}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}+\frac{\sqrt{3}}{3}\right)}{157464}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x**3+27)/(-64*x**6+729)**2,x)
```

```
[Out] x/(34992*x**3 + 118098) - log(x - 3/2)/157464 + 7*log(x + 3/2)/472392 - 7*l
og(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2 + 9/4)/314928 + 7*sqrt(3)*
atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(
3)/3)/157464
```

Giac [A]
time = 1.53, size = 89, normalized size = 0.79

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x+3)\right)+\frac{7}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)+\frac{x}{4374\,(8\,x^3+27)}+\frac{1}{314928}\log\left(4\,x^2+6\,x+9\right)-\frac{7}{944784}\log\left(4\,x^2-6\,x+9\right)+\frac{7}{472392}\log\left(|2\,x+3|\right)-\frac{1}{157464}\log\left(|2\,x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1/
9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x + 9
) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(abs(2*x + 3)) - 1/157464*l
og(abs(2*x - 3))
```

Mupad [B]
time = 0.17, size = 102, normalized size = 0.90

$$\frac{7\ln\left(x+\frac{3}{2}\right)}{472392}-\frac{\ln\left(x-\frac{3}{2}\right)}{157464}+\frac{x}{34992\left(x^3+\frac{27}{8}\right)}-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right)-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{7}{944784}+\frac{\sqrt{3}\,7\mathrm{i}}{944784}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{7}{944784}+\frac{\sqrt{3}\,7\mathrm{i}}{944784}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(8*x^3 - 27)/(64*x^6 - 729)^2,x)
```

```
[Out] (7*log(x + 3/2))/472392 - log(x - 3/2)/157464 + x/(34992*(x^3 + 27/8)) - lo
g(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/314928 - 1/314928) + log(x + (3^(
1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/314928 + 1/314928) - log(x - (3^(1/2)*3i)/4
 - 3/4)*((3^(1/2)*7i)/944784 + 7/944784) + log(x + (3^(1/2)*3i)/4 - 3/4)*((
3^(1/2)*7i)/944784 - 7/944784)
```

3.574 $\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=131

$$\frac{1}{26244(3-2x)}-\frac{3-2x}{26244(9-6x+4x^2)}-\frac{11\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{7\log(3-2x)}{157464}+\frac{\log(3+2x)}{472392}+\cdot$$

```
[Out] 1/26244/(3-2*x)+1/26244*(-3+2*x)/(4*x^2-6*x+9)-7/157464*ln(3-2*x)+1/472392*
ln(3+2*x)+17/944784*ln(4*x^2-6*x+9)+1/314928*ln(4*x^2+6*x+9)-11/472392*arct
an(1/9*(3-4*x)*3^(1/2))*3^(1/2)-1/472392*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2
)
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1600, 2099, 652, 632, 210, 648, 642}

$$-\frac{11\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{3-2x}{26244(4x^2-6x+9)}+\frac{17\log(4x^2-6x+9)}{944784}+\frac{\log(4x^2+6x+9)}{314928}+\frac{1}{26244(3-2x)}-\frac{7\log(3-2x)}{157464}+\frac{\log(2x+3)}{472392}$$

Antiderivative was successfully verified.

```
[In] Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(26244*(3 - 2*x)) - (3 - 2*x)/(26244*(9 - 6*x + 4*x^2)) - (11*ArcTan[(3 -
 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(15746
4*Sqrt[3]) - (7*Log[3 - 2*x])/157464 + Log[3 + 2*x]/472392 + (17*Log[9 - 6*
x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^2]/314928
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = \int \frac{1}{(27-36x+24x^2-8x^3)^2(27+36x+24x^2+8x^3)}\,dx$$
$$= \int \left(\frac{1}{13122(-3+2x)^2} - \frac{7}{78732(-3+2x)} + \frac{1}{236196(3+2x)} + \frac{3+}{4374(9-6}\right.$$
$$= \frac{1}{26244(3-2x)} - \frac{7\log(3-2x)}{157464} + \frac{\log(3+2x)}{472392} + \frac{\int \frac{3+17x}{9-6x+4x^2}\,dx}{118098} + \frac{\int \frac{x}{9+6x+4x}}{39360}$$
$$= \frac{1}{26244(3-2x)} - \frac{3-2x}{26244(9-6x+4x^2)} - \frac{7\log(3-2x)}{157464} + \frac{\log(3+2x)}{472392} + \int$$
$$= \frac{1}{26244(3-2x)} - \frac{3-2x}{26244(9-6x+4x^2)} - \frac{7\log(3-2x)}{157464} + \frac{\log(3+2x)}{472392} + \frac{1}{}$$
$$= \frac{1}{26244(3-2x)} - \frac{3-2x}{26244(9-6x+4x^2)} - \frac{11\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{\tan^{-1}\left(\frac{3+4}{3\sqrt{}}\right.}{157464\sqrt{}}$$

Mathematica [A]
time = 0.04, size = 111, normalized size = 0.85

$$\frac{\frac{216x}{27-36x+24x^2-8x^3} + 22\sqrt{3}\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right) - 2\sqrt{3}\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right) - 42\log(3-2x) + 2\log(3+2x) + 17\log(9-6x+4x^2) + 3\log(9+6x+4x^2)}{944784}$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] ((216*x)/(27 - 36*x + 24*x^2 - 8*x^3) + 22*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqr
t[3])] - 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 42*Log[3 - 2*x] + 2*Log[
3 + 2*x] + 17*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/944784
```

Maple [A]
time = 0.40, size = 102, normalized size = 0.78

method	result
risch	$-\frac{x}{34992\left(x^3-3x^2+\frac{9}{2}x-\frac{27}{8}\right)} - \frac{7\ln(-3+2x)}{157464} + \frac{\ln(2x+3)}{472392} + \frac{17\ln(484x^2-726x+1089)}{944784} + \frac{11\sqrt{3}\arctan\left(\frac{2\left(22x-\frac{33}{2}\right)\sqrt{3}}{99}\right)}{472392}$
default	$\frac{\frac{9x}{4}-\frac{27}{8}}{118098x^2-177147x+\frac{531441}{2}} + \frac{17\ln(4x^2-6x+9)}{944784} + \frac{11\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392} + \frac{\ln(4x^2+6x+9)}{314928} - \frac{\sqrt{3}\arctan\left(\frac{(8x+6}{}\right.}{472392}$

meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(\cdots}{}\right.}{}\right.}{6\left(x^6\right)^{\frac{1}{6}}}\right.}{78732}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/118098*(9/4*x-27/8)/(x^2-3/2*x+9/4)+17/944784*ln(4*x^2-6*x+9)+11/472392*3
^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/314928*ln(4*x^2+6*x+9)-1/472392*3^(1/
2)*arctan(1/18*(8*x+6)*3^(1/2))+1/472392*ln(2*x+3)-1/26244/(-3+2*x)-7/15746
4*ln(-3+2*x)
```

Maxima [A]
time = 0.52, size = 95, normalized size = 0.73

$$-\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{11}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374(8x^3-24x^2+36x-27)}+\frac{1}{314928}\log\left(4x^2+6x+9\right)+\frac{17}{944784}\log\left(4x^2-6x+9\right)+\frac{1}{472392}\log\left(2x+3\right)-\frac{7}{157464}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] -1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(8*x^3 - 24*x^2 + 36*x - 27) + 1/314928*l
og(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(2*x + 3
) - 7/157464*log(2*x - 3)
```

Fricas [A]
time = 0.38, size = 187, normalized size = 1.43

$$-\frac{2\sqrt{3}(8x^3-24x^2+36x-27)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-22\sqrt{3}(8x^3-24x^2+36x-27)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-3(8x^3-24x^2+36x-27)\log\left(4x^2+6x+9\right)-17(8x^3-24x^2+36x-27)\log\left(4x^2-6x+9\right)-2(8x^3-24x^2+36x-27)\log\left(2x+3\right)+42(8x^3-24x^2+36x-27)\log\left(2x-3\right)+216x}{944784(8x^3-24x^2+36x-27)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] -1/944784*(2*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x +
 3)) - 22*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x - 3)
) - 3*(8*x^3 - 24*x^2 + 36*x - 27)*log(4*x^2 + 6*x + 9) - 17*(8*x^3 - 24*x^
2 + 36*x - 27)*log(4*x^2 - 6*x + 9) - 2*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*
x + 3) + 42*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*x - 3) + 216*x)/(8*x^3 - 24*
x^2 + 36*x - 27)
```

Sympy [A]
time = 0.29, size = 119, normalized size = 0.91

$$-\frac{x}{34992x^3-104976x^2+157464x-118098}-\frac{7\log\left(x-\frac{3}{2}\right)}{157464}+\frac{\log\left(x+\frac{3}{2}\right)}{472392}+\frac{17\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{314928}+\frac{11\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{472392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(34992*x**3 - 104976*x**2 + 157464*x - 118098) - 7*log(x - 3/2)/157464 +
 log(x + 3/2)/472392 + 17*log(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2
 + 9/4)/314928 + 11*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 - sqrt(3
)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392
```

Giac [A]
time = 1.51, size = 99, normalized size = 0.76

$$-\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{11}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374(4x^2-6x+9)(2x-3)}+\frac{1}{314928}\log\left(4x^2+6x+9\right)+\frac{17}{944784}\log\left(4x^2-6x+9\right)+\frac{1}{472392}\log\left(|2x+3|\right)-\frac{7}{157464}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] -1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x - 3)) + 1/314928*
log(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(abs(2*
x + 3)) - 7/157464*log(abs(2*x - 3))
```

Mupad [B]
time = 0.19, size = 111, normalized size = 0.85

$$\frac{\ln\left(x+\frac{3}{2}\right)}{472392}-\frac{7\ln\left(x-\frac{3}{2}\right)}{157464}-\frac{x}{34992\left(x^3-3x^2+\frac{9x}{2}-\frac{27}{8}\right)}+\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3i}{4}\right)\left(\frac{1}{314928}+\frac{\sqrt{3}\,1i}{944784}\right)-\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3i}{4}\right)\left(-\frac{1}{314928}+\frac{\sqrt{3}\,1i}{944784}\right)-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3i}{4}\right)\left(-\frac{17}{944784}+\frac{\sqrt{3}\,11i}{944784}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3i}{4}\right)\left(\frac{17}{944784}+\frac{\sqrt{3}\,11i}{944784}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((36*x + 24*x^2 + 8*x^3 + 27)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/472392 - (7*log(x - 3/2))/157464 - x/(34992*((9*x)/2 - 3*x^2 +
 x^3 - 27/8)) + log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 + 1/3149
28) - log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 - 1/314928) - log(
x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*11i)/944784 - 17/944784) + log(x + (3^(
1/2)*3i)/4 - 3/4)*((3^(1/2)*11i)/944784 + 17/944784)
```

3.575 $\int \frac{x(27-2x^3)}{729-64x^6}\,dx$

Optimal. Leaf size=99

$$-\frac{5\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}}-\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{32\sqrt{3}}-\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(3+2x)+\frac{5}{576}\log\left(9-6x+4x^2\right)+\frac{1}{192}\log\left(9+6\right.$$

```
[Out] -1/96*ln(3-2*x)-5/288*ln(3+2*x)+5/576*ln(4*x^2-6*x+9)+1/192*ln(4*x^2+6*x+9)
-5/288*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)-1/96*arctan(1/9*(3+4*x)*3^(1/2))
*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1525, 298, 31, 648, 632, 210, 642}

$$-\frac{5\text{ArcTan}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{4x+3}{3\sqrt{3}}\right)}{32\sqrt{3}}+\frac{5}{576}\log\left(4x^2-6x+9\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)-\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
```

```
[Out] (-5*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(96*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[
3])]/(32*Sqrt[3]) - Log[3 - 2*x]/96 - (5*Log[3 + 2*x])/288 + (5*Log[9 - 6*x
 + 4*x^2])/576 + Log[9 + 6*x + 4*x^2]/192
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1525

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[-(e/2 + c*(d/(2*q))), Int[(f*
x)^m/(q - c*x^n), x], x] + Dist[e/2 - c*(d/(2*q)), Int[(f*x)^m/(q + c*x^n),
 x], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(27-2x^3)}{729-64x^6}\,dx &= 3\int \frac{x}{216-64x^3}\,dx+5\int \frac{x}{216+64x^3}\,dx\\ &= \frac{1}{24}\int \frac{1}{6-4x}\,dx-\frac{1}{24}\int \frac{6-4x}{36+24x+16x^2}\,dx-\frac{5}{72}\int \frac{1}{6+4x}\,dx+\frac{5}{72}\int \frac{6+4x}{36-24x+1\cdots}\\ &= -\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(3+2x)+\frac{1}{192}\int \frac{24+32x}{36+24x+16x^2}\,dx+\frac{5}{576}\int \frac{-24+\cdots}{36-24x\cdots}\\ &= -\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(3+2x)+\frac{5}{576}\log\left(9-6x+4x^2\right)+\frac{1}{192}\log\left(9+6x+4x^{2}\cdots\right.\\ &= -\frac{5\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}}-\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{32\sqrt{3}}-\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(3+2x)+\frac{5}{576}\log\left(\cdots\right.\end{aligned}$$

Mathematica [A]
time = 0.01, size = 91, normalized size = 0.92

$$\frac{1}{576}\left(10\sqrt{3}\tan^{-1}\left(\frac{-3+4x}{3\sqrt{3}}\right)-6\sqrt{3}\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)-6\log(3-2x)-10\log(3+2x)+5\log\left(9-6x+4x^2\right)+3\log\left(9+6x+4x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
```

```
[Out] (10*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*Sqrt[3]*ArcTan[(3 + 4*x)/(3*
Sqrt[3])] - 6*Log[3 - 2*x] - 10*Log[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3*L
og[9 + 6*x + 4*x^2])/576
```

Maple [A]
time = 0.39, size = 76, normalized size = 0.77

method	result
default	$\frac{5\ln(4x^2-6x+9)}{576}+\frac{5\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{288}+\frac{\ln(4x^2+6x+9)}{192}-\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{96}-\frac{5\ln(2x+3)}{288}-\frac{\ln(-3+\ldots}{96}$
risch	$\frac{5\ln(16x^2-24x+36)}{576}+\frac{5\sqrt{3}\,\arctan\left(\frac{(-3+4x)\sqrt{3}}{9}\right)}{288}-\frac{\ln(-3+2x)}{96}+\frac{\ln(4x^2+6x+9)}{192}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{96}-$
meijerg	$\frac{x^5\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}+\sqrt{\ldots}\right.}{288(x^6)^{\frac{5}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*x^3+27)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

```
[Out] 5/576*ln(4*x^2-6*x+9)+5/288*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/192*ln(4
*x^2+6*x+9)-1/96*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-5/288*ln(2*x+3)-1/96*
ln(-3+2*x)
```

Maxima [A]
time = 0.51, size = 75, normalized size = 0.76

$$-\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)+\frac{5}{576}\log\left(4x^2-6x+9\right)-\frac{5}{288}\log\left(2x+3\right)-\frac{1}{96}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt
(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9) -
5/288*log(2*x + 3) - 1/96*log(2*x - 3)
```

Fricas [A]
time = 0.37, size = 75, normalized size = 0.76

$$-\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)+\frac{5}{576}\log\left(4x^2-6x+9\right)-\frac{5}{288}\log\left(2x+3\right)-\frac{1}{96}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt
(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9) -
5/288*log(2*x + 3) - 1/96*log(2*x - 3)
```

Sympy [A]
time = 0.19, size = 102, normalized size = 1.03

$$-\frac{\log\left(x-\frac{3}{2}\right)}{96}-\frac{5\log\left(x+\frac{3}{2}\right)}{288}+\frac{5\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{576}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{192}+\frac{5\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{288}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*x**3+27)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/96 - 5*log(x + 3/2)/288 + 5*log(x**2 - 3*x/2 + 9/4)/576 + log
(x**2 + 3*x/2 + 9/4)/192 + 5*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/288 -
sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/96
```

Giac [A]
time = 0.88, size = 69, normalized size = 0.70

$$-\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\log\left(x^2+\frac{3}{2}x+\frac{9}{4}\right)+\frac{5}{576}\log\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)-\frac{5}{288}\log\left(\left|x+\frac{3}{2}\right|\right)-\frac{1}{96}\log\left(\left|x-\frac{3}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt
(3)*(4*x - 3)) + 1/192*log(x^2 + 3/2*x + 9/4) + 5/576*log(x^2 - 3/2*x + 9/4
) - 5/288*log(abs(x + 3/2)) - 1/96*log(abs(x - 3/2))
```

Mupad [B]
time = 5.10, size = 91, normalized size = 0.92

$$-\frac{\ln\left(x-\frac{3}{2}\right)}{96}-\frac{5\ln\left(x+\frac{3}{2}\right)}{288}+\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{192}+\frac{\sqrt{3}\,1\mathrm{i}}{192}\right)-\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{192}+\frac{\sqrt{3}\,1\mathrm{i}}{192}\right)-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{5}{576}+\frac{\sqrt{3}\,5\mathrm{i}}{576}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{5}{576}+\frac{\sqrt{3}\,5\mathrm{i}}{576}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*x^3 - 27))/(64*x^6 - 729),x)
```

```
[Out] log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/192 + 1/192) - (5*log(x + 3/2))
/288 - log(x - 3/2)/96 - log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/192 -
1/192) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*5i)/576 - 5/576) + log(x +
 (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*5i)/576 + 5/576)
```

3.576 $\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{a+bx^n} dx$

Optimal. Leaf size=162

$$\frac{(bf-ag)x^{1+n}(cx)^m}{b^2(1+m+n)}+\frac{gx^{1+2n}(cx)^m}{b(1+m+2n)}+\frac{(b^2e-abf+a^2g)(cx)^{1+m}}{b^3c(1+m)}+\frac{(b^3d-ab^2e+a^2bf-a^3g)(cx)^{1+m}\,{}_2F_1(1}{ab^3c(1+m)}$$

```
[Out] (-a*g+b*f)*x^(1+n)*(c*x)^m/b^2/(1+m+n)+g*x^(1+2*n)*(c*x)^m/b/(1+m+2*n)+(a^2
*g-a*b*f+b^2*e)*(c*x)^(1+m)/b^3/c/(1+m)+(-a^3*g+a^2*b*f-a*b^2*e+b^3*d)*(c*x
)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/b^3/c/(1+m)
```

Rubi [A]
time = 0.12, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1858, 20, 30, 371}

$$\frac{(cx)^{m+1}(a^2g-abf+b^2e)}{b^3c(m+1)}+\frac{(cx)^{m+1}(a^3(-g)+a^2bf-ab^2e+b^3d)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ab^3c(m+1)}+\frac{x^{n+1}(cx)^m(bf-ag)}{b^2(m+n+1)}+\frac{gx^{2n+1}(cx)^m}{b(m+2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n),x]
```

```
[Out] ((b*f - a*g)*x^(1 + n)*(c*x)^m)/(b^2*(1 + m + n)) + (g*x^(1 + 2*n)*(c*x)^m)
/(b*(1 + m + 2*n)) + ((b^2*e - a*b*f + a^2*g)*(c*x)^(1 + m))/(b^3*c*(1 + m)
) + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*(c*x)^(1 + m)*Hypergeometric2F1[1,
 (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*b^3*c*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(cx)^m\left(d+ex^n+fx^{2n}+gx^{3n}\right)}{a+bx^n}\,dx=\int\left(\frac{\left(b^2e-abf+a^2g\right)(cx)^m}{b^3}+\frac{(bf-ag)x^n(cx)^m}{b^2}+\frac{gx^{2n}(cx)^m}{b}+\right.$$
$$=\frac{\left(b^2e-abf+a^2g\right)(cx)^{1+m}}{b^3c(1+m)}+\frac{g\int x^{2n}(cx)^m\,dx}{b}+\frac{(bf-ag)\int x^n(cx)}{b^2}$$
$$=\frac{\left(b^2e-abf+a^2g\right)(cx)^{1+m}}{b^3c(1+m)}+\frac{\left(b^3d-ab^2e+a^2bf-a^3g\right)(cx)^{1+m}\,{}_2F}{ab^3c(1+m)}$$
$$=\frac{(bf-ag)x^{1+n}(cx)^m}{b^2(1+m+n)}+\frac{gx^{1+2n}(cx)^m}{b(1+m+2n)}+\frac{\left(b^2e-abf+a^2g\right)(cx)^{1+m}}{b^3c(1+m)}$$

Mathematica [A]
time = 0.44, size = 150, normalized size = 0.93

$$x(cx)^m\left(\frac{a^2g}{b^3(1+m)}+\frac{e}{b+bm}+\frac{fx^n}{b(1+m+n)}+\frac{gx^{2n}}{b+bm+2bn}-\frac{a\left(\frac{f}{1+m}+\frac{gx^n}{1+m+n}\right)}{b^2}+\frac{\left(b^3d-ab^2e+a^2bf-a^3g\right)\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{ab^3(1+m)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n),x]
```

```
[Out] x*(c*x)^m*((a^2*g)/(b^3*(1 + m)) + e/(b + b*m) + (f*x^n)/(b*(1 + m + n)) +
(g*x^(2*n))/(b + b*m + 2*b*n) - (a*(f/(1 + m) + (g*x^n)/(1 + m + n)))/b^2 +
 ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*Hypergeometric2F1[1, (1 + m)/n, (1 +
m + n)/n, -((b*x^n)/a)])/(a*b^3*(1 + m)))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m\left(d+e\,x^n+f\,x^{2n}+g\,x^{3n}\right)}{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x)
```

```
[Out] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="max
ima")
```

```
[Out] (b^3*c^m*d + a^2*b*c^m*f - a^3*c^m*g - a*b^2*c^m*e)*integrate(x^m/(b^4*x^n
+ a*b^3), x) + ((m^2 + m*(n + 2) + n + 1)*b^2*c^m*g*x*e^(m*log(x) + 2*n*log
(x)) - ((m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*a*b*c^m*f - (m^2 + m*(3*n + 2
) + 2*n^2 + 3*n + 1)*a^2*c^m*g - (m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*b^2*
c^m*e)*x*x^m + ((m^2 + 2*m*(n + 1) + 2*n + 1)*b^2*c^m*f - (m^2 + 2*m*(n + 1
) + 2*n + 1)*a*b*c^m*g)*x*e^(m*log(x) + n*log(x)))/((m^3 + 3*m^2*(n + 1) +
(2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n + 1)*b^3)
```

Fricas [F]
time = 0.37, size = 38, normalized size = 0.23

$$\text{integral}\left(\frac{(gx^{3n} + fx^{2n} + ex^n + d)(cx)^m}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="fri
cas")
```

```
[Out] integral((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 23.93, size = 654, normalized size = 4.04

$$\frac{c^m dmxx^m\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{m}{n}+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+\frac{1}{n}\right)}{an^2\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)} + \frac{c^m dxx^m\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{m}{n}+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+\frac{1}{n}\right)}{an^2\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)} + \frac{c^m emxx^mx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{m}{n}+1+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}{an^2\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)} + \frac{c^m exx^mx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{m}{n}+1+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}{an\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)} + \frac{c^m exx^mx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{m}{n}+1+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}{an^2\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)} + \frac{c^m fmxx^mx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{m}{n}+2+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}{an^2\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right)} + \frac{2c^m fxx^mx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{m}{n}+2+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}{an\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right)} + \frac{c^m fxx^mx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{m}{n}+2+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}{an^2\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right)} + \frac{c^m gmxx^mx^{3n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{m}{n}+3+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right)}{an^2\Gamma\left(\frac{m}{n}+4+\frac{1}{n}\right)} + \frac{3c^m gxx^mx^{3n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{m}{n}+3+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right)}{an\Gamma\left(\frac{m}{n}+4+\frac{1}{n}\right)} + \frac{c^m gxx^mx^{3n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{m}{n}+3+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right)}{an^2\Gamma\left(\frac{m}{n}+4+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n),x)
```

```
[Out] c**m*d*m*x*x**m*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n
+ 1/n)/(a*n**2*gamma(m/n + 1 + 1/n)) + c**m*d*x*x**m*lerchphi(b*x**n*exp_po
lar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*n**2*gamma(m/n + 1 + 1/n)) +
 c**m*e*m*x*x**m*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1 + 1/n)*
gamma(m/n + 1 + 1/n)/(a*n**2*gamma(m/n + 2 + 1/n)) + c**m*e*x*x**m*x**n*ler
chphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(a*n
*gamma(m/n + 2 + 1/n)) + c**m*e*x*x**m*x**n*lerchphi(b*x**n*exp_polar(I*pi)
/a, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(a*n**2*gamma(m/n + 2 + 1/n)) +
```

```
c**m*f*m*x*x**m*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 2 + 1/
n)*gamma(m/n + 2 + 1/n)/(a*n**2*gamma(m/n + 3 + 1/n)) + 2*c**m*f*x*x**m*x**
(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 2 + 1/n)*gamma(m/n + 2 +
1/n)/(a*n*gamma(m/n + 3 + 1/n)) + c**m*f*x*x**m*x**(2*n)*lerchphi(b*x**n*ex
p_polar(I*pi)/a, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(a*n**2*gamma(m/n +
 3 + 1/n)) + c**m*g*m*x*x**m*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
 m/n + 3 + 1/n)*gamma(m/n + 3 + 1/n)/(a*n**2*gamma(m/n + 4 + 1/n)) + 3*c**m
*g*x*x**m*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 3 + 1/n)*gam
ma(m/n + 3 + 1/n)/(a*n*gamma(m/n + 4 + 1/n)) + c**m*g*x*x**m*x**(3*n)*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 3 + 1/n)*gamma(m/n + 3 + 1/n)/(a*n**
2*gamma(m/n + 4 + 1/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="gia
c")
```

```
[Out] integrate((g*x^(3*n) + f*x^(2*n) + x^n*e + d)*(c*x)^m/(b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(c\,x)}^m\,\left(d+e\,x^n+f\,x^{2\,n}+g\,x^{3\,n}\right)}{a+b\,x^n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n),x)
```

```
[Out] int(((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n), x)
```

3.577 $\int \left(c+dx^{-1+n}\right)(a+bx^n)^3\,dx$

Optimal. Leaf size=84

$$a^3cx+\frac{3a^2bcx^{1+n}}{1+n}+\frac{3ab^2cx^{1+2n}}{1+2n}+\frac{b^3cx^{1+3n}}{1+3n}+\frac{d(a+bx^n)^4}{4bn}$$

```
[Out] a^3*c*x+3*a^2*b*c*x^(1+n)/(1+n)+3*a*b^2*c*x^(1+2*n)/(1+2*n)+b^3*c*x^(1+3*n)
/(1+3*n)+1/4*d*(a+b*x^n)^4/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1905, 250, 267}

$$a^3cx+\frac{3a^2bcx^{n+1}}{n+1}+\frac{3ab^2cx^{2n+1}}{2n+1}+\frac{d(a+bx^n)^4}{4bn}+\frac{b^3cx^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

```
[Out] a^3*c*x + (3*a^2*b*c*x^(1 + n))/(1 + n) + (3*a*b^2*c*x^(1 + 2*n))/(1 + 2*n)
 + (b^3*c*x^(1 + 3*n))/(1 + 3*n) + (d*(a + b*x^n)^4)/(4*b*n)
```

Rule 250

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1905

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx^{-1+n}\right)(a+bx^n)^3\,dx &= c\int (a+bx^n)^3\,dx + d\int x^{-1+n}(a+bx^n)^3\,dx\\ &= \frac{d(a+bx^n)^4}{4bn} + c\int \left(a^3+3a^2bx^n+3ab^2x^{2n}+b^3x^{3n}\right)dx\\ &= a^3cx + \frac{3a^2bcx^{1+n}}{1+n} + \frac{3ab^2cx^{1+2n}}{1+2n} + \frac{b^3cx^{1+3n}}{1+3n} + \frac{d(a+bx^n)^4}{4bn}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 162, normalized size = 1.93

$$\frac{4a^3(1+6n+11n^2+6n^3)\,(cnx+dx^n)+6a^2b(1+5n+6n^2)\,x^n(2cnx+d(1+n)x^n)+4ab^2(1+4n+3n^2)\,x^{2n}(3cnx+d(1+2n)x^n)+b^3(1+3n+2n^2)\,x^{3n}(4cnx+d(1+3n)x^n)}{4n(1+n)(1+2n)(1+3n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

```
[Out] (4*a^3*(1 + 6*n + 11*n^2 + 6*n^3)*(c*n*x + d*x^n) + 6*a^2*b*(1 + 5*n + 6*n^
2)*x^n*(2*c*n*x + d*(1 + n)*x^n) + 4*a*b^2*(1 + 4*n + 3*n^2)*x^(2*n)*(3*c*n
*x + d*(1 + 2*n)*x^n) + b^3*(1 + 3*n + 2*n^2)*x^(3*n)*(4*c*n*x + d*(1 + 3*n
)*x^n))/(4*n*(1 + n)*(1 + 2*n)*(1 + 3*n))
```

Maple [A]
time = 0.34, size = 118, normalized size = 1.40

method	result
risch	$a^3cx+\frac{b^3d\,x^{4n}}{4n}+\frac{b^2(nbcx+3adn+ad)x^{3n}}{n(1+3n)}+\frac{3ab(2nbcx+2adn+ad)x^{2n}}{2n(1+2n)}+\frac{a^2(3nbcx+adn+ad)x^n}{n(1+n)}$
norman	$a^3cx+\frac{a^3d\,\mathrm{e}^{n\ln(x)}}{n}+\frac{a\,b^2d\,\mathrm{e}^{3n\ln(x)}}{n}+\frac{b^3cx\,\mathrm{e}^{3n\ln(x)}}{1+3n}+\frac{b^3d\,\mathrm{e}^{4n\ln(x)}}{4n}+\frac{3d\,a^2b\,\mathrm{e}^{2n\ln(x)}}{2n}+\frac{3ac\,b^2x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{3c\,a^2bx\,\mathrm{e}^{n\ln}}{1+n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*c*x+1/4*b^3*d/n*(x^n)^4+b^2*(b*c*n*x+3*a*d*n+a*d)/n/(1+3*n)*(x^n)^3+3/2
*a*b*(2*b*c*n*x+2*a*d*n+a*d)/n/(1+2*n)*(x^n)^2+a^2*(3*b*c*n*x+a*d*n+a*d)/n/
(1+n)*x^n
```

Maxima [A]
time = 0.29, size = 118, normalized size = 1.40

$$a^3cx+\frac{b^3dx^{4\,n}}{4\,n}+\frac{ab^2dx^{3\,n}}{n}+\frac{3\,a^2bdx^{2\,n}}{2\,n}+\frac{b^3cx^{3\,n+1}}{3\,n+1}+\frac{3\,ab^2cx^{2\,n+1}}{2\,n+1}+\frac{3\,a^2bcx^{n+1}}{n+1}+\frac{a^3dx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] a^3*c*x + 1/4*b^3*d*x^(4*n)/n + a*b^2*d*x^(3*n)/n + 3/2*a^2*b*d*x^(2*n)/n +
 b^3*c*x^(3*n + 1)/(3*n + 1) + 3*a*b^2*c*x^(2*n + 1)/(2*n + 1) + 3*a^2*b*c*
x^(n + 1)/(n + 1) + a^3*d*x^n/n
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(82) = 164$.
time = 0.38, size = 305, normalized size = 3.63

$$\frac{4\,{\left(6\,a^{3}cn^{4}+11\,a^{3}cn^{3}+6\,a^{3}cn^{2}+a^{3}cn\right)}x+{\left(6\,b^{3}dn^{3}+11\,b^{3}dn^{2}+6\,b^{3}dn+b^{3}d\right)}x^{4\,n}+4\,{\left(6\,ab^{2}dn^{3}+11\,ab^{2}dn^{2}+6\,ab^{2}dn+ab^{2}d+{\left(2\,b^{3}cn^{3}+3\,b^{3}cn^{2}+b^{3}cn\right)}x\right)}x^{3\,n}+6\,{\left(6\,a^{2}bdn^{3}+11\,a^{2}bdn^{2}+6\,a^{2}bdn+a^{2}bd+2\,{\left(3\,ab^{2}cn^{3}+4\,ab^{2}cn^{2}+ab^{2}cn\right)}x\right)}x^{2\,n}+4\,{\left(6\,a^{3}dn^{3}+11\,a^{3}dn^{2}+6\,a^{3}dn+a^{3}d+3\,{\left(6\,a^{2}bcn^{3}+5\,a^{2}bcn^{2}+a^{2}bcn\right)}x\right)}x^{n}}{4\,{\left(6\,n^{4}+11\,n^{3}+6\,n^{2}+n\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(6*a^3*c*n^4 + 11*a^3*c*n^3 + 6*a^3*c*n^2 + a^3*c*n)*x + (6*b^3*d*n^
3 + 11*b^3*d*n^2 + 6*b^3*d*n + b^3*d)*x^(4*n) + 4*(6*a*b^2*d*n^3 + 11*a*b^2
*d*n^2 + 6*a*b^2*d*n + a*b^2*d + (2*b^3*c*n^3 + 3*b^3*c*n^2 + b^3*c*n)*x)*x
^(3*n) + 6*(6*a^2*b*d*n^3 + 11*a^2*b*d*n^2 + 6*a^2*b*d*n + a^2*b*d + 2*(3*a
*b^2*c*n^3 + 4*a*b^2*c*n^2 + a*b^2*c*n)*x)*x^(2*n) + 4*(6*a^3*d*n^3 + 11*a^
3*d*n^2 + 6*a^3*d*n + a^3*d + 3*(6*a^2*b*c*n^3 + 5*a^2*b*c*n^2 + a^2*b*c*n)
*x)*x^n)/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1251 vs. $2(75) = 150$.
time = 1.00, size = 1251, normalized size = 14.89

$$\begin{cases} a^{3}cx - \frac{a^{3}d}{x} + 3a^{2}bc\log{\left(x \right)} - \frac{3a^{2}bd}{2x^{2}} - \frac{3ab^{2}c}{x} - \frac{ab^{2}d}{x^{3}} - \frac{b^{3}c}{2x^{2}} - \frac{b^{3}d}{4x^{4}} & \text{for}\: n = -1 \\ a^{3}cx - \frac{2a^{3}d}{\sqrt{x}} + 6a^{2}bc\sqrt{x} - \frac{3a^{2}bd}{x} + 3ab^{2}c\log{\left(x \right)} - \frac{2ab^{2}d}{x^{\frac{3}{2}}} - \frac{2b^{3}c}{\sqrt{x}} - \frac{b^{3}d}{2x^{2}} & \text{for}\: n = - \frac{1}{2} \\ a^{3}cx - \frac{3a^{3}d}{\sqrt[3]{x}} + \frac{9a^{2}bcx^{\frac{2}{3}}}{2} - \frac{9a^{2}bd}{2x^{\frac{2}{3}}} + 9ab^{2}c\sqrt[3]{x} - \frac{3ab^{2}d}{x} + b^{3}c\log{\left(x \right)} - \frac{3b^{3}d}{4x^{\frac{4}{3}}} & \text{for}\: n = - \frac{1}{3} \\ \left(a + b\right)^{3} \left(cx + d\log{\left(x \right)}\right) & \text{for}\: n = 0 \\ \frac{24a^{3}cn^{4}x}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{44a^{3}cn^{3}x}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{24a^{3}cn^{2}x}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{4a^{3}cnx}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{24a^{3}dn^{3}x^{n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{44a^{3}dn^{2}x^{n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{24a^{3}dnx^{n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{4a^{3}dx^{n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{72a^{2}bcn^{3}xx^{n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{60a^{2}bcn^{2}xx^{n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{12a^{2}bcnxx^{n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{36a^{2}bdn^{3}x^{2n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{66a^{2}bdn^{2}x^{2n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{36a^{2}bdnx^{2n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{6a^{2}bdx^{2n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{36ab^{2}cn^{3}xx^{2n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{48ab^{2}cn^{2}xx^{2n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{12ab^{2}cnxx^{2n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{24ab^{2}dn^{3}x^{3n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{44ab^{2}dn^{2}x^{3n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{24ab^{2}dnx^{3n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{4ab^{2}dx^{3n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{8b^{3}cn^{3}xx^{3n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{12b^{3}cn^{2}xx^{3n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{4b^{3}cnxx^{3n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{6b^{3}dn^{3}x^{4n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{11b^{3}dn^{2}x^{4n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{6b^{3}dnx^{4n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} + \frac{b^{3}dx^{4n}}{24n^{4} + 44n^{3} + 24n^{2} + 4n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*c*x - a**3*d/x + 3*a**2*b*c*log(x) - 3*a**2*b*d/(2*x**2) -
3*a*b**2*c/x - a*b**2*d/x**3 - b**3*c/(2*x**2) - b**3*d/(4*x**4), Eq(n, -1)
), (a**3*c*x - 2*a**3*d/sqrt(x) + 6*a**2*b*c*sqrt(x) - 3*a**2*b*d/x + 3*a*b
**2*c*log(x) - 2*a*b**2*d/x**(3/2) - 2*b**3*c/sqrt(x) - b**3*d/(2*x**2), Eq
(n, -1/2)), (a**3*c*x - 3*a**3*d/x**(1/3) + 9*a**2*b*c*x**(2/3)/2 - 9*a**2*
b*d/(2*x**(2/3)) + 9*a*b**2*c*x**(1/3) - 3*a*b**2*d/x + b**3*c*log(x) - 3*b
**3*d/(4*x**(4/3)), Eq(n, -1/3)), ((a + b)**3*(c*x + d*log(x)), Eq(n, 0)),
(24*a**3*c*n**4*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**3*c*n**3*x/(2
4*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*c*n**2*x/(24*n**4 + 44*n**3 + 2
4*n**2 + 4*n) + 4*a**3*c*n*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*
d*n**3*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**3*d*n**2*x**n/(24*n
**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*d*n*x**n/(24*n**4 + 44*n**3 + 24*n
**2 + 4*n) + 4*a**3*d*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 72*a**2*b*
c*n**3*x*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 60*a**2*b*c*n**2*x*x**n
```

```
/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 12*a**2*b*c*n*x*x**n/(24*n**4 + 44*n
**3 + 24*n**2 + 4*n) + 36*a**2*b*d*n**3*x**(2*n)/(24*n**4 + 44*n**3 + 24*n*
*2 + 4*n) + 66*a**2*b*d*n**2*x**(2*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) +
 36*a**2*b*d*n*x**(2*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*a**2*b*d*x*
*(2*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a*b**2*c*n**3*x*x**(2*n)/(2
4*n**4 + 44*n**3 + 24*n**2 + 4*n) + 48*a*b**2*c*n**2*x*x**(2*n)/(24*n**4 +
44*n**3 + 24*n**2 + 4*n) + 12*a*b**2*c*n*x*x**(2*n)/(24*n**4 + 44*n**3 + 24
*n**2 + 4*n) + 24*a*b**2*d*n**3*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n
) + 44*a*b**2*d*n**2*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a*b*
*2*d*n*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 4*a*b**2*d*x**(3*n)/(
24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 8*b**3*c*n**3*x*x**(3*n)/(24*n**4 + 44
*n**3 + 24*n**2 + 4*n) + 12*b**3*c*n**2*x*x**(3*n)/(24*n**4 + 44*n**3 + 24*
n**2 + 4*n) + 4*b**3*c*n*x*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6
*b**3*d*n**3*x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 11*b**3*d*n**2*
x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*b**3*d*n*x**(4*n)/(24*n**4
 + 44*n**3 + 24*n**2 + 4*n) + b**3*d*x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2
+ 4*n), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(82) = 164.
time = 0.82, size = 392, normalized size = 4.67

$$\frac{24\,a^3cn^4x+8\,b^3cn^3xx^{3\,n}+36\,ab^2cn^3xx^{2\,n}+72\,a^2bcn^3xx^n+44\,a^3cn^3x+6\,b^3dn^3x^{4\,n}+24\,ab^2dn^3x^{3\,n}+12\,b^3cn^2xx^{3\,n}+36\,a^2bdn^3x^{2\,n}+48\,ab^2cn^2xx^{2\,n}+24\,a^3dn^3x^n+60\,a^2bcn^2xx^n+24\,a^3cn^2x+11\,b^3dn^2x^{4\,n}+44\,ab^2dn^2x^{3\,n}+4\,b^3cnxx^{3\,n}+66\,a^2bdn^2x^{2\,n}+12\,ab^2cnxx^{2\,n}+44\,a^3dn^2x^n+12\,a^2bcnxx^n+4\,a^3cnx+6\,b^3dnx^{4\,n}+24\,ab^2dnx^{3\,n}+36\,a^2bdnx^{2\,n}+24\,a^3dnx^n+b^3dx^{4\,n}+4\,ab^2dx^{3\,n}+6\,a^2bdx^{2\,n}+4\,a^3dx^n}{4\,(6\,n^4+11\,n^3+6\,n^2+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] 1/4*(24*a^3*c*n^4*x + 8*b^3*c*n^3*x*x^(3*n) + 36*a*b^2*c*n^3*x*x^(2*n) + 72
*a^2*b*c*n^3*x*x^n + 44*a^3*c*n^3*x + 6*b^3*d*n^3*x^(4*n) + 24*a*b^2*d*n^3*
x^(3*n) + 12*b^3*c*n^2*x*x^(3*n) + 36*a^2*b*d*n^3*x^(2*n) + 48*a*b^2*c*n^2*
x*x^(2*n) + 24*a^3*d*n^3*x^n + 60*a^2*b*c*n^2*x*x^n + 24*a^3*c*n^2*x + 11*b
^3*d*n^2*x^(4*n) + 44*a*b^2*d*n^2*x^(3*n) + 4*b^3*c*n*x*x^(3*n) + 66*a^2*b*
d*n^2*x^(2*n) + 12*a*b^2*c*n*x*x^(2*n) + 44*a^3*d*n^2*x^n + 12*a^2*b*c*n*x*
x^n + 4*a^3*c*n*x + 6*b^3*d*n*x^(4*n) + 24*a*b^2*d*n*x^(3*n) + 36*a^2*b*d*n
*x^(2*n) + 24*a^3*d*n*x^n + b^3*d*x^(4*n) + 4*a*b^2*d*x^(3*n) + 6*a^2*b*d*x
^(2*n) + 4*a^3*d*x^n)/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

Mupad [B]
time = 5.13, size = 115, normalized size = 1.37

$$a^3\,c\,x+\frac{a^3\,d\,x^n}{n}+\frac{b^3\,d\,x^{4\,n}}{4\,n}+\frac{b^3\,c\,x\,x^{3\,n}}{3\,n+1}+\frac{3\,a^2\,b\,d\,x^{2\,n}}{2\,n}+\frac{a\,b^2\,d\,x^{3\,n}}{n}+\frac{3\,a\,b^2\,c\,x\,x^{2\,n}}{2\,n+1}+\frac{3\,a^2\,b\,c\,x\,x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^(n - 1))*(a + b*x^n)^3,x)
```

```
[Out] a^3*c*x + (a^3*d*x^n)/n + (b^3*d*x^(4*n))/(4*n) + (b^3*c*x*x^(3*n))/(3*n +
1) + (3*a^2*b*d*x^(2*n))/(2*n) + (a*b^2*d*x^(3*n))/n + (3*a*b^2*c*x*x^(2*n)
)/(2*n + 1) + (3*a^2*b*c*x*x^n)/(n + 1)
```

3.578 $\int \left(c+dx^{-1+n}\right) \left(a+bx^n\right)^2 \, dx$

Optimal. Leaf size=61

$$a^2cx+\frac{2abcx^{1+n}}{1+n}+\frac{b^2cx^{1+2n}}{1+2n}+\frac{d(a+bx^n)^3}{3bn}$$

```
[Out] a^2*c*x+2*a*b*c*x^(1+n)/(1+n)+b^2*c*x^(1+2*n)/(1+2*n)+1/3*d*(a+b*x^n)^3/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1905, 250, 267}

$$a^2cx+\frac{2abcx^{n+1}}{n+1}+\frac{d(a+bx^n)^3}{3bn}+\frac{b^2cx^{2n+1}}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]
```

```
[Out] a^2*c*x + (2*a*b*c*x^(1 + n))/(1 + n) + (b^2*c*x^(1 + 2*n))/(1 + 2*n) + (d*
(a + b*x^n)^3)/(3*b*n)
```

Rule 250

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1905

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx^{-1+n}) (a+bx^n)^2 \, dx &= c\int (a+bx^n)^2 \, dx + d\int x^{-1+n}(a+bx^n)^2 \, dx \\ &= \frac{d(a+bx^n)^3}{3bn} + c\int (a^2+2abx^n+b^2x^{2n}) \, dx \\ &= a^2cx + \frac{2abcx^{1+n}}{1+n} + \frac{b^2cx^{1+2n}}{1+2n} + \frac{d(a+bx^n)^3}{3bn}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 99, normalized size = 1.62

$$\frac{3a^2(1+3n+2n^2)(cnx+dx^n)+3ab(1+2n)x^n(2cnx+d(1+n)x^n)+b^2(1+n)x^{2n}(3cnx+d(1+2n)x^n)}{3n(1+n)(1+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]
```

```
[Out] (3*a^2*(1 + 3*n + 2*n^2)*(c*n*x + d*x^n) + 3*a*b*(1 + 2*n)*x^n*(2*c*n*x + d
*(1 + n)*x^n) + b^2*(1 + n)*x^(2*n)*(3*c*n*x + d*(1 + 2*n)*x^n))/(3*n*(1 +
n)*(1 + 2*n))
```

Maple [A]
time = 0.34, size = 80, normalized size = 1.31

method	result	size
risch	$a^2cx+\frac{b^2d\,x^{3n}}{3n}+\frac{b(nbcx+2adn+ad)x^{2n}}{n(1+2n)}+\frac{a(2nbcx+adn+ad)x^n}{n(1+n)}$	80
norman	$a^2cx+\frac{a^2d\,e^{n\ln(x)}}{n}+\frac{abd\,e^{2n\ln(x)}}{n}+\frac{b^2cx\,e^{2n\ln(x)}}{1+2n}+\frac{b^2d\,e^{3n\ln(x)}}{3n}+\frac{2abcx\,e^{n\ln(x)}}{1+n}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c*x+1/3*b^2*d/n*(x^n)^3+b*(b*c*n*x+2*a*d*n+a*d)/n/(1+2*n)*(x^n)^2+a*(2*
b*c*n*x+a*d*n+a*d)/n/(1+n)*x^n
```

Maxima [A]
time = 0.29, size = 78, normalized size = 1.28

$$a^2cx+\frac{b^2dx^{3n}}{3n}+\frac{abdx^{2n}}{n}+\frac{b^2cx^{2n+1}}{2n+1}+\frac{2abcx^{n+1}}{n+1}+\frac{a^2dx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] a^2*c*x + 1/3*b^2*d*x^(3*n)/n + a*b*d*x^(2*n)/n + b^2*c*x^(2*n + 1)/(2*n +
1) + 2*a*b*c*x^(n + 1)/(n + 1) + a^2*d*x^n/n
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(59) = 118$.
time = 0.39, size = 160, normalized size = 2.62

$$\frac{3\left(2\,a^2cn^3+3\,a^2cn^2+a^2cn\right)x+\left(2\,b^2dn^2+3\,b^2dn+b^2d\right)x^{3\,n}+3\left(2\,abdn^2+3\,abdn+abd+\left(b^2cn^2+b^2cn\right)x\right)x^{2\,n}+3\left(2\,a^2dn^2+3\,a^2dn+a^2d+2\left(2\,abcn^2+abcn\right)x\right)x^n}{3\left(2\,n^3+3\,n^2+n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(2*a^2*c*n^3 + 3*a^2*c*n^2 + a^2*c*n)*x + (2*b^2*d*n^2 + 3*b^2*d*n +
 b^2*d)*x^(3*n) + 3*(2*a*b*d*n^2 + 3*a*b*d*n + a*b*d + (b^2*c*n^2 + b^2*c*n
)*x)*x^(2*n) + 3*(2*a^2*d*n^2 + 3*a^2*d*n + a^2*d + 2*(2*a*b*c*n^2 + a*b*c*
n)*x)*x^n)/(2*n^3 + 3*n^2 + n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(53) = 106$.
time = 0.58, size = 552, normalized size = 9.05

$$\begin{cases} a^2cx - \frac{a^2d}{x} + 2abc\log(x) - \frac{abd}{x^2} - \frac{b^2c}{x} - \frac{b^2d}{3x^3} & \text{for}\ n = -1 \\ a^2cx - \frac{2a^2d}{\sqrt{x}} + 4abc\sqrt{x} - \frac{2abd}{x} + b^2c\log(x) - \frac{2b^2d}{3x^{\frac{3}{2}}} & \text{for}\ n = -\frac{1}{2} \\ (a+b)^2\,(cx + d\log(x)) & \text{for}\ n = 0 \\ \frac{6a^2cn^3x}{6n^3+9n^2+3n} + \frac{9a^2cn^2x}{6n^3+9n^2+3n} + \frac{3a^2cnx}{6n^3+9n^2+3n} + \frac{6a^2dn^2x^n}{6n^3+9n^2+3n} + \frac{9a^2dnx^n}{6n^3+9n^2+3n} + \frac{3a^2dx^n}{6n^3+9n^2+3n} + \frac{12abcn^2xx^n}{6n^3+9n^2+3n} + \frac{6abcnxx^n}{6n^3+9n^2+3n} + \frac{6abdn^2x^{2n}}{6n^3+9n^2+3n} + \frac{9abdnx^{2n}}{6n^3+9n^2+3n} + \frac{3abdx^{2n}}{6n^3+9n^2+3n} + \frac{3b^2cn^2xx^{2n}}{6n^3+9n^2+3n} + \frac{3b^2cnxx^{2n}}{6n^3+9n^2+3n} + \frac{2b^2dn^2x^{3n}}{6n^3+9n^2+3n} + \frac{3b^2dnx^{3n}}{6n^3+9n^2+3n} + \frac{b^2dx^{3n}}{6n^3+9n^2+3n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))*(a+b*x**n)**2,x)
```

```
[Out] Piecewise((a**2*c*x - a**2*d/x + 2*a*b*c*log(x) - a*b*d/x**2 - b**2*c/x - b
**2*d/(3*x**3), Eq(n, -1)), (a**2*c*x - 2*a**2*d/sqrt(x) + 4*a*b*c*sqrt(x)
- 2*a*b*d/x + b**2*c*log(x) - 2*b**2*d/(3*x**(3/2)), Eq(n, -1/2)), ((a + b)
**2*(c*x + d*log(x)), Eq(n, 0)), (6*a**2*c*n**3*x/(6*n**3 + 9*n**2 + 3*n) +
 9*a**2*c*n**2*x/(6*n**3 + 9*n**2 + 3*n) + 3*a**2*c*n*x/(6*n**3 + 9*n**2 +
3*n) + 6*a**2*d*n**2*x**n/(6*n**3 + 9*n**2 + 3*n) + 9*a**2*d*n*x**n/(6*n**3
 + 9*n**2 + 3*n) + 3*a**2*d*x**n/(6*n**3 + 9*n**2 + 3*n) + 12*a*b*c*n**2*x*
x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*c*n*x*x**n/(6*n**3 + 9*n**2 + 3*n) + 6
*a*b*d*n**2*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 9*a*b*d*n*x**(2*n)/(6*n**3 +
 9*n**2 + 3*n) + 3*a*b*d*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*c*n**2*x
*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*c*n*x*x**(2*n)/(6*n**3 + 9*n**2
+ 3*n) + 2*b**2*d*n**2*x**(3*n)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*d*n*x**(3*
n)/(6*n**3 + 9*n**2 + 3*n) + b**2*d*x**(3*n)/(6*n**3 + 9*n**2 + 3*n), True)
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(59) = 118$.
time = 0.91, size = 196, normalized size = 3.21

$$\frac{6\,a^2cn^3x+3\,b^2cn^2xx^{2\,n}+12\,abcn^2xx^n+9\,a^2cn^2x+2\,b^2dn^2x^{3\,n}+6\,abdn^2x^{2\,n}+3\,b^2cnxx^{2\,n}+6\,a^2dn^2x^n+6\,abcnxx^n+3\,a^2cnx+3\,b^2dnx^{3\,n}+9\,abdnx^{2\,n}+9\,a^2dnx^n+b^2dx^{3\,n}+3\,abdx^{2\,n}+3\,a^2dx^n}{3\left(2\,n^3+3\,n^2+n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^2*c*n^3*x + 3*b^2*c*n^2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 9*a^2*c*n
^2*x + 2*b^2*d*n^2*x^(3*n) + 6*a*b*d*n^2*x^(2*n) + 3*b^2*c*n*x*x^(2*n) + 6*
a^2*d*n^2*x^n + 6*a*b*c*n*x*x^n + 3*a^2*c*n*x + 3*b^2*d*n*x^(3*n) + 9*a*b*d
*n*x^(2*n) + 9*a^2*d*n*x^n + b^2*d*x^(3*n) + 3*a*b*d*x^(2*n) + 3*a^2*d*x^n)
/(2*n^3 + 3*n^2 + n)
```

Mupad [B]
time = 5.06, size = 76, normalized size = 1.25

$$a^2\,c\,x+\frac{a^2\,d\,x^n}{n}+\frac{b^2\,d\,x^{3\,n}}{3\,n}+\frac{b^2\,c\,x\,x^{2\,n}}{2\,n+1}+\frac{a\,b\,d\,x^{2\,n}}{n}+\frac{2\,a\,b\,c\,x\,x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^(n - 1))*(a + b*x^n)^2,x)
```

```
[Out] a^2*c*x + (a^2*d*x^n)/n + (b^2*d*x^(3*n))/(3*n) + (b^2*c*x*x^(2*n))/(2*n +
1) + (a*b*d*x^(2*n))/n + (2*a*b*c*x*x^n)/(n + 1)
```

3.579 $\int (c+dx^{-1+n}) (a+bx^n) \, dx$

Optimal. Leaf size=41

$$acx + \frac{adx^n}{n} + \frac{bdx^{2n}}{2n} + \frac{bcx^{1+n}}{1+n}$$

```
[Out] a*c*x+a*d*x^n/n+1/2*b*d*x^(2*n)/n+b*c*x^(1+n)/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1905, 14}

$$acx + \frac{adx^n}{n} + \frac{bcx^{n+1}}{n+1} + \frac{bdx^{2n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))*(a + b*x^n),x]
```

```
[Out] a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x^(1 + n))/(1 + n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1905

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx^{-1+n}) (a+bx^n) \, dx &= c\int (a+bx^n) \, dx + d\int x^{-1+n}(a+bx^n) \, dx \\
&= acx + \frac{bcx^{1+n}}{1+n} + d\int \left(ax^{-1+n}+bx^{-1+2n}\right) \, dx \\
&= acx + \frac{adx^n}{n} + \frac{bdx^{2n}}{2n} + \frac{bcx^{1+n}}{1+n}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 42, normalized size = 1.02

$$\frac{2a(cnx+dx^n)+bx^n\left(\frac{2cnx}{1+n}+dx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n),x]
[Out] (2*a*(c*n*x + d*x^n) + b*x^n*((2*c*n*x)/(1 + n) + d*x^n))/(2*n)
```

Maple [A]
time = 0.03, size = 43, normalized size = 1.05

method	result	size
risch	$acx+\frac{bd\,x^{2n}}{2n}+\frac{(nbcx+adn+ad)x^{n}}{n(1+n)}$	43
norman	$acx+\frac{ad\,{e}^{n\ln(x)}}{n}+\frac{bcx\,{e}^{n\ln(x)}}{1+n}+\frac{bd\,{e}^{2n\ln(x)}}{2n}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))*(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] a*c*x+1/2*b*d/n*(x^n)^2+(b*c*n*x+a*d*n+a*d)/n/(1+n)*x^n
```

Maxima [A]
time = 0.31, size = 39, normalized size = 0.95

$$acx+\frac{bdx^{2\,n}}{2\,n}+\frac{bcx^{n+1}}{n+1}+\frac{adx^{n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="maxima")
[Out] a*c*x + 1/2*b*d*x^(2*n)/n + b*c*x^(n + 1)/(n + 1) + a*d*x^n/n
```

Fricas [A]
time = 0.37, size = 56, normalized size = 1.37

$$\frac{2\,(acn^{2}+acn)x+(bdn+bd)x^{2\,n}+2\,(bcnx+adn+ad)x^{n}}{2\,(n^{2}+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="fricas")
[Out] 1/2*(2*(a*c*n^2 + a*c*n)*x + (b*d*n + b*d)*x^(2*n) + 2*(b*c*n*x + a*d*n + a
*d)*x^n)/(n^2 + n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(36) = 72.
time = 0.31, size = 163, normalized size = 3.98

$$\begin{cases} acx-\frac{ad}{x}+bc\log{(x)}-\frac{bd}{2x^{2}} & \text{for}\: n=-1 \\ (a+b)\,(cx+d\log{(x)}) & \text{for}\: n=0 \\ \frac{2acn^{2}x}{2n^{2}+2n}+\frac{2acnx}{2n^{2}+2n}+\frac{2adnx^{n}}{2n^{2}+2n}+\frac{2adx^{n}}{2n^{2}+2n}+\frac{2bcnxx^{n}}{2n^{2}+2n}+\frac{bdnx^{2n}}{2n^{2}+2n}+\frac{bdx^{2n}}{2n^{2}+2n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))*(a+b*x**n),x)
```

```
[Out] Piecewise((a*c*x - a*d/x + b*c*log(x) - b*d/(2*x**2), Eq(n, -1)), ((a + b)*
(c*x + d*log(x)), Eq(n, 0)), (2*a*c*n**2*x/(2*n**2 + 2*n) + 2*a*c*n*x/(2*n*
*2 + 2*n) + 2*a*d*n*x**n/(2*n**2 + 2*n) + 2*a*d*x**n/(2*n**2 + 2*n) + 2*b*c
*n*x*x**n/(2*n**2 + 2*n) + b*d*n*x**(2*n)/(2*n**2 + 2*n) + b*d*x**(2*n)/(2*
n**2 + 2*n), True))
```

Giac [A]
time = 1.43, size = 65, normalized size = 1.59

$$\frac{2\,acn^2x+2\,bcnxx^n+2\,acnx+bdnx^{2\,n}+2\,adnx^n+bdx^{2\,n}+2\,adx^n}{2\,(n^2+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*c*n^2*x + 2*b*c*n*x*x^n + 2*a*c*n*x + b*d*n*x^(2*n) + 2*a*d*n*x^n
+ b*d*x^(2*n) + 2*a*d*x^n)/(n^2 + n)
```

Mupad [B]
time = 5.06, size = 38, normalized size = 0.93

$$a\,c\,x+\frac{a\,d\,x^n}{n}+\frac{b\,d\,x^{2\,n}}{2\,n}+\frac{b\,c\,x\,x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^(n - 1))*(a + b*x^n),x)
```

```
[Out] a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x*x^n)/(n + 1)
```

3.580 $\int (c+dx^{-1+n}) \, dx$

Optimal. Leaf size=12

$$cx+\frac{dx^n}{n}$$

[Out] c*x+d*x^n/n

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$cx+\frac{dx^n}{n}$$

Antiderivative was successfully verified.

[In] Int[c + d*x^(-1 + n),x]

[Out] c*x + (d*x^n)/n

Rubi steps

$$\int (c+dx^{-1+n}) \, dx = cx+\frac{dx^n}{n}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$cx+\frac{dx^n}{n}$$

Antiderivative was successfully verified.

[In] Integrate[c + d*x^(-1 + n),x]

[Out] c*x + (d*x^n)/n

Maple [A]
time = 0.02, size = 13, normalized size = 1.08

method	result	size
default	$cx+\frac{d\,x^n}{n}$	13
risch	$cx+\frac{dx\,x^{-1+n}}{n}$	16

norman	$cx + \frac{dx\,e^{(-1+n)\ln(x)}}{n}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c+d*x^(-1+n),x,method=_RETURNVERBOSE)
```

```
[Out] c*x+d*x^n/n
```

Maxima [A]
time = 0.29, size = 12, normalized size = 1.00

$$cx + \frac{dx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x^(-1+n),x, algorithm="maxima")
```

```
[Out] c*x + d*x^n/n
```

Fricas [A]
time = 0.39, size = 17, normalized size = 1.42

$$\frac{cnx + dxx^{n-1}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x^(-1+n),x, algorithm="fricas")
```

```
[Out] (c*n*x + d*x*x^(n - 1))/n
```

Sympy [A]
time = 0.01, size = 12, normalized size = 1.00

$$cx + d\left(\begin{cases} \frac{x^n}{n} & \text{for}\ n \neq 0 \\ \log(x) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x**(-1+n),x)
```

```
[Out] c*x + d*Piecewise((x**n/n, Ne(n, 0)), (log(x), True))
```

Giac [A]
time = 1.68, size = 12, normalized size = 1.00

$$cx + \frac{dx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x^(-1+n),x, algorithm="giac")
```

```
[Out] c*x + d*x^n/n
```

Mupad [B]
time = 5.01, size = 12, normalized size = 1.00

$$c\,x+\frac{d\,x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c + d*x^(n - 1),x)
```

```
[Out] c*x + (d*x^n)/n
```

3.581 $\int \frac{c+dx^{-1+n}}{a+bx^n} dx$

Optimal. Leaf size=42

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

```
[Out] c*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a+d*ln(a+b*x^n)/b/n
```

Rubi [A]

time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1905, 251, 266}

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))/(a + b*x^n),x]
```

```
[Out] (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*Log[a +
 b*x^n])/(b*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1905

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx &= c\int \frac{1}{a+bx^n}\,dx+d\int \frac{x^{-1+n}}{a+bx^n}\,dx\\ &= \frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 48, normalized size = 1.14

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}+\frac{d(n\log(x)+\log\left(a-ax^n\right))}{bn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^(-1 + n))/(a + b*x^n),x]
```

```
[Out] (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*(n*Log[
x] + Log[a - a*x^n]))/(b*n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{c+d\,x^{-1+n}}{a+b\,x^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))/(a+b*x^n),x)
```

```
[Out] int((c+d*x^(-1+n))/(a+b*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] d*log(x)/b + integrate((b*c*x - a*d)/(b^2*x*x^n + a*b*x), x)
```

Fricas [F]
time = 0.38, size = 21, normalized size = 0.50

$$\mathrm{integral}\left(\frac{dx^{n-1}+c}{bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d*x^(n - 1) + c)/(b*x^n + a), x)
```

Sympy [A]
time = 6.20, size = 65, normalized size = 1.55

$$d\left(\begin{cases}\frac{\log(x)}{a} & \text{for } b=0\wedge n=0\\ \frac{x^n}{an} & \text{for } b=0\\ \frac{\log(x)}{a+b} & \text{for } n=0\\ \frac{\log\left(\frac{a}{b}+x^n\right)}{bn} & \text{otherwise}\end{cases}\right)+\frac{cx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))/(a+b*x**n),x)
```

```
[Out] d*Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x**n/(a*n), Eq(b, 0)), (log(x
)/(a + b), Eq(n, 0)), (log(a/b + x**n)/(b*n), True)) + c*x*lerchphi(b*x**n*
exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a), x)
```

Mupad [B]
time = 5.33, size = 43, normalized size = 1.02

$$\frac{c\,x\,{}_2\mathrm{F}_1\left(1,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a}+\frac{d\,\ln\left(a+b\,x^n\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^(n - 1))/(a + b*x^n),x)
```

```
[Out] (c*x*hypergeom([1, 1/n], 1/n + 1, -(b*x^n)/a))/a + (d*log(a + b*x^n))/(b*n)
```

3.582 $\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=44

$$-\frac{d}{bn\,(a+bx^n)}+\frac{cx\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}$$

```
[Out] -d/b/n/(a+b*x^n)+c*x*hypergeom([2, 1/n],[1+1/n],-b*x^n/a)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1905, 251, 267}

$$\frac{cx\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}-\frac{d}{bn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))/(a + b*x^n)^2,x]
```

```
[Out] -(d/(b*n*(a + b*x^n))) + (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b
*x^n)/a)])/a^2
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1905

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx = c\int \frac{1}{(a+bx^n)^2}\,dx + d\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx$$
$$= -\frac{d}{bn\,(a+bx^n)} + \frac{cx\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}$$

Mathematica [A]
time = 0.10, size = 56, normalized size = 1.27

$$\frac{\frac{a(-ad+bcx)}{b(a+bx^n)} + c(-1+n)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^2,x]
[Out] ((a*(-(a*d) + b*c*x))/(b*(a + b*x^n)) + c*(-1 + n)*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{c+d\,x^{-1+n}}{(a+b\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))/(a+b*x^n)^2,x)
[Out] int((c+d*x^(-1+n))/(a+b*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="maxima")
[Out] c*(n - 1)*integrate(1/(a*b*n*x^n + a^2*n), x) + (b*c*x - a*d)/(a*b^2*n*x^n
+ a^2*b*n)
```

Fricas [F]
time = 0.39, size = 34, normalized size = 0.77

$$\text{integral}\left(\frac{dx^{n-1}+c}{b^2x^{2\,n}+2\,abx^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^(n - 1) + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 16.61, size = 299, normalized size = 6.80

$$c\left(\frac{nx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}+\frac{nx\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}-\frac{x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}+\frac{bnxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a^2\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}-\frac{bxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a^2\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}\right)+d\left(\begin{cases}\frac{\log\left(x\right)}{a^2} & \text{for}\: b=0\wedge n=0\\\frac{x^n}{a^2n} & \text{for}\: b=0\\\frac{\log\left(x\right)}{\left(a+b\right)^2} & \text{for}\: n=0\\-\frac{1}{abn+b^2nx^n} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))/(a+b*x**n)**2,x)
```

```
[Out] c*(n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gam
ma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + n*x*gamma(1/n)/(a*(a*n**3*gamm
a(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - x*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 +
1/n))) + b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(
a**2*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - b*x*x**n*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a**2*(a*n**3*gamma(1 + 1/
n) + b*n**3*x**n*gamma(1 + 1/n)))) + d*Piecewise((log(x)/a**2, Eq(b, 0) & E
q(n, 0)), (x**n/(a**2*n), Eq(b, 0)), (log(x)/(a + b)**2, Eq(n, 0)), (-1/(a*
b*n + b**2*n*x**n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a)^2, x)
```

Mupad [B]
time = 5.35, size = 49, normalized size = 1.11

$$\frac{c\,x\,{}_2\mathrm{F}_1\left(2,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a^2}-\frac{a\,d}{b\,\left(a^2\,n+a\,b\,n\,x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^(n - 1))/(a + b*x^n)^2,x)
```

```
[Out] (c*x*hypergeom([2, 1/n], 1/n + 1, -(b*x^n)/a))/a^2 - (a*d)/(b*(a^2*n + a*b*
n*x^n))
```

3.583 $\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=46

$$-\frac{d}{2bn\,(a+bx^n)^2}+\frac{cx\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}$$

```
[Out] -1/2*d/b/n/(a+b*x^n)^2+c*x*hypergeom([3, 1/n],[1+1/n],-b*x^n/a)/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {1905, 251, 267}

$$\frac{cx\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}-\frac{d}{2bn\,(a+bx^n)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))/(a + b*x^n)^3,x]
```

```
[Out] -1/2*d/(b*n*(a + b*x^n)^2) + (c*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1),
-((b*x^n)/a)])/a^3
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1905

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx = c\int \frac{1}{(a+bx^n)^3}\,dx + d\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx$$
$$= -\frac{d}{2bn\,(a+bx^n)^2} + \frac{cx\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. $2(46) = 92$.
time = 0.11, size = 108, normalized size = 2.35

$$\frac{x(c+dx^{-1+n})\left(\frac{a^2n(-ad+bcx)}{b(a+bx^n)^2}+\frac{ac(-1+2n)x}{a+bx^n}+c(1-3n+2n^2)\,x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\right)}{2a^3n^2\,(cx+dx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^3,x]
```

```
[Out] (x*(c + d*x^(-1 + n))*((a^2*n*(-(a*d) + b*c*x))/(b*(a + b*x^n)^2) + (a*c*(-
1 + 2*n)*x)/(a + b*x^n) + c*(1 - 3*n + 2*n^2)*x*Hypergeometric2F1[1, n^(-1)
, 1 + n^(-1), -((b*x^n)/a)]))/(2*a^3*n^2*(c*x + d*x^n))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{c+d\,x^{-1+n}}{(a+b\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

```
[Out] int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] (2*n^2 - 3*n + 1)*c*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b^2*
c*(2*n - 1)*x*x^n + a*b*c*(3*n - 1)*x - a^2*d*n)/(a^2*b^3*n^2*x^(2*n) + 2*a
^3*b^2*n^2*x^n + a^4*b*n^2)
```

Fricas [**F**]
time = 0.35, size = 47, normalized size = 1.02

$$\text{integral}\left(\frac{dx^{n-1}+c}{b^3x^{3n}+3ab^2x^{2n}+3a^2bx^n+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x^(n - 1) + c)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a
^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))/(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a)^3, x)
```

Mupad [**B**]
time = 5.41, size = 59, normalized size = 1.28

$$\frac{c\,x\,{}_2\mathrm{F}_1\left(3,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a^3}-\frac{d}{2\,b\,\left(a^2\,n+b^2\,n\,x^{2\,n}+2\,a\,b\,n\,x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^(n - 1))/(a + b*x^n)^3,x)
```

```
[Out] (c*x*hypergeom([3, 1/n], 1/n + 1, -(b*x^n)/a))/a^3 - d/(2*b*(a^2*n + b^2*n*
x^(2*n) + 2*a*b*n*x^n))
```

3.584 $\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=305

$$\frac{d(cx)^{1+m}\sqrt{1+\frac{bx^n}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)\sqrt{a+bx^n}}+\frac{ex^{1+n}(cx)^m\sqrt{1+\frac{bx^n}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{1+m+n}{n};\frac{1+m+2n}{n};-\frac{bx^n}{a}\right)}{(1+m+n)\sqrt{a+bx^n}}+fx^{1+}$$

```
[Out] d*(c*x)^(1+m)*hypergeom([1/2, (1+m)/n],[(1+m+n)/n],-b*x^n/a)*(1+b*x^n/a)^(1
/2)/c/(1+m)/(a+b*x^n)^(1/2)+e*x^(1+n)*(c*x)^m*hypergeom([1/2, (1+m+n)/n],[(
1+m+2*n)/n],-b*x^n/a)*(1+b*x^n/a)^(1/2)/(1+m+n)/(a+b*x^n)^(1/2)+f*x^(1+2*n)
*(c*x)^m*hypergeom([1/2, (1+m+2*n)/n],[(1+m+3*n)/n],-b*x^n/a)*(1+b*x^n/a)^(
1/2)/(1+m+2*n)/(a+b*x^n)^(1/2)+g*x^(1+3*n)*(c*x)^m*hypergeom([1/2, (1+m+3*n
)/n],[(1+m+4*n)/n],-b*x^n/a)*(1+b*x^n/a)^(1/2)/(1+m+3*n)/(a+b*x^n)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1858, 372, 371, 20}

$$\frac{d(cx)^{m+1}\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{c(m+1)\sqrt{a+bx^n}}+\frac{ex^{n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{(m+n+1)\sqrt{a+bx^n}}+\frac{fx^{2n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{m+2n+1}{n};\frac{m+3n+1}{n};-\frac{bx^n}{a}\right)}{(m+2n+1)\sqrt{a+bx^n}}+\frac{gx^{3n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{m+3n+1}{n};\frac{m+4n+1}{n};-\frac{bx^n}{a}\right)}{(m+3n+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/Sqrt[a + b*x^n],x]
```

```
[Out] (d*(c*x)^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m)/n, (1 +
 m + n)/n, -((b*x^n)/a)])/(c*(1 + m)*Sqrt[a + b*x^n]) + (e*x^(1 + n)*(c*x)^
m*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + n)/n, (1 + m + 2*n)/n
, -((b*x^n)/a)])/((1 + m + n)*Sqrt[a + b*x^n]) + (f*x^(1 + 2*n)*(c*x)^m*Sqr
t[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 2*n)/n, (1 + m + 3*n)/n, -
((b*x^n)/a)])/((1 + m + 2*n)*Sqrt[a + b*x^n]) + (g*x^(1 + 3*n)*(c*x)^m*Sqrt
[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 3*n)/n, (1 + m + 4*n)/n, -(
(b*x^n)/a)])/((1 + m + 3*n)*Sqrt[a + b*x^n])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^m\left(d+ex^n+fx^{2n}+gx^{3n}\right)}{\sqrt{a+bx^n}}\,dx &= \int \left(\frac{d(cx)^m}{\sqrt{a+bx^n}}+\frac{ex^n(cx)^m}{\sqrt{a+bx^n}}+\frac{fx^{2n}(cx)^m}{\sqrt{a+bx^n}}+\frac{gx^{3n}(cx)^m}{\sqrt{a+bx^n}}\right)dx \\
&= d\int \frac{(cx)^m}{\sqrt{a+bx^n}}\,dx+e\int \frac{x^n(cx)^m}{\sqrt{a+bx^n}}\,dx+f\int \frac{x^{2n}(cx)^m}{\sqrt{a+bx^n}}\,dx+g\int \\
&= \left(ex^{-m}(cx)^m\right)\int \frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx+\left(fx^{-m}(cx)^m\right)\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx+ \\
&= \frac{d(cx)^{1+m}\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)\sqrt{a+bx^n}}+\frac{\left(ex^{-m}(cx)^m\sqrt{1\right.}{} \\
&= \frac{d(cx)^{1+m}\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)\sqrt{a+bx^n}}+\frac{ex^{1+n}(cx)^m\sqrt{1}}{(1}
\end{aligned}
$$

Mathematica [A]
time = 1.57, size = 399, normalized size = 1.31

$$
\frac{x(cx)^m\left(2(1+m)\left(a+bx^n\right)\left(4a^2g(1+m^2+3n+2n^2+m(2+3n))-2ab(f(2+2m^2+7n+5n^2+m(4+7n))+g(2+2m^2+5n+2n^2+m(4+5n))x^n)+b^2(e(4+4m^2+16n+15n^2+8m(1+2n))+(2+2m+n)x^n(f(2+2m+5n)+g(2+2m+3n)x^n))\right)+\left(-2ab^2e(1+m)\left(4+4m^2+16n+15n^2+8m(1+2n)\right)-8a^3g(1+m)\left(1+m^2+3n+2n^2+m(2+3n)\right)+4a^2bf(1+m)\left(2+2m^2+7n+5n^2+m(4+7n)\right)+b^3d(8+8m^3+36n+46n^2+15n^3+12m^2(2+3n)+m(24+72n+46n^2))\right)\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)\right)}{b^3(1+m)(2+2m+n)(2+2m+3n)(2+2m+5n)\sqrt{a+bx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/Sqrt[a + b*x^n],x]
```

```
[Out] (x*(c*x)^m*(2*(1 + m)*(a + b*x^n)*(4*a^2*g*(1 + m^2 + 3*n + 2*n^2 + m*(2 +
3*n)) - 2*a*b*(f*(2 + 2*m^2 + 7*n + 5*n^2 + m*(4 + 7*n)) + g*(2 + 2*m^2 + 5
*n + 2*n^2 + m*(4 + 5*n))*x^n) + b^2*(e*(4 + 4*m^2 + 16*n + 15*n^2 + 8*m*(1
 + 2*n)) + (2 + 2*m + n)*x^n*(f*(2 + 2*m + 5*n) + g*(2 + 2*m + 3*n)*x^n)))
+ (-2*a*b^2*e*(1 + m)*(4 + 4*m^2 + 16*n + 15*n^2 + 8*m*(1 + 2*n)) - 8*a^3*g
*(1 + m)*(1 + m^2 + 3*n + 2*n^2 + m*(2 + 3*n)) + 4*a^2*b*f*(1 + m)*(2 + 2*m
^2 + 7*n + 5*n^2 + m*(4 + 7*n)) + b^3*d*(8 + 8*m^3 + 36*n + 46*n^2 + 15*n^3
 + 12*m^2*(2 + 3*n) + m*(24 + 72*n + 46*n^2)))*Sqrt[1 + (b*x^n)/a]*Hypergeo
metric2F1[1/2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)]))/(b^3*(1 + m)*(2 +
2*m + n)*(2 + 2*m + 3*n)*(2 + 2*m + 5*n)*Sqrt[a + b*x^n])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m \left(d + e\,x^n + f\,x^{2n} + g\,x^{3n}\right)}{\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((g*x^(3*n) + f*x^(2*n) + x^n*e + d)*(c*x)^m/sqrt(b*x^n + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorith
m="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [C] Result contains complex when optimal does not.
time = 26.53, size = 274, normalized size = 0.90

$$\frac{c^m dxx^m\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{a}\,n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}+\frac{c^m exx^mx^n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{n}+1+\frac{1}{n}\\ \frac{m}{n}+2+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{a}\,n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}+\frac{c^m fxx^mx^{2n}\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{n}+2+\frac{1}{n}\\ \frac{m}{n}+3+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{a}\,n\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right)}+\frac{c^m gxx^mx^{3n}\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{n}+3+\frac{1}{n}\\ \frac{m}{n}+4+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{a}\,n\Gamma\left(\frac{m}{n}+4+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n)**(1/2),x)
```

```
[Out] c**m*d*x*x**m*gamma(m/n + 1/n)*hyper((1/2, m/n + 1/n), (m/n + 1 + 1/n,), b*
x**n*exp_polar(I*pi)/a)/(sqrt(a)*n*gamma(m/n + 1 + 1/n)) + c**m*e*x*x**m*x*
*n*gamma(m/n + 1 + 1/n)*hyper((1/2, m/n + 1 + 1/n), (m/n + 2 + 1/n,), b*x**
n*exp_polar(I*pi)/a)/(sqrt(a)*n*gamma(m/n + 2 + 1/n)) + c**m*f*x*x**m*x**(2
*n)*gamma(m/n + 2 + 1/n)*hyper((1/2, m/n + 2 + 1/n), (m/n + 3 + 1/n,), b*x*
*n*exp_polar(I*pi)/a)/(sqrt(a)*n*gamma(m/n + 3 + 1/n)) + c**m*g*x*x**m*x**(
3*n)*gamma(m/n + 3 + 1/n)*hyper((1/2, m/n + 3 + 1/n), (m/n + 4 + 1/n,), b*x
**n*exp_polar(I*pi)/a)/(sqrt(a)*n*gamma(m/n + 4 + 1/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((g*x^(3*n) + f*x^(2*n) + x^n*e + d)*(c*x)^m/sqrt(b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x\right)}^m\,\left(d+e\,x^n+f\,x^{2\,n}+g\,x^{3\,n}\right)}{\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n)^(1/2),x)
```

```
[Out] int(((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n)^(1/2), x)
```

3.585 $\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}$$

```
[Out] -2*(a*g+2*a*h*x^(1/4*n)-b*f*x^(1/2*n))/a/n/(a+b*x^n)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {6873, 1830}

$$-\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(-(a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-1 + n) + b*h*x^(-1 +
(5*n)/4))/(a + b*x^n)^(3/2),x]
```

```
[Out] (-2*(a*g + 2*a*h*x^(n/4) - b*f*x^(n/2)))/(a*n*Sqrt[a + b*x^n])
```

Rule 1830

```
Int[((x_)^(m_.)*((e_) + (h_.)*(x_)^(n_.) + (f_.)*(x_)^(q_.) + (g_.)*(x_)^(r
_.)))/((a_) + (c_.)*(x_)^(n_.))^(3/2), x_Symbol] :> Simp[-(2*a*g + 4*a*h*x^
(n/4) - 2*c*f*x^(n/2))/(a*c*n*Sqrt[a + c*x^n]), x] /; FreeQ[{a, c, e, f, g,
 h, m, n}, x] && EqQ[q, n/4] && EqQ[r, 3*(n/4)] && EqQ[4*m - n + 4, 0] && E
qQ[c*e + a*h, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\begin{aligned}\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx &= \int \frac{x^{-1+\frac{n}{4}}\left(-ah+bfx^{n/4}+bgx^{3n/4}+bhx^n\right)}{(a+bx^n)^{3/2}}\,dx\\ &= -\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A]
time = 0.67, size = 45, normalized size = 1.00

$$\frac{2bfx^{n/2}-2a\left(g+2hx^{n/4}\right)}{an\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-1 + n) + b*h*x^
(-1 + (5*n)/4))/(a + b*x^n)^(3/2),x]
```

```
[Out] (2*b*f*x^(n/2) - 2*a*(g + 2*h*x^(n/4)))/(a*n*Sqrt[a + b*x^n])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{-ah\,x^{-1+\frac{n}{4}}+bf\,x^{-1+\frac{n}{2}}+bg\,x^{-1+n}+bh\,x^{-1+\frac{5n}{4}}}{\left(a+b\,x^{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n))/(a+b
*x^n)^(3/2),x)
```

```
[Out] int((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n))/(a+b
*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n)
)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(1
/4*n - 1))/(b*x^n + a)^(3/2), x)
```

Fricas [A]
time = 0.38, size = 66, normalized size = 1.47

$$\frac{2\sqrt{bx^4x^{n-4}+a}\left(bfx^2x^{\frac{1}{2}n-2}-2\,ahxx^{\frac{1}{4}n-1}-ag\right)}{abnx^4x^{n-4}+a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n)
)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x^4*x^(n - 4) + a)*(b*f*x^2*x^(1/2*n - 2) - 2*a*h*x*x^(1/4*n - 1)
- a*g)/(a*b*n*x^4*x^(n - 4) + a^2*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x**(-1+1/4*n)+b*f*x**(-1+1/2*n)+b*g*x**(-1+n)+b*h*x**(-1+5/
4*n))/(a+b*x**n)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n)
)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(1
/4*n - 1))/(b*x^n + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{b\,f\,x^{\frac{n}{2}-1}-a\,h\,x^{\frac{n}{4}-1}+b\,h\,x^{\frac{5\,n}{4}-1}+b\,g\,x^{n-1}}{{\left(a+b\,x^n\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*f*x^(n/2 - 1) - a*h*x^(n/4 - 1) + b*h*x^((5*n)/4 - 1) + b*g*x^(n - 1
))/(a + b*x^n)^(3/2),x)
```

```
[Out] int((b*f*x^(n/2 - 1) - a*h*x^(n/4 - 1) + b*h*x^((5*n)/4 - 1) + b*g*x^(n - 1
))/(a + b*x^n)^(3/2), x)
```

3.586 $\int (cx)^m \left(d+ex+fx^2+gx^3\right) (a+bx^n)^p \, dx$

Optimal. Leaf size=273

$$\frac{d(cx)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{n},-p;\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)}+\frac{e(cx)^{2+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{2+m}{n},-\right.}{c^2(2+m)}$$

```
[Out] d*(c*x)^(1+m)*(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/c/(
1+m)/((1+b*x^n/a)^p)+e*(c*x)^(2+m)*(a+b*x^n)^p*hypergeom([-p, (2+m)/n],[(2+
m+n)/n],-b*x^n/a)/c^2/(2+m)/((1+b*x^n/a)^p)+f*(c*x)^(3+m)*(a+b*x^n)^p*hyper
geom([-p, (3+m)/n],[(3+m+n)/n],-b*x^n/a)/c^3/(3+m)/((1+b*x^n/a)^p)+g*(c*x)^
(4+m)*(a+b*x^n)^p*hypergeom([-p, (4+m)/n],[(4+m+n)/n],-b*x^n/a)/c^4/(4+m)/(
(1+b*x^n/a)^p)
```

Rubi [A]

time = 0.13, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1858, 372, 371}

$$\frac{g(cx)^{m+4} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+4}{n},-p;\frac{m+n+4}{n};-\frac{bx^n}{a}\right)}{c^4(m+4)}+\frac{f(cx)^{m+3} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+3}{n},-p;\frac{m+n+3}{n};-\frac{bx^n}{a}\right)}{c^3(m+3)}+\frac{e(cx)^{m+2} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+2}{n},-p;\frac{m+n+2}{n};-\frac{bx^n}{a}\right)}{c^2(m+2)}+\frac{d(cx)^{m+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+1}{n},-p;\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(d + e*x + f*x^2 + g*x^3)*(a + b*x^n)^p,x]
```

```
[Out] (d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)
/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*(c*x)^(2 + m)*(a + b*
x^n)^p*Hypergeometric2F1[(2 + m)/n, -p, (2 + m + n)/n, -((b*x^n)/a)])/(c^2*
(2 + m)*(1 + (b*x^n)/a)^p) + (f*(c*x)^(3 + m)*(a + b*x^n)^p*Hypergeometric2
F1[(3 + m)/n, -p, (3 + m + n)/n, -((b*x^n)/a)])/(c^3*(3 + m)*(1 + (b*x^n)/a
)^p) + (g*(c*x)^(4 + m)*(a + b*x^n)^p*Hypergeometric2F1[(4 + m)/n, -p, (4 +
 m + n)/n, -((b*x^n)/a)])/(c^4*(4 + m)*(1 + (b*x^n)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (cx)^m \left(d+ex+fx^2+gx^3\right)(a+bx^n)^p\,dx = \int \left(d(cx)^m(a+bx^n)^p + \frac{e(cx)^{1+m}(a+bx^n)^p}{c} + \frac{f(cx)^{2+m}(a}{c^2}\right.$$

$$= d\int (cx)^m(a+bx^n)^p\,dx + \frac{e\int (cx)^{1+m}(a+bx^n)^p\,dx}{c} + \frac{f\int(c}{}$$

$$= \left(d(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int (cx)^m\left(1+\frac{bx^n}{a}\right)^p dx + \frac{(e(}{}$$

$$= \frac{d(cx)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{n},-p;\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)}$$

Mathematica [A]
time = 0.24, size = 178, normalized size = 0.65

$$x(cx)^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(\frac{d\,{}_2F_1\left(\frac{1+m}{n},-p;\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{1+m}+x\left(\frac{e\,{}_2F_1\left(\frac{2+m}{n},-p;\frac{2+m+n}{n};-\frac{bx^n}{a}\right)}{2+m}+x\left(\frac{f\,{}_2F_1\left(\frac{3+m}{n},-p;\frac{3+m+n}{n};-\frac{bx^n}{a}\right)}{3+m}+\frac{gx\,{}_2F_1\left(\frac{4+m}{n},-p;\frac{4+m+n}{n};-\frac{bx^n}{a}\right)}{4+m}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(d + e*x + f*x^2 + g*x^3)*(a + b*x^n)^p,x]
```

```
[Out] (x*(c*x)^m*(a + b*x^n)^p*((d*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n
, -((b*x^n)/a)])/(1 + m) + x*((e*Hypergeometric2F1[(2 + m)/n, -p, (2 + m +
n)/n, -((b*x^n)/a)])/(2 + m) + x*((f*Hypergeometric2F1[(3 + m)/n, -p, (3 +
m + n)/n, -((b*x^n)/a)])/(3 + m) + (g*x*Hypergeometric2F1[(4 + m)/n, -p, (4
 + m + n)/n, -((b*x^n)/a)])/(4 + m)))))/(1 + (b*x^n)/a)^p
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (cx)^m\left(g\,x^3+f\,x^2+ex+d\right)(a+b\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x)
```

```
[Out] int((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((g*x^3 + f*x^2 + x*e + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Fricas [F]
time = 0.39, size = 32, normalized size = 0.12

$$\text{integral}\left(\left(gx^3+fx^2+ex+d\right)(bx^n+a)^p(cx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="fricas")
[Out] integral((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 110.51, size = 248, normalized size = 0.91

$$\frac{a^pc^mdxx^m\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}+\frac{a^pc^mex^2x^m\Gamma\left(\frac{m}{n}+\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{n}+\frac{2}{n}\\ \frac{m}{n}+1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{2}{n}\right)}+\frac{a^pc^mfx^3x^m\Gamma\left(\frac{m}{n}+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{n}+\frac{3}{n}\\ \frac{m}{n}+1+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{3}{n}\right)}+\frac{a^pc^mgx^4x^m\Gamma\left(\frac{m}{n}+\frac{4}{n}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{n}+\frac{4}{n}\\ \frac{m}{n}+1+\frac{4}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{4}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(g*x**3+f*x**2+e*x+d)*(a+b*x**n)**p,x)
[Out] a**p*c**m*d*x*x**m*gamma(m/n + 1/n)*hyper((-p, m/n + 1/n), (m/n + 1 + 1/n,)
, b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1 + 1/n)) + a**p*c**m*e*x**2*x**
m*gamma(m/n + 2/n)*hyper((-p, m/n + 2/n), (m/n + 1 + 2/n,), b*x**n*exp_pola
r(I*pi)/a)/(n*gamma(m/n + 1 + 2/n)) + a**p*c**m*f*x**3*x**m*gamma(m/n + 3/n
)*hyper((-p, m/n + 3/n), (m/n + 1 + 3/n,), b*x**n*exp_polar(I*pi)/a)/(n*gam
ma(m/n + 1 + 3/n)) + a**p*c**m*g*x**4*x**m*gamma(m/n + 4/n)*hyper((-p, m/n
+ 4/n), (m/n + 1 + 4/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1 + 4/n)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f*x^2 + x*e + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^m\,(a+b\,x^n)^p\,\left(g\,x^3+f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x^n)^p*(d + e*x + f*x^2 + g*x^3),x)
[Out] int((c*x)^m*(a + b*x^n)^p*(d + e*x + f*x^2 + g*x^3), x)
```

3.587 $\int (cx)^m (a+bx^n)^p (d+ex^n+fx^{2n}+gx^{3n})\,dx$

Optimal. Leaf size=297

$$\frac{d(cx)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{n},-p;\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)}+\frac{ex^{1+n}(cx)^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{n}\right.}{1+m+n}$$

```
[Out] d*(c*x)^(1+m)*(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/c/(
1+m)/((1+b*x^n/a)^p)+e*x^(1+n)*(c*x)^m*(a+b*x^n)^p*hypergeom([-p, (1+m+n)/n
],[(1+m+2*n)/n],-b*x^n/a)/(1+m+n)/((1+b*x^n/a)^p)+f*x^(1+2*n)*(c*x)^m*(a+b*
x^n)^p*hypergeom([-p, (1+m+2*n)/n],[(1+m+3*n)/n],-b*x^n/a)/(1+m+2*n)/((1+b*
x^n/a)^p)+g*x^(1+3*n)*(c*x)^m*(a+b*x^n)^p*hypergeom([-p, (1+m+3*n)/n],[(1+m
+4*n)/n],-b*x^n/a)/(1+m+3*n)/((1+b*x^n/a)^p)
```

Rubi [A]
time = 0.14, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1858, 372, 371, 20}

$$\frac{d(cx)^{m+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{n},-p;\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{c(m+1)}+\frac{ex^{n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{n},-p;\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{m+n+1}+\frac{fx^{2n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2n+1}{n},-p;\frac{m+3n+1}{n};-\frac{bx^n}{a}\right)}{m+2n+1}+\frac{gx^{3n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+3n+1}{n},-p;\frac{m+4n+1}{n};-\frac{bx^n}{a}\right)}{m+3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x]
```

```
[Out] (d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)
/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*x^(1 + n)*(c*x)^m*(a
+ b*x^n)^p*Hypergeometric2F1[(1 + m + n)/n, -p, (1 + m + 2*n)/n, -((b*x^n)/
a)])/((1 + m + n)*(1 + (b*x^n)/a)^p) + (f*x^(1 + 2*n)*(c*x)^m*(a + b*x^n)^p
*Hypergeometric2F1[(1 + m + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/((1
 + m + 2*n)*(1 + (b*x^n)/a)^p) + (g*x^(1 + 3*n)*(c*x)^m*(a + b*x^n)^p*Hyper
geometric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -((b*x^n)/a)])/((1 + m +
 3*n)*(1 + (b*x^n)/a)^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right) dx &= \int \left(d(cx)^m (a+bx^n)^p + ex^n(cx)^m (a+bx^n)^p + fx^{2n}(cx)^m\right.\\ &= d\int (cx)^m (a+bx^n)^p \, dx + e\int x^n(cx)^m (a+bx^n)^p \, dx + f\\ &= \left(ex^{-m}(cx)^m\right)\int x^{m+n}(a+bx^n)^p \, dx + \left(fx^{-m}(cx)^m\right)\int x^m\\ &= \frac{d(cx)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{n},-p;\frac{1+m+n}{n};-\frac{b}{}\right.}{c(1+m)}\\ &= \frac{d(cx)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{n},-p;\frac{1+m+n}{n};-\frac{b}{}\right.}{c(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.76, size = 204, normalized size = 0.69

$$x(cx)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(\frac{d\,{}_2F_1\left(\frac{1+m}{n},-p;\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{1+m} + x^n\left(\frac{e\,{}_2F_1\left(\frac{1+m+n}{n},-p;\frac{1+m+2n}{n};-\frac{bx^n}{a}\right)}{1+m+n} + x^n\left(\frac{f\,{}_2F_1\left(\frac{1+m+2n}{n},-p;\frac{1+m+3n}{n};-\frac{bx^n}{a}\right)}{1+m+2n} + \frac{gx^n\,{}_2F_1\left(\frac{1+m+3n}{n},-p;\frac{1+m+4n}{n};-\frac{bx^n}{a}\right)}{1+m+3n}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x]
```

```
[Out] (x*(c*x)^m*(a + b*x^n)^p*((d*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n
, -((b*x^n)/a)])/(1 + m) + x^n*((e*Hypergeometric2F1[(1 + m + n)/n, -p, (1
+ m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n) + x^n*((f*Hypergeometric2F1[(1 + m
 + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/(1 + m + 2*n) + (g*x^n*Hyper
geometric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -((b*x^n)/a)])/(1 + m +
3*n)))))/(1 + (b*x^n)/a)^p
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (cx)^m (a + b x^n)^p \left(d + e x^n + f x^{2n} + g x^{3n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x)
[Out] int((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="m
axima")
[Out] integrate((g*x^(3*n) + f*x^(2*n) + x^n*e + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Fricas [F]
time = 0.38, size = 38, normalized size = 0.13

$$\text{integral}\left(\left(gx^{3n} + fx^{2n} + ex^n + d\right)(bx^n + a)^p (cx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="f
ricas")
[Out] integral((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(a+b*x**n)**p*(d+e*x**n+f*x**(2*n)+g*x**(3*n)),x)
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARx near
 0Simplification assuming sageVARc near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^m\,(a+b\,x^n)^p\,\left(d+e\,x^n+f\,x^{2\,n}+g\,x^{3\,n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x)
```

```
[Out] int((c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)), x)
```

3.588 $\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=162

$$\frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{abn\,(a+bx^n)}-\frac{(bd(2-n)-af(2+n))x^{\frac{2+n}{2}}\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-\frac{bx^n}{a}\right)}{a^2bn(2+n)}+\frac{(ae-bc(1-$$

```
[Out] x*(b*c-a*e+(-a*f+b*d)*x^(1/2*n))/a/b/n/(a+b*x^n)-(b*d*(2-n)-a*f*(2+n))*x^(1
+1/2*n)*hypergeom([1, 1/2+1/n],[3/2+1/n],-b*x^n/a)/a^2/b/n/(2+n)+(a*e-b*c*(
1-n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b/n
```

Rubi [A]
time = 0.09, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1906, 1432, 251, 371}

$$\frac{x(ae-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}-\frac{x^{\frac{n+2}{2}}(bd(2-n)-af(n+2))\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-\frac{bx^n}{a}\right)}{a^2bn(n+2)}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{abn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2,x]
```

```
[Out] (x*(b*c - a*e + (b*d - a*f)*x^(n/2)))/(a*b*n*(a + b*x^n)) - ((b*d*(2 - n) -
 a*f*(2 + n))*x^((2 + n)/2)*Hypergeometric2F1[1, (1 + 2/n)/2, (3 + 2/n)/2,
-((b*x^n)/a)])/(a^2*b*n*(2 + n)) + ((a*e - b*c*(1 - n))*x*Hypergeometric2F1
[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
```

```
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1906

```
Int[(P3_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{A = Coeff[P3, x
^(n/2), 0], B = Coeff[P3, x^(n/2), 1], C = Coeff[P3, x^(n/2), 2], D = Coeff
[P3, x^(n/2), 3]}, Simp[-(x*(b*A - a*C + (b*B - a*D)*x^(n/2))*(a + b*x^n)^(
p + 1))/(a*b*n*(p + 1)), x] - Dist[1/(2*a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*Simp[2*a*C - 2*b*A*(n*(p + 1) + 1) + (a*D*(n + 2) - b*B*(n*(2*p + 3) +
 2))*x^(n/2), x], x], x]] /; FreeQ[{a, b, n}, x] && PolyQ[P3, x^(n/2), 3] &
& ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx = \frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{abn\,(a+bx^n)} + \frac{\int \frac{2(ae-bc(1-n))-(bd(2-n)-af(2+n))x^{n/2}}{a+bx^n}\,dx}{2abn}$$

$$= \frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{abn\,(a+bx^n)} + \frac{(ae-bc(1-n))\int \frac{1}{a+bx^n}\,dx}{abn} - \frac{(bd(2-n)-$$

$$= \frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{abn\,(a+bx^n)} - \frac{(bd(2-n)-af(2+n))x^{\frac{2+n}{2}}\,{}_2F_1\left(1,\tfrac{1}{2}(1+\right.}{a^2bn(2+n)}$$

Mathematica [A]
time = 0.26, size = 151, normalized size = 0.93

$$\frac{x\left((bd(-2+n)+af(2+n))x^{n/2}(a+bx^n)\,{}_2F_1\left(1,\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-\frac{bx^n}{a}\right)+(2+n)\left(a\left(b\left(c+dx^{n/2}\right)-a\left(e+fx^{n/2}\right)\right)+(ae+bc(-1+n))\,(a+bx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\right)\right)}{a^2bn(2+n)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2,x]
```

```
[Out] (x*((b*d*(-2 + n) + a*f*(2 + n))*x^(n/2)*(a + b*x^n)*Hypergeometric2F1[1, 1
/2 + n^(-1), 3/2 + n^(-1), -((b*x^n)/a)] + (2 + n)*(a*(b*(c + d*x^(n/2)) -
a*(e + f*x^(n/2))) + (a*e + b*c*(-1 + n))*(a + b*x^n)*Hypergeometric2F1[1,
n^(-1), 1 + n^(-1), -((b*x^n)/a)])))/(a^2*b*n*(2 + n)*(a + b*x^n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{c+d\,x^{\frac{n}{2}}+e\,x^{n}+f\,x^{\frac{3n}{2}}}{\left(a+b\,x^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="maxim
a")
```

```
[Out] ((b*d - a*f)*x*x^(1/2*n) + (b*c - a*e)*x)/(a*b^2*n*x^n + a^2*b*n) + integra
te(1/2*(2*b*c*(n - 1) + (a*f*(n + 2) + b*d*(n - 2))*x^(1/2*n) + 2*a*e)/(a*b
^2*n*x^n + a^2*b*n), x)
```

Fricas [F]
time = 0.38, size = 46, normalized size = 0.28

$$\mathrm{integral}\left(\frac{fx^{\frac{3}{2}\,n}+dx^{\frac{1}{2}\,n}+ex^{n}+c}{b^{2}x^{2\,n}+2\,abx^{n}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="frica
s")
```

```
[Out] integral((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n +
 a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(1/2*n)+e*x**n+f*x**(3/2*n))/(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="giac"
)
```

```
[Out] integrate((f*x^(3/2*n) + d*x^(1/2*n) + x^n*e + c)/(b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+e\,x^n+d\,x^{n/2}+f\,x^{\frac{3\,n}{2}}}{{\left(a+b\,x^n\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + e*x^n + d*x^(n/2) + f*x^((3*n)/2))/(a + b*x^n)^2,x)
```

```
[Out] int((c + e*x^n + d*x^(n/2) + f*x^((3*n)/2))/(a + b*x^n)^2, x)
```

3.589 $\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=24

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

```
[Out] x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {1604}

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
,x]
```

```
[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Mathematica [A]
time = 5.75, size = 24, normalized size = 1.00

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c + d
*x^2]),x]
```

```
[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

Maple [A]
time = 0.39, size = 21, normalized size = 0.88

method	result	size
gosper	$x\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}$	21
default	$x\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}$	21
risch	$x\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}$	21
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,x\sqrt{bd\,x^{4}+ad\,x^{2}+c\,x^{2}b+ac}}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
```

Maxima [A]
time = 0.33, size = 20, normalized size = 0.83

$$\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x
, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x
```

Fricas [A]
time = 0.37, size = 20, normalized size = 0.83

$$\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x
, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac+2adx^2+2bcx^2+3bdx^4}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+2*(a*d+b*c)*x**2+3*b*d*x**4)/(b*x**2+a)**(1/2)/(d*x**2+c)**(
1/2),x)
```

```
[Out] Integral((a*c + 2*a*d*x**2 + 2*b*c*x**2 + 3*b*d*x**4)/(sqrt(a + b*x**2)*sqr
t(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((3*b*d*x^4 + 2*(b*c + a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2
 + c)), x)
```

Mupad [B]
time = 5.59, size = 20, normalized size = 0.83

$$x\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + 2*x^2*(a*d + b*c) + 3*b*d*x^4)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1
/2)),x)
```

```
[Out] x*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)
```

3.590 $\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

```
[Out] 1/4*arctan(2^(1/4)*x/(x^4+1)^(1/4))*2^(3/4)-1/4*arctan(1/2*(x^4+1)^(1/4)*2^
(3/4))*2^(3/4)+1/4*arctanh(2^(1/4)*x/(x^4+1)^(1/4))*2^(3/4)+1/4*arctanh(1/2
*(x^4+1)^(1/4)*2^(3/4))*2^(3/4)
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1913, 385, 218, 212, 209, 455, 65, 304}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) - ArcTan[(1 + x^4)^(1/4)/2^
(1/4)]/(2*2^(1/4)) + ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) + Arc
Tanh[(1 + x^4)^(1/4)/2^(1/4)]/(2*2^(1/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1913

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)
*(x_)^(n_))^(q_.), x_Symbol] :> Dist[A, Int[(a + b*x^n)^p*(c + d*x^n)^q, x]
, x] + Dist[B, Int[x^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b,
c, d, A, B, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx &= \int \frac{1}{(1-x^4)\sqrt[4]{1+x^4}}\,dx + \int \frac{x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt[4]{1+x}}\,dx, x, x^4\right) + \text{Subst}\left(\int \frac{1}{1-2x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right. \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx, x, \sqrt[4]{1+\right. \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\sqrt[4]{\right.}{2\sqrt[4]{2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.13, size = 93, normalized size = 0.90

$$
\frac{1}{4}x^4F_1\left(1;\frac{1}{4},1;2;-x^4,x^4\right) + \frac{2\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right) - \log\left(1-\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right) + \log\left(1+\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[4]{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)),x]
```

```
[Out] (x^4*AppellF1[1, 1/4, 1, 2, -x^4, x^4])/4 + (2*ArcTan[(2^(1/4)*x)/(1 + x^4)
^(1/4)] - Log[1 - (2^(1/4)*x)/(1 + x^4)^(1/4)] + Log[1 + (2^(1/4)*x)/(1 + x
^4)^(1/4)])/(4*2^(1/4))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$
\int \frac{x^3+1}{(-x^4+1)\,(x^4+1)^{\frac{1}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x)
```

```
[Out] int((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{x}{x^{3}\sqrt[4]{x^{4}+1}-x^{2}\sqrt[4]{x^{4}+1}+x\sqrt[4]{x^{4}+1}-\sqrt[4]{x^{4}+1}}\right)dx-\int\frac{x^{2}}{x^{3}\sqrt[4]{x^{4}+1}-x^{2}\sqrt[4]{x^{4}+1}+x\sqrt[4]{x^{4}+1}-\sqrt[4]{x^{4}+1}}\,dx-\int\frac{1}{x^{3}\sqrt[4]{x^{4}+1}-x^{2}\sqrt[4]{x^{4}+1}+x\sqrt[4]{x^{4}+1}-\sqrt[4]{x^{4}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(-x**4+1)/(x**4+1)**(1/4),x)
```

```
[Out] -Integral(-x/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1
)**(1/4) - (x**4 + 1)**(1/4)), x) - Integral(x**2/(x**3*(x**4 + 1)**(1/4) -
 x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(1/4) - (x**4 + 1)**(1/4)), x) - In
tegral(1/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(
1/4) - (x**4 + 1)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-(x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^3+1}{\left(x^4-1\right)\,{\left(x^4+1\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 + 1)/((x^4 - 1)*(x^4 + 1)^(1/4)),x)
```

```
[Out] int(-(x^3 + 1)/((x^4 - 1)*(x^4 + 1)^(1/4)), x)
```

3.591 $\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} (ac-bdx^{2n})\, dx$

Optimal. Leaf size=28

$$x(a+bx^n)^{-1/n} (c+dx^n)^{-1/n}$$

```
[Out] x/((a+b*x^n)^(1/n))/((c+d*x^n)^(1/n))
```

Rubi [A]
time = 0.07, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {1912}

$$x(a+bx^n)^{-1/n} (c+dx^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(2*n)),x
]
```

```
[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

Rule 1912

```
Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(p_.)*((e_) +
 (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[e*x*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(p + 1)/(a*c)), x] /; FreeQ[{a, b, c, d, e, g, n, p}, x] && EqQ[n2, 2*n] &
& EqQ[n*(p + 1) + 1, 0] && EqQ[a*c*g - b*d*e*(2*n*(p + 1) + 1), 0]
```

Rubi steps

$$\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} (ac-bdx^{2n})\, dx = x(a+bx^n)^{-1/n} (c+dx^n)^{-1/n}$$

Mathematica [A]
time = 0.28, size = 28, normalized size = 1.00

$$x(a+bx^n)^{-1/n} (c+dx^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(2
*n)),x]
```

```
[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int (a+b\,x^{n})^{\frac{-1-n}{n}} (c+d\,x^{n})^{\frac{-1-n}{n}} \left(ac-bd\,x^{2n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x)
```

```
[Out] int((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, al
gorithm="maxima")
```

```
[Out] -integrate((b*d*x^(2*n) - a*c)/((b*x^n + a)^((n + 1)/n)*(d*x^n + c)^((n + 1
)/n)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(28) = 56$.
time = 0.39, size = 61, normalized size = 2.18

$$\frac{bdxx^{2\,n}+acx+(bc+ad)xx^{n}}{(bx^{n}+a)^{\frac{n+1}{n}}(dx^{n}+c)^{\frac{n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, al
gorithm="fricas")
```

```
[Out] (b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)/((b*x^n + a)^((n + 1)/n)*(d*x^n
 + c)^((n + 1)/n))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**((-1-n)/n)*(c+d*x**n)**((-1-n)/n)*(a*c-b*d*x**(2*n)),
x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(28) = 56.
time = 0.81, size = 228, normalized size = 8.14

$$bdxx^{2n}e^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)}+bcxx^ne^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)}+adxx^ne^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)}+acxe^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, al
gorithm="giac")
```

```
[Out] b*d*x*x^(2*n)*e^(-(n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c)
 + log(d*x^n + c))/n) + b*c*x*x^n*e^(-(n*log(b*x^n + a) + log(b*x^n + a))/n
 - (n*log(d*x^n + c) + log(d*x^n + c))/n) + a*d*x*x^n*e^(-(n*log(b*x^n + a)
 + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n + c))/n) + a*c*x*e^(-(
n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n + c))/
n)
```

Mupad [B]
time = 5.20, size = 95, normalized size = 3.39

$$\frac{\frac{a\,c\,x}{{\left(a+b\,x^n\right)}^{\frac{n+1}{n}}}+\frac{x\,x^n\,\left(a\,d+b\,c\right)}{{\left(a+b\,x^n\right)}^{\frac{n+1}{n}}}+\frac{b\,d\,x\,x^{2\,n}}{{\left(a+b\,x^n\right)}^{\frac{n+1}{n}}}}{{\left(c+d\,x^n\right)}^{\frac{n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*d*x^(2*n))/((a + b*x^n)^((n + 1)/n)*(c + d*x^n)^((n + 1)/n)),x
)
```

```
[Out] ((a*c*x)/(a + b*x^n)^((n + 1)/n) + (x*x^n*(a*d + b*c))/(a + b*x^n)^((n + 1)
/n) + (b*d*x*x^(2*n))/(a + b*x^n)^((n + 1)/n))/(c + d*x^n)^((n + 1)/n)
```

3.592 $\int (hx)^{-1-n-np} (a+bx^n)^p (c+dx^n)^p (ac-bdx^{2n})\,dx$

Optimal. Leaf size=45

$$-\frac{(hx)^{-n(1+p)} (a+bx^n)^{1+p} (c+dx^n)^{1+p}}{hn(1+p)}$$

```
[Out] -(a+b*x^n)^(1+p)*(c+d*x^n)^(1+p)/h/n/(1+p)/((h*x)^(n*(1+p)))
```

Rubi [A]
time = 0.10, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {1863}

$$-\frac{(hx)^{-n(p+1)} (a+bx^n)^{p+1} (c+dx^n)^{p+1}}{hn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2*n)),x]
```

```
[Out] -(((a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n*(1 + p)*(h*x)^(n*(1 + p)))
)
```

Rule 1863

```
Int[((h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(
n_.))^(p_.)*((e_) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[e*(h*x)^(m + 1)*(
a + b*x^n)^(p + 1)*((c + d*x^n)^(p + 1)/(a*c*h*(m + 1))), x] /; FreeQ[{a, b
, c, d, e, g, h, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[m + n*(p + 1) + 1, 0]
&& EqQ[a*c*g*(m + 1) - b*d*e*(m + 2*n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int (hx)^{-1-n-np} (a+bx^n)^p (c+dx^n)^p \left(ac-bdx^{2n}\right) dx = -\frac{(hx)^{-n(1+p)} (a+bx^n)^{1+p} (c+dx^n)^{1+p}}{hn(1+p)}$$

Mathematica [A]
time = 0.34, size = 46, normalized size = 1.02

$$-\frac{(hx)^{-n-np} (a+bx^n)^{1+p} (c+dx^n)^{1+p}}{hn+hnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2*
n)),x]
```

```
[Out] -(((h*x)^(-n - n*p)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n + h*n*p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.72, size = 138, normalized size = 3.07

method	result
risch	$-\frac{(a+b\,x^{n})^{p}\mathrm{e}^{-\frac{(np+n+1)\left(-i\pi\mathrm{csgn}(ihx)^{3}+i\pi\mathrm{csgn}(ihx)^{2}\mathrm{csgn}(ih)+i\pi\mathrm{csgn}(ihx)^{2}\mathrm{csgn}(ix)-i\pi\,\mathrm{csgn}(ihx)\mathrm{csgn}(ih)\mathrm{csgn}(ix)+2\ln(h)+2\ln(x)\right)}{2}}(bd\,x^{2n}+}{n(1+p)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x,method=_RE
TURNVERBOSE)
```

```
[Out] -(a+b*x^n)^p*exp(-1/2*(n*p+n+1)*(-I*Pi*csgn(I*h*x)^3+I*Pi*csgn(I*h*x)^2*csg
n(I*h)+I*Pi*csgn(I*h*x)^2*csgn(I*x)-I*Pi*csgn(I*h*x)*csgn(I*h)*csgn(I*x)+2*
ln(h)+2*ln(x)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*x/n/(1+p)*(c+d*x^n)^p
```

Maxima [A]
time = 0.42, size = 77, normalized size = 1.71

$$-\frac{(bdx^{2n}+ac+(bc+ad)x^{n})h^{-np-n-1}e^{(-np\log(x)+p\log(bx^{n}+a)+p\log(dx^{n}+c)-n\log(x))}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, alg
orithm="maxima")
```

```
[Out] -(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n)*h^(-n*p - n - 1)*e^(-n*p*log(x) + p*
log(b*x^n + a) + p*log(d*x^n + c) - n*log(x))/(n*(p + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(46) = 92.
time = 0.39, size = 119, normalized size = 2.64

$$-\frac{\left(bdxx^{2n}e^{(-(np+n+1)\log(h)-(np+n+1)\log(x))}+acxe^{(-(np+n+1)\log(h)-(np+n+1)\log(x))}+(bc+ad)xx^{n}e^{(-(np+n+1)\log(h)-(np+n+1)\log(x))}\right)(bx^{n}+a)^{p}(dx^{n}+c)^{p}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] -(b*d*x*x^(2*n)*e^(-(n*p + n + 1)*log(h) - (n*p + n + 1)*log(x)) + a*c*x*e^
(-(n*p + n + 1)*log(h) - (n*p + n + 1)*log(x)) + (b*c + a*d)*x*x^n*e^(-(n*p
```

```
 + n + 1)*log(h) - (n*p + n + 1)*log(x)))*(b*x^n + a)^p*(d*x^n + c)^p/(n*p
+ n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)**(-n*p-n-1)*(a+b*x**n)**p*(c+d*x**n)**p*(a*c-b*d*x**(2*n)),
x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(46) = 92.
time = 0.89, size = 237, normalized size = 5.27

$$-\frac{(bx^n+a)^p(dx^n+c)^pbdxx^{2n}e^{(-np\log(h)-np\log(x)-n\log(h)-n\log(x)-\log(h)-\log(x))}+(bx^n+a)^p(dx^n+c)^pbcxx^ne^{(-np\log(h)-np\log(x)-n\log(h)-n\log(x)-\log(h)-\log(x))}+(bx^n+a)^p(dx^n+c)^padxx^ne^{(-np\log(h)-np\log(x)-n\log(h)-n\log(x)-\log(h)-\log(x))}+(bx^n+a)^p(dx^n+c)^pacxe^{(-np\log(h)-np\log(x)-n\log(h)-n\log(x)-\log(h)-\log(x))}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, alg
orithm="giac")
```

```
[Out] -((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e^(-n*p*log(h) - n*p*log(x) - n
*log(h) - n*log(x) - log(h) - log(x)) + (b*x^n + a)^p*(d*x^n + c)^p*b*c*x*x
^n*e^(-n*p*log(h) - n*p*log(x) - n*log(h) - n*log(x) - log(h) - log(x)) + (
b*x^n + a)^p*(d*x^n + c)^p*a*d*x*x^n*e^(-n*p*log(h) - n*p*log(x) - n*log(h)
 - n*log(x) - log(h) - log(x)) + (b*x^n + a)^p*(d*x^n + c)^p*a*c*x*e^(-n*p*
log(h) - n*p*log(x) - n*log(h) - n*log(x) - log(h) - log(x)))/(n*p + n)
```

Mupad [**B**]
time = 5.37, size = 124, normalized size = 2.76

$$-(c+d\,x^n)^p\,\left(\frac{a\,c\,x\,{(a+b\,x^n)}^p}{n\,{(h\,x)}^{n+n\,p+1}\,(p+1)}+\frac{x\,x^n\,(a\,d+b\,c)\,{(a+b\,x^n)}^p}{n\,{(h\,x)}^{n+n\,p+1}\,(p+1)}+\frac{b\,d\,x\,x^{2\,n}\,{(a+b\,x^n)}^p}{n\,{(h\,x)}^{n+n\,p+1}\,(p+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*c - b*d*x^(2*n))*(a + b*x^n)^p*(c + d*x^n)^p)/(h*x)^(n + n*p + 1),x
)
```

```
[Out] -(c + d*x^n)^p*((a*c*x*(a + b*x^n)^p)/(n*(h*x)^(n + n*p + 1)*(p + 1)) + (x*
x^n*(a*d + b*c)*(a + b*x^n)^p)/(n*(h*x)^(n + n*p + 1)*(p + 1)) + (b*d*x*x^(
2*n)*(a + b*x^n)^p)/(n*(h*x)^(n + n*p + 1)*(p + 1)))
```

3.593 $\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1-\right.$

Optimal. Leaf size=31
$$\frac{ex(a+bx^n)^{1+p}(c+dx^n)^{1+p}}{ac}$$

```
[Out] e*x*(a+b*x^n)^(1+p)*(c+d*x^n)^(1+p)/a/c
```

Rubi [A]
time = 0.14, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}} = 0.014$, Rules used = {1911}

$$\frac{ex(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^n)/(a*c
) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)),x]
```

```
[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

Rule 1911

```
Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(p_.)*((e_) +
 (f_.)*(x_)^(n_.) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[e*x*(a + b*x^n)^(
p + 1)*((c + d*x^n)^(p + 1)/(a*c)), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}
, x] && EqQ[n2, 2*n] && EqQ[a*c*f - e*(b*c + a*d)*(n*(p + 1) + 1), 0] && Eq
Q[a*c*g - b*d*e*(2*n*(p + 1) + 1), 0]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx = \frac{ex(a+bx^n)^{1+p}(c-}{ac}$$

Mathematica [A]
time = 0.62, size = 31, normalized size = 1.00

$$\frac{ex(a+bx^n)^{1+p}(c+dx^n)^{1+p}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^n
)/(a*c) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)),x]
```

```
[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

Maple [A]
time = 0.51, size = 52, normalized size = 1.68

method	result	size
risch	$\frac{(a+b\,x^{n})^{p} (b d\,x^{2 n}+a d\,x^{n}+b c\,x^{n}+a c) e x (c+d\,x^{n})^{p}}{a c}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2*n*p+2
*n+1)*x^(2*n)/a/c),x,method=_RETURNVERBOSE)
```

```
[Out] (a+b*x^n)^p*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a/c*(c+d*x^n)^p
```

Maxima [A]
time = 0.37, size = 64, normalized size = 2.06

$$\frac{\left(acxe + bdxe^{(2\,n\log(x)+1)} + (bc+ad)xe^{(n\log(x)+1)}\right)e^{(p\log(bx^n+a)+p\log(dx^n+c))}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="maxima")
```

```
[Out] (a*c*x*e + b*d*x*e^(2*n*log(x) + 1) + (b*c + a*d)*x*e^(n*log(x) + 1))*e^(p*
log(b*x^n + a) + p*log(d*x^n + c))/(a*c)
```

Fricas [A]
time = 0.40, size = 54, normalized size = 1.74

$$\frac{(bdexx^{2\,n} + acex + (bc+ad)exx^n)(bx^n+a)^p(dx^n+c)^p}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="fricas")
```

```
[Out] (b*d*e*x*x^(2*n) + a*c*e*x + (b*c + a*d)*e*x*x^n)*(b*x^n + a)^p*(d*x^n + c)
^p/(a*c)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+n+1)*x**n/a/c+b*d
*e*(2*n*p+2*n+1)*x**(2*n)/a/c),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(32) = 64.
time = 0.87, size = 115, normalized size = 3.71

$$\frac{(bx^n+a)^p(dx^n+c)^p bdxx^{2n}e+(bx^n+a)^p(dx^n+c)^p bcxx^n e+(bx^n+a)^p(dx^n+c)^p adxx^n e+(bx^n+a)^p(dx^n+c)^p acxe}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="giac")
```

```
[Out] ((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e + (b*x^n + a)^p*(d*x^n + c)^p*
b*c*x*x^n*e + (b*x^n + a)^p*(d*x^n + c)^p*a*d*x*x^n*e + (b*x^n + a)^p*(d*x^
n + c)^p*a*c*x*e)/(a*c)
```

Mupad [B]
time = 5.30, size = 76, normalized size = 2.45

$$(c+d\,x^n)^p\,\left(e\,x\,{(a+b\,x^n)}^p+\frac{e\,x\,x^n\,\left(a\,d+b\,c\right)\,{(a+b\,x^n)}^p}{a\,c}+\frac{b\,d\,e\,x\,x^{2\,n}\,{(a+b\,x^n)}^p}{a\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(c + d*x^n)^p*(e + (e*x^n*(a*d + b*c)*(n + n*p + 1))/(a*c
) + (b*d*e*x^(2*n)*(2*n + 2*n*p + 1))/(a*c)),x)
```

```
[Out] (c + d*x^n)^p*(e*x*(a + b*x^n)^p + (e*x*x^n*(a*d + b*c)*(a + b*x^n)^p)/(a*c
) + (b*d*e*x*x^(2*n)*(a + b*x^n)^p)/(a*c))
```

3.594 $\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)}{ac(1+m)}\right.$

Optimal. Leaf size=45

$$\frac{e(hx)^{1+m}(a+bx^n)^{1+p}(c+dx^n)^{1+p}}{ach(1+m)}$$

```
[Out] e*(h*x)^(1+m)*(a+b*x^n)^(1+p)*(c+d*x^n)^(1+p)/a/c/h/(1+m)
```

Rubi [A]
time = 0.38, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 86, $\frac{\text{number of rules}}{\text{integrand size}} = 0.012$, Rules used = {1862}

$$\frac{e(hx)^{m+1}(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ach(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + m + n + n*
p)*x^n)/(a*c*(1 + m)) + (b*d*e*(1 + m + 2*n + 2*n*p)*x^(2*n))/(a*c*(1 + m))
),x]
```

```
[Out] (e*(h*x)^(1 + m)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*h*(1 + m))
```

Rule 1862

```
Int[((h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(
n_.))^(p_.)*((e_) + (f_.)*(x_)^(n_.) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Sim
p[e*(h*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(p + 1)/(a*c*h*(m + 1)))
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[
a*c*f*(m + 1) - e*(b*c + a*d)*(m + n*(p + 1) + 1), 0] && EqQ[a*c*g*(m + 1)
- b*d*e*(m + 2*n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)}+\frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = \frac{e}{-}$$

Mathematica [A]
time = 0.98, size = 41, normalized size = 0.91

$$\frac{ex(hx)^m(a+bx^n)^{1+p}(c+dx^n)^{1+p}}{ac(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + m +
n + n*p)*x^n)/(a*c*(1 + m)) + (b*d*e*(1 + m + 2*n + 2*n*p)*x^(2*n))/(a*c*(1
 + m))),x]
```

```
[Out] (e*x*(h*x)^m*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*(1 + m))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.79, size = 136, normalized size = 3.02

method	result
risch	$\frac{(a+b\,x^n)^p \mathrm{e}^{\frac{m\left(-i\pi \operatorname{csgn}(ihx)^3+i\pi \operatorname{csgn}(ihx)^2 \operatorname{csgn}(ih)+i\pi \operatorname{csgn}(ihx)^2 \operatorname{csgn}(ix)-i\pi \operatorname{csgn}(ihx)\operatorname{csgn}(ih)\operatorname{csgn}(ix)+2\ln(h)+2\ln(x)\right)}{2}} \left(bd\,x^{2n}+ad\,x^n+bc\,x\right.}{ac(1+m)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/c/(1+m
)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x,method=_RETURNVERBOSE)
```

```
[Out] (a+b*x^n)^p*exp(1/2*m*(-I*Pi*csgn(I*h*x)^3+I*Pi*csgn(I*h*x)^2*csgn(I*h)+I*P
i*csgn(I*h*x)^2*csgn(I*x)-I*Pi*csgn(I*h*x)*csgn(I*h)*csgn(I*x)+2*ln(h)+2*ln
(x)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a/c/(1+m)*(c+d*x^n)^p
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(46) = 92$.
time = 0.40, size = 94, normalized size = 2.09

$$\frac{\left(bdh^m x e^{(m\log(x)+2n\log(x)+1)} + ach^m x e^{(m\log(x)+1)} + (bch^m + adh^m)xe^{(m\log(x)+n\log(x)+1)}\right)e^{(p\log(bx^n+a)+p\log(dx^n+c))}}{ac(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/
c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="maxima")
```

```
[Out] (b*d*h^m*x*e^(m*log(x) + 2*n*log(x) + 1) + a*c*h^m*x*e^(m*log(x) + 1) + (b*
c*h^m + a*d*h^m)*x*e^(m*log(x) + n*log(x) + 1))*e^(p*log(b*x^n + a) + p*log
(d*x^n + c))/(a*c*(m + 1))
```

Fricas [A]
time = 0.40, size = 88, normalized size = 1.96

$$\frac{\left(bdexx^{2n}e^{(m\log(h)+m\log(x))} + acexe^{(m\log(h)+m\log(x))} + (bc+ad)exx^n e^{(m\log(h)+m\log(x))}\right)(bx^n+a)^p(dx^n+c)^p}{acm+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/
c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="fricas")
```

```
[Out] (b*d*e*x*x^(2*n)*e^(m*log(h) + m*log(x)) + a*c*e*x*e^(m*log(h) + m*log(x))
+ (b*c + a*d)*e*x*x^n*e^(m*log(h) + m*log(x)))*(b*x^n + a)^p*(d*x^n + c)^p/
(a*c*m + a*c)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)**m*(a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x
**n/a/c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x**(2*n)/a/c/(1+m)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(46) = 92.
time = 1.44, size = 155, normalized size = 3.44

$$\frac{(bx^n+a)^p(dx^n+c)^p bdxx^{2n}e^{(m\log(h)+m\log(x)+1)}+(bx^n+a)^p(dx^n+c)^p bcxx^n e^{(m\log(h)+m\log(x)+1)}+(bx^n+a)^p(dx^n+c)^p adxx^n e^{(m\log(h)+m\log(x)+1)}+(bx^n+a)^p(dx^n+c)^p acxe^{(m\log(h)+m\log(x)+1)}}{acm+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/
c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="giac")
```

```
[Out] ((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e^(m*log(h) + m*log(x) + 1) + (b
*x^n + a)^p*(d*x^n + c)^p*b*c*x*x^n*e^(m*log(h) + m*log(x) + 1) + (b*x^n +
a)^p*(d*x^n + c)^p*a*d*x*x^n*e^(m*log(h) + m*log(x) + 1) + (b*x^n + a)^p*(d
*x^n + c)^p*a*c*x*e^(m*log(h) + m*log(x) + 1))/(a*c*m + a*c)
```

Mupad [B]
time = 5.64, size = 106, normalized size = 2.36

$$(c+d\,x^n)^p\,\left(\frac{e\,x\,{\left(h\,x\right)}^m\,{\left(a+b\,x^n\right)}^p}{m+1}+\frac{e\,x\,x^n\,{\left(h\,x\right)}^m\,\left(a\,d+b\,c\right)\,{\left(a+b\,x^n\right)}^p}{a\,c\,\left(m+1\right)}+\frac{b\,d\,e\,x\,x^{2\,n}\,{\left(h\,x\right)}^m\,{\left(a+b\,x^n\right)}^p}{a\,c\,\left(m+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + (e*x^n*(a*d + b*c)*(m + n + n*
p + 1))/(a*c*(m + 1)) + (b*d*e*x^(2*n)*(m + 2*n + 2*n*p + 1))/(a*c*(m + 1))
),x)
```

```
[Out] (c + d*x^n)^p*((e*x*(h*x)^m*(a + b*x^n)^p)/(m + 1) + (e*x*x^n*(h*x)^m*(a*d
+ b*c)*(a + b*x^n)^p)/(a*c*(m + 1)) + (b*d*e*x*x^(2*n)*(h*x)^m*(a + b*x^n)^
p)/(a*c*(m + 1)))
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
      return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```